

**PRICE ELASTICITIES OF DEMAND
ARE MINUS ONE-HALF**

by

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Abstract

As an empirical regularity for broad commodity groups, we show that price elasticities of demand are scattered around the value of minus one-half. We also show that this finding is not inconsistent with the utility-maximising theory of the consumer under the conditions of preference independence. When nothing is known about the price-sensitivity of a good, a reasonable first approximation to its price elasticity is thus minus one-half.

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The value of the price elasticity of demand for a product is needed in a vast array of problems in microeconomics. When detailed econometric estimates of this elasticity are unavailable, for products that are broad aggregates, the value of minus one-half is a useful first approximation. This paper establishes this rule of thumb on two grounds, (i) as an empirical regularity and (ii) on the basis of the utility-maximising theory of consumer demand under the conditions of preference independence (or additive utility). This rule could be of use in CGE models which can require hundreds of price elasticities, as well as in the economic analysis of illicit drugs where data are usually insufficient to estimate elasticities.¹

1. Evidence on Price Elasticities

Table 1 and Figure 1 present information from reviews regarding estimated price elasticities for several commodities that have been analysed extensively in the literature. Panels 1-3 of the table show that the major alcoholic beverages have mean/median elasticities not too away from the value minus one-half. The means from Selvanathan and Selvanathan (2005a) are beer -0.37 (with standard error 0.09), wine -0.46 (0.08) and spirits -0.57 (0.12), so that the maximum distance from $-\frac{1}{2}$ is a mere 1.4 standard errors. The next two panels of the table, Panels 4 and 5, show that more or less the same result holds for cigarettes and residential water. For petrol, the long-run elasticities tend to be closer to minus one-half than the short-run values, but the reverse seems to be true for electricity (panels 6 and 7). Finally, Panel 8 of the table shows that the mean price elasticity of “branded products” is -1.76 , which is substantially different from $-\frac{1}{2}$. But this is not unexpected as there are many good substitutes for a branded product -- other brands of the same basic product. In this sense then, branded products are fundamentally different to the others in the table: Branded products are much more narrowly defined than are products such as alcoholic beverages, cigarettes, water, petrol and electricity. In what follows, we develop a theory that is applicable to broader products only, not more narrowly defined goods like branded products.

2. Differential Demand Equations

The differential approach to consumption theory was introduced by Theil (1980) as a way to analyse the pattern of consumer demand with minimal a priori restrictions on behaviour. This section sets out the approach, with the underlying technical material presented in the Appendix.

Let p_i , q_i be the price and quantity consumed of good i , so that if n is the number of goods, $M = \sum_{i=1}^n p_i q_i$ is total expenditure (“income” for short) and $w_i = p_i q_i / M$ is the i^{th} budget

¹ Regarding the import of the price elasticity for drugs, Caulkins (1999, p. 175) comments “...the elasticity of demand is perhaps the single most important parameter for evaluating the efficacy of US drug control policy.”

share. The budget constraint of the consumer is $\sum_{i=1}^n p_i q_i = M$, and its total differential is $dM = \sum_{i=1}^n (q_i dp_i + p_i dq_i)$. Using the identity $d(\log x) = dx/x$, we can write the budget constraint as $d(\log M) = d(\log P) + d(\log Q)$, where $d(\log P) = \sum_{i=1}^n w_i d(\log p_i)$ and $d(\log Q) = \sum_{i=1}^n w_i d(\log q_i)$ are price and volume indexes defined as budget-share weighted averages of the n price and quantity log-changes. As the volume index can be expressed as $d(\log Q) = d(\log M) - d(\log P)$, the excess of the change in nominal income over the price index, we can identify $d(\log Q)$ as the change in the consumer's real income. Under general conditions, we can write the utility-maximising demand equation for good i in terms of the changes in real income and the n relative prices as

$$(2.1) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n v_{ij} [d(\log p_j) - d(\log P')].$$

The variable on the left-hand side of this equation, the logarithmic change in the quantity demanded of good i weighted by its budget share, has two interpretations. First, it is the quantity component of the change in the i^{th} budget share, which can be confirmed by taking the differential of $w_i = p_i q_i / M$, to yield $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log M)$. As prices and income are taken to be exogenous in consumption theory, the quantity component of the change in the budget share is the endogenous part of the overall change and thus is a reasonable choice for the dependent variable of a demand equation. The second interpretation of term $w_i d(\log q_i)$ is the contribution of good i to the change in real income, $d(\log Q) = \sum_{i=1}^n w_i d(\log q_i)$. The right-hand side of equation (2.1) is made up of an income term, $\theta_i d(\log Q)$, and a relative price term, $\sum_{j=1}^n v_{ij} [d(\log p_j) - d(\log P')]$. In the income term, $\theta_i = \partial(p_i q_i) / \partial M$ is the marginal share of good i which answers the question, if income rises by one dollar by how much does expenditure on i increase? As all of income is assumed to be spent, these marginal shares have a unit sum. Accordingly, the income term of the i^{th} demand equation is a fraction θ_i of the change in real income.

