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THE ROTTERDAM DEMAND MODEL HALF A CENTURY ON

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DISCUSSION PAPER 14.34

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Abstract

Half a century ago, Barten (1964) and Theil (1965) formulated what is now known as the Rotterdam model. A path-breaking innovation, this system of demand equations allowed for the first time rigorous testing of the theory of the utility-maximising consumer. This has led to a vibrant, on-going strand of research on the theoretical underpinnings of the model, extensions and numerous applications. But perhaps due to its European heritage and unorthodox derivation, there is still misunderstanding and a tendency for the Rotterdam model to be regarded with reservations and/or uncertainties (if not mistrust). This paper marks the golden jubilee of the model by clarifying its economic foundations, highlighting its strengths and weaknesses, elucidating its links with other models of consumer demand, and dealing with some recent developments that have their roots in Barten and Theil's pioneering research of the 1960s.

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1. INTRODUCTION

Few papers in economics have a working life, in terms of citations and influence, longer than a decade or so. It is thus a very rare event for a paper to continue to be read, cited, taught and followed after almost half a century. Two such papers are Anton Barton's "Consumer Demand Equations Under Conditions of Almost Additive Preferences" and Henri Theil's "The Information Approach to Demand Analysis" that were published in Econometrica in 1964 and 1965, respectively (Barten, 1964, Theil, 1965). These related papers introduced what has become known as the "Rotterdam model". For the first time, this model combined generality and operational tractability so that it became a prominent vehicle for the econometric analysis of the pattern of consumer demand and the rigorous testing of utility-maximisation theory. To this day, the model still appears in leading journals, and the general approach has formed the basis for a rich class of models known as the "differential approach" to demand analysis that is used in both time-series and cross-country contexts. The Rotterdam model has spawned an extensive literature and occupies a similar status in consumer demand to the linear expenditure system (LES, Stone, 1954), the translog (Christensen et al., 1975) and the almost ideal demand system (Deaton and Muellbauer, 1980a). Figure 1 provides some evidence for this claim in terms of citations.

The beguilingly simplicity, transparency and apparent generality of the Rotterdam model have led to its prominent position in demand analysis. Interestingly (ironically?), these very features have led to controversy, misunderstanding and a feeling that perhaps the Rotterdam system was "too good to be true" and its simplicity deceptive. Non-Rotterdam approaches such as the LES start with the algebraic form of the consumer's utility function and then derive the corresponding demand functions, which obviously contain (most of) the information embodied in the utility function. This sequence is reversed in the Rotterdam approach. It starts with demand functions, takes the total differential, uses utility-maximisation theory to give restrictions on the demand functions and then, as the last step, takes certain transformations of the slopes of the demand functions to be constants. The important distinction is that the utility function is not specified explicitly, but lies behind the demand equations in the background. Preferences are not ignored as the utility function provides restrictions on (transforms of) the slopes of the demand equations. In this sense, the Rotterdam system can be considered to be consistent with a variety of utility functions.

As the underlying theory can confront the data only via demand functions, there is the merit in *directness* in the Rotterdam approach that starts and ends with demand functions. Expressing the consumer's preferences in the form of the utility function or the cost function involves a construct that is unobservable; in this sense, competitive approaches that start by specifying preferences are *indirect* in the theory-data matching process. A related issue is the question of parameterisation of the Rotterdam, which contains a mixture of constant slopes of Engel functions and a type of semi-elasticity for the prices. This formulation is unconventional and has been the source of some difficulties and resistance. For the non-Rotterdam approaches the question of parameterisation is simple: Choose the algebraic form of the utility function (direct or indirect, or the cost function), which then determines the form of the corresponding demand equations. The reverse engineering methodology makes the Rotterdam appear different and distinct and, arguably, is at the heart of the controversy and misunderstanding. Rotterdam critics also argue the microeconomic foundations of the model are questionable and have claimed it implies Cobb-Douglas preferences, so that all income elasticities are unity and price elasticities are -1, which, if true, would be a devastating weakness.

Why does the Rotterdam model continue to be enthusiastically used and endure in view of such apparent trenchant criticism? Some of the controversy will possibly never be completely settled, and, of course, it is not productive to be dogmatic in matters that are inherently unresolvable if they simply reflect different preferences (of researchers). The fact is the Rotterdam passes the Darwinian test of survival by a sizable margin, and it seems it will continue to be used for some time in the future. It is thus appropriate to mark the first half century of the Rotterdam model's existence with a review that sets out the model, its advantages and disadvantages and reveals its surprisingly close links with other popular models, with the objective of clarifying at least some of the misunderstanding that still surrounds the model. Relatedly, it is also appropriate to discuss the developments that the Rotterdam directly lead to – the differential approach and the emerging area of cross-county demand analysis.

2. THE ORIGINAL FORMULATION

This section presents the original foundation and formulation of the Rotterdam model by Barten (1964) and Theil (1965). In the interests of focusing on what is important and making it as accessible as possible, the presentation is selective and gives a simplified exposition of the original papers. Two additional "basic sources" are Theil (1967, Chap 5-6) and Theil (1975/76).[1]

Basic Concepts

Let p_i and q_i be the price and quantity demanded of good i, $i = 1,\ldots,n$. The consumer is taken to choose the basket $q_1,\ldots,q_n$ to maximise the utility function

$$(2.1) \qquad u(q_1,\ldots,q_n)$$

[1] For reviews of the broader area of applied demand analysis, see Barnett and Serletis (2008), Barten (1977), Bewley (1986). Blundell (1988), Brown and Deaton (1972), Deaton (1986), Deaton and Muellbauer (1980b), Goldberger (1987), Phlips (1974), Pollak and Wales (1992), Powell (1974), Theil (1980) and Theil and Clements (1987).

subject to the budget constraint

(2.2) $$M=\sum_{i=1}^{n} p_i q_i,$$

where M is total expenditure (“income” for short). This leads to a demand equation for good i of the form $q_i = q_i(M, p_1, \ldots, p_n)$. As money income is held constant, this is a Marshallian demand equation. The differential of this demand equation is

$$dq_i = \frac{\partial q_i}{\partial M} dM + \sum_{j=1}^{n} \frac{\partial q_i}{\partial p_j} dp_j.$$

The total effect on the consumption of good i of a change in the price of good j $\partial q_i / \partial p_j$, can be decomposed into income and substitution effects according to the Slutsky equation, $\partial q_i / \partial p_j = s_{ij} - q_j \, \partial q_i / \partial M$, where s_{ij} is the substitution effect that holds real income constant. Thus, the above can be expressed as

$$dq_i = \frac{\partial q_i}{\partial M}\left(dM - \sum_{j=1}^{n} q_j dp_j \right) + \sum_{j=1}^{n} s_{ij} dp_j.$$

The term in brackets is the change in money income deflated of the income effects of the n price changes, which represents the change in real income. Using the identity $d(\log x) = dx/x$, the above can be expressed logarithmically as

(2.3) $$d(\log q_i) = \frac{M}{q_i}\frac{\partial q_i}{\partial M}\left[d(\log M) - \sum_{j=1}^{n} \frac{p_j q_j}{M} d(\log p_j) \right] + \sum_{j=1}^{n} \frac{p_j}{q_i} s_{ij} d(\log p_j).$$

To simplify the above, let $\eta_i = \partial(\log q_i)/\partial(\log M) = (M/q_i)(\partial q_i/\partial M)$ be the income elasticity of good i, $\eta_{ij} = \partial(\log q_i)/\partial(\log p_j) = (p_j/q_i)s_{ij}$ be the $(i, j)^{th}$ price elasticity (income compensated), $w_j = p_j q_j / M$ the budget share of j and $d(\log Q) = d(\log M) - \Sigma_{j=1}^{n} w_j d(\log p_j)$ be the change in real income. Using these concepts, equation (2.3) then becomes

(2.4) $$d(\log q_i) = \eta_i d(\log Q) + \sum_{j=1}^{n} \eta_{ij} d(\log p_j).$$

For $i = 1, \ldots, n$, (2.4) is a system of n demand equations, the parameters of which satisfy the adding-up constraints

(2.5) $$\sum_{i=1}^{n} w_i \eta_i = 1, \quad \sum_{i=1}^{n} w_i \eta_{ij} = 0, \ j = 1, \ldots, n,$$

which follow from the budget constraint (2.2). As real income is controlled for in equation (2.4), demand homogeneity means that an equiproportional increase in all prices has no effect on quantities consumed, which implies

(2.6) $$\sum_{j=1}^{n} \eta_{ij} = 0, \; i = 1, \dots, n.$$

Finally, Slutsky symmetry states that the substitution effects are symmetric in i and j, that is, $s_{ij} = s_{ji}$, $i, j = 1, \dots, n$. As $\eta_{ij} = \left(p_j / q_i\right) s_{ij}$, symmetry in terms of elasticities takes the form $\left(q_i / p_j\right) \eta_{ij} = \left(q_j / p_i\right) \eta_{ji}$, or, multiplying by $p_i \, p_j / M$,

(2.7) $$w_i \eta_{ij} = w_j \eta_{ji}, \; i, j = 1, \dots, n.$$

Transforming Log-Linear Demands

To apply model (2.4) to time-series data, Barten (1964) replaces infinitesimal changes with finite changes and takes the elasticities to be constants:

(2.8) $$Dq_{it} = \eta_i DQ_t + \sum_{j=1}^{n} \eta_{ij} Dp_{jt} + \varepsilon_{it}, \; i = 1, \dots, n \text{ goods}, \; t = 1, \dots, T \text{ observations},$$

where D denotes the log-change operator $\left(Dx_t = \log x_t - \log x_{t-1}\right)$ and ε_{it} is a disturbance term. This is an n-equation system that is linear in the parameters, the elasticities. Furthermore, the homogeneity constraint (2.6) is also linear. These are attractive features of the model, and Barten (1964, p. 8) justifies its use "for reasons that are largely pragmatic". Not so attractive is the appearance of the budget shares in constraints (2.5) and (2.7). Barten (1964) treats the shares as constants in these constraints and replaces them with their sample means.

Theil (1965) dealt with this difficulty by multiplying both sides of (2.8) by the budget share and then reparameterising. As the model refers to changes over time from period t-1 to t, the share in either of these periods would seem to have equal merits. But to choose one leads to an asymmetry that is best avoided. To treat both periods symmetrically, Theil (1965) proposed a two-period moving average of the share over t-1, t, $\bar{w}_{it} = 1/2\left(w_{i,t-1} + w_{it}\right)$. The reformulated version of (2.8) is

(2.9) $$\bar{w}_{it} Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{n} \pi_{ij} Dp_{jt} + \mu_{it}, \; i = 1, \dots, n \text{ goods}, \; t = 1, \dots, T \text{ observations},$$

The new parameters are $\theta_i = \bar{w}_{it}\eta_i$ and $\pi_{ij} = \bar{w}_{it}\eta_{ij}$, which are taken to be constants, while $\mu_i = \bar{w}_{it}\varepsilon_{it}$ is the new disturbance. Thus, we move from a constant elasticity formulation to something else; that something else, model (2.9), has come to be known as the Rotterdam model.[2]

Interpretations and Implications

The Rotterdam model describes the budget-share weighted change in the quantity consumed of good i, $\bar{w}_{it}Dq_{it}$, as a linear function of the change in real income, DQ_t, and the change in each of the n prices, $Dp_{1t},\ldots,Dp_{nt}$. There is one equation for each good, so (2.9) comprises a system of n equations; and this system refers to the changes over successive periods for each of T periods. The parameters of the model are θ_i, $i=1,\ldots,n$, and π_{ij}, $i, j=1,\ldots,n$, which have the following interpretations. As the budget share is defined as $w_i = p_i q_i / M$ and as the income elasticity is $\eta_i = (M/q_i)(\partial q_i/\partial M)$, it follows that their product $w_i\eta_i$ is $p_i\partial q_i/\partial M = \partial(p_i q_i)/\partial M$. Thus, the parameter attached to the real income variable in (2.9), θ_i, is just this product, so it follows that $\theta_i = \partial(p_i q_i)/\partial M$. Accordingly, θ_i answers the question, if income rises by one dollar, how much of this is spent on good i? This parameter is known as the marginal share of good i. An inferior good has a negative marginal share, while for a normal good the share is positive. As the additional income is taken to be completely spent, the marginal shares have a unit sum:

$$\sum_{i=1}^{n}\theta_i = 1. \tag{2.10}$$

The income elasticity of good i implied by model (2.9) is simply the ratio of the marginal share to its budget share, $\eta_i = \theta_i / w_i$, so that luxuries $(\eta_i > 1)$ have a marginal share greater than the budget share and vice versa for necessities $(\eta_i < 1)$. Constraint (2.10) implies that a budget-share weighted average of the income elasticities is unity, which is the first member of (2.5). It can be seen that one of the attractions of taking the marginal shares to be constants is that one of the adding-up constraints is satisfied as a parametric restriction without any reference to the (variable) budget shares.

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[2] The name, introduced by Parks (1969), reflects the location of Barten and Theil when the model was first formulated. Theil moved from The Netherlands School of Economics (now Erasmus University Rotterdam) to The University of Chicago in 1966, then in 1981, to The University of Florida, Gainesville and retired in 1994. He died in 2000. Barten, Theil's former student, moved from Rotterdam to The Catholic University of Louvain and then later to Tilburg University. For biographical information on these two pioneers of econometrics, see Barnett (2003), Bewley (2000), Blaug (1999), Koerts (1992), Raj (1992) and Seale and Moss (2003).

Next, consider the coefficients of the prices in (2.9). The coefficient of the change in the j^{th} price in the i^{th} equation is π_{ij}. As this is the product of the budget share of i, $\bar{w}_{it}$, and the $(i, j)^{th}$ price elasticity, η_{ij}, it follows that the Rotterdam model (2.9) implies that this price elasticity takes the form $\eta_{ij} = \pi_{ij}/\bar{w}_{it}$. Multiplying both sides of the homogeneity constraint (2.6) by $\bar{w}_{it}$ gives

(2.11) $$\sum_{j=1}^{n} \pi_{ij} = 0, \; i = 1, \ldots, n.$$

Similarly, the symmetry constraints (2.7) now become

(2.12) $$\pi_{ij} = \pi_{ji}, \; i, j = 1, \ldots, n.$$

As π_{ij} refers to the substitution effect of a change in the price of good j on the demand for good i when real income is held constant, π_{ij} is known as the $(i, j)^{th}$ Slutsky coefficient. As can be seen, the Rotterdam specification solves the problem with the budget shares being part of the constraints of the constant elasticity model (2.8). The issue not yet discussed is the second member of (2.5). But as now will be shown, with the Rotterdam, this also becomes a simple matter.

