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## **ECONOMICS**

# **BUFFER EFFECT AND PRICE EFFECT OF A PERSONAL CARBON TRADING SCHEME**

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**DISCUSSION PAPER 15.07**

# BUFFER EFFECT AND PRICE EFFECT OF A PERSONAL CARBON TRADING SCHEME

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## DISCUSSION PAPER 15.07

### ABSTRACT:

Personal carbon trading (PCT) is a downstream cap-and-trade scheme used to reduce carbon emissions from the household sector. It is argued that the PCT scheme could provide a buffer between the energy price and the total energy price, and thus energy demand remains stable. However these effects have never been verified. To fill in this gap in the literature, a price effect analysis is conducted. Firstly, a general utility optimization (GUO) model is proposed to obtain the general formulae of the price effect, substitution effect and income effect under the PCT scheme. Secondly, a specific version of the GUO model, namely a Cobb-Douglas utility function model, is employed to obtain the specific effect formulae to verify the buffer effect. Finally, a numerical example and a sensitive analysis are presented to demonstrate these effects. The results indicate that, under the PCT scheme, the total energy price and energy demand are less sensitive to the energy price changes. Thus, when energy prices fluctuate, the PCT scheme is capable of providing certainty in emissions reduction and is more effective than carbon taxes. On the basis of these results, implications of this research are discussed and suggestions for future research are provided.

**Keywords:** Personal Carbon Trading; Buffer effect; Price effect; Energy demand; Energy price; Allowance price

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## **1. Introduction**

Carbon emissions caused by household energy consumption have become a major source of total emissions and have attracted worldwide attention (Jaehn and Letmathe, 2010; Rout et al., 2011; Feng et al., 2011). For instance, at present about 30% of carbon emissions in China are generated in the household sector (NBSC, 2013). There is no doubt that these emissions will continue to grow due to the rapid economic growth and urbanization in the coming decades (Liu et al., 2011; Rout et al., 2011; Fan et al., 2012).

Personal carbon trading (PCT) has been proposed as a radical policy to start translating the consumption perspective into carbon reduction practice at the household level (Fleming, 1996). It is a downstream cap-and-trade scheme which could be used to reduce carbon emissions from the household sector, and which is analogous to the upstream emissions trading system operating in the industrial sector (Harwatt et al., 2011; Roberts and Thumin, 2006; Fleming, 1996). In fact, there are links between these two schemes. In an upstream carbon trading scheme (such as the EU Emission Trading Scheme), generators surrender allowances for the carbon contained in the electricity sales. In a PCT scheme, consumers surrender allowances for their electricity consumption. These two trading schemes will interfere with each other and produce two interaction results: double regulation and double counting (Sorrell, 2010). Namely, consumers would face two sets of carbon prices (i.e. the EU ETS price and the PCT price) and they would also surrender two separate carbon allowances (Sorrell and Sijm, 2003). Establishing links between the upstream trading scheme and PCT scheme will improve liquidity and reduce the risk of participants using their market power to influence the allowance prices (Sorrell, 2010). At the same time, to avoid double regulation and double counting, the PCT scheme would need to ensure that the energy and fuel purchased by consumers would not include the price of the carbon allowance in the upstream trading scheme (Sorrell, 2010).

Under the PCT scheme, emission allowances are allocated equally and freely to each adult by the government and the allowances allocation is reduced year by year

(Harwatt et al., 2011; Fawcett et al., 2007; Meyer, 2000). However, taking into account that some low income consumers with greater energy need may use older and energy-intensive appliances, for instance living in lower insulated house, a supporting system is proposed to address this equity issue. Bristow et al. (2010) argued that equity is addressed through a supporting system with higher allocations or financial support to those with greater need. This system is similar to the Clean Development Mechanism (CDM) foreseen in Kyoto Protocol to close the gap between the low economical class and middle/high class. Carbon allowance trading among consumers is the key feature of the PCT scheme. In this scheme, the over-emitters (allowance buyers) who emit more than their initial allowances have to buy permits in the market from the under-emitters (allowance sellers) who emit less than their initial allowances (Cohen, 2011). Although no one is compelled to sell or buy allowances in this system, one's utility would be improved if one chooses to do so.

Compared with an upstream carbon trading scheme, PCT is a progressive scheme in which the poorer consumers are mostly 'winners', as their levels of emissions are generally lower. To some extent, the capacity to reduce carbon emissions under the PCT scheme is much larger. Weitzman et al. (1974) noted that carbon taxes and tradable permits are theoretically equivalent in terms of efficiency and effectiveness. However, a carbon tax policy is mainly a price-based environmental regulation which fixes the allowance price and lets the market determine the amount of carbon emissions emitted, whilst a PCT scheme is mainly a quantity-based one which fixes the quantity emitted and lets the allowance price to be determined by the market. The emission cap under the PCT scheme can directly affect the carbon reduction targets. If there is uncertainty over the cost function, it is better to fix the price through a tax policy, and if there is uncertainty over the damage function, fixing the quantity through a tradable system is more appropriate (Pizer, 2002; Stranlund and Ben-Haim, 2008).

PCT has attracted widespread attention in both research and policy domains. In 2010, the *Climate Policy* journal published a special issue on PCT with 10 articles. Most research on PCT focuses on equity, effectiveness, public acceptability, barriers

to implementation and its potential advantages over the existing climate policy instruments, such as upstream “cap and trade” systems and carbon taxes (Harwatt, 2008; Jagers, 2010; Eyre, 2010; Bristow et al., 2010; Parag and Eyre, 2010; Fawcett et al., 2007; Wallace et al., 2010; Sorrell, 2010; Parag et al., 2011; Wadud, 2011; Starkey, 2012). Compared with a simple tax system the costs of PCT, which include implementation cost, participation cost and transaction cost, may be higher (Fawcett, 2010). However, the short-term elasticities of demand for vehicle fuel and domestic energy are low, which means that the effect of a carbon tax will be fairly limited (Lockwood, 2010). At the same time, the attitude of the public is another factor that needs to be considered. It is argued that consumers are more likely to oppose the carbon tax and accept the PCT (Harwatt, 2008).

Lane et al. (2008) argued that the allowance price significantly influences energy price and furthermore affects energy demand and associated lifestyles. Wadud (2011) noted that one of the advantages of a personal tradable permit approach over the carbon tax is the buffer it provides when the price of oil in the world market changes suddenly. When the price of oil increases, the price of the permits would fall. Then the total amount (price of the permits + price of pre-tax gasoline) paid by consumers would remain stable, and thus the gasoline demand would also remain stable. However this conclusion is based on qualitative judgments. A more rigorous investigation is thus required.