The term in square brackets on the right-hand side of equation (2.1), $d(\log p_j) - d(\log P')$, is the change in the relative price of good j , with the nominal price change deflated by the price index $d(\log P') = \sum_{i=1}^n \theta_i d(\log p_i)$. This index uses as weights the marginal shares, and is thus a marginal price index, which is known as the Frisch index. Deflation by this index serves to hold the marginal

utility of income constant. The coefficient attached to the j^{th} relative price change in equation (2.1) is v_{ij} , which is defined as $(\lambda p_i p_j / M) u^{ij}$, where $\lambda > 0$ is the marginal utility of income and u^{ij} is the $(i, j)^{\text{th}}$ element of the inverse of the Hessian matrix of the utility function. If $v_{ij} > 0$ (< 0), then an increase in the relative price of good j causes consumption of i to increase (decrease), and the two goods are said to be Frisch substitutes (complements); and if $v_{ij} = 0$, the two goods are independent. These v_{ij} are known as Frisch price coefficients and satisfy

$$(2.2) \quad \sum_{j=1}^n v_{ij} = \phi \theta_i, \quad i = 1, \dots, n,$$

where $\phi = (\partial \log \lambda / \partial \log M)^{-1}$ is the reciprocal of the income elasticity of the marginal utility of income, which is known as the “income flexibility” for short. A sufficient condition for a budget-constrained utility maximum is that the $n \times n$ Hessian matrix of the utility function and its inverse $[u^{ij}]$ are both symmetric negative definite; this means that the $n \times n$ matrix of Frisch price coefficients $[v_{ij}]$ is also symmetric negative definite. The pattern of the matrix $[v_{ij}]$ is a convenient way of organising prior ideas about the manner in which the n goods interact in consumption, as will be illustrated below. To understand further the workings of equation (2.1), divide both sides by w_i to yield

$$(2.3) \quad d(\log q_i) = \eta_i d(\log Q) + \sum_{j=1}^n \eta_{ij} [d(\log p_j) - d(\log P')].$$

where $\eta_i = \theta_i / w_i$ is the i^{th} income elasticity and $\eta_{ij} = v_{ij} / w_i$ is the elasticity of demand for good i with respect to the j^{th} relative price. As this relative price uses the Frisch index as the deflator, this elasticity holds constant the marginal utility of income and is known as the $(i, j)^{\text{th}}$ Frisch price elasticity.

3. Preference Independence

Next, suppose the n goods are all sufficiently broad aggregates so that they are unlikely to exhibit much substitutability. This would be consistent with tastes that exhibit preference independent, whereby the consumer’s utility function is (some increasing function of) the sum of n sub-utility functions, one for each good: $u(q_1, \dots, q_n) = \sum_{i=1}^n u_i(q_i)$, with $u_i(q_i)$ the i^{th} sub-utility function that depends only on the consumption of good i . Preference independence (PI) implies that each marginal utility depends only on the consumption of the good in question, not the others, so that all second-order cross derivatives of the utility function vanish. PI means that as commodities do not interact in the utility function, utility is derived from the consumption of good 1

and good 2 and good 3, and so on, where the word “and” is underlined to emphasise the additive nature of preferences. Such a hypothesis about tastes is clearly more applicable to broader aggregates than to more finely distinguished goods such as “branded products”.

Under PI, the Hessian of the utility function is diagonal, so that all cross-price Frisch coefficients are zero, $v_{ij} = 0, i \neq j$. Moreover, equation (2.2) implies that $v_{ii} = \phi\theta_i$, so that the relative price term of demand equation (2.1) takes a simplified form

$$\sum_{j=1}^n v_{ij} [d(\log p_j) - d(\log P')] = \phi\theta_i [d(\log p_i) - d(\log P')],$$

and equation (2.3) becomes

$$(3.1) \quad d(\log q_i) = \eta_i d(\log Q) + \phi\eta_i [d(\log p_i) - d(\log P')].$$

As only the own-relative price appears in the demand equation for good i under PI, so that all goods are pairwise independent, this characterisation of tastes clearly restricts the substitution possibilities. Thus as the broader aggregates have few substitutes, this is another way of understanding why it is that these goods are more suitable for the application of PI. Another implication of PI is that inferior goods are ruled out, which is also not unreasonable for broad aggregates.

