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ECONOMICS

PATTERNS IN WORLD METALS PRICES

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DISCUSSION PAPER 12.08

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Abstract

This paper uses a descriptive statistical approach to identify longer-term patterns, or empirical regularities, in the prices of 16 prominent metals from 1950 to 2010. We also examine some related aspects of the behaviour of the corresponding volumes. Our approach is to summarise the data in the form of price and volume indexes and comparison matrices that provide a convenient way of making pairwise comparisons of different metals. Finally, we also present some evidence on the sensitivity of metal prices to variations in supplies.

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1. INTRODUCTION

In recent years, major fluctuations in international commodity markets have once again focused attention on the nature and functioning of these markets. Major issues include the following questions. How long can high prices be sustained? Is there excessive price volatility? Do prices reflect underlying fundamentals? To what extent has the role of commodities as financial assets changed the way in which they are priced? What is the role of speculators; do they smooth or amplify price fluctuations? These issues are of direct importance to commodity producers and consumers, as well as to governments in large producing countries that rely on commodities for a substantial part of their revenue. In addition, those who consume food, energy and metal products – that is, everyone – are also indirectly affected by developments in international commodity markets.

The above issues can only be properly addressed once there is a clear understanding of exactly what has occurred in commodity markets. In this paper we make an initial attempt to gain such an understanding using a descriptive statistical approach to identify longer-term patterns, or empirical regularities, in commodity data. In particular, we consider the price behaviour of metals, an important class of commodities, from 1950 to 2010; to avoid being overwhelmed with detail, we confine attention to the 16 metals that comprise the bulk of global mineral trade. Of these 16 metals, 10 are traded on the London Metal Exchange, a well-organised, deep market. We also examine some related aspects of the behaviour of the corresponding volumes. Our approach is to summarise the data in the form of price and volume indexes and comparison matrices that provide a convenient way of making pairwise comparisons of different metals. Finally, we also present some evidence on the sensitivity of metal prices to variations in supplies.

2. SIXTEEN IMPORTANT METALS

We consider the 16 metals listed in column 1 of Table 1. The price/volume data are annual for the 61-year period 1950–2010 from the US Geological Survey (USGS)[1]. These metals represent the most valuable among the 38 metal commodities included in the USGS data in 2010.[2] Prices are expressed in terms of US dollars per metric tonne (which is equivalent to

[1] The USGS provides time-series data for approximately 90 minerals from more than 18,000 mineral producers and consumers around the world. Prices are annual averages of apparent consumption prices, obtained from international trade statistics, while volumes refer to world production. See <http://minerals.usgs.gov/ds/2005/140/>.

[2] The term "most valuable" refers to volumes multiplied by prices $p_{it}q_{it}$ for t = 2010 in the notation introduced below. There are altogether 43 metals commodities listed on the USGS online database. However, due to

1,000 kilograms), while volumes are in metric tonnes. Plots of the data are given in Appendix A1.

We define $Dx_t = \log x_t - \log x_{t-1} = \log(x_t/x_{t-1})$ as the log change in any variable $x_t > 0$ from year t-1 to t, which, for small changes, is approximately the annual percentage change when multiplied by 100. Then, if p_{it} and q_{it} are the price and volume of metal i (i = 1,…,16) in year t (t = 1,…,61), Dp_{it} and Dq_{it} are the corresponding log changes. These changes over the whole period are summarised in Table 1 and several patterns are evident. First, for the majority of the metals, the average rate of price increases is the same order of magnitude as that for volumes; for all the metals, prices increase by an average of approximately 4.4 percent p.a. and volumes by 3.4 percent (last entries of columns 2 and 7, respectively). Second, the distributions of price changes tend to be skewed to the right, as the mean exceeds the median in all but three cases. This pattern does not apply to volumes. Third, the standard deviations in columns 6 and 11 indicate that, except for one instance (magnesium), price changes are more volatile than volumes; averaging over all metals, the standard deviation for price changes is approximately 18.0 percent, while that for volumes is 4.8 percent.

The economic importance of metal i in year t is its value $p_{it}q_{it}$. If $M_t = \Sigma_{i=1}^{16} p_{it}q_{it}$ is the total value of the 16 metals, then the relative value of i is the share $w_{it} = p_{it}q_{it}/M_t$, which we shall refer to as the value share of i. The 16 value shares at the beginning and end of the period are given in columns 2 and 3 of Table 2, where, for convenience, metals are now ordered in terms of their shares in 2010. Thus, we see that the most important are iron ore, copper, gold and aluminium, and that the value of iron ore is now more than twice that of copper. Furthermore, column 4 of the table reveals some rather substantial changes in the value shares over the 61 years: For example, zinc accounted for 10.5 percent of the total in 1950, but this share decreased to 3.6 percent over the ensuing six decades. Sulfur, tin and lead also experienced similar large decreases. The largest increase was for iron ore, for which the value share increased by 15.5 percentage points to 35.2 percent in 2010. The last three columns of Table 2, which deal with a decomposition of the changes in shares, will be discussed later in the paper.

missing price or volume data in the beginning or ending of the period, five metals (boron, bromine, columbium, silicon and tellurium) are discarded, leaving 38.

3. INDEXES OF PRICES AND VOLUMES

If there are n metals, then $M=\sum_{i=1}^{n} p_i q_i$ is their value and the value share of i is $w_i = p_i q_i / M$. The differential of the value identity is $dM=\sum_{i=1}^{n} p_i dq_i + \sum_{i=1}^{n} q_i dp_i$, or, using $d(\log x)=dx/x$, $d(\log M)=\sum_{i=1}^{n} w_i d(\log p_i)+\sum_{i=1}^{n} w_i d(\log q_i)$. We write this as

(1) $$d(\log M)=d(\log P)+d(\log Q),$$

where

(2) $$d(\log P)=\sum_{i=1}^{n} w_i d(\log p_i), \qquad d(\log Q)=\sum_{i=1}^{n} w_i d(\log q_i)$$

are price and volume indexes. Thus, the logarithmic change in value can be conveniently decomposed into price and volume indexes. The price (volume) index is a share-weighted average of the n price (volume) changes and is of the Divisia form. These indexes have an attractively simple sampling interpretation (Theil, 1967, pp. 136–137). We write the price change of metal i, $d(\log p_i)$, as x_i and consider a discrete random variable X that can take the n possible values $x_1,\ldots,x_n$. To derive the probabilities for these n realisations, suppose that the names of the metals are drawn at random from this distribution such that each dollar of the total value has an equal chance of being selected. This means that the probability of drawing x_i is w_i, the value share of i. Accordingly, the expected value of the random variable X is $E(X)=\sum_{i=1}^{n} w_i x_i$, which coincides with $d(\log P)$, so that the index can be interpreted as the expected value of the distribution of price changes. The volume index $d(\log Q)$ has a similar interpretation.

To apply the indexes to discrete data, we replace (i) the value share w_i with its arithmetic average over years t-1 and t, $\bar{w}_{it}=1/2\left(w_{it}+w_{i,t-1}\right)$; and (ii) $d(\log p_i)$ with the corresponding log change Dp_{it}, and similarly for volumes. Thus, the discrete versions of the continuous-time indexes (2) for the n = 16 metals are

(3) $$DP_t=\sum_{i=1}^{16} \bar{w}_{it} Dp_{it}, \qquad DQ_t=\sum_{i=1}^{16} \bar{w}_{it} Dq_{it}.$$

The results are contained in the top panel of Figure 1. The same information is also displayed in panel A of Figure 2 and columns 3 and 4 of panel A of Table 3 in the form of decade averages. It is evident from Figure 2 that prices surged in the 1970s, slumped in the 1990s and accelerated again in the 2000s, while volumes exhibited smoother growth.[3] Fisher's factor reversal test requires that the product of the price and volume indexes equal the observed value. In the context of the log-change formulation, this becomes $DM_t = DP_t + DQ_t$, where $DM_t = \log(M_t/M_{t-1})$. It is evident from column 5 of Table 3 that while the indexes (3) do not satisfy this test exactly, the approximation errors are on the whole modest. Finally, Figure 3 presents indexes (3) in level form, obtained by setting them to 100 in the first year and then accumulating the changes, as well as the corresponding level of value. This shows that over the six-decade period, average volumes increased by a factor of approximately 9, prices by a factor of 13 and values by 120 $(\approx 9 \times 13)$.

4. A DECOMPOSITION OF VALUE SHARES AND VOLATILITIES

This section considers a decomposition of the value shares into price and volume components and the volatilities of the prices and volumes. Using $w_i = p_i q_i / M$, we have $d(\log w_i) = d(\log p_i) + d(\log q_i) - d(\log M)$ or $dw_i = w_i \left[d(\log p_i) + d(\log q_i) - d(\log M) \right]$. In view of equation (1), $dw_i = w_i \left[d(\log p_i) - d(\log P) + d(\log q_i) - d(\log Q) \right]$. This shows that the change in the value share is made up of the sum of two terms, a relative price component and a relative volume component, which can be written as

$$(4) \qquad dw_i = w_i d\left(\log \frac{p_i}{P}\right) + w_i d\left(\log \frac{q_i}{Q}\right).$$

Using the relative price in this way has the advantage of avoiding monetary units, so that both terms on the right-hand side of equation (4) are pure numbers, as is the change in the share.

Using the same approach as above, finite-change data can be applied to decomposition (4):

$$(5) \qquad \Delta w_{it} = \overline{w}_{it} D\left(\frac{p_{it}}{P_t}\right) + \overline{w}_{it} D\left(\frac{q_{it}}{Q_t}\right) + \text{approximation error}_{it},$$

where $\Delta w_{it} = w_{it} - w_{i,t-1}$ and $D(p_{it}/P_t) = Dp_{it} - DP_t$, and similarly for the volume term. The approximation error in equation (5) is analogous to that discussed previously. Each element in equation (5) when summed over the n metals is zero:

[3] The bottom panel of Figure 1 and panel B of Figure 2 are discussed later in the paper.

$$\sum_{i=1}^{n} \Delta w_{it} = \sum_{i=1}^{n} \bar{w}_{it} D\left(\frac{p_{it}}{P_t}\right) = \sum_{i=1}^{n} \bar{w}_{it} D\left(\frac{q_{it}}{Q_t}\right) = \sum_{i=1}^{n} \text{approximation error}_{it} = 0,$$

which reflects the "within metals" nature of the decomposition.

To apply (5) to the 16 metals, we now measure time in 60-year units, so all changes in this equation are to be interpreted as referring to the transition from 1950 to 2010; the arithmetic average of the value share, $\bar{w}_{it}$, now becomes $1/2\left(w_{i,2010} + w_{i,1950}\right)$. The results are given in columns 5–7 of Table 2. Consider the case of iron ore (row 16), the share of which increased by 15.5 percentage points over the whole period. This is made up of an increase in its relative price, accounting for a 7.2-percentage point increase, a relative volume growth component of 9.7 points and an approximation error of -1.4 points. The largest decrease in the volume component is for gold (-13.4 points), which is partially offset by its price growth of 11.8 points. Interestingly, this price term for gold is by far the largest among the 16 metals, while the volume term is by far the smallest (or the largest decline). There is a weak tendency for the price and volume components to move in opposite directions, and this is further discussed later in the paper.

Indexes (3) are weighted first-order moments of the distributions of the price and volume log changes. The corresponding second-order moments are

(6) $$\Pi_t = \sum_{i=1}^{16} \bar{w}_{it} \left(Dp_{it} - DP_t\right)^2, \quad K_t = \sum_{i=1}^{16} \bar{w}_{it} \left(Dq_{it} - DQ_t\right)^2.$$

These are measures of dispersion in that if each of the 16 prices move proportionately, for example, then $\Pi_t = 0$; otherwise, $\Pi_t > 0$. As before, prices and volumes are weighted by value shares to recognise the relative economic importance of different metals. These measures are known as Divisia variances. In (6) and subsequently, the average of the value share, $\bar{w}_{it}$, should be interpreted as the arithmetic average of the share in years t-1 and t, while the price and volume log changes refer to 1-year transitions.

