

$\mathcal{N} = 2$ supersymmetric higher spin gauge theories and current multiplets in three dimensions

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We describe several families of primary linear supermultiplets coupled to three-dimensional $\mathcal{N} = 2$ conformal supergravity and use them to construct topological BF -type terms. We introduce conformal higher-spin gauge superfields and associate with them Chern-Simons-type actions that are constructed as an extension of the linearized action for $\mathcal{N} = 2$ conformal supergravity. These actions possess gauge and super-Weyl invariance in any conformally flat superspace and involve a higher-spin generalization of the linearized $\mathcal{N} = 2$ super-Cotton tensor. For massless higher-spin supermultiplets in (1,1) anti-de Sitter (AdS) superspace, we propose two off-shell Lagrangian gauge formulations, which are related to each other by a duality transformation. Making use of these massless theories allows us to formulate consistent higher-spin supercurrent multiplets in (1,1) AdS superspace. Explicit examples of such supercurrent multiplets are provided for models of massive chiral supermultiplets. Off-shell formulations for massive higher-spin supermultiplets in (1,1) AdS superspace are proposed.

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I. INTRODUCTION

In four spacetime dimensions (4D), there exist two off-shell formulations for pure $\mathcal{N} = 1$ anti-de Sitter (AdS) supergravity: minimal (see, e.g., [1,2] for pedagogical reviews) and nonminimal [3].¹ These supergravity theories are related to each other by a superfield duality transformation [3] and possess a single maximally supersymmetric solution, the famous $\mathcal{N} = 1$ AdS superspace [4–6], which is the simplest member of the family of \mathcal{N} -extended AdS superspaces

$$\text{AdS}^{4|4\mathcal{N}} = \frac{\text{OSp}(\mathcal{N}|4)}{\text{SO}(3, 1) \times \text{SO}(\mathcal{N})}. \quad (1.1)$$

These supergravity theories are also intimately related to the two dually equivalent series of massless gauge supermultiplets of half-integer superspin $s + \frac{1}{2} \geq \frac{3}{2}$ [describing two ordinary massless spin- $(s + \frac{1}{2})$ and spin- $(s + 1)$ fields

on the mass shell] in AdS_4 , which were proposed in [7] as a natural extension of the formulations in Minkowski space constructed earlier in [8,9]. Specifically, for the lowest superspin value corresponding to $s = 1$, one series yields the linearized action for minimal AdS supergravity, while the other leads to the linearized nonminimal AdS supergravity.

In the 3D case, the AdS group is reducible,

$$\text{SO}(2, 2) \cong (\text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{R}))/\mathbb{Z}_2,$$

and so are its simplest supersymmetric extensions, $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$. This implies that \mathcal{N} -extended AdS supergravity exists in several incarnations [10]. These are known as the (p, q) AdS supergravity theories, where the non-negative integers $p \geq q$ are such that $\mathcal{N} = p + q$. For any allowed values of p and q , the pure (p, q) AdS supergravity was constructed in [10] as a Chern-Simons theory with the gauge group $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$. The Chern-Simons construction is not particularly useful when one is interested in coupling AdS supergravity to supersymmetric matter. This is one of the reasons why off-shell formulations for 3D \mathcal{N} -extended conformal supergravity have been developed [11–13].

Within the off-shell supergravity framework of [12], (p, q) AdS superspace

$$\text{AdS}^{(3|p,q)} = \frac{\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})}{\text{SL}(2, \mathbb{R}) \times \text{SO}(p) \times \text{SO}(q)} \quad (1.2)$$

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¹It was believed for almost 30 years that there is no non-minimal formulation for $\mathcal{N} = 1$ AdS supergravity [1]. However, such a formulation was constructed in [3].

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originates as a maximally symmetric conformally flat supergeometry with covariantly constant torsion and curvature generated by a tensor $S^{IJ} = S^{JI}$ [14], with the $SO(\mathcal{N})$ indices I, J taking values from 1 to \mathcal{N} . It turns out that the symmetric matrix $S = (S^{IJ})$ is nonsingular, and the parameters p and $q = \mathcal{N} - p$ determine its signature. The ordinary AdS space

$$\text{AdS}_3 = \frac{\text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{R})}{\text{SL}(2, \mathbb{R})} \quad (1.3)$$

is the bosonic body of $\text{AdS}^{(3|p,q)}$. The curvature of AdS_3 is proportional to $\text{tr}(S^2)$. The Killing vector fields of $\text{AdS}^{(3|p,q)}$ can be shown to generate the isometry group $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$; see [14] for the technical details.

The 3D $\mathcal{N} = 2$ supersymmetry is a natural cousin of the 4D $\mathcal{N} = 1$ one. This is the lowest value of \mathcal{N} for which there are at least two inequivalent AdS superspaces, $\text{AdS}^{(3|1,1)}$ and $\text{AdS}^{(3|2,0)}$, which were thoroughly studied in [15]. The former is the 3D counterpart of the 4D $\mathcal{N} = 1$ AdS superspace, while the latter has no 4D analog. The superspaces $\text{AdS}^{(3|1,1)}$ and $\text{AdS}^{(3|2,0)}$ are maximally symmetric solutions of the known off-shell $\mathcal{N} = 2$ AdS supergravity theories presented in [15]. $\text{AdS}^{(3|1,1)}$ is the unique maximally symmetric solution of the two dually equivalent (1,1) AdS supergravity theories, minimal and nonminimal ones. $\text{AdS}^{(3|2,0)}$ is the unique maximally symmetric solution of the (2,0) AdS supergravity, which was originally formulated in [11] in the component setting. The early superspace descriptions of the minimal (1,1) supergravity were given in [16,17].

Since there are three off-shell $\mathcal{N} = 2$ AdS supergravity theories [15], one might expect existence of three series of massless higher-spin gauge supermultiplets. In this paper we present two series of massless higher-spin actions, which are associated with the minimal and the nonminimal (1,1) AdS supergravity theories, respectively, and which generalize similar constructions in the super-Poincaré case [18]. Off-shell higher-spin actions with (2,0) AdS supersymmetry will be described in a separate work [19].

Similar to the pure gravity and simple supergravity theories in three dimensions, pure $\mathcal{N} = 2$ supergravity (massless superspin-3/2 multiplet) and its higher-spin extensions have no propagating degrees of freedom. Nevertheless, there are at least two nontrivial applications of the massless higher-spin gauge supermultiplets. Firstly, one can follow the pattern of topologically massive (super) gravity [20–23] and construct massive higher-spin supermultiplets by combining a massless action with a higher-spin extension of the action for linearized conformal supergravity. This has been achieved in [18] in the $\mathcal{N} = 2$ super-Poincaré case, and similar ideas have been implemented in the frameworks of $\mathcal{N} = 1$ Poincaré and

AdS supersymmetry [24,25]. Secondly, making use of the off-shell formulations for massless higher-spin supermultiplets in AdS_3 , one can define consistent higher-spin supercurrent multiplets in AdS superspace (i.e., higher-spin extensions of the supercurrent) that contain ordinary bosonic and fermionic conserved currents in AdS_3 . One can then look for explicit realizations of such higher-spin supercurrents in concrete supersymmetric theories in AdS_3 . Such a program in the 4D $\mathcal{N} = 1$ Poincaré and AdS supersymmetric cases has been described in a series of papers [26–29]. Alternatively, one can develop a 3D extension of the approach advocated in [30–32] and based on the use of superfield Noether procedures [33,34].

In principle, all massless higher-spin supermultiplets in 3D (1,1) AdS superspace may be obtained from their counterparts in 4D $\mathcal{N} = 1$ AdS superspace [7] by dimensional reduction. In practice, however, carrying out such a reduction proves to be a nontrivial technical task. To explain this, it suffices to consider the longitudinal formulation for massless superspin- $(s + \frac{1}{2})$ multiplets, with $s = 1, 2, \dots$, in four and three dimensions.² In the 4D $\mathcal{N} = 1$ AdS case [7], the massless superspin- $(s + \frac{1}{2})$ multiplet is described by a real unconstrained gauge superfield $H_{\alpha_1 \dots \alpha_s \dot{\alpha}_1 \dots \dot{\alpha}_s} = H_{(\alpha_1 \dots \alpha_s)(\dot{\alpha}_1 \dots \dot{\alpha}_s)}$, a complex longitudinal linear compensator $G_{\alpha_1 \dots \alpha_{s-1} \dot{\alpha}_1 \dots \dot{\alpha}_{s-1}} = G_{(\alpha_1 \dots \alpha_{s-1})(\dot{\alpha}_1 \dots \dot{\alpha}_{s-1})}$ and its conjugate. The dimensional reduction of $H_{\alpha_1 \dots \alpha_s \dot{\alpha}_1 \dots \dot{\alpha}_s}$ leads to a family of real unconstrained symmetric superfields $H_{\alpha_1 \dots \alpha_{2s}}, H_{\alpha_1 \dots \alpha_{2s-2}}, \dots, H$. Next, the dimensional reduction of $G_{\alpha_1 \dots \alpha_{s-1} \dot{\alpha}_1 \dots \dot{\alpha}_{s-1}}$ leads to a family of constrained 3D superfields, which include a complex longitudinal linear compensator $G_{\alpha_1 \dots \alpha_{2s-2}}$ and some lower-spin supermultiplets. As will be demonstrated in Sec. IV B, in three dimensions the massless superspin- $(s + \frac{1}{2})$ multiplet in (1,1) AdS superspace is described by the gauge superfield $H_{\alpha_1 \dots \alpha_{2s}}$, the compensator $G_{\alpha_1 \dots \alpha_{2s-2}}$ and its conjugate. The above consideration makes it clear that the naive $4\text{D} \rightarrow 3\text{D}$ dimensional reduction leads to the massless superspin- $(s + \frac{1}{2})$ multiplet intertwined with lower-superspin multiplets. The nontrivial technical task is to disentangle the pure superspin- $(s + \frac{1}{2})$ multiplet from the rest. This was explicitly done in [15] for linearized supergravity, $s = 1$, for which dimensional reduction leads to two supermultiplets in (1,1) AdS superspace, which are a massless superspin- $\frac{3}{2}$ multiplet and a massless vector supermultiplet. Instead of carrying out dimensional reduction, it proves more efficient to recast the 4D gauge principle of [7] in a 3D form and use it to construct gauge-invariant actions. This is our approach in the present paper. Of course, our topologically massive higher-spin supermultiplets described by the actions (9.1) cannot be derived from 4D by dimensional reduction.

²The $s = 1$ case corresponds to linearized supergravity.

Before we turn to the main body of this work, a few comments are in order about maximally supersymmetric backgrounds in the off-shell $\mathcal{N} = 2$ supergravity theories, since the superspaces $\text{AdS}^{(3|1,1)}$ and $\text{AdS}^{(3|2,0)}$ are special examples of such supermanifolds. The most general maximally supersymmetric backgrounds are characterized by several conditions [35,36] on the torsion superfields \mathcal{R} , \mathcal{S} and \mathcal{C}_a , which determine the superspace geometry of $\mathcal{N} = 2$ conformal supergravity (see Sec. II for the technical details). These requirements are as follows:

$$\mathcal{R}\mathcal{S} = 0, \quad \mathcal{R}\mathcal{C}_a = 0, \quad (1.4a)$$

$$\begin{aligned} \mathcal{D}_A\mathcal{R} &= 0, & \mathcal{D}_A\mathcal{S} &= 0, \\ \mathcal{D}_a\mathcal{C}_b &= 0 \Rightarrow \mathcal{D}_a\mathcal{C}_b = 2\varepsilon_{abc}\mathcal{C}^c\mathcal{S}. \end{aligned} \quad (1.4b)$$

The (1,1) AdS superspace is singled out by the conditions $\mathcal{S} = 0$ and $\mathcal{C}_a = 0$, with \mathcal{R} and its conjugate $\bar{\mathcal{R}}$ having nonzero constant values. The (1,1) AdS superspace is characterized by the following algebra of covariant derivatives [15]:

$$\begin{aligned} \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= -2\bar{\mathcal{R}}(\gamma_a)_{\alpha\beta}\varepsilon^{abc}M_{bc}, \\ \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} &= 2\mathcal{R}(\gamma_a)_{\alpha\beta}\varepsilon^{abc}M_{bc}, \end{aligned} \quad (1.5a)$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2i(\gamma^c)_{\alpha\beta}\mathcal{D}_c, \quad (1.5b)$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = i(\gamma_a)_{\beta\gamma}\bar{\mathcal{R}}\bar{\mathcal{D}}_\gamma, \quad (1.5c)$$

$$[\mathcal{D}_a, \bar{\mathcal{D}}_\beta] = -i(\gamma_a)_{\beta\gamma}\mathcal{R}\mathcal{D}_\gamma, \quad (1.5d)$$

$$[\mathcal{D}_a, \mathcal{D}_b] = -4\bar{\mathcal{R}}\mathcal{R}M_{ab}, \quad (1.5e)$$

where M_{ab} denotes the Lorentz generator. The (2,0) AdS superspace belongs to the family of all maximally supersymmetric backgrounds with $\mathcal{R} = 0$. These backgrounds are characterized by the following algebra of covariant derivatives [35,36]:

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0, \quad \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0, \quad (1.6a)$$

$$\begin{aligned} \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} &= -2i(\gamma^c)_{\alpha\beta}(\mathcal{D}_c - 2SM_c - i\mathcal{C}_cJ) \\ &\quad + 4\varepsilon_{\alpha\beta}(\mathcal{C}^cM_c - iSJ), \end{aligned} \quad (1.6b)$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = i\varepsilon_{abc}(\gamma^b)_{\beta\gamma}\mathcal{C}^c\mathcal{D}_\gamma + (\gamma_a)_{\beta\gamma}S\mathcal{D}_\gamma, \quad (1.6c)$$

$$[\mathcal{D}_a, \bar{\mathcal{D}}_\beta] = -i\varepsilon_{abc}(\gamma^b)_{\beta\gamma}\mathcal{C}^c\bar{\mathcal{D}}_\gamma + (\gamma_a)_{\beta\gamma}S\bar{\mathcal{D}}_\gamma, \quad (1.6d)$$

$$[\mathcal{D}_a, \mathcal{D}_b] = 4\varepsilon_{abc}(\mathcal{C}^c\mathcal{C}_d + \delta^c_d S^2)M^d. \quad (1.6e)$$

Here J is the generator of the $\mathcal{N} = 2$ R -symmetry group, $U(1)_R$, and $M^a := \frac{1}{2}\varepsilon^{abc}M_{bc}$. The solution with $\mathcal{C}_a = 0$ and $\mathcal{S} \neq 0$ corresponds to (2,0) AdS superspace [15]. It may be

shown that the $U(1)_R$ connection is flat if and only if $\mathcal{S} = 0$ [12]. The nonvanishing $U(1)_R$ curvature is the main reason why the structure of massless higher-spin gauge supermultiplets in (2,0) AdS superspace [19] considerably differs from their counterparts with (1,1) AdS supersymmetry.

This paper is organized as follows. In Sec. II, primary linear supermultiplets coupled to $\mathcal{N} = 2$ conformal supergravity are described and then used to construct topological BF -type terms. Given a positive integer $n > 0$, we introduce a conformal gauge superfield $\mathfrak{H}_{\alpha(n)}$ and show that, for every conformally flat superspace, there exists a unique primary gauge-invariant descendant $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ of $\mathfrak{H}_{\alpha(n)}$ with the properties (2.25). In terms of $\mathfrak{H}_{\alpha(n)}$ and $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ we construct a higher-spin extension of the linearized action for $\mathcal{N} = 2$ conformal supergravity. Section III provides a brief summary of the key results concerning the (1,1) AdS superspace and superfield representations of the corresponding isometry group. In Secs. IV and V, we present two dually equivalent off-shell Lagrangian formulations for every massless higher-spin supermultiplet in (1,1) AdS superspace. Making use of these massless theories allows us to formulate, in Sec. VI, consistent higher-spin supercurrent multiplets. Explicit examples of such supercurrents are provided in Secs. VII and VIII for models described by chiral supermultiplets. In Sec. IX we discuss several extensions of the constructions obtained. The paper is concluded with two appendices. Appendix A describes our notation, conventions and several important identities involving the spinor covariant derivatives of (1,1) AdS superspace. Appendix B describes the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction of the massless integer superspin model (5.6) in Minkowski superspace.

II. SUPERCONFORMAL HIGHER-SPIN MULTIPLETS

Before presenting superconformal higher-spin multiplets, we give a succinct review of 3D $\mathcal{N} = 2$ conformal supergravity following [11,12]. There exists more general formulation for conformal supergravity [13] known as the $\mathcal{N} = 2$ conformal superspace. For our purposes it suffices to use the formulation of [12], which is obtained from the $\mathcal{N} = 2$ conformal superspace by partially fixing the gauge freedom.

A. Conformal supergravity

All known off-shell formulations for 3D $\mathcal{N} = 2$ supergravity [12,15] can be realized in curved superspace $\mathcal{M}^{3|4}$ with structure group $SL(2, \mathbb{R}) \times U(1)_R$, where $SL(2, \mathbb{R})$ and $U(1)_R$ stand for the spin group and the R -symmetry group, respectively. The superspace is parametrized by local bosonic (x^m) and fermionic ($\theta^\mu, \bar{\theta}_\mu$) coordinates $z^M = (x^m, \theta^\mu, \bar{\theta}_\mu)$, where the variables θ^μ and $\bar{\theta}_\mu$ are related to each other by complex conjugation: $\bar{\theta}^\mu = \bar{\theta}^\mu$.

The superspace covariant derivatives have the form

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^\alpha) = E_A + \Omega_A + i\Phi_A J. \quad (2.1)$$

Here E_A and Ω_A denote the inverse supervielbein and the Lorentz connection, respectively,

$$\begin{aligned} E_A &= E_A^M \frac{\partial}{\partial z^M}, \\ \Omega_A &= \frac{1}{2} \Omega_A^{bc} M_{bc} = -\Omega_A^b M_b = \frac{1}{2} \Omega_A^{\beta\gamma} M_{\beta\gamma}. \end{aligned} \quad (2.2)$$

The Lorentz generators with two vector indices ($M_{ab} = -M_{ba}$), with one vector index (M_a) and with two spinor indices ($M_{\alpha\beta} = M_{\beta\alpha}$) are defined in Appendix A. The $U(1)_R$ generator J in (2.1) is defined to act on the covariant derivatives as follows:

$$[J, \mathcal{D}_a] = \mathcal{D}_a, \quad [J, \bar{\mathcal{D}}^\alpha] = -\bar{\mathcal{D}}^\alpha, \quad [J, \mathcal{D}_\alpha] = 0. \quad (2.3)$$

In order to describe $\mathcal{N} = 2$ conformal supergravity, the torsion has to obey the covariant constraints proposed in [11]. Solving the constraints leads to the following algebra of covariant derivatives [12,15]

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{\mathcal{R}} M_{\alpha\beta}, \quad (2.4a)$$

$$\begin{aligned} \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} &= -2i(\gamma^c)_{\alpha\beta} \mathcal{D}_c - 2\mathcal{C}_{\alpha\beta} J - 4i\varepsilon_{\alpha\beta} S J \\ &\quad + 4iS M_{\alpha\beta} - 2\varepsilon_{\alpha\beta} \mathcal{C}^{\gamma\delta} M_{\gamma\delta}, \end{aligned} \quad (2.4b)$$

$$\begin{aligned} [\mathcal{D}_a, \mathcal{D}_\beta] &= i\varepsilon_{abc} (\gamma^b)_\beta{}^\gamma \mathcal{C}^c \mathcal{D}_\gamma + (\gamma_a)_\beta{}^\gamma S \mathcal{D}_\gamma - i(\gamma_a)_{\beta\gamma} \bar{\mathcal{R}} \bar{\mathcal{D}}^\gamma \\ &\quad + i(\gamma_a)_\beta{}^\gamma \mathcal{D}_\gamma \mathcal{C}_{\delta\rho} M^{\delta\rho} - \frac{1}{3} (2\mathcal{D}_\beta S + i\bar{\mathcal{D}}_\beta \bar{\mathcal{R}}) M_a \\ &\quad - \frac{2}{3} \varepsilon_{abc} (\gamma^b)_\beta{}^\alpha (2\mathcal{D}_\alpha S + i\bar{\mathcal{D}}_\alpha \bar{\mathcal{R}}) M^c \\ &\quad + \frac{i}{2} \left((\gamma_a)^{\alpha\gamma} \mathcal{D}_{(\alpha} \mathcal{C}_{\beta\gamma)} + \frac{1}{3} (\gamma_a)_\beta{}^\gamma (8i\mathcal{D}_\gamma S - \bar{\mathcal{D}}_\gamma \bar{\mathcal{R}}) \right) J, \end{aligned} \quad (2.4c)$$

where the $U(1)_R$ charges of the torsion superfields \mathcal{R} , $\bar{\mathcal{R}}$ and $\mathcal{C}_{\alpha\beta}$ are -2 , $+2$ and 0 , respectively. They also satisfy the Bianchi identities

$$\begin{aligned} \mathcal{D}_\alpha \bar{\mathcal{R}} &= 0, \quad (\bar{\mathcal{D}}^2 - 4\mathcal{R}) S = 0 \\ \mathcal{D}^\beta \mathcal{C}_{\alpha\beta} &= -\frac{1}{2} (\bar{\mathcal{D}}_\alpha \bar{\mathcal{R}} + 4i\mathcal{D}_\alpha S), \end{aligned} \quad (2.5)$$

Throughout this paper, we make use of the definitions $\mathcal{D}^2 := \mathcal{D}^\alpha \mathcal{D}_\alpha$ and $\bar{\mathcal{D}}^2 := \bar{\mathcal{D}}_\alpha \bar{\mathcal{D}}^\alpha$.

