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Abstract

Divisia indexes and the less-well-known Frisch counterparts both involve weighted logarithmic changes of prices/quantities. This paper shows how they can be used in tandem to understand interesting aspects of consumer behaviour. The indexes can be used for measurement of quality and its price; the estimation of the income elasticity of the marginal utility of income, which is related to the overall degree of substitution among goods in the consumer's basket; and for the analysis of the bias in the consumer price index due to the neglect of substitution effects. In this sense, Divisia and Frisch (indexes) are friends.

Keywords: Divisia index numbers; Frisch index numbers; measuring quality; the income flexibility; the CPI bias

JEL Codes: C43, D11, D12

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1. Introduction

Laspeyres and Paasche are the most popular index numbers of prices and quantities in existence. If $\boldsymbol{p}_t$ and $\boldsymbol{q}_t$ denote price and quantity vectors in the base and current periods $t = 0, 1$, the Laspeyres and Paasche price and volume indexes are

$$L_P = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_0}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_0}, \quad L_Q = \frac{\boldsymbol{p}_0 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_0}, \quad P_P = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_1}, \quad P_Q = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_1 \cdot \boldsymbol{q}_0}.$$

These indexes are related in four ways:

1. *Factor reversal test*. This relates to internal consistency and states that the product of the price and volume indexes equal the value ratio, the ratio of observed expenditures in the two periods, $\frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_0}$. While neither Laspeyres nor Paasche by themselves satisfy the test, crossing them does so:
$$L_P \cdot P_Q = L_Q \cdot P_P = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_0}.$$
Factor reversal also holds for the geometric means of Laspeyres and Paasche: $F_P \cdot F_Q = \frac{\boldsymbol{p}_1 \cdot \boldsymbol{q}_1}{\boldsymbol{p}_0 \cdot \boldsymbol{q}_0}$, where $F_P = \sqrt{L_P \cdot P_P}$, $F_Q = \sqrt{L_Q \cdot P_Q}$ are Fisher's (1922) ideal indexes.
2. *Functional form.* Let p_{it} be i^th^ price in period t $(i = 1, \cdots, n, t = 0, 1)$ and q_{it} the corresponding quantity consumed, so that $w_{it} = \frac{p_{it} q_{it}}{\sum_{j=1}^{n} p_{jt} q_{jt}}$ is the budget share of the good. Then, Laspeyres and Paasche indexes can be expressed in terms of a related functional form,
$$L_P = \sum_{i=1}^{n} w_{i0} \left(\frac{p_{i1}}{p_{i0}}\right), \qquad L_Q = \sum_{i=1}^{n} w_{i0} \left(\frac{q_{i1}}{q_{i0}}\right),$$
$$P_P = \left(\sum_{i=1}^{n} w_{i1} \left(\frac{p_{i1}}{p_{i0}}\right)^{-1}\right)^{-1}, \quad P_Q = \left(\sum_{i=1}^{n} w_{i1} \left(\frac{q_{i1}}{q_{i0}}\right)^{-1}\right)^{-1}.$$
In words, Laspeyres is a base-period budget-share weighted arithmetic average of the price/quantity relatives, while Paasche is a current-period budget-share weighted harmonic average.
3. *Bounds to the true cost of living.* Let $C(u_t, \boldsymbol{p}_{t'})$ be the minimum cost of achieving the level of utility in period *t*, u_t, at prices in period t' for $t, t' = 0, 1$. When prices change from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$, the true-cost-of-living index of utility in the reference-period *t* is $T(u_t) = \frac{C(u_t, \boldsymbol{p}_1)}{C(u_t, \boldsymbol{p}_0)}, t = 0, 1$. Then, Laspeyres and Paasche form upper and lower bounds to the true cost-of-living index in the sense$T(u_0) \leq L_P$ and $T(u_1) \geq P_P$ (Samuelson and Swamy, 1974).
4. *The difference.* The proportional difference between the Paasche and Laspeyres price index is equal to (i) the same difference in the volume indexes; and (ii) a weighted quantity-price covariance:
$$\frac{P_P - L_P}{L_P} = \frac{P_Q - L_Q}{L_Q} = \sum_{i=1}^{n} w_{i0} \left\{\frac{q_{i1}/q_{i0}}{L_Q} - 1\right\} \left\{\frac{p_{i1}/p_{i0}}{L_P} - 1\right\},$$
where w_{i0} is the budget share of good i in period 0. In words, if, on average, the quantities of goods that grow faster than L_Q also experience increasing relative prices, then the Paasche index exceeds Laspeyres.

The above relationships show that Laspeyres and Paasche indexes are closely linked – they can be described as “friends”. The objective of this paper is to show that indexes of Divisia (1926) and the less-well-known indexes of Frisch (1932) are also sufficiently closely linked to be thought of as friendly. The nature of the linkage is not the same as in the Laspeyres-Paasche case, but one that will be shown to be useful in highlighting certain fundamental patterns in consumption behaviour. This includes the measurement of quality and its price; characterising the income sensitivity of the marginal utility of income, and how it varies over the income distribution; and the substitution bias in the consumer price index.

Divisia indexes are familiar to most from the Solow (1957) residual whereby the growth in total factor productivity is the excess of output growth over a factor-share-weighted average of input growth; the latter component is an index of input growth of the Divisia form. Other influential research on Divisia indexes includes Balk (2005), Barnett et al. (1984), Diewert (1976), Hulten (1973), Jorgenson and Griliches (1967), Star and Hall (1976) and Theil (1967). Frisch indexes are also weighted averages of growth in prices/quantities, but there are two differences to Divisia. First, Frisch indexes employ as weights marginal, rather than average, shares, and so measure a different dimension of behaviour. Second, as there is much less prior research on Frisch, it is probably safe to include these indexes are less familiar to most than are Divisia. A summary picture of the key literature is given in Figure 1.1. On the basis of citations, Samuelson’s most important paper on index numbers was with Swamy (Samuelson and Swamy, 1974) and the four columns on the far left of the figure represent its citation counts in each of the last four decades. The other columns of Figure 1.1 contain citations to major publications on index numbers by four other influential authors (the actual publications are listed in Table 1.1). As can be seen, citations for both Divisia and Frisch are substantially lower than those for Fisher and Diewert. It is somewhat surprising that citations for Frisch exceeds Divisia in each of the four decades and even approximate those of Samuelson and Swamy. It could be that Divisia indexes are so commonly used that explicit citation is no longer felt necessary. On the other hand, however, Divisia’s citations have grown more rapidly than Frisch’s, but neither is as fast as those of Fisher, Diewert and Balk.

A related perspective on Frisch indexes is from the attention they receive in major surveys. Frisch himself published a major survey on index numbers in 1936. The treatment of his work in subsequent surveys is mixed. The Frisch price index was rehabilitated by Barten (1964) and Theil (1965, 1967) in the context of demand analysis. They use the index as a price deflator for relative prices on the left-hand side of demand equations. The coefficient of the j^{th} relative price in the i^{th} demand equation measures the so-called “specific substitution effect”, which in holding constant the marginal utility of income, measures the interactions of goods i and j in the utility function. Chapter 2 of Goldberger’s famous 1967 monograph, published 20 years later (Goldberger, 1987), contains substantial material on the Barten-Theil approach with its implicit references to Frisch. Frisch's work continued to receive attention in Theil's later work, as summarised in Theil (1980). In their advanced textbook, Deaton and Muellbauer (1980) devote a chapter to index numbers. However, the only mention of Frisch (1936) is at the end of the chapter in the (small font size) bibliographic notes (p. 189), where the paper is described as "retain[ing] it classic status". In his influential survey, Diewert (1981) refers several times to Frisch (1936), and once also to Frisch (1930). The book Price Level Measurement, edited Diewert (1990a), contains only one reference to Frisch -- in footnote 14 of their chapter, Jorgenson and Slesnick (1990, p. 226) cite Frisch (1936), along with references to several other surveys of index-number theory. Diewert (1990b), in a chapter of the same book, does not refer to Frisch. More recently, Diewert (2008) comes back to referring Frisch (1936), as do Diewert and Hill (2010). All in all, it is probably fair to conclude that the Frisch index no longer occupies a prominent position in the literature; and it is now not as well-known as the Divisia index.

The paper is structured as follows. The next section contains details of Divisia and Frisch indexes and their properties. Section 3 shows how the two indexes can be used to measure quality and its dual price, while Section 4 deals with the foundations in utility-maximisation theory and demand equations. Section 5 is about the income elasticity of the marginal utility of income, how to estimate the elasticity with Divisia and Frisch indexes, and the so-called "Frisch conjecture" regarding the income-dependence of the elasticity. Section 6 shows how under certain conditions, the substitution bias of the consumer price index can be formulated in terms of a weighted variance of price changes and as the weights are marginal, this is a Frisch variance. Recent data from the OECD and the International Comparisons Program of the World Bank are used to illustrate the application of the concepts. Section 7 contains concluding comments.

2. Divisia and Frisch Indexes

Let p_i be the price of good i $(i = 1, \dots, n)$ and q_i be the corresponding quantity demanded, so that $M = \sum_{i=1}^{n} p_i\, q_i$ is total expenditure (to be referred to as "income" for short) and $w_i = p_i q_i / M$ is budget share of i. The change in income is $dM = \sum_{i=1}^{n} p_i\, dq_i + \sum_{i=1}^{n} q_i\, dp_i$ or, using $d(\log x) = dx/x\,, x > 0$,

$$(2.1) \qquad d(\log M) = d(\log P) + d(\log Q),$$

where

$$(2.2) \qquad d(\log P) = \sum_{i=1}^{n} w_i d(\log p_i), \quad d(\log Q) = \sum_{i=1}^{n} w_i d(\log q_i),$$

are Divisia (1926) price and volume indexes. These indexes weight each price (quantity) change by its relative economic importance as measured by its budget share, so it contributes to the index in a representative manner. From (2.1), these indexes satisfy the factor reversal test that their sum equals the change in income, so there is a clean split of the nominal change in income into price and volume components.

The indexes (2.2) are formulated in continuous change form and there are many ways of making discrete approximations to compare, say, period t with t-1. A popular approach, advocated by Theil (1967) in particular, is to replace (i) budget shares with their arithmetic averages over the two periods, $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{i,t-1})$; and (ii) infinitesimal changes in logarithms with log-changes, $\log p_{it} - \log p_{i,t-1} = \log \frac{p_{it}}{p_{i,t-1}}$, and similarly for quantity changes. Defining the log-change operator, $Dx_t = \log \frac{x_t}{x_{t-1}}$, the finite-change indexes are then

$$\text{(2.3)} \qquad DP_t = \sum_{i=1}^{n} \bar{w}_{it} Dp_{it} \quad \text{and} \quad DQ_t = \sum_{i=1}^{n} \bar{w}_{it} Dq_{it}.$$

In an influential paper (see Table 1.1), Diewert (1976) shows that the price index DP is exact for the nonhomogeneous translog cost function evaluated at the geometric mean of utility in the two periods. As the translog is a second-order approximation to an arbitrary twice-differentiable cost function, Diewert terms the price index DP "superlative". The index DQ possesses analogous properties. For related approximation results, see Kloek (1967) and Theil (1967, 1975/76). Diewert tends to refer to DP and DQ as Törnqvist-Theil indexes, after Törnqvist (1936) and Theil (1965, 1967). As the meaning will be clear from the context, for simplicity in what follows we shall use the terms "Divisia index of prices" and "Divisia index of quantities" to refer to both (2.2) and (2.3); we also use the term the "Divisia volume index" to describe $d(\log Q)$ and DQ.

A different interpretation/justification of the price index DP is along sampling lines (Theil, 1967, pp. 136-37). Consider a discrete random variable X_t which can take the values $Dp_{it}, \cdots, Dp_{nt}$. Suppose prices are drawn at random from this distribution such that each dollar of expenditure at a time mid-way between t and t-1 has an equal chance of being selected. This means that the probability of drawing Dp_{it} is $\bar{w}_{it}$, and so the expected value of X_t is $\mathrm{E}(X_t) = \sum_{i=1}^{n} \bar{w}_{it} Dp_{it}$, which is the index DP. In words, the Törnqvist-Theil index has the interpretation as the expected value of the distribution of logarithmic price relatives.

The budget share w_i is the proportion of income spent on good i. Contrast this with the marginal share of the good, defined as $\theta_i = \partial\,(p_i q_i)/\partial M$, which answers the question, what fraction of a one-dollar rise in income is spent on i? As income is totally spent, both the budget and marginal shares have a unit sum. Rather than using budget shares as weights in the indexes (2.2), we can equally well define analogous measures that are marginally weighted:

$$\text{(2.4)} \qquad d(\log P') = \sum_{i=1}^{n} \theta_i d(\log p_i), \quad d(\log Q') = \sum_{i=1}^{n} \theta_i d(\log q_i).$$

These are Frisch (1932) price and volume indexes. Similar to (2.3), finite-change versions are

$$(2.5) \qquad DP_t' = \sum_{i=1}^{n} \theta_{it} Dp_{it}, \quad DQ_t' = \sum_{i=1}^{n} \theta_{it} Dq_{it},$$

where the precise specification of θ_{it} will be described subsequently.

Divisia and Frisch indexes are both weighted averages of price and quantity changes. But there are two related differences. First, as mentioned, the nature of the weights differs. The ratio of the marginal to budget share is the income elasticity, $\eta_i = \theta_i / w_i = \partial(\log q_i)/\partial(\log M)$. Luxuries have $\eta_i > 1$ or $\theta_i > w_i$, and so are more heavily weighted in Frisch indexes than Divisia, while necessities $(\eta_i < 1)$ are less heavily weighted. A second difference is that Divisia weights are directly observed, while the weights for Frisch have to be estimated econometrically

3. Quality Measurement

The quality of a product is difficult to define in theory and even more so in practice. There are challenges even in what might be considered the easy case of gold. Purity, weight, the current physical location, and even the security of ownership title (was it stolen?) might be all conceivably be part of what constitutes the quality of gold bars. For gold nuggets, things are still harder with shape, authenticity (was it created by nature or man?) and history (who previously owned it?) possibly additional factors determining "quality". Measuring the quality of most other goods, and especially services, pose their own problems. Probably the hedonic approach (Griliches, 1961) is the most popular way of measuring quality. Other approaches include identifying higher quality goods with higher prices paid (Houthakker, 1952, Theil, 1952) and the matched model method used by statistical agencies (see, e. g., Silver and Heravi, 2005).

When quality is formulated in a certain way, it can be conveniently measured with Divisia and Frisch indexes. The quality measures used derive from Theil (1975/76) and Clements and Gao (2012), which involve the revealed preference of the consumer as indicated by the change in the consumption/prices of goods and their co-relationship with the income elasticities. As the consumer becomes more affluent, relatively more is spent on luxuries $(\eta_i > 1)$, less on necessities $(\eta_i < 1)$. This is enshrined in Engel's (1857) law that the food budget share declines as income rises. As a budget-share weighted-average of the income elasticities is unity, this is a natural dividing line between those goods that are more preferred, or of higher quality, and those less preferred, or of lower quality. This formulation of quality is based on the observed behaviour of the consumer and requires no pre-specified "objective" criteria of what is beneficial for the consumer such as the memory capacity and speed of a computer, safety/comfort features of a car or the number of bathrooms in a house.

An attraction of this measure of quality is that as the income elasticity is a pure number, it can be applied to all goods and services, not only those with identifiable physical characteristics. Suppose good i is a luxury $(\eta_i > 1)$, and its consumption increases $[d(\log q_i) > 0]$. Then, as

$(\eta_i - 1)d(\log q_i) > 0$, we could say this good contributes positively to the growth in quality. The growth in consumption of a necessity diminishes quality, as does a fall in a luxury $[(\eta_i - 1)d(\log q_i) > 0]$. For the basket comprising the n goods, we simply take a budget-share weighted average of $(\eta_i - 1)d(\log q_i)$: $\sum_{i=1}^{n} w_i(\eta_i - 1)d(\log q_i)$. As $\sum_{i=1}^{n} w_i\eta_i = 1$, $\sum_{i=1}^{n} w_i(\eta_i - 1)d(\log Q) = 0$, this is equivalent to

$$(3.1) \qquad y_{\eta q} = \sum_{i=1}^{n} w_i[\eta_i - 1][d(\log q_i) - d(\log Q)],$$

which is a weighted covariance between the *n* income elasticities and the *n* quantity changes. This is the index of the <u>quality of consumption</u>. Thus, when $y_{\eta q} > 0$ (< 0), on average the consumption of luxuries has grown (fallen) relative to that of necessities and the quality of consumption has improved (diminished). The covariance interpretation means that when the income elasticities are positively correlated with the quantity changes, quality has increased. As $\eta_i = \theta_i/w_i$, equations (2.2), (2.4) and (3.1) imply

$$y_{\eta q} = d(\log Q') - d(\log Q),$$

so the quality of consumption is equivalently the excess of the Frisch volume index over its Divisia counterpart.

Consider a similar weighted covariance between the n income elasticities $\eta_1, \ldots, \eta_n$ and the changes in expenditure on the goods, $d(\log p_1 q_1), \ldots, d(\log p_n q_n)$:

$$y_{\eta,pq} = \sum_{i=1}^{n} w_i[\eta_i - 1][d(\log p_i q_i) - d(\log M)],$$

where $d(\log M) = \sum_{i=1}^{n} w_i d(\log p_i q_i)$ is the Divisia mean of expenditure changes. This covariance is positive when the pattern of expenditures moves, on average, towards luxuries, away from necessities and is a measure of the <u>quality of expenditure</u>. As the budget share $w_i = p_i q_i/M$, $d(\log w_i) = d(\log p_i q_i) - d(\log M)$, and so the above expression for $y_{\eta,pq}$ is equivalent to a weighted covariance of the income elasticities and the budget shares:

$$(3.2) \qquad y_{\eta,pq} = \sum_{i=1}^{n} w_i[\eta_i - 1]d(\log w_i).$$

Similarly, a weighted covariance of the income elasticities and prices,

$$(3.3) \qquad y_{\eta p} = \sum_{i=1}^{n} w_i[\eta_i - 1][d(\log p_i) - d(\log P)],$$

measures the change in the structure of relative prices. This $y_{\eta p}$ is positive if, on average, the prices of luxuries rise relative to those of necessities. Compared to the poor, the rich consume relatively more (less) luxuries (necessities), so $y_{\eta p} > 0$ means price changes erode the real incomes of the rich, or have a progressive effect on the distribution of income. The measures (3.1)-(3.3) satisfy a type of4factor reversal property, viz., $y_{\eta,pq} = y_{\eta p} + y_{\eta q}$, or $y_{\eta,pq} - y_{\eta p} = y_{\eta q}$, and so it can be seen that $y_{\eta p}$ acts as the deflator that transforms the quality of expenditures, a nominal variable, into the real variable, the quality of consumption. This established that $y_{\eta p}$ can be identified as the price of quality. As before with $y_{\eta q}$, the price of quality can also be expressed in terms of Frisch and Divisia indexes as

$$y_{\eta p} = d(\log P') - d(\log P).$$ [1]

To illustrate the workings of the indexes we use annual data pertaining to the 37 OECD countries listed in Table 3.1 and distinguish $n = 9$ commodities. These are fairly recent data and go back 20-25 years for the majority of countries, while in several instances the period is much longer. Table 3.2 gives the means over years and countries of the shares and income elasticities. As can be seen from column 4, food and alcohol and housing and utilities are necessities; furnishings, equipment, recreation and culture, and education are luxuries; while the remaining commodities are near-borderline cases. Figure 3.1 presents the distributions of the quality indexes. Thus, on average, the quality of consumption improves by about 0.4 percent p. a. (panel A), while the price of quality falls by about one-quarter of one percent p. a. (panel B). In both cases, while the mode seems to be reasonably well defined, there is considerable dispersion (the distributions also have a tendency to be slightly negatively skewed). Also worth noting is the considerable more dispersion of the consumption index than that of the price index. When the 0.4-percent average quality increase is compared to the average (measured) increase in per capita incomes of something of the order of 2.0-2.5 percent p. a., quality improvement can be seen to be nontrivial. The average fall in the price of quality means that the price of luxuries falls relative to necessities, or that the structure of price is becoming more regressive. But at only at only about one-quarter of one percent per year, this effect is quite mild.

Panel A of Figure 3.2 indicates that the quality of consumption increases with income, as is to be expected. The income elasticity is significant but quite low at 0.15. A ten-percent rise in income leads to a one-and-one-half-percent increase in quality -- in other words, a noticeable improvement

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[1] A further interesting result is as follows. Consider the excess of the (change in) the price of quality over the quality of consumption,

$$y_{\eta p} - y_{\eta q} = [d(\log P') - d(\log P)] - [d(\log Q') - d(\log Q)].$$

This equals the covariance between the income elasticities and the excess of the changes in the relative prices of the individual commodities over and that of quantities:

$$\sum_{i=1}^{n} w_i[\eta_i - 1][\{d(\log p_i) - d(\log P)\} - \{d(\log q_i) - d(\log Q)\}].$$

Consequently, when, on average, the relative prices of luxuries grow faster than quantities, and necessity prices grow slower, then the covariance is positive. In such cases, the price of quality grows relative to the quality of consumption, $y_{\eta p} > y_{\eta q}$. In other cases, the reverse occurs and $y_{\eta p} < y_{\eta q}$. If rather than treating prices and quantities as independent variables, they are taken to be negatively correlated (due to the law of demand, when income is held constant), the term $\{d(\log p_i) - d(\log P)\} - \{d(\log q_i) - d(\log Q)\}$ has the same sign as the price change $\{d(\log p_i) - d(\log P)\}$. Then, the covariance is positive (negative) when, on average, the prices of luxuries rise (fall) relative to necessities; it is also positive (negative) when the prices of necessities falls (rises) relative to luxuries. This latter condition amounts to the same interpretation as that of the price of quality $y_{\eta p}$ of equation (3.3).

in quality only comes about with a large rise in income, other things remaining unchanged.[2] Consistent with the average fall in the price of quality, it can be seen from panel B of Figure 3.2 that this price also falls with income. But the coefficient of income in insignificantly different from zero. These findings refer to changes over time. We can apply the approach equally to changes from one country to another using data from the 2011 round of the International Comparisons Program (ICP). Panel A of Figure 3.3 shows the low income elasticity of quality is also apparent with these data. Interestingly, the elasticity here for the "rich" countries (those with per capita incomes above the world median) is close to those for the OECD. Panel B of Figure 3.3 shows that the price of quality for poor is more or less unrelated to income, while now prices fall significantly for the rich. For these countries, the prices of luxuries relative to necessities fall as income rises, so there is a regressive bias in the structure of prices.