The consumer's budget is given by equation (2.2). The differential is $dM = \Sigma_{i=1}^{n} \left(p_i dq_i + q_i dp_i\right)$, or $d\left(\log M\right) = d\left(\log Q\right) + d\left(\log P\right)$, with

$$d\left(\log Q\right) = \sum_{i=1}^{n} w_i d\left(\log q_i\right) \text{ and } d\left(\log P\right) = \sum_{i=1}^{n} w_i d\left(\log p_i\right)$$

the Divisia volume and price indexes. A frequently employed discrete approximation of this volume index is

(2.13) $$DQ_t = \sum_{i=1}^{n} \bar{w}_{it} Dq_{it}.$$

The Rotterdam model uses this index to measure the change in real income, which is the reason for using the notation DQ_t in both (2.9) and (2.13). This means if we add both sides of equation (2.9) over $i = 1, \ldots, n,$ we obtain DQ_t on the left-hand side. To ensure that the right-hand side is also DQ_t, we need the cross-equation restrictions on the parameters given by (2.10) and

(2.14) $$\sum_{i=1}^{n} \pi_{ij} = 0, \;\; j = 1, \ldots, n,$$

as well the disturbances having a zero sum, $\Sigma_{i=1}^{n}\mu_{it} = 0.$ Constraint (2.14) coincides with the second member of (2.5), $\Sigma_{i=1}^{n} w_i \eta_{ij} = 0, j = 1,\ldots,n.$ The vanishing of the budget shares here again underscores the attractiveness of the Rotterdam formulation.[3]

As mentioned above, the dependent variable of the model, $\bar{w}_{it}Dq_{it}$, is a budget-share weighted change in the quantity consumed of good i. Something more can be said about the interpretation of this variable. First, from equation (2.13), $\bar{w}_{it}Dq_{it}$ is the contribution of good i to the change in real income. A second interpretation derives from the differential of the budget share, $dw_i = w_i d(\log q_i) + w_i d(\log p_i) - w_i d(\log M)$, which shows that the change in the share is made up of price, quantity and income components. If for the transition from period t-1 to t, this is approximated by $w_{it} - w_{i,t-1} \approx \bar{w}_{it}Dq_{it} + \bar{w}_{it}Dp_{it} - \bar{w}_{it}DM_t$, it can then be seen that the dependent variable of the model is also interpreted as the quantity component of the change in the budget share of good i.

Further Issues

The above presentation of the Rotterdam model emphasises its interpretation and the simplicity of the constraints on the parameters. These features have contributed to its widespread use, as well as some of the controversy surrounding the model, to be discussed in the next section. But before concluding this section, there are other aspects of Rotterdam that should be at least mentioned. One is the extensive econometric analysis that has been developed to estimate and test the model. As model (2.9) is linear in the parameters, when income and the prices are taken as exogenous it can be estimated as a seemingly unrelated regression system. Moreover, as the homogeneity and symmetry constraints are also linear, their testing would also seem to be straightforward. But as Laitinen (1978) and Meisner (1979) showed with Monte Carlo studies, the conventional tests based on asymptotic theory (that is, as $T \rightarrow \infty$) break down when the number of goods n is large relative to the number of time periods, but these tests can be modified appropriately.[4]

The structure of preferences within the Rotterdam framework should also be mentioned. The consumer's preferences are described by the utility function (2.1). If this general form is made more

[3] Defining real income as the sum of the left-hand variables means that model (2.9) is an allocation system. That is, given the price changes and the values of the disturbances, the n equations of the model allocate the change in real income to each of the n goods. Knowledge of n-1 allocations is sufficient to determine that of the n^{th}; in other words, the Rotterdam model is a singular system in which one equation is redundant. This means that the adding up constraints are automatically satisfied and not testable.

[4] For a comprehensive account of these matters, see Theil (1987).

specific there are additional restrictions on the demand equations. We could, for example, suppose that the n goods are unrelated in the sense that the marginal utility of each good is independent of the consumption of all others. This means that $u(q_1,\dots,q_n)=\Sigma_{i=1}^{n}u_i(q_i)$, where $u_i(q_i)$ is the i^{th} sub-utility function that involves only q_i. This case is known as preference independence (or strong separability) and, as discussed in Section 4, the $(i, j)^{th}$ price elasticity takes the form $\eta_{ii}=\phi\eta_i(1-w_i\eta_i)$ if $i=j$ (denoting the own-price elasticity) and $\eta_{ij}=-\phi\eta_i w_j\eta_j$ if $i\neq j$ (the cross-price elasticity), where $\phi<0$ is a factor of proportionality (known as the "income flexibility", the reciprocal of the income elasticity of the marginal utility of income), w_i is the budget share of good i and η_i is the income elasticity of i. Defining δ_{ij} as the Kronecker delta $\left(\delta_{ij}=1 \text{ if } i=j,\ 0 \text{ otherwise}\right)$ and using $\pi_{ij}=w_i\eta_{ij}$, the substitution effect in (2.9) then becomes

(2.15) $$\sum_{j=1}^{n}\pi_{ij}Dp_{jt}=\phi w_i\eta_i\sum_{j=1}^{n}\left(\delta_{ij}-w_j\eta_j\right)Dp_{jt}=\phi\theta_i\sum_{j=1}^{n}\left(\delta_{ij}-\theta_j\right)Dp_{jt}=\phi\theta_i\left(Dp_{it}-DP_t'\right),$$

where $DP'=\sum_{i=1}^{n}\theta_i Dp_{it}$ is the Frisch (1932) price index.

Thus, the preference independence version of model (2.9) takes the form

(2.16) $$\bar{w}_{it}Dq_{it}=\theta_i DQ_t+\phi\theta_i\left(Dp_{it}-DP_t'\right)+\mu_{it}.$$

The term in brackets in this equation is the change in the relative price of good i, with the Frisch index acting as a deflator of the change in the nominal price. Accordingly, under preference independence, consumption of good i depends on income and its own relative price, not the prices of the other goods. In words, the absence of any cross effects in the utility function translates directly into the absence of cross-price effects in the demand equations. It follows from equation (2.15) that the preference independence hypothesis takes the form of simple parametric restrictions on the Slutsky coefficients: $\pi_{ij}=\phi\theta_i\left(\delta_{ij}-\theta_j\right)$, for $i, j=1,\dots,n$.

As preference independence rules out all utility interactions among goods, this condition is strong and is often thought to be more applicable to the broad aggregates such as food, clothing, housing, etc., where the substitution possibilities are likely to be modest. But as it is the simplest form of preferences, it is a natural starting point for testing. According to the weaker condition of block independence (or weak separability), groups of goods are additive in the utility function, rather than individual goods. This leads to group demand equations and systems of conditional demands, which express the demand for each member of a group in terms of the total consumption of the group and prices within the group. An example is the alcoholic beverages group comprising beer, wine and spirits. When alcohol is block independent, there exists a group demand for alcohol as a whole, which depends on real income and the price of alcohol. Then, given the demand for total alcohol, there is a system of three conditional demand equations. The linear structure of the Rotterdam framework means that the group demands and conditional demands have the same basic Rotterdam forms and are consistent in aggregation. Conditional demand equations allow the focus to be on sub-groups of goods that are of particular interest. Block independence implies parametric restrictions on the Slutsky coefficients that are analogous to those of preference independence. Thus, the Rotterdam model allows ideas regarding the nature of goods, and how closely related goods might be grouped together, to be mapped straightforwardly into observed demand behaviour via simple parametric restrictions.

3. CRITICISMS

Like all models, the Rotterdam system can at best be considered as only an approximation to reality and is not perfect. In this section we consider three criticisms of the Rotterdam model.

Constant Marginal Shares

The infinitesimal version of equation (2.9) is $w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j)$. If prices are held constant, then the substitution term vanishes and $w_i d(\log q_i) = \theta_i d(\log Q)$. As the change in real income $d(\log Q)$ coincides with the change in money income $d(\log M)$, this becomes $w_i d(\log q_i) = \theta_i d(\log M)$, or $p_i dq_i = \theta_i dM$. Integration yields a linear Engel curve:

$$p_i q_i = \alpha_i + \theta_i M, \tag{3.1}$$

where α_i is a constant of integration. By definition, the income elasticity of i is

$$\eta_i = \frac{\partial(\log q_i)}{\partial(\log M)} = \frac{\partial q_i / q_i}{\partial M / M} = \frac{p_i \partial q_i / \partial M}{p_i q_i / M} = \frac{\partial(p_i q_i)/\partial M}{w_i}, \tag{3.2}$$

where the last step follows when the price is held constant and where $w_i = p_i q_i / M$ is the budget share of i, as before. Equation (3.2) shows that the income elasticity is the ratio of the marginal share to the corresponding budget share; geometrically, the elasticity is the ratio of the slope of the Engel curve to that of the ray from the origin. Thus, for the linear Engel curve (3.1) that is implied by the Rotterdam model, the elasticity takes the form $\eta_i = \theta_i / w_i$, with the marginal share, θ_i, a constant.

The constant marginal share means that the income elasticity is inversely proportional to the budget share, that is $d(\log \eta_i) = -d(\log w_i)$. If the price is held constant, it follows from $w_i = p_i q_i / M$ and equation (3.2) that

(3.3) $$d(\log w_i) = d(\log q_i) - d(\log M) = (\eta_i - 1)\, d(\log M),$$

so that

(3.4) $$d(\log \eta_i) = -(\eta_i - 1)\, d(\log M).$$

This shows that when the marginal share is constant, the income elasticity of the income elasticity is $-(\eta_i - 1)$. Accordingly, for luxuries $(\eta_i > 1)$, income growth causes the income elasticity to fall, while the reverse is true for necessities. This type of behaviour is problematic for food, the leading necessity, as it means that its income elasticity for the rich is larger than that for the poor: Food is less of a necessity, or more of a luxury, for a more affluent consumer! This makes no economic sense and is contradicted by several studies.[5]

Fortunately, a simple modification to the Rotterdam model solves the problem: Working's (1943) model states that the budget share is a linear function of the logarithm of income:

(3.5) $$w_i = \alpha_i + \beta_i \log M,$$

where α_i and β_i are new parameters.[6] The implied marginal share is

(3.6) $$\frac{\partial(p_i q_i)}{\partial M} = w_i + \beta_i.$$

Rather than being a constant, now the marginal share differs from the corresponding budget share by a constant, β_i. The income elasticity is

(3.7) $$\eta_i = 1 + \frac{\beta_i}{w_i},$$

so that the good is a luxury (necessity) if its $\beta_i > 0$ (< 0). The differential of (3.7) can be expressed as $d(\log \eta_i) = -\left[(\eta_i - 1)/\eta_i\right] d(\log w_i)$, so that, in view of (3.3),

(3.8) $$d(\log \eta_i) = -\left[\frac{(\eta_i - 1)^2}{\eta_i}\right] d(\log M).$$

Now, the income elasticity always falls as income grows (as long as $\eta_i > 0$): The food income elasticity for the rich is now lower than that for the poor, which solves the previous problem.

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[5] Gao (2012) uses the 2005 ICP data for a large number of countries to provide evidence on this matter by plotting the log of the food budget share (w) against the log of income (M). There is clear evidence of a downward-sloping relationship that gets steeper as income rises. As $\partial(\log w)/\partial(\log M) = \eta - 1$, where η is the food income elasticity, the negative slope means that $\eta < 1$, while the increasing slope means that η falls as income rises. For other evidence that η declines with income, see, e. g., Clements and S. Selvanathan (1994), Deaton and Paxson (1998), Hymans and Shapiro (1976), Lluch et al. (1977) and Timmer and Alderman (1979).

[6] This model is also associated with Leser (1963).

The income term in the Rotterdam model (2.9) is $\theta_i DQ_t$ where $\theta_i = \text{constant}$. Alternatively, if we use the variable marginal share implied by Working's model, (3.6), the income term then becomes $(\bar{w}_{it} + \beta_i) DQ_t$, where to be consistent with the left-hand-side of Rotterdam, $\bar{w}_{it}$ is used. Then, after minor rearrangements, the demand equation for good i takes the form

$$\bar{w}_{it}\left(Dq_{it} - DQ_t\right) = \beta_i DQ_t + \sum_{j=1}^{n} \pi_{ij} Dp_{jt} + \mu_{it},$$

where β_i, the coefficient of income, has exactly the same interpretation of β_i in Working's model. This shows that the Rotterdam model can be modified in a simple manner to make the income responses more satisfactory.

<u>What's Constant, What's Variable</u>?

The i^{th} equation of the Rotterdam model is $\bar{w}_{it} Dq_{it} = \theta_i DQ_t + \Sigma_{j=1}^{n} \pi_{ij} Dp_{jt} + \mu_{it}$. Here, the quantity component of the change in the i^{th} budget share is a linear function of the change in real income, the n price log-changes and a random disturbance term. The constant parameters are the good's marginal share, θ_i, and the Slutsky coefficients, $\pi_{ij}, j = 1, \ldots, n$, defined as

$$\text{(3.9)} \qquad \theta_i = \frac{\partial\left(p_i q_i\right)}{\partial M}, \quad \pi_{ij} = \frac{p_i p_j}{M} \cdot s_{ij}, \text{ with } s_{ij} = \left.\frac{\partial q_i}{\partial p_j}\right|_{DQ=0}.$$

These expressions have been the source of some confusion: At first glance, they look complex as they involve derivatives of the demand function, as well as prices and income. If the derivatives are constant, while prices and income are variable, how can the terms in equation (3.9) possibly be constant parameters?

Clearly, θ_i and π_{ij} cannot be constants if one insists on constant derivatives. It may be common practice to think of derivatives as being more or less constant, but this is just a matter of convenience that serves as a simplification, when, for example, comparative statics is used to determine a sign. But there is no fundamental reason for these derivatives to be constants. The issue is the choice of the functional form of the equation to be estimated. Take again the case of the Engel curve for good i: We could consider the two alternatives, the linear specification (3.1) or a double-log form,

$$p_i q_i = \alpha_i + \beta_i M, \qquad \log\left(p_i q_i\right) = \alpha_i' + \beta_i' \log M.$$

This involves choosing between specifying a constant slope, β_i, or a constant elasticity, β_i'. If the slope is constant, then elasticity is variable, and vice versa, as $\beta_i' = \beta_i \cdot (M/p_i q_i)$. The goodness of fit of the two specifications is the obvious way to choose between them.[7]

The Slutsky coefficient π_{ij} in equation (3.9) involves prices, income and preferences in the form of the response to the j^{th} price when the consumer remains on the same indifference curve.[8] An objection to treating π_{ij} as a constant is that this mixture of the subjective (preferences) and the objective (prices and income) is somehow mixed up. This point has been heard (but apparently not written down) at The University of Chicago and possibly reflects the influence of Milton Friedman, who forcefully argued that preferences are distinct from the budget constraint and those who mix the two do so at their peril. While this is no doubt a good guiding principle of applied price theory, the objection has little merit in the context of the Rotterdam model which is used for applied demand analysis. In essence, this is similar to the issue discussed above parameterisation on the basis of goodness-of-fit considerations.

