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A MULTI-MARKET APPROACH TO MEASURING THE CYCLE

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DISCUSSION PAPER 13.16

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Abstract

At any given instance there are many indicators of the state of the economy or the state of a sector and frequently the signals are mixed – some may point to an expansion, others to a contraction, for example. We consider how best to combine this conflicting information into an overall index of economic conditions. This index plays the role of the "underlying cycle" that has the property of minimizing the distortionary impact of noise in the n individual signals. This is essentially the panel regression approach of Stock and Watson (2010). We elaborate and evaluate this rich approach, note its links to stochastic-index-number theory and suggest new interpretations, modifications and extensions. The approach is illustrated with the world prices of six important metals.

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1. Introduction

Consider a sector of the economy made up of a number of distinct markets. An example is the metals sector made up of aluminium, copper, lead, nickel, tin and zinc. Each market is subject to a large number of shocks, some common to all markets, others market specific. Common shocks to the metals sector could be a surge in construction in China that substantially adds to world demand for metals, major central banks embarking on coordinated quantitative easing leading to a large increase in global liquidity, or a rise in energy prices that depresses the world economy. Market-specific shocks could include technological break throughs that make lower-grade ore deposits commercially viable, strikes in major supplying countries or natural disasters that disrupt production of certain metals. A reoccurring pattern of common shocks is likely to result in price cycles.

The question we consider is, how can we best use this disaggregated price information to identify the underlying cycle that is common for the sector as a whole? The problem of measuring the cycle in a multi-market context can be thought of as a statistical one: The underlying cycle is unobservable, but each market provides a noisy reading on it; the noise comes about because market-specific shocks lead to departures from the underlying cycle. The statistical problem is to minimise the distortionary impact of the noise to estimate the underlying cycle by employing some form of averaging. This averaging can be conveniently formulated as a type of panel regression model using data across markets and time. This is a particularly rich framework as it leads to econometric *estimates* of the characteristics of the cycle such as duration. The regression provides a point estimate of duration, its standard error and, with some additional assumptions, the whole probability distribution. The dispersion of this distribution is directly related to the extent to which the individual markets are idiosyncratic. When, for example, some prices are experiencing a lengthy expansion, while others rise for only a shorter period, there is substantial diversity in duration across markets. Here, the standard error of estimated duration of the underlying boom will be large, reflecting the greater uncertainty regarding the nature of the cycle for the sector as a whole. This probabilistic approach means that hypothesis testing can be carried out in the normal way to test questions such as: Is the duration of one boom longer than another; are cycles symmetric around turning points; and following a peak, do prices initially collapse and then tail off as they approach the trough? At a fundamental level, this approach is equivalent to the stochastic approach to index numbers, which emphasises index numbers as means of prices (or quantities) and applies the theory of the sampling distribution of the mean to the index. In other words, the multimarket approach to measuring the cycle is a branch of index-number theory, something that has not previously been recognised. We show how this is a useful new perspective on the cycle.

As GDP is the sum of value added in each sector, the multi-market approach is applicable to measuring the cycle for the economy as a whole. In a pioneering study, Stock and Watson (2010) develop this idea to dating the business cycle. We elaborate their approach by examining in detail the workings of the panel regression model, and highlight as the source of the estimation error of the underlying cycle the extent to which the disaggregated variables display disparate cyclical behaviour (as measured by the dispersion of their turning points). We deal with the estimation of the turning points of the cycle (the dates of the peaks and troughs), as do Stock and Watson, but the approach is equally applicable to other characteristics of the cycle such as amplitude and the nature of the path between turning points.

In addition to Stock and Watson (2010), of relevance to our work is the paper by Harding and Pagan (2006) on the degree of synchronization of cycles across sectors and countries. They develop tests for the degree of comovement of variables, as well as an algorithm to extract a common cycle based on non-parametric techniques. This approach contrasts with that of Stock and Watson (2010), which, as mentioned above, is based on a regression model. It is also relevant to mention dynamic factor models as a related branch of the literature (for surveys, see Bai and Ng, 2008, and Stock and Watson, 2011). Here, a large number of macroeconomic variables are related to a smaller number of contemporaneous and lagged latent factors that represent the state of the underlying economy. When there is only one factor in such a model, its interpretation is clearly the common factor that codes the state of the whole economy; that factor could be the latent trend growth if the original variables are expressed as growth rates. But when there are multiple factors, their interpretation becomes more challenging.[2]

The next section of the paper deals with the meaning of the underlying cycle and the Stock and Watson approach, followed by an econometric analysis in Sections 3. Section 4 is an illustrative application to the metals sector comprising the prices of the six metals mentioned above. Concluding comments are given in Section 5.

[2] The forecasting of turning points in economic variables discussed by Helperin (2010), Kling (1987), Wecker (1979), Zellner et al. (1990) and Zellner et al. (1991), among others, is also related to the dating and measurement of business cycles, but explicit links are still to be fully developed.

2. The Underlying Cycle and Stochastic Index Numbers

The business cycle could be dated on the basis of the trajectory of a single variable such as GDP whereby an algorithm is used to identify local turning points as peaks and troughs. An alternative approach, used by the NBER, is to examine a number of disaggregated macroeconomic measures of real economic activity (personal income, employment, industrial production, etc.) for individual peaks and troughs and then aggregate the dates to date the cycle. As GDP is the aggregation of the underlying macro variables, the two approaches could be described as (i) aggregate and then date; and (ii) date then aggregate. Stock and Watson (2010) compare these two approaches and in this section we describe, interpret and extend their work.

Suppose we have monthly data for n = 3 disaggregated macroeconomic variables. In the simplest possible case, over the course of the cycle the three variables all peak in the same month, so that it is clear that this common peak is the peak for the economy as a whole. This situation is illustrated as "Scenario 1" in column 2 of Table 2.1, where the common peak occurs in month number 5. There can be no uncertainty regarding this peak. Next, consider the slightly more complex case in which variable 1 peaks in month 3, variable 2 continues to peak in month 5 and 3 peaks in month 7, so that the mean of the peaks continues to occur in month 5 (column 3 of the table). The important difference is that now as there is some dispersion around this mean, as summarised by the standard deviation of 2 months. There is now estimation uncertainty regarding the mean peak that is measured by the standard error of the mean of 1.2 months. A rough 95-percent confidence interval for the peak is [3, 7] months, which is substantial. Clearly, the diversity of behaviour in scenario 2 represents an essentially different economic situation than the first, where there is no diversity. Scenario 2 thus requires more caution, and perhaps even further investigation, in declaring month 5 to be the peak. In scenario 3 (column 4) there is even more dispersion and more uncertainty regarding when the peak occurs (although the point estimate remains the same at month 5).

The model underlying the above analysis can be expressed as

(2.1) $y_i = \alpha + \varepsilon_i$, $i = 1, \ldots, n$ disaggregated variables,

where y_i is the peak (measured in terms of the month number) for variable i, α is the underlying peak, which is the same for all variables, and ε_i is a random disturbance term with zero expectation and constant variance. According to this model, the peak for each variable is made up of the common peak α plus a random term ε_i. Thus, there are n noisy readings on the common peak, so the estimation problem is to combine these n readings so as to minimise the noise. Under the stated properties of the disturbance term, the mean of the n peaks of the disaggregated variables is the best linear unbiased estimator of the common peak.

Model (2.1) applies to one cycle in which each of the n variables has one peak. When we have a series of cycles, each with its own peak, (2.1) can be written for cycle c as $y_{ic} = \alpha_c + \varepsilon_{ic},\ i = 1,\ldots,n$. It is possible that some of the underlying variables might lead or lag the common cycle in a consistent manner. If the peak for variable i occurs $\beta_i > 0$ months after the common peak, then to provide an unbiased reading on the common peak, we need to "shift" i's peak back β_i months by replacing y_{ic} on the left-hand side of the above equation with $y_{ic} - \beta_i$, so that it is now "in phase". If variable j leads, then its $\beta_j < 0,$ and we continue to subtract, so the net effect is to add the lead to synchronise it with the common trend. Thus, model (2.1) can be extended to deal with the "out-of-phase" variables by using $y_{ic} - \beta_i = \alpha_c + \varepsilon_{ic}$, or

(2.2) $y_{ic} = \alpha_c + \beta_i + \varepsilon_{ic},\ i = 1,\ldots,n$ variables, $c = 1,\ldots,C$ cycles.

Each peak is now made up of the sum of three terms – a peak that is common to all n variables, a phase parameter specific to variable i and a random disturbance. As stated above, variables that lead (lag) have a negative (positive) value of β_i. As this model is subject to one additive degree of freedom, a natural identifying assumption is that the phase parameters sum to zero, $\Sigma_{i=1}^{n}\beta_i = 0,$ so that the leading variables are balanced by the laggers, or on average variables are synchronised. This same approach can equally be applied to dating the trough in each cycle.

Stock and Watson (2010) use model (2.2) with n = 270 monthly variables and demonstrate that their date-then-aggregate approach gives results that are reasonably close to the dates of peaks and troughs for the US economy obtained by traditional methods. The 95-percent confidence intervals range from ±0.8 to ±1.8 months around the point estimates, so the estimates of the turning points are precise. Stock and Watson emphasise that the key benefit of their approach is that it provides standard errors for turning points, which is new.

The Stock and Watson treat model (2.2) as a fixed effects panel regression to be estimated by OLS. The problem they consider can also be thought of as an index-number problem, viz., how to best combine the turning points of the disaggregated variables into one number, the peak/trough of the overall cycle. Traditionally, index-number theory has been mostly deterministic as it involves the application of an aggregation formula (Paasche, Laspeyres, Fisher, etc.) to yield one number, such as the overall rate of inflation. By contrast, the index-number perspective of the Stock and Watson approach is a statistical one, which leads an estimate of the overall turning point that has a sampling distribution. This approach has the advantage of being able to make probabilistic statements regarding the turning points and to carry out hypothesis testing, which opens up new possibilities. At a formal level, the Stock and Watson approach is an application of stochastic index-number theory and some of the material that follows is motivated by that theory. There is now a revival of interest in the stochastic approach and according to Diewert (2007), it is one of the four main approaches to index-number theory (the fixed basket approach, the test approach and the economic approach are the other three).[3]

3. The Econometrics of the Cycle

In the Appendix A1 we show that the least-squares estimators of the common peak and the phase parameters of model (2.2) are

$$(3.1)\qquad \hat{\alpha}_c = \frac{1}{n}\sum_{i=1}^{n} y_{ic},\ c = 1,\dots,C, \qquad \hat{\beta}_i = \frac{1}{C}\sum_{c=1}^{C}\left(y_{ic} - \hat{\alpha}_c\right),\ i = 1,\dots,n,$$

which satisfy $\Sigma_{i=1}^{n}\hat{\beta}_i = 0$. These expressions are attractively simple: The estimated common peak, $\hat{\alpha}_c$, continues to be the mean of the n peaks and the estimate of the i^{th} phase-adjustment parameter, $\hat{\beta}_i$, is just that variable's phase discrepancy, $y_{ic} - \hat{\alpha}_c$, averaged over cycles $c = 1,\dots,C$. When the disturbance term in equation (2.2), ε_{ic}, has a constant variance, σ^2, then

$$(3.2)\qquad \text{var}\,\hat{\alpha}_c = \frac{\sigma^2}{n}, \quad \text{var}\,\hat{\beta}_i = \frac{\sigma^2}{C}\left(1 - \frac{1}{n}\right).$$

The first expression states that the sampling variance of the estimate of the common peak for cycle c is proportional to the variance of the disturbance term in (2.2), so that it is more difficult to estimate this peak precisely when there is more dispersion among the peaks of the disaggregated variables, which makes perfect sense.[4] It is also established in Appendix A1 that

$$\text{cov}\left(\hat{\alpha}_c, \hat{\alpha}_d\right) = 0,\ c \neq d; \text{cov}\left(\hat{\beta}_i, \hat{\beta}_j\right) = -\frac{\sigma^2}{Cn}, i \neq j; \text{cov}\left(\hat{\alpha}_c, \hat{\beta}_i\right) = 0,\ c = 1,\dots,C,\ i = 1,\dots,n.$$

In words, the estimated peaks are independent of each other, while the correlation between two phase parameter estimates is $-\left(\sigma^2/Cn\right)\Big/\left[\sigma^2/Cn/(n-1)\right] = -1/(n-1)$.