Figure 3.4 is a price-quantity scatter for quality for the OECD. The points are scattered around a downward-sloping straight line and the slope of -1.3 is interpreted as a rough estimate of the price elasticity of demand for quality. The demand for quality is price elastic, which seems reasonable. The rich countries of the ICP also have an elastic demand, while this is inelastic for the poor (Figure 3.5). This could be interpreted as saying relative to the poor, the rich have more alternatives when it comes to quality. As the demand curve for the poor lies everywhere below that for the rich, the consumption of quality by the rich exceeds the poor's, reflecting the positive income effect.

4. Utility Foundations

Using the differential approach to consumption theory (Theil, 1980, Theil and Clements, 1987), this section establishes the important roles of Divisia and Frisch indexes in demand analysis and their foundations in utility-maximisation theory.

The Marshallian demand equation for good *i* depends on income *M* and the *n* prices: $q_i = q_i(M, p_1, \dots, p_n)$. The differential is $dq_i = (\partial q_i/\partial M)dM + \sum_{j=1}^{n}(\partial q_i/\partial p_j)\,dp_j$, or using the Slutsky equation, $dq_i = (\partial q_i/\partial M)\big(dM - \sum_{j=1}^{n} q_j dp_j\big) + \sum_{j=1}^{n} s_{ij}\,dp_j$, where s_{ij} is the $(i,j)^{th}$ substitution effect. Multiplying by p_i/M and using $w_i = p_i q_i/M$, as well as $dx/x = d(\log x), x > 0$, we have

$$w_i d(\log q_i) = \theta_i \left[d(\log M) - \sum_{j=1}^{n} w_j\, d(\log p_j) \right] + \sum_{j=1}^{n} \pi_{ij}\, d(\log p_j),$$

where $\theta_i = \partial\,(p_i q_i)/\partial M$ is the i^{th}marginal share and $\pi_{ij} = (p_i p_j/M) s_{ij}$ is the $(i,j)^{th}$ Slutsky coefficient. The term in square brackets shows how the income effects of the *n* price changes,

[2] This low income elasticity is consistent with the findings of Clements and Si (2017a) regarding the quality of food consumption.

$\sum_{j=1}^{n} w_j\, d(\log p_j)$, act as a price deflator that converts the change in money income, $d(\log M)$, into the change in real income, $d(\log M) - \sum_{j=1}^{n} w_j\, d(\log p_j)$. As discussed above, the differential of the budget constraint is $d(\log M) = \sum_{j=1}^{n} w_j\, d(\log p_j) + \sum_{j=1}^{n} w_j\, d(\log q_j)$, so that $d(\log M) - \sum_{j=1}^{n} w_j\, d(\log p_j) = \sum_{j=1}^{n} w_j\, d(\log q_j)$, which is the Divisia volume index $d(\log Q)$ of equation (2.2). Thus, the above equation can be written in an equivalent, more compact form as

(4.1) $$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij}\, d(\log p_j).$$

The variable on the left of equation (4.1), $w_i d(\log q_i)$, has the dual interpretation of (i) the contribution of good i to the Divisia volume index; and (ii) the quantity component of the change in the budget share of i, $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log M)$. The Divisia volume index $d(\log Q)$ has a well-defined utility foundation. Let $u(q_1, \cdots, q_n)$ be the utility function, the differential of which is $du = \sum_{i=1}^{n} (\partial u/\partial q_i)\, dq_i$. Or, using $\partial u/\partial q_i = \lambda p_i$, where λ is the marginal utility of income (the Lagrange multiplier), $du = \lambda \sum_{i=1}^{n} p_i\, dq_i = \lambda M \sum_{i=1}^{n} w_i\, d(\log q_j)$. Defining $\lambda^* = 1/\lambda = \partial C/\partial u$ as the marginal cost of utility, we have $\sum_{i=1}^{n} w_i\, d(\log q_j) = \lambda^* du/M$. Thus, the Divisia volume index is equal to the change in utility, valued at marginal cost, as a fraction of income.

Next, suppose that preferences are such that the marginal utility of consumption of each good depends only on the consumption of the good in question, so utility is additive, $u(q_1, \cdots, q_n) = \sum_{i=1}^{n} u_i(q_i)$. This case is known as preference independence, and it and variants were championed by Frisch (1959) as a simplification that could promote progress in applied demand analysis. In this context, a key concept is the income elasticity of the marginal utility of income in reciprocal form, defined as $\phi = [\partial(\log \lambda)/\partial(\log M)]^{-1}$, which is referred to as the "income flexibility" for short. Under preference independence, the substitution terms are not zero, but take a structured form that can be expressed in terms of Slutsky coefficients as:

$$\pi_{ij} = \begin{cases} \phi\theta_i(1-\theta_i) & i = j \\ -\phi\theta_i\theta_j & i \neq j. \end{cases}$$

This equation for $i, = 1, \cdots, n$ can be written as

(4.2) $$\boldsymbol{\Pi} = \phi\boldsymbol{\Theta}(\boldsymbol{I} - \boldsymbol{\iota}\boldsymbol{\theta}'),$$

where $\boldsymbol{\Pi}$ is the $n \times n$ matrix of Slutsky coefficients $[\pi_{ij}]$, $\boldsymbol{\Theta} = diag[\boldsymbol{\theta}]$, $\boldsymbol{\theta} = [\theta_i]$, the n-vector of marginal shares, $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{\iota} = [1, \ldots, 1]'$. Accordingly, the income flexibility acts as a scaling factor of the Slutsky matrix. The $(i,j)^{th}$Slutsky price elasticity (η_{ij}) holds real income constant and under preference independence takes the form:

$$\eta_{ij} = \begin{cases} \phi\eta_i(1-w_i\eta_i) & i = j \\ -\phi\eta_i w_j\eta_j & i \neq j. \end{cases}$$

This shows that the values of the income elasticities (η_i), the budget shares (w_i) and ϕ are all that is needed to construct the own- and cross-price elasticities. Frequently income elasticities are readily available either from prior studies or from considerations of the luxury/necessity nature of the commodities. But where does the value of the income flexibility come from? This is dealt with in the subsequent section.

Define the Frisch-deflated change in the relative price of good i as $d\left(\log\frac{p_j}{P\prime}\right) = d(\log p_i) - d(\log P')$, where $d(\log P')$ is the Frisch price index of equation (2.4). Then, under preference independence, the demand equation (4.1) becomes

(4.3) $$w_i d(\log q_i) = \theta_i d(\log Q) + \phi\theta_i d\left(\log\frac{p_i}{P'}\right),$$

where equation (4.2) has been used. The substitution term is now simplified considerably as it contains only the own good's relative price change. It is also to be noted that the Frisch own-price elasticity is $\phi\theta_i/w_i$, which is proportional to the income elasticity θ_i/w_i, with the income flexibility ϕ as the proportionality factor (Deaton, 1974). As $\sum_{i=1}^{n}\theta_i = 1$, it follows that $\phi = \sum_{i=1}^{n} w_i(\phi\theta_i/w_i)$. In words, the income flexibility is interpreted as a budget-share weighted-average of the price elasticities, and so is an average measure of the degree of substitutability among goods. More will be said about the Frisch price elasticity at the end of this section.

Demand equation (4.3) contains two price indexes on the right-hand side – Divisia and Frisch. While Divisia is not explicit, it is still there, embedded in real income, $d(\log Q) = d(\log M) - d(\log P)$. Thus, money income is transformed into real income by deflating by the Divisia price index, while Frisch transforms the nominal into the relative price change.

Finally, we present some results on the marginal utility of income, mostly due to Theil (1975/76) using the fundamental-matrix-equation approach of Barten (1964). The marginal utility of income λ depends on income and the prices, which we express as $\lambda = \lambda(M, p_1, \cdots, p_n)$, with

$$\frac{\partial\lambda}{\partial p_i} = -\lambda\frac{\partial q_i}{\partial M} - \frac{\partial\lambda}{\partial M}q_i.$$

Thus, the change in λ is $d(\log\lambda) = \frac{1}{\phi}d(\log M) + \sum_{j=1}^{n}\left(\frac{\partial\lambda}{\partial p_j}\frac{p_j}{\lambda}\right)d(\log p_j)$ or, using the above,

$$d(\log\lambda) = \frac{1}{\phi}d(\log M) + \sum_{j=1}^{n}\left(-\theta_j - \frac{1}{\phi}w_j\right)d(\log p_j).$$

Using equations (2.2) and (2.4), this becomes

(4.4) $$d(\log\lambda) = \frac{1}{\phi}[d(\log M) - d(\log P)] - d(\log P') = \frac{1}{\phi}d(\log Q) - d(\log P').$$

This states that the marginal utility of income depends on real income with elasticity $1/\phi$ and the Frisch price index in inverse proportion (that is, with elasticity -1).

The expression on the far right of equation (4.4) involves a combination of real and nominal variables. Things are clearer if this mixture is avoided by deflating $d(\log\lambda)$ by the Divisia index. As

λ falls as prices rise (at least, when they rise in the sense of $d(\log P) > 0$), deflation entails *adding* $d(\log P)$ to $d(\log \lambda)$:

(4.5) $$d(\log \lambda) + d(\log P) = \frac{1}{\phi} d(\log Q) - [d(\log P') - d(\log P)].$$

Equation (4.5) expresses the attractive result that the real value of the marginal utility of income depends on real income and the (relative) price of quality. Most estimates of ϕ are negative fractions (see, e. g., Brown and Deaton, 1972, Clements and Zhao, 2009, pp. 228-29), so equation (4.5) implies that the real value of λ is more sensitive to real income changes than to changes in the relative price of quality.[3]

From equation (4.4), $d(\log Q) = \phi[\, d(\log \lambda) + d(\log P')]$, so that the right-hand side of demand equation (4.3) can be expressed as

$$\theta_i \phi[\, d(\log \lambda) + d(\log P')\,] + \phi\theta_i[d(\log p_i) - d(\log P')].$$

As $1/\lambda = \lambda^*$ is the marginal cost of utility, $d(\log \lambda) = -\, d(\log \lambda^*)$ and the above is equal to $\phi\theta_i[d(\log p_i) - d(\log \lambda^*)]$. Accordingly, the demand equation for good i under preference independence is

(4.6) $$w_i d(\log q_i) = \phi\theta_i[d(\log p_i) - d(\log \lambda^*)].$$

Now $d(\log \lambda^*)$ is the deflator of $d(\log p_i)$. Dividing both sides of equation (4.6) by w_i and stetting $d(\log \lambda^*) = 0$, it can be easily seen that $\phi\theta_i / w_i$ is the price elasticity under the condition that the marginal cost of utility is held constant. This is known as the Frisch price elasticity, which was used earlier in this section.

5. The Income Flexibility

The income flexibility ϕ, the reciprocal of the income elasticity of the marginal utility of income, is an important parameter in three contexts: The computation of benefits in project evaluation, choice under uncertainty and a summary measure of the overall degree of substitution among goods in the consumption basket. This section shows that Divisia and Frisch indexes can be used to obtain estimates of ϕ in a simple manner.

Frisch (1959) discusses how the income flexibility might vary with income. He states:

> We may, perhaps, assume that in most cases [ϕ] has values of the order of magnitude given below.
>
> -0.1 for the extremely poor and apathetic part of the population.
> -0.25 for the slightly better off but still poor part of the population with a fairly pronounced desire to become better off.
> -0.5 for the middle income bracket, "the median part" of the population.
> -1.4 for the better off part of the population.

[3] A further interpretation of equation (4.5) is also possible. Suppose the n prices change by $d(\log p_1), \ldots, d(\log p_n)$, leading to certain values of $d(\log P)$ and $d(\log P')$. By how much would real income need to change in order to keep the real value of the marginal utility of income unchanged? From equation (4.5) with $d(\log \lambda \cdot P) = 0$, the answer is $\phi[d(\log P') - d(\log P)]$. In words, the relative price of quality times the income flexibility holds constant the marginal utility of income in real terms.

-10 for the rich part of the population with ambitions towards "conspicuous consumption".

It would be a very promising research project to determine $[\phi]$ for different countries and for different types of populations. A universal "atlas" of the values of $[\phi]$ should be constructed. It would serve an extremely useful purpose in demand analysis.[4]

It can readily be understood from Section 4 how such an atlas of ϕ-values would be extremely useful in the estimation of price elasticities in situations in which the information base was less than perfect. The hypothesis that the income flexibility increases in absolute value as the consumer becomes more affluent as set out in the above quotation is known as the "Frisch conjecture".

In Barten (1964) and Theil's (1965) Rotterdam demand model the income flexibility is parametrised as a constant.[5] While this is a matter of simplicity and convenience, this approach contradicts the Frisch conjecture. Thus, it is natural to test the hypothesis that ϕ is a constant. Previous tests of the dependence of ϕ on income have yielded mixed findings. One group of studies finds little or no dependence. See Clements and Theil (1979), S. Selvanathan (1993, Secs. 4.8 and 6.5), Selvanathan and Selvanathan (2003, Secs. 3.6 and 4.6), Theil (1975/76, Sec. 15.4), Theil (1987, Sec. 2.13) and Theil and Brooks (1970/71). In contrast, more support for the Frisch conjecture is found by DeJanvry et al. (1972) and Lluch et al. (1977). As the issue is still not closed, it seems sensible to shed new light on it with recent data. In what follows, we present three types of tests of the Frisch conjecture.

Our starting point is a finite-change version of demand equation (4.3):

(5.1) $$\bar{w}_{it} Dq_{it} = \theta_i DQ_t + \phi\theta_i (Dp_{it} - DP'_t) + \varepsilon_{it}.$$

This refers to the change in the demand for good i from year *t-1* to *t*, where $\bar{w}_{it} = \frac{1}{2}(w_{it} + w_{it-1})$; $Dq_{it} = \log q_{it} - \log q_{i,t-1}$; $DQ_t = \sum_{i=1}^{n} \bar{w}_{it} Dq_{it}$ is the finite-change Divisia volume index [previously defined in equation (2.3)]; $Dp_{it} = \log p_{it} - \log p$; $DP'_t = \sum_{i=1}^{n} \theta_{it} Dp_{it}$ is the analogous Frisch price index [equation (2.5)]; and ε_{it} is a zero-mean disturbance term. When the marginal shares θ_i and ϕ are treated as constant parameters, equation (5.1), for $i = 1, \cdots, n$, is the preference independence version of Rotterdam model of Barten (1964) and Theil (1965).[6] Taking the marginal shares as constants means linear Engel curves. To avoid this possibly restrictive assumption, we merge (5.1) with Working's model (1943), so the marginal shares take the form $\theta_{it} = \beta_i + \bar{w}_{it}$, with β_i a constant.[7] Estimates of ϕ for each of the 37 countries are given in Table 5.1. All estimates are

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[4] In his paper, Frisch actually refers to ϕ^{-1}, rather than ϕ. In order to be consistent with the previous discussion, we have made the appropriate changes to the quotation.

[5] The Rotterdam model is a finite-change version of equation (4.1) for $i = 1, \cdots, n$ with constant marginal shares and Slutsky coefficients. As set out subsequently in equation (5.1), under preference independence, this takes the form of equation (4.3) for $i = 1, \cdots, n$ with constant marginal shares and ϕ. Clements and Gao (2015) show the Rotterdam model continues to be prominent in the field as measured by citations.

[6] For a recent review of this model and its application, see Clements and Gao (2015).

[7] Linear Engel curves mean that income elasticities are inversely proportional to budget shares. For food, the budget share falls as income rises (Engel's law), and so the food income elasticity is higher for the rich than the poor. This

negative, all bar two are negative fractions, most are estimated reasonably precisely and the mean over countries is -0.5. [8]

Figure 5.1 plots the above ϕ-estimates against GDP per capita, with the vertical scale inverted. The three panels include/exclude some countries and in each case, there is a significant relationship between the income flexibility and income. Panel A of Table 5.2 reproduced the regressions from the figure. Panel B contains weighted regressions with weights reflecting the sampling errors of the ϕ-estmates, and the results are similar to those in the panel A, the unweighted case. The coefficient of the logarithm of income is negative and greater than twice its standard error in all cases, indicating support for the Frisch conjecture that the absolute value of ϕ increases with income.[9]

The above is a cross-country test of the Frisch conjecture. A within-country test is also possible first defining

$$\mathrm{V}'_{pt} = \sum_{i=1}^{n} \theta_i (Dp_{it} - DP'_t)^2$$

as a weighted variance of the price changes. As the weights are marginal shares, this is a Frisch variance, which is a measure of price dispersion. Further, let

$$C_{pqt} = \sum_{i=1}^{n} \bar{w}_{it} (Dp_{it} - DP'_t)\,(Dq_{it} - DQ_t).$$

This is a budget-share weighted price-quantity covariance of the Divisia form. This is expected to be negative as consumers tend to substitute away from those goods with above-average price increases.

In demand equation (5.1) the income flexibility is specified as a constant. We now suppose it varies from one year to another, and denote its value in year t by ϕ_t. Setting the disturbance term at its expected value of zero, equation (5.1) now becomes

(5.2) $$Dq_{it} = \theta_i DQ_t + \phi_t \theta_i (Dp_{it} - DP'_t).$$

There are $i = 1, \dots, n$ equations of the above form with the same ϕ_t appearing in each. Accordingly, if the marginal shares are known, this system can be used with Divisia and Frisch concepts to provide an estimate of the income flexibility for each year. Multiply both sides of equation (5.2) by the Frisch relative price $(Dp_{it} - DP'_t)$ and then sum over $i = 1, \dots, n$. This yields for the left-hand side $\sum_{i=1}^{n} \bar{w}_{it} Dq_{it}(Dp_{it} - DP'_t) = C_{pqt} - (DP'_t - DP_t)DQ_t$. On the right-hand side there is an income term and a substitution term. The income term is zero as $\sum_{i=1}^{n} \theta_i DQ_t (Dp_{it} - DP'_t) = 0$,

violates economic intuition. Working's model rectifies this problem. In order not to overburden the notation, we omit the *t* subscript from θ_i on the understanding that when it is derived from Working's model it is time dependent.

[8] The underlying data are summarised in Table 3.1; the number of goods is $n = 9$, as before. For further details, including all parameter estimates and elasticities for each country, see Clements and Si (2017b).

[9] As $\log(y_c/y_{US}) = 0$ for $c = \mathrm{US}$, the intercept is interpreted as the estimated income flexibility for that country. The intercept estimates seem reasonable, and although they vary from -0.5 to -0.8, the differences are unlikely to be highly significant. Additionally, these values are not too far from the ϕ-estimate for the US in Table 5.1 of -0.6.

while the substitution term is $\phi_t V'_{pt}$. Hence, $C_{pqt} - (DP'_t - DP_t)DQ_t = \phi_t V'_{pt}$, so the estimator of ϕ_t is

(5.3) $$\hat{\phi}_t = \frac{C_{pqt} - (DP'_t - DP_t)DQ_t}{V'_{pt}}.$$

The numerator is a price-quantity covariance corrected for changes in the price of quality, $DP'_t - DP_t$, scaled by the income change DQ_t. The denominator is the Frisch price variance. This is the OLS estimator of ϕ_t from a weighted cross-commodity regression, by regressing $Dq_{it} - \eta_i DQ_t$ on $\eta_i(Dp_{it} - DP'_t)$, $i = 1, \cdots, n$, where η_i is the income elasticity of i, using budget shares as weights.[10]

Figure 5.2 plots the income flexibilities estimated by (5.3) with the OECD data. Table 5.3 presents tests of the dependence of the income flexibility on income for each country. The coefficients of log income are positive on average, but insignificant for the vast majority of countries in both the unweighted and weighted cases. The conclusion would seem to be that this time-series evidence does not support the Frisch conjecture. Probably there is insufficient year-to-year variability in income to for there to be much appreciable effect.

A third test of Frisch uses data pooled over countries and time. Table 5.4 reveals that once country fixed effects are allowed for, the coefficient of income while positive, is insignificantly different from zero.[11] Again, this is not supportive of the Frisch conjecture.

Figure 5.3 gives the corresponding distribution of the income flexibilities for the ICP countries. These have mean of -0.52, not too far from that of the OECD of -0.41 (Figure 5.2), but there is more dispersion. As discussed in the Appendix, as there now is less evidence in favour of the income dependence of ϕ, we have to reject Frisch with the broader group of countries. Consequently, it would not be unreasonable to treat the income flexibility as a constant equal to about -1/2 (at least as a first approximation).

The results of this section can be summarised as follows:

1. The principle of diminishing returns means that the marginal utility of income falls as income rises. Frisch argues that the marginal utility of income falls at a decreasing rate when things are measured logarithmically. That is, the income elasticity of the marginal utility falls in absolute value as income rises.

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[10] Note that under the (very) special case of unity income elasticities, the marginal shares coincide with the budget shares and equation (5.3) becomes

(*) $$\hat{\phi}_t = \frac{\sum_{i=1}^{n} \bar{w}_{it}(Dp_{it} - DP_t)(Dq_{it} - DQ_t)}{\sum_{i=1}^{n} \bar{w}_{it}(Dp_{it} - DP_t)^2} = \rho_t \frac{\sigma_{qt}}{\sigma_{pt}},$$

where ρ_t is the Divisia price-quantity correlation and σ_{qt} (σ_{pt}) is the Divisia standard deviation of quantities (prices), defined as $\sigma_{qt} = [\sum_{i=1}^{n} \bar{w}_{it}(Dq_{it} - DQ_t)^2]^{1/2}$ and $\sigma_{pt} = [\sum_{i=1}^{n} \bar{w}_{it}(Dp_{it} - DP_t)^2]^{1/2}$. If, for example, $\rho_t = -0.4$ and quantities are 50-percent more variable than prices, then $\hat{\phi}_t = -0.6$. As (*) involves no unknown parameters, this might be a reasonable starting point when little or nothing is known.

[11] The country fixed effects measure a combination of country-specific income flexibilities (relative to the US, the base country) and differing base years of income. Because of the differing base years, it does not make sense to estimate a restricted version of the pooled regression without country fixed-effects.

2. The *inverse* of this elasticity, known as the income flexibility, is a key parameter in demand analysis as it measures the average degree of substitutability among goods. Frisch's argument amounts to the absolute value of the income flexibility *increasing* with income. This is the Frisch conjecture.
3. Using a Divisia-Frisch-index-number approach to obtain estimates of ϕ, the evidence reveals support for the Frisch conjecture when we compare incomes across OECD countries. But with the larger group of countries in the ICP, there is little or no such support.
4. Setting $\phi = -1/2$ is not unreasonable.