Columns 6 and 7 of Table 3 give a summary of the square roots of the variances (6). These show that prices are substantially more variable than volumes; on average over the whole period, the standard deviation is approximately 140 percent p.a. for prices and 50 percent for volumes. This reflects the well-known volatility of prices. The decade averages of the standard deviations are also plotted in panel B of Figure 2, which shows a substantial increase in price volatility in the 2000s. In addition to the variances (6), there is also the corresponding price–volume correlation:

$$\frac{\Gamma_t}{\sqrt{\Pi_t K_t}}, \text{with } \Gamma_t = \sum_{i=1}^{16} \bar{w}_{it} \left(Dp_{it} - DP_t\right)\left(Dq_{it} - DQ_t\right).$$

The last column of Table 3 reveals that at most times the correlation is close to zero. While prices and volumes are more or less uncorrelated in terms of growth rates, as established in Section 6, there is a striking negative relationship between the levels of the two variables.

As mentioned above, the upper panel of Figure 1 plots the price and volume indexes against time. The lower panel of this figure gives the corresponding volatilities, $\sqrt{\Pi_t}$ and $\sqrt{K_t}$. The interesting pattern that emerges is that when there is a large change in the price index DP_t in either direction, the dispersion of prices increases. This pattern is illustrated by the vertical lines in the figure that identify years of large price changes. This comovement, which does not occur at all times of large changes, but does in the majority of cases, is real in that it is not simply an artefact of the way in which the indexes and volatilities are constructed.[4]

5. MULTI-METAL MATRIX (MMM) COMPARISONS

This section systematically compares one metal with another. For n metals, there are $1/2 \cdot n\left(n-1\right)$ distinct pairwise comparisons, which can be conveniently arranged in the form of an $n \times n$ matrix, $\mathbf{X} = \left[x_{ij}\right]$. We thus term these multi-metal matrix (MMM) comparisons. One specific way to formulate these comparisons would be the dollar value of metal i minus that of metal j, $p_i q_i - p_j q_j$. Obviously, when a metal is compared with itself, the comparison yields zero, so that $x_{ii} = 0,\ i = 1,\ldots,n.$ Furthermore, as i compared to j is identical to the comparison of j with i, except for the sign, all pairwise comparisons satisfy a skew symmetric property, that is, $x_{ij} = -x_{ji},\ i, j = 1,\ldots,n.$ This means that the comparison matrix **X** is skew symmetric, $\mathbf{X} = -\mathbf{X}'$.[5]

It is more convenient to use a logarithmic formulation, which yields a comparison matrix for year t, $\mathbf{X}_t$, that has $x_{ijt} = \log\left(p_{it}q_{it}\right) - \log\left(p_{jt}q_{jt}\right)$ as the $\left(i, j\right)^{th}$ element, or

[4] A similar pattern has been found in consumer prices whereby higher inflation is associated with increased price dispersion. See, for example, Balk (1978), Clements and Nguyen (1981), Foster (1978), Glejser (1965), Parks (1978) and Vining and Eltwertowski (1976).

(7) $$x_{ijt} = \log\left(\frac{p_{it}q_{it}}{p_{jt}q_{jt}}\right) = \log\left(\frac{p_{it}}{p_{jt}}\right) + \log\left(\frac{q_{it}}{q_{jt}}\right).$$

This shows that each value comparison can be decomposed into corresponding price and volume components. As we have a comparison matrix for each of the 61 years, to keep things manageable we average them to give the average comparison matrix $\bar{\mathbf{X}} = 1/61 \cdot \left[\Sigma_{t=1}^{61} x_{ijt}\right]$. For convenience, the 16 metals are ordered from the most to the least valuable, where, as before, value is interpreted as the product of price and volume. Table 4 contains the upper triangle of this matrix, bordered by an additional row and column. The diagonal elements are suppressed as they are all zero, while the elements below the diagonal are to be interpreted as the negative of those above the diagonal. The first row of the table refers to iron ore and the elements are 31, 39, 72, …, 379. These numbers are all positive and increasing, which reflects the ordering and the fact that iron ore is the most valuable metal. As the elements are logarithmic differences multiplied by 100, the first number in the row, 31, means that iron ore is approximately 31 percent more valuable than aluminium (the second most valuable metal) 39 percent more valuable than copper, 72 percent more valuable than gold, and so on.

The last element in the first row of Table 4, 204, is the average of all elements in the row including the suppressed zero first element. To interpret this row average, average equation (7) over j = 1,…,16:

(8) $$x_{i\bullet t} = \frac{1}{16}\sum_{j=1}^{16}\log\left(\frac{p_{it}q_{it}}{p_{jt}q_{jt}}\right) = \log\left(p_{it}q_{it}\right) - \frac{1}{16}\sum_{j=1}^{16}\log\left(p_{jt}q_{jt}\right).$$

This $x_{i\bullet t}$ is the logarithmic difference between the value of metal i and the log of the geometric mean of the 16 values; equivalently, $\exp\left(x_{i\bullet t}\right)$ is the ratio of the value of i to the geometric mean of the value of all metals. The differences $x_{i\bullet t}$ have a zero sum over the 16 metals, $\Sigma_{i=1}^{16} x_{i\bullet t} = 0$. The last column of Table 4 presents the 61-year averages of these differences for each of the 16 metals, $\bar{x}_{i\bullet} = 1/61 \cdot \Sigma_{t=1}^{61} x_{i\bullet t}$. Thus, the first entry in this column, for example, states that on average for the period, the value of iron ore is approximately 204 percent greater than average for all metals, that of aluminium is 173 greater, that of copper is 165 percent greater, and so on. Since the metals are ordered from the most to the least valuable, the elements in column 18 always decrease as we move down the column and are positive (negative) for above-average (below-average) metals. Manganese and lead are

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[5] Clements and Izan (2012) use an analogous matrix comparison approach to analyse the structure of pay schedules. See Appendix A2 for some details of the comparison matrix approach.

located near the average. The elements in the last column of Table 4 are plotted in Figure 4. Finally, the last row of Table 4 contains the column averages, which are the negatives of the row averages because of the skew symmetry.

We use a similar procedure to construct comparison matrices for prices and volumes and these are summarised in Table 5. This table has three panels that refer to values, prices and volumes. The last row of panel A reproduces the row averages from the last column of Table 4. The corresponding decade averages are given in the other six rows of that panel. The value of iron ore, for example, was 226 percent greater than average in the 1960s and 180 percent greater than in the 1990s. The values are reasonably stable for the more valuable metals, but are more variable for some of the others, such as tin, sulfur and platinum. The standard deviation of these values, given in column 18, decreased slightly over the whole period, from approximately 126 percent at the beginning to 123 percent at the end.

Panels B and C of Table 5 compare prices and volumes and are interpreted analogously to panel A. As everything is in logs, the elements in the three panels satisfy the identity value = price + volume, which is a reflection of (7). In the vast majority of cases, for a given metal, prices and volumes have opposite signs, with magnesium the major exception to the rule. Thus, a metal with an above-average price has a below-average volume. This negative correlation refers to levels of prices and volumes and is different to the finding above for changes over time, for which there was little or no relationship.

6. A SIMPLE METALS PRICING MODEL

Expression (8) gives for year t the average of the i^{th} row of the comparison matrix $\mathbf{X}_t$ in terms of values; this is the logarithmic deviation of the value of metal i from the average value of all 16 metals. We define the analogous price and volume concepts as

(9) $$x^{p}_{i\bullet t} = \log p_{it} - \frac{1}{16}\sum_{j=1}^{16}\log p_{jt}, \quad x^{q}_{i\bullet t} = \log q_{it} - \frac{1}{16}\sum_{j=1}^{16}\log q_{jt},$$

which satisfy $x^{p}_{i\bullet t} + x^{q}_{i\bullet t} = x_{i\bullet t}$, where $x_{i\bullet t}$ is the value concept defined by equation (8).

Next, consider a regression of prices on volumes

(10) $$x^{p}_{i\bullet t} = \beta x^{q}_{i\bullet t} + \varepsilon_{it}, i = 1,\ldots,16; t = 1,\ldots,T,$$

where ε_{it} is a zero-mean disturbance term and T is the number of observations. This equation has no intercept as prices and volumes are expressed as deviations from the mean. The logarithmic formulation means that the slope β is the elasticity of price with respect to volume, $\beta = d(\log p_i)/d(\log q_i)$, which is also known as the price flexibility. The least-squares estimator of this flexibility is $\hat{\beta} = \sigma_{p,q}/\sigma_q^2$, where

$$\sigma_{p,q} = \frac{1}{16\times T}\sum_{i=1}^{16}\sum_{t=1}^{T}\left(\log p_{it} - \frac{1}{16}\sum_{j=1}^{16}\log p_{jt}\right)\left(\log q_{it} - \frac{1}{16}\sum_{j=1}^{16}\log q_{jt}\right),$$

$$\sigma_q^2 = \frac{1}{16\times T}\sum_{i=1}^{16}\sum_{t=1}^{T}\left(\log q_{it} - \frac{1}{16}\sum_{j=1}^{16}\log q_{jt}\right)^2$$

are the price–volume covariance and volume variance, respectively.

Figure 5 is a scatter plot of $x_{i\bullet t}^{p}$ against $x_{i\bullet t}^{q}$ for $i = 1,\ldots,16,\ t = 1,\ldots,61$. The vast majority of the points are scattered around a downward-sloping line with slope of approximately -0.9. Rather than pooling the data over the 61 years, we can also estimate model (10) separately for each year, and Table 6 summarises these results. It is evident that the estimated slope is reasonably stable and tends to fall in the range between -0.8 and -0.9. Thus, if as an approximation we set the price flexibility to -1 and the random disturbance ε_{it} to its expected value of zero, then model (10) takes a very simple form:

(11) $$\log p_{it} = \log \bar{M}_t - \log q_{it},$$

where

$$\log \bar{M}_t = \frac{1}{16}\sum_{i=1}^{16}\log\left(p_{it}q_{it}\right) = \log P_t + \log Q_t.$$

The terms $\log P_t$ and $\log Q_t$ are price and volume indexes, defined as $\log P_t = 1/16 \cdot \sum_{i=1}^{16}\log p_{it}$ and $\log Q_t = 1/16 \cdot \sum_{i=1}^{16}\log q_{it}$. Thus, $\log \bar{M}_t$ is the log of the geometric mean of values in year t, and the sum of (logarithmic) price and volume indexes.

According to equation (11), the price of metal i depends on two factors. The first is $\log \bar{M}_t$, which reflects the overall state of the metals market, as measured by values; this indicator of the state of the market contains both aggregate price and volume components. The elasticity of each price with respect to the market is unity, so prices move in proportion to the market. The second term is $-\log q_{it}$, which measures the impact of changes in the volume of metal i on its price; as the corresponding elasticity is -1, the price of a metal is inversely proportional to its volume. If, for example, the overall metals market grows by 10 percent in a year and the volume of metal i also increases by 10 percent, so that $\Delta \log \bar{M} = \Delta \log q_i \approx 0.10$, then the price of i will remain unchanged. It will increase (decrease) if its volume increases at a slower (faster) rate than that of the overall market. In other words, according to equation (11), the price of a metal is a simple sum of a market-wide factor and a metal-specific factor. Alternatively, (11) can be written as

$$\log p_{it} - \log P_t = -\left(\log q_{it} - \log Q_t\right),$$

which expresses the relative price of metal i, $\log p_{it} - \log P_t$, in terms of the corresponding relative volume, $\log q_{it} - \log Q_t$. This shows that the relative price of i decreases (increases) if the relative volume increases (decreases).