The buffer effect would influence energy demand under the PCT scheme. Wadud’s (2007) argued that the buffer effect would help stabilise energy demand. However, this conclusion does not take into account the allowance price change and its further influence on consumers’ full budget (money budget plus the product of the allowance price and initial allowance allocation). Due to the buffer effect, an increase of the energy price would lead to the fall in the allowance price and hence a decrease in the full budget (Varian, 1992). Therefore energy demand would become uncertain.

To illustrate this, a price effect (PE) analysis might be appropriate. According to economic theory when consumer’s income is constant, changes in commodity prices will lead to changes in quantity demanded (Klein and Rubin, 1947; Varian, 1992).

This is known as the price effect. The price effect is determined by the substitution effect (SE) and income effect (IE) (William, 1973; Neary and Roberts, 1980). On the one hand, changes in commodity prices will lead to relative changes in other prices, and thus lead to changes in the quantities demanded. This phenomenon is known as the substitution effect. On the other hand, changes in commodity prices will also lead to changes in real income levels, and thus lead to changes in quantities demanded. This phenomenon is called the income effect (William, 1973). For different commodities (normal goods, inferior goods and Giffen goods), the influences of price effect, substitution effect and income effect on quantities demanded are quite different (Varian, 1992).

However, whether there is a buffer effect or not remains a question. It is also unclear whether or not energy demand is stable under the PCT scheme. In the following sections, nonlinear optimization models will be proposed to explore these questions. The remainder of this paper is organized as follows. Section 2 introduces a general utility optimization (GUO) model to obtain the general formulae of the price effect, substitution effect and income effect under the PCT scheme. Section 3 presents a specific version of the GUO model and a Cobb-Douglas utility function model is employed to obtain the specific formulae of these effects under the with-PCT and without-PCT scenarios. Section 4 describes the data calibration results. Finally Section 5 concludes the paper and points out the implications and limitations of the results.

## **2. A GUO model under the PCT scheme**

We assume that commodities can be classified into two categories, namely energy commodities  $X$  and non-energy commodities  $Y$ . The general utility function can be expressed as  $U = U(X, Y)$ . Given that consumers need to pay money as well as surrender certain carbon allowances when they purchase energy, their utility optimization is subject to two constraints, namely, the money budget and the carbon allowance allocation (Absi et al., 2013). Furthermore the rationale for the allocation

of these allowances is critical in the design of the PCT scheme. Most authors have focused on an equal per capita allowance allocation which is based on the argument that everyone has equal rights to the environment (Pezzey, 2003; Wadud, 2011). For example, Harwatt et al. (2011) noted that the initial carbon allowance could be allocated equally to individuals free of charge. Thus, we assume an equal initial allowance  $\omega$  for all consumers.

Under the PCT scheme, consumers should solve the following utility maximization problem.

$$\begin{aligned} \text{Max } U &= U(X, Y) \\ \text{s.t. } &\begin{cases} p'X + q'Y + p_c\psi \leq I \\ c_x'X - \psi \leq \omega \end{cases} \end{aligned} \quad (1)$$

where  $X = (x_1, x_2, \dots, x_m)^T$  is the energy consumption vector and  $Y = (y_1, y_2, \dots, y_n)^T$  is the non-energy consumption vector.  $p' = (p_1, p_2, \dots, p_m)$  is the energy price vector and  $q' = (q_1, q_2, \dots, q_n)$  is the non-energy commodity price vector.  $c_x' = (c_{x_1}, c_{x_2}, \dots, c_{x_m})$  is the vector of energy emission rates.  $p_c$  is the carbon allowance price.  $\psi$  is the allowances bought (or sold) by consumers.  $I$  is the money budget. According to the constraint conditions of equation (1), we have

$$(p' + p_c c_x')X + q'Y = I + p_c \omega \quad (2)$$

Let  $P = p' + p_c c_x'$  and  $Y' = I + p_c \omega$ , and thus we have

$$PX + q'Y = Y' \quad (3)$$

where  $P$  is the total price vector of energy and  $Y'$  is the full budget. We define equation (3) as a full price constraint.

According to equation (1), we define the indirect utility function,  $v(P, q', Y')$ , as the highest level of utility the consumer could reach given prices  $P$  and  $q'$ , and budget  $Y'$ . According to the duality theorem (Yano, 2012), we have

$$X_i(P, q', Y') = h_i \left[ P, v(P, q', Y') \right] \quad (4)$$

and hence the partial derivative of equation (4) with respect to  $Y'$  and  $p_i$

$$X_{iY'}(P, q', Y') = h_{iv} v_{Y'} \quad (5)$$

$$X_{ip_i}(P, q', Y') = h_{ii} \left(1 + \frac{\partial p_c}{\partial p_i} c_{xi}'\right) + h_{iv} \left[ v_{P_i} \left(1 + \frac{\partial p_c}{\partial p_i} c_i\right) + v_{Y'} \frac{\partial Y'}{\partial p_c} \frac{\partial p_c}{\partial p_i} \right] \quad (6)$$

According to Roy's identity (Cowan, 2012), we obtain

$$\frac{\partial v}{\partial P_i} = v_{P_i} = -v_{Y'} X_i \quad (7)$$

Substituting equations (5) and (7) into equation (6), we have

$$X_{ip_i}(P, q', Y') = h_{ii} \left(1 + \frac{\partial p_c}{\partial p_i} c_i\right) - X_{iY'}(P, q', Y') \left[ X_i \left(1 + \frac{\partial p_c}{\partial p_i} c_i\right) - \frac{\partial Y'}{\partial p_c} \frac{\partial p_c}{\partial p_i} \right] \quad (8)$$

According to the definition of price effect, we know that  $X_{ip_i}(P, q', Y')$  is regarded as the price effect. Since  $PE = SE + IE$ , the substitution effect and the income effect can be represented as

$$SE = h_{ii} \left(1 + \frac{\partial p_c}{\partial p_i} c_i\right) \quad (9)$$

$$IE = -X_{iY'}(P, q', Y') \left[ X_i \left(1 + \frac{\partial p_c}{\partial p_i} c_i\right) - \frac{\partial Y'}{\partial p_c} \frac{\partial p_c}{\partial p_i} \right] \quad (10)$$

### 3. A specific utility function model of GUO model

#### 3.1. Scenarios under the PCT

To calculate the price effect, income effect and substitution effect, the Marshallian demand function and the Hicksian demand function under the PCT scheme are considered first.