A comparison of equations (2.3) and (3.1) reveals that the under PI, the own-price elasticity η_{ii} is proportional to the corresponding income elasticity with the income flexibility playing the role of the proportionality factor:

$$(3.2) \quad \eta_{ii} = \phi\eta_i, \quad i=1, \dots, n.$$

Accordingly, luxuries (goods with $\eta_i > 1$) are more price elastic than necessities ($\eta_i < 1$). Deaton (1974) refers to a variant of this proportionality relationship as “Pigou’s (1910) law”. The proportionality relationship (3.2) agrees with the intuitive idea that necessities (luxuries) tend to be essential (discretionary) goods, which have few (many) substitutes.

4. The Rule of -1/2

The budget constraint implies that a budget-share weighted average of the income elasticities is unity, that is, $\sum_{i=1}^n w_i \eta_i = 1$, so that an “average” commodity has an income elasticity of unity. This, together with equation (3.2), means that for such a commodity, the own-price elasticity $\eta_{ii} = \phi$. An alternative way to establish the same result is to consider an average commodity in price elasticity space; the price elasticity of this good is given by a budget-share weighted average of the n price elasticities, $\sum_{i=1}^n w_i \eta_{ii}$. If we multiply both sides of the proportionality relationship

(3.2) by w_i , sum over $i=1, \dots, n$, and use $\eta_i = \theta_i/w_i$ and $\sum_{i=1}^n \theta_i = 1$, we then obtain $\sum_{i=1}^n w_i \eta_{ii} = \phi$.

This reveals that the average price elasticity is also equal to ϕ . A substantial body of research points to the value of the income flexibility ϕ being approximately equal to minus one half.² This means for an average commodity, the price elasticity will also take the value of minus one half. As not all goods will coincide with the “average” exactly, we have to modify the above statement to the weaker form that the n price elasticities will be *approximately* equal to minus one half.³ It is to be noted that the above price elasticities are of the Frisch variety, whereby the marginal utility of income is held constant. As the more common Slutsky (or compensated) own-price elasticity

² Selvanathan (1993) uses time-series data to estimate a differential demand system for 15 OECD countries. When the data are pooled over the 15 countries, the estimate of ϕ is -0.45 , with asymptotic standard error 0.02 (Selvanathan, 1993, p. 198). Using a related approach, Selvanathan (1993, Sec. 6.4) obtains 322 estimates of ϕ , one for each year in the sample period for each of 18 OECD countries; the weighted mean of these estimates is very similar to the above value at -0.46 (ASE = 0.03). Two other cross-country estimates of ϕ are also relevant: Using the ICP data for 30 countries from Kravis et al. (1982), Theil (1987, Sec. 2.8) obtains a ϕ -estimate of -0.53 (0.04). Chen (1999, p. 171) estimates a demand system for 42 countries and obtains an estimate of ϕ of -0.42 (0.05), when there are intercepts in his differential demand equations, which play the role of residual trends in consumption, and -0.29 (0.05) when there are no such intercepts. The final element of support for $\phi = -0.5$ is the earlier, but still influential, survey by Brown and Deaton (1972, p. 1206) who review previous findings and conclude that “there would seem to be fair agreement on the use of a value for ϕ around minus one half”.

It should be noted that treating the income flexibility as a constant parameter is at variance with Frisch’s (1959) famous conjecture that ϕ should increase in absolute value as the consumer becomes more affluent. However, most tests of the Frisch conjecture tend to reject it; see, e.g., Clements and Theil (1996), Selvanathan (1993, Secs. 4.8 and 6.5), Theil (1975/76, Sec. 15.4), Theil (1987, Sec. 2.13) and Theil and Brooks (1970/71). Such a finding is not surprising as the Frisch conjecture refers to the third-order derivative of the utility function, and most consumption data could not be expected to be very informative about the nature of this higher-order effect. On the other hand however, evidence supporting Frisch has been reported by DeJanvry et al. (1972) and Lluch et al. (1977). Note also that according to Frisch (1959, p. 189) a ϕ -value of -0.5 would pertain to the “middle income bracket, ‘the median part’ of the population”.