Nutting (1977) used the following metal-pricing model

$$\log p_{it} = \alpha_t + \beta' \log q_{it} + \varepsilon'_{it}, \tag{12}$$

where ε'_{it} is a disturbance term. Using data for 14 metals, he obtained an estimated slope coefficient of approximately -0.7. Nutting's work occupies a reasonably prominent place in the literature on metals pricing and the log-linear model (12) is known as Nutting's Law. In view of definition (9), models (10) and (12) are the same, with

$$\alpha_t = \frac{1}{16}\sum_{i=1}^{16} \log p_{jt} + \beta \frac{1}{16}\sum_{j=1}^{16} \log q_{jt}, \quad \beta = \beta', \quad \varepsilon_{it} = \varepsilon'_{it}.$$

This accounts for the similarity between Nutting's result of $\hat{\beta}' \approx -0.7$ and ours of $\hat{\beta}$ falling in the range -0.8 to -0.9.

Returning to Figure 5, one notable pattern is the clustering of observations for each metal. This suggests that model (10) should be extended by adding a dummy variable for each metal to account for fixed effects:

$$x^p_{i\bullet t} = \alpha_i + \beta x^q_{i\bullet t} + \varepsilon_{it}, i = 1,\ldots,16; t = 1,\ldots,T, \tag{13}$$

where α_i is the metal-specific intercept. As $\Sigma_{i=1}^{16} x^p_{i\bullet t} = \Sigma_{i=1}^{16} x^q_{i\bullet t} = 0$, the intercepts and disturbances of (13) satisfy $\Sigma_{i=1}^{16}\alpha_i = \Sigma_{i=1}^{16}\varepsilon_{it} = 0$.[6] Table 7 contains the results for the whole

[6] The least-squares estimates of α_i sum over metals to zero. To show this, it is convenient to write (13) as $y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$, i=1,… n, t=1,… T. Defining $\mathbf{y} = [y_{11},\ldots,y_{1T},\ldots,y_{n1},\ldots,y_{nT}]'$, $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_n]$, $\mathbf{x} = [x_{11},\ldots,x_{1T},\ldots,x_{n1},\ldots,x_{nT}]'$, and $\boldsymbol{\varepsilon} = [\varepsilon_{11},\ldots,\varepsilon_{1T},\ldots,\varepsilon_{n1},\ldots,\varepsilon_{nT}]'$, we have $\mathbf{y} = \mathbf{D}\boldsymbol{\alpha} + \mathbf{x}\beta + \boldsymbol{\varepsilon}$, where $\mathbf{D} = \boldsymbol{\iota}_T \otimes \mathbf{I}_n$ is an $nT \times n$ matrix, $\boldsymbol{\iota}_T$ is a $T \times 1$ column vector of unit elements and $\mathbf{I}_n$ is an $n \times n$ identity matrix. The LS estimators are (Greene, 2008, p. 195) $\hat{\boldsymbol{\alpha}} = [\mathbf{D}'\mathbf{D}]^{-1}\mathbf{D}'\left(\mathbf{y} - \mathbf{x}\hat{\beta}\right)$, and $\hat{\beta} = [\mathbf{x}'\mathbf{M}\mathbf{x}]^{-1}\mathbf{x}'\mathbf{M}\mathbf{y}$, where $\mathbf{M} = \mathbf{I}_{nT} - \mathbf{D}\left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'$. As $\mathbf{D}'\mathbf{D} = T \cdot \mathbf{I}_n$, we have $\hat{\boldsymbol{\alpha}} = T^{-1}\mathbf{D}'\left(\mathbf{y} - \mathbf{x}\hat{\beta}\right)$. In scalar terms,

$$\hat{\alpha}_i = T^{-1}\sum_{t=1}^{T}\left(y_{it} - \hat{\beta}x_{it}\right) = \overline{y}_i - \hat{\beta}\overline{x}_i, \; i = 1,\ldots,n,$$

where $\overline{y}_i$ and $\overline{x}_i$ are means over time. As $\Sigma_{i=1}^{n} y_{it} = \Sigma_{i=1}^{n} x_{it} = 0$, the estimated fixed effects have a zero sum:

$$\sum_{i=1}^{n}\hat{\alpha}_i = T^{-1}\left[\sum_{i=1}^{n}\sum_{t=1}^{T} y_{it} - \hat{\beta}\sum_{i=1}^{n}\sum_{t=1}^{T} x_{it}\right] = 0.$$

period. It is evident that adding the fixed effects causes the estimated slope coefficient to become nearly zero and insignificant. Owing to the relatively limited variability of the data over time for each metal (which is evident in the clustering in Figure 5), the fixed effects act as a substitute for the volume variable, so that when both sets of variables are included, volumes play little or no role in price determination.

Before concluding this section, some additional comments are appropriate. Regressing prices on volumes treats volumes as exogenous. This is usually thought to be to a satisfactory approach for agricultural products with lengthy gestation periods, so that current supplies on the market are more or less unrelated to current prices. For a sampling interval of 1 year, a similar argument is also possibly applicable to metals. In such a case, equations (10)–(13) are interpreted as inverse demand models that give the price needed to sell a given volume of metal. However, they are a special type of inverse demand function as the slope (the price flexibility) is the same for each of the 16 metals. For a rigorous analysis of this issue, see Chen (2012). If we consider the reciprocal case of regressing volumes on prices, the estimated slope coefficient, $\hat{\lambda}$ say, would be different to the inverse of $\hat{\beta}$ from (10) or $\hat{\beta}'$ from (12), but the two regressions would have the same R^2 values and the slopes would satisfy $\hat{\lambda} \times \hat{\beta} = R^2$. Thus, the better the fit, the closer one slope would approximate the inverse of the other. See Berndt (1976) for details on these issues.

APPENDIX A1

THE DATA

Figure A1 plots the price and volume data for each of the 16 metals. To facilitate comparisons across metals, the same scale is used for all price and volume log-changes. These plots reveal several features. First, for a given metal, the volume tends to increase more steadily than the price. In other words, prices are usually more volatile than volumes, as mentioned in the text. Second, there is a tendency for prices to be more volatile in the second half of the period, a pattern previously identified by Chen (2010). Third, towards the end of the period (2010), the prices of most metals were at or near their peak. Finally, in the last several years, there were large spikes in sulfur prices.

Figure A2 shows histograms of the price and volume changes for the individual metals, while Figure A3 contains corresponding histograms for the two indexes.

APPENDIX A2

MATRIX COMPARISONS

Suppose we have n numbers ranked in descending order, $y_1,\ldots,y_n$. This appendix, which is based on Clements and Izan (2012), considers a matrix approach that systematically compares each of these n numbers with all others. Let us compare one value with another in terms of the difference $y_i - y_j$. We can express all pairwise differences in the form of an $n \times n$ matrix:

(A1) $$\mathbf{X} = \mathbf{y}\boldsymbol{\iota}' - \boldsymbol{\iota}\mathbf{y}',$$

where $\mathbf{y} = [y_1,\ldots,y_n]'$ and $\boldsymbol{\iota} = [1,\ldots,1]'$ is a vector of n unit elements. Equation (A1) defines a comparison matrix.

Consider the i^{th} row of **X**, $[x_{i1},\ldots,x_{in}]$. One way in which the information contained in this row can be summarised in terms of one number, call it x_i, is by the value that minimises the sum of squared deviations, $\sum_{j=1}^{n}(x_{ij} - x_i)^2$. This leads to x_i being the mean of the row, which we denote by $\bar{x}_i = (1/n)\sum_{j=1}^{n} x_{ij}$, or $\bar{\mathbf{x}} = (1/n)\mathbf{X}\boldsymbol{\iota}$ for the corresponding vector of n row means. The vector $\bar{\mathbf{x}}$ is a desirable centre-of-gravity measure of the **X** matrix in a least-squares sense. Denoting the mean of $y_1,\ldots,y_n$ by

$$\bar{y} = \frac{1}{n}\sum_{j=1}^{n} y_j = \frac{1}{n}\boldsymbol{\iota}'\mathbf{y},$$

it then follows from definition (A1) that the row averages of **X** take the form

$$\text{(A2)} \qquad \bar{\mathbf{x}} = \frac{1}{n}\mathbf{X}\boldsymbol{\iota} = \frac{1}{n}\left(\mathbf{y}\boldsymbol{\iota}' - \boldsymbol{\iota}\mathbf{y}'\right)\boldsymbol{\iota} = \mathbf{y} - \bar{y}\boldsymbol{\iota},$$

which shows that the averages of the rows of **X** are just the deviations of each element of **y** from the overall mean. Equation (A2) can be expressed more compactly as

$$\text{(A3)} \qquad \bar{\mathbf{x}} = \mathbf{M}\mathbf{y}, \text{ with } \mathbf{M} = \mathbf{I} - \tfrac{1}{n}\boldsymbol{\iota}\boldsymbol{\iota}',$$

where **M** is a symmetric idempotent matrix $\left(\mathbf{M}^2 = \mathbf{M}\right)$ of order $n \times n$ that satisfies $\mathbf{M}\boldsymbol{\iota} = \mathbf{0}$. As $\mathbf{M}$ is symmetric, $\boldsymbol{\iota}'\mathbf{M} = \mathbf{0}'$, which implies that $\boldsymbol{\iota}'\bar{\mathbf{x}} = \boldsymbol{\iota}'\mathbf{M}\mathbf{y} = 0$. Thus, the sum over all deviations from the mean is zero.

The variance of the elements of **y** is one measure of dispersion:

$$\text{(A4)} \qquad \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2 = \frac{1}{n}\left(\mathbf{y} - \bar{y}\boldsymbol{\iota}\right)'\left(\mathbf{y} - \bar{y}\boldsymbol{\iota}\right) = \frac{1}{n}\bar{\mathbf{x}}'\bar{\mathbf{x}} = \frac{1}{n}\sum_{i=1}^{n}\bar{x}_i^2,$$

where the third step follows from (A2). Accordingly, the variance of $y_1, \ldots, y_n$ is the average of the sum of the squared row averages of the comparison matrix **X**. Note also that equations (A3) and (A4) imply that the variance can also be expressed as $\sigma^2 = (1/n)\mathbf{y}'\mathbf{M}'\mathbf{M}\mathbf{y}$, or, since **M** is idempotent,

$$\text{(A5)} \qquad \sigma^2 = \frac{1}{n}\mathbf{y}'\mathbf{M}\mathbf{y}.$$

Next, consider the dispersion of the elements of the i^{th} row of **X** about their centre of gravity, as well as their comovement with the elements of some other row. The following variance and covariance provide convenient ways to measure these concepts:

$$\sigma_{ii} = \frac{1}{n}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_i\right)^2, \qquad \sigma_{ik} = \frac{1}{n}\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_i\right)\left(x_{kj} - \bar{x}_k\right).$$

This σ_{ii} is the variance of the i^{th} row of the **X** matrix, while σ_{ik} is the covariance between rows i and k. The matrix $\mathbf{X} - \bar{\mathbf{x}}\boldsymbol{\iota}'$ is **X** expressed as a deviation from the mean vector $\bar{\mathbf{x}}$. The covariance matrix $(1/n)(\mathbf{X}-\bar{\mathbf{x}}\boldsymbol{\iota}')(\mathbf{X}-\bar{\mathbf{x}}\boldsymbol{\iota}')'$ contains on the diagonal the n row variances $\sigma_{11},\ldots,\sigma_{nn}$ and the cross-row covariances σ_{ik} as the off-diagonal elements. In view of equations (A3) and (A5), as well as the idempotence of **M**, the covariance matrix takes the form

$$\frac{1}{n}(\mathbf{X}-\bar{\mathbf{x}}\boldsymbol{\iota}')(\mathbf{X}-\bar{\mathbf{x}}\boldsymbol{\iota}')' = \sigma^2\boldsymbol{\iota\iota}', \text{or } \sigma_{ii} = \sigma_{ik} = \sigma^2, \quad i,k = 1,\ldots,n.$$

In words, each row of **X** has a common variance σ^2, and each of the distinct $1/2[n(n-1)]$ covariances also takes this value. The corresponding correlation matrix is $\boldsymbol{\iota\iota}'$, so that each correlation is unity.

