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Money and Inflation in a Macroeconomic Model with Indexed Bonds

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This book starts with three central questions that have preoccupied monetary economists since the emergence of paper money in the 18th century: Why is there inflation? Why is inflation costly? Why is inflation beneficial? William Coleman observes tongue-in-cheek: “There is inflation, but no one is quite sure how. Inflation is usually bad, but no one really knows why. Inflation is sometimes good, but no one has a confident answer when.” The main strength of this book is the coherent analytic framework that is constructed step-by-step in a clear and systematic way in the first nine chapters. Following a practice that has become almost universal in contemporary macroeconomics, the model is based on explicit microeconomic foundations. Still, it is appropriate to review this book in a history of thought journal because the author is inspired by the work of Hicks (1939/46), Kaldor (1970), Patinkin (1955/65), Samuelson (1947, pp. 117-122) and other mid-20th century economists. Although side-stepping the contributions of real business cycle macroeconomists and new-Keynesian macroeconomists, William Coleman arrives at an analytic framework that is close to a modern dynamic macroeconomic model with microeconomic foundations. That it is possible to construct a modern macroeconomic model building directly upon the work of famous mid-20th century economists indicates a degree of continuity in macroeconomic thought that is generally not appreciated.

Inflation in a Risk Free World

In Chapters 2 to 5, it is assumed that the future is perfectly known. Consumers can substitute between four assets: money, nominal bonds, real bonds and capital goods. Nominal bonds earn the nominal interest rate; real bonds, which are inflation indexed, yield the real interest rate; and capital goods are held because they produce a real return in the production process, the profit rate. Money, which does not earn an explicit return, is held because it serves as means of payment in a monetary economy and it is costly to convert nonmonetary assets into money. For this reason, money is included in the representative consumer’s utility function. More on this will be said in the section on inflation and uncertainty. Outside money which is created by the government and inside money which is supplied by the banks are perfect substitutes. The distinction between outside money and inside money is now rarely employed in macroeconomic models. However, it is important because the current financial crisis affects the supply of inside money by the ailing financial sector in the United States and elsewhere.

Equations (1) to (4) present the optimum conditions of a utility maximising consumer in a model with two time periods. Somewhat unconventionally, C indicates consumption in the current time period and $C1$ is consumption in the next period. The utility function has the form

$U(C, C1, h)$, where h measures the amount of real money holdings in the current time period. This is a more general utility function than the one usually used in macroeconomic models. The subjective discount rate is not explicitly shown because the marginal utility of future consumption U_{C1}, which is measured in the initial time period, directly captures time preference. Similarly, Fisher (1930, Chapter II, Mathematical Appendix) subsumed time preference in the marginal utility of future consumption.

Money: $$U_C = U_h + \frac{U_{C1}}{1+\pi} \quad (1)$$

Nominal Bonds: $$U_C = \frac{1+i}{1+\pi} U_{C1} \quad (2)$$

Real Bonds: $$U_C = (1+r) U_{C1} \quad (3)$$

Capital: $$U_C = (1+\rho) U_{C1} \quad (4)$$

Equation (1) states that the marginal utility of a dollar spent on current consumption U_C must be equal to the sum of marginal utility of hoarding the dollar U_h and spending it in the next period, where it yields marginal utility $U_{C1}/(1+\pi)$. π is the inflation rate. A dollar that is saved buys $1/(1+\pi)$ consumer goods in the next period and multiplying this with U_{C1} yields marginal utility $U_{C1}/(1+\pi)$. Equations (2) to (4) represent the optimum conditions for the demand for nominal bonds, real bonds and physical capital. Optimal behaviour requires that is not possible to raise utility by shifting spending from one period to the next, using one of the three assets as vehicle of saving. Therefore, the marginal utility of current consumption U_C equals the marginal utility of future consumption U_{C1}, taking account of the real return on saving. The real gross return equals $(1+i)/(1+\pi)$ for nominal bonds that pay the nominal interest rate i, it is $(1+r)$ for indexed bonds that yield the real interest rate r, and it is $(1+\rho)$ for physical capital that earns the profit rate ρ.