³ Deaton (1974) examines whether price and income elasticities are (approximately) proportional, as predicted by preference independence. On the basis of UK data, he finds no such relationship and concludes that “*the assumption of additive preferences [preference independence] is almost certain to be invalid in practice and the use of demand models based on such an assumption will lead to severe distortion of measurement*” (his emphasis). Deaton’s rejection of preference independence (PI) on the basis of indirect evidence (the lack of proportionality of price and income elasticities) is consistent with first-generation direct tests of the implied parametric restrictions on the demand equations; see Barten (1977) for a survey. These results can be responded to in two ways. First, Selvanathan (1993) examines elasticities from 18 OECD countries, and finds the evidence not inconsistent with the proportionality relationship, indicating that Deaton may have been premature in declaring the invalidity of PI. Second, as the first-generation tests of the hypothesis of PI have only an asymptotic justification, it is appropriate to exercise caution in taking the results at face value when the underlying sample sizes are not large. To avoid potential problems with asymptotics associated with modest sample sizes, Selvanathan (1987, 1993) develops a Monte Carlo test of PI, and the results reject the hypothesis much less frequently. For example, in applications of the methodology, Selvanathan and Selvanathan (2005b) reject PI in 9 countries out of a total of 45; Clements et al. (1997) are unable to reject PI for beer, wine and spirits in all of the 7 countries they consider; and Selvanathan and Selvanathan (2005a, p. 235) are unable to reject the hypothesis for the three alcoholic beverages in 10 out of 10 countries. While there is still scope for differing views on this matter, it now seems safe to conclude that the assumption of PI should not be rejected out of hand, or at least as rapidly as the older studies might suggest.

differs from its Frisch counterpart by a term of order $1/n$, in most cases the differences will be small, so the Slutsky elasticities will also be scattered around the value minus one-half.^{4,5}

5. Summary

The argument of this paper can be summarised as follows:

- On the basis of a large number of studies (of the order of several hundred) of the price-sensitivity of consumption of broad commodities, price elasticities of demand are scattered around the value minus one-half.
- This empirical regularity is not inconsistent with the utility maximisation under the conditions of preference independence (or additive utility), whereby price elasticities are proportional to income elasticities, with factor of proportionality the income flexibility (the reciprocal of the income elasticity of the marginal utility of income). As a number of studies point to the value of the income flexibility being in the vicinity of minus one-half, when the income elasticity is unity (as it is on average), the corresponding price elasticity is also minus one-half.
- When nothing is known about the price-sensitivity of a good, a reasonable first approximation to its price elasticity is thus minus one-half.

⁴ The relationship between the Slutsky (η'_{ii}) and Frisch (η_{ii}) own-price elasticities is $\eta'_{ii} = \eta_{ii}(1 - \theta_i)$, where θ_i is the marginal share of i . As $\sum_{i=1}^n \theta_i = 1$, the order of the marginal shares is $1/n$, as is the difference between η'_{ii} and η_{ii} .

⁵ Note that in the general case constraint (2.2) implies that $\eta_{i\bullet} = \phi \eta_i$, where $\eta_{i\bullet} = \sum_{j=1}^n \eta_{ij}$ is the sum of the own- and cross-price elasticities involving good i and η_i is the i^{th} income elasticity. Thus whereas PI implies that the own-price elasticities are proportional to the corresponding income elasticities, when we give up the assumption of PI the sums of own- and cross-price elasticities are proportional to income elasticities. Application of the argument in this section then shows that these sums are approximately equal to minus one-half.

It is worthwhile to note that Powell (1992) shows that under PI the average elasticity of substitution $\sigma \approx -\phi$. Powell has pointed out to me that in an early application of demand analysis, Leser (1960) actually set each pairwise σ (and thus their average) at $1/2$ on the basis that this was about midway between his two initial estimates of 0.60 and 0.39.

APPENDIX 1

ELEMENTS OF CONSUMPTION THEORY

The objective of this appendix is to use the utility-maximising theory of the consumer to derive the demand equation (2.1) for $i = 1, \dots, n$. This involves applying the method for comparative statics to obtain information about the nature of general Marshallian demand equations. For more details of this material, see Clements (1987) and Theil (1980).

Marshallian Demand Equations

It is assumed that the consumer chooses the quantities of the n goods, q_1, \dots, q_n to maximise the utility function $u(q_1, \dots, q_n)$, subject to the budget constraint $\sum_{i=1}^n p_i q_i = M$, where M is total expenditure (“income” for short). If we write $\mathbf{q} = [q_i]$ and $\mathbf{p} = [p_i]$ for the vectors of n quantities and prices, the utility function and the budget constraint become $u(\mathbf{q})$ and $\mathbf{p}'\mathbf{q} = M$. It is assumed that each marginal utility is positive, so that $\partial u / \partial \mathbf{q} > \mathbf{0}$; and that there is generalised diminishing marginal utility, so that the Hessian matrix of the utility function $\mathbf{U} = \partial^2 u / \partial \mathbf{q} \partial \mathbf{q}'$ is negative definite. This latter condition is sufficient to ensure an interior maximum.