Definition (A1) implies that the mean of the n^2 elements of **X** is zero: $1/n\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij} = 1/n\cdot(\boldsymbol{\iota}'\mathbf{X}\boldsymbol{\iota}) = 1/n\cdot(\boldsymbol{\iota}'\mathbf{y}\boldsymbol{\iota}'\boldsymbol{\iota}-\boldsymbol{\iota}'\boldsymbol{\iota}\mathbf{y}'\boldsymbol{\iota}) = 0.$ Thus, the average sum of squares of these elements is their variance and it can be shown that this takes the form $(1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij}^2 = 2\sigma^2$. The multiple 2 here derives from the structure of **X**, which involves all the bivariate comparisons (y_i, y_j), i, j = 1,…,n. This means that y_i is compared to y_j, and reciprocally, y_j is compared with y_i, so that the whole matrix contains $x_{ij} = y_i - y_j$ and $x_{ji} = y_j - y_i = -x_{ij}$ for i, j =1,…,n. Thus, when we square the elements of **X**, the minus signs disappear and in essence each distinct pair (i,j) is included twice in the average sum of squares $(1/n^2)\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij}^2$.

For some additional results on smoothness, see Clements and Izan (2012).

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Table 1 Summary of logarithmic changes in prices and volumes for 16 metals, 1950–2010

Metal		Prices					Volumes				
		Mean	Median	Minimum	Maximum	S.D.	Mean	Median	Minimum	Maximum	S.D.
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1.	Aluminium	2.96	2.86	-41.87	49.00	17.44	5.52	5.91	-11.94	18.90	5.76
2.	Chromium	5.96	4.99	-54.19	60.36	23.73	3.86	5.85	-23.91	38.26	12.15
3.	Cobalt	3.97	1.39	-69.31	88.49	29.19	4.21	5.36	-36.41	43.22	14.60
4.	Copper	4.63	3.59	-33.11	59.44	18.15	3.19	2.84	-5.20	13.86	3.62
5.	Gold	5.94	0.46	-28.60	68.91	18.27	1.78	1.71	-7.70	11.91	3.84
6.	Iron ore	5.03	4.57	-15.20	22.92	9.26	3.89	3.90	-9.40	18.08	6.67
7.	Lead	3.51	3.82	-35.93	48.91	18.26	1.54	0.68	-9.84	17.73	5.07
8.	Magnesium	3.79	0.00	-31.40	67.16	14.66	4.68	3.06	-61.31	59.66	19.81
9.	Manganese	5.09	4.98	-55.23	69.31	20.23	2.85	3.59	-18.54	33.38	9.90
10.	Molybdenum	4.65	2.55	-90.17	113.47	35.04	4.69	4.51	-39.81	42.61	14.23
11.	Nickel	5.15	4.48	-56.70	104.78	24.72	3.99	4.51	-24.43	30.47	9.82
12.	Platinum	3.93	4.65	-72.18	53.90	23.49	5.36	4.24	-39.46	22.75	9.80
13.	Silver	5.49	0.75	-67.36	71.99	23.09	2.16	2.70	-7.26	10.18	3.79
14.	Sulfur	2.29	0.60	-504.53	372.50	89.31	3.07	2.91	-7.41	31.10	5.77
15.	Tin	3.83	1.38	-43.84	55.45	19.54	0.79	1.51	-26.83	14.41	7.41
16.	Zinc	3.33	4.43	-55.08	86.07	21.54	2.87	2.45	-6.56	12.17	3.79
	All metals	4.35	3.22	-504.53	372.50	18.03	3.40	3.32	-61.31	59.66	4.75

Notes: All entries are to be divided by 100.

Table 2 Value shares for 16 metals, 1950–2010

Metal		Share			Component of change		
		1950	2010	Change	Price	Volume	Approximation error
(1)		(2)	(3)	(4)	(5)	(6)	(7)=(4)–(5)–(6)
1.	Cobalt	0.42	0.48	0.06	-0.17	0.24	-0.02
2.	Magnesium	0.40	0.55	0.15	-0.23	0.39	-0.02
3.	Sulfur	3.07	0.65	-2.42	-2.57	-0.26	0.41
4.	Tin	5.79	0.78	-5.01	-1.52	-4.94	1.45
5.	Molybdenum	0.50	1.14	0.64	0.02	0.68	-0.06
6.	Platinum	0.52	1.16	0.64	-0.34	1.04	-0.06
7.	Lead	7.67	1.34	-6.33	-2.95	-4.75	1.38
8.	Silver	2.40	2.00	-0.40	1.18	-1.51	-0.07
9.	Chromium	0.85	2.60	1.75	1.41	0.58	-0.24
10.	Manganese	2.85	2.83	-0.02	0.84	-0.77	-0.10
11.	Zinc	10.50	3.64	-6.86	-5.40	-1.85	0.39
12.	Nickel	2.29	4.67	2.38	1.15	1.44	-0.22
13.	Aluminium	9.27	12.65	3.38	-10.79	14.57	-0.40
14.	Gold	15.71	13.63	-2.08	11.79	-13.37	-0.49
15.	Copper	18.07	16.66	-1.41	0.38	-1.20	-0.59
16.	Iron ore	19.70	35.22	15.52	7.18	9.70	-1.36
Total		100.00	100.00	0.00	0.00	0.00	0.00

Note: All entries are to be divided by 100.

Table 3 Summary of price and volume indexes and volatilities for metals, 1950–2010 (logarithmic change)

Period	Index of Values DM_t	Index of Prices DP_t	Index of Volumes DQ_t	Approximation error (2)–(3)–(4)	Second-order moment Prices $\sqrt{\Pi_t}$	Second-order moment Volumes $\sqrt{K_t}$	Price–volume correlation
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				A. Average by decade			
1950–59	8.43	2.96	5.46	0.01	91.48	72.26	0.04
1960–69	7.40	2.22	5.19	0.00	66.37	39.86	0.02
1970–79	14.12	11.51	2.63	-0.02	136.99	51.54	-0.04
1980–89	4.23	1.86	2.38	-0.01	181.43	43.13	0.01
1990–99	-1.84	-3.38	1.54	0.00	118.29	45.52	0.08
2000–10	14.76	10.07	4.41	0.29	231.71	52.63	-0.02
				B. Summary statistics over 1950–2010			
Mean	7.96	4.32	3.58	0.05	140.05	50.49	0.02
Median	8.56	3.24	3.13	0.00	123.39	46.81	0.04
Minimum	-16.84	-19.02	-5.22	-0.52	35.05	25.29	-0.62
Maximum	41.06	35.58	11.58	4.24	659.77	89.76	0.63

Note: All entries except those in column 8 are to be divided by 100.

Table 4 Comparison of average metal values, 1950–2010 (logarithmic difference ×100)

Metal	Metal																Row average
	Iron ore	Aluminium	Copper	Gold	Zinc	Nickel	Manganese	Lead	Tin	Sulfur	Silver	Chromium	Platinum	Molybdenum	Magnesium	Cobalt	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
1. Iron ore		31	39	72	140	172	206	219	244	246	253	276	298	334	357	379	204
2. Aluminium			8	41	109	141	175	188	214	215	222	245	268	303	326	348	173
3. Copper				32	101	133	167	180	205	207	214	237	259	295	318	340	165
4. Gold					68	100	135	147	173	174	181	205	227	263	285	307	133
5. Zinc						32	66	79	105	106	113	137	159	195	217	239	64
6. Nickel							35	47	73	74	81	105	127	163	185	207	32
7. Manganese								13	38	40	47	70	92	128	150	173	-2
8. Lead									26	27	34	57	80	115	138	160	-15
9. Tin										1	8	32	54	90	112	135	-40
10. Sulfur											7	30	53	88	111	133	-42
11. Silver												23	46	81	104	126	-49
12. Chromium													22	58	80	103	-72
13. Platinum														36	58	81	-94
14. Molybdenum															22	45	-130
15. Magnesium																22	-153
16. Cobalt																	-175
Column average	-204	-173	-165	-133	-64	-32	2	15	40	42	49	72	94	130	153	175	0

Table 5 Summary of average values, prices and volume comparisons, 1950–2010 (logarithmic difference ×100)

Decade	Metal																S.D.
	Iron ore	Aluminium	Copper	Gold	Zinc	Nickel	Manganese	Lead	Tin	Sulfur	Silver	Chromium	Platinum	Molybdenum	Magnesium	Cobalt	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
A. Values																	
1950–59	205	142	176	122	88	-11	28	71	26	-2	-51	-115	-193	-153	-157	-175	126
1960–69	226	178	182	102	75	24	2	29	9	5	-53	-129	-145	-128	-157	-220	128
1970–79	211	174	169	93	63	45	-22	-4	-5	-18	-48	-88	-104	-101	-169	-195	116
1980–89	199	171	126	158	45	15	-16	-65	-49	22	-21	-58	-64	-156	-143	-163	111
1990–99	180	188	162	172	65	41	1	-54	-97	-71	-66	-32	-42	-164	-137	-146	118
2000–10	203	187	174	147	52	76	-4	-61	-118	-173	-54	-17	-25	-83	-152	-152	123
Average	204	173	165	133	64	32	-2	-15	-40	-42	-49	-72	-94	-130	-153	-175	115
B. Prices																	
1950–59	-522	-97	-69	678	-152	8	-264	-139	56	-405	309	-258	717	70	-64	130	329
1960–69	-505	-97	-63	669	-160	23	-289	-163	75	-408	337	-253	715	88	-60	90	329
1970–79	-515	-128	-74	707	-162	28	-295	-180	89	-458	363	-222	713	92	-75	118	342
1980–89	-513	-141	-117	771	-178	1	-268	-223	74	-414	378	-207	734	38	-62	128	351
1990–99	-534	-138	-96	755	-158	13	-224	-189	35	-496	326	-192	742	23	-53	186	351
2000–10	-523	-148	-84	757	-170	50	-225	-175	22	-581	346	-178	755	103	-86	136	361
Average	-519	-125	-84	723	-164	21	-260	-178	58	-462	343	-218	730	70	-67	131	343
C. Volumes																	
1950–59	727	238	244	-556	240	-19	291	210	-31	403	-359	142	-910	-223	-93	-306	390
1960–69	731	275	244	-567	235	1	291	192	-66	413	-390	124	-860	-217	-98	-310	387
1970–79	725	302	243	-614	226	17	273	176	-94	440	-412	135	-818	-193	-94	-313	388
1980–89	712	312	243	-613	223	13	252	158	-123	435	-399	149	-798	-194	-81	-290	382
1990–99	715	326	258	-583	223	28	225	135	-132	424	-392	160	-784	-186	-84	-332	378
2000–10	726	335	258	-611	221	27	221	114	-140	407	-400	161	-780	-186	-66	-288	378
Average	723	299	249	-591	228	11	258	163	-98	420	-392	146	-824	-200	-86	-306	383

Table 6 Price flexibility for metals, 1950–2010

$$x^{p}_{i\bullet t} = \beta_t x^{q}_{i\bullet t} + \varepsilon_{it}, i = 1, \ldots, 16$$

Period (1)	Price flexibility β (2)	R^2 (3)
	A. Average by decade	
1950–59	-0.80	0.91
1960–69	-0.81	0.90
1970–79	-0.84	0.91
1980–89	-0.88	0.91
1990–99	-0.88	0.90
2000–10	-0.90	0.89
	B. Summary statistics over 1950–2010	
Mean	-0.85	0.90
Median	-0.86	0.90
Minimum	-0.94	0.81
Maximum	-0.77	0.94

Note: The regression equation given at the top of the table is estimated separately for each year. Panel A gives the decade averages of the estimated slope coefficients and R^2 values, while panel B summarises the 61 estimates of the slopes and R^2 values. For estimates when the data are pooled over the 61 years, see Figure 5.

