Joint Optimization of Successive Manufacturing Processes and Cycle Service Levels in the Paper Supply Chain

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DEDICATIONS

To my parents, Farid and Yasmin

All I do in my life is a reflection of my upbringing and because of your prayers.

To my wife, Humaira

Thank you for what you are.

To my bundle of joy, my daughter Hayaa

ABSTRACT

The objective of the thesis is to integrate three core production considerations in an optimization model namely: cycle service level, trim loss when cutting stock, and grade changeover costs associated with lot-sizing. Various industries encounter the cutting stock and lot-sizing problems in successive manufacturing processes. The lot-sizing problem (LSP) finds a trade-off between setup and inventory holding costs, whereas the cutting stock problem (CSP) involves cutting large objects into smaller ones while minimizing the trim loss. These two processes are strongly interlinked because the latter is governed by the customer demand and the demand for production lots is derived from it. Particularly for the paper industry, the end demand for smaller rolls of different grades drives the production schedules for the paper machine producing jumbo reels. However, the literature has mostly dealt with these two processes separately, which has important repercussions especially for cycle service levels. A separate optimization approach restricts cycle service levels by putting an upper bound on the total number of different grades of jumbo reels to be produced on the paper machine.

This study jointly optimizes the two successive manufacturing processes of lot-sizing at the paper machine and determining the cutting pattern during paper conversion with cycle service level considerations. Initially, an integrated formulation is developed as a conventional single objective function embracing the costs of trim loss, grade changeover and inventory holding as well as the tardiness penalty incurred whenever an order fails to meet its due date. Standard genetic algorithm is used as the solution method for the joint problem of simultaneously solving the two NP-hard combinatorial problems. The results reveal that the service levels are maximized by simultaneously solving the trim loss and lot-sizing problem. However, there is a substantial increase in production costs. On the other hand, a least cost production plan for the paper manufacturing and conversion stages results in poor cycle service levels where there is a failure to meet many of

the customer order due dates; this leads to the joint production problem being treated as a multiple objective problem.

A two step solution approach is proposed to the bi-objective production planning problem for simultaneously minimizing production costs and maximizing cycle service levels. In the first step, a set of non-dominated solutions is obtained by employing the epsilon constraint method. The standard genetic algorithm is used as the solution approach. In the second step, the epsilon constraint method solutions are used as part of an initial population for the non dominated sorting algorithm (NSGA-II) to improve the Pareto frontier. The two step approach has been necessitated because the NSGA-II is unable to obtain feasible solutions when initiated by a randomized population. No such issue arises when standard GA is used as the resolution algorithm for the epsilon constraint method. Therefore, the epsilon constraint method was used as the first step to generate an approximation to the Pareto frontier which was improved by resorting to NSGA-II in the second step.

The results show that higher levels of cycle service can be achieved if additional trim loss is incurred and are pertinent for all the industrial settings where the cutting stock and lot-sizing problems are successive production processes. The proposed multi-objective joint optimization approach is particularly relevant to the paper industry as it gives the production managers an analytical tool with a complete range of possibilities for production cost, flexibility and customer service. It also helps to put a premium on the higher cycle service requirements of its customers which can be a crucial element in price negotiations.

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In the end, I would like to acknowledge and thank the Australian Development Scholarship Program that gave me the opportunity to study at this prestigious university.

DECLARATION

I hereby declare that except where specific reference is made in the text to the work of others, the contents of this thesis are original and have not been submitted to any other university.

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CHAPTER 1. INTRODUCTION

The objective of this study is to optimize the scheduling of production processes in the paper industry while at the same time achieving higher levels of service to customers. This leads to the question of how much cost saving must be sacrificed to achieve a range of service levels.

1.1 Supply Chains in the Paper Industry

The Paper industry operates with a long and integrated supply chain starting from forests to various products for daily use such as tissues, papers, newspapers and packaging materials. The pulp and paper supply chain is characterized by the divergent structure of its output and its varying degree of vertical integration. Paper mills may have their own plantations or wood yards and may also own the facilities such as chip mills, saw mills, and pulp mills that transform the wood logs into pulp - the raw material used in the paper machines. The jumbo reels produced at the paper machine are cut and sheeted according to customer specifications at the company's own conversion facilities or sold to customers who specialize in further transforming the reels by cutting, sheeting or printing or sometimes making corrugated boxes for their clients (Figure 1-1). The differences in the supply and distribution networks depend upon the type of finished products manufactured and also on the competitive strategy adopted by the mill's management. In all cases, significant issues arise for the optimization of the pulp and paper supply chain.

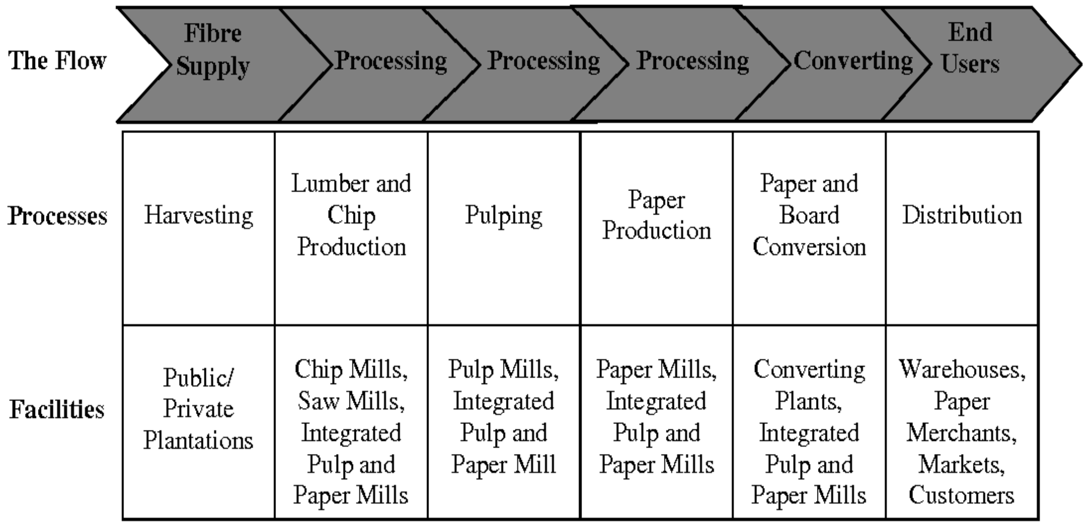


Figure 1–1: The Pulp and Paper Supply Chain Flow

1.2 Paper Making: The Sequence of Manufacturing Operations

Figure 1-1 also shows the sequence of operations necessary for delivering the final products of the paper industry to customers. The dominant source of cellulose fibres used in paper products is obtained from wood, accounting for more than 80% of the content of paper by weight. Therefore, the properties of cellulose fibres are crucial for the desired characteristics of paper constituting a strong link backwards in the supply chain to the very origin of the fibre (Carlsson et al. 2006). In other words, the decision to harvest particular wood species or a mix of species is determined by the type of paper being produced at the paper machine. For example, softwoods such as pine and spruce with their long fibres are used for producing high strength paper and packaging material whereas the softness of tissues necessitates the use of short fibres of hardwood like eucalyptus, birch and aspen. For some products like toilet papers and photocopy papers, softwood is combined with hardwood to give strength.

Logs are cut, debarked, chipped and pulped to separate the fibre from other plant materials. In solid wood, fibres are bound together by a chemical substance called ‘lignin’. Pulping is the process of breaking the lignin bonds to separate the fibres.

Chemical, semi-chemical, chemi-mechanical and mechanical pulping are the four main categories of pulping processes in the order of decreasing reliance on chemical reactions and increasing use of mechanical energy to separate fibres. As was the case with the fibres, the type of paper produced at the paper machine dictates the use of a particular pulping process. For example, mechanical and semi-mechanical pulping methods are used when high opacity, bulky and low strength paper is required, such as newsprint. Chemical Pulping is more suited for high strength requirements (kraft paper, cardboard, fine paper) because lignin which interferes with hydrogen bonding is largely removed (Biermann 1996).

The pulp produced can be used for making paper in the same premises or it can be sold as market pulp to paper mills that don't produce it. A mill which produces both paper and pulp required to produce that paper is called an 'integrated pulp and paper mill'. An integrated pulp and paper mill may also house chipping and sawing facilities to provide the raw materials for pulping (Figure 1-1). Though an integrated mill comprises a range of manufacturing processes, the general trend is to treat these separately. An ever growing orientation towards make to order (MTO) production has resulted in a paradigm shift towards a holistic view of the planning activities at the paper machine and conversion processes. But even so, the customer orders only penetrate up to the paper machine and all earlier manufacturing processes like pulp making generally produce to stock with sizeable inventories.

The paper machine uses pulp as a raw material and performs forming, pressing and drying processes to produce jumbo reels of paper. The dilute fibre slurry is passed on to a moving mesh fabric, where most of the water drains away, leaving a wet fibre mat. The separation of water from fibres is crucial to paper making, and is done in several stages. After the fibres have formed the wet sheet, more water is drained away by vacuum suction, leaving a mat that is about 85 per cent water. After as much water as possible has been removed by vacuum, the still-wet sheet is pressed between rollers or between pressure 'blankets' to remove more water. The sheet is now around 60 per cent water. The only way water can be removed further is by heating to evaporate the water. This is done by passing the sheet over a number of steam-heated rollers. It results in a sheet of about 8–10 per cent moisture, depending on the grade. According to the grade, the sheet may now be compressed, smoothed and even ‘polished’ by passing it between two or more rollers with high pressure between them. It is then wound on a large reel to form a ‘jumbo reel’ of paper.

The jumbo reel is unwound across a series of slitter knives during the conversion stage, and cut into customer specified smaller sizes called ‘cut rolls’. The cutting of jumbo reels into smaller rolls is referred to as a ‘one dimensional cutting stock problem (1D-CSP)’. The rolls may be further cut into sheets of various lengths and widths. The sheeting processes are classified as two dimensional cutting stock problem (2D-CSP). Depending upon the grade, further finishing operations can also be performed like coating and printing.

1.3 Manufacturing Supply Chains

Traditionally, the manufacturing sector has been classified into either discrete manufacturing or process industry categories. Process industries are businesses that add value to materials by mixing, separating, forming or by chemical reactions. The process industry may further be classified into continuous or batch production systems. Batch processing implies a manufacturing technique where parts are accumulated and processed together in a lot while continuous production constitutes a lot-less and continuous flow of products. Paper manufacturing is categorized as a batching production process where different grades of paper are processed in lots. The relative position of paper making process in an adaptation of the Taylor et al. (1981)’s classification of various industries is shown on a product process graph in Figure 1-2.

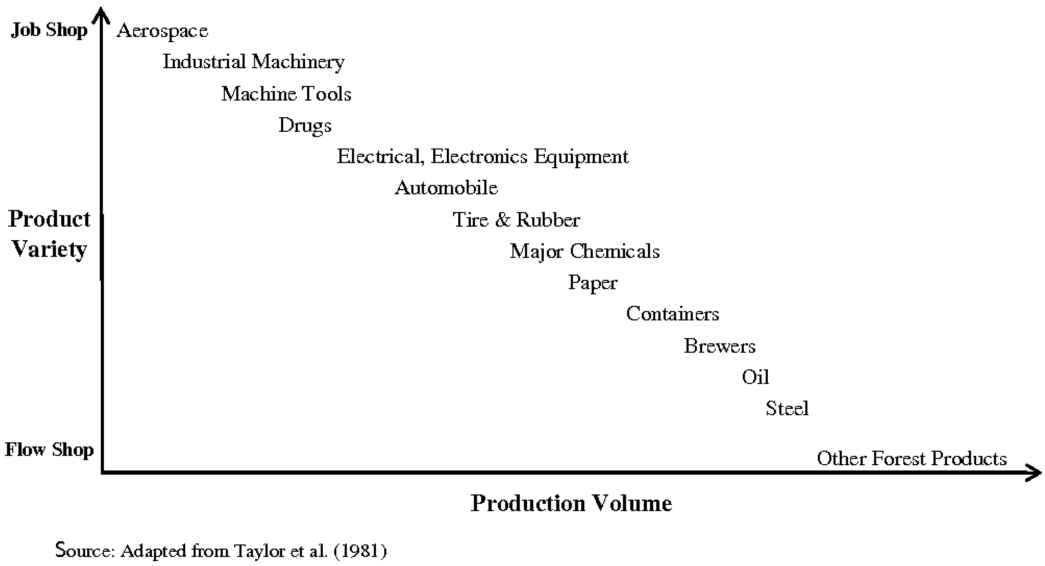


Figure 1–2: Approximate Industry Classification on a Product Process Graph

The paper making and other process industries are located towards the bottom right of Figure 1-2 characterized by high production volume and comparatively low product variety relative to discrete manufacturing. However, the advent of supply chain management has changed the focus from individual facilities to the network of all activities that delivers a service or a product to the customer. A supply chain may consist of discrete manufacturing facilities only or the process production processes or a supply chain can be a hybrid one that contains both discrete and process industry characteristics. Apart from the paper making process, pulping and paper converting operations are the other two core manufacturing activities of the pulp and paper supply chain. On the suggested classification, pulping also belongs to the process industry whereas the finishing operations of the supply chain such as reel cutting and sheeting belong to discrete manufacturing. The hybrid supply chain complicates the optimizations of operations because the characteristics of process and discrete manufacturing differ substantially. The production processes of the process industry are characterized by fixed routings, high degree of automation, low flexibility but high volumes, specialized equipment with narrow product assortment, low labour intensity, capital intensiveness, substantial product changeover times and mostly with high environmental considerations (Ashayeri & Selen 1996; Crama 2001).

The process industries supposedly lag behind the discrete industries in the identification and implementation of effective planning and scheduling techniques (Dennis & Meredith 2000). Shah (2005) reviewed the supply chain issues confronting process industries including pulp and paper and explained that process industries were lagging behind in supply chain benchmarks when compared with the discrete production supply chains. The analysis revealed that stock levels in the supply chain of process industries amount to 30–90% of annual demand and 4–24 weeks' worth of finished goods stocks, figures that indicate poor supply chain integration.

As discussed above, there are substantial differences in the planning and control requirements for process and discrete manufacturing industries and therefore, the planning and scheduling algorithms vary across hybrid supply chains such as in the paper industry. The discrete part of the supply chain i.e the conversion processes have long been planned by mathematical programming based algorithms (Goulimis 1990; Haessler & Sweeney 1991; Harjunkoski et al. 1996; Menon & Schrage 2002; Correia 2004). However production planning for the paper machine lagged behind. De Treville, Shapiro & Hameri (2004) narrate the supply chain issues confronted by a paper mill that utilized rules of thumb for its production planning. Large lot sizes caused excessive inventory and long lead times caused frequent stockouts, even in the presence of large inventories because customer orders did not match the available stocks of inventories. The last decade has witnessed the extension of operations research to paper manufacturing with different lot-sizing models being proposed for the paper machine (Rizk, Martel & and D'Amours 2004; Gupta & Magnusson 2005; Bouchriha, Ouhimmou & D'Amours 2007). However, all these optimization models have been aimed at solving the isolated production processes. Despite its great applicability in several industries, the cutting stock and lot-sizing problems need more attention as a joint problem because of the associated complexity. There is a strong case for believing that optimal solutions to the joint problem differ substantially from the solutions to the cutting stock and lot-sizing problems considered separately (Gramani & França 2006; Gramani, França & Arenales 2009; Gramani, França & Arenales 2010).

In the next Section, the planning approach to jointly optimize the two aforementioned processes encountered in the successive stages of the pulp and paper supply chain is discussed.

1.4 Joint Optimization of the Lot-Sizing and Cutting Stock Processes

The lot-sizing problem (LSP) finds a trade-off between setup and inventory holding costs, whereas the cutting stock problem (CSP) involves cutting large objects into smaller ones while minimizing the trim loss. The literature has mostly dealt with these two problems separately. However, in industries like paper, aluminium, copper and furniture, the lot-sizing and cutting stock problems are encountered in successive stages. For such instances, a separate approach to the lot-sizing and cutting stock problems will yield locally good solutions but may conflict with the overall production objectives like joint or total costs and cycle service levels (CSL). Cycle Service Level (CSL) is defined as the probability that the cycle time for the customer's order will be less than the quoted lead time (Hopp & Spearman 2008).

For example, Gramani, França & Arenales (2009) noted that solving the lot-sizing problem before the cutting stock problem in the furniture industry may find infeasible solutions with respect to production capacity. Similarly, in the paper industry, if the cutting stock problem is solved before lot sizing, the cycle service levels may suffer because the solution to the LSP is bounded by the CSP solution. Moreover, if the production exceeds demand, it may give cutting patterns with lesser trim loss along with reduced setup costs. However, inventory holding costs will increase in this instance. Therefore, in order to prepare good production schedules, there are strong grounds for an integrated approach where both problems are solved simultaneously instead of being dealt with separately.

There have been some attempts, reported in the literature, to encompass the intimate relationship between the lot-sizing problem (LSP) and the cutting stock problem (CSP). Hendry et al. (1996), Krichagina et al. (1998), Respicio et al (2002), Nonas and Thorstenson (2000), Nonas and Thorstenson (2008) and Malik et al. (2009) sought to incorporate the cutting stock problem into the planning of production schedules. However, only Malik et al. (2009) minimized the changeover, inventory holding and trim loss costs simultaneously. Harjunkoski and Westerlund (1998) and Westerlund and Isaksson (1998) extended the trim loss problem in the paper industry to include the paper production but the only decision variable added to the cutting stock formulation was the number of jumbo rolls. It did not consider the grade changeovers at the paper machine and the inventory holding costs of the finished products. However, the changeover costs associated with the cutting patterns at the conversion processes were included in the objective function. Instead of constraining the finished products inventory, the formulation penalized the overproduction by selling it at a lower price.

Gramani and Franca (2006) and Gramani et al.(2009) proposed a mathematical model for coupling the lot-sizing and cutting stock problems in the furniture industry. Because of a different industrial context, the sequence of operations was also different from the current study. The cutting stock operations were followed by lot-sizing, whereas in the paper industry the lot-sizing problem precedes the conversion processes. The sequence of operations in the furniture industry implies that the solution of the cutting stock problem is unknown a priori and dependent upon the solution of the lot-sizing problem. The problem discussed in this thesis pertains to the pulp and paper supply chain where the cutting stock problem is in the context of an independent demand and the demand for jumbo rolls is unknown beforehand but derived through the CSP.

Poltroniere et al. (2008) formulated the joint lot-sizing and cutting stock problem for the paper industry and solved it with the help of two separate iterative heuristics in a decomposed manner. The first heuristic called lot-cutting heuristic (LCH) is based on a lagrangian relaxation approach wherein the LSP-CSP coupling constraint is added to the objective function while solving the LSP ahead of the CSP. As already discussed, in a paper mill with production and conversion process in successive stages, the customer orders are for the finished products only i.e the smaller rolls. Therefore, the demand for jumbo reels is dependent upon the demand for the smaller cut rolls and is only determined through solution of the cutting stock problem. Solving the LSP ahead of the CSP is only possible with an estimated dependent demand i.e the total number of jumbo reels required to fulfill the end demand of cut rolls. The authors dealt with this by adding an estimated incremental factor to the dependent demand. The CSP is solved after each LSP is updated with different estimated factors for the dependent demand. The coupling between the two processes is achieved by updating the lagrangian multiplier in each iteration.

The objective function included the production costs at the paper machine, grade changeover costs, inventory holding costs for the jumbo rolls, trim loss and inventory holding costs for the finished products i.e the cut rolls. However only relevant objectives were included in the decomposed and separate solution method i.e trim loss and end item inventory holding costs were considered for the CSP whereas production costs, grade changeovers and jumbo reels holding cost was minimized for the LSP.

In the alternative heuristic called cutting-lot (CLH), the cutting stock problem is solved first, followed by the LSP. The results obtained showed that CLH consistently performed better than the LCH (Poltroniere et al. 2008). Different results for the two procedures also show that the order of solving these two problems has a bearing on costs; therefore, the idea of simultaneously solving the two processes gains merit. However, it appears that the joint optimization of the lot-sizing and cutting stock problems has only received limited attention in the literature but its relevance in various industrial settings and the potential benefits makes it an important research topic.

1.5 Failure to integrate Cycle Service Levels into Joint Optimization

The joint optimization of the cutting stock and lot-sizing problem also has a bearing on the resulting cycle service levels of the paper supply chain but the relationship between all three appears to have received little attention. There have been studies that included the due date requirements for the cutting stock problem but few if any incorporated the lot-sizing problem. For example, Johnston & Sadinlija (2002; 2004) amended the classical cutting stock formulation to include the due date considerations for the paper industry but their model did not include the paper machine scheduling. Velasques et al. (2007) added a variety of optimization criteria including the due date performance in their multi-objective formulation for only the cutting operations of a corrugated box manufacturer. Similarly, Reinertsen & Vossen (2010) applied the column generation technique to solve a one dimensional cutting stock problem with due dates for the steel industry.

The joint optimization of the two manufacturing processes and its integration with cycle service level is both important and difficult for the paper industry because the customer orders may comprise different grades and a planning disconnect between the two processes will necessitate inventory holding of different grades of jumbo reels for punctual deliveries. A joint optimization model for the cutting stock and lot-sizing problem will enable the paper supply chain to produce most orders in time but it will have implications for costs. Conversely, a least cost production plan for the paper manufacturing and conversion stages results in poor cycle service levels where many deliveries of customer orders fail to meet the due dates. This study explicitly deals with the two conflicting aims of minimizing production costs and maximizing cycle service levels.

1.6 Research Objectives

The aim is to develop an integrated formulation focused on cycle service level for the joint optimization of the lot-sizing problem at the paper machine and the cutting stock problem encountered during the paper conversion process. The integrated model for the two successive manufacturing stages of the paper mill - operating under a make-to-order production strategy and with due date considerations - will synchronize the product flow along these two stages and could be the first building block for the synchronization of the product flow along the entire supply chain. The final question to be addressed is how to balance production cost against cycle service levels.

1.7 Data Used

Paper mills in two Australian states, Victoria and Western Australia, were visited. The intention was to use production data from one of the mills to validate the integrated cutting stock and lot-sizing model and also to give the research project a practical orientation. The visits provided useful insights into industrial practices but for two reasons they did not serve the purpose of acquiring the entire production data to run a model. Firstly, the paper mills were only willing to share a limited amount of data because of confidentiality. Secondly, it would have been difficult to generalize the findings obtained by using data for a particular production line. Instead, the thesis uses a systematically generated random data set in conjunction with the technical specifications of a paper machine from an actual paper mill.

The paper machine's speed determining its capacity was provided by an Australian manufacturer along with the grade changeover times. Trade journals were consulted for cost data for different grades of kraft paper. Only the details of customer orders for the finished products were unknown and randomly generated data was used to represent these. The random generation of test data was inspired by Gau & Wascher (1995) who introduced a test set generator (CUTGEN) for the cutting stock problem which has been followed widely by the operations research community (Wascher & Gau 1996; Umetani, Yagiura & Ibaraki 2006; Poldi & Arenales 2009).

Excel was used to generate random test instances based on CUTGEN for evaluating the effectiveness of genetic algorithms as a solution technique for the classical cutting stock problem. However, it was modified considerably for the integrated cutting stock and lot-sizing model to include the industry provided data and also because of the different production context. The details are as follows

The customer orders are usually for cut rolls or sheets obtained during the conversion stage of a paper mill and are characterized by paper grades, roll width or sheet dimensions, number of rolls required and order due date. Cut roll widths 'l_i' were randomly generated from a uniform distribution so that the simulated widths 'l_i' represent possible values from 20% to 80% of jumbo reel width. In a joint optimization, the number of cut rolls required is determined by machine capacity because it is the bottleneck resource in the paper manufacturing supply chain. Its capacity is determined by the machine speed which in turn determines the quantity of customer orders it can handle in one week. Therefore the randomly generated roll widths were spread across the planning horizon to match the paper machine capacity. The order due dates were also randomly generated from a uniform distribution of the five working days in a week which was deemed sufficient to make the point regarding service level.

The simulated data not only made up for lack of observed data but in-fact it was more useful because of its ability to represent a wide range of possibilities. Five data sets were generated to test the proposed model. Details are given in Sections 4.4.3 and 5.3.3.

1.8 Contributions

The thesis contributions are in operational planning and scheduling. New optimization paradigms approaches to the successive manufacturing processes in the paper industry, paper making and conversion, have been developed in conjunction with optimized customer service. Specific contributions are:

- The traditional aims of minimizing grade changeover costs at the paper machine and the trim loss for the cutting stock problem are integrated with cycle service level in a new joint optimization framework (Section 5.2.1).

- The experiments performed in this study show that the joint optimization approach maximizes the cycle service level when compared to the existing planning approaches but additional costs are incurred (Section 5.5).

- The joint optimization is identified as a bi-objective problem with conflicting objectives of minimum production costs and maximum cycle service levels. It is demonstrated that a single criterion of either least cost maximum service level maximization yielded a good solution from one perspective but gave poor results for the other objective (Section 5.5, 6.1).

- The study tests the appropriateness of two multi-objective resolution methods namely the epsilon constraint and Non-Dominated Sorting Algorithm-II (NSGA-II) to approximate the tradeoff curve between the conflicting objectives of cost minimization and cycle service level maximization (Section 6.4).

- The experimental outcomes obtained in this thesis indicate superior performance of the standard genetic algorithm's 'penalty function' based constraint mechanism over the Non-Dominated Sorting Algorithm-II (NSGA-II)'s constraint principle that prefers all feasible solutions over infeasible solutions for the problem under study (Section 6.5).

- This thesis implements a novel two step model for the bi-objective production problem. In the first step, a set of non-dominated solutions is obtained by employing the epsilon constraint method. The standard genetic algorithm is used as the solution approach. In the second step, the epsilon constraint method solutions were used as part of an initial population for the non dominated sorting algorithm (NSGA-II) to improve the Pareto frontier (Section 6.6.2).

- The results demonstrate the bi-objective formulation's ability to represent different lot-sizing models in a single experiment by a prior determination of its entire feasible search space (Section 6.6.1)

- The investigation into the relationship of initial population with the approximated Pareto frontier concludes that the prior knowledge of the two extremes (a least cost solution and a maximum service level plan) leads to the integration of the separate and joint models in a single experiment (Section 6.7).

- The study exemplifies the significance of knowledge of ideal and nadir objective vectors for determining the entire feasible search space which in turn ensures a true representation of the decision context (Section 6.7).

- Single and two chromosomes genetic representations are applied to the classical cutting stock problem and compared with an exact solution approach. The range of input parameters for which GA can match exact solutions is determined for the two representations (Section 4.4).

- It is shown that different genetic representations result in dissimilar performances and that a genetic algorithm with fewer genes is more suited to the classical cutting stock problem (Section 4.4).

1.9 Thesis Layout

The thesis outline is as follows:

Chapter 2 discusses the pulp and paper supply chain within global and Australian industrial contexts. It also reviews the body of work on the operational planning and optimization of the supply chain in the paper industry, highlighting the need for integration of planning processes. The supply chain issues confronting the industry are discussed along with apparently unexplored areas.

Chapter 3 reviews the cutting stock problem (CSP) and its solution approaches including column generation and branch and price. The variants of the classical cutting stock problem such as CSP with knife setups, the pattern minimization problem, ordered cutting stock problem, ordered spread minimization problem, minimization of open stacks, CSP with contiguity, CSP with due dates, multi-objective CSP and integration of the cutting stock problem with other production processes are reviewed with an emphasis on the associated complexity that may warrant heuristic solution.

Chapter 4 analyses genetic algorithms (GA) as a solution method for the cutting stock problem. A single item per gene representation of the cutting stock problem is proposed and compared with the earlier GA representations of the problem. The proposed GA representations are also compared with the exact solution approach, using systematically generated random data. This chapter builds on the premise that if one of the two NP-hard problems embedded in a cutting stock variant is solved satisfactorily by a metaheuristic such as genetic algorithm (GA), it may be reasonable to assume that the entire problem could also be treated effectively in this way.

Chapter 5 develops an integrated planning model for the joint optimization of the two successive manufacturing processes in the paper supply chain i.e paper making and conversion. The determination of the cutting patterns and their frequencies in the conversion process, the allocation of individual cutting patterns to different time periods and necessary grade changeovers at the paper machine are solved simultaneously. The integrated planning approach is compared with traditional planning in the paper industry to highlight its utility.

Chapter 6 introduces multi-objective optimization and demonstrates its advantages over conventional single objective optimization. A case is made for a multi-objective perspective for the joint optimization of the paper making and conversion process. The joint model proposed in Chapter 5 is modified to be solved by multi-objective algorithms. The results are compared to determine the efficacy of the two approaches.

Chapter 7 summarizes the main findings in the thesis and provides a discussion of the ramifications of the proposed approach for the industry. The relevance of the proposed approach for other industrial contexts is also discussed along with recommendations for future research.

CHAPTER 2. PLANNING AND SCHEDULING IN THE PULP AND PAPER SUPPLY CHAIN

2.1 Pulp and Paper Products

As discussed in Section 1.1, the pulp and paper industry's supply chain starts from the forests and plantations which provide the industry with its raw material i.e wood logs. The chip mills convert logs into chips which are fed into the pulp mills along with chips and sawdust from sawmills. Paper mills consume pulp to produce big reels of paper (jumbo reels) which are trimmed to smaller rolls (cut rolls) during the conversion process. The trimmed rolls can be directly sold to the market or sheeted into printing and writing papers products. The principal products can be classified into five main families:

- Market Pulp
- Printing and writing paper
- Newsprint
- Paperboards
- Household and Sanitary paper

Market pulp is the pulp produced by independent pulp mills for the purpose of selling it to paper mills. The printing and writing paper category covers a variety of products such as photocopier paper, printing paper, computer stationery, magazines, brochures, catalogues, calendars, books, annual reports. Newsprint is an uncoated mechanical grade of paper used for printing newspapers. Paper boards are used for packaging purposes like folding cartons for foodstuffs, soap, detergents, cigarettes and liquid packaging for fresh milk and juices as well as corrugated cardboard boxes and packing cartons. Household and sanitary paper is made for a wide variety of uses, but primarily involving personal hygiene i.e tissues, toilet papers, serviettes and paper towelling.

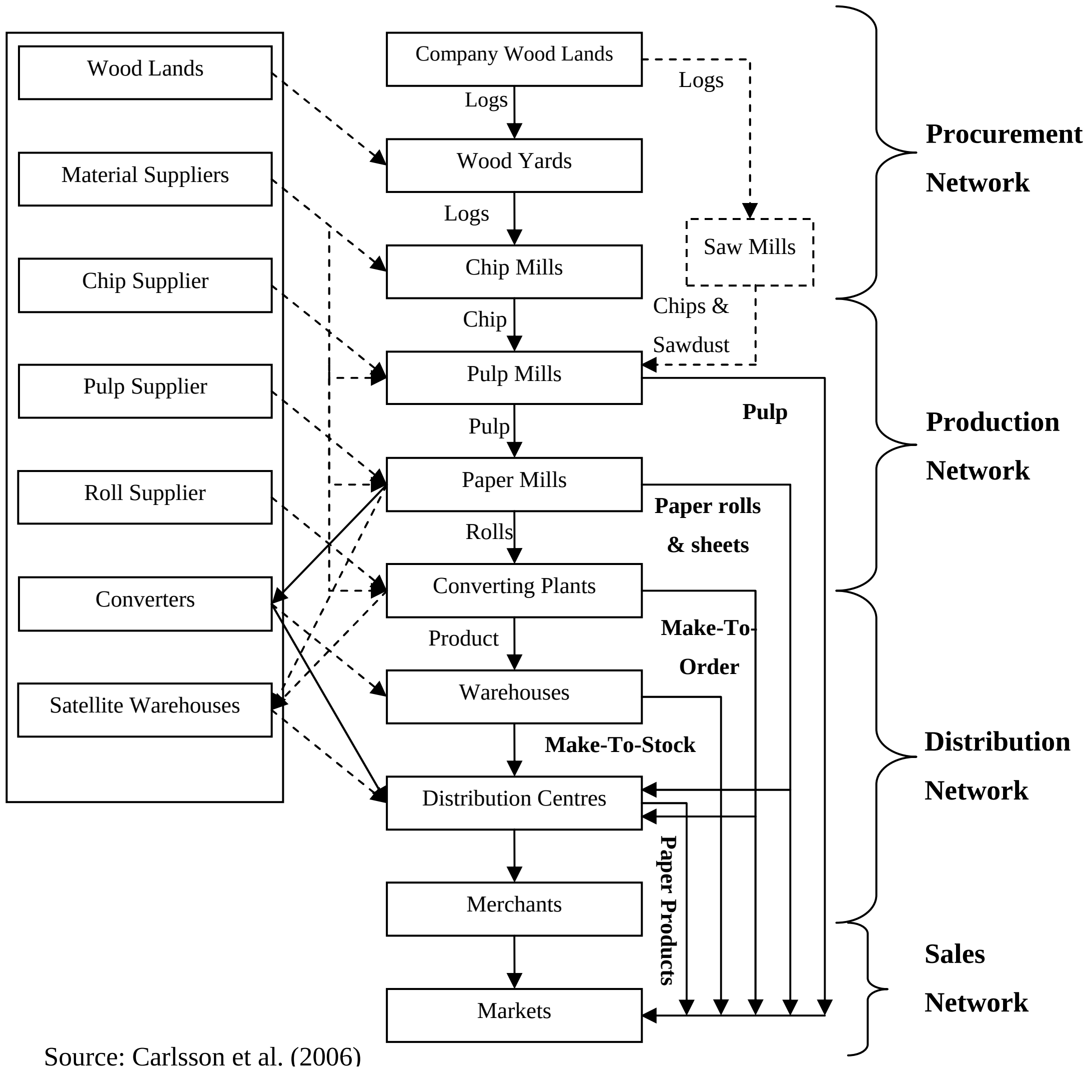


Figure 2–1: A schematic of the pulp and paper supply chain

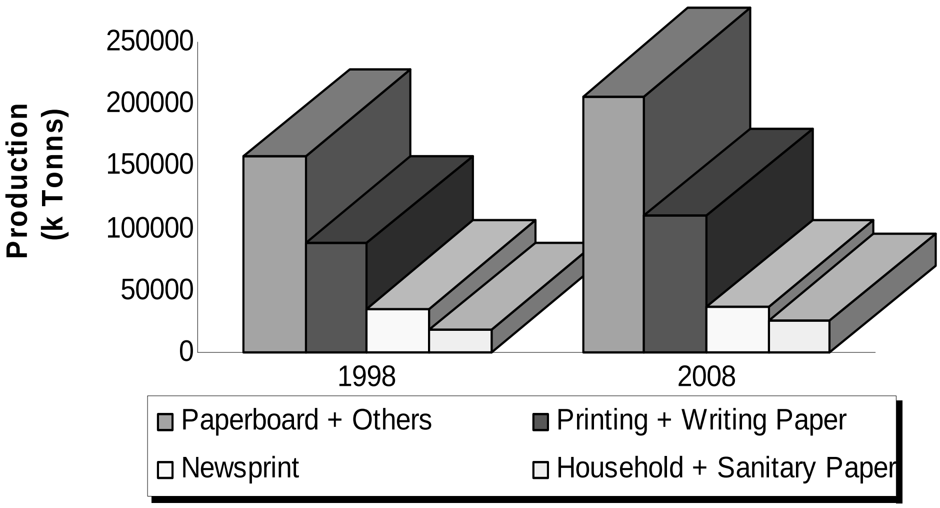
The supply and distribution networks for the pulp and paper supply chain vary and depend primarily on the type of product being manufactured. The networks may bypass a stage of the supply chain, as depicted in Fig 2-1, depending upon the individual needs of the mill. The finished products are distributed either directly or through the network of wholesalers, merchants and distributors. Traditionally, the pulp and paper mills tend to produce to order and ship large tonnage directly to customers like market pulp to paper mills, newsprint to publishers and linerboard to corrugated container manufacturers. For intermediate or small tonnages, mills ship to company owned distribution centres or paper merchants (Martel, Barek & D'Amours 2005). All transportation modes are used in the pulp and paper supply chain with trucks and trains being used inland and overseas shipments being made on vessels.

A typical company in the pulp and paper industry would own many mills. A mill which produces both paper and the pulp required to produce that paper is said to be an integrated mill. 72 % of the world's pulp production is integrated (Carlsson et al. 2006). The schematic in Figure: 2-1 includes pulp for paper production from an external network as well as in-house pulp production.

2.2 Market Context

2.2.1 The Economics of the Pulp and Paper Industry

The global market shares of the final products of the paper industry have not significantly changed over the last ten years, contrary to what was widely anticipated following the emergence of e-paper and the internet (Fig 2-2). Newsprint appears to have been most seriously affected as its share of the paper products was reduced to 9.71% in 2008 from 12% in 1998. While the printing and writing paper market share is still hovering around 29%, the "paperboard + others" and household/sanitary paper have recorded increases of 2.10% and 0.64%, respectively. The world production data of the Food and Agriculture Organization (FAO) of the United Nations has been used to analyse any shift in the market share of paper products from 1998 to 2008. The "paperboard + other" term used by FAO combines the "other papers" category with the paperboard such as construction paper and paperboard, special thin paper, wrapping and packaging paper, and other paper and paperboard not elsewhere specified

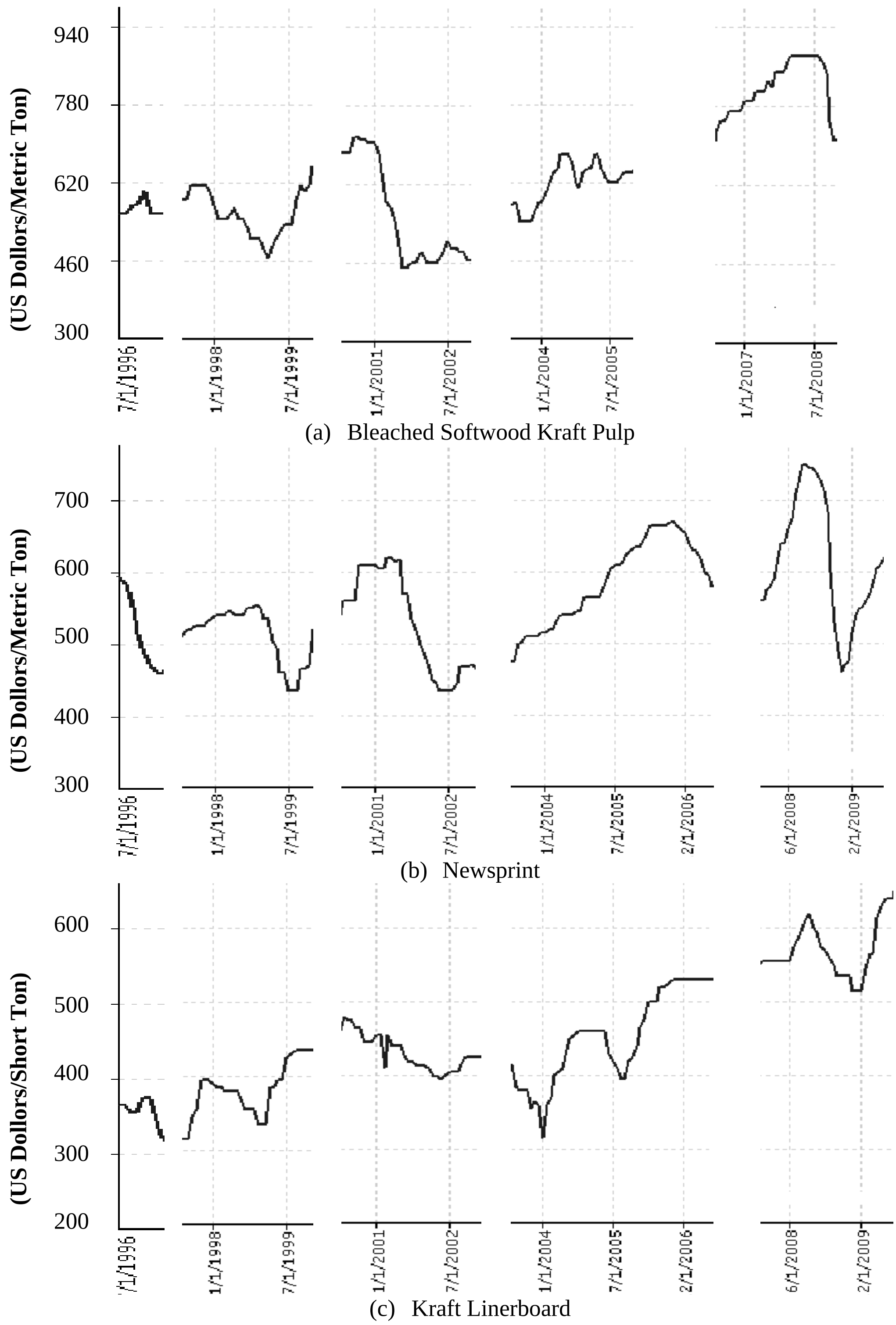


Source: UN Data @ http://faostat.fao.org/site/626/default.aspx#ancor

Figure 2–2: Global Production of Paper Products (1998-2008)

An examination of global prices for the pulp and paper supply chain reveals fluctuations (Pulp&Paper.Net 2011; P&PN 2011). The fluctuating prices are more relevant in the earlier stages of the supply chain where standardized products are manufactured which act like commodities. Three selected products, bleached softwood kraft pulp, linerboard and newsprint are shown in Figure 2-3 with irregular dips in prices followed by periods of growth and stability. The graph breaks in Figure 2-3 indicate unavailability of data for that particular year.

Raw material prices and changes in the world economy are seen as major determinants of the price fluctuations because of their effects on industry capacity and output levels. Haarla (2003) attributed the price fluctuations for communication paper and newsprint to the increase or decrease of producer inventories. He cited evidence from the literature showing that fluctuations are lower for businesses closer to the end user and vice versa.



Source: www.pulpandpaper.net

Note: The price graphs have been published with missing years. To make this clear in Figure 2-3, graphs have been broken at the missing years.

Figure 2–3: Fluctuations in International Prices for Selected Products of Pulp and Paper Industry

Li & Luo (2008) analysed empirical data for the United States linerboard industry to understand the effect of market concentration and other factors contributing to price fluctuations. The perception was that producers respond to low demands by price discounting which is usually matched by competitors pushing prices down. However, during the recession, producers tried to take voluntary downtime in order to offset weak demand and maintain higher prices. Li & Luo's (2008) analyses of price data from 1982 to 2003 suggested that the impact of market concentration on linerboard prices was statistically insignificant. The demand elasticity of linerboard with respect to total industrial production was 0.43; this was highly significant statistically implying a strong corelation with overall economic conditions. Amongst other inputs such as labour and energy costs, only pulpwood prices were statistically significant in explaining demand for linerboard.

Diesen (1998) linked standardized quality requirements and large volumes of bulk products to the commodity cycles exhibited by the bulk products of the pulp and paper supply chain such as market pulp, newsprint and corrugated raw material. The standardized quality requirements allow the end user to interchange the products of different suppliers. Therefore, the bulk products are susceptible to fluctuating prices because the sale price is the single most competitive factor. However, further along the supply chain, the product differentiation increases for specialty and packaging products with higher value added and with other considerations such as quality levels, delivery time, and technical service competing with price to win orders.

The price volatility in the paper industry has also been studied by Whitehead, Lentz & Bonomo (1999). They believed that standardized production with low value addition is to be blamed for the poor financial performances of half of the US paper mills in 1997 as the return on investment was less than the long term US average bond rate. The authors argued that the US paper industry was close to the point where there were diminishing marginal benefits from cost reductions; instead, to remain profitable, the mills have to rely on value creation with a customer centered approach. The authors identified many problems for the capital intensive production approach of the paper industry such as reduced production flexibility resulting in standardized and high-volume segments of demand. Also, any further capacity enhancement would have to buy its way into the market, with obvious implications for pricing. Above all, the capital intensive competitive strategy was often emulated because producers have little or no proprietary process technology. A mill's capacity enhancement can be superseded by a competitor's latest investment. The authors concluded that one of the solutions for asset-intensive paper mills lay in a better understanding of their markets which might provide opportunities to move away from price as the ultimate order winner.

The customer centred approach starts with a focus on segmenting and targeting customers to differentiate the product range by creating a distinctive good or service through promotion, packaging, delivery, customer service, availability and other marketing factors. Product differentiation is deemed crucial for maintaining competitive advantage; it either improves sales at the same price or brings the option to charge a premium. Analysing the dwindling profit margins of Canadian newsprint manufacturers, Oinonen & Malashenko (2000) predicted that the only way to improve revenues was to replace standard newsprint grades with value-added grades. Similarly, Haarla (2003) studied differentiation strategies of newsprint and communication paper producers and observed that the trend to replace standard printing grades by value added grades and increased number of products had been profitable. In addition to product differentiation, diversification has also been identified as a competitive strategy to overcome the strong corelation between overall economic condition and prices. Mills that originally had a focus on one or two grades evolved to include other related products (Lamberg 2007).

Differentiation can also be time based wherein firms develop an understanding of customers' preferences with respect to time and their willingness to pay. Different customer segments can be served profitably through careful time and price-based market segmentation (Boyaci & Ray 2003; Xuying, Stecke & Prasad 2006;

Jayaswal, Jewkes & Ray 2011). Boyaci & Ray (2003) mentioned a case in the pulp and paper supply chain that was catering for the time sensitive demand for printed packaging and publishing materials. Similarly, Boston consulting group carried out a survey of the customers for a corrugated paperboard manufacturer regarding the implementation of a customer relationship management program and found out that punctual product delivery was the top priority (Lange & Andersson 2004).

The above analyses of the market context of pulp and paper suggest that the mills manufacturing standardized products have low profit potential because of substitution threats. However, further along the supply chain, there are opportunities for higher returns by differentiating products with value addition, variety, quality, customer service, technical assistance, delivery speed and reliability. Meeting delivery time requirements has become a major marketing factor.

2.2.2 Australian Paper Industry

Australia is a substantial consumer of paper and paperboard products, its national per capita consumption being recorded as the 14th highest (EarthTrends 2005). Moreover, the annual per capita consumption has been steadily increasing from 177 kilograms (kgs) in 1995 to 202 kgs in 2006 (ABS 2008). The Australian pulp and paper industry includes all sectors of paper manufacturing, from primary processing of raw fibre to produce pulp, to production of printing and communication paper, newsprint, tissue, packaging and paperboard products. The industry had an annual turnover of AUD$8.6 billion in 2005-06 and made a $2.7 billion contribution to GDP. In 2006, Australia produced 0.59% of world pulp which fulfilled only three quarters of domestic demand. The global share of Australian paper production is 1.07% which is also less than the domestic demand (A3P 2008). Australia is only self sufficient in the production of paperboard or packaging material where production exceeds consumption. Newsprint, printing and writing paper and household and sanitary paper have to be imported to meet the domestic demand (Figure 2-4).

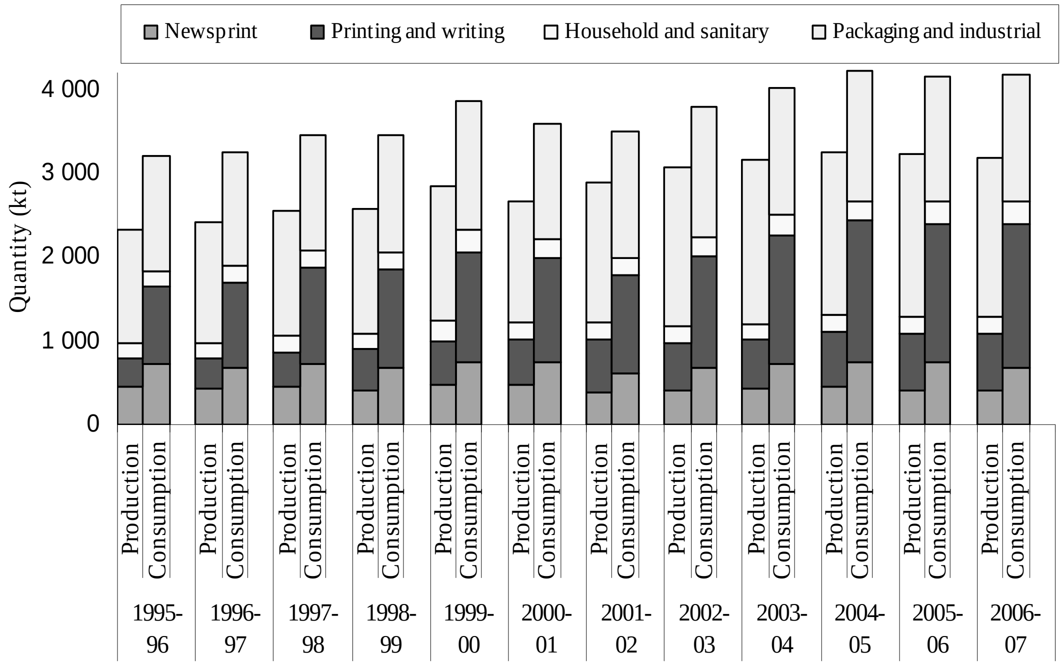


Figure 2–4: Paper and Paperboard Production and Apparent Consumption in Australia (1995-2007)

The Australian pulp and paper industry has some competitive advantages like abundance of inexpensive fibre, fairly low cost energy resources, a highly skilled labour force with best practice working conditions, an attractive investment and business environment, and a regulatory framework that ensures world-class, sustainable environmental outcomes. However, the Australian pulp and paper industry is facing considerable international competition. The Australian industry is one of the more open pulp, paper and paperboard manufacturing industries in the world. It has been estimated that the effective tariff rate for paper and paper products fell from 16 per cent in 1998 to 1.8 per cent in 2008, resulting in significantly increased competitive pressure for Australia's paper producers. There are barriers to the import of cheaper printing, writing and packaging paper because of a 5% tariff but tissues can be imported without any extra cost (P&PISG 2009).