The algebra of covariant derivatives given by (2.4) does not change under the super-Weyl transformation [12,15]

$$\mathcal{D}'_\alpha = e^{\frac{1}{2}\sigma} (\mathcal{D}_\alpha + \mathcal{D}^\gamma \sigma M_{\gamma\alpha} - \mathcal{D}_\alpha \sigma J), \quad (2.6a)$$

$$\bar{\mathcal{D}}'_\alpha = e^{\frac{1}{2}\sigma} (\bar{\mathcal{D}}_\alpha + \bar{\mathcal{D}}^\gamma \sigma M_{\gamma\alpha} + \bar{\mathcal{D}}_\alpha \sigma J), \quad (2.6b)$$

$$\begin{aligned} \mathcal{D}'_a &= e^\sigma \left(\mathcal{D}_a - \frac{i}{2} (\gamma_a)^{\gamma\delta} \mathcal{D}_\gamma \sigma \bar{\mathcal{D}}_\delta - \frac{i}{2} (\gamma_a)^{\gamma\delta} \bar{\mathcal{D}}_\gamma \sigma \mathcal{D}_\delta \right. \\ &\quad \left. + \varepsilon_{abc} \mathcal{D}^b \sigma M^c - \frac{i}{2} (\mathcal{D}^\gamma \sigma) \bar{\mathcal{D}}_\gamma \sigma M_a \right. \\ &\quad \left. - \frac{i}{24} (\gamma_a)^{\gamma\delta} e^{-3\sigma} [\mathcal{D}_\gamma, \bar{\mathcal{D}}_\delta] e^{3\sigma} J \right), \end{aligned} \quad (2.6c)$$

which induces the following transformation of the torsion tensors:

$$S' = e^\sigma \left(S + \frac{i}{4} \mathcal{D}^\gamma \bar{\mathcal{D}}_\gamma \sigma \right), \quad (2.6d)$$

$$\mathcal{C}'_a = \left(\mathcal{C}_a + \frac{1}{8} (\gamma_a)^{\gamma\delta} [\mathcal{D}_\gamma, \bar{\mathcal{D}}_\delta] \right) e^\sigma, \quad (2.6e)$$

$$\mathcal{R}' = -\frac{1}{4} e^{2\sigma} (\bar{\mathcal{D}}^2 - 4\mathcal{R}) e^{-\sigma}, \quad (2.6f)$$

where the parameter σ is an arbitrary real scalar superfield. The super-Weyl invariance (2.6) is intrinsic to conformal supergravity. For every supergravity-matter system, its action is required to be a super-Weyl invariant functional of the supergravity multiplet coupled to certain conformal compensators; see [12,15] for more details.

The $\mathcal{N} = 2$ supersymmetric extension of the Cotton tensor [37] is given by

$$\mathcal{W}_{\alpha\beta} = -\frac{i}{4} [\mathcal{D}^\gamma, \bar{\mathcal{D}}_\gamma] \mathcal{C}_{\alpha\beta} + \frac{1}{2} [\mathcal{D}_{(\alpha}, \bar{\mathcal{D}}_{\beta)}] S + 2S \mathcal{C}_{\alpha\beta}. \quad (2.7)$$

It may be checked that $\mathcal{W}_{\alpha\beta}$ transforms homogeneously,

$$\mathcal{W}'_{\alpha\beta} = e^{2\sigma} \mathcal{W}_{\alpha\beta}, \quad (2.8)$$

under (2.6). The super-Cotton tensor obeys the Bianchi identities [13]

$$\bar{\mathcal{D}}^\beta \mathcal{W}_{\alpha\beta} = \mathcal{D}^\beta \mathcal{W}_{\alpha\beta} = 0. \quad (2.9)$$

The curved superspace is conformally flat if and only if $\mathcal{W}_{\alpha\beta} = 0$ [13].

B. Primary superfields

Let $T_{\alpha(n)} := T_{\alpha_1 \dots \alpha_n} = T_{(\alpha_1 \dots \alpha_n)}$ be a symmetric rank- n spinor superfield of $U(1)_R$ charge q ,

$$JT_{\alpha(n)} = qT_{\alpha(n)}. \quad (2.10)$$

The $T_{\alpha(n)}$ is said to be super-Weyl primary of dimension d if its infinitesimal super-Weyl transformation law is

$$\delta_\sigma T_{\alpha(n)} = d\sigma T_{\alpha(n)}. \quad (2.11)$$

As follows from (2.8), the super-Cotton tensor is super-Weyl primary of dimension $+2$. We now introduce several types of primary superfields that will be important for our subsequent consideration.

A symmetric rank- n spinor superfield $G_{\alpha(n)}$ is called longitudinal linear if it obeys the following first-order constraint:

$$\bar{\mathcal{D}}_{(\alpha_1} G_{\alpha_2 \dots \alpha_{n+1})} = 0, \quad (2.12)$$

which implies

$$(\bar{\mathcal{D}}^2 + 2n\mathcal{R})G_{\alpha(n)} = 0. \quad (2.13)$$

If $G_{\alpha(n)}$ is super-Weyl primary, then the constraint (2.12) is consistent provided the dimension $d_{G(n)}$ and $U(1)_R$ charge $q_{G(n)}$ of $G_{\alpha(n)}$ are related to each other as follows:

$$d_{G(n)} = -\frac{n}{2} - q_{G(n)}. \quad (2.14)$$

In the scalar case, $n = 0$, the constraint (2.12) becomes the condition of covariant chirality, $\bar{\mathcal{D}}_\alpha G = 0$. The dimension d_G and $U(1)_R$ charge q_G of any primary chiral scalar superfield G are related as $d_G + q_G = 0$, in accordance with [12].

The longitudinal linear superfields form a ring. Given two such superfields $G_{\alpha(n)}$ and $\tilde{G}_{\alpha(m)}$, their product $G_{\alpha(n+m)} := G_{(\alpha_1 \dots \alpha_n} \tilde{G}_{\alpha_{n+1} \dots \alpha_{n+m})}$ is longitudinal linear. If $G_{\alpha(n)}$ and $\tilde{G}_{\alpha(m)}$ are super-Weyl primary superfields, their product $G_{\alpha(n+m)}$ is also super-Weyl primary.

Given a positive integer n , a symmetric rank- n spinor superfield $\Gamma_{\alpha(n)}$ is called transverse linear if it obeys the first-order constraint

$$\bar{\mathcal{D}}^\beta \Gamma_{\beta\alpha_1 \dots \alpha_{n-1}} = 0, \quad n \neq 0, \quad (2.15)$$

which implies

$$(\bar{\mathcal{D}}^2 - 2(n+2)\mathcal{R})\Gamma_{\alpha(n)} = 0. \quad (2.16)$$

If $\Gamma_{\alpha(n)}$ is super-Weyl primary, then the constraint (2.15) is consistent provided the dimension $d_{\Gamma(n)}$ and $U(1)_R$ charge $q_{\Gamma(n)}$ of $\Gamma_{\alpha(n)}$ are related to each other as follows:

$$d_{\Gamma(n)} = 1 + \frac{n}{2} - q_{\Gamma(n)}. \quad (2.17)$$

In the $n = 0$ case, the constraint (2.15) is not defined. However its corollary (2.16) is perfectly consistent,

$$(\bar{\mathcal{D}}^2 - 4\mathcal{R})\Gamma = 0, \quad (2.18)$$

and defines a covariantly linear scalar superfield Γ . The dimension d_Γ and $U(1)_R$ charge q_Γ of any primary linear scalar Γ are related as $d_\Gamma + q_\Gamma = 1$, in accordance with [12].

In the case of 4D $\mathcal{N} = 1$ AdS supersymmetry, longitudinal linear and transverse linear superfields were pioneered by Ivanov and Sorin [6] who studied the superfield representations of the AdS isometry group $O\text{Sp}(1|4)$. In the framework of 4D $\mathcal{N} = 1$ conformal supergravity, primary longitudinal linear and transverse linear supermultiplets were introduced for the first time by Kugo and Uehara [38]. Such superfields were used in [7–9, 18, 29] for the description of off-shell massless gauge theories in four and three dimensions.

The constraints (2.12) and (2.15) are solved in terms of prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$ as follows:

$$G_{\alpha(n)} = \bar{\mathcal{D}}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_n)}, \quad (2.19a)$$

$$\Gamma_{\alpha(n)} = \bar{\mathcal{D}}^\beta \Phi_{(\beta\alpha_1 \dots \alpha_n)}. \quad (2.19b)$$

Provided the constraints (2.12) and (2.15) are the only conditions imposed on $G_{\alpha(n)}$ and $\Gamma_{\alpha(n)}$ respectively, the prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$ can be chosen to be unconstrained complex, and are defined modulo gauge transformations of the form

$$\delta_\zeta \Psi_{\alpha(n-1)} = \bar{\mathcal{D}}_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{n-1})}, \quad (2.20a)$$

$$\delta_\xi \Phi_{\alpha(n+1)} = \bar{\mathcal{D}}^\gamma \xi_{(\gamma\alpha_1 \dots \alpha_{n+1})}, \quad (2.20b)$$

with the gauge parameters $\zeta_{\alpha(n-2)}$ and $\xi_{\alpha(n+2)}$ being unconstrained. If the linear superfields $G_{\alpha(n)}$ and $\Gamma_{\alpha(n)}$ are super-Weyl primary, then their prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$ can also be chosen to be super-Weyl primary.

Given two linear superfields $G_{\alpha(n+1)}$ and $\Gamma_{\alpha(n)}$ such that their $U(1)_R$ charges are constrained by $q_{G(n+1)} + q_{\Gamma(n)} = -1$, we can define a gauge-invariant and super-Weyl-invariant BF term

$$\begin{aligned} I_{BF}^{(n)} &= \int d^{3|4}z E \Psi^{\alpha(n)} \Gamma_{\alpha(n)} \\ &= -(-1)^n \int d^{3|4}z E \Phi^{\alpha(n+1)} G_{\alpha(n+1)}, \end{aligned} \quad (2.21)$$

where the superspace integration measure is $d^{3|4}z := d^3x d^2\theta d^2\bar{\theta}$ and $E^{-1} := \text{Ber}(E_A^M)$.

In the $n = 0$ case, the prepotential solution (2.19b) is still valid. The prepotential Φ_α can be chosen to be

unconstrained complex provided the constraint (2.18) is the only condition imposed on Γ . However, if we are dealing with a real linear superfield,

$$(\bar{D}^2 - 4\mathcal{R})L = 0, \quad \bar{L} = L, \quad (2.22)$$

then the constraints are solved [12] in terms of an unconstrained real prepotential V ,

$$L = i\bar{D}^\alpha \mathcal{D}_\alpha V, \quad \bar{V} = V, \quad (2.23)$$

which is defined modulo gauge transformations of the form

$$\delta V = \lambda + \bar{\lambda}, \quad \mathcal{J}\lambda = 0, \quad \bar{D}_\alpha \lambda = 0. \quad (2.24)$$

If L is super-Weyl primary, then Eq. (2.17) tells us that the dimension of L is $+1$. In this case it is consistent to consider the gauge prepotential V to be inert under the super-Weyl transformations [12], $\delta_\sigma V = 0$.

Let us assume that the background curved superspace allows the existence of a real transverse linear superfield $\mathfrak{B}_{\alpha(n)} = \bar{\mathfrak{B}}_{\alpha(n)}$,

$$\bar{D}^{\beta\gamma} \mathfrak{B}_{\beta\alpha_1 \dots \alpha_{n-1}} = 0, \quad \mathcal{D}^\beta \mathfrak{B}_{\beta\alpha_1 \dots \alpha_{n-1}} = 0. \quad (2.25)$$

Then it is automatically conserved,

$$\mathcal{D}^{\beta\gamma} \mathfrak{B}_{\beta\gamma\alpha_1 \dots \alpha_{n-2}} = 0. \quad (2.26)$$

in accordance with (2.4b). The super-Cotton tensor $\mathcal{W}_{\alpha\beta}$ is an example of such supermultiplets. If $\mathfrak{B}_{\alpha(n)}$ is super-Weyl primary, then its dimension is equal to $(1 + n/2)$, in accordance with (2.17). As will be shown in the next subsection, a solution to (2.25) in terms of an unconstrained prepotential exists for every conformally flat superspace.

C. Conformal gauge superfields

Let n be a positive integer. A real symmetric rank- n spinor superfield $\mathfrak{H}_{\alpha(n)}$ is said to be a conformal gauge supermultiplet if (i) it is super-Weyl primary of dimension $(-n/2)$,

$$\delta_\sigma \mathfrak{H}_{\alpha(n)} = -\frac{n}{2} \sigma \mathfrak{H}_{\alpha(n)}; \quad (2.27)$$

and (ii) it is defined modulo gauge transformations of the form

$$\delta_\lambda \mathfrak{H}_{\alpha(n)} = \bar{D}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_n)} - (-1)^n \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_n)}, \quad (2.28)$$

with the gauge parameter $\lambda_{\alpha(n-1)}$ being unconstrained complex. The dimension of $\mathfrak{H}_{\alpha(n)}$ in (2.27) is uniquely fixed by requiring the longitudinal linear superfield $g_{\alpha(n)} = \bar{D}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_n)}$ in the right-hand side of (2.28) to be super-Weyl primary. Indeed, the gauge parameter $g_{\alpha(n)}$ must be

neutral with respect to the R -symmetry group $U(1)_R$ since $\mathfrak{H}_{\alpha(n)}$ is real, and then the dimension of $g_{\alpha(n)}$ is equal to $(-n/2)$, in accordance with (2.14).

Starting with $\mathfrak{H}_{\alpha(n)}$ one can construct its real descendant $\mathfrak{B}_{\alpha(n)}(\mathfrak{H}) = \mathcal{A} \mathfrak{H}_{\alpha(n)}$, where \mathcal{A} is a linear differential operator involving \mathcal{D}_A , the torsion superfields and their covariant derivatives, with the following properties:

- (1) $\mathfrak{B}_{\alpha(n)}$ is super-Weyl primary of dimension $(1 + n/2)$,

$$\delta_\sigma \mathfrak{B}_{\alpha(n)} = \left(1 + \frac{n}{2}\right) \sigma \mathfrak{B}_{\alpha(n)}. \quad (2.29)$$

- (2) The gauge variation of $\mathfrak{B}_{\alpha(n)}$ vanishes if the superspace is conformally flat,

$$\delta_\lambda \mathfrak{B}_{\alpha(n)} = O(\mathcal{W}_{(2)}), \quad (2.30)$$

where $\mathcal{W}_{(2)}$ is the super-Cotton tensor (2.7).

- (3) $\mathfrak{B}_{\alpha(n)}$ is divergenceless if the superspace is conformally flat,

$$\begin{aligned} \bar{D}^{\beta\gamma} \mathfrak{B}_{\beta\alpha(n-1)} &= O(\mathcal{W}_{(2)}), \\ \mathcal{D}^\beta \mathfrak{B}_{\beta\alpha(n-1)} &= O(\mathcal{W}_{(2)}). \end{aligned} \quad (2.31)$$

Here $O(\mathcal{W}_{(2)})$ stands for contributions containing the super-Cotton tensor and its covariant derivatives. In general, $\mathfrak{B}_{\alpha(n)}(\mathfrak{H})$ is uniquely defined modulo a normalization and contributions involving the super-Cotton tensor (2.7). The primary superfields $\mathfrak{B}_{\alpha(n)}(\mathfrak{H})$ may be interpreted as linearized higher-spin super-Cotton tensors in supergravity; see also the next subsection.

D. Linearized conformal higher-spin supergravity in conformally flat backgrounds

So far the supergravity background has been completely arbitrary. In the remainder of this section we assume that the background curved superspace $\mathcal{M}^{3|4}$ is conformally flat,

$$\mathcal{W}_{\alpha\beta} = 0. \quad (2.32)$$

Then each primary superfield $\mathfrak{B}_{\alpha(n)}(\mathfrak{H})$, with $n = 1, 2, \dots$, is gauge invariant,

$$\delta_\lambda \mathfrak{B}_{\alpha(n)} = 0, \quad (2.33)$$

and obeys the conservation equations (2.25). These properties and the super-Weyl transformation laws (2.27) and (2.29) imply that the action³

³The super-Weyl transformation of the superspace density is $\delta_\sigma E = -\sigma E$.

$$S_{\text{SCS}}[\mathfrak{H}_{(n)}] = -\frac{i^n}{2^{\lfloor n/2 \rfloor + 1}} \int d^{3|4} z E \mathfrak{H}^{\alpha(n)} \mathfrak{W}_{\alpha(n)}(\mathfrak{H}) \quad (2.34)$$

is gauge and super-Weyl invariant,

$$\delta_\lambda S_{\text{SCS}}[\mathfrak{H}_{(n)}] = 0, \quad \delta_\sigma S_{\text{SCS}}[\mathfrak{H}_{(n)}] = 0. \quad (2.35)$$

In accordance with the results of [37,39], it is natural to think of $\mathfrak{H}_{\alpha\beta}$ and $\mathfrak{W}_{\alpha\beta}(\mathfrak{H})$ as the linearized prepotential for $\mathcal{N} = 2$ conformal supergravity and the linearized super-Cotton tensor respectively. It is worth recalling that (2.32) is the equation of motion for conformal supergravity.

For $n = 2$ the functional (2.34) is proportional to the linearized action for conformal supergravity, which is obtained by linearising the nonlinear action for $\mathcal{N} = 2$ conformal supergravity [40,41] around a stationary point defined by (2.32). We can interpret $\mathfrak{W}_{\alpha(n)}$ to be a linearized higher-spin super-Cotton tensor. We now turn to constructing $\mathfrak{W}_{\alpha(n)}$ on a conformally flat superspace.

In Minkowski superspace, the linearized higher-spin super-Cotton tensors were constructed in [18], and here we reproduce these results. Associated with a real prepotential $H_{\alpha(n)} = H_{\alpha_1 \dots \alpha_n}$ is the following real symmetric rank- n spinor descendant

$$W_{\alpha(n)}(H) = \frac{1}{2^{n-1}} \sum_{j=0}^{\lfloor n/2 \rfloor} \left\{ \binom{n}{2j} \Delta^{\square j} \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2j}}^{\beta_{n-2j}} H_{\alpha_{n-2j+1} \dots \alpha_n) \beta_1 \dots \beta_{n-2j}} + \binom{n}{2j+1} \Delta^2 \square^j \partial_{(\alpha_1}^{\beta_1} \dots \partial_{\alpha_{n-2j-1}}^{\beta_{n-2j-1}} H_{\alpha_{n-2j} \dots \alpha_n) \beta_1 \dots \beta_{n-2j-1}} \right\}, \quad (2.36)$$

where

$$\Delta = \frac{i}{2} D^\alpha \bar{D}_\alpha, \quad (2.37)$$

and $D_A = (\partial_a, D_\alpha, \bar{D}^{\dot{\alpha}})$ are the covariant derivatives for Minkowski superspace,

$$\begin{aligned} \partial_a &= \frac{\partial}{\partial x^a}, & D_\alpha &= \frac{\partial}{\partial \theta^\alpha} + i\bar{\theta}^\beta (\gamma^a)_{\alpha\beta} \partial_a, \\ \bar{D}_\alpha &= -\frac{\partial}{\partial \bar{\theta}^\alpha} - i\theta^\beta (\gamma^a)_{\alpha\beta} \partial_a. \end{aligned} \quad (2.38)$$

The field strength (2.36) is invariant,

$$\delta_\lambda W_{\alpha(n)} = 0, \quad (2.39)$$

under the gauge transformations

$$\delta_\lambda H_{\alpha(n)} = \bar{D}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_n)} - (-1)^n D_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_n)}, \quad (2.40)$$

where the gauge parameter $\lambda_{\alpha(n-1)}$ is unconstrained complex. The field strength (2.36) is conserved,

$$D^\beta W_{\beta\alpha_1 \dots \alpha_{n-1}} = \bar{D}^\beta W_{\beta\alpha_1 \dots \alpha_{n-1}} = 0. \quad (2.41)$$

Making use of $W_{\alpha(n)}$ allows us to construct the higher-spin super-Cotton tensor $\mathfrak{W}_{\alpha(n)}$ in any conformally flat superspace $\mathcal{M}^{3|4}$.