Table 7 Price flexibility for metals and metal-specific intercepts, 1950–2010

$$x^{p}_{i\bullet t} = \alpha_i + \beta x^{q}_{i\bullet t} + \varepsilon_{it}, i = 1,\ldots,16; t = 1,\ldots,61$$

Variable (1)	Coefficient (2)	S.E. (3)	*t*-value (4)	*p*-value (5)
Volume, β	-0.07	0.05	-1.45	0.15
Intercept α_i				
Aluminium	-1.06	0.14	-7.37	0.00
Chromium	-2.08	0.08	-25.78	0.00
Cobalt	1.11	0.15	7.59	0.00
Copper	-0.67	0.12	-5.52	0.00
Gold	6.84	0.27	25.13	0.00
Iron ore	-4.71	0.33	-14.21	0.00
Lead	-1.67	0.09	-19.14	0.00
Magnesium	-0.73	0.06	-12.02	0.00
Manganese	-2.43	0.13	-19.29	0.00
Molybdenum	0.56	0.10	5.54	0.00
Nickel	0.22	0.05	4.67	0.00
Platinum	6.76	0.38	17.91	0.00
Silver	3.17	0.18	17.25	0.00
Sulfur	-4.34	0.20	-22.11	0.00
Tin	0.51	0.06	7.99	0.00
Zinc	-1.49	0.11	-13.10	0.00
R^2	0.99			

Figure 1. Price and volume indexes and dispersion for metals, 1950–2010, changes

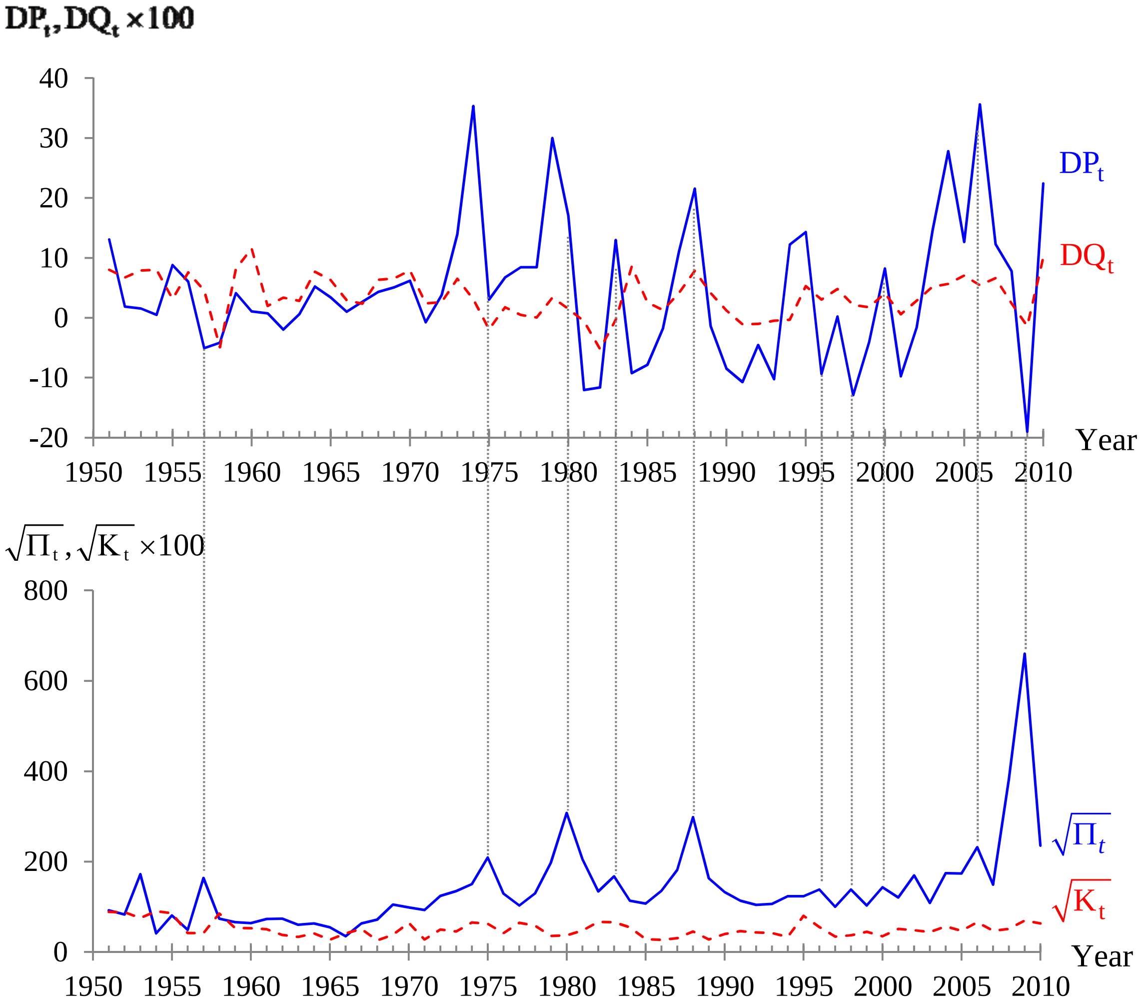
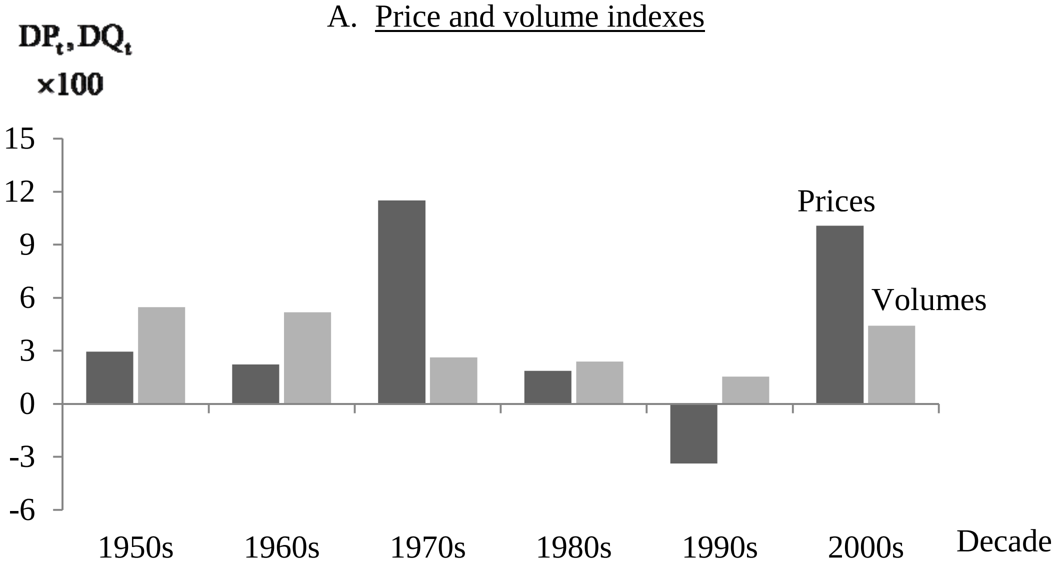


Figure 2. Summary of price and volume indexes and dispersion for metals, 1950–2010

A. Price and volume indexes



B. Standard deviation for prices and volumes

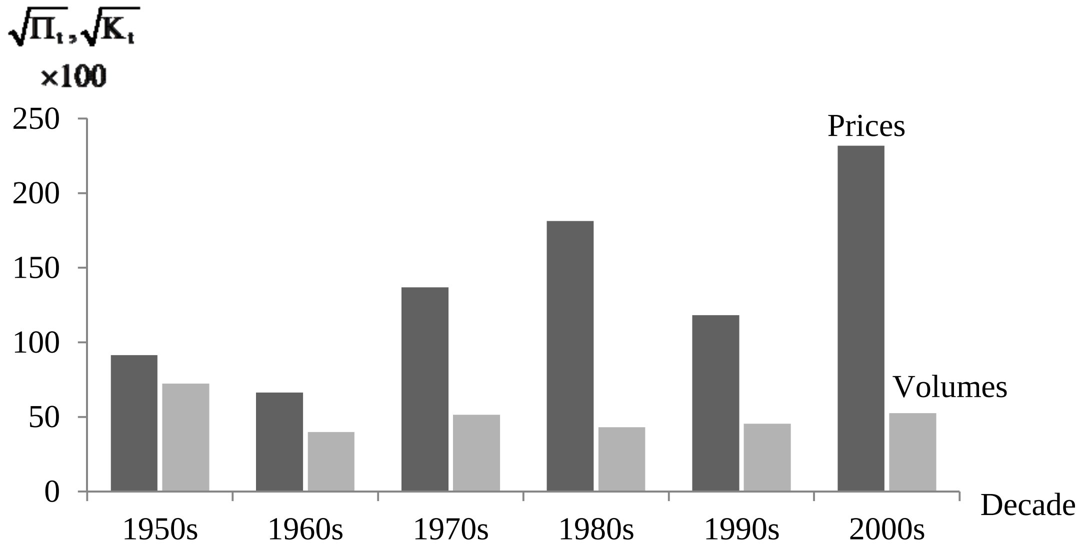


Figure 3. Value, price and volume indexes for metals, 1950–2010

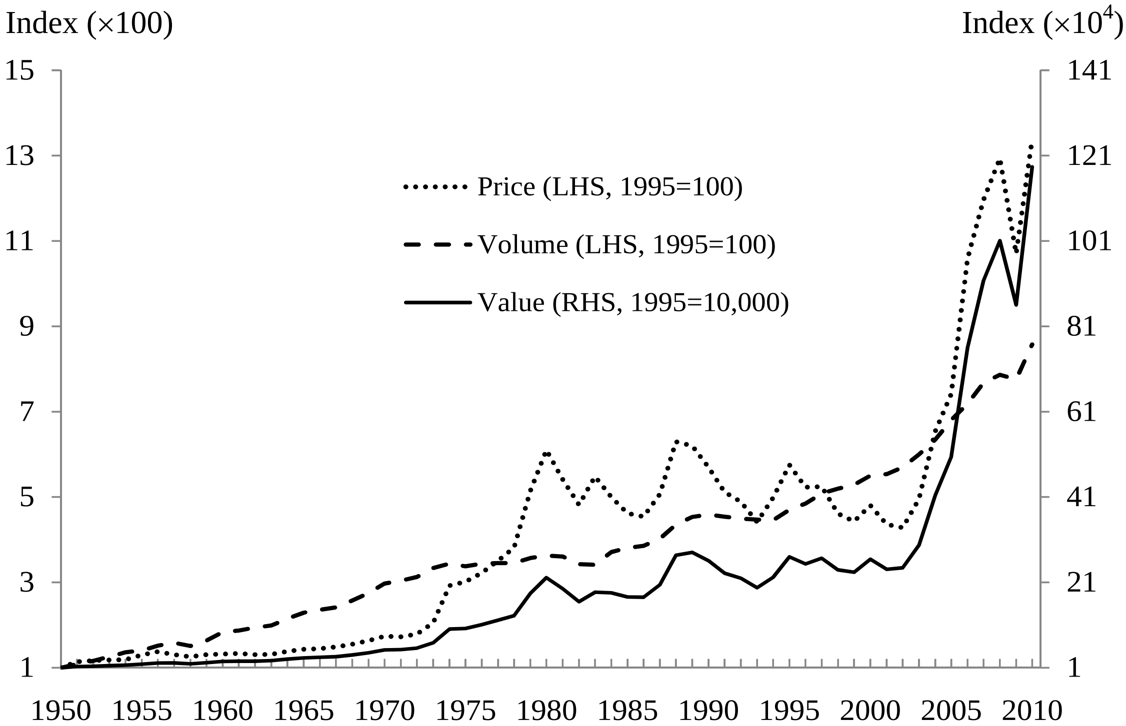


Figure 4. Differences in average metal values, 1950–2010

(logarithmic deviation from mean ×100)