The financial performance of Australian pulp and paper industry reveals dwindling average profit margins from 4.5 % to 2.8 % despite constant capital investment rates around 20% for 2006-07 to 2009-10. The number of profitable businesses in the paper industry also fell from 68.4% to 57.2% during the same period (ABS 2011). These indicators tend to imply that the Australian pulp and paper industry is faced with market dynamics such as described in Section 2.2.1. The emphasis on cost advantage by investing in capacity enhancements has not increased the profitability. Instead, it appears that coupled with increased competition from international producers, the pulp and paper industry is struggling to provide value to its shareholders.

The pulp and paper industry in Australia may face a threat to its already low profitability form the proposed carbon tax (Leslie Feb 24, 2011). The industry is also fairly exposed to fluctuating international prices and is a ‘price-taker’ for its standardized and bulk products. The Australian manufacturers may be able to utilize the concepts discussed in Section 2.1.2 to move away from the price as the only competitive strategy to survive by employing a complete customer centred production approach with emphasis on flexibility and product differentiation with regards to product variety, quality, customer service, technical assistance, delivery speed and reliability.

Incorporation of some of the above factors in an operational planning model has been examined in this thesis such as production of multiple grades on a single paper machine and the subsequent cutting of different grades of jumbo reels according to customer specified dimensions and delivery schedules.

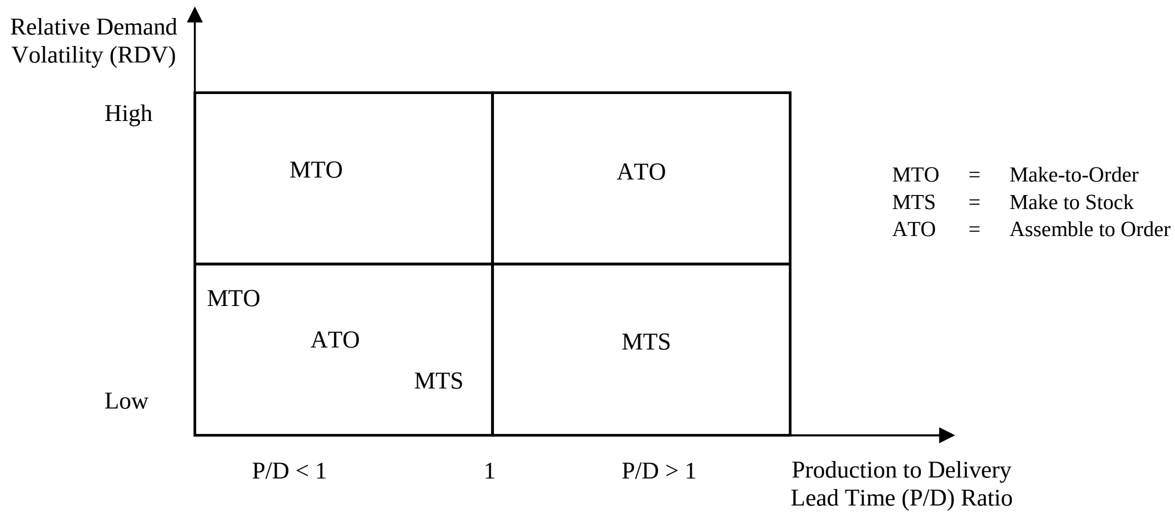
2.3 Supply Chain Issues in the Pulp and Paper Industry

The traditional emphasis in the pulp and paper supply chain has been on scale economies for cost advantage but the intensified competition and increased customer requirements have forced companies to switch to more flexible production planning and shorter production runs. The challenge now is to obtain a balance between flexibility, customer satisfaction and high capacity utilization which is only possible through advanced planning (Porkka 2010). Failure to adapt to ‘customer-centric’ planning may result in unwarranted inventory and stockouts leading to poor financial performance (Whitehead, Lentz & Bonomo 1999; Oinonen & Malashenko 2000; De Treville, Shapiro & Hameri 2004). Similarly, a study of the supply chain issues confronted by the top five paper producers in North Europe revealed that despite significant investment with an average of 45 days of inventory in the distribution system, the delivery lead time were long irrespective of the distance to markets (Koskinen & Hilmola 2008).

2.3.1 Order Penetration Point Location

The Order Penetration Point (OPP) location is probably the first step towards orientation to customer requirements. It is the interface between the Make-To-Stock (MTS) and Make-To-Order (MTO) production philosophies and determines ‘the reach’ of a customer in the production line. The OPP is defined as the point at which the semifinished product (e.g pulp or parent roll) inventory decouples a push system (MTS) from a pull system (MTO). The semifinished product at the OPP is built up based on matching demand forecasts (MTS) whereas the rest of the production distribution is planned just in time and pulled by firm orders.

The positioning of the Order Penetration Point (OPP) in a supply chain is a strategic decision mainly depending upon the response time accepted by customers. The operational planning models may vary substantially with respect to OPP positioning. Moreover, location of an OPP is a product specific decision depending critically on the market pressures, production situation, customer requirements and sometimes location of the mill. Therefore, a mill producing a variety of products may have several different OPPs (Lehtonen & Holmstrom 1998).



Source: Olhager (2003)

Figure 2–5: A Model for Positioning the Order Penetration Point (OPP)

Olhager (2003) identified the production to delivery lead time (P/D) ratio and the relative demand volatility (RDV) as two major factors that affect the strategic positioning of the OPP (Figure 2-5). The RDV is defined as the coefficient of variation, i.e. the standard deviation of demand relative to the average demand. Possible scenarios with different RDV and P/D ratio are shown in Figure 2-5.

Olhager (2003) listed four possible OPP locations in a general manufacturing concern, namely, make-to-stock, assemble-to-order, make-to-order and engineer-to-order. In an assemble-to-order manufacturing environment, parts are manufactured based on demand forecasts while the final assembly is carried out on receipt of the customer's orders. When customer orders dictate the design of a product, the product delivery strategy is said to be engineer-to-design. An adaptation of Olhager's classification of different product delivery strategies for different OPPs in the paper industry is shown in Figure 2-6.

Product Delivery Strategy	Paper Production	Converting Processes		Distribution
	Paper Machine	Jumbo Reel Conversion	Sheeting Processes	Shipment
Make-to-stock	·········	·········	·········	►OPP ——►
Sheet-to-order	·········	·········	► OPP ——	——►
Cut-to-order	·········	► OPP ——	——	——►
Make-to-order	OPP ——	——	——	——►

Figure 2–6: An Adaptation of Olhager's (2003) Product Delivery Strategies to the Paper Industry

Carlsson et al. (2006) maintained that the OPP can be set at three different locations in the paper industry: before the paper machine, at the conversion process or at the warehouse. The order penetration point before the paper machine means a make to order (MTO) production environment and an OPP at the warehouse is classified as a pure make to stock (MTS) production strategy. However, as indicated in Figure 2-6, the order penetration point at the conversion process can be further classified into cut to order (CTO) and sheet to order (STO). In a CTO strategy, jumbo rolls are cut into smaller ones according to customer orders whereas in STO, smaller cut rolls are sheeted to customer specifications (Figure 2-7). The sheet to order strategy can be found in fine paper producers where an inventory of smaller cut rolls is kept to fulfil end demand for sheeted items (Lehtonen 1998; Chauhan, Martel & D'Amour 2008).

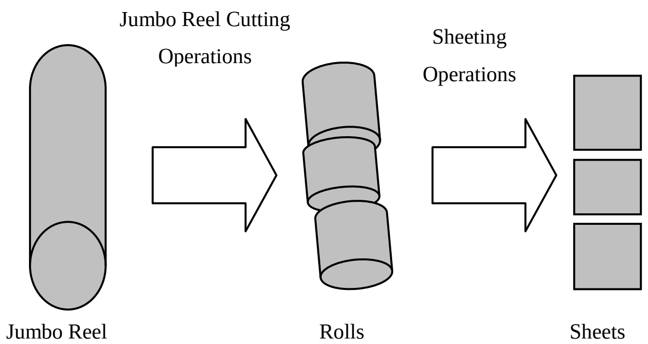


Figure 2–7: Cutting and Sheeting Operations in Paper Mills

Lehtonen (1998) analysed alternative order penetration points under different situational factors through simulation of six Nordic and European paper mills that delivered the finished product to the customers as paper sheets. The study revealed that as the OPP is moved later in the supply chain, the customer delivery time is reduced at the cost of increased inventory. Mill location was also found to have a direct influence on the choice of order penetration point because of delivery time considerations. The case studies revealed that the delivery time was significantly reduced for sheet deliveries if a sheet to order (STO) strategy was employed. For jumbo reels delivery, either an MTS or MTO strategy could be employed depending upon the customer delivery time (Lehtonen 1998).

Hameri & Nikkola (2001) analysed deliveries of three European fine paper mills for one year and came to the conclusion that there was still potential for improving efficiency by better management of OPPs. Traditionally, pulp and paper mills tend to produce to order for high tonnage. For intermediate and small tonnages, a mill tends to satisfy the customer’s demand through its distribution centre or paper merchants. However, there is a trend in paper mills to move closer to their end customers (Martel, Barek & D'Amours 2005).

Although order penetration point location is a strategic decision, it greatly affects the operational planning and scheduling decisions. For example, in a pure MTO environment with the OPP located before paper machines and the end demand being smaller rolls, all the customer orders are grouped together. The consequent demand for jumbo reels is derived from the final demand which initiates the production of jumbo reels at the paper machine. Here the aim is to meet the quoted due date and the resulting plan may yield higher trim loss or grade-changeover cost at the paper machine. Besides, it will also increase the delivery lead time, which will be the sum of paper production at paper machine, paper converting time and the transportation time. However, if a make to stock strategy is applied in the same paper mill, it will give the planner an opportunity to group together the end items in a way that either minimizes the trim loss or reduces the number of grade changeovers in a production cycle even though additional holding costs are incurred. Moreover, it also reduces the delivery lead time which is now only the transport time.

As depicted in Figure 2-5, the ultimate factors in determining the OPP are the time a customer is willing to wait and the demand volatility. Because the final products of the paper industry are subject to different market pressures, writers have recorded a variety of OPP strategies for the pulp and paper supply chain. An MTO strategy is used for integrated planning and scheduling in Murthy et al. (1999), Akkiraju et al. (2001), and Keskinocak et al. (2002) whereas CTO is the production environment preferred by Sweeney (1990), Goulimis (1990), Haessler & Sweeny (1991), Harjunkoski et al. (1996), Giannelos & Georgiadis (2001), Respicio, Captivo & Rodrigues (2002), Menon & Schrage (2002) and Johnston & Sadinlija (2002; 2004). Similarly, a make-to-stock (MTS) approach was employed by Krichagina et al (1998), Rizk, Martel & D'Amours (2004), Bouchriha, Ouhimmou & D'Amours (2007) and Rizk, Martel & D'Amours (2008).

The sheet to order (STO) strategy is employed to deal with the randomness in cut-sheet demand where safety buffers must be created either by stocking jumbo reels, cut rolls, finished products or a combination. Since reels are too large to stock, this option is not practical. When cut rolls are used as a buffer the paper cutting process is decoupled from the paper machine and finished products are sheeted to order. Chauhan et al. (2008) developed an optimization model corresponding to the STO strategy where an assortment of cut roll sizes to stock must be selected so that when a customer order for sheets arrives, cut rolls are taken out of stock and sheeted to the required size on a sheeter. The advantage of this order penetration strategy is that the delivery lead time is reduced significantly: it now includes only the paper sheeting time and the transport, giving a competitive edge.

2.3.2 Planning and Scheduling

De Treville, Shapiro & Hameri (2004) describe the supply chain issues confronted by a paper mill that utilized rules of thumb for its production planning. Large lot sizes caused excessive inventory and long lead times caused frequent stock-outs, even in the presence of large inventories because customer orders did not match the available stocks. An Enterprise Resource Planning (ERP) system was implemented to improve supply chain performance of the mill. Since ERP systems do not perform optimal planning and are an information transactional tool only, a stand alone application of ERP did not achieve the desired results but made the manufacturing inefficiencies worse. In response to the problem, the company acquired production facilities in each major market, creating shorter and more manageable supply chains. This solution, though expensive, solved the immediate problem of poor service levels and excessive inventory investment.

The traditional approach for planning in a pulp and paper mill has been to schedule each stage independently with long production runs. While planning and scheduling for the pulp and paper making processes was done manually as in De Treville, Shapiro & Hameri (2004), operations research has been reportedly applied to plan conversion processes for some time (Goulimis 1990; Sweeney 1990; Haessler & Sweeney 1991; Harjunkoski et al. 1996). However, during the last few years, there has been a gradual paradigm shift towards incorporation of decision support and application of operations research to other processes of the industry in order to improve the supply chain efficiency (Shaw 1998; D'Amours, Ronnqvist & Weintraub 2008).

The optimization attempts for supply chain operational planning in the pulp and paper industry can be categorized according to the two main manufacturing processes of pulp making and then the paper manufacturing and conversion processes.

2.3.2.1 Supply Chain Optimization for Pulp Mills

The collaboration of Linkoping University, Sweden and Sodra Cell AB, a manufacturer of market pulp, resulted in a large body of work on supply chain optimization for the Swedish pulp industry. Bredstrom et al. (2004) developed an integrated mathematical model for production planning and scheduling, transportation and distribution. Carlsson & Ronqvist (2005) mentioned the development of mathematical models for integrating pulp log sorting decisions with transport back haulage and combining facility location with ship routing

2.3.2.2 Supply Chain Optimization for Paper Mills

Paper Production Planning

A paper machine typically switches between different grades of paper, each having different basis weights (g/m2). Every time a paper machine switches its production to a different grade, there are losses of paper sheets that vary depending on which grade was being produced before. Therefore, the changeover costs are sequence dependent and an estimate of changeover costs is based on the paper sheet losses. The production planning problem for paper machines involves a tradeoff between inventory holding costs and grade changeover costs and is identified in the literature as a 'Lot Sizing Problem'. The lot-sizing problem (LSP) for the paper machine has been modelled in a number of ways depending upon the production strategy and the detailed planning requirements (Rizk & Martel 2001).

The details of different lot-sizing models are discussed in Chapter 5 but the important thing to note here is that all the lot-sizing models are known to be NP-Hard (Drexl & Kimms 1997; Rizk & Martel 2001; Karimi, Fatemi Ghomi & Wilson 2003; Tempelmeier & Buschkühl 2008; Quadt & Kuhn 2008; Mateus et al. 2010; Buschkühl et al. 2010). The complexity of lot-sizing models is further increased, as in the case of the paper machine, by the sequence dependency of setups. The industry generally uses a predetermined sequence of grade changes from the lowest in basis weight (g/m^2) to the heaviest and then decreasing the basis weight to the lowest as shown in Figure 2-8 (Viitamaki 2004). Rizk, Martel & D'Amours (2004) called this predetermined sequence a technological constraint for the paper industry, however, it also reduces the complexity of the production problem because of the relative ease in estimating changeover costs. The pre-determined sequence relaxes the sequence dependent setup constraints, allowing the setup costs to be calculated a-priori and to be used as an input in the aggregate planning model.

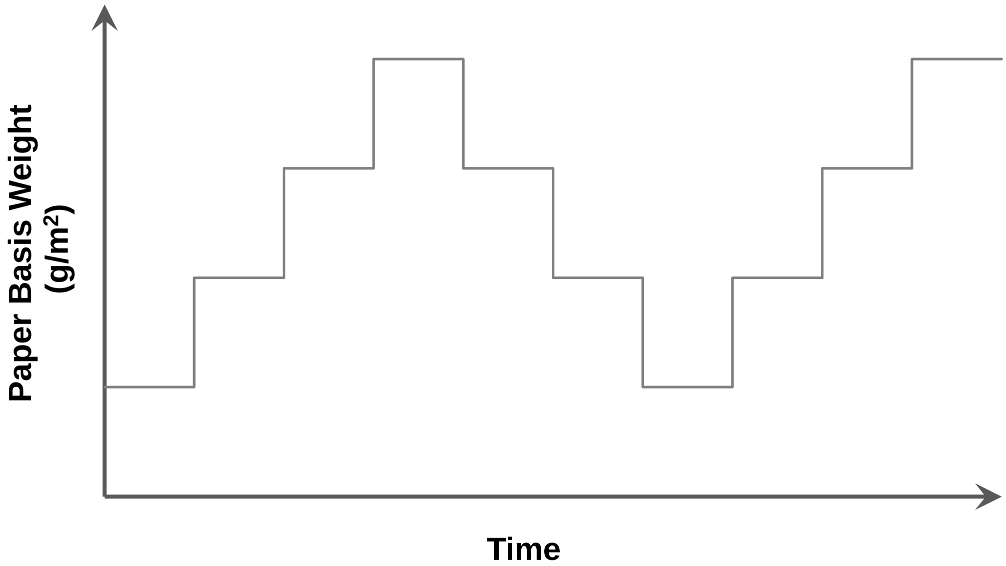


Figure 2–8: Pre-determined Sequence of Grade Changes on a Paper Machine (Hypothetical)

Traditionally, scheduling of paper grades at the paper machine was mostly done manually, resulting in poor service levels and high inventories (De Treville, Shapiro & Hameri 2004; Porkka 2010). Pickard & Yeager (1997) reported one of the first implementations of a decision support system for paper machine scheduling that resulted in $US 700,000 cost savings per year over manual scheduling. Shaw (1998), Murthy et al. (1999), Akkiraju et al. (2001) and Keskinocak et al. (2002) documented the implementation of a decision support system for paper machine scheduling, paper conversion and vehicle loading processes at an American paper mill. Porkka et al (2003) dealt with a paper production planning problem by introducing a setup carryover condition in their Capacitated Lot-Sizing Problem (CLSP) formulation. Rizk, Martel & D’Amours (2004) solved the capacitated lot-sizing problem (CLSP) for multiple paper machines in an exact manner by strengthening the model formulation with valid inequality cuts. Such cuts to a proposed mixed integer formulation allow optimal solution of some problems that previously appeared insoluble. The pre-determined sequence was modelled by assuming at most one setup per planning period. Gupta and Magnusson (2005) modelled the sandpaper production planning problem as a Capacitated Lot-Sizing Problem with sequence dependent setups (CLSPD). Bouchriha, Ouhimmou & D’Amours (2007) evaluated the production planning practice in a Canadian paper mill where plant managers produced volumes or lot sizes within ‘campaigns’ of fixed duration called production cycle length. The ideal production cycle length was first calculated based on the analytical model of Hanssmann (1962) and a production planning model was developed based on the calculated cycle length with a predetermined sequence of products. The same planning problem was also solved ignoring the cycle constraint and it was observed that there were potential cost savings when the cyclic production approach was jettisoned both for one week and two week production cycle lengths.

Planning the Conversion Process

During the conversion stage, the large paper rolls called jumbo reels produced on the paper machine, are cut into different combinations of smaller rolls - called cutting patterns – to meet the end demand. The cutting patterns may or may not match the exact length of the jumbo roll. Trim loss or waste is generated whenever a cutting pattern’s length is less than the jumbo’s length. For example, a paper machine producing jumbo rolls of 10 metres to be cut into three smaller rolls of 4, 3 and 2.5 metres will generate a trim loss of 0.5 metres (Figure 2-9). The decision problem is to minimize the trim loss while fulfilling the finished products demand.

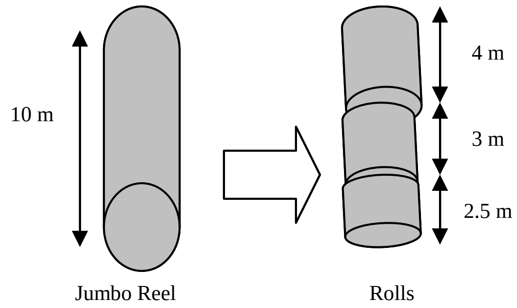


Figure 2–9: Paper Conversion Process

The complexity of the problem stems from an exponential increase in the number of possible cutting patterns with an increase in number and quantity of smaller rolls. For small scale problems, all possible combinations of the cutting patterns can be enumerated and then solving the resulting formulation is straightforward. However, enumerating all possible cutting patterns for most real life problems is hard. The optimization problem for planning conversion processes is to determine the frequencies of cutting patterns that minimize the trim loss. The literature identifies it as a one dimensional cutting stock problem (CSP); the seminal work to solve the CSP to optimality, called the 'column generation approach', was done by Gilmore & Gomory (1961; 1963). Instead of enumerating all possible cutting patterns, attractive columns (patterns) are generated with the help of an auxiliary model and then added to the linear programming formulation of the cutting stock problem. The iterative process stops when the generated columns do not improve the solution.

Column generation virtually guarantees optimal solution of the classical CSP, however, its effectiveness suffers when variants of the classical cutting stock problem are encountered such as CSP with due dates or with knives setup considerations or for a cutting stock problem that contains additional factors in its formulation such as minimization of open stacks (orders already open), constraints on knife arrangements, penalizing excess production etc. Sequential heuristic approaches and meta-heuristics such as genetic algorithms, evolutionary programming, and simulated annealing are employed in the literature to deal with the full complexity of the decision problem. The details of the cutting stock problem, its variants and the solution approaches are treated in Chapter 3 but its relevance for the paper industry and the problem under study is briefly described below

Minimizing trim loss during paper conversion has long been seen as crucial to competitive advantage because cutting patterns with excessive waste will increase the number of jumbo rolls required to fulfil the final products demand, thus pushing costs higher. Goulimis (1990) solved the cutting stock problem for a paperboard mill by enumerating all the possible cutting patterns, and solving the associated integer program. Sweeney (1990) utilized a two stage sequential heuristic to solve the cutting stock problem for a paper mill. Harjunkoski et al. (1996; 1998) tackled a more complex problem by adding knife setups to trim minimization and presented a linear transformation for the originally non linear formulation. Giannelos & Georgiadis (2001) put forward a linear formulation for the cutting stock problem wherein the decision variables corresponded to the items in the cutting pattern whose number increased sharply with the size of the problem. Johnston & Sadinlija (2002; 2004) used an approach that does not necessitate the pre-enumeration of all possible cutting patterns but instead is based on pre-specification of cutting pattern usage levels. This novel approach turned out to be the seminal work on allocating the cutting patterns to planning periods, therefore solving CSP with due dates. Menon & Scharge (2002) combined order allocation with the cutting stock problem to minimize the total cost of production. Correia (2004) used a two phase approach in solving a cutting stock problem for a Portuguese paper mill by first enumerating all possible cutting patterns and then applying a linear programming formulation. Respicio & Captivo (2005) solved a bi-objective sequencing problem for the conversion stage of a paper mill.

As discussed in Section 1.3 (Chapter 1), the conversion stage of the paper supply chain exhibits the characteristics of discrete production with many applications of operations research techniques. However, it appears that there are still variants of the cutting stock problem for the paper industry that need the attention of researchers, such as the joint optimization of the integrated cutting stock and lot-sizing problem with cycle service levels. In the paper supply chain, customer orders specify grade types, roll or sheet sizes, the quantity required, and the due dates. The way to meet customer demand is decided first by solving the cutting stock problem and the solution decides the quantity of different grades of jumbo reels to be produced at the paper machine. The frequency of grade changeovers is determined by the solution to the lot-sizing problem at the paper machine which has implications for the cycle service levels. Therefore there is a strong case for simultaneous treatment of the cutting stock and lot-sizing problems in the paper industry.

2.3.3 Integrated Planning in the Paper Supply Chain

The planning criteria for the different stages of the paper supply chain are variable and at times, conflicting. An optimum plan for the paper machine tends to produce jumbo reels as late as possible while minimizing the grade changeover cost whereas the planning objective for the conversion process is to minimize the trim loss. A good plan for the paper machine may result in higher trim loss and vice versa. Similarly, the distribution stage has its own goals. Therefore, for the optimized operations of the paper mill, it is imperative that a holistic approach to the supply chain planning is adopted. Planning integration for the paper industry and the overall manufacturing sector is a growing research area. A summary of studies integrating either paper making, paper converting or distribution processes is listed in Table 2.1.

Table 2-1: Analyses of Planning Integration in the Pulp and Paper Supply Chain

Authors	**Processes Integrated**
Shaw (1998), Murthy et al. (1999), Akkiraju et al. (2001), Keskinocak et al. (2002)	Paper grades sequencing, paper cutting plans, vehicle load and distribution planning.
Krichagina et al (1998)	Mill shut downs, trim loss and grade sequencing
Respecio (2002)	Paper cutting and paper making without grade changeovers
Rizk et al(2008)	Lot-sizing problem at the paper machine, paper distribution
Poltroniere et al.(2008)	Lot-sizing problem at the paper machine, paper conversion process
Chauhan et al. (2008)	Roll assortment problem, sheet cutting

IBM developed a decision support system for Madison Paper Inc. USA that prepared a list of schedules for the paper machine and would present the corresponding trimming solutions to the operator (Shaw 1998). Vehicle loading and order allocation were also developed and added to the decision support system (Murthy et al. 1999; Akkiraju et al. 2001; Keskinocak et al. 2002). Krichagina et al. (1998) minimized the expected shutdown, trim loss, back ordering and inventory holding cost for a paper manufacturing and converting plant. A two step approach of linear programming and ‘brownian control’ was used to solve the optimization problem to a sub-optimal level.

Respecio et al. (2002) developed an integrated model for paper manufacturing and conversion processes in a make-to-order production environment. However, grade changeover costs at the paper machine were not considered. Rizk et al. (2008) integrated the production planning decisions for the paper machine with the distribution process by developing a mathematical model; however, the model did not include the conversion process. Poltroniere et al.(2008) sought to integrate the lot-sizing problem at the paper machine with the cutting stock problem at the conversion stage by developing an integrated planning model but solved it in a two phase decomposed approach. The objective was minimization of three sets of costs, namely, inventory holding costs, grade changeover costs and trim loss, with no due date or service level considerations. The model was tested against a variety of data and the results showed that solving the cutting stock problem before the lot-sizing problem minimizes total costs.

Chauhan et al. (2008) tackled the problem of selecting the assortment of cut rolls to stock and later on, upon receipt of customer orders, assignment of cut rolls to finished products, i.e sheets, in order to minimize expected trim and inventory holding costs. The proposed solution approaches include a branch and price algorithm based on column generation and a fast pricing heuristic and a greedy marginal cost heuristic. The greedy marginal cost heuristic proposed is simple to implement and very fast, but does not guarantee optimality. However, its simplicity and ease of application prompted the mill to choose it as its decision support system.

Although there have been advances in the integration of supply chain processes in the paper industry, none of the reported studies encompassed the joint optimization of the lot-sizing problem at the paper machine and the cutting stock problem with cycle service levels - the main issue addressed in this thesis.

2.3.4 Service Levels in the Paper Supply Chain

Integrated planning to jointly optimize the cutting stock and lot-sizing problems has a direct effect on the resulting cycle service levels for paper product customers. The customer orders are usually for cut rolls or sheets obtained during the conversion stage of a paper mill and are characterized by paper grade, roll width or sheet dimension, number of rolls required and order due date. As discussed in section 1.3, these orders dictate the production of jumbo reels of different grades at the paper machine. When separately optimizing the two successive manufacturing processes, the solution of the cutting stock problem i.e the minimum number of jumbo reels of different grades required to fulfill the end demand of various cut rolls and sheets determines the production schedule of the paper machine.

Traditionally, the optimization criterion for the pulp and paper supply chain has been minimum total costs. However different final products of the paper industry are subject to different market pressures. High service level is an important consideration for packaging material where meeting the customer’s due date is important. Boston consulting group carried out a survey of the customers for a corrugated box manufacturer in order to implement a customer relationship management program and found that punctual delivery was the top priority (Lange & Andersson 2004).

One of the earliest studies of the effect of different scheduling rules on service levels was by Bookbinder & Higginson (1986). They studied different scheduling rules for a corrugated box manufacturing mill and observed a non-linear relationship between the service levels and trim waste. The scheduling rules that minimized the trim loss resulted in lower service levels and conversely, scheduling rules that gave priority to increased service levels had higher trim loss. The simulation study revealed that a trade-off between service levels and trim loss can be achieved by scheduling with both criteria.

The classical cutting stock problem aggregates the total demand over the entire planning horizon (which may be only one week) and gives a solution based on least trim loss. The cutting stock problem formulation has to be amended to incorporate the due dates of customer orders. As discussed in Section 2.3.2, the cutting stock problem with due dates was tackled by Johnston & Sadinlija (2002) for the paper industry. Johnston & Sadinlija (2004) improved their earlier model by removing the condition of pre-specifying the cutting pattern's usage level or run lengths. Similarly for scheduling a corrugator machine, Velasques et al. (2007) added a variety of objectives such as due date performance, client related importance and finished machine's queue management to the cost minimization criterion.

These two studies focused on solving the cutting stock problem with due dates and it appears that the cutting stock problem with service level has not been extended to paper machine production. This is especially important for the paper industry because the customer orders may comprise different grades and a planning disconnect between the two processes may necessitate holding inventory of different grades of jumbo reels for punctual deliveries.

2.3.5 Multi–Objective Optimization for the Paper Industry

Most of the planning and scheduling studies for paper supply chain discussed so far have treated the problem as conventional single objective optimization. There is a parallel stream of research that maintains that whenever a problem is faced with more than one conflicting objective, the notion of a global optimum does not hold any more. Instead of formulating a single cost function for the conflicting objectives, a multi-objective optimization problem is solved to obtain a Pareto frontier or a set of efficient solutions for the decision maker to choose from. Respicio & Captivo (2005) reported one of the first multiple-objective optimization studies for the conversion stage. Once a least trim loss solution was obtained by the column generation technique, the multi-objective optimization approach was applied to meet production aims of minimizing work in progress and to minimize the time taken to complete work on a customer order.

The study by Velasques et al. (2007) applied multi-objective evolutionary algorithms to solve a scheduling problem for a corrugated box manufacturer. The objectives were due date performance, client related importance, finished machine queue management and the usual criterion of minimum cost.

Matsumoto, Umetani & Nagamochi (2010) solved the cutting stock problem for the paper tube industry with multiple objectives by tabu search. The multiple objectives included minimizing parent roll used, knife setups, open stacks and the combination of cut lengths in the open stacks; these were considered simultaneously. This complex goal facilitated manual handling procedures and took account of shipment lot sizes in the cutting stock problem.

2.3.6 Flexible Lead Times and Supply Chain Efficiency

Bjork & Carlsson (2007) introduced the concept of flexible lead time for the producer in their project to reduce the working capital for all actors in a North European tissue supply chain. Models for combined production planning and delivery decisions for the tissue producer were presented to show the effects of flexible lead times. Because of extensive production setup costs, the tissue producer was accorded flexibility in delivering products to distributors. A mixed integer linear optimization problem (MILP) was formulated to minimize the inventory holding, back ordering and production costs with and without flexibility in the delivery lead times. A commercial MILP solver was used to solve the reduced version of the optimizing problem whereas genetic algorithms (GAs) were used to solve the industrial case study problem. Possible producer savings up to 24% were recorded when the producer was given the flexibility of only one day. The study measured the useful effects of flexible lead time on the tissue production costs but did not analyse its repercussion for the distributors or for entire supply chain.

Bjork, Sialiauka & Carlsson (2008) built on the earlier study by using simulation to measure the effects on the supply chain of flexible lead time for the tissue producer. Total supply chain costs were simulated against the following variables: the lead time between manufacturer and distributor, flexibility in the lead time, variation in the end demand, utilization of machine capacity, service level (distributor to consumer and manufacturer to distributor). The study found that the existing fixed lead time agreements between the distributors and producers yield sub optimal results for the entire supply chain. If the producer is given some flexibility in fulfilling the distributor's order, the total supply chain costs may be reduced by a significant margin. For example, the possible savings in total supply chain costs were found to be more than 5% when the producer is allowed one day flexibility on a two day lead time agreement.

For a few paper products customers may be happy to be flexible with their delivery times for the sake of cost advantage but these studies cannot be extended to the entire supply chain. As discussed in Section 2.3.4, higher service level is an important consideration for packaging material and meeting the customer's due date is important.

2.3.7 Just-in-time (JIT) Philosophy and the Paper Supply Chain

Just-in-time philosophy has been responsible for an overhaul of manufacturing practices around the world because of its contribution to the successes of the Japanese automotive sector. JIT has two components - JIT manufacturing and JIT purchasing. JIT manufacturing is characterized by pull demand, quick setups, small lot-sizes, flexible resources, uniform plant loading and a supply based on JIT purchasing. The supply procedure based on JIT purchasing ensures inventories are delivered by reliable suppliers in time for the manufacturing process and inventories held on hand are at a minimum. The successful execution of JIT components leads to improvements in company performance. Addressing both internal operations and supplier-customer relationships; its implementation at one of the stages is likely to contribute to a general improvement in the supply chain.

JIT has attracted a great deal of attention in discrete production systems but the process industry, including the pulp and paper supply chain appears to lag behind. Lehtonen & Holmstrom (1998) is one study testing the applicability of JIT to paper mills. Their four case studies simulation exercise was aimed at finding the effects of reducing the production cycle and varying the location of the order penetration point (OPP).

The usual strategy in paper mills of running long production runs to maximize capacity utilization has been diametrically opposed to JIT principles of pull based production and smaller lot-sizes to ensure production flexibility and improve customer service. However, intense competition and customer requirements have forced mills to switch to pull based demand, smaller production runs and more frequent grade changeovers, thus, moving towards JIT. The location of the order penetration point (OPP) is an important determinant of the degree of conformity to JIT. A production system is well adapted to JIT if a customer order triggers the start of production; in the context of a paper mill an order for finished products pulls the production of the corresponding grade of jumbo reel. This could only be possible with the OPP placed before the paper machine and with the planning approach indicated in this thesis of jointly considering the lot-sizing problem at the paper machine and the cutting stock problem at the conversion stage.

2.3.8 Synchronizing Product Flow

Synchronization is the process of prioritizing independent demand from customer orders or demand forecasts and of deriving all dependent demand for the necessary components and materials accordingly. Synchronized product flow ensures a continuous and coordinated flow of material and helps to reduce safety buffers between different stages of a supply chain by decreasing the variance of production and distribution quantities (Rohde & Wagner 2005). One of the major issues confronting the paper industry is synchronizing product flow in the manufacturing processes. The production processes are generally de-coupled and even in a pure make-to-order environment the dependent demand, i.e the quantities of jumbo reels to be produced on the paper machine, is not synchronized with the subsequent production or distribution processes. This approach results in substantial inventories of intermediate products. For example, Poltroniere et al. (2008) developed a coupled planning model for paper machine scheduling and for the cutting stock problem, however the planning model included holding inventories of both the jumbo rolls and the finished products i.e the cut rolls. Synchronized product flow in a truly joint optimization of the lot-sizing and cutting stock problem would do away with the inventory of intermediate products because only those jumbo reels would be produced at the paper machine which would be cut to smaller rolls to fulfil the end demand. It would also help to bring the supply chain closer to the just in time (JIT) principle where the in-process inventory is reduced.

2.4 Achievements of Earlier Work and Unexplored Areas of Research

This chapter’s review reveals that the emphasis in the paper industry on maximizing machine utilization and minimizing cost has had adverse effects on supply chain benchmarks such as over capacity, long lead times, excessive inventory and poor customer service (Ranta, Ollus & Leppänen 1992; Hameri & Holmström 1997; Hameri & Lehtonen 2001; De Treville, Shapiro & Hameri 2004; Malik & Qiu 2008). This led to a gradual shift to a more flexible production strategy with shorter production cycle times and increased number of grade changeovers for better customer service. While the capacity driven strategy may still be valid for a few standard products with high volume, the increased customization of paper products warrants a focus on meeting customer requirements through a flexible production approach.

The literature review also establishes that there have been substantial advances in implementing operations research techniques and decision support system for managing paper supply processes. However, as pointed out by Koskinen & Hilmola (2008), there are numerous supply chain issues confronted by the paper industry which are reflected in the high levels of in-process inventory. This is partly explained by the apparent planning disconnect between processes. The key issue is to synchronize product flow along the supply chain and joint optimize the manufacturing processes but most of the literature deals with individual processes. Section 2.3.3 highlighted the need for planning integration and a gradual paradigm shift in the industry towards it; however, the studies reviewed did not achieve synchronized product flow through the paper manufacturing and conversion stages by joint optimization.

The processes of paper making and conversion are strongly interrelated because the dependent demand for jumbo reels at the paper machine is determined by the schedule prepared for the conversion stage, which is faced with the final demand. However these processes are subject to conflicting objectives. A good plan for the paper machine will produce jumbo rolls as late as possible and with least changeovers but it may give higher trim loss. Conversely, a good schedule for the conversion plant may require more grade changes than when considered in isolation. Integration of these two processes is imperative for a good composite plan.

As already noted, Poltroniere et al. (2008) sought to integrate the lot-sizing problem at the paper machine and the cutting stock problem at the paper converting process but the solution approach was two phased and decomposed and, in fact, they concluded that solving the cutting stock problem before the lot-sizing problem minimized the total costs. The relevant cost parameters namely grade changeover, inventory holding costs and trim loss were considered but the due date was not included. Also, the planning model assumed an order penetration point (OPP) before the converting machine and both the intermediate and finished products incurred inventory holding costs. Similarly the planning models with due date considerations have mostly included grade changeover and inventory holding costs or trim loss and its variants but all these criteria have not been included in a single planning model. This can be attributed to the complexity of the joint problem which combines two well known NP-hard problems. The lot sizing problem with setup considerations and the cutting stock problem when combined together is a non-convex optimization problem and due date considerations further add to the complexity. The individual cutting stock problem with due date is itself a very difficult problem and has only been recently addressed for the steel industry (Reinertsen & Vossen 2010). There have been few similar precedents in the paper industry but the impediments discussed in Section 2.3.2.2 indicate why progress has been slow.

Integration of these features into an optimizing model where multiple paper grades are produced and cut strictly to customer orders is at the core of this thesis. This is achieved by developing a new method of joint optimization with a customer focus as reported in Chapter 5. The integrated planning model for the lot-sizing and cutting stock activities in two successive stages of the paper mill - operating under a make-to-order production strategy and with due date considerations – will synchronize the product flow through these two stages and it would provide the first building block for synchronizing the product flow along the entire supply chain. Before the formulation of the joint optimization, individual processes of cutting stock and lot-sizing are considered in detail. The lot-sizing model is deferred to Chapter 5 whereas the cutting stock problem (CSP) is discussed in the next chapter which is at the heart of the joint optimizing approach because the integrated cutting stock and lot-sizing model is essentially an extension of it with added complexities.

CHAPTER 3. THE CUTTING STOCK PROBLEM (CSP)

3.1 Introduction

In Chapter two, the literature on the planning and optimization of the pulp and paper supply chain was reviewed and paper manufacturing and conversion were identified as the core production processes. Paper manufacturing comprises production of different grades on the paper machine whereas during the conversion stage the larger jumbo reels are efficiently cut into smaller rolls to meet the customer demand. The literature classifies the conversion stage as the cutting stock problem (CSP) and it is crucial in the development of an integrated cutting stock and lot-sizing model because it determines the demand for the jumbo reels to be produced at the paper machine. The cutting stock problem on its own is an NP Hard combinatorial problem but if the lot-sizing decisions are integrated with it, as has been done in this thesis, the associated complexity increases greatly. The goal of minimizing trim loss is now joined by other objectives such as minimizing grade changeovers and inventory holding costs and minimizing late orders. Most real industrial cutting stock problems include similar additional constraints and objectives and are called variants of the cutting stock problem. For example, restricting the jobs in process along with the minimization of trim loss is an integration of two NP–hard problems of pattern generation and sequencing (Becceneri, Yanasse & Soma 2004; Alves 2005). In such scenarios, application of exact algorithms such as column generation or branch and price has had limited success and there have been instances where heuristic solutions have been preferred.

This chapter reviews the classical cutting stock problem (CSP) and the exact solution approaches, such as column generation and branch and price. Variants of the cutting stock problem are reviewed to assess the appropriateness of solution approaches.

The literature credits Kantorovich (1960) with the formal identification of the cutting stock problem (CSP) as an optimizing problem. The larger intermediate products are cut according to different combinations of smaller finished products, called cutting patterns, which may or may not match the exact length of the intermediate products. Scrap or waste is generated whenever a cutting pattern's length is less than the intermediate product's length. Kantorovich (1960) expressed the optimal production plan as the one that minimizes the scrap.

Minimize	$\sum_{j=1}^{n} y_j$				(3.1)
Subject to	$\sum_{j=1}^{n} x_{ij}$	$\geq$	d_i	$i = 1, \ldots, m$	(3.2)
	$\sum_{i=1}^{m} x_{ij} l_i$	$\leq$	$L\, y_j$	$j = 1, \ldots, n$	(3.3)
Where	y_j	$\in$	$\{0,1\}$	$j = 1, \ldots, n$	(3.4)
	x_{ij}	$\geq$	*0 and integer*	$i = 1, \ldots, m,$ $j = 1, \ldots, n$	(3.5)

The objective function (3.1) has a binary objective function where y_j represents the choice of roll j $(j=1,\ldots, n)$, $y_j = 1$, if roll j is used, and $y_j = 0$, otherwise. The number of items of size l_i where $i = 1,\ldots, m,$ assigned to roll j is given by x_{ij}. The demand of finished products is denoted by d_i and L is the length of jumbo reel. Constraint (3.2) stipulates the fulfillment of end demand whereas constraint (3.3) enforces the jumbo length constraint.

Although a watershed in optimization, the Kantorovich formulation has some weaknesses which restrict its use. Mainly, the lower bound on the corresponding linear relaxation solution is given by $\sum_{i=1}^{m} l_i/L$ which can be very weak especially for instances when there is a large amount of waste (Vance 1994; Ben Amor & Valerio de Carvalho 2005). Vance (1994) cited examples where the ratio of LP bound to the optimal integer values approached 1/2. This establishes the poor LP bound as a major drawback of the Kantorovich formulation.

Other issues associated with the Kantorovich model are the need to obtain a priori an upper bound 'n' on the total number of rolls to be used and symmetry of its solution space. The upper bound 'n' on the number of rolls can be calculated by employing heuristics. The symmetry of solution space implies that different solutions to the model, with the same cutting patterns swapped in different rolls, will correspond to the same global cutting solution. Ostrowski (2009) identified symmetry (the existence of multiple equivalent solutions) as a main problem for this formulation because it identifies specific rolls from which a pattern is to be cut. Moreover, Ben Amor & Valerio de Carvalho (2005) highlighted the link between solution space symmetry of the Kantorovich model with the difficulty in applying branching rules to obtain integer values.

An alternate formulation proposed by Gilmore and Gomory (1961; 1963) overcomes all shortcomings of the Kantorovich model, such as the poor LP bound, and it does not contain any symmetry (Ostrowski 2009). The Gilmore and Gomory formulation also minimizes the total number of jumbo rolls to be used in order to fulfill a given demand but it relaxes the binary integer constraint. Mathematically, let x and j denote a jumbo roll and a cutting pattern respectively, then, x_j describes the number of times the *jth* pattern is used on the intermediate products. Also, let A_{ij} be the number of times the order width i is produced on pattern j. If length of the jumbo reel is given by L whereas finished product i's demand and width is given by d_i and l_i, then, the cutting stock problem can be formulated as follows:

Minimize $$\sum_{j \in J} x_j \tag{3.6}$$

Subject to

$$\sum_{j \in J} A_{ij} x_j \geq d_i \tag{3.7}$$

Where $$x_j \geq 0, \textit{Integer} \tag{3.8}$$

For a valid cutting pattern, following condition has to be met:

$$\sum_{i \in I} A_{ij} l_i \leq L \tag{3.9}$$

The cutting stock problem (CSP) belongs to a wider category of cutting and packing (C&P) problems - which have attracted a great deal of attention from both academics and practitioners. Bin packing, strip packing, vehicle loading, pallet and container loading, ship scheduling, nesting and partitioning problems are other C&P problems. Dyckhoff (1990) gave a typology of C&P problems and classified the cutting stock problem as (1/V/I/R) with ‘1’ referring to the number of geometric dimensions which are to be cut, ‘V’ refers to the assignment of all smaller items to the larger objects, ‘I’ refers to several but identical large objects and R refers to many end items of relatively few different sizes. Wascher, Haussner & Schumann (2007) modified Dyckhoff’s typology by replacing ‘V’ with input minimization and ‘R’ with a weakly heterogeneous assortment of small items for the classification of the one dimensional cutting stock problem.

Bin packing and cutting stock problems are NP-hard problems i.e these are considered to be as hard as the hardest in the Non Deterministic Polynomial Time (NP) family of decision problems (Garey & Johnson 1979; Gau & Wascher 1995; Chengbin & Antonio 1999; Alves 2005; Mukhacheva & Mukhacheva 2006; Yang et al. 2009). Bin packing problems are similar to cutting stock problems except for the low demand for its end items. Dyckhoff classified it as (1/V/I/M) where M represents many end items of many different shapes. The modified typology of Wascher, Haussner & Schumann (2007) replaces the weak heterogeneous assortment of small items by a strongly heterogeneous assortment of small items meaning that only very few end elements are of identical shape and size. If that happens, the end items are treated as individual elements and the demand for each item is equal to one.

The complexity associated with the CSP arises from the presence of different possible combinations through which an intermediate product (IP) can be cut to meet the demand. For small scale problems, all possible combinations of the cutting patterns can be enumerated and then solving the resulting problem is straightforward. However, the number of cutting patterns grows exponentially with an increase in the number of items to be cut from the jumbo reel. In most practical cases, the number of cutting patterns J is prohibitively large and since many of these cutting patterns may never be used in the optimal plan because of excessive waste Gilmore & Gomory (1961; 1963) devised a method through which only those cutting patterns were used in the LP relaxation of the above model that improved the results. The iterative process of adding new cutting patterns or columns is called the column generation method. Along with the column generation method, sequential heuristic procedures and meta-heuristic techniques are the other two main solution approaches applied to the one dimensional cutting stock problem.

3.2 Column Generation for Cutting Stock Problems

Column generation is a method to solve large scale linear programming models with a large number of variables by using explicitly only a fraction of the formulation's data. It does not necessitate the prior enumeration of all possible cutting patterns but instead, through an iterative exchange of information between two sub-models – restricted master problem and pricing sub model, obtains an optimal solution by considering only attractive columns i.e the columns that improve the results. The restricted master problem (RMP) is the LP relaxation of the original integer cutting stock problem with only a few cutting patterns and is formulated as:

Minimize	$\sum_{j \in J'} x_{j'}$			(3.10)
Subject to	$\sum_{i' \in I'} A_{i\,j'}\, x_{j'}$	$\geq$	d_i	(3.11)
Where	$x_{j'}$	$\geq$	0	(3.12)

Let y_f be a cutting pattern that improves RMP results but has not been added to it. If λ_f are the shadow prices associated with RMP solution that generates y_f, then, the formulation of pricing sub-model can be given as :

Maximize	$\sum_f \lambda_f y_f$			(3.13)
Subject to	$\sum_f y_f\, l_f$	$\leq$	L	(3.14)
Where	y_f	$\geq$	$0, Integer$	(3.15)

There are two main differences between the restricted master problem (RMP) and the original formulation of the cutting stock problems. The initial number of cutting patterns *J'* is far smaller than the *J* cutting patterns in (3.6). Constraint (3.9) is imposed in the pricing sub-model as strictly negative (3.14) so that the generated columns do not exceed the length of the jumbo rolls *L*. The pricing sub-model utilizes the shadow prices associated with the RMP to find columns with negative reduced costs in an iterative procedure as shown in Figure: 3-1.

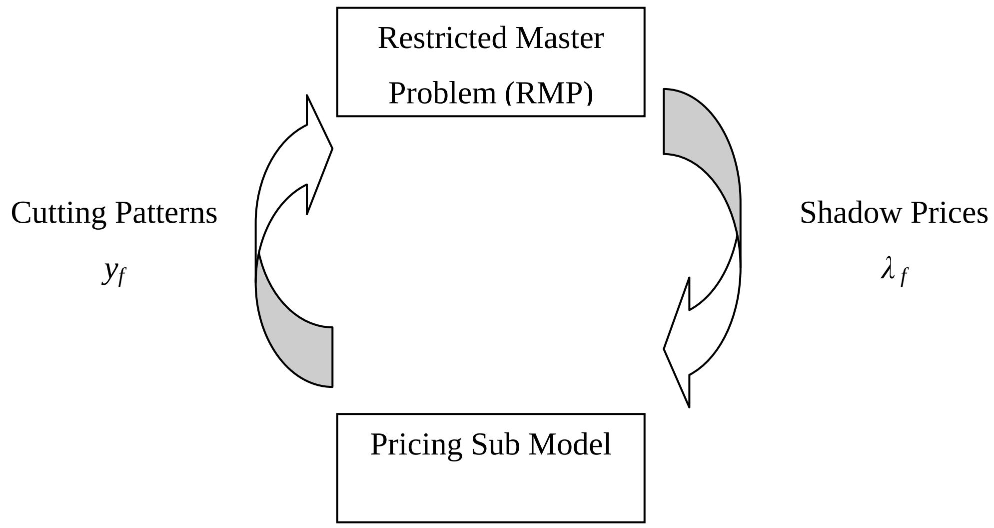


Figure 3–1: Schematic of Column Generation Approach

These columns are added to the RMP and it is re-optimized. Shadow prices with new patterns included are again used to derive columns with negative reduced cost in the pricing model. If no column with negative reduced cost is found, that implies that there are no other columns that can improve the result of the restricted master problem and, hence, the solution is an optimum one. This is called the reduced cost criterion of the simplex method. Any feasible solution to the pricing sub-model corresponds to a feasible cutting pattern in the cutting-stock problem. If the optimal solution for the pricing sub-model is > 1, its reduced cost is given as (Bisschop 2008):

$$0 > 1 - \sum_f \lambda_f y_f \qquad (3.16)$$

Whenever the term $\sum_f \lambda_f y_f$ is greater than one, equation (3.16) is satisfied. This implies that $\sum_f \lambda_f y_f$ as an objective function has to be maximized in the pricing sub-model (3.13). Whenever the optimal value of the pricing sub-model is > 1, the generated cutting patterns are of interest and would be added to the restricted master problem (RMP). The iterative process (Figure 3-1) will be stopped when the optimal solution is less than or equal to 1 because the inequality (3.16) is not met signifying that there does not exist a cutting pattern with negative reduced cost.