[3] On the stochastic approach to index numbers, see, for example, Aldrich (1992), Clements et al. (2006), Diewert (2007) and Rao and Selvanathan (1994).

[4] As noted by Harding and Pagan (2006), in their influential work Burns and Mitchell (1946, p. 70) were aware of the dispersion of turning points: They observed that at any point in time "some activities [are] in an expanding phase, some beginning to recede from their peaks, some contracting, and some beginning to revive from their troughs", but "at any one time one phase is dominant".

The above results mean that hypothesis testing is straight forward in this framework. For example, the estimated the peak-to-peak duration of the cycle and its sampling variance is

$$\hat{\alpha}_c - \hat{\alpha}_{c-1} = \frac{1}{n}\sum_{i=1}^{n}\left(y_{ic} - y_{i,c-1}\right), \quad \text{var}\left(\hat{\alpha}_c - \hat{\alpha}_{c-1}\right) = \frac{2\sigma^2}{n}, \ c = 2,\ldots,C.$$

The average duration is

$$\frac{1}{C-1}\sum_{c=2}^{C}\left(\hat{\alpha}_c - \hat{\alpha}_{c-1}\right) = \frac{1}{(C-1)n}\sum_{c=2}^{C}\sum_{i=1}^{n}\left(y_{ic} - y_{i,c-1}\right),$$

with variance

$$\text{var}\left(\frac{1}{C-1}\sum_{c=2}^{C}\left(\hat{\alpha}_c - \hat{\alpha}_{c-1}\right)\right) = \text{var}\left(\frac{1}{C-1}\left(\hat{\alpha}_C - \hat{\alpha}_1\right)\right) = \frac{2\sigma^2}{n(C-1)}.$$

In the above, duration is the time between peaks. Alternatively, we could first compute the durations of the n disaggregated cycles, $\left(y_{1,c} - y_{1,c-1}\right),\ldots,\left(y_{n,c} - y_{n,c-1}\right)$, and then estimate the common duration. To analyse the workings of this approach, we return to model (2.2), $y_{ic} = \alpha_c + \beta_i + \varepsilon_{ic}$, and difference it across cycles: $y_{i,c} - y_{i,c-1} = \alpha_c - \alpha_{c-1} + \varepsilon_{i,c} - \varepsilon_{i,c-1}$. Defining $\Delta_c y_{i,c} = y_{i,c} - y_{i,c-1}$ as duration for i, $\gamma_{c,c-1} = \alpha_c - \alpha_{c-1}$ underlying duration and $\eta_{i,c} = \varepsilon_{i,c} - \varepsilon_{i,c-1}$ the duration disturbance, we have $\Delta_c y_{i,c} = \gamma_{c,c-1} + \eta_{i,c}$, $\text{var}\left(\eta_{i,c}\right) = 2\sigma^2$. Clearly, the OLS estimator of underlying duration is the sample mean:

$$\hat{\gamma}_{c,c-1} = \frac{1}{n}\sum_{i=1}^{n}\Delta_c y_{i,c} = \frac{1}{n}\sum_{i=1}^{n}\left(y_{i,c} - y_{i,c-1}\right) = \hat{\alpha}_c - \hat{\alpha}_{c-1}, \text{ with } \text{var}\left(\hat{\gamma}_{c,c-1}\right) = \frac{2\sigma^2}{n},$$

which is exactly the same as before. Accordingly, the two approaches are identical. However, if the disturbances in the equation for the peaks, ε_{ic}, in equation (2.2), are serially independent, then the duration disturbances are not as $\eta_{i,c} = \varepsilon_{i,c} - \varepsilon_{i,c-1}$. In particular, we have $\text{cov}\left(\eta_{i,c}, \eta_{i,c-k}\right) = \sigma^2\left(2\delta_{c,c-k} \text{ -}\delta_{c,c-k+1}\right)$, where δ_{ij} is the Kronecker delta, so that the first-order autocorrelation coefficient is -1/2. Finally, as $\text{var}\left(\hat{\beta}_i - \hat{\beta}_j\right) = 2\sigma^2/C$, if we wished to investigate whether variable i had the same phase as j, we would test: $H_0 : \beta_i = \beta_j$ and, under the additional assumption of normality, use a t-test of the difference: $\hat{\beta}_i - \hat{\beta}_j / SE\left(\hat{\beta}_i - \hat{\beta}_j\right) = \Sigma_{c=1}^{C}\left(y_{ic} - y_{jc}\right)/\sqrt{2C\sigma^2}$.

The above assumption of homoscedasticity means that the variance (3.2) is the same for each estimate of the c = 1, …,C common peaks. In some cycles, the behaviour of the disaggregated variables could be similar in that many peak at roughly similar times; such a period is one of more or less uniform expansion/contraction and in this sense there is a certain amount of tranquillity and predictability. But in other times, there may be more diversity in the peaks, less uniformity, more volatility and less predictability. As it restrictive to rule out this type of diversity, we need to allow for heteroscedasticity.

Define the vectors $\mathbf{y} = [y_{11},\ldots,y_{1C},\ldots,y_{n1},\ldots,y_{nC}]'$, $\boldsymbol{\varepsilon} = [\varepsilon_{11},\ldots,\varepsilon_{1C},\ldots,\varepsilon_{n1},\ldots,\varepsilon_{nC}]'$, $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_C]'$ and $\boldsymbol{\beta} = [\beta_1,\ldots,\beta_{n-1}]'$. After substituting out the constraint $\Sigma_{i=1}^{n}\beta_i = 0$, model (2.2) can be expressed as

$$\text{(3.3)} \qquad \mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\mathbf{X}$ and $\mathbf{Z}$ are matrices of cycle and group (or variable) dummies, respectively. To allow for heteroscedasticity across cycles, now suppose the variance of the disturbance term ε_{ic} takes the form $\sigma^2\omega_c, \omega_c > 0.$ If we define $\boldsymbol{\Omega} = \operatorname{diag}[\omega_1,\ldots,\omega_C]$ and $\boldsymbol{\Sigma} = \mathbf{I}_{n\times n} \otimes \boldsymbol{\Omega},$ the covariance matrix of the disturbance vector $\boldsymbol{\varepsilon}$ is $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\boldsymbol{\Sigma}$. If $\hat{\omega}_c$ is an unbiased estimator of ω_c, then the GLS estimator of α_c and β_i in model (2.5) is

$$\text{(3.4)} \qquad \hat{\alpha}_c = \frac{1}{n}\sum_{i=1}^{n} y_{ic}, \quad \hat{\beta}_i = \sum_{c=1}^{C}\lambda_c\left(y_{ic} - \hat{\alpha}_c\right), \text{ with } \lambda_c = \frac{1/\hat{\omega}_c}{\sum_{c=1}^{C}\left(1/\hat{\omega}_c\right)}.$$

As can be seen, while the estimator of the peak is as before, the phase parameter is now estimated as a weighted average over cycles of the relevant phase discrepancy $y_{ic} - \hat{\alpha}_c$. The weights $\lambda_1,\ldots,\lambda_C$ are inversely proportional to the error variances in the cycles, so that more noisy cycles are accorded less weight. The corresponding variances are

$$\operatorname{var}\left(\hat{\alpha}_c\right) = \frac{\sigma^2\hat{\omega}_c}{n}, \quad \operatorname{var}\left(\hat{\beta}_i\right) = \frac{\sigma^2}{\sum_{c=1}^{C}\left(1/\hat{\omega}_c\right)n/(n-1)}.$$

All covariances are zero except for

$$\operatorname{cov}\left(\hat{\beta}_i, \hat{\beta}_j\right) = -\frac{\sigma^2}{\sum_{c=1}^{C}\left(1/\hat{\omega}_c\right)n}, \ i \neq j.$$ [5]

[5] For derivations of the above results, see Appendix A1. The nature of the dependent variable in model (2.2), y_{ic} could give rise to another issue of heteroscedasticity. This y_{ic} is the peak for variable i in cycle c is measured as the number of the month from the start of the overall sample period, which always increases as time advances. Accordingly, it is conceivable that the dispersion of the disturbance also rises with time. Thus, if error variance were proportional to the cycle index c, so that $\sigma_c^2 = \phi c, \ \phi > 0,$ it would be appropriate to weight observations by $c^{-1/2}$.

4. Application to Metals

As an illustrative example, we use monthly price data for six major non-ferrous metals, aluminium, copper, lead, nickel, tin and zinc, from 1989/06 to 2012/04. The prices are expressed in US dollars of 2005 by deflating by the US Producer Price Index.[6] Let p_{it} be the price of metal i in month t and q_{it} be the corresponding volume. Then, $M_t = \sum_{i=1}^{6} p_{it} q_{it}$ is the total value and $w_{it} = p_{it} q_{it} / M_t$ is the value share of i. If we write $Dp_{it} = \log p_{it} - \log p_{i,t-1}$ for the log-change in the i^{th} price, then the Divisia price index is

$$(4.1) \qquad DP_t = \sum_{i=1}^{6} \overline{w}_{it} Dp_{it},$$

where $\overline{w}_{it} = 1/2 \cdot \left(w_{it} + w_{i,t-1} \right)$ is the average share over months t and t-1.[7] This index weights prices according to the relative economic importance of the metals. As can be seen from Table 4.1, there is a fair degree of correlation among the prices, with correlations averaging about one-half.

We use the Bry-Boschan (1971) algorithm to date the turning points in the prices.[8] For convenience, we shall refer to the phase of the cycle from a peak to the next trough as a "slump" in prices and the subsequent recovery to the next peak as a "boom". Figure 4.1 gives the results in graphical form by shading the periods of slumps. The expansion that commenced in the early 2000s, known as the "Millennium Boom", was unusually long, and is an important feature of the whole period. The average duration of the phases are given in Table 4.2. From columns 3 and 5, it can be seen that even the average duration of phases differ substantially across metals, which points to the role of uncertainty in the underlying cycle. The dates of peaks of the price index are contained in columns 2 and 3 of panel A of Table 4.3 and the peak-to-peak durations of the cycle are contained in column 4. Panel B contains similar information for the troughs.

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[6] The US PPI is from http://stats.oecd.org/Index.aspx?DataSetCode=REFSERIES. The metal prices are from Thompson-Reuters DataStream and refer to the last trading day for the month. For prior studies on the cyclical behaviour of metal prices, see Cashin et al. (2002), Davutyan and Roberts (1994), Labys et al. (1998) and Roberts (2009).

[7] The volume of turnover on the London Metals Exchange is used as a measure of q_{it}. To reduce the large amount of noise, turnover is smoothed using a 7-point unweighted centred moving average. Prices are not smoothed. For a discussion of this issue, see Pagan and Sossounov (2003) and Cashin et al (2002). The turnover data are from Thompson-Reuters DataStream.

[8] The algorithm involves the following steps: (i) The identification of possible peaks (troughs) as local maximum (minimum) using a window comprising the previous five and the next five months. (ii) Censoring of the peaks and troughs with three rules. (a) Peaks and troughs must alternate – when there are two consecutive peaks (troughs), the higher (lower) of the two is kept. (b) Peaks and troughs in the last 6 months and the first 6 months of the sample period are eliminated. (c) A phase (that is, a boom or a slump) must last for at least 6 months, and a cycle (the period of the boom and slump) must last at least 15 months. We use Adrian Pagan's MATLAB program to implement the BB algorithm, available at http://www.ncer.edu.au/data/. The amplitude threshold parameter is not used for the dating in this paper.