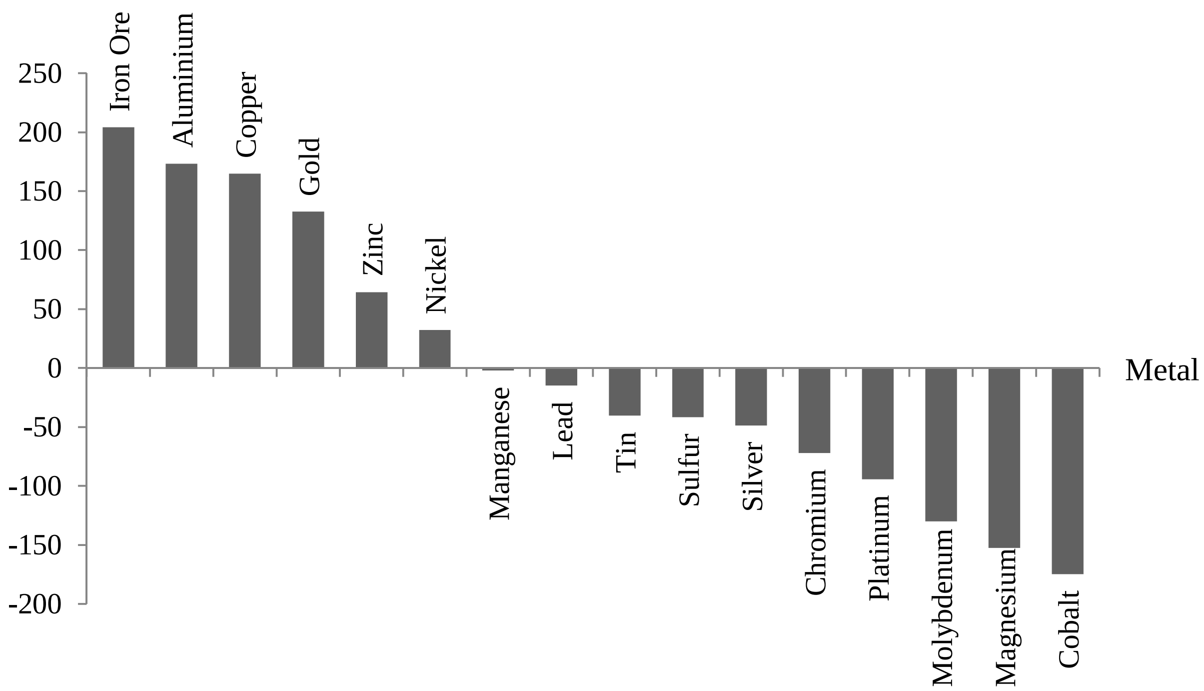
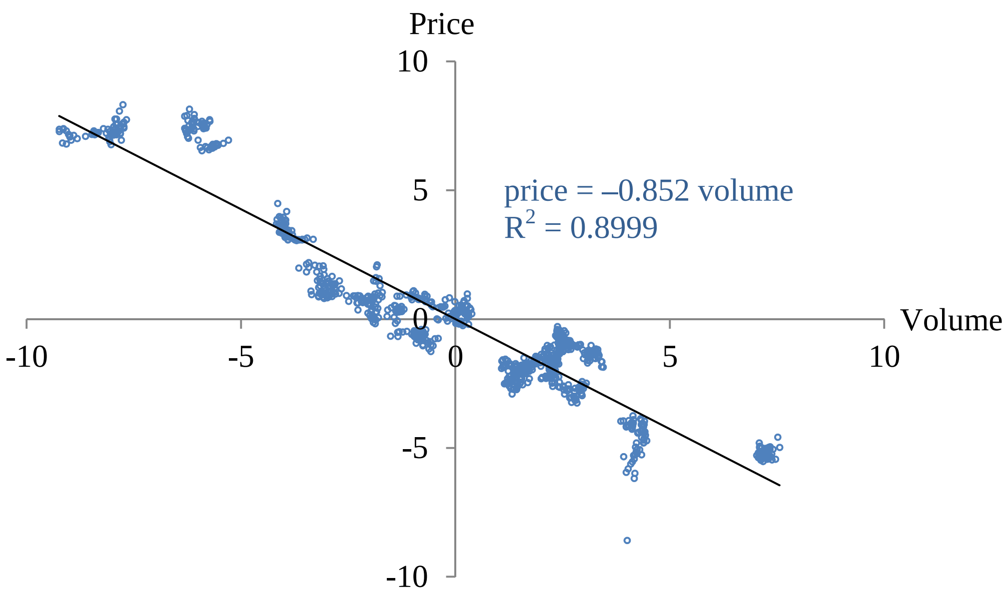


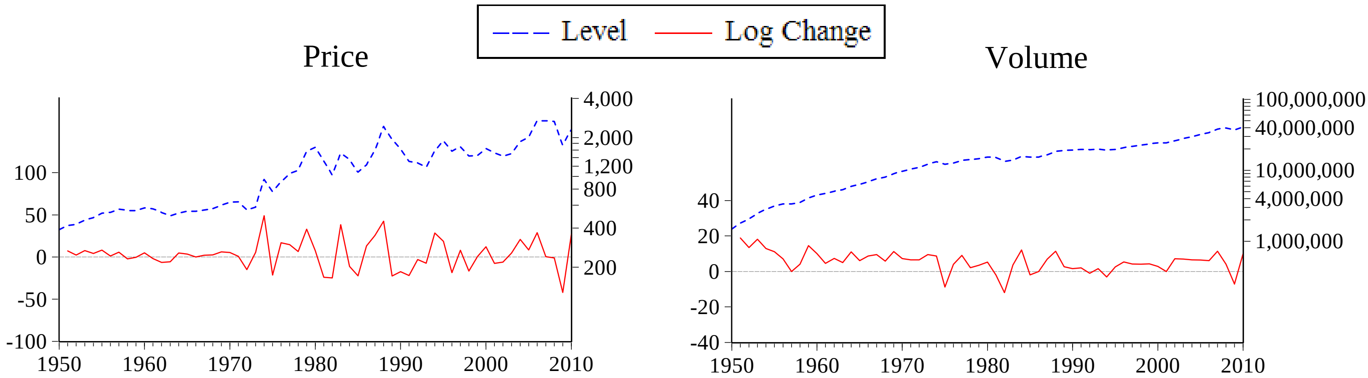
Figure 5. Prices and volumes of 16 metals, 1950–2010



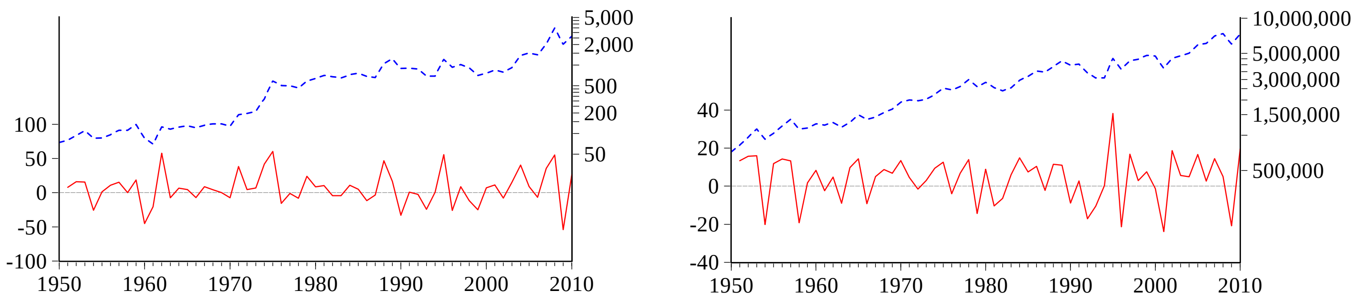
Notes: This is a scatter plot of prices against volumes for 16 metals in each of 61 years, with both variables measured as the logarithmic difference from the mean. That is, the variable on the vertical axis is $\log p_{it} - 1/16 \cdot \Sigma_{j=1}^{16} \log p_{jt}$, while that on the horizontal axis is $\log q_{it} - 1/16 \cdot \Sigma_{j=1}^{16} \log q_{jt}$, for $i = 1,\ldots,16$, $t = 1,\ldots,61$.

