Comment on "Measuring a Photonic Qubit without Destroying It"

Recently, Pryde *et al.* reported the demonstration of a quantum nondemolition (QND) scheme for single-photon polarization states with linear optics and projective measurements [1]. In this experiment, a single photon with a specific polarization in the signal mode interacts on a beam splitter with a second polarized photon in the meter mode. A destructive polarization measurement of the meter photon then sometimes reveals the polarization of the signal photon *without* the need for direct detection of the signal mode. This allows the signal photon to propagate freely. Hence the interpretation of this experiment as a single-photon QND measurement.

To give a quantitative characterization of their QND scheme, Pryde *et al.* introduced a *measurement fidelity* F_M, which measures the overlap between the signal input and the measurement distributions. This fidelity is based on the probabilities $P_{sm} = P_{HH}$, P_{HV}, P_{VH}, and P_{VV}. Here, P_{jk} is the probability of finding a j-polarized photon in the signal mode and a k-polarized photon in the meter mode. In other words, F_M is determined solely by *coincidence counting*.

However, for the protocol to work in true QND fashion, the signal photon should propagate freely after the measurement. This means that the proper fidelity measure of a QND protocol cannot be based on the coincidence probabilities P_{sm} alone: Using only coincidence counting *necessarily* implies destructive photodetection of the signal mode. This is incompatible with the definition of a quantum nondemolition measurement. A proper measurement fidelity must take into account P_{k0} and P_{0k}, where 0 denotes the absence of a detector count and $k \in \{H, V\}$.

Physically, when the circuit is operated in proper QND fashion, the signal mode is not detected. This means that we have only the output of the meter mode to tell us what the polarization state of the signal mode is. But when the detectors have imperfections (low quantum efficiency and lack of single-photon resolution), the meter-mode detection might tell us there was only one horizontally polarized photon, when, in fact, there was a second photon that failed to trigger the detector. In that case, the signal mode is in the vacuum state, while we believe it has a horizontally polarized photon. The probability that we mistake the output vacuum for a horizontally polarized photon is given by $1 - F_{\mathrm{QND}}$, where we define the fidelity of the QND device F_{QND} as the overlap between the ideal output state and the physical output state when photodetection of the signal mode is omitted (for a detailed discussion on the interpretation of the fidelity, see Ref. [2]). This leads to $F_{\mathrm{QND}} = \mathrm{Tr}[\hat{E}_k^{(1)} \hat{E}_{\perp k}^{(0)} \otimes |k\rangle_s\langle k| \hat{\rho}_{sm}]$, where $\hat{\rho}_{sm}$ is the density operator of the state before detection, $|k\rangle$ is a single-photon polarization state, and $\hat{E}_k^{(\ell)}$ is the positive operator valued measure (POVM) that models the (imperfect) detection of ℓ photons with polarization k in the meter mode.

According to Pryde *et al.*, the photodetectors can distinguish only between the vacuum state and nonvacuum states. Such detectors cannot tell the difference between one and two photons. Furthermore, the detectors have a probability $\zeta < 1$ of detecting a photon [1]. The POVMs that correspond to such detectors are derived in Ref. [2], and can be written as

$$\begin{aligned} \hat{E}_k^{(0)} &= |0\rangle_k\langle 0| + (1-\zeta)|1\rangle_k\langle 1| + (1-\zeta)^2|2\rangle_k\langle 2|, \\ \hat{E}_k^{(1)} &= \zeta|1\rangle_k\langle 1| + \zeta(2-\zeta)|2\rangle_k\langle 2|, \end{aligned} \tag{1}$$

where $|n\rangle_k\langle n|$ is the projector onto the n-photon Fock state in polarization mode k, and $\hat{E}_k^{(0)} + \hat{E}_k^{(1)} = 1$ on the truncated Fock space $\{|0\rangle_k, |1\rangle_k, |2\rangle_k\}$. The fidelity of the QND circuit is then given by

$$F_{\mathrm{QND}} = \frac{1}{2-\zeta}. \tag{2}$$

The nonpostselected fidelity F_{QND} with a typical detector efficiency of $\zeta = 65\%$ is approximately 0.74. This is significantly less than the fidelity $F_M > 0.99$, measured by Pryde *et al.*

The fidelity F_M may be a good measure for a similar experiment with perfect photodetectors and single-photon sources, but it is meaningful only when the circuit is conditioned on coincidence counting. Such a mode of operation is inconsistent with a QND measurement. To characterize the QND mode of operation, a different fidelity measure such as F_{QND} must be used. With current detectors, it is not clear whether F_{QND} can be made sufficiently large for quantum information processing [3].

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