##### 3.1.1. The Marshallian demand function

For simplicity, we assume that there is only one energy commodity  $x$  and one non-energy commodity  $y$  in our model. In a perfectly competitive market, to



determine the purchases of energy and non-energy commodities and the trading of allowances, the utility maximization for both under-emitters and over-emitters should be achieved simultaneously. Heffetz (2007) noted that the consumer choice models with the Cobb-Douglas utility function are simple and tractable, generating clear and testable empirical predictions. Based on the same reason, we also use a Cobb-Douglas utility function  $x^\alpha y^\beta$  ( $\alpha$  and  $\beta$  are the exponents of utility function and  $\alpha + \beta = 1$ ) as our objective function (Rosenzweig and Schultz, 1983).

We assume that there are  $m$  buyers and  $n$  sellers in the trading market. To obtain the equilibrium price for allowances, we need to know the market demand and supply curves. We know that the market demand and supply curves are based on the individual demand and supply curves. Firstly we need to obtain the demand and supply curves of over-emitter (buyer)  $i$  and the under-emitter (seller)  $j$ . Under the scheme, emission allowances could be tradable so that the under-emitter  $j$  could sell redundant allowances  $\psi_j$  to gain extra income, whereas the over-emitter  $i$  would purchase allowances  $\psi_i$  at the market prices  $p_c$ .

Based on equation (1), the utility maximization problem consumers faced can be transformed to the following form:

$$\begin{aligned} \text{Max } U(x, y) &= x^\alpha y^\beta \\ \text{s.t. } \begin{cases} px + qy + p_c \psi \leq I & (\text{shadow price } \partial_1) \\ c_x x - \psi \leq \omega & (\text{shadow price } \partial_2) \end{cases} \end{aligned} \quad (\text{A0})$$

The shadow prices  $\partial_1$  and  $\partial_2$  are defined as the marginal changes in the objective function with respect to an increase in the right-hand side of the constraint conditions. According to Hobbs et al. (2010), the constraints in equation (A0) imply that the shadow prices are positive. Solving the problem in equation (A0) involves a linear program whose Karush-Kuhn-Tucker (KKT) optimality conditions are shown

in Appendix A.

According to the KKT conditions we have the optimization results of allowance sales, energy demand, and non-energy commodity demand as follows:

$$\psi = \frac{\alpha c_x (I + p_c \omega)}{p + p_c c_x} - \omega = \frac{\alpha c_x Y}{P} - \omega \quad (11)$$

$$x = \frac{\psi + \omega}{c_x} = \alpha \frac{I + p_c \omega}{p + p_c c_x} = \alpha \frac{Y}{P} \quad (12)$$

$$y = \frac{-px - p_c \psi + I}{q} = \beta \frac{I + p_c \omega}{q} = \beta \frac{Y}{q} \quad (13)$$

Thus, the indirect utility function is expressed as

$$v(P, q, Y) = x^\alpha y^\beta = \left(\alpha \frac{Y}{P}\right)^\alpha \left(\beta \frac{Y}{q}\right)^\beta = \frac{\beta}{q} \left(\frac{\alpha q}{\beta P}\right)^\alpha Y \quad (14)$$

In addition, according to equation (11), the allowances purchased by the over-emitter and sold by the under-emitter can be expressed as follows:

$$\psi_i = \frac{\alpha c_x I_i - p\omega - \beta \omega c_x p_c}{p_c c_x + p} \quad (15)$$

$$\psi_j = \frac{-\alpha c_x I_j + p\omega + \beta \omega c_x p_c}{p_c c_x + p} \quad (16)$$

As there are  $m$  buyers and  $n$  sellers in the trading market, the allowance price is determined by the market clearing conditions at which demand must be equal to supply (Lane et al., 2008; Benz and Trück, 2009; Fan et al., 2014), that is,

$$\sum_{i=1}^m \psi_i = \sum_{j=1}^n \psi_j \quad (17)$$

Then the equilibrium price for allowances is

$$p_c = \frac{\alpha \left( \sum_{i=1}^m I_i + \sum_{j=1}^n I_j \right)}{\omega \beta (m+n)} - \frac{p}{\beta C_x} \quad (18)$$

According to equations (A7), (15), (16) and (18), we obtain the Marshallian demand functions as follows:

$$x_i = \left( \frac{\beta c_x (I_i - I_j)}{c_x (I_i + I_j) - 2p\omega} + 1 \right) \cdot \frac{\omega}{c_x} \quad (19)$$

$$x_j = \left( \frac{\beta c_x (I_j - I_i)}{c_x (I_i + I_j) - 2p\omega} + 1 \right) \cdot \frac{\omega}{c_x} \quad (20)$$

According to the Slutsky's equation and the definition of price effect (Varian, 1992), the price effects of the over-emitter and the under-emitter are

$$PE_i = \frac{\alpha(\alpha c_x I_i - p\omega - \beta\omega c_x p_c)}{\beta C_x (p_c c_x + p)^2} \quad (21)$$

$$PE_j = \frac{\alpha(-\alpha c_x I_j + p\omega + \beta\omega c_x p_c)}{\beta C_x (p_c c_x + p)^2} \quad (22)$$

### 3.1.2. The Hicksian demand function

To calculate the substitution effect, we need to obtain the Hicksian demand function. The optimization model for consumers can be represented as follows:

$$\begin{aligned} \text{Min } C(x, y) &= px + qy + p_c \psi \\ \text{s.t. } &\begin{cases} -x^\alpha y^\beta \leq -U(\text{shadow price } \phi_1) \\ c_x x - \psi \leq \omega(\text{shadow price } \phi_2) \end{cases} \end{aligned} \quad (B0)$$

The KKT optimality conditions for the inequality constrained optimization problems are shown in Appendix B. According to the KKT conditions we obtain the Hicksian demand function which can be expressed as (The detailed derivation is shown in Appendix B)

$$x_h(P, q, U) = \frac{U(\alpha q)^\beta}{[\beta(p + c_x p_c)]^\beta} = \frac{U(\alpha q)^\beta}{(\beta P)^\beta} \quad (23)$$

Therefore, the Hicksian demand function for the over-emitter is

$$x_{hi} = \frac{U_i(\alpha q)^\beta}{(\beta P)^\beta} \quad (24)$$

The Hicksian demand function for the under-emitter is

$$x_{hj} = \frac{U_j(\alpha q)^\beta}{(\beta P)^\beta} \quad (25)$$

### 3.1.3. The substitution effect and the income effect

Based on equations (9), (18) and (24), the substitution effect of the over-emitter is represented as

$$SE_i = -\beta \frac{U_i(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1} \left(1 - \frac{1}{\beta}\right) = \alpha \frac{U_i(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1} \quad (26)$$

Similarly, based on equations (9), (18) and (25), the substitution effect of the under-emitter is represented as

$$SE_j = -\beta \frac{U_j(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1} \left(1 - \frac{1}{\beta}\right) = \alpha \frac{U_j(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1} \quad (27)$$

According to equations (10), (12) and (18), the income effect of the over-emitter is represented as

$$IE_i = -\frac{\alpha}{P} \left[ x_i \left(1 - \frac{1}{\beta}\right) + \frac{\omega}{\beta c_x} \right] \quad (28)$$

Similarly, the income effect of the under-emitter is represented as

$$IE_j = -\frac{\alpha}{P} \left[ x_j \left(1 - \frac{1}{\beta}\right) + \frac{\omega}{\beta c_x} \right]$$

(29)

The price effect, substitution effect and income effect under the PCT scenarios are shown in Table 1.