Figure A1. Metal prices and volumes, 1950–2010

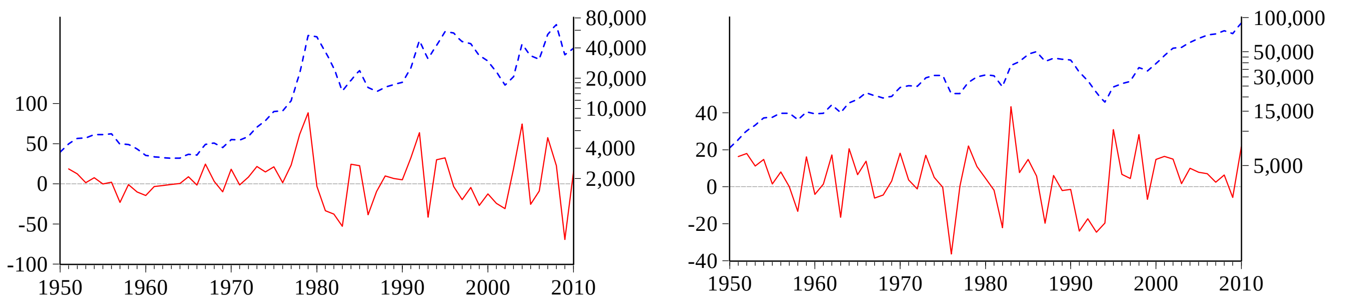
A. Aluminium



B. Chromium



C. Cobalt



D. Copper

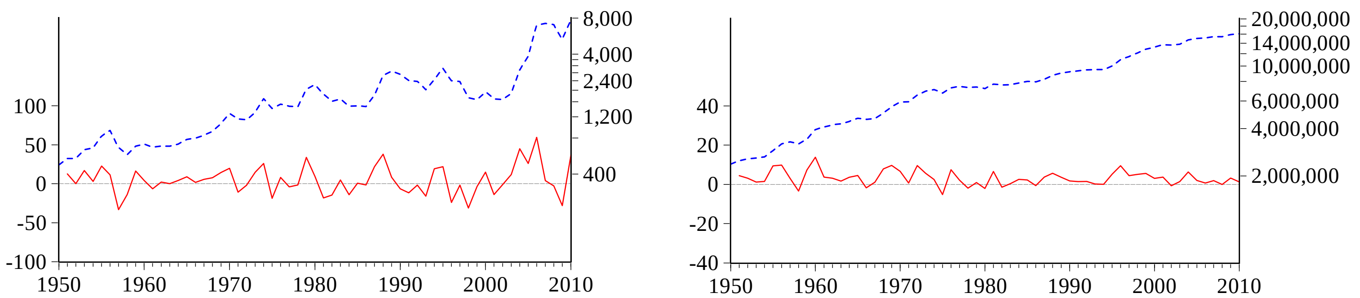
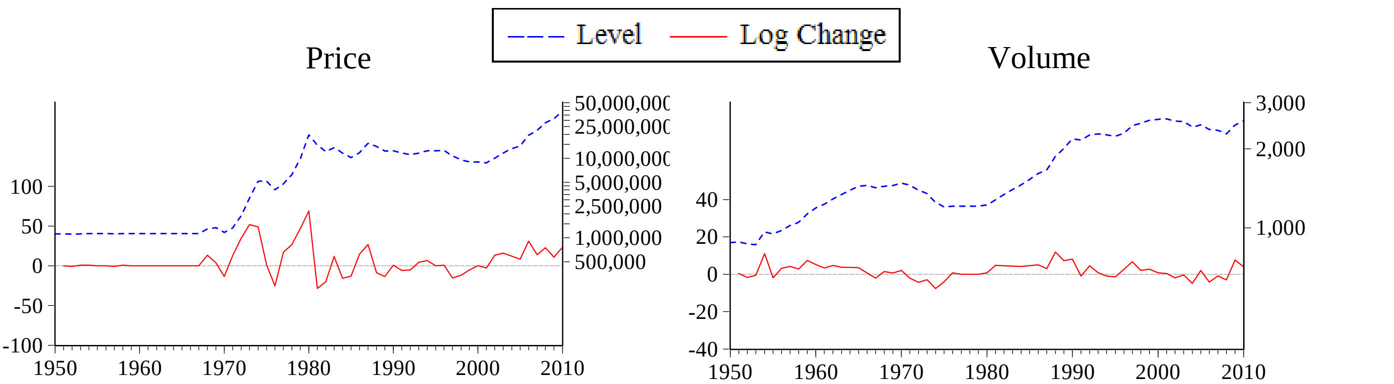
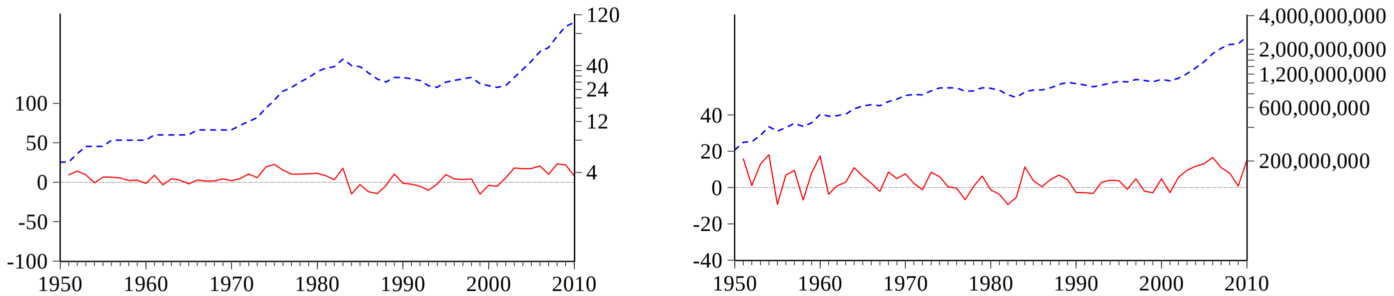


Figure A1 (continued)

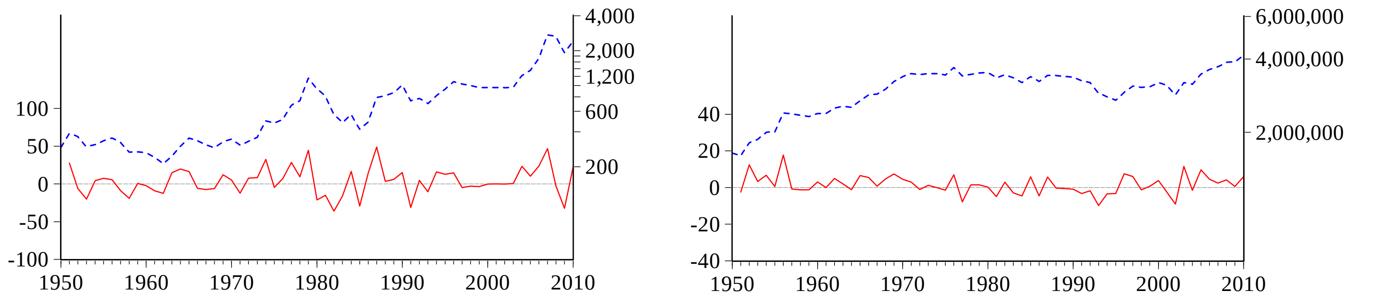
E. Gold



F. Iron ore



G. Lead



H. Magnesium

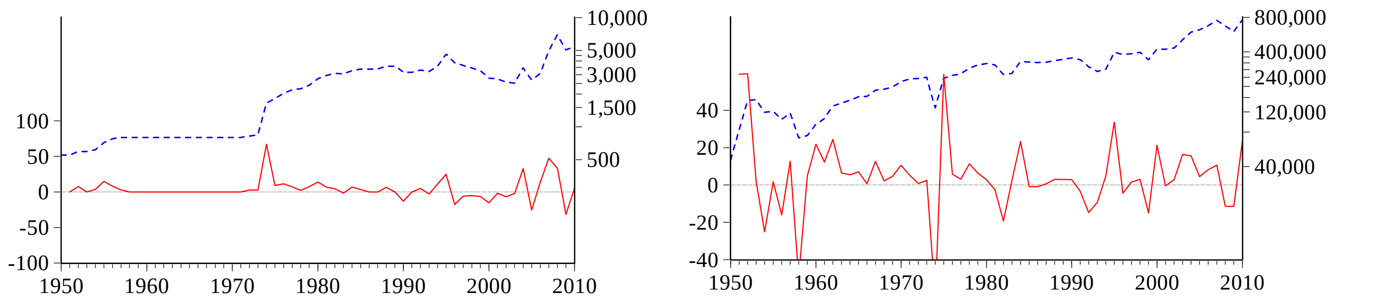
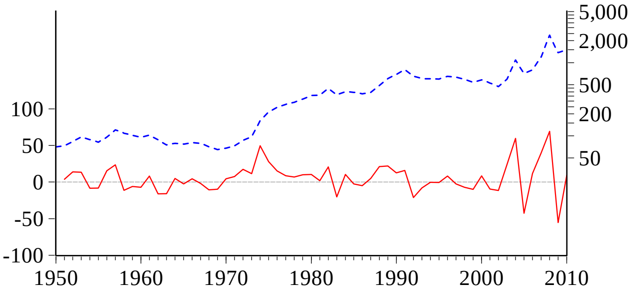


Figure A1 (continued)

