Environmental policy evaluation under uncertainty through application of robust nonlinear programming[*]

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Abstract. Environmental policy evaluation is characterized by a paucity of information. The novel technique of robust mathematical programming is introduced as a means to proactively account for this uncertainty in policy analysis. The procedure allows identification of expected bounds on the range of abatement costs associated with environmental policy. It also has the advantage of not limiting conclusions to realizations of specific point estimates or probability distributions. Empirical insights are provided in an application to a New Zealand inland lake threatened by nitrate pollution from dairy farming. Overall, this novel framework is demonstrated to have several advantages, including explicit treatment of uncertainty, capacity to bound the range of expected abatement costs accruing to a given policy instrument, and the opportunity to identify robust plans that are immune to parametric variation.

Key words. Nonpoint pollution, policy evaluation, water quality.

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1. Introduction

Pollution of the world's aquatic environments is now primarily attributable to nonpoint pollution (United Nations Environment Program, 2008) since point sources are generally more easily identified and regulated. Eutrophication of lakes and rivers following nutrient pollution is widespread, with more than three-quarters of fresh water bodies in the United States of America exceeding safe thresholds for total nitrogen and phosphorus, imposing a cost of around 2.2 billion U. S. dollars annually (Dodds et al., 2009). Efficient regulation of nonpoint pollution is often problematic given the ambiguity that characterizes the formulation of policy instruments. This uncertainty stems from the diffuse nature of pollution, high number of polluters, non-market characteristics of most regulatory benefits, presence of complex production relationships (e.g. factor substitution), and the response of economic agents to market and production uncertainty.

There is a substantial literature exploring the implications of risk for nonpoint pollution policy (Shortle and Horan, 2001; Kampas and White, 2004). However, the definition of specific probability distributions for model parameters is difficult when modeling complex systems, as (a) adequate information may be unavailable to guide their estimation, (b) additional data can be costly to obtain, (c) information gathering can be complicated by measurement error, and (d) future values (e.g. for market prices) are often difficult to estimate (Doole and Kingwell, 2010). Indeed, the evaluation of nonpoint pollution policies is characterised by uncertainty that can raise doubts about the validity of using standard expected-value analysis considering risk (Shaw and Woodward, 2008).

Mathematical programming (MP) is widely used for policy analysis given its capacity to optimize; its ability to provide a consistent, coherent, and flexible framework for describing systems; and its ability to efficiently solve large, complex problems. Inclusion of risk aversion in policy models has long been recognised as an important means to inject greater realism and dampen the elastic behaviour of linear optimization frameworks (Hazell and Norton, 1986). Indeed, applications of MP incorporating this feature, especially using MOTAD (Minimisation of the Total Absolute Deviation) and target MOTAD formulations, have been numerous over the last thirty years (e.g. Adesina and Ouattara, 2000; Acs et al., 2009). Economists have also widely applied the methods of stochastic programming (Rae, 1971; Kingwell et al., 1993), chance-constrained programming (Zhu et al., 1994), and structured sensitivity analysis (Pannell, 1997) to gain insight into the impacts of stochastic features on decision problems when distributional information is available. Nevertheless, the treatment of pure uncertainty—where the distribution of key parameters incorporated in the decision model is unknown—has not been considered in empirical policy models given a lack of a suitable methodological framework.

Hansen and Sargent (2001) introduced robust control as an elegant means to consider ambiguity in conceptual economic models. This approach was subsequently adopted in the analysis of natural resource problems, with applications to water management (Roseta-Palma and Xepapadeas, 2004) and species preservation (Woodward and Shaw, 2008). Robust control is a powerful technique for small and weakly-nonlinear models, particularly those typically used by economic theorists. However, this method does not naturally extend to large empirical models of the type generally used for environmental policy analysis. Thus, "development of appropriate frameworks for decision making in light of [ambiguity] is an important challenge to economists today" (Woodward and Shaw, 2008, p. 603).

In light of this challenge, this study introduces and provides an assessment of a novel technique of MP—robust nonlinear programming (RNP) (Doole and Kingwell, 2010)—that proactively deals with ambiguity. This is the first empirical application of RNP for environmental policy analysis and offers practitioners an additional tool for analytical and numerical analysis. It offers several benefits, including the representation of uncertainty aversion, solution using standard MP algorithms, and capacity to represent complex systems in models incorporating thousands of equations. The utility of the procedure is assessed in the context of a case study concerning the mitigation of nitrate pollution of a New Zealand inland lake. Economic analysis is pertinent because factor substitution and manipulation of the productive characteristics of livestock can potentially offset abatement costs associated with regulation.

The paper is structured as follows. Section 2 describes the modeling framework. Section 3 describes the model used to evaluate various policy options for the case study. Section 4 presents an empirical application of this model. Section 5 concludes.

2. Robust nonlinear programming

This section presents a concise description of RNP. The discussion follows Doole and Kingwell (2010), which contains a more thorough treatment.

A closed interval is denoted $C = [c^L, c^U]$, where c^L and c^U are respectively the lower and upper bounds of the interval. The *midpoint* of an interval is denoted $C^M = 0.5(c^L + c^U)$, while its *range* (a measure of its spread) is defined $C^R = 0.5(c^U - c^L)$. Superscripts *M* and *R* are used to denote the midpoint and range, respectively, from here onwards. The range or half-length represents the difference between the midpoint and either bound. No further information on the probability distribution of parameters is assumed.

A generic non-linear programming (NLP) problem can be defined: $\max_{\mathbf{x}} J = \pi(\mathbf{x})$, subject to $g(\mathbf{x}) \leq 0$ and $\mathbf{x} \geq 0$, where $\pi(\mathbf{x})$ is a profit function and $g(\mathbf{x})$ denotes the constraint functions. A pedagogical example is: $\max_{x} J = 2x^2$, subject to $3x - 10 \leq 0$ and $x \geq 0$.

An interval-valued non-linear programming problem (**IVP**) can be defined: $\max_{\mathbf{x}} J^{iv} = [\pi^L(\mathbf{x}), \pi^U(\mathbf{x})]$, subject to $[g^L(\mathbf{x}), g^U(\mathbf{x})] \leq 0$ and $\mathbf{x} \geq 0$, where J^{iv} is the interval-valued objective function. This problem states that the objective and constraint functions are bounded given the use of closed intervals to define uncertain parameters. Its practical solution is discussed in the next paragraph. Assume that the simple example above is subject to interval-valued uncertainty. This could be represented by the problem: $\max_{x} J = [1,3]x^2$, subject to $[2,4]x - [9,11] \leq 0$ and $x \geq 0$. Here, the midpoint of each parameter is that from the deterministic example and the range from the midpoint is unity.