An important idea from index-number theory is that goods be weighted to reflect their economic importance. We do this for metals by using the value of their production. As the value of production of metal i at time t is proportional to its value share w_{it}, we apply weighted least squares by multiplying both sides of model (2.2) by $\sqrt{w_{i,y_{ic}}}$ and reinterpret y_{ic} of Sections 2 and 3 as $\sqrt{w_{i,y_{ic}}}y_{ic}$. Panel A of column 9 of Table 4.3 contains the estimated peaks. These are estimates of model (2.2) obtained with the heteroscedasticity adjustment, as set out in the previous section. The estimated peaks are quite close to those from the price index contained in column 3, and the standard errors fall in the range 0.3-2.6 months (column 10), indicating that these turning points are estimated quite precisely. Similar comments apply to the estimates of troughs contained in panel B, except for two cases: The second trough in December 1991, denoted by T2, and T6 in October 2002.[9] The corresponding estimates of durations are contained in column 11, which are reasonably close to those in column 4, except for the two cycles before and after trough 6. The standard errors of duration (column 12) have a mean of approximately 2 months and are all substantially less than the corresponding point estimate.

A comparison of columns 6 and 10 of Table 4.3 reveals a sizable fall in the standard errors in most cases as a result of the heteroscedasticity adjustment. The situation is similar for the duration estimates (compare columns 8 and 12 of Table 4.3).[10] Figure 4.3 underscores the importance of heteroscedasticity by presenting box plots of the residuals from model (2.2) without the heteroscedasticity adjustment. These plots clearly reveal that there is substantial time-wise variation in the error variance.

The estimates of the phase parameters are given in column 3 of Table 4.4. These show that aluminium reaches its price peak/trough 1.2 months later than the underlying market, while lead and zinc lead the underlying cycle by about one month. The right-hand side of Figure 4.2 is a scatter of the dates of the turning points for the price index against those for aluminium. The regression line through these points has a horizontal intercept of approximately 1 month, in agreement with the estimated 1-month lag of Table 4.4. Similarly for zinc on the left-hand side of the figure, where the intercept is approximately -1 month.

The turning points y_{ic} for metal i in cycle c in model (2.2) are obtained by applying the Bry-Boschan (BB) algorithm to the price of metal i. These y_{ic}'s are then used to estimate the underlying

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[9] As discussed in Appendix A2, prices tended to be quite flat around the time of these two troughs, so the turning points are not too distinct.
[10] Table 4.4 reveals a similar reduction in the standard errors of the estimates of the phase parameters.

cycle. Accordingly, there are two types of uncertainties involved in the estimates: The prices are random, which can be described as "data uncertainty", and there is also "model uncertainty" in the form of the disturbance term in equation (2.2). To analyse the reliability of the approach, the two sources of randomness need to be recognised. To do this, we proceed in two steps. Step one involves simulating the prices. The relative price of metal i at time t, r_{it}, defined as $\log\left(p_{it}/P_t\right)$, can be represented by a random walk, a model that has appealing foundations in market efficiency and has the additional advantage of having no unknown parameters.[11] Accordingly, $r_{it} = r_{i,t-1} + \mu_{it}$, where μ_{it} is an iid disturbance with zero expectation, so the model is $\log p_{it} = \log P_t + r_{i,t-1} + \mu_{it}$, which we simulate by bootstrapping the residuals. Applying the BB algorithm to the simulated prices from a given trial, we obtain a new set of turning points, which are then used to estimate model (2.2). Step two in the procedure involves simulating model (2.2) by bootstrapping its residuals.[12]

We use 1,000 trails for each of the above two steps, so there is a total of $1,000^2$ realisations of the estimates of model (2.2), written as $\hat{\alpha}_c^{(s)}$ and $\hat{\beta}_i^{(s)}$, $s = 1,\ldots,10^6$. As can be seen from columns 3 and 5 of Table 4.5, the means of $\hat{\alpha}_c^{(s)}$ are almost the same as the "true" values. The root-mean-squared errors (RMSEs) of these simulated estimates in column 6 are of the same order as standard errors of the estimates obtained with the observed data as shown in column 10 of Table 4.3. Averaging the squared standard errors of each $\hat{\alpha}_c^{(s)}$ over the 1 million trials and then taking the square root, we have the root-mean-squared standard errors (RMSSEs) in column 7 of Table 4.5. The ratios of the RMSE to the RMSSE are listed in column 8, which are close to 1 in most cases. Similar conclusions emerge for durations (see columns 9-12) and phase parameters for six metals (Table 4.6). These results point to the feasibility and reliability of the approach of the paper.

To further illustrate the possibilities of the approach, Figure 4.4 shows the distributions of duration of phases of the cycle from the 10^6 simulations. As can be seen, the histogram has two distinct parts with the one on the right referring to the Millennium Boom (MB). The average simulated duration of the MB is 70 months with standard deviation of 3 months, while the mean duration of the other 6 booms and 7 slumps is 15 months and the standard deviation is 6 months. The fact that the MB distribution is so far away from that of the others and that there is virtually no overlap of the two parts of histogram clearly shows that this boom was atypically long. During this period (2002-2008) there was an unusual confluence of events – the world business cycle booming,

[11] For details, see Appendix A3.
[12] For precise details, see Appendix A3.

the rise of China and its resource imports, and supply constraints associated with low levels of investment in resource industries in the past – all of which contributed to this long boom in metals.

5. Concluding Comments

A basic measurement problem is to determine the current state of an economy or sector – is it expanding or contracting, or are prices in general increasing or decreasing? In a data-rich environment with multiple indicators of the current state, the question is how to combine potentially conflicting indicators into one measure. In a recent paper, Stock and Watson (2010) introduced a new approach to measuring the business cycle in a multiple indicator context. They start with n indicators of the state of the macroeconomy and date their turning points using conventional methods, so that there are n peaks and n troughs. These dates are viewed as reflecting the combined effects of the behaviour of the underlying cycle of the economy, factors specific to individual indicators and random events. Thus, Stock and Watson then aggregate the individual dates into the peak and trough of the underlying cycle using a panel regression procedure with time and indicator effects. This procedure has the advantage of recasting cycle measurement into an econometric framework, so there are *estimates* of the dates of the turning points and these have standard errors reflecting estimation uncertainty. These standard errors of the turning points of the common cycle are new to the cycle measurement literature.

In this paper, we examined in detail this rich approach and point to its link with stochastic-index-number theory, which also provides the standard error of the value of the index (Clements et al., 2006, Selvanathan and Rao, 1994). This unification between cycle measurement and index-number theory could open up new possibilities for both areas. For example, one branch of index numbers examines the moments of the distributions of the n price and quantity changes in the form of Divisia indexes (the first moments), variances and the price-quantity covariance (Theil, 1967, Chap. 5). A similar approach could be used to analyse the distributions of duration of phases of the cycle and their amplitude. We elaborated the Stock and Watson approach and illustrated how it can be extended and enhanced. This involved a detailed analysis of the framework and recognition that some phases of the cycle are more volatile than others so that a heteroscadasticity adjustment is required. Our approach was illustrated with the cyclical behaviour of the world metal prices.

Appendix A1
DERIVATIONS

A1.1 Estimating Equation (2.2)

Equation (2.2) is a simple time and group (variable) fixed effect model, which we reproduce for convenience:

(2.2) $y_{ic} = \alpha_c + \beta_i + \varepsilon_{ic}$, $i = 1,\ldots,n$ variables, $c = 1,\ldots,C$ cycles.

For identification, the group effects β_i are normalised to have zero mean, $\sum_{i=1}^{n}\beta_i = 0$. This restriction can be imposed by substituting $\beta_j = -\sum_{i\neq j}\beta_i$. As the choice of j has no impact on the results, we eliminate β_n by replacing it with $-\sum_{i=1}^{n-1}\beta_i$.

Matrix Formulation

Write all peaks for the n variables in one vector as $\mathbf{y} = [y_{11},\ldots,y_{1C},\ldots,y_{n1},\ldots,y_{nC}]'$, the disturbances as $\boldsymbol{\varepsilon} = [\varepsilon_{11},\ldots,\varepsilon_{1C},\ldots,\varepsilon_{n1},\ldots,\varepsilon_{nC}]'$, and the parameters as $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_C]'$ and $\boldsymbol{\beta} = [\beta_1,\ldots,\beta_{n-1}]'$. In terms of these vectors, equation (2.2) can be expressed as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\mathbf{X}$ is the matrix of the reference cycle dummies and $\mathbf{Z}$ is the matrix of group dummies:

$$\mathbf{X} = \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \boldsymbol{\iota}_n \otimes \mathbf{I}_{C\times C} \text{ and } \mathbf{Z} = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ -1 & \cdots & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} = \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}'_{n-1} \end{bmatrix} \otimes \boldsymbol{\iota}_C,$$

where $\boldsymbol{\iota}_n$ represents a $n\times 1$ unit vector and $\mathbf{I}_{C\times C}$ is the $C\times C$ identity matrix.

The LS Estimator

Defining $\mathbf{D} = [\mathbf{X}\ \ \mathbf{Z}]$ and $\boldsymbol{\gamma} = [\boldsymbol{\alpha}'\ \ \boldsymbol{\beta}']'$, the ordinary least squares estimator of $\boldsymbol{\gamma}$ is

$$\hat{\boldsymbol{\gamma}} = (\mathbf{D}'\mathbf{D})^{-1}\mathbf{D}'\mathbf{y} = \left(\begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix}[\mathbf{X}\ \ \mathbf{Z}]\right)^{-1}\begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix}\mathbf{y} = \begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix}\mathbf{y},$$

and the variance of $\hat{\boldsymbol{\gamma}}$ is $\text{var}(\hat{\boldsymbol{\gamma}}) = \sigma^2(\mathbf{D}'\mathbf{D})^{-1}$, where σ^2 is the variance of the disturbance term.

Since $\mathbf{X}' = \boldsymbol{\iota}'_n \otimes \mathbf{I}_{C\times C}$ and $\mathbf{Z}' = [\mathbf{I}_{(n-1)\times(n-1)}\ \ -\boldsymbol{\iota}_{n-1}] \otimes \boldsymbol{\iota}'_C$ and using the Kronecker product rule that $(A\otimes B)(C\otimes D) = AC\otimes BD$, we have

$$\mathbf{X}'\mathbf{Z} = \left(\boldsymbol{\iota}_n' \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}_{n-1}' \end{bmatrix} \right) \otimes \left(\mathbf{I}_{C\times C}\boldsymbol{\iota}_C \right) = \mathbf{0}_{1\times(n-1)} \otimes \boldsymbol{\iota}_C = \mathbf{0}_{C\times(n-1)},$$

$$\mathbf{Z}'\mathbf{X} = \left(\begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} & -\boldsymbol{\iota}_{n-1} \end{bmatrix} \boldsymbol{\iota}_n \right) \otimes \left(\boldsymbol{\iota}_C'\mathbf{I}_{C\times C} \right) = \mathbf{0}_{(n-1)\times 1} \otimes \boldsymbol{\iota}_C' = \mathbf{0}_{(n-1)\times C}.$$

Therefore, the moment matrix $\mathbf{D}'\mathbf{D}$ is block diagonal with inverse

$$\left(\mathbf{D}'\mathbf{D}\right)^{-1} = \begin{bmatrix} \left(\mathbf{X}'\mathbf{X}\right)^{-1} & \mathbf{0}_{C\times(n-1)} \\ \mathbf{0}_{(n-1)\times C} & \left(\mathbf{Z}'\mathbf{Z}\right)^{-1} \end{bmatrix}.$$

Post-multiplying $\left(\mathbf{D}'\mathbf{D}\right)^{-1}$ by $\mathbf{D}'$, we have

$$\left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}' = \begin{bmatrix} \left(\mathbf{X}'\mathbf{X}\right)^{-1} & \mathbf{0}_{C\times(n-1)} \\ \mathbf{0}_{(n-1)\times C} & \left(\mathbf{Z}'\mathbf{Z}\right)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix} = \begin{bmatrix} \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}' \\ \left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\mathbf{Z}' \end{bmatrix}.$$

Therefore, the estimators of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\hat{\boldsymbol{\alpha}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{y} \text{ and } \hat{\boldsymbol{\beta}} = \left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\mathbf{Z}'\mathbf{y}.$$