The variable on the left-hand side of the Rotterdam equation (2.9) is $\bar{w}_{it} Dq_{it}$. Thus, if we set the disturbance term in that equation at its expected value of zero and then divide both sides by the budget share, we obtain a demand equation with the quantity change on the left:

$$Dq_{it} = \left(\frac{\theta_i}{\bar{w}_{it}}\right) DQ_t + \sum_{j=1}^{n} \left(\frac{\pi_{ij}}{\bar{w}_{it}}\right) Dp_{jt} = \eta_{it} \cdot DQ_t + \sum_{j=1}^{n} \eta_{ijt} \cdot Dp_{jt},$$

where $\eta_{it} = \theta_i / \bar{w}_{it}$ is the income elasticity of demand for good i and $\eta_{ijt} = \pi_{ij} / \bar{w}_{it}$ is the $(i, j)^{th}$ compensated price elasticity. In this form, it is clear that the Rotterdam approach leads to income and price elasticities that are variable. These elasticities always satisfy the adding-up constraints (2.5); homogeneity (2.6) if $\Sigma_{j=1}^{n} \pi_{ij} = 0;$ and symmetry (2.7) if $\pi_{ij} = \pi_{ji}, i, j = 1, \ldots, n.$ As the Slutsky coefficients are constant, $d(\log \eta_{ijt}) = -d(\log \bar{w}_{it})$. With income growth, the budget shares of necessities fall, so that the price elasticities of these goods rise; and vice versa for luxuries. In some situations this may be an unpalatable consequence of the Rotterdam parameterisation.

A related misunderstanding of the Rotterdam model comes from the unfamiliar way in which it is derived. Most other demand systems are derived from an algebraic specification of the utility or

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[7] Although there is the additional consideration mentioned above regarding the counter-intuitive implications of the linear form when applied to food. Another issue is that the linear form satisfies the budget constraint, while the double-log does not. These points are secondary here to the main point: What is taken to be a constant – the slope or elasticity? It is natural to answer this question with reference to the data.

[8] As indicated in equation (3.9), the Slutsky coefficient holds real income constant. For small changes, this is equivalent to holding utility constant.

cost function, so the parameters of the demand equations (or some transformations thereof) are from the underlying objective function. The Rotterdam model follows a different path involving a three-step process:

1. Start with a general system of differential demand equations.
2. Constrain these equations so that they satisfy homogeneity and symmetry. These constraints come out of the solution to the budget-constrained utility maximisation problem, but the specific functional form is unspecified. As at this stage, the "coefficients" of the demand equation are not constant, the approach is general and consistent with any algebraic form of the utility or cost function. This generality will be illustrated in the next section by presenting other popular demand systems in a form that resembles the Rotterdam model.
3. Finally, infinitesimal changes are replaced with finite changes and the model is parameterised by taking the marginal shares and Slutsky coefficients to be constants.

Accordingly, the coefficients in the Rotterdam equations are not coefficients from the utility function as the algebraic form of that function is not specified in the above process. While it can take some mental "gear changing" to fully appreciate the workings of this approach, the logic is compelling and the three steps clarify the underpinnings of the model.[9]

The McFadden Critique

The Rotterdam model is formulated in terms of changes over time. What are the implications of the model for the demand equation for good i in terms of levels, $q_i = q_i(M, p_1, \ldots, p_n)$? McFadden (1964) showed that the implications are rather drastic: The Rotterdam is only consistent with a levels demand equation for all values of income and prices when $q_i(M, p_1, \ldots, p_n) = \alpha_i(M/p_i)$, where α_i is a constant. This means that each budget share is constant, $p_i q_i/M = \alpha_i$, so that all income elasticites are equal to 1, own-price elasticites -1 and cross-price elasticities 0. As these features violate Engel's

[9] Reviews of Theil's (1975/76) two-volume book give an appreciation of some of the less-than-enthusiastic critical reaction to the Rotterdam model. Muellbauer (1978) writes "original, idiosyncratic, highly specialised, massive are some of the terms brought to mind by the 850 pages in these two volumes". He goes on to acknowledge the overall quality of the work even if he is not completely enamoured with the approach. He concludes with a restrained summary evaluation that "[the books] represent coherent theorising and econometrics taken to their furthest limits in a narrowly defined area and will appeal to researchers in this area rather than to economists in general". A second review by Horowitz (1978) is even harsher: "I found both books to be tedious reading that I wouldn't recommend to my worst enemy, and certainly not to the readership of Interfaces. On the other hand, neither I, my worst enemy, nor the overwhelming majority of our readership are intensely interested in the theory and measurement of consumer demand." After expressing some grudging admiration ("an absolute must for scholars with a compulsion to turn their talents towards the estimation of consumer demand"), Horowitz states "[v]olume 1 contains an awful lot of mathematical manipulation, little of which is motivated, none of which is especially difficult or interesting, and almost all of which is presented in a cut-and-dried fashion that makes virtually no attempt to lure the reader into subsequent sections." The terms "idiosyncratic", "highly specialised", "tedious", uninteresting "mathematical manipulation" can be interpreted as reflecting uncertainty, possible reservations and/or unfamiliarity with the Rotterdam model and the way in which it is derived.

law, economic intuition, and are not observed in practice, the McFadden critique would seem to greatly diminish the usefulness of the Rotterdam model.

There have been two main responses to this criticism. The first is to regard all models as approximations that hold over some limited range of data only, rather than globally. Without experimental data that span the entire range of possible values of prices and income, it is impossible to observe all conceivable consumption behaviour, so this would seem to be a defensible pragmatic position to take. Theil (1967, p. 203) writes that:

> [The] constancy [of the coefficients of the Rotterdam model (2.9)] is restrictive, of course. It implies, as far as [$\theta_i = \partial(p_i q_i)/\partial M$] is concerned, that the optimal quantities are linear functions of income. Although this is probably not too serious when real income and relative prices are subject to moderate changes, it should be realised that equation [(2.9)] -- or, for that matter, any other form of demand equation -- is a Procrustean bed which fits empirical observations imperfectly. The main justification of [(2.9)] is it simplicity... Since our starting point was formulated in terms of first-order effects (infinitesimal changes), the implications of the demand equations should also be confined to first-order changes.

This approach to applied work is also exemplified in Theil's (1971, p. vi) declaration that "models are to be used but not to be believed".[10] Goldberger (1987, p. 96) expresses the issue slightly differently:

> If one is to assess the fruitfulness of the Rotterdam [model], it is important to recognise that no stigma attaches to [its] being approximate rather than exact. With the true utility function being unknown, there is after all no guarantee that any of the "exact" consumer demand models will be exact in fact. [The Rotterdam model], quite possibly, provides an adequate approximation to utility-maximising behaviour over a range of conceivable true utility functions; this without being exactly appropriate for any particular one. Such robustness is naturally attractive.

A second response to the McFadden critique is to realise that strictly speaking, the economic theory of the consumer, which gives rise to the homogeneity and symmetry constraints, applies to the individual consumer only. Thus, we might start with micro demand equations for each individual and then inquire about the nature of the aggregated, or macro, demand equations. Barnett (1979b), Theil (1971, 1975/76) and Selvanathan (1991) use the convergence approach (or random coefficients) to aggregate micro-level differential demand equations. Under not unreasonable conditions, the Rotterdam model emerges from this analysis as a Taylor-series approximation to the aggregated

[10] But note that this statement comes with a qualifier at the start of the full sentence: "It does require maturity to realise that models are to be used but not to be believed." Those who struggle with this rule suffer from immaturity! A variation on this theme is the open-minded adage that enjoyed currency among Chicago PhD students during the time Theil was there: "Fall in love with your data, not your model."

demand equations. This response would seem to reinforce the approach of treating models as approximating reality, not mirroring it.[11]

4. THE DIFFERENTIAL APPROACH

Partly in response to the above criticisms, Theil (1980) introduced a more general formulation that represents a new class of model, or perhaps a new "mode" of analysis – the differential approach, which can be thought of as a way of conducting comparative statics. This section discusses the essentials of the approach.[12]

The Rotterdam model is given by equation (2.9) above. Setting the disturbance term at its expected value of zero, the i^{th} equation is $\overline{w}_{it} Dq_{it} = \theta_i DQ_t + \Sigma_{j=1}^{n} \pi_{ij} Dp_{jt}$. This is formulated in terms of finite changes. The corresponding infinitesimal version is

$$(4.1) \qquad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j).$$

As discussed above, this is a transformation of the differential of the conventional Marshallian demand equation in which the quantity demanded depends on money income (M) and the n prices, $q_i = q_i(M, p_1, \ldots, p_n)$. That is, $dq_i = (\partial q_i / \partial M) dM + \sum_{j=1}^{n} (\partial q_i / \partial p_j) dp_j$. As in comparative statics, the differentials here refer to any displacement, not necessarily ones that are over time. For what follows, it is important to set out in detail the nature and interpretation of this transform and how it forms the basis for the differential approach. It is helpful to start with a brief recapitulation.

<u>The Price Term</u>

Holding money income constant, the total effect of a change in the j^{th} price on consumption of good i is $dq_i = (\partial q_i / \partial p_j) dp_j$. According to the Slutsky equation, this can be decomposed into the substitution and income effects, $dq_i = s_{ij}(dp_j) - q_j(\partial q_i / \partial M)(dp_j)$, where s_{ij} is the substitution effect. Multiplying through by p_i / M and using $dx/x = d(\log x)$, we have

$$\frac{p_i}{M}(dq_i) = \left(\frac{p_i p_j}{M} s_{ij}\right) d(\log p_j) - w_j \frac{\partial(p_i q_i)}{\partial M} d(\log p_j),$$

where $w_i = p_i q_i / M$ is the budget share of good i, the proportion of income spend on the good. Thus, when the n prices change, the effect on consumption of good i is just the sum over $j = 1, \ldots, n$ of the above equation, which we write as

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[11] For further research on the Rotterdam model as an approximation, see Barnett (1984), Byron (1984) and Mountain (1988).

[12] Barnett and Serletis (2009) study the differential approach in the context of the Rotterdam model.

$$(4.2)\qquad w_i d(\log q_i)\big|_{dM=0} = \sum_{j=1}^{n}\left[\left(\frac{p_i p_j}{M}s_{ij}\right) - w_j\frac{\partial(p_i q_i)}{\partial M}\right]d(\log p_j) = \sum_{j=1}^{n}\left[\pi_{ij} - w_j\theta_i\right]d(\log p_j),$$

where $\pi_{ij} = (p_i p_j/M)s_{ij}$ is the $(i, j)^{th}$ Slutsky coefficient, which refers to the substitution effect, and $\theta_i = \partial(p_i q_i)/\partial M$ is the marginal share of i, which measures the additional expenditure on the good following a one-dollar increase in income.

The Income and Price Terms Combined

The effect on the demand for good i of a change in income is $dq_i = (\partial q_i/\partial M)dM$, which by again multiplying by p_i/M and using $dx/x = d(\log x)$, can be expressed as

$$(4.3)\qquad w_i d(\log q_i)\big|_{dp_1=\ldots=dp_n=0} = \frac{\partial(p_i q_i)}{\partial M}d(\log M) = \theta_i d(\log M),$$

where θ_i is the same marginal share as in (4.2).

When prices and income change simultaneously, the impact on consumption is given by the sum of equations (4.2) and (4.3):

$$(4.4)\qquad \begin{cases} w_i d(\log q_i)\big|_{dM=0} + w_i d(\log q_i)\big|_{dp_1=\ldots=dp_n=0} = \theta_i d(\log M) + \sum_{j=1}^{n}\left[\pi_{ij} - w_j\theta_i\right]d(\log p_j) \\ \qquad = \theta_i\left[d(\log M) - \sum_{j=1}^{n} w_j d(\log p_j)\right] + \sum_{j=1}^{n}\pi_{ij} d(\log p_j), \end{cases}$$

The term in square brackets in the second line of this equation, $d(\log M) - \Sigma_{j=1}^{n} w_j d(\log p_j)$, shows how the income effects of the n price changes, $\Sigma_{j=1}^{n} w_j d(\log p_j)$, act as a deflator that converts the change in money income, $d(\log M)$, into the change in real income, $d(\log M) - \Sigma_{j=1}^{n} w_j d(\log p_j)$. If we denote by $w_i d(\log q_i)$ the sum $w_i d(\log q_i)\big|_{dM=0} + w_i d(\log q_i)\big|_{dp_1=\ldots=dp_n=0}$ and by $d(\log Q)$ the change in real income, then we obtain directly equation (4.1), the differential version of the Rotterdam.

Interpreting Real Income

Another way of analysing the real income term in (4.4) is to go back to the budget constraint, $M = \Sigma_{i=1}^{n} p_i q_i$, and take the differential:

$$d(\log M) = \sum_{j=1}^{n} w_j d(\log q_j) - \sum_{j=1}^{n} w_j d(\log p_j).$$

This reveals that the difference $d(\log M)-\Sigma_{j=1}^{n}w_j d(\log p_j)$ is equivalent to $\Sigma_{j=1}^{n}w_j d(\log q_j)$, which is a volume index of income. This equivalence confirms the above interpretation of $d(\log M)-\Sigma_{j=1}^{n}w_j d(\log p_j)$ as the change in real income. A further confirmation is available from the utility function $u=u(q_1,\ldots,q_n)$ in differential form, $du=\Sigma_{i=1}^{n}(\partial u/\partial q_i)dq_i$. For a budget-constrained utility maximum, each marginal utility is proportional to the corresponding price, $\partial u/\partial q_i=\lambda p_i$, where the proportionality factor λ is the marginal utility of income. Accordingly, $du=\lambda\Sigma_{i=1}^{n}p_i dq_i=\lambda M\Sigma_{i=1}^{n}w_i d(\log q_i)$, so the change in utility is proportional to the budget-share weighted average of the n quantity changes. Further, as du/λ is the change in utility measured in terms of dollars, it can be seen that $M\Sigma_{i=1}^{n}w_i d(\log q_i)$ is interpreted as the change in this measure of utility. Thus, $\Sigma_{i=1}^{n}w_i d(\log q_i)$ is the proportionate change in real income.

It is worthwhile to note that the two budget-share weighted averages used above have a prominent place in index number theory. These are Divisia (1925) indexes of (the change in) prices and real income, $\Sigma_{j=1}^{n}w_j d(\log p_j)$, $\Sigma_{j=1}^{n}w_j d(\log q_j)$.