Solutions belong to $\Re$ in standard NLP and thus may be ordered using inequality notation. Closed intervals may not be ordered equivalently. However, a nondominated solution may be identified if neither bound of the interval-valued objective function can be feasibly improved without decreasing the other (in a maximisation problem). Doole and Kingwell (2010) outline that this solution may be identified through the formulation (**RNP**):

$\max_{\mathbf{x}} J = (\boldsymbol{\pi}^{M}{}' - \boldsymbol{\Omega}\boldsymbol{\pi}^{R}{}')\mathbf{x} + (\boldsymbol{\pi}^{M}{}' + \boldsymbol{\Omega}\boldsymbol{\pi}^{R}{}')\mathbf{x}$, subject to $(g^{M}(\mathbf{x}) + \boldsymbol{\Lambda} g^{R}(\mathbf{x})) \leq 0$ and $\mathbf{x} \geq 0$, where Ω and Λ are exogenous trade-off parameters ($\Omega = [0,1]$ and $\Lambda = [0,1]$) defined for the objective and constraint functions, respectively, and the superscripts *M* and *R* denote the midpoint and range, respectively. A feature of RNP is that any number of coefficients may be defined as uncertain in a straightforward manner. This is done through extension of the standard process—that discussed in this section and applied in Section 3—to the relevant number of parameters. The pedagogical example in this format is defined: $\max_{x} J = (2 - \Omega \cdot 1)x^2 + (2 + \Omega \cdot 1)x^2$, subject to $(3 + \Lambda \cdot 1)x - (10 + \Lambda \cdot 1) \leq 0$, and $x \geq 0$. RNP is consistent with parametric uncertainty, but can easily be extended to study problems in which the functional forms for the lower and upper bounds are dissimilar through the use of convex combinations.

The objective function for RNP incorporates the summation of its lower and upper bound. It is linear additive in the case of parametric uncertainty, in which case $J = 2(\boldsymbol{\pi}^{M}{}')\mathbf{x}$ in the optimization. This ensures the identification of a nondominated solution to the maximisation problem. However, it is important to retain $(\boldsymbol{\pi}^{M}{}' - \boldsymbol{\Omega}\boldsymbol{\pi}^{R}{}')\mathbf{x}$ and $(\boldsymbol{\pi}^{M}{}' + \boldsymbol{\Omega}\boldsymbol{\pi}^{R}{}')\mathbf{x}$ since these identify lower and upper bounds of the objective function for the optimal solution. This yields the expected range of optimal outcomes upon solution, conditional on the uncertainty defined in the model and the tolerances of decision makers to this uncertainty.

Trade-off parameters specify the proportion of the variation in the uncertain parameter (i.e. the difference between the midpoint and the range) that is considered in the determination of the robust plan. Note that the addition of the range to the midpoint in the less-than constraint in RNP is consistent with maintaining feasibility to the degree determined by the trade-off parameter. For example, the plan is robust to all expected outcomes if the trade-off parameter is set to unity. Trade-off parameters are a simple measure of uncertainty aversion since they represent the degree of conservatism that a decision maker wishes to consider in formulating the optimal plan. The worst-case outcome is bounded when $\boldsymbol{\Lambda} = \boldsymbol{\Omega} = \mathbf{1}$ and the constraint functions are positive.[‡] This inherent conservativeness is justified in a state of uncertainty (Hansen and Sargent, 2001;

[‡] Whether ranges are added to or subtracted from the midpoint for the constraint functions depends on their sign and the direction of the constraint. See Doole and Kingwell (2010) for more information and examples.

Woodward and Shaw, 2008), particularly given the possibility of irreversible environmental degradation. However, the worst-case approach can be relaxed through the inclusion of trade-off parameters. For example, the standard deterministic model involving no uncertainty is recovered when $\mathbf{\Lambda} = \mathbf{\Omega} = \mathbf{0}$. (In a geometric sense, these trade-off parameters determine the placement of the constraints that delineate the feasible region between their lower and upper bounds.)

Trade-off parameters should be estimated to provide a robust plan that meets the requirements of decision makers. Indeed, results in Sections 4.4 and 4.5 show that the use of a trade-off parameter of unity for all uncertain coefficients in the empirical model incurs a substantial cost. This highlights the need to estimate trade-off parameters using the best information possible. Trade-off parameters may be estimated through identification of those values that minimise the differences between observed data and model output. Such a calibration procedure is used in the empirical model presented in this paper (see Section 3.3). Trade-off parameters may also be estimated through surveys or focus groups. The focus of these activities should be to identify what proportion of outcomes decision makers want to insure against when robust solutions are implemented. It is also important to conduct sensitivity analysis to determine how model output is affected with different trade-off parameters.

3. Application

3.1 Nitrate pollution of New Zealand freshwater resources

The New Zealand dairy industry is the country's dominant agricultural industry, with dairy products valued at $7.5 billion comprising 21 per cent of total merchandise exports in the year ending June 2007 (Statistics New Zealand, 2007). The high prices received for dairy products over the last decade have promoted significant intensification of what historically was a low-input, pasture-based system. Accordingly, national milk production increased by 33 per cent and stocking rates and per cow production both increased by 12.5 per cent between 1997 and 2007 (Livestock Improvement Corporation, 2008). Augmented production intensity follows increasing use of supplementary feeds, particularly maize silage, and nitrogenous fertilizer. Indeed, mean use increased by more than 375 and 300 per cent, respectively, in the study region between 1997 and 2007 (Environment Waikato, 2008a). However, intensification has led to greater nitrate leaching and subsequent nutrient pollution of freshwater bodies (see Monaghan et al. (2007) and references therein).

Lakes Karapiro and Arapuni are hydroelectric dams on the Waikato River, New Zealand's longest watercourse. As well as electricity generation, these lakes are important for recreation and tourism, and have cultural value to local Maori, the indigenous people of New Zealand. Algal blooms have been observed in the lakes in recent years, as nitrate discharges from dairy farms in the surrounding catchment have decreased water quality (Environment Waikato, 2008a, 2008b). Dairy farming currently covers 42,938 ha of the catchment for these two lakes (AsureQuality, 2008), nearly three-quarters of agricultural land in this area. Accordingly, there is a need for the regional environmental agency to establish appropriate regulatory tools to manage nutrient pollution.

This analysis contributes to this goal through the use of RNP to identify the potential costs of emissions standards. This study is also of international relevance given the increasing global awareness of the environmental impacts of dairy production, especially in China and India. RNP is applied to proactively deal with uncertainty surrounding nitrate leaching, variable costs, and pasture production (see Section 3.2). Variable costs are estimated using positive mathematical programming (Henseler et al., 2009), but are subject to variation across years.

3.2 Model description

This section presents the RNP model used in the case study. The model extends the individual farm model used by Doole (2010) to the catchment scale and to represent uncertainty. Lakes Karapiro and Arapuni are henceforth referred to collectively as "the lake".

The model describes a management year consisting of 26 fortnightly periods ($i = [1,2,...,26]$) beginning on 1 July. The first time period follows the last time period in a cyclical fashion. Feed supplies are measured using tonnes of dry matter (DM).