Similifications

Using the Kronecker product rules: $\left(A\otimes B\right)\left(C\otimes D\right) = AC\otimes BD$, and $\left(A\otimes B\right)^{-1} = A^{-1}\otimes B^{-1}$, we have

$$\left(\mathbf{X}'\mathbf{X}\right)^{-1} = \left[\left(\boldsymbol{\iota}_n'\otimes\mathbf{I}_{C\times C}\right)\left(\boldsymbol{\iota}_n\otimes\mathbf{I}_{C\times C}\right)\right]^{-1} = \left(\boldsymbol{\iota}_n'\boldsymbol{\iota}_n\otimes\mathbf{I}_{C\times C}\right)^{-1} = \frac{1}{n}\mathbf{I}_{C\times C}, \text{ and}$$

$$\begin{aligned}\left(\mathbf{Z}'\mathbf{Z}\right)^{-1} &= \left(\left(\begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} & -\boldsymbol{\iota}_{n-1} \end{bmatrix} \otimes \boldsymbol{\iota}_C' \right) \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}_{n-1}' \end{bmatrix} \otimes \boldsymbol{\iota}_C \right)^{-1} = \left(\begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} & -\boldsymbol{\iota}_{n-1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}_{n-1}' \end{bmatrix} \otimes \boldsymbol{\iota}_C'\boldsymbol{\iota}_C \right)^{-1} \\ &= \frac{1}{C}\left(\mathbf{I}_{(n-1)\times(n-1)} + \boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}'\right)^{-1} = \frac{1}{C}\left(\mathbf{I}_{(n-1)\times(n-1)} - \frac{1}{n}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}'\right).\end{aligned}$$

Substituting $\left(\mathbf{X}'\mathbf{X}\right)^{-1}$ and $\left(\mathbf{Z}'\mathbf{Z}\right)^{-1}$ into the expressions for $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$, we have

$$\hat{\boldsymbol{\alpha}} = \frac{1}{n}\left(\boldsymbol{\iota}_n'\otimes\mathbf{I}_{C\times C}\right)\mathbf{y}, \quad \hat{\boldsymbol{\beta}} = \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} - n^{-1}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}' & -n^{-1}\boldsymbol{\iota}_{n-1} \end{bmatrix} \otimes \left(C^{-1}\boldsymbol{\iota}_C'\right)\mathbf{y},$$

or, in scalar terms,

$$\hat{\alpha}_c = \frac{1}{n}\sum_{i=1}^{n} y_{ic}, c = 1,\ldots,C, \text{ and } \hat{\beta}_i = C^{-1}\sum_{c=1}^{C}\left(y_{ic} - n^{-1}\sum_{j=1}^{n} y_{jc}\right), \; i = 1,\ldots,n-1.$$

For $i = n$,

$$\hat{\beta}_n = -\sum_{i=1}^{n-1}\hat{\beta}_i = C^{-1}\sum_{c=1}^{C}\left(\frac{n-1}{n}\sum_{j=1}^{n} y_{jc} - \sum_{i=1}^{n-1} y_{ic}\right) = C^{-1}\sum_{c=1}^{C}\left(y_{nc} - n^{-1}\sum_{j=1}^{n} y_{jc}\right).$$

The last expression confirms that the choice of which β_i to eliminate is immaterial.

The Variances

Assuming that the disturbance terms are independent with zero mean and variance σ^2, the variances of $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ are

$$\text{var}(\hat{\boldsymbol{\alpha}}) = \sigma^2 (\mathbf{X}'\mathbf{X})^{-1} = \frac{\sigma^2}{n}\mathbf{I}_{C\times C} \text{ or } \text{var}(\hat{\alpha}_c) = \frac{\sigma^2}{n}, c = 1,\ldots,C, \text{ and}$$

$$\text{var}(\hat{\boldsymbol{\beta}}) = \sigma^2 (\mathbf{Z}'\mathbf{Z})^{-1} = \frac{\sigma^2}{C}\left(\mathbf{I}_{(n-1)\times(n-1)} - \frac{1}{n}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}'_{n-1}\right), \text{ or } \text{var}(\hat{\beta}_i) = \frac{\sigma^2}{C}\left(1-\frac{1}{n}\right), \; i = 1,\ldots,n-1.$$

For $i = n$,

$$\text{var}(\hat{\beta}_n) = \text{var}(-\boldsymbol{\iota}'_{n-1}\hat{\boldsymbol{\beta}}) = \sigma^2\boldsymbol{\iota}'_{n-1}(\mathbf{Z}'\mathbf{Z})^{-1}\boldsymbol{\iota}_{n-1} = \frac{\sigma^2}{C}\left(n-1-\frac{(n-1)^2}{n}\right) = \frac{\sigma^2}{C}\left(1-\frac{1}{n}\right).$$

As the moment matrix $\mathbf{D}'\mathbf{D}$ is block diagonal, all covariances between the elements of $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$ vanish. Moreover, it follows directly from the above expressions that

$$\text{cov}(\hat{\alpha}_c, \hat{\alpha}_d) = 0, \; c \neq d, c = 1,\ldots,C,; \text{cov}(\hat{\beta}_i, \hat{\beta}_j) = -\frac{\sigma^2}{Cn}, i \neq j, i, j = 1,\ldots,n.$$

A1.2 A Two-Step Estimation Procedure

To allowing for heteroscedasticity across cycles, we now consider the case when the disturbance term ε_{ic} has a variance of the form $\sigma^2\omega_c$, where $\omega_c, c = 1,\ldots,C$, are a set of positive constants. We continue to assume that the ε_{ic} are independent with zero mean. If we define $\boldsymbol{\Omega} = \text{diag}[\omega_1,\ldots,\omega_C]$ and $\boldsymbol{\Sigma} = \mathbf{I}_{n\times n} \otimes \boldsymbol{\Omega}$, the covariance matrix of $\boldsymbol{\varepsilon}$ is $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\boldsymbol{\Sigma}$. Since $\boldsymbol{\Omega}$ is unknown, a two-step estimation procedure is applied.

Step One

The first step is the estimation of equation (2.2), which in vector form is $\mathbf{y} = \mathbf{X}\boldsymbol{\alpha} + \mathbf{Z}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, and use the residuals $\hat{\boldsymbol{\varepsilon}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\alpha}} + \mathbf{Z}\hat{\boldsymbol{\beta}}$ to estimate the matrix $\boldsymbol{\Omega}$. The cth diagonal element of $\boldsymbol{\Omega}$ is estimated as $\hat{\omega}_c = 1/n \cdot \sum_{i=1}^{n}\hat{\varepsilon}_{ic}^2$, where $\hat{\varepsilon}_{ic}$ is the corresponding element of $\hat{\boldsymbol{\varepsilon}}$.

Step Two

Define $\hat{\mathbf{\Omega}}^{-1/2} = \text{diag}\left[\hat{\omega}_1^{-1/2},\ldots,\hat{\omega}_C^{-1/2}\right]$ and $\hat{\mathbf{\Sigma}}^{-1/2} = \mathbf{I}_{n\times n} \otimes \hat{\mathbf{\Omega}}^{-1/2}$, which is the weight matrix in the second step regression:

$$\hat{\mathbf{\Sigma}}^{-1/2}\mathbf{y} = \hat{\mathbf{\Sigma}}^{-1/2}\mathbf{X\alpha} + \hat{\mathbf{\Sigma}}^{-1/2}\mathbf{Z\beta} + \mathbf{e},$$

where $\mathbf{e} = \hat{\mathbf{\Sigma}}^{-1/2}\boldsymbol{\varepsilon}$ is new disturbance vector with $\mathrm{E}\left(\mathbf{ee}'\right) = \mathrm{E}\left(\boldsymbol{\varepsilon}\hat{\mathbf{\Sigma}}^{-1}\boldsymbol{\varepsilon}'\right) = \sigma^2\mathbf{I}$. The second step estimator of $\boldsymbol{\gamma} = \left[\boldsymbol{\alpha}' \;\; \boldsymbol{\beta}'\right]'$ is

$$\hat{\boldsymbol{\gamma}} = \begin{bmatrix} \mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X} & \mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{Z} \\ \mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X} & \mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{Z} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{y} \\ \mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{y} \end{bmatrix}.$$

Applying Kronecker product rules, we have $\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1} = \boldsymbol{\iota}_n' \otimes \hat{\mathbf{\Omega}}^{-1}$ and $\mathbf{Z}'\mathbf{\Sigma}^{-1} = \left[\mathbf{I}_{(n\text{-}1)\times(n\text{-}1)} \quad -\boldsymbol{\iota}_{n-1}\right] \otimes \left(\boldsymbol{\iota}_C'\hat{\mathbf{\Omega}}^{-1}\right)$, $\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{Z} = \mathbf{0}_{c\times(n\text{-}1)}$ and $\mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X} = \mathbf{0}_{(n\text{-}1)\times c}$. Therefore, the second-step estimators are

$$\hat{\boldsymbol{\alpha}} = \left(\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{y} \text{ and } \hat{\boldsymbol{\beta}} = \left(\mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{Z}\right)^{-1}\mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{y}.$$

The inverses of the weighted moment matrices are

$$\left(\mathbf{X}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{X}\right)^{-1} = \left(\boldsymbol{\iota}_n'\boldsymbol{\iota}_n \otimes \hat{\mathbf{\Omega}}^{-1}\right)^{-1} = \frac{1}{n}\hat{\mathbf{\Omega}}\text{ , and}$$

$$\left(\mathbf{Z}'\hat{\mathbf{\Sigma}}^{-1}\mathbf{Z}\right)^{-1} = \frac{1}{\boldsymbol{\iota}_C'\hat{\mathbf{\Omega}}^{-1}\boldsymbol{\iota}_C}\left(\mathbf{I}_{(n\text{-}1)\times(n\text{-}1)} + \boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}'\right)^{-1} = \frac{1}{\boldsymbol{\iota}_C'\hat{\mathbf{\Omega}}^{-1}\boldsymbol{\iota}_C}\left(\mathbf{I}_{(n\text{-}1)\times(n\text{-}1)} - \frac{1}{n}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}'\right).$$

Substituting into the expressions for $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\beta}}$, we have

$$\hat{\boldsymbol{\alpha}} = \frac{1}{n}\hat{\mathbf{\Omega}}\cdot\left(\boldsymbol{\iota}_n' \otimes \hat{\mathbf{\Omega}}^{-1}\right)\mathbf{y} = \frac{1}{n}\left(\boldsymbol{\iota}_n' \otimes \mathbf{I}_{C\times C}\right)\mathbf{y},$$

$$\hat{\boldsymbol{\beta}} = \left[\mathbf{I}_{(n\text{-}1)\times(n\text{-}1)} - n^{-1}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}' \quad -n^{-1}\boldsymbol{\iota}_{n-1}\right] \otimes \left(\frac{1}{\boldsymbol{\iota}_C'\hat{\mathbf{\Omega}}^{-1}\boldsymbol{\iota}_C}\boldsymbol{\iota}_C'\hat{\mathbf{\Omega}}^{-1}\right)\mathbf{y}.$$

Scalar Expressions

From the above, scalar expressions for the two-step estimators are

$$\hat{\alpha}_c = \frac{1}{n}\sum_{i=1}^{n} y_{ic}, c = 1,\ldots,C, \quad \text{and } \hat{\beta}_i = \frac{1}{\sum_{c=1}^{C}\hat{\omega}_c^{-1}}\sum_{c=1}^{C}\hat{\omega}_c^{-1}\left(y_{ic} - n^{-1}\sum_{j=1}^{n} y_{jc}\right),\; i = 1,\ldots,n-1.$$

This estimator of α_c is the same as that from the one-step procedure. However, when the data are unbalanced (that is, when the number of turning points differs across cycles), the two sets of estimators of the β_i are not the same.