A Decomposition of the Substitution Effect

In the above, the total effect of a price change was split into income and substitution effects. Barten (1964) went further to split the substitution effect into two distinct components, a “specific” effect and a “general” effect. The specific effect measures the degree of interactions between goods i and j in the utility function, while the general effect comes from the workings of the budget constraint and reflects the competition of all goods for the consumer’s dollar. This was a breakthrough in consumer demand analysis as it provided for the first time a practical way to test for the degree of utility interactions.

Barten (1964) applied the method of comparative statics to the first-order conditions for the utility-maximisation problem to formulate his Fundamental Matrix Equation. He then showed that the solution to this equation meant that the substitution effect of a change in the price of good j and the demand for good i can be expressed as

$$(4.5)\qquad s_{ij}=\lambda u^{ij}-\frac{\lambda}{\partial\lambda/\partial M}\frac{\partial q_i}{\partial M}\frac{\partial q_j}{\partial M}.$$

Here, λ is the marginal utility of income and u^{ij} is the $(i,j)^{th}$ element of the inverse of the $n\times n$ Hessian matrix of the utility function, that is, $u^{ij}=\left[\partial^2 u/\partial q_i\,\partial q_j\right]^{-1}$. The elements of this matrix describe how the marginal utility of each good changes as consumption of all others vary.

To illustrate the workings of equation (4.5), consider, for example, the demand for broad aggregates such as food, clothing, housing, etc. As there are likely to be only limited possibilities to substitute one of these goods for another, it might be reasonable to suppose there are no utility interactions, as mentioned previously. In such a case, utility is additive in each of the goods, $u(q_1,\ldots,q_n)=\Sigma_{i=1}^{n}u_i(q_i)$, where $u_i(\cdot)$ is the i^{th} sub-utility function that depends only on consumption of good i. Thus, each marginal utility in independent of the consumption of all other goods, the Hessian is diagonal and so it its inverse. This implies that for each pair of goods $i \neq j$, $u^{ij}=0$, so that the first term on the right-hand side of equation (4.5) vanishes. Because u^{ij} measures the interactions of the goods in utility and as this term cannot be decomposed into separate parts involving i and j, λu^{ij} is called the <u>specific substitution effect</u> (Houthakker, 1960). As the Hessian matrix of the utility function and its inverse are both symmetric, it follows that the specific effect is symmetric in i and j. The specific substitution effect holds constant the marginal utility of income.

The second term in equation (4.5), $-(\lambda/\partial\lambda/\partial M)(\partial q_i/\partial M)(\partial q_j/\partial M)$, is always present regardless of the nature of the utility function. This term is proportional to the product of the income slopes of the demand functions for the two goods in question, and thus can be decomposed. This second term is the <u>general substitution effect</u> (Houthakker, 1960), which is clearly symmetric in i and j. The sum of the specific and general effects is the "total substitution effect", s_{ij}, which is also symmetric.

<u>A Relative-Price Formulation</u>

The demand equation (4.1) contains the Slutsky coefficients that refer to the total substitution effects. This equation can be reformulated to identify the specific and general substitution effects.

As the Slutsky coefficient is defined as $\pi_{ij}=(p_ip_j/M)s_{ij}$. we multiply both sides of (4.5) by p_ip_j/M to yield

(4.6) $\pi_{ij}=\nu_{ij}-\phi\theta_i\theta_j$,

where $\nu_{ij}=(\lambda/M)p_i\,p_j\,u^{ij}$ and $\phi=(\partial\log\lambda/\partial\log M)^{-1}<0$ is the reciprocal of the income elasticity of the marginal utility of income, which is known as the <u>income flexibility</u> for short. The ν_{ij} are obviously symmetric in i and j and satisfy

(4.7) $\sum_{j=1}^{n}\nu_{ij}=\phi\theta_i,\ i=1,\ldots,n.$

The $n \times n$ matrix of price coefficients is $[\nu_{ij}] = (\lambda/M)P[\partial^2 u/\partial q_i \partial q_j]^{-1}P$, where P is a diagonal matrix with $p_1,\ldots,p_n$ on the main diagonal. The inverse of the price coefficients matrix is thus $[\nu_{ij}]^{-1} = (M/\lambda)P^{-1}[\partial^2 u/\partial q_i \partial q_j]P^{-1} = (M/\lambda)[\partial^2 u/\partial(p_i q_i)\partial(p_j q_j)]$, which shows that $[\nu_{ij}]$ is inversely proportional to (that is, proportional to the inverse of) the Hessian matrix in expenditure terms.

Using (4.6), the substitution term in equation (4.1) becomes

$$\sum_{j=1}^{n}\pi_{ij}\,d(\log p_j) = \sum_{j=1}^{n}\nu_{ij}d(\log p_j) - \phi\theta_i\sum_{j=1}^{n}\theta_j d(\log p_j) = \sum_{j=1}^{n}\nu_{ij}\left[d(\log p_j) - d(\log P')\right],$$

where the second step is based on constraint (4.7) and $d(\log P') = \Sigma_{i=1}^{n}\theta_i\,d(\log p_i)$ is the Frisch price index that uses marginal shares as weights. Thus, the demand equation in relative prices takes the form

(4.8) $$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n}\nu_{ij}\left[d(\log p_j) - d(\log P')\right],$$

This shows that the ν_{ij}, which measure the specific substitution effects, are interpreted as coefficients of the relative prices. Here, the general substitution effect acts as the deflator of $d(\log p_j)$ to isolate the specific effect. This formulation is the relative price version of equation (4.1).

<u>More Interpretations</u>

As discussed above, the Divisia price index, $\Sigma_{j=1}^{n} w_j d(\log p_j)$, acts as the deflator that transforms money income into real income. Now in demand equation (4.8) we have the Frisch price index, $d(\log P') = \Sigma_{i=1}^{n}\theta_i\,d(\log p_i)$. Like its Divisia counterpart, the Frisch index is a weighted average of prices, but now the weights are marginal shares, rather than budget shares. This distinction can be clarified by considering the income elasticity of demand for good i, $\partial(\log q_i)/\partial(\log M) = \theta_i/w_i$. As the marginal share of a luxury (income elasticity >1) is greater than its budget share, it can be seen that relative to Divisia, luxuries are more heavily weighted in the Frisch index, and vice versa for necessities.

Equation (4.8) forms the basis for what Theil (1980) calls the <u>differential approach</u>.[13] The variable on the left, $w_i d(\log q_i)$, is interpreted as the quantity component of the change in the budget share of good i, $dw_i = w_i d(\log q_i) + w_i d(\log p_i) - w_i d(\log M)$. This quantity component is a

[13] As earlier forerunners of the approach, Theil (1980, p. xi) acknowledges the work of Divisia (1925) on index numbers, Frisch (1959) on want independence (and some generalisations) and Gorman (1959, 1968) on separable utility.

weighted sum of the change in real income, $d(\log Q)$, and the n relative prices, $d\log(p_1/P'),\ldots,d\log(p_n/P')$. The marginal share θ_i is the coefficient of income, while the ν_{ij} are the coefficients attached to the relative prices. Dividing both sides of equation (4.8) by w_i, it can be seen that θ_i/w_i is the income elasticity of demand for good i and ν_{ij}/w_i is the elasticity of demand for good i with respect to the j^{th} relative price. This price elasticity holds constant the marginal utility of income and is known as a Frisch elasticity.

There are some further interesting aspects of the Frisch elasticites: It follows from equation (4.7) that $\Sigma_{j=1}^{n} w_i(\nu_{ij}/w_i) = \phi\theta_i$, so that a budget-share weighted average of the own- and cross-price Frisch elasticities of a good is proportional to its marginal share. Furthermore, as the marginal shares have a unit sum, summing both sides of this last equation over $i = 1,\ldots,n$ gives $\Sigma_{i=1}^{n}\Sigma_{j=1}^{n} w_i(\nu_{ij}/w_i) = \phi$, which shows that the income flexibility ϕ is also interpreted as a weighted average of all these price elasticites. In other words, ϕ is a measure of the overall degree of substitutability among the n goods.

As $\nu_{ij} = (\lambda/M)p_i p_j u^{ij}$, restrictions on preferences imply restrictions on the price coefficients. Under additive utility, for example, $u^{ij} = 0,\ i \neq j$, so the corresponding price coefficients $\nu_{ij} = 0$, and, in view of (4.7), the substitution term in (4.8) becomes

$$(4.9)\qquad \sum_{j=1}^{n}\nu_{ij}\left[d(\log p_j) - d(\log P')\right] = \phi\theta_i\left[d(\log p_i) - d(\log P')\right].$$

This shows that if the marginal utility of each good is independent of the consumption of all others, then the substitution part of each good's demand equation contains only the own relative price, not the others. This is an attractively simple result that links observed demand behaviour to the consumer's preferences. Generalisations involve utility being additive in (or some increasing function of) groups of goods. If, for example, food and alcoholic beverages constitute separate commodity groups in the utility function, then the demand for bread is independent of the price of beer.

The appeal of equation (4.8) is its transparent link to a general utility function, its clean split between the income and substitution effects (of both the specific and general varieties) and the ease of interpretation of its coefficients. The differential approach is general in the sense that the "coefficients" of demand equation (4.8) are not necessarily constant and that equation is consistent with any utility/cost function. The parameterisation decision (what is taken to be constant and what is variable) is delayed until the last moment – when the demand equations are estimated with actual data. The flexibility of the approach promotes constructive interaction between the data and the functional form of the demand equations. The differential approach is also a way of unifying other approaches, as is illustrated in the next section.

5. A UNIFICATION OF ALTERNATIVE FUNCTIONAL FORMS

In this section, we illustrate the workings of the differential approach as a general mode of analysis by expressing five other demand systems as special cases of equation (4.8). The detailed derivations are contained in the Appendix.

Working's Model

Working's (1943) model was originally proposed to analyse cross-sectional household data, where all households face the same set of prices. According to this model, the budget share of good i is a linear function of the logarithm of income,

$$(5.1) \quad w_i = \alpha_i + \beta_i \log M,$$

where α_i and β_i are constant parameters. In Section 3 above we showed how this model could be used to replace the constant marginal shares of the Rotterdam model. Here, we show how equation (5.1) is consistent with the more general differential approach. The simplicity of this case makes it a useful starting point for the discussion of other functional forms.

The differential of equation (5.1) is $dw_i = \beta_i d(\log M)$. When prices are constant, the differential of the budget share is $dw_i = w_i d(\log q_i) - w_i d(\log M)$. Combining these two equations, we have $w_i d(\log q_i) = (\beta_i + w_i) d(\log M)$. But when prices remain unchanged, the change in money income coincides with the change in real income, $d(\log M) = d(\log Q)$, so the differential version of Working's model is $w_i d(\log q_i) = (w_i + \beta_i) d(\log Q)$. This can be expressed as

$$(5.2) \quad w_i d(\log q_i) = \theta_i d(\log Q), \quad \text{with } \theta_i = \beta_i + w_i.$$

When prices are constant, the substitution term equation (4.8) of the differential approach vanishes, so it takes the form of equation (5.2). But when we use model (5.1), the "coefficient" θ_i in equation (5.2) is defined as $\beta_i + w_i$. Equations (5.1) and (5.2) contain exactly the same information regarding the income sensitivity of consumption, that is, the latter equation is just the differential version of the former.

The Linear Expenditure System

Next, a role for prices can be introduced via the linear expenditure system. Write utility derived from the consumption basket $q_1, \ldots, q_n$ as $u(q_1, \ldots, q_n)$. The Stone (1954)-Geary (1949-50), or Klein-Rubin (1948), specification of this utility function is $u(q_1,\ldots,q_n)=\Sigma_{i=1}^{n}\beta_i \log(q_i-\gamma_i)$, where β_i and $\gamma_i < q_i$ are constant parameters. Maximising this utility function subject to the budget constraint gives the demand function for good i that can be expressed in expenditure form as

$$(5.3) \qquad p_i q_i = p_i\gamma_i + \beta_i\left(M-\sum_{j=1}^{n} p_j\gamma_j\right).$$

For i = 1,..., n, this is known as Stone's (1954) linear expenditure system (LES).

When the γ_i parameters are positive, they are sometimes interpreted as "subsistence" quantities as utility is not defined until consumption of each good exceeds this amount. Model (5.3) can then be interpreted as saying the consumer proceeds in two steps: First, expenditure is allocated to purchasing the subsistence basket $\gamma_1,\ldots,\gamma_n$ at a cost of $\Sigma_{j=1}^{n}p_j\gamma_j$. The second step deals with the allocation of the remaining income, $M-\Sigma_{j=1}^{n}p_j\gamma_j$, which can be called supernumerary income. A fixed proportion β_i of supernumerary income is allocated to each good in this second step. The parameters β_i here are the marginal shares. In the linear expenditure system, the marginal shares are constants; the Rotterdam model shares this property (as discussed above). Model (5.3) has a simple linear form, but as it is based on an additive utility function, it is restrictive as will be shown below.

Define the n-vectors $\mathbf{p}=[p_i]$ and $\boldsymbol{\gamma}=[\gamma_i]$, so that $\mathbf{p}'\boldsymbol{\gamma}=\Sigma_{j=1}^{n}p_j\gamma_j$ is the cost of subsistence and $(M-\mathbf{p}'\boldsymbol{\gamma})/M>0$ is supernumerary income measured as a fraction of income, to be called the "supernumerary ratio"; this ratio plays a prominent role in the way the substitution term is formulated in what follows. Dividing both sides of equation (5.3) by M, we have a budget share equation, which leads to the following differential form:

$$(5.4) \qquad \begin{cases} w_i d(\log q_i) = \beta_i d(\log Q) + \left(\dfrac{M-\mathbf{p}'\boldsymbol{\gamma}}{M}\right)\beta_i \displaystyle\sum_{j=1}^{n}(\beta_j-\delta_{ij})\, d(\log p_j) \\ \qquad\qquad\quad = \theta_i d(\log Q) + \displaystyle\sum_{j=1}^{n}\pi_{ij} d(\log p_j). \end{cases}$$

Here, δ_{ij} is the Kronecker delta ($\delta_{ij}=1$ if i = j, 0 otherwise), and the marginal shares and Slutsky coefficients are defined as

$\theta_i=\beta_i$, a set of constant coefficients,

$\pi_{ij}=\left(\dfrac{M-\mathbf{p}'\boldsymbol{\gamma}}{M}\right)\beta_i(\beta_j-\delta_{ij})$, not constants.