In accordance with (2.6), for a conformally flat superspace $\mathcal{M}^{3|4}$ we can choose a local frame in which the covariant derivatives have the form

$$\mathcal{D}_\alpha = e^{\frac{1}{2}\sigma} (D_\alpha + D^\gamma \sigma M_{\gamma\alpha} - D_\alpha \sigma J), \quad (2.42a)$$

$$\bar{\mathcal{D}}_\alpha = e^{\frac{1}{2}\sigma} (\bar{D}_\alpha + \bar{D}^\gamma \sigma M_{\gamma\alpha} + \bar{D}_\alpha \sigma J), \quad (2.42b)$$

$$\begin{aligned} \mathcal{D}_a &= e^\sigma \left(\partial_a - \frac{i}{2} (\gamma_a)^{\gamma\delta} D_{(\gamma} \sigma \bar{D}_{\delta)} - \frac{i}{2} (\gamma_a)^{\gamma\delta} \bar{D}_{(\gamma} \sigma D_{\delta)} \right. \\ &\quad \left. + \varepsilon_{abc} \partial^b \sigma M^c + \frac{i}{2} (D_\gamma \sigma) \bar{D}^\gamma \sigma M_a \right. \\ &\quad \left. - \frac{i}{24} (\gamma_a)^{\gamma\delta} e^{-3\sigma} [D_\gamma, \bar{D}_\delta] e^{3\sigma} J \right), \end{aligned} \quad (2.42c)$$

for some real scale factor σ . Then, in accordance with (2.29), the higher-spin super-Cotton tensor $\mathfrak{W}_{\alpha(n)}$ in $\mathcal{M}^{3|4}$ is related to the flat-space one, Eq. (2.36), by the rule

$$\mathfrak{W}_{\alpha(n)} = e^{(1+\frac{n}{2})\sigma} W_{\alpha(n)}. \quad (2.43)$$

Similarly, Eq. (2.27) tells us that the prepotentials $\mathfrak{H}_{\alpha(n)}$ and $H_{\alpha(n)}$ can be chosen to be related to each other by

$$\mathfrak{H}_{\alpha(n)} = e^{-\frac{n}{2}\sigma} H_{\alpha(n)}. \quad (2.44)$$

In general, it is a difficult technical problem to express $\mathfrak{W}_{\alpha(n)}$ in terms of the covariant derivatives \mathcal{D}_A and the gauge prepotential $\mathfrak{H}_{\alpha(n)}$, for arbitrary n .

There exists a refined version of the representation (2.42) for those conformally flat superspaces which are characterized by the condition

$$S = 0. \quad (2.45)$$

This family includes the (1,1) AdS superspace defined by the (anti)commutation relations (1.5). If (2.45) holds, then Eq. (2.6d) tells that the scale factor in (2.42) is constrained,

$$D^\gamma \bar{D}_\gamma \sigma = 0 \Rightarrow \sigma = \eta + \bar{\eta}, \quad \bar{D}_\alpha \eta = 0, \quad (2.46)$$

with the chiral scalar η being, in principle, arbitrary. Now, applying a local R -symmetry transformation

$$\mathcal{D}_A \rightarrow \mathcal{D}_A = e^{-(\eta - \bar{\eta})J} \mathcal{D}_A e^{(\eta - \bar{\eta})J} \quad (2.47)$$

leads to covariant derivatives without $U(1)_R$ connection. The resulting covariant derivatives are

$$\mathcal{D}_\alpha = e^{\frac{1}{2}(3\bar{\eta} - \eta)} (D_\alpha + D^\gamma \eta M_{\gamma\alpha}), \quad (2.48a)$$

$$\bar{\mathcal{D}}_\alpha = e^{\frac{1}{2}(3\eta - \bar{\eta})} (\bar{D}_\alpha + \bar{D}^\gamma \bar{\eta} M_{\gamma\alpha}), \quad (2.48b)$$

$$\begin{aligned} \mathcal{D}_a = e^{\eta + \bar{\eta}} & \left(\partial_a - \frac{i}{2} (\gamma_a)^{\alpha\beta} D_\alpha \eta \bar{D}_\beta - \frac{i}{2} (\gamma_a)^{\alpha\beta} \bar{D}_\alpha \bar{\eta} D_\beta \right. \\ & \left. + \varepsilon_{abc} \partial^b (\eta + \bar{\eta}) M^c + \frac{i}{2} (D_\gamma \eta) (\bar{D}^\gamma \bar{\eta}) M_a \right). \end{aligned} \quad (2.48c)$$

In the case of (1,1) AdS superspace, the scale factor η was computed in [15].

III. (1,1) ADS SUPERSPACE

In this section we give a brief summary of the key results concerning the (1,1) AdS superspace [15], as well as elaborate on superfield representations of the (1,1) AdS isometry group. The covariant derivatives of $\text{AdS}^{(3|1,1)}$ satisfy the following algebra [15]:

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2i\mathcal{D}_{\alpha\beta}, \quad (3.1a)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{\mu}M_{\alpha\beta}, \quad \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 4\mu M_{\alpha\beta}, \quad (3.1b)$$

$$[\mathcal{D}_{\alpha\beta}, \mathcal{D}_\gamma] = -2i\bar{\mu}\varepsilon_{\gamma(\alpha}\bar{\mathcal{D}}_{\beta)}, \quad [\mathcal{D}_{\alpha\beta}, \bar{\mathcal{D}}_\gamma] = 2i\mu\varepsilon_{\gamma(\alpha}\mathcal{D}_{\beta)}, \quad (3.1c)$$

$$[\mathcal{D}_{\alpha\beta}, \mathcal{D}_{\gamma\delta}] = 4\bar{\mu}\mu(\varepsilon_{\gamma(\alpha}M_{\beta)\delta} + \varepsilon_{\delta(\alpha}M_{\beta)\gamma}), \quad (3.1d)$$

with $\mu \neq 0$ being a complex parameter. As compared with (1.5), we have denoted $\mathcal{R} = \mu$. This notation will be used in the remainder of this paper.

The covariantly transverse linear and longitudinal linear superfields on an arbitrary supergravity background were described in the previous section. In the case of (1,1) AdS superspace, such superfields play an important role. One can define projectors P_n^\perp and P_n^\parallel on the spaces of transverse linear and longitudinal linear superfields, respectively. The projectors are

$$P_n^\perp = \frac{1}{4(n+1)\mu} (\bar{\mathcal{D}}^2 + 2n\mu), \quad (3.2a)$$

$$P_n^\parallel = -\frac{1}{4(n+1)\mu} (\bar{\mathcal{D}}^2 - 2(n+2)\mu), \quad (3.2b)$$

with the properties

$$\begin{aligned} (P_n^\perp)^2 &= P_n^\perp, & (P_n^\parallel)^2 &= P_n^\parallel, & P_n^\perp P_n^\parallel &= P_n^\parallel P_n^\perp = 0, \\ P_n^\perp + P_n^\parallel &= 1. \end{aligned} \quad (3.3)$$

Given a complex tensor superfield $V_{\alpha(n)}$ with $n \neq 0$, it can be represented as a sum of transverse linear and longitudinal linear multiplets,

$$\begin{aligned} V_{\alpha(n)} &= -\frac{1}{2\mu(n+2)} \bar{\mathcal{D}}^\gamma \bar{\mathcal{D}}_{(\gamma} V_{\alpha_1 \dots \alpha_n)} \\ &\quad - \frac{1}{2\mu(n+1)} \bar{\mathcal{D}}_{(\alpha_1} \bar{\mathcal{D}}^{|\gamma|} V_{\alpha_2 \dots \alpha_n)\gamma}. \end{aligned} \quad (3.4)$$

Choosing $V_{\alpha(n)}$ to be longitudinal linear ($G_{\alpha(n)}$) or transverse linear ($\Gamma_{\alpha(n)}$), the above identity gives the relations (2.19a) and (2.19b) for some prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$, respectively.

In accordance with the general formalism of [2], the isometries of $\text{AdS}^{(3|1,1)}$ are generated by those real super-vector fields $\lambda^A E_A$ which obey the Killing equation

$$\left[\Lambda + \frac{1}{2} l^{ab} M_{ab}, \mathcal{D}_C \right] = 0, \quad (3.5)$$

where

$$\Lambda = \lambda^A \mathcal{D}_A = \lambda^\alpha \mathcal{D}_\alpha + \lambda^\alpha \bar{\mathcal{D}}_\alpha + \bar{\lambda}^{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}}, \quad \bar{\lambda}^{\dot{\alpha}} = \lambda^{\dot{\alpha}} \quad (3.6)$$

and l^{ab} is some local Lorentz parameter. As demonstrated in [15], this equation implies that the parameters λ^α and l^{ab} are uniquely expressed in terms of the vector λ^α ,

$$\lambda_\alpha = \frac{i}{6} \bar{\mathcal{D}}^\beta \lambda_{\alpha\beta}, \quad l_{\alpha\beta} = 2\mathcal{D}_{(\alpha} \lambda_{\beta)}, \quad (3.7)$$

and the vector parameter obeys the equation

$$\mathcal{D}_{(\alpha} \lambda_{\beta\gamma)} = 0 \Leftrightarrow \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta\gamma)} = 0. \quad (3.8)$$

In comparison with the 3D $\mathcal{N} = 2$ Minkowski superspace, the specific feature of $\text{AdS}^{(3|1,1)}$ is that any two of the three parameters $\{\lambda_{\alpha\beta}, \lambda_\alpha, l_{\alpha\beta}\}$ are expressed in terms of the third parameter, in particular

$$\lambda_{\alpha\beta} = \frac{i}{\mu} \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)}, \quad \lambda_\alpha = -\frac{1}{12\bar{\mu}} \mathcal{D}^\beta l_{\alpha\beta}. \quad (3.9)$$

From (3.7) and (3.9) we deduce

$$\bar{D}^\alpha \lambda_\alpha = \mathcal{D}_\alpha \lambda^\alpha = 0. \quad (3.10)$$

Every solution λ^A of the above relations is called a Killing supervector field of $\text{AdS}^{(3|1,1)}$. These supervector fields can be shown to generate the isometry group of $\text{AdS}^{(3|1,1)}$, $\text{OSp}(1|2; \mathbb{R}) \times \text{OSp}(1|2; \mathbb{R})$.

In Minkowski superspace $\mathbb{M}^{3|4}$, there are two ways to generate supersymmetric invariants, one of which corresponds to the integration over the full superspace and the other over its chiral subspace. In (1,1) AdS superspace, every chiral integral can always be recast as a full superspace integral. Associated with a scalar superfield \mathcal{L} is the following supersymmetric invariant:

$$\int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L} = -\frac{1}{4} \int d^3x d^2\theta \mathcal{E} (\bar{D}^2 - 4\mu) \mathcal{L},$$

$$E^{-1} = \text{Ber}(E_A{}^M), \quad (3.11)$$

where \mathcal{E} denotes the chiral integration measure. Let \mathcal{L}_c be a covariantly chiral scalar Lagrangian, $\bar{D}_\alpha \mathcal{L}_c = 0$. It generates a supersymmetric invariant of the form $\int d^3x d^2\theta \mathcal{E} \mathcal{L}_c$. The specific feature of (1,1) AdS superspace is that the chiral action can be equivalently written as an integral over the full superspace [15]

$$\int d^3x d^2\theta \mathcal{E} \mathcal{L}_c = \frac{1}{\mu} \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L}_c. \quad (3.12)$$

Unlike the flat superspace case, the integral on the right does not vanish in AdS.

Supersymmetric invariant (3.11) can be reduced to component fields by the rule [35]

$$\int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L} = \frac{1}{16} \int d^3x e (\mathcal{D}^2 - 16\bar{\mu}) (\bar{D}^2 - 4\mu) \mathcal{L}|, \quad (3.13)$$

with $e^{-1} := \det(e_a{}^m)$. Here $e_a{}^m$ is the inverse vielbein, which determines the torsion-free covariant derivative of AdS space

$$\nabla_a = e_a + \frac{1}{2} \omega_a{}^{bc}(e) M_{bc}, \quad e_a := e_a{}^m \partial_m. \quad (3.14)$$

In general, the $\theta, \bar{\theta}$ -independent component, $T|_{\theta=\bar{\theta}=0}$, of a superfield $T(x, \theta, \bar{\theta})$ is denoted $T|$. To complete the formalism of component reduction, we only need the following relation:

$$(\mathcal{D}_a T)| = \nabla_a T|. \quad (3.15)$$

In what follows, we will work with full superspace integrals only and make use of the notation $d^{3|4}z := d^3x d^2\theta d^2\bar{\theta}$.

IV. MASSLESS HALF-INTEGERS SUPERSPIN GAUGE THEORIES IN (1,1) ADS SUPERSPACE

The superconformal higher-spin action (2.34) in a conformally flat superspace is formulated in terms of the conformal gauge superfields $\mathfrak{H}_{\alpha(n)}$. The same gauge superfield, at least for $n = 2s$, with $s = 1, 2, \dots$, can be used to construct massless actions in two of the three $\mathcal{N} = 2$ maximally symmetric backgrounds, which are Minkowski superspace and (1,1) AdS superspace. Such actions, however, involve not only $\mathfrak{H}_{\alpha(n)}$ but also some compensators.

In Minkowski space, there are two off-shell formulations for the massless $\mathcal{N} = 2$ multiplet of half-integer superspin $(s + 1/2)$, with $s = 2, 3, \dots$, which are dual to each other [18]. They are referred to as transverse and longitudinal. Here we extend these gauge theories to (1,1) AdS superspace.

A. Transverse formulation

The transverse formulation for the massless superspin $(s + 1/2)$ multiplet is realized in terms of the following dynamical variables:

$$\mathcal{V}_{(s+1/2)}^\perp = \{\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}\}. \quad (4.1)$$

Here $\mathfrak{H}_{\alpha(2s)} = \mathfrak{H}_{(\alpha_1 \dots \alpha_{2s})}$ is an unconstrained real superfield, and the complex superfield $\Gamma_{\alpha(2s-2)} = \Gamma_{(\alpha_1 \dots \alpha_{2s-2})}$ is transverse linear, Eq. (2.15). In accordance with (2.19b), the constraint on $\Gamma_{\alpha(2s-2)}$ is solved in terms of an unconstrained prepotential $\Phi_{\alpha(2s-1)}$,

$$\Gamma_{\alpha(2s-2)} = \bar{D}^\beta \Phi_{(\beta \alpha_1 \dots \alpha_{2s-1})}, \quad (4.2)$$

which is defined modulo gauge transformations of the form

$$\delta_\xi \Phi_{\alpha(2s-1)} = \bar{D}^\beta \xi_{(\beta \alpha_1 \dots \alpha_{2s-1})}, \quad (4.3)$$

with the gauge parameter $\xi_{\alpha(2s)}$ being unconstrained.

The dynamical superfields $\mathfrak{H}_{\alpha(2s)}$ and $\Gamma_{\alpha(2s-2)}$ are postulated to be defined modulo gauge transformations of the form

$$\delta_\lambda \mathfrak{H}_{\alpha(2s)} = \bar{D}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{2s})} - \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_{2s})} \equiv g_{\alpha(2s)} + \bar{g}_{\alpha(2s)}, \quad (4.4a)$$

$$\delta_\lambda \Gamma_{\alpha(2s-2)} = -\frac{1}{4} \bar{D}^\beta (\mathcal{D}^2 + 2(2s-1)\bar{\mu}) \bar{\lambda}_{\beta \alpha(2s-2)}$$

$$= \frac{s}{2s+1} \bar{D}^\beta \mathcal{D}^\gamma \bar{g}_{(\beta \gamma \alpha_1 \dots \alpha_{2s-2})}, \quad (4.4b)$$

where the complex gauge parameter $\lambda_{\alpha(2s-1)}$ is unconstrained. The gauge transformation of $\mathfrak{S}_{\alpha(2s)}$ coincides with (2.28) for $n = 2s$. From $\delta_\lambda \Gamma_{\alpha(2s-2)}$ we read off the gauge transformation of the prepotential $\Phi_{\alpha(2s-1)}$ defined by Eq. (4.2), which is

$$\delta_\lambda \Phi_{\alpha(2s-1)} = -\frac{1}{4} (\mathcal{D}^2 + 2(2s-1)\bar{\mu}) \bar{\lambda}_{\alpha(2s-1)}. \quad (4.5)$$

Modulo an overall normalization factor, there is a unique quadratic action which is invariant under the gauge transformations (4.4). It is given by

$$S_{(s+\frac{1}{2})}^\perp = \left(-\frac{1}{2}\right)^s \int d^3|4 z E \left\{ \frac{1}{8} \mathfrak{S}^{\alpha(2s)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\beta \mathfrak{S}_{\alpha(2s)} + 2s(s-1) \bar{\mu} \mu \mathfrak{S}^{\alpha(2s)} \mathfrak{S}_{\alpha(2s)} \right. \\ \left. + \mathfrak{S}^{\alpha(2s)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} \Gamma_{\alpha_3 \dots \alpha_{2s}} - \bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} \bar{\Gamma}_{\alpha_3 \dots \alpha_{2s}}) + \frac{2s-1}{s} \bar{\Gamma}^{\alpha(2s-2)} \Gamma_{\alpha(2s-2)} + \frac{2s+1}{2s} (\Gamma^{\alpha(2s-2)} \Gamma_{\alpha(2s-2)} + \bar{\Gamma}^{\alpha(2s-2)} \bar{\Gamma}_{\alpha(2s-2)}) \right\}. \quad (4.6)$$

In the flat superspace limit, this action reduces to the one derived in [18].

The $s = 1$ choice was excluded from the above consideration, since the constraint (2.15) is not defined for $n = 0$. However, as discussed in Sec. II, the corollary (2.16) of (2.15) is perfectly consistent for $n = 0$ and defines a covariantly transverse linear scalar superfield (2.18),

$$(\bar{\mathcal{D}}^2 - \mu) \Gamma = 0. \quad (4.7)$$

We therefore postulate Γ and its conjugate $\bar{\Gamma}$ to be the compensators in the $s = 1$ case. Choosing $s = 1$ in the gauge transformation law (4.4) gives

$$\delta_\lambda \mathfrak{S}_{\alpha\beta} = \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)} - \mathcal{D}_{(\alpha} \bar{\lambda}_{\beta)}, \quad (4.8a)$$

$$\delta_\lambda \Gamma = -\frac{1}{4} \bar{\mathcal{D}}^\beta (\mathcal{D}^2 + 2\bar{\mu}) \bar{\lambda}_\beta. \quad (4.8b)$$

The variation $\delta_\lambda \Gamma$ is compatible with the constraint (4.7), that is $(\bar{\mathcal{D}}^2 - \mu) \delta_\lambda \Gamma = 0$. Finally, choosing $s = 1$ in (4.6) gives the linearized action for nonminimal (1,1) AdS supergravity, which was originally derived in section 9.2 of [15].

B. Longitudinal formulation

The longitudinal formulation for the massless superspin- $(s + \frac{1}{2})$ multiplet is described in terms of the following variables:

$$\mathcal{V}_{(s+\frac{1}{2})}^\parallel = \{ \mathfrak{S}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)} \}. \quad (4.9)$$

Here $\mathfrak{S}_{\alpha(2s)}$ is the same as in (4.1), and the complex superfield $G_{\alpha(2s-2)}$ is longitudinal linear, Eq. (2.12). In accordance with (2.19a), the constraint (2.12) can be solved in terms of an unconstrained complex prepotential $\Psi_{\alpha(2s-3)}$,

$$G_{\alpha(2s-2)} = \bar{\mathcal{D}}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_{2s-2})}, \quad (4.10)$$

which is defined modulo gauge transformations of the form

$$\delta_\zeta \Psi_{\alpha(2s-3)} = \bar{\mathcal{D}}_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s-3})}, \quad (4.11)$$

with the gauge parameter $\zeta_{\alpha(2s-4)}$ being unconstrained complex.