**Table 1**

The price effect, substitution effect and income effect (with-PCT scenarios)

Effects	<i>PE</i>	<i>SE</i>	<i>IE</i>
<b>Consumers</b>			
Over-emitter	$\frac{\alpha(\alpha c_x I_i - p\omega - \beta\omega c_x p_c)}{\beta C_x (p_c c_x + p)^2}$	$\alpha \frac{U_i(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1}$	$-\frac{\alpha}{P} \left[ x_i \left(1 - \frac{1}{\beta}\right) + \frac{\omega}{\beta c_x} \right]$
Under-emitter	$\frac{\alpha(-\alpha c_x I_j + p\omega + \beta\omega c_x p_c)}{\beta C_x (p_c c_x + p)^2}$	$\alpha \frac{U_j(\alpha q)^\beta}{(\beta)^\beta} P^{-\beta-1}$	$-\frac{\alpha}{P} \left[ x_j \left(1 - \frac{1}{\beta}\right) + \frac{\omega}{\beta c_x} \right]$

Source: Authors' own calculation

Table 1 implies that the substitution effects are always positive for the consumers. This is contrary to the traditional economic theory of consumer choice (Varian, 1992). However the sign and size of the price effect and income effect are uncertain and need further investigation.

### 3.2. Scenarios without PCT

Without the PCT scheme, emission allowances could not be tradable between the consumers. The allowance price is  $p_c = 0$ . According to equations (12) and (13), the energy and non-energy commodities demanded of the over-emitter and under-emitter are represented as

$$x'_i = \alpha \frac{I_i + p_c \omega}{p + p_c c_x} = \alpha \frac{I_i}{p} \quad (30)$$

$$y'_i = \beta \frac{I_i + p_c \omega}{q} = \beta \frac{I_i}{q} \quad (31)$$

$$x'_j = \alpha \frac{I_j + p_c \omega}{p + p_c c_x} = \alpha \frac{I_j}{p} \quad (32)$$

$$y'_j = \beta \frac{I_j + p_c \omega}{q} = \beta \frac{I_j}{q} \quad (33)$$

Hence, the price effects can be expressed as

$$PE'_i = \frac{dx'_i}{dp} = -\alpha I_i p^{-2} \quad (34)$$

$$PE'_j = \frac{dx'_j}{dp} = -\alpha I_j p^{-2} \quad (35)$$

Based on equations (24) and (25), the Hicksian demand functions are

$$x'_{hi} = \frac{U'_i (\alpha q)^\beta}{(\beta p)^\beta} \quad (36)$$

$$x'_{hj} = \frac{U'_j (\alpha q)^\beta}{(\beta p)^\beta} \quad (37)$$

where  $U'_i = (x'_i)^\alpha (y'_i)^\beta$  and  $U'_j = (x'_j)^\alpha (y'_j)^\beta$ .

Therefore, based on equations (26) and (27), the substitution effects are

$$SE'_i = -\beta \frac{U'_i (\alpha q)^\beta}{(\beta)^\beta} p^{-\beta-1} \quad (38)$$

$$SE'_j = -\beta \frac{U'_j (\alpha q)^\beta}{(\beta)^\beta} p^{-\beta-1} \quad (39)$$

Based on equations (28) and (29), the income effects are represented as

$$IE'_i = -\frac{\alpha}{p} x'_i \quad (40)$$

$$IE'_j = -\frac{\alpha}{p} x'_j \quad (41)$$

The price effect, substitution effect and income effect under the scenarios without-PCT are shown in Table 2.

**Table 2**

The total effect, substitution effect and income effect (without-PCT)

Effects	$PE'$	$SE'$	$IE'$
Consumers			
Over-emitter	$-\alpha I_i p^{-2}$	$-\beta \frac{U'_i (\alpha q)^\beta}{(\beta)^\beta} p^{-\beta-1}$	$-\frac{\alpha}{p} x'_i$
Under-emitter	$-\alpha I_j p^{-2}$	$-\beta \frac{U'_j (\alpha q)^\beta}{(\beta)^\beta} p^{-\beta-1}$	$-\frac{\alpha}{p} x'_j$

Source: Authors' own calculation

Table 2 implies that the price effects, substitution effects and income effects are always negative for consumers. These are consistent with traditional economic theory and it can be concluded that energy is a normal good under the scenarios without-PCT (Varian, 1992). In the following section, a numerical example will be employed to further illustrate the relationship between price effect, substitution effect and income effect and their influences on energy demand.

## 4. Data Calibration

In this section the influence of the buffer effect on the energy price is analyzed through numerical simulation. For this purpose, some model parameters are based on Chinese statistics. In China major energy sources include coal, petroleum and natural gas (Huang and Yan, 2009). For simplicity we take gasoline as an example to conduct the data calibration. The current price of gasoline is about \$4.46/gallon and the emission rate of gasoline is about 9.84 kg/gallon in China.<sup>1</sup> In this paper, we specify that the energy price ranges from \$4.50/gallon to \$9.00/gallon and the emission rate is  $c_x = 10.00$  kg/gallon. According to the distribution of per capita disposable income in China, we specify the consumption budget as  $I_i = \$12,000$  and  $I_j = \$4,500$ , and the exponents of the utility function as  $\alpha = 0.10$  and  $\beta = 0.90$  (NBSC, 2013). Other parameters, such as the price of non-energy commodities and the initial allowance were specified as  $q = \$1.00$ /unit and  $\omega = 800$  (kg/year). In addition, we assume that there are 400 buyers and 500 sellers in the trading market, namely  $m=400$  and  $n=500$ .

In addition the system's fixed operational cost  $\eta$ , which includes audit, verification and reporting requirements, should be considered in the PCT scheme. This is because the portion of this cost as part of the household annual consumption cost is expected to be higher than that of the industrial sector in relation to its turnover. This cost has a significant influence on the PCT scheme (Fawcett, 2010). Based on Harwatt et al. (2011) and the statistics of OECD<sup>2</sup>, we specify the fixed operational cost  $\eta$  as \$0.05/gallon. Thus, the total energy price of gasoline can be represented as  $P = p + p_c c_x + \eta = p + p_c c_x + 0.05$ . In section 4.1 to section 4.3, five propositions will be proposed. These propositions are concerned with the relationships between the energy price, allowance price, total energy price and total energy demand.