Equation (5.4) shows how LES can be expressed in the format of the differential approach, that is, in exactly the form of equation (4.1). The marginal shares are constants, while the Slutsky coefficients are dependent on the values of prices and income and so are variable. Each Slutsky coefficient is proportional to the supernumerary ratio $\left(\mathrm{M}-\mathbf{p}'\boldsymbol{\gamma}\right)/\mathrm{M}$.[14]

As shown in the Appendix, LES can also be expressed in terms of the relative price version of the differential approach as

$$(5.5)\quad \begin{cases} w_i d(\log q_i) = \beta_i d(\log Q) + \left(\dfrac{\mathbf{p}'\boldsymbol{\gamma}-\mathrm{M}}{\mathrm{M}}\right)\beta_i\left[d(\log p_i) - \sum_{j=1}^{n}\beta_j d(\log p_j)\right] \\ \qquad = \theta_i d(\log Q) + \phi\theta_i\left[d(\log p_j) - d(\log P')\right], \end{cases}$$

where the income flexibility and the Frisch price index take the form

$$\phi = \frac{\mathbf{p}'\boldsymbol{\gamma}-\mathrm{M}}{\mathrm{M}} < 0, \qquad d(\log P') = \sum_{j=1}^{n}\beta_j d(\log p_j).$$

The second line of equation (5.5) is exactly the same as the preference independence version of the differential demand equation (4.8) with the substitution term specified as in (4.9). Here only the own-deflated price forms the substitution part of the equation; this is due to the additive utility function underlying the LES that rules out any utility interactions among goods. Note again the important role played by the supernumerary ratio in the substitution term in (5.5); in fact, the negative of this ratio is the income flexibility ϕ.

Quadratic Utility

Suppose the utility function is approximated by a quadratic:

$$u(q_1,\ldots,q_n) = \sum_{i=1}^{n} a_i q_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{=1}^{n} u_{ij} q_i q_j = \mathbf{a}'\mathbf{q} + \frac{1}{2}\mathbf{q}'\mathbf{U}\mathbf{q},$$

where $\mathbf{a} = [a_i]$ and $\mathbf{q} = [q_i]$ are n-vectors and $\mathbf{U} = [u_{ij}]$ is the $n\times n$ Hessian matrix, which is negative definite and taken to be constant. The utility-maximising quantity vector is

$$(5.6)\quad \mathbf{q} = -\mathbf{U}^{-1}\mathbf{a} + \frac{\mathrm{M}+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{U}^{-1}\mathbf{p},$$

where $\mathbf{U}^{-1}$ is the inverse of $\mathbf{U}$, $\mathbf{p} = [p_i]$ is the price vector and M is income.

Substituting for $\mathbf{q}$ back into the utility function, gives the indirect utility function,

$$u_I(\mathrm{M},p_1,\ldots,p_n) = -\frac{1}{2}\mathbf{a}'\mathbf{U}^{-1}\mathbf{a} + \frac{1}{2}\frac{\left(\mathrm{M}+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}\right)^2}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}},$$

so the marginal utility of income is

[14] Deaton (1974) was the first to express LES in this form.

(5.7) $\lambda = \dfrac{M + \mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}},$

which is linear in M. As **U** is negative definite the marginal utility of income declines as income rises, that is, $\partial\lambda/\partial M = 1/\mathbf{p}'\mathbf{U}^{-1}\mathbf{p} < 0.$ The income flexibility takes the form

(5.8) $\phi = \left(\dfrac{\partial \log\lambda}{\partial \log M}\right)^{-1} = \dfrac{\mathbf{p}'\mathbf{U}^{-1}\mathbf{a} + M}{M}.$

Equations (5.6) and (5.7) can be combined to give the simpler form $\mathbf{q} = \mathbf{U}^{-1}(\lambda\mathbf{p} - \mathbf{a}),$ or, in scalar terms, $q_i = \sum_{j=1}^{n} u^{ij}(\lambda p_j - a_j).$ As shown in the Appendix, the log differential of this form of the demand equation, multiplied by the budget share, is

$$\begin{aligned} w_i d(\log q_i) &= \frac{\sum_{j=1}^{n} u^{ij} p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d(\log Q) + \sum_{j=1}^{n}\left[\frac{\lambda u^{ij} p_i p_j}{M} - \phi \frac{\sum_{k=1}^{n} u^{ik} p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \cdot \frac{\sum_{k=1}^{n} u^{jk} p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\right] d(\log p_j). \\ &= \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j), \end{aligned}$$

with ϕ as defined in equation (5.8). This establishes that quadratic utility implies marginal shares and Slutsky coefficients of the form

$$\theta_i = \frac{\sum_{j=1}^{n} u^{ij} p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \text{ and } \pi_{ij} = \frac{\lambda u^{ij} p_i p_j}{M} - \phi \frac{\sum_{k=1}^{n} u^{ik} p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} \cdot \frac{\sum_{k=1}^{n} u^{jk} p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}.$$

These are not constant -- the marginal shares depend on prices, while the Slutsky coefficients depend on income and prices. It follows from equations (5.6) and (5.7) that the Slutsky coefficients can be expressed in the simpler for as $\pi_{ij} = (\lambda p_i p_j / M)(u^{ij} - \mathbf{p}'U^{-1}U^{-1}\mathbf{p}/\mathbf{p}'U^{-1}\mathbf{p}).$

The above is the absolute price version of the differential demand equation for good i implied by the quadratic. The corresponding relative price version is

$$\begin{aligned} w_i d(\log q_i) &= \frac{\sum_{j=1}^{n} u^{ij} p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d(\log Q) + \sum_{j=1}^{n} \frac{\lambda u^{ij} p_i p_j}{M}\left[d(\log p_j) - \sum_{j=1}^{n} \frac{\sum_{k=1}^{n} u^{jk} p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d(\log p_j)\right] \\ &= \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij}\left[d(\log p_j) - d(\log P')\right]. \end{aligned}$$

Here, $\nu_{ij} = \lambda u^{ij} p_i p_j / M$ is the $(i, j)^{th}$ price coefficient and

$$d(\log P') = \sum_{j=1}^{n} \frac{\sum_{k=1}^{n} u^{jk} p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d(\log p_j) = \sum_{j=1}^{n} \theta_j d(\log p_j)$$

is the Frisch price index.

The Translog

In the previous subsection, we considered the case when the direct utility function is a quadratic function of the quantities. Alternatively, suppose indirect utility is quadratic in the logarithms of prices and income:

$$\log V = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log\left(\frac{p_i}{M}\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} \log\left(\frac{p_i}{M}\right)\log\left(\frac{p_j}{M}\right),$$

where α_0, α_i and β_{ij} are constants and p_i/M is the price of good i deflated by income. The matrix $\left[\beta_{ij}\right]$ is symmetric. It can be shown that the income flexibility is

$$\phi = \left(\frac{\partial \log \lambda}{\partial \log M}\right)^{-1} = \frac{\sum_{i=1}^{n} \beta_{ij}}{-A} - A - 1, \quad \text{where } A = \sum_{i=1}^{n}\left[\alpha_i + \sum_{j=1}^{n} \beta_{ij} \log\left(\frac{p_j}{M}\right)\right].$$

Applying Roy's theorem yields the translog demand system (Christensen et al., 1975)

$$\text{(5.9)} \qquad w_i = \frac{\alpha_i + \sum_{j=1}^{n} \beta_{ij} \log\left(p_j/M\right)}{A}, \quad i=1,\ldots,n.$$

The implied marginal shares are

$$\theta_i = \frac{\partial\left(w_i M\right)}{\partial M} = w_i + \frac{w_i \sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} - \sum_{j=1}^{n} \beta_{ij}}{A}, \quad i = 1,\ldots,n.$$

Taking the total differential of (5.9), and subtracting from both sides by $w_i d\left(\log p_i\right) - w_i d\left(\log M\right)$, we have

$$\text{(5.10)} \quad \begin{cases} w_i d\left(\log q_i\right) = \theta_i\, d\left(\log Q\right) + \sum_{j=1}^{n}\left[\theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A}\right] d\left(\log p_j\right) \\ \qquad = \theta_i d\left(\log Q\right) + \sum_{j=1}^{n} \pi_{ij} d\left(\log p_j\right), \end{cases}$$

where the Slutsky coefficients take the form

$$\pi_{ij} = \theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A}.$$

Although not self-evident from this expression, it can be shown that these π_{ij}'s satisfy homogeneity and symmetry. The corresponding relative price version is

$$w_i d\left(\log q_i\right) = \theta_i\, d\left(\log Q\right) + \sum_{j=1}^{n}\left(\theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A} + \phi \theta_i \theta_j\right)\left[d\left(\log p_j\right) - d\log P'\right]$$

$$= \theta_i d\left(\log Q\right) + \sum_{j=1}^{n} \nu_{ij}\left[d\left(\log p_j\right) - d\left(\log P'\right)\right],$$

where

$$\nu_{ij} = \theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A} + \phi \theta_i \theta_j \quad \text{and} \quad d(\log P') = \sum_{j=1}^{n} \theta_j d(\log p_j)$$

are the price coefficients and the Frisch price index, respectively.

The Almost Ideal Demand System

The cost function refers to the minimum cost of obtaining a specified level of utility (u) given the price vector, written as $c(u, \mathbf{p})$. Deaton and Muellbauer (1980a) suggested a specific form of the logarithmic cost function

$$\log c = a(\mathbf{p}) + ub(\mathbf{p}),$$

where $a(\mathbf{p}) = \sum_{i=1}^{n} \alpha_i \log p_i + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij}^{*} \log p_i \cdot \log p_j$ and $b(\mathbf{p}) = \gamma_0 \Pi_{i=1}^{n} p_i^{\gamma_i}$ with constants α_i, β_{ij}^{*}, γ_0 and γ_i. As the indirect utility function is $u_I(M, \mathbf{p}) = [\log M - a(\mathbf{p})] / b(\mathbf{p})$, the income flexibility ϕ is -1. Thus, like the Rotterdam model, ϕ is specified as a constant; but unlike the Rotterdam, now this parameter takes the prespecified value of -1.

Using Shephard's lemma, the almost ideal demand system is:

$$\text{(5.11)} \qquad w_i = \alpha_i + \gamma_i \log\left(\frac{M}{P^*}\right) + \sum_{j=1}^{n} \beta_{ij} \log p_j, \quad i=1,\ldots,n,$$

where $\beta_{ij} = \left(\beta_{ij}^{*} + \beta_{ji}^{*}\right)/2$, and $\log P^* = \sum_{i=1}^{n} \alpha_i \log p_i + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \log p_i \cdot \log p_j$. The implied marginal shares are

$$\theta_i = \frac{\partial w_i M}{\partial M} = w_i + \gamma_i, \qquad i=1,\ldots n.$$

These have the same form as those for Working's model because both models have the same log-linear income term; see equation (5.2).

Following a similar approach to that used for the translog (see Appendix for details), the differential version of (5.11) in absolute prices is

$$\text{(5.12)} \qquad \begin{cases} w_i d(\log q_i) = (\gamma_i + w_i) d \log Q + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i \gamma_j \log\left(\frac{M}{P^*}\right) \right] d(\log p_j) \\ \qquad = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j), \end{cases}$$

where $\pi_{ij} = \beta_{ij} + w_i (w_j - \delta_{ij}) + \gamma_i \gamma_j \log(M/P^*)$ are the Slutsky coefficients. The model in relative prices is

$$(5.13)\quad \begin{cases} w_i d(\log q_i) = \theta_i\, d(\log Q) + \sum_{j=1}^{n}\left[\beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i\gamma_j \log\left(\frac{M}{P^*}\right) - \theta_i\theta_j\right]\left[d(\log p_j) - d\log P'\right] \\ \qquad = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij}\left[d(\log p_j) - d(\log P')\right], \end{cases}$$

where the $(i, j)^{th}$ price coefficient is $\nu_{ij} = \beta_{ij} + w_i(w_j - \delta_{ij}) + \gamma_i\gamma_j \log(M/P^*) - \theta_i\theta_j$ and $d(\log P')$ is the corresponding Frisch price index.

The Rotterdam Special Case

The Rotterdam model is the simplest example of the general differential demand system: According to this model, the coefficients of equations (4.1) and (4.8) -- the marginal shares, income flexibility, Slutsky coefficients and price coefficients -- are all constants. This has the attraction of transparency, but as discussed above in Section 3, the constancy of the marginal shares can be problematic, especially for food, when there are large variations in income. What of the constancy of ϕ, the income flexibility, which is a measure of the overall substitutability among goods in the basket (see Section 4)? This could be considered to be another weakness of the Rotterdam system as Frisch (1959, p. 189) famously conjectured that ϕ would increase in absolute value with income, from -0.1 for "the extremely poor and apathetic part of the population", to -0.5 for "the middle income bracket, 'the median part' of the population", and end up at -10 for "the rich part of the population with ambitions towards 'conspicuous consumption' ". This conjecture has been tested in a several studies and found to be wanting, so it seems not unreasonable to treat the income flexibility as a constant.[15]

Table 1 provides a convenient summary of this section. It draws together the expressions for the key coefficients of the differential system that are implied by the six models that have been considered.[16]

6. FROM TIME TO SPACE: CROSS-COUNTRY DEMAND ANALYSIS

The Rotterdam model deals with the change in consumption over time. But as mentioned above, the underlying differential approach deals with any kind of displacement, no matter what the source; thus, the approach has also been used for the analysis of consumption patterns in different countries. Compared with time-series data, one difference with cross-country data is that they have

[15] For tests of the Frisch conjecture, see, for example, Clements and Theil (1979), S. Selvanathan (1993), S. Selvanathan and E. A. Selvanathan (2003), Theil (1975/76), Theil (1987) and Theil and Brooks (1970/71). As the Frisch conjecture refers to the third-order derivative of the utility function, it is unsurprising that evidence on such a higher-order effect is difficult to find. But this issue is still not completely closed as DeJanvry et al. (1972) and Lluch et al. (1977) find evidence in favour of Frisch.

[16] For more on differential demands and functional form, see Barten (1993), Keller (1984), Keller and van Driel (1985), Neves (1994) and E. A. Selvanathan (1985).

no natural ordering, so a pairwise country comparison, analogous to the change from one period to the next, is awkward. A second difference is that international data exhibit substantially more variability in income and prices. For example, the poorest countries can devote substantially more than one-half of income to food consumption, while in the richest, food absorbs less than 10 percent. This diversity provides the opportunity for more precise estimates of key demand parameters. But it also presents a challenge: Can the utility-maximisation model possibly explain these vast differences without resorting to the ad hocery of "special factors" that may influence the pattern of demand in one or more countries? Relatedly, can tastes be taken to be sufficiently similar so that the same demand model can be applied to all countries, no matter how they differ in affluence and/or in the structure of prices? The key references regarding the adaption of the Rotterdam approach to cross-country comparisons are Clements and Theil (1979), Theil and Suhm (1981), Theil (1987), Theil et al. (1989) and Seale and Regmi (2006).[17] This section, which is based on Theil et al. (1989), is a brief overview of some of these developments.