The Variances

The variances of the estimators are

$$\operatorname{var}(\hat{\boldsymbol{\alpha}}) = \frac{\sigma^2}{n}\hat{\boldsymbol{\Omega}} \text{ and } \operatorname{var}(\hat{\boldsymbol{\beta}}) = \sigma^2\left(\mathbf{Z}'\boldsymbol{\Sigma}^{-1}\mathbf{Z}\right)^{-1} = \frac{\sigma^2}{\boldsymbol{\iota}_C'\hat{\boldsymbol{\Omega}}^{-1}\boldsymbol{\iota}_C}\left(\mathbf{I}_{(n-1)\times(n-1)} - \frac{1}{n}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}_{n-1}'\right),$$

where σ^2 is the scaling factor of the disturbance covariance matrix.

The Omitted β_n

As the constraint $\sum_{i=1}^{n}\beta_i = 0$ is imposed by substituting $\beta_n = -\sum_{i\neq j}\beta_i$, the estimator and variance of β_n are

$$\hat{\beta}_n = -\boldsymbol{\iota}_{n-1}'\hat{\boldsymbol{\beta}} = \left[-n^{-1}\boldsymbol{\iota}_{n-1}' \quad 1-n^{-1}\right]\otimes\left(\frac{1}{\boldsymbol{\iota}_C'\hat{\boldsymbol{\Omega}}^{-1}\boldsymbol{\iota}_C}\boldsymbol{\iota}_C'\hat{\boldsymbol{\Omega}}^{-1}\right)\mathbf{y} = \frac{1}{\sum_{c=1}^{C}\hat{\omega}_c^{-1}}\sum_{c=1}^{C}\hat{\omega}_c^{-1}\left(y_{nc} - n^{-1}\sum_{j=1}^{n}y_{jc}\right)$$

and

$$\operatorname{var}\left(\hat{\beta}_n\right) = \operatorname{var}\left(-\boldsymbol{\iota}_{n-1}'\boldsymbol{\beta}\right) = \sigma^2\boldsymbol{\iota}_{n-1}'\left(\mathbf{Z}'\Sigma^{-1}\mathbf{Z}\right)^{-1}\boldsymbol{\iota}_{n-1} = \frac{\sigma^2}{\boldsymbol{\iota}_C'\boldsymbol{\Omega}^{-1}\boldsymbol{\iota}_C}\left(1-\frac{1}{n}\right).$$

A1.3 <u>A System of Two Sets of Turning Points</u>

Applying equation (2.2) to peaks and troughs separately, we have two sets of estimates, $\left(\hat{\boldsymbol{\alpha}}^P, \hat{\boldsymbol{\beta}}^P\right)$ and $\left(\hat{\boldsymbol{\alpha}}^T, \hat{\boldsymbol{\beta}}^T\right)$. Pooling peaks and troughs together, we have, for $c = 1,\ldots,C$ and $i = 1,\ldots,n$,

$$y_{ic} = \delta\alpha_c^P + (1-\delta)\alpha_c^T + \delta\beta_i^P + (1-\delta)\beta_i^T + \varepsilon_{ic},$$

where $\delta = 1$ if y_{ic} is a peak and 0 otherwise. The specific-metal effects β_i $(i = 1,\ldots,n)$ are now different for the two phases of the cycle: β_i^P represents the lead/lag for metal i for peaks (measured in terms of months) and β_i^T is the lead/lag for troughs, which satisfy $\sum_{i=1}^{n}\beta_i^P = \sum_{i=1}^{n}\beta_i^T = 0$. Combining all peaks and troughs for the n variables, we have

$$\mathbf{y} = \mathbf{X}^*\boldsymbol{\alpha} + \mathbf{Z}^*\boldsymbol{\beta} + \boldsymbol{\varepsilon},$$

where $\boldsymbol{\alpha} = \left[\alpha_1^P, \ldots, \alpha_C^P, \alpha_1^T, \ldots, \alpha_C^T\right]$ and $\boldsymbol{\beta} = \left[\beta_1^P, \ldots, \beta_{n-1}^P, \beta_1^T, \ldots, \beta_{n-1}^T\right]'$. The two matrices of dummy variables are

$$\mathbf{X}^* = \mathbf{I}_{2\times2} \otimes \left(\boldsymbol{\iota}_n \otimes \mathbf{I}_{C\times C}\right) \text{ and } \mathbf{Z}^* = \mathbf{I}_{2\times2} \otimes \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}'_{n-1} \end{bmatrix} \otimes \boldsymbol{\iota}_C.$$

Testing Parallel Phase Displacement

Define the vectors $\hat{\boldsymbol{\beta}}^P = \left[\hat{\beta}_1^P, \ldots, \hat{\beta}_{n-1}^P\right]'$ and $\hat{\boldsymbol{\beta}}^T = \left[\hat{\beta}_1^T, \ldots, \hat{\beta}_{n-1}^T\right]'$. Suppose the specific-metal effects are the same for peaks and troughs, that is, $\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T = \mathbf{0}$. We referred to this in the text as the case of "parallel phase displacement (PPD)". The validity of this assumption can be tested by the Wald statistic:

$$W = \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right)' \left[\operatorname{var}\left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right)\right]^{-1} \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right).$$

As $\hat{\boldsymbol{\beta}}^P$ and $\hat{\boldsymbol{\beta}}^T$ are independent, $\operatorname{var}\left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right) = \operatorname{var}\left(\hat{\boldsymbol{\beta}}^P\right) + \operatorname{var}\left(\hat{\boldsymbol{\beta}}^T\right)$. If $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \operatorname{diag}\left[\boldsymbol{\Sigma}_P, \boldsymbol{\Sigma}_T\right]$, where $\boldsymbol{\Sigma}_k = \mathbf{I}_{n\times n} \otimes \boldsymbol{\Omega}_k$, $k = P$ or T, and it can be shown that the estimators are the same as those from the previous approach with separate regressions for peaks and troughs. The Wald statistic can be simplified to

$$\begin{aligned} W &= \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right)' \sigma^{-2} \left[\left(\mathbf{Z}'\boldsymbol{\Sigma}_P^{-1}\mathbf{Z}\right)^{-1} + \left(\mathbf{Z}'\boldsymbol{\Sigma}_T^{-1}\mathbf{Z}\right)^{-1}\right]^{-1} \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right) \\ &= \sigma^{-2} \left(\frac{1}{\boldsymbol{\iota}'_C\boldsymbol{\Omega}_P^{-1}\boldsymbol{\iota}_C} + \frac{1}{\boldsymbol{\iota}'_C\boldsymbol{\Omega}_T^{-1}\boldsymbol{\iota}_C}\right)^{-1} \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right)' \left(\mathbf{I}_{(n-1)\times(n-1)} - \frac{1}{n}\boldsymbol{\iota}_{n-1}\boldsymbol{\iota}'_{n-1}\right) \left(\hat{\boldsymbol{\beta}}^P - \hat{\boldsymbol{\beta}}^T\right) \\ &= \sigma^{-2} \left(\frac{1}{\sum_{c=1}^C \left(\hat{\omega}_c^P\right)^{-1}} + \frac{1}{\sum_{c=1}^C \left(\hat{\omega}_c^T\right)^{-1}}\right)^{-1} \left[\sum_{i=1}^{n-1}\left(\hat{\beta}_i^P - \hat{\beta}_i^T\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n-1}\hat{\beta}_i^P - \sum_{i=1}^{n-1}\hat{\beta}_i^T\right)^2\right]. \end{aligned}$$

Since σ^2 is unknown, the unbiased estimator $s^2 = \hat{\boldsymbol{\varepsilon}}'\hat{\boldsymbol{\varepsilon}} \big/ \left((n-1)(2C-2)\right)$ can be used in its place. The Wald statistic asymptotically follows a chi-square distribution with the degree of freedom $n-1$. (Under homoscedasticity, $\hat{\omega}_c^P = \hat{\omega}_c^T = 1$ for all c in the above.) A significant value of W leads to a rejection of the null hypothesis. The Wald test of the hypothesis of PDD in our model yields a p-value of 0.38, so there is no strong evidence against the null.

Imposing PPD

When $\hat{\boldsymbol{\beta}}^{\mathrm{P}} - \hat{\boldsymbol{\beta}}^{\mathrm{T}} = \mathbf{0}$ the model can be written

$$\mathbf{y} = \mathbf{X}^{*}\boldsymbol{\alpha} + \mathbf{Z}^{**}\boldsymbol{\beta}^{*} + \boldsymbol{\varepsilon}^{*},$$

where $\boldsymbol{\beta} = \left[\beta_1, \ldots, \beta_{n-1}\right]'$ and

$$\mathbf{Z}^{**} = \boldsymbol{\iota}_2 \otimes \begin{bmatrix} \mathbf{I}_{(n-1)\times(n-1)} \\ -\boldsymbol{\iota}'_{n-1} \end{bmatrix} \otimes \boldsymbol{\iota}_{\mathrm{C}} = \begin{bmatrix} \mathbf{Z}_{\mathrm{P}} \\ \mathbf{Z}_{\mathrm{T}} \end{bmatrix}.$$

Allowing for heteroscedasticity, the new estimator of $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}}^{*} = \left(\mathbf{Z}^{**\prime}\boldsymbol{\Sigma}^{*-1}\mathbf{Z}^{**}\right)^{-1}\mathbf{Z}^{**\prime}\boldsymbol{\Sigma}^{*-1}\mathbf{y},$$

where $\boldsymbol{\Sigma}^{*} = \mathrm{diag}\left(\boldsymbol{\Sigma}_{\mathrm{P}}, \boldsymbol{\Sigma}_{\mathrm{T}}\right)$. Since this covariance matrix is block diagonal, we have

$$\mathbf{Z}^{**\prime}\boldsymbol{\Sigma}^{*-1}\mathbf{Z}^{**} = \left[\mathbf{Z}'_{\mathrm{P}} \quad \mathbf{Z}'_{\mathrm{T}}\right]\begin{bmatrix} \boldsymbol{\Sigma}_{\mathrm{P}}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\mathrm{T}}^{-1} \end{bmatrix}\begin{pmatrix} \mathbf{Z}_{\mathrm{P}} \\ \mathbf{Z}_{\mathrm{T}} \end{pmatrix} = \mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{Z}_{\mathrm{P}} + \mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{Z}_{\mathrm{T}}.$$

If the data are balanced, we have

$$\hat{\boldsymbol{\beta}}^{*} = \left(\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{Z}_{\mathrm{P}} + \mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{Z}_{\mathrm{T}}\right)^{-1}\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{y}_{\mathrm{P}} + \left(\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{Z}_{\mathrm{P}} + \mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{Z}_{\mathrm{T}}\right)^{-1}\mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{y}_{\mathrm{T}}$$

As $\hat{\beta}^{\mathrm{P}} = \left(\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{Z}_{\mathrm{P}}\right)^{-1}\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{y}_{\mathrm{P}}$ and $\hat{\beta}^{\mathrm{T}} = \left(\mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{Z}_{\mathrm{T}}\right)^{-1}\mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{y}_{\mathrm{T}}$, the estimator of $\boldsymbol{\beta}^{*}$ can be written as

$$\hat{\boldsymbol{\beta}}^{*} = \lambda\hat{\boldsymbol{\beta}}^{\mathrm{P}} + (1-\lambda)\hat{\boldsymbol{\beta}}^{\mathrm{T}}, \quad \mathrm{i}=1,\ldots,\mathrm{n}-1,$$

where

$$\lambda_{\mathrm{p}} = \left(\mathbf{Z}'_{\mathrm{P}}\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\mathbf{Z}_{\mathrm{P}} + \mathbf{Z}'_{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{T}}^{-1}\mathbf{Z}_{\mathrm{T}}\right)^{-1}\left(\mathbf{Z}'_{\mathrm{P}}\left(\boldsymbol{\Sigma}_{\mathrm{P}}^{-1}\right)\mathbf{Z}_{\mathrm{P}}\right) = \frac{\boldsymbol{\iota}'_{\mathrm{C}}\hat{\Omega}_{\mathrm{P}}^{-1}\boldsymbol{\iota}_{\mathrm{C}}}{\boldsymbol{\iota}'_{\mathrm{C}}\left(\hat{\Omega}_{\mathrm{P}}^{-1} + \hat{\Omega}_{\mathrm{T}}^{-1}\right)\boldsymbol{\iota}_{\mathrm{C}}}\mathbf{I}_{(n-1)\times(n-1)}$$

is a measure of that part of the total variance attributable to peaks.