6. The Substitution Bias

In many countries the consumer price index is of the Laspeyres form, the cost of last-period's basket at this period's prices $(\boldsymbol{p_1} \cdot \boldsymbol{q}_0)$, expressed as a fraction of last period's cost $(\boldsymbol{p_0} \cdot \boldsymbol{q}_0)$, that is, $\frac{\boldsymbol{p_1} \cdot \boldsymbol{q}_0}{\boldsymbol{p_0} \cdot \boldsymbol{q}_0} = \sum_{i=1}^{n} \left(\frac{p_{i0} q_{i0}}{\boldsymbol{p_0} \cdot \boldsymbol{q}_0}\right) \frac{p_{i1}}{p_{i0}}$, a weighted average of the price relatives. As is well known, this fixed-weight index does not permit substitution and represents an upper bound to "true" inflation. In a major study of the US when CPI inflation averaged about 3 percent p. a., Boskin et al. (1996, 1998) estimated that the CPI overstated inflation by about 0.4 percentage points due to the neglect of substitution. This section demonstrates how existing measures of the CPI can be easily adjusted to allow for substitution by using the Frisch price variance. It should be acknowledged that Diewert's (1976) class of superlative index numbers is now the leading way to deal with substitution.[12] The main objective of this section is thus not so much to offer a serious alternative way to measure the true cost of living, as to simply provide another illustration of the utility of the Divisia-Frisch approach. Some insight into the nature of the substitution bias is also provided in this section.

The cost function, $C(u, \boldsymbol{p})$, gives the minimum cost of achieving utility u given the price vector $\boldsymbol{p}$. Thus, the cost of base-period utility u_0 at current-period prices $\boldsymbol{p}_1 = [p_{i1}]$ is $C(u_0, \boldsymbol{p}_1)$, with

$$\frac{\partial C(u_0, \boldsymbol{p}_1)}{\partial p_{i1}} = q_i(u_0, \boldsymbol{p}_1), i = 1, \cdots, n, \quad \frac{\partial^2 C(u_0, \boldsymbol{p}_1)}{\partial p_{i1} \partial p_{j1}} = s_{ij}, i, j = 1, \cdots, n.$$

Here, $q_i(u_0, \boldsymbol{p}_1)$ is the utility-constant demand for good i and $s_{ij} = \partial q_i(u_0, \boldsymbol{p}_1)/\partial p_{j1}$ is the $(i,j)^{th}$ substitution term, $s_{ij} = s_{ji}, i, j = 1, \cdots, n.$ A second-order Taylor's series approximation around base-period prices $\boldsymbol{p}_0 = [p_{i0}]$ is

$$C(u_0, \boldsymbol{p}_1) \approx C(u_0, \boldsymbol{p}_0) + \sum_{i=1}^{n} \frac{\partial C(u_0, \boldsymbol{p}_0)}{\partial p_{i0}} (p_{i1} - p_{i0}) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 C(u_0, \boldsymbol{p}_0)}{\partial p_{i0} \partial p_{j0}} (p_{i1} - p_{i0})(p_{j1} - p_{j0})$$

$$= \sum_{i=1}^{n} p_{i1} q_i(u_0, \boldsymbol{p}_0) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij} (p_{i1} - p_{i0})(p_{j1} - p_{j0}).$$

[12] As discussed in Section 2, the Divisia price index is exact for the translog cost function and as the translog is a flexible functional form, Divisia is a superlative index. Consequently, the Divisia index would likely to be a better choice than Laspeyres for measuring the true cost of living.

Thus, the true-cost-of-living index of maintaining base-period utility when prices change from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$ is

$$(6.1)\qquad \frac{C(u_0,\boldsymbol{p}_1)}{C(u_0,\boldsymbol{p}_0)} \approx \frac{\sum_{i=1}^{n} p_{i1}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{ij}Dp_{i1}Dp_{j1},$$

where $\pi_{ij} = \frac{p_{i0}p_{j0}s_{ij}}{\sum_{i=1}^{n} p_{i0}q_{i0}}$ is a Slutsky coefficient and $Dp_{i1} = \log\frac{p_{i1}}{p_{i0}} \approx \frac{p_{i1}-p_{i0}}{p_{i0}}$ is the log-change in the price of i from period 0 to period 1.

The first term on the right of equation (6.1) is the Laspeyres index L_P, while the second term is the "CPI bias". Write the bias as

$$B = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{ij}\,Dp_iDp_j \geq 0,$$

where the nonnegativity follows from the $n \times n$ matrix $[\pi_{ij}]$ being negative semidefinite. Accordingly,

$$\frac{C(u_0,\boldsymbol{p}_1)}{C(u_0,\boldsymbol{p}_0)} \approx L_P - B,$$

so the Laspeyres is never less than the true index.

Demand homogeneity means that the row sums of the Slutsky matrix are zero, $\sum_{j=1}^{n}\pi_{ij} = 0, i = 1,\cdots,n$. If each price changes proportionately, then $Dp_i = k,\ i = 1,\cdots,n,$ and the bias is zero: $B = -\frac{k^2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{ij} = 0$. This means that as only relative prices matter, the bias can be expressed in terms of changes in relative prices

$$B = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{ij}(Dp_i - DP)(Dp_j - DP).$$

where $DP = \sum_{i=1}^{n}\overline{w}_{ii}Dp_i$ is the Divisia price index. The above expression can be written as

$$(6.2)\qquad B = \frac{1}{2}\left[\sum_{i=1}^{n} -\pi_{ii}(Dp_i - DP)^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n} -\pi_{ij}(Dp_i - DP)(Dp_j - DP)\right].$$

The first term in the squares brackets is a weighted sum of squared relative prices with weights the negative of the own-price Slutsky coefficients, $-\pi_{ii} > 0$. This term is always positive and becomes more larger as the dispersion of prices grows, when there is more change in the structure of relative prices. The more price-sensitive commodities make a larger contribution to this component of the bias. The second term in the square brackets of equation (6.2) involves cross-price effects. The weights are negative for substitutes,$-\pi_{ij} < 0, i \neq j$. If, on average, the prices of such goods are negatively correlated – if apples are expensive when bananas cheap and vice versa -- there is an additional effect that reinforces the impact of the price changes, resulting in larger substitution effects and a larger bias. The same is true when the prices of complementary goods $(-\pi_{ij} > 0)$ are positively correlated. In all other cases, the second term is negative and reduces the overall bias.

The bias can also be expressed in terms of the Frisch variance as follows. Define the vector of n price log-changes as $\boldsymbol{Dp} = [Dp_i]$ and $\boldsymbol{\theta} = [\theta_i]$ as the vector of marginal shares, so that the Frisch price index is $DP' = \sum_{i=1}^{n} \theta_i Dp_i = \boldsymbol{\theta' Dp}$, The Frisch variance can be expressed as a quadratic form:

(6.3) $$V_p' = \sum_{i=1}^{n} \theta_i \, (Dp_i - DP')^2 = (\boldsymbol{Dp})' \boldsymbol{\Theta} (\boldsymbol{I} - \boldsymbol{\iota\theta'}) \boldsymbol{Dp},$$

where $\boldsymbol{\Theta} = diag[\boldsymbol{\theta}]$, $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{\iota} = [1, \ldots, 1]'$.[13] Write the bias as $B = -\frac{1}{2}(\boldsymbol{Dp})' \boldsymbol{\Pi Dp}$, where $\boldsymbol{\Pi} = [\pi_{ij}]$ is the Slutsky matrix. Under preference independence, this matrix is given by equation (4.2), $\boldsymbol{\Pi} = \phi \boldsymbol{\Theta}(\boldsymbol{I} - \boldsymbol{\iota\theta'})$, so that, in view of equation (6.3),

(6.4) $$B = -\frac{1}{2}\phi(\boldsymbol{Dp})' \boldsymbol{\Theta}(\boldsymbol{I} - \boldsymbol{\iota\theta'})\boldsymbol{Dp} = -\frac{1}{2}\phi V_p'.$$

In words, the CPI bias is proportional to the Frisch price variance with $-\frac{1}{2}\phi$ the factor of proportionality. This attractively simple result holds under the condition of preference independence. In general terms, the assumption of preference independence is restrictive as it rules out specific interactions of goods in generating utility and so limits the degree of substitution. But when goods are relatively broad aggregates such as the nine we use -- food, clothing, housing, etc. (see Table 3.2) -- there is unlikely to be substantial substitution between them. Therefore, preference independence may be satisfactory, although needless to say, iron-glad guarantees cannot be given.

As can be seen from Figure 6.1 for the OECD, the Frisch variances tend to be very small, and so multiplying by $0 < -\frac{1}{2}\phi < 1$, the bias is even smaller. This could possibly reflect the annual-change nature of the underlying data, but even if we accumulate to the decade level, the bias would still only amount to about 0.5 percentage points at most. There is considerably more difference in the relative-price structure across countries, as is evident from Figure 6.2, which gives the Frisch variances for the ICP countries. Using the median variance of 7.4×10^{-2} and $\phi = -1/2$ in equation (6.4), the bias is slightly less than 2 percent. A bias of this magnitude could be of concern.

Finally, there is a different interpretation of the above analysis. The term $-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{ij}(Dp_i - DP)(Dp_j - DP)$ is a quadratic form in the price changes. If the price changes result from distortions in the economy (due, for example, to monopoly, import tariffs and so on), then the quadratic form is one of Harberger's (1964, Sec. II) measures of the welfare cost of distortions. This is a generalised measure of lost consumer surplus not offset by transfers from consumers to elsewhere in the economy, all expressed as a proportion of total expenditure.[14] Consequently, if tastes are characterised by preference independence, Harberger's measure also simplifies to $-\frac{1}{2}\phi V_p'$, one half the income flexibility times the Frisch price variance, with the sign ignored.

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[13] If $\boldsymbol{M} = (\boldsymbol{I} - \boldsymbol{\iota\theta'})$, then equation (6.3) follows from a type of weighted idempotent property of the $\boldsymbol{M}$ matrix, $\boldsymbol{M'\Theta M} = \boldsymbol{\Theta M}$. This matrix is also idempotent in the usual sense, $\boldsymbol{MM} = \boldsymbol{M}$.

[14] This measure is a generalised one as it accounts for multiple distortions and all own- and cross-price effects.

7. Concluding Comments

Suppose we have *n* prices $p_1, \cdots, p_n$. The Divisia (1926) index summarises this distribution in logarithmic change form by taking a budget-share-weighted average of the price changes, that is, $d(\log P) = \sum_{i=1}^{n} w_i d(\log p_i)$, where $w_i = p_i q_i / M$ is the budget share of good i, the share of total expenditure, $M = \sum_{i=1}^{n} p_i\, q_i$, devoted to the i^{th}good. For finite-change data from period *t-1* to *t*, the discrete approximation $DP_t = \sum_{i=1}^{n} \bar{w}_{it} Dp_{it}$, where $\bar{w}_{it} = \frac{1}{2}\left(w_{it} + w_{i,t-1}\right)$ is the share averaged over the two periods and $Dp_{it} = \log p_{it} - \log p_{i,t-1}$, is frequently used. In an influential paper, Diewert (1976) calls DP the Törnqvist (1936)-Theil (1965, 1967) index. There is also an analogous volume index $DQ_t = \sum_{i=1}^{n} \bar{w}_{it} Dq_{it}$, where $Dq_{it} = \log q_{it} - \log q_{i,t-1}$ is the quantity log-change, which measures the change in the consumer's real income. Divisia indexes have been shown to have good approximation properties (Diewert, 1976).

The Divisia index is well known from total factor productivity measurement. The Frisch (1932) index is less well known. Like Divisia, it is also a weighted average of the component changes, but rather than budget shares, it uses marginal shares as weights. The marginal share of good i is $\theta_i = \partial\, (p_i q_i)/\partial M$, the fraction of a one-dollar rise in income spent on the good. The Frisch price index is, thus, $d(\log P') = \sum_{i=1}^{n} \theta_i d(\log p_i)$, or for finite-change data, $DP_t' = \sum_{i=1}^{n} \theta_{it} Dp_{it}$, where θ_{it} is the marginal share for the transition from t-1 to t. The income elasticity of good i is just the ratio of the two shares θ_i / w_i, and as the elasticity exceeds unity for luxuries, these goods are more heavily weighted in the Frisch index as compared to Divisia; and the reverse is true for necessities. The Frisch volume index is $DQ_t' = \sum_{i=1}^{n} \theta_{it} Dq_{it}$.

This paper showed Divisia and Frisch indexes have interesting relationships that can be used to illuminate certain aspects of consumer behaviour. First, using the income elasticities to measure quality (luxuries are typically of higher quality), the excess of the Frisch volume index over its Divisia counterpart, $DQ' - DQ$, has the interpretation as the change in the quality of the overall consumption basket, while $DP' - DP$ is the dual price of quality (Clements and Gao, 2012, Theil, 1975/76). Second, Divisia and Frisch indexes can be used in a simple estimator of the income flexibility, the reciprocal of Frisch's (1959) "money flexibility" (the income elasticity of the marginal utility of income). This measures the overall degree of substitution among all goods in the basket, as well as being a key parameter for applied welfare economics. Third, the indexes can be used in the analysis of the nature and extent of the substitution bias in the consumer price index. As the CPI typically uses a fixed-weight index (such as Laspeyres) with no allowance for substitution effects, it overestimates "true" inflation when there are substantial changes in relative prices. The-understatement of inflation is the substitution bias. The paper showed that under certain simplifying assumptions, the substitution bias is proportional to a weighted variance of prices, with marginal shares as weights, that is, the Frisch variance. The link with Harberger's (1964) generalised measure of the cost of distortions was also noted.

Because of these interesting and useful relationships, Divisia and Frisch (indexes) can be described as "friends".

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Table 1.1 Selected Index-Numbers Publications and Citations

Item (1)	Citations (2)
A. Balk	
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Caves, D. W., L. R. Christensen and **W. E. Diewert** (1982b). "The Economic Theory of Index Numbers and the Measurement of Input, Output, and Productivity." Econometrica 50: 1393-1414.	4,227
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C. Divisia	
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D. Fisher	
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E. Frisch	
Frisch, R. (1930). "Necessary and Sufficient Conditions Regarding the Form of an Index Number which shall meet certain of Fisher's Tests." Journal of the American Statistical Association 25: 397-406.	99
Frisch, R. (1932). New Methods of Measuring Marginal Utility. Tübingen: J. C. B. Mohr.	315
Frisch, R. (1936). "Annual Survey of General Economic Theory: The Problem of Index Numbers." Econometrica 4: 1-38.	381
F. Samuelson and Swamy	
Samuelson, P. A. and S. Swamy (1974). "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis." American Economic Review 64: 566-93.	591

Note: These items are used to construct Figure 1.1. They represent the most-cited works, on a lifetime basis, on index numbers by the respective author. Citations are from a January 2018 search using Google Scholar (https://scholar.google.com.au/). Diewert's items are restricted to those with a minimum of 200 lifetime number citations. **Embolden** names indicate the authors referred to in Figure 1.1.

Table 3.1 The OECD Data

Country	Period	Number of observations	Per capita GDP, 2010	
			International dollars	US = 100
(1)	(2)	(3)	(4)	(5)
1. Luxembourg	1995 - 2015	21	85,621	177.3
2. Norway	1970 - 2014	45	58,054	120.2
3. Switzerland	1995 - 2013	19	53,132	110.0
4. United States	1970 - 2015	46	48,291	100.0
5. Netherlands	1995 - 2014	20	44,542	92.2
6. Denmark	1995 - 2015	21	43,057	89.2
7. Ireland	1996 - 2014	19	42,803	88.6
8. Australia	1985 - 2014	30	42,350	87.7
9. Austria	1995 - 2015	21	41,944	86.9
10. Sweden	1993 - 2015	23	41,649	86.2
11. Belgium	1995 - 2014	20	40,003	82.8
12. Germany	1995 - 2014	20	39,918	82.7
13. Canada	1981 - 2015	35	39,885	82.6
14. Finland	1975 - 2015	41	38,781	80.3
15. Iceland	1995 - 2014	20	38,411	79.5
16. France	1959 - 2015	57	37,209	77.1
17. UK	1995 - 2015	21	35,769	74.1
18. Japan	1994 - 2013	20	35,203	72.9
19. Italy	1995 - 2014	20	34,893	72.3
20. Spain	1995 - 2014	20	31,968	66.2
21. New Zealand	1987 - 2014	28	31,133	64.5
22. Korea	1970 - 2015	46	30,664	63.5
23. Israel	1995 - 2014	20	29,646	61.4
24. Greece	1995 - 2015	21	28,061	58.1
25. Slovenia	1995 - 2015	21	27,740	57.4
26. Czech Rep.	1995 - 2015	21	27,636	57.2
27. Portugal	1995 - 2015	21	27,331	56.6
28. Slovak Rep.	1995 - 2015	21	24,939	51.6
29. Estonia	1995 - 2015	21	21,613	44.8
30. Hungary	1995 - 2015	21	21,435	44.4
31. Poland	1995 - 2015	21	20,798	43.1
32. Lithuania	1995 - 2014	20	19,965	41.3
33. Latvia	1995 - 2014	20	17,652	36.6
34. Turkey	1998 - 2014	17	17,464	36.2
35. Mexico	2003 - 2014	12	14,586	30.2
36. South Africa	1975 - 2014	40	11,652	24.1
37. Colombia	2000 - 2014	15	10,680	22.1

Notes: Data are from the OECD website.

Table 3.2 Budget Shares, Marginal Shares and Income Elasticities, OECD Countries

Commodity	Budget share $w_i \times 100$	Marginal share $\theta_i \times 100$	Income elasticity η_i
(1)	(2)	(3)	(4)
1. Food and alcohol	20.5	12.0	0.57
2. Clothing and footwear	5.3	5.5	1.01
3. Housing and utilities	20.9	12.4	0.60
4. Furnishings, equipment	5.7	7.3	1.30
5. Health	4.1	4.1	0.98
6. Recreation and culture	8.5	11.6	1.38
7. Education	1.4	1.1	1.41
8. Restaurants and hotels	7.4	7.6	1.06
9. Misc goods and services	26.1	38.5	1.48

Notes:

1. The budget share of good i is $w_i = p_i q_i / M$, the share of i in total expenditure M.
2. The marginal share is $\theta_i = \partial(p_i q_i)/\partial M$, which is estimated from the differential demand system that is specific to each of the 37 countries. The i[th] equation of this system $(i = 1,\ldots,9)$ is $\bar{w}_{it}(Dq_{it} - DQ_t) = \beta_i DQ_t + \phi(\beta_i + \bar{w}_i)(Dp_{it} - DP'_t) + \varepsilon_{it}$, where $\bar{w}_{it} = ½(w_{it} + w_{i,t-1})$; D is the log-change operator, defined as $Dx_{it} = \log x_{it} - \log x_{i,t-1}$; $DQ_t = \Sigma_{i=1}^{9} \bar{w}_{it} Dq_{it}$; $\bar{w}_i$ is the mean over time of w_{it}; $DP'_t = \Sigma_{i=1}^{9}(\beta_i + \bar{w}_i)Dp_{it}$; and ε_{it} is a disturbance term. Here, β_i is Working's (1943) income coefficient and ϕ the income flexibility, the reciprocal of the income elasticity of the marginal utility of income. This model implies that $\theta_i = \beta_i + \bar{w}_{it}$. For details, see Clements and Si (2017b).
3. The income elasticity of i is $\eta_i = \theta_i / w_i$.
4. All elements of this table are means over time and countries.

Table 5.1 Income Flexibilities, 37 OECD Countries
(Standard errors in parentheses)

1. Australia	-0.414 (0.054)	21. Lithuania	-0.394 (0.089)
2. Austria	-0.463 (0.062)	22. Luxembourg	-0.848 (0.143)
3. Belgium	-0.050 (0.047)	23. Mexico	-0.830 (0.000)
4. Canada	-0.350 (0.048)	24. Netherlands	-0.282 (0.066)
5. Colombia	-0.082 (0.045)	25. New Zealand	-0.624 (0.067)
6. Czech Rep.	-0.438 (0.067)	26. Norway	-0.906 (0.082)
7. Denmark	-0.530 (0.087)	27. Poland	-0.606 (0.071)
8. Estonia	-0.321 (0.055)	28. Portugal	-0.612 (0.083)
9. Finland	-0.486 (0.059)	29. Slovak Rep.	-0.477 (0.058)
10. France	-0.444 (0.046)	30. Slovenia	-0.336 (0.084)
11. Germany	-0.300 (0.051)	31. South Africa	-0.376 (0.046)
12. Greece	-0.148 (0.134)	32. Spain	-0.219 (0.062)
13. Hungary	-0.524 (0.052)	33. Sweden	-0.515 (0.068)
14. Iceland	-0.806 (0.095)	34. Switzerland	-1.706 (0.031)
15. Ireland	-2.459 (0.053)	35. Turkey	-0.183 (0.039)
16. Israel	-0.017 (0.059)	36. UK	-0.812 (0.078)
17. Italy	-0.526 (0.048)	37. United States	-0.601 (0.047)
18. Japan	-0.605 (0.160)		
19. Korea	-0.209 (0.046)	Mean	-0.538 (0.067)
20. Latvia	-0.423 (0.106)		

Source: Clements and Si (2017b).

Table 5.2 First Test of Frisch Conjecture:
Between 37 OECD Countries

$$\phi_c = \alpha + \beta \log\left(\frac{y_c}{y_{US}}\right) + \varepsilon_c$$

(Standard errors in parentheses)

Sample (1)	Intercept α (2)	Slope β (3)	SEE × 100 (4)	R^2 (5)
		A. Unweighted		
1. 37 countries	-0.69 (0.10)	-0.35 (0.16)	42.32	0.12
2. 36 countries	-0.70 (0.10)	-0.42 (0.17)	41.74	0.16
3. 34 countries	-0.54 (0.05)	-0.23 (0.08)	20.24	0.20
		B. Weighted		
4. 36 countries	-0.84 (0.12)	-0.59 (0.19)	1.47	0.63
5. 34 countries	-0.47 (0.05)	-0.17 (0.07)	0.59	0.82

Notes: The ϕ_c-values are from Table 5.1. The variable y_c is GDP per capita of country c in 2010 in $US using PPP exchange rates; and y_{US} is US GDP per capita. The weights in panel B are proportional to reciprocals of standard errors of the estimates of ϕ_c. The "37 countries" group refers to the full set of countries; "36 countries" omits Mexico due to near-zero standard error of its ϕ_c-estimate; and the "34 countries" group further eliminates Ireland and Switzerland as outliers as their ϕ_c-estimates seem large in absolute value.