The regulator is assumed to manage a catchment, or proportion of a catchment, consisting of a hectares. New Zealand dairy farms are typically rotationally grazed. Intermittent grazing at high stocking rates improves pasture quality, utilization, and usually production. Producers may spell fields from grazing during periods of substantial pasture growth and harvest them for grass silage. The area of pasture grazed at time *t* that has not been grazed since period *i* is represented by $A_{i,t}^{G}$. Similarly, $A_{i,t}^{SM}$ denotes the area harvested for silage production at time *t* that has not been grazed since period *i*. In addition, $A_{i,t}^{X}$ represents the area of pasture grazed at time *t* that was ensiled in period *i*. Total land use at time *t* is described by:

$$a \geq \sum_{i=1}^{26}(A_{i,t}^{G} + A_{i,t}^{SM} + A_{i,t}^{X}) + \sum_{i}\sum_{t\#}(A_{i,t\#}^{G} + A_{i,t\#}^{SM} + A_{i,t\#}^{X})_{\forall i \neq t, t > i, t\# > t} + \sum_{i}\sum_{t\#}(A_{i,t\#}^{G} + A_{i,t\#}^{SM} + A_{i,t\#}^{X})_{\forall i \neq t, i > t, t\# > t} \,. \quad (2)$$

The first term on the RHS describes land used at time *t* that has been spelled since period *i*. The second term describes the land spelled at an earlier time period *i* that is to be used at a later period *t#*, where $t\# > t$. The third term modifies the second term to include those periods *i* that occur later than *t#* given the definition of time as a cyclical process.

Total feed production in period *t* (P_t^j for $j = \{G, SM, X\}$) is defined through: $P_t^j = \sum_{i=1}^{26} A_{i,t}^{j}(r_i^j + \sum_{g=i+1}^{t}(b_g^M - \Lambda^P b_g^R) - r_t^j) \;\; \forall t \neq i$, where r is residual biomass, b_g^M (b_g^R) represents the midpoint (range) of pasture biomass growth in period *g*, and Λ^P is the trade-off parameter representing the degree of uncertainty in pasture production that the decision maker wishes to insure against. The range is subtracted from the midpoint in this supply equation, consistent with the conservative approach inherent to RNP.[§]

This relationship is conditioned by two bounds: $A_{i,t}^{j}\alpha_t^j \leq A_{i,t}^{j}(r_i^j + \sum_{g=i+1}^{t}(b_g^M - \Lambda^P b_g^R)) \;\forall t \neq i$ and $A_{i,t}^{j}(r_i^j + \sum_{g=i+1}^{t}(b_g^M + \Lambda^P b_g^R)) \leq A_{i,t}^{j}\beta_t^j \;\forall t \neq i$, where α and β are minimum and maximum biomass levels, respectively. Here, the range is subtracted (added) to the midpoint in the first (second) relationship, consistent with a conservative approach to defining the production environment.

[§] In contrast, addition of the range to the midpoint would make the model highly optimistic as, in this alternative formulation, pasture is expected to grow at its highest rate. This is consistent with a maximax formulation.

Also, pasture growth may be promoted using nitrogen fertilizer through $P_t^N = \sum_{i=1}^{26} f_{i,t} F_i$, where P_t^N is the pasture biomass (t ha^{-1}) produced through nitrogen fertilization in period *t*, $f_{i,t}$ is the yield response (t DM ha^{-1}) in time *t* following application of one tonne of nitrogen fertilizer in period *i*, and F_i is the amount of nitrogen fertilizer (t ha^{-1}) applied during period *i*.

Possible herd configurations differ by calving date, lactation length, herd status, and productivity. Calving can begin on July 1, July 15, and August 1. There are five possible lactation lengths: 180, 210, 240, 270, and 300 days. There are two herd classifications: cull or standard. Cull herds can be milked for any of the five lactation lengths, with all cows culled at the end of lactation. Conversely, standard herds can only be milked for 240, 270, and 300 days. There are three possible productivity levels: low, medium, and high. Individual cows hence may belong to one of 45 cull herds (the product of 5 lactation lengths, 3 calving dates, and 3 productivity levels) or 27 standard herds (the product of 3 lactation lengths, 3 calving dates, and 3 productivity levels).

Metabolizable energy (ME) is that available for livestock growth and maintenance after the digestion of feed. Milk production increases with productivity level (represented by bodyweight) and lactation length for a given calving date. However, the cost of increased production is additional energy demand. The demand and supply of energy is calculated for each fortnightly period through:

$$\sum_{h=1}^{72} D_h E_{h,t} \leq (P_t^G + P_t^X + P_t^N) u_t^P q_t^P + P_t^{SF} u^S q_t^S + V_t u^V q^V + K_t u^K q^K , \qquad (3)$$

where D_h represents the number of cows in herd *h*, $E_{h,t}$ represents the energy requirement (measured in MJ of ME per fortnightly period) of a cow in herd *h* at time *t*, u_t represents the proportion of the feed that is consumed by livestock (e.g. u_t^P represents pasture utilization), q_t is the energy content of each feed at time *t* (MJ ME^{-1} t^{-1} DM), P_t^{SF} is the total amount of grass silage fed to cows, it is required that $P_t^{SM} \geq P_t^{SF}$, V_t is the amount of maize silage (t DM) fed to cows at time *t*, and K_t is the amount of concentrate (t DM) fed to cows at time *t*.

The feed intake of cows is constrained so herds do not consume an unrealistic quantity through:

$$\sum_{h=1}^{72} D_h I_t^P \geq (P_t^G + P_t^X + P_t^N) u_t^P + P_t^{SF} \Gamma^S u^S + \Gamma^S V_t u^V + \Gamma^K K_t u^K \,, \qquad (4)$$

where I_t^P is the maximum per cow intake of pasture dry matter at time t (t DM cow^{-1}), Γ^S is the substitution rate of pasture to forage supplements (grass and maize silage), and Γ^K is the substitution rate of pasture to concentrate.

Production impacts nitrate levels in the lake through:

$$N = (\chi^M + \Lambda^N \chi^R) + (\phi^M + \Lambda^N \phi^R) a^{-1} \sum_{t=1}^{26} F_t + (\eta^M + \Lambda^N \eta^R) a^{-1} \sum_{h=1}^{72} D_h - (\tau^M + \Lambda^N \tau^R) a^{-1} \sum_{t=1}^{26} V_t \,, \quad (5)$$

where N is the total amount of nitrate leaching (kg ha^{-1}), superscripts of M and R denote midpoints and ranges respectively, Λ^N is the trade-off parameter representing the degree of uncertainty in nitrate leaching that the decision maker wishes to consider in the determination of optimal plans, χ is a constant term, and $\{\phi, \eta, \tau\}$ are slope coefficients representing the relationship between nitrate leaching and nitrogen fertilizer application, stocking rate, and maize silage use, respectively. Total nitrate leaching per farm is recovered where a^{-1} is removed from eq. 5.