Illustrative Example: An Excel Implementation

The column generation procedure is best explained with the following example.

Consider a paper machine that produces jumbo reels which are to be cut into four items. The data used in the following discussion (Table 3-1) has been adapted from Chvatal (1983).

Table 3-1: Example Data

Jumbo Length *(L)*	10 Feet	
No of Final Product *(i)*	4	
#	**Finals Width l_i *(ft)***	**Quantity Required d_i**
1	4.5	97
2	3.6	610
3	3.1	395
4	1.4	211

Source: Adapted from Chvatal (1983)

As discussed in Section 3.1, the complexity of a cutting stock problem stems from the simultaneous determination of the best possible cutting patterns and its frequencies. If all possible cutting patterns can be enumerated then solving the resulting problem through linear or integer programming is straightforward. However the enumeration of cutting patterns is a hard combinatorial problem because the number of possible patterns increases exponentially with the number of finals to be cut from the jumbo reel and also because most of those will never be used owing to high trim loss. The example data with only four finals to be cut has 37 possible cutting patterns which are enumerated in Table 3-2. With an increase in the number of finals required, the possible cutting patterns may run into hundreds or even more.

Column generation solves the linear relaxation of the cutting stock problem with a few cutting patterns only and the resulting shadow prices drive the process to find attractive cutting patterns that improve the solution quality. The procedure is explained with the help of the example data.

Table 3-2: Enumeration of all Possible Cutting Patterns

Pattern J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
x_j																			
A_{1j}	2	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
A_{2j}	0	1	1	0	0	0	0	0	0	2	2	2	1	1	1	1	1	1	1
A_{3j}	0	0	0	1	1	0	0	0	0	0	0	0	2	1	1	1	0	0	0
A_{4j}	0	1	0	1	0	3	2	1	0	2	1	0	0	2	1	0	4	3	4
$A_{1j}\,l_1$	9	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	0	0	0	0	0	0	0	0	0	0
$A_{2j}\,l_2$	0	3.6	3.6	0	0	0	0	0	0	7.2	7.2	7.2	3.6	3.6	3.6	3.6	3.6	3.6	3.6
$A_{3j}\,l_3$	0	0	0	3.1	3.1	0	0	0	0	0	0	0	6.2	3.1	3.1	3.1	0	0	0
$A_{4j}\,l_4$	0	1.4	0	1.4	0	4.2	2.8	1.4	0	2.8	1.4	0	0	2.8	1.4	0	5.6	4.2	5.6
$\sum A_{ij}\,l_i$	9	9.5	8.1	9	7.6	8.7	7.3	5.9	4.5	10	8.6	7.2	9.8	9.5	8.1	6.7	9.2	7.8	9.2
$L-\sum A_{ij}\,l_i$	1	0.5	1.9	1	2.4	1.3	2.7	4.1	5.5	0	1.4	2.8	0.2	0.5	1.9	3.3	0.8	2.2	0.8
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\sum A_{ij}\,x_j$																			

Pattern J	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
x_j																		
A_{1j}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A_{2j}	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A_{3j}	0	0	3	2	2	2	1	1	1	1	1	0	0	0	0	0	0	0
A_{4j}	1	0	0	2	1	0	4	3	2	1	0	7	6	5	4	3	2	1
$A_{1j}\,l_1$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{2j}\,l_2$	3.6	3.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{3j}\,l_3$	0	0	9.3	6.2	6.2	6.2	3.1	3.1	3.1	3.1	3.1	0	0	0	0	0	0	0
$A_{4j}\,l_4$	1.4	0	0	2.8	1.4	0	5.6	4.2	2.8	1.4	0	9.8	8.4	7	5.6	4.2	2.8	1.4
$\sum A_{ij}\,l_i$	5	3.6	9.3	9	7.6	6.2	8.7	7.3	5.9	4.5	3.1	9.8	8.4	7	5.6	4.2	2.8	1.4
$L-\sum A_{ij}\,l_i$	5	6.4	0.7	1	2.4	3.8	1.3	2.7	4.1	5.5	6.9	0.2	1.6	3	4.4	5.8	7.2	8.6
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$A_{1j}\,x_j$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\sum A_{ij}\,x_j$																		

Notations

x_j = Number of times the jth pattern is used on the jumbo reel

L = Length of Jumbo reel

l_i = Order width of roll i

A_{ij} = No of times the order width 'i' is produced on pattern j

$\sum A_{ij}\,l_i$ = Total width of a cutting pattern (For a cutting pattern to be feasible, it should always be less than L)

$L-\sum A_{ij}\,l_i$ = Trim Loss

$\sum A_{ij}\,x_j$ = Total number of final products produced by the use of the jth pattern

The restricted master problem is formulated by considering four cutting patterns. The initial cutting patterns do not have to be very efficient because the subsequent iterations will guide the process towards the optimal cutting patterns. However, the only concern is that all final cuts are represented in the initial cutting patterns so that a feasible solution is obtained by meeting the final demand 'd_i'. The linear relaxation can easily be solved with the help of Excel solver. The results of the first iteration are shown below (Table 3-3):

Table 3-3: Column Generation Procedure Illustrated - First Iteration

Restricted Master Problem (RMP)

Iteration 1

Minimize 1005

Pattern J	1	2	3	4	5	6	7	8	9	10	Sum		
x_j	0	610	0	395							1005		
A_{1j}	2	1	1	1	***0***								
A_{2j}	0	1	1	0	***1***								
A_{3j}	0	0	0	1	***2***								
A_{4j}	0	1	0	1	***0***								
$A_{1j}\,l_1$	9	4.5	4.5	4.5									
$A_{2j}\,l_2$	0	3.6	3.6	0									
$A_{3j}\,l_3$	0	0	0	3.1									
$A_{4j}\,l_4$	0	1.4	0	1.4									
$\sum A_{ij}\,l_i$	9	9.5	8.1	9							35.6		
$L-\sum A_{ij}\,l_i$	1	0.5	1.9	1							$\sum A_{ij}\,x_j$		d_i
$A_{1j}\,x_j$	0	610	0	395							1005	>=	97
$A_{1j}\,x_j$	0	610	0	0							610	>=	610
$A_{1j}\,x_j$	0	0	0	395							395	>=	395
$A_{1j}\,x_j$	0	610	0	395							1005	>=	211

λ_f y_f

Pricing Sub Model (PSM)

Maximize 3

f	1	2	3	4
λ_f	***0***	***1***	***1***	***0***
y_f	0	1	2	0
l_f	4.5	3.6	3.1	1.4
$y_f\,l_f$	0	1	1	0
$\Sigma\,y_f\,l_f$	9.8	<=	10	

The information exchange *(y_f & λ_f)* between restricted master problem (RMP) and pricing sub model (PSM) is shown above in italics, bold and bigger fonts. The pricing sub model (PSM) is solved with the shadow prices λ_f obtained from the restricted master problem (RMP) which results in the cutting pattern y_f that is likely to improve the RMP solution.

For the second iteration, the restricted master problem (RMP) is solved with an additional pattern obtained in the previous iteration and improved results are noted. The number of jumbo reels required to fulfil the given demand has been reduced from 1005 to 610 (Table 3-4). The shadow prices are again used to solve the pricing sub model which will generate a new cutting pattern that will further improve the results. The results of iteration 2 are as follows (Table 3-4):

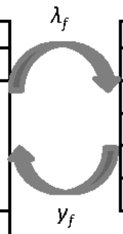
Table 3-4: Column Generation Procedure Illustrated - Second Iteration

Restricted Master Problem (RMP)

Iteration 2

Minimize 610

Pattern J	1	2	3	4	5	6	7	8	9	10	Sum		
x_j	0	412.5	0	0	197.5						610		
A_{1j}	2	1	1	1	0	***0***							
A_{2j}	0	1	1	0	1	***2***							
A_{3j}	0	0	0	1	2	***0***							
A_{4j}	0	1	0	1	0	***0***							
$A_{1j}\, l_1$	9	4.5	4.5	4.5	0								
$A_{2j}\, l_2$	0	3.6	3.6	0	3.6								
$A_{3j}\, l_3$	0	0	0	3.1	6.2								
$A_{4j}\, l_4$	0	1.4	0	1.4	0								
$\sum A_{ij}\, l_i$	9	9.5	8.1	9	9.8						45.4		
$L - \sum A_{ij}\, l_i$	1	0.5	1.9	1	0.2						$\sum A_{ij}\, x_j$		d_i
$A_{1j}\, x_j$	0	412.5	0	0	0						412.5	>=	97
$A_{1j}\, x_j$	0	412.5	0	0	197.5						610	>=	610
$A_{1j}\, x_j$	0	0	0	0	395						395	>=	395
$A_{1j}\, x_j$	0	412.5	0	0	0						412.5	>=	211



Pricing Sub Model (PSM)

Maximize 2

f	1	2	3	4
λ_f	***0***	***1***	***0***	***0***
y_f	0	2	0	0
l_f	4.5	3.6	3.1	1.4
$y_f\, l_f$	0	7.2	0	0
$\sum y_f\, l_f$	7.2	<=	10	

The iterative process is repeated till no further cutting pattern is obtained that improves the existing results. The number of jumbo reels required was reduced to 509.25 in the third iteration, 468.54 in the fourth iteration and the best value is 452.25 obtained in the fifth iteration (Table 3-5). While covering the theoretical background of the column generation approach, it was noted that the pricing sub model can only result in a cutting pattern that improves the restricted master problem if its objective function value is greater than 1 (Result 3.18). Therefore, the fifth iteration gives us the optimal values with the best cutting pattern identified and the corresponding frequencies determined. The noticeable advantage of column generation is that it required only eight patterns to reach the optimal solution. The other alternative could have been to enumerate the 37 possible cutting patterns (Table 3-2) and then apply a solution algorithm. That approach is impractical for real world cutting stock problems because it may require prior enumeration of hundreds or thousands of cutting patterns.

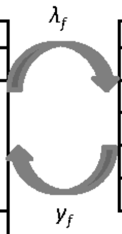
Table 3-5: Column Generation Procedure Illustrated – Third, Fourth and Fifth Iterations

Restricted Master Problem (RMP) **Pricing Sub Model (PSM)**

Iteration 3

Minimize 509.25

Pattern J	1	2	3	4	5	6	7	8	9	10	Sum		
x_j	0	211	0	0	197.5	100.75					509.25		
A_{1j}	2	1	1	1	0	0	***0***						
A_{2j}	0	1	1	0	1	2	***0***						
A_{3j}	0	0	0	1	2	0	***0***						
A_{4j}	0	1	0	1	0	0	***7***						
$A_{1j}\,l_1$	9	4.5	4.5	4.5	0	0							
$A_{2j}\,l_2$	0	3.6	3.6	0	3.6	7.2							
$A_{3j}\,l_3$	0	0	0	3.1	6.2	0							
$A_{4j}\,l_4$	0	1.4	0	1.4	0	0							
$\sum A_{ij}\,l_i$	9	9.5	8.1	9	9.8	7.2							
$L-\sum A_{ij}\,l_i$	1	0.5	1.9	1	0.2	2.8					$\sum A_{ij}\,x_j$		d_i
$A_{1j}\,x_j$	0	211	0	0	0	0					211	>=	97
$A_{2j}\,x_j$	0	211	0	0	197.5	201.5					610	>=	610
$A_{3j}\,x_j$	0	0	0	0	395	0					395	>=	395
$A_{4j}\,x_j$	0	211	0	0	0	0					211	>=	211



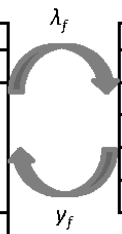
Maximize 3.5

f	1	2	3	4
λ_f	***0***	***0.5***	***0.25***	***0.5***
y_f	0	0	0	7
l_f	4.5	3.6	3.1	1.4
$y_f\,l_f$	0	0	0	9.8
$\sum y_f\,l_f$	9.8	<=	10	

Iteration 4

Minimize 468.54

Pattern J	1	2	3	4	5	6	7	8	9	10	Sum		
x_j	0	97	0	0	197.5	157.75	16.29				468.54		
A_{1j}	2	1	1	1	0	0	0	***0***					
A_{2j}	0	1	1	0	1	2	0	***2***					
A_{3j}	0	0	0	1	2	0	0	***0***					
A_{4j}	0	1	0	1	0	0	7	***2***					
$A_{1j}\,l_1$	9	4.5	4.5	4.5	0	0	0						
$A_{2j}\,l_2$	0	3.6	3.6	0	3.6	7.2	0						
$A_{3j}\,l_3$	0	0	0	3.1	6.2	0	0						
$A_{4j}\,l_4$	0	1.4	0	1.4	0	0	9.8						
$\sum A_{ij}\,l_i$	9	9.5	8.1	9	9.8	7.2	9.8						
$L-\sum A_{ij}\,l_i$	1	0.5	1.9	1	0.2	2.8	0.2				$\sum A_{ij}\,x_j$		d_i
$A_{1j}\,x_j$	0	97	0	0	0	0	0				97	>=	97
$A_{2j}\,x_j$	0	97	0	0	197.5	315.5	0				610	>=	610
$A_{3j}\,x_j$	0	0	0	0	395	0	0				395	>=	395
$A_{4j}\,x_j$	0	97	0	0	0	0	114.03				211.03	>=	211



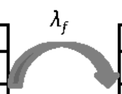
Maximize 1.284

f	1	2	3	4
λ_f	***0.36***	***0.5***	***0.25***	***0.14***
y_f	0	2	0	2
l_f	4.5	3.6	3.1	1.4
$y_f\,l_f$	0	7.2	0	2.8
$\sum y_f\,l_f$	10	<=	10	

Iteration 5

Minimize 452.25

Pattern J	1	2	3	4	5	6	7	8	9	10	Sum		
x_j	48.5	0	0	0	197.5	100.75	0	105.5			452.25		
A_{1j}	2	1	1	1	0	0	0	0					
A_{2j}	0	1	1	0	1	2	0	2					
A_{3j}	0	0	0	1	2	0	0	0					
A_{4j}	0	1	0	1	0	0	7	2					
$A_{1j}\,l_1$	9	4.5	4.5	4.5	0	0	0	0					
$A_{2j}\,l_2$	0	3.6	3.6	0	3.6	7.2	0	7.2					
$A_{3j}\,l_3$	0	0	0	3.1	6.2	0	0	0					
$A_{4j}\,l_4$	0	1.4	0	1.4	0	0	9.8	2.8					
$\sum A_{ij}\,l_i$	9	9.5	8.1	9	9.8	7.2	9.8	10					
$L-\sum A_{ij}\,l_i$	1	0.5	1.9	1	0.2	2.8	0.2	0			$\sum A_{ij}\,x_j$		d_i
$A_{1j}\,x_j$	97	0	0	0	0	0	0	0			97	>=	97
$A_{2j}\,x_j$	0	0	0	0	197.5	201.5	0	211			610	>=	610
$A_{3j}\,x_j$	0	0	0	0	395	0	0	0			395	>=	395
$A_{4j}\,x_j$	0	0	0	0	0	0	0	211			211	>=	211



Maximize 1

f	1	2	3	4
λ_f	***0.5***	***0.5***	***0.25***	***0***
y_f	0	1	2	0
l_f	4.5	3.6	3.1	1.4
$y_f\,l_f$	0	3.6	6.2	0
$\sum y_f\,l_f$	9.8	<=	10	

The restricted master problem is the linear relaxation and can easily be solved by Excel's Solver. The pricing sub model is an integer formulation and can also be solved by the branch and bound algorithm of Excel's solver. The overall problem is modelled in Excel and a Visual Basic for Application (VBA) macro is written to automate the repeated applications of solver and also to automate the information exchange between the two models. With a convenient user interface, the column generation procedure is carried out in few seconds by just a click to start the process. The column generation approach has the capacity to obtain an optimal solution for problems of any size. Industrial cutting stock problems with millions of possible cutting patterns are easily and instantly solved because only those patterns are used which improve the solution. The column generation's Excel implementation developed for this thesis can easily be modified to handle a variety of input data such as demand, number of final products etc. A variety of test problems for the classical cutting stock problems are randomly generated and solved by this Excel column generation model in Chapter 4.

The cutting stock example highlights the usefulness of the column generation approach for optimization problems with a lot of possibilities. It has been widely used for combinatorial problems that involve a large number of variables for example air line crew scheduling, vehicle routing problems, travelling salesman problem with time windows, air network design for express shipment service, fleet assignment and aircraft routing and scheduling, ship routing and scheduling, job grouping for flexible manufacturing systems, grouping and packaging of electronic circuits, bandwidth packing in telecommunication networks, traffic assignment in satellite communication systems, graph colouring, generalized assignment problems and graph partitioning problems (Desrosiers & Lubbecke 2005).

The column generation method works on the linear relaxation of the original integer optimization problem and the fractional solutions are to be converted to integers for meaningful interpretation. In its simplest form, the fractional values can be rounded up to the nearest integer values but it may compromise the optimality. Various ways to generate integer optimal or near optimal solutions from the continuous output of the column generation method are discussed in the next section.

3.3 Integers and the Column Generation Approach

The column generation approach involves LP relaxation of the integer formulation and the pricing model is driven through shadow prices of the restricted master problem. Therefore, obtaining integer solutions to the cutting stock problem in an exact manner is an issue that was perceived previously to be intractable (Gilmore. 1979). Appropriate rounding was considered to be the only way and the same was suggested by the authors pioneering the use of the column generation approach for cutting stock problems (Gilmore & Gomory 1961; Gilmore & Gomory 1963). However, the last two decades have witnessed extensive work on utilizing branch and bound techniques in conjunction with column generation approach to obtain an integer solution, called 'Branch and Price'. At each node of the branch-and-bound tree, branching is carried out i.e LP relaxation is solved using column generation with additional constraints in order to remove the fractional values.

Moreover, reformulation and decomposition of the integer models to obtain 'tighter relaxations' has also been suggested so that the difference between the optimal solutions of the integer and corresponding linear relaxation is minimal which facilitates the successful application of 'branch and price' solution technique. A tighter relaxation of the integer programming problem approximates the convex hull of feasible solutions in such a way that the continuous solution is not far from the integer solution whereas a weak formulation may result in a substantial difference between the two. The Kantrovich formulation for the binary cutting stock problem is known to have a weak formulation (Vance 1994) i.e the difference between the solutions of the binary cutting stock problem and its linear relaxation may be significant. Therefore column generation may not be a good solution technique because it only solves the linear relaxation of a given integer problem. In such scenarios, integer problems are reformulated and with decomposition techniques, tighter relaxations are obtained so that the continuous solutions are closer to the integer solution (P. Vance 1998; Holthaus 2002; Vanderbeck & Savelsbergh 2006; Vanderbeck & Wolsey 2010).

Unlike the Kantrovich binary cutting stock problem, the LP relaxation of the Gilmore-Gomory model is considered to be very tight (Ben Amor & Valerio de Carvalho 2005). Several classes of the cutting stock problems using the Gilmore-Gomory's formulation were shown to have an optimal value given by the least integer greater than or equal to the optimal value of its linear programming relaxation. This characteristic of the CSP is called the Integer Round-Up Property (IRUP), which the authors argued, is exhibited by most real world cutting stock problems (Marcotte 1985). Similarly, it was shown through theoretical investigations and extensive experiments that the cutting stock problems that do not exhibit IRUP conform to the Modified integer Round Up Property (MIRUP). An integer programming problem is said to possess MIRUP if the optimal integer solution is within two units of the optimal value of the corresponding linear programming relaxation. In other words, the modified integer round-up property for a linear integer minimization problem means that the optimal value of this problem is not greater than the optimal value of the corresponding LP relaxation rounded up plus one (Scheithauer & Terno 1995; Scheithauer & Terno 1997; Nitsche, Scheithauer & Terno 1998; Rietz & Dempe 2008; Reinertsen & Vossen 2010).

Most of the earlier research on obtaining the integer solutions for the cutting stock problem has focused on using rounding heuristics in conjunction with the column generation approach to bridge the small gap between the LP relaxation and the integer solutions. Different reformulation and decomposition approaches have also been used to obtain tighter relaxations to facilitate the branching operations during the branch and price procedure. Details are given in the following sub sections

3.3.1 Rounding Heuristics

The earliest method of obtaining integer solutions from the linear relaxation of the cutting stock problem has been through the use of appropriate rounding heuristics. Extensive work has been reported in the literature on the difference between the optimal solution of the integer and the corresponding continuous relaxation. Wascher & Gau (1996) evaluated the solution quality of different rounding heuristics for cutting stock problems. The class of heuristics that was considered started from an initial optimal solution for the continuous relaxation of the cutting stock problem, obtained by column generation, and then proceeded to an integer solution in the neighbourhood of the linear optimum solution. 4000 instances of randomly generated test problems were solved and it was revealed that two of the heuristics performed exceptionally well by finding an optimal solution for 98.0 % and 92.7 % of the test instances. For all the test instances, the deviation of the heuristic from the optimal value was just one unit.

The effectiveness of Washer & Gau (1996)'s residual rounding heuristics was also confirmed when Poldi & Arenales (2009) used these heuristics to for solve the integer one-dimensional CSP with multiple stock lengths, a variant of the classical cutting stock problem with added complexities The experimental results showed that the residual rounding heuristics performed as well as the exact methods.

Reinertsen & Vossen (2010) applied the column generation technique to solve the one dimensional cutting stock problem with due dates and used a simple but effective way to obtain an integer solution on the presumption that their model conforms to the IRUP or MIRUP which is fine because they used Gilmore and Gomory's formulation known to have a tight relaxation. Instead of carrying out branching operations during the column generation iterations as is customary in the branch and price algorithm, the resulting integer programming model was solved using all the patterns that were found with the LP relaxation. This yielded a good feasible integer solution to the original integer cutting stock problem with due dates.

3.3.2 Decomposition Techniques for Tighter Relaxations

The second approach, decomposition and reformulation, helps to define the problem in an alternate way that facilitates efficient solution approaches for linear and integer programming problems. Decomposition techniques can be advantageous for solving the cutting stock problem in the following ways:

— Decomposition techniques can minimize the difference between the integer programming solution and the corresponding linear relaxation especially for the binary cutting stock problem. With the resulting tighter formulation, the rounding heuristics of Section 3.3.1 can be justified as solution approaches to remove the fractional values obtained by the column generation technique.

— Decomposition can also be useful for problems with tighter relaxations because the reformulated problem is more suitable for the implementation of the branch and price algorithm to obtain an integer solution. (Barnhart et al. 1998).

The application of decomposition methods to the binary cutting stock problem results in as tight a formulation as the Gilmore and Gomory (1961; 1963) formulation. Even for tight relaxations such as the Gilmore and Gomory model, reformulation helps to obtain the integer solutions by facilitating the branch and price algorithm (Villeneuve et al. 2005). The general integer programming reformulation and the decomposition principle is explained as follows:

Let a polyhedron Q contain the feasible set of integer vectors for an integer programming problem in such a way that

$$Q = \{ x \in R^n | A x \geq b\} \tag{3.17}$$

The constraint matrix $A \in Q^{m \times n}$ and $b \in Q^m$ is the vector of constraint requirements. Let $F = Q \cap Z^n$ be the feasible set of solutions and P the convex hull of F. The integer optimization problem for P is given as:

$$Z_{IP} = min_{x \in Z^n}\{c^T x \,| A x \geq b\} = min_{x \in F}\{c^T x\} = min_{x \in P}\{c^T x\} \tag{3.18}$$

The continuous approximation or linear relaxation of the original integer problem is the most common method for approximating the convex hull. Mathematically:

$$Z_{LP} = min_{x \in R^n}\{c^T x \,| A x \geq b\} = min_{x \in Q}\{c^T x\} \tag{3.19}$$

The resulting solution of the continuous approximation can be far from the integer optimum, i.e

$$Z_{IP} \gg Z_{LP} \tag{3.20}$$

The formulation leading to such an approximation is referred to as a 'weak formulation'. Weak formulations can be improved by dynamically generating additional polyhedral information that can be used to augment the approximation. An improved formulation that approximates well the convex hull is called a 'tighter relaxation'. Moreover, it also facilitates the subsequent application of the branching procedures to derive the integer solutions.

Cutting plane methods, Dantzig–Wolfe (D-W) reformulation and lagrangian relaxation are popular decomposition approaches that can be used to obtain models for integer and combinatorial optimization problems with tighter LP relaxations (Vanderbeck & Wolsey 2010).

The dynamic construction of an additional polyhedron which can be intersected with Q to form a better approximation is explained as follows

$$min_{x \in Z^n}\{c^T x \mid A' x \geq b'\} = min_{x \in F'}\{c^T x\} = min_{x \in P'}\{c^T x\} \qquad (3.21)$$

Equation (3.19) is the continuous relaxation of (3.18) where $F' = \{ x \in Z^n | A'x \geq b'\}$ for some $A'x \in Q^{m' \times n}, b' \in Q^{m'}$ and P' is the convex hull of F' (also, $F \subset F'$).

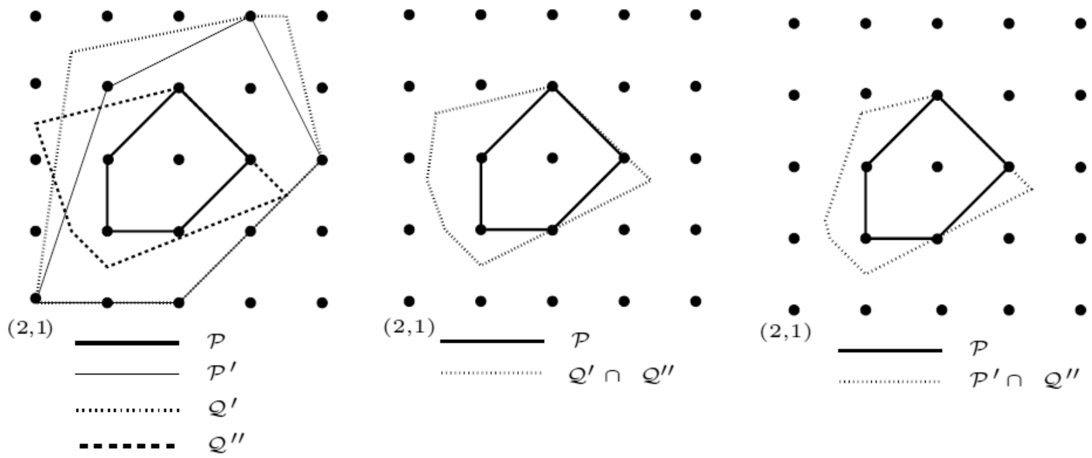
Side constraints $[A'', b''] \in Q^{m \times (n+1)}$ are associated with P' in such a way that $Q = \{ x \in R^n | A'x \geq b', A'' x \geq b''\}$.

Let Q' denote the polyhedron described by inequalities $[A', b']$ and Q'' denote the polyhedron described by the inequalities $[A'', b'']$. Therefore, $Q = Q' \cap Q'' and\ F = \{x \in Z^n |\ x \in P' \cap Q''\}$.

For the decomposition to be effective, $P' \cap Q'' \subset Q$ so that the bound obtained by optimizing over $P' \cap Q''$ is as good as the LP bound and strictly better for some other objective functions (Ralphs & Galati 2006). Therefore,

$$Z_D = min_{x \in p'}\{c^T x \mid A'' x \geq b''\} = min_{x \in F' \cap Q''}\{c^T x\} \quad (3.22)$$
$$= min_{x \in P' \cap Q''}\{c^T x\}$$

The useful effects of decomposition can also be illustrated by the following general integer optimization problem. Let $P = conv\{x \in R^2 \mid A'x \geq b', A''x \geq b''\}$, $Q' = \{x \in R^2 \mid A'x \geq b'\}$, $Q'' = \{x \in R^2 \mid A''x \geq b''\}$ and $P' = conv(Q' \cap Z^2)$. Figure 3-2 shows the additional polyhedron generation and shows that the optimization over $P' \cap Q''$ leads to an improvement over the continuous approximation $Q' \cap Q''$ (Ralphs & Galati 2006).



Source: Ralphs & Galati (2006)

Figure 3–2: Additional Polyhedra and Convex Hull Approximation

The Dantzig–Wolfe (D-W) decomposition approach has been universally applied as a reformulation of the cutting stock problem for the successful implementation of the branch and price algorithm (Vance 1994; P. Vance 1998; Vanderbeck 1999; Vanderbeck 2000b; Holthaus 2002; Belov & Scheithauer 2002; Valerio de

Carvalho 2002; Villeneuve et al. 2005; Ben Amor & Valerio de Carvalho 2005; Alves 2005; Alves & Valério de Carvalho 2008).

3.3.3 Branch and Price

The Dantzig-Wolfe decomposition is just a reformulation technique that only gives an alternate description of the problem to help the implementation of the actual optimization method i.e 'branch and price' devised to obtain the integer solutions. Branch and price (B&P) is a general approach that uses branch and bound (B&B) in conjunction with the column generation procedure to remove the fractional values obtained by solving the linear relaxation of the original integer problem. Using the branch and bound technique to obtain an integer solution for the cutting stock problem is not challenging if all the possible cutting patterns are enumerated first. The resulting formulation is solved by LP and then, the B&B algorithm is applied to obtain integers. However, this approach is only feasible for small scale problems. Goulimis (1990) employed the same technique to remove the fractional values from his solution. As discussed earlier, the column generation technique helps to tackle large scale problems with exponential variables by pricing out only the variables that improve the results. Applying branch-and-bound to a continuous formulation that utilizes a column generation technique is not trivial. The challenge lies in finding a branching rule that excludes the current fractional solution, validly partitions the solution space of the problem, and provides a sub-problem that is tractable (Vance et al. 1994).

Earlier applications of a customized branch and price algorithm included real world problems that could be modelled as set covering or partitioning problems with tight formulations, for example, the binary bin packing problem (Vance et al. 1994), vehicle routing problems (Bramel & Simchi-Levi 1997), generalized assignment and crew scheduling problems (Barnhart et al. 1998). The customized branching algorithm of Vance et al. (1994) developed for the binary bin packing problem was found to be inefficient for the general integer cutting stock problem. In subsequent work, Vance (1998) developed separate branching algorithms for the two formulations of the cutting stock problem and it was observed that both algorithms were successful at solving real-world instances. However, the B&P algorithm for general integer formulation struggled against randomly generated test data. The time required to solve the binary cutting stock problem also increased exponentially with an increase in problem size. Vance et al. (1994; 1998) employed a convexification approach for the Dantzig-Wolfe decomposition wherein the knapsack constraint was replaced by a convex combination of the extreme points. Vanderbeck (2000b) proposed Dantzig-Wolfe decomposition of the CSP on the 'discretization' of the integer polyhedron associated with a subsystem of constraints and showed that it performs just as well or better when compared with Vance's B&P algorithms. The discretization approach is based on Nemhauser & Wolsey (1988) and is particularly useful for the general integer cutting stock problem where the integral restriction of the original problem naturally translates into integral restrictions for the restricted master problem. The convexification approach enforces the integer constraint through the convex combination that arises in the master problem (Ben Amor & Valerio de Carvalho 2005).

Lubbecke & Desrosiers (2005) reviewed the developments in column generation and concluded that the implementation of a column-generation-based integer programming code is still an issue, mainly because of the vast possibilities for tuning each of them.

Puchinger et al.(2008) developed software capable of solving integer optimization problems with branch and price. The first step of the B&P algorithm is the application of Dantzig-Wolfe (D-W) decomposition, followed by the column generation and the branching procedures. The software is claimed to have provisions to tackle certain issues peculiar to the cutting stock problem like symmetries due to identical sub problems and customized branching procedures. However these provisions require the user to implement an application specific branching scheme. Similarly Vanderbeck (2010) highlighted various shortcomings of available generic and customized branch and price algorithms and introduced his own generic scheme. The results of computational testing of his generic scheme for the cutting stock problem and the bin packing problem indicate that the proposed scheme is a practical way to get to integer values while not modifying the structure of the pricing problem and avoiding symmetry. Both Puchinger's software and Vanderbeck's generic scheme could only solve the classical one dimensional cutting stock problem whereas, as discussed in Chapter 2, the cutting stock problem encountered in various industrial settings including the pulp and paper supply chain are complicated by additional considerations and constraints.

3.3.4 Limitations of the Branch and Price Algorithm

All the preceding efforts pertain to the classical cutting stock problem wherein minimization of trim loss is the only objective which can also be accomplished by minimizing the maximum number of jumbo reels required to meet the end demand (Objective 3.6); both these formulations yield identical results. Erjavec, Gradisar & Trkman (2009) contended that the CSP has been tackled too narrowly and decrease in trim loss is only part of the optimization goal in many industrial settings. Industrial cutting stock problems encountered in the real world include additional constraints and objectives and are called variants of the cutting stock problem. CSP in itself is NP hard and additional constraints or objectives add to its complexity. For example, restricting the jobs in process along with minimization of trim loss is an integration of two NP–hard problems of pattern generation and sequencing (Becceneri, Yanasse & Soma 2004; Alves 2005). In such scenarios, application of exact algorithms such as column generation or branch and price has had limited success and there have been various instances where heuristic solutions have been preferred.

The next section examines the literature on variants of the cutting stock problem and also classifies the work of various authors on the basis of the CSP variant treated.

3.4 Variants of the CSP and their Solutions

The classical cutting stock problem with only a trim minimization goal is best solved with the column generation method. The fractional values in the solution can be removed by resorting to rounding heuristics or by the branch and price algorithm. However, most industrial cutting stock problems are extensions of the classical CSP with added constraints and objectives. For example, Haessler (1975) identified knife setup cost as an important factor during the conversion stage of a paper mill. Similarly, for a two dimensional steel cutting problem, Wascher (1990) solved it as a vector optimization problem against multiple criteria of material cost, trim loss, storage and handling costs and over run costs. Since then, most of the cutting stock problems tackled in the literature are variants of the classical cutting stock problem with additional objectives such as cutting knife setups, minimization of open orders or contiguity, minimization of open stacks, minimization of open stacks and setups, due dates considerations, service level constraints, CSP with multiple objectives and integration of CSP with other production processes. The effectiveness of an exact solution approach like branch and price is limited for such complex problems leaving heuristic solutions as the preferred option. Details are given in the following seven subsections

3.4.1 CSP with Knife Setups and Pattern Minimization

The first variant is the CSP with knives setups which has also been modelled as the pattern minimization problem. Different cutting patterns in a cutting plan correspond to different arrangement of the knives and a setup cost is incurred whenever the arrangement of knives is to be changed for a new cutting pattern. The knife setup cost is substantial for industrial settings with scarce capacities because changing knives results in unproductive or idle time and also the transitions may induce errors. Therefore, minimization of knife setups needed to fulfill the customer demand is an important consideration for some industrial processes and has attracted a great deal of attention. For solving the cutting stock problem, knives setup cost can be included in the following four different ways:

- As a primary objective: Knife setup considerations can be used as the main optimization objective instead of the traditional trim loss criterion. This is somewhat unrealistic and is only possible when the cost of input material is far less than the knife changeover costs.
- In a combined minimization of cost objective: The most frequently reported approach in the literature has been to minimize combined costs that include both the knife setup and trim loss.
- As a secondary optimization objective: Minimization of Knife setup cost can also be used as a secondary optimization objective to the trim loss criterion. Once an optimal plan is obtained, the number of distinct cutting patterns is to be minimized. This is known as a pattern minimization problem (PMP).
- Multi-objective optimization: Instead of formulating a single cost function for knife setup and trim loss, a bi-objective optimization problem is solved to obtain a Pareto frontier or a set of efficient solutions.

Haessler (1975) made one of the earliest proposals that recognized the tradeoff between knife changeover and the trim loss by developing a sequential heuristic procedure. The heuristic first generated a list of candidate patterns which were sequentially added to the solution based on the low waste and high frequency criteria. Chen, Hart & Tham (1996) solved a combined objective function comprising trim loss and knife setup cost with simulated annealing. Harjunkoski et al. (1996; 1998) studied an optimization problem for the conversion stage of a paper mill which also included minimization of trim loss along with the knife changeover. The heuristic solution pre-generated an accepted set of cutting patterns with a specified trim loss and then, a joint optimization problem for the knife setup cost reduction and trim loss was solved by integer programming.

In a pattern minimizing problem, minimization of knife setup cost is carried out as a secondary optimization objective to the trim loss criterion. Once an optimal plan is obtained, the number of distinct cutting patterns is to be minimized. McDiarmid (1999) classified the pattern minimization problem as 'strongly NP-hard' even if the trim loss solution was trivial implying PMP's enhanced complexity. Forester & Wáscher (2000) used lexicographic search to minimize the number of setups or different patterns by first obtaining the traditional trim loss solution and then the number of setups was reduced by repeatedly replacing subsets of the optimal cutting patterns by other cutting pattern sets which also fulfil the given demand for smaller rolls. Vanderbeck (2000a) sought to solve the pattern minimization problem in an exact manner by formulating it as a quadratic integer non linear optimization problem with an upper bound on the number of jumbo reels given by the classical cutting stock problem. The weak linear relaxation was strengthened by applying Dantzig-Wolfe reformulation and was checked for 16 real world problem instances. Computational results showed that an optimal solution was obtained for 12 relatively small instances but as the problem size increased, the optimality of the solutions suffered. Moreover, the effectiveness of Vanderbeck's algorithm was unclear against a variety of input data such as randomly generated test data (Gau & Wascher 1995) which has been extensively used in the literature. Gau & Wäscher (1995) generated random instances for number of order lengths, average demand and the length factor which is the ratio of the order lengths to the jumbo length. For an effective CSP algorithm, the results must hold for different combinations of ordered lengths, average demand and the length factor.

Alves (2005) showed in his PhD thesis that Vanderback's model even after reformulation had a poor continuous bound i.e the integer solution is far from the linear approximation ; he strengthened it by additional cutting planes. The resulting price and cut algorithm slightly improved Vanderbeck's results but when checked against randomly generated test data, it could not solve many instances to optimality. Belov & Scheithauer (2003) developed a combined optimization model for trim loss and setup minimization and used a branch and price algorithm to solve it. Computational experiments showed that the algorithm improved Foerster & Wáscher (2000) results but its performance was worse than the Vanderbeck (2000a).

Umetani, Yagiura & Ibaraki (2003) proposed an iterated local search to minimise the number of jumbo reels used by restricting the number of cutting pattern to a user defined number "n". It was observed that by increasing "n", trim loss was reduced and therefore, a tradeoff curve was obtained for a range of "n" values. Umetani, Yagiura & Ibaraki (2006) improved their earlier algorithm by using a local search that alternated between two types of local search processes and also used linear programming to reduce the number of solutions in each neighbourhood. Results showed that the hybrid approach not only improved their earlier results but also outperformed Foerster & Wáscher (2000) and Belov & Scheithauer (2003). Similarly, Cui et al. (2008) presented a sequential heuristic algorithm for the pattern minimization problem which differed from the traditional heuristic procedure as instead of using all remaining items to generate the current pattern, it uses a selected subset of remaining items. The subset is selected such that the number of patterns is reduced.

Golfeto & Neto (2009) is the only approach where multi-objective evolutionary optimization algorthim was used to solve the cutting stock problem with knife setup considerations. A 'symbiotic relationship', between the population of solutions and the population of cutting patterns, was used to obtain an approximation to the Pareto-frontier.

3.4.2 CSP with Contiguity, Open Stacks & Order Spread Considerations

The second of variants, CSP with contiguity, open stacks or order spread are related cases that simultaneously minimize the trim loss and the number of incomplete orders. Most of these problems have been solved with heuristic solutions to complement few implementations of exact algorithms for small to medium sized problems.

Hinterding & Khan (1995) formulated a cutting stock problem which had a joint optimization objective of minimizing both trim loss and incomplete orders. The formulation was termed a cutting stock problem with contiguity and was solved with genetic algorithm. The ordered CSP and minimization of open stacks or order spread are similar concepts and have been deemed necessary optimization criteria because of material handling and storage space constraints. Solving the ordered CSP and the CSP with contiguity, open stacks or order spread considerations is particularly challenging because it integrates two NP-hard problems of pattern generation and sequencing (Becceneri, Yanasse & Soma 2004; Alves 2005).

Foerster & Wascher (1998) used simulated annealing to sequence the cutting patterns in a way that also minimized the open order spread along with the trim loss. Similarly, Becceneri, Yanasse & Soma (2004) developed a heuristic solution for the same problem experienced in a sawmill whereas Yanasse & Pinto Lamosa (2007) used lagrangian relaxation to decompose the minimization of order spread problem into two sub problems, the classical cutting stock problem and the pattern sequencing problem. The classical cutting stock problem was solved with column generation while the pattern sequencing was handled by an enumeration scheme.

Ragsdale & Zobel (2004) dealt with the ordered cutting stock problem for a window and door manufacturer restricting the number of open jobs at any given time to one while minimizing the trim loss. This had important repercussions for the manufacturing firm as it greatly reduced the in-process inventory and simplified the material handling operations. The solution method was genetic algorithm with a customized heuristic. Alves (2005), as part of his PhD project, tried to develop an exact solution to the ordered cutting stock problem with the help of a branch-price-cut algorithm. The computational results showed that for bigger problems, the quality of solution greatly deteriorated.

Respicio & Captivo (2005) solved a bi-objective sequencing problem for a paper mill. The objectives were minimization of the maximum number of open stacks and the minimization of the average order spread. The Pareto optimal set was approximated by SPEA 2, a multi-objective evolutionary algorithm hybridized with a local search procedure.

3.4.3 CSP with Minimization of Open stacks and Setups

The third variant simultaneously minimized the number of open stacks i.e number of incomplete customer orders and the number of knives setups. Belov & Scheithauer (2007) developed a stepwise sequential heuristic for a combined problem of minimizing trim loss, open stacks and setup cost. As a first step, the number of open stacks was restricted to two and a sequential heuristic was developed to minimize the trim loss. Then, the heuristic was extended to restrict any given number of open stacks. In the last step, the sequential heuristic was integrated into a setup minimization problem.

3.4.4 CSP with Multiple Stock Lengths

The fourth CSP variant extends the single machine problem to multiple machines. As discussed in introductory Section 3.1, the classical cutting stock problems where a single size of stock or jumbo length is available to be cut into smaller order lengths is classified as the ‘Single Stock Size Cutting Stock Problem (SSSCSP)’.

However, in various industrial settings, multiple machines are available to cut the larger objects into smaller ones and the corresponding cutting stock problem is called the 'Multiple Stock Size Cutting Stock Problem (MSSCSP)' (Wascher, Haussner & Schumann 2007). The problem typically arises in the pulp and paper supply chain where a company may own multiple plants or may have multiple paper machines in a single plant. The production problem is again a composite problem with two NP-hard problems, the cutting stock problem and the order allocation to different stock size machines, combined together.

Most of the earlier solution approaches to the Multiple Stock Size Cutting Stock Problem MSSCSP were heuristic. Gilmore & Gomory (1963) did extend their integer cutting stock formulation to multiple stock lengths but unlike the single size stock cutting stock formulation, the MSSCSP formulation has a weak linear relaxation and the integer roundup property (IRUP) does not apply (Belov & Scheithauer 2002; Alves 2005). Roodman (1986) found near optimal integer solutions using a procedure based on the solution of the LP relaxation complemented by a heuristic. Gradisar, Resinovic & Kljajic (1999) and Gradisar & Trkman (2005) hybridised the sequential heuristic procedure with the linear programming based methods to tackle the MSSCSP. Holthaus (2002) solved MSSCSP by applying specific rounding heuristics to the column generation solutions. Belov and Scheithauer (2002) proposed an exact method that combined the 'Chvatal-Gomory' cutting planes and column generation to solve the one-dimensional cutting stock problem with multiple stock lengths. Alves (2005) and Alves & Valério de Carvalho (2008) also solved the multiple stock length cutting problem with an exact solution. Poldi & Arenales (2009) reviewed some heuristics from the literature and proposed others for a slightly modified multiple stock length cutting stock problem with limited supply and observed that the rounding heuristics proposed by Wascher and Gao (1996) matched the exact results of Belov & Scheithauer (2002). Araujo, Constantino & Poldi (2011) proposed an evolutionary algorithm for the same problem where multiple stock lengths are in limited supply.

3.4.5 CSP with Due Dates and Service Level Considerations

The fifth CSP variant integrates due date and service level consideration with the traditional aim of minimum trim loss. Boston consulting group survey of customers for a corrugated paperboard manufacturer found that punctual product delivery was one of the top priorities (Lange & Andersson 2004). Many other industries may also have to put more emphasis on timely deliveries, thus, making the classical cutting stock problem an inappropriate modelling approach. The classical cutting stock problem aggregates total demand for the entire planning horizon and gives a solution based on least trim loss; it needs to be amended to incorporate the due dates of customer orders.

The cutting stock problem with due date was tackled by Johnston & Sadinlija (2002) for the paper industry with an approach that does not necessitate the pre-enumeration of all possible cutting patterns but instead is based on pre-specification of cutting pattern usage levels. The method struggles with large problems and also the solution quality is heavily dependent upon the selection of the pre-specified usage levels. Johnston & Sadinlija (2004) improved their earlier model by removing the need to pre-specify the usage of cutting patterns or run lengths. The robustness of the formulation was checked against a few small problems. Coupled with the weak linear programming formulation, this may not be an effective planning tool in a variety of industrial situations.

Reinertsen & Vossen (2010) applied the column generation technique to solve a one dimensional cutting stock problem with due dates and used a simple but effective way to obtain an integer solution on the presumption that their model conforms to the IRUP or MIRUP which is fine because they used Gilmore and Gomory’s formulation which has a tight relaxation. Instead of carrying out branching operations during the column generation iterations as is customary in the branch and price algorithm, the resulting integer programming model was solved using all the patterns that were found with the LP relaxation. This yielded a feasible and apparently good integer solution to the original integer cutting stock problem with due dates.

3.4.6 Multi-Objective Optimization applied to the CSP Variants

The sixth variant treats the cutting stock problem as a multi-objective problem. Most of the cutting stock variants discussed so far were treated as conventional single objective problems. There is a parallel stream of research that maintains that whenever a problem is faced with multiple and conflicting objectives the notion of global optimization is no longer valid. Instead of formulating a single cost function for the conflicting objectives, a multi-objective optimization problem is solved to obtain a Pareto frontier or a set of efficient solutions for the decision maker to choose from. Respicio & Captivo (2005) applied a bi-objective sequencing problem to the column generation solution obtained for the conversion stage of a paper mill. The objectives were to minimize the maximum number of open stacks and to minimization of the average order spread in order to improve the material handling at the shop floor. The Pareto optimal set was approximated by SPEA 2, a multi-objective evolutionary algorithm hybridized with a local search procedure.

Velasques et al. (2007) added a variety of objectives such as due date performance, client related importance and finished machine queue management to the traditional criterion of cost minimization for a corrugated box manufacturer. The problem was solved with evolutionary algorithms.

Similarly, Varela et al.(2009b) modelled a variant of the cutting stock problem for a plastic roll manufacturer with additional constraints and objectives. The multiple objectives are to minimize the setup cost, the amount of stock generated, the completion times weighted by order' priority and the number of open stacks. The problem was solved by developing a Sequential Heuristic Randomized Procedure (SHRP) which tackled the problem in a hierarchical manner. The computational results showed that SHRP was mainly effective in optimizing the two main objectives. In a subsequent paper, multi-objective genetic algorithms were employed to improve the initial solution obtained by SHRP (Varela et al. 2009a).

Matsumoto, Umetani & Nagamochi (2010) solved the cutting stock problem for the paper tube industry with multiple objectives by tabu search. The multiple objectives considered simultaneously included minimizing parent roll used, knife setups, open stacks and the combination of cut lengths in the open stacks. This complex objective facilitated the manual handling procedures and took into account shipment lot sizes during the cutting stock problem.

3.4.7 Integration of CSP with other Production Processes

The seventh and most relevant CSP variant to the problem under study is the integration of the cutting stock problem with other production processes. As discussed in Chapter 2, the lot-sizing problem (LSP) finds a trade-off between setup and inventory holding costs, whereas the cutting stock problem (CSP) involves cutting large objects into smaller ones while minimizing the trim loss. The setup costs of the lot sizing problem are different to the knife setup discussed in Section 3.4.1. Setup in a lot-sizing problem refers to the grade changeover cost incurred whenever a new product is being manufactured in a production facility that manufactures multiple products. For example, in a paper mill, the lot sizing problem is encountered at the paper machine which manufactures multiple grades. Every time the paper machine switches its production to another grade of paper, paper sheets are wasted and therefore, setup or grade changeover costs are incurred. The paper machine is usually the bottleneck resource and therefore the knife setup cost encountered during conversion can be ignored.