The first-order conditions for a maximum are the budget constraint

$$(A1) \quad \mathbf{p}'\mathbf{q} = M,$$

and the proportionality conditions, $\partial u / \partial q_i = \lambda p_i$, $i = 1, \dots, n$, where $\lambda > 0$ is the marginal utility of income. We write these as

$$(A2) \quad \frac{\partial u}{\partial \mathbf{q}} = \lambda \mathbf{p}.$$

These are $n+1$ scalar equations in (A1) and (A2), which can in principle be solved for the $n+1$ endogenous variables q_1, \dots, q_n, λ in terms of the $n+1$ endogenous variables M, p_1, \dots, p_n . We write this solution as

$$(A3) \quad \mathbf{q} = \mathbf{q}(M, \mathbf{p}),$$

which is a system of n Marshallian demand equations.

Comparative Statics

We use equation (A1) and (A2) to ask, How do \mathbf{q} and λ respond to changes in M and \mathbf{p} ? To answer this question, we proceed in three steps.

First, we differentiate (A2) with respect to M and \mathbf{p} to yield

$$(A4) \quad \mathbf{U} \frac{\partial \mathbf{q}}{\partial M} = \frac{\partial \lambda}{\partial M} \mathbf{p}, \quad \mathbf{U} \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = \lambda \mathbf{I} + \mathbf{p} \frac{\partial \lambda}{\partial \mathbf{p}'},$$

where \mathbf{U} is the Hessian matrix; $\partial \mathbf{q} / \partial M = [\partial q_i / \partial M]$ is a vector of n income slopes of the demand equations (A3); $\partial \mathbf{q} / \partial \mathbf{p}' = [\partial q_i / \partial p_j]$ is the $n \times n$ matrix of price derivatives; \mathbf{I} is the $n \times n$ identity matrix; and $\partial \lambda / \partial \mathbf{p}' = [\partial \lambda / \partial p_i]$. *Second*, we differentiate (A1) with respect to M and \mathbf{p} :

$$(A5) \quad \mathbf{p}' \frac{\partial \mathbf{q}}{\partial M} = 1, \quad \mathbf{p}' \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} = -\mathbf{q}'.$$

Third, to solve (A4) and (A5) simultaneously, we combine them to yield Barten's (1964) fundamental matrix equation of consumption theory:

$$(A6) \quad \begin{bmatrix} \mathbf{U} & \mathbf{p} \\ \mathbf{p}' & 0 \end{bmatrix} \begin{bmatrix} \partial \mathbf{q} / \partial M & \partial \mathbf{q} / \partial \mathbf{p}' \\ -\partial \lambda / \partial M & -\partial \lambda / \partial \mathbf{p}' \end{bmatrix} = \begin{bmatrix} 0 & \lambda \mathbf{I} \\ 1 & -\mathbf{q}' \end{bmatrix}.$$

The second matrix on the left-hand side of equation (A6) contains the derivatives of all the endogenous variables with respect to all the endogenous variables. This equation can be solved for the derivatives of the demand equations (see Clements, 1987, Sec. 1.11, for details) to yield

$$\begin{aligned} \frac{\partial \mathbf{q}}{\partial M} &= \frac{\partial \lambda}{\partial M} \mathbf{U}^{-1} \mathbf{p} \\ \frac{\partial \mathbf{q}}{\partial \mathbf{p}'} &= \lambda \mathbf{U}^{-1} - \frac{\lambda}{\partial \lambda / \partial M} \frac{\partial \mathbf{q}}{\partial M} \frac{\partial \mathbf{q}'}{\partial M} - \frac{\partial \mathbf{q}}{\partial M} \mathbf{q}'. \end{aligned}$$

Writing u^{ij} for the $(i, j)^{\text{th}}$ element of \mathbf{U}^{-1} , the above two matrix equations can be expressed in scalar form as

$$(A7) \quad \frac{\partial q_i}{\partial M} = \frac{\partial \lambda}{\partial M} \sum_{j=1}^n u^{ij} p_j, \quad i = 1, \dots, n,$$

$$(A8) \quad \frac{\partial q_i}{\partial p_j} = \lambda u^{ij} - \frac{\lambda}{\partial \lambda / \partial M} \frac{\partial q_i}{\partial M} \frac{\partial q_j}{\partial M} - \frac{\partial q_i}{\partial M} q_j, \quad i, j = 1, \dots, n.$$