Stocking rate is the primary driver of nitrate leaching in New Zealand dairy-farming systems since grazed pastures typically provide more nitrogen than cows require and this is expelled in urine (Monaghan et al., 2007). Nitrogen fertilizer plays an indirect role, increasing pasture production and hence stocking rate. In contrast, the low N content of maize silage decreases the N excreted by cows.

The objective function is:

$$\max \pi = p^{milk}\sum_{h=1}^{72} D_h z_h + p^{cull}\sum_{h=1}^{45} D_h + p^{calf}(\sum_{h=1}^{72} D_h\psi - \sum_{h=1}^{45} D_h\omega) - c^D\sum_{h=1}^{72} D_h$$
$$- 0.5(Q^M - \Psi Q^R)\left(\sum_{h=1}^{72} D_h\right)^2 - 0.5(Q^M + \Psi Q^R)\left(\sum_{h=1}^{72} D_h\right)^2 \quad (6)$$
$$- c^S\sum_{i=1}^{26} P_i^{SM} - c^V\sum_{i=1}^{26} V_i - c^K\sum_{i=1}^{26} K_i - c^F\sum_{i=1}^{26} F_i - c^{FC}a,$$

where p^{milk} is the price received for milk solids (MS) ($ t^{-1}), z_h is annual milk production (t cow^{-1}) of a cow in herd h, p^{cull} is the price received for one cull cow ($ cow^{-1}), p^{calf} is the price received for one calf ($ $calf^{-1}$), ψ is the calving rate, ω is the replacement rate, c^D is the variable cost associated with a single cow ($ cow^{-1}), Q^M and Q^R are the midpoint and range of the quadratic cost parameter, Ψ is the trade-off parameter representing the degree of variation in the quadratic cost function that the decision maker wishes to consider in the optimization problem, c^S is the cost of conserving grass silage ($ t^{-1} DM), c^V is the cost of maize silage ($ t^{-1} DM), c^K is the cost of concentrate ($ t^{-1} DM), c^F is the cost of nitrogen fertilizer ($ t^{-1}), and c^{FC} is the fixed cost of production ($ ha^{-1}).

Eq. 6 is maximized subject to the constraints described in this section, with all decision variables constrained to be non-negative. The second line of eq. 6 describes the bounded cost function. Thus, the interval-valued profit function $\pi(\mathbf{x}) = [\pi^L(\mathbf{x}), \pi^U(\mathbf{x})]$ associated with a given production plan can be recovered through $\pi^L(\mathbf{x}) = F(\mathbf{x}) - 0.5(Q^M + \Psi Q^R)\left(\sum_{h=1}^{72} D_h\right)^2$ and $\pi^U(\mathbf{x}) = F(\mathbf{x}) - 0.5(Q^M - \Psi Q^R)\left(\sum_{h=1}^{72} D_h\right)^2$, where $F(\mathbf{x})$ is all of those terms on the first and third lines in eq. 6. Illustrations of these functions are provided in Section 4.

3.3 Parameter values

Nitrogen fertilizer responses and minimum, maximum, and residual pasture masses are taken from McCall et al. (1999). Feed energy, substitution, and utilization rates are taken from McCall et al. (1999) and Dexcel (2008). Pasture production for 1986–2006 is determined using meteorological data from NZCD (2008) and a variant of the model of Moir et al. (2000).

Temporal milk production in each herd is described with the widely-used gamma function. Shape parameters from Johnson (2008) are employed, while the coefficient specifying maximum daily milk production is determined for each herd using data from McCall et al. (1999) and a Generalised Reduced Gradient (GRG) method (Bazaraa et al., 2006) to perform root-finding. Energy demand as a function of grazing, milk production, and pregnancy is taken from Dexcel (2008).

The relationship between production decisions and nitrate leaching described in Eq. 5 is determined from the OVERSEER model (Monaghan et al., 2007). Leachate burdens are calculated for multiple combinations of nitrogen fertilizer, stocking rate, and maize silage for those soil types found on farms in the study region. OVERSEER output is regressed to form a metamodel using SHAZAM econometric software (Whistler et al., 2004). The response surfaces are bounded to represent spatial differences in soil type.

Prices for calibration are taken from Livestock Improvement Corporation (2008). The value of supplementary feeds, calves, and cull cows are drawn from different editions of the New Zealand Financial Budget Manual (e.g. Chaston, 2008). Variable and fixed costs are calculated from the Economic Survey (ES) of New Zealand Dairy Farmers (e.g. Dexcel, 2006). Nitrogen fertilizer prices are taken from fertilizer company records.

Defining every model parameter using bounded sets increases the chance that conclusions will be impractical due to overconservatism since there is a low probability that all variables will simultaneously take their worst-case values given the law of large numbers. Thus, selection of appropriate parameters to bound is critical. The parameters bounded in this model are selected based on their critical importance to the problem, discussions with experienced modellers of New Zealand dairy farming systems, previous modelling work, goals of the application, and *a priori* computation of sensitivity indices (Pannell, 1997) to highlight the correlation between model output and perturbations of different coefficients. Those relationships represented as uncertain in the model are nitrate leaching, the quadratic cost function, and pasture production (Figure 1). Note there are two surfaces depicted in Figure 1a; some variation is present, but only small differences have been estimated using the OVERSEER model and the metamodelling approach. The quadratic cost function is subject to much larger ambiguity (Figure 1b). The annual variability of pasture growth changes with season, becoming more regular between late August and the end of September (early spring) (Figure 1c).

[Insert Figure 1 near here]

Trade-off parameters for the pollution meta-model and quadratic cost function are set to unity. This indicates that all expected parametric variation in nitrate leaching and variable costs are considered in the formulation of the optimal plan. This is relevant for a number of reasons. First, there is broad uncertainty surrounding both variable costs and nitrate leaching and lower values of uncertainty aversion would disregard this ambiguity in the optimisation. Second, conservativeness is justified in a state of uncertainty (Woodward and Shaw, 2008). Third, the focus of the study is estimating the potential range of abatement costs. Last, a worst-case approach is taken with respect to environmental degradation (ie. nitrate leaching) given the possibility of irreversible environmental degradation (Pindyck, 2007).

The trade-off parameter for pasture production requires careful estimation, as high values lead to unrealistic management plans and/or render the calibration constraints infeasible. It is therefore estimated using a combinatorial search algorithm. The trade-off parameter for pasture production is treated as an unknown and a simulated-annealing procedure (Doole and Pannell, 2008) is coded in GAMS Distribution 22.8 (Brooke et al., 2008)[**] to identify that value which minimises the absolute difference between observed (AsureQuality, 2006; AsureQuality, 2008; Livestock Improvement Corporation, 2008) and optimal levels of herd size and milk production. This identifies a value consistent with historical management and also reduces some of the burden placed on the quadratic-cost function as a calibration instrument. The estimated trade-off parameter for pasture production is 0.8. This signifies that the average producer constructs their management plan such that it would be expected to remain feasible in eight years of each decade. A value of zero indicates that the range of uncertain coefficients is not considered at all in the

[**] A combinatorial search algorithm is required since the presence of numerous logical conditions in the complex constraint set of the model precludes the identification of this parameter using mathematical programming in GAMS.

optimisation, while values between zero and 0.8 indicate consideration of less ambiguity than the base case.