Table 5.3 Second Test of Frisch Conjecture:
Within 37 OECD Countries

$$\phi_t = \alpha + \beta \log Q_t + \varepsilon_t$$

(Standard errors in parentheses)

Country	Unweighted		Weighted	
	Intercept	Slope	Intercept	Slope
(1)	(2)	(3)	(4)	(5)
1. Australia	-0.63 (0.13)	0.87 (0.38)	-0.63 (0.12)	0.87 (0.40)
2. Austria	-0.37 (0.26)	-0.02 (1.80)	-0.05 (0.32)	-2.10 (2.08)
3. Belgium	-0.81 (0.96)	9.62 (8.32)	-	-
4. Canada	-0.47 (0.25)	1.28 (0.84)	-0.27 (0.37)	1.83 (1.26)
5. Colombia	-0.31 (0.16)	0.45 (0.78)	-0.43 (0.23)	0.67 (1.01)
6. Czech Rep.	-1.00 (0.24)	2.33 (0.82)	-1.21 (0.25)	3.23 (0.80)
7. Denmark	-0.95 (0.49)	2.13 (3.87)	-1.05 (0.53)	1.76 (4.29)
8. Estonia	-0.77 (0.46)	0.50 (0.66)	-2.11 (0.68)	2.29 (0.93)
9. Finland	-0.48 (0.22)	0.37 (0.49)	-0.38 (0.33)	0.44 (0.66)
10. France	-0.36 (0.16)	0.02 (0.18)	-0.51 (0.21)	0.21 (0.23)
11. Germany	-0.16 (0.26)	-2.42 (2.48)	-0.28 (0.28)	0.63 (2.60)
12. Greece	-0.08 (0.67)	0.73 (2.74)	-0.65 (1.10)	3.40 (3.85)
13. Hungary	-0.49 (0.28)	1.02 (1.02)	-0.64 (0.40)	1.93 (1.42)
14. Iceland	-0.60 (0.50)	-0.09 (1.51)	-0.97 (0.90)	1.62 (2.64)
15. Ireland	-2.41 (0.29)	0.41 (0.26)	-2.23 (0.40)	0.38 (0.35)
16. Israel	-0.05 (0.27)	-0.05 (1.42)	0.06 (0.27)	-0.49 (1.41)
17. Italy	-0.82 (0.45)	6.21 (4.85)	-1.76 (0.33)	18.01 (4.62)
18. Japan	-1.40 (0.88)	7.14 (11.16)	-8.45 (1.46)	82.88 (46.37)
19. Korea	-0.01 (0.21)	-0.12 (0.16)	0.47 (0.41)	-0.42 (0.26)
20. Latvia	0.00 (0.42)	-0.18 (0.68)	-0.24 (0.46)	0.40 (0.77)
21. Lithuania	-0.03 (0.32)	-0.29 (0.43)	0.83 (0.62)	-0.97 (0.74)
22. Luxembourg	-1.70 (0.76)	7.44 (4.86)	-1.91 (0.78)	6.78 (4.94)
23. Mexico	-0.45 (0.32)	-0.25 (2.76)	-0.40 (0.42)	-0.41 (3.37)
24. Netherlands	1.12 (0.93)	-9.42 (5.64)	0.75 (0.90)	-10.14 (6.35)
25. New Zealand	-0.40 (0.17)	0.58 (0.79)	-0.38 (0.17)	1.26 (0.81)
26. Norway	-1.00 (0.22)	0.58 (0.39)	-0.91 (0.26)	0.62 (0.47)
27. Poland	-0.07 (0.29)	-1.03 (0.62)	0.37 (0.31)	-1.97 (0.62)
28. Portugal	-0.92 (0.38)	1.98 (1.39)	-1.00 (0.79)	2.11 (2.68)
29. Slovak Rep.	-0.14 (0.38)	-0.96 (0.72)	-0.18 (0.70)	-1.01 (1.12)
30. Slovenia	-0.05 (0.25)	-0.86 (0.87)	-0.01 (0.23)	-0.89 (0.80)
31. South Africa	-0.51 (0.13)	0.42 (0.47)	-0.56 (0.13)	0.52 (0.39)
32. Spain	0.65 (0.38)	-3.44 (2.01)	1.05 (0.40)	-4.43 (2.47)
33. Sweden	-0.28 (0.29)	0.15 (1.11)	-0.10 (0.39)	-0.26 (1.35)
34. Switzerland	-2.19 (0.14)	1.05 (0.44)	-2.18 (0.21)	0.85 (0.63)
35. Turkey	-0.26 (0.13)	0.33 (0.62)	-0.34 (0.18)	0.53 (0.74)
36. UK	-0.65 (0.14)	-0.21 (0.52)	-0.60 (0.16)	-0.40 (0.58)
37. United States	-0.27 (0.16)	-0.11 (0.27)	0.08 (0.15)	-0.73 (0.28)
Mean	-0.52 (0.35)	0.71 (1.85)	-0.75 (0.44)	3.03 (2.90)
Median	-0.45	0.37	-0.42	0.53
SD	0.66	3.07	1.52	14.04
% significant	51.35	8.11	47.22	22.22

Notes: The ϕ_t -values are from equation (5.3) with n = 9. The variable $\log Q_t$ is the log of real income, defined as the cumulative value the logarithmic income changes, that is, $\log Q_t = DQ_t + \log Q_{t-1}$ for t > 1, with $\log Q_0 = 0$. For the weighted regressions (columns 4 and 5), the weights are inversely proportional to the square roots of the denominator on the right-hand side of equation (5.3) with n = 9, that is, weight in year $t = k \cdot V_{pt}^{-1/2}$, where $V_{pt} = \sum_{i=1}^{9} \theta_i (Dp_{it} - DP_t')^2$ and $k^{-1} = \sum_{t=1}^{T} V_{pt}^{-1/2}$.

Table 5.4 Third Test of Frisch Conjecture:
Between and Within 37 OECD Countries

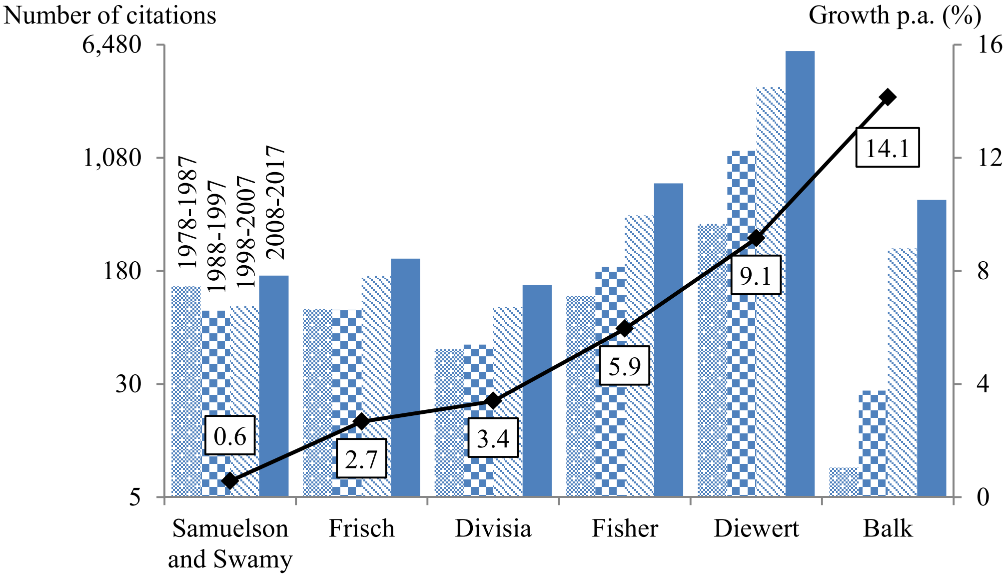
$$\phi_{ct} = \alpha + \sum_{d=1}^{36} \gamma_d I_{dt}(c = d) + \beta \log Q_{ct} + \varepsilon_{ct}$$

(Standard errors in parentheses)

Measure (1)	Coefficient (2)	Measure (3)	Coefficient (4)
Slope β	0.10 (0.11)		
Common intercept α	-0.38 (0.12)		
Country dummy γ_d (Base = US)			
1. Australia	-0.03 (0.18)	19. Korea	0.11 (0.17)
2. Austria	-0.01 (0.20)	20. Latvia	0.23 (0.20)
3. Belgium	0.61 (0.21)	21. Lithuania	0.08 (0.20)
4. Canada	0.20 (0.17)	22. Luxembourg	-0.24 (0.20)
5. Colombia	0.13 (0.23)	23. Mexico	-0.11 (0.25)
6. Czech Rep.	0.00 (0.20)	24. Netherlands	-0.01 (0.20)
7. Denmark	-0.34 (0.20)	25. New Zealand	0.05 (0.18)
8. Estonia	-0.13 (0.20)	26. Norway	-0.38 (0.16)
9. Finland	0.00 (0.16)	27. Poland	-0.16 (0.20)
10. France	-0.05 (0.15)	28. Portugal	-0.05 (0.20)
11. Germany	-0.02 (0.21)	29. Slovak Rep.	-0.29 (0.20)
12. Greece	0.44 (0.20)	30. Slovenia	0.08 (0.20)
13. Hungary	0.12 (0.20)	31. South Africa	-0.07 (0.16)
14. Iceland	-0.28 (0.20)	32. Spain	0.41 (0.20)
15. Ireland	-1.71 (0.21)	33. Sweden	0.11 (0.19)
16. Israel	0.30 (0.20)	34. Switzerland	-1.55 (0.21)
17. Italy	0.08 (0.21)	35. Turkey	0.16 (0.22)
18. Japan	-0.57 (0.21)	36. UK	-0.36 (0.20)

Notes: The dependent variable ϕ_{ct} is the income flexibility in country c (c = 1,…, 37 OECD countries) in year t. The term $I_{dt}(c = d)$ is an indicator function that equals 1 when c = d, 0 otherwise. The variable $\log Q_{ct}$ is the log of income in c for year t, defined as the cumulative change. The equation is estimated as a pooled regression. See notes to Table 5.3.

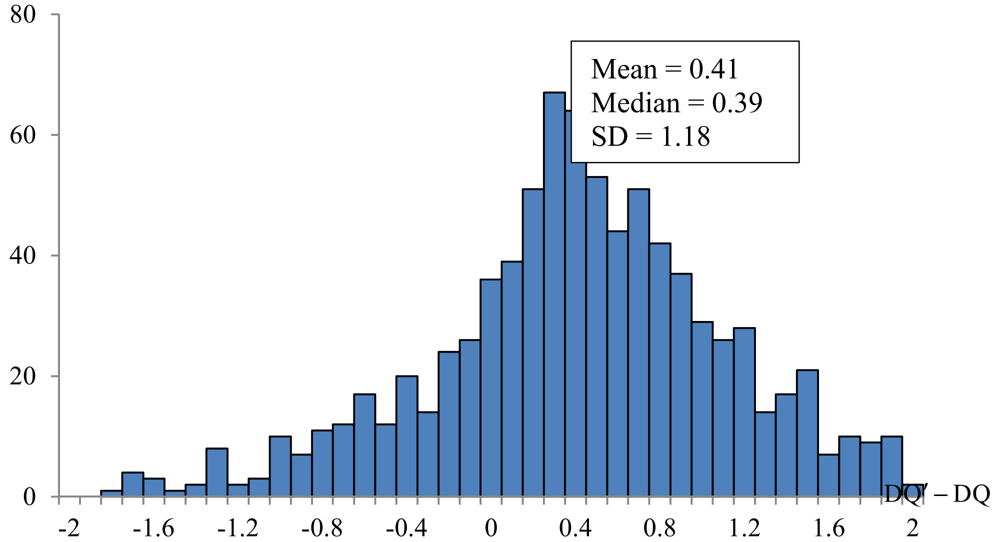
Figure 1.1 Citations of Index-Number Scholars



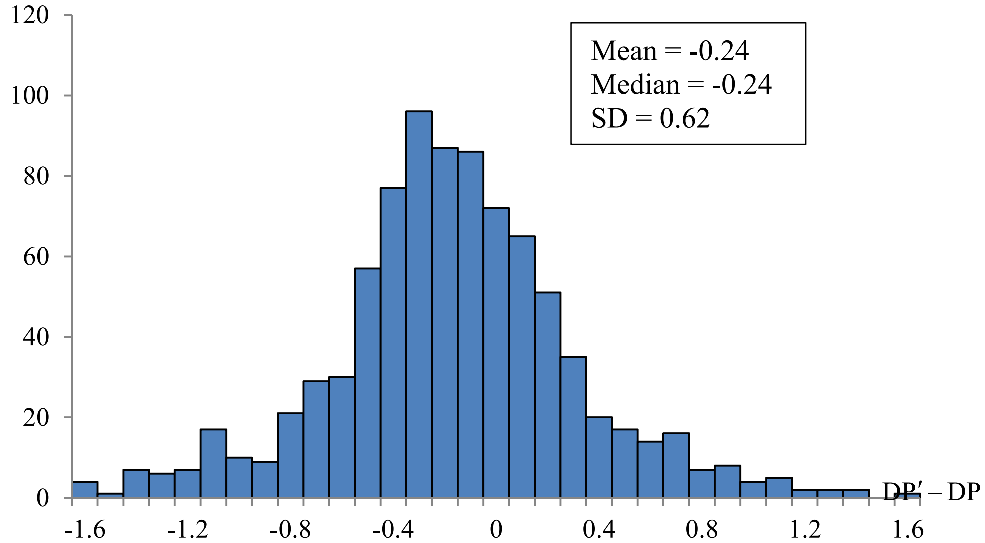
Note: This figure gives the results of keyword searches conducted on January 2018 using Google Scholar (https://scholar.google.com.au/). The search terms are selected works on index numbers by the authors listed on the horizontal axis. The height of each column represents the number of citations to these items in the prescribed decade, measured against the geometric scale on the left-hand axis. The right-hand axis gives the annual growth rate in the number of these citations, calculated as 100 times the logarithm of the ratio of the number in the period 2008-2017 to that in 1978-1987, divided by 30 (the number of years in the three decades). A list of the selected works is given in Table 1.1.

Figure 3.1 Quality Indexes, OECD Countries

A. Consumption



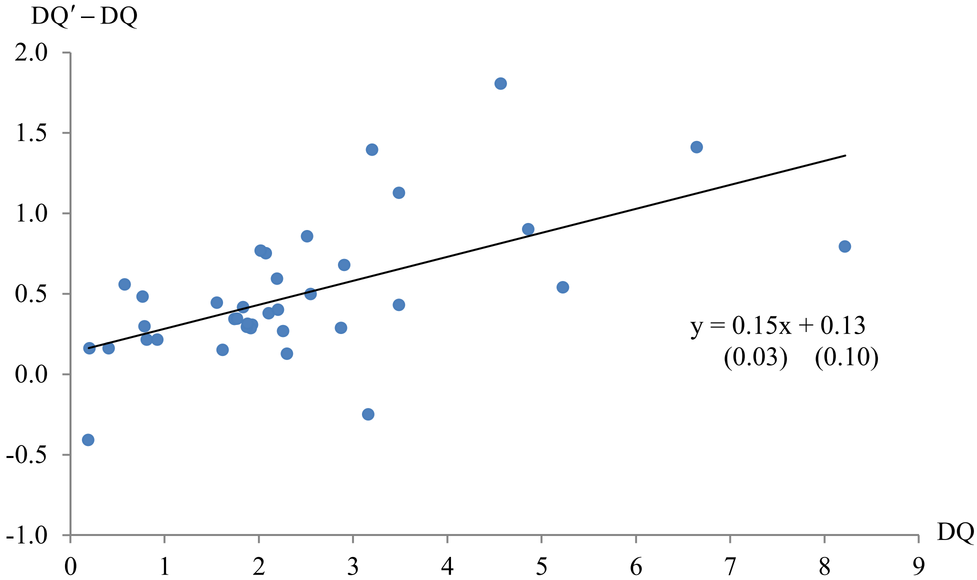
B. Prices



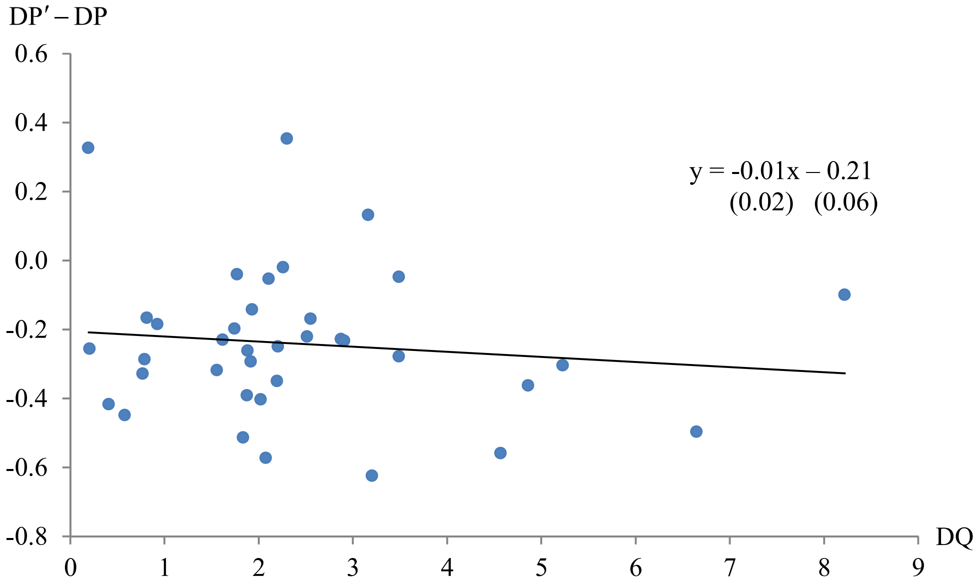
Note: Panel A contains the index of the quality of consumption $DQ' - DQ$, where DQ' is the Frisch volume index and DQ its Divisia counterpart. The index of the price of quality in panel B is defined analogously. To ease readability, panels A and B have been truncated to [-2, 2] and [-1.6, 1.6], respectively. All values are ×100.

Figure 3.2 Quality, Prices and Incomes, OECD Countries
(Medians × 100)

A. Consumption



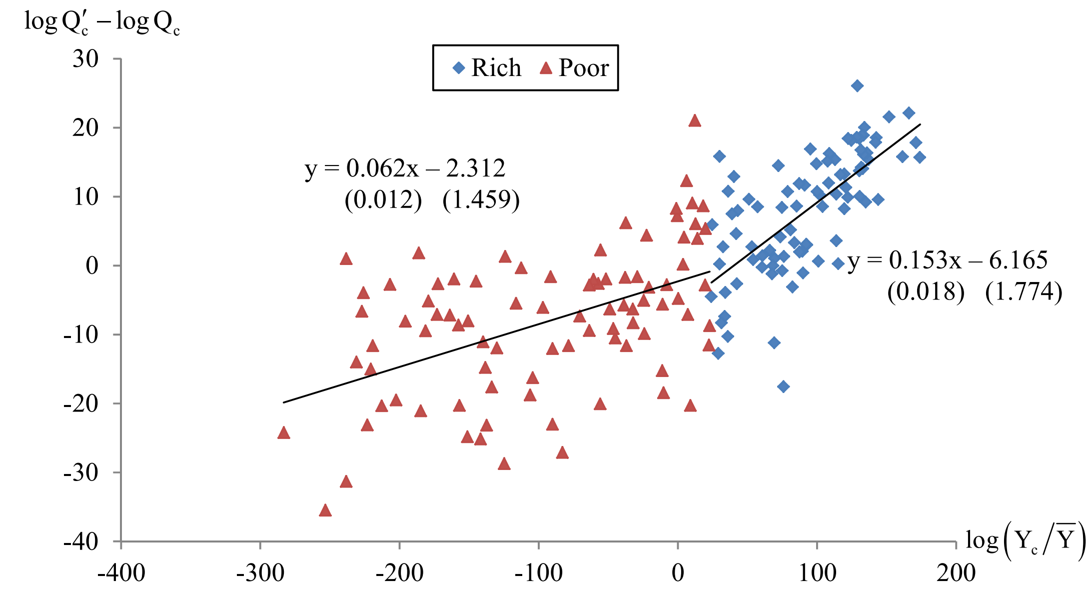
B. Prices



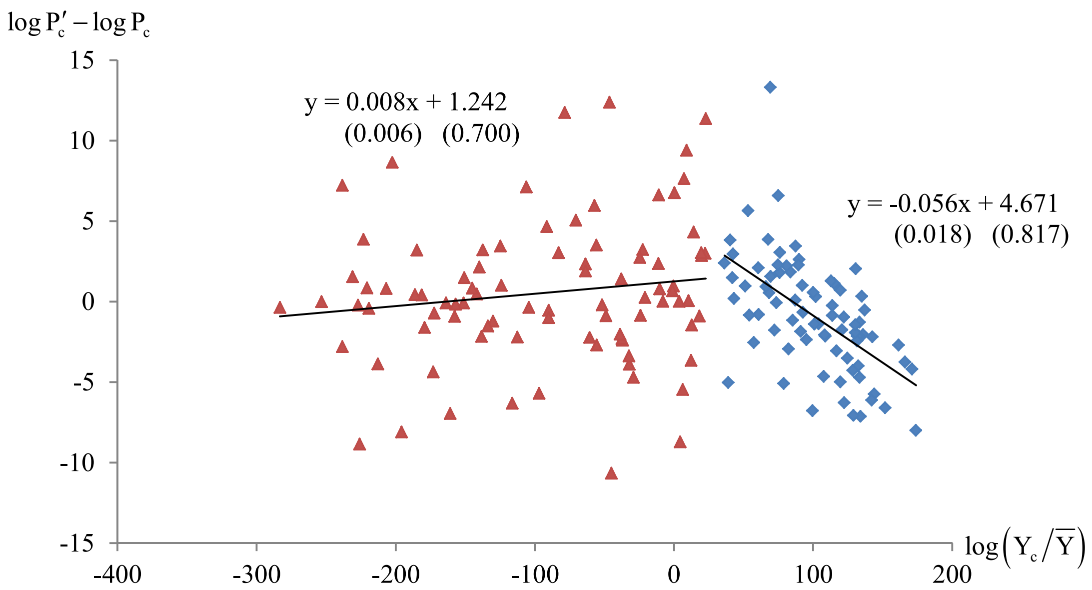
Notes: This figure plots each of the 37 countries' median quality of consumption, $DQ' - DQ$, and median price of quality, $DP' - DP$, against median income, DQ. All values are ×100.