Similar optimum conditions can be found in recent real business cycle models and new-Keynesian macroeconomic models. The Ramsey-Solow model, which dominates advanced macroeconomics courses, includes nominal bonds and physical capital, and the Sidrauski model adds money. Coleman distinguishes between nominal bonds and real bonds, which is an interesting extension of these standard models, although it may be premature to include real bonds in a macroeconomic model. Indexed government bonds were introduced in the United Kingdom in 1981, in Australia in 1985 and in the United States in 1997. Today, many countries issue indexed government bonds, but quantities are small and there are no indexed corporate bonds. CPI futures, which can be used to hedge the inflation risk of a nominal bond, are even less common than indexed bonds. CPI futures were traded at the Coffee, Sugar and Cocoa Exchange in New York from 1985 to 1989 and they were reintroduced by the Chicago Mercantile Exchange in 2004.

Indexed bonds and CPI futures are unpopular because they provide incomplete protection against inflation and they may even increase monetary uncertainty. A practical difficulty arises because official inflation statistics are occasionally revised in some countries.[1] What should be done if the inflation rate that was used to settle a contract is revised at a later date? More importantly, different groups of consumers experience different inflation rates if their spending patterns differ. For example, the inflation rate that applies to the poor exceeds the official inflation rate if food prices increase more than other prices, and the inflation rate of the elderly exceeds the official inflation rate if an increase in the price of health care accounts for much of the official rate. Since the riskfree real interest rate is crucial in the model, these issues should have been discussed earlier than in footnote 1 on page 225.

A central bank may either control the quantity of outside money or it may set the nominal interest rate in a manner that was advocated by Knut Wicksell. The book provides comparative static results for both monetary policy regimes. A transitory increase in the supply of outside money leads to a less than proportional increase in the price level, whereas a permanent increase in the supply of outside money produces a proportional increase in the price level. Thus, the quantity theory of money holds if the supply of outside money is increased permanently. The current financial crisis in the United States amounts to a negative shock to the supply of inside money, which reduces the price level. Other comparative static results concern shocks to money demand and shocks to the profit rate on physical capital.

The Wicksellian approach to monetary policy, which has been used by most central banks since the early 1980s, focuses on the credit market. Equations (2) and (4), the optimum conditions for nominal bonds and physical capital, yield the demand price for credit:

$$i = \rho + \pi + \rho\pi \approx \rho + \pi \qquad (5)$$

[1] The Consumer Price Index is not subject to revisions in Australia.

Optimal behaviour implies that the nominal interest rate is equal to the real return on capital plus the inflation rate. This is the Fisher equation. Since perfect foresight is assumed, there is no distinction between the actual inflation rate and expected inflation.

In Wicksell's model the supply price of credit is determined by the central bank. Coleman proposes that the central bank implements the following interest rate rule:

$$i = \bar{i} + \phi(\ln P - \ln P_R) \tag{6}$$

The central bank raises the nominal interest rate above the benchmark rate $\bar{i}$ if the actual price level P exceeds the reference value P_R. The parameter ϕ determines the strength of the central bank's policy response. The gap between the actual price level and the target level is measured in percent, using the logarithmic approximation. In Wicksell's model the central bank must satisfy any demand for outside money at the target interest rate. Therefore, equation (1) has no important function, although it can be used to calculate the endogenous supply of outside money, which lacks policy significance.

The comparative static analysis of the Wicksellian model yields no surprises. An increase in the benchmark interest rate reduces the price level, increases in the target price level and the profit rate both increase the price level, and a higher future price level increases the current price level. In a multiperiod model, the current price level positively depends on the price level in the next period, which positively depends on the price level one period further ahead, and so on. Therefore, a higher benchmark interest rate reduces current prices, irrespective of whether the interest rate is raised now or in any future time period. In the same way, increases in the target price level and the profit rate raise current prices, whatever the date of the increases in the target price level and the profit rate. As it is unlikely that there are strong trends in the profit rate or the target interest rate, the cause of long-run inflation is a sustained increase in the central bank's target price level.