3.4 Solution of model with robust nonlinear programming

Estimates of the lower and upper bound for the total dairy cow population in the catchment are 113,356 (AsureQuality, 2006) and 123,232 (AsureQuality, 2008) in 2006 and 2008, respectively. Unsurprisingly, the linear model does not naturally calibrate to either of these magnitudes. A reasonable instrument for calibration is a (convex) quadratic variable-cost function associated with herd size. Marginal costs may increase with herd size *ceteris paribus* for many reasons, including inefficiencies associated with fixed capital and detrimental impacts on soil properties.

The RNP problem contains 4,349 variables and 6,407 constraints. The corresponding GAMS program is available from the authors on request.

The base solution contains output for the standard parameter values used in the model. Environment Waikato currently uses emissions controls elsewhere to improve water quality in a lake. So, the primary focus of the study is exploring the abatement cost of emissions standards defined between 0–50 per cent of current levels under different circumstances. The model is used to investigate a number of scenarios:

1. Abatement costs are determined for the base case.
2. The impact of decreasing/increasing the milk price by \$500 t^{-1} is explored.
3. The implications of defining trade-off parameters for pasture growth of $\Lambda = \{0, 0.5, 0.8, 1\}$ are investigated.
4. Monte Carlo simulation is used to explore the implications of subsequent variability in annual pasture growth for the profitability of cow herd compositions formulated through RNP. Hedging against this source of uncertainty is costly because supplementary feeding is required. For each level of the trade-off parameter listed for the third scenario, this involves:
    a. Optimizing the model. The quadratic cost function is defined at its midpoint to allow generation of a histogram in Step c.
    b. Fixing cow herds at their optimal levels.

c. Re-optimizing the remaining decision variables for 100 scenarios in which pasture growth in each period is represented as a uniform random variable defined between the lower and upper bounds identified in Section 3.3.

4. Results and Discussion

4.1 Base solution

The base solution closely describes production behaviour in the study region. The optimal stocking rate is 2.69 cows ha^{-1}, 2 per cent higher than the 2006/07 stocking rate consistent with the lower calibration bound (AsureQuality, 2006). Milk production in the optimal solution is 334 kg cow^{-1}, 5 per cent higher than mean New Zealand production over 2003–2008 (Livestock Improvement Corporation, 2008). Also, lactation length is 276 days, only 3 per cent longer than national mean days in production in the 2006/07 season (Livestock Improvement Corporation, 2008).[††] Production results in nitrate leaching of 33.1 kg N ha^{-1} yr^{-1}, a typical load observed in New Zealand dairy systems (Monaghan et al., 2007). Hence the model provides a sufficient description of reality to allow useful insight into the value of alternative environmental policies. These results also demonstrate the effectiveness of the calibration procedure incorporating the bounded cost function and trade-off parameter for pasture growth.

4.2 Restriction of nitrate emissions

The optimal stocking rate and the level of nitrogen fertilization decrease linearly with the stringency of the emission standards (Table 1). The level of maize silage used fluctuates; however, it is used in very low amounts. For example, in the base case, total feed per hectare consists of only 0.008 per cent of maize silage and this proportion varies little with regulation. Thus, although low-protein feeds can decrease leaching load, their overall impact is insufficient to warrant significant factor substitution to increase environmental mitigation. This is reinforced by recent research indicating the significant leaching losses arising from the soils where such low-N supplementary feeds are produced (Basset-Mens et al., 2009).

[††] The 2007/08 season is not a worthwhile comparison since most farmers in the Waikato region decreased lactation length in response to a severe drought.

[Insert Table 1 near here]

Conceptually, a producer could reduce abatement cost through increasing per-cow production. Nonetheless, model output suggests that manipulation of production characteristics for a given stocking rate and level of nitrogen fertilization offers little benefit. Milk production varies, but is never more than 3 per cent of its base value (Table 1). Furthermore, lactation length is never adjusted by more than 2 per cent of its standard magnitude (data not reported). We suspected that this behaviour could have arisen from definition of the quadratic cost function, as the reduction in cow numbers made in response to emission standards would directly reduce variable costs (see eq. 6). However, the model displays equivalent behaviour once this function is removed, with milk production (lactation length) changing by a maximum of 4.4 (6.6) per cent and with no apparent relationship with the stringency of regulation.

Thus, the best response of producers to emission standards is to unequivocally decrease production intensity without manipulating per-cow production, at least given the agronomic, economic, and technical reality described by the model. Production on most dairy farms in New Zealand is constrained by the inability of more-productive cows to derive sufficient nutrition from pastures (Clark, 2005). Likewise, in the model, the retention of a predominantly pasture-based diet prevents the use of such increases in production to offset the costs of environmental regulation.

The interval-valued function delineating the trade-off between optimal profit and the stringency of emissions standards is shown in Figure 2. The interval-valued quadratic cost function represents the uncertainty surrounding the precise description of the catchment in the model. The bounded profit function represents all expected realisations of profit for a given set of trade-off parameter values. In contrast to a stochastic MP model, no probabilistic statement can be made regarding the realization of a given value, apart from membership in the set of expected outcomes. This arises directly from the description of input data using bounded uncertainty sets, rather than standard probability measures.

[Insert Figure 2 near here]

The trade-off between environmental improvement and producer profit is not large, especially for decreases in nitrate leaching below 30 percent. For example, a 25 per cent decrease in nitrate leaching lowers optimal profit from [$1211, $1478] to [$1117, $1382] or by 7.8–12.4 per cent (Figure 2). However, the profit function is convex, with abatement costs increasing markedly at emissions standards approaching 50 per cent of unregulated levels. Importantly, model output shows that nitrate regulation will incur a cost, even in the long-run when the hypothetical producer has had sufficient time to adjust their farming system in response to regulatory policy.

4.3 Impact of different milk prices

A standard 2008/09 milk payment of $5000 t^{-1} milk solids (MS) is used in the base model. This could be bounded, but a single value is used because the inclusion of a conservative range of $500 t^{-1} MS leads to substantial ambiguity surrounding the abatement-cost curve (Figure 3b), compared with the use of a point estimate (Figure 3a). The abatement-cost curves for $4500 t^{-1} MS and $5500 t^{-1} MS scenarios have a similar breadth and curvature to that computed in the standard solution (Figure 3a). This arises because the optimal management plan derived for the base case is very robust to output price uncertainty. For example, stocking rate and milk production change by less than 1 per cent, relative to the base case, for each scenario. Furthermore, use of nitrogen fertiliser, a key productive input, varies by only 7–8 per cent.