Figure 3.3 Quality and Income,
176 ICP Countries, 2011

A. Consumption

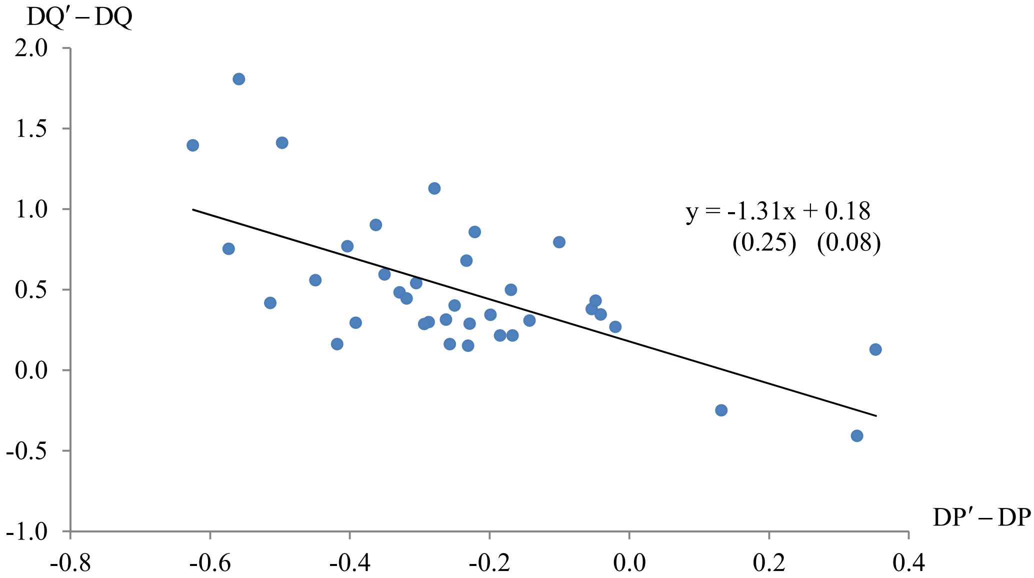


B. Prices



Notes: The quality of consumption for country c is the excess of the Frisch over the Divisia volume index $\log Q_c' - \log Q_c$. The price of quality is defined analogously. Income is real per capita consumption of country c, relative to the cross-country geometric mean, $\bar{Y}$, $\log\left(Y_c/\bar{Y}\right)\times 100$ Rich countries are those in the upper two income quartiles; poor countries are in the lower two quartiles. See Appendix for details. All values are $\times 100$.

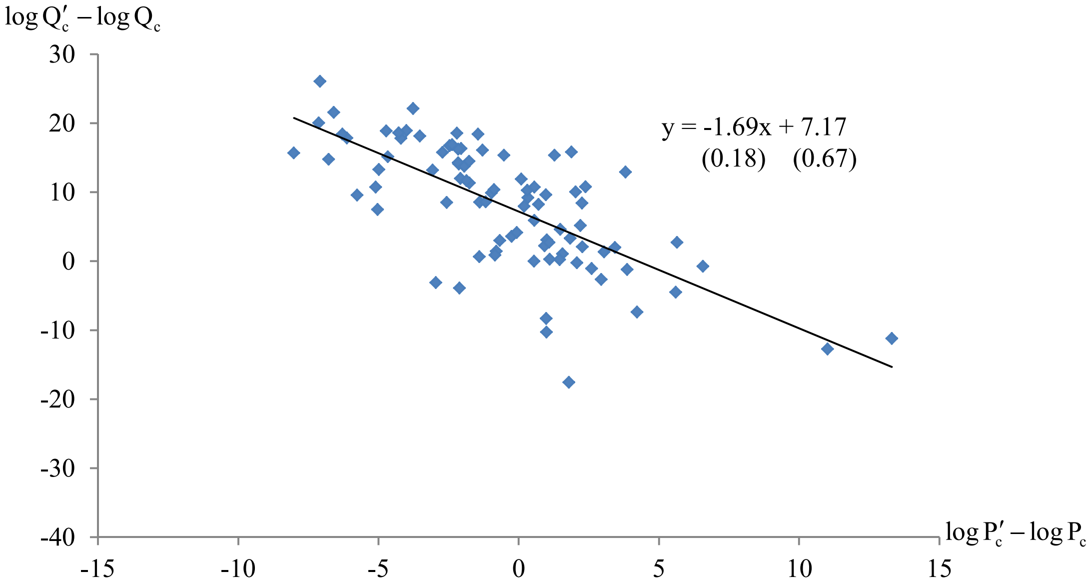
Figure 3.4 Price-Quantity Scatter of Quality, OECD Countries
(Medians × 100)



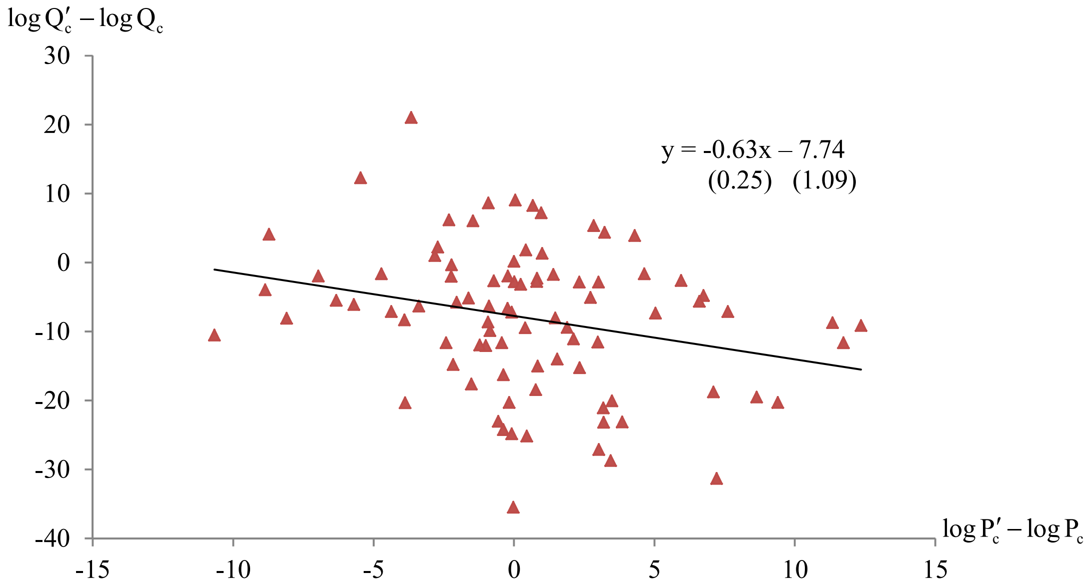
Note: The term $DQ' - DQ$ is the quality of consumption; and $DP' - DP$ is the price of quality. This figure contains the medians for each of the 37 OECD countries. All values are ×100.

Figure 3.5 Price-Quantity Scatter of Quality,
176 ICP Countries, 2011

A. Rich

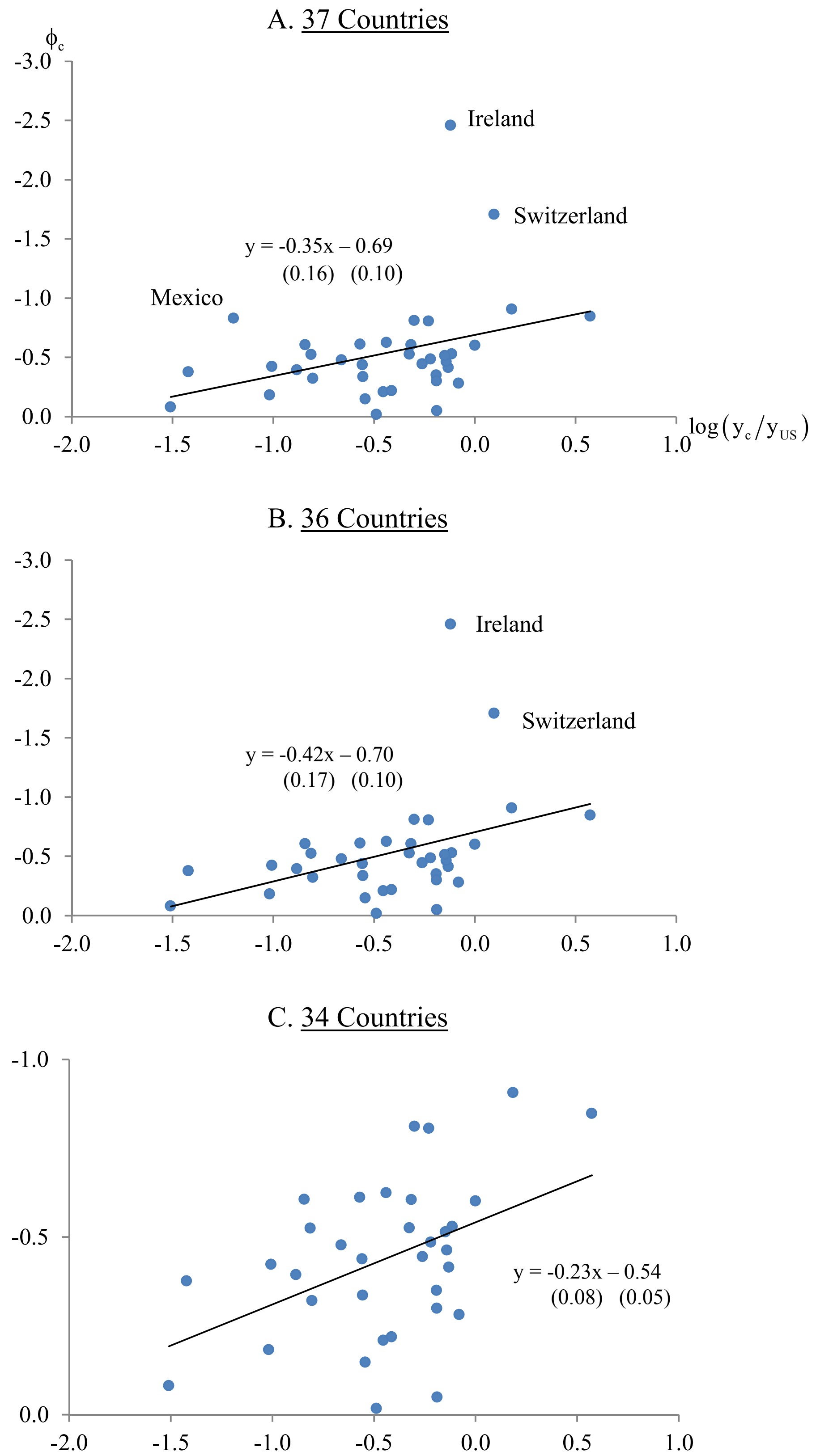


B. Poor



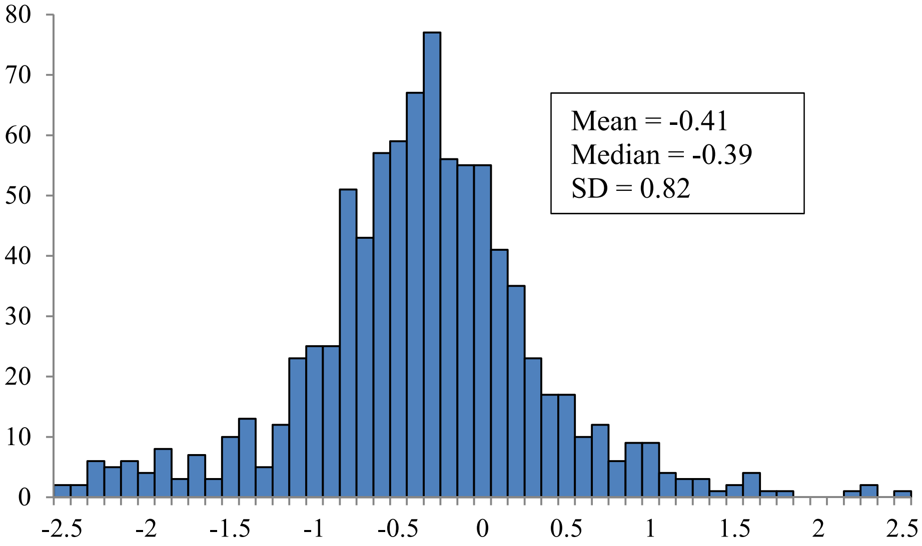
Note: The quality of consumption is $\log Q'_c - \log Q_c$, the excess of the Frisch volume index over the Divisia counterpart; and $\log P'_c - \log P_c$ is the price of quality, the excess of the Frisch price index over the Divisia counterpart. Panels A and B together contain observations pertaining to the 176 ICP countries, split by rich (the upper two income quartiles) and poor (the lower two quartiles). See Appendix for details. All values are ×100.

Figure 5.1 Income Flexibilities and GDP per capita,
OECD Countries



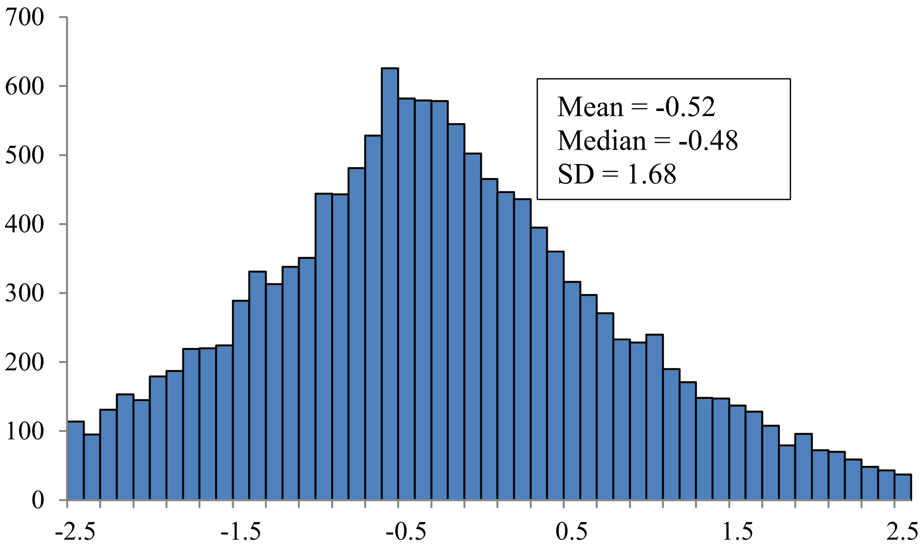
Note: The variable y_c is GDP per capita of country c in 2010 in $US using PPP exchange rates; and y_{US} is US GDP per capita. Note that the scale on the vertical axis is inverted.

Figure 5.2 Income Flexibilities,
Each Year and Country, OECD Countries



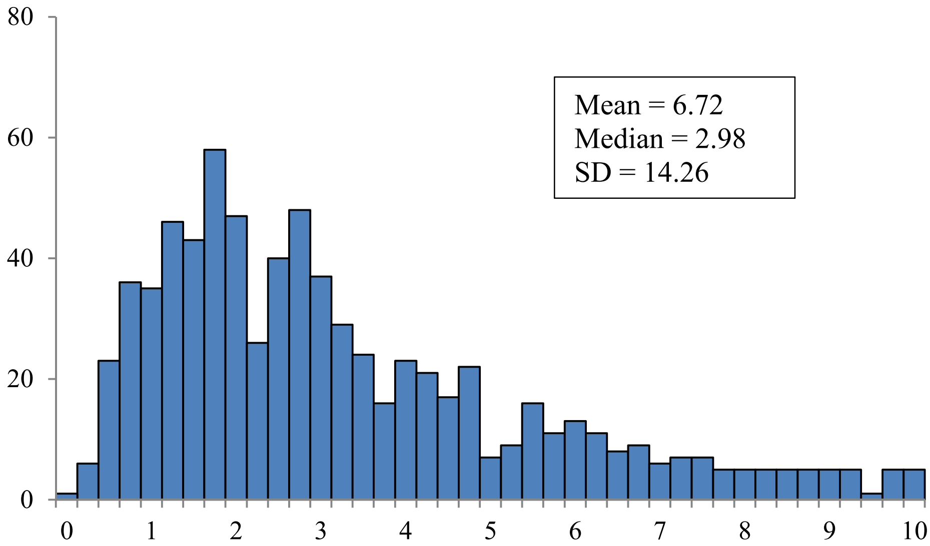
Note: Truncated at [-2.5, 2.5].

Figure 5.3 Income Flexibilities,
176 ICP Countries, 2011



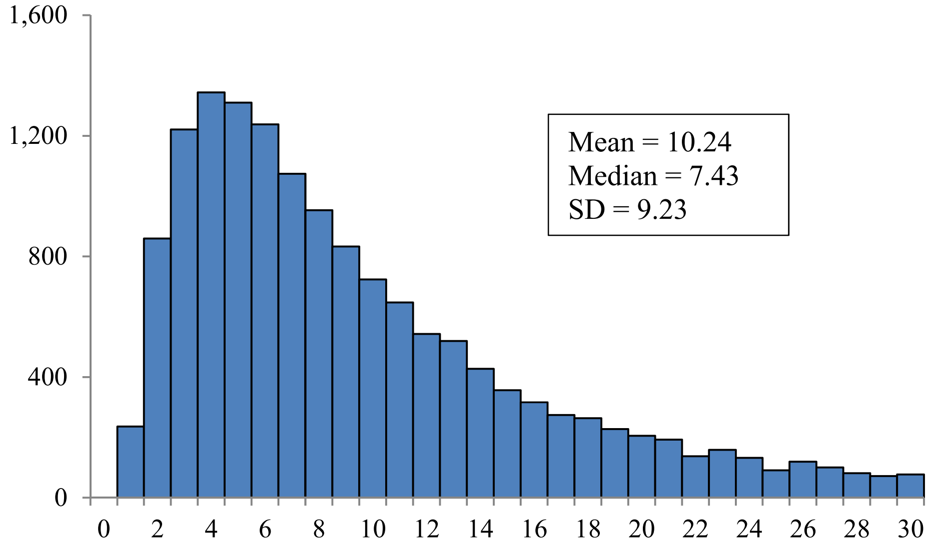
Note: Truncated at [-2.5, 2.5]. See Appendix for details.

Figure 6.1 Frisch Variances,
Each Year and Country, OECD Countries



Note: The Frisch variance is $\Sigma_{i=1}^{9}\theta_{it}\left(Dp_{it}-DP_t'\right)^2$, where $\theta_{it}=\beta_i+\overline{w}_{it}$ is the marginal share of good i in year t and $DP_t'=\Sigma_{i=1}^{9}\theta_{it}Dp_{it}$ is the Frisch price index. The term β_i is Working's (1943) income coefficient for good i and is country-specific; see Clements and Si (2017b) for details. The horizontal axis is truncated at [0, 10]. All values are $\times\ 10^4$.

Figure 6.2 Frisch Price Variances,
176 ICP Countries, 2011



Note: The bilateral Frisch price variance for countries c and d is $\Sigma_{i-1}^{9}\theta_{icd}\left(\log p_{icd}-\log P_{cd}'\right)^2$, where $\theta_{icd}=\left(1/2\right)\left(\theta_{ic}+\theta_{id}\right)$ is the average marginal share of i in c and d; $\log p_{icd}=\log p_{ic}-\log p_{id}$ is the price of item i in c compared to d; and $\log P_{cd}'=\Sigma_{i=1}^{9}\theta_{icd}\log p_{icd}$ is the Frisch price index. See Appendix for details. All values are $\times\ 10^2$.

APPENDIX
THE 176 ICP COUNTRIES

The Data

The data from the 2011 round of the International Comparison Program (ICP) contain disaggregated expenditures and PPP prices in 182 countries for 19 broad categories that are the components of GDP. These are publically available data on the World Bank's ICP website http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html. We are particularly interested in total consumption expenditure, which is defined as the sum of the first 12 categories in this database; this follows the ICP's definition of "Actual Household Consumption", which is the total value of the individual consumption expenditures of households, non-profit institutions serving households, and general government at purchasers' prices.

We edit the data as follows. Firstly, we remove duplicate entries for three countries (Russia, Sudan and Egypt), each of which is a dual participant in the ICP, leaving 179 unique countries. Next, Cuba and Bonaire do not have complete data and are omitted. Lastly, Algeria is omitted as there are issues with the marginal shares for this country; see the last section of this appendix for details. Our final sample thus contains 182 (the starting number of countries) – 3 (duplicates) – 2 (incomplete data) – 1 (Algeria) = 176 countries.

Summary of Results

Table A1 provides the key measures for each country. Countries are ranked according to decreasing affluence, defined as per capita income (strictly speaking, real per capita consumption), and are divided into quartiles. The ICP countries can be compared to the 37 OECD countries of the main text of the paper. From column 5 of Table 3.1 of the main text, income in most OECD countries is at least 50 percent that of the US. Using 50 percent as a cut-off, from columns 2 and 9 of Table A1 it can be concluded that most of the OECD countries would fall in the first quartile (the richest) of the ICP countries.

The other main points that can be drawn from Table A1 are:

- In going from the richest countries to poorest, on average, the quality of consumption decreases (columns 3 and 10), whilst there is a weak tendency for the price of quality to increase (columns 4 and 11). Both these measures have a zero mean, so the values for an individual country are interpreted as being relative to the world average. Thus, for example, from row 91 of column 3, the quality of consumption for Thailand is 5.4;

this means that the quality of the Thai consumption basket is about 5.4 percent greater than the world average.

- From columns 5 and 12, there seems to be no easily identifiable movement in the value of the income flexibility. In contrast, the income flexibility for the OECD countries was found to increase in absolute value as income rose.
- Price dispersion, as measured by the Divisia standard deviation (SD) of columns 6 and 13 seems roughly the same across income quartiles. The same is true of the Frisch SD (columns 7 and 14); and for a given country, the Frisch and Divisia SDs are not too far apart.

A more extensive discussion of these measures follows.