Aluminium, copper and furniture are other industries where lot-sizing and cutting stock problems are encountered in successive stages and are interlinked. One of these two processes is usually faced with independent final demand whereas demand for the other process is derived from the independent demand. In the paper industry, the end demand is expressed in smaller rolls which are the output from the cutting stock whereas the demand for the jumbo rolls to be produced at the paper machine is derived via the demand for smaller rolls. Similarly, in the furniture industry, the cutting stock operations are followed by lot-sizing problem and the sequence implies that the demand addressed by the cutting stock problem is unknown a priori and dependent upon the demand handled by lot-sizing. For such instances, separate approaches to the lot-sizing and cutting stock may yield good local solutions but may conflict with the global production objectives like joint costs and cycle service levels (CSL).

Gramani & França (2006) and Gramani, Franca & Arenales (2009) proposed a mathematical model for coupling the lot-sizing and cutting stock problems in the furniture industry. A decomposition heuristic based on lagrangian relaxation was proposed as a solution technique to the integrated optimization problem. Poltroniere et al.(2008) proposed to integrate the lot-sizing problem at the paper machine with the cutting stock problem at the conversion stage by developing an integrated planning model. The objective was to minimize three sets of costs, inventory holding costs, grade changeover costs and trim loss, with no due date or service level considerations. The model was solved with the help of two decomposition heuristics which deal with the two problems separately one after the other and in an iterative manner.

3.5 Conclusion and Further Research Directions

The assessment in this chapter of work carried out on the cutting stock problem leads to the conclusion that the trim minimization problem has an efficient exact solution approach in branch and price (B&P) algorithm but the effectiveness of B&P is limited when variants of cutting stock problem are encountered in various industries, leaving heuristics as the preferred option to obtain good solutions.

Metaheuristics have been used extensively to solve the variants of cutting stock problem but as has been the case with approximate algorithms, the quality of the solutions is unknown. In most cases, the global optimum for the CSP variants is also unknown, therefore, the appropriateness and effectiveness of metaheuristic is not assured. The selection of a metaheuristic as the preferred solution method for the CSP variant is given some justification if it is found to be efficient in solving part of the original variant which has a known optimum.

The next chapter builds on the same argument and tests the use of genetic algorithms (GA) as the solution method for the trim minimization problem which has a known optimum obtained by the exact solution techniques mentioned in Section 3.2. The same test problems are solved by an exact method and by genetic algorithms to determine the range of input parameters for which GA can match exact solutions. This is done with a view to establish reasonable justification for taking the next step in Chapter 5 of using GA to search for a joint optimum for the integrated cutting stock and lot-sizing problem combined with penalties for failing to meet due dates. In Chapter 6 the added step is taken of treating customer service as a separate goal.

CHAPTER 4. GENETIC ALGORITHMS AND THE CUTTING STOCK PROBLEM

4.1 Introduction

In Chapter three column generation based algorithms such as branch and price were shown to be the preferred solution techniques for the classical cutting stock problem (CSP). However, most cutting stock problems encountered in the real world are variants of the classical CSP with many more complexities. It was noted that exact algorithms have been found wanting for these variant problems such as cutting stock with knives setup considerations, pattern minimization, ordered cutting stock, order spread minimization, minimization of open stacks, CSP with contiguity, CSP with due dates and service level considerations, CSP with multiple objectives and integration of the cutting stock problem with other production processes.

In such scenarios, various heuristics and meta-heuristics have been used as solution approaches. The exact or global solution is not pursued because it is perceived to be intractable or too difficult to obtain and instead, good solutions but not necessarily the best ones are deemed to suffice. Genetic algorithm is one such meta-heuristic which is widely known to solve very difficult or nearly intractable problems but the optimality of resulting solutions is not guaranteed. Since, in most these cases, the global optimal is also unknown, the effectiveness of genetic algorithms in solving difficult problems can not be quantitatively measured.

Most of the computationally challenging cutting stock variants mentioned above are essentially two NP-hard problems solved simultaneously. It can be argued that if GAs can efficiently solve one of the two NP-hard problems or part of a difficult joint optimization problem, it may be reasonable to assume that GAs will also be effective in obtaining a very good solution to the combined problem. Also, the robustness of GAs in solving a particular type of optimization problem can be tested by comparing the results with exact solutions which are relatively easy to obtain for the cutting stock problem. In this chapter, based on this premise, a variety of classical cutting stock problems are solved first by an exact algorithm and then by genetic algorithms in order to measure GAs' effectiveness. The results give clues to whether genetic algorithms are capable of obtaining good results for the joint problem tackled in this thesis where the lot-sizing problem at the paper machine and the cutting stock problem at the converting machines are solved simultaneously.

In the next Section, genetic algorithms are introduced with a description of their working principles. Section 4.3 deals with the application of genetic algorithm to the cutting stock problem and reviews the relevant literature. Section 4.4 narrates the experiments performed to compare the GA solutions and the exact solutions to a randomly generated test problems followed by discussion in Section 4.5.

4.2 Genetic Algorithms

GA is a meta-heuristic search technique that works with a set of solutions called a 'population' mimicking biological evolution to identify continuously improving solutions. Each solution is represented in a single chromosome and is known as an 'individual'. Individuals are chosen through a 'selection process' and are given reproductive opportunities so that the resulting chromosomes or 'offspring' are different to the 'parent' individuals. The reproduction opportunities are provided through two genetic operators: crossover and mutation. The offspring which are better than the parents with regard to a criterion called 'fitness' tend to replace the existing individuals in the population. The process continues till a stopping criterion is reached. The flowchart of genetic algorithms' working principle is shown in Figure 4-1 and the detailed specifications of the genetic algorithm used in this case are presented in the following four sub-sections:

4.2.1 Selection Method

The first step or element is Selection in which individuals are chosen from the population and genetic operators are applied to them to produce offspring. The criterion chosen for the selection process is critical and is considered to be the driving force behind the convergence of a solution. The seminal work of Holland (1975) selected individuals in proportion to their fitness value. Similarly, many other selection methods are more likely to choose parents with better fitness because better parent individuals are likely to produce better offspring. But, selecting only the best chromosomes has one major disadvantage; all chromosomes in a population will start to look the same very quickly. This narrows the exploration space, pushes the genetic algorithm into a local optimum, and prevents the genetic algorithm from finding possibly better solutions that reside in inaccessible areas of the search space. Ideally, the population should contain a diverse set of solutions so that a wider search space is explored. To preserve the diversity of solutions in the population, selection operations usually introduce randomness in the selection process so that some not very good 'parents' are also selected ensuring diversity among solutions (Goldberg & Deb 1991; Hancock 1994).

A scaling mechanism for the fitness value has also been used to avoid a consistent selection of the individuals with better fitness. Scaling lowers the difference between a dominant individual and the rest of the population, allowing selection of non dominant solutions as well (Goldberg 1989).

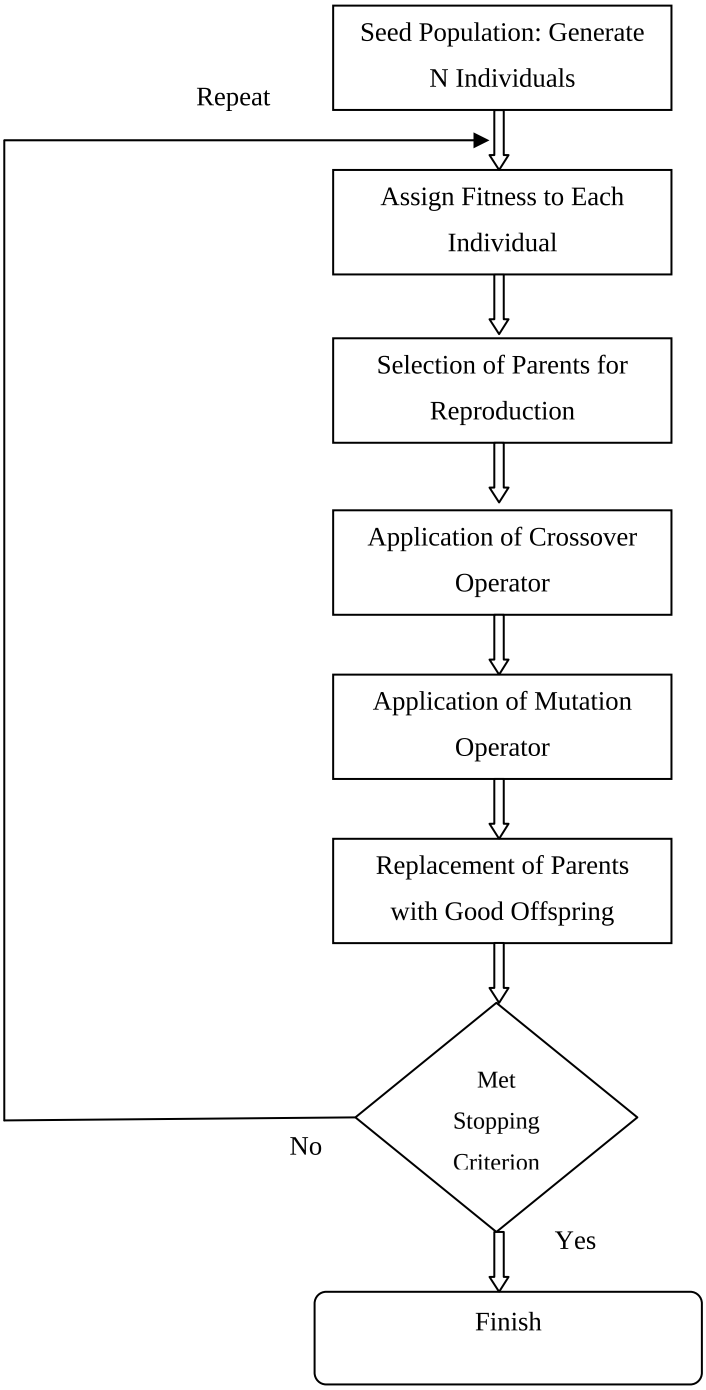


Figure 4–1: How Genetic Algorithms Work?

If the dominant individual has an extreme fitness value, the effectiveness of the scaling mechanism becomes limited. To overcome the shortcoming of reliance on an extreme value, a fitness ranking method has been used with success (Beasley, Bull & Martin 1993). In a fitness ranking method introduced by Baker (1985), individuals are sorted according to their fitness value and then they are assigned an offspring count that is solely a function of their rank. A Rank based mechanism is used in the package used to solve the problem at hand.

4.2.2 The Genetic Operators

The second major step is to apply the genetic operators. Once parent individuals from the population have been selected, the reproduction process begins with the help of genetic operators to produce offspring. These offspring will be part of the next generation and hopefully will have a better fitness value. The two most common genetic operators are crossover and mutation which are explained in the following paragraphs:

4.2.2.1 Crossover Operator

Crossover is the process of taking two parent solutions and producing offspring from them. One form of genetic algorithm uses a single point crossover, where the two mating chromosomes are cut once at corresponding points and the Sections after the cuts are exchanged. Two point and multi point crossover GAs are also used. Instead of x-point crossover, the GA used in this Chapter employs uniform crossover as a principal recombination genetic operator. Each gene in the offspring is created by copying the corresponding gene from one or the other parent chosen according to a randomly generated binary crossover mask of the same length as the chromosome. When there is a 1 in the crossover mask, the gene is copied from the first parent and when there is a 0 in the mask, the gene is copied from the second parent (Sivanandam & Deepa 2008). In Figure 4-2, bits in darker shade are copied to the first offspring as per the mask coding whereas the lighter shade bits are copied to the second offspring. In the traditional x-point crossover, irrelevant variable positions may bias the search process, whereas the uniform crossover method is considered better at preserving schema, and can generate any schema from the two parents (Palisade 2009a).

Parent 1	**1**	**0**	1	**1**	0	**0**	**1**	1
Parent 2	0	0	**0**	1	**1**	0	1	**0**
Crossover Mask	*1*	*1*	*0*	*1*	*0*	*1*	*1*	*0*
Offspring 1	**1**	**0**	**0**	**1**	**1**	**0**	**1**	**0**
Offspring 2	0	0	1	1	0	0	1	1

Figure 4–2: The Uniform Crossover Operator

4.2.2.2 Mutation Operator

Mutation is the genetic operator that works on the chromosomes after the crossover has been applied to introduce further variation in the chromosome (Figure 4-3). The further variation is necessary to enable the algorithm to break out of any local optimum and to recover lost genetic material as well as to randomly disturb the genetic information. It is considered to be an insurance against irreversible loss of genetic material and ensures exploration of the whole search space (Sivanandam & Deepa 2008). There are various ways of implementing mutation and at its simplest it randomly alters each gene with a small probability. The employed GA mutation method works by looking at each variable individually. A random number between 0 and 1 is generated for each of the variables in the organism, and if a variable gets a number that is less than or equal to the mutation rate (for example, 0.06), then that variable is mutated. The amount and nature of the mutation is automatically determined by a proprietary algorithm. Mutating a variable involves replacing it with a randomly generated value (within its valid min-max range) (Palisade 2009a).

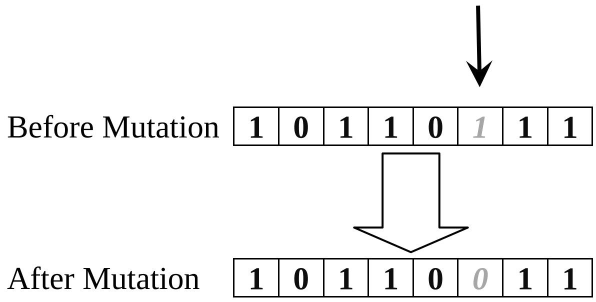


Figure 4–3: The Mutation OperatorReplacement Strategy

The third element is the replacement process which is necessitated because reproduction results in offspring which makes the number of possible solutions more than the initial population. Since most genetic algorithms have a fixed population, the new and possibly better solutions must ‘replace’ the existing solutions. The two most common replacement methods are the generational and the steady state replacement methods. Most of the early genetic algorithms employed pure generational replacement with the entire population of parents replaced by new individuals or offspring in every reproductive cycle. In order to safeguard against loss of potentially good solutions, elitism has been used so that the one or a few individuals with the best fitness are copied to the next generation (Goldberg 1989).

The steady state approach used in this study has a strategy of replacing just one individual at a time from the population rather than an entire generation. If a good new offspring is created in a steady state genetic algorithm, it is immediately available for use but in the case of generational genetic algorithms, the good new offspring only becomes available to use in the next generation. This is a disadvantage because by the time generational GA starts using the new good offspring, a steady state replacement mechanism would have already thoroughly integrated it into the population (Syswerda 1991). The steady state technique has been shown to work as well or better than the generational replacement method

(Vavak & Fogarty 1996; Beasley, Bull & Martin 1993; ShiZhen & ZhouYang 2004).

With the steady state replacement method, a decision has also to be made regarding which member of the population is to be replaced by a new offspring. Various alternatives have been used, replacing a randomly selected individual or the oldest individual or the most similar individual or the worst individual. There have been a few recent studies comparing these replacement options but no “one size fits all” option has been singled out mainly because different replacement strategies show different relative rankings according to the choice of performance metric (Smith 2007; Uyar 2007; Lozano, Herrera & Cano 2008). However, it has been reported that replacing worst converges the fastest and is much more reliable (Smith 2007). Furthermore, when paired with a selection method that selects both weak and fit parents for reproduction, the strategy of replacing the worst individual improves the overall fitness of the population considerably (Sivanandam & Deepa 2008). The genetic algorithm used here replaces the worst performing individuals; combined with the ranking based selection method for ensuring diversity of solution quality in the population, this makes it a very efficient algorithm.

4.2.3 Constraint Handling Mechanism

The fourth element is the specification of constraints which is not straightforward because the application of the two genetic operators’ mutation and crossover on a parent chromosome may result in offspring that violate the constraints. There are a number of standard ways of handling constraints in GAs such as to use a representation that automatically ensures all solutions are feasible, to separate the evaluation of fitness and infeasibility, to design a heuristic operator or repair operator which guarantees to transform any infeasible solution into a feasible solution and to apply a penalty function to penalise the fitness of any infeasible solution without distorting the fitness landscape (Chu & Beasley 1998; Craenen, Eiben & and Marchiori 2001).

GA used here utilizes two types of constraints: hard and soft based on aforementioned mechanisms. Hard constraints are implemented with a "backtracking" technology where the search process is not allowed to go to infeasible regions. If a new offspring violates some externally imposed constraint, the algorithm backtracks towards one of the parents of the child, changing the child until it falls within the valid solution space. Soft constraints do not completely reject the infeasible cases because the algorithm allows the new offspring to violate the constraints with an associated penalty. The use of soft constraints are also known as the Pro-life approach (Michalewicz 1995) and is considered to be a suitable option for highly constrained problems such as the case under study, when finding a solution is as hard as finding the optimum (Goldberg 1989). The pro-life approach allows the presence of infeasible solutions by penalizing them to reflect the degree of violation of constraints (Qiu 2000).

A related topic to the incorporation of a constraint handling mechanism in a GA is the impact of tightness of the constraints on its performance. 'Tightness Ratio' is the most common parameter used in the literature to investigate the behaviour of a particular type of constraint (Raidl 1999; Aguirre & Tanaka 2002; Kaparis & Letchford 2008; Mezura Montes & Coello Coello 2011). The procedure is to relax the actual constraint by percentage intervals called tightness ratio and the corresponding performance of the algorithm is observed to note any effects. A computational exercise is carried out in Section 4.4.5 to examine the impact of tightness ratio on the behaviour of the GA used in this thesis.

In the next section, the suitability of genetic algorithms as a solution method is evaluated by reviewing instances from the literature related to the cutting stock problem.

4.3 Genetic Algorithms and the Cutting Stock Problem

The Cutting stock problem involves optimal allocation or grouping of a finite set of items into a number of categories subject to constraints and, has been referred to as a 'grouping optimization problem'. Genetic Algorithms have been used as a solution technique for the cutting stock problem and similar 'grouping optimization problems' such as the Bin Packing Problems (BPP), timetabling problems, knapsack problems and vehicle loading problems. However, there are divergent views on the effectiveness of a standard genetic algorithm in solving grouping optimization problems. Falkenauer (1992; 1994; 1996) believed that the standard crossover may disrupt good combinations obtained for bin packing problems and consequently convergence to the optimum or near optimum would be slow and, in most cases, the initial GA generations would result in an improved solution with no or little improvement thereafter. He proposed a different GA mapping scheme called Grouping Genetic Algorithm (GGA) wherein each gene hosted a group of candidate solutions. It was deemed that with this arrangement, traditional genetic operators would be less disruptive. The improved experimental results were also backed by Gen and Cheng (2000) in their analyses of genetic algorithms for bin packing problems. On the contrary, Reeves (1996) did not approve of Falkenauer's grouping genetic algorithm and instead used the standard genetic algorithm hybridized with a local heuristic to reduce the size of the bin packing problem and found it to be effective.

Hinterding & Khan (1995) employed Falkenauer's grouping genetic algorithm to solve the cutting stock problem with and without contiguity. As discussed in Chapter three, CSP with contiguity involves non-linearity and therefore, the column generation approach is inapplicable but can be easily applied to the CSP without contiguity. A total number of ten test problems were solved to validate results. However, it has to be said that the chosen test data set was limited in its variety with low demand and only two variations of the number of items to be cut from the jumbo rolls. It is felt that for any novel CSP solution technique to be considered effective, it should be tested against a variety of input data such as that produced by the random CSP test input data generator introduced by Gau & Wascher (1995). This has become a popular test generator for comparing CSP solution techniques because of its ability to represent a wide range of input data. The test data is randomly generated by systematically varying CSP parameters such as items to be cut from the jumbo reel (sometimes referred to as number of ordered lengths), demand and the roll widths.

Shahin & Salem (2004) and Khalifa & Shahin (2006) also reported application of genetic algorithms for solving one dimensional cutting stock problems for the construction industry in the United States where steel bars, sections and lumber cutting was required. Different combinations of good cutting patterns were generated a priori and genetic algorithms were used to determine which patterns to select and their frequency. Therefore, the genetic representation of the cutting stock problem comprised just the pattern number and its frequency. Effectiveness of this representation depends largely on the pre-generated cutting patterns. All cutting patterns that generated waste less than the smallest finished product length were accepted to be part of the second stage when GA was to be employed. This GA representation reduced the size of the problem but also compromised on the solution optimality because the challenge in a cutting stock problem is not only to find a frequency of different cutting patterns to fulfil a given demand but also to find different combinations of cutting patterns that minimize the waste.

Peng & Chu (2010) utilized a two chromosome genetic representation for the classical cutting stock problem and the CSP with contiguity. The first chromosome consisted of different cutting patterns with each cutting pattern being represented by a single gene while the second chromosome represented the frequency of cutting patterns. Experimental results showed that their mapping scheme showed better or equally good results compared with those obtained by Liang et al. (2002) who used mutation as the only genetic operator for their cutting stock problem. The work of Liang follows a parallel line of research that considers crossover as having a major disruptive influence on the grouping optimization problems. Other applications of evolutionary algorithms that utilized mutation as the only genetic operator include Chiong & Beng (2007) for the cutting stock problem and Stawowy (2008) for the bin packing problem. All the authors reported the superiority of their approach on the basis of experimental results but it can be argued that the randomness associated with mutation can also be disruptive at times. Nevertheless, the utility of both crossover and mutation in obtaining improved solutions has been confirmed by Peng & Chu (2010) when their genetic algorithm performed equally as well as or better than Liang's evolutionary algorithm. Therefore, it is reasonable to assume that instead of jettisoning crossover altogether, a better genetic representation coupled with careful parametric settings is more likely to succeed.

As discussed in Chapter 3, the most effective solution technique for the classical cutting stock problem is column generation where optimality of the solution is guaranteed. However, genetic algorithms and other heuristic solution approaches come into play when dealing with the real world cutting stock problems that have added complexities. Most of the genetic algorithm approaches discussed so far dealt with both the classical cutting stock problem and the cutting stock problem with contiguity. The CSP with contiguity involves simultaneous minimization of trim loss and partly finished orders which greatly increases its computational complexity. For such a problem, genetic algorithm is likely to be a better approach. Reports of genetic algorithm for the CSP variants include Wagner (1999) for the bundled cutting stock problem, Ragsdale and Zobel (2004) for the ordered cutting stock problem and Golfeto & Neto (2009) for the pattern minimization problem. However, in all these cases, the global optimal is unknown and multiple runs of genetic algorithms are believed to give good solutions.

Also, as discussed earlier in this section, different genetic representations have yielded different degrees of success. This seems to be in consonance with Reeves (1996) who long ago appears to have dismissed the criticism of genetic algorithm by maintaining that most of its reported poor performances were a result of application in a naïve way. Therefore, it makes sense to apply a particular GA representation to the classical cutting stock problem before it can be used to solve CSP variants. In this way, the effectiveness of a particular GA representation can be compared with the optimal solutions of the classical cutting stock problem and if it performs well, the same representation can be extended to take in other complexities in real world production environments.

4.4 Comparison of GA and Exact Solutions to the Classical CSP

In the following sub-sections, separate genetic representations are applied to the classical cutting stock problem to ascertain their usefulness in solving the variants of the cutting stock problem. The rationale behind testing multiple representations for suitability stems from the findings of the previous section that different GA representations have corresponded to different results.

4.4.1 GA Representation for the Classical Cutting Stock Problem

The first issue is to find a representation of the problem. As discussed earlier, the challenge is to find the combination of cutting patterns and their frequencies simultaneously to meet a given demand. Therefore, the most natural genetic representation is the allocation of each ordered length of the cutting pattern to a single gene i.e a single prospective cutting pattern is represented by multiple genes. Also, in many cases, a cutting pattern may be repeated many a times to produce the required quantity of ordered lengths i.e its frequency will be more than one. The usage of each cutting pattern can be represented either in a single chromosome or a two chromosome representation with one chromosome representing the cutting patterns and the second representing the frequency of the cutting patterns. Details are given in the following paragraphs.

4.4.1.1 Single Chromosome Representation

The single chromosome representation for five ordered lengths i.e m = 5 is shown in Figure 4-4 where each gene represents an ordered length. If a cutting pattern is used more than once, it is treated as a new pattern i.e if a pattern is used ‘n’ times it appears ‘n’ times in the chromosome with the same combination of ordered lengths. Each pattern represents a possible combination of ordered lengths to be cut from the jumbo reel, therefore, the number of patterns in the Figure 4-4 represent the required number of jumbo reels to meet the customer demand. The Figure represents a relatively small problem where seven jumbo reels are sufficient to fulfil the demand.

Although easy to model, this representation’s major flaw is that the length of the chromosome increases considerably with increasing problem size, thus accentuating the effects of crossover.

Pattern 1					Pattern 2					Pattern 3					Pattern 4					Pattern 5					Pattern 6					Pattern 7				
2	0	0	0	1	0	0	0	3	0	0	1	2	0	0	0	0	1	1	1	0	0	0	1	2	1	1	0	0	1	0	0	1	0	1
Single Chromosome																																		

Figure 4–4: Single Chromosome Representation of the Cutting Stock Problem

4.4.1.2 Two Chromosome Representation

The two chromosome representation models the usage of each cutting pattern and its combination in two separate chromosomes as shown in Figure 4-5. Possible combinations of ordered lengths are represented in the chromosome 1 and the second chromosome represents the frequencies of patterns; that is different from the single chromosome representation where a repeat pattern was treated as a new pattern. In Figure 4-5, pattern 1 is used six times, pattern two seven times, pattern three eight times, pattern four nine times, pattern five not used at all, pattern six is used three times and pattern seven is used four times. The total number of jumbo reels used to meet customer demand is the sum of all the frequencies and for the example shown in Figure 4-5, the required number of jumbo reels is forty three. By comparing the examples shown in Figure 4-4 and 4-5, it becomes evident that the two chromosome representation can handle much bigger problems with the additional chromosome being only a few genes.

The strength of the two chromosome representation is its ability to significantly reduce the size of the problem and is therefore expected to outperform the single chromosome. This representation is similar to Peng & Chu (2010) except for the fact that they assigned a group of ordered lengths to a single gene and here, each gene corresponds to a single ordered length.

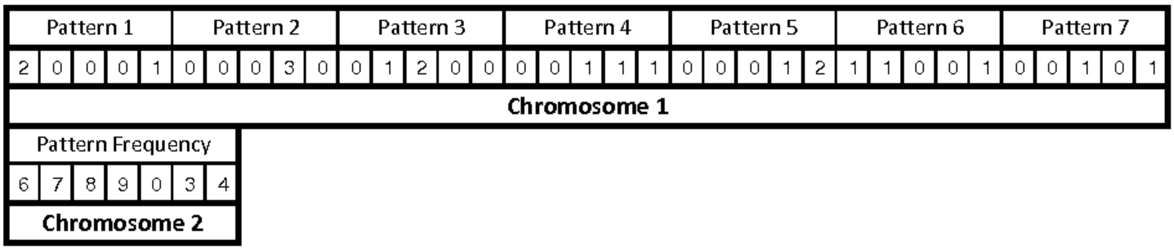


Figure 4–5: Two Chromosome Representation of the Cutting Stock Problem

4.4.1.3 GA Implementation

Evolver, a GA application in Palisade Decision Tools, is used to solve the problem. Evolver is an Excel add-in and has become increasingly popular in the research community because of its ease of use and the flexibility that Excel brings with it. Recent relevant applications include Carter & Ragsdale (2002), Ragsdale & Zobel (2004), Dooley, Parker & Blair (2005), Grigoryan & He (2005), Wang & Hsu (2008), Carter & Ragsdale (2009), Pech (2009), Sheremetov (2009), Torng & Lee

(2009), Torng et al. (2009), Yu-Su, June-Chung & Dah-Chuan (2009) and Chaudhry, Mahmood & Ahmad (2010).

Evolver has the capability to deal with the two chromosome representation by treating its chromosome as comprising two distinct parts i.e the Evolver chromosome will have n+m genes with n representing the cutting patterns and m the frequencies. All the genetic operators are applied separately to these two parts. Evolver also comes with different ‘solving methods’, each of which is a different type of genetic algorithm with customized attributes. The Recipe solving method is a genetic algorithm that treats each decision variable as an ingredient in a recipe, trying to find the best mix by changing each decision variable independently. Grouping solving method is a special type of recipe method with a reduced search space; it involves multiple variables being grouped together in sets. The number of different groups that Evolver creates will be equal to the number of unique values present in the adjustable cells at the start of an optimization (Palisade 2009b). The Grouping method is applied on the cutting patterns chromosome whereas the Recipe method is suitable for the frequency chromosome. A uniform crossover value of 0.5 is used across all experiments and auto mutation is used. Auto-mutation rate adjustment allows Evolver to increase the mutation rate automatically when an organism "ages" significantly; that is, it has remained in place over an extended number of trials. For many models, especially where the optimal mutation rate is not known, selecting Auto can give better results faster (Palisade 2009b). The initial population of 2500 gave good results without significant increase in the computational load and was therefore selected for all the experiments. The global known optimum was used as the stopping criterion and in case optimal solutions were not obtained, experiments showed that GAs converged before 200 equivalent GA generations or 500,000 iterations; therefore, it was used as the stopping criterion.

4.4.2 Comparison of GA Representations

The second issue is to compare the three GA representations of the cutting stock problem. As discussed in Section 4.3, grouping GA has been proved to be an effective representation of the grouping optimization problems for test problems with limited variety. Hinterding & Khan (1995) applied the grouping GA to the cutting stock problem for a restricted set of test data (Table 4-1). As an initial comparison, Hinterding and Khan’s five test problems that corresponded to the classical cutting stock problem were solved by the two proposed representations and also by the column generation method to determine the effectiveness of the three GAs i.e the grouping GA, the single chromosome and the two chromosome representations. The grouping GA matched the solutions obtained by the column generation method, therefore, Hinterding and Khan’s results for the five test problems were used as a benchmark. Although, it is treated as a benchmark here, grouping GA is unable to handle a wider range of problems. Nevertheless, results of the two proposed approaches have to match the grouping GA results for the five test problems in order to be classified as successful solution approaches.

The results obtained (Table 4-2) show that the single chromosome representation only solved one of the five problems as well as the Hinterding and Khan’s method whereas the two chromosome representation gave equally good results for the first four data sets but failed by a small margin on the fifth problem.

Table 4-1: Hinterding and Khan Five Test Problems

Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
Jumbo Length	14	**Jumbo Length**	15	**Jumbo Length**	25	**Jumbo Length**	25	**Jumbo Length**	4300
Ordered Lengths	8	**Ordered Lengths**	8	**Ordered Lengths**	8	**Ordered Lengths**	8	**Ordered Lengths**	18
Demand	**Lengths**	**Demand**	**Lengths**	**Demand**	**Lengths**	**Demand**	**Lengths**	**Demand**	**Lengths**
5	3	4	3	6	3	7	5	2	2350
2	4	8	4	12	4	12	6	4	2250
1	5	5	5	6	5	15	7	4	2220
2	6	7	6	5	6	7	8	15	2100
4	7	8	7	15	7	4	9	6	2050
2	8	5	8	6	8	6	10	11	2000
1	9	5	9	4	9	8	11	6	1950
3	10	8	10	6	10	1	12	15	1900
								13	1850
								5	1700
								2	1650
								9	1350
								3	1300
								6	1250
								10	1200
								4	1150
								8	1100
								3	1050

Source: Hinterding and Khan (1995)

While proposing the two representations in Section 4.4.1, it was noted that the major difference between the two was the size of chromosome with the two chromosome representation reducing the length substantially. The first four problems require eight ordered lengths to be cut from the main stock length while eighteen ordered lengths are to be cut for the fifth problem. All the problems can be classified as facing low demand and the length factor i.e ratio of the order lengths to jumbo length is low too.

Table 4-2: Estimated Minimum Number of Jumbo Reels: Three Alternate GA Methods of Solving Five Different Test Problems

Problem	Solutions Obtained by Three Different Genetic Representations Objective : Minimize the Number of Jumbo Reels				
	Grouping GA	Single Chromosome Representation		Two Chromosome Representation	
	Solution: Number of Jumbo Reels	Solution: Number of Jumbo Reels	Number of Genes to Represent the Problem	Solution: Number of Jumbo Reels	Number of Genes to Represent the Problem
1	9	9	72	9	56
2	23	25	184	23	62
3	15	16	120	15	72
4	19	21	152	19	64
5	53	No Feasible Solutions	954	56	324

Source: Test problems and Grouping GA Results from Hinterding and Khan (1995)

The single chromosome representation's total number of genes for the first four problems varied from a minimum of 72 to 152 whereas for the fifth problem, it requires atleast 954 genes. The exponential increase in the chromosome length appears to explain the deteriorating performance of this representation to the extent that it failed to obtain feasible solutions for the fifth problem. The length of the chromosome for the two-chromosome representation varies from 56 for the first problem to 324 for problem 5, showing substantial reduction in the length of chromosome. The effect is evident as the two representations' results drifted with increasing size of the problem. This seems to second the argument of Falkenauer (1996) that with an increase in the chromosome length, the probability that a crossing site during a crossover operation will fall between good combinations increases and that this has a disruptive influence on convergence.

The comparison shows that the grouping scheme used by Hinterding & Khan (1995) where a single gene may contain more than one ordered length is the most effective of the three representations because it minimizes the size of the genetic representation of the cutting stock problem, thus mitigating the adverse effects of crossover. The two chromosome representation also performed well for the first four problems when the number of ordered lengths was eight. It fell just short of matching the grouping GA for the fifth problem when the number of required ordered lengths increased to eighteen. It appears that the required number of ordered lengths has an impact on the GA performance because of its implications for the size of chromosome. But, as discussed earlier, the five test problems of Hinterding and Khan do not represent a full range of possibilities for the different input parameters such as number of ordered lengths and it is difficult to generalize the above findings.

In order to do that, different classes of test data have been systematically generated in the manner of the CUTGEN problem generator of Washer and Guo (1995). Details are given in the next Section. A thorough examination of the performance of the two proposed representations across a variety of input data also helps determine the effectiveness of each representation across a particular range of input data. This will be useful when modelling certain CSP variants that do not allow grouping GA and one has to use a one gene per ordered length genetic representation.

4.4.3 Random Test Instances Generator

The third issue is how to generate test data set that encompasses wide ranging input parameters. In this Section, different classes of data set are randomly generated and solved in an exact manner by the column generation approach. Integer solutions are obtained with a rounding heuristic. The same problem (a data set) is then solved with genetic algorithms in order to compare the two solution approaches. The previous section details different genetic algorithm implementations of the cutting stock problem which were evaluated and compared against test instances to prove their effectiveness, but the test data used was limited in its variety. Hinterding & Khan (1995) used five test instances corresponding to the classical cutting stock problem but had only two different sets of required ordered lengths (Table 4-1). Four test instances involved eight finished products i.e eight items were to be cut from the jumbo reel while the number of finished products required for the fifth test problem was eighteen. The quantity demanded for all test instances was low, thus, the sizes of the problems could be categorized as small. Realizing the apparent inadequacies of Hinterding & Khan (1995)'s test instances, Liang et al. (2002) added more test instances of relatively bigger magnitude but again the test instances did not reflect a systematic variation in the important input parameters of the cutting stock problem. Peng & Chu (2010) confined themselves to the test instances used by Liang et al (2002). It is felt that the robustness of an algorithm can only be confirmed when tested against a range of test instances with systematic variation in the input data. Gau & Wascher (1995) introduced a test set generator (CUTGEN) for the cutting stock problem wherein three important input parameters, number of cuts or number of ordered lengths (m), demand factor (d) and length factor (b) are varied one at a time to randomly generate several classes of test instances. Although no genetic algorithm representation has been tested against CUTGEN, it has been used as a test data generator for various other solution approaches to the CSP (Wascher & Gau 1996; Umetani, Yagiura & Ibaraki 2006; Poldi & Arenales 2009). The proposed genetic representation will be compared with exact solutions using the test instances generated by an approach similar to CUTGEN. Details are as follows.

4.4.3.1 Generation of Test Sets

In order to generate several classes of problem instances the parameters of the one dimensional cutting stock problem (1DCSP) have been varied in the following ways.

Number of ordered lengths (m)

The number of ordered item lengths to be cut from the jumbo reel is an important input parameter for the cutting stock problem. Different values of m used in the comparison are:

$$m = 3, 5, 7, 10 \text{ and } 15. \quad (4.1)$$

Demand for ordered lengths (d_i)

The third input parameter for the cutting stock problem is the individual demand for ordered lengths. Average demand per order ($\bar{d}$) is the determinant of the total demand for ordered lengths. The GA implementations discussed in Section 4.3 were evaluated against test data characterized by their low demand of ordered lengths but here, both low and high demand test instances are generated. Two values of $\tilde{d}$ = (10, 50) are used to differentiate between the low and high demand cutting stock problems. The total demand of all ordered lengths is given by:

$$D = m * \tilde{d} \quad (4.2)$$

The individual demand (d_i) of each ordered length (l_i) is represented by the following equation:

$$d_i = (D \times R_i)/(R_1 + R_2 + \cdots + R_m) \text{ where } i = 1, \ldots, m-1 \quad (4.3)$$

where R_i is a random variable drawn from a uniform distribution [0,1]

The demand for mth ordered length is determined by subtracting the individual random demands obtained from the above equation from the total demand D.

$$d_m = D - \sum_{i=1}^{m-1} d_i \qquad (4.4)$$

<u>Length Factor (*b)*</u>

The ratio of ordered lengths *(l_i)* to the jumbo length *(L)* is defined as the length factor *(b)*. It is believed that length factor is an important input parameter which may affect the solution quality. Consideration of length factor for checking the effectiveness of different GA representations appears to have been ignored in previous studies.

The jumbo length for all problem instances is fixed at 10,000 length units, while different cases have been distinguished with respect to the order lengths. The order lengths are modelled as uniformly distributed integer random variables which were allowed to vary between one and a certain percentage *b* of *L*, i.e.

$$l_i = [1000, b \times L] \qquad (4.5)$$

The values of the length factor *b* that have been investigated were *b*= 0.25, 0.5, 0.75 and 1.0.

<u>An Example</u>

The entire procedure for generation of random data sets is best illustrated with the following example:

From (4.1) $Let\ m_2 = 5$

From (4.2) $D_1 = 5 \times 10 = 50\ for\ \tilde{d}_1$= 10 &

$D_2 = 5 \times 50 = 250\ for\ \tilde{d}_2$= 50

From (4.3) $d_{i1} = 10$ & $d_{i2} = 51$ $i = 1, 2, 3, 4, 5$

$d_{i1} = 4$ & $d_{i2} = 21$ $d_{i1}\ corresponds\ to\ \tilde{d}_1 = 10$

$d_{i1} = 5$ & $d_{i2} = 25$ $d_{i2}\ corresponds\ to\ \tilde{d}_2 = 50$

$d_{i1} = 18$ & $d_{i2} = 92$

From (4.4) $d_{51} = 13$ & $d_{52} = 61$

From (4.5) $l_i = 2371, 1373, 1849, 1567$& 1951 $i = 1, 2, 3, 4, 5$

$for\ b_1 = 0.25$

$l_i = 4635, 3022, 4078, 4155,$ & 4565

$for\ b_2 = 0.5$

$l_i = 6914, 4170, 5034, 6818,$ & 1104

$for\ b_3 = 0.75$

$l_i = 9276, 1755, 5051, 8801$ & 1953

$for\ b_4 = 1.0$

Similarly, the three input parameters m, b and $\tilde{d}$ are systematically varied one at a time and for each ordered length, eight test problems are generated as shown in Figure 4-6. For five different ordered lengths, 40 test problems were created which represented a wide range of cutting stock problem. The 40 test problems are attached as Appendix A. Comparing solution techniques on these 40 test problems will give a measure of effectiveness.

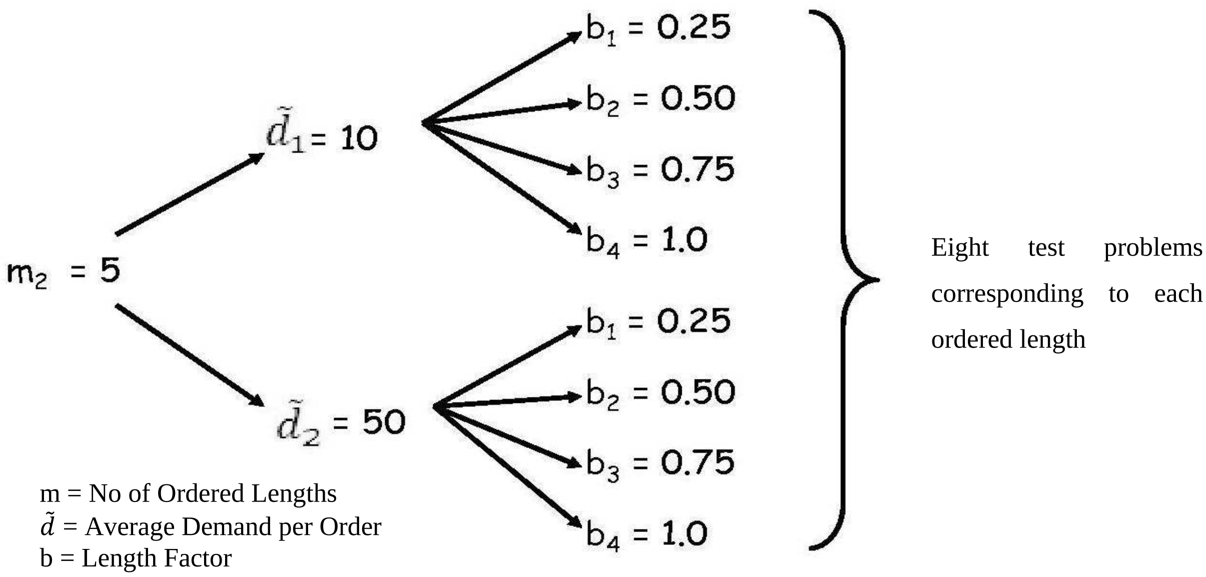


Figure 4–6: Generation of Test Instances

4.4.4 Exact Solutions

The fourth step in testing the GA is to generate exact solutions for comparison. The exact solutions were obtained by solving the continuous relaxation of the original integer problem with the column generation approach. With Excel built-in functions and a Visual Basic for Application (VBA) program, solver was automated for repeated exchange of information between the restricted master and its dual problem of Chapter 3, Section 3.2. Once an optimal solution to the linear relaxation was obtained, the RSUC rounding heuristic proposed by Wascher & Gau (1996) was applied to obtain integer solutions. Wascher & Gau (1996) evaluated the solution quality of different rounding heuristics for 4000 randomly generated cutting stock test problems. They found that one of the ‘residual problem’ heuristics named RSUC was the most effective as it found an optimal solution for 98.0 % of the test problems. For the remaining 2%, the RSUC integer solutions were just one unit away from the optimal values. Therefore, RSUC was selected as the rounding heuristic to obtain the integer solution from the continuous relaxation solutions. RSUC works by rounding down the fractional solutions obtained during the column generation problem which results in a residual because rounding down results in partially unfulfilled demand. The column generation method is again applied to the residual problem. The rounding down procedure is repeated and the residual problem again solved until no fractional solutions are obtained. The simplicity of the procedure coupled with a high success rate makes it an attractive alternative to the branch and price algorithm which has its own issues as discussed in Chapter 3.

4.4.5 Experimental Results

The detailed experimental results are shown in Table 4-3. In 33 of the 40 test problem, GA's two chromosome representation was able to match the exact solution. With smaller problems as determined by the required number of jumbo reels, the two chromosome representation was efficient in matching exact solutions even with a random initial population but as the problem size increased, the GA was initiated with feasible solutions for further improvements. However the single chromosome representation was only effective for smaller problems; as the minimum number of jumbo reels needed to satisfy the end demand increased, its performance deteriorated even with an initial population of feasible solutions. The two chromosome representation was effective for all the problems characterized by low demand ($\tilde{d} = 10$) because of lower number of jumbo reels required. However, with higher demand ($\tilde{d} = 50$), the two chromosome representation could only match the exact solutions when the number of ordered lengths was 7 or less.

Generally, the higher length factor increased the problem size and therefore, the difference between the exact and GA solution increased with higher length factor for m=10 and d=50. However, for m = 15 and d = 50, the GA solution was close to

Table 4-3: Comparison of Two GA Representations with Exact Solutions,

Objective: Minimization of Jumbo Reels Required

(Full Set of 40 Problems as shown in Figure 4-6)

		Data Set 1			Data Set 2			Data Set 3			Data Set 4			Data Set 5			Data Set 6			Data Set 7			Data Set 8		
Test Data		m_1	$\tilde{d}_1$	b_1	m_1	$\tilde{d}_1$	b_1	m_1	$\tilde{d}_1$	b_1	m_1	$\tilde{d}_1$	b_1	ml	$\tilde{d}_2$	b1	ml	$\tilde{d}_2$	b2	ml	$\tilde{d}_2$	b3	ml	$\tilde{d}_2$	b4
		3	10	0.25	3	10	0.5	3	10	0.75	3	10	1	3	50	0.25	3	50	0.5	3	50	0.75	3	50	1
Exact Solution		6			7			12			12			26			30			60			59		
GA	Two Chromosome	6			7			12			12			26			30			60			59		
	Single Chromosome	6			7			12			12			29			34			64			63		
(GA Best - Exact)		0			0			0			0			0			0			0			0		

		Data Set 9			Data Set 10			Data Set 11			Data Set 12			Data Set 13			Data Set 14			Data Set 15			Data Set 16		
Test Data		m_2	$\tilde{d}_1$	b_1	m_2	$\tilde{d}_1$	b_2	m_2	$\tilde{d}_1$	b_3	m_2	$\tilde{d}_1$	b_4	m_2	$\tilde{d}_2$	b_1	m_2	$\tilde{d}_2$	b_2	m_2	$\tilde{d}_2$	b_3	m_2	$\tilde{d}_2$	b_4
		5	10	0.25	5	10	0.5	5	10	0.75	5	10	1	5	50	0.25	5	50	0.5	5	50	0.75	5	50	1
Exact Solution		10			25			33			35			47			123			168			175		
GA	Two Chromosome	10			25			33			35			47			123			168			175		
	Single Chromosome	10			26			35			36			51			140			184			196		
(GA Best - Exact)		0			0			0			0			0			0			0			0		

		Data Set 17			Data Set 18			Data Set 19			Data Set 20			Data Set 21			Data Set 22			Data Set 23			Data Set 24		
Test Data		m_3	$\tilde{d}_1$	b_1	m_3	$\tilde{d}_1$	b_2	m_3	$\tilde{d}_1$	b_3	m_3	$\tilde{d}_1$	b_4	m_3	$\tilde{d}_2$	b_1	m_3	$\tilde{d}_2$	b_2	m_3	$\tilde{d}_2$	b_3	m_3	$\tilde{d}_2$	b_4
		7	10	0,25	7	10	0.5	7	10	0.75	7	10	1	7	50	0.25	7	50	0.5	7	50	0.75	7	50	1
Exact Solution		13			19			34			58			57			90			166			282		
GA	Two Chromosome	13			19			34			58			57			90			166			282		
	Single Chromosome	13			22			39			61			77			107			197			299		
(GA Best - Exact)		0			0			0			0			0			0			0			0		

		Data Set 25			Data Set 26			Data Set 27			Data Set 28			Data Set 29			Data Set 30			Data Set 31			Data Set 32		
Test Data		m_4	$\tilde{d}_1$	b_1	m_4	$\tilde{d}_1$	b_2	m_4	$\tilde{d}_1$	b_3	m_4	$\tilde{d}_1$	b_4	m_4	$\tilde{d}_2$	b_1	m_4	$\tilde{d}_2$	b_2	m_4	$\tilde{d}_2$	b_3	m_4	$\tilde{d}_2$	b_4
		10	10	0.25	10	10	0.5	10	10	0.75	10	10	1	10	50	0.25	10	50	0.5	10	50	0.75	10	50	1
Exact Solution		21			26			62			56			95			123			306			274		
GA	Two Chromosome	21			26			62			56			95			125			308			277		
	Single Chromosome	26			37			79			86			138			199			424			447		
(GA Best - Exact)		0			0			0			0			0			**2**			**2**			**3**		

		Data Set 33			Data Set 34			Data Set 35			Data Set 36			Data Set 37			Data Set 38			Data Set 39			Data Set 40		
Test Data		m_5	$\tilde{d}_1$	b_1	m_5	$\tilde{d}_1$	b_2	m_5	$\tilde{d}_1$	b_3	m_5	$\tilde{d}_1$	b_4	m_5	$\tilde{d}_2$	b_1	m_5	$\tilde{d}_2$	b_2	m_5	$\tilde{d}_2$	b_3	m_5	$\tilde{d}_2$	b_4
		15	10	0.25	15	10	0.5	15	10	0.75	15	10	1	15	50	0.25	15	50	0.5	15	50	0.75	15	50	1
Exact Solution		29			43			67			99			132			209			335			493		
GA	Two Chromosome	29			43			67			99			137			216			338			501		
	Single Chromosome	50			76			98			149			No Improvements											
(GA Best - Exact)		0			0			0			0			**5**			**7**			**3**			**8**		

m = No of Ordered Lengths = 3, 5, 7, 10, 15 $\tilde{d}$ = Average Demand per Order = 10, 50 b = Length Factor = 0.25, 0.5, 0.75, 1.0

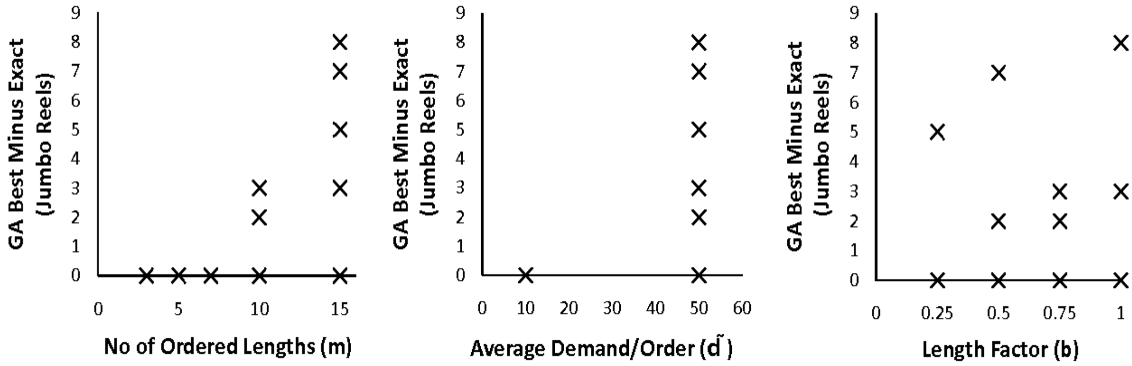


Figure 4–7: GA Performance against Three Input Variables (Differences in Jumbo Reels)

the exact solution for b = 0.75. This could be attributed to the randomness in the data generation. Although, the number of jumbo reels required was higher for b = 0.75 than b = 0.25 and b = 0.50 the randomly generated ordered widths l_i might have led to good combinations for the cutting patterns.