The longitudinal formulation may be obtained from the transverse one, developed in the previous subsection, by performing a superfield duality transformation. Starting from the action $S_{(s+\frac{1}{2})}^\perp = S_{(s+\frac{1}{2})}^\perp[\mathfrak{S}, \Gamma, \bar{\Gamma}]$, Eq. (4.6), we introduce a first-order model described by the action

$$S[\mathfrak{S}, V, \bar{V}, G, \bar{G}] = \left(-\frac{1}{2}\right)^s \int d^3|4 z E \left\{ \frac{1}{8} \mathfrak{S}^{\alpha(2s)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\beta \mathfrak{S}_{\alpha(2s)} + 2s(s-1) \bar{\mu} \mu \mathfrak{S}^{\alpha(2s)} \mathfrak{S}_{\alpha(2s)} \right. \\ \left. + \mathfrak{S}^{\alpha(2s)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} V_{\alpha(2s-2)} - \bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} \bar{V}_{\alpha(2s-2)}) + \frac{2s-1}{s} \bar{V}^{\alpha(2s-2)} V_{\alpha(2s-2)} \right. \\ \left. + \frac{2s+1}{2s} (V^{\alpha(2s-2)} V_{\alpha(2s-2)} + \bar{V}^{\alpha(2s-2)} \bar{V}_{\alpha(2s-2)}) \right. \\ \left. - \frac{2}{s} (G^{\alpha(2s-2)} V_{\alpha(2s-2)} + \bar{G}^{\alpha(2s-2)} \bar{V}_{\alpha(2s-2)}) \right\}. \quad (4.12)$$

Here $V_{\alpha(2s-2)}$ is an unconstrained complex superfield, and $G_{\alpha(2s-2)}$ is given by (4.10). The first-order action is invariant under the gauge transformation (4.4a) accompanied with

$$\delta_\lambda V_{\alpha(2s-2)} = \delta_\lambda \Gamma_{\alpha(2s-2)}, \quad (4.13a)$$

$$\begin{aligned} \delta_\lambda G_{\alpha(2s-2)} &= -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4s\mu)\mathcal{D}^\beta \lambda_{\alpha(2s-2)\beta} \\ &\quad + i(s-1)\bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}^{|\beta\gamma|} \lambda_{\alpha_2 \dots \alpha_{2s-2})\beta\gamma} \\ &= s \left(\frac{1}{2s+1} \mathcal{D}^\beta \bar{\mathcal{D}}^\gamma + i\mathcal{D}^{\beta\gamma} \right) g_{\beta\gamma\alpha(2s-2)}, \end{aligned} \quad (4.13b)$$

where $\delta_\lambda \Gamma_{\alpha(2s-2)}$ is given by (4.4b). From (4.13b) we read off the transformation law of the prepotential $\Psi_{\alpha(2s-3)}$, Eq. (4.10), which is

$$\delta_\lambda \Psi_{\alpha(2s-3)} = -\frac{1}{2}(\bar{\mathcal{D}}^\beta \mathcal{D}^\gamma - 2i(s-1)\mathcal{D}^{\beta\gamma})\lambda_{\beta\gamma\alpha(2s-3)}. \quad (4.14)$$

Varying the action (4.12) with respect to $\Psi_{\alpha(2s-3)}$ implies that $V_{\alpha(2s-2)} = \Gamma_{\alpha(2s-2)}$, and then $S[\mathfrak{S}, V, \bar{V}, G, \bar{G}]$ reduces to the transverse action (4.6). This means that the theories (4.6) and (4.12) are equivalent. On the other hand, $V_{\alpha(2s-2)}$ and its conjugate $\bar{V}_{\alpha(2s-2)}$ are auxiliary since they appear in the action (4.12) without derivatives. Integrating out these auxiliary superfields leads to the following dual theory:

$$\begin{aligned} S_{(s+\frac{1}{2})}^{\parallel} &= \left(-\frac{1}{2}\right)^s \int d^{3|4}z E \left\{ \frac{1}{8} \mathfrak{S}^{\alpha(2s)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\beta \mathfrak{S}_{\alpha(2s)} + 2s(s-1)\mu\bar{\mu} \mathfrak{S}^{\alpha(2s)} \mathfrak{S}_{\alpha(2s)} \right. \\ &\quad - \frac{1}{16} ([\mathcal{D}_\beta, \bar{\mathcal{D}}_\gamma] \mathfrak{S}^{\beta\gamma\alpha(2s-2)}) [\mathcal{D}^\delta, \bar{\mathcal{D}}^\rho] \mathfrak{S}_{\delta\rho\alpha(2s-2)} + \frac{s}{2} (\mathcal{D}_{\beta\gamma} \mathfrak{S}^{\beta\gamma\alpha(2s-2)}) \mathcal{D}^{\delta\rho} \mathfrak{S}_{\delta\rho\alpha(2s-2)} \\ &\quad + \frac{2s-1}{2s} \left[i(\mathcal{D}_{\beta\gamma} \mathfrak{S}^{\beta\gamma\alpha(2s-2)}) (G_{\alpha(2s-2)} - \bar{G}_{\alpha(2s-2)}) + \frac{1}{s} \bar{G}^{\alpha(2s-2)} G_{\alpha(2s-2)} \right] \\ &\quad \left. - \frac{2s+1}{4s^2} (G^{\alpha(2s-2)} G_{\alpha(2s-2)} + \bar{G}^{\alpha(2s-2)} \bar{G}_{\alpha(2s-2)}) \right\}. \end{aligned} \quad (4.15)$$

This action is invariant under the gauge transformations

$$\delta_\lambda \mathfrak{S}_{\alpha(2s)} = \bar{\mathcal{D}}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{2s})} - \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_{2s})}, \quad (4.16a)$$

$$\begin{aligned} \delta_\lambda G_{\alpha(2s-2)} &= -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4s\mu)\mathcal{D}^\beta \lambda_{\alpha(2s-2)\beta} \\ &\quad + i(s-1)\bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}^{|\beta\gamma|} \lambda_{\alpha_2 \dots \alpha_{2s-2})\beta\gamma}. \end{aligned} \quad (4.16b)$$

In the flat superspace limit, this action reduces to the one derived in [18].

In the $s = 1$ case, the compensator G becomes covariantly chiral, $\bar{\mathcal{D}}_\alpha G = 0$. Choosing $s = 1$ in (4.15) gives the linearized action for minimal (1,1) AdS supergravity, which was originally derived in Sec. 9.1 of [15], provided we identify $G = 3\sigma$. Choosing $s = 1$ in the gauge transformation law (4.16) gives

$$\delta_\lambda \mathfrak{S}_{\alpha\beta} = \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)} - \mathcal{D}_{(\alpha} \bar{\lambda}_{\beta)}, \quad (4.17a)$$

$$\delta_\lambda G = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}^\beta \lambda_\beta. \quad (4.17b)$$

It is clear that the variation $\delta_\lambda G$ is covariantly chiral.

V. MASSLESS INTEGER SUPERSPIN GAUGE THEORIES IN (1,1) ADS SUPERSPACE

When attempting to develop a Lagrangian formulation for a massless multiplet of superspin s , where $s = 1, 2, \dots$, a naive expectation is that the dynamical variables of such

a theory should consist of a conformal gauge superfield $\mathfrak{S}_{\alpha(2s-1)} = \bar{\mathfrak{S}}_{\alpha(2s-1)}$, introduced in Sec. II C, in conjunction with some compensator(s). Instead, our approach in this section will be based on developing 3D $\mathcal{N} = 2$ analogs of the two dually equivalent off-shell formulations, the so-called longitudinal and transverse ones, for the massless $\mathcal{N} = 1$ multiplets of integer superspin in AdS₄ [7]. Then we will provide a reformulation of the longitudinal formulation derived in the next subsection in a way similar to the one proposed in the 4D $\mathcal{N} = 1$ AdS case [29]. Such a reformulation naturally leads to the appearance of a conformal gauge superfield $\mathfrak{S}_{\alpha(2s-1)}$.

A. Longitudinal formulation

Given an integer $s \geq 1$, the longitudinal formulation for the massless superspin- s multiplet is realized in terms of the following dynamical variables:

$$\mathcal{V}_{(s)}^{\parallel} = \{U_{\alpha(2s-2)}, G_{\alpha(2s)}, \bar{G}_{\alpha(2s)}\}. \quad (5.1)$$

Here, $U_{\alpha(2s-2)}$ is an unconstrained real superfield, and the complex superfield $G_{\alpha(2s)}$ is longitudinal linear, Eq. (2.12). In accordance with (2.19), the constraint (2.12) can be solved in terms of an unconstrained complex prepotential $\Psi_{\alpha(2s-1)}$,

$$G_{\alpha_1 \dots \alpha_{2s}} := \bar{\mathcal{D}}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_{2s})}, \quad (5.2)$$

which is defined modulo gauge transformations of the form

$$\delta_\zeta \Psi_{\alpha(2s-1)} = \bar{\mathcal{D}}_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s-1})}, \quad (5.3)$$

with the gauge parameter $\zeta_{\alpha(2s-2)}$ being unconstrained complex.

We postulate the dynamical superfields $U_{\alpha(2s-2)}$ and $G_{\alpha(2s)}$ to be defined modulo gauge transformations of the form

$$\begin{aligned} \delta_L U_{\alpha(2s-2)} &= \mathcal{D}^\beta L_{\beta\alpha_1 \dots \alpha_{2s-2}} - \bar{\mathcal{D}}^{\beta\bar{\gamma}} \bar{L}_{\beta\alpha_1 \dots \alpha_{2s-2}} \\ &\equiv \bar{\gamma}_{\alpha(2s-2)} + \gamma_{\alpha(2s-2)}, \end{aligned} \quad (5.4a)$$

$$\begin{aligned} \delta_L G_{\alpha(2s)} &= -\frac{1}{2} \bar{\mathcal{D}}_{(\alpha_1} (\mathcal{D}^2 - 2(2s+1)\bar{\mu}) L_{\alpha_2 \dots \alpha_{2s})} \\ &= \bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}_{\alpha_2} \bar{\gamma}_{\alpha_3 \dots \alpha_{2s})}. \end{aligned} \quad (5.4b)$$

Here the gauge parameter $L_{\alpha(2s-1)}$ is an unconstrained complex superfield, and $\gamma^{\alpha(2s-2)} := \bar{\mathcal{D}}_{\beta\bar{\gamma}} \bar{L}^{\beta\alpha(2s-2)}$ is transverse linear. From (5.4b) we read off the gauge transformation law of the prepotential,

$$\begin{aligned} \delta_L \Psi_{\alpha(2s-1)} &= -\frac{1}{2} (\mathcal{D}^2 - 2(2s+1)\bar{\mu}) L_{\alpha(2s-1)} \\ &= \mathcal{D}_{(\alpha_1} \mathcal{D}^{|\beta|} L_{\alpha_2 \dots \alpha_{2s-1})\beta}. \end{aligned} \quad (5.5)$$

Modulo an overall normalization factor, there is a unique quadratic action which is invariant under the gauge transformations (5.4). The action is

$$\begin{aligned} S_{(s)}^{\parallel} &= \left(-\frac{1}{2}\right)^s \int d^3|4_z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^\gamma (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\gamma U_{\alpha(2s-2)} + \frac{s}{2s+1} U^{\alpha(2s-2)} (\mathcal{D}^\beta \bar{\mathcal{D}}^\gamma G_{\beta\gamma\alpha(2s-2)} - \bar{\mathcal{D}}^{\beta\bar{\gamma}} \mathcal{D}^\gamma \bar{G}_{\beta\gamma\alpha(2s-2)}) \right. \\ &\quad \left. + \frac{s}{2s-1} \bar{G}^{\alpha(2s)} G_{\alpha(2s)} + \frac{s}{2(2s+1)} (G^{\alpha(2s)} G_{\alpha(2s)} + \bar{G}^{\alpha(2s)} \bar{G}_{\alpha(2s)}) + 2s(s+1)\mu\bar{\mu} U^{\alpha(2s-2)} U_{\alpha(2s-2)} \right\}. \end{aligned} \quad (5.6)$$

The special $s=1$ case, which corresponds to the massless gravitino multiplet, will be studied in more detail in Sec. VD.

B. Transverse formulation

The transverse formulation for the massless superspin- s multiplet is realized in terms of the following dynamical variables:

$$\mathcal{V}_{(s)}^\perp = \{U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}\}. \quad (5.7)$$

Here, $U_{\alpha(2s-2)}$ is the same as in (5.1), and the complex superfield $\Gamma_{\alpha(2s)}$ is transverse linear, Eq. (2.15). In accordance with (2.19b), the constraint on $\Gamma_{\alpha(2s)}$ is solved in terms of an unconstrained prepotential $\Phi_{\alpha(2s+1)}$,

$$\Gamma_{\alpha(2s)} = \bar{\mathcal{D}}^\beta \Phi_{(\beta\alpha_1 \dots \alpha_{2s})}, \quad (5.8)$$

which is defined modulo gauge transformations of the form

$$\delta_\xi \Phi_{\alpha(2s+1)} = \bar{\mathcal{D}}^\beta \xi_{(\beta\alpha_1 \dots \alpha_{2s+1})}, \quad (5.9)$$

with the gauge parameter $\xi_{\alpha(2s+2)}$ being unconstrained.

The transverse formulation for the massless superspin- s multiplet is obtained from the longitudinal one developed in the previous subsection by performing a superfield duality transformation. The first step is to replace the gauge-invariant action (5.6) with the following first-order action:

$$\begin{aligned} S_s[U, V, \bar{V}, \Gamma, \bar{\Gamma}] &= \left(-\frac{1}{2}\right)^s \int d^3|4_z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^\gamma (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\gamma U_{\alpha(2s-2)} + 2s(s+1)\mu\bar{\mu} U^{\alpha(2s-2)} U_{\alpha(2s-2)} \right. \\ &\quad \left. + \frac{s}{2s+1} U^{\alpha(2s-2)} (\mathcal{D}^\beta \bar{\mathcal{D}}^\gamma V_{\beta\gamma\alpha(2s-2)} - \bar{\mathcal{D}}^{\beta\bar{\gamma}} \mathcal{D}^\gamma \bar{V}_{\beta\gamma\alpha(2s-2)}) + \frac{s}{2s-1} \bar{V}^{\alpha(2s)} V_{\alpha(2s)} \right. \\ &\quad \left. + \frac{s}{2(2s+1)} (V^{\alpha(2s)} V_{\alpha(2s)} + \bar{V}^{\alpha(2s)} \bar{V}_{\alpha(2s)}) + \frac{4s}{(2s+1)(2s-1)} (V^{\alpha(2s)} \Gamma_{\alpha(2s)} + \bar{V}^{\alpha(2s)} \bar{\Gamma}_{\alpha(2s)}) \right\}, \end{aligned} \quad (5.10)$$

in which $V_{\alpha(2s)}$ is an unconstrained complex superfield, and $\Gamma_{\alpha(2s)}$ is given by (5.8). This action is invariant under the gauge transformation (5.4a) accompanied with

$$\delta_L V_{\alpha(2s)} = \delta_L G_{\alpha(2s)}, \quad (5.11a)$$

$$\begin{aligned} \delta_L \Gamma_{\alpha(2s)} &= -\frac{1}{4}(\bar{\mathcal{D}}^2 + 4s\mu)\mathcal{D}_{(\alpha_1}\bar{L}_{\alpha_2\dots\alpha_{2s})} \\ &\quad + \frac{i}{2}(2s+1)\bar{\mathcal{D}}^\gamma\mathcal{D}_{(\gamma\alpha_1}\bar{L}_{\alpha_2\dots\alpha_{2s})} \\ &= \frac{1}{2}\mathcal{D}_{(\alpha_1}\bar{\mathcal{D}}_{\alpha_2}\gamma_{\alpha_3\dots\alpha_{2s})} \\ &\quad - \frac{i}{2}(2s-1)\mathcal{D}_{(\alpha_1\alpha_2}\gamma_{\alpha_3\dots\alpha_{2s})}, \end{aligned} \quad (5.11b)$$

where $\gamma_{\alpha(2s-2)} = -\bar{\mathcal{D}}^\beta\bar{L}_{\beta\alpha_1\dots\alpha_{2s-2}}$, and $\delta_L G_{\alpha(2s)}$ is given by (5.4b). The first-order model described by action (5.10) is equivalent to the longitudinal theory (5.6). Indeed, varying $S_s[U, V, \bar{V}, \Gamma, \bar{\Gamma}]$ with respect to the prepotential $\Phi_{\alpha(2s+1)}$, Eq. (5.8), gives $V_{\alpha(2s)} = G_{\alpha(2s)}$, and then the action (5.10) reduces to the longitudinal one, Eq. (5.6). On the other hand, we can integrate out the auxiliary superfield $V_{\alpha(2s)}$ and its conjugate $\bar{V}_{\alpha(2s)}$ from (5.11b) using their equations of motion. This leads to the transverse action

$$\begin{aligned} S_{(s)}^\perp &= \left(-\frac{1}{2}\right)^s \int d^4z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^\gamma (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\gamma U_{\alpha(2s-2)} - \frac{2s-1}{16(2s+1)} (8s \mathcal{D}^{\alpha_1\alpha_2} U^{\alpha_3\dots\alpha_{2s}} \mathcal{D}_{(\alpha_1\alpha_2} U_{\alpha_3\dots\alpha_{2s})} \right. \\ &\quad + [\mathcal{D}^{\alpha_1}, \bar{\mathcal{D}}^{\alpha_2}] U^{\alpha_3\dots\alpha_{2s}} [\mathcal{D}_{(\alpha_1}, \bar{\mathcal{D}}_{\alpha_2)} U_{\alpha_3\dots\alpha_{2s})} + 2s(s+1)\mu\bar{\mu} U^{\alpha(2s-2)} U_{\alpha(2s-2)} - i U^{\alpha_1\dots\alpha_{2s-2}} \mathcal{D}^{\alpha_{2s-1}\alpha_{2s}} (\Gamma_{\alpha(2s)} - \bar{\Gamma}_{\alpha(2s)}) \\ &\quad \left. - \frac{2}{2s-1} \bar{\Gamma}^{\alpha(2s)} \Gamma_{\alpha(2s)} + \frac{1}{2s+1} (\Gamma^{\alpha(2s)} \Gamma_{\alpha(2s)} + \bar{\Gamma}^{\alpha(2s)} \bar{\Gamma}_{\alpha(2s)}) \right\}. \end{aligned} \quad (5.12)$$

The action is invariant under (5.4a) and (5.11b).

C. Reformulation of the longitudinal theory

In this subsection we consider a reformulation of the longitudinal theory that is similar to the one proposed in the 4D $\mathcal{N} = 1$ AdS case [29]. It is obtained by enlarging the gauge freedom (5.4) at the cost of introducing new purely gauge superfield variables in addition to $U_{\alpha(2s-2)}$, $\Psi_{\alpha(2s-1)}$ and $\bar{\Psi}_{\alpha(2s-1)}$. In such a setting, the gauge freedom of $\Psi_{\alpha(2s-1)}$ coincides with that of a complex conformal gauge superfield. Given a positive integer $s \geq 2$, a massless superspin- s multiplet can be described in AdS^(3|1,1) by using the following superfield variables: (i) an unconstrained prepotential $\Psi_{\alpha(2s-1)}$ and its complex conjugate $\bar{\Psi}_{\alpha(2s-1)}$; (ii) a real superfield $U_{\alpha(2s-2)} = \bar{U}_{\alpha(2s-2)}$; and (iii) a complex superfield $\Sigma_{\alpha(2s-3)}$ and its conjugate $\bar{\Sigma}_{\alpha(2s-3)}$, where $\Sigma_{\alpha(2s-3)}$ is constrained to be transverse linear,

$$\bar{\mathcal{D}}^\beta \Sigma_{\beta\alpha(2s-4)} = 0. \quad (5.13)$$

The constraint (5.13) is solved in terms of an unconstrained complex prepotential $Z_{\alpha(2s-2)}$ by the rule

$$\Sigma_{\alpha(2s-3)} = \bar{\mathcal{D}}^\beta Z_{(\beta\alpha_1\dots\alpha_{2s-3})}. \quad (5.14)$$

This prepotential is defined modulo gauge transformations

$$\delta_\xi Z_{\alpha(2s-2)} = \bar{\mathcal{D}}^\beta \xi_{(\beta\alpha_1\dots\alpha_{2s-2})}, \quad (5.15)$$

with the gauge parameter $\xi_{\alpha(2s-1)}$ being unconstrained.