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<sup>1</sup> See detail at <http://www.cngold.org/crude/qiyou.html> and [http://urbanian.org/infor\\_news.asp?sid=34&nid=35&lid=79&id=320](http://urbanian.org/infor_news.asp?sid=34&nid=35&lid=79&id=320)

<sup>2</sup> See detail at <http://www.oecd.org/statistics/>

## 4.1. Buffer effect and price effect under the PCT scheme

### 4.1.1. Buffer effect

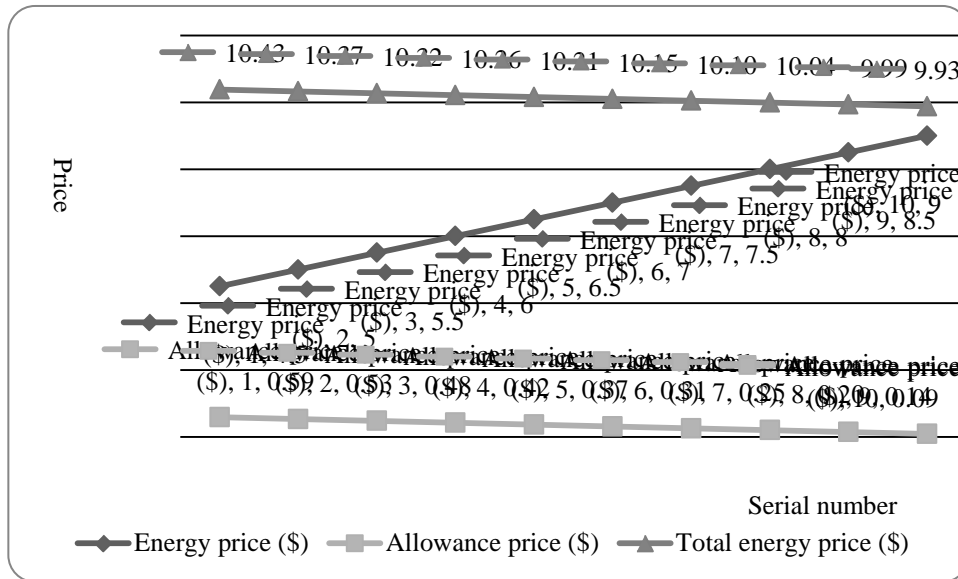
As mentioned above, the buffer effect has been proposed by researchers but has never been verified. To fill in this gap, in this section we will propose several propositions and prove them using our models.

**Proposition 1.** The allowance price would fall when the energy price increases.

**Proof.** As  $p_c = \frac{\alpha(\sum_{i=1}^m I_i + \sum_{j=1}^n I_j)}{\omega\beta(m+n)} - \frac{p}{\beta C_x}$ , totally differentiating the equation with

respect to  $p$  yields  $\frac{dp_c}{dp} = -\frac{1}{\beta C_x} < 0$ . Thus, the allowance price would fall if energy

price increases. The trend is vividly depicted in Fig.1 by exploiting the numerical calibration method.



**Fig.1.** Energy price, allowance price and the total energy price

Source: Authors' own calculation

**Proposition 2.** When the energy price rises, the total energy price would decrease modestly.

**Proof.** As mentioned above the total energy price is the sum of the energy price and allowance price, that is  $P = p + p_c c_x + 0.05$ . Totally differentiating the equation



yields  $dP = dp + c_x dp_c$ . As  $\frac{dp_c}{dp} = -\frac{1}{\beta c_x}$ , we have  $dP = dp - \frac{1}{\beta} dp$ . Where  $-\frac{1}{\beta} dp$  is

the buffer effect (BE) of energy price on total energy price.  $\beta$  is a constant parameter that measures the share of non-energy commodities in total consumption.

$-\frac{1}{\beta}$  represents the buffer multiplier. Since  $0 < \beta < 1$ , we have  $\left| -\frac{1}{\beta} dp \right| > |dp|$ , which

means that the BE dominates the energy price changes. As the energy price rises, the allowance price will decrease and the total energy price will also decrease only modestly (See Fig.1). This phenomenon is consistent with the works of Wadud (2011) and Gittell (2008) and it could be explained by the buffer effect.

#### 4.1.2. Price effect

**Proposition 3.** When the initial allowance allocation is given, the total energy demand remains stable under the PCT scheme.

**Proof.** We now provide a numerical example to demonstrate Proposition 3. Table 3 reports on the price effect, substitution effect, income effect and energy demand under the PCT scheme. The results in Table 3 are based on the above parameter choices, equations and formulae. As shown in Table 3, the substitution effect of an energy price increase on energy demand is positive, which has to do with the fact that the total energy price will decrease with the energy price. However the income effect is negative, which follows the demand theory of Normal goods (Varian, 1992).

For the over-emitter, the substitution effect dominates the income effect. Thus, the price effect is positive. The over-emitter's energy demand will increase as the energy price rises. However for the under-emitter, the income effect dominates the substitution effect. Thus, the price effect is negative. The energy demand of the under-emitter will decrease with the energy price. In addition, it is found that the total energy demand ( $x_i + x_j$ ) is approximately equal to 168. It is indicated that when the initial allowance allocation is given, the total energy demand remains stable even if the energy price fluctuates drastically. Therefore the PCT scheme can actually achieve

emission reduction targets.

**Table 3**

Price effects and energy demand under the PCT scheme.

$P$	$p_c$	$P$	Over-emitter				Under-emitter				$x_i + x_j$
			$x_i$	$PE_i$	$SE_i$	$IE_i$	$x_j$	$PE_j$	$SE_j$	$IE_j$	
4.50	0.59	10.43	119.95	0.42	1.15	-0.72	47.66	-0.35	0.46	-0.80	167.61
5.00	0.53	10.37	120.16	0.43	1.16	-0.73	47.48	-0.35	0.46	-0.81	167.65
5.50	0.48	10.32	120.38	0.43	1.16	-0.73	47.31	-0.35	0.46	-0.81	167.69
6.00	0.42	10.26	120.60	0.44	1.17	-0.74	47.13	-0.36	0.46	-0.82	167.73
6.50	0.37	10.21	120.82	0.44	1.18	-0.74	46.95	-0.36	0.46	-0.82	167.77
7.00	0.31	10.15	121.04	0.45	1.19	-0.74	46.77	-0.37	0.46	-0.82	167.81
7.50	0.25	10.10	121.27	0.45	1.20	-0.75	46.59	-0.37	0.46	-0.83	167.86
8.00	0.20	10.04	121.50	0.46	1.21	-0.75	46.40	-0.37	0.46	-0.83	167.90
8.50	0.14	9.99	121.73	0.46	1.22	-0.75	46.22	-0.38	0.46	-0.84	167.95
9.00	0.09	9.93	121.96	0.47	1.22	-0.76	46.03	-0.38	0.46	-0.84	167.99

*Source:* Authors' own calculation

*Note:*  $p$  represents energy price,  $p_c$  represents allowance price,  $P$  represents the total energy price,  $PE$  represents the price effect,  $SE$  represents the substitution effect,  $IE$  represents the income effect,  $x$  represents the energy demand and  $x_i + x_j$  represents the total energy demand.