Appendix A2

THE CASE OF TROUGHS 2 AND 6

The estimates of the dates of troughs 2 and 6 in Table 4.3 are four-five months away from the corresponding troughs of the price index. In this appendix, we study the reason for this.

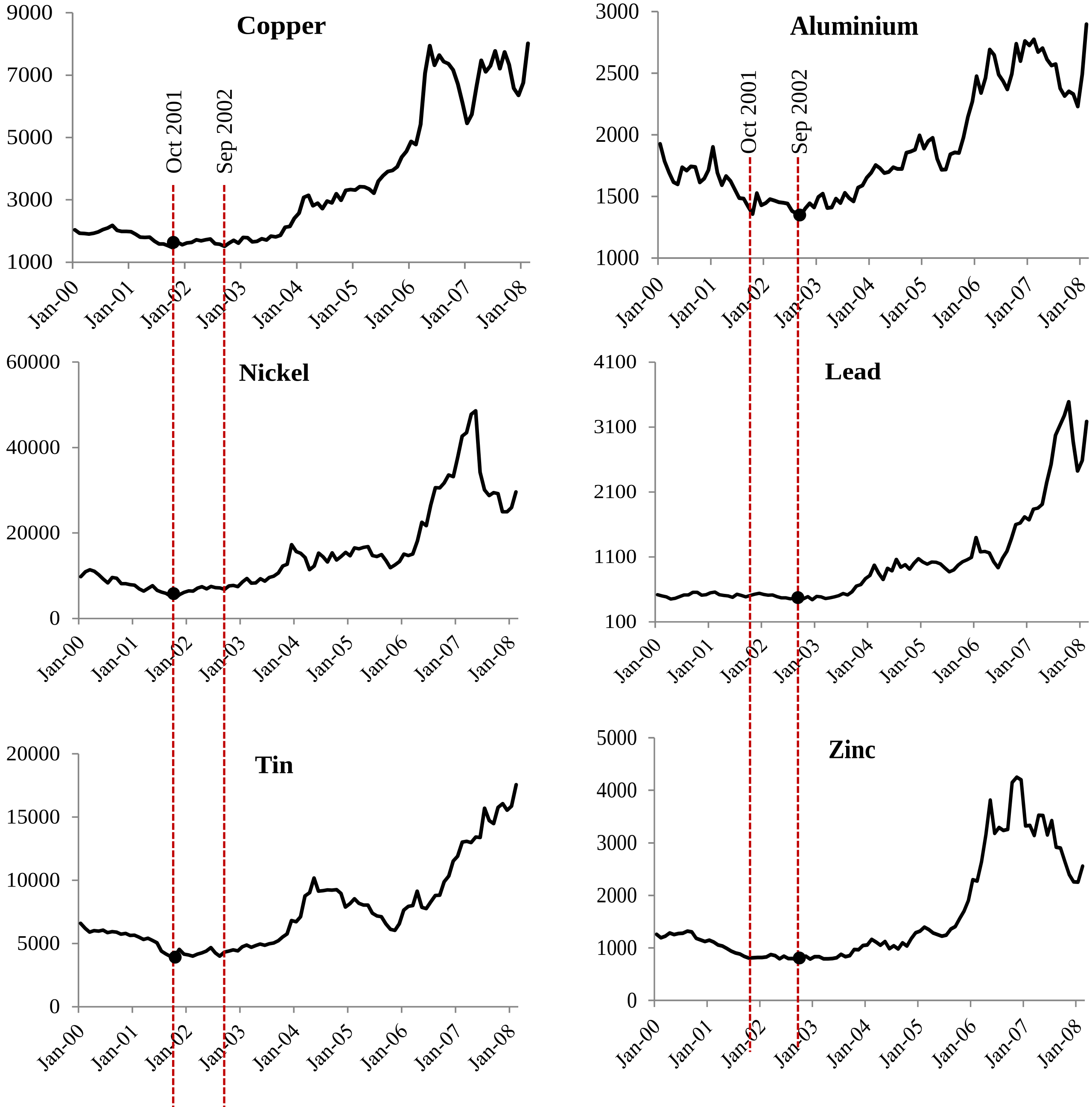
It is convenient to start with trough 6. This lies in the episode from 2000/01 to 2008/02, which is defined by peak 5 and peak 6 of the price index, as shown in Table 4.3. The prices during this episode are shown in Figure A2.1. Copper, nickel and tin all fall to their trough in October 2001, while the other three metals experience a trough in September 2002, which is 11 months later. The two dates are indicated in the figure by the broken vertical lines. The price index also reaches the trough in October 2001. Taking the weighted average of these six trough dates, the estimate of α_c for this episode in model (2.2) is March 2002, which necessarily lies between the two extremes, October 2001 and September 2002. Thus, this estimate is 5 months away from the trough of the price index (March 2002 – October 2001 = 5 months). This "bias" is not as bad as might seem to be the case for the following reason. The prices of the six metals are all low over the period from mid-2001 to mid-2003. Thus, the price index is also low during that period – at the trough in October 2001 the index took the value of 40.2 and it only increased marginally over the next 11 months to reach 41.1 by September 2002. In other words, the flatness of prices means that the date of this trough of the index is not particularly well defined; this, of course, is reflected in the high standard error of the estimated date (2.4 months).

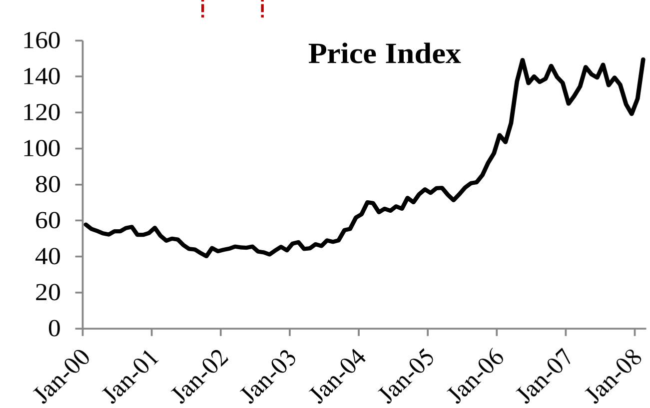
Prices were similarly flat around trough 2. Copper reached the bottom in May 1991, while the troughs of the other metals occurred after October 1991. The estimate of α_c from model (2.2) for this episode is August 1991. As the price of copper stays low for the whole year 1991, and as this metal is heavily weighted in the index, the price index reaches its trough in December 1991. The apparent bias of 5 months (December – August) is again understandable in terms of a period of sluggish prices.

To conclude, model (2.2) gives rise to reliable estimates of the dates of underlying cycles when prices behave in a clear cyclical manner. A bias only occurs when the turning points are not particularly distinct – when the market levels off around a peak or a trough and remains flat for a substantial period thereafter.

FIGURE A2.1

METALS PRICES, 2000-2008





Note: The vertical broken line on the left represents the date of the trough for copper, nickel and tin, and the price index, viz., Oct 2001. The vertical line on the right is the trough for aluminium, lead and zinc, Sep 2002.

Appendix A3

THE SIMULATIONS

There are two types of randomness involved in this approach to dating the cycle. First, there is the disturbance term in model (2.2), $y_{ic} = \alpha_c + \beta_i + \varepsilon_{ic}$, which measures "model uncertainty". As each cycle contains a boom and a slump, it is convenient to now move from measuring time in terms of cycles to "episodes", which are periods that contain turning points (either a peak or trough), to be denoted by e =1,…,E. Thus, the dates of the n turning points occurring in episode e are denoted by $y_{1e},\ldots,y_{ne}$, α_e is the date of the turning point of the underlying cycle, β_i is the phase parameter for metal i and ε_{ie} the disturbance. Using the price index (4.1) to define episodes, we apply this model to the 6 metals and E = 15 episodes.[13] That is,

$$\text{(A3.1)} \qquad y_{ie} = \alpha_e + \beta_i + \varepsilon_{ie}, \quad i = 1,\ldots,6,\ e = 1,\ldots,15.$$

By bootstrapping the estimates of disturbance term ε_{ie}, a distribution of durations of episodes are obtained.

The turning points dates used to estimate model (A3.1) are obtained by applying the Bry-Boschan (BB) algorithm to the prices. Thus, the second form of randomness in the approach stems from the prices. Let the relative price of metal i in month t be $r_{it} = \log\left(p_{it}/P_t\right)$, where P_t is the level of the index (4.1). Consider the following model for this relative price: $\Delta r_{it} = \theta_{i0} + \theta_{i1} t + \theta_{i2} r_{i,t-1} + \sum_{k=2}^{K} \gamma_{ik} \Delta r_{i,t-k+1} + \mu_{it}$, On the basis of the SIC, the lag length K=0 for all six metals. We also find that the drift parameter θ_{i0} and the time trend add very little in the majority of cases. Thus, the model becomes $\Delta r_{it} = \theta_{i2} r_{i,t-1} + \mu_{it}$. ADF unit root tests on the relative prices of six metals are shown in column 2 of Table A1. As the p-values are all greater than 0.3, the null hypothesis that the relative prices are unit-root process cannot be rejected. As shown in column 4, this unit-root model, $r_{it} = r_{i,t-1} + \mu_{it}$, explains a substantial part of the prices. On the basis of the above results, we use as the data generating process

$$\text{(A3.2)} \qquad \log p_{it} = \log P_t + \log\left(p_{i,t-1}/P_{t-1}\right) + \mu_{it}.$$

where μ_{it} is an iid disturbance.

[13] The price index contains 7 peaks and 7 troughs, plus the first slump.

TABLE A3.1

MODELLING METAL PRICES

Metals	ADF test (p-value)		$\Delta r_{it} = \theta_i r_{i,t-1} + \mu_{it}$		
			$\hat{\theta}_i$ (SE)		R^2 with $\theta_i = 0$
(1)	(2)		(3)		(4)
Aluminium	-0.809	(0.365)	-0.001	(0.420)	0.972
Copper	0.800	(0.885)	0.000	(0.425)	0.970
Lead	0.037	(0.694)	0.000	(0.970)	0.898
Nickel	-0.240	(0.599)	0.000	(0.811)	0.876
Tin	0.208	(0.746)	0.000	(0.835)	0.930
Zinc	-0.684	(0.420)	-0.001	(0.494)	0.904

Notes: 1. For the SIC, the maximum lag length is set at 15 and the result is that no lagged terms should be included.
2. The p-values in column 2 are MacKinnon (1996) one-sided p-values.

The algorithm for the simulation is as follows:

1. Calculate the residuals from (A3.2) as

$$\hat{\mu}_{it} = \log\left(p_{it}/P_t\right) - \log\left(p_{i,t-1}/P_{t-1}\right),\ i = 1,\ldots,6,\ t = 4,\ldots,273.$$

2. Permute the elements of $\hat{\boldsymbol{\mu}} = [\hat{\boldsymbol{\mu}}_4,\ldots,\hat{\boldsymbol{\mu}}_{273}]$, where $\hat{\boldsymbol{\mu}}_t = \left[\hat{\mu}_{1,t},\ldots,\hat{\mu}_{6,t}\right]$ to give a new residual vector $\hat{\boldsymbol{\mu}}^{(1)} = \left[\hat{\boldsymbol{\mu}}_4^{(1)},\ldots,\hat{\boldsymbol{\mu}}_{273}^{(1)}\right]$, and replace the disturbance μ_{it} in (A3.2) with the new value $\mu_{it}^{(1)}$ to give the simulated value of $\log p_{it}$, written as $\log p_{it}^{(1)}$.[14]

3. Use the BB algorithm to locate the turning points in the time series $\left\{\log p_{it}^{(1)}\right\}$, i=1,…6, t=1,…274, to be written as $\left\{y_{ie}^{(1)}\right\}$, $i = 1,\ldots,6$, $e = 1,\ldots,15$ episodes.