The starting point is equation (4.1), the differential demand equation for good i in absolute prices, $w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n \pi_{ij} d(\log p_j)$. For country c, when income is fixed at $\log Q_c$, the first term on the right vanishes. Combining this with the change in the budget share for c, $dw_{ic} = w_{ic}[d(\log p_{ic}) - d(\log P_c)] + w_{ic} d(\log q_{ic})$, gives

$$(6.1) \qquad dw_{ic} = w_{ic}\left[d(\log p_{ic}) - d(\log P_c)\right] + \sum_{j=1}^{n} \pi_{ij}\, d(\log p_{jc}),$$

where $d(\log P_c) = \sum_{j=1}^n w_{jc} d(\log p_{jc})$ is the Divisia price index. Suppose there are $c = 1,\ldots,C$ countries. Define the world price of good i as the geometric mean over countries (to be denoted by $\bar{p}_i$), $\tilde{q}_{ic}$ as the quantity of good i consumed in country c evaluated at c's real income and these world prices, and $\tilde{w}_{ic}$ as the corresponding budget share. If dw_{ic} is interpreted as $w_{ic} - \tilde{w}_{ic}$, and $d(\log p_{ic})$ as $\log p_{ic} - \log \bar{p}_i$, then from the mean value theorem of differential calculus, equation (6.1) becomes

$$(6.2) \qquad w_{ic} - \tilde{w}_{ic} = \tilde{w}_{ic}\left[\log\left(\frac{p_{ci}}{\bar{p}_i}\right) - \sum_{j=1}^{n} \tilde{w}_{jc} \log\left(\frac{p_{cj}}{\bar{p}_j}\right)\right] + \sum_{j=1}^{n} \pi_{ij} \log\left(\frac{p_{cj}}{\bar{p}_j}\right).$$

[17] Other related research includes Barten (1989), Chen (1999), Clements and S. Selvanathan (1994), Gao (2012), Meade et al. (2014), Muhammad et al. (2011), Regmi and. Seale (2010), Seale et al. (1991), Seale at al. (2003), Seale and Regmi (2006, 2009), Seale and Solano (2012), S. Selvanathan (1991, 1993), S. Selvanathan and E. A. Selvanathan (1994) and Theil (1996). Consumption economics has, of course, long had an international flavour; influential cross-country studies that use other approaches to demand modelling include Goldberger and Gamaletsos (1970), Houthakker (1957, 1965), Kravis et al. (1982, Chap 9), Lluch and Powell (1975), Lluch et al. (1977), Pollak and Wales (1987), Parks and Barten (1973) and Rimmer and Powell (1992).

The term $\log(p_{ic}/\overline{p}_i)$ is the price of good i in country c compared with the world price. The first term on the right-hand side of this equation recognises that a higher relative price raises expenditure on the good when the consumer buys the same quantity despite its higher price; this leads to an increase in the budget share. The second term deals with the substitution effect whereby the consumer buys less of good i following a price increase, and more other goods (on average, at least).

Equation (6.2) holds real income constant. The budget share $\tilde{w}_{ic}$ here is evaluated at country c's real income and the world average prices. Theil et al. (1989) allow income to differ across countries by using Working's (1943) model, $\tilde{w}_{ic} = \alpha_i + \beta_i \log Q_c$.[18] Combining this with equation (6.2) gives

$$w_{ic} = \alpha_i + \beta_i \log Q_c + \tilde{w}_{ic}\left[\log\left(\frac{p_{ci}}{\overline{p}_i}\right) - \sum_{j=1}^{n} \tilde{w}_{jc} \log\left(\frac{p_{cj}}{\overline{p}_j}\right)\right] + \sum_{j=1}^{n} \pi_{ij} \log\left(\frac{p_{cj}}{\overline{p}_j}\right).$$

Finally, to avoid over-parameterisation, preference independence is assumed. As noted in Section 2, this means that the Slutsky coefficients take the form $\pi_{ij} = \phi\theta_i(\delta_{ij} - \theta_j)$. Then, as the marginal shares in Working's model are $\theta_{ic} = w_{ic} + \beta_i$, the above becomes

$$\text{(6.3)} \qquad w_{ic} = \alpha_i + \beta_i \log Q_c + \tilde{w}_{ic}\left[\log\left(\frac{p_{ci}}{\overline{p}_i}\right) - \sum_{j=1}^{n} \tilde{w}_{jc} \log\left(\frac{p_{cj}}{\overline{p}_j}\right)\right] + \phi(\tilde{w}_{ic} + \beta_i)\left[\log\left(\frac{p_i}{\overline{p}_i}\right) - \sum_{j=1}^{n}(\tilde{w}_{jc} + \beta_j)\log\left(\frac{p_j}{\overline{p}_j}\right)\right].$$

Equation (6.3) is known as the Working-PI model, where PI stands for preference independence.[19] This model contains Working's α_i and β_i plus and a constant income flexibility ϕ, a total of 2(n-1) + 1 parameters. This modest number of parameters reflects the assumption of preference independence. As the coefficients are taken to be the same in different countries, tastes are assumed to be the same internationally. Theil and his co-authors argue that tastes are less likely to be substantially different when broad commodity groups are used. Thus, for example, Theil and Suhm (1981, pp. 1-2) state:

> The underlying assumption is that "all countries are the same" in the sense that their per capita consumption of goods and services can be viewed as being generated by essentially the same consumer preferences. If this assumption is acceptable, the only (but major!) differences between countries result from their differences in income and prices, apart possibly from random effects. But this assumption immediately raises the objection that different nations have different

[18] Other functional forms, such as the logistic and reciprocal, are analysed in Gao (2012), who places particular emphasis on the form of the Engel curve for food.

[19] More recently, this has been referred to as the Florida-PI model (Muhammad et al., 2011), in recognition of the location of much research on cross-country consumption comparisons in general and the development of the model in particular.

cultures. We should expect that Indians spend little on meat and, indeed, the data produced by Kravis et al. [1982] confirm that this is so. The appropriate answer is that a cross-country demand system should be constructed, not for a very large number of detailed consumption categories, but only for a modest number of much broader categories. Accordingly, meat is part of a "good" called Food …[20]

7. CONCLUDING COMMENTS

Most economists are now familiar with and Deaton and Muellbauer's (1980a) Almost Ideal Demand System. This is a system of n demand equations whose form is obtained from an algebraic specification of the consumer's cost function, which can be thought of as an approximation to the unknown true form. Similarly, the translog system (Christensen et al., 1975) to an approximation to the unknown true form. Similarly, the translog system (Christensen et al., 1975) comes from a specification of the indirect utility function, while the linear expenditure system (Stone, 1954) derives from a specific direct utility function. Less familiar is the approach of the Rotterdam model (Barten, 1964, Theil, 1965). Here, the precise form of the function to be optimised (the cost or utility function) is left unspecified, but the general constraints of the utility-maximisation problem of homogeneity (the absence of money illusion) and symmetry of the substitution effects are incorporated into the demand equations. Working with a straightforward transform of demand equations expressed in terms of differences in the logarithms (weighting them by the corresponding budget shares), these constraints take the convenient form of linear parametric restrictions. The question of functional form is therefore left until the last step in the Rotterdam approach, rather than the first as in other approaches. The Rotterdam demand equations are thus interpreted as approximations to the true, unknown ones. These demand equations provide a simple way to test the theory of consumer demand, as well as the separability of preferences. The transparency of the approach, and its generality, has contributed to the prominence of the Rotterdam model over the past half-century. Some of the major events in the development of the model and its descendants are listed in Table 2.

The 1960s were a period of rapid developments in econometrics with many breakthroughs, one of which was the Rotterdam demand model. Interestingly, this occurred in Rotterdam, not at either of the two Cambridges, that, at the time, laid claims to global leadership in economic research.

[20] The assumption that tastes are constant is a major element of Chicago economics as practiced by Becker, Freidman and Stigler (Freidman, 1962, Stigler and Becker, 1977). Interestingly, while at Chicago, Theil's approach to this issue seemed to evolve as earlier he argued that it was perfectly acceptable for tastes to be different in different countries: When describing the results from two independent applications of the Rotterdam model to Dutch and British time-series data with n = 4 goods, Theil (1975/76, vol.1, p. 208) writes "[t]he income elasticities of food agree with each other and they are of the order of magnitude which one would expect them to be, but the elasticities of the three other commodity groups show larger differences. There is no reason to be apologetic about this result, since different nations may have different preferences." S. Selvanathan (1991, 1993) provides evidence that points towards the similarity of tastes for broad aggregates.

Why/how did this occur in the second city of a small, non-English-speaking country still recovering from the ravages of war? Whatever the reason for this seemingly unlikely event, Barten and Theil had the prescience to come up with an elegantly simple model that has stood the test of time and their Rotterdam model is now one of the workhorses of applied demand analysis.[21]

There is still some lingering resistance to the model due to doubts about its microeconomic foundations and the unconventional nature of its derivation. In marking the model's golden jubilee, this paper has emphasised its strong link with preferences and utility maximisation, highlighted its strengths and weaknesses and shown how it has opened up new opportunities for creative research in the form of the differential approach and cross-country consumption economics. By devoting considerable attention to the derivation and interpretation of the model, our objective has been to enhance understanding and make the Rotterdam system more accessible to economists. The paper also emphasised that like all model, the Rotterdam is not perfect and involves familiar trade-offs between generality, tractability and ease of interpretation. In addition to consumer demand, the model (or variants thereof) have been applied to a range of other areas, including labour economics, international trade, marketing, the economics of advertising, forecasting, regional economics, monetary economics and the analysis of market structure in industrial organisation.[22] It seems likely that the Rotterdam model will continue to play an influential role in the future.

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[21] In discussion with Clements, Paul Frijters has pointed out that as Rotterdam was a long-standing centre of international trade, English was widely spoken there in the 1960s and its scholars were well connected with the rest-of-the-world academic community. He argues that research networks at that time centred more around individuals rather than locations. This can be taken to say that the physical/cultural aspects of 1960s Rotterdam were not particularly different from those of other centres of economic research. Perhaps the creation of the Rotterdam model in the context of Rotterdam of the 1960s was not as unlikely as implied by the text.

[22] A selection of studies in these areas is as follows: In labour economics, Barnett (1979a, 1981) and Kiefer (1977); international trade, Berner (1977), Marquez (1994), Seale et al. (1992) and Theil and Clements (1978); marketing, Clements and E. A. Selvanathan (1988) and Vilcassim (1989); advertising, Duffy (1987) and E. A. Selvanathan (1989, 1995a,b); forecasting, E. A. Selvanathan (1995c); regional economics, S. Selvanathan (1991); monetary economics, Clements and Nguyen (1980), Fayyad (1986) and Offenbacher (1980); and industrial organisation, Capps et al. (2003). There have also been a numerous applications of the Rotterdam system to the demand for agricultural commodities.

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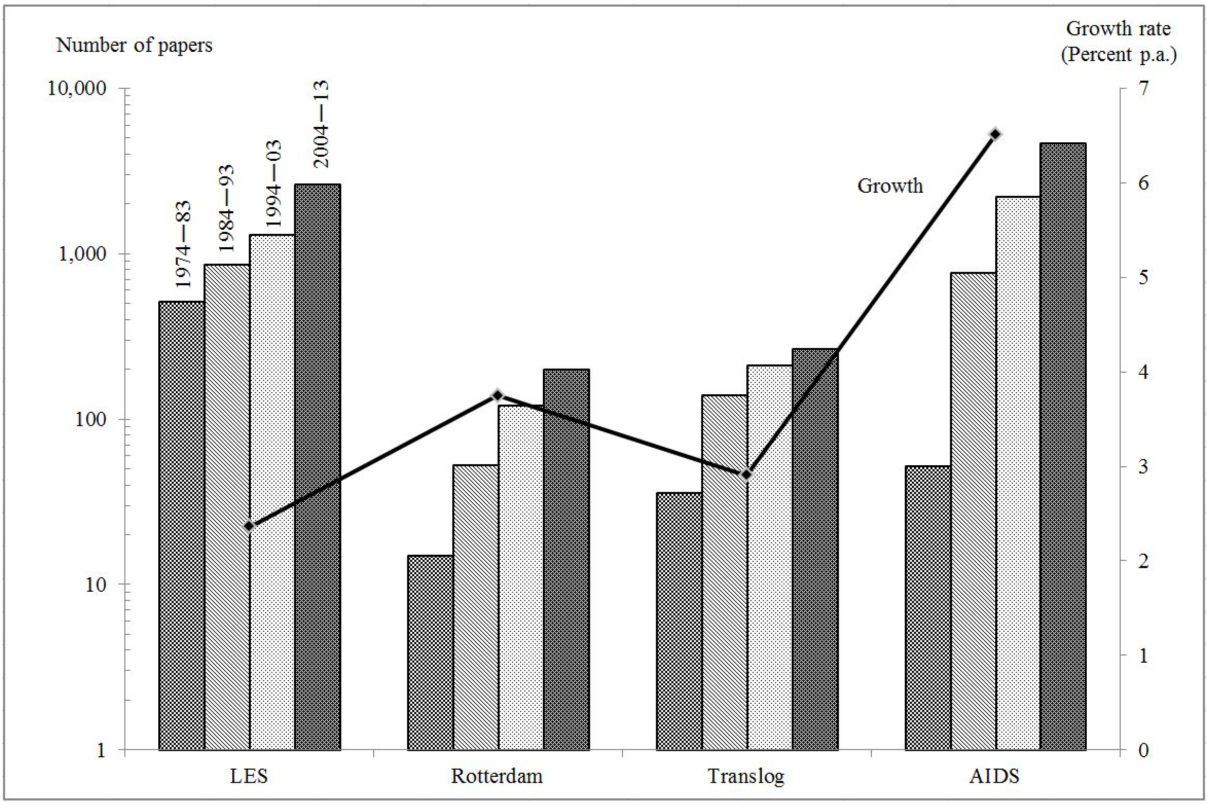
FIGURE 1

CITATIONS OF FOUR DEMAND MODELS



Note: This figure gives the results of keyword searches conducted in September 2014 using Google Scholar (http://scholar.google.com.au). Search terms are "linear expenditure system", "Rotterdam demand", "translog demand", and "almost ideal demand" for the categories on the horizontal axis. Duplicate articles were removed. The height of each column represents the number of citations, measured against the left-hand axis (labelled "Number of papers" as the vast majority of citations are by papers). The right-hand axis gives the annual growth rate in citations, calculated as 100 times the logarithm of the ratio of the number of citations in the period 2004-13 to that in 1974-83, divided by 30 (the number of years between the first and last periods).