The gauge freedom of $\Psi_{\alpha_1\dots\alpha_{2s-1}}$ is given by

$$\delta_{\mathfrak{B},\zeta} \Psi_{\alpha_1\dots\alpha_{2s-1}} = \mathcal{D}_{(\alpha_1} \mathfrak{B}_{\alpha_2\dots\alpha_{2s-1})} + \bar{\mathcal{D}}_{(\alpha_1} \zeta_{\alpha_2\dots\alpha_{2s-1})}, \quad (5.16a)$$

with unconstrained gauge parameters $\mathfrak{B}_{\alpha(2s-2)}$ and $\zeta_{\alpha(2s-2)}$. The \mathfrak{B} -transformation is defined to act on the superfields $U_{\alpha(2s-2)}$ and $\Sigma_{\alpha(2s-3)}$ as follows:

$$\delta_{\mathfrak{B}} U_{\alpha(2s-2)} = \mathfrak{B}_{\alpha(2s-2)} + \bar{\mathfrak{B}}_{\alpha(2s-2)}, \quad (5.16b)$$

$$\delta_{\mathfrak{B}} \Sigma_{\alpha(2s-3)} = \bar{\mathcal{D}}^\beta \bar{\mathfrak{B}}_{\beta\alpha(2s-3)} \Rightarrow \delta_{\mathfrak{B}} Z_{\alpha(2s-2)} = \bar{\mathfrak{B}}_{\alpha(2s-2)}. \quad (5.16c)$$

The longitudinal linear superfield defined by (5.2) is invariant under the ζ -transformation (5.16) and varies under the \mathfrak{B} -transformation as

$$\delta_{\mathfrak{B}} G_{\alpha_1\dots\alpha_{2s}} = \bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}_{\alpha_2} \mathfrak{B}_{\alpha_3\dots\alpha_{2s})}. \quad (5.17)$$

The gauge-invariant action is given by

$$\begin{aligned}
S_{(s)}^{\parallel} = & \left(-\frac{1}{2} \right)^s \int d^3|4 z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^{\beta} (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_{\beta} U_{\alpha(2s-2)} + \frac{s}{2s+1} U^{\alpha(2s-2)} (\mathcal{D}^{\beta} \bar{\mathcal{D}}^{\gamma} G_{\beta\gamma\alpha(2s-2)} - \bar{\mathcal{D}}^{\beta} \mathcal{D}^{\gamma} \bar{G}_{\beta\gamma\alpha(2s-2)}) \right. \\
& + 2s(s+1) \bar{\mu} \mu U^{\alpha(2s-2)} U_{\alpha(2s-2)} + \frac{s}{2s-1} \bar{G}^{\alpha(2s)} G_{\alpha(2s)} + \frac{s}{2(2s+1)} (G^{\alpha(2s)} G_{\alpha(2s)} + \bar{G}^{\alpha(2s)} \bar{G}_{\alpha(2s)}) \\
& + \frac{1}{2} \frac{s-1}{2s-1} U^{\alpha(2s-2)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}^2 \bar{\Sigma}_{\alpha_2 \dots \alpha_{2s-2}} - \bar{\mathcal{D}}_{\alpha_1} \mathcal{D}^2 \Sigma_{\alpha_2 \dots \alpha_{2s-2}}) + \frac{1}{2s-1} \Psi^{\alpha(2s-1)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} - 2i(s-1) \mathcal{D}_{\alpha_1 \alpha_2}) \Sigma_{\alpha_3 \dots \alpha_{2s-1}} \\
& + \frac{1}{2s-1} \bar{\Psi}^{\alpha(2s-1)} (\bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} - 2i(s-1) \bar{\mathcal{D}}_{\alpha_1 \alpha_2}) \bar{\Sigma}_{\alpha_3 \dots \alpha_{2s-1}} - \mu(s+3) U^{\alpha(2s-2)} \mathcal{D}_{\alpha_1} \bar{\Sigma}_{\alpha_2 \dots \alpha_{2s-2}} \\
& + \bar{\mu}(s+3) U^{\alpha(2s-2)} \bar{\mathcal{D}}_{\alpha_1} \Sigma_{\alpha_2 \dots \alpha_{2s-2}} + \frac{s-1}{4(2s-1)} (\Sigma^{\alpha(2s-3)} \mathcal{D}^2 \Sigma_{\alpha(2s-3)} - \bar{\Sigma}^{\alpha(2s-3)} \bar{\mathcal{D}}^2 \bar{\Sigma}_{\alpha(2s-3)}) \\
& - \frac{1}{2s-1} \bar{\Sigma}^{\alpha(2s-3)} \left((2s^2 - s + 1) \mathcal{D}^{\beta} \bar{\mathcal{D}}_{\alpha_1} + 2i \frac{(s-1)(2s-3)}{2s-1} \mathcal{D}^{\beta}{}_{\alpha_1} \right) \Sigma_{\beta \alpha_2 \dots \alpha_{2s-3}} \\
& \left. + \mu(s+3) \bar{\Sigma}^{\alpha(2s-3)} \bar{\Sigma}_{\alpha(2s-3)} + \bar{\mu}(s+3) \Sigma_{\alpha(2s-3)} \Sigma^{\alpha(2s-3)} \right\}, \tag{5.18}
\end{aligned}$$

with gauge symmetries (5.16) and, by construction, (5.15). The above action is real due to the identity (A12).

The \mathfrak{B} -gauge freedom (5.16) allows us to gauge away $\Sigma_{\alpha(2s-3)}$,

$$\Sigma_{\alpha(2s-3)} = 0. \tag{5.19}$$

In this gauge, the action (5.18) reduces to that describing the longitudinal formulation for the massless superspin- s multiplet (5.6). The gauge condition (5.19) does not fix completely the \mathfrak{B} -gauge freedom. The residual gauge transformations are generated by

$$\mathfrak{B}_{\alpha(2s-2)} = \mathcal{D}^{\beta} L_{(\beta \alpha_1 \dots \alpha_{2s-2})}, \tag{5.20}$$

with $L_{\alpha(2s-1)}$ being an unconstrained superfield. With this expression for $\mathfrak{B}_{\alpha(2s-2)}$, the gauge transformations (5.16a) and (5.16b) coincide with (5.4b). Thus, the action (5.18) indeed provides an off-shell formulation for the massless superspin- s multiplet in (1,1) AdS superspace.

The action (5.18) includes a single term which involves the “naked” gauge field $\bar{\Psi}_{\alpha(2s-1)}$ and not the field strength $\bar{G}_{\alpha(2s)}$, the latter being defined by (5.2) and invariant under the ζ -transformation (5.16). This is actually a BF term, for it can be written in two different forms

$$\begin{aligned}
& \int d^3|4 z E \bar{\Psi}^{\alpha(2s-1)} (\bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} - 2i(s-1) \mathcal{D}_{\alpha_1 \alpha_2}) \Sigma_{\alpha_3 \dots \alpha_{2s-1}} \\
& = -\frac{2s}{2s+1} \int d^3|4 z E \bar{G}^{\alpha(2s)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} + i(2s+1) \mathcal{D}_{\alpha_1 \alpha_2}) \bar{Z}_{\alpha_3 \dots \alpha_{2s}}. \tag{5.21}
\end{aligned}$$

The former makes the gauge symmetry (5.15) manifestly realized, while the latter turns the ζ -transformation (5.16a) into a manifest symmetry.

Making use of (5.21) leads to a different representation for the action (5.18). It is

$$\begin{aligned}
S_{(s)}^{\parallel} = & \left(-\frac{1}{2} \right)^s \int d^3|4 z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^{\beta} (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_{\beta} U_{\alpha(2s-2)} + \frac{s}{2s+1} U^{\alpha(2s-2)} (\mathcal{D}^{\beta} \bar{\mathcal{D}}^{\gamma} G_{\beta\gamma\alpha(2s-2)} - \bar{\mathcal{D}}^{\beta} \mathcal{D}^{\gamma} \bar{G}_{\beta\gamma\alpha(2s-2)}) \right. \\
& + 2s(s+1) \bar{\mu} \mu U^{\alpha(2s-2)} U_{\alpha(2s-2)} + \frac{s}{2s-1} \bar{G}^{\alpha(2s)} G_{\alpha(2s)} + \frac{s}{2(2s+1)} (G^{\alpha(2s)} G_{\alpha(2s)} + \bar{G}^{\alpha(2s)} \bar{G}_{\alpha(2s)}) \\
& + \frac{1}{2} \frac{s-1}{2s-1} U^{\alpha(2s-2)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}^2 \bar{\Sigma}_{\alpha_2 \dots \alpha_{2s-2}} - \bar{\mathcal{D}}_{\alpha_1} \mathcal{D}^2 \Sigma_{\alpha_2 \dots \alpha_{2s-2}}) + \frac{2s}{(2s-1)(2s+1)} G^{\alpha(2s)} (\bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} + i(2s+1) \mathcal{D}_{\alpha_1 \alpha_2}) Z_{\alpha_3 \dots \alpha_{2s}} \\
& - \frac{2s}{(2s-1)(2s+1)} \bar{G}^{\alpha(2s)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} + i(2s+1) \mathcal{D}_{\alpha_1 \alpha_2}) \bar{Z}_{\alpha_3 \dots \alpha_{2s}} - \mu(s+3) U^{\alpha(2s-2)} \mathcal{D}_{\alpha_1} \bar{\Sigma}_{\alpha_2 \dots \alpha_{2s-2}} \\
& + \bar{\mu}(s+3) U^{\alpha(2s-2)} \bar{\mathcal{D}}_{\alpha_1} \Sigma_{\alpha_2 \dots \alpha_{2s-2}} + \frac{s-1}{4(2s-1)} (\Sigma^{\alpha(2s-3)} \mathcal{D}^2 \Sigma_{\alpha(2s-3)} - \bar{\Sigma}^{\alpha(2s-3)} \bar{\mathcal{D}}^2 \bar{\Sigma}_{\alpha(2s-3)}) \\
& - \frac{1}{2s-1} \bar{\Sigma}^{\alpha(2s-3)} \left((2s^2 - s + 1) \mathcal{D}^{\beta} \bar{\mathcal{D}}_{\alpha_1} + 2i \frac{(s-1)(2s-3)}{2s-1} \mathcal{D}^{\beta}{}_{\alpha_1} \right) \Sigma_{\beta \alpha_2 \dots \alpha_{2s-3}} \\
& \left. + \mu(s+3) \bar{\Sigma}^{\alpha(2s-3)} \bar{\Sigma}_{\alpha(2s-3)} + \bar{\mu}(s+3) \Sigma_{\alpha(2s-3)} \Sigma^{\alpha(2s-3)} \right\}. \tag{5.22}
\end{aligned}$$

Before concluding this section, it is worth discussing the structure of the dynamical variable $\Psi_{\alpha(2s-1)}$. This superfield is unconstrained complex, and its gauge transformation law is given by Eq. (5.16a). Comparing (5.16a) with the gauge transformation law (2.28) $n = 2s - 1$, which corresponds to the conformal gauge superfield $\mathfrak{S}_{\alpha(2s-1)}$, we see that $\Psi_{\alpha(2s-1)}$ may be interpreted as a complex conformal gauge superfield.

D. Massless gravitino multiplet

The massless gravitino multiplet, which corresponds to the $s = 1$ case, was excluded from our consideration of the previous subsection. Here we will fill the gap.

The (generalized) longitudinal formulation for the gravitino multiplet is described by the action

$$\begin{aligned}
 S_{\text{GM}}^{\parallel} = & -\frac{1}{2} \int d^3|4zE \left\{ \frac{1}{8} U \mathcal{D}^{\beta} (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_{\beta} U \right. \\
 & + \frac{1}{3} U (\mathcal{D}^{\alpha} \bar{\mathcal{D}}^{\beta} G_{\alpha\beta} - \bar{\mathcal{D}}^{\alpha} \mathcal{D}^{\beta} \bar{G}_{\alpha\beta}) + \bar{G}^{\alpha\beta} G_{\alpha\beta} \\
 & + \frac{1}{6} (G^{\alpha\beta} G_{\alpha\beta} + \bar{G}^{\alpha\beta} \bar{G}_{\alpha\beta}) + |\mu|^2 \left(2U - \frac{\Phi}{\mu} - \frac{\bar{\Phi}}{\bar{\mu}} \right)^2 \\
 & \left. + 2 \left(\frac{\Phi}{\mu} + \frac{\bar{\Phi}}{\bar{\mu}} \right) (\mu \mathcal{D}^{\alpha} \Psi_{\alpha} + \bar{\mu} \bar{\mathcal{D}}_{\alpha} \bar{\Psi}^{\alpha}) \right\}, \quad (5.23)
 \end{aligned}$$

where Φ is a covariantly chiral scalar superfield, $\bar{\mathcal{D}}_{\alpha} \Phi = 0$, and

$$G_{\alpha\beta} = \bar{\mathcal{D}}_{(\alpha} \Psi_{\beta)}, \quad \bar{G}_{\alpha\beta} = -\mathcal{D}_{(\alpha} \bar{\Psi}_{\beta)}. \quad (5.24)$$

This action is invariant under gauge transformations of the form

$$\delta U = \mathfrak{B} + \bar{\mathfrak{B}}, \quad (5.25a)$$

$$\delta \Psi_{\alpha} = \mathcal{D}_{\alpha} \mathfrak{B} + \bar{\mathcal{D}}_{\alpha} \zeta, \quad (5.25b)$$

$$\delta \Phi = -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4\mu) \bar{\mathfrak{B}}, \quad (5.25c)$$

where the gauge parameters \mathfrak{B} and ζ are unconstrained complex superfields.

The gauge \mathfrak{B} -freedom (5.25) allows us to impose the condition $\Phi = 0$. In this gauge the action (5.23) turns into (5.6) with $s = 1$, and the residual gauge \mathfrak{B} -freedom is described by $\mathfrak{B} = \mathcal{D}^{\beta} L_{\beta}$, where the spinor gauge parameter L_{α} is unconstrained complex.

The action (5.23) involves the chiral scalar Φ and its conjugate only in the combination $(\varphi + \bar{\varphi})$, where $\varphi = \Phi/\mu$. This means that the model (5.23) possesses a dual formulation realized in terms of a real linear superfield subject to the constraint (2.22).

VI. HIGHER-SPIN SUPERCURRENTS IN (1,1) ADS SUPERSPACE

Inspired by the analysis of Dumitrescu and Seiberg [42], the most general supercurrent multiplets for theories with (1,1) AdS or (2,0) AdS supersymmetry were introduced in [15], with the (1,1) AdS case being a natural extension of the 4D $\mathcal{N} = 1$ AdS supercurrents classified in [3,43]. Here we will formulate higher-spin supercurrents in (1,1) AdS superspace by making use of the off-shell formulations for massless supersymmetric higher-spin gauge theories in (1,1) AdS superspace, which have been constructed in the previous two sections. Our analysis will be analogous to the one recently given in the 4D $\mathcal{N} = 1$ case [29].

A. Nonconformal supercurrents: Half-integer superspin

The two off-shell formulations for the massless super-spin- $(s + \frac{1}{2})$ multiplet, which we reviewed in Secs. IV A and IV B, lead to different higher-spin supercurrent multiplets. In this subsection we first describe the explicit structure of these supermultiplets and then show how they are related to each other.

1. Longitudinal supercurrent

In the framework of the longitudinal formulation (4.15), let us couple the prepotentials $\mathfrak{S}_{\alpha(2s)}$, $\Psi_{\alpha(2s-3)}$ and $\bar{\Psi}_{\alpha(2s-3)}$, to external sources

$$\begin{aligned}
 S_{\text{source}}^{(s+\frac{1}{2})} = & \int d^3|4zE \{ \mathfrak{S}^{\alpha(2s)} J_{\alpha(2s)} + \Psi^{\alpha(2s-3)} T_{\alpha(2s-3)} \\
 & + \bar{\Psi}_{\alpha(2s-3)} \bar{T}^{\alpha(2s-3)} \}. \quad (6.1)
 \end{aligned}$$

Requiring $S_{\text{source}}^{(s+\frac{1}{2})}$ to be invariant under (4.11) gives

$$\bar{\mathcal{D}}^{\beta} T_{\beta\alpha(2s-4)} = 0, \quad (6.2a)$$

and therefore $T_{\alpha(2s-3)}$ is a transverse linear superfield.

Requiring $S_{\text{source}}^{(s+\frac{1}{2})}$ to be invariant under the gauge transformations (4.4a) and (4.14) gives the following conservation equation:

$$\bar{\mathcal{D}}^{\beta} J_{\beta\alpha(2s-1)} + \frac{1}{2} (\mathcal{D}_{(\alpha_1} \bar{\mathcal{D}}_{\alpha_2} - 2i(s-1) \mathcal{D}_{(\alpha_1\alpha_2)}) T_{\alpha_3 \dots \alpha_{2s-1}}) = 0. \quad (6.2b)$$

For completeness, we also give the conjugate equation

$$\mathcal{D}^{\beta} J_{\beta\alpha(2s-1)} - \frac{1}{2} (\bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}_{\alpha_2} - 2i(s-1) \bar{\mathcal{D}}_{(\alpha_1\alpha_2)}) \bar{T}_{\alpha_3 \dots \alpha_{2s-1}}) = 0. \quad (6.2c)$$

As in [29], it is useful to introduce auxiliary real variables ζ^α . Given a tensor superfield $U_{\alpha(m)}$, we associate with it the following field:

$$U_{(m)}(\zeta) := \zeta^{\alpha_1} \dots \zeta^{\alpha_m} U_{\alpha_1 \dots \alpha_m}, \quad (6.3)$$

which is homogeneous of degree m in the variables ζ^α . We introduce operators that increase the degree of homogeneity in the variable ζ^α ,

$$\mathcal{D}_{(1)} := \zeta^\alpha \mathcal{D}_\alpha, \quad (6.4a)$$

$$\bar{\mathcal{D}}_{(1)} := \zeta^\alpha \bar{\mathcal{D}}_\alpha, \quad (6.4b)$$

$$\mathcal{D}_{(2)} := i\zeta^\alpha \zeta^{\beta} \mathcal{D}_{\alpha\beta} = -\frac{1}{2} \{ \mathcal{D}_{(1)}, \bar{\mathcal{D}}_{(1)} \}. \quad (6.4c)$$

We also introduce two operators that decrease the degree of homogeneity in the variable ζ^α , specifically

$$\mathcal{D}_{(-1)} := \mathcal{D}^\alpha \frac{\partial}{\partial \zeta^\alpha}, \quad (6.5a)$$

$$\bar{\mathcal{D}}_{(-1)} := \bar{\mathcal{D}}^\alpha \frac{\partial}{\partial \zeta^\alpha}. \quad (6.5b)$$

Making use of the above notation, the transverse linear condition (6.2) and its conjugate become

$$\bar{\mathcal{D}}_{(-1)} T_{(2s-3)} = 0, \quad (6.6a)$$

$$\mathcal{D}_{(-1)} \bar{T}_{(2s-3)} = 0. \quad (6.6b)$$

The conservation equations (6.2b) and (6.2c) turn into

$$\frac{1}{2s} \bar{\mathcal{D}}_{(-1)} J_{(2s)} - \frac{1}{2} A_{(2)} T_{(2s-3)} = 0, \quad (6.7a)$$

$$\frac{1}{2s} \mathcal{D}_{(-1)} J_{(2s)} - \frac{1}{2} \bar{A}_{(2)} \bar{T}_{(2s-3)} = 0. \quad (6.7b)$$

where

$$\begin{aligned} A_{(2)} &:= -\mathcal{D}_{(1)} \bar{\mathcal{D}}_{(1)} + 2(s-1) \mathcal{D}_{(2)}, \\ \bar{A}_{(2)} &:= \bar{\mathcal{D}}_{(1)} \mathcal{D}_{(1)} - 2(s-1) \mathcal{D}_{(2)}. \end{aligned} \quad (6.8)$$

Since $(\bar{\mathcal{D}}_{(-1)})^2 J_{(2s)} = 0$, the conservation equation (6.7a) is consistent provided

$$\bar{\mathcal{D}}_{(-1)} A_{(2)} T_{(2s-3)} = 0. \quad (6.9)$$

This is indeed true, as a consequence of the transverse linear condition (6.6a).

2. Transverse supercurrent

One can also make use of the transverse formulation (4.6) and couple the prepotentials $\mathfrak{H}_{\alpha(2s)}$, $\Phi_{\alpha(2s-1)}$ and $\bar{\Phi}_{\alpha(2s-1)}$ to external sources

$$\begin{aligned} S_{\text{source}}^{(s+\frac{1}{2}),tr} &= \int d^3|4_z E \{ \mathfrak{H}^{\alpha(2s)} \mathbb{J}_{\alpha(2s)} + \Phi_{\alpha(2s-1)} \bar{\mathbb{F}}^{\alpha(2s-1)} \\ &\quad + \bar{\Phi}^{\alpha(2s-1)} \mathbb{F}_{\alpha(2s-1)} \}. \end{aligned} \quad (6.10)$$

Requiring that the action (6.10) be invariant under the gauge transformations (4.4a), (4.5), and (4.3) leads to the following conditions on the transverse supercurrent multiplet:

$$\bar{\mathcal{D}}_{(\alpha_1} \bar{\mathbb{F}}_{\alpha_2 \dots \alpha_{2s})} = 0, \quad (6.11a)$$

$$\bar{\mathcal{D}}^\beta \mathbb{J}_{\beta\alpha(2s-1)} - \frac{1}{4} (\bar{\mathcal{D}}^2 + 2\mu(2s-1)) \mathbb{F}_{\alpha(2s-1)} = 0. \quad (6.11b)$$

Thus, the trace multiplet $\bar{\mathbb{F}}_{\alpha(2s-1)}$ is longitudinal linear.

3. Improvement transformation

We now construct a well-defined improvement transformation which converts the higher-spin supercurrent (6.2) to (6.11), thus showing that they are indeed equivalent.

The transverse linearity condition (6.2a) implies that there exists a well-defined complex tensor operator $X_{\alpha(2s-2)}$ such that

$$T_{\alpha(2s-3)} = \bar{\mathcal{D}}^\beta X_{\beta\alpha(2s-3)}. \quad (6.12)$$

Let us split $X_{\alpha(2s-2)}$ into its real and imaginary parts,

$$X_{\alpha(2s-2)} = U_{\alpha(2s-2)} + iV_{\alpha(2s-2)}. \quad (6.13)$$

Then one may check that the operators

$$\begin{aligned} \mathbb{J}_{\alpha(2s)} &:= J_{\alpha(2s)} + \frac{s}{2} [\mathcal{D}_{(\alpha_1}, \bar{\mathcal{D}}_{\alpha_2}] U_{\alpha_3 \dots \alpha_{2s}} + s \mathcal{D}_{(\alpha_1 \alpha_2} V_{\alpha_3 \dots \alpha_{2s}}), \\ &\quad (6.14a) \end{aligned}$$

$$\mathbb{F}_{\alpha(2s-1)} := \mathcal{D}_{(\alpha_1} \{ 2s U_{\alpha_2 \dots \alpha_{2s-1}} - iV_{\alpha_2 \dots \alpha_{2s-1}} \} \quad (6.14b)$$

satisfy the conservation equation (6.11b) and the longitudinal linear condition (6.11a).

The improvement transformation (6.14) turns the higher-spin supercurrent (6.2) to (6.11). It is also not difficult to construct an inverse improvement transformation converting the higher-spin supercurrent (6.11) to (6.2). Therefore the higher-spin supercurrents (6.2) and (6.11) are equivalent, and it suffices to work with one of them, say, the longitudinal supermultiplet (6.2). The situation proves to be analogous in the integer superspin case, for which we

will formulate in the next subsection a higher-spin supercurrent associated with the new gauge formulation (5.18).