[Insert Figure 3 near here]

Profit decreases by 46–53 per cent, 41–48 per cent, and 39–45 per cent as emission standards are increased from 0 to 50 per cent for output prices of $4500 t^{-1} MS, $5000 t^{-1} MS, and $5500 t^{-1} MS, respectively. Thus, though the breadth and curvature of the abatement-cost curves are similar, higher output prices intuitively dampen the cost of environmental regulation, *ceteris paribus*.

4.4 Manipulation of the trade-off parameter

A trade-off parameter of unity for pasture growth specifies a robust solution that remains feasible in light of all uncertainty regarding biomass production. This greatly reduces profit relative to the base case (Figure 4), highlighting the inherent conservatism of the worst-case formulation.

Moreover, the computed range is narrow since variable cost declines as total herd size almost halves, compared with the standard solution. Profit increases, relative to the base case, as the trade-off parameter is reduced to 0.5 and 0 (Figure 4), as these latter scenarios incorporate more optimistic specifications of pasture growth compared to the worst-case scenario. Nonetheless, the range of the profit function increases in response to inflation of the stocking rate from 2.69 cows ha^{-1} in the base case to 3.29 cows ha^{-1} when the trade-off parameter is 0.5 and 3.61 cows ha^{-1} when the trade-off parameter is 0. Model output is very sensitive to the magnitude of the trade-off parameter defined for pasture growth. This highlights the importance of careful consideration of the level to which decision makers give weight to parametric uncertainty.

[Insert Figure 4 near here]

4.5 Stability of income associated with robust plans

Figure 5 highlights a trade-off between the level and stability of income. The {mean, standard deviation} of profit is {1937, 422}, {2020, 195}, {1819, 155}, and {714.6, 0.5} for trade-off parameters (Λ^P) of 0 (deterministic solution), 0.5, 0.8, and 1 (fully robust solution), respectively. These yield coefficients of variation (the ratio of the standard deviation and the mean) of 0.22, 0.1, 0.09, and 0.0007 for trade-off parameters of 0, 0.5, 0.8, and 1, respectively.

Mean profit for the deterministic scenario is only 6 per cent higher than that for the base case ($\Lambda^P = 0.8$), mainly due to the high variance of the former (Figure 5, compare Scenarios 5b and 5d). The stocking rate for the deterministic scenario is 25 per cent higher than that for the base case. Thus, profit varies substantially (Figure 5, Scenario 5b) since high levels of supplementary feeding are required when pasture growth is lower than expected. Furthermore, the midpoint of the distribution obtained for $\Lambda^P = 0.8$ is greater than that computed for the deterministic case and its range is 2.7 times smaller. The cost of the robust formulation used in the standard model is therefore negligible, primarily reflecting its greater stability in light of subsequent uncertainty. In contrast, the mean and midpoint of the base case is lower than that generated for a trade-off parameter of $\Lambda^P = 0.5$. Nevertheless, the base value is retained, as it provides a better description of observed production given the use of formal calibration of this parameter, as described in Section 3.3.

[Insert Figure 5 near here]

The fully robust solution, immunized against all uncertainty, is characterised by a minimal variance, but a mean well below that of the deterministic scenario (Figure 5, Scenario 5a). Moreover, the output generated for this production plan is first-degree stochastically dominated by that of the base and $\Lambda^{P} = 0.5$ solutions. This illustrates the severe conservatism, and hence limited utility, of the worst-case formulation in this application.

5. Conclusions

Policy evaluation conducted using economic optimization models suffers from parametric uncertainty given the large size of models, cost of information acquisition, measurement error, and a weak correlation between historical and future states. Economists are slowly beginning to consider such ambiguity in the analysis of natural resource issues (e.g. Roseta-Palma and Xepapadeas, 2004). However, the development of appropriate decision frameworks remains an important challenge (Woodward and Shaw, 2008), particularly given the limited capacity of robust control to model large, complex systems.

This paper presents a framework that allows the explicit treatment of bounded uncertainty in empirical policy models. It has multiple benefits, including (a) removal of the assumption that decision makers base their plans on certain knowledge, (b) provision of a precautionary approach to natural resource management, (c) capacity to bound the range of expected abatement costs accruing to a given policy instrument, (d) chance to identify robust plans that are immune to parametric variation within the specified bounds, (e) straightforward solution in a MP context, and (f) endogenous stability that can provide more realistic simulation behaviour. Robust optimisation does not naturally incorporate correlations between random variables and distributional information. Nonetheless, these can be incorporated in a robust optimisation model using stochastic programming if sufficient information is available.

Nonetheless, there is a direct relationship between the conservativeness of the optimal solution and the magnitude of the trade-off parameter(s) that describe the maximum specification error that decision makers are willing to tolerate. This highlights the need to carefully estimate these parameters through calibration or qualitative methods.

This method is applied to an illustrative example involving regulation of nitrate pollution of two New Zealand lakes. New Zealand dairy producers possess a number of management options to reduce nitrate leaching. Use of low-protein supplementary feed can reduce nitrate emissions and the negative impact of reducing livestock density, the primary driver of leaching in these systems, can be buffered through switching to high-producing animals and/or extending lactation length. This analysis highlights that these strategies are of little or no value in offsetting the financial impact of nitrate regulation. Consequently, a cautious approach to policy formulation is recommended.

A number of extensions of this analysis are worthy of further research. First, using robust mathematical programming to calibrate individual farms in a catchment context could provide insights into the value of spatially differentiating environmental policy. Second, the trade-off parameter has implicit linkages with the concept of uncertainty aversion and Choquet expected utility theory (Epstein, 1999). Formalising these relationships with a focus on estimating the trade-off parameter for empirical work is conceptually interesting and may be practically important.

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Table 1. Key model output for proportional reductions in nitrate leaching load.