TABLE 1

DIFFERENTIAL REPRESENTATION OF SIX DEMAND SYSTEMS

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij} \left[d(\log p_j) - d(\log P') \right], \quad i = 1, \ldots, n$$

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij}\, d(\log p_j), \quad i = 1, \ldots, n$$

Function to be optimised	Budget share equation	Marginal share θ_i	Income flexibility ϕ	Price coefficient ν_{ij}	Slutsky coefficient π_{ij}
(1)	(2)	(3)	(4)	(5)	(6)
A. Working's Model					
–	$w_i = \alpha_i + \beta_i \log M$	$w_i + \beta_i$	–	–	–
B. Linear Expenditure System					
Direct utility $\sum_{i=1}^{n} \beta_i \log(q_i - \gamma_i)$	$w_i = \frac{p_i \gamma_i}{M} + \beta_i \left(\frac{M - \mathbf{p}'\boldsymbol{\gamma}}{M} \right)$, $\mathbf{p}'\boldsymbol{\gamma} = \Sigma_{j=1}^{n} p_j \gamma_j$	$\beta_i =$ constant	$\frac{\mathbf{p}'\boldsymbol{\gamma} - M}{M}$	$\left(\frac{\mathbf{p}'\boldsymbol{\gamma} - M}{M} \right) \beta_i \delta_{ij}$, $\delta_{ij} = 1$ if $i = j$, 0 otherwise	$\left(\frac{M - \mathbf{p}'\boldsymbol{\gamma}}{M} \right) \beta_i (\beta_j - \delta_{ij})$
C. Quadratic Utility System					
Direct utility $\sum_{i=1}^{n} \alpha_i q_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} u_{ij} q_i q_j = \mathbf{a}'\mathbf{q} + \frac{1}{2} \mathbf{q}'\mathbf{U}\mathbf{q}$	$w_i = \frac{p_i}{M} \sum_{j=1}^{n} u^{ij} (\lambda p_j - a_j)$, $\lambda =$ marginal utility of income	$\frac{1}{\mathbf{p}'U^{-1}\mathbf{p}} \sum_{j=1}^{n} u^{ij} p_i p_j$	$\frac{\mathbf{p}'\mathbf{U}^{-1}\mathbf{a} + M}{M}$	$\frac{\lambda}{M} u^{ij} p_i p_j$	$\frac{\lambda p_i p_j}{M} \left(u^{ij} - \frac{\mathbf{p}'U^{-1}U^{-1}\mathbf{p}}{\mathbf{p}'U^{-1}\mathbf{p}} \right)$

(Continued next page)

TABLE 1 (Continued)

DIFFERENTIAL REPRESENTATION OF SIX DEMAND SYSTEMS

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij}\left[d(\log p_j) - d(\log P')\right], \quad i = 1,\ldots,n$$

$$w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij}\, d(\log p_j), \quad i = 1,\ldots,n$$

Function to be optimised (1)	Budget share equation (2)	Marginal share θ_i (3)	Income flexibility ϕ (4)	Price coefficient ν_{ij} (5)	Slutsky coefficient π_{ij} (6)
			D. Translog		
Indirect utility $\alpha_0 + \sum_{i=1}^{n}\alpha_i \log\frac{p_i}{M} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\log\frac{p_i}{M}\log\frac{p_j}{M}$	$w_i = \frac{\alpha_i + \sum_{j=1}^{n}\beta_{ij}\log\left(\frac{p_j}{M}\right)}{A}$	$w_i + \frac{w_i\beta_{..} - \beta_{i.}}{A}$	$\frac{\beta_{.j}}{-A} - A - 1$	$\theta_i w_j - w_i\delta_{ij} + \phi\theta_i\theta_j + \left(\beta_{ij} - w_i\beta_{.j}\right)/A$	$\theta_i w_j - w_i\delta_{ij} + \left(\beta_{ij} - w_i\beta_{.j}\right)/A$
			E. Almost Ideal Demand System		
Cost function $c = e^{a(\mathbf{p})+ub(\mathbf{p})}$	$w_i = \alpha_i + \gamma_i \log\left(\frac{M}{P^*}\right) + \sum_{j=1}^{n}\beta_{ij}\log p_j$	$w_i + \gamma_i$	-1	$\beta_{ij} + w_i\left(w_j - \delta_{ij}\right) + \gamma_i\gamma_j \log\left(M/P^*\right) - \theta_i\theta_j$	$\beta_{ij} + w_i\left(w_j - \delta_{ij}\right) + \gamma_i\gamma_j\log\left(M/P^*\right)$
			F. Rotterdam Model		
Direct utility $u(\mathbf{q})$	$\bar{w}_i Dq_{it} = \theta_i DQ_t + \sum_{j=1}^{n}\pi_{ij}Dp_{jt}$	$\theta_i =$ constant	$\phi =$ constant	$\nu_{ij} =$ constant	$\pi_{ij} =$ constant

Note: In panel D, $A = \sum_{i=1}^{n}\left[\alpha_i + \sum_{j=1}^{n}\beta_{ij}\log\left(p_j/M\right)\right]$, $\beta_{i.} = \sum_{j=1}^{n}\beta_{ij} = \sum_{i=1}^{n}\beta_{ij} = \beta_{.j}$ and $\beta_{..} = \sum_{i=1}^{n}\beta_{i.}$. In panel E, $a(\mathbf{p}) = \sum_{i=1}^{n}\alpha_i \log p_i + (1/2)\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\log p_i \cdot \log p_j = \log P^*$ and $b(\mathbf{p}) = \gamma_0 \Pi_{i=1}^{n} p_i^{\gamma_i}$.

TABLE 2

MILESTONES IN DEVELOPMENT OF THE ROTTERDAM AND RELATED MODELS OF CONSUMER DEMAND

Topic	Reference
Initial formulation of Rotterdam model	Barten (1964) "Consumer Demand Functions" Theil (1965) "Information Approach" Theil (1967) Economics and Information Theory
Estimation and testing	Barten (1967) "Evidence on Slutsky" Barten (1968) "Estimating Demand Equations" Barten and Geyskens (1975) "Negativity Condition"
Comprehensive presentation and extensions of model	Theil (1975/76) Theory and Measurement of Consumer Demand
More testing	Laitinen (1978) "Demand Homogeneity" Meisner (1979) "Slutsky Symmetry"
Cobb-Douglas Utility?	McFadden (1964) "Existence of Theil-Type Preferences"
Aggregation over consumers	Barnett (1979b) "Theoretical Foundations of Rotterdam Model" Selvanathan (1991) "Aggregation of Differential Demand Equations"
Introduction of labour supply	Barnett (1981) Consumer Demand and Labour Supply
Approximation properties of Rotterdam	Byron (1984) "Flexibility of Rotterdam" Barnett (1984) "Flexibility of Rotterdam" Mountain (1988) "Approximation in Variable Space"
Inverse Rotterdam model	Barten and Bettendorf (1989) "Inverse Demand System"
Cross-country consumption analysis	Clements and Theil (1979) "Cross-Country Consumption" Theil and Suhm (1981) International Consumption Comparisons Theil, Chung and Seale (1989) International Consumption Patterns Seale, Walker and Kim (1991) "Florida Model" Seale and Regmi (2006) "Modelling International Consumption"
Theory, econometric methodology and applications	Theil and Clements (1987) Applied Demand Analysis

Note: Items are listed in approximate chronological order and grouped by broad topic. Titles of works are abbreviated; for full details, see list of references at end.

Appendix

DERIVATIONS

The Linear Expenditure System

The ith equation of LES is

(A1) $$p_i q_i = p_i \gamma_i + \beta_i \left(M - \sum_{j=1}^{n} p_j \gamma_j \right),$$

or, dividing by income M,

(A2) $$w_i = s_i + \beta_i r,$$

where $w_i = p_i q_i / M$ is the budget share of goods i, $s_i = p_i \gamma_i / M$ is cost of subsistence of that good expressed as a fractions of income, and $r = \left(M - \sum_{j=1}^{n} p_j \gamma_j \right) / M$ is the supernumerary ratio.

The logarithmic differential of (A1) is

$$d(\log p_i) + d(\log q_i) = \frac{s_i}{w_i} d(\log p_i) + \frac{\beta_i}{w_i} d(\log M) - \frac{\beta_i}{w_i} \sum_{j=1}^{n} s_j d(\log p_j).$$

Multiplying both sides by w_i and collecting terms, we have

$$w_i d(\log q_i) = \beta_i d(\log M) + \sum_{j=1}^{n} \left[\delta_{ij}(s_i - w_i) - \beta_i s_j \right] d(\log p_j),$$

where δ_{ij} is the Kronecker delta. From (A2), $s_i - w_i = -\beta_i r$, so the term in square brackets above is $-\beta_i (r\delta_{ij} + s_j)$. Using the Divisia decomposition of the charge in money income into real income and price components, $d(\log M) = d(\log Q) + d(\log P)$, where $d(\log P) = \sum_{i=1}^{n} w_i d(\log p_i)$, the above then becomes

$$w_i d(\log q_i) = \beta_i \left[d(\log Q) + \sum_{j=1}^{n} w_j d(\log p_j) \right] - \beta_i \sum_{j=1}^{n} (r\delta_{ij} + s_j) d(\log p_j)$$

$$= \beta_i d(\log Q) + r\beta_i \sum_{j=1}^{n} (\beta_j - \delta_{ij}) d(\log p_j),$$

where the second step follows from (A2). Thus, LES can be expressed in term of the differential demand equation as $w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{i=1}^{n} \pi_{ij} d(\log p_j)$, with marginal share $\theta_i = \beta_i$, a constant, and Slutsky coefficients $\pi_{ij} = r\beta_i (\beta_j - \delta_{ij})$, variable. This verifies equation (5.4) of the text.

The utility function underlying LES is

(A3) $u(q_1,\ldots,q_n)=\sum_{i=1}^{n}\beta_i \log(q_i-\gamma_i)$,

with $u_i \equiv \partial u/\partial q_i = \beta_i/(q_i-\gamma_i)$ and $u_{ii} \equiv \partial^2 u/(\partial q_i)^2 = -\beta_i/(q_i-\gamma_i)^2$. Dividing both sides of (A1) by p_i gives $q_i=\gamma_i+(\beta_i/p_i)(M-\Sigma_{j=1}^{n} p_j\gamma_j)$, which, when substituted into (A3), gives the indirect utility function:

$$u_I(M,p_1,\ldots,p_n)=\sum_{i=1}^{n}\beta_i \log\left[\frac{\beta_i}{p_i}\left(M-\sum_{j=1}^{n}p_j\gamma_j\right)\right]$$

$$=\sum_{i=1}^{n}\beta_i\log\beta_i-\sum_{i=1}^{n}\beta_i\log p_i+\sum_{i=1}^{n}\beta_i\log\left(M-\sum_{j=1}^{n}p_j\gamma_j\right).$$

Thus, the marginal utility of income is $\lambda \equiv \partial u_I/\partial M = 1/(M-\Sigma_{j=1}^{n}p_j\gamma_j)$, with $\partial\lambda/\partial M=-1/(M-\Sigma_{j=1}^{n}p_j\gamma_j)^2$, so that

$$\frac{\partial\log\lambda}{\partial\log M}=\frac{-1}{\left(M-\sum_{j=1}^{n}p_j\gamma_j\right)^2}\cdot\frac{M}{1/\left(M-\sum_{j=1}^{n}p_j\gamma_j\right),}=\frac{-M}{\left(M-\sum_{j=1}^{n}p_j\gamma_j\right)}=-r^{-1}.$$

This shows that the income flexibility, $\phi=(\partial\log\lambda/\partial\log M)^{-1}$, in LES equals the negative of the supernumerary ratio.

For a utility maximum, each marginal utility is proportional to the corresponding price, with the marginal utility of income the proportionality factor, $u_i=\lambda p_i$, or $p_i=u_i/\lambda=(M-\Sigma_{j=1}^{n}p_j\gamma_j)\beta_i/(q_i-\gamma_i)$. Thus, the $(i,i)^{th}$ price coefficient associated with LES takes the form

$$\nu_{ii}=\lambda\frac{p_ip_i}{M}u^{ii}=-\frac{1}{(M-\Sigma_{j=1}^{n}p_j\gamma_j)}\frac{(M-\Sigma_{j=1}^{n}p_j\gamma_j)\beta_i}{(q_i-\gamma_i)}\frac{(M-\Sigma_{j=1}^{n}p_j\gamma_j)\beta_i}{(q_i-\gamma_i)}\frac{1}{M}\frac{(q_i-\gamma_i)^2}{\beta_i}=\phi\beta_i,$$

where $u^{ii}=1/u_{ii}$. As (A3) is additive, all second-order cross derivatives vanish, so that $\nu_{ij}=0, i\neq j$. This confirms equation (5.5) of the text.

<u>Quadratic Utility</u>

The quadratic utility function is $u(q_1,\ldots,q_n)=\mathbf{a'q}+1/2\cdot\mathbf{q'Uq}$, and the corresponding demand system is

(A4) $\mathbf{q}=-\mathbf{U}^{-1}\mathbf{a}+\dfrac{M+\mathbf{p'U}^{-1}\mathbf{a}}{\mathbf{p'U}^{-1}\mathbf{p}}\mathbf{U}^{-1}\mathbf{p}$

Substituting, the indirect utility function is

$$-\mathbf{a}'\mathbf{U}^{-1}\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{a}'\mathbf{U}^{-1}\mathbf{p}+\frac{1}{2}\left(-\mathbf{U}^{-1}\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{U}^{-1}\mathbf{p}\right)'\mathbf{U}\left(-\mathbf{U}^{-1}\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{U}^{-1}\mathbf{p}\right)$$

$$=-\mathbf{a}'\mathbf{U}^{-1}\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{a}'\mathbf{U}^{-1}\mathbf{p}+\frac{1}{2}\left(-\mathbf{a}'\mathbf{U}^{-1}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{p}'\mathbf{U}^{-1}\right)\left(-\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{p}\right)$$

$$=-\mathbf{a}'\mathbf{U}^{-1}\mathbf{a}+\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{a}'\mathbf{U}^{-1}\mathbf{p}+\frac{1}{2}\left(\mathbf{a}'\mathbf{U}^{-1}\mathbf{a}-\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{a}'\mathbf{U}^{-1}\mathbf{p}-\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}+\frac{\left(M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}\right)^2}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\right)$$

$$=-\frac{1}{2}\mathbf{a}'\mathbf{U}^{-1}\mathbf{a}+\frac{1}{2}\frac{\left(M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}\right)^2}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}.$$

The marginal utility of income is and the income flexibility are

(A5) $$\lambda=\frac{\partial u}{\partial M}=\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}},\qquad \phi=\left(\frac{\partial\log\lambda}{\partial\log M}\right)^{-1}=\frac{\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}+M}{M}.$$

The demand equation for good i can also be expressed as (see text):

(A6) $$q_i=\sum_{j=1}^{n}u^{ij}\left(\lambda p_j-a_j\right).$$

The total differential is $dq_i=\sum_{j=1}^{n}u^{ij}\left(\lambda dp_j+p_j d\lambda\right)$. Multiplying both sides by p_i/M, we have

(A7) $$w_i d\left(\log q_i\right)=\frac{\lambda p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\left(\log p_j\right)+\frac{p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\lambda.$$

From (A5), the differential of λ is

$$d\lambda=d\left(\frac{M+\mathbf{p}'\mathbf{U}^{-1}\mathbf{a}}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\right)$$
$$=\frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left(dM+\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}a_j dp_i\right)-\frac{2\lambda}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}p_i p_j d\left(\log p_j\right)$$
$$=\frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left[dM+\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}\left(a_i-2\lambda p_i\right)p_j d\left(\log p_j\right)\right]$$
$$=\frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left[dM-\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}\lambda p_i p_j d\left(\log p_j\right)-\sum_{j=1}^{n}p_j q_j d\left(\log p_j\right)\right],$$

where the last step follows from (A6).