4. Use $\left\{y_{ie}^{(1)}\right\}$ to estimate model (A3.1) in the form

(A3.3) $y_{ie}^{(1)} = \alpha_e + \beta_i + \varepsilon_{ie}^{(1)},\quad i = 1,\ldots,6,\ e = 1,\ldots,15,$

where $\varepsilon_{ie}^{(1)}$ is the new disturbance associated with $y_{ie}^{(1)}$. We then proceed in four sub-steps:

i) Write the residuals $\hat{\varepsilon}_{ie}^{(1)}$ in vector form as $\hat{\boldsymbol{\varepsilon}}^{(1)} = \left[\hat{\varepsilon}_{ie}^{(1)}\right]$. Permute the elements of residual vector, to give a new residual series $\hat{\boldsymbol{\varepsilon}}^{(1,1)} = \left[\hat{\varepsilon}_{ie}^{(1,1)}\right]$, i=1,…,6, e=1,…,15.

ii) Replace the disturbance in model (A3.3) with $\hat{\varepsilon}_{ie}^{(1,1)}$ to give a new value of $y_{ie}^{(1)}$, written as $y_{ie}^{(1,1)}$.

[14] As the price index for the first three months are not available as a result of the moving average in volumes, as an approximation the observed prices $\log p_{it}$ are used as $\log p_{it}^{(1)}$ for $t = 1,\ 2,\ 3$ in the simulation.

iii) Re-estimate model (A3.3) in the form

$$y_{ie}^{(1,1)} = \alpha_e + \beta_i + \varepsilon_{ie}'^{(1,1)},\ \ i = 1,\ldots,6,\ e = 1,\ldots,15,$$

and write the estimates as $\hat{\alpha}_e^{(1,1)}$ and $\hat{\beta}_i^{(1,1)}$.

iv) Repeat steps 2-3 1,000 times to give $\{\hat{\alpha}_e^{(1,k)}\}$ and $\{\hat{\beta}_i^{(1,k)}\}$, k=1,…,1,000.

The procedures of this step are to be interpreted as referring to the weighted counterparts to allow for heteroscadesticity as discussed in the text.

5. Repeat steps 2-4 1,000 times to give $\{\hat{\alpha}_e^{(j,k)}\}$ and $\{\hat{\beta}_i^{(j,k)}\}$, j, k=1,…,1,000.

The simulation provides 1×10^6 realisations of turning points and durations. Thus, there are about 14×10^6 episodes (booms and slumps) so obtained.

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TABLE 2.1

PEAKS OF THREE VARIABLES

(Month number)

Disaggregated variable	Scenario		
	1	2	3
(1)	(2)	(3)	(4)
1	5	3	1
2	5	5	5
3	5	7	9
Mean	5	5	5
Standard deviation	0	$\sqrt{8/(3-1)} = 2$	$\sqrt{32/(3-1)} = 4$
Standard error of mean	0	$2/\sqrt{3} = 1.2$	$4/\sqrt{3} = 2.4$

TABLE 4.1

CORRELATION COEFFICIENTS FOR METAL PRICES

(Log-changes)

Metal	Copper	Lead	Nickel	Tin	Zinc	Index
Aluminium	0.64	0.39	0.50	0.49	0.49	0.82
Copper		0.45	0.51	0.47	0.59	0.94
Lead			0.39	0.38	0.54	0.54
Nickel				0.45	0.46	0.64
Tin					0.35	0.55
Zinc						0.70

TABLE 4.2

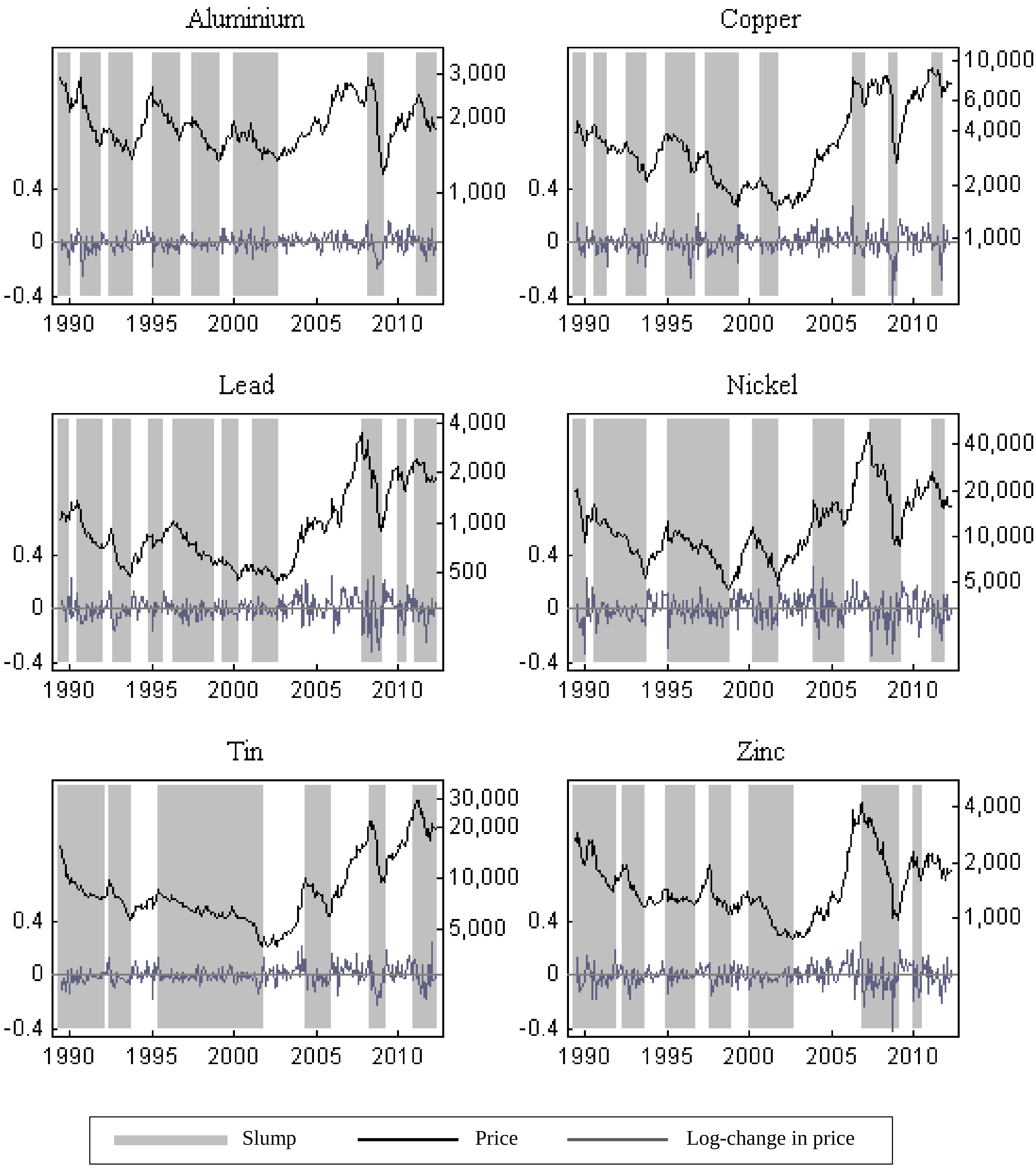
SUMMARY OF DURATION PHASES IN METAL PRICE CYCLES

	Slumps		Booms	
Metal	No. of episodes	Duration (No. of months)	No. of episodes	Duration (No. of months)
(1)	(2)	(3)	(4)	(5)
Aluminium	7	19	7	20
Copper	8	13	9	20
Lead	9	15	9	15
Nickel	6	26	7	18
Tin	5	30	5	22
Zinc	6	20	7	18
Mean				
All	6.83	21	7.33	19
No MB	5.33	22	5.66	12

Notes: Columns 3 and 5 are averages. The mean of the last row excludes the atypically long Millennium Boom (MB).

FIGURE 4.1

CYCLES IN METALS PRICES



Note: The dark lines are the prices, which refer to the right-hand side axes (in US dollars of 2005 per tonne, log scale). The grey lines are the monthly price log-changes, which refer to the left-hand axes. Shaded areas are the peak-to-trough slumps.

TABLE 4.3

THREE SETS OF TURNING POINTS FOR METAL PRICES

	Price index			Estimates of turning points and duration under							
				Homoscedasticity, $\sigma_{ic}^2 = \sigma^2$				Heteroscedasticity, $\sigma_{ic}^2 = \sigma^2\omega_c$			
	Turning point		Duration (No. of months)	Turning point		Duration (No. of months)		Turning point		Duration (No. of months)	
Event	Date	Month number		Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
				A. Peaks							
P1	1990/08	15	-	15	1.81	-		15	0.28	-	
P2	1992/07	38	23	37	1.74	23	2.51	37	0.57	22	0.64
P3	1995/01	68	30	67	1.70	30	2.43	68	1.12	31	1.25
P4	1997/05	96	28	97	1.83	30	2.49	97	1.42	28	1.81
P5	2000/01	128	32	130	1.69	33	2.49	129	1.83	32	2.32
P6	2008/02	225	97	224	1.65	94	2.36	224	2.56	95	3.15
P7	2011/02	261	36	260	1.64	36	2.33	261	1.49	37	2.97
Mean			41			41	2.44			41	2.02
				B. Troughs							
T1	1990/01	8	-	7	1.86	-		7	0.31	-	
T2	1991/12	31	23	26	1.82	18	2.60	27	1.66	20	1.69
T3	1993/11	54	23	53	1.78	27	2.54	52	0.24	25	1.67
T4	1996/09	88	34	87	1.86	35	2.58	87	1.07	35	1.09
T5	1999/01	116	28	117	1.81	30	2.60	116	1.00	29	1.46
T6	2001/10	149	33	155	1.72	37	2.50	154	2.43	38	2.63
T7	2009/01	236	87	236	1.72	81	2.43	236	0.69	82	2.52
T8	2011/09	268	32	268	2.21	32	2.80	268	0.81	32	1.06
Mean			37			40	2.57			40	1.74

Notes: 1. If during a given cycle of the price index there is more than one turning point for a given metal, the one corresponding to the highest price is selected for the peak and the lowest for the trough.

2. Duration in columns 4, 7 and 11 is defined as the length of the peak-to-peak cycle in panel A and the trough-to-trough cycle in panel B.

TABLE 4.4

TWO SETS OF LEADS/LAGS FOR SIX METALS

(Months)

Metal	Homoscedasticity $\sigma_{ic}^2 = \sigma^2$		Heteroscedasticity $\sigma_{ic}^2 = \sigma^2 \omega_c$	
(1)	(2)		(3)	
Aluminium	1.22	(1.07)	1.28	(0.25)
Copper	0.54	(1.01)	0.57	(0.25)
Lead	-0.13	(2.32)	-1.03	(0.47)
Nickel	-0.46	(1.81)	0.13	(0.32)
Tin	1.52	(3.21)	0.19	(0.47)
Zinc	-2.69	(1.45)	-1.14	(0.75)

FIGURE 4.2

TWO SCATTERS OF TURNING POINTS

(Dates of turning points – number of months)