## 4.2. Comparative analysis for the over-emitter

**Proposition 4.** For the over-emitter, the energy demand is more stable under the PCT scheme.

**Proof.** We now provide a numerical example to demonstrate Proposition 4. As shown in Table 4, when the PCT scheme is not introduced (without-PCT scenarios) for the over-emitter, the substitution effect and income effect are both negative and

thus the price effect is negative. The over-emitter's energy demand will decrease as the energy price increases (Fig.2). However under the with-PCT scenarios, the substitution effect dominates the income effect. Thus the price effect is positive, and the energy demand will increase with the energy price (Fig. 2).

In addition it is found that the price effect will decrease dramatically when the PCT scheme is not introduced. The proportions of the price effect under the with- and without-PCT scenarios range from -0.71% to -3.15%. The reason can be attributed to the influence of the buffer effect. Under the PCT scheme the buffer effect can keep the price effect, substitution effect and income effect from fluctuating dramatically. Thus, the over-emitter's energy demand under the PCT scheme is stable (Fig.2).

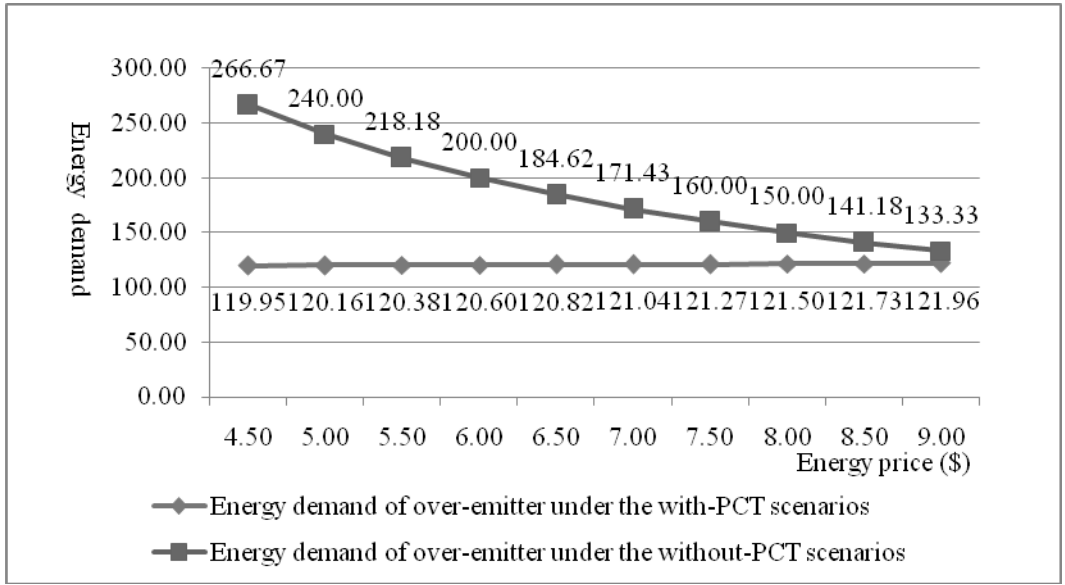
**Table 4**

Comparative analysis for the over-emitter.

$p$	Without-PCT			With-PCT			$PE_i / PE_i'$
	$PE_i'$	$SE_i'$	$IE_i'$	$PE_i$	$SE_i$	$IE_i$	
4.50	-59.26	-53.33	-5.93	0.42	1.15	-0.72	-0.71%
5.00	-48.00	-43.20	-4.80	0.43	1.16	-0.73	-0.89%
5.50	-39.67	-35.70	-3.97	0.43	1.16	-0.73	-1.09%
6.00	-33.33	-30.00	-3.33	0.44	1.17	-0.74	-1.31%
6.50	-28.40	-25.56	-2.84	0.44	1.18	-0.74	-1.55%
7.00	-24.49	-22.04	-2.45	0.45	1.19	-0.74	-1.82%
7.50	-21.33	-19.20	-2.13	0.45	1.20	-0.75	-2.11%
8.00	-18.75	-16.88	-1.88	0.46	1.21	-0.75	-2.43%
8.50	-16.61	-14.95	-1.66	0.46	1.22	-0.75	-2.78%
9.00	-14.81	-13.33	-1.48	0.47	1.22	-0.76	-3.15%-3.20%

Source: Authors' own calculation

Note:  $p$  represents energy price,  $PE$  represents the price effect,  $SE$  represents the substitution effect and  $IE$  represents the income effect.



**Fig.2.** Energy demand of the over-emitter

*Source:* Authors' own calculation

### 4.3. Comparative analysis for the under-emitter

**Proposition 5.** For the under-emitter, the energy demand is more stable under the PCT scheme.

**Proof.** We now provide a numerical example to demonstrate Proposition 5. Similarly as shown in Table 5, under the without-PCT scenarios, the under-emitter's substitution effect and income effect are both negative. Thus the price effect is negative, which is similar to that of the over-emitter's. However when the PCT scheme is introduced, the income effect dominates the substitution effect. Thus the price effect is negative. Furthermore it is also found that the under-emitter's price effects decrease dramatically when the PCT scheme is not introduced and their relative proportions under the with- and without-PCT scenarios range from 1.56% to 6.86%. The buffer effect can keep the price effect, substitution effect and income effect from fluctuating dramatically. Thus the energy demand is stable when the PCT scheme is introduced (Fig.3).