A Differential Demand System

In this section, we use the above results to derive a general differential demand system. Consider the i^{th} equation of the Marshallian demand system (A3), $q_i = q_i(M, p_1, \dots, p_n)$. The total differential of this equation is

$$dq_i = \frac{\partial q_i}{\partial M} dM + \sum_{j=1}^n \frac{\partial q_i}{\partial p_j} dp_j.$$

We transform this into logarithmic-differential form by multiplying both sides by p_i/M and using the identity for any positive variable x , $dx/x = d(\log x)$,

$$(A9) \quad w_i d(\log q_i) = \frac{\partial(p_i q_i)}{\partial M} d(\log M) + \sum_{j=1}^n \frac{p_i p_j}{M} \frac{\partial q_i}{\partial p_j} d(\log p_j),$$

where $w_i = p_i q_i / M$ is the budget share of good i . Using equation (A8), the second term on the right-hand side of (A9) can be expressed as

$$\sum_{j=1}^n \frac{p_i p_j}{M} \left(\lambda u^{ij} - \frac{\lambda}{\partial \lambda / \partial M} \frac{\partial q_i}{\partial M} \frac{\partial q_j}{\partial M} - \frac{\partial q_i}{\partial M} q_j \right) d(\log p_j),$$

so that, after rearrangement, equation (A9) becomes

$$(A10) \quad w_i d(\log q_i) = \frac{\partial(p_i q_i)}{\partial M} \left[d(\log M) - \sum_{j=1}^n w_j d(\log p_j) \right] + \sum_{j=1}^n \left(\frac{\lambda p_i p_j u^{ij}}{M} - \frac{\lambda / M}{\partial \lambda / \partial M} \frac{\partial(p_i q_i)}{\partial M} \frac{\partial(p_j q_j)}{\partial M} \right) d(\log p_j).$$

Equation (A10) can be simplified as follows. The total differential of the budget constraint (A1) is $\sum_i p_i dq_i + \sum_i q_i dp_i = dM$, or, if we divide total sides by M , and use $w_i = p_i q_i / M$ and $dx/x = d(\log x)$, $\sum_i w_i d(\log q_i) + \sum_i w_i d(\log p_i) = d(\log M)$. As $d(\log M) - \sum_i w_i d(\log p_i) = \sum_i w_i d(\log q_i)$, the term in square brackets on the right-hand side of equation (A10) is interpreted as the Divisia index of the change in the consumer's real income; that is, the excess of the growth in money income, $d(\log M)$, over a budget-share weighted average of the n price changes, $\sum_i w_i d(\log p_i)$. This change in real income is also known as the Divisia volume index and written as $d(\log Q)$. In equation (A10), this $d(\log Q)$ is multiplied by $\partial(p_i q_i) / \partial M$, which is the marginal share of good i , to be denoted by θ_i . The marginal share answers the question, If

income increases by one dollar, how much of this is spent on good i ? As the additional income is entirely spent, it follows that $\sum_i \theta_i = 1$.

To simplify the price substitution term of (A10), we define

$$\phi = \frac{\lambda/M}{\partial \lambda / \partial M} = \left(\frac{\partial \log \lambda}{\partial \log M} \right)^{-1} < 0,$$

as the reciprocal of the income elasticity of the marginal utility of income, or the income flexibility for short. We also define

$$v_{ij} = \frac{\lambda}{M} p_i p_j u^{ij}, \quad i, j = 1, \dots, n,$$

which, in view of equation (A7), satisfy

$$(A11) \quad \sum_{j=1}^n v_{ij} = \phi \theta_i, \quad i = 1, \dots, n,$$

which is equation (2.2) of the text. With the above definitions, the price substitution term of equation (A10) becomes

$$\begin{aligned} \sum_{j=1}^n \left(\frac{\lambda p_i p_j u^{ij}}{M} - \frac{\lambda/M}{\partial \lambda / \partial M} \frac{\partial (p_i q_i)}{\partial M} \frac{\partial (p_j q_j)}{\partial M} \right) d(\log p_j) &= \sum_{j=1}^n (v_{ij} - \phi \theta_i \theta_j) d(\log p_j) \\ &= \sum_{j=1}^n v_{ij} \left[d(\log p_j) - \sum_{k=1}^n \theta_k d(\log p_k) \right] \\ &= \sum_{j=1}^n v_{ij} \left[d(\log p_j) - d(\log P') \right], \end{aligned}$$

where the second step is based on constraint (A11), while in the third step the Frisch price index, $d(\log P') = \sum_k \theta_k d(\log p_k)$, has been introduced. This index uses marginal shares as weights.