Using the above expression for $d\lambda$, the right-hand side of (A7) becomes

$$\frac{\lambda p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\left(\log p_j\right)+\frac{p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\lambda$$

$$=\frac{\lambda p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\left(\log p_j\right)+\frac{p_i}{M}\sum_{j=1}^{n}u^{ij}p_j\frac{1}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left[dM-\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}\lambda p_i p_j d\left(\log p_j\right)-\sum_{j=1}^{n}p_j q_j d\left(\log p_j\right)\right]$$

$$=\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log M\right)+\frac{\lambda p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\left(\log p_j\right)-\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}\lambda\frac{p_i p_j}{M}d\left(\log p_j\right)+\sum_{j=1}^{n}w_j d\left(\log p_j\right)\right]$$

$$=\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log M\right)+\frac{\lambda p_i}{M}\sum_{j=1}^{n}u^{ij}p_j d\left(\log p_j\right)-\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}u^{ij}\lambda\frac{p_i p_j}{M}d\left(\log p_j\right)+d\left(\log P\right)\right]$$

$$=\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log Q\right)+\sum_{j=1}^{n}\left[\frac{\lambda u^{ij}p_i p_j}{M}-\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{k=1}^{n}u^{kj}\lambda\frac{p_k p_j}{M}\right]d\left(\log p_j\right).$$

The term in square brackets in the last line of the above can be simplified by noting that equation (A5) implies

(A8) $$\frac{\lambda}{M}=\frac{\phi}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}},$$

so that term becomes

$$\left[\frac{\lambda u^{ij}p_i p_j}{M}-\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{k=1}^{n}u^{kj}\lambda\frac{p_k p_j}{M}\right]=\frac{\lambda u^{ij}p_i p_j}{M}-\phi\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{k=1}^{n}\frac{u^{kj}p_k p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}$$

Equation (A7) can be formulated in absolute (or undeflated) prices as

$$w_i d\left(\log q_i\right)=\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log Q\right)+\sum_{j=1}^{n}\left[\frac{\lambda u^{ij}p_i p_j}{M}-\phi\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\cdot\frac{\sum_{k=1}^{n}u^{jk}p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\right]d\left(\log p_j\right).$$

This is the differential demand equation in absolute prices in the text.

To derive the relative price version, note that from equation (A8)

$$\frac{\lambda u^{ij}p_i p_j}{M}=\frac{\phi}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\cdot u^{ij}p_i p_j,$$

or, summing over we have $j=1,\ldots,n$,

$$\sum_{j=1}^{n}\frac{\lambda u^{ij}p_i p_j}{M}=\phi\frac{\sum_{j=1}^{n}u^{ij}p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}.$$

Using this, the substitution term in the last line of the equation above (A8) can be written as

$$\sum_{j=1}^{n}\left[\frac{\lambda u^{ij}p_i p_j}{M}-\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{k=1}^{n}u^{kj}\lambda\frac{p_k p_j}{M}\right]d\left(\log p_j\right)$$

$$=\sum_{j=1}^{n}\frac{\lambda u^{ij}p_i p_j}{M}d\left(\log p_j\right)-\phi\frac{\sum_{k=1}^{n}u^{ik}p_i p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{u^{kj}p_k p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log p_j\right)$$

$$=\sum_{j=1}^{n}\frac{\lambda u^{ij}p_i p_j}{M}\left[d\left(\log p_j\right)-\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{u^{kj}p_k p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}}d\left(\log p_j\right)\right].$$

Retracing our steps, equation (A7) becomes

$$w_i d\left(\log q_i\right) = \frac{\sum_{j=1}^{n} u^{ij} p_i p_j}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d\left(\log Q\right) + \sum_{j=1}^{n} \frac{\lambda u^{ij} p_i p_j}{M} \left[d\left(\log p_j\right) - \sum_{j=1}^{n} \frac{\sum_{k=1}^{n} u^{jk} p_j p_k}{\mathbf{p}'\mathbf{U}^{-1}\mathbf{p}} d\left(\log p_j\right) \right].$$

This is the differential demand equation for good i expressed in terms of relative prices that corresponds to quadratic utility.

<u>The Translog</u>

The translog system is equation (5.9):

$$w_i = \frac{\alpha_i + \sum_{j=1}^{n} \beta_{ij} \log\left(p_j/M\right)}{A}, \quad i=1,\ldots,n.$$

Writing this as $w_i = A_i/A$, the total differential is

$$dw_i = \frac{1}{A}\left(dA_i - w_i \sum_{k=1}^{n} dA_k \right),$$

Using $dA_i = \sum_{j=1}^{n} \beta_{ij}\, d\left(\log p_j/M\right)$, we have

$$\begin{aligned} dw_i &= \frac{1}{A}\left[\sum_{j=1}^{n} \beta_{ij}\, d\left(\log p_j\right) - \sum_{j=1}^{n} \beta_{ij}\, d\left(\log M\right) - w_i \sum_{k=1}^{n} \left(\sum_{j=1}^{n} \beta_{kj}\, d\left(\log p_j\right) - \sum_{j=1}^{n} \beta_{kj}\, d\left(\log M\right) \right) \right] \\ &= \frac{1}{A} \sum_{j=1}^{n} \left(\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj} \right) d\left(\log p_j\right) - \frac{1}{A} \sum_{j=1}^{n} \left(\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj} \right) d\left(\log M\right) \\ &= \left(\frac{w_i \beta_{..} - \beta_{i.}}{A} \right) d\left(\log M\right) + \frac{1}{A} \sum_{j=1}^{n} \left(\beta_{ij} - w_i \beta_{.j} \right) d\left(\log p_j\right) \end{aligned}$$

where $\beta_{i.} = \sum_{j=1}^{n} \beta_{ij}$, $\beta_{.j} = \sum_{k=1}^{n} \beta_{kj}$ and $\beta_{..} = \sum_{i=1}^{n} \beta_{i.}$.

Next, combine the above expression for dw_i with the differential of the budget share, $dw_i = w_i d\left(\log q_i\right) + w_i d\left(\log p_i\right) - w_i d\left(\log M\right)$, to give:

$$w_i d\left(\log q_i\right) = \left(\frac{w_i \beta_{..} - \beta_{i.}}{A} + w_i \right) d\left(\log M\right) + \frac{1}{A} \sum_{j=1}^{n} \left(-A w_i \delta_{ij} + \beta_{ij} - w_i \beta_{.j} \right) d\left(\log p_j\right),$$

As $d\left(\log M\right) = d\left(\log P\right) + d\left(\log Q\right)$, with $d\left(\log P\right) = \sum_{j=1}^{n} w_j d\left(\log p_j\right)$ and $d\left(\log Q\right) = \sum_{j=1}^{n} w_j d\left(\log q_j\right)$, the above equation becomes

(A9) $$\begin{cases} w_i d\left(\log q_i\right) = \theta_i\, d\left(\log Q\right) + \sum_{j=1}^{n} \left[\theta_i w_j - w_i \delta_{ij} + \dfrac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A} \right] d\left(\log p_j\right) \\ \qquad\qquad\qquad = \theta_i d\left(\log Q\right) + \sum_{j=1}^{n} \pi_{ij} d\left(\log p_j\right). \end{cases}$$

Here, the marginal shares and the Slutsky coefficients take the form

$$\theta_i = \frac{w_i \beta_{..} - \beta_{i.}}{A} + w_i, \qquad \pi_{ij} = \theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A}.$$

To obtain the differential form of the translog in relative prices, add and subtract $\phi \theta_i \sum_{j=1}^{n} \theta_j\, d\left(\log p_j\right)$ from the right-hand side of the first equation in (A9):

$$w_i d(\log q_i) = \theta_i\, d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j) + \phi\theta_i \sum_{j=1}^{n} \theta_j\, d(\log p_j) - \phi\theta_i \sum_{j=1}^{n} \theta_j\, d(\log p_j)$$

$$= \theta_i\, d(\log Q) + \sum_{j=1}^{n} \left(\pi_{ij} + \phi\theta_i\theta_j\right) d(\log p_j) - \phi\theta_i\, d(\log P')$$

$$= \theta_i\, d(\log Q) + \sum_{j=1}^{n} \left(\pi_{ij} + \phi\theta_i\theta_j\right) d(\log p_j) - \sum_{j=1}^{n} \left(\pi_{ij} + \phi\theta_i\theta_j\right) d(\log P')$$

$$= \theta_i\, d(\log Q) + \sum_{j=1}^{n} \left(\pi_{ij} + \phi\theta_i\theta_j\right) \left[d(\log p_j) - d(\log P') \right]$$

$$= \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij} \left[d(\log p_j) - d(\log P') \right].$$

The third line of the above follows from equations (4.6) and (4.7) in the form $\sum_{j=1}^{n}\left(\pi_{ij} + \phi\theta_i\theta_j\right) = \phi\theta_i$. The $(i, j)^{th}$ price coefficient in the above takes the form

$$\nu_{ij} = \theta_i w_j - w_i \delta_{ij} + \frac{\beta_{ij} - w_i \sum_{k=1}^{n} \beta_{kj}}{A} + \phi\theta_i\theta_j.$$

<u>The Almost Ideal Demand System (AIDS)</u>

According to equation (5.11), the i^{th} equation of AIDS is

$$w_i = \alpha_i + \gamma_i \log\left(\frac{M}{P^*}\right) + \sum_{j=1}^{n} \beta_{ij} \log p_j,$$

so that $dw_i = \gamma_i\, d(\log M) - \gamma_i\, d(\log P^*) + \sum_{j=1}^{n} \beta_{ij}\, d(\log p_j)$. Subtracting from both sides $w_i d(\log p_i) - w_i d(\log M)$ gives the quantity component of the change in the share:

$$w_i d(\log q_i) = (\gamma_i + w_i) d(\log M) - \gamma_i\, d(\log P^*) + \sum_{j=1}^{n} \left(\beta_{ij} - w_i \delta_{ij}\right) d(\log p_j).$$

Substituting $d(\log P^*) = \sum_{i=1}^{n} \alpha_i\, d(\log p_i) + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} d(\log p_i) \cdot d(\log p_j)$, and replacing $d(\log M)$ by $d(\log P) + d(\log Q)$, where $d(\log P) = \sum_{j=1}^{n} w_j d \log p_j$, gives

$$w_i d(\log q_i) = (\gamma_i + w_i) d(\log Q) + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + (w_i + \gamma_i) w_j - \gamma_i \alpha_j - \frac{\gamma_i}{2} \sum_{k=1}^{n} \left(\beta_{kj} + \beta_{jk}\right) d(\log p_k) \right] d(\log p_j).$$

$$= (\gamma_i + w_i) d(\log Q) + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + (w_i + \gamma_i) w_j - \gamma_i \alpha_j - \gamma_i \sum_{k=1}^{n} \beta_{kj} d(\log p_k) \right] d(\log p_j),$$

since $\beta_{kj} = \beta_{jk}$. It follows from equation (5.11) that $\alpha_i + \sum_{j=1}^{n} \beta_{ij} \log p_j = w_i - \gamma_i \log\left(M/P^*\right)$, so the substitution term in the above becomes

$$\sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + (w_i + \gamma_i) w_j - \gamma_i \alpha_j - \gamma_i \sum_{k=1}^{n} \beta_{kj} d(\log p_k) \right] d(\log p_j)$$

$$= \sum_{j=1}^{n} \left\{ \beta_{ij} - \delta_{ij} w_i + (w_i + \gamma_i) w_j - \gamma_i \left[w_j - \gamma_j \log\left(M/P^*\right) \right] \right\} d(\log p_j)$$

$$= \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i \gamma_j \log\left(M/P^*\right) \right] d(\log p_j).$$

Thus, the differential version of AIDS, in absolute prices, is

$$
\text{(A10)} \quad \begin{cases} w_i d(\log q_i) = (\gamma_i + w_i) d \log Q + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i \gamma_j \log\left(\frac{M}{P^*}\right) \right] d(\log p_j) \\ \qquad = \theta_i d(\log Q) + \sum_{j=1}^{n} \pi_{ij} d(\log p_j), \end{cases}
$$

where $\pi_{ij} = \beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i \gamma_j \log(M/P^*)$ are the implied Slutsky coefficients. The above is equation (5.12) of the text.

As noted in the previous section, $\sum_{j=1}^{n} (\pi_{ij} + \phi\theta_i\theta_j) = \phi\theta_i$. Thus, adding and then subtracting $\phi\theta_i \sum_{j=1}^{n} \theta_j d(\log p_j)$ from the right-hand side of the first equation in (A10) gives the relative price version of AIDS in differential form:

$$
\begin{aligned}
w_i d(\log q_i) &= (\gamma_i + w_i) d(\log Q) \\
&\quad + \sum_{j=1}^{n} \left[\beta_{ij} - \delta_{ij} w_i + w_i w_j + \gamma_i \gamma_j \log\left(\frac{M}{P^*}\right) - (w_i + \gamma_i)(w_j + \gamma_j) \right] \left[d(\log p_j) - d\log P' \right] \\
&= \theta_i d(\log Q) + \sum_{j=1}^{n} \nu_{ij} \left[d(\log p_j) - d(\log P') \right],
\end{aligned}
$$

where the $(i, j)^{th}$ price coefficient is $\nu_{ij} = \beta_{ij} + w_i (w_j - \delta_{ij}) + \gamma_i \gamma_j \log(M/P^*) - \theta_i \theta_j$. This is equation (5.13) of the text.

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