N leaching reduction (%)	**Profit range ($ ha^{-1})**	**Stocking rate (cows ha^{-1})**	**N fert. (kg N ha^{-1} yr^{-1})**	**Maize silage (kg ha^{-1})**	**Milk solids production (kg cow^{-1})**
0	[1211, 1578]	2.69	120	132	334
5	[1210, 1557]	2.62	106	156	333
10	[1195, 1523]	2.54	92	160	332
15	[1178, 1484]	2.45	80	155	330
20	[1154, 1442]	2.38	65	149	327
25	[1117, 1382]	2.28	54	144	325
30	[1066, 1301]	2.15	47	110	325
35	[1008, 1213]	2.01	40	67	325
40	[936, 1110]	1.85	37	17	327
45	[833, 975]	1.68	36	0	330
50	[711, 827]	1.51	34	0	333

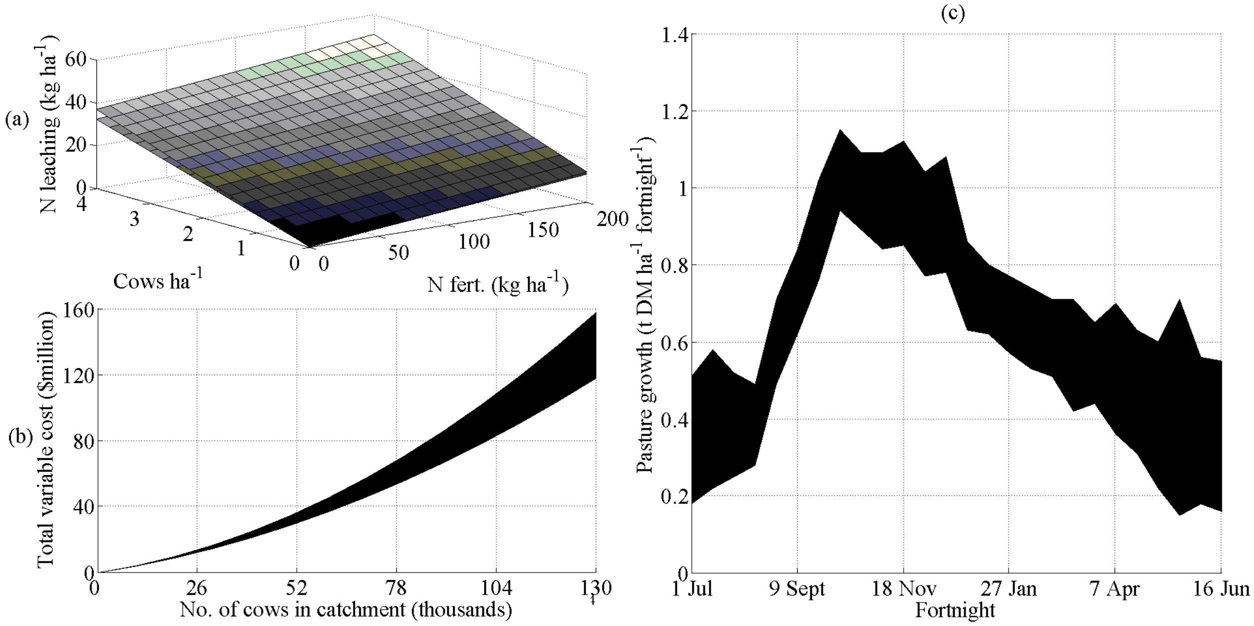

Figure 1. Interval-valued input relationships for the (a) metamodel for nitrate leaching, (b) the quadratic variable-cost function, and (c) fortnightly pasture production defined in the base model.

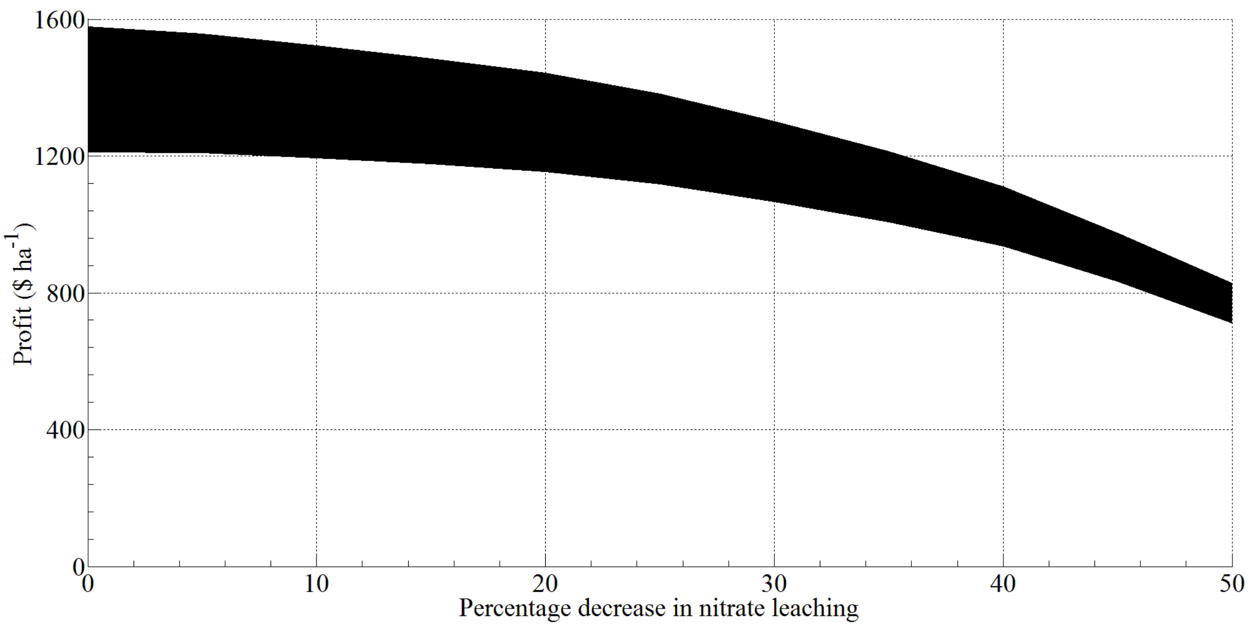

Figure 2. Interval-valued profit function derived for given reductions in nitrate emissions for the standard parameter values.