B. Nonconformal supercurrents: Integer superspin

We now make use of the new gauge formulation (5.18), or equivalently (5.22), for the integer superspin- s multiplet to derive the 3D analog of the nonconformal higher-spin supercurrents proposed in [29].

Let us couple the prepotentials $U_{\alpha(2s-2)}$, $Z_{\alpha(2s-2)}$ and $\Psi_{\alpha(2s-1)}$ to external sources

$$\begin{aligned} \mathcal{S}_{\text{source}}^{(s)} = & \int d^3{}^4z E \{ \Psi^{\alpha(2s-1)} J_{\alpha(2s-1)} - \bar{\Psi}^{\alpha(2s-1)} \bar{J}_{\alpha(2s-1)} \\ & + U^{\alpha(2s-2)} S_{\alpha(2s-2)} + Z^{\alpha(2s-2)} T_{\alpha(2s-2)} \\ & + \bar{Z}^{\alpha(2s-2)} \bar{T}_{\alpha(2s-2)} \}. \end{aligned} \quad (6.15)$$

In order for $\mathcal{S}_{\text{source}}^{(s)}$ to be invariant under the ζ -transformation in (5.16a), the source $J_{\alpha(2s-1)}$ must satisfy

$$\bar{\mathcal{D}}^\beta J_{\beta\alpha(2s-2)} = 0 \Leftrightarrow \mathcal{D}^\beta \bar{J}_{\beta\alpha(2s-2)} = 0. \quad (6.16)$$

Next, requiring $\mathcal{S}_{\text{source}}^{(s)}$ to be invariant under the transformation (5.15) leads to

$$\bar{\mathcal{D}}_{(\alpha_1} T_{\alpha_2 \dots \alpha_{2s-1})} = 0 \Leftrightarrow \mathcal{D}_{(\alpha_1} \bar{T}_{\alpha_2 \dots \alpha_{2s-1})} = 0. \quad (6.17)$$

We see that the superfields $J_{\alpha(2s-1)}$ and $T_{\alpha(2s-2)}$ are transverse linear and longitudinal linear, respectively. Finally, requiring $\mathcal{S}_{\text{source}}^{(s)}$ to be invariant under the \mathfrak{B} -transformation (5.16) gives the following conservation equation:

$$-\mathcal{D}^\beta J_{\beta\alpha(2s-2)} + S_{\alpha(2s-2)} + \bar{T}_{\alpha(2s-2)} = 0 \quad (6.18a)$$

as well as its conjugate

$$\bar{\mathcal{D}}^\beta \bar{J}_{\beta\alpha(2s-2)} + S_{\alpha(2s-2)} + T_{\alpha(2s-2)} = 0. \quad (6.18b)$$

Taking the sum of (6.18a) and (6.18b) leads to

$$\mathcal{D}^\beta J_{\beta\alpha(2s-2)} + \bar{\mathcal{D}}^\beta \bar{J}_{\beta\alpha(2s-2)} + T_{\alpha(2s-2)} - \bar{T}_{\alpha(2s-2)} = 0. \quad (6.19)$$

As a consequence of (6.17), the conservation equation (6.19) implies

$$\mathcal{D}_{(\alpha_1} \{ \mathcal{D}^{|\beta|} J_{\alpha_2 \dots \alpha_{2s-1})\beta} + \bar{\mathcal{D}}^\beta \bar{J}_{\alpha_2 \dots \alpha_{2s-1})\beta} \} + \mathcal{D}_{(\alpha_1} T_{\alpha_2 \dots \alpha_{2s-1})} = 0. \quad (6.20)$$

Using our notation introduced in the previous subsection, the transverse linear condition (6.16) turns into

$$\bar{\mathcal{D}}_{(-1)} J_{(2s-1)} = 0, \quad (6.21)$$

while the longitudinal linear condition (6.17) takes the form

$$\bar{\mathcal{D}}_{(1)} T_{(2s-2)} = 0. \quad (6.22)$$

The conservation equation (6.18a) becomes

$$-\frac{1}{(2s-1)} \mathcal{D}_{(-1)} J_{(2s-1)} + S_{(2s-2)} + \bar{T}_{(2s-2)} = 0 \quad (6.23)$$

and (6.20) takes the form

$$\frac{1}{(2s-1)} \mathcal{D}_{(1)} \{ \mathcal{D}_{(-1)} J_{(2s-1)} + \bar{\mathcal{D}}_{(-1)} \bar{J}_{(2s-1)} \} + \mathcal{D}_{(1)} T_{(2s-2)} = 0. \quad (6.24)$$

VII. HIGHER-SPIN SUPERCURRENTS FOR CHIRAL MATTER IN (1,1) ADS SUPERSPACE: HALF-INTEGERSUPERSPIN

In the remainder of this paper we will study explicit realizations of the higher-spin supercurrents introduced above in supersymmetric field theories in AdS.

A. Superconformal model for a chiral superfield

Let us consider the superconformal theory of a single chiral scalar superfield

$$S = \int d^3{}^4z E \bar{\Phi} \Phi, \quad (7.1)$$

where Φ is covariantly chiral, $\bar{\mathcal{D}}_\alpha \Phi = 0$. We construct the following conformal supercurrent $J_{(2s)}$, which is a minimal extension of the conserved supercurrent constructed in flat $\mathcal{N} = 2$ Minkowski superspace [44]:

$$\begin{aligned} J_{(2s)} = & \sum_{k=0}^s (-1)^k \left\{ \frac{1}{2} \binom{2s}{2k+1} \mathcal{D}_{(2)}^k \bar{\mathcal{D}}_{(1)} \bar{\Phi} \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi \right. \\ & \left. + \binom{2s}{2k} \mathcal{D}_{(2)}^k \bar{\Phi} \mathcal{D}_{(2)}^{s-k} \Phi \right\}. \end{aligned} \quad (7.2)$$

Making use of the massless equations of motion, $(\mathcal{D}^2 - 4\bar{\mu})\Phi = 0$, one may check that $J_{(2s)}$ satisfies the conservation equation

$$\mathcal{D}_{(-1)} J_{(2s)} = 0 \Leftrightarrow \bar{\mathcal{D}}_{(-1)} J_{(2s)} = 0. \quad (7.3)$$

The calculation of (7.3) in AdS is much more complicated than in flat superspace due to the fact that the algebra of covariant derivatives (3.1) is nontrivial. Let us sketch the main steps in evaluating the left-hand side of Eq. (7.3) with $J_{(2s)}$ given by (7.2). We start with the obvious relations

$$\frac{\partial}{\partial \zeta^\alpha} \mathcal{D}_{(2)} = 2i\zeta^\beta \mathcal{D}_{\alpha\beta}, \quad (7.4a)$$

$$\frac{\partial}{\partial \zeta^\alpha} \mathcal{D}_{(2)}^k = \sum_{n=1}^k \mathcal{D}_{(2)}^{n-1} 2i\zeta^\beta \mathcal{D}_{\alpha\beta} \mathcal{D}_{(2)}^{k-n}, \quad k > 1. \quad (7.4b)$$

To simplify Eq. (7.4b), we may push $\zeta^\beta \mathcal{D}_{\alpha\beta}$, say, to the left provided that we take into account its commutator with $\mathcal{D}_{(2)}$:

$$[\zeta^\beta \mathcal{D}_{\alpha\beta}, \mathcal{D}_{(2)}] = -4i\bar{\mu}\mu\zeta_\alpha \zeta^\beta \zeta^\gamma M_{\beta\gamma}. \quad (7.5)$$

Associated with the Lorentz generators are the operators

$$M_{(2)} := \zeta^\alpha \zeta^\beta M_{\alpha\beta}, \quad (7.6)$$

where $M_{(2)}$ appears in the right-hand side of (7.5). These operators annihilate every superfield $U_{(m)}(\zeta)$ of the form (6.3),

$$M_{(2)} U_{(m)} = 0. \quad (7.7)$$

From the above consideration, it follows that

$$[\zeta^\beta \mathcal{D}_{\alpha\beta}, \mathcal{D}_{(2)}^k] U_{(m)} = 0, \quad (7.8a)$$

$$\left(\frac{\partial}{\partial \zeta^\alpha} \mathcal{D}_{(2)}^k \right) U_{(m)} = 2ik\zeta^\beta \mathcal{D}_{\alpha\beta} \mathcal{D}_{(2)}^{k-1} U_{(m)}. \quad (7.8b)$$

We also state some other properties which we often use throughout our calculations:

$$\mathcal{D}_{(1)}^2 = -2\bar{\mu}M_{(2)}, \quad (7.9a)$$

$$[\mathcal{D}_{(1)}, \mathcal{D}_{(2)}] = [\bar{\mathcal{D}}_{(1)}, \mathcal{D}_{(2)}] = 0, \quad (7.9b)$$

$$[\mathcal{D}^\alpha, \mathcal{D}_{(2)}] = -2\bar{\mu}\zeta^\alpha \bar{\mathcal{D}}_{(1)}, \quad (7.9c)$$

$$[\mathcal{D}^\alpha, \mathcal{D}_{(2)}^k] = -2\bar{\mu}k\zeta^\alpha \mathcal{D}_{(2)}^{k-1} \bar{\mathcal{D}}_{(1)}, \quad (7.9d)$$

$$[\mathcal{D}^\alpha, \zeta^\beta \mathcal{D}_{\alpha\beta}] = 3i\bar{\mu} \bar{\mathcal{D}}_{(1)}. \quad (7.9e)$$

The above identities suffice to prove that the supercurrent (7.2) does obey the conservation equation (7.3).

B. Nonsuperconformal model for a chiral superfield

Let us now add the mass term to (7.1) and consider the following action:

$$S = \int d^3|_4 z E \bar{\Phi} \Phi + \left\{ \frac{1}{2} \int d^3|_4 z E \frac{m}{\mu} \Phi^2 + \text{c.c.} \right\}, \quad (7.10)$$

with m a complex mass parameter. In the massive case $J_{(2s)}$ satisfies a more general conservation equation (6.7b) for some superfield $\bar{T}_{(2s-3)}$. Making use of the equations of motion

$$\begin{aligned} -\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\Phi + \bar{m}\bar{\Phi} &= 0, \\ -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar{\Phi} + m\Phi &= 0, \end{aligned} \quad (7.11)$$

we obtain

$$\mathcal{D}_{(-1)} J_{(2s)} = \bar{F}_{(2s-1)}, \quad (7.12a)$$

where we have denoted

$$\begin{aligned} \bar{F}_{(2s-1)} &= \bar{m}(2s+1) \sum_{k=0}^{s-1} (-1)^k \binom{2s}{2k+1} \\ &\times \left\{ (-1)^s + \frac{2k+1}{2s-2k+1} \right\} \mathcal{D}_{(2)}^k \bar{\Phi} \mathcal{D}_{(2)}^{s-k-1} \bar{\mathcal{D}}_{(1)} \bar{\Phi}. \end{aligned} \quad (7.12b)$$

We now look for a superfield $\bar{T}_{(2s-3)}$ such that (i) it obeys the transverse antilinear constraint (6.6b); and (ii) it satisfies the equation

$$\bar{F}_{(2s-1)} = s\bar{A}_{(2)} \bar{T}_{(2s-3)}. \quad (7.13)$$

Our analysis will be similar to the one performed in [29] in the case of four-dimensional AdS. We consider a general ansatz

$$\bar{T}_{(2s-3)} = \bar{m} \sum_{k=0}^{s-2} c_k \mathcal{D}_{(2)}^k \bar{\Phi} \mathcal{D}_{(2)}^{s-k-2} \bar{\mathcal{D}}_{(1)} \bar{\Phi} \quad (7.14)$$

with some coefficients c_k which have to be determined. For $k = 1, 2, \dots, s-2$, condition (i) implies that the coefficients c_k must satisfy

$$kc_k = (s-k-1)c_{s-k-1}, \quad (7.15a)$$

while (ii) gives the following equation:

$$\begin{aligned} c_{s-k-1} + sc_k + (s-1)c_{k-1} &= -\frac{2s+1}{2s} (-1)^k \binom{2s}{2k+1} \\ &\times \left\{ (-1)^s + \frac{2k+1}{2s-2k+1} \right\}. \end{aligned} \quad (7.15b)$$

Condition (ii) also implies that

$$(s-1)c_{s-2} + c_0 = (2s+1) \left\{ 1 + (-1)^s \frac{2s-1}{3} \right\}, \quad (7.15c)$$

$$c_0 = -\frac{1}{s} (1 + (-1)^s (2s+1)). \quad (7.15d)$$

It turns out that the equations (7.15) lead to a unique expression for c_k given by

$$c_k = (-1)^{s+k-1} \frac{(2s+1)(s-k-1)}{2s(s-1)} \sum_{l=0}^k \frac{1}{s-l} \binom{2s}{2l+1} \times \left\{ 1 + (-1)^s \frac{2l+1}{2s-2l+1} \right\}, \quad k = 0, 1, \dots, s-2. \quad (7.16)$$

If the parameter s is odd, $s = 2n + 1$, with $n = 1, 2, \dots$, one can check that Eqs. (7.15a)–(7.15c) are identically satisfied. However, if the parameter s is even, $s = 2n$, with $n = 1, 2, \dots$, there appears an inconsistency: the right-hand side of (7.15c) is positive, while the left-hand side is negative, $(s-1)c_{s-2} + c_0 < 0$. Therefore, our solution (7.16) is only consistent for $s = 2n + 1, n = 1, 2, \dots$

Relations (7.2), (7.14), (7.15d) and (7.16) determine the nonconformal higher-spin supercurrents in the massive chiral model (7.10). Unlike the conformal higher-spin supercurrents (7.2), the nonconformal ones exist only for the odd values of s , $s = 2n + 1$, with $n = 1, 2, \dots$

C. Superconformal model with N chiral superfields

In this subsection we will generalize the superconformal model (7.1) to the case of N covariantly chiral scalar superfields Φ^i , $i = 1, \dots, N$,

$$S = \int d^3{}^4z E \bar{\mathcal{D}}^i \Phi^i, \quad \bar{\mathcal{D}}_\alpha \Phi^i = 0. \quad (7.17)$$

There exist two different types of conformal supercurrents, which are

$$J_{(2s)}^+ = S^{ij} \sum_{k=0}^s (-1)^k \left\{ \frac{1}{2} \binom{2s}{2k+1} \mathcal{D}_{(2)}^k \bar{\mathcal{D}}_{(1)} \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi^j + \binom{2s}{2k} \mathcal{D}_{(2)}^k \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k} \Phi^j \right\}, \quad S^{ij} = S^{ji} \quad (7.18)$$

and

$$J_{(2s)}^- = iA^{ij} \sum_{k=0}^s (-1)^k \left\{ \frac{1}{2} \binom{2s}{2k+1} \mathcal{D}_{(2)}^k \bar{\mathcal{D}}_{(1)} \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi^j + \binom{2s}{2k} \mathcal{D}_{(2)}^k \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k} \Phi^j \right\}, \quad A^{ij} = -A^{ji} \quad (7.19)$$

Here S and A are arbitrary real symmetric and antisymmetric constant matrices, respectively. We have put an overall factor $\sqrt{-1}$ in Eq. (7.19) in order to make $J_{(2s)}^-$ real. One can show that the currents (7.18) and (7.19) are conserved on-shell:

$$\mathcal{D}_{(-1)} J_{(2s)}^\pm = 0 \Leftrightarrow \bar{\mathcal{D}}_{(-1)} J_{(2s)}^\pm = 0. \quad (7.20)$$

The above results can be recast in terms of the matrix conformal supercurrent $J_{(2s)} = (J_{(2s)}^{ij})$ with components

$$J_{(2s)}^{ij} := \sum_{k=0}^s (-1)^k \left\{ \frac{1}{2} \binom{2s}{2k+1} \mathcal{D}_{(2)}^k \bar{\mathcal{D}}_{(1)} \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi^j + \binom{2s}{2k} \mathcal{D}_{(2)}^k \bar{\Phi}^i \mathcal{D}_{(2)}^{s-k} \Phi^j \right\}, \quad (7.21)$$

which is Hermitian, $J_{(2s)}^\dagger = J_{(2s)}$. The chiral action (7.17) possesses rigid $U(N)$ symmetry acting on the chiral column-vector $\Phi = (\Phi^i)$ by $\Phi \rightarrow g\Phi$, with $g \in U(N)$, which implies that the supercurrent (7.21) transforms as $J_{(2s)} \rightarrow g J_{(2s)} g^{-1}$.

VIII. HIGHER-SPIN SUPERCURRENTS FOR CHIRAL MATTER IN (1,1) ADS SUPERSPACE: INTEGER SUPERSPIN

In this section we provide explicit realizations for the fermionic higher-spin supercurrents (integer superspin) in a model of a single massive chiral scalar superfield.

We start by considering the massive action

$$S = \int d^3{}^4z E \bar{\Psi} \Psi + \left\{ \frac{1}{2} \int d^3{}^4z E \frac{m}{\mu} \Psi^2 + \text{c.c.} \right\}, \quad (8.1)$$

where the superfield Ψ is covariantly chiral, $\bar{\mathcal{D}}_\alpha \Psi = 0$ and m is a complex mass parameter. By a change of variables it is possible to make m real. Let us introduce a new chiral superfield Φ , $\bar{\mathcal{D}}_\alpha \Phi = 0$, related to Ψ by phase transformations,

$$\Phi = e^{i\alpha/2} \Psi, \quad m = M e^{i\alpha}, \quad \bar{M} = M. \quad (8.2)$$

Then the action (8.1) turns into

$$S = \int d^3{}^4z E \bar{\Phi} \Phi + \left\{ \frac{1}{2} \int d^3{}^4z E \frac{M}{\mu} \Phi^2 + \text{c.c.} \right\}. \quad (8.3)$$

We emphasize that the mass parameter M is now real.

In the massless case, $M = 0$, the conserved fermionic supercurrent $J_{\alpha(2s-1)}$ is given by

$$J_{(2s-1)} = \sum_{k=0}^{s-1} (-1)^k \left\{ \binom{2s-1}{2k+1} \mathcal{D}_{(2)}^k \mathcal{D}_{(1)} \Phi \mathcal{D}_{(2)}^{s-k-1} \Phi - \binom{2s-1}{2k} \mathcal{D}_{(2)}^k \Phi \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi \right\}. \quad (8.4)$$

By changing the summation index in (8.4), it is not difficult to see that $J_{(2s-1)}$ is zero for odd values of s . Making use of the massless equations of motion, $-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\Phi = 0$, one may check that $J_{(2s-1)}$ obeys, for $s > 1$, the conservation equations

$$\mathcal{D}_{(-1)} J_{(2s-1)} = 0, \quad \bar{\mathcal{D}}_{(-1)} J_{(2s-1)} = 0. \quad (8.5)$$

We will now construct fermionic higher-spin supercurrents corresponding to the massive model (8.3). Making use of the massive equation of motion

$$-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\Phi + M\bar{\Phi} = 0, \quad (8.6)$$

we obtain

$$\mathcal{D}_{(-1)} J_{(2s-1)} = 8Ms \sum_{k=0}^{s-1} (-1)^{k+1} \binom{2s-1}{2k} \left\{ \mathcal{D}_{(2)}^k \Phi \mathcal{D}_{(2)}^{s-k-1} \bar{\Phi} + \frac{k}{2k+1} \mathcal{D}_{(2)}^{k-1} \bar{\mathcal{D}}_{(1)} \bar{\Phi} \mathcal{D}_{(2)}^{s-k-1} \mathcal{D}_{(1)} \Phi \right\}. \quad (8.7)$$

It can be shown that the massive supercurrent $J_{(2s-1)}$ also obeys (6.21).

We now look for a superfield $T_{\alpha(2s-2)}$ such that (i) it obeys the longitudinal linear constraint (6.22), and (ii) it satisfies (6.24), which is a consequence of the conservation equation (6.23). For this we consider a general ansatz

$$T_{(2s-2)} = \sum_{k=0}^{s-1} c_k \mathcal{D}_{(2)}^k \Phi \mathcal{D}_{(2)}^{s-k-1} \bar{\Phi} + \sum_{k=1}^{s-1} d_k \mathcal{D}_{(2)}^{k-1} \mathcal{D}_{(1)} \Phi \mathcal{D}_{(2)}^{s-k-1} \bar{\mathcal{D}}_{(1)} \bar{\Phi}. \quad (8.8)$$

Condition (i) implies that the coefficients must be related by

$$c_0 = 0, \quad c_k = 2d_k, \quad (8.9a)$$

while for $k = 1, 2, \dots, s-2$, condition (ii) gives the following recurrence relations:

$$d_k + d_{k+1} = -\frac{8Ms}{2s-1} (-1)^{k+1} \binom{2s-1}{2k} \frac{4ks + 3s - 1 - 2s^2}{(2k+1)(2k+3)}. \quad (8.9b)$$

Condition (ii) also implies that

$$d_1 = -\frac{8}{3} Ms(s-1), \quad d_{s-1} = -\frac{8}{2s-1} Ms(s-1). \quad (8.9c)$$

The above conditions lead to a simple expression for d_k :

$$d_k = \frac{8Ms}{2s-1} \frac{k}{2k+1} (-1)^k \binom{2s-1}{2k}, \quad (8.10)$$

where $k = 1, 2, \dots, s-1$ and the parameter s is even for $J_{(2s-1)}$ to be nonzero.