Of the two proposed single item per gene representations, the two chromosome representation performance is far superior; its deviations from the optima are plotted in Figure 4-7 against variations in the three input parameters. Figure 4-7 shows that GA performed very well when the number of ordered lengths was less than 10, irrespective of demand and length factor variations. With m = 10, GA was still able to reach the global optimum for five of the eight test instances belonging to the same class but when faced with high demand and high ratio of length factor, the optimal values obtained by GA were two to three units (jumbo reels) away from the exact solution. When the number of ordered lengths increased to 15, GA was still able to match the exact solution approach when the demand and length factor were low but it fell short when the demand and length factor increased.

Tightness of GA Constraints

As discussed in Section 4.2.3, effects of the tightness of GA constraints on its performance are best explained by tightness ratio (α) which is obtained by relaxing the original constraint by percentage intervals. In the standard Gilmore-Gomory formulation of Chapter 3, different levels of constraint tightness are obtained by setting the right hand side of constraint (3.6) i.e the demand of *i* finished products d_i as follows:

$$\sum_{j \in J} A_{ij} x_j \geq \alpha \sum_{j \in J} A_{ij} \tag{4.6}$$

Where $\alpha = 0.25, 0.5, 0.75, 1$

The four values of tightness ratio (α) introduced in constraint (4.6) corresponds to four levels of constraint tightness. For a minimization problem such as the cutting stock problem, the tightness ratio (α) increases with the constraint tightness whereas for a maximization problem, the lower values of α correspond to tighter constraints. For the problem under study, the constraints are being relaxed with a decreasing 'α' value.

The test data randomly generated in Section 4.4.3 is also used to gauge the effects of constraint tightness on the genetic algorithm used in this thesis. Instead of all forty test data sets (Table 4-3), only seven sets of the generated problems are used because of gaps obtained between the GA best and exact solution. For the remaining 33 sets, the best GA solutions matched optimal values therefore it is felt that with a relaxed constraint, the algorithm is also likely to match the exact solution. A computational exercise for Data set 29 confirmed this.

Eight data sets (29-32 and 37-40, Table 4-3) are solved with different values of tightness ratio (α) for the two chromosome GA representation to observe any improvements because of relaxed constraints. The results are shown in Table 4-4.

Table 4-4: The Impact of Tightness Ratio on GA Performance

Test Data	Data Set 29				Data Set 30				Data Set 31				Data Set 32			
	α				α				α				α			
	0.25	0.5	0.75	1	0.25	0.5	0.75	1	0.25	0.5	0.75	1	0.25	0.5	0.75	1
GA Best Minus Exact	**0**	**0**	**0**	**0**	**0**	**1**	**2**	**2**	**1**	**2**	**2**	**2**	**1**	**2**	**2**	**3**
Test Data	**Data Set 37**				**Data Set 38**				**Data Set 39**				**Data Set 40**			
	α				α				α				α			
	0.25	0.5	0.75	1	0.25	0.5	0.75	1	0.25	0.5	0.75	1	0.25	0.5	0.75	1
GA Best Minus Exact	**2**	**3**	**5**	**5**	**3**	**3**	**5**	**7**	**2**	**3**	**3**	**3**	**3**	**3**	**5**	**8**

α = Tightness Ratio

Table 4-4 reveals a positive correlation between the tightness ratio (α) and the quality of obtained solutions which makes sense. Constraints get tighter with higher values of 'α' which also increases the problem difficulty. The lower values of 'α' entail a reduction in problem size because now a fraction of end items demand is required. This also means a lower number of minimum rolls required to fulfil the reduced demand. Because of a decrease in problem size, the difference in GA best and exact solution is less for lower values of tightness ratio. However, the gap between the two increases with higher values of tightness ratio implying that as constraints get tighter so does the problem difficulty.

4.4.6 Discussion

When the two different genetic representations were tested against several classes of cutting stock problem, it was found that as the length of the chromosome increased the proposed GA representation's ability to match the exact solutions deteriorated. This was more severe in the single chromosome representation than in the two chromosome representation because of the involvement of the genetic operators with a greater number of genes. This also suggests that the grouping GA introduced by Hinterding and Khan (1995) and further transformed by Peng and

Chu (2010) into a two chromosome representation is likely to perform better because of its much reduced size because each gene hosts an entire pattern as compared to the proposed approach where each element of a pattern is hosted by a single gene. However, the use of a grouping two chromosome representation has shortcomings. Firstly, unlike the proposed representation, the results reported have not been tested against several classes of test instances with systematic variations in the three input parameters. Whether, the grouping GA will be effective for a similar variety of data is unknown. Secondly, the objective of the entire exercise was to get an indication of GA's performance when applied to a joint problem where the cutting stock problem and the lot-sizing problem at the paper machine to achieve desired service levels are to be solved simultaneously. The modelling environment of the joint problem and other CSP variants would not allow the multiple items per gene representation. Therefore, only the single item per gene representation can be used in this chapter. For such scenarios, the comparison carried out gives some confidence that GA is a good choice to solve CSP variants against certain classes of input data. If the number of ordered lengths is equal to or less than ten, the proposed representation is likely to perform as well as the exact solution technique with an ability to tackle the complexities of the joint problem.

Another important consideration is the type of input data encountered in real world situations or the magnitude of customer orders. The answer to this question depends largely on the industry, segment of the supply chain and production environment. The nature of demand will vary for different industries and but it can also vary within the industry under focus i.e the paper supply chain. As shown in Figure 2-1 (Chapter 2) while discussing planning and scheduling in the paper industry, there are many finished products, each facing different market pressures. Also the manufacturing stage of the pulp and paper supply chain may have different production facilities. If it is only a paper converting facility, it is likely to face different kinds of orders with a great deal of variety. However, if it is an integrated facility where paper manufacturing and converting are two successive stages, the workload at the converting machine is relaxed as the bottleneck resource is the paper machine. Moreover, if the order penetration point is located before the paper machine, implying no inventory of jumbo reels as is the case in the joint problem encountered in this thesis, the number of order arrivals for the conversion process is determined by the paper machine speed. It will be seen in Chapter 5 that the supply chain constituting paper making and cutting processes operating under a make-to-order production environment will be faced with a pattern of customer orders similar to that for which the GA was effective.

4.5 Conclusion

The classical cutting stock problem is best solved with the help of column generation or branch and price algorithms but the real world scenarios often involve non-linearities and added complexities which can only be captured by variants of the cutting stock problem. Application of exact solution approaches is limited in such scenarios where approximate methods such as heuristics and meta-heuristics can give good solutions. However, the optimality of solutions is not guaranteed because the global optimum is unknown. For such instances, applying the approximate method to the classical cutting stock problem with a readily available exact solution helps to determine the performance of the approximate method for a bigger and more complex CSP variant.

In this chapter, two genetic algorithm representations were applied to the classical cutting stock problem and it was found that GA can match exact solutions over a considerable range of input parameters, providing a strong basis for the selection of genetic algorithms as the solution technique for the integrated cutting stock and lot-sizing problem in the next Chapter.

CHAPTER 5. JOINT OPTIMIZATION OF THE CUTTING STOCK AND LOT SIZING PROBLEM WITH CYCLE SERVICE LEVELS

5.1 Introduction

In Chapter 4, genetic algorithm was shown to be a robust way of solving the problem of cutting paper rolls in a variety of sample cases where the number of ordered lengths is less than 15. This number is common in industrial situations and the results give some confidence that GA is an appropriate means of obtaining good solutions to the problem of jointly optimizing lot sizes, cutting stock and service level for customers, which is the full objective of the study. The focus of Chapters 3 and 4 has been on cutting jumbo reels into smaller rolls as required by customers. Achieving the best or a very efficient way of doing this is known as the cutting stock problem, one of the important production problems in the pulp and paper supply chain. The literature review has covered its variants encountered in different industrial settings and the solution approaches.

However, in the discussion of planning and scheduling for the paper industry in Chapter 2, the lot-sizing problem at the paper machine was identified as the principal production problem for paper manufacturing. The cutting stock problem (CSP) involves cutting large objects into smaller ones while minimizing the trim loss whereas the lot-sizing problem (LSP) finds a trade-off between setup and inventory holding costs. The literature has mostly dealt with these two problems separately which meant that the optimum or near optimal solutions obtained for the individual processes may conflict with the supply chain’s overall objectives. Particularly, as discussed in Chapter 3, the cutting stock problem has focused mainly on the reduction of trim loss and in some instances minimizing cutting patterns and open stacks has also been considered along with trim minimization.

But, all these objectives pertain to the cutting processes only and do not take into account other supply chain processes.

The isolated planning for conversion processes in the paper supply chain was identified as a weakness by Bookbinder and Higginson (1986) in their analyses of the empirical production data for a Canadian packaging mill. They noted a non-linear relationship between trim waste and service levels and predicted poor service levels for all the planning solutions based on a single objective of trim minimization. Trkman and Gradisar (2007) also argued that the cutting stock problem should be considered in the context of total supply chain objectives. They preferred to obtain good trim loss solutions instead of the global optimum (least trim) by returning waste material above a given threshold so that it could be used in a later time period. The results showed that the long run costs were reduced even if an immediate suboptimal trim loss was obtained. The authors cited the findings to predict that companies would broaden their optimization objectives for the cutting stock problem to include other supply chain processes. Similarly, Erjavec et al (2009) emphasized a holistic approach to the cutting stock problem to achieve the total organizational goals with total company costs taken into consideration instead of costs associated only with the conversion processes. The authors suggested development of a mathematical model that optimized the combination of cutting, warehousing and purchasing costs so that the total benefits are maximized. The core manufacturing activity of the case study was only the cutting stock problem; an adaptation to the paper industry - where the supply chain includes other production processes too - would be an extension of the cutting stock problem to include the lot-sizing problem at the paper machine, other finishing processes and the distribution of the finished products.

Lot-sizing and cutting stock problems are also encountered in successive stages of various other industries like aluminium, copper, furniture, steel and auto parts manufacturing. For such cases, as discussed above, a separate approach to the lot-sizing and cutting stock problems may yield locally good solutions that conflict with the overall production objectives like joint or total costs and cycle service levels (CSL). For example, Gramani et al. (2009) noted that solving the lot-sizing problem before the cutting stock problem in the furniture industry may find infeasible solutions with respect to the production capacity. In the paper industry, the customary practice is to solve the cutting stock problem before lot sizing which may restrict cycle service levels for customers because the solution to the lot-sizing problem is bounded by the cutting stock solution. Moreover, if production at the paper machine exceeds demand, it may give cutting patterns with lesser trim loss along with the reduced setup costs. However, inventory holding costs will increase in this instance. Therefore, in order to prepare good production schedules, there are strong grounds for an integrated or joint approach where the two problems are solved simultaneously instead of being dealt with separately.

There have been a limited number of attempts reported in the literature to encompass the intimate relationship between the lot-sizing problem (LSP) and the cutting stock problem (CSP). Hendry et al. (1996), Krichagina et al. (1998), Respício et al (2002), Nonas and Thorstenson (2000) and Nonas and Thorstenson (2008) tried to incorporate the cutting stock problem into the planning of production schedules. However, none of these minimized the changeover, inventory holding and trim loss costs simultaneously. Hendry et al. (1996) developed a two stage solution technique for the production planning problem at a copper foundry. The final product of copper billets is produced by first melting scrap metal, then casting it as 'logs' and in the final step, the logs are cut to billets of required dimensions. In the first stage, the cutting stock problem finds the least trim loss solution and the second stage minimizes the total time required to produce the logs, including the setup time required when there is a change from one log diameter to another. Similarly, Krichagina et al. (1998) minimized the expected shutdown, trim loss, back ordering and inventory holding cost for a paper manufacturing and converting plant. A two step approach of linear programming and ‘brownian control’ was used to solve the optimization problem to a sub-optimal level. Respecio et al. (2002) developed an integrated model for paper manufacturing and conversion processes in a make-to-order production environment. However, grade changeover costs at the paper machine were not considered. Nonas and Thorstenson (2000) and Nonas and Thorstenson (2008) solved the combined lot-sizing and cutting stock problem for steel parts used in a truck manufacturing company but the setup costs considered were associated with the knife changeovers only.

Similarly, Harjunkoski and Westerlund (1998) and Westerlund and Isaksson (1998) extended the trim loss problem in the paper industry to include the paper production but the only decision variable added to the cutting stock formulation was the number of jumbo rolls. It did not consider the grade changeovers at the paper machine and the inventory holding costs of the finished products. However, the changeover costs associated with the cutting patterns at the conversion processes were included in the objective function. Instead of holding the finished products inventory, the formulation penalized the overproduction by selling it on a lower cost.

Gramani and Franca (2006) and Gramani et al.(2009) proposed a mathematical model for coupling the lot-sizing and cutting stock problems in the furniture industry. Because of a different industrial context, the sequence of operations was also different from the current case study. The cutting stock operations were followed by the lot-sizing problem, whereas in the paper industry the lot-sizing problem precedes the conversion processes. The sequence of operations in the furniture industry implies that the solution of the cutting stock problem is unknown a priori and dependent upon the solution of the lot-sizing problem. The problem discussed in this chapter pertains to the pulp and paper supply chain where the cutting stock problem is faced with an independent demand and the demand for jumbo rolls is unknown beforehand but derived through the CSP.

Poltroniere et al. (2008) formulated the joint lot-sizing and cutting stock problem for the paper industry and solved it with the help of two separate iterative heuristic solutions in a decomposed manner. The first heuristic called Lot-Cutting Heuristic (LCH) is based on Lagrangian relaxation approach wherein the LSP-CSP coupling constraint is added to the objective function while solving the LSP ahead of the CSP. As already discussed, in a paper mill with production and conversion process in successive stages, the customer orders are for the finished products only i.e the smaller rolls. Therefore, the demand for jumbo reels is dependent upon the demand for the smaller or cut rolls and is only determined through solution of the cutting stock problem. Solving the LSP ahead of the CSP is only possible with an estimated dependent demand for the jumbos. The authors dealt with this by adding an estimated incremental factor to the dependent demand. The CSP is solved after each LSP updates the LSP and the process is repeated with different estimated factors for the dependent demand. The coupling between the two processes is achieved by updating the lagrangian multiplier in each iteration. The objective function included the production costs at the paper machine, grade changeover costs: inventory holding costs for the jumbo reels, trim loss and inventory holding costs for the finished products i.e the cut rolls. However, during the decomposed solution approach, the lot sizing and cutting stock problems were solved with an objective function that only included the relevant cost parameters i.e trim loss and end item inventory holding costs were considered for the CSP and production cost, grade changeovers and holding cost for the jumbo rolls was considered for the LSP. In the second heuristic called cutting-lot (CLH), the cutting stock problem is solved first, followed by the LSP. The results obtained showed that CLH consistently performed better than the LCH. Different results for the two heuristics also show that the order of solving these two problems had a bearing on costs, therefore, the idea of simultaneously solving the two processes gains merit.

Apart from the aforementioned attempts, it appears that the integrated lot-sizing and cutting stock problem has only received limited attention in the literature but its relevance in various industrial settings and the potential benefits makes it an important research topic. Typical issues associated with the pulp and paper industry were highlighted in Chapter 2 and it was noted that generally, there is no synchronization of product flow along the supply chain which often results in poor cycle service levels and unwarranted inventory. An integrated lot-sizing and cutting stock model will synchronize the product flow through two stages namely, paper manufacturing and conversion processes.

In this chapter, an integrated model for a paper mill is developed for the joint optimization of lot-sizing and cutting stock with service level considerations for a paper mill. The changeover cost, finished product holding costs, trim loss and tardiness costs are optimized simultaneously. A sequence of different grades of paper on the paper machine is determined by the model while meeting the customer specified due dates for the finished products. It also gives the sequence of cutting patterns and allocates them to different time periods depending upon customer requirements. The relationship between the consequent cycle service levels and the total joint costs is also studied. The integrated model is formulated along with a discussion of the problem definition in Section 5.2. Then Section 5.3 describes the experimental setting and Section 5.4 presents the computational results as well as the measures for evaluating the quality of results. The comparison with other optimization approaches is undertaken in Section 5.5. The chapter concludes with findings and suggested directions of future research in Section 5.6.

5.2 Model Formulation

5.2.1 The Components of the Joint Problem

5.2.1.1 The Cutting Stock Problem

The first element of the joint problem is the cutting stock problem which was discussed at length in Chapters 3 and 4 but its essentials for the comprehensive problem formulation are briefly described. The conversion stage of the pulp and paper supply chain is faced with the independent demand i.e demand for finished products (FP) which is dynamic i.e deterministic and time varying and is established through customers' orders which are grouped together on a weekly basis. During the conversion stages, the large paper rolls (jumbo reels) produced on the paper machine, are cut according to different combinations of smaller rolls - called cutting patterns – to meet the end demand. The cutting patterns may or may not match the exact length of the jumbo roll. Trim loss or waste is generated whenever a cutting pattern's length is less than the jumbo's length. For example, a paper machine produces jumbo rolls of 10 m which are to be cut into three smaller rolls of 4, 3, 2.5 metres will generate a trim loss of 0.5 metres (Figure 5-1). The decision problem is to minimize the trim loss while fulfilling the finished products demand.

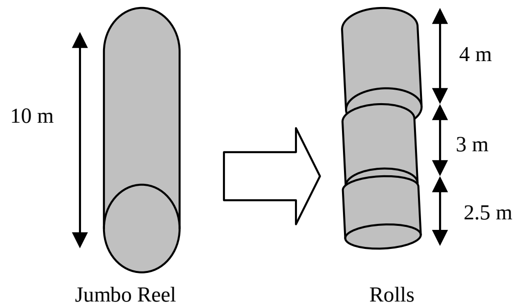


Figure 5–1: The Cutting Stock Problem

The customer orders for the finished products have the following characteristics:

- Paper grade
- Roll width
- Number of rolls required

- Order due dates which can be a particular day of the week long planning horizon. It is assumed that in case the roll requires further finishing activities the quoted due date includes the necessary time buffer.

It is assumed that the cutting stage is unconstrained because the rate of cutting jumbo reels is much faster than the production rate at the paper machine. It is a reasonable assumption because the paper machine is usually the bottleneck resource in the paper supply chain (Martel et al. 2005). The knives setup need not to be taken into account because of the un-capacitated nature of the conversion process; there has been a recent trend to ignore the knives because of their little overall impact due to automated knives changeover (Reinertsen & Vossen 2010).

The finished product (FP) demand over the entire planning horizon has to be met; however, if an order cannot be delivered in time, it incurs a tardiness cost 'M'. Cycle Service Level (CSL) is defined as the probability that the cycle time for the customer's order will be less than the quoted lead time (Hopp & Spearman 2008). Mathematically,

$$CSL = Probability\ \{Cycle\ Time < Lead\ Time\} \qquad (5.1)$$

5.2.1.2 The Lot-Sizing Problem

The second element of the joint problem is the planning activity for the paper machine which involves a tradeoff between inventory holding cost and setup cost. Different grades of jumbo reels are produced at the paper machine and a setup cost is incurred whenever production of a new grade is started. In other words, different paper grades require a common production resource and whenever a production switch to a new grade is made, production time is lost in setting up the machinery. In the paper industry, setup costs are important as the machine keeps making paper but it takes time to adjust to the quality settings of the new grade. The paper produced in the transition time is rejected. Therefore, apart from the opportunity costs (i.e. lost production time), significant material losses are also encountered. Another aspect of sharing resources is that different grades cannot be produced at the same time, therefore, customer orders must be sequenced which has repercussions for the cycle service levels.

Apart from the order sequencing issue, inventory holding costs are also an important consideration for planning purposes. Grade changeovers can be minimized for a particular production plan by scheduling each grade only once during the planning horizon till the demand is met, however, the opportunity costs of capital tied up in inventory, the direct costs of storing goods and holding items prohibit large stacks of inventory. Furthermore, in some instances, securing additional capital may also be a concern and therefore, another reason to limit inventory holding costs. Additionally, minimizing setup costs alone may also limit the cycle service levels.

In short, the lot-sizing problem finds a production schedule that achieves a balance between low setup costs (favouring large production lots) and low holding costs (favouring a lot-for-lot-like production where sequence decisions have to be made due to sharing common resources). Therefore, the problem of short-term production planning turns out to be a lot sizing and scheduling problem (Drexl & Kimms 1997). The production planning at the paper machine is classified as single or two level multi-item capacitated dynamic lot-sizing problem and has been modelled in a number of ways (Rizk & Martel 2001).

Literature Review on Lot-Sizing Models

Different lot-sizing models have been used in the literature depending upon the production processes and detailed planning requirements. The major difference between various lot-sizing models is the time horizon. Two terms, big bucket and small bucket lot-sizing models, have been commonly used to differentiate between medium term and short term production planning (Figure 5-2).

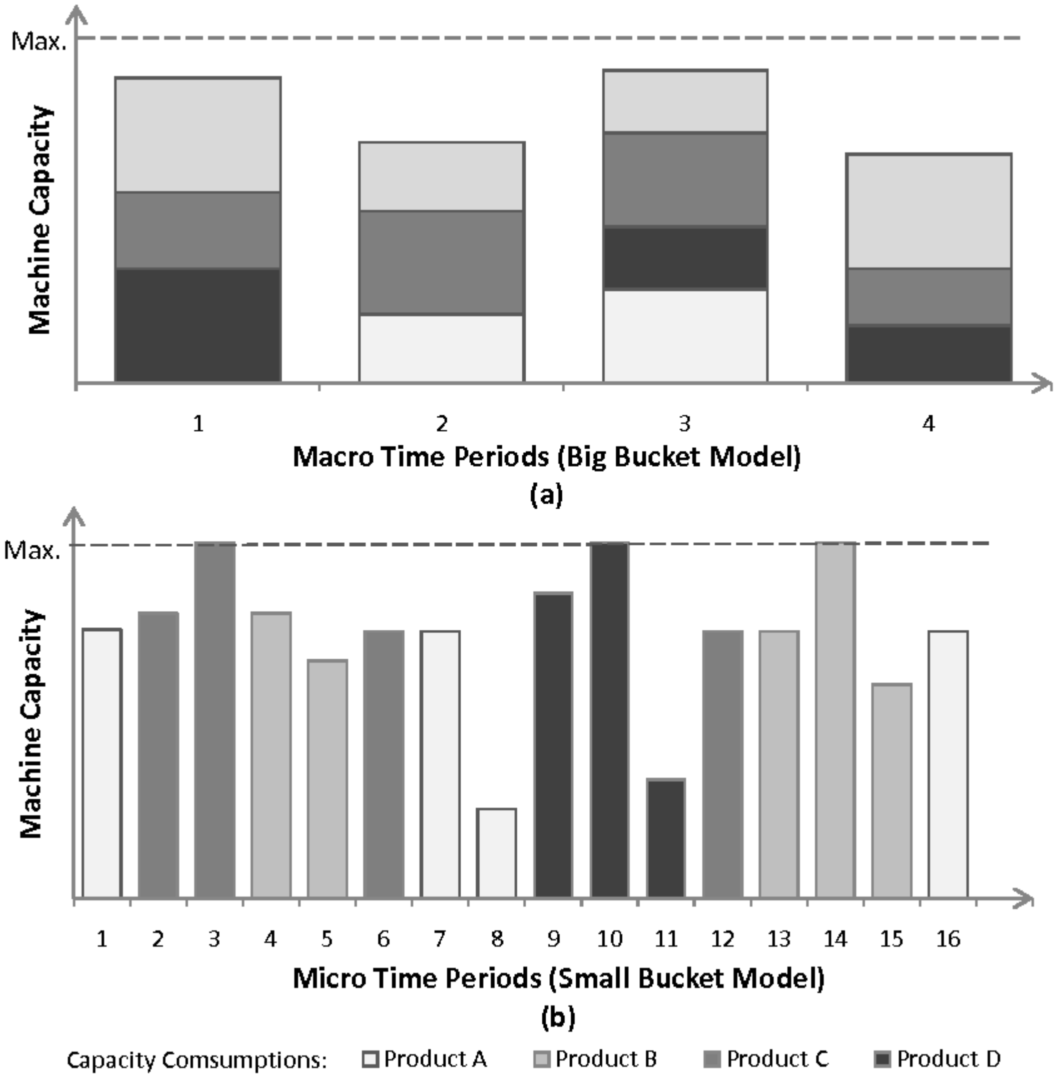


Figure 5–2: Classification of Lot-Sizing Problems (a) Big Bucket Model (b) Small Bucket Model

The planning horizon for the big bucket planning models may vary from a month to six months. The length of individual planning periods is typically a week or two with production of multiple products. The small bucket lot-sizing models planning horizon may vary from a week to a month and individual planning periods may comprise hours, shifts or days. Generally, the number of products produced in a single planning period is low.

Figure 5-2 shows the differences between the big bucket and small bucket lot-sizing models. Multiple products can be manufactured in the macro time periods of the big bucket models whereas the standard small bucket planning model can only manufacture one product per planning period which may result in unutilized capacity as shown in Figure 5-2 (b). Whenever a new product is manufactured, it requires a setup which reduces the machine utilization. However, if the same product is allowed to carry over into the next planning period, there is no need for separate setup and the whole planning period is dedicated to the production process to meet the demand requirements (Figure 5-2 (b), products C, B and D). The standard big bucket model does not permit the setup carryover but its extensions do. Similarly, the unutilized capacity has also been used in the subsequent proposed improvements of the small bucket problem. The details of the standard big and small bucket lot-sizing models and the extensions are discussed in the next section and a summary of important lot-sizing models is presented in Table 5-1.

Big Bucket Lot-Sizing Models

The Capacitated Lot-Sizing Problem (CLSP) is the standard big bucket planning model wherein several products can be produced in each period and a setup cost is incurred whenever a new product is set up for production. The individual planning period length is usually a week or two and the planning horizon may be as long as six months. Solving a standard formulation of CLSP gives the number of different products to be produced in one planning period but does not define the sequence of production within the planning period. Therefore, the sequencing decisions are to be determined separately. The usual approach is to solve the CLSP first, and to solve a scheduling problem for each period separately afterwards (Drexl & Kimms 1997).

Table 5-1: Lot-Sizing Models and their Characteristics

	Lot – Sizing Models	**Characteristics**
1.	Capacitated Lot-Sizing Problem (CLSP)	o Several products per planning period i.e multiple setups in a single planning period. o Big bucket planning problem - Up to six months planning horizon - Typically weekly or biweekly planning periods. o Lot-sizing and scheduling carried out separately.
2.	Capacitated Lot-Sizing Problem with Setup Carryover or with Linked Lot-Sizes (CLSPSCO)	o CLSP extension because it requires the sequence of first and last product of each planning period to be known. o A two level lot-sizing model with a planning horizon constituting macro time periods and each macro period is subdivided into a number of micro periods. The macro period lot-sizing problem is an extension of CLSP whereas the micro-period lot-sizing problem is similar to the CSLP. o Lot-sizing and sequencing decisions are carried out simultaneously.
3.	Capacitated Lot-Sizing Problem with Sequence Dependent Setups (CLSPSD)	o CLSPSCO extension because it requires sequencing of all products within a planning period. o A two level lot-sizing model with a planning horizon constituting macro time periods and each macro period is subdivided into a number of micro periods. The macro period lot-sizing problem is an extension of CLSP whereas the micro-period lot-sizing problem is similar to the CSLP. o Lot-sizing and sequencing decisions are carried out simultaneously.
4.	Continuous Setup Lot-Sizing Problem (CSLP)	o One product per planning period i.e at-most one setup in a single planning period. o Small bucket planning problem - Shorter planning periods such as hours, shifts, days - Weekly or biweekly planning horizon o Sequencing decision imbedded in the lot-sizing solution
5.	Discrete Lot-Sizing and Scheduling Problem (DLSP)	o An extension of the Continuous Setup Lot-Sizing Problem (CSLP) but unlike CSLP, it only supports either zero or full production capacity.
6.	Proportion Lot-Sizing and Scheduling Problem (PLSP)	o An extension of the Continuous Setup Lot-Sizing Problem (CSLP) but unlike CSLP, it allows at most two products in a single planning period.

The Capacitated Lot-Sizing Problem (CLSP) does not permit the production of products across adjacent planning time periods i.e the setup state of a product cannot be carried over to the next planning period. This happens because the sequencing order of the products is unknown within a CLSP solution and the last product to be produced in the time period '*t*' that may be carried over to 't+1' is unknown. The setup carryover capability is important in the process industry where the setup costs are substantial and the necessity of a new setup at the beginning of each planning period gives costly schedules. The setup carryover capability in a CLSP model can be achieved if the sequencing decisions are also incorporated. Haase (1994) did the pioneering work that allowed the setup carryovers and termed it CLSP with linked lot-sizes because the lots were linked in the adjacent time periods. Other applications include Sox and Guo (1999) , Suerie and Stadtler (2003), Porkka et al. (2003), Suerie (2006), and Tempelmeier and Buschkühl (2009).

Another sequencing related issue with the standard CLSP formulation is with regard to the sequence dependency of the setups which are encountered in many industries like the paper manufacturing. The sequence dependency implies that the expenditures for the setups of a machine depend on the sequence in which different items are scheduled on the machine. Especially if a machine produces items of different family types i.e the product range produced on a machine varies substantially, setups between items of different families can be substantially more costly than setups between items of the same family. Also, the objective of lot-sizing is to minimize the sum of setup and holding costs. In order to compute the sum of sequence dependent setup costs accurately, the sequencing order of the products has to be ascertained first. Therefore, a Capacitated Lot-Sizing Problem with sequence dependent setup time (CLSPD) also needs to integrate sequencing decision within the lot-sizing decisions. The difference between the sequence dependent setups and setup carryover condition is that that the setups carryover condition requires a decision regarding the first and last products in each period whereas the former requires the identification of the complete sequence. Haase (1996) provided the first implementation of the Capacitated Lot-Sizing Problem with sequence dependent setup time (CLSPD) on a single machine with a heuristic solution. Recent applications are Gupta and Magnusson (2005) and Mateus et al. (2010).

The General Lot Sizing and Scheduling Problem (GLSP) introduced by Fleischmann and Meyr (1997) is closely related to the Capacitated Lot-Sizing Problem with sequence dependent setup time (CLSPD). It is similar to the CLSPD with regard to the two levels of decision making and allows the lot-sizing and sequencing decisions to be taken simultaneously; it eases restrictive assumptions of CLSPD by allowing a product to be produced more than once within a planning period and by relaxing strict setup cost calculation conditions (Koclar 2005). Transchel et al. (2011) is one of the most recent implementations of GLSP.

Small Bucket Lot-Sizing Models

The Continuous Setup Lot-Sizing Problem (CSLP) is the standard small bucket lot-sizing model and differs from the CLSP on a number of counts. Firstly, CSLP is a small bucket problem with the planning periods being much smaller such as hours, shifts or days and the planning horizon is generally limited to a week or two. The other differences between the CLSP and the CSLP are that in the latter, at most one product is produced in a period and setup carryover is it allowed i.e a changeover cost is incurred only in the periods when the production of a new item starts. This implies that the maximum number of setups allowed in a single period is one but a setup state may carry over to the next planning period if the same product is being manufactured (Figure 5-2 (b)). This seems a reasonable restriction as the length of the planning period has been substantially reduced and a shorter period may be dedicated entirely to a single product. Another advantage of this approach is that the sequencing decisions are imbedded because the resulting solution determines both the quantity and sequence of the products to be produced in each planning period.

Other small bucket lot-sizing models are in-fact variants of the Continuous Setup Lot-Sizing Problem (CSLP) and include Discrete Lot Sizing and Scheduling Problem (DLSP) and Proportion Lot Sizing Problem (PLSP). DLSP was first introduced by Fleischmann (1990) and is similar to CSLP because it also assumes at most one item to be produced per period. The difference is that in the DLSP, the quantity produced in each period is either zero or the full production capacity. In other words, if a setup is performed, the entire time interval must be devoted to the setup. That is, setups and production runs comprise an integer number of time intervals. Later on, sequence dependent setups were also studied with Discrete Lot Sizing and Scheduling Problem (DLSPSD) by Fleischmann (1994) and Salomon et al. (1997). Industrial applications of DLSP include Jans and Degraeve (2004) for tire manufacturing and Brodkorb and Dangelmaier (2010) for the automotive sector.

The proportional lot sizing and scheduling problem (PLSP) relaxes the restriction of allowing production for only one product in each time period but at most only two different items can be produced in each time period. There is still atmost one setup in each period, but the setup from the previous period can be carried over to the next period. The basic idea behind the PLSP is to use the remaining capacity for scheduling a second item in the particular period if the capacity of a period is not used in full. The unutilized capacity is infact the shortcoming of the CSLP (Drexl & Haase 1995). Recent industrial applications include Tempelmeier and Buschkühl (2008) for the automotive sector, Stadtler (2011) for pharmaceuticals and Kaczmarczyk (2011) for a general parallel machine modelling problem.

Computational Complexity of the Lot-Sizing Models

Different ways to model the single level multi-item capacitated lot-sizing problem have been discussed above and it has been noted that all these differ only by virtue of additional constraints or by the difference in the lengths of planning periods. Capacitated Lot Sizing Problem (CLSP) and its variants like CSLP, DLSP, PLSP, and GLSP are known to be NP hard even if no setups are involved i.e it is a single product planning problem (Bitran & Yanasse 1982). Its complexity is further increased when setups are considered or for a multi item planning problem (Trigeiro 1989; Drexl & Kimms 1997; Fleischmann & Meyr 1997; Rizk & Martel

2001; Karimi, Fatemi Ghomi & Wilson 2003). All these models have been extensively used in the literature depending upon the specific needs of individual problems. Small to moderate problems have been solved to optimality by mathematical approaches whereas in other cases, meta-heuristics as well as problem specific heuristics have been applied as solution approaches (Jans & Degraeve 2007; Quadt & Kuhn 2008; Buschkühl et al. 2010)

Lot-Sizing Model for Paper Manufacturing

Rizk et al (2004), Martel et al. (2005) and Rizk et al. (2008) used the small bucket Continuous Setup Lot-Sizing Problem (CSLP) formulation for production planning for paper machines where at most one grade was produced in a single planning period. The formulation is likely to come up with minimum setup costs but the resulting solution may be found wanting on the cycle service level issue and also could yield increased holding costs. Instead of using a small bucket planning problem, Bouchriha et al. (2007) used an iterative process to determine the optimum length of a single planning period. A two week planning period was found out to be the one giving the minimum cost wherein all the products were to be manufactured once with variable tonnage and therefore, the modelling approach used was close to the Capacitated Lot-Sizing Problem (CLSP) with an additional constraint of sequence dependent setups. However, as discussed earlier, this approach focused only on the minimization of cost assuming that the customer can wait up to two weeks for the delivery of their orders which may not be acceptable. Similarly, Gupta and Magnusson (2005) modelled the sandpaper production planning problem as a Capacitated Lot-Sizing Problem with sequence dependent setups (CLSPD). Porkka et al (2003)'s paper production planning problem focused on introducing a setup carryover condition in their Capacitated Lot-Sizing Problem (CLSP) formulation.

The lot sizing problem modelling approach to be used in this chapter will be twofold. First, a small bucket approach will be used with a daily planning period and a planning horizon of one week but it will differ from the Continuous Setup Lot-Sizing Problem (CSLP) because it will not restrict the maximum number of setups incurred in a single planning period. Then the problem will also be solved by enforcing the maximum number of setups per planning period to be one i.e CLSP will be used. The results will be used to gauge the effectiveness of the planning approaches against cycle service levels along with the traditional aim of cost minimizing.

5.2.1.3 Integrated Cutting Stock and Lot-Sizing Problem with Service Levels

The production problem is essentially to plan the production and inventory levels of multiple finished products (FP) i.e rolls and intermediate products (IP) i.e jumbo reels over a finite planning horizon in a paper mill, where paper production and conversion are two successive stages. The large jumbo reels of paper are produced on paper machines are cut into smaller rolls according to customer specifications during the conversion process. No inventory of jumbo reels is kept i.e reels are instantly converted to FP rolls. Each planning period represents a 24-hour shift whereas the planning horizon comprises seven days. A schematic of the two processes is shown in Figure 5-3:

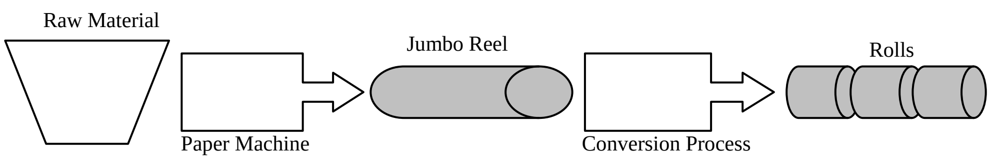


Figure 5–3: A Schematic of the Production Problem under Consideration

The demand for intermediate products i.e jumbo reels is unknown but derived through the independent demand i.e FP demand. Although, no inventory of jumbo reels is kept, finished products can be stored at the manufacturing facility. Changeover costs are sequence dependent and are incurred whenever a different grade of paper is manufactured on a paper machine. The lot-sizing problem here determines the tradeoff between changeover and inventory holding costs, whereas the cutting stock problem defines the relationship between the finished and intermediate products. It also defines the demand for jumbo rolls. The integrated lot-sizing and cutting stock problem will jointly optimize the changeover, FP holding costs, the waste incurred during the conversion stage and the tardiness penalty accrued on each late order. The industry generally uses a predetermined sequence of grade changes from an incremental increase in bases weight (g/m^2) to the heaviest and then decreasing the basis weight to the lowest. The pre-determined sequence relaxes the sequence dependent setup constraints, allowing the setup costs to be calculated a-priori and to be used as an input to the aggregate planning model.

The proposed optimization approach is tested by considering a production case where a paper machine produces different grades of paper sack or kraft paper. Paper sacks are known for their high strength and are used for wrapping and packaging purposes in the food products, flour, sugar, chemicals and cement industry. A closely associated product is linerboard with relatively higher basis weight and is used to manufacture corrugated packaging materials.

5.2.2 Mathematical Formulation for the Joint Problem

The objective function consists of four sets of costs associated with the changeover, FP inventory holding, the trim loss and the tardiness penalty, which are to be optimized simultaneously. Mathematically, the problem is specified as the following:

Minimize

$$\sum_{t}^{T}\sum_{i\in IP} K_{it}\rho_{it} + \sum_{t}^{T}\sum_{i'\in FP} h_{i't}I_{i't} + \sum_{j\in J} c_j w_j + \sum_{t}^{T}\sum_{i'\in FP} M\, y_{i't} \quad (5.2)$$

Subject to

$$C_t \geq a_{it}Q_{it} + k_{it}\rho_{it} \quad (5.3)$$

$$d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't} \quad (5.4)$$

$$\sum_{t}^{T} Q_{it} - \sum_{j\in J} x_{ij} = 0 \quad (5.5)$$

$$\sum_{j\in J} A_{i'j}\, x_{ij} \geq d_{i'} \quad (5.6)$$

$$\rho, \pi \in (0,1) \quad (5.7)$$

$$Q_{i't}, Q_{it} \geq 0, Integer \quad (5.8)$$

$$d_{i't}, A_{i'j} \geq 0, Integer \quad (5.9)$$

$$y_{i't}, w_j \geq 0 \quad (5.10)$$

Constraint (5.3) enforces the capacity constraint for the paper machine whereas equation (5.4) is the inventory balancing constraint for the finished products. The link between the intermediate products and selected cutting patterns is established through equation (5.5). Constraint (5.6) stipulates that the demand of finished products has to be met for the conversion process. Production and setup indicators are assigned binary values through constraint (5.7). Constraints (5.8), (5.9) and (5.10) specify the non-negative and integer values of IP, FP, $d_{i't}$, $A_{i'j}$ and non-negative values of $y_{i't}$ and w_j.

The potential cutting patterns should also not exceed the jumbo length which is achieved as follows:

$$\sum_{i' \in FP} A_{i'j}\, l_{i'} \leq L \qquad (5.11)$$

The notations used are listed in Table 5-2.

Table 5-2: Notations

T	=	Length of the planning Horizon
t	=	A single planning period
i	ϵ	Intermediate products (IP)
K_{it}	=	Grade changeover cost for IP i (hours)
$\rho_{i\,t}$	=	Setup Indicator for IP i in period t
i'	ϵ	Finished products (FP)
$h_{i't}$	=	Inventory holding cost of item i' in period t.
$I_{i't}$	=	Inventory of FP i' at the end of period t
j	ϵ	A cutting pattern.
c_i	=	Cost of waste incurred on jumbo reel i
w_{ij}	=	Waste incurred by using pattern j on jumbo reel i
M	=	Tardiness penalty ($)
$y_{i't}$	=	FP quantity i' that is not delivered within due date.
C_t	=	Paper machine's production capacity (hours)
a_{it}	=	Capacity consumption rate of IP i (hours/metric ton)
$Q_{i\,t}$	=	Quantity of IP i produced during period t
k_{it}	=	Grade changeover time for IP i (hours)
$d_{i't}$	=	Demand for the FP i' in period t
$Q_{i't}$	=	Quantity of FP i' produced during period t
x_{ij}	=	Number of times the *jth* pattern is use don IP i to generate FP i'
$A_{i'j}$	=	No of times the order width i' is produced on pattern j
$l_{i'}$	=	Order Width of FP i' (m)
L	=	Jumbo Length (m)

In the objective function (5.2) the first three terms cover grade changeover cost, inventory holding cost and the cost of material wasted in the cutting process, all being normal production costs expressed in dollars. The fourth term measures the cost of failure to meet due delivery dates but the cost 'M' attributed to such a failure is set at the high value of $5,000. Although it affects the result this amount is not a normal production cost and is deducted from the final solution cost.

The assumption of the constant and highest value of tardiness penalty is rather arbitrary but it facilitates the prioritization of the customer cycle service level (CSL) over other objectives. However, in Chapter 6, this issue is resolved because of the use of multi-objective optimization algorithm which utilizes the Pareto rank or dominance based approach where all objectives are given equal importance.

The above formulation integrates the two successive production processes of paper making and cutting with the cycle service level considerations in a joint optimization framework greatly enhancing the computational complexity. The lot-sizing problem with setups describes the production process of paper making whereas the paper cutting is achieved through the cutting stock problem. The simultaneous determination of the cutting patterns to be used on the jumbo reels and the lot-size for the paper machine so that the trim loss, grade changeovers and inventory holding costs are minimized is actually an integration of two NP-hard problems. The addition of cycle service level further increases the difficulty of the composite problem. Now various decisions such as generation of cutting pattern and their allocation to different planning periods, which also triggers the production of jumbo reels of different grades at the paper machine, are carried out concurrently in order to minimize an aggregated objective function (5.2).

5.3 Experimental Testing

5.3.1 The Solution Approach

Genetic Algorithms were chosen as the search tool because of the complexity of the problem. The lot-sizing problem and the cutting stock problem with due date considerations are both NP-hard problems and solving them simultaneously greatly increases the complexity. The presence of a large number of integers and complex relationships between the intermediate products and finished products can easily be handled by genetic algorithms. Evolver, a GA application from the Palisade Decision Tools, is used to solve the problem. Evolver is an Excel add-in and has been increasingly becoming popular in the research community because of its ease of use and the flexibility that Excel brings with it. Recent application include Carter & Ragsdale (2002), Ragsdale & Zobel (2004), Dooley, Parker & Blair (2005), Grigoryan & He (2005), Wang & Hsu (2008), Carter & Ragsdale (2009), Pech (2009), Sheremetov (2009), Torng & Lee (2009), Torng et al. (2009), Yu-Su, June-Chung & Dah-Chuan (2009) and Chaudhry, Mahmood & Ahmad (2010). The specifications of Evolver's genetic algorithm have been described in Chapter 4.

5.3.1.1 The Genetic Representation

A two chromosome GA representation scheme has been adopted for the joint problem. The two chromosome representation for an individual solution has two strings of length 'n' and 'm' respectively with m genes representing the production of jumbo reels with a domain range of zero to sixty and 'n' genes represent the potential cutting patterns having a domain range of zero to three or four depending upon the test data.

5.3.2 Experimental Setting

An Excel-based model is developed for the problem at hand. Excel's built-in functions and user friendly interface greatly facilitate the modelling process. Its macros or Visual Basic for Applications (VBA) capabilities allow the user to write its own code enabling Excel to capture even the most intricate details of a complex problem such as the integrated cutting stock and lot-sizing problem. The VBA routine captures the relationship between the cutting stock and lot-sizing problem within each iteration; together with Evolver, a genetic algorithm add-in for MS

Excel; it searches for near optimal solutions for the problem. The genetic algorithm performs following main functions:

- Calculation of the objective function for each solution
- Searching for the near optimal solutions, and
- Imposition of constraints

The VBA routine uses minimum waste as the criterion to allocate potential cutting patterns to different time windows and to the jumbo reels, making it an efficient algorithm. The Excel model allowed the imposition of constraint (5.5) through local domain knowledge which is an advantage because it reduced the number of constraints to be handled by the GA. Michalewicz (1995)'s 'Pro-life approach' was used to apply all other constraints.

Evolver comes with different 'solving methods', each of which is a different type of genetic algorithm with customized attributes. The recipe solving method is a genetic algorithm that treats each decision variable as an ingredient in a recipe, trying to find the best mix by changing each decision variable independently. The grouping solving method is a special type of recipe genetic algorithm with a reduced search space and it involves multiple variables to be grouped together in sets. The number of different groups that Evolver creates will be equal to the number of unique values present in the adjustable cells at the start of an optimization (Palisade 2009b). The lot-sizing problem warrants the "Recipe method", whereas the "Grouping method" is found to be more appropriate to the cutting stock problem (CSP) because CSP is essentially a grouping problem that allocates multiple variables to sets or in other words, it determines the arrangement of cutting patterns that optimizes a given criterion.

A uniform crossover value of 0.5 is used across all experiments and auto-mutation is used. Auto-mutation rate adjustment allows Evolver to increase the mutation rate automatically when an organism "ages" significantly; that is, it has remained in place over an extended number of trials. For many models, especially where the optimal mutation rate is not known, selecting Auto can give better results faster (Palisade 2009b).Experiments with initial populations of 50, 200, 500 and 1000 were performed and it was noted that the convergence pattern improved with the 500 population size but no improvements were recorded with a 1000 despite considerable increase in computational workload. Therefore, the population size of 500 was chosen. Similarly, experiments showed that GAs converged before 200 equivalent GA generations or 100,000 iterations; therefore, 200 generations was selected as the stopping criterion for all the experiments.

5.3.3 Generation of Test Data

The technical specifications of a paper machine from an Australian mill have been used in the modelling process. The changeover times for different grades were also provided by the mill. However, the cost and demand data were not provided due to confidentiality. Trade journals were consulted for cost data of different grades of paper Kraft. Now, only the details of customer orders for the finished products is unknown which also determines the demand of the jumbo reels at the paper machine. The following method was used to randomly generate customer orders.

The proposed optimization approach is tested by considering a production case where a paper machine produces four different grades of paper sack or sack kraft without any intermediate storage i.e the jumbos produced were immediately cut in four or less rolls depending upon customer requirements. A week long planning horizon was selected as the planning horizon and a single day was chosen as the individual planning periods. These parameters were deemed sufficient to demonstrate the effectiveness of the joint optimization approach. The model could be modified to deal with more grades at the paper machine or more types of finished products and a longer planning horizon but it would increase the computational complexity. As discussed in Section 5.2.1.1, apart from the paper grade, the customer order encompasses the following three characteristics:

<u>Order Widths ($l_{i'}$)</u>

Roll widths '$l_{i'}$' were randomly generated from a uniform distribution similar to the 'length factor' criterion for the generation of test data for the classical cutting stock problem in Chapter 4 in the following manner:

$$l_{i'} = [\, b_1 * L,\ b_2 * L] \qquad (5.12)$$

The length factor '*b*' is the ratio of ordered length '$l_{i'}$' to the jumbo length '*L*'. Here, the randomly generated roll widths are kept within $b_1 = 0.20$ and $b_2 = 0.80$ to avoid extreme widths of rolls.