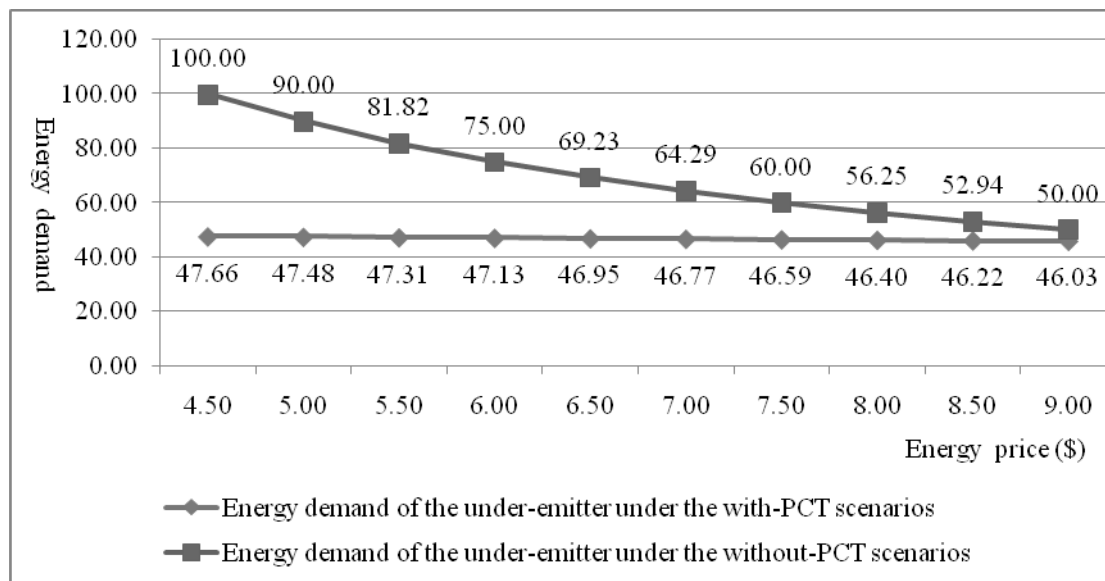
**Table 5**

Comparative analysis for the under-emitter.

$p$	Without-PCT			With-PCT			$PE_j / PE_j'$
	$PE_j'$	$SE_j'$	$IE_j'$	$PE_j$	$SE_j$	$IE_j$	
4.50	-22.22	-20.00	-2.22	-0.35	0.46	-0.80	1.56%
5.00	-18.00	-16.20	-1.80	-0.35	0.46	-0.81	1.95%
5.50	-14.88	-13.39	-1.49	-0.35	0.46	-0.81	2.38%
6.00	-12.50	-11.25	-1.25	-0.36	0.46	-0.82	2.86%
6.50	-10.65	-9.59	-1.07	-0.36	0.46	-0.82	3.39%
7.00	-9.18	-8.27	-0.92	-0.37	0.46	-0.82	3.98%
7.50	-8.00	-7.20	-0.80	-0.37	0.46	-0.83	4.61%
8.00	-7.03	-6.33	-0.70	-0.37	0.46	-0.83	5.30%
8.50	-6.23	-5.61	-0.62	-0.38	0.46	-0.84	6.05%
9.00	-5.56	-5.00	-0.56	-0.38	0.46	-0.84	6.86%

*Source:* Authors' own calculation

*Note:*  $p$  represents energy price,  $PE$  represents the price effect,  $SE$  represents the substitution effect and  $IE$  represents the income effect.

**Fig.3.** Energy demand of the under-emitter*Source:* Authors' own calculation

#### 4.4. Sensitivity analysis

According to the results mentioned above, it can be seen that the total energy price and the energy demand are less sensitive to the energy price changes under the PCT scheme. To further illustrate this phenomenon we conduct the following sensitivity analyses.

As shown in Table 6, upward adjustments in the energy price of 10% to 50% result in a fall in total energy price of -0.48% to -2.40%, a decrease in energy demand of the under-emitter of -0.33% to -1.67%, and an increase in energy demand of the over-emitter of 0.16% to 0.82%, respectively. The downward adjustments in the energy price of -10% to -50% result in an increase in the total energy price of 0.48% to 2.40%, an increase in energy demand of the under-emitter of 0.32% to 1.59%, and a decrease in energy demand of the over-emitter of -0.16% to -0.78%, respectively. The sensitivity test indicates that the variation in energy price only slightly influences the total energy price and energy demand are less sensitive to the energy price changes under the PCT scheme.

**Table 6** Results of sensitivity analysis of changes in the energy price.

Change rate of energy price	Change rate of total energy price	Change rate of energy demand	
		Over-emitter	Under-emitter
+50%	-2.40%	0.82%	-1.67%
+40%	-1.92%	0.65%	-1.33%
+30%	-1.44%	0.49%	-0.99%
+20%	-0.96%	0.32%	-0.66%
+10%	-0.48%	0.16%	-0.33%
-10%	0.48%	-0.16%	0.32%
-20%	0.96%	-0.32%	0.64%
-30%	1.44%	-0.47%	0.96%
-40%	1.92%	-0.63%	1.28%
-50%	2.40%	-0.78%	1.59%

*Source:* Authors' own calculation

## 5. Conclusions and implications

PCT scheme is a new and effective policy proposal to reduce carbon emissions at the household level by using carbon rationing and tradable emission allowances (Parag, 2008). PCT is an extension of the ‘cap and trade’ scheme in production sectors, with the aim to provide market signals and incentives for consumers to adapt to lower-carbon consumption and lifestyles. However the influence of the PCT scheme on low-carbon behavior seems to be complex and uncertain.

This paper mainly focuses on the buffer effect on energy price. Our analysis reflects that the PCT scheme provides a buffer between energy price and total energy price. The allowance price will fall when the energy price increases, keeping the total energy price stable and will thus help stabilizing the energy market and keeping the economic development running smoothly. Furthermore under the influence of the buffer effect, the price effect, substitution effect, income effect and energy demand in the PCT scheme are less sensitive to the energy price changes. The policy effect of the PCT scheme is certain and easier to control.

In addition we found that under the PCT scheme the substitution effect of an energy price increase on energy demand is positive, which is contrary to the traditional economic theory of consumer choice. However the buffer effect can explain this result. The total energy price will decrease with the energy price due to the fact that the buffer effect is negative and dominates the energy price increase. Furthermore the relationship between the substitution effect and the income effect is different for the over-emitter and the under-emitter. For the over-emitter, the substitution effect dominates the income effect. Thus the price effect is positive. The over-emitter’s energy demand will increase as energy price increases. In fact the energy demand of the over-emitter is determined by the total energy price. Under the PCT scheme, when the total energy price increases, the energy demand will decrease. This is the control function of PCT scheme. For the under-emitter the income effect dominates the substitution effect. Thus the price effect is negative. The energy demand of the under-emitter will decrease with the energy price.