Inflation in a Risky World

In Chapters 6 to 9, the model is extended to a world in which consumers have imperfect knowledge of the future. Two sources of uncertainty are considered: technological uncertainty affects the profit rate of physical capital and monetary uncertainty influences the inflation rate. Consumers exchange real bonds and physical capital until an efficient allocation of technological uncertainty is attained, and they trade nominal bonds and capital until the allocation of monetary risk is efficient. Chapters 6 and 7 are excellent, providing the microeconomic foundations for a novel treatment of asset demand in macroeconomic models that stresses the optimal allocation of risk among consumers.

The analysis of money demand, however, has shortcomings. In Chapter 2, it was reasoned that money yields utility because money must be used as means of payment and it is costly to convert nonmonetary assets into money. This rationale for the inclusion of real money holdings in the utility function works better when there is uncertainty than when there is perfect foresight. Brunner and Meltzer (1971) convincingly demonstrated that money is not needed if there is perfect foresight. If there are no costs of acquiring information, a barter-credit economy arises "in which differences in the timing of receipts and payments are adjusted by issuing verbal promises in exchange for goods and, later, delivering goods" (Brunner and Meltzer, 1971, footnote 4). This implies that the marginal utility of money is really zero in the perfect foresight model in Chapters 2 to 5. The unsatisfactory treatment of money demand is a missed opportunity in this book. Still, the harm is limited because, starting with Sidrauski (1967), it has become common practice to include money in the utility function in macroeconomic models, even if the reason for doing it is obscure.

Equations (7) to (10) are the adjusted optimum conditions for the four asset markets in an uncertain world.

Money: $$U_C = U_h + E\left[\frac{U_{C1}}{1+\pi}\right] = U_h + E\left[\frac{1}{1+\pi}\right]E\left[U_{C1}\right] + \text{cov}\left[\frac{1}{1+\pi}, U_{C1}\right] \quad (7)$$

Nominal Bonds: $$U_C = (1+i)E\left[\frac{U_{C1}}{1+\pi}\right] = (1+i)\left[E\left[\frac{1}{1+\pi}\right]E\left[U_{C1}\right] + \text{cov}\left[\frac{1}{1+\pi}, U_{C1}\right]\right] \quad (8)$$

Real Bonds: $$U_C = (1+r)E\left[U_{C1}\right] \quad (9)$$

Capital: $$U_C = E\left[(1+\rho)U_{C1}\right] = E\left[1+\rho\right]E\left[U_{C1}\right] + \text{cov}\left[(1+\rho), U_{C1}\right] \quad (10)$$

Three quantities are uncertain: the inflation rate, the profit rate and future consumption. Therefore, the expectation operator E is applied to these three quantities. For example, $E[U_{C1}/(1+\pi)]$ is used in the money market equation because U_{C1} and π are uncertain, and $E[(1+\rho)U_{C1}]$ is used in the capital market equation because ρ is also uncertain. The issuers of nominal bonds must pay the agreed nominal interest rate irrespective of monetary shocks and technological shocks, and the issuers of real bonds must pay the real interest rate whatever the state of the world in the next period. Since the return on nominal bonds and the return on indexed bonds are certain, (1+*i*) and (1+*r*) are factored out from the expectation term in equations (8) and (9). The second equality sign in equations (7), (8) and (10) uses the product rule for expectations.[2] In the money market equation and the equation for nominal bonds, $1/(1+\pi)$ and U_{C1} are random. Using the product rule yields:

$$E\left[\frac{1}{1+\pi}U_{C1}\right]=E\left[\frac{1}{1+\pi}\right]E\left[U_{C1}\right]+\operatorname{cov}\left[\frac{1}{1+\pi},U_{C1}\right] \qquad (11)$$

Similarly, in the capital market equation the product rule for expectations is applied for the random variables $(1+\rho)$ and U_{C1}.