<u>Number of rolls required</u>

The bottleneck resource is the paper machine and its capacity is determined by the machine speed which in turn determines the quantity of customer orders it can handle in one week. Also the randomly generated roll widths affect the required number of jumbo reels because of different combinations of cutting patterns. These two parameters restrict the required quantity; therefore, the number of rolls required is spread across all roll widths to match the paper machine capacity.

<u>Order due dates</u>

The order due dates were also randomly generated from a uniform distribution of the five working days in a week long planning horizon which was considered enough to make the point regarding service level considerations. The due dates could have included all seven days of the week at the cost of increased complexity.

Based on above parameters, five data sets were generated to test the proposed model and to compare it with other approaches. The data sets are shown in Appendix B.

5.4 Experimental Results

5.4.1 The Quality of the Solutions

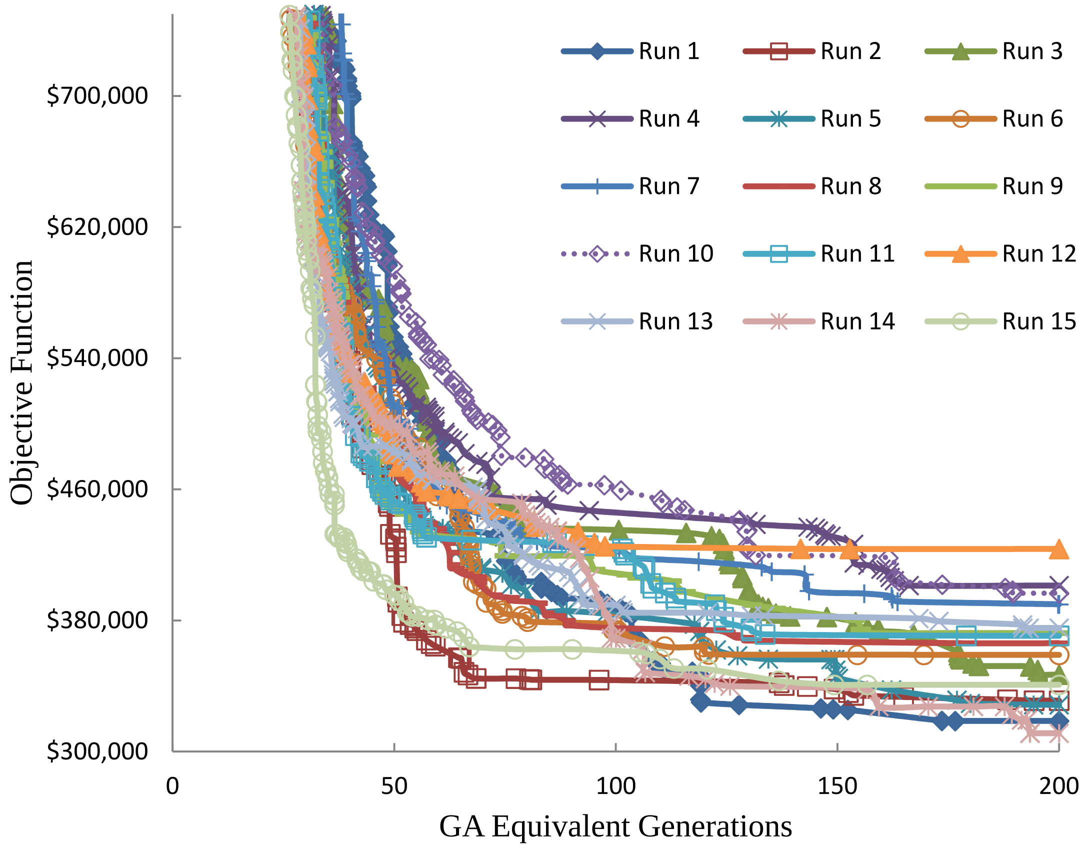
Genetic algorithm has been extensively used for engineering and supply chain optimization problems because of the ability to capture most real world intricacies and because, it always comes up with improvements. However, like other stochastic optimization techniques, the success of GA in finding the best solutions is not guaranteed. A single GA experiment or run may converge on a local optimum solution especially in multi-modal search spaces. Running GA experiments independently a number of times in a sense to some extent offsets this shortcoming. Rudolph (1998) showed that if the probability of NOT obtaining optimal solutions P in a single run is a polynomial time function than by repeated GA runs, P decreases exponentially. Although, repeat GA runs improve the probability of reaching an optimal solution, still there is no guarantee that by running a fixed number of GA runs, the best possible solution would be achieved.

Yuen et al (2001) introduced a method to estimate the probability of attaining optimality by using repeated runs of GA. The term success was used for reaching an optimum and the core of the procedure was based on the estimation of the probability of success for a single GA run *(P_{SGA})*. If P_{SGA} can be estimated, the authors proposed a procedure that determined the number of GA experiments '*N*' necessary to obtain a *95%* success probability i.e a probability of success by repeated application of GA (P_{RGA}). The relationship between *N* and P_{SGA} based on a *95%* value for P_{RGA} is an exponential and decreasing curve which essentially means that as P_{SGA} decreases, the number of GA experiments required for a 95% success increases exponentially. For example, with a $P_{SGA} > 0.18$, $N = 15$ will suffice whereas for $P_{SGA} < 0.05$, N increases to more than *58* for a *95%* P_{RGA}.

Yeun et al (2001) and Cheung et al (2004) were able to estimate P_{SGA} for their object detection problem with the help of historic data; because the case under study is an altogether different optimization problem, P_{SGA} cannot be estimated. However, past studies give some confidence that with repeated GA experiments, the probability of obtaining a nearly optimal solution increases exponentially. The issue to consider here is the computational load associated with repeated runs. A GA experiment for the integrated cutting stock and lot-sizing problem took nearly five hours to complete 200 generations on an Intel Core 2 Duo CPU (2.26 GHz processor speed and 2 GB of RAM), therefore, running GA experiments more than 15 times ($N > 15$) would have been computationally expensive. Comparison of the integrated formulation with existing planning practices, both industrial and academic also required more experiments. Therefore, fifteen GA experiments have been carried out to search for the production schedule for the integrated problem with reasonable assurance of the quality of solutions.

5.4.2 Convergence of Solutions

The objective function which is the sum of grade changeover costs, inventory holding costs, trim loss and tardiness penalty is plotted against GA generations for the fifteen experiments in Figure 5-4. The convergence pattern is somewhat similar for all the experiments with most of the solutions improvements occurring before the 150th generation. However, in a few runs, improvements were also recorded after the 150th generation but it appears that by the 200th generation, all of the solutions had converged. However, the final objective function value is different for all the fifteen experiments, giving a multi peak fitness landscape (Figure 5-5).



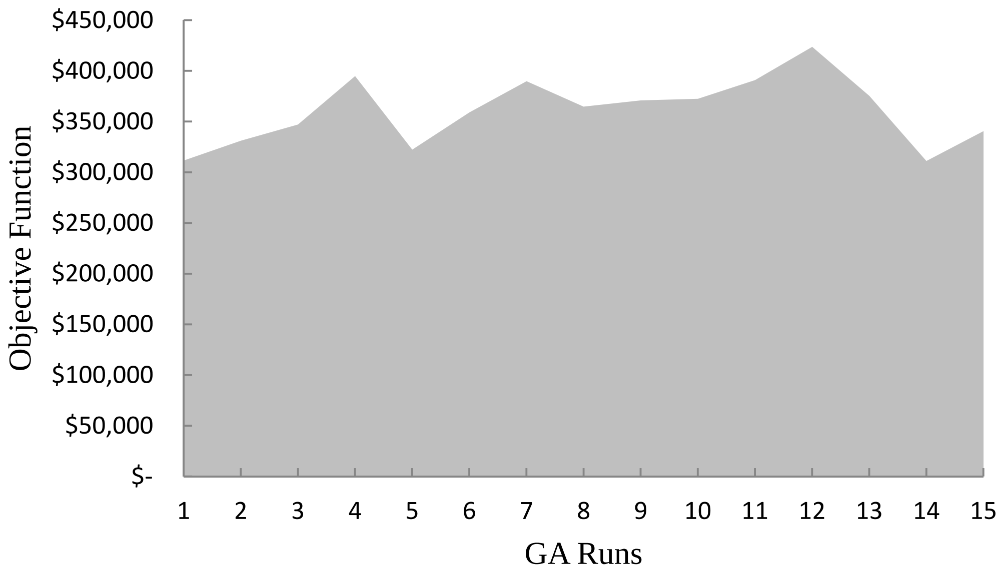
Note: The objective function comprises production costs and the cost of failure to meet due dates (tardiness penalty)

Figure 5–4: The Convergence Pattern for Multiple GA Runs

Figure 5-4 plots the objective function (5.2) which includes tardiness penalties for late orders along with the production cost. Run 14 results in the lowest objective value followed by Run 1 indicating the two maximum cycle service plans.

The presence of multiple optima in the fitness landscape indicates the multi-modal nature of the problem and, typical of multi-modal optimization, each of the fifteen GA runs is a potential solution in itself and could be useful in the real world because the decision maker has the option of choosing between different solutions based on the decision context. If an unforeseen event like a late priority order renders the chosen solution as impractical then the decision maker could switch to another solution.

The presence of multiple peaks in Figure 5-5 could be seen as a merit of genetic algorithm as the solution approach. Because it is q population based stochastic search process, GA has been long regarded as a suitable solution approach for multi-modal optimization problems. Keane (1995) argued that GAs are fundamentally good for multi-peak optimization problems but advocated multiple runs for obtaining a range of possible solutions.



Note: The objective function comprises production costs and the cost of failure to meet due dates (tardiness penalty)

Figure 5–5: Multiple Peak Fitness Landscape

5.4.3 Euclidean Distance between Solutions

GA individuals at the last generation are not identical in the fifteen experiments and the distribution of better GA individuals in the search space gives an indication to the quality of solutions. If they consistently cluster together, it is likely that there is a single peak in the vicinity. Euclidean distance between solutions, a numerical measurement of differences between solutions, has been used as a mechanism for investigating the shape of the search space (Qiu 2000). Mathematically, Euclidean distance d_{xy} between two vectors X and Y is defined as:

$$d_{xy} = \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2} \tag{5.13}$$

Where

$\boldsymbol{X} = (x_1, x_2, ..., x_n)$

$\boldsymbol{Y} = (y_1, y_2, ..., y_n)$

Table 5-3: Euclidian Distance between Solutions

Run	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1															
2	43.65														
3	48.68	46.62													
4	47.71	44.26	44.92												
5	47.68	43.22	45.09	44.33											
6	46.56	45.33	45.14	42.76	45.31										
7	46.81	46.73	46.57	45.13	46.09	45.79									
8	46.49	44.77	46.49	45.02	43.79	43.74	47.75								
9	49.66	48.09	48.70	51.42	48.59	50.26	45.18	49.16							
10	44.12	44.16	44.10	46.81	45.69	45.51	46.67	47.10	45.02						
11	50.22	47.95	47.81	46.97	45.77	48.58	48.98	48.73	51.63	48.92					
12	48.01	47.22	44.60	42.67	41.23	45.40	41.01	45.03	47.34	43.79	48.16				
13	47.18	45.90	44.70	42.85	43.92	44.56	43.78	45.11	46.35	50.95	47.12	42.63			
14	48.28	47.03	45.13	42.46	45.89	46.36	47.75	47.12	52.75	45.69	48.03	46.73	44.82		
15	44.75	44.65	47.38	46.12	47.10	46.77	46.22	45.59	51.00	47.90	47.61	49.52	47.80	46.82	
Production Cost*	256.70	246.32	237.22	259.90	237.40	274.20	269.98	269.83	240.92	267.52	256.02	298.83	275.37	261.24	260.73
CSL (%)	99.17	98.71	98.33	97.95	98.71	98.70	98.18	98.56	98.25	98.41	98.19	98.11	98.48	99.24	98.79

Euclidean distances between GA individuals of all fifteen solutions are calculated and presented in Table 5-3. The total number of decision variables is 988. 28 decision variables associated with the lot-sizing problem have a domain range of *[0, 60]* whereas the domain range of the remaining 960 decision variables of the cutting stock problem is *[0, 3]*. The domain range of the Euclidean distance is as follows:

$$Max.\ Value = \sqrt{960 \times 3^2 + 28 \times 60^2} = 330.81 \qquad (5.14)$$

$$Min.\ Value = \sqrt{960 \times 0^2 + 28 \times 0^2} = 0 \qquad (5.15)$$

As indicated in Table 5-3, the Euclidean distance measure, indicating dispersion of GA individuals in the search space with values being in a close range of 41.01 to 51.63. This shows that the GA is consistently converging to solutions in close proximity which means that the global minimum is in the vicinity if not already achieved.

Among the fifteen experiments reported in Table 5-3, Run 3 and 5 are solutions of interest because these two yielded the two least cost plans yet obtained relatively high cycle service levels. These two solutions are relatively close to each other with a Euclidean distance of 45.09. Similarly, Run 9 is also close with the 3rd lowest production cost and a CSL in excess of 98% and is located at an Euclidean distance of 48.59 from Run 5 and 48.70 from Run 3.

5.4.4 Impact on Cycle Service Level

The last term in the objective function (5.2) shows that meeting due date contributes to the objective. As discussed in the previous sections, all the fifteen runs are potentially good solutions with service levels attained being in the range from 97.95% to 99.24 %. The service level is plotted against total joint production costs, trim loss and grade changeover costs in Figure 5-6 but it is emphasized that the high penalty of $5000 on each later delivery has forced the solutions in to the narrow range of service levels.

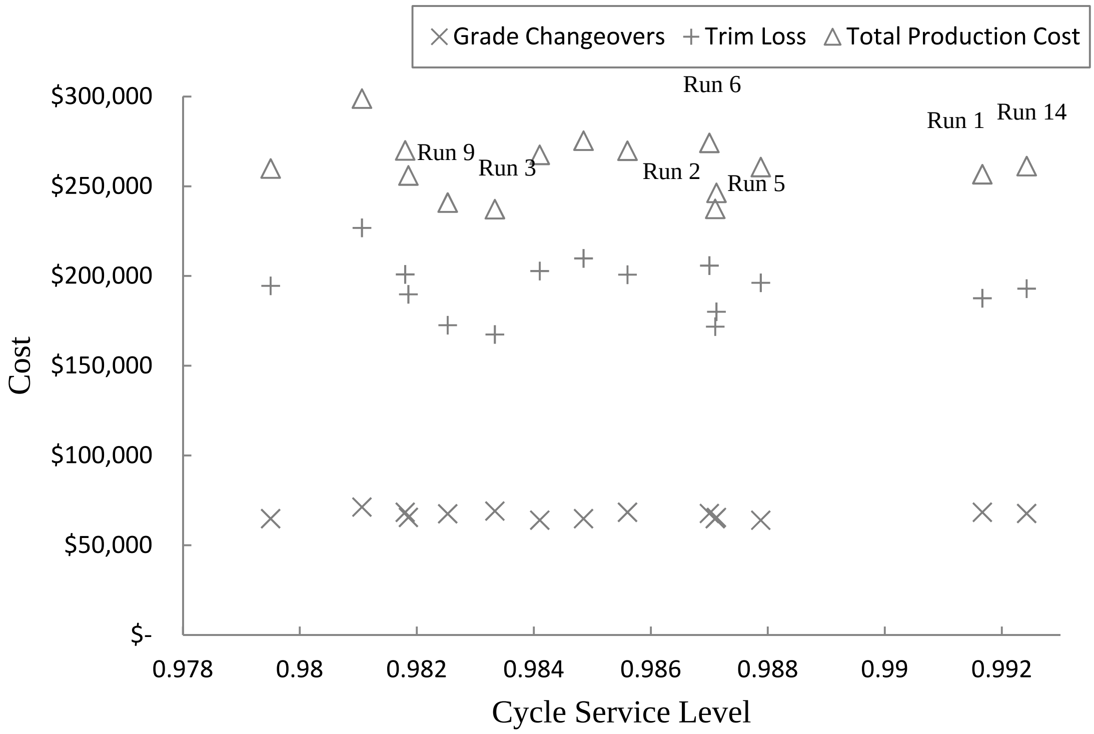


Figure 5–6: Relationship between Trim Loss, Grade Changeovers and Cycle Service Levels

Over the limited range, total joint production costs and trim loss appear to follow an irregular pattern whereas grade changeover cost varies slightly with service levels. The irregular relationship of production cost with the service level is due to the trim loss, the difference being grade changeover cost. A non linear relationship between the trim loss and cycle service levels has also been earlier highlighted by Bookbinder and Higginson (1986). However, their approach did not involve any optimization attempt; they only analysed production data to compare different scheduling rules for a corrugated box manufacturer.

5.4.5 Cost and Cycle Service Level

Table 5-3 and Figure 5-6 can also help a decision maker to choose among different potential solutions depending upon the tradeoff between the production costs and the service levels. The lowest service level obtained is 97.95% for Run 4 and the corresponding production costs are $ 259,901. Due to the non symmetrical relationship between the two ordinates in Figure 5-6, improved cycle service levels are observed in other experiments with lower costs. Therefore, results from Run 4 can be ignored. Similarly, the cycle service level obtained in Run 6 is almost equal to Run 3 and Run2 but the production cost are much higher. A careful analysis of the tradeoff between production costs and cycle service level helps to narrow the focus down to six experiments i.e Run 1, 2, 3, 5, 9 and 14. The highest service level 99.24 % is obtained for experiment 14, closely followed by run 1 with 99.16%. The corresponding production costs are of $261,240 and $256,700 respectively. The other experiments of interest offer significantly lower production cost but the CSL is also slightly reduced. For example, Run 3 and 5 are the two least cost solutions with $237,220 and $237,400 but the CSL is reduced to 98.33% and 98.71% respectively. Based on the above discussion, it seems likely that the decision maker is likely to choose results from one of these experiments.

The preference for individual customers may also be an overriding factor in selection of a final production plan. Typically in the paper industry, some customers enjoy considerable leverage on the paper mill and will insist on having their orders delivered in time because of their own constraints. Although, the customer importance was not modelled explicitly in the joint optimization formulation, the availability of multiple production plans (six experiments of interest) enables the production manager to have an option of choosing a particular Run that ensures timely delivery to the preferred customers.

5.5 Comparison with Relevant Optimization Approaches

The proposed approach of simultaneously minimizing the grade changeover cost at the paper machine, the inventory holding of finished products i.e rolls, trim loss for the cutting stock problem and the tardiness penalty is compared with the following two optimization approaches to gauge the effectiveness:

- Separate Cutting Stock and Lot Sizing with multiple products per planning period (SCL–MP).

- Integrated Cutting Stock and Lot-Sizing with single product per planning period (ICL–SP)

These two planning approaches and the variants have been used both in the literature and in the industry. It is imperative that the proposed approach discussed in this chapter is compared with the above models for a variety of test data. Therefore, the five data sets, generated by the procedure described in Section 5.3.3, have been used to carry out experiments the comparison purposes. The summary of results for the five data sets is shown in Table 5-4.

In the past, the paper industry has tended to optimize manufacturing processes separately. The production planning for the paper machine and the cutting stock problem is disconnected, with stacks of inventory in between. The paper machine scheduling is carried out to minimize the grade changeover costs and the cutting stock problem's optimization criterion is trim loss only. This production philosophy suits the make-to-stock manufacturing strategy because customer orders and its delivery is not a concern. However, over the years, the paper industry has undergone a paradigm shift towards orienting itself to customer needs (Koskinen 2009). Particularly, the packaging products of the pulp and paper supply chain are now only made to order with an emphasis on delivery punctuality (Lange & Andersson 2004). The separate planning approach does result in schedules both for the paper machine and the conversion stages that are aligned to the customer orders but the two step solution tends to restrict the ability to achieve higher cycle service levels. The details are discussed in Section 5.5.1.

The other optimizing approach pertains to the lot-size modelling technique. As discussed in Section 5.2.1.2, the small bucket lot-sizing models have restrictions on the number of products being manufactured in each planning period. In the absence of service level requirements, these restrictions make sense because of reduced setup costs. However, the make to order production environment for packaging paper manufacturing and cutting necessitates the multiple products per planning period approach proposed in Section 5.4. Nevertheless, the integrated approach is also tested for a case where production of only one product is allowed in an individual planning period. The comparative results of the two approaches are presented in Section 5.5.2.

5.5.1 Separate Cutting Stock and Lot-Sizing Problem with Multiple Products per Planning Period (SCL-MP)

The separate cutting stock and lot-sizing problem follows a two step decomposed solution technique where the cutting stock problem is solved first with a minimization of trim loss only and then the allocation of each cutting pattern to the individual planning periods that triggers the production of jumbo reels at the paper machine is carried out in the second step. Mathematically,

Step 1

$$Minimize \quad \sum_{j \in J} c_j w_j \tag{5.16}$$

Subject to

$$\sum_{j \in J} A_{i'j} \; x_{ij} \geq d_{i'} \tag{5.17}$$

$$A_{i'j} \geq 0, Integer \tag{5.18}$$

Step 2

$$Minimize \sum_{t}^{T} \sum_{i \in IP} K_{it}\rho_{it} + \sum_{t}^{T} \sum_{i' \in FP} h_{i't} I_{i't} + \sum_{t}^{T} \sum_{i' \in FP} M y_{i'} \tag{5.19}$$

Subject to

$$C_t \geq a_{it} Q_{it} + k_{it} \rho_{it} \tag{5.20}$$

$$d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't} \quad (5.21)$$

$$\sum_{t}^{T} Q_{it} - \sum_{j \in J} x_{ij} = 0 \quad (5.22)$$

$$\rho \in (0,1) \quad (5.23)$$

$$d_{i't}, Q_{it}, Q_{i't} \geq 0, Integer \quad (5.24)$$

$$y_{i't}, w_j \geq 0 \quad (5.25)$$

The notations used here are similar to Table 5-2. All the constraints are same as in the integrated model presented in Section 5.2. The original problem has been decomposed into two sub problems and is solved separately in two steps. The objective function (5.16) minimizes trim loss only and can be obtained by the column generation based exact solution approach used in Chapter 4. Once the composition of cutting patterns and their frequency is determined in the first step, the lot-sizing problem determines when to produce a particular grade on the paper machine so that a sum of inventory holding cost for the finished products, cost of delaying orders and grade changeover cost is minimized. Five randomly generated data sets for a paper machine of a given speed are used to compare the separate approach with the integrated approach. Fifteen experiments are preformed each for the separate and integrated approached for Data Set 1 while six runs are performed for each of the other data sets. The average values for the multiple runs are shown in Table 5-4 while the details are as follows:

For Data Set 1, fifteen runs of both approaches confirm the superiority of the integrated approach for achieving higher service levels. All experiments for the integrated approach yielded service levels in excess of 97.95 % with the average being 98.52 %. The maximum service level obtained was 99.24 % with total production cost of $ 261,235. The maximum service level obtained by the separate approach was 95.68 % whereas the average value was 94.05 % (Table 5-4). Although, the integrated approach resulted in consistently higher service levels, it also resulted in substantially higher costs, particularly the trim loss. The average trim loss for the integrated approach was 40 % higher than the trim loss obtained through the separate approach.

Table 5-4: Comparison of Three Optimization Approaches

Data Set	Optimization Approach	Average Results for Multiple Runs				
		Grade Change overs	Trim Loss	Holding Costs	Total Production Costs	Service Level
1	Integrated Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (ICL-MP)	$ 66,780	$ 193,308	$ 762	$ 260,850	98.52%
	Separate Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (SCL-MP)	$ 64,811	$ 114,516	$ 781	$ 180,108	94.05%
	Integrated Cutting Stock and Lot Sizing with Single-Product Per Planning Period (ICL-SP)	$ 15,326	$ 162,254	$ 667	$ 178,247	83.09%
2	Integrated Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (ICL-MP)	$ 68,515	$ 166,150	$ 1,059	$ 235,725	97.47%
	Separate Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (SCL-MP)	$ 65,730	$ 92,139	$ 1,044	$ 158,913	91.12%
	Integrated Cutting Stock and Lot Sizing with Single-Product Per Planning Period (ICL-SP)	$ 12,653	$ 137,396	$ 959	$ 151,008	78.34%
3	Integrated Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (ICL-MP)	$ 68,069	$ 155,911	$ 847	$ 224,827	97.98%
	Separate Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (SCL-MP)	$ 64,568	$ 58,191	$ 827	$ 123,587	88.85%
	Integrated Cutting Stock and Lot Sizing with Single-Product Per Planning Period (ICL-SP)	$ 16,119	$ 131,910	$ 644	$ 148,673	79.98%
4	Integrated Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (ICL-MP)	$ 70,240	$ 170,111	$ 919	$ 241,270	96.03%
	Separate Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (SCL-MP)	$ 62,532	$ 61,344	$ 977	$ 124,853	87.69%
	Integrated Cutting Stock and Lot Sizing with Single-Product Per Planning Period (ICL-SP)	$ 15,202	$ 151,663	$ 852	$ 167,717	84.70%
5	Integrated Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (ICL-MP)	$ 69,973	$ 123,918	$ 786	$ 194,677	95.30%
	Separate Cutting Stock and Lot Sizing with Multi-Products Per Planning Period (SCL-MP)	$ 66,064	$ 81,282	$ 822	$ 148,169	90.29%
	Integrated Cutting Stock and Lot Sizing with Single-Product Per Planning Period (ICL-SP)	$ 14,423	$ 135,077	$ 738	$ 150,238	80.77%

It shows that the trim loss obtained by the integrated approach is primarily focused on maximization of service levels because of the highest cost parameter ‘M’ being the tardiness penalty. The grade changeover costs for the integrated approach are also slightly higher with an average of 2.95 %. This is because of the higher number of jumbo reels produced at the paper machine as a result of higher trim loss. The results obtained for all the other data sets follow a similar pattern. The integrated approach consistently resulted in higher service levels at the expense of additional trim loss. The difference in the service levels obtained by the two approaches was 6.52 % with a 32.59 % more expensive production plan for data set 2, 9.32 % with 45.03 % additional costs for data set 3, 8.69 % with an increase of 48.25 % in total costs for data set 4 and 5.25 % with 23.85 % higher costs for data set 5.

The results reveal that when there is no restriction on the number of products produced in each planning period, the boost to service levels is achieved by an additional trim loss. It suggests that, since the only objective for the cutting stock problem is minimum trim loss, it comes up with a very cost effective solution but also imposes an upper bound on the lot-sizing problem i.e the maximum number of jumbo reels to be produced. The upper bound of the jumbo rolls brings the costs down as compared to the integrated model where the quantity of jumbo reels produced is consistently higher. However, it performs poorly in satisfying customers order on a daily basis, restricting the CSL. The higher number of jumbo rolls for the integrated model enables the production plan to generate better cycle service levels.

5.5.2 Integrated Cutting Stock and Lot Sizing Problem with a Single Products in Each Planning Period (ICL-SP)

Simultaneously tackling the lot-sizing and cutting stock problems with multiple-products in a single planning period has proved to be effective for the attainment of high cycle service levels. Multiple products in a single planning period of a small bucket problem have been given little attention by the paper industry primarily because of the probably high grade changeover costs. Rizk et al.(2004), Martel et al.(2005) and Rizk et al.(2008) allowed only one product per planning period for their small bucket planning problem whereas Bouchriha et al. (2007) used a big bucket problem for production planning at the paper machine with a single planning period of a week or two. All these efforts aimed to minimize production costs only and did not take into account the service level. .

In this section, the integrated formulation for the joint optimization is amended to correspond to the continuous setup lot-sizing model (CSLP) of Section 5.2.1.1 that only allows production of one product in each planning period. The addition of the following constraint transforms the integrated cutting stock and lot-sizing with multiple products model (ICL-MP) into an integrated cutting stock and lot-sizing with single products (ICL-SP):

$$\sum_{i \in IP} \pi_{it} = 1 \qquad (5.26)$$

Where

$\pi_{i\,t}$ = Binary production indicator for IP i in period t

Constraint (5.26) stipulates the number of intermediate products to be produced in a single planning period to be 1. This implies that a particular grade can only be produced once in each micro planning time period with a possibility of carrying over the production setup to the subsequent planning periods. As earlier discussed in the literature review of lot-sizing models (Section 5.2.1.2), the setup carryover capability is important in certain industrial contexts because of substantial associated costs but it may also have adverse affects on cycle service levels. This restriction was not imposed in the models formulated in Section 5.2.2 and Section

5.5.2. The number of setups for micro planning periods in those models was unconstrained and subject to the customer orders.

Again, multiple runs are preformed for the multiple products and single product integrated approaches for the five data sets and the results are averaged for the comparison (Table 5-4). Fifteen experiments are carried out for Data Set 1 while five runs are performed for all other data sets.

For Data Set 1, the integrated cutting stock and lot-sizing with single-product per planning period (ICL-SP) gave many unpunctual order deliveries with the an average service levels of 83% which is considerably lower than both the integrated cutting stock and lot-sizing with multi-products per planning period (ICL-MP) and Separate Cutting Stock and Lot-Sizing with multiple-products per planning periods (SCL-MP). However, it is also the least costly production plan with the substantially lower grade changeover costs. This is understandable as the number of grade changeover for the ICL-MP varied from 20 to 28 in the weekly planning horizon whereas for the ICL-SP it was only seven. As in the integrated formulation for the ICL-MP, the ICL-SP generated cutting plans with a preference for meeting customer specified due dates that resulted in higher trim loss than the SCL-MP. Despite that, the service levels were poor to the extent that not all the demand for the jumbo reels could be met in the weekly time horizon and it would have to be transferred to the next planning horizon bringing the service level further down. The lot-sizing approach in ICL-SP is similar to the continuous lot-sizing model (CSLP) discussed in Section 5.2.1.2 and it was noted that one of the major drawbacks of the approach is the unutilized capacity (Figure 5-2). The same has been observed here (Table 5-5) as the machine utilization which is defined as the actual time spent by the machine in producing a product excluding the grade changeovers and the unutilized capacity. For Data Set 1, the machine utilization for the ICL-SP is only 70.39 % compared to the 90.76% for the ICL-MP.

The results obtained for all the other data sets follow a similar pattern. ICL-SP resulted in consistently lower service levels but there was also a reduction in grade changeover costs. The grade changeover costs in the paper industry are substantial and by restricting the number of products, the cost savings are significant but with ramifications for the service level and for machine utilization. While the single product planning approach may not be the most advantageous choice for packaging materials with a requirement of punctual deliveries, it may also struggle for the other products of the pulp and paper supply chain because of the unutilized capacity.

Table 5-5: Average Machine Utilization for the two Integrated Cutting Stock and Lot-Sizing Models

Data Set	1		2		3		4		5	
Optimization Approach	ICL-MP	ICL-SP	ICL-MP	ICL-SP	ICL-MP	ICL-SP	ICL-MP	ICL-SP	ICL-MP	ICL-SP
Machine Utilization	90.76%	70.39%	89.81%	72.85%	88.25%	71.17%	90.19%	79.12%	88.55%	76.91%

ICL-MP = Integrated Cutting Stock and Lot Sizing with Multiple Products

ICL-SP = Integrated Cutting Stock and Lot Sizing with Single Product

Tables 5-5 reports the paper machine utilization levels for CLSP are significantly lower than the proposed approach. The paper industry is capital intensive and a paper machine usually runs continuously with only maintenance breaks. The proposed approach results in near capacity running in all the experiments whereas, depending upon the initial search space, the GA can be restricted to very low utilization levels because of limiting production to a single product in each planning period. There is also symmetry in the results of Tables 5-4 and 5-5: whenever machine utilization is low, the corresponding cycle service levels also suffer and with an improvement, the service levels also improved..

5.6 Conclusion

The model has successfully integrated decision making for the paper manufacturing processes at the paper machine and for the conversion processes. The synchronization of product flow for the two stages has resulted in much improved customer service levels. The high cycle service levels in the integrated model are attributed to the new approach which not only allows multiple setups in a single planning period of small bucket lot-sizing problems but also solves the joint cutting stock and lot-sizing problem simultaneously while minimizing four sets of costs, namely, grade changeovers, inventory holding costs for finished products and trim loss as well as the tardiness penalty.

The integrated cutting stock and lot-sizing model is also compared with other planning approaches or variants that have been used in the literature and in the industry. The separate cutting stock and lot-sizing problem (SCL-MP) follows a two step decomposed solution approach where the cutting stock problem is solved first with a minimization of trim loss only. Then, the allocation of each cutting pattern to the individual planning periods that triggers the production of jumbo reels at the paper machine is carried out in the second step. Experiments showed that although the separate approach results in lower production cost, it fails to match the cycle service levels obtained through the integrated approach. When the integrated formulation is amended to restrict the number of production setups in each planning period to one, the integrated cutting stock and lot-sizing approach with single product in each planning period (ICL-SP) results in consistently lower service levels to the extent that the entire end demand could not be met. Moreover, it also restricted the machine utilization. However, there was considerable reduction in grade changeover costs.

The experimental results also point out the conflicting nature of the relationship between the total production costs and service level maximization. It was shown that a least cost production plan by either restricting the number of grade changeovers or trim loss for the paper manufacturing and conversion stages resulted in poor cycle service levels where many of the customer orders failed to meet the due dates. The situation may be acceptable for few final products in the supply chain but for packaging materials, meeting customer specified due dates is important. On the other hand, the proposed integrated approach maximizes the cycle service levels but the costs have been increased to the extent of being prohibitive. In most real world situations, a decision maker may not opt for maximum service level because it is too expensive and he may prefer a range of values in between the two extremes of a least cost plan and a maximizes service level solution. The trade-off between conflicting objectives has been most effectively captured with the help of a widely known concept of Pareto optimality or dominance wherein solutions are sought from which it is impossible to improve one objective without deterioration in another objective. The Pareto optimal solutions provide the entire range of non dominated solution giving a clear picture of the effects of a least cost solution on the cycle service levels and vice versa. This is the topic of the next chapter where multi-objective optimization approach is applied to the problem under study.

Apart from reflecting the decision context in a better way by giving the entire range of values between the two extremes, the multi-objective optimization approach is also useful in the situations where it is hard to estimate the cost coefficients associated with the objective. The conventional optimization works by aggregating different objectives into a single cost function and the search process to optimality is influenced by the associated cost parameters. For example, the tardiness penalty in this penalty was given the highest value i.e $5000, forcing the search direction of the genetic algorithm towards maximum service level. That penalty was an estimate and may not have correctly captured the decision maker’s preference for punctual deliveries. A multi-objective optimization approach removes the bias towards a particular objective by either normalizing the coefficients of the aggregated objective function, using only one objective at a time or by incorporating the Pareto rank or dominance based approach where all objectives are given equal importance during the pair-wise comparison for dominance. Multi-objective optimization is the topic of the next chapter.

CHAPTER 6. MULTI-OBJECTIVE OPTIMIZATION: MANUFACTURING PROCESSES AND THE CYCLE SERVIC LEVELS

6.1 Introduction – From Single Objective to Multiple Objective Optimization

It was noted in Chapter 5 that high service level and minimum production cost are conflicting objectives in the paper supply chain. It was shown that a least cost production plan for the paper manufacturing and conversion stages resulted in poor cycle service levels where many deliveries of customer orders failed to meet the due dates. The situation may be acceptable for a few final products in the supply chain but for packaging materials, meeting customer specified due dates is important. The traditional production focus in the paper industry has been on maximizing machine utilization and minimizing cost but it has had adverse effects on the overall supply chain benchmarks such as overcapacity, long lead times, excessive inventory and low customer service (Ranta, Ollus & Leppänen 1992; Hameri & Holmström 1997; Hameri & Lehtonen 2001; De Treville, Shapiro & Hameri 2004). This led to a gradual shift to a more flexible production strategy with shorter production cycle times and increased number of grade changeovers for better customer service. While the capacity driven strategy may still be valid for a few standard products with high volumes, the increased product customization in the paper industry warrants a focus on meeting customer requirements that is only possible through a flexible production approach. Hameri & Lehtonen (2001) described a transition in the production strategy for five Nordic paper mills manufacturing paperboard, speciality, standard and standard fine paper. The volume driven strategy with emphasis on utilization and low cost was replaced by a flexible approach that focused on small lot sizes, shorter lead times and punctual delivery. Small lot sizes essentially mean higher frequency of grade changeovers and it was shown in Chapter 5 that an increase in the number of grade changeovers improves cycle service levels. It was also shown that a single criterion of either minimum cost or maximum service levels yields a good solution from one perspective but is likely to give poor results for the conflicting objective. Therefore production planning in the paper industry is faced with more than one optimization criterion which transforms the traditional waste or least cost objective into a multiple objective problem with consideration for meeting customer requirements, particularly due dates.

With multiple and conflicting objectives, the usual meaning of the optimum does not suffice in the decision making context because a solution optimizing all objectives simultaneously generally does not exist. The identification of a best solution requires a trade-off or compromise between the conflicting objectives. As already noted, the tradeoff between conflicting objectives is effectively captured by the concept of Pareto optimality or dominance wherein solutions are sought from which it is impossible to improve one objective without deterioration in another. The Multi-objective optimization approach utilizes the Pareto dominance concept to tackle conflicting objectives and is different to the conventional single objective optimization approach on the following counts:

- There are at least two distinct objectives instead of one.
- It results in multiple solutions giving a range of values between the extreme possibilities for each objective.
- It possesses two different search spaces: objective space and decision space.
- The search process is not influenced by the magnitude of the cost coefficients associated with each objective.

The usefulness of a multi-objective optimization is accentuated in the situations where it is hard to estimate the cost coefficients associated with the objectives because the search process is unaffected by their magnitude. Even if these coefficients are estimated, their magnitude represents a bias that guides the search process in a specific direction. Thus, in Chapter 5, meeting the customer's quoted due dates was incorporated in a single objective function as a tardiness penalty that also minimized the joint production cost. The tardiness penalty was applied whenever an order was not delivered by the due date and it was set at the high cost of $5000. As happens in aggregated single objective optimization, the search process was guided by the sum of the costs. Since the tardiness penalty was high, the search process favoured the least tardiness penalty solutions. Multi-objective optimization removes the bias towards a particular objective by either normalizing the coefficients of the aggregated objective function or by using only one objective at a time or by incorporating the Pareto rank/dominance based approach where all objectives are given equal importance during the pair-wise comparison for dominance.

Although most optimization problems have been solved through a single objective approach, over the years a parallel line of research has evolved by taking a new perspective on the combinatorial optimization problems hitherto treated as single objective problems. For example, vehicle routing, travelling salesman, timetabling, machine scheduling, airline crew scheduling problems and cutting stock problems have long been optimized using single objectives but there is a growing realization that most real world problems need to satisfy more than one criterion for optimization. Routing problems such as travelling salesman and vehicle routing are generally optimized by minimizing the total travelled distance but Ombuki, Ross & Hanshar (2006) identified minimization of the number of vehicles used as another objective and argued that vehicle routing is intrinsically a multi objective optimization problem. Jozefowiez, Semet & Talbi (2008) carried out a survey of multi objective optimization methods applied to routing problems and noted that depending upon the problem context, the optimum considerations included minimization of criteria like travelled distance, vehicles, vehicle waiting times, merchandize deterioration, mean transit time, variance in transit time, individual perceived risk, the actual risk, individual disutility, unused working hours, the length of the longest tour, route balancing and maximization of capacity utilization as well as size of the population covered. Similarly, for timetabling problems, Datta, Deb & Fonseca (2007) proposed two conflicting minimization objectives of average number of weekly free time slots between two classes for the students and average number of weekly consecutive classes for the teachers. For machine scheduling, Li et al. (2010) used minimization of makespan, completion time and tardiness as optimization criteria. For crew scheduling, total cost, delays, and unbalanced utilization have been simultaneously minimized (Lucic & Teodorovic 2007). Ghoseiri, Szidarovszky & Asgharpour (2004) used a dual objective scheduling approach for train operations by considering lower fuel cost for the railway company as one objective while shortening total passenger-time was the other objective.

The integrated cutting stock and lot sizing problem that has been tackled in Chapter 5 aggregated four objectives into a single objective function. Apart from minimization of the production costs of trim loss and grade changeovers, inventory holding costs of finished products and a penalty function for late deliveries was included. The planning premise was that the due dates of all the customer orders were to be respected and therefore, the high tardiness penalty forced the search direction of the genetic algorithms towards maximization of service levels with achievement in excess of 99 %. However, all high service levels solutions were expensive. In most real world situations, a decision maker may not opt for maximum service level because it is too expensive. Instead, the decision maker is more likely to be interested in solutions that a give a range of values in between the two extremes obtained by the multi-objective optimization.

In this chapter a multi-objective optimization approach of the integrated cutting stock and lot-sizing problem is used to obtain a range of compromise solutions between the two conflicting objectives of low production cost minimization and high cycle service levels. The next section briefly describes the theoretical background and introduces concepts which are applied to the production problem at hand. Section 6.3 reviews the available multi objective algorithms and analyses the merits and drawbacks of popular approaches. The bi-objective optimization formulations for the two successive manufacturing processes of the paper industry are presented in Section 6.4. The solution approaches are explained in Section 6.5. The results are described in Section 6.6 and the implications are in Section 6.7.

6.2 Multi-Objective Optimization – Theoretical Background

Mathematically, a multi-objective optimization problem is stated as follows:

Find	$X = [x_1, x_2, x_3, \ldots, x_n]$		(6.1)
to Minimize	$f_i(x)$	where $i = 1, 2, 3, \ldots, k$	(6.2)
Subject to	$g_j(x) \leq 0$	where $j = 1, 2, 3, \ldots, m$	(6.3)

The decision problem here is to find *'n'* decision variables that simultaneously minimize *'i'* objective functions subject to *'j'* constraints. If $i=1$, the above multiple-objective problem becomes a conventional single objective problem with a unique optimum. However, if $i > 1$, a solution that concurrently minimizes all objective function generally does not exist. In such a case, Pareto optimal solutions are obtained which give a trade-off between the conflicting objectives. In a Pareto optimal solution, no further improvement is possible in an objective without deterioration in another objective. Formal definition of the Pareto-dominance concept is as follows:

A feasible solution X^* in a decision space S is said to dominate another feasible solution X if the following two conditions are satisfied:

I. The solution X^* is no worse than X in all *'k'* objectives.

⇒ For a minimization problem,

$f_i(x^*) \leq f_i(x)$ for all $i = 1, 2, 3, \ldots, k$. (6.4)

II. The solution X^* is strictly better than X in at least one objective.

⇒ For a minimization problem,

$f_i(x^*) < f_i(x)$ for at least one $i = 1, 2, 3, \ldots, k$. (6.5)

If both the above conditions are satisfied than the solution X^* is said to dominate solution X or X^* is non-dominated by X. Mathematically, Solution $X^* \preceq$ Solution X and the same concept is used to define a Pareto optimal solution in the objective space $f(x)$ where quality of solution X^* is evaluated. If $y = f(x)$, then the objective vector $y^* \in Y$ is said to be Pareto optimal if there does not exist any other y such that $y_i^* \leq y_i$ for all $i = 1, 2, 3, \ldots, k$ and $y_i^* < y_i$ for at least one $i = 1, 2, 3, \ldots, k$. The set of optimal solutions in the decision space $\boldsymbol{S}$ is referred to as the Pareto Optimal Set $X^* \in \boldsymbol{S}$ whereas an optimal objective vector is known as Pareto front $y^* = f(x^*) \in Y$.

Figure 6-1 illustrates the dominance and Pareto frontier concept with the help of two graphs representing the decision and objective space of an optimization problem. There are two minimizing objectives whereas the decision space could have been for n variables $X = [x_1, x_2, x_3, \ldots, x_n]$ but for ease of visualization, the example used here only contains two variables x_1 and x_2 in the decision space S. Seven possible solutions are plotted in the objective space Y and will be checked for dominance. The innermost curve comprises three solutions (1, 3), (1, 2) and (2, 1). For now, two of these three solution (1, 2) and (2, 1) will be used to explain the dominance concept. According to the first dominance condition, solution (1, 2) is less than or equal to all the other solutions. Similarly, solution (2, 1) is also no worse than all the other solutions. Similarly, for the second dominance condition, one of the two objectives for solution (1, 2) is better than all other solutions except solution (2, 1) and solution (2, 1) has at least one objective better than all other solutions except for solution (1, 2).

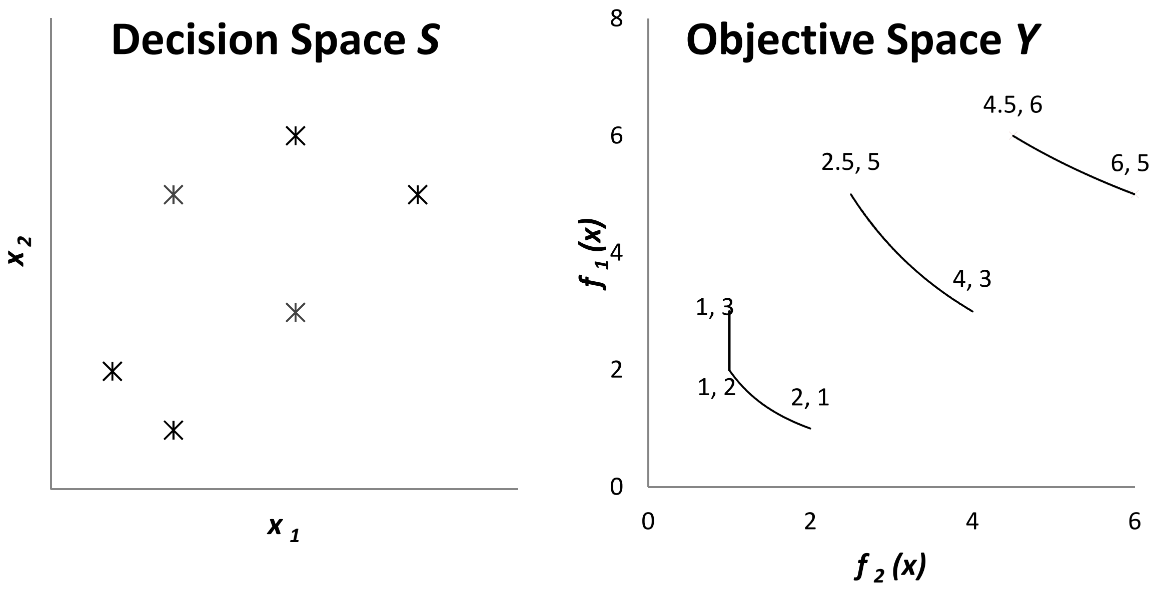


Figure 6–1: Two Different Search Spaces of the Multi-Objective Optimization Problem

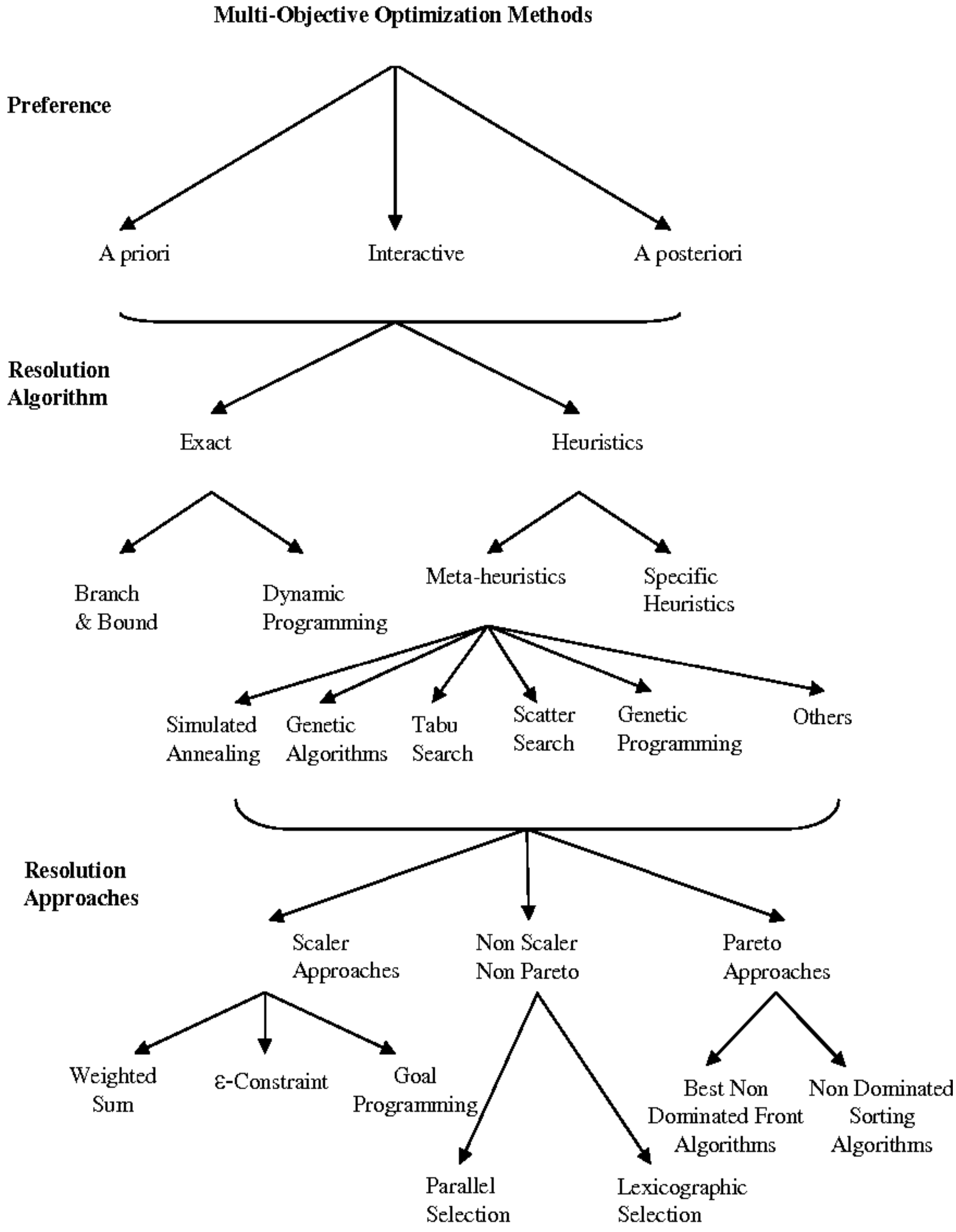
This shows that that solutions (2, 1) and (1, 2) dominate all other solutions but are non-dominated by each other and are joined by a curve – a non dominated or Pareto frontier. The other two local non dominated fronts are also plotted showing the dominance of each front to the others and non dominance of individual solutions contained in each curve. A solution which is Pareto-optimal in only a small neighbourhood is referred to as a *local Pareto-solution* whereas the non dominated front of the entire search space is called the *global Pareto front* of the optimization problem. Mathematically, a decision vector $X^* \in S$ is a *locally Pareto optimal* if there exists $\delta > 0$ such that X^* is Pareto optimal in $S \cap B(X^*,\delta)$ and the corresponding objective vector $y^* \in Y$ is called a *locally Pareto Optimal.* Weak Pareto optimality is another concept associated with Pareto dominance and the relationship between the two is that the Pareto dominant solutions are a subset of weakly Pareto optimal solutions. Mathematically, a decision vector X^* is a *weakly Pareto optimal solution* if there does not exist any other vector X such that $f_i(x) < f_i(x^*)$ for all i objectives where $i = 1,2,3,.., k$ and the corresponding objective vector Y^* is called *weakly Pareto optimal* (Miettinen 1999b; Deb 2005; Ehrgott

2005a). Based on this definition, it is noted that all the three solutions (1, 3), (1, 2) and (2, 1) on the inner most curve of Figure 6-1 are weakly Pareto optimal solution but only solutions (1, 2) and (2, 1) fulfil the two standard conditions of Pareto dominance.