The findings in the research could be useful for decision makers to introduce and implement PCT in the future. As stated above the PCT has the potential to act as a buffer to smooth out the energy price fluctuations. At the same time under the influence of buffer effect, the energy demand will also remain stable, even when facing a sudden energy price spikes. Therefore the PCT can also be effective when the energy price fluctuates considerably. For comparison, a carbon tax will always add a constant amount (or proportion) to the energy price, even if the total energy price is higher than that required to achieve a target reduction. Thus a tax policy is susceptible to the energy price in its effectiveness and is unable to provide certainty in emissions reduction (Wadud, 2011). When the underlying energy prices fluctuate, the PCT scheme is more effective than carbon taxes. In the past year, the average Brent crude oil price decreased by about 50%.<sup>3</sup> Considering the fluctuation of energy price for the past few years, the PCT could become an attractive option in the presence of uncertainty in future prices (Wadud, 2011).

Although the research has some interesting findings, we are mindful of the fact that the model has ignored a number of other factors that are also likely to be of first-order importance in practice. For instance, the paper has ignored the banking and borrowing of allowances (Parsons et al., 2009). Establishing a model which allows for banking and borrowing would bring our analysis a significant step closer to the realities of the PCT system. Additionally we assumed that the trading market is perfectly competitive and that all the participants are price-takers. If we relax this assumption, the model could yield more interesting results, which would be more informative for decision-makers. Thus, these two aspects need to be considered in future research and these are what we set out to do in the following studies.

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<sup>3</sup> See detail at <http://finance.sina.com.cn/futures/quotes/OIL.shtml>.



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## Appendix A

The Karush-Kuhn-Tucker (KKT) optimality conditions are

$$0 \leq x \perp \alpha x^{\alpha-1} y^\beta - p\hat{\partial}_1 - c_x \hat{\partial}_2 \geq 0 \quad (\text{A1})$$

$$0 \leq y \perp \beta x^\alpha y^{\beta-1} - q\hat{\partial}_1 \geq 0 \quad (\text{A2})$$

$$0 \leq \psi \perp -p_c \hat{\partial}_1 + \hat{\partial}_2 \geq 0 \quad (\text{A3})$$

$$0 \leq \hat{\partial}_1 \perp px + qy + p_c \psi - I \geq 0 \quad (\text{A4})$$

$$0 \leq \hat{\partial}_2 \perp c_x x - \psi - \omega \geq 0 \quad (\text{A5})$$

where  $\perp$  indicates orthogonality between two vectors, which in this case simply expresses the complementary slackness condition in linear programming (Zhao et al., 2010; Chen et al., 2011).

According to the Karush-Kuhn-Tucker (KKT) conditions we have

$$\frac{\alpha}{\beta} \cdot \frac{y}{x} = \frac{p}{q} + \frac{c_x}{q} \frac{\hat{\partial}_2}{\hat{\partial}_1} \quad (\text{A6})$$

$$x = \frac{\psi + \omega}{c_x} \quad (\text{A7})$$

$$y = \frac{-px - p_c \psi + I}{q} \quad (\text{A8})$$

$$-p_c \partial_1 + \partial_2 = 0 \quad (\text{A9})$$

Letting  $P = p + p_c c$  and  $Y = I + p_c \omega$ . Substituting equations (A7), (A8) and (A9) into equation (A6), we have

$$\psi = \frac{\alpha c_x (I + p_c \omega)}{p + p_c c_x} - \omega = \frac{\alpha c_x Y}{P} - \omega \quad (\text{A10})$$

Substituting equation (A10) into equation (A7), we have

$$x = \frac{\psi + \omega}{c_x} = \alpha \frac{I + p_c \omega}{p + p_c c_x} = \alpha \frac{Y}{P} \quad (\text{A11})$$

Substituting equations (A10) and (A11) into equation (A8), we have

$$y = \frac{-px - p_c \psi + I}{q} = \beta \frac{I + p_c \omega}{q} = \beta \frac{Y}{q} \quad (\text{A12})$$

The indirect utility function is

$$v(P, q, Y) = x^\alpha y^\beta = \left(\alpha \frac{Y}{P}\right)^\alpha \left(\beta \frac{Y}{q}\right)^\beta = \frac{\beta}{q} \left(\frac{\alpha q}{\beta P}\right)^\alpha Y \quad (\text{A13})$$

In addition, according to equation (A10), we have

$$\psi = \frac{\alpha c_x (I + p_c \omega)}{p + p_c c_x} - \omega = \frac{\alpha c_x I - p\omega - \beta \omega p_c c_x}{p + p_c c_x}$$

If  $\psi > 0$ , the allowance purchased by over-emitter  $i$  is

$$\psi_i = \frac{\alpha c_x I_i - p\omega - \beta \omega c_x p_c}{p_c c_x + p} \quad (\text{A14})$$

If  $\psi < 0$ , the allowance sold by under-emitter  $j$  is

$$\psi_j = \frac{-\alpha c_x I_j + p\omega + \beta \omega c_x p_c}{p_c c_x + p} \quad (\text{A15})$$

When the market demand equals market supply, that is

$$\sum_{i=1}^m \psi_i = \sum_{j=1}^n \psi_j \quad (\text{A16})$$

Then the equilibrium price for carbon allowance is

$$p_c = \frac{\alpha(\sum_{i=1}^m I_i + \sum_{j=1}^n I_j)}{\omega\beta(m+n)} - \frac{p}{\beta C_x} \quad (\text{A17})$$

## Appendix B

The Karush-Kuhn-Tucker (KKT) optimality conditions for the inequality constrained optimization problems are

$$0 \leq x \perp p - \alpha x^{\alpha-1} y^\beta \phi_1 + c_x \phi_2 \geq 0 \quad (\text{B1})$$

$$0 \leq y \perp q - \beta x^\alpha y^{\beta-1} \phi_1 \geq 0 \quad (\text{B2})$$

$$0 \leq \psi \perp p_c - \phi_2 \geq 0 \quad (\text{B3})$$

$$0 \leq \phi_1 \perp x^\alpha y^\beta - U \geq 0 \quad (\text{B4})$$

$$0 \leq \phi_2 \perp c_x x - \psi - \omega \geq 0 \quad (\text{B5})$$

According to equations (B1), (B2) and (B3), we have

$$\left(\frac{y}{x}\right)^\beta = \left[\frac{\beta(p + c_x p_c)}{\alpha q}\right]^\beta \quad (\text{B6})$$

According to equations (B6) and (B4), the Hicksian demand function for consumer is

$$x_h(P, q, U) = \frac{U(\alpha q)^\beta}{[\beta(p + c_x p_c)]^\beta} = \frac{U(\alpha q)^\beta}{(\beta P)^\beta} \quad (\text{B7})$$

According to equation (B7), the Hicksian demand function for the over-emitter is

$$x_{hi} = \frac{U_i(\alpha q)^\beta}{(\beta P)^\beta} \quad (\text{B8})$$

Similarly, the Hicksian demand function for the under-emitter is

$$x_{hj} = \frac{U_j(\alpha q)^\beta}{(\beta P)^\beta} \quad (\text{B9})$$

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