Retracing our steps, it can be seen that demand equation (A10) can be written as

$$(A12) \quad w_i d(\log q_i) = \theta_i d(\log Q) + \sum_{j=1}^n v_{ij} \left[d(\log p_j) - d(\log P') \right],$$

which is equation (2.1) of the text. The variable of the left of (A12) has the dual interpretation as (i) the quantity component of the change in w_i , as $dw_i = w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log M)$; and (ii) the contribution of good i to the Divisia volume index. The right-hand side of (A12) comprises a real income term and a relative price term, which deals with the substitution effects of changes in the n prices. The income term is a multiple θ_i of the change in real income as measured by the Divisia volume index $d(\log Q)$. As $d(\log Q) = d(\log M) - d(\log P)$, with

$d(\log P) = \sum_i w_i d(\log p_i)$, the Divisia price index, it can be seen that this price index transforms the change in money income into the change in real income. Furthermore, as the Divisia price index uses budget shares as weights, this index measures the income effect of the n price changes on the demand for good i .

In the relative price term on the right-hand side of equation (A12), $\sum_j v_{ij} [d(\log p_j) - d(\log P)]$, the Frisch price index $d(\log P)$ acts as the deflator of the nominal price change $d(\log p_j)$ to give the change in the relative price of good j . In the demand equation for good i , the j^{th} relative price change is multiplied by v_{ij} , which is known as the $(i, j)^{\text{th}}$ Frisch price coefficient. It follows from the definition of v_{ij} given above equation (A11) that the price coefficients are symmetric in i and j , and that the $n \times n$ matrix $[v_{ij}]$ is negative definite. Note that as $\sum_i \theta_i = 1$, equation (A11) implies that $\sum_i \sum_j v_{ij} = \phi$; as the left-hand side of this equation is a quadratic form with matrix $[v_{ij}]$ and vector $[1, \dots, 1]'$, the negative definiteness of $[v_{ij}]$ implies that $\phi < 0$. Under preference independence, the matrix $[v_{ij}]$ is diagonal with v_{11}, \dots, v_{nn} on the diagonal; as this matrix is negative definite, $v_{ii} < 0$. Preference independence also implies that equation (A11) takes the form $v_{ii} = \phi \theta_i$, which means that the marginal share θ_i must be positive as $\phi < 0$. This shows that under PI inferior goods are ruled out.

The major attractions of demand equation (A12) are its generality and its elegant simplicity. As the “coefficients” of this equation, θ_i and v_{ij} , are not necessarily constant, (A12) is consistent with (almost) any form of the utility function. In contrast to other approaches to generating demand equations, a feature of the differential approach is that it requires no algebraic specification of the utility function, the indirect utility function or the cost function. The elegance of equation (A12) revolves around its transparent link to a general utility function, its clean split between income and substitution effects (of both the specific and general varieties) and the ease of interpretation of its coefficients.