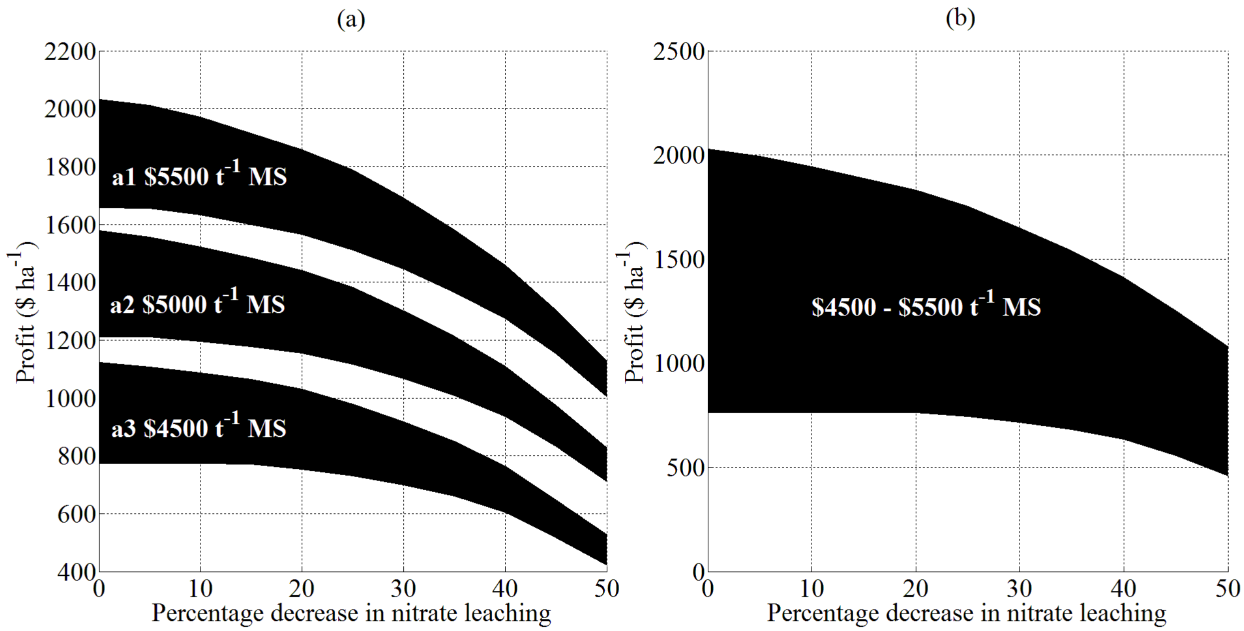

Figure 3. Interval-valued profit functions derived for given reductions in nitrate emissions with (a) discrete prices of milk solids (MS), and (b) a range of prices. Note scenario a2 is the standard case presented in Figure 2, but shown in the context of other discrete prices and the wider price range.

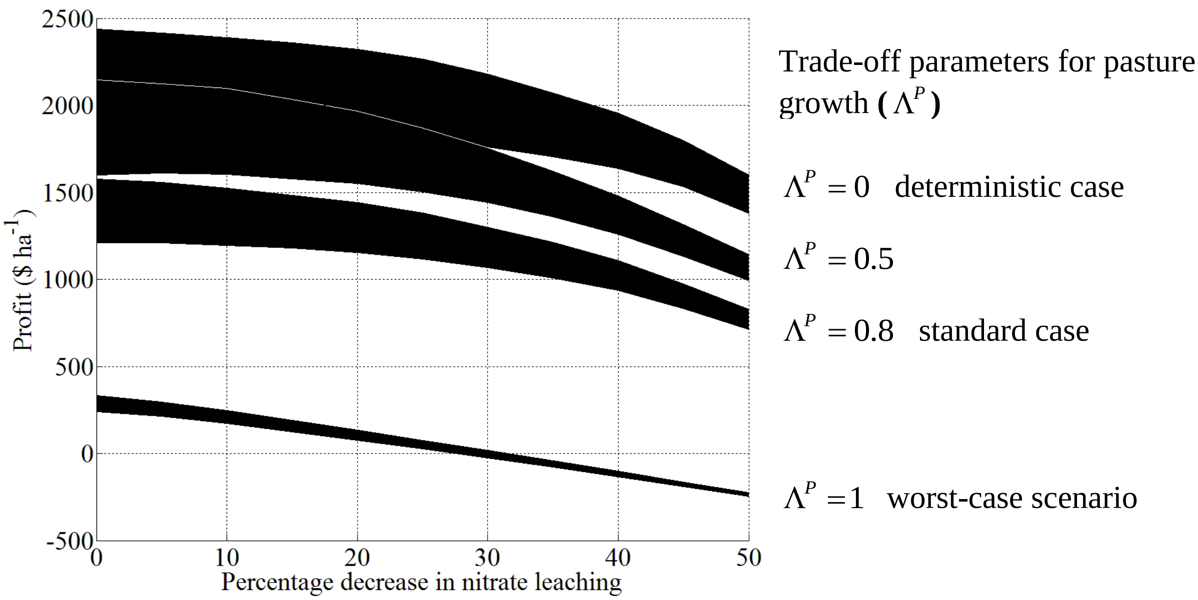

Figure 4. Interval-valued profit functions given trade-off parameters for pasture growth.

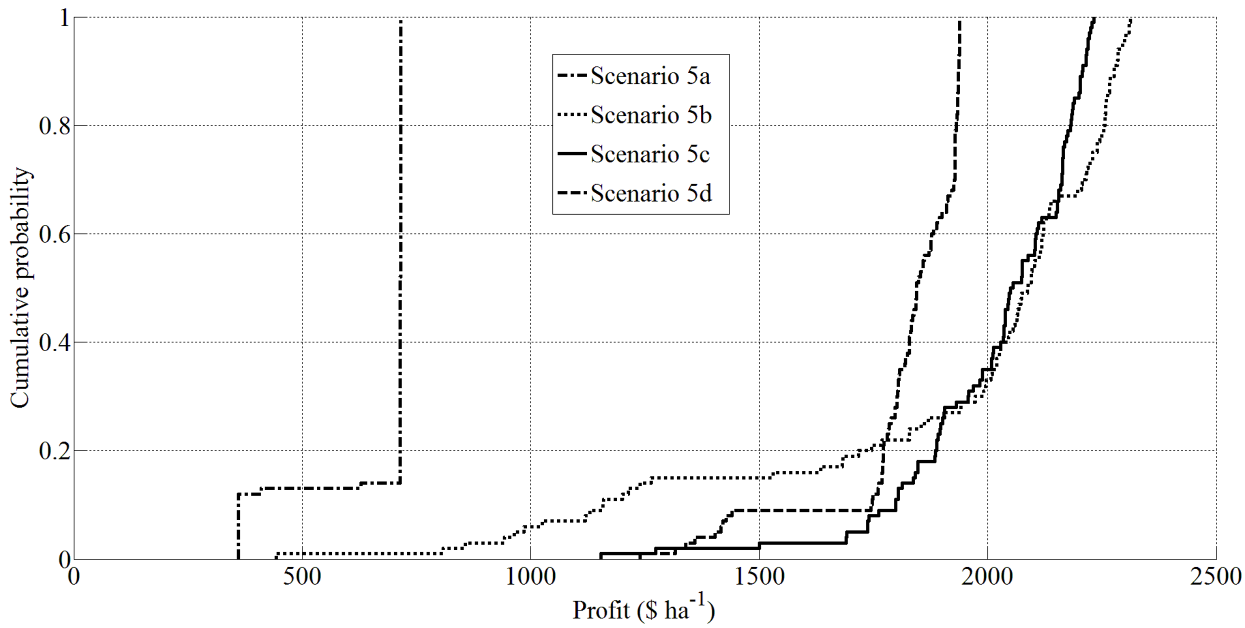

Figure 5. Cumulative distributions of profit given stochastic pasture growth when cow herds are fixed at levels from the optimal plans determined for trade-off parameters for pasture growth (Λ^P) of (a) 1 (consistent with the worst-case model), (b) 0 (consistent with a deterministic model), (c) 0.5, and (d) 0.8 (standard case). [#It would be easier to relate the graph to the text if the scenarios were listed and numbered in the same order in the figure as they are discussed in the text (i.e. 0, 0.5, 0.8, and 1).]