<u>Quality</u>

Figure A1 plots the distribution of the bilateral indexes of the quality of consumption and its price. The key features here are:

- The dispersion of the quality of consumption is 16.9 percent, while that of prices is 5.8 percent. From Figure 3.1, the corresponding values for the OECD are 1.2 percent for consumption and 0.6 percent for prices. That is, in both cases, quantities are more volatile than prices.
- Relatedly, the dispersion of the quality of consumption for the ICP is substantially larger than that for the OECD (Figure 3.1). The same is true of the price of quality.[15]

Figure A2 plots the quality of consumption and its price against income, with countries in the top two income quartiles (referred to in the figure as the "rich") distinguished from those in the bottom two quartiles (the "poor"). The following points can be noted:

- From panel A, for most of the poor countries, the quality of consumption is negative, which means it is below the world average. For most rich countries, the reverse is true, so they are above, as is to be expected. But within each country group, the quality of consumption increases with income. This effect is much stronger for the rich than the poor. Still, however, the income elasticity (the coefficient of log income) for the rich countries is low at 0.15. A further point is that the larger dispersion of observations around the regression line for the poor countries than for the rich.
- Panel B of Figure A2, regarding the price of quality, provides a contrast to panel A:

[15] As mentioned above, the means of the multilateral versions of these measures in Table A1 are zero. The means of the bilateral versions in Figure A1 are not zero because they refer to just the upper triangle of the corresponding matrices; see the notes to Figure A1 for details. The means of all elements of the matrices (in both the upper and lower triangles) are zero as the matrices are skew symmetric.

1. The poor countries are scattered around zero and the regression line is near horizontal. So, the prices are about at world average and there is no systematic relation with income.
2. For the rich, the prices are mostly less than world average, so quality is cheaper in these countries, and becomes increasingly cheaper as income rises. The income elasticity of the price in the rich countries is significantly negative, but low at -0.06.

- Taken as a whole, on average, the price of quality in rich countries is less than that in the poor, so luxuries relative to necessities are cheaper in the rich world. There is thus a regressive bias in the structure of prices. All else being the same, given your consumption basket, if you are rich, and therefore consume luxuries intensively, it is better to locate in a rich country, where luxuries are relatively cheaper; and the richer the country, the lower the relative price of luxuries and the better off you will be. Similarly, as necessities are relatively cheaper in poor countries, and as the poor are intensive consumers of these goods, there is an incentive for the poor to locate in poor countries. But it is to be emphasised that (i) this commodity-price effect is only modest; and (ii) the difference in the structure of prices is obviously only one factor in the where-to-locate decision.
- The income elasticity for the rich countries in panel A is 0.15, the same as that for the OECD (Figure 3.2, panel A).

  An abbreviated version of Figure A2 appears as Figure 3.3 of the text.

<u>A Quality Demand Curve</u>

Figure A3 contains price-quantity scatter plots for quality. The regressions lines for both the rich and poor countries slope down and the slope coefficients are significant. The price elasticity of the demand for quality in the rich countries is -1.7 (panel A), while that for the OECD countries in Figure 3.4 of the text is not too far from this value at -1.3. It is unlikely these two estimates are significantly different from each other, and they point to a quite high price-responsiveness of the demand for quality. From panel B of Figure A3, the price elasticity for the poor is substantially lower at -0.6. The difference in the elasticities might reasonably be interpreted as saying that as compared to the poor, the rich face more alternatives, including a wider array of goods associated with more variety, and have more discretion in spending as the cost of the necessities of life is more easily covered with their higher incomes.

Comparing the two panels of Figure A3, it can be seen that the demand curve for the poor lies everywhere below that for the rich. That is, for a given price, the demand for quality is higher in the rich countries, reflecting the positive income elasticity.

Figure A3 is Figure 3.5 of the text.

The Income Flexibilities

Equation (5.3) of the text defines an estimator of the income flexibility in year t for a certain country:

$$(5.3) \qquad \hat{\phi}_t = \frac{C_{pqt} - (DP'_t - DP_t)DQ_t}{V'_{pt}}.$$

This is implemented in the text with the data for the 37 OECD countries and the results for each year in the sample and all countries together are plotted in the histogram of Figure 5.2. The mean is -0.41 and the standard deviation is 0.82. As set out in detail in the notes to Table A1, application of the same approach to the 176 ICP countries involves a comparison of each country with all others, giving a value of the income flexibility for each country pair c and d, written as $\phi_{cd}, c, d = 1, \ldots, 176,$ where for notational simplicity the circumflex is omitted. These satisfy $\phi_{cc} = 0,\ \phi_{cd} = \phi_{dc}, c, d = 1, \ldots, 176,$ so to avoid duplicates and zero values, Figure A4 plots $\phi_{cd}, c < d = 1, \ldots, 176.$ As can be seen, the mean is -0.52, which is tolerably close to the OECD value (-0.41). But now there is considerably more dispersion -- the standard deviation is 1.68, more than twice as high as the OECD's. Higher ICP dispersion was encountered previously and is probably to be expected in view of the substantially larger differences in incomes across the ICP countries. Figure A4 is reproduced as Figure 5.3 of the text.

Next, a single value of the income flexibility for country c is obtained by averaging ϕ_{cd} over all countries, $\phi_c = (1/176)\Sigma_{d=1}^{176}\phi_{cd}, c = 1, \ldots, 176.$ These values are contained in columns 5 and 12 of Table A1 and summarised in Table A2 by income quartile. Three comments can be made:

- Panel A of Table A2 shows that on the basis of all 176 countries, the income flexibility becomes, on average, more negative as income increases, except for the first quartile (the richest countries).
- When the 24 positive value of ϕ_c are omitted (panel B), the above pattern seems to be reversed. Now the income flexibility seems to fall (in absolute value) with higher

income. This pattern can also be seen from panel B of Figure A5 (note that the scale on the vertical axis is inverted).

- Column 3 of Table A2 reveals that weighting ϕ_c by the reciprocal of the square root of the Frisch price variance, as a rough way to adjust for possible heteroscedasticity, does not substantially affect the results. The reason is this variance is broadly similar across countries (see columns 7 and 14 of Table A1).

The role of income in determining the value of the income flexibility is further examined in Figure A6. As any income dependence would seem to be fragile, the conclusion from these plots would seem to be that ϕ_c is more or less unaffected by the country's income.

<u>Price Dispersion</u>

Price dispersion is measured by the Divisia (budget-share weighted) and Frisch (marginal-share weighted) variances. These are given in standard deviation (SD) form in columns 6, 7, 13 and 14 of Table A1. Judging by the quartile means, price dispersion does not vary greatly over the world income distribution. Figure A7 contains the distribution of the Frisch variance. The mean is 10.24×100^{-1}, so the SD is $\sqrt{10.24\times100^{-1}}=0.3200$, or about 32.0 percent, which is in satisfactory agreement with the grand mean of column 7 of Table A1 of 29.3 percent. The plot in Figure A7 has a long tail and resembles a χ^2 distribution. About two-thirds of countries have a variance less than the mean, the remaining one-third above. A few countries have a high variance of twice or three times the mean. This figure is reproduced as Figure 6.2 in the text.

<u>The Marginal Shares</u>

The marginal share of a good is the fraction of a one-dollar rise in income that is spent on good i. We use Working's (1943) model which implies that for $i = 1,\cdots,9$ goods and $c = 1,\cdots,177$ countries, these shares take the form $\theta_{ic} = w_{ic} + \beta_i$, where β_i are the median estimates from the 37 OECD countries. These median values are (all × 100): Food and alcohol -9.8; clothing and footwear 0.3; housing and utilities -9.0; furnishings, equipment 1.8; health 0.1; recreation and culture 3.8; education -0.3; restaurants and hotels 0.4; and miscellaneous goods and service 13.1. (These values are then normalised to they have a zero sum, as required by Working's model.)

Consider the marginal share for i = food: $\theta_{ic} = w_{ic} - 0.098$. Engel's law states that the food budget share, w_{ic}, falls as income rises. Thus, Working's model implies that the marginal share also falls for a rise in income. For the richest countries, if spending on food becomes less than 9.8 percent of income (that is, total consumption expenditure), then the marginal share turns negative and food becomes an inferior good. But such goods are ruled out by the assumption of want independence as a negative marginal share means the Slutsky substitution matrix is no longer negative semi-definite. This problem arises for five rich ICP, as shown in panel A of Figure A8. The negative shares are small and there are only 5 problems out of a total of 177 countries. This is not such a terrible outcome given that the set of β_i have been imported from the OECD countries and applied to the ICP.

Panel B of Figure A8 shows that there is a further problem with housing and utilities, the good with the second lowest β_i. In 18 instances, $\theta_{ic} < 0$ for this good. For housing and utilities, $\beta_i < 0$, making it a necessity, so the budget share should fall as income rises, as in the food case. But problem is different to food because as indicated in the figure, the 18 countries with negative marginal shares are all poor, not rich. The "resolution" to the problem is that for the ICP, the budget share increases with income, so prima facie it is a luxury, not a necessity. That is to say, the two sources of information regarding this commodity are inconsistent. While this seems to be an issue for this one commodity out of nine, it should be kept in mind as a qualification to the ICP results. It is worth mentioning that no country has a negative marginal share for both food and housing and utilities simultaneously.

Negative marginal shares can give rise to a negative Frisch price variance, $V'_{cd} = \Sigma_{i=1}^{9} \theta_{icd} \left(\log p_{icd} - \log P'_{cd} \right)^2$, but this will not always be the case. The only ICP country to have a negative variance is Algeria. As a negative variance is nonsensical -- and gives rise to problems for estimator (5.3), for example -- this country is dropped from the analysis.

Table A1 Income, Quality, the Income Flexibility and Price Dispersion, 176 ICP Countries, 2011

Country	Income (US = 100)	Quality of consumption	Price of quality	Income flexibility	Price dispersion		Country	Income (US = 100)	Quality of consumption	Price of quality	Income flexibility	Price dispersion	
					Divisia	Frisch						Divisia	Frisch
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
A. First quartile							B. Second quartile						
1. Bermuda	108.1	15.7	-8.0	0.715	46.2	39.8	45. Slovakia	47.8	3.1	1.0	-0.508	25.7	23.9
2. Luxembourg	105.1	17.8	-4.2	-0.585	60.2	53.5	46. Trinidad and Tobago	47.2	11.6	-1.9	-0.464	25.2	23.8
3. United States	100.0	22.1	-3.8	0.108	45.2	42.5	47. St. Kitts and Nevis	46.7	-1.1	2.6	-1.237	34.8	33.8
4. Cayman Islands	95.4	15.7	-2.7	0.457	28.4	26.5	48. Poland	46.5	2.0	2.3	-0.864	28.9	26.8
5. Hong Kong SAR	86.6	21.5	-6.6	-0.406	35.0	32.4	49. Lithuania	45.5	1.9	3.4	-0.886	27.4	25.8
6. UAE	80.2	9.6	-5.8	-0.421	35.3	34.2	50. Brunei Darussalam	45.4	11.9	0.1	-0.288	27.6	26.5
7. Norway	79.0	18.5	-2.2	-0.075	34.3	32.6	51. Bahamas, The	44.6	8.6	-1.2	-0.012	27.6	26.1
8. Switzerland	78.7	17.8	-6.1	-0.123	50.8	45.5	52. Croatia	43.9	3.3	1.8	-0.548	25.6	23.6
9. Austria	74.9	15.3	-0.5	-0.519	38.0	34.9	53. Barbados	43.2	-3.1	-3.0	1.373	28.1	26.6
10. Germany	73.9	16.3	-2.1	-0.123	34.5	29.8	54. Hungary	42.7	5.1	2.2	-0.709	27.2	26.0
11. Aruba	73.3	9.2	0.3	-0.334	25.8	24.8	55. Oman	41.8	10.7	-5.1	-0.250	28.3	27.1
12. Australia	72.7	20.0	-7.1	-0.014	42.6	39.4	56. Turkey	40.7	1.3	3.0	-0.722	31.5	30.6
13. Taiwan, China	72.1	18.8	-4.7	-0.106	29.0	26.2	57. Seychelles	40.6	-17.6	1.8	-1.577	30.9	29.6
14. Sweden	72.0	16.1	-1.3	-0.213	41.3	37.7	58. Russian Federation	40.2	-0.8	6.6	-1.364	32.1	26.7
15. Finland	71.8	14.0	-2.1	-0.121	37.1	32.3	59. Montserrat	40.2	8.4	2.3	-0.944	26.9	24.2
16. Canada	71.5	18.9	-4.0	-0.039	35.9	33.9	60. Estonia	39.7	4.1	-0.1	-0.531	28.5	25.1
17. France	71.0	14.2	-2.2	-0.107	36.6	31.9	61. Chile	39.1	14.5	-1.8	-0.230	26.4	23.8
18. Denmark	70.8	16.7	-2.5	0.038	40.2	35.8	62. Latvia	38.1	1.0	1.6	-0.554	27.1	25.5
19. Iceland	70.1	10.0	2.0	-0.399	32.6	30.0	63. Belarus	37.9	-11.2	13.3	-1.931	49.1	38.6
20. Belgium	70.0	13.7	-1.9	-0.150	39.8	35.6	64. Kazakhstan	37.6	0.0	0.6	-0.628	28.7	27.1
21. Netherlands	70.0	18.4	-1.4	-0.129	39.7	34.3	65. Montenegro	37.4	-1.2	3.9	-0.925	30.0	28.5
22. Singapore	69.0	26.0	-7.1	-0.583	36.6	33.8	66. Antigua and Barbuda	36.8	2.2	0.9	-0.982	25.2	24.0
23. United Kingdom	68.8	18.6	-4.3	-0.148	41.2	35.9	67. Uruguay	34.8	1.4	-0.8	-0.270	28.1	27.1
24. Macao SAR	66.0	18.1	-3.5	-0.890	26.4	23.9	68. Mauritius	34.8	-0.2	2.1	-1.018	26.7	25.7
25. Japan	64.5	18.4	-6.3	0.087	30.4	27.4	69. Mexico	33.7	8.5	-2.6	-0.527	40.1	34.4
26. Cyprus	64.4	9.9	-1.0	-0.319	35.0	32.9	70. Romania	32.6	0.8	-0.8	-0.678	33.1	30.6
27. Italy	63.5	11.3	-1.8	-0.174	32.1	28.5	71. Bulgaria	32.3	2.7	5.6	-1.035	31.5	31.0
28. Curaçao	62.9	8.2	0.7	-0.758	25.6	24.3	72. Malaysia	31.7	9.6	1.0	-0.889	25.4	23.9
29. New Zealand	62.9	13.3	-5.0	0.169	32.2	28.0	73. Panama	29.2	7.9	0.2	-0.669	27.5	25.8
30. Ireland	61.2	13.2	-3.1	0.268	34.9	32.4	74. Serbia	29.0	-2.7	2.9	-0.757	29.2	28.0
31. Kuwait	60.1	0.3	1.1	-0.680	36.0	34.1	75. Grenada	28.9	4.6	1.5	-1.159	26.5	24.8
32. Spain	59.4	10.3	-0.9	-0.597	36.0	31.1	76. Venezuela, RB	28.4	12.9	3.8	-0.689	50.3	44.4
33. Greece	59.2	3.6	-0.2	-0.406	29.6	26.5	77. Virgin Islands, British	28.0	7.5	-5.0	-0.170	31.4	28.6
34. Malta	58.8	15.3	1.3	-0.829	25.6	23.2	78. Brazil	27.3	10.8	2.4	-0.506	33.0	29.3
35. Anguilla	56.4	16.2	-2.2	-0.611	24.0	22.5	79. Jordan	27.2	-10.3	1.0	-1.233	28.6	25.6
36. Israel	56.1	12.0	-2.1	-0.049	30.2	27.7	80. Iran, Islamic Rep.	26.7	-3.9	-2.1	-0.612	40.3	38.5
37. Qatar	55.7	15.1	-4.7	-0.018	41.6	40.5	81. Macedonia, FYR	26.6	-7.4	4.2	-0.779	31.8	31.3
38. Sint Maarten	53.7	8.5	-1.4	0.967	25.3	23.7	82. St. Vin. and the Gren.	26.3	2.7	1.1	-0.777	25.0	23.7
39. Slovenia	52.7	10.2	0.3	-0.376	32.1	28.8	83. Egypt, Arab Rep.	26.0	-8.3	1.0	-0.710	36.9	31.1
40. Saudi Arabia	52.1	0.6	-1.4	-0.300	34.8	33.1	84. Costa Rica	25.6	15.8	1.9	0.130	34.6	31.2
41. Portugal	51.6	10.7	0.6	-0.571	32.0	28.2	85. Dominican Republic	25.6	0.2	1.5	-0.872	25.2	23.5
42. Bahrain	51.4	14.7	-6.8	-0.053	35.4	35.6	86. Ukraine	25.4	-12.8	11.0	-1.529	42.5	35.0
43. Korea, Rep.	49.3	16.9	-2.4	-0.597	31.4	29.6	87. Dominica	24.3	5.9	0.6	-0.810	25.5	24.2
44. Czech Republic	48.0	3.0	-0.7	-0.516	31.2	27.7	88. Bosnia and Herz.	24.2	-4.5	5.6	-0.843	31.9	29.8
Mean	68.5	14.0	-2.7	-0.217	35.2	32.1	Mean	35.3	2.2	1.6	-0.709	30.6	28.2
Median	68.9	15.2	-2.2	-0.162	34.9	32.4	Median	35.8	2.1	1.5	-0.716	28.6	26.8
SD	13.9	5.5	2.5	0.375	7.0	6.2	SD	7.5	7.4	3.4	0.512	5.9	4.5

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Table A1 Income, Quality, the Income Flexibility and Price Dispersion, 176 ICP Countries, 2011 (continued)

Country	Income (US = 100)	Quality of consumption	Price of quality	Income flexibility	Price dispersion	
					Divisia	Frisch
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		C. Third quartile				
89. Azerbaijan	23.8	-8.7	11.3	-1.402	44.4	33.4
90. Albania	23.7	-11.6	3.0	-0.710	45.3	44.0
91. Thailand	23.2	5.4	2.8	-0.903	27.6	25.6
92. St. Lucia	23.1	-2.9	3.0	-1.147	29.5	28.7
93. South Africa	22.8	8.6	-0.9	-0.301	26.4	24.2
94. Colombia	21.9	3.9	4.3	-0.363	28.9	25.9
95. Tunisia	21.6	6.1	-1.5	-0.664	26.3	24.5
96. T. and Caicos Islds	21.5	21.0	-3.7	-0.827	31.7	26.8
97. Jamaica	21.1	9.0	0.0	-0.432	23.8	21.3
98. Armenia	20.8	-20.3	9.4	-1.206	47.0	35.8
99. Georgia	20.4	-7.1	7.6	-1.423	42.9	34.5
100. Botswana	20.2	12.3	-5.5	-1.241	29.0	24.9
101. Namibia	19.8	4.1	-8.7	-0.589	33.1	30.3
102. El Salvador	19.7	0.1	0.0	-0.282	30.2	28.7
103. Belize	19.1	-4.8	6.8	-1.625	33.7	30.1
104. Ecuador	18.9	7.2	1.0	-0.326	27.0	25.2
105. Peru	18.8	8.3	0.7	-0.249	25.4	23.6
106. Mongolia	17.5	-2.8	0.0	-1.011	33.9	32.2
107. Iraq	17.1	-18.4	0.8	-1.196	28.3	26.1
108. Moldova	17.0	-5.7	6.6	-0.648	33.7	31.1
109. Sri Lanka	17.0	-15.2	2.3	-0.585	29.0	28.0
110. Guatemala	15.4	-3.2	0.2	-0.381	25.6	23.7
111. Paraguay	15.2	4.4	3.2	-0.158	31.7	28.9
112. Swaziland	14.9	-9.9	-0.9	-0.560	26.0	24.1
113. Suriname	14.9	-5.1	2.7	-1.360	30.6	26.0
114. Gabon	14.2	-1.6	-4.7	-0.207	25.0	23.3
115. Cape Verde	13.8	-8.3	-3.9	-0.898	25.1	22.6
116. Fiji	13.7	-6.3	-3.4	-0.356	32.7	28.2
117. Indonesia	13.1	-11.6	-2.4	-0.553	24.8	23.3
118. China	13.1	6.2	-2.3	-0.680	26.6	25.5
119. Morocco	13.0	-1.8	1.4	-0.514	28.4	24.4
120. Equatorial Guin.	12.9	-5.8	-2.0	-0.371	26.8	25.2
121. Maldives	12.1	-10.6	-10.7	-0.236	60.6	53.4
122. Kyrgyzstan	11.9	-9.1	12.4	-0.854	56.0	41.3
123. Bhutan	11.6	-6.3	-0.9	-0.592	24.9	23.3
124. Philippines	11.3	-2.0	-0.2	-0.657	23.2	21.4
125. Palestinian T.	10.9	2.3	-2.7	-1.206	26.4	23.7
126. Pakistan	10.9	-20.1	3.5	-0.842	30.4	29.1
127. Nicaragua	10.7	-2.6	6.0	-0.427	33.4	28.6
128. Angola	10.4	-2.0	-2.2	-0.700	24.6	23.6
129. Honduras	10.1	-2.8	2.3	-0.131	29.2	27.0
130. Vietnam	10.1	-9.4	1.9	-0.823	42.0	43.0
131. Bolivia	9.4	-7.3	5.0	-0.249	33.2	28.0
132. Tajikistan	8.7	-11.7	11.7	-0.277	61.2	47.2
Mean	16.2	-3.1	1.2	-0.685	32.4	28.9
Median	15.3	-3.0	0.7	-0.620	29.1	26.4
SD	4.6	8.6	4.9	0.389	9.3	7.0