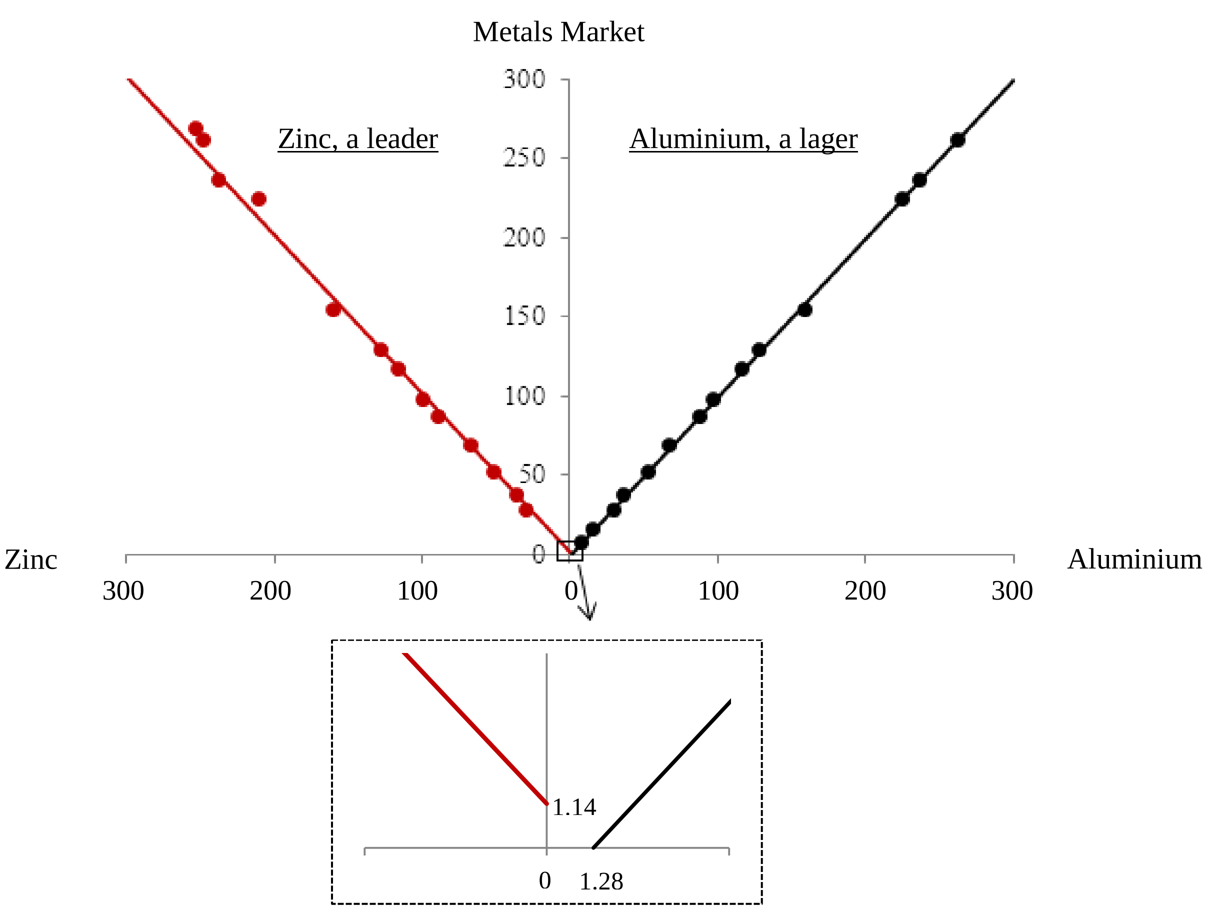
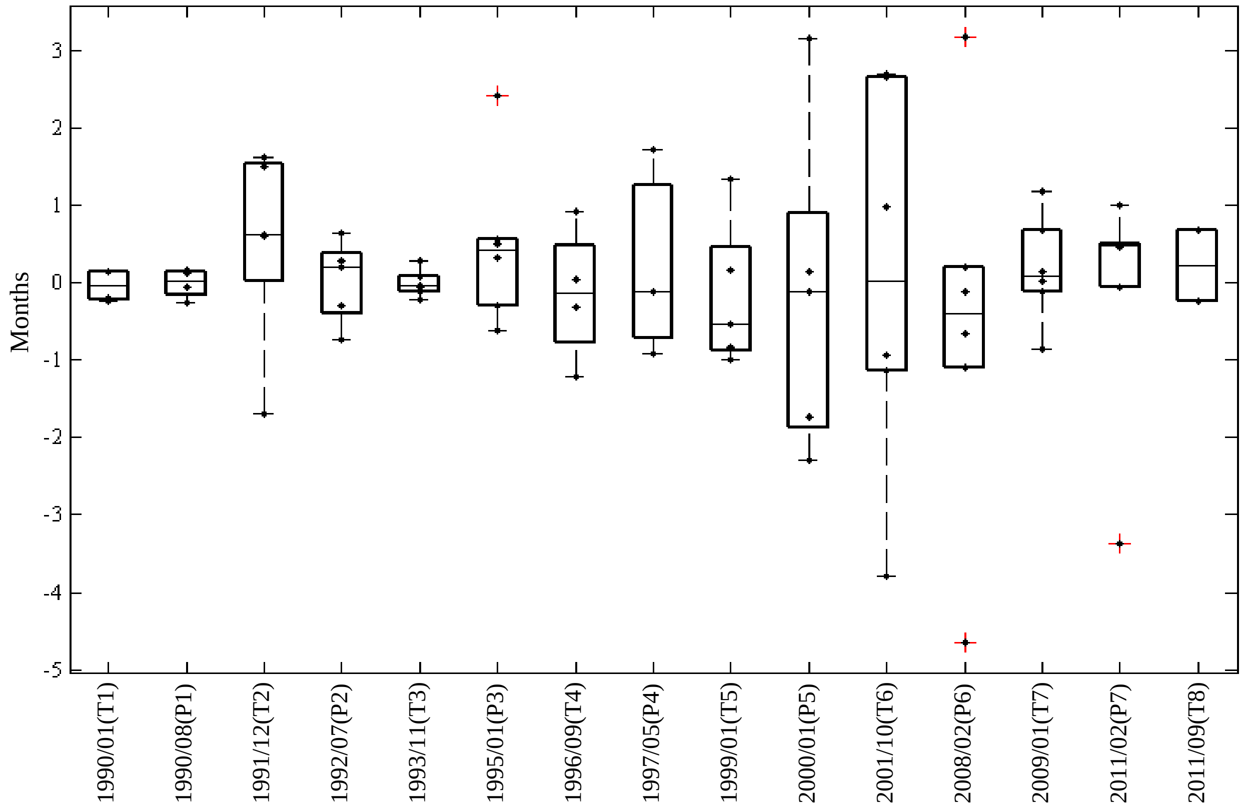


FIGURE 4.3

BOXPLOT OF RESIDUALS



Note: The residuals are from the regression $y_{ic} = \alpha_c + \beta_i + \varepsilon_{ic}$ without the heteroscadesticity adjustment.

TABLE 4.5

SIMULATION OF TURNING POINTS AND DURATIONS OF UNDERLYING CYCLE

Event	True Value			Turing Point				Duration			
	Turning point		Duration (No. of months)	Mean (Month number)	RMSE (Months)	RMSSE (Months)	RMSSE / RMSE	Mean (Month number)	RMSE (Months)	RMSSE (Months)	RMSSE / RMSE
	Date	Month number									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
T1	1989/12	7		7	0.32	0.27	0.84				
P1	1990/08	15	8	15	0.46	0.24	0.53	7	0.65	0.27	0.42
T2	1991/08	27	12	27	1.38	1.42	1.03	13	1.49	1.40	0.94
P2	1992/06	37	10	37	0.54	0.49	0.91	10	1.52	1.48	0.97
T3	1993/09	52	15	52	0.45	0.21	0.46	16	0.84	0.51	0.60
P3	1995/01	68	16	68	0.93	0.96	1.02	16	0.97	0.94	0.97
T4	1996/08	87	19	87	0.89	0.91	1.03	19	1.33	1.31	0.98
P4	1997/06	97	10	97	1.24	1.22	0.99	10	1.53	1.51	0.98
T5	1999/01	116	19	116	0.86	0.86	0.99	19	1.48	1.48	1.00
P5	2000/02	129	13	129	1.52	1.57	1.03	13	1.79	1.77	0.99
T6	2002/03	154	25	154	2.03	2.08	1.03	25	2.61	2.59	0.99
P6	2008/01	224	70	224	2.11	2.20	1.04	70	3.04	3.03	1.00
T7	2009/01	236	12	236	0.73	0.59	0.81	12	2.27	2.23	0.98
P7	2011/02	261	25	261	1.25	1.28	1.03	24	1.55	1.39	0.89
T8	2011/09	268	7	269	0.85	0.69	0.81	8	1.61	1.42	0.88
Mean			18.6		1.04	1.00	0.90	18.7	1.62	1.52	0.90

Notes: 1. The true values in column 3 are the estimates of α_c of model (2.2), from column 9 of Table 4.3.

2. Duration is the length of a boom or a slump.

3. RMSE is the root-mean-squared error. RMSSE is the root-mean-squared standard error.

TABLE 4.6

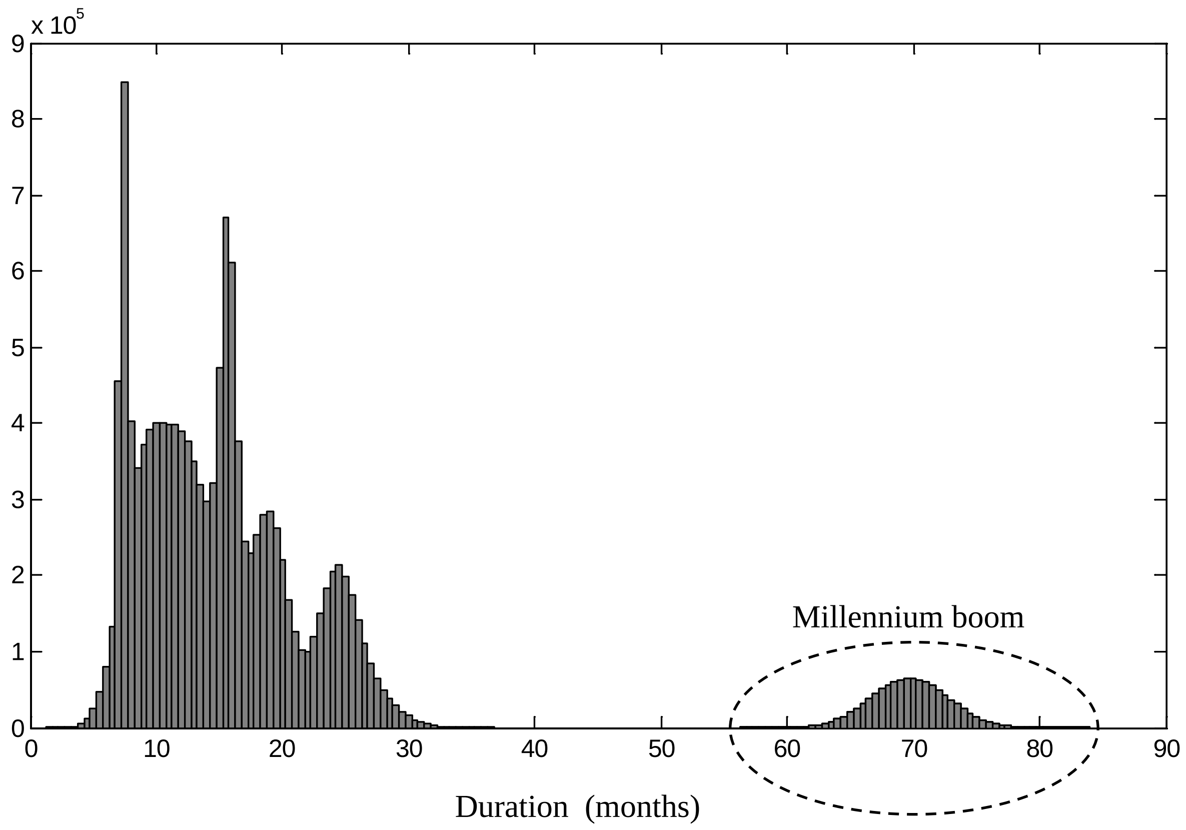
SIMULATION OF LEADS/LAGS FOR SIX METALS

Metal	True value (Months)	Simulated value				
		Mean (Months)	RMSE (Months)	RMSSE (Months)	RMSSE / RMSE
(1)	(2)	(3)	(4)	(5)	
Aluminium	1.28	1.27	0.21	0.21	1.00
Copper	0.57	0.56	0.21	0.21	0.99
Lead	-1.03	-1.02	0.40	0.40	1.01
Nickel	0.13	0.12	0.27	0.27	1.01
Tin	0.19	0.18	0.41	0.41	1.00
Zinc	-1.14	-1.11	0.63	0.64	1.02

Note: The true values in column 2 are the estimates of β_i of model (2.2), from column 3 of Table 4.4.

FIGURE 4.4

HISTOGRAM OF DURATION OF BOOMS



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