A special feature of this model is that it includes indexed bonds. Since consumers can earn a riskless profit by buying indexed bonds, they hold money and nominal bonds only if they are compensated for monetary or inflation risk. Therefore, the covariance between inflation and consumption affects both money demand and the demand for nominal bonds in equations (7) and (8). Coleman assumes that inflation and consumption are negatively correlated. Then, $\operatorname{cov}[1/(1+\pi),U_{C1}]$ is negative because an increase in inflation *reduces* the ratio $1/(1+\pi)$ and a fall in consumption *raises* marginal utility U_{C1}. A negative correlation between inflation and consumption implies that nominal bonds carry monetary risk because nominal bonds yield a low real return when inflation is high and consumption is low. An asset whose return is low when consumption is low is risky because the low return occurs when extra income enhances utility strongly. For this reason, monetary risk reduces the demand for nominal bonds. Similarly, a negative covariance between inflation and consumption reduces money demand.

However, business cycle history does not support the view that inflation and consumption are necessarily negatively correlated. For example, Romer (2006, Table 4.3) shows that there is no strong relationship between inflation and the business cycle in the United States. If inflation and consumption are positively correlated, the covariance term in equations (7) and (8) is positive. This increases the demand for money and nominal bonds because nominal assets provide a hedge against monetary risk.

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[2] The expectation of the product of two random variables x_1 and x_2 is:

$$E\left[x_1\cdot x_2\right]=E\left[x_1\right]\cdot E\left[x_2\right]+\operatorname{cov}\left[x_1,x_2\right]$$

The demand for physical capital depends on the covariance between the profit rate and consumption. Diminishing marginal utility implies that $\mathrm{cov}[1+\rho, U_{C1}]$ is negative if the profit rate and consumption are positively correlated. Then, the holders of physical capital incur technological risk because profits are high when consumption is high and profits are low when consumption is low. Thus, extra profit income is earned when it is not really needed, and profit income is low when it would help more. As a consequence, technological risk reduces the demand for physical capital. Note that covariances matter for all asset demands, and not variances. The co-movement between asset returns and consumption matters, whereas the volatility of asset returns is irrelevant. This is a standard finding of the consumption-based asset pricing model, which has been developed by Lucas (1978) and Breeden (1979).

The comparative static analysis generally supports the findings of the perfect-foresight model. Indeed, Coleman concludes that the model with risk "emerges as a superior vehicle for presenting the Quantity Theory's ideas" and that "Wicksellian ideas in some ways lend themselves better to a model with risk than to a model with perfect foresight." Yet, the analysis of the model with risk is incomplete. Kydland and Prescott (1977) and Barro and Gordon (1983) pointed out that monetary policy rules lack credibility. How can the central bank convince the public that it will keep the inflation rate low when it is politically expedient to raise inflation in order to reduce unemployment? Coleman pays no attention to the issue of credibility of monetary policy rules. This omission affects the discussion of the effect of a pre-announced future change in monetary policy on current economic outcomes. How can the central bank convince the public that it will indeed follow some time path of outside money in the Quantity Theory Model or of the target price level in the Wicksellian approach? The issue of credibility lies at the heart of the current monetary policy debate in Japan. Starting with Krugman (1998), many economists have suggested that the Bank of Japan should promise to expand the money supply in the future in order to overcome current deflation, but nobody knows how such a commitment can be made credible.

Presentation

This is a well structured book that develops a coherent analytic framework in a systematic way, but it is quite hard to read because derivations often do not refer to earlier results that are needed for the understanding of a line of argument, and few hints are given when algebraic results are derived. It is likely that the reader will take many results on trust. As a service to the reader, the adjusted Fisher equation (8.10) is derived here. The adjusted Fisher equation determines the demand price for nominal bonds in the Wicksellian model with uncertainty.

$$i \approx E[\rho] + E[\pi] + \operatorname{cov}\left[1+\rho, \frac{U_{C1}}{E[U_{C1}]}\right] - \operatorname{cov}\left[\frac{1}{1+\pi}, \frac{U_{C1}}{E[U_{C1}]}\right] \tag{12}$$

Optimal behaviour requires that the nominal interest rate must be approximately equal to the sum of the expected profit on physical capital and the expected inflation rate, where both expectations are adjusted for consumption risk that is measured by the respective covariances. Here, the covariances are scaled by expected marginal utility in order to render their measuring unit comparable with an interest rate.