Country	Income (US = 100)	Quality of consumption	Price of quality	Income flexibility	Price dispersion	
					Divisia	Frisch
(8)	(9)	(10)	(11)	(12)	(13)	(14)
		D. Fourth quartile				
133. Myanmar	8.3	-27.1	3.0	-0.883	45.2	45.1
134. Yemen	7.7	-23.0	-0.6	0.153	34.3	30.1
135. São Tomé and P.	7.7	-12.0	-1.0	-0.137	27.4	25.2
136. India	7.6	-1.7	4.6	0.006	30.6	30.0
137. Ghana	7.2	-6.1	-5.7	-1.334	36.1	33.8
138. Cambodia	6.7	-16.3	-0.4	-1.034	34.6	33.2
139. Lao PDR	6.6	-18.8	7.1	-0.031	44.8	42.7
140. Lesotho	6.2	-0.4	-2.2	-0.666	26.5	24.9
141. Nigeria	5.9	-5.5	-6.3	-1.501	32.0	29.7
142. Kenya	5.5	1.3	1.0	0.552	26.5	24.2
143. Bangladesh	5.5	-28.7	3.4	-0.328	26.9	26.6
144. Sudan	5.2	-12.0	-1.2	-0.090	29.4	28.0
145. Mauritania	5.0	-17.6	-1.5	-0.391	25.2	23.4
146. Nepal	4.8	-23.2	3.2	-0.425	29.9	30.7
147. Senegal	4.8	-14.8	-2.2	-0.581	24.4	22.8
148. Cameroon	4.7	-11.1	2.1	0.053	22.5	20.0
149. Djibouti	4.6	-25.2	0.5	-0.613	23.9	22.8
150. Côte d'Ivoire	4.5	-2.3	0.8	-0.088	23.9	22.1
151. Congo, Rep.	4.2	-8.1	1.5	-0.372	27.5	24.2
152. Zambia	4.2	-24.8	-0.1	-0.061	25.5	23.6
153. Haiti	4.0	-20.3	-0.2	-0.327	29.7	27.9
154. Uganda	3.9	-8.6	-0.9	-0.972	25.2	23.2
155. Gambia, The	3.8	-2.0	-7.0	-1.912	34.0	32.2
156. Sierra Leone	3.7	-7.2	-0.1	-1.034	33.3	29.9
157. Togo	3.4	-2.7	-0.7	-0.795	29.3	27.4
158. Benin	3.4	-7.1	-4.4	-1.257	24.3	22.4
159. Chad	3.2	-5.2	-1.6	-0.955	25.0	22.6
160. Zimbabwe	3.1	-9.5	0.4	-1.010	22.8	20.8
161. Rwanda	3.0	-21.1	3.2	0.198	26.7	25.6
162. Madagascar	3.0	1.8	0.4	-0.994	31.6	31.4
163. Ethiopia	2.7	-8.1	-8.1	-1.693	32.8	31.0
164. Malawi	2.5	-19.5	8.6	0.800	36.9	32.4
165. Mali	2.4	-2.8	0.8	-0.816	25.8	24.3
166. Tanzania	2.3	-20.4	-3.9	-1.748	27.3	24.2
167. Burkina Faso	2.1	-11.6	-0.4	0.117	25.1	23.1
168. Guinea-Bissau	2.1	-15.0	0.8	0.056	30.9	29.1
169. Guinea	2.0	-23.1	3.9	0.355	46.9	36.8
170. Liberia	2.0	-4.0	-8.9	-2.739	36.6	32.5
171. Mozambique	2.0	-6.6	-0.2	0.712	27.0	24.2
172. C. African Rep.	1.9	-14.0	1.5	1.690	31.4	26.9
173. Burundi	1.8	-31.3	7.2	0.778	35.8	35.0
174. Niger	1.8	1.0	-2.8	-1.387	30.5	28.3
175. Comoros	1.5	-35.5	0.0	0.253	25.3	24.4
176. Congo, D. Rep.	1.1	-24.3	-0.4	0.072	32.5	29.2
Mean	4.1	-13.1	-0.1	-0.463	30.1	27.9
Median	3.9	-11.8	-0.1	-0.382	29.4	27.1
SD	1.9	9.6	3.7	0.830	5.8	5.4

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Table A1 Income, Quality, the Income Flexibility and Price Dispersion,176 ICP Countries, 2011 (continued)

	Income (US = 100)	Quality of consumption	Price of quality	Income flexibility	Price dispersion	
					Divisia	Frisch
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Grand mean	31.0	0.0	0.0	-0.518	32.1	29.3
Grand median	24.0	0.2	-0.1	-0.539	30.4	28.0
Grand SD	25.7	12.6	4.1	0.592	7.4	6.1

Notes:

1. *Column 1*: Countries are ranked in terms of per capita income and are divided into 4 quartiles.
2. *Columns 2 and 9:* Income is defined as total real consumption per capita at PPP prices with US = 100. More precisely, for country c, income is the sum of real consumption expenditures on nine broad items: $Y_c = \Sigma_{i=1}^{9} q_{ic}$, where $q_{ic} = M_{ic}/p_{ic}$ is the per capita consumption of item i; and M_{ic} and p_{ic} denote expenditure on and the PPP price of item i.
3. *Columns 3 and 10*: The index of the quality of consumption for country c is defined as excess of the Frisch over the Divisia volume index, $100\times(\log Q'_c - \log Q_c)$. Here, $\log Q'_c = (1/176)\Sigma_{d=1}^{176} \log Q'_{cd}$ is the multilateral Frisch volume index for c; $\log Q'_{cd} = \Sigma_{i=1}^{9} \theta_{icd}(\log q_{ic} - \log q_{id})$ is the bilateral counterpart that is a marginal-share weighted average of quantities consumed in c as compared to d; $\theta_{icd} = (1/2)(\theta_{ic} + \theta_{id})$ is the average of the marginal share of i in c and d (more subsequently); $\log Q_c = (1/176)\Sigma_{d=1}^{176} \log Q_{cd}$ is the multilateral Divisia volume index for c; $\log Q_{cd} = \Sigma_{i=1}^{9} w_{icd}(\log q_{ic} - \log q_{id})$ the bilateral counterpart, a budget-share weighted average of the quantities in c compared to d; and $w_{icd} = (1/2)(w_{ic} + w_{id})$ is the average of the budget share of i in c and d, with $w_{ic} = p_{ic}q_{ic}/M_c$ the budget share of i in c and $M_c = \Sigma_{i=1}^{9} p_{ic}q_{ic}$ total expenditure on the 9 goods.
4. *Marginal shares*: The marginal share of i in c is the increase in expenditure on the good resulting from a one-dollar rise in total expenditure: $\theta_{ic} = \partial(p_{ic}q_c)/\partial M_c$. The marginal shares are parametrised according to Working's (1943) model, viz., $\theta_{ic} = w_{ic} + \beta_i$, where β_i are the median estimates from 37 OECD countries (Clements and Si, 2017). These values are (all × 100): Food and alcohol -9.8; clothing and footwear 0.3; housing and utilities -9.0; furnishings, equipment 1.8; health 0.1; recreation and culture 3.8; education -0.3; restaurants and hotels 0.4; and miscellaneous goods and service 13.1. These values are then normalised to they have a zero sum.
5. *Columns 4 and 11*: The index of the price of quality for country c is defined as excess of the Frisch over the Divisia price index, $100\times(\log P'_c - \log P_c)$. Here, $\log P'_c = (1/176)\Sigma_{d=1}^{176} \log P'_{cd}$ is the multilateral Frisch price index for c; $\log P'_{cd} = \Sigma_{i=1}^{9} \theta_{icd}(\log p_{ic} - \log p_{id})$ is the bilateral Frisch price index; and $\log P_c = (1/176)\Sigma_{d=1}^{176} \log P_{cd}$ is the multilateral Divisia price index for c, with $\log P_{cd} = \Sigma_{i=1}^{9} w_{icd}(\log p_{ic} - \log p_{id})$ the bilateral Divisia index of prices in c as compared to d.
6. *Columns 5 and 12*: The income flexibility is the inverse of the income elasticity of the marginal utility of income, $\{\partial(\log\lambda)/\partial(\log M)\}^{-1}$, where λ is the marginal utility of income. The income flexibility for country c is $\phi_c = (1/176)\Sigma_{d=1}^{76}\phi_{cd}$. Here, $\phi_{cd} = \{C_{cd} - (\log P'_{cd} - \log P_{cd})\log Q_{cd}\}/V'_{cd}$ is the income flexibility for c and d, where $C_{cd} = \Sigma_{i=1}^{9} w_{icd}(\log p_{icd} - \log P'_{cd})(\log q_{icd} - \log Q_{cd})$ is a type of price-quantity covariance, with $\log p_{icd} = \log p_{ic} - \log p_{id}$ and $\log q_{icd} = \log q_{ic} - \log q_{id}$ the price and quantity of i in c relative to d; $\log P'_{cd} - \log P_{cd}$ is the price of quality, as defined above; $\log Q_{cd}$ is the Divisia volume index defined above; $V'_{cd} = \Sigma_{i=1}^{9} \theta_{icd}(\log p_{icd} - \log P'_{cd})^2$ is the bilateral Frisch price variance; and $\log P'_{cd}$ is the bilateral Frisch price index also defined above.
7. *Columns 6 and 13*: The Divisia measure of price dispersion is the budget-share weighted standard deviation, defined for country c as $100\times(V_c)^{1/2} = 100\times(1/176)\Sigma_{d=1}^{176}(V_{cd})^{1/2}$, where $V_{cd} = \Sigma_{i=1}^{9} w_{icd}(\log p_{icd} - \log P_{cd})^2$ is the bilateral variance for c and d; and $\log P_{cd}$ is the bilateral Divisia index of prices, as defined above.
8. *Columns 7 and 14*: The Frisch measure of price dispersion is the marginal-share weighted standard deviation. This is defined in a similar manner to its Divisia counterpart, except that the budget share, w_{icd}, is replaced by the corresponding marginal share, θ_{icd}. Accordingly, the Frisch standard deviation for country c is $100\times(V'_c)^{1/2} = 100\times(1/176)\Sigma_{d=1}^{176}(V'_{cd})^{1/2}$, where V'_{cd} is the bilateral variance as defined above.
9. *Last three rows*: "Grand" mean, median and standard deviation refer to the 176 countries.

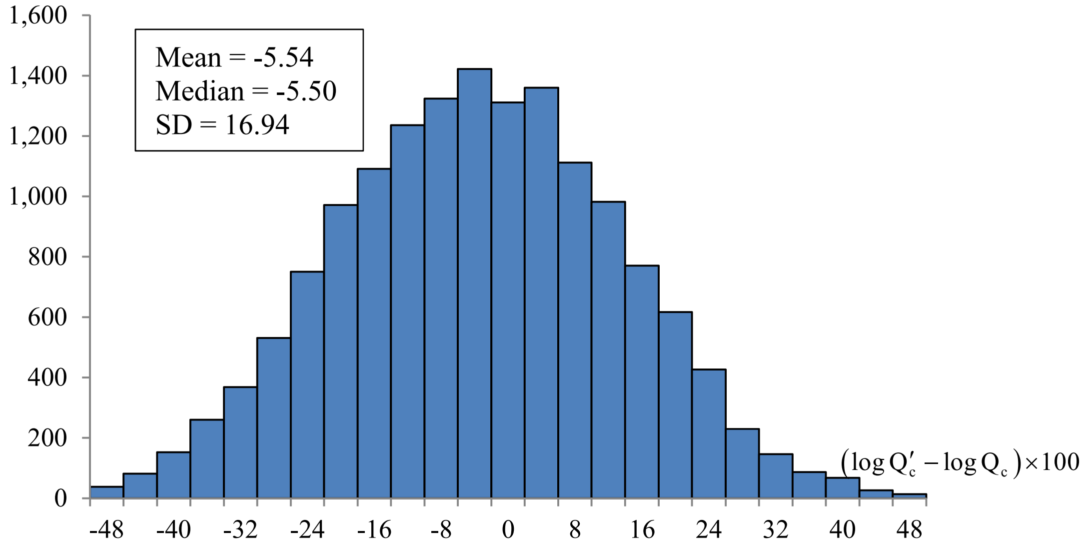
Table A2 Income Flexibilities by Income Quartile, 176 ICP Countries, 2011

Income quartile	Unweighted mean	Weighted mean	Number ≥ 0
(1)	(2)	(3)	(4)
	A. All 155 countries		
First	-0.217	-0.230	8
Second	-0.709	-0.695	2
Third	-0.685	-0.674	0
Fourth	-0.463	-0.456	14
All countries	-0.518	-0.522	24
	B. Observations ≥ 0 omitted		
First	-0.343	-0.365	-
Second	-0.778	-0.763	-
Third	-0.685	-0.674	-
Fourth	-0.872	-0.851	-
All countries	-0.667	-0.535	-

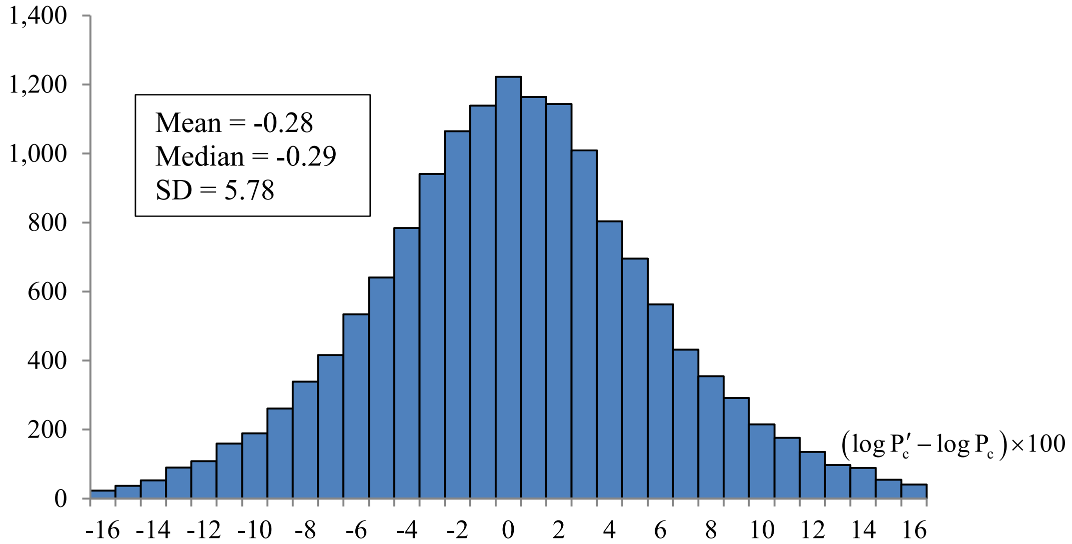
Note: For the weighted means, weights are inversely proportional to the square roots of the multilateral Frisch price variance (V_c'). That is, the weight for country c is $(V_c')^{-1/2} / \Sigma_{c=1}^{C} (V_c')^{-1/2}$. This variance is an approximate measure of the dispersion of the estimator of the income flexibility.

Figure A1 Quality Indexes,
176 ICP Countries, 2011

A. Consumption



B. Prices

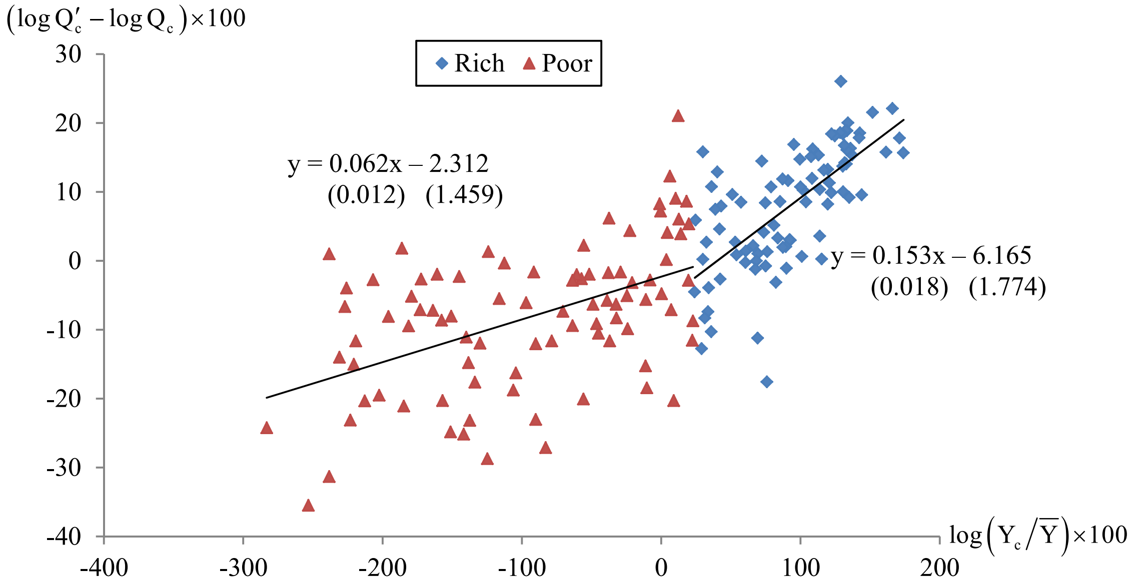


Notes:

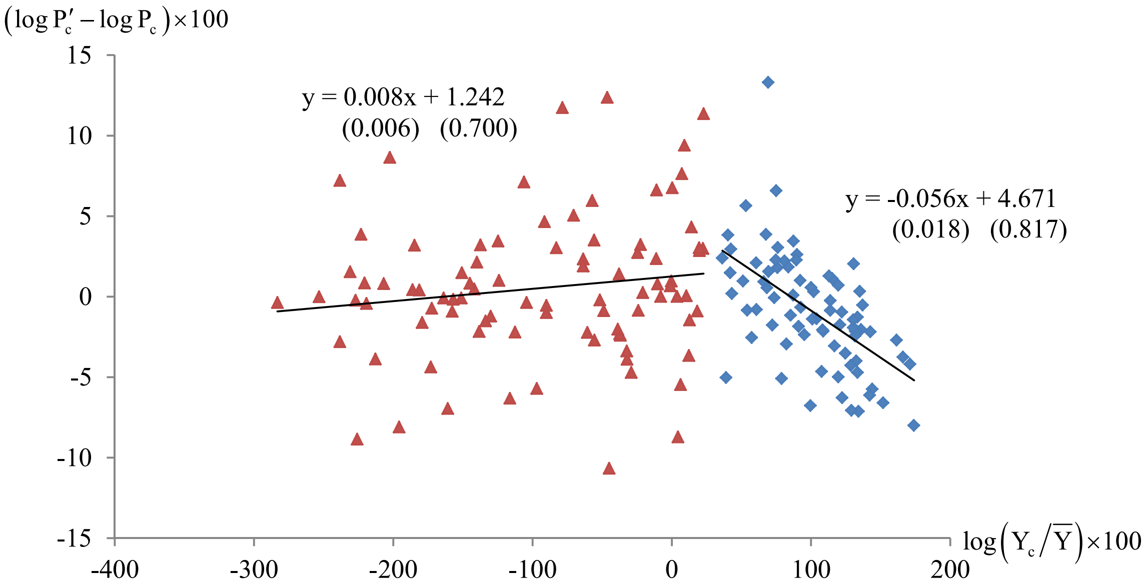
1. *Panel A*: The index of the quality of consumption for country c is the excess of the Frisch over Divisia volume index, $\log Q'_c - \log Q_c$. Here, $\log Q'_c = (1/176)\Sigma_{d=1}^{176} \log Q'_{cd}$ is the multilateral Frisch volume index for country c, with $\log Q'_{cd} = \Sigma_{i=1}^{9} \theta_{icd} (\log q_{ic} - \log q_{id})$ the bilateral counterpart; $\theta_{icd} = (1/2)(\theta_{ic} + \theta_{id})$ is the average of the marginal share of i in c and d; $\log Q_c = (1/176)\Sigma_{d=1}^{176} \log Q_{cd}$ is the multilateral Divisia volume index for c, with $\log Q_{cd} = \Sigma_{i=1}^{9} w_{icd} (\log q_{ic} - \log q_{id})$ the bilateral version and $w_{icd} = (1/2)(w_{ic} + w_{id})$ the average of the budget share of i in c and d.
2. *Panel B*: The index of the price of quality index for country c is the excess of the Frisch over Divisia price index, $\log P'_c - \log P_c$. Here, $\log P'_c = (1/176)\Sigma_{d=1}^{176} \log P'_{cd}$ is the multilateral Frisch price index for country c, with $\log P'_{cd} = \Sigma_{i=1}^{9} \theta_{icd} (\log p_{ic} - \log p_{id})$ the bilateral counterpart; and $\log P_c = (1/176)\Sigma_{d=1}^{176} \log P_{cd}$ is the multilateral Divisia price index for c, with $\log P_{cd} = \Sigma_{i=1}^{9} w_{icd} (\log p_{ic} - \log p_{id})$ the bilateral version.
3. *Both panels*: To avoid duplication, the displayed values in each panel are the upper triangle of a 176×176 skew symmetric matrix, $[\log P'_{cd} - \log P_{cd}]$ or $[\log Q'_{cd} - \log Q_{cd}]$, excluding the diagonal elements (all zeros), so the number of values is $(176^2 - 176)/2 = 15,400$.