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TABLE 1

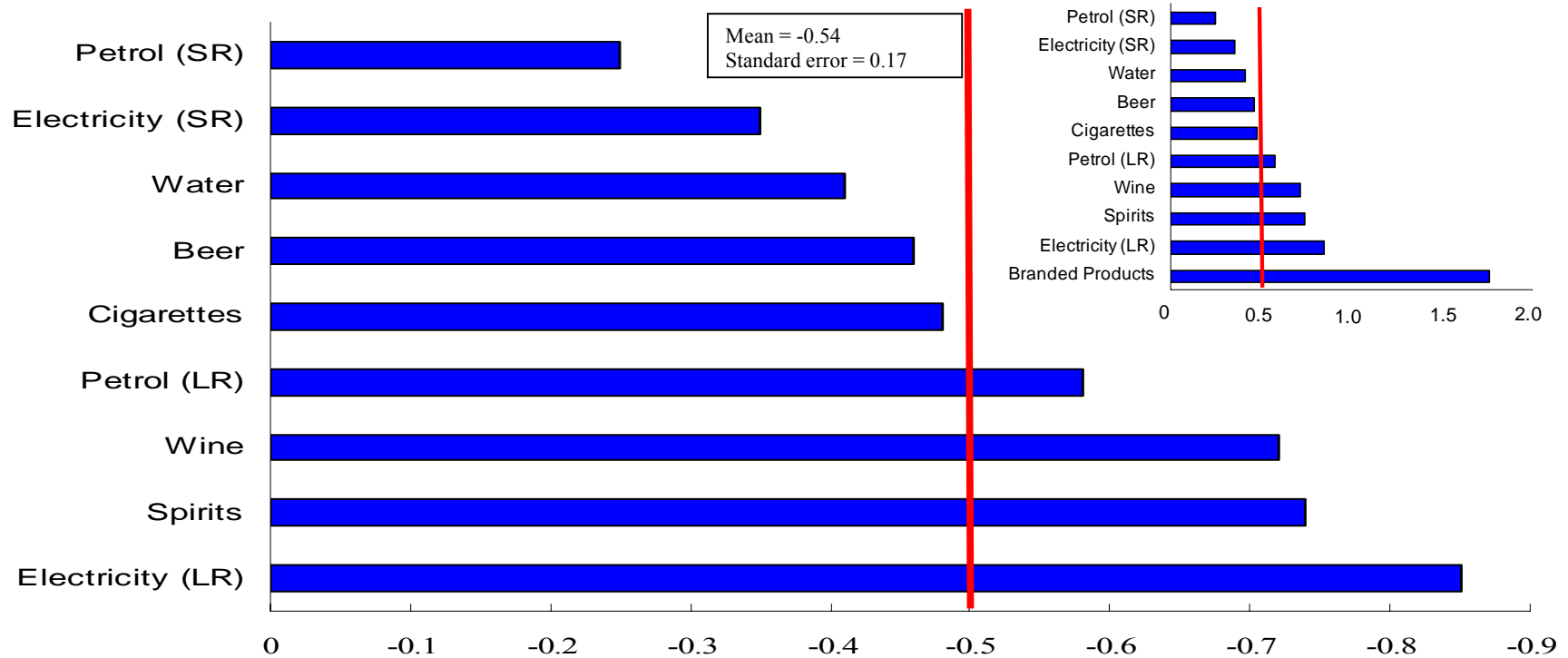
PRICE ELASTICITIES OF DEMAND FOR SELECTED PRODUCTS

Product	Mean	Median	Number of observations	Length of run	Source
1. Beer	-0.46 -0.37	-0.35	139 10		Fogarty (2005, Chapter 2) Selvanathan and Selvanathan (2005a, p. 232)
2. Wine	-0.72 -0.46	-0.58	141 10		Fogarty (2005, Chapter 2) Selvanathan and Selvanathan (2005a, p. 232)
3. Spirits	-0.74 -0.57	-0.68	136 10		Fogarty (2005, Chapter 2) Selvanathan and Selvanathan (2005a, p. 232)
4. Cigarettes	-0.48	-0.40 -0.44	523 368 155	Short run Long run	Gallet and List (2003) Gallet and List (2003) Gallet and List (2003)
5. Residential water	-0.41 -0.51	-0.35 -0.38 -0.64	314 124	Short run Long run	Dalhuisen et al. (2003) Espey et al. (1997) Espey et al. (1997) Espey et al. (1997)
6. Petrol	-0.26 -0.25 -0.25 -0.58 -0.64 -0.53 -0.77 -0.35	-0.23 -0.21 -0.43	363 46 387 277 51 70 213 52	Short run Short run Short run Long run Long run Long run Long run Intermediate	Espey (1998) Goodwin et al. (2004) Graham and Glaister (2002, p. 48) Espey (1998) Goodwin et al. (2004) Espey (1996) Graham and Glaister (2002, p. 48) Graham and Glaister (2002, p. 54)
7. Residential electricity	-0.35 -0.85	-0.28 -0.81	123 125	Short run Long run	Espey and Espey (2004) Espey and Espey (2004)
8. Branded products	-1.76		337		Tellis (1988)

- Notes
1. The other average elasticities of road traffic and fuel consumption reported in Goodwin et al. (2004) and Graham and Glaister (2002) are excluded as they are not confined to the demand by final consumers.
 2. Although the elasticities reported by Goodwin et al. (2004) and Graham and Glaister (2002) refer to “fuel” used by motor vehicles, which is broader than “petrol” (“gasoline”), for simplicity of presentation of the table we list these under the product “petrol”.

FIGURE 1

PRICE ELASTICITIES OF DEMAND



Note: The elasticity values in this figure are the means from Table 1. In those cases where these are multiple means for the same product, we use those with the largest number of observations. The elasticity value for petrol (SR) of -0.25 is from the third entry in Panel 6 of the table; electricity (SR) of -0.35 is from the first entry of Panel 7; water of -0.41 is from first entry of Panel 5; beer of -0.46 is from the first entry of Panel 1; cigarettes of -0.48 is from first entry of Panel 4; petrol (LR) of -0.58 is from the fourth entry of Panel 6; wine of -0.72 is from the first entry of Panel 2; spirits of -0.74 is from the first entry of Panel 3; electricity (LR) of -0.85 is from the second entry of Panel 7; and branded goods (included only in the “mini version” of the figure in the top right-hand corner) of -1.76 is from Panel 8. SR denotes short run, and LR denotes long run