There are related concepts that play an important role in understanding and applying multi-objective optimization methods such as the notion of ideal and Nadir objective vectors. An ideal objective vector represents the lower bound of all the objective functions in the entire feasible search space whereas the Nadir objective vector comprises the upper bound. Estimation of ideal and nadir objective vectors is crucial for effective application of classical multi objective optimization algorithms such as the weighted sum and epsilon constraint methods. Various normalization techniques used in the weighted sum method necessitate prior calculations of ideal and nadir vectors. Similarly, the epsilon constraint method requires this information to estimate the range for the epsilon intervals. Usually, the ideal vector is estimated by solving each objective separately and payoff tables have been extensively used to approximate the corresponding Nadir vector (Miettinen 1999a; Deb 2005; Ehrgott 2005b; Grodzevich & Romanko 2006).

6.3 Multi-Objective Optimization Methods

The ultimate goal of all the multi objective optimization techniques is to obtain Pareto optimal solutions but these methods may differ on account of the approach employed: the criterion for Pareto optimality, obtaining solutions one at time or simultaneously, incorporation of decision maker’s preferences, types of algorithms used, enforcement of constraints and many more. Alternatives are illustrated in Figure 6-2 – an adaptation of Basseur et al (2006). However, in the context of this thesis, the classification on the basis of resolution approaches is reviewed with an emphasis on the scaler approaches and Pareto approaches.



Source: Adapted from Basseur et al. (2006)

Figure 6–2: Classification of Multi-Objective Combinatorial Optimization Methods

6.3.1 Scaler Approaches

Most conventional optimization algorithms require a scaler fitness function for the evaluation of the search process which naturally leads to the aggregation of multiple objectives into a single objective. Once multiple objectives are combined in a scaler function, existing solution methods can easily be applied to generate Pareto optimal solutions by systematically repeating the optimizimg process. Although computationally expensive, scaler methods have been extensively used for multi-objective optimization mainly because the basic concept is simple to apply. A scaler approach can be used either as an a priori method or as an a posteriori method of articulation of a decision maker's preferences. An a priori approach allows the user to specify preferences before the optimizating process which may be articulated in terms of goals or the relative importance of different objectives. However, in many cases, it is not easy for the users to specify their preferences beforehand or to model the preferences in an optimizing model. In such scenarios, the optimization process tries to find a range of values representing the Pareto frontier from which the decision maker can choose a solution. This is called a posteriori articulation of preferences. The following two sub-sections outline the main scaler approaches:

6.3.1.1 Weighted Sum Method

In a weighted sum method, all the objectives are aggregated into a single objective function with an associated weight $\boldsymbol{w_i}$ reflecting the relative importance of each objective. Mathematically, Minimize $\sum_{i}^{k} w_i f_i(x)$ *where i = 1, 2, 3, …., k* (6.6)

Subject to $g_j(x) \leq 0$ *where j = 1, 2, 3, …., m* (6.7)

$$0 \leq w_i \leq 1 \ \& \ \sum_{i}^{k} w_i = 1 \quad (6.8)$$

The relationship of Pareto optimal solutions and the combination of weights has been studied and it has been shown mathematically that under a certain conditions, a weighted sum method generates Pareto optimal solutions (Miettinen 1999a; Ehrgott 2005b). The weighted sum has been used both as an a priori and as an a posteriori method of articulating decision maker preferences. When used a priori, the weighted sum method and its variants incorporate parameters which are coefficients or exponents set to reflect preferences and usually these provide a single solution point. Though the idea of using weights to represent the decision maker's preferences in an aggregated objective function appears straightforward, the real challenge lies in the mathematical modelling of the preferences. In many cases, there will not be enough information regarding the underlying value or utility function representing preferences in the objective space and if there is, its validity is not certain (Miettinen 1999b). Similarly, Marler & Arora (2010) showed that the preferences function can only be linearly approximated and that the weighted sum method is fundamentally incapable of incorporating complex preference information. These issues restrict the a priori use of the weighted sum method, however, it has been widely used as an a posteriori method that generates multiple Pareto optimal solutions by varying weights in a systematic way representing the varied importance of each objective. Each result corresponding to a particular combination of weights provides an optimal solution and the resulting range of Pareto optimal solutions represents the Pareto frontier.

Seminal work on the weighted sum method was done by Zadeh (1963) and it continues to be popular multi objective optimization tool. Despite its widespread use, the weighted sum method has some limitations which need to be addressed for its successful implementation. Das & Dennis (1997) noted following two shortcomings of the weighted sum method:

- The solutions are not uniformly distributed on the Pareto front.
- The solutions lying on the non convex part of the Pareto front, if any, can not be identified by the weighted sum method.

Das & Dennis (1997) introduced an algorithm that overcame these deficiencies and it is called the 'normal boundary intersection' (NBI) method. The algorithm aimed at getting a good diversity of solutions on the efficient frontier by starting from normal directions to the ideal plane. Similarly, Kim & de Weck (2005) introduced the concept of adaptive weighted sum methods wherein additional constraints were introduced based on the location of an optimal solution on the Pareto frontier and showed that their algorithm was capable of obtaining an equi-spaced Pareto frontier for some convex and non-convex test problems. However, it remains to be seen if these improvements are effective across a range of practical problems.(Das & Dennis 1997)

Despite its limitations, the weighted sum method has been popular for multi objective optimization primarily because existing single objective algorithms can be easily applied to generate the Pareto frontier. This is particularly useful for hard problems such as highly constrained combinatorial problems in which the complexity only allows a limited number of solution approaches. For example, meta-heuristic such as genetic algorithms have been used for the weighted sum formulation of the multi-objective combinatorial problems (Syswerda & Palmucci 1991; Hajela & Lin 1992; Jakob & Blume 1992; Jones & Glen 1993; Liu, Begg & Fishwick 1998; Ombuki, Ross & Hanshar 2006). Other recent applications of weighted sum methods are Bufu et al. (2006), Loukil et al. (2007), Jong-hyun et al. (2009), Sameeullah Khan and Cemal Ardil (2009), Xiang et al. (2010) and Iori et al. (2010).

6.3.1.2 Epsilon Constraint Method

The Epsilon constraint method is a technique wherein a multi objective optimization is converted into a single objective problem by keeping one criterion as an objective whereas all other criteria are modelled as constraints by setting an upper bound ε to each of them. The value of ε is systematically changed and the optimization process is repeated over a number of times to generate the Pareto frontier. Mathematically,

Minimize	$f_l(x)$		(6.9)
Subject to	$f_i(x) \leq \varepsilon_i$	*where* $i = 1, 2, 3, \ldots, k$ & $i \neq l$	(6.10)
	$g_j(x) \leq 0$	*where* $j = 1, 2, 3, \ldots, m$	(6.11)
	$x \in S$		(6.12)

The Epsilon constraint method was described by Haimes, Lasdon & Wismer (1971) and since then, it has been extensively used as a multi objective optimization technique, as in the weighted sum method, it allows the application of the existing single objective optimizing approaches. It is also known not to exhibit the shortcomings of the weighted sum method with respect to Pareto front generation for non convex objective space and also results in equi-spaced Pareto optimal points. Miettinen (1999b) showed that the solutions of epsilon constraint methods are always weakly Pareto optimal and to generate one Pareto optimal solution, the problem has to be solved k times where k is the number of objectives added as constraints. Therefore, the Epsilon constraint method can be easily applied to a bi objective optimization problem because it has to be solved only once to obtain a single Pareto optimal solution and by systematically changing ε, a Pareto frontier can be generated easily. However, the challenge lies in determining the ε vector which is usually estimated through the knowledge of nadir and ideal objective vectors.

Ranji Ranjithan, Kishan Chetan & Dakshina (2001) developed an evolutionary algorithm based on the ε-constraint method to generate a Pareto frontier by incrementally changing ε in a single run. Once the population converged to a non-dominated solution according to a stopping criterion, the solution is stored and the constraint value is incremented to gradually shift the population towards the non-dominated solution corresponding to the updated constraint value. The process is repeated till the non dominated solutions to all the ε-constraint increments are found. The algorithm was named the 'Constrained Method based Evolutionary Algorithm' (CMEA) and was tested on two test problems to compare its performance against some other state of the art multi-objective evolutionary algorithms. It was shown that the CMEA performed as well or even better for the two objective test problems. Kumar & Ranjithan (2002) extended the comparative study to a test problem with three objectives and confirmed the effectiveness of CMEA.

Traditionally, ε-constraint methods employ a fixed number of increments to the objective functions bounds that are used as constraints. The increment size is essentially a trade-off between computational effort and the non-dominated spread on the Pareto frontier. However, there have been studies that select the increment size by utilizing the information regarding the search space during optimization and the whole process is known as an adaptive ε-constraint method. The main advantage has been a decreased computational effort required to approximate the Pareto frontier (Reiter & Gutjahr 2010).

6.3.2 Pareto Multi Objective Evolutionary Methods

Multi Objective Evolutionary Algorithms (MOEA) utilize the Pareto based dominance concept in finding a set of non-dominated solutions. MOEA can be categorized on the basis of the mechanism to find non-dominated solutions. Some MOEA only find the best non dominated solutions of a population while some algorithms sort a population according to different non domination levels or Pareto ranks (Deb 2005).

The algorithms that find the best non-dominated frontier carry out a pairwise dominance comparison of each solution *i* from the population *P* with a partially filled population ***P****. If *i* dominates any member *j* of P*, then *j* is removed from *P** and if *i* is not dominated by any member of *P**, *i* is added to P*. If *i* is neither dominated nor dominates any member of *P** then *i* is ignored. The process is repeated for all the solutions contained in *P* and at the end, all members of partial population *P** represent the best non-dominated or Pareto frontier. The concept has been employed for the development of multi-objective optimization algorithms such as Pareto Archived Evolution Strategy (PAES) by Knowles & Corne (2000) and Strength Pareto Evolutionary Algorithm (SPEA) by Zitzler & Thiele (1999).

The non-domination sorting algorithms rank the whole population of solutions according to the domination count n_i i.e number of solutions that dominate solution *i*. The best Pareto frontier will correspond to $n_i = 0$; for each of these solutions, a set of solutions S_i being dominated by *i* is also calculated. S_i is used to find out all the other non-dominated fronts by increasing the domination count by one. The process continues till the whole population is ranked. One of the earliest implementations of a non-dominated sorting mechanism was by Sirinivas and Deb (1993) who developed the non-dominated Sorting Genetic Algorithm (NSGA). Later on, NSGA-II was introduced to improve upon the three known deficiencies of NSGA i.e high computational complexity of non-domination sorting, lack of elitism and the use of a user specified sharing parameter for ensuring diversity of solutions to inhibit early convergence (Deb et al. 2002). Similarly, the Pareto ranking concept has also been used in Fonseca & Fleming's (1993) Multi-Objective Genetic Algorithm (MOGA), in the Niched-Pareto Genetic Algorithm (NPGA) developed by Horn, Nafpliotis & Goldberg (1994) and its subsequent improvement NPGA-2 (Erickson, Mayer & Horn 2001).

There have been various studies comparing the above and some other state of the art multi-objective algorithms for effectiveness in approximating the true Pareto frontier. However, NSGA-II is widely believed to be one of the best and it has been shown that it outperforms most other algorithms (Circu & Leon 2010). Therefore, NSGA-II is used as one of the solution approaches in this chapter. Apart from incorporating elitism and reducing the computational complexity, NSGA-II replaced the sharing method with the crowding distance measure for diversity preservation. The algorithm starts by creating a population Q_t from an initial population P_t and the two populations are combined to form R_t with size 2N. Then, the non dominated sorting is carried out for the population R_t. Since, R_t is twice the size of the initial population P_t, the solutions belonging to poorer fronts are simply deleted. However, some of the solutions from a single front may not be included in the non-dominated sorting list because the list must equal the size of P_t as shown in the Figure 6-3. Only a limited number of solutions from front F_3 can be included in the non-dominated list and the crowding distance selection method is used to pick those solutions with a large distance so that diversity in the candidate solutions is maintained.

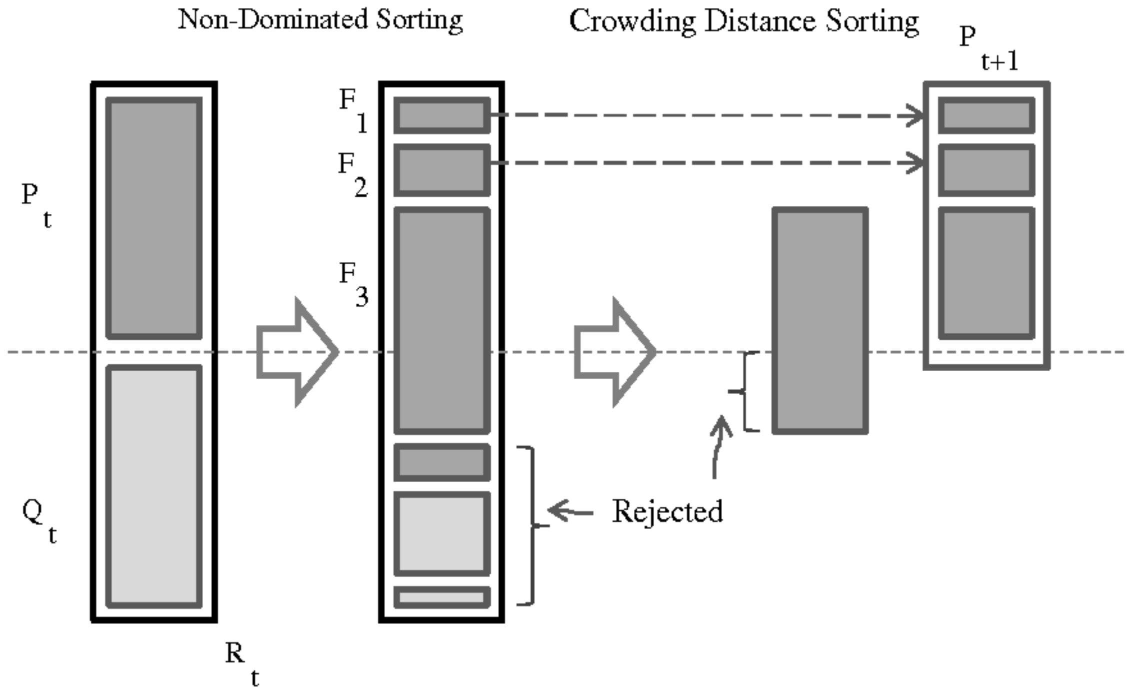


Figure 6–3: Non Dominated Sorting Procedure

The Crowded Tournament selection is similar to the to the standard tournament selection because the two solutions are chosen at random from the population and compared for inclusion into the mating pool. The solution with lower rank wins. The crowding distance 'd_i' is used to differentiate them when the two solutions have the same rank and is defined as the measure of the search space around solution *i* which is not occupied by any other solution in the population (Deb 2001). The solution with larger crowding distance wins implying that if the two solutions have the same rank, the solution lying farther out in the search space is preferred. In case both solutions have the same rank and the same crowding distance (d_i) then the winner is chosen randomly. The crowded tournament is believed to improve the diversity of the solutions obtained (Circu & Leon 2010).

NSGA-II is a constrained multi-objective optimization tool that uses the following mechanism for constraint handling (Deb 2000):

- Any feasible solution is preferred to any infeasible solution,
- Among two feasible solutions, the one having the better objective function value is preferred.
- Among two infeasible solutions, the one having the smaller constraint violation is preferred.

NSGA-II is widely considered a state of art multi-objective optimization tool and has been extensively used (Coello 2010). However, a few issues have been raised regarding its constraint handling mechanism. The preference for a feasible solution over an infeasible solution as a constraint handling technique is different to the commonly used penalty function but is considered an advantage over the traditional penalty function constraint used in GA because the penalty function parameter setting is mostly problem specific and is set by the user (Coello 2002). There are also researchers who have questioned the greedy preference of NSGA-II for feasible solutions over infeasible solutions and believe that when faced with a discrete feasible space the population might quickly converge to a feasible region of the space abandoning all infeasible solutions (Young 2005). Similarly, Geng et al. (2006) argued that the preference for feasible solutions over infeasible solutions may hamper the search ability of NSGA-II when there are multiple and discrete feasible regions in the search space. It is unlikely that the NSGA-II will be able to traverse the infeasible regions to reach new feasible solutions because of its constraint handling mechanism. However, it may be argued that the problem can be handled by ensuring a variety of initial feasible solutions. Similarly, the stochastic nature of GA may also help it to hit on feasible areas of interest even if the search space is discrete.

The number of objectives in a multi-objective evolutionary algorithm (MOEA) is also an important consideration because the Pareto dominance based algorithms have been found wanting for more than two objectives (Wagner, Beume & Naujoks 2007; Purshouse & Fleming 2007; Praditwong & Xin 2007; Ishibuchi et al. 2008). When the number of objectives increases, almost all solutions become non-dominated with respect to each other, severely weakening the Pareto dominance-based selection pressure toward the Pareto frontier. Therefore, the algorithm's ability to converge is seriously affected. Also, the increase in number of objectives tends to raise the computational requirements substantially (Ishibuchi, Tsukamoto & Nojima 2008). The scalability of MOEA to many objectives without affecting its performance is a topic of active research (Schutze, Lara & Coello Coello 2010).

6.4 Multi-objective Optimization for Paper Production

The analyses of the results obtained by conventional optimization of the combined production problem of the two successive manufacturing processes in the paper industry revealed that considerable improvements in cycle service levels were obtained by producing multiple grades in each planning period. The cycle service level improvements were made at the expense of increased grade changeover costs. However, for all data sets, it was noted that the cycle service levels could not be improved beyond a certain level when a separate planning approach for the conversion and paper machine scheduling was used. When an integrated planning was used for the two successive production process considerable improvements in service levels were recorded. This was achieved by the generation of cutting patterns that tolerated extra trim loss but resulted in cuts that met the customer quoted due dates. These results suggested that improvement of cycle service levels (CSL) can be thought as a two pronged process depending upon the requirements. For example, for Data Set 1, if a cycle service level below 96 % suffices, increasing the grade changeover frequency in each planning period with a least trim loss solution is the better option. However, if the CSL is to be improved further, an integrated approach that simultaneously generates cutting patterns and varies the paper machine schedule should be the preferred choice. Therefore, the following two alternate bi-objective formulations of the production problem are presented here to obtain a trade-off curve between cycle service levels and costs.

I. A separate planning approach with grade changeovers and service levels as the two optimization objectives.
II. An integrated or joint planning approach with the total production cost comprising trim loss and grade changeovers cost as one objective and the cycle service levels as the other objective.

6.4.1 Separate Planning Approach: Grade Changeover Cost Vs Service Levels

In this section, a trade-off curve between the two conflicting aims of grade changeover cost minimization and improves cycle service levels is obtained by employing a bi-objective formulation of the manufacturing processes which utilizes a separate or sequential two step planning approach:

- Step 1: The conversion process is solved with a single objective of minimization of trim loss.
- Step 2: Allocation of cutting patterns to different planning periods triggering the production of jumbo reels of the respective grades with a bi objective optimization criterion namely, grade changeover cost and cycle service level. Mathematically,

Objective to be minimized (Notations in Table 6-1)

$$f_1 = \sum_{t}^{T} \sum_{i \in IP} K_{it} \rho_{it} \tag{6.13}$$

$$f_2 = \sum_{t}^{T} \sum_{i' \in FP} y_{i't} \tag{6.14}$$

Constraints

$$C_t \geq a_{it} Q_{it} + k_{it} \rho_{it} \tag{6.15}$$

$$d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't} \tag{6.16}$$

$$\sum_{t}^{T} Q_{it} - \sum_{j \in J} x_{ij} = 0 \tag{6.17}$$

$$\rho \in (0,1) \tag{6.18}$$

$$Q_{it}, Q_{i't}, y_{i't}, d_{i't} \geq 0, Integer \tag{6.19}$$

The indexes, parameters, sets and decision variables used in the above formulation are explained in Table 6-1. The planning problem has been formulated as a bi-objective minimization problem with f_1 representing the grade changeover costs (6.13); and service level improvements have been indirectly formulated in f_2 as minimization of late orders $y_{i't}$ (6.14). Customer orders for the finished product *i'* that will not be delivered by the customer specified due date are to be minimized along with the grade changeover costs incurred on the paper machine subject to the capacity constraint (6.15) and the material balancing constraints (6.16) and (6.17). While the constraint (6.16) ensures that the end demand of finished products is met, constraint (6.16) stipulates that the cut finished products are equal to the number of jumbo reels of a particular grade (IP), not allowing any inventory of intermediate products. Constraints (6.18) and (6.19) ensure integer and non-negative solution to the planning problem.

Table 6-1: Notations

T	=	Length of the planning Horizon
t	=	A single planning period
i	ϵ	Intermediate Products (IP)
K_{it}	=	Grade changeover cost for IP i (hours)
$\rho_{i\,t}$	=	Setup Indicator for IP i in period t
i'	ϵ	Finished Products (FP)
$y_{i't}$	=	FP quantity i' that are not delivered within due date.
C_t	=	Paper machine's production capacity (hours)
a_{it}	=	Capacity consumption rate of IP i (hours/metric ton)
$Q_{i\,t}$	=	Quantity of IP i produced during period t
k_{it}	=	Grade changeover time for IP i (hours)
$d_{i't}$	=	Demand for the FP i' in period t
$Q_{i't}$	=	Quantity of FP i' produced during period t
$I_{i't}$	=	Inventory of FP i' at the end of period t
j	ϵ	A cutting pattern.
x_{ij}	=	Number of time the *jth* pattern is used on IP i
c_i	=	Cost of waste incurred on jumbo reel i
w_{ij}	=	Waste incurred by using pattern j on jumbo reel i
$A_{i'j}$	=	No of times the order width i' is produced on pattern j
$l_{i'}$	=	Order Width of FP i' (m)
L	=	Jumbo Length (m)

While discussing the lot-sizing issues at the paper machine in Chapter 5, it was noted that the continuous setup lot-sizing model (CSLP) restricts the product setups in each planning period, resulting in lower grade changeover costs but also adversely affecting the service levels. It was experimentally shown that the cycle service levels can be increased by allowing a higher number of grade changeovers per planning period. Separate single objective optimization experiments were performed in Chapter 5 for the two different lot-sizing models each corresponding to a different number of grade changeovers per planning period (Section 5.5.2). However, a significant advantage of multi-objective optimization applied to the planning of paper manufacturing and converting is that it integrates both the single setup and multiple setups lot-sizing models into a single planning problem. The bi-objective formulation just presented will result in a Pareto frontier that gives a trade-off between the changeover costs and the cycle service levels with the lower grade changeover cost corresponding to the single product per planning period approach whereas the solutions with higher service levels are only possible by manufacturing multiple products in each planning period.

6.4.2 Joint Optimization: Total Production Cost Vs Service Levels

The second bi-objective formulation utilizes the integrated or joint optimizing approach presented in Chapter 5 wherein determination of the cutting patterns for the conversion process, allocation of the cutting patterns to individual planning periods and scheduling of different grades of jumbo reels on the paper machine is carried out simultaneously. Trim loss and grade changeovers are combined to yield total production cost as one of the two optimization criteria f_1 and minimization of late orders f_2 is the other objective. Mathematically,

Objective to be minimized (Notations in Table 6-1)

$$f_1 = \sum_{t}^{T} \sum_{i \in IP} K_{it} \rho_{it} + \sum_{i \in IP} \sum_{j \in J} c_i w_{ij} \qquad (6.20)$$

$$f_2 = \sum_{t}^{T} \sum_{i' \in FP} y_{i't} \qquad (6.21)$$

Constraints

$$C_t \geq a_{it} Q_{it} + k_{it} \rho_{it} \qquad (6.22)$$

$$d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't} \qquad (6.23)$$

$$\sum_{t}^{T} Q_{it} - \sum_{j \in J} x_{ij} = 0 \qquad (6.24)$$

$$\sum_{j \in J} A_{i'j} x_{ij} \geq d_{i'} \qquad (6.25)$$

$$\rho \in (0,1) \qquad (6.26)$$

$$Q_{it}, Q_{i't}, y_{i't}, d_{i't}, A_{i'j} \geq 0, Integer \qquad (6.27)$$

$$w_j \geq 0 \tag{6.28}$$

Table 6-1 also contains additional notations used in the integrated formulation that minimizes the delayed orders and the production costs simultaneously. Corresponding to the separate planning formulation, constraint (6.22) deals with capacity restrictions whereas constraints (6.23) and (6.24) ensure material balancing. Constraint (6.25) stipulates that the entire demand of finished products is to be fulfilled. As in the earlier formulation, integer and non negative solutions for the integrated problem are obtained by enforcing constraints (6.26), (6.27) and (6.28). The generated cutting patterns are constrained to be less than the jumbo reel in the following manner:

$$\sum_{i' \in FP} A_{i'j} \, l_{i'} \leq L \tag{6.29}$$

The bi-objective model could have been formulated as a three objective problem but the findings of Chapter 5 forced us adherence to the bi-objective formulation. The experimental results in Chapter 5 showed that the two sets of costs, grade changeovers and trim loss combined into one objective as ‘total production cost’ are in-fact acting together against the conflicting objective of improved cycle service levels. Also, while analyzing the literature on multi-objective evolutionary algorithms (MOEA) in Section 6.3.2, it was noted that the MOEAs are most efficient with two objectives and their performance deteriorates with increase in objectives (Ishibuchi, Tsukamoto & Nojima 2008; Ishibuchi et al. 2008; Schutze, Lara & Coello Coello 2010; Coello 2010; Circu & Leon 2010).

6.5 Solution Approaches

As discussed in Section 6.3, there have been various ways to apply multi-objective optimization but the scaler and Pareto approaches are the main ones. Due to the fundamental difference between the methods employed to approximate the Pareto frontier, these two approaches may differ substantially from each other with regard to suitability for application to a specific decision context and with regard to the results obtained. Therefore, it is deemed prudent to test both solution approaches for the production problem of the two successive stages of paper manufacturing. The Epsilon constraint method is selected as the scaler approach whereas the non-dominated sorting algorithm-II (NSGA-II) is chosen as the preferred Pareto or multi-objective evolutionary algorithm (MOEA).

6.5.1 Epsilon Constraint Method with Standard GA

Table 6-2: Epsilon Constraint Method Applied to the Production Planning Problem of a Paper Mill

Separate Planning Approach*(Section 6.4.1)*	***Joint Optimization Approach****(Section 6.4.2)*
Objective to be minimized *(Notations in Table 6-1)*	
$f = \sum_{t}^{T} \sum_{i \in IP} K_{it} \rho_{it}$	$f = \sum_{t}^{T} \sum_{i \in IP} K_{it} \rho_{it} + \sum_{i \in IP} \sum_{j \in J} c_i w_{ij}$
Constraints	
$\sum_{t}^{T} \sum_{i' \in FP} y_{i't} \leq \varepsilon^k$ [Epsilon Constraint] $C_t \geq a_{it} Q_{it} + k_{it} \rho_{it}$ $d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't}$ $\sum_{t}^{T} Q_{it} - \sum_{j \in J} x_{ij} = 0$ $\rho \in (0,1)$ $Q_{it}, Q_{i't} \geq 0, Integer$	$\sum_{t}^{T} \sum_{i' \in FP} y_{i't} \leq \varepsilon^k$ [Epsilon Constraint] $C_t \geq a_{it} Q_{it} + k_{it} \rho_{it}$ $d_{i't} = Q_{i't} + I_{i'(t-1)} - I_{i't}$ $\sum_{t}^{T} Q_{it} - \sum_{j \in J} x_{ij} = 0$ $\sum_{i' \in FP} A_{i'j} \, l_{i'} \leq L$ $\sum_{j \in J} A_{i'j} \, x_{ij} \geq d_{i'}$ $\rho \in (0,1)$ $Q_{i't}, Q_{it} \geq 0, Integer$ $A_{i'j} \geq 0, Integer$

The relative advantages of the epsilon constraint method over other scaler approaches were discussed in Section 6.3.2 and it is reasonable to assume that it is likely to perform at least as well as the other scaler approaches. In both separate and integrated formulations, f_1 is kept as a single objective function f whereas f_2, the objective minimizing late orders, is converted to an epsilon constraint as shown in Table 6-2. The subscript k denotes the number of epsilon intervals used to approximate the Pareto frontier.

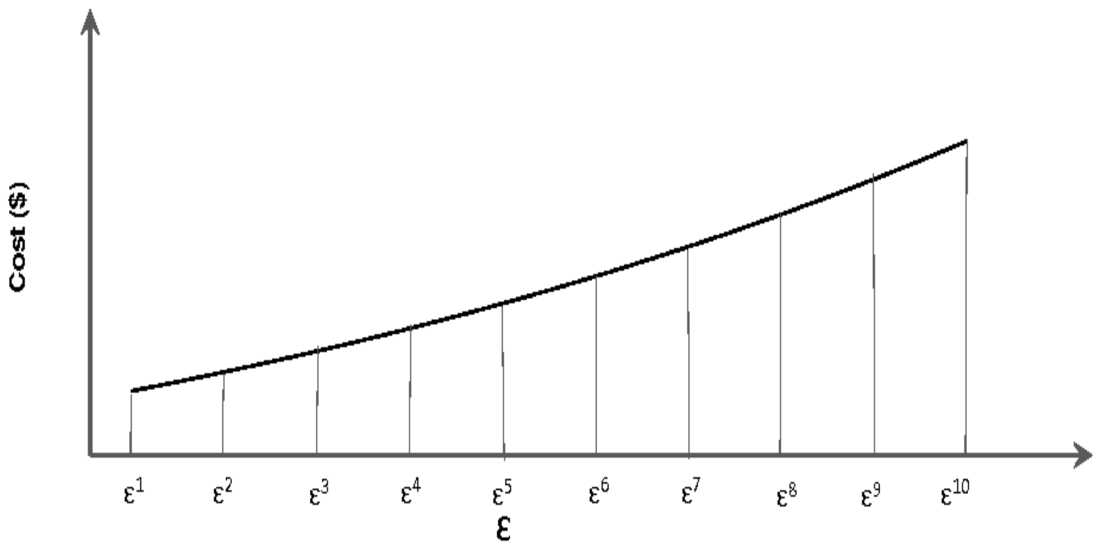


Figure 6–4: Epsilon Constraint Method Conceptually Applied to Paper Manufacturing

The epsilon values are changed systematically and corresponding values of the objective function f are obtained and plotted to approximate the Pareto frontier (Figure 6-4). For the separate planning approach, minimum grade changeover cost is maintained as the only optimizing objective whereas the minimization of late orders becomes the epsilon constraint. Similarly, for the integrated approach, minimum total production cost that includes the grade changeover cost and trim loss is one criterion whereas the minimization of late orders is converted into the epsilon constraint. Deb (2005) highlighted the importance of the initial and final bounds of epsilon i.e $\varepsilon^1$and ε^{10} (Figure 6-4) because it determines the feasible search space for the optimizing problem. As discussed in Section 6.2, knowledge of ideal and nadir objective vectors in a payoff table has been mostly used to determine these bounds. The ideal vector for both bi-objective formulations is obtained by separately solving the single objective problem for maximizing the service levels and then the cost minimizing problem. The corresponding upper bounds represent the nadir objective vector.

The spacing of the epsilon interval is another important decision that has to be made prior to the experiments. It is obvious that closely spaced epsilon intervals are preferable for a better approximation of the Pareto front but it comes at an increased computational cost because each epsilon interval corresponds to a separate optimizing process. For the problem under study, ten epsilon values are used to approximate the Pareto frontier which means that the solution process has to be repeated ten times (Figure 6-4).

The Resolution Algorithm

As discussed in Section 6.3.1, the epsilon constraint method is a multi-objective optimization converted into a single objective problem and solved through conventional algorithms. It was also noted that different resolution algorithms ranging from exact to meta-heuristics, depending upon the problem context, have been used in conjunction with the epsilon constraint method. The standard genetic algorithm used in Chapter four for the cutting stock problem and in Chapter five for the conventional optimization of the successive manufacturing processes of the paper industry has been maintained as the resolution algorithm for both of the bi-objective epsilon constraint formulations with modifications to the existing Excel based model. The specifications of the model and the genetic algorithm employed are detailed in Chapter 4. The experimental settings for the GA parameters were as follows:

A uniform crossover value of 0.5 is used across all experiments and auto-mutation is used. The latter allows the genetic algorithm to increase the mutation rate automatically when an organism "ages" significantly; that is, it has remained in place over an extended number of trials. Experiments with initial populations of 500 were performed and in most cases, GAs converged before 200 GA equivalent generations or 100,000 iterations; therefore, it was selected as the stopping criterion for all the experiments.

The standard GA has been previously used as the resolution algorithms for the epsilon constraint method but it appears that the proposed approach is a first for real world problems. Ranji Ranjithan, Kishan Chetan & Dakshina (2001) and Kumar & Ranjithan (2002) developed an evolutionary algorithm based on the ε-constraint method the 'Constrained Method based Evolutionary Algorithm' (CMEA) and applied it to unconstrained and constrained test problems. The incremental changes to ε were carried out in a single run to generate the Pareto frontier. Once the population converged to a non-dominated solution according to a stopping criterion, the solution was stored and the constraint values were incremented to gradually shift the population towards the non-dominated solution corresponding to the updated constraint value. The process was repeated until the non dominated solutions to all the ε-constraint increments were found. Unlike the CMEA, the repeated runs for increments in the epsilon value were carried out as separate experiments for this study.

6.5.2 Non Dominated Sorting Genetic Algorithm (NSGA) - II

NSGA-II was selected as the Pareto based multi objective evolutionary algorithm. GANetXL, a software platform that utilizes NSGA-II for multi-objective optimization, has been used. It is written in C++ and exploits a component object model (COM) interface to interact with Excel (Savic, Bicik & Morley 2011). Its interface with Excel facilitated model development with the help of Visual Basic for Application (VBA) macros.

6.5.3 Test Data

Five data sets were generated in Chapter 5 and Data Set 1 has been used here to evaluate the effectiveness of the proposed bi-objective formulations and to compare the results obtained through two different optimizations paradigms i.e the conventional and multi-objective optimization approaches.

6.6 Results

6.6.1 Separate Planning Approach: Grade Changeover Cost Vs Service Levels

6.6.1.1 Epsilon Constraint Method with Standard Genetic Algorithm

In a two step process, the first step involves cutting jumbo reels with a minimum trim loss criterion and the second step allocates the cutting patterns to different planning periods triggering the production of jumbo reels of the respective grades with two minimization objectives namely grade changeover cost and number of late deliveries. The total production costs included a fixed trim loss for the given problem along with the grade changeover costs, which is one of the two optimizing criteria. The ideal and nadir objective vector were obtained respectively as (0.959, \$11,698.97) and (0.838, \$51,044.8). These two vectors determined the extreme values of the Pareto frontier and with ten epsilon increments, the Pareto frontier was approximated in Figure 6-5.

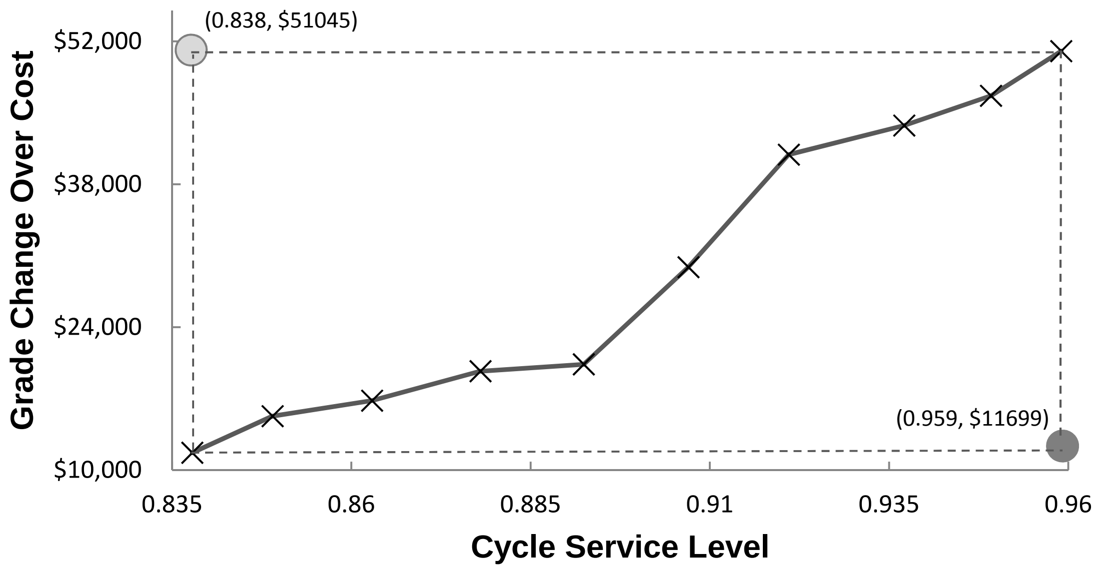


Figure 6–5: Approximated Pareto Frontier - Separate Planning Approach (Epsilon Constraint Method)

A well spread Pareto frontier is obtained between the Nadir and Ideal vectors represented in Figure 6-5 by the light and dark shaded circles respectively. The Pareto frontier for the separate production planning approach gives the decision maker a range of solutions to choose from.

A least cost solution of $11699 for grade changeover costs results in a cycle service level of 0.838 but as the grade changeover cost increased, the service level also improved. This is because the solutions resulting in lower grade changeover cost correspond to the continuous setup lot-sizing model (CSLP) discussed in Chapter 5 that only allowed at most one setup in one planning period with the possibility of carrying over the setup state to the next planning period. For example, the solution resulting in grade changeover cost of $11,699 and a cycle service level of 0.838 had only 5 setups in the week long planning horizon. The number of setups in the planning horizon gradually increased to 8, 9, 12, 17, 18, 19, 21and the cycle service levels increased correspondingly. The maximum cycle service level of 0.959

resulted from 21 setups in the one week production schedule but incurred much higher costs of $51,045.

In a sense, the bi-objective optimization of grade changeovers and corresponding cycle service levels simplifies the lot-sizing decisions as detailed in Chapter 5. In the conventional single objective optimization, one of the important considerations for the selection of an appropriate lot-sizing model is the maximum number of products that can be manufactured in a single planning period. It was shown in Chapter 5 that the lot-sizing model that corresponded to a single product per planning period gave poor results with regard to the cycle service levels but there were cost savings. When multiple products were allowed in each planning period, the cycle service levels were maximized but with increased costs. In bi-objective optimization, there is no need to make a prior decision regarding number of products in each planning period. Multiple lot-sizing models have been integrated into one experiment and the resulting Pareto frontier gives the production manager a range of options to choose from.

The important consideration is whether the estimated Pareto frontier is global or local, i.e can the solutions be improved further? The answer to this question lies in the resolution algorithm. If an exact algorithm were used as the solution approach, the estimated Pareto frontier would have been global and could not have been improved any further. Genetic algorithm was used as the solution approach and being a stochastic search algorithm, the optimality of the solutions cannot be guaranteed in a single run. As discussed in Chapter 5, repeated genetic algorithm runs enhances the probability of obtaining close to optimal solutions. However, it would have been computationally prohibitive in this case because each of the ten solutions obtained would have to be re-run a number of times. Nevertheless, determining the solution quality is important and another measure is to solve the same problem by a multi-objective evolutionary algorithm such as NSGA-II and to compare the results.

6.6.1.2 Non Dominated Sorting Genetic Algorithm- NSGA II

The state of the art Non Dominated Sorting Genetic Algorithm NSGA II discussed in Section 6.3.2 was also applied to the same problem. The initial population was generated randomly. However, it was noted that no individual among the initial population was a feasible solution. The algorithm was allowed to run for 5000 generation with a 500 population. After nearly 50 hours of run time, the algorithm was unable to generate a single feasible solution. Different GA parameters were tried but the generated solutions were always infeasible. The possibility of obtaining feasible solutions after 5000 generations cannot be ruled out but the computational cost was prohibitive.

The other alternative is to start with a population of feasible solutions; this approach has been reported in the literature for similarly hard combinatorial problems. Datta, Deb & Fonseca (2007) also encountered infeasibility of NSGA II generated solutions for a highly constrained university timetabling problem and when they used feasible solutions as the initial population, considerable improvement was recorded. Similarly, Fangguo & Huan (2008) ensured feasibility of all solutions for their dominance based multi-objective genetic algorithm by using an initial feasible population. Sathe, Schenk & Burkhart (2009) employed a clustering algorithm along with NSGA II in order to always generate feasible solutions for a multi-constraint bin packing problem. Li & Hamzaoui (2009) also used initial feasible solutions for their NSGA II implementation. Varela et al.(2009a) improved the heuristic solution obtained for a variant of the cutting-stock problem by using it as the initial population for their multi-objective genetic algorithm. Craig, While & Barone (2009) improved their hockey league scheduling with the help of multi-objective evolutionary algorithm by using the previous year’s schedules as the initial population. Reiter & Gutjahr (2010) implemented NSGA-II for a bi-objective vehicle routing problem by using a separate algorithm to generate feasible solutions to be used as an initial population.

The same approach of injecting feasible solutions, including the results obtained by the epsilon constraint method (Section 6.6.1.1), into the randomized population was used to solve the problem under study. However, only 30% of the initial population was filled with the feasible solutions while the rest of the population comprised the randomly generated infeasible solutions. This was done to ensure diversity among solutions. Different GA parameter settings were tested and the algorithm did return improved feasible results this time.

The ability of the standard genetic algorithm when applied as an epsilon constraint method to obtain feasible solutions and the inability of NSGA-II to do the same from an initial random population can be explained by the different constraint handling mechanisms for the two multi-objective algorithms employed. The standard GA uses the penalty function to handle constraints. The aim is to transform a constrained optimization problem into an unconstrained one by penalizing the objective function by a value based on the constraint violation. This is particularly useful for NP-Hard combinatorial optimization like the problem under study because the feasibility of solutions is gradually achieved by reducing the sum of the 'soft penalties' to zero. On the contrary, NSGA-II's constraint handling mechanism explained in Section 6.3.2 has proved to be ineffective if the entire initial population is infeasible. However, injecting 30% feasible solution as part of the initial solution did improve the results. Injecting good solutions obtained from the weighted sum method to NSGA-II's initial population was also used by Dasgupta et al. (2008; 2009) but their case is different because they modified the selection scheme for NSGA-II to increase the likelihood of selecting the injected solutions which shows that their randomly generated initial population did contain feasible results. Details are as follow.

Earlier, in order to generate feasible solutions from the initial random population, a population size of 500 was used which resulted in high computational cost, fifty hours being required for a 5000 generations run. This was because of the computational complexity of NSGA II which is exponentially related to the population size i.e $O(MN^2)$ where M is the number of objectives and N is the population size. With feasible solutions in the initial population, the computational load can be reduced by resorting to smaller populations especially because multiple runs are essential to ensure that quality solutions are obtained by stochastic optimizers such as GA. Initially, a population size of 100 was chosen with simple crossover probability of 0.95 and a mutation by gene probability of 0.05. The adaptive mutation probability of 0.01 was also resorted to after 1000 generations. After a four hour run and 2000 generations, there were nine improved feasible solutions and all the rest were copies. Thus, a stopping criterion of 2000 generations was selected.

With the same initial feasible solutions and the same proportion of randomized initial infeasible solutions, the algorithm was run ten times. All the solutions from the ten runs were then combined and the top hundred were used as the initial population to generate the best Pareto frontier for the problem (Figure 6-6). The individuals in the last generation of all the ten NSGA II runs were different; therefore, the combination of all solutions to obtain the final frontier makes sense. As expected, the combined run that includes the best 100 solutions turns out to give the best estimated global Pareto frontier for the problem.

In Figure 6-6, the Pareto frontiers obtained by Run 5 and Run 7 appear to cross the global Pareto frontier obtained by the Combined Run. However, it does not happen actually because all the points obtained by Run 5 and Run 7 are dominated by the Combined Run. It is just that the Run 5 and Run 7 contain fewer solutions and the additional solutions of the Combined Run at the exact locations of the overlaps give the false impression that Run 5 and Run 7 are better.

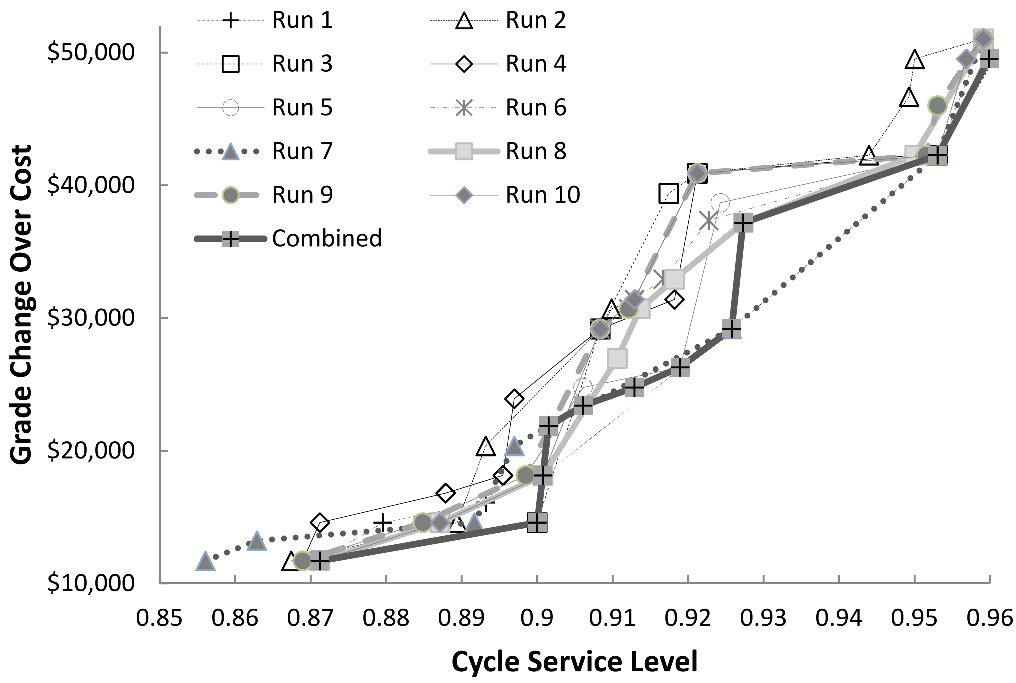


Figure 6–6: Separate Planning Approach - Multiple NSGA-II Runs

The comparison of the Pareto frontier obtained by the Epsilon constraint method which is also the initial Pareto frontier is compared with the improvements recorded by the NSGA-II solutions in Figure 6-7. All the NSGA-II solutions are equally good or better than the epsilon constraint results highlighting the fact that the Epsilon constraint method did not result in a global Pareto frontier. This is not surprising because the GA experiments corresponding to one epsilon value were only performed once. While evaluating the quality of GA solutions in Chapter 5, it was noted that only repeated GA experiments can ensure good solutions. Figure 6-7 also shows that the NSGA-II's Pareto front stayed within the bounds obtained by the ideal and nadir objective vectors. The overall shape of the Pareto frontier also did not change much suggesting that NSGA-II was only able to find improved solution in the vicinity of the existing feasible solutions. Therefore, it appears that the initial Pareto front tended to dictate NSGA-II's search.