IX. CONCLUDING COMMENTS

Let us give a succinct coherent summary of the results obtained in this paper. In Sec. II B we proposed the families of primary linear supermultiplets coupled to $\mathcal{N} = 2$ conformal supergravity and used them to construct topological BF -type terms. In Sec. II C we introduced the conformal higher-spin gauge superfields coupled to $\mathcal{N} = 2$ conformal supergravity. In Sec. II D we constructed the linearized conformal higher-spin supergravity actions in arbitrary conformally flat backgrounds. The results in Secs. IV–VIII concern supersymmetric field theories in (1,1) AdS superspace. In Secs. IV and V we constructed the massless higher-spin supermultiplets in (1,1) AdS superspace. Making use of these massless theories, in Sec. VI we formulated consistent higher-spin supercurrent multiplets in (1,1) AdS superspace. In Secs. VII and VIII we provided the explicit examples of these supercurrent multiplets for models of massive chiral supermultiplets.

The constructions presented in this paper have several interesting extensions, some of which are briefly discussed below.

Our results can be used to construct off-shell formulations for massive higher-spin supermultiplets in (1,1) AdS superspace.⁴ This is readily achieved in the case of a half-integer superspin by considering two dually equivalent gauge-invariant actions

$$S_{\text{massive}}^{\perp} = \kappa S_{\text{SCS}}[\mathfrak{H}_{(2s)}] + m^{2s-1} S_{(s+\frac{1}{2})}^{\perp}[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}], \quad (9.1a)$$

$$S_{\text{massive}}^{\parallel} = \kappa S_{\text{SCS}}[\mathfrak{H}_{(2s)}] + m^{2s-1} S_{(s+\frac{1}{2})}^{\parallel}[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}]. \quad (9.1b)$$

Here the parameter κ is dimensionless, while m has dimension of mass. The superconformal action $S_{\text{SCS}}[\mathfrak{H}_{(2s)}]$ is obtained

⁴Two different Lagrangian formulations for massive higher-spin $\mathcal{N} = 1$ supermultiplets in AdS₃ were developed in [25,45].

from (2.34) by setting $n = 2s$. The massless actions $S_{(s+\frac{1}{2})}^\perp$ and $S_{(s+\frac{1}{2})}^\parallel$ are given by Eqs. (4.6) and (4.15), respectively. In the flat-superspace limit, the actions (9.1a) and (9.1b) reduce to those proposed in [18].

We expect that the equations of motion in the topologically massive models (9.1a) and (9.1b) describe a subclass of the irreducible on-shell massive supermultiplets in (1,1) AdS superspace proposed in [39]. This is indeed the case in Minkowski superspace, as demonstrated in [18]. However, analysis of the equations of motion in (1,1) AdS superspace is more complicated since we still do not have a closed-form expression for the higher-spin super-Cotton tensor $\mathfrak{W}_{\alpha(n)}$, Eq. (2.43), in terms of the prepotential $\mathfrak{S}_{\alpha(n)}$ and the covariant derivatives \mathcal{D}_A of (1,1) AdS superspace. Here we simply recall the explicit structure of irreducible on-shell massive higher-spin supermultiplets in (1,1) AdS superspace [39]. Given a positive integer $n > 0$, such a supermultiplet is realized in terms of a real symmetric rank- n spinor $T_{\alpha(n)}$ constrained by

$$\mathcal{D}^\beta T_{\alpha_1 \dots \alpha_{n-1} \beta} = \bar{\mathcal{D}}^\beta T_{\alpha_1 \dots \alpha_{n-1} \beta} = 0, \quad (9.2a)$$

$$\left(\frac{i}{2} \mathcal{D}^\gamma \bar{\mathcal{D}}_\gamma + m \right) T_{\alpha_1 \dots \alpha_n} = 0. \quad (9.2b)$$

It can be shown that

$$\left(\frac{i}{2} \mathcal{D}^\gamma \bar{\mathcal{D}}_\gamma \right)^2 T_{\alpha_1 \dots \alpha_n} = (\mathcal{D}^a \mathcal{D}_a + 2(n+2)|\mu|^2) T_{\alpha_1 \dots \alpha_n}. \quad (9.3)$$

New duality transformations were introduced in [46] for theories formulated in terms of the linearized higher-spin super-Cotton tensor $W_{\alpha(n)}$ in Minkowski superspace, Eq. (2.36). These duality transformations can readily be generalized to arbitrary conformally flat backgrounds by replacing $W_{\alpha(n)}$ with $\mathfrak{W}_{\alpha(n)}$ given by Eq. (2.43).

It is worth studying in more detail the higher-derivative Chern-Simons theory (2.34) on conformally flat superspace backgrounds. It is a reducible gauge theory (following the terminology of the Batalin-Vilkovisky quantization [47]) since one and the same gauge transformation (2.28) is generated by two gauge parameters, $\lambda_{\alpha(n-1)}$ and $\tilde{\lambda}_{\alpha(n-1)}$, such that their difference is longitudinal linear,

$$\delta_\lambda \mathfrak{S}_{\alpha(n)} = \delta_{\tilde{\lambda}} \mathfrak{S}_{\alpha(n)}, \quad \tilde{\lambda}_{\alpha(n-1)} := \lambda_{\alpha(n-1)} + \bar{\mathcal{D}}_{(\alpha_1} \rho_{\alpha_2 \dots \alpha_{n-1})}, \quad (9.4)$$

for arbitrary $\rho_{\alpha(n-2)}$. It would be interesting to quantize the topological theory (2.34) and compute its partition function on topologically nontrivial backgrounds such as $S^1 \times S^2$.

Following [14], we can introduce a real basis for the spinor covariant derivatives which is obtained by replacing the complex operators \mathcal{D}_α and $\bar{\mathcal{D}}_\alpha$ with ∇_α^I , where $I = 1, 2$, defined by

$$\mathcal{D}_\alpha = \frac{e^{i\varphi}}{\sqrt{2}} (\nabla_\alpha^1 - i\nabla_\alpha^2), \quad \bar{\mathcal{D}}_\alpha = -\frac{e^{-i\varphi}}{\sqrt{2}} (\nabla_\alpha^1 + i\nabla_\alpha^2), \quad (9.5)$$

where we have represented $\mu = -ie^{2i\varphi}|\mu|$. The new covariant derivatives can be shown to obey the following algebra:

$$\begin{aligned} \{\nabla_\alpha^1, \nabla_\beta^1\} &= 2i\nabla_{\alpha\beta} - 4i|\mu|M_{\alpha\beta}, \\ \{\nabla_\alpha^2, \nabla_\beta^2\} &= 2i\nabla_{\alpha\beta} + 4i|\mu|M_{\alpha\beta}, \end{aligned} \quad (9.6a)$$

$$\{\nabla_\alpha^1, \nabla_\beta^2\} = 0, \quad (9.6b)$$

$$\begin{aligned} [\nabla_a, \nabla_\beta^1] &= |\mu|(\gamma_a)_\beta{}^\gamma \nabla_\gamma^1, \\ [\nabla_a, \nabla_\beta^2] &= -|\mu|(\gamma_a)_\beta{}^\gamma \nabla_\gamma^2, \end{aligned} \quad (9.6c)$$

$$[\nabla_a, \nabla_b] = -4|\mu|^2 M_{ab}. \quad (9.6d)$$

The graded commutation relations for the operators ∇_a and ∇_α^I have the following properties: (i) they do not involve ∇_α^2 , and (ii) they are identical to those defining the $\mathcal{N} = 1$ AdS superspace, AdS^{3|2}; see [14] for the details. These properties mean that AdS^{3|2} is naturally embedded in (1,1) AdS superspace as a subspace. The Grassmann variables $\theta_I^\mu = (\theta_1^\mu, \theta_2^\mu)$ may be chosen in such a way that AdS^{3|2} corresponds to the surface defined by $\theta_2^\mu = 0$. Every supersymmetric field theory in (1,1) AdS superspace may be reduced to AdS^{3|2}. Such $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ AdS superspace reduction may be carried out for all the higher-spin supersymmetric theories constructed in this paper. Implementation of this program will be described elsewhere. Here we only point out that reducing the longitudinal model for the massless superspin- s multiplet (presented in Sec. VA) to AdS^{3|2} leads to a new massless higher-spin gauge theory that was not described in [25]. Appendix B provides the technical details of such a reduction in the flat-superspace case.

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APPENDIX A: NOTATION, CONVENTIONS AND ADS IDENTITIES

We follow the notation and conventions adopted in [12]. In particular, the Minkowski metric is $\eta_{ab} = \text{diag}(-1, 1, 1)$. The spinor indices are raised and lowered using the $\text{SL}(2, \mathbb{R})$ invariant tensors

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon^{\alpha\gamma} \varepsilon_{\gamma\beta} = \delta_{\beta}^{\alpha} \quad (\text{A1})$$

by the standard rule:

$$\psi^{\alpha} = \varepsilon^{\alpha\beta} \psi_{\beta}, \quad \psi_{\alpha} = \varepsilon_{\alpha\beta} \psi^{\beta}. \quad (\text{A2})$$

We make use of real gamma-matrices, $\gamma_a := ((\gamma_a)_{\alpha}^{\beta})$, which obey the algebra

$$\gamma_a \gamma_b = \eta_{ab} \mathbb{1} + \varepsilon_{abc} \gamma^c, \quad (\text{A3})$$

where the Levi-Civita tensor is normalized as $\varepsilon^{012} = -\varepsilon_{012} = 1$. The completeness relation for the gamma-matrices reads

$$(\gamma^a)_{\alpha\beta} (\gamma_a)^{\rho\sigma} = -(\delta_{\alpha}^{\rho} \delta_{\beta}^{\sigma} + \delta_{\alpha}^{\sigma} \delta_{\beta}^{\rho}). \quad (\text{A4})$$

Here the symmetric matrices $(\gamma_a)^{\alpha\beta}$ and $(\gamma_a)_{\alpha\beta}$ are obtained from $\gamma_a = (\gamma_a)_{\alpha}^{\beta}$ by the rules (A2). Some useful relations involving γ -matrices are

$$\varepsilon_{abc} (\gamma^b)_{\alpha\beta} (\gamma^c)_{\gamma\delta} = \varepsilon_{\gamma(\alpha} (\gamma_a)_{\beta)\delta} + \varepsilon_{\delta(\alpha} (\gamma_a)_{\beta)\gamma}, \quad (\text{A5a})$$

$$\text{tr}[\gamma_a \gamma_b \gamma_c \gamma_d] = 2\eta_{ab} \eta_{cd} - 2\eta_{ac} \eta_{db} + 2\eta_{ad} \eta_{bc}. \quad (\text{A5b})$$

Given a three-vector x_a , it can be equivalently described by a symmetric second-rank spinor $x_{\alpha\beta}$ defined as

$$x_{\alpha\beta} := (\gamma^a)_{\alpha\beta} x_a = x_{\beta\alpha}, \quad x_a = -\frac{1}{2} (\gamma_a)^{\alpha\beta} x_{\alpha\beta}. \quad (\text{A6})$$

In the 3D case, an antisymmetric tensor $F_{ab} = -F_{ba}$ is Hodge-dual to a three-vector F_a , specifically

$$F_a = \frac{1}{2} \varepsilon_{abc} F^{bc}, \quad F_{ab} = -\varepsilon_{abc} F^c. \quad (\text{A7})$$

Then, the symmetric spinor $F_{\alpha\beta} = F_{\beta\alpha}$, which is associated with F_a , can equivalently be defined in terms of F_{ab} :

$$F_{\alpha\beta} := (\gamma^a)_{\alpha\beta} F_a = \frac{1}{2} (\gamma^a)_{\alpha\beta} \varepsilon_{abc} F^{bc}. \quad (\text{A8})$$

These three algebraic objects, F_a , F_{ab} and $F_{\alpha\beta}$, are in one-to-one correspondence to each other, $F_a \leftrightarrow F_{ab} \leftrightarrow F_{\alpha\beta}$.

The corresponding inner products are related to each other as follows:

$$-F^a G_a = \frac{1}{2} F^{ab} G_{ab} = \frac{1}{2} F^{\alpha\beta} G_{\alpha\beta}. \quad (\text{A9})$$

The Lorentz generators with two vector indices ($M_{ab} = -M_{ba}$), one vector index (M_a) and two spinor indices ($M_{\alpha\beta} = M_{\beta\alpha}$) are related to each other by the rules $M_a = \frac{1}{2} \varepsilon_{abc} M^{bc}$ and $M_{\alpha\beta} = (\gamma^a)_{\alpha\beta} M_a$. These generators act on a vector V_c and a spinor Ψ_{γ} as follows:

$$M_{ab} V_c = 2\eta_{c[a} V_{b]}, \quad M_{\alpha\beta} \Psi_{\gamma} = \varepsilon_{\gamma(\alpha} \Psi_{\beta)}. \quad (\text{A10})$$

The covariant derivatives of (1,1) AdS superspace obey various identities, which can be readily derived from the covariant derivatives algebra (3.1). We have made use of the following identities:

$$\begin{aligned} \mathcal{D}_{\alpha} \mathcal{D}_{\beta} &= \frac{1}{2} \varepsilon_{\alpha\beta} \mathcal{D}^2 - 2\bar{\mu} M_{\alpha\beta}, \\ \bar{\mathcal{D}}_{\alpha} \bar{\mathcal{D}}_{\beta} &= -\frac{1}{2} \varepsilon_{\alpha\beta} \bar{\mathcal{D}}^2 + 2\mu M_{\alpha\beta}, \end{aligned} \quad (\text{A11a})$$

$$\begin{aligned} \mathcal{D}_{\alpha} \mathcal{D}^2 &= 4\bar{\mu} \mathcal{D}^{\beta} M_{\alpha\beta} + 4\bar{\mu} \mathcal{D}_{\alpha}, \\ \mathcal{D}^2 \mathcal{D}_{\alpha} &= -4\bar{\mu} \mathcal{D}^{\beta} M_{\alpha\beta} - 2\bar{\mu} \mathcal{D}_{\alpha}, \end{aligned} \quad (\text{A11b})$$

$$\begin{aligned} \bar{\mathcal{D}}_{\alpha} \bar{\mathcal{D}}^2 &= 4\mu \bar{\mathcal{D}}^{\beta} M_{\alpha\beta} + 4\mu \bar{\mathcal{D}}_{\alpha}, \\ \bar{\mathcal{D}}^2 \bar{\mathcal{D}}_{\alpha} &= -4\mu \bar{\mathcal{D}}^{\beta} M_{\alpha\beta} - 2\mu \bar{\mathcal{D}}_{\alpha}, \end{aligned} \quad (\text{A11c})$$

$$\begin{aligned} [\bar{\mathcal{D}}^2, \mathcal{D}_{\alpha}] &= 4i \mathcal{D}_{\alpha\beta} \bar{\mathcal{D}}^{\beta} + 6\mu \mathcal{D}_{\alpha} \\ &= 4i \bar{\mathcal{D}}^{\beta} \mathcal{D}_{\alpha\beta} - 6\mu \mathcal{D}_{\alpha}, \end{aligned} \quad (\text{A11d})$$

$$\begin{aligned} [\mathcal{D}^2, \bar{\mathcal{D}}_{\alpha}] &= -4i \mathcal{D}_{\beta\alpha} \mathcal{D}^{\beta} + 6\bar{\mu} \bar{\mathcal{D}}_{\alpha} \\ &= -4i \mathcal{D}^{\beta} \mathcal{D}_{\beta\alpha} + 6\bar{\mu} \bar{\mathcal{D}}_{\alpha}, \end{aligned} \quad (\text{A11e})$$

where $\mathcal{D}^2 = \mathcal{D}^{\alpha} \mathcal{D}_{\alpha}$, and $\bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\alpha} \bar{\mathcal{D}}^{\alpha}$. These relations imply the identity

$$\mathcal{D}^{\alpha} (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_{\alpha} = \bar{\mathcal{D}}_{\alpha} (\mathcal{D}^2 - 6\bar{\mu}) \bar{\mathcal{D}}^{\alpha}, \quad (\text{A12})$$

which guarantees the reality of the actions considered in the main body of the paper.

APPENDIX B: $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ SUPERSPACE REDUCTION

In this appendix we carry out the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction [24] of the massless integer-superspin model (5.6). For simplicity our analysis is restricted to flat superspace. An extension to the AdS case will be discussed elsewhere.

In order to be consistent with the previous work [24], in which the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction of the

massless half-integer-superspin models of [18] was studied, we denote by \mathbb{D}_α and $\bar{\mathbb{D}}_\alpha$ the spinor covariant derivatives⁵ of $\mathcal{N} = 2$ Minkowski superspace $\mathbb{M}^{3|4}$. They obey the anti-commutation relations

$$\{\mathbb{D}_\alpha, \bar{\mathbb{D}}_\beta\} = -2i\partial_{\alpha\beta}, \quad \{\mathbb{D}_\alpha, \mathbb{D}_\beta\} = \{\bar{\mathbb{D}}_\alpha, \bar{\mathbb{D}}_\beta\} = 0. \quad (\text{B1})$$

In order to carry out the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction, it is useful to introduce real Grassmann coordinates θ_I^α for $\mathbb{M}^{3|4}$, where $I = \underline{1}, \underline{2}$. We define these coordinates by choosing the corresponding spinor covariant derivatives D_α^I as in [48]:

$$\mathbb{D}_\alpha = \frac{1}{\sqrt{2}}(D_\alpha^{\underline{1}} - iD_\alpha^{\underline{2}}), \quad \bar{\mathbb{D}}_\alpha = -\frac{1}{\sqrt{2}}(D_\alpha^{\underline{1}} + iD_\alpha^{\underline{2}}). \quad (\text{B2})$$

From (B1) we deduce

$$\{D_\alpha^I, D_\beta^J\} = 2i\delta^{IJ}(\gamma^m)_{\alpha\beta}\partial_m, \quad I, J = \underline{1}, \underline{2}. \quad (\text{B3})$$

Given an $\mathcal{N} = 2$ superfield $U(x, \theta_I)$, we define its $\mathcal{N} = 1$ bar-projection

$$U| := U(x, \theta_I)|_{\theta_{\underline{2}}=0}, \quad (\text{B4})$$

which is a superfield on $\mathcal{N} = 1$ Minkowski superspace $\mathbb{M}^{3|2}$ parametrized by real Cartesian coordinates $z^A = (x^a, \theta^\alpha)$, where $\theta^\alpha := \theta_{\underline{1}}^\alpha$. The spinor covariant derivative of $\mathcal{N} = 1$ Minkowski superspace $D_\alpha := D_\alpha^{\underline{1}}$ obeys the anticommutation relation

$$\{D_\alpha, D_\beta\} = 2i(\gamma^m)_{\alpha\beta}\partial_m. \quad (\text{B5})$$

Finally, the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ superspace reduction of the $\mathcal{N} = 2$ supersymmetric action is carried out using the rule [24]

$$S = \int d^3|4_z L_{(\mathcal{N}=2)} = \int d^3|2_z L_{(\mathcal{N}=1)}, \quad L_{(\mathcal{N}=1)} := -\frac{i}{4}(D^{\underline{2}})^2 L_{(\mathcal{N}=2)}|. \quad (\text{B6})$$

Given an integer $s \geq 1$, the longitudinal formulation for the massless superspin- s multiplet is realized in terms of the following dynamical variables:

$$\mathbb{V}_{(s)}^\parallel = \{\mathbb{U}_{\alpha(2s-2)}, \mathbb{G}_{\alpha(2s)}, \bar{\mathbb{G}}_{\alpha(2s)}\}. \quad (\text{B7})$$

Here $\mathbb{U}_{\alpha(2s-2)}$ is an unconstrained real superfield, and the complex superfield $\mathbb{G}_{\alpha(2s)}$ is longitudinal linear,

⁵The operators \mathbb{D}_α and $\bar{\mathbb{D}}_\alpha$ coincide with D_α and \bar{D}_α given in Eq. (2.38). However, it is advantageous here to use the different notation for these covariant derivatives.

$$\bar{\mathbb{D}}_{(\alpha_1} \mathbb{G}_{\alpha_2 \dots \alpha_{2s+1})} = 0. \quad (\text{B8})$$

The dynamical superfields are defined modulo gauge transformations of the form

$$\delta \mathbb{U}_{\alpha(2s-2)} = \bar{\gamma}_{\alpha(2s-2)} + \gamma_{\alpha(2s-2)}, \quad (\text{B9a})$$

$$\delta \mathbb{G}_{\alpha(2s)} = \bar{\mathbb{D}}_{(\alpha_1} \mathbb{D}_{\alpha_2} \bar{\gamma}_{\alpha_3 \dots \alpha_{2s})}, \quad (\text{B9b})$$

where the gauge parameter $\gamma_{\alpha(2s-2)}$ is an arbitrary transverse linear superfield,

$$\bar{\mathbb{D}}^\beta \gamma_{\beta\alpha_1 \dots \alpha_{2s-3}} = 0. \quad (\text{B10})$$