Figure A2 Quality and Income,
176 ICP Countries, 2011

A. Consumption



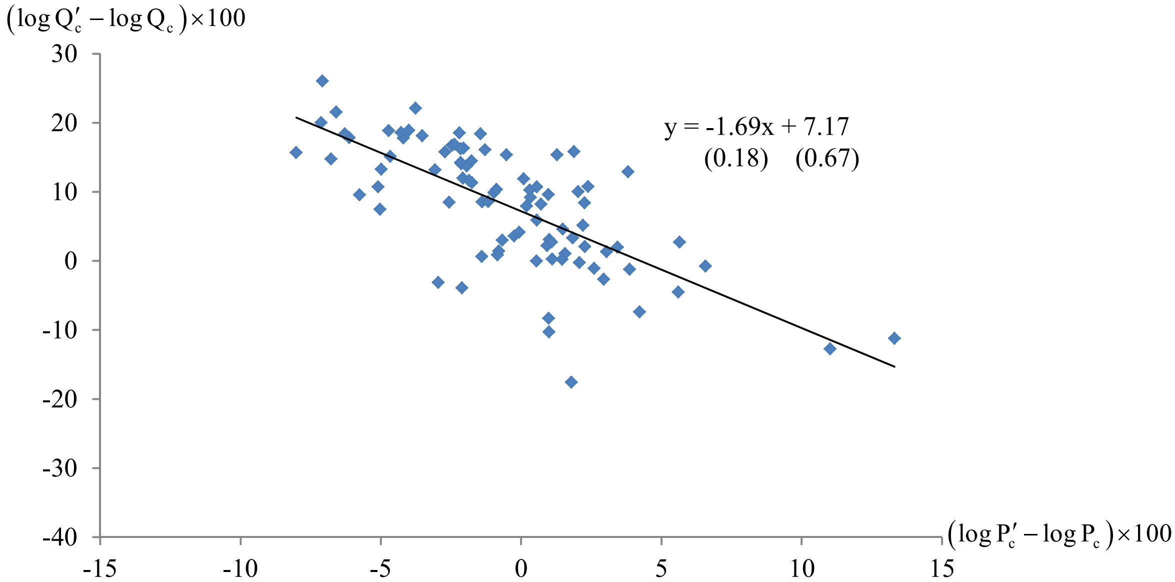
B. Prices



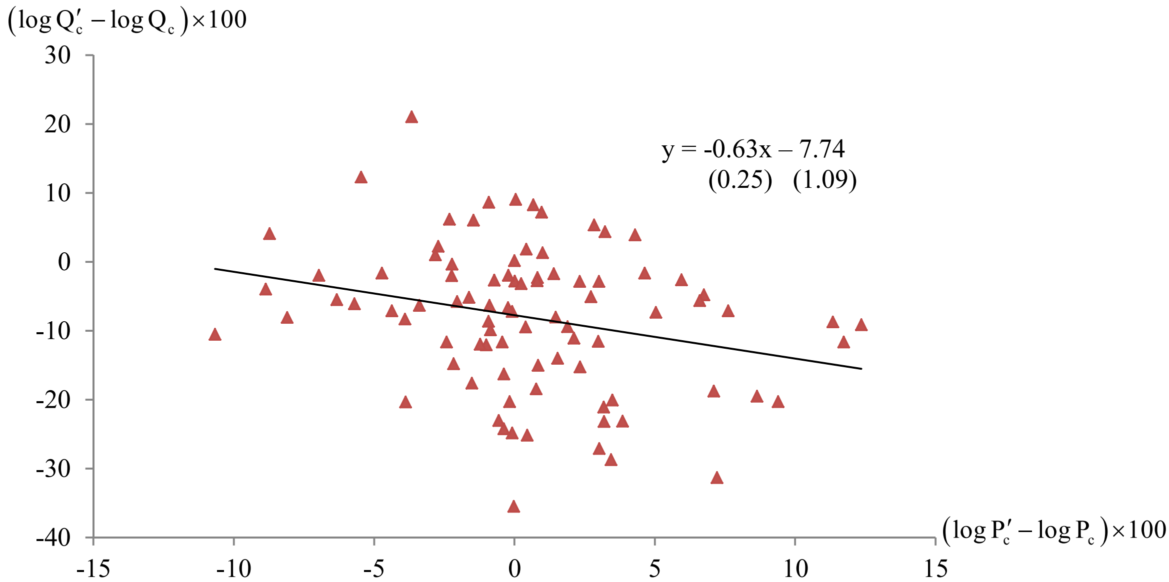
Notes: The index of the quality of consumption for country c is defined as excess of the Frisch over the Divisia volume index $\log Q_c' - \log Q_c$. Here, $\log Q_c' = (1/176)\Sigma_{d=1}^{176} \log Q_{cd}'$ is the multilateral Frisch volume index for country c and $\log Q_{cd}' = \Sigma_{i=1}^{9} \theta_{icd} (\log q_{ic} - \log q_{id})$ is the bilateral counterpart, with θ_{icd} the marginal share of good i averaged over countries c and d; and $\log Q_c = (1/176)\Sigma_{d=1}^{176} \log Q_{cd}$ is the multilateral Divisia volume index for c and $\log Q_{cd} = \Sigma_{i=1}^{9} w_{icd} (\log q_{ic} - \log q_{id})$ is the bilateral counterpart, with w_{icd} the budget share of good i averaged over countries c and d. The price of quality for country c is defined analogously. Income is defined as real per capita consumption of country c, $Y_c = \Sigma_{i=1}^{9} q_{ic}$, relative to the cross-country geometric mean, $\bar{Y}$; so the logarithmic measure of relative income is $\log(Y_c/\bar{Y}) \times 100$ Rich countries are those in the upper two income quartiles; poor countries are in the lower two quartiles.

Figure A3 Scatter Plot of the Quality of Consumption and its Price, 176 ICP Countries, 2011

A. Rich

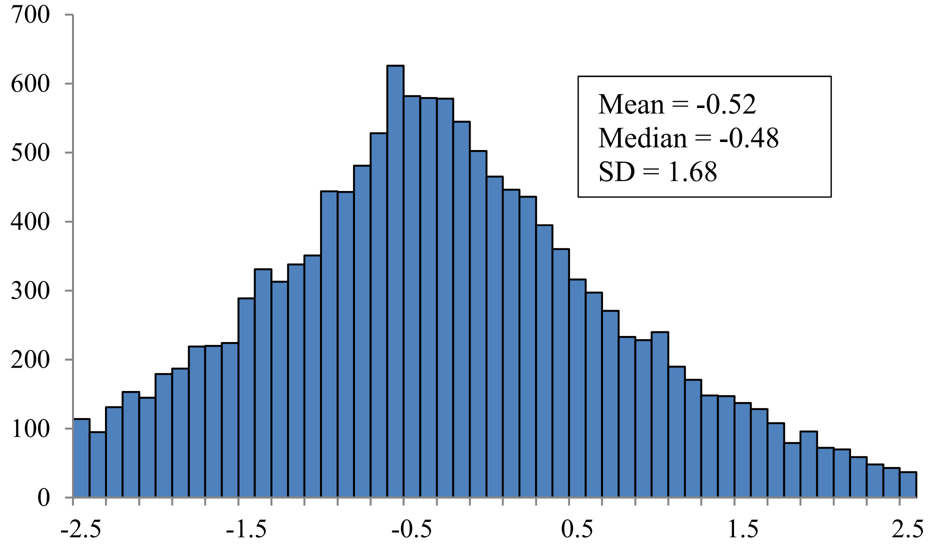


B. Poor



Note: The quality of consumption is $\log Q_c' - \log Q_c$, the excess of the Frisch volume index over the Divisia counterpart; and $\log P_c' - \log P_c$ is the price of quality, the excess of the Frisch price index over the Divisia counterpart. Panels A and B together contain observations pertaining to the 176 ICP countries, split by rich (the upper two income quartiles) and poor (the lower two quartiles).

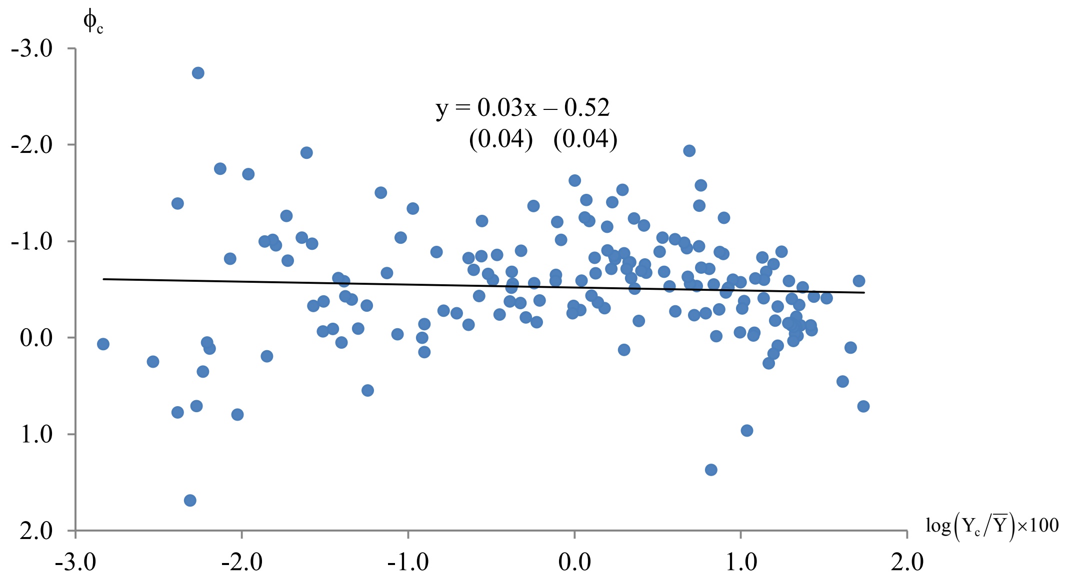
Figure A4 Income Flexibilities,
176 ICP Countries, 2011



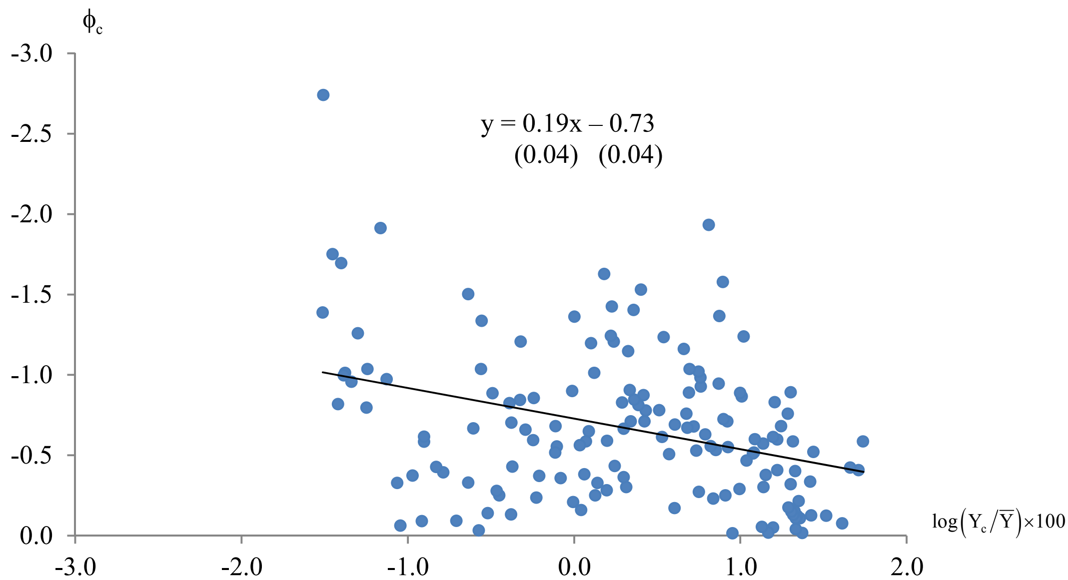
Note: Truncated at [-2.5, 2.5]. See text of appendix for details.

Figure A5 Income Flexibilities and Income,
176 ICP Countries, 2011

A. All 176 countries



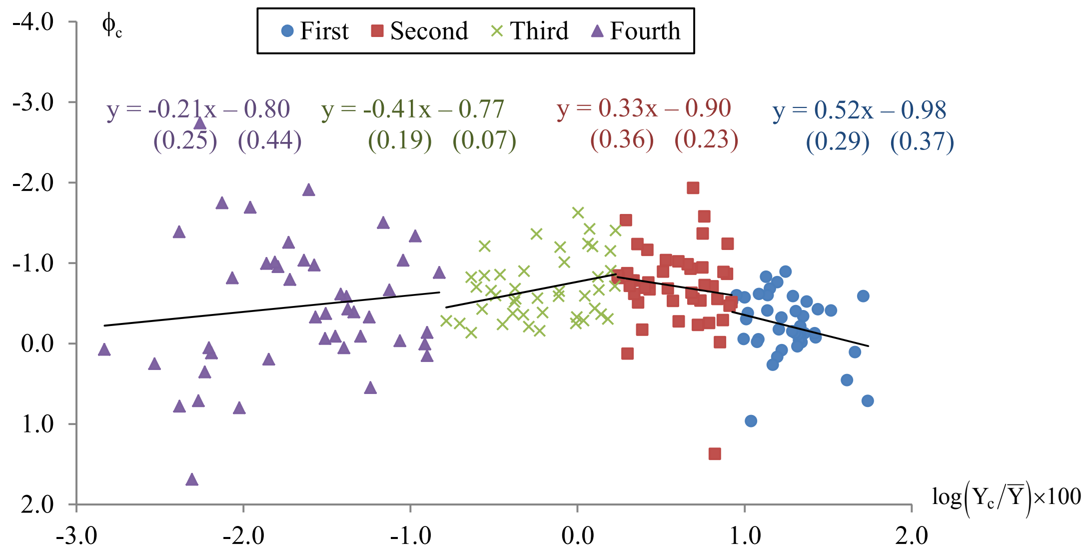
B. Observations ≥ 0 omitted



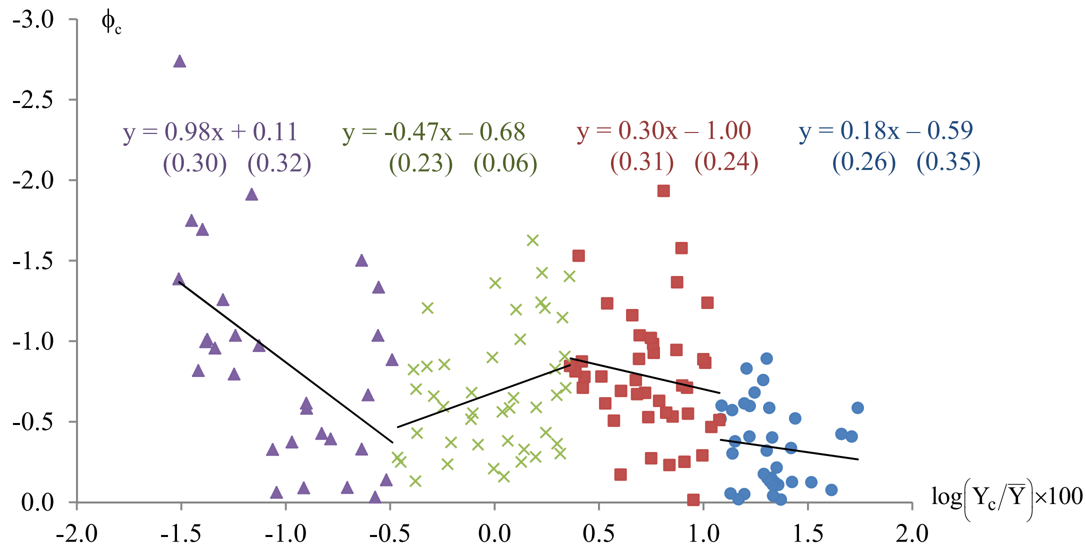
Notes: ϕ_c is the income flexibility for country c. Income of c is defined as real per capita consumption, $Y_c = \Sigma_{i=1}^{9} q_{ic}$, relative to the cross-country geometric mean, $\overline{Y}$; so the logarithmic measure of relative income is $\log\left(Y_c/\overline{Y}\right)\times 100$ Rich countries are those in the upper two income quartiles; poor countries are in the lower two quartiles. Note that the scales on the vertical axis of both panels are inverted.

Figure A6 Income Flexibilities and Income by Quartile, 176 ICP Countries, 2011

A. All 176 countries

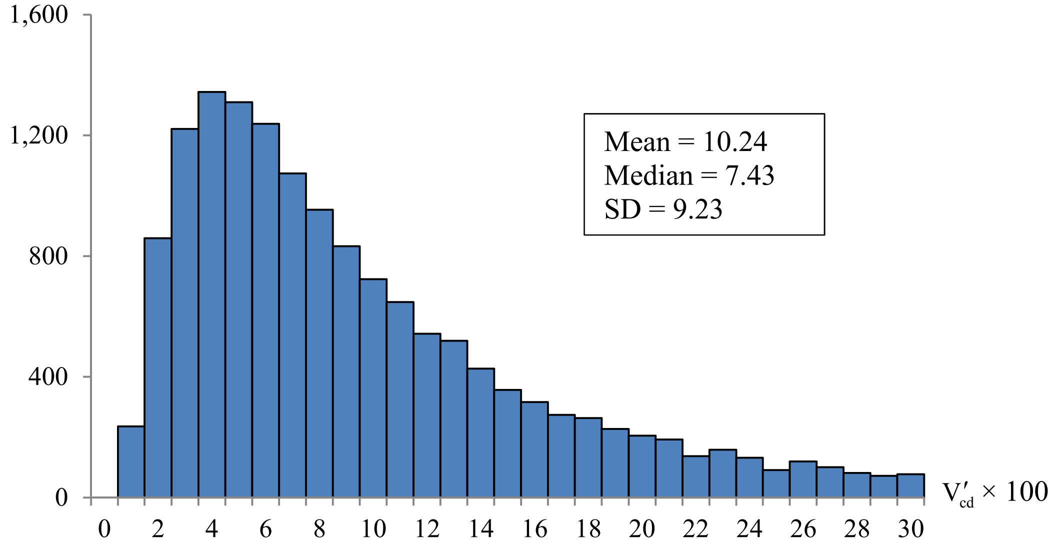


B. Observations ≥ 0 omitted



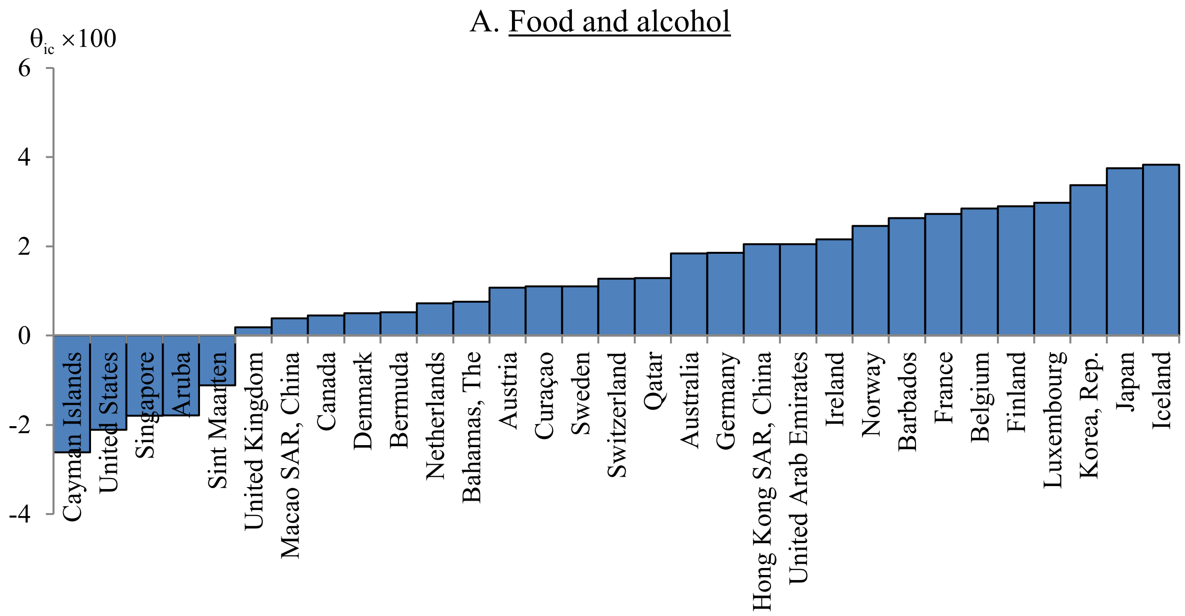
Notes: ϕ_c is the income flexibility for country c. Income of c is defined as real per capita consumption, $Y_c = \Sigma_{i=1}^{9} q_{ic}$, relative to the cross-country geometric mean, $\bar{Y}$, so the logarithmic measure of relative income is $\log\left(Y_c/\bar{Y}\right)\times 100$ Rich countries are those in the upper two income quartiles; poor countries are in the lower two quartiles. The various markers represent the four income quartiles. Note that the scales on the vertical axis of both panels are inverted.

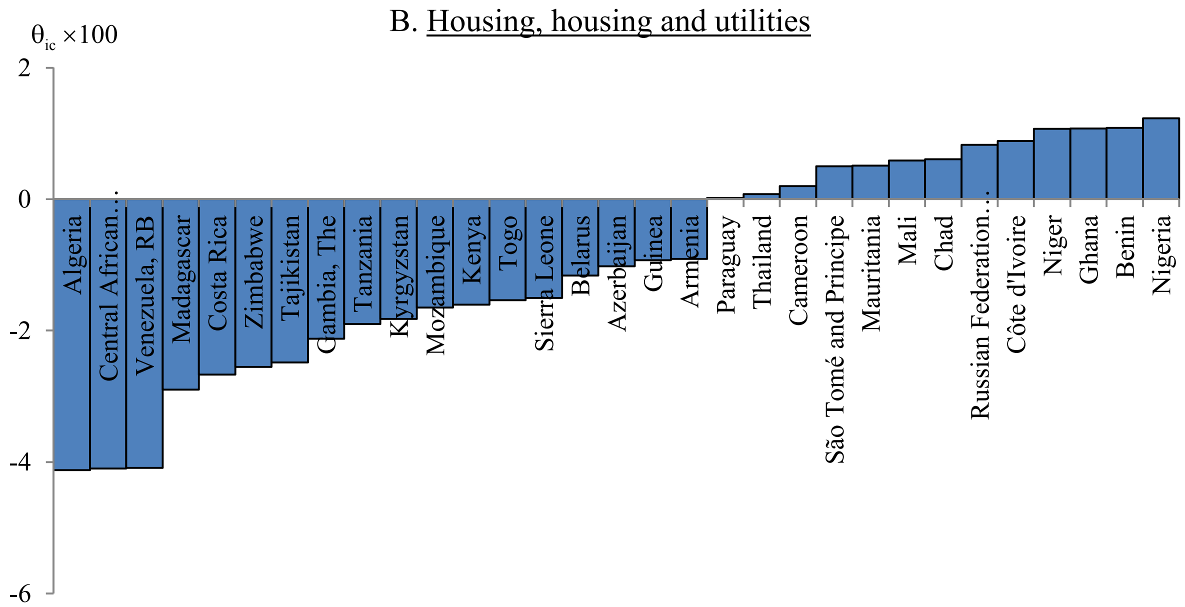
Figure A7 Frisch Price Variances, 176 ICP Countries, 2011



Note: This figure plots the bilateral Frisch price variances. For countries c and d, this variance is $V'_{cd} = \Sigma_{i-1}^{9} \theta_{icd} \left(\log p_{icd} - \log P'_{cd} \right)^2$, where $\theta_{icd} = \left(1/2 \right) \left(\theta_{ic} + \theta_{id} \right)$ is the average marginal share of i in c and d; $\log p_{icd} = \log p_{ic} - \log p_{id}$ is the price of item i in c compared to d; and $\log P'_{cd} = \Sigma_{i=1}^{9} \theta_{icd} \log p_{icd}$ is the Frisch index of prices in c as compared to d. The displayed values are the upper triangle of the 176×176 symmetric matrix $\left[V'_{cd} \right]$ excluding the diagonal elements (all zeros), so the number of values is $\left(176^2 - 176 \right) / 2 = 15,400$.

Figure A8 Selected Marginal Shares, 177 ICP Countries, 2011





Note: Countries with the 30 smallest marginal shares are displayed, in ascending order, in each panel. There are 177 countries here as Algeria is included.

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