The adjusted Fisher equation (12) can be derived from equations (8) and (10) in the following way. First, equate the two equations:

$$(1+i)E\left[\frac{1}{1+\pi}\right]E[U_{C1}] + (1+i)\operatorname{cov}\left[\frac{1}{1+\pi}, U_{C1}\right] = E[1+\rho]E[U_{C1}] + \operatorname{cov}[1+\rho, U_{C1}] \tag{13}$$

Then, divide both sides of the equation by $E[U_{C1}]$. This expected value is a constant that can be multiplied into the covariance terms.

$$(1+i)E\left[\frac{1}{1+\pi}\right] + (1+i)\operatorname{cov}\left[\frac{1}{1+\pi}, \frac{U_{C1}}{E[U_{C1}]}\right] = E[1+\rho] + \operatorname{cov}\left[1+\rho, \frac{U_{C1}}{E[U_{C1}]}\right] \tag{14}$$

Ignoring products of small decimal fractions, an approximation of the first term on the left-hand-side is:

$$\begin{aligned}(1+i)E\left[\frac{1}{1+\pi}\right] &= E\left[\frac{1+i}{1+\pi}\right] = E\left[\frac{(1+i)(1-\pi)}{(1+\pi)(1-\pi)}\right] = E\left[\frac{1+i-\pi-i\pi}{1-\pi^2}\right] \\ &\approx E[1+i-\pi] = 1+i-E[\pi]\end{aligned} \tag{15}$$

It follows:

$$1+i-E[\pi] + (1+i)\operatorname{cov}\left[\frac{1}{1+\pi}, \frac{U_{C1}}{E[U_{C1}]}\right] \approx E[1+\rho] + \operatorname{cov}\left[1+\rho, \frac{U_{C1}}{E[U_{C1}]}\right] \tag{16}$$

On the left-hand-side, use another approximation, $(1+i)\,\text{cov}[...] \approx \text{cov}[...]$. This is legitimate if the absolute value of the covariance is small, yielding a small product $i \times \text{cov}[...]$. No estimate for this covariance is provided, but it is probably small because marginal utility is scaled by expected marginal utility. Finally, calculating the expectation $E[1+\rho]=1+E[\rho]$ on the right-hand-side of the equation and subtracting 1 on both sides produces the adjusted Fisher equation (12).

Clearly, the derivation of the adjusted Fisher equation (12) is not trivial and it should not be left to the reader. Also note that the author uses the symbols ω_ρ and $\omega_{-\pi}$ for the *negative* of the covariances. Therefore, the signs before the covariances are the opposite in equation (12) from those in equation (8.10) of the book under review.

Costs and Benefits of Inflation

In the first nine chapters the first question is answered: Why is there inflation? The remaining chapters deal with the other two questions: Why is inflation costly? Why is inflation beneficial? Chapter 10 considers the costs and benefits of inflation that arise in the money market. Inflation reduces real money demand, which lowers welfare because money is an argument in the utility function. At the same time, inflation increases the supply of inside money by the banks. Inside money is wasteful because the government can create outside money at virtually no cost. Therefore, inflation increases the social loss from the use of inside money, although the individual issuer benefits at the cost of nonissuers of inside money. These costs of inflation are offset by the role that money plays in a system of optimal taxation. Phelps (1973) proposed that seigniorage, which is the revenue of the government from supplying outside money, may involve a smaller welfare loss than other forms of taxation. Overall, the conclusion in this chapter is that the net welfare loss of money growth and inflation that arises in the money market is small and that it provides only a weak argument for price stability.