The gauge-invariant action is

$$S_{(s)}^\parallel = \left(-\frac{1}{2}\right)^s \int d^3|4_z \left\{ \frac{1}{8} \mathbb{U}^{\alpha(2s-2)} \mathbb{D}^\gamma \bar{\mathbb{D}}^2 \mathbb{D}_\gamma \mathbb{U}_{\alpha(2s-2)} + \frac{s}{2s+1} \mathbb{U}^{\alpha(2s-2)} (\mathbb{D}^\beta \bar{\mathbb{D}}^\gamma \mathbb{G}_{\beta\gamma\alpha(2s-2)} - \bar{\mathbb{D}}^\beta \mathbb{D}^\gamma \bar{\mathbb{G}}_{\beta\gamma\alpha(2s-2)}) + \frac{s}{2s-1} \bar{\mathbb{G}}^{\alpha(2s)} \mathbb{G}_{\alpha(2s)} + \frac{s}{2(2s+1)} (\mathbb{G}^{\alpha(2s)} \mathbb{G}_{\alpha(2s)} + \bar{\mathbb{G}}^{\alpha(2s)} \bar{\mathbb{G}}_{\alpha(2s)}) \right\}. \quad (\text{B11})$$

Making use of the representation (B2), the transverse linear constraint (B10) takes the form

$$D^{2\beta} \gamma_{\beta\alpha_1 \dots \alpha_{2s-3}} = iD^{\underline{1}\beta} \gamma_{\beta\alpha_1 \dots \alpha_{2s-3}}. \quad (\text{B12})$$

It follows that $\gamma_{\alpha(2s-2)}$ has two independent $\theta_{\underline{2}}$ -components, which are

$$\gamma_{\alpha(2s-2)}|, \quad D_{(\alpha_1}^{\underline{2}} \gamma_{\alpha_2 \dots \alpha_{2s-1})}|. \quad (\text{B13})$$

The gauge transformation of $\mathbb{U}_{\alpha(2s-2)}$, Eq. (B9), allows us to impose two conditions

$$\mathbb{U}_{\alpha(2s-2)}| = 0, \quad D_{(\alpha_1}^{\underline{2}} \mathbb{U}_{\alpha_2 \dots \alpha_{2s-1})}| = 0. \quad (\text{B14})$$

In this gauge we define the following unconstrained real $\mathcal{N} = 1$ superfields:

$$U_{\alpha(2s-3)} := \frac{i}{s} D^{2\beta} \mathbb{U}_{\beta\alpha(2s-3)}|, \quad (\text{B15a})$$

$$U_{\alpha(2s-2)} := -\frac{i}{4s} (D^{\underline{2}})^2 \mathbb{U}_{\alpha(2s-2)}|. \quad (\text{B15b})$$

The residual gauge freedom, which preserves the gauge conditions (B14), is described by unconstrained real $\mathcal{N} = 1$ superfield parameters $\zeta_{\alpha(2s-2)}$ and $\lambda_{\alpha(2s-1)}$ defined by

$$\gamma_{\alpha(2s-2)}| = \frac{i}{2}\zeta_{\alpha(2s-2)}, \quad \bar{\zeta}_{\alpha(2s-2)} = \zeta_{\alpha(2s-2)}, \quad (\text{B16a})$$

$$D_{(\alpha_1}^2 \gamma_{\alpha_2 \dots \alpha_{2s-1})}| = \frac{1}{2}\lambda_{\alpha(2s-1)}, \quad \bar{\lambda}_{\alpha(2s-1)} = \lambda_{\alpha(2s-1)}, \quad (\text{B16b})$$

The gauge transformation laws of the superfields (B15) are

$$\delta U_{\alpha(2s-3)} = -\frac{i}{s}D^\beta \zeta_{\beta\alpha(2s-3)}, \quad (\text{B17a})$$

$$\delta U_{\alpha(2s-2)} = \frac{1}{2s}D^\beta \lambda_{\beta\alpha(2s-2)}, \quad (\text{B17b})$$

We now turn to reducing $\mathbb{G}_{\alpha(2s)}$ to $\mathcal{N} = 1$ superspace. From the point of view of $\mathcal{N} = 1$ supersymmetry, $\mathbb{G}_{\alpha(2s)}$ is equivalent to two unconstrained complex superfields, which we define as follows:

$$\mathbb{G}_{\alpha(2s)}| = -\frac{1}{2}(G_{\alpha(2s)} + iH_{\alpha(2s)}), \quad (\text{B18a})$$

$$iD^{2\beta}\mathbb{G}_{\beta\alpha(2s-1)}| = \Phi_{\alpha(2s-1)} + i\Psi_{\alpha(2s-1)}. \quad (\text{B18b})$$

Making use of the gauge transformation (B9) gives

$$\delta\mathbb{G}_{\alpha(2s)} = -i\partial_{(\alpha_1\alpha_2}\bar{\gamma}_{\alpha_3\dots\alpha_{2s})} + iD_{(\alpha_1}^1 D_{\alpha_2}^2 \bar{\gamma}_{\alpha_3\dots\alpha_{2s})}, \quad (\text{B19a})$$

$$\begin{aligned} iD^{2\beta}\delta\mathbb{G}_{\beta\alpha(2s-1)} = i\left\{ -i\frac{2s-1}{2s}\partial^\beta{}_{\alpha_1} D_{(\beta}^2 \bar{\gamma}_{\alpha_2\dots\alpha_{2s-1})} \right. \\ \left. + \frac{s-1}{s}\partial_{(\alpha_1\alpha_2} D^\beta \bar{\gamma}_{\alpha_3\dots\alpha_{2s-1})\beta} \right. \\ \left. - 2D^\beta \partial_{\beta(\alpha_1} \bar{\gamma}_{\alpha_2\dots\alpha_{2s-1})} \right. \\ \left. + \frac{2s+1}{4s}D^2 D_{(\alpha_1}^2 \bar{\gamma}_{\alpha_2\dots\alpha_{2s-1})} \right\}, \quad (\text{B19b}) \end{aligned}$$

At this stage one should recall that upon imposing the $\mathcal{N} = 1$ supersymmetric gauge conditions (B14) the residual gauge freedom is described by the gauge parameters (B16a) and (B16b). From (B19) we read off the gauge transformations of the $\mathcal{N} = 1$ complex superfields (B18)

$$\delta\mathbb{G}_{\alpha(2s)}| = -\frac{1}{2}\{\partial_{(\alpha_1\alpha_2}\zeta_{\alpha_3\dots\alpha_{2s})} + iD_{(\alpha_1}\lambda_{\alpha_2\dots\alpha_{2s})}\}, \quad (\text{B20a})$$

$$\begin{aligned} iD^{2\beta}\delta\mathbb{G}_{\beta\alpha(2s-1)}| = -\frac{2s-1}{4s}\partial^\beta{}_{(\alpha_1}\lambda_{\alpha_2\dots\alpha_{2s-1})\beta} \\ - i\frac{2s+1}{8s}D^2\lambda_{\alpha(2s-1)} \\ + \frac{s-1}{2s}\partial_{(\alpha_1\alpha_2} D^\beta \zeta_{\alpha_3\dots\alpha_{2s-1})\beta} \\ - D^\beta \partial_{\beta(\alpha_1}\zeta_{\alpha_2\dots\alpha_{2s-1})}. \quad (\text{B20b}) \end{aligned}$$

In the $\mathcal{N} = 1$ supersymmetric gauge (B14), $\mathbb{U}_{\alpha(2s-2)}$ is described by two unconstrained real superfields $U_{\alpha(2s-3)}$ and $U_{\alpha(2s-2)}$ defined according to (B15), and their gauge transformation laws are given by Eqs. (B17a) and (B17b), respectively. It follows from the gauge transformations (B17a), (B17b) and (B20) that in fact we are dealing with two different gauge theories. One of them is formulated in terms of the unconstrained real gauge superfields

$$\{G_{\alpha(2s)}, U_{\alpha(2s-3)}, \Psi_{\alpha(2s-1)}\}, \quad (\text{B21})$$

which are defined modulo gauge transformations of the form

$$\delta G_{\alpha(2s)} = \partial_{(\alpha_1\alpha_2}\zeta_{\alpha_3\dots\alpha_{2s})}, \quad (\text{B22a})$$

$$\delta U_{\alpha(2s-3)} = -\frac{i}{s}D^\beta \zeta_{\beta\alpha(2s-3)}, \quad (\text{B22b})$$

$$\begin{aligned} \delta\Psi_{\alpha(2s-1)} = -i\frac{s-1}{2s}\partial_{(\alpha_1\alpha_2} D^\beta \zeta_{\alpha_3\dots\alpha_{2s-1})\beta} \\ + iD^\beta \partial_{\beta(\alpha_1}\zeta_{\alpha_2\dots\alpha_{2s-1})}, \quad (\text{B22c}) \end{aligned}$$

where the gauge parameter $\zeta_{\alpha(2s-2)}$ is unconstrained real. The other theory is described by the gauge superfields

$$\{H_{\alpha(2s)}, U_{\alpha(2s-2)}, \Phi_{\alpha(2s-1)}\} \quad (\text{B23})$$

with the following gauge freedom:

$$\delta H_{\alpha(2s)} = D_{(\alpha_1}\lambda_{\alpha_2\dots\alpha_{2s})}, \quad (\text{B24a})$$

$$\delta U_{\alpha(2s-2)} = \frac{1}{2s}D^\beta \lambda_{\beta\alpha(2s-2)}, \quad (\text{B24b})$$

$$\begin{aligned} \delta\Phi_{\alpha(2s-1)} = -\frac{1}{8s}\{(4s-2)\partial^\beta{}_{(\alpha_1}\lambda_{\alpha_2\dots\alpha_{2s-1})\beta} \\ + i(2s+1)D^2\lambda_{\alpha(2s-1)}\}. \quad (\text{B24c}) \end{aligned}$$

Applying the reduction rule (B6) to the action (B11) gives two decoupled $\mathcal{N} = 1$ supersymmetric actions, which are described in terms of the dynamical variables (B21) and (B23), respectively. In the former case, the superfield $\Psi_{\alpha(2s-1)}$ is auxiliary. Integrating it out, we arrive at the following action:

$$\begin{aligned}
 S = & - \left(-\frac{1}{2} \right)^s \frac{s^2(s-1)}{2s-1} \frac{i}{2} \int d^3|z \left\{ \frac{1}{2s} G^{\alpha(2s)} D^2 G_{\alpha(2s)} - \frac{i}{s-1} G^{\alpha(2s-1)\beta} \partial_{\beta}^{\gamma} G_{\alpha(2s-1)\gamma} - 2i U^{\alpha(2s-3)} \partial^{\beta\gamma} D^{\delta} G_{\beta\gamma\delta\alpha(2s-3)} \right. \\
 & \left. + 2U^{\alpha(2s-3)} \square U_{\alpha(2s-3)} + \frac{(2s-3)(s-2)}{2s-1} \partial_{\delta\lambda} U^{\delta\lambda\alpha(2s-5)} \partial^{\beta\gamma} U_{\beta\gamma\alpha(2s-5)} - \frac{1}{2} \frac{2s-3}{2s-1} D_{\beta} U^{\alpha(2s-4)\beta} D^2 D^{\gamma} U_{\gamma\alpha(2s-4)} \right\}. \quad (\text{B25})
 \end{aligned}$$

This action is invariant under the gauge transformations (B22a) and (B22b).

In the latter case, the superfield $\Phi_{\alpha(2s-1)}$ is auxiliary. Integrating it out, we obtain the following gauge-invariant action:

$$\begin{aligned}
 S = & \left(-\frac{1}{2} \right)^s \frac{s}{2s-1} i \int d^3|z \left\{ \frac{1}{2} H^{\alpha(2s)} D^2 H_{\alpha(2s)} + i H^{\alpha(2s-1)\beta} \partial_{\beta}^{\gamma} H_{\alpha(2s-1)\gamma} \right. \\
 & + 2i(2s-1) U^{\alpha(2s-2)} \partial^{\beta\gamma} H_{\beta\gamma\alpha(2s-2)} + (2s-1) U^{\alpha(2s-2)} D^2 U_{\alpha(2s-2)} \\
 & \left. + 2(2s-1)(s-1) D_{\beta} U^{\beta\alpha(2s-3)} D^{\gamma} U_{\gamma\alpha(2s-3)} \right\}. \quad (\text{B26})
 \end{aligned}$$

This action is invariant under the gauge transformations (B24a) and (B24b). Modulo an overall normalization factor, (B26) coincides with the off-shell $\mathcal{N} = 1$ supersymmetric action for massless superspin- s multiplet [24] in the form given in [25].

The action (B25) defines a new $\mathcal{N} = 1$ supersymmetric higher-spin theory which did not appear in the analysis of [24]. It may be shown that at the component level it reduces, upon imposing a Wess-Zumino gauge and eliminating the auxiliary fields, to a sum of two massless actions, one of which is the bosonic Fronsdal-type spin- s model and the other is the fermionic Fang-Fronsdal-type spin- $(s + \frac{1}{2})$ model.

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- [1] S. J. Gates, Jr., M. T. Grisaru, M. Roček, and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry* (Benjamin/Cummings, Reading, MA, 1983).
- [2] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk through Superspace* (IOP, Bristol, 1995) (Revised Edition: 1998).
- [3] D. Butter and S. M. Kuzenko, A dual formulation of supergravity-matter theories, *Nucl. Phys.* **B854**, 1 (2012).
- [4] B. W. Keck, An alternative class of supersymmetries, *J. Phys. A* **8**, 1819 (1975).
- [5] B. Zumino, Nonlinear realization of supersymmetry in de Sitter space, *Nucl. Phys.* **B127**, 189 (1977).
- [6] E. A. Ivanov and A. S. Sorin, Superfield formulation of $\text{OSp}(1,4)$ supersymmetry, *J. Phys. A* **13**, 1159 (1980).
- [7] S. M. Kuzenko and A. G. Sibiryakov, Free massless higher-superspin superfields on the anti-de Sitter superspace, *Yad. Fiz.* **57**, 1326 (1994) [*Phys. At. Nucl.* **57**, 1257 (1994)].
- [8] S. M. Kuzenko, V. V. Postnikov, and A. G. Sibiryakov, Massless gauge superfields of higher half-integer superspins, *Pis'ma Zh. Eksp. Teor. Fiz.* **57**, 521 (1993) [*JETP Lett.* **57**, 534 (1993)].
- [9] S. M. Kuzenko and A. G. Sibiryakov, Massless gauge superfields of higher integer superspins, *Pis'ma Zh. Eksp. Teor. Fiz.* **57**, 526 (1993) [*JETP Lett.* **57**, 539 (1993)].
- [10] A. Achúcarro and P. K. Townsend, A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories, *Phys. Lett. B* **180**, 89 (1986).
- [11] P. S. Howe, J. M. Izquierdo, G. Papadopoulos, and P. K. Townsend, New supergravities with central charges and Killing spinors in $2 + 1$ dimensions, *Nucl. Phys.* **B467**, 183 (1996).
- [12] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Off-shell supergravity-matter couplings in three dimensions, *J. High Energy Phys.* **03** (2011) 120.
- [13] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in three dimensions: New off-shell formulation, *J. High Energy Phys.* **09** (2013) 072.
- [14] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Three-dimensional (p,q) AdS superspaces and matter couplings, *J. High Energy Phys.* **08** (2012) 024.
- [15] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Three-dimensional $N = 2$ (AdS) supergravity and associated supercurrents, *J. High Energy Phys.* **12** (2011) 052.
- [16] B. M. Zupnik and D. G. Pak, Superfield formulation of the simplest three-dimensional gauge theories and conformal supergravities, *Teor. Mat. Fiz.* **77**, 97 (1988) [*Theor. Math. Phys.* **77**, 1070 (1988)].
- [17] H. Nishino and S. J. Gates, Jr., Chern-Simons theories with supersymmetries in three dimensions, *Int. J. Mod. Phys. A* **08**, 3371 (1993).
- [18] S. M. Kuzenko and D. X. Ogburn, Off-shell higher spin $N = 2$ supermultiplets in three dimensions, *Phys. Rev. D* **94**, 106010 (2016).

- [19] J. Hutomo and S. M. Kuzenko, Higher spin supermultiplets in three dimensions: (2,0) AdS supersymmetry, [arXiv:1809.00802](#).
- [20] S. Deser, R. Jackiw, and S. Templeton, Three-Dimensional Massive Gauge Theories, *Phys. Rev. Lett.* **48**, 975 (1982).
- [21] S. Deser, R. Jackiw, and S. Templeton, Topologically massive gauge theories, *Ann. Phys. (N.Y.)* **140**, 372 (1982); *Erratum* **185**, 406 (1988).
- [22] S. Deser and J. H. Kay, Topologically massive supergravity, *Phys. Lett.* **120B**, 97 (1983).
- [23] S. Deser, Cosmological topological supergravity, in *Quantum Theory of Gravity*, edited by S. M. Christensen (Adam Hilger, Bristol, 1984), pp. 374–381.
- [24] S. M. Kuzenko and M. Tsulaia, Off-shell massive $N = 1$ supermultiplets in three dimensions, *Nucl. Phys.* **B914**, 160 (2017).
- [25] S. M. Kuzenko and M. Ponds, Topologically massive higher spin gauge theories, [arXiv:1806.06643](#).
- [26] S. M. Kuzenko, R. Manvelyan, and S. Theisen, Off-shell superconformal higher spin multiplets in four dimensions, *J. High Energy Phys.* **07** (2017) 034.
- [27] J. Hutomo and S. M. Kuzenko, Non-conformal higher spin supercurrents, *Phys. Lett. B* **778**, 242 (2018).
- [28] J. Hutomo and S. M. Kuzenko, The massless integer super-spin multiplets revisited, *J. High Energy Phys.* **02** (2018) 137.
- [29] E. I. Buchbinder, J. Hutomo, and S. M. Kuzenko, Higher spin supercurrents in anti-de Sitter space, *J. High Energy Phys.* **09** (2018) 027.
- [30] I. L. Buchbinder, S. J. Gates, Jr., and K. Koutrolikos, Higher spin superfield interactions with the chiral supermultiplet: Conserved supercurrents and cubic vertices, *Universe* **4**, 6 (2018).
- [31] K. Koutrolikos, P. Koči, and R. von Unge, Higher spin superfield interactions with complex linear supermultiplet: Conserved supercurrents and cubic vertices, *J. High Energy Phys.* **03** (2018) 119.
- [32] I. L. Buchbinder, S. J. Gates, Jr., and K. Koutrolikos, Conserved higher spin supercurrents for arbitrary spin massless supermultiplets and higher spin superfield cubic interactions, *J. High Energy Phys.* **08** (2018) 055.
- [33] H. Osborn, $N = 1$ superconformal symmetry in four-dimensional quantum field theory, *Ann. Phys. (N.Y.)* **272**, 243 (1999).
- [34] M. Magro, I. Sachs, and S. Wolf, Superfield Noether procedure, *Ann. Phys. (N.Y.)* **298**, 123 (2002).
- [35] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs, and G. Tartaglino-Mazzucchelli, Three-dimensional $N = 2$ supergravity theories: From superspace to components, *Phys. Rev. D* **89**, 085028 (2014).
- [36] S. M. Kuzenko, Supersymmetric spacetimes from curved superspace, *Proc. Sci., CORFU2014* (**2015**) 140.
- [37] S. M. Kuzenko, Prepotentials for $N = 2$ conformal supergravity in three dimensions, *J. High Energy Phys.* **12** (2012) 021.
- [38] T. Kugo and S. Uehara, $N = 1$ superconformal tensor calculus: Multiplets with external Lorentz indices and spinor derivative operators, *Prog. Theor. Phys.* **73**, 235 (1985).
- [39] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Higher derivative couplings and massive supergravity in three dimensions, *J. High Energy Phys.* **09** (2015) 081.
- [40] M. Roček and P. van Nieuwenhuizen, $N \geq 2$ supersymmetric Chern-Simons terms as $D = 3$ extended conformal supergravity, *Classical Quantum Gravity* **3**, 43 (1986).
- [41] D. Butter, S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Conformal supergravity in three dimensions: Off-shell actions, *J. High Energy Phys.* **10** (2013) 073.
- [42] T. T. Dumitrescu and N. Seiberg, Supercurrents and brane currents in diverse dimensions, *J. High Energy Phys.* **07** (2011) 095.
- [43] D. Butter and S. M. Kuzenko, $N = 2$ AdS supergravity and supercurrents, *J. High Energy Phys.* **07** (2011) 081.
- [44] A. A. Nizami, T. Sharma, and V. Umesh, Superspace formulation and correlation functions of 3d superconformal field theories, *J. High Energy Phys.* **07** (2014) 022.
- [45] I. L. Buchbinder, T. V. Snegirev, and Y. M. Zinoviev, Lagrangian description of massive higher spin supermultiplets in AdS₃ space, *J. High Energy Phys.* **08** (2017) 021.
- [46] S. M. Kuzenko, Higher spin super-Cotton tensors and generalisations of the linear-chiral duality in three dimensions, *Phys. Lett. B* **763**, 308 (2016).
- [47] I. A. Batalin and G. A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, *Phys. Rev. D* **28**, 2567 (1983).
- [48] S. M. Kuzenko, J.-H. Park, G. Tartaglino-Mazzucchelli, and R. Unge, Off-shell superconformal nonlinear sigma-models in three dimensions, *J. High Energy Phys.* **01** (2011) 146.