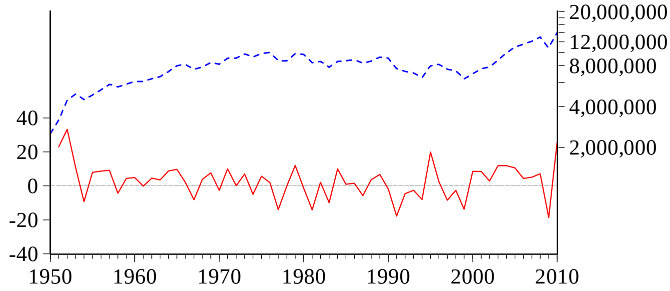
I. Manganese

Level — Log Change

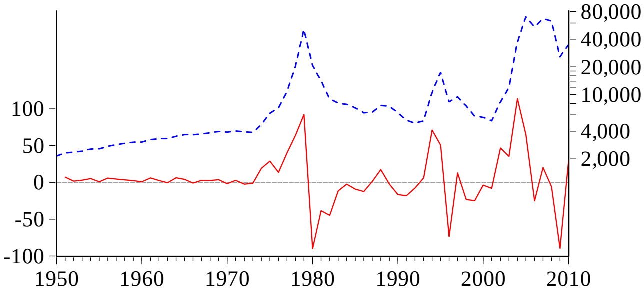
Price

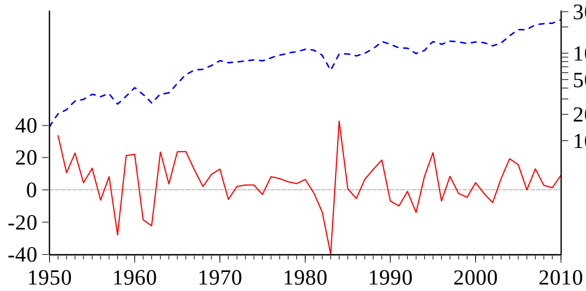


Volume

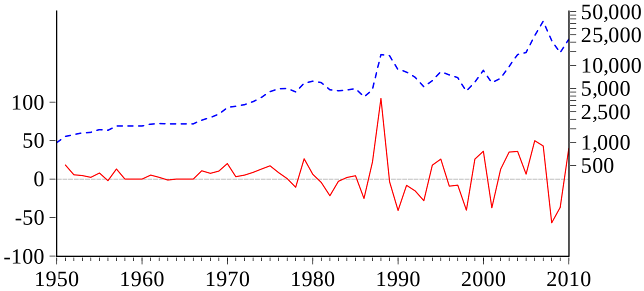


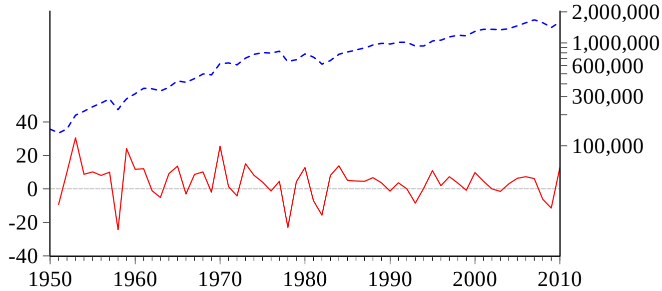
J. Molybdenum



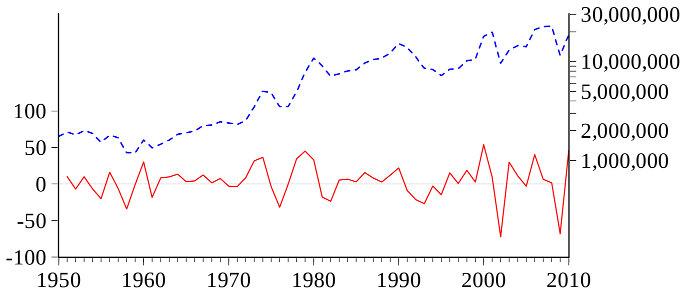


K. Nickel





L. Platinum



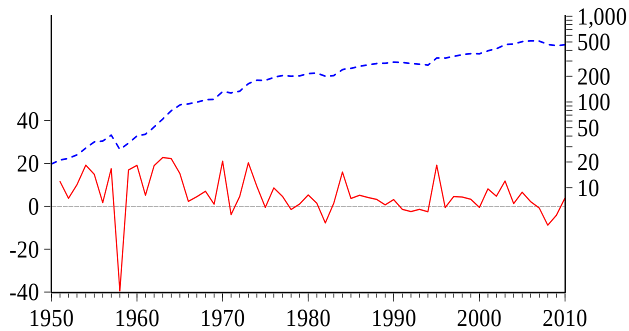
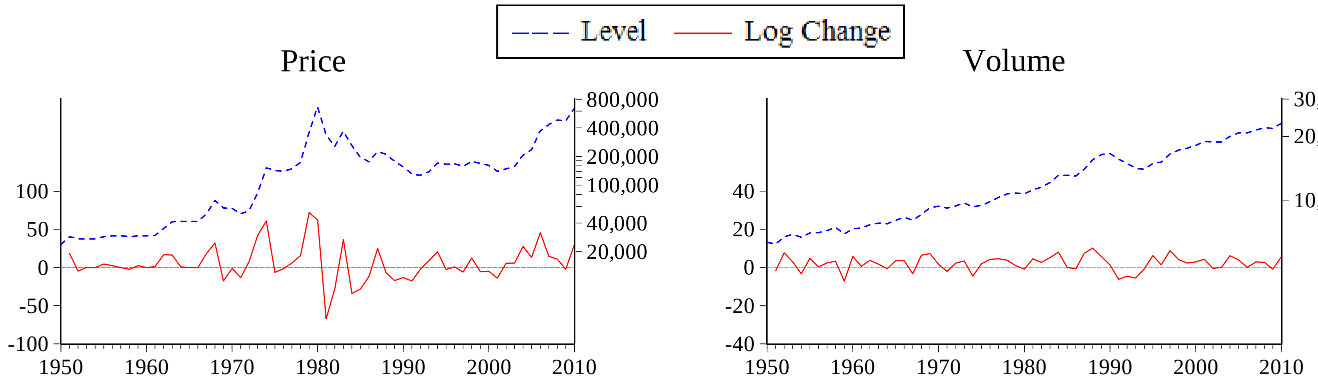
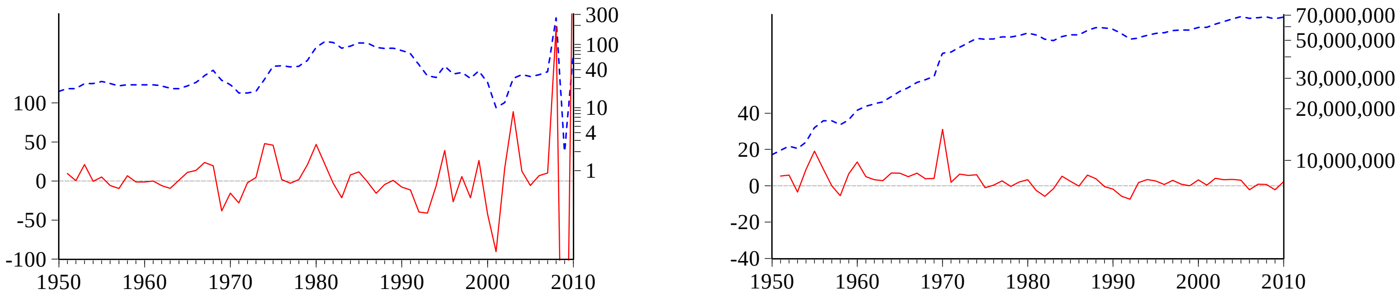


Figure A1 (continued)

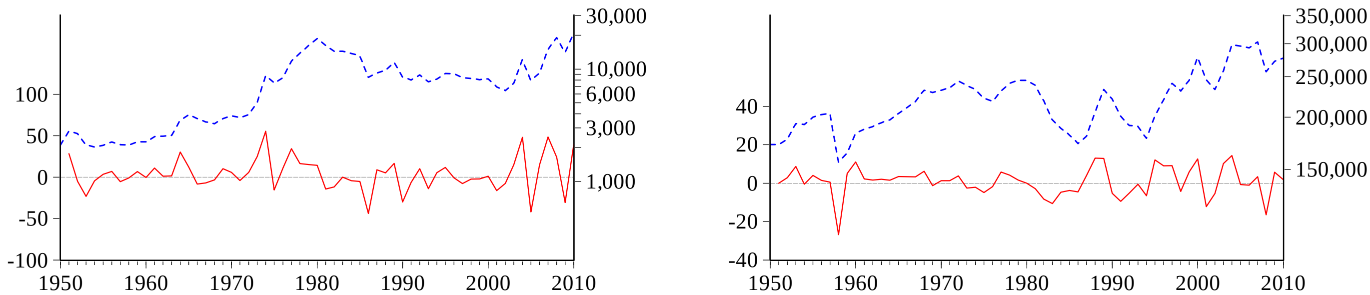
M. Silver



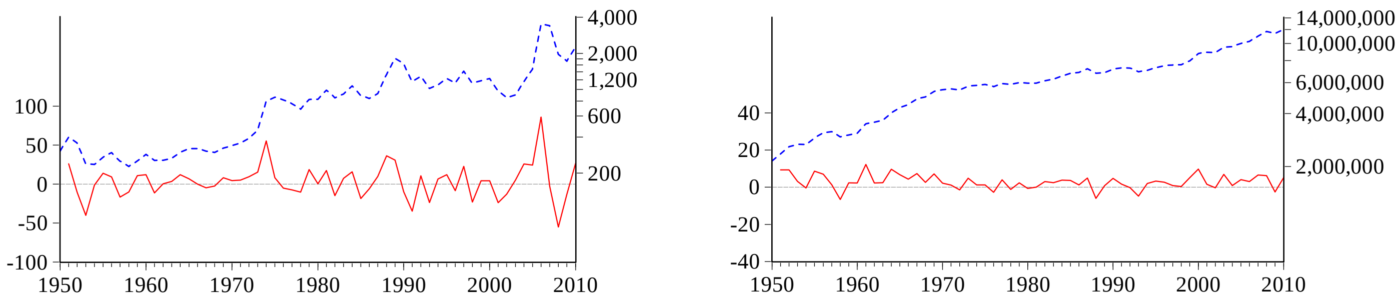
N. Sulfur



O. Tin

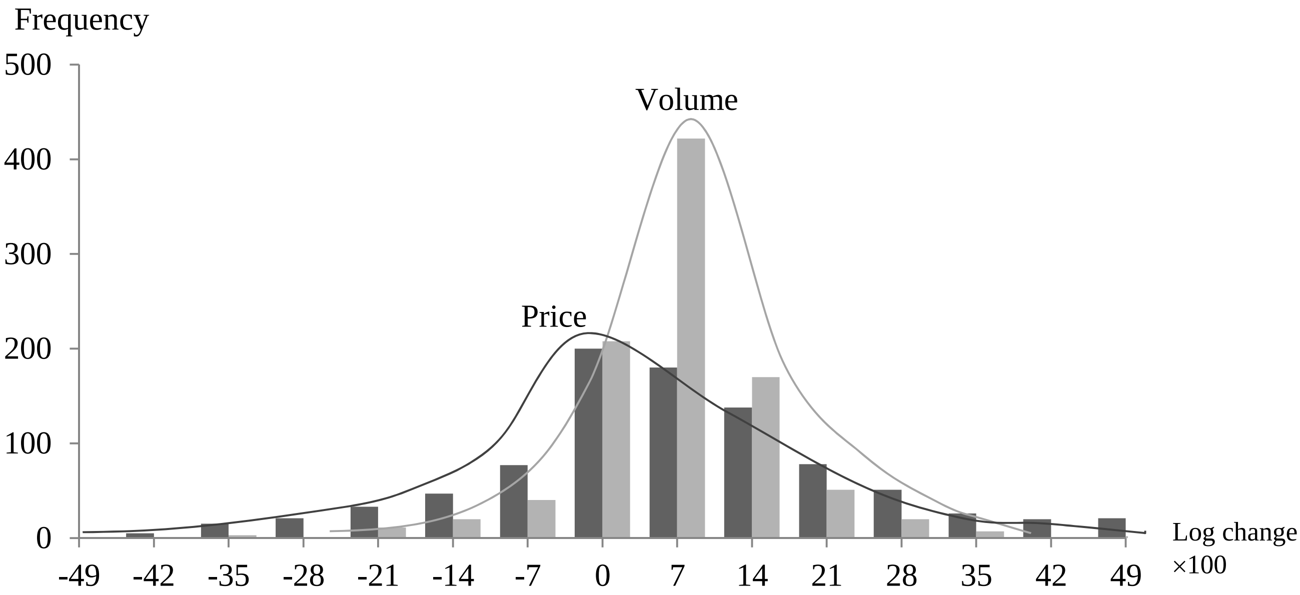


P. Zinc



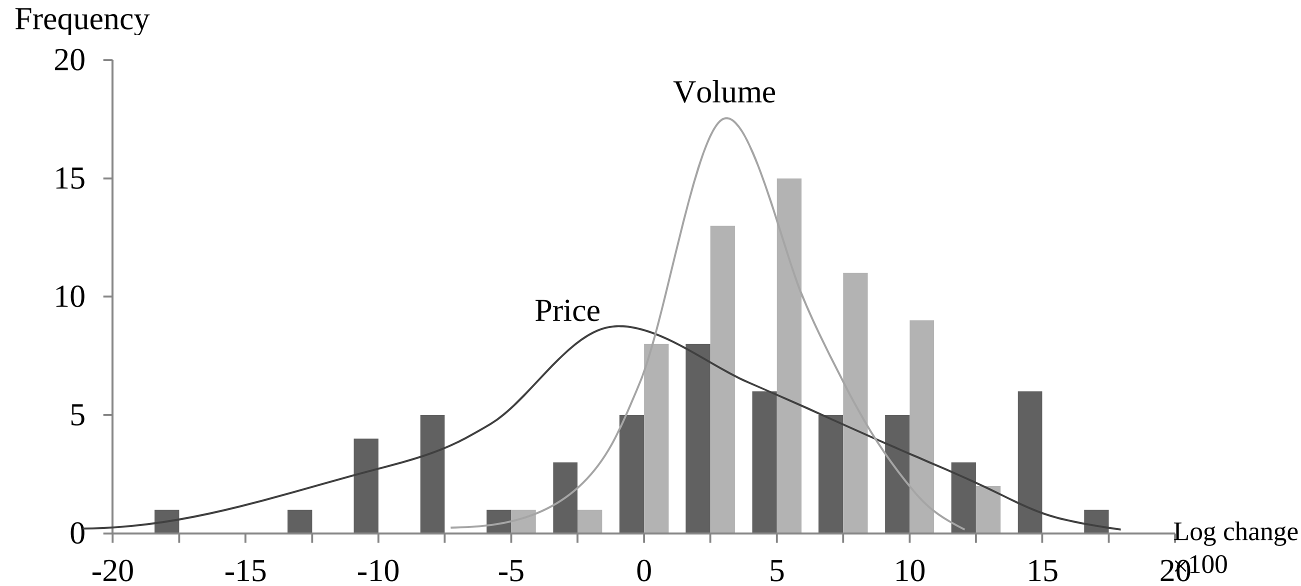
Notes:
1. Prices are nominal in terms of $/tonne and volumes refer to world production in tonnes.
2. Right-hand axis (log scale) refers to levels; left-hand axis refers to annual log-changes ×100.
3. To facilitate presentation, observations with annual logarithmic changes (×100) in price and volume lying outside the ranges [-100, 100] and [-40, 40], respectively, are omitted.

Figure A2. Histogram of logarithmic changes in prices and volumes for 16 metals, 1950–2010



Notes: To facilitate presentation, observations with annual logarithmic changes (×100) lying outside the range [-49, 49] are omitted. As a result, 48 price and five volume observations are excluded.

Figure A3. Histogram of logarithmic changes in price and volume indexes, 1950–2010



Notes: To facilitate presentation, observations with annual logarithmic changes (×100) lying outside the range [-20, 20] are omitted. As a result, six observations are excluded for the price index.

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