The analysis of the costs and benefits of inflation that occur in the credit market, which is provided in Chapter 11, is less convincing. In a model that includes real bonds, inflation does not affect the credit market because borrowers and lenders who use indexed bonds face no inflation risk. For this reason, the analytic framework that has been used in the preceding ten chapters is suddenly abandoned and it is assumed that there are no indexed bonds. Then, uncertain inflation interferes with the efficient allocation of risk in the credit market. This is a promising start to the analysis, but it leads to the totally unrealistic conclusion that in the long-run inflation produces a more equitable distribution of wealth because workers who are initially endowed with a small amount of capital save more. This is reminiscent of the doctrine of forced saving, whose classical origins are discussed in Hayek (1932). The difference between the classical doctrine of inflation-induced saving and Coleman's approach is that in the former inflation forces consumers to save more, whereas in the latter it is optimal to save more. Irrespective of this difference, rapid inflation has never produced a more equitable distribution of wealth, certainly not during the German hyperinflation after World War I which tore apart the fabric of German society by creating a massive redistribution of wealth away from the middle class, and also not during the current hyperinflation in Zimbabwe which benefits a small group of cronies.

It is difficult to give justice to this intriguing book within the confines of a short review. The book follows a tradition in economics that stresses the advancement of economic theory over empirical research - an approach that was favoured for example by Niehans (1993). The microeconomic analysis is skilful, combining analytic rigour with a high degree of economic intuition. The optimal allocation of risk among consumers is viewed as central for macroeconomic outcomes. The use of microeconomic reasoning in macroeconomics gives the book a distinctly modern flavour, and yet virtually no attention is paid to macroeconomic research during the past 20 to 30 years. For this reason, the book cannot be used as an introduction to modern macroeconomics. However, it provides in its own right a coherent macroeconomic framework with strong microeconomic foundations that is influenced by the work of mid-20th century economists.

References

Barro, R.J. and D.B. Gordon. 1983. 'A Positive Theory of Monetary Policy in a Natural-Rate Model', *Journal of Political Economy* 91, pp. 589-610.

Breeden, D.T. 1979. 'An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities', *Journal of Financial Economics* 7, pp. 265-296.

Brunner, K. and A.H. Meltzer. 1971. 'The Uses of Money: Money in the Theory of an Exchange Economy', *American Economic Review* 61, pp. 784-805.

Fisher, I. 1930. *The Theory of Interest*. New York: Macmillan.

Hayek, F.A. 1932. 'A Note on the Development of the Doctrine of "Forced Saving"', *Quarterly Journal of Economics* 47(1), pp. 123-133.

Hicks, J.R. 1939/46. *Value and Capital*. London: Oxford University Press.

Kaldor, N. 1970. 'The New Monetarism', *Lloyd's Bank Review* 95, pp. 1-18.

Krugman, P. 1998. 'It's Baaack! Japan's Slump and the Return of the Liquidity Trap', *Brookings Papers on Economic Activity* 2, pp. 137-187.

Kydland, F.E. and E.C. Prescott. 1977. 'Rules Rather Than Discretion: The Inconsistency of Optimal Plans', *Journal of Political Economy* 85, pp. 473-491.

Lucas, R.E. 1978. 'Asset Prices in an Exchange Economy', *Econometrica* 46, pp. 1429-45.

Niehans, J. 1993. 'Revolution and Evolution in Economic Theory', Bateman Memorial Lecture, University of Western Australia, *Australian Quarterly* 65(1), pp. 498-515.

Patinkin, D. 1955/65. *Money, Interest and Prices*. New York: Harper and Row.

Phelps, E.S. 1973. 'Inflation in the Theory of Public Finance', *Scandinavian Journal of Economics* 75, pp. 67-82.

Romer, D. 2006. *Advanced Macroeconomics*. 3rd ed., New York: McGraw-Hill.

Samuelson, P.A. 1947. *Foundations of Economic Analysis*. Cambridge, MA: Harvard University Press.

Sidrauski, M. 1967. 'Rational Choice and Patterns of Growth in a Monetary Economy', *American Economic Review* 57(2), pp. 534-544.