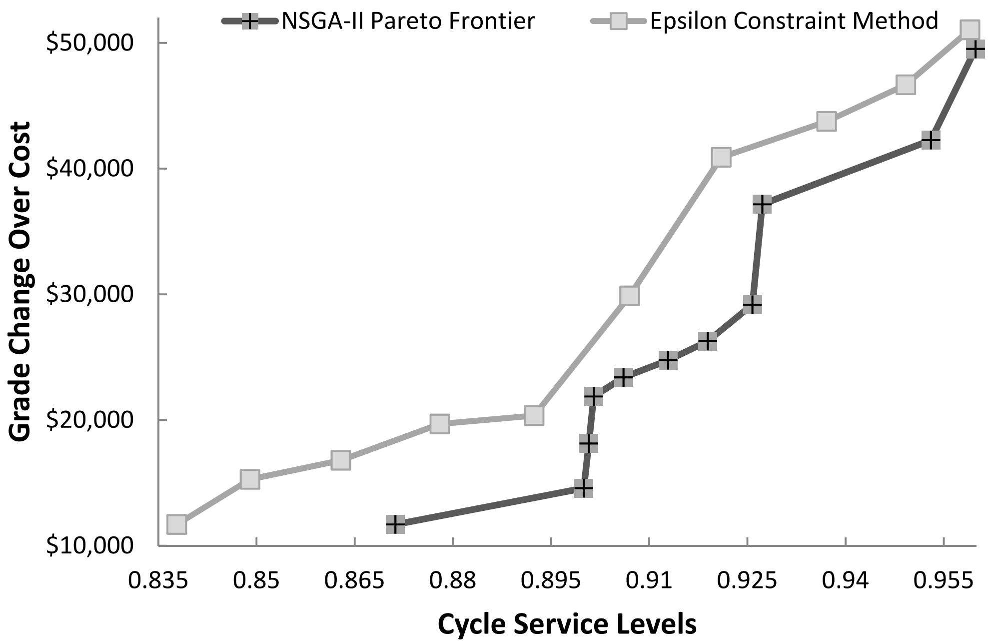


Figure 6–7: Separate Planning: NSGA II Comparison with the Epsilon Constraint Results

6.6.2 Joint Optimization: Total Production Cost and Cycle Service Levels

Whereas Section 6.6.1 dealt with the cycle service levels versus grade changeovers only, this section extends the multi-objective planning model to the joint problem which simultaneously optimizes two conflicting objectives; minimum total production cost and maximum cycle service level. The total production cost here comprises both grade changeovers and trim loss. The usefulness of joint optimization to attain high cycle service levels was discussed in Chapter 5. It was noted that in the separate planning approach, the cutting stock problem solved with least trim loss resulted in an upper bound on the number of jumbo reels to be used in the lot-sizing model, thus restricting the cycle service levels. Joint optimization wherein the cutting stock and lot-sizing models are solved simultaneously does not restrict the cycle service level and, with additional trim loss, it is possible to maximize the service level. The experiments for five different data sets showed that separate planning results in limited cycle service level and that further improvements in service levels is only possible by simultaneously generating the cutting patterns and allocating those patterns to different planning periods triggering production of jumbo reels.

6.6.2.1 Inadequacy of the Epsilon Constraint Method for the Joint Problem

In view of the non-linear conflicting relationship between service level and total production costs observed in chapter 5, it becomes difficult for the classical multi-objective optimization algorithms such as the scaler approaches to approximate the Pareto frontier because their working principle involves systematically changing the weights or epsilon intervals to generate one non-dominated point for each experiment. Because of the competing but non-linear relationship, non dominating results may not be obtained at each epsilon interval. The same is observed when the epsilon constraint method was applied to joint planning because the solution did not meet all epsilon interval constraints i.e the service levels requirements. Therefore, it appears that the epsilon constraint method indicated in the right hand column of Table 6-2 may not be the best approach to obtain a uniformly spread Pareto frontier for the joint problem. However, it was able to generate the extreme ends of the Pareto frontier by utilizing the knowledge of ideal and nadir objective vectors.

Although the epsilon constraint method is not able to generate an equi-spaced Pareto frontier, it provides some good solutions that can be used as part of an initial feasible population for NSGA-II if the random initial population does not converge to feasible solutions as has been the case for the separate approach. The initial Pareto frontier approximated from the epsilon constraint solutions can be also useful in measuring the improvements obtained by NSGA-II (Figure 6-9).

6.6.2.2 NSGA-II Applied to the Joint Problem

As discussed in Chapter 5, the complexity of the joint planning problem is increased because two NP-Hard problems have been integrated; the generation of cutting patterns, allocation of those patterns to different planning periods and production of jumbo reels on the paper machine is carried out simultaneously. The joint problem is much harder than separate planning; there are additional constraints and the number of decision variables is increased to 988. Consequently, it is not likely that the initial random population will converge to a feasible solution: when the joint problem was allowed to run for 7000 generations with an initial random population of 500, no feasible solutions were obtained. Therefore, feasible epsilon constraint solutions together with solutions from Chapter 5 were injected into the initial random population in order to obtain solutions to the NSGA-II joint problem.

It was noted in Section 6.6.1.2 that the NSGA-II's final Pareto frontier stayed within the bounds defined by the initial Pareto frontier and that the general shape of both the frontiers did not change much. This finding has serious implications for the successful implementation of NSGA-II for the joint problem because the initial population plays an important role in determining the final Pareto frontier. The injected initial feasible solutions should represent the entire range of feasible solutions for the two objectives so that the Pareto frontier obtained truly reflects all possibilities.

While discussing the general application of the epsilon constraint method in Section 6.4.2, it was noted that the knowledge of ideal and nadir objective vector is useful in determining the feasible search space for an optimization problem. Although, the epsilon constraint method struggled to obtain a uniformly spaced Pareto frontier for the joint problem, it did obtain the ideal vector (0.99, $159,793) by separately solving for minimum production cost and maximum service level. The corresponding upper bounds (0.87, $235,724) represented the nadir objective vector. A few other solutions between the two bounds were also obtained to be used as part of an initial population for the NSGA-II.

The calculation and incorporation of the extremes of the Pareto frontier as part of the initial NSGA-II population also helps to combine the separate and joint planning approaches of Section 6.4.1 and Section 6.4.2 into a single framework. The ideal and nadir objective vectors correspond to the extremes of least production cost and maximum service levels. A least production plan allows least number of grade changeovers whereas the maximum service plan means many grade changeovers and excess trim loss. Therefore, an NSGA-II implementation initialized with ideal and nadir objective vectors is likely to yield a Pareto frontier that gives an entire range of possibilities for grade changeover cost, trim loss and cycle service levels. An example of this effect is shown Figure 6-11(Section 6.7).

The filling of the entire initial population with feasible solutions was considered detrimental to NSGA-II's convergence and therefore, to ensure diversity, only 30% of the population was replaced with feasible solutions, rest of the population being randomly generated individuals. Also, to reduce computational load, a population size of 100 was chosen to start with. Different GA settings were tried and it was found that the mutation by gene rate had to be reduced to 0.005 and the adaptive mutation probability was also reduced to 0.001 which was to come into effect after 1000 generations. These parameters vary substantially from the separate planning problem because of a much harder joint problem that integrates cutting stock and lot-sizing decisions. The simple crossover probability of 0.95 was used. It was noted that there were not much improvements till 2000 generations, the stopping criterion for the separate planning problem; therefore, NSGA-II was allowed to run further. After thirteen hours of computation and 5000 generations, there was considerable improvement and it was selected as the stopping criterion. The process was repeated to obtain multiple NSGA-II runs with same initial feasible solutions and 70% of randomized initial solutions. Solutions from all ten runs were combined together and unlike the separate planning approach, the combined number of non-dominated solutions from all ten runs was much greater than the initial population of 100 individuals. Therefore, only the top hundred solutions were used as the initial population to generate the best Pareto frontier for the combined problem (Figure 6-8).

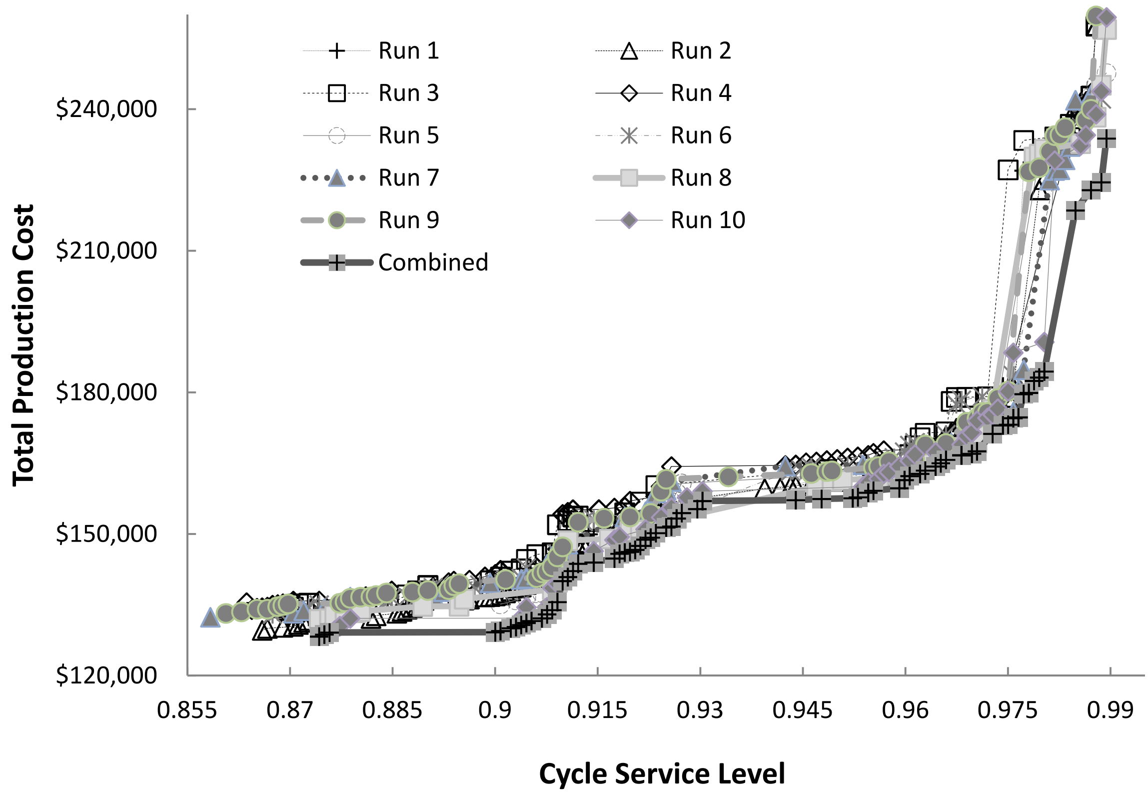
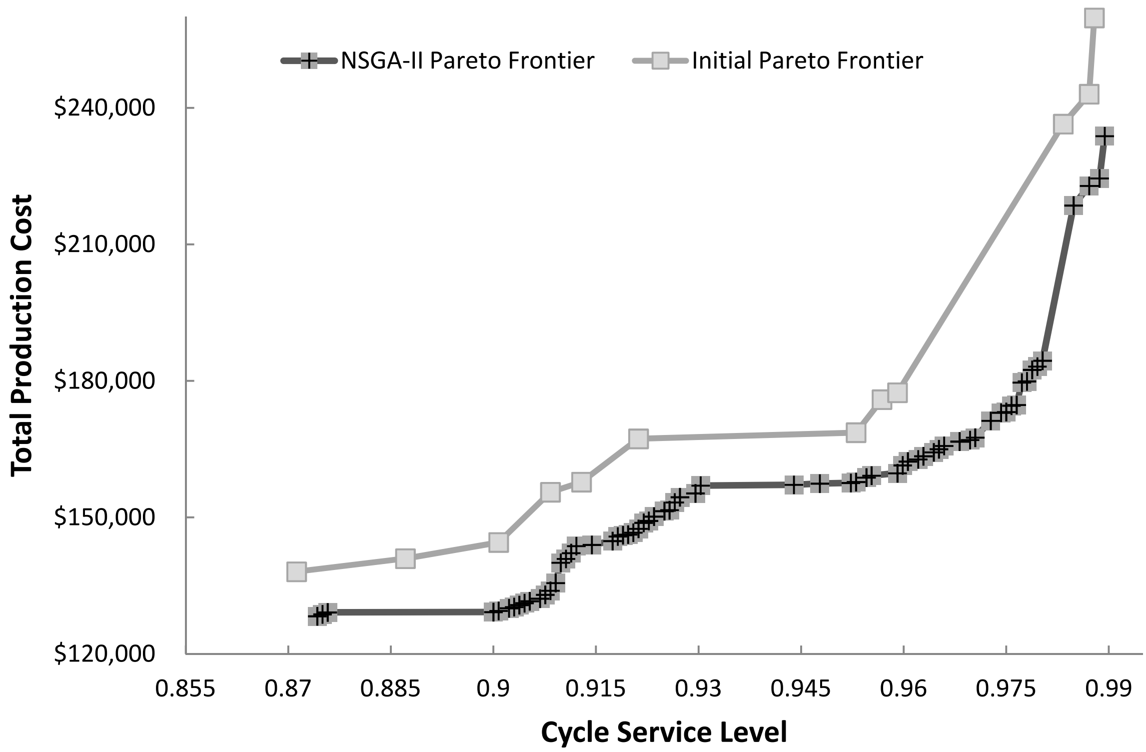


Figure 6–8: Integrated Planning Approach – Multiple NSGA-II Runs

Although the initial population contained 30 feasible solutions, only 12 solutions constituted the Pareto frontier. Not only was the NSGA-II able to improve the Pareto frontier but also the number of Pareto optimal solutions also improved considerably (Figure 6-9). However, the shape of the Pareto frontier did not change much suggesting that NSGA-II was only able to find improved solution in the vicinity of the existing feasible solutions which is in line with what was found in the separate planning problem.



Note: The initial Pareto frontier is an approximation from the epsilon constraint method that does not result in all non-dominated solutions at equal intervals combined with solutions from chapter 5.

Figure 6–9: Integrated Planning: Comparison of NSGA-II Results with the Initial Pareto Frontier

The final Pareto frontier obtained in Figure 6-9 is extremely useful for the decision maker because it shows that a cycle service level improvement from 0.87 to 0.90 is virtually achieved with out any further cost. Similarly, the cost incurred to move from 0.93 service level to 0.96 is low as compared to the CSL gains. Conversely, substantial cost is incurred to achieve a CSL of 0.98 and beyond which may not be economically viable. Within these significant objective values, the joint optimization was able to achieve a wide range of compromise solutions for the two conflicting objectives of least production cost and maximum cycle service level. NSGA-II's initialization with extreme solutions not only enabled the joint problem to incorporate the characteristics of the separate planning problem but also the simultaneous minimization of trim loss and grade changeover costs along with maximum cycle service increased both the number and quality of solutions. There is a definite pattern in the solutions as shown by Figure 6-10 which shows the relationship between trim loss and cycle service levels for the variable grade changeover costs.

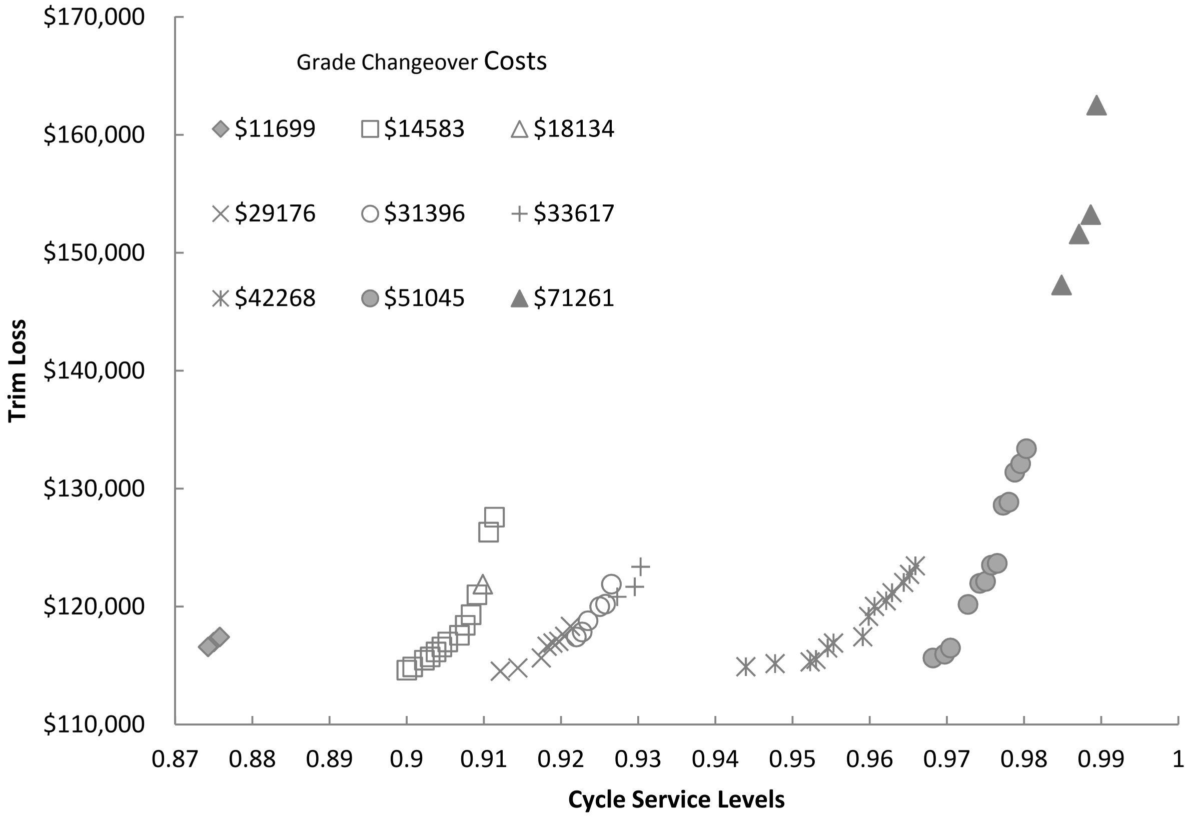


Figure 6–10: The relationship between trim loss and cycle service levels for grade changeover costs

The algorithm tried to obtain higher cycle service levels by increasing trim loss for each value of grade changeovers. As discussed in Section 6.5.4.1, the increase in grade changeover cost represents the higher number of setups in the week long planning horizon. The minimum grade changeover cost of $11,699 corresponds to 5 production setups and the maximum grade changeover cost incurred was $71,261 because of 28 setups in the seven day planning horizon. Figure 6-10 also highlights an important aspect of the joint optimization: when there is ‘informed initialization’ or extreme values of the Pareto frontier are used as an input, an increase in trim loss may yield cycle service levels that could alternatively be achieved by carrying out additional setups. For example, at the $14,583 grade changeover cost cycle service level could be increased from 0.90 to 0.91 by incurring additional trim loss of $13,000. Alternatively, the same cycle service level could be obtained by carrying out extra setups, thus, increasing the grade changeover cost to $29,176 with lower trim loss but the total production costs would be almost unchanged.

6.7 Discussion

In this chapter, multi-objective optimization has been employed for planning successive manufacturing processes in the paper industry. However, the following issues were crucial for the successful implementation of multi-objective optimization.

- The NSGA-II was unable to obtain feasible solutions when initiated by a randomized population comprising only infeasible solutions. No such issue arose when standard GA was used as the solution algorithm for the epsilon constraint method.
- Once feasible solutions were injected as part of the initial population, NSGA-II found the improved solution in the vicinity of existing solutions. This characteristic of NSGA-II may have crucial ramifications for finding the entire range of possibilities.
- The separate and joint optimization approaches can be combined together when the knowledge of ideal and nadir objective vectors is utilized to obtain the extreme ends of the Pareto frontier for a scaler multi-objective optimization method. The solutions obtained are then used as an initial population for NSGA-II which yields a Pareto frontier for the entire range of possibilities from a least grade changeover and least trim loss production plan to a production plan that yields maximum service levels at additional costs of grade changeover and trim loss.

Multiple experiments had been carried out to compare different lot-sizing models in Chapter 5 but the use of a bi-objective formulation for the same problem resulted in a Pareto frontier that gave a trade-off between the changeover costs and the cycle service levels with the lower grade changeover cost corresponding to the single product per planning period approach whereas the solutions with higher service levels are only possible by manufacturing multiple products in each planning period. Therefore, two different lot-sizing models were combined in a single experiment by resorting to multi-objective optimization.

The constraint handling mechanism of NSGA-II works by preferring feasible solutions over the infeasible solutions. Its ability to traverse the infeasible regions has been questioned previously (Geng et al. 2006; Young 2005) and the experiments performed in this chapter confirm that there is a problem. On the other hand, the standard genetic algorithm when applied as an epsilon constraint method to obtain feasible and improved solution from the initial randomized population did not encounter this problem because of a different constraint handling specification. The standard GA uses a penalty function to handle constraints which transforms a constrained optimization problem into an unconstrained one by penalizing the objective function by a value based on the constraint violation. Feasible solutions are gradually achieved by reducing the ‘soft penalties’ to zero; this worked well in the problem under study.

The superior constraint handling performance of standard GA when applied as a scaler multi-objective optimization approach may also raise questions about NSGA-II as an alternative solution method. However, it was shown in Section 6.6.1.2 that NSGA-II has the ability to improve the Pareto frontier obtained through the epsilon constraint method. The greatest advantage of NSGA-II is that potentially, each individual of the population can be part of the Pareto frontier which is obtained by performing just a single experiment. But, for all this to happen, the NSGA-II has to be initiated by a population which contains some feasible solutions. Therefore, the solution approach to the production planning problem of the two successive manufacturing processes in the paper industry is a two step process. The first step generates a set of feasible solutions by the epsilon constraint method which is then used to initiate the NSGA-II for further improvements.

The NSGA-II search tends to obtain improved solutions in the vicinity of existing solutions as confirmed by the similar shapes of the initial and improved Pareto fronts in Section 6.6.1.2 and 6.6.2.2. This may have serious implications for NSGA-II implementation for real world problems. If the solutions injected as part of the initial population do not represent the entire range of possibilities, the resulting Pareto frontier may not be truly representative of the decision context.

Figure 6-11 explains this point: (a) and (c) representing the experiments performed in the previous section where the initial population had a wide range of grade changeover and trim loss solutions obtained by the ideal and nadir objective vectors. Figure 6-11 (b) and (d) represents a separate experiment with an initial population that had only five different grade changeovers as compared to nine grade changeovers for (a) and (c). The final Pareto frontiers in both cases drew heavily from the respective initial solutions. The ideal and nadir objective vector represent the extreme positions of a Pareto frontier and for the initial Pareto frontier of Figure 6-11 (a) & (c), their values are (0.99, $128,290) and (0.87, $233,800) respectively.

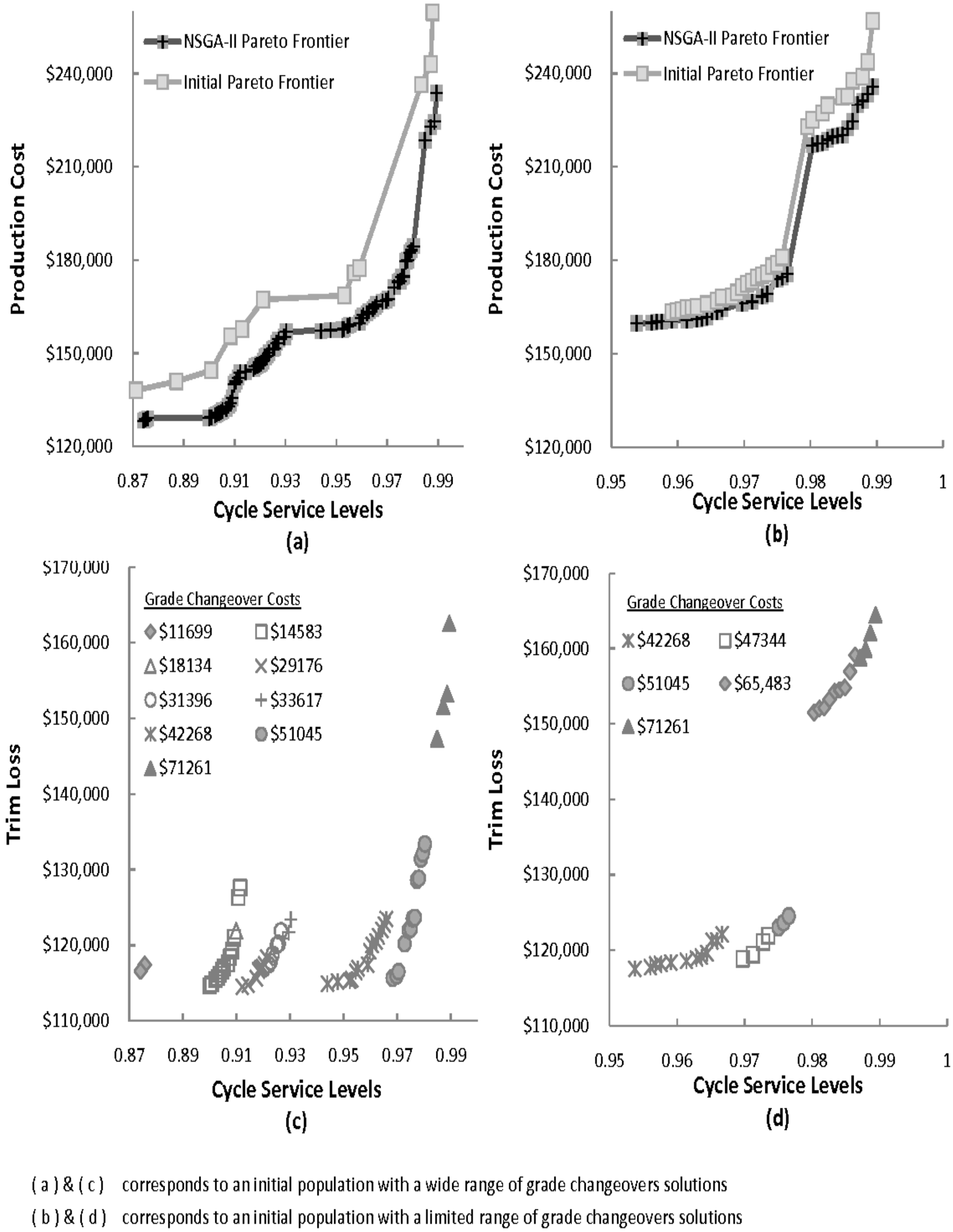


Figure 6–11: Effect of the 'Informed Initial Population' on the Final Pareto Front

For the experiment shown in Figure 6-11 (b) & (d), the values for ideal and nadir objectives are (0.99, $159,793) and (0.953, $235,724). It is apparent that the initial Pareto front for (b) and (d) approximates only part of the initial Pareto frontier for (a) and (c) and the same is true for the corresponding final Pareto frontiers. It shows that the ideal and nadir objective vectors for (b) and (d) did not represent the lower and upper bounds over the entire feasible search space, which was expected because there was no methodical estimation of the two objective vectors. As discussed in Section 6.2, the estimation of nadir and ideal objective vectors is crucial for the scaler multi-objective optimization approaches such as the weighted sum and epsilon constraint methods. If the feasible solutions that become part of the initial population of NSGA-II are generated with proper estimation of the nadir and ideal objective vector by one of the two scaler approaches then there is a high probability that the resulting Pareto frontier covers the entire feasible search space. Therefore, a multi-objective implementation for hard combinatorial problems can be thought of as a two step process. In the first step, extreme and some internal values of the Pareto frontier are generated by a scaler method. The generated solutions are then used as part of an initial solution for further improvements through NSGA-II.

The knowledge of ideal and nadir objective vectors also helps to incorporate the separate planning problem within the joint planning problem because the initial Pareto frontier used for NSGA-II now contains solutions within the entire range of possibilities from a least grade changeover and least trim loss production plan to a production plan that yields maximized service levels at the cost of additional grade changeover and trim loss. A single run now determines a range of solutions as were obtained by various experiments in Chapter 5, further emphasizing the usefulness of multi-objective optimization.

Figure 6-11 also highlights the importance of multiple runs because (a) corresponds to a case where multiple NSGA-II runs were performed. The best solutions were combined together and re-run to obtain substantial improvements over the initial

Pareto frontier. The Pareto frontier shift for Figure 6-11 (b) is comparatively much smaller because it corresponds to just a single experiment.

6.7.1 Choosing a Single Optimal Solution

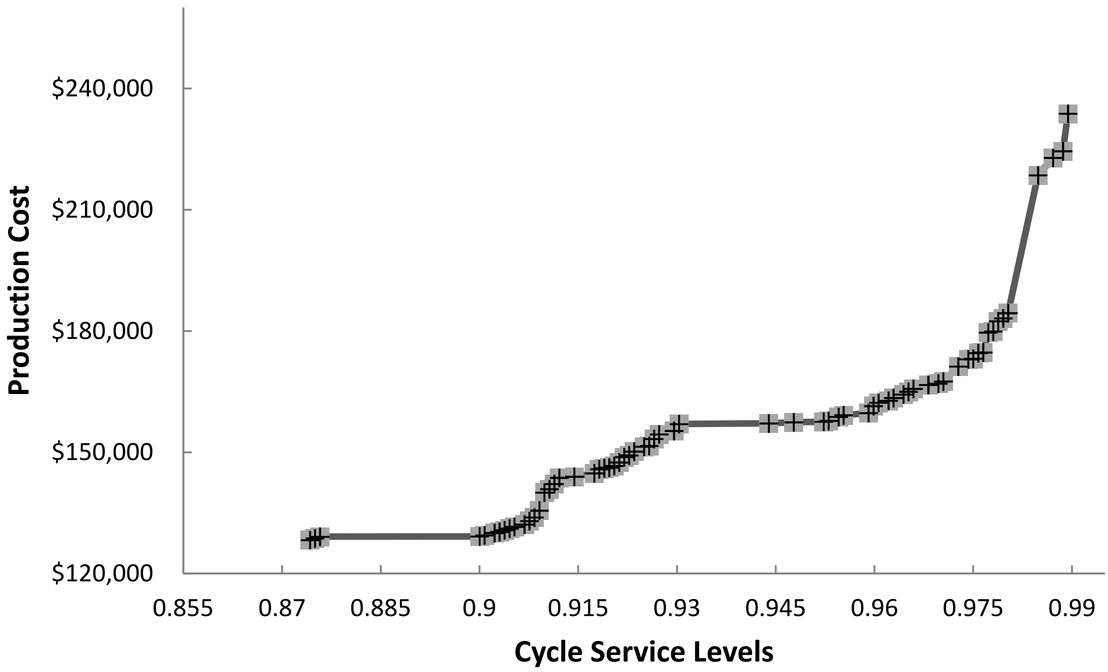


Figure 6–12: The Final Pareto Frontier and the Selection of a Single Optimal Solution

The multi-objective optimization methods used in this chapter provide for a posteriori articulation of preferences, requiring a decision maker to identify the final Pareto optimal solution from the entire range of possibilities as shown in Figure 6.12. The decision maker's choice depends upon the value function representing his preferences. The aim here is to truly describe the decision context and give all possible options to choose from. The Pareto curve has made it easy for the production manager because of the visual representation that reveals patterns where considerable cycle service level are achievable by minimal increase in production cost. For example, a cycle service level improvement from 0.87 to 0.90 or from 0.93 to 0.96 could be obtained with insignificant additional cost expenditure. For the Pareto curve segments between 0.9 to 0.93 and 0.96 to 0.98

cycle service levels, the objective tradeoff between production cost f_1 and CSL f_2 can be estimated by the following equation:

$$Objective\ Tradeoff = \frac{f_1(x^2) - f_1(x^1)}{f_2(x^2) - f_2(x^1)} \tag{6.30}$$

6.8 Conclusion

Paper mills generally like to run long production runs with minimum grade changeovers and manufacture jumbo reels to stock in anticipation of customer demand. This was the decision context prevailing previously but ever increasing customer requirements and market pressures now warrant a trade-off between production cost, flexibility and customer service. Typically, in the paper industry, while some customers enjoy considerable leverage on the paper mill and will insist on having their orders delivered in time, the paper mill can also afford to delay some orders by being flexible with a few of its customers for order delivery, therefore, saving on production costs. The cost reduction by compromising on the service levels can be an advantage for both the supplier and customers. For the customers insisting on punctual delivery, the bi-objective formulation discussed in this chapter gives a useful tool to the mill manager because it can help to quantify the associated extra costs.

In this chapter, multi objective optimization is demonstrated for the successive manufacturing processes of the paper industry supply chain. A two step solution approach is proposed for bi-objective production planning. In the first step, a set of non-dominated solutions is obtained by employing the epsilon constraint method which is then used as part of an initial population for the NSGA-II in the second step. NSGA-II not only improves the quality of epsilon constraint solutions but also increases the number of solutions on the Pareto frontier. Joint optimization is preferred over the separate planning approach because it gives the entire range of possibilities from a least grade changeover and a least trim loss solution to the maximized service level plan with additional production costs. Issues associated with the successful implementation of the multi-objective optimization algorithms have been discussed and the importance of estimating the ideal and nadir objective vector to reflect the entire set of feasible search space has been highlighted.

CHAPTER 7. CONCLUSIONS, SUMMARY AND RECOMMENDATIONS

7.1 Summary

Over the years, the paper industry has undergone a paradigm shift towards customer oriented production practices. The traditional objectives of minimizing costs and fully utilizing capacity have to be balanced now with flexibility and customer focus. All this requires sophisticated planning and a new perspective on scheduling. The planning activities now have to be seen in a holistic manner unlike the traditional practices where planning disconnects existed between the successive production stages generating sizeable inventory and yet the customer orders did not match inventory at hand. The focus is now on the integration and joint optimization of all the activities in the supply chain because the likely outcome of firms optimizing individual operations separately is an overall suboptimal supply chain. There is a strong case for integration or joint optimization of successive paper making and converting processes in a paper mill. The integrated cutting stock and lot-sizing model (ICL-MP) proposed in this thesis would enable customer orders to penetrate right up to the paper machine and achieve synchronized product flow within the manufacturing processes of the paper supply chain with the following outcomes:

- Consistently higher cycle service levels
- Higher manufacturing flexibility
- Increased customer responsiveness
- No inventory holding of jumbo reels

- The benefits of JIT manufacturing concepts such as pull demand, smaller lots and reduced inventory holding
- Overcoming the planning disconnect between the paper manufacturing and conversion processes.

The integrated cutting stock and lot-sizing model (ICL-MP) model would not only integrate the decision making for the paper machine and the conversion processes but also the synchronized product flow would result in much improved customer service levels. The high cycle service levels in the integrated model are attributed to the new approach which not only allows multiple setups in a single planning period of small bucket lot-sizing problems but also solves the joint cutting stock and lot-sizing problem simultaneously while minimizing four sets of costs, namely, grade changeovers, inventory holding costs for finished products, trim loss and tardiness penalty.

When the integrated cutting stock and lot-sizing model (ICL-MP) is compared with the traditional approaches which either solve the joint problem separately or restrict the number of setups in each planning period, the service levels are consistently higher in all test problems. The traditional single product planning model (ICL-SP) performs poorly for cycle service levels even to the extent that it would not meet the end demand. However, the ICL-SP reduces grade changeover costs which are substantial in the paper industry by restricting the number of products in each planning period. The cost savings are significant but the ramifications for service level and for machine utilization make this model an unsuitable choice for punctual deliveries. The unutilized capacity also raises questions for the appropriateness of ICL-SP as a planning method when higher cycle service level is not a strict requirement.

The separate planning approach (SCL-MP) results in least trim loss but the grade changeovers are consistently on the high side because there are no restrictions on the number of grades produced on the paper machine in each planning period. The resulting cycle service levels are higher than the integrated cutting stock and lot-sizing model with restricted setups (ICL-SP) but consistently lower than the integrated approach without restriction on the setups (ICL-MP). This indicates that without any restriction on the number of products produced in each planning period, the boost to service levels is associated with additional trim loss.

Since the only objective for the pure cutting stock problem is to minimize trim loss, it achieves a very cost effective solution but also imposes an upper bound on the lot size i.e the maximum number of jumbo reels to be produced. The upper bound on the jumbo reels brings the costs down as compared to the integrated model where the quantity of jumbo reels produced is consistently higher. However, it performs poorly in satisfying customer orders. The higher number of jumbo rolls for the integrated model enables the production plan to generate better cycle service levels.

The comparison of the proposed approach with existing planning methods has demonstrated that high service level and minimum production cost are conflicting objectives in the paper industry. It has been shown that a least cost production plan results in poor cycle service levels where many customer deliveries fail to meet the due dates.

The joint production problem for paper manufacturing and conversion has been solved by multi-objective optimization. In the first step of bi-objective production planning a set of non-dominated solutions is obtained by employing the epsilon constraint method. A standard genetic algorithm solution is used. In the second step, the epsilon constraint solutions become part of an initial population for the non dominated sorting algorithm (NSGA-II) to improve the Pareto frontier. The two step approach is necessary because the NSGA-II was unable to obtain feasible solutions when initiated by a randomized population. No such issue arises when standard GA is used as the resolution algorithm for the epsilon constraint method.

Therefore the epsilon constraint method has been used as the first step to generate an approximation to the Pareto frontier which is then improved by resorting to NSGA-II in the second step. The Epsilon constraint method explicitly uses the knowledge of ideal and nadir objective vectors to define the feasible search space. This characteristic helps to integrate the three different planning approaches discussed in Chapter 5 into a single experiment. The ideal and nadir objective vectors represent the two extremes of the Pareto frontier with the separate planning approach combined with restricted number of setups yielding lowest service levels but also lowest costs whereas the integrated formulation with unrestricted setups achieves high service levels at the expense of increased production costs.

7.2 Implications for the Industry

The paper industry is in the midst of orienting itself to customer satisfaction and the integrated model developed for this thesis can meet the emerging need. Ideally, the paper mill would like to have long duration paper production runs to minimize grade changeovers and also would like to minimize the trim loss during the conversion stages by grouping together the customer orders without regard to service level. This approach was the norm in past years but the intensification of competition and customer requirements have forced paper mills to switch to more flexible and customer focused production strategies. Now, the emphasis on capacity utilization has to be traded off with flexibility and customer satisfaction.

As discussed in Chapter 2, Section 2.2, the international paper market is becoming increasingly competitive so that mills have to focus on customer service while trying to maintain low production costs. The change in Australia is even more severe because high levels of import protection have gone. This means that the previous focus on production is being replaced by customer service in order to meet import competition.

The proposed multi-objective optimization approach to the successive paper manufacturing processes gives the entire range of possibilities between least cost production plans and maximized service levels. The production manager in consultation with the sales team can choose the production plan depending upon the emphasis on delivery punctuality. It also enables the production manager to bargain with the customers because prompt delivery may warrant a premium which can be easily obtained from the trade-off curve.

7.3 Wider Implications

The 'thesis' presented in this thesis is that most of the problems encountered in the supply chain processes are multi-faceted and therefore, should be treated by multi-objective resolution algorithms. This concept could easily be extended to other real world situations. The set of non-dominated solutions obtained through multi-objective optimization gives the entire range of possibilities to the decision maker to make an informed decision. The conventional aggregated objective optimization method yields just one best solution, not adequately capturing the trade-off dynamics. A multi-objective approach removes the bias towards a particular objective by either normalizing the coefficients of the aggregated objective function, using only one objective at a time or by incorporating the Pareto rank or dominance based approach where all objectives are given equal importance during the pair-wise comparison for dominance. Moreover, the utility of multi-objective optimization is highlighted when the estimation of parameters for the objectives in the conventional optimization is challenging. For example, in Chapter 5, maximum cycle service level was achieved by assigning highest value to the tardiness penalty and (as happens in aggregated single objective optimization) the search process was guided by the respective coefficients of each objective. Since the tardiness penalty had the highest magnitude, the search process favoured the least tardiness penalty solutions. However, all high service levels solutions were expensive. In most real world situations, a decision maker may not opt for maximum service level because it is too expensive and be interested in solutions that give a range of values between the two extremes obtained by multi-objective optimization.

Another conclusion drawn from the study is that additional service is possible by jointly optimizing lot-sizing at the paper machine and cutting at the conversion stage. In principle, the integrated cutting and lot-sizing could be applied to all the industrial settings where these two processes are encountered in successive stages. Whenever multiple products are produced from a common resource - implying production setups – in one stage and preceded or followed by the cutting process to correspond to customer requirements, joint optimization is a candidate approach. However, implementation may have to account for specific industrial settings. For example, in the furniture industry, the sequence of operations is different from the current case study, with cutting stock operations being followed by lot-sizing. Consequently the solution of the cutting stock problem is unknown a priori and dependent upon the solution of the lot-sizing problem.

7.4 Recommendations for Future Research

The production environment considered in the study involved a single paper machine but it is not uncommon in the paper industry for a single production facility to host more than one paper machine or the company may own multiple plants. The production problem in such scenarios is a composite problem with two NP-hard problems, cutting stock problem and the order allocation to different stock size machines, combined together. One of the logical extensions of the study is to include multiple paper machines in both the conventional and multi-objective optimization models.

The conventional optimization approach used in Chapter 5 used a constant tardiness penalty for all the orders that could not be delivered in time. Similarly, in the multi-objective optimization of the production problem, the total number of late orders was jointly minimized along with the cost minimization. All the customer orders were treated as equally important but in most real world situations, not all the customers will be treated alike by a manufacturer. While some customers may enjoy considerable leverage on the paper mill and will insist on having their orders delivered in time because of their own constraints, the paper mill can also afford to delay some orders by being flexible with a few of its customers, thereby saving on production costs. Certain production environments may warrant incorporation of customer related information in the problem formulation so that order deliveries are prioritized.

In this study, an entire trade-off between the conflicting objectives of production cost and maximization of cycle service levels was obtained and the selection of final solution was left to the decision maker. This is typical of a posteriori articulation of preferences where all possible solutions are generated and it is assumed that the decision maker will make a decision based on his requirements. This is fine for many situations because the decision makers are usually experienced and well versed in the decision context. However, there is an emerging stream of literature that advocates provision of one or only a few solutions presented to the decision maker instead of the entire Pareto frontier. This could be achieved by interactively incorporating the decision maker’s preferences in the modelling process, leading to the final solution by the repeated exchange of information between the decision maker and the results. However, this approach can only be implemented with active participation of the industry personnel and would be specific to an individual mill’s requirements. An alternate approach is to obtain a single or few most suitable solutions from the already generated Pareto frontier by one of the a posteriori methods such as the one used in this thesis. Frequently used ‘narrowing down’ approaches are focusing on ‘knee points’ in the Pareto frontier; a large sacrifice may be required in one objective to achieve a small gain in another and the decision would require preference information such as utility functions and marginal rate of return.

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APPENDIX A Test Problems for Chapter 4

Test Data		Data Set 1		Data Set 2		Data Set 3		Data Set 4		Data Set 5		Data Set 6		Data Set 7		Data Set 8	
		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$	
		l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i2}		d_{i2}	l_i	d_{i2}	l_i	d_{i2}
m = 3	i=1	1954	16	1268	16	4508	16	4808	16	1954	82	1268	82	4508	82	4808	82
	i=2	1801	3	3745	3	7498	3	5345	3	1801	13	3745	13	7498	13	5345	13
	i=3	1214	11	2440	11	1343	11	1655	11	1214	55	2440	55	1343	55	1655	55
Test Data		Data Set 9		Data Set 10		Data Set 11		Data Set 12		Data Set 13		Data Set 14		Data Set 15		Data Set 16	
		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$	
		l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i2}		d_{i2}	l_i	d_{i2}	l_i	d_{i2}
m = 5	i=1	2371	10	4635	10	6914	10	9276	10	2371	51	4635	51	6914	51	9276	51
	i=2	1373	4	3022	4	4170	4	1755	4	1373	21	3022	21	4170	21	1755	21
	i=3	1849	5	4078	5	5034	5	5051	5	1849	25	4078	25	5034	25	5051	25
	i=4	1567	18	4155	18	6818	18	8801	18	1567	92	4155	92	6818	92	8801	92
	i=5	1951	13	4565	13	1104	13	1953	13	1951	61	4565	61	1104	61	1953	61
Test Data		Data Set 17		Data Set 18		Data Set 19		Data Set 20		Data Set 21		Data Set 22		Data Set 23		Data Set 24	
		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$	
		l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i2}	l_i	d_{i2}	l_i	d_{i2}	l_i	d_{i2}
m= 7	i=1	1170	23	2263	23	4030	23	3771	23	1170	115	2263	115	4030	115	3771	115
	i=2	1214	15	2035	15	6747	15	6988	15	1214	76	2035	76	6747	76	6988	76
	i=3	2250	7	3929	7	2704	7	9307	7	2250	33	3929	33	2704	33	9307	33
	i=4	2184	1	3576	1	5174	1	5153	1	2184	7	3576	7	5174	7	5153	7
	i=5	1178	10	2751	10	1515	10	8616	10	1178	48	2751	48	1515	48	8616	48
	i=6	1595	3	1109	3	1839	3	5593	3	1595	14	1109	14	1839	14	5593	14
	i=7	2278	11	2419	11	4000	11	9131	11	2278	57	2419	57	4000	57	9131	57

b = Length Factor = 0.25, 0.5, 0.75, 1.0

l_i = Order width

d_{i1} = Demand for order length *i* for $\tilde{d}_1 = 10$

d_{i2} = Demand for order length *i* for $\tilde{d}_2 = 50$

$\tilde{d}$ = Average Demand per Order = 10, 50

m = No of Ordered Lengths = 3, 5, 7, 10, 15

Test Data		Data Set 25		Data Set 26		Data Set 27		Data Set 28		Data Set 29		Data Set 30		Data Set 31		Data Set 32	
		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$	
		l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i2}		d_{i2}	l_i	d_{i2}	l_i	d_{i2}
m = 10	i=1	2405	8	1824	8	3531	8	9817	8	2405	39	1824	39	3531	39	9817	39
	i=2	2132	23	1308	23	7244	23	2216	23	2132	116	1308	116	7244	116	2216	116
	i=3	1912	7	3808	7	6555	7	9345	7	1912	37	3808	37	6555	37	9345	37
	i=4	2016	1	2075	1	2011	1	2576	1	2016	5	2075	5	2011	5	2576	5
	i=5	1880	1	1753	1	3600	1	1498	1	1880	3	1753	3	3600	3	1498	3
	i=6	1344	21	2174	21	3212	21	3543	21	1344	107	2174	107	3212	107	3543	107
	i=7	1302	11	3533	11	6916	11	7499	11	1302	55	3533	55	6916	55	7499	55
	i=8	2358	12	1795	12	7033	12	1964	12	2358	59	1795	59	7033	59	1964	59
	i=9	1589	11	4712	11	1955	11	9222	11	1589	55	4712	55	1955	55	9222	55
	i=10	2400	5	1640	5	2256	5	7405	5	2400	24	1640	24	2256	24	7405	24

Test Data		Data Set 33		Data Set 34		Data Set 35		Data Set 36		Data Set 37		Data Set 38		Data Set 39		Data Set 40	
		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$		$b_1 = 0.25$		$b_2 = 0.5$		$b_3 = 0.75$		$b_4 = 1.0$	
		l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i1}	l_i	d_{i2}	l_i	d_{i2}	l_i	d_{i2}	l_i	d_{i2}
m = 15	i=1	2127	4	4445	4	2870	4	9100	4	2127	20	4445	20	2870	20	9100	20
	i=2	1521	14	3307	14	1120	14	6325	14	1521	69	3307	69	1120	69	6325	69
	i=3	2112	9	3691	9	7339	9	7617	9	2112	47	3691	47	7339	47	7617	47
	i=4	2015	11	3893	11	3287	11	8192	11	2015	57	3893	57	3287	57	8192	57
	i=5	1503	16	1596	16	3837	16	2855	16	1503	82	1596	82	3837	82	2855	82
	i=6	1316	11	3367	11	6348	11	3665	11	1316	57	3367	57	6348	57	3665	57
	i=7	1609	15	2572	15	6664	15	1288	15	1609	76	2572	76	6664	76	1288	76
	i=8	1620	9	2872	9	6150	9	2406	9	1620	43	2872	43	6150	43	2406	43
	i=9	1672	7	2328	7	3457	7	7683	7	1672	33	2328	33	3457	33	7683	33
	i=10	1480	10	2731	10	2704	10	8232	10	1480	49	2731	49	2704	49	8232	49
	i=11	1275	8	2429	8	5017	8	5680	8	1275	38	2429	38	5017	38	5680	38
	i=12	2371	6	2003	6	2398	6	6215	6	2371	31	2003	31	2398	31	6215	31
	i=13	1661	12	2119	12	1830	12	7672	12	1661	60	2119	60	1830	60	7672	60
	i=14	2320	15	2191	15	5643	15	7526	15	2320	74	2191	74	5643	74	7526	74
	i=15	1222	3	3791	3	3919	3	9052	3	1222	14	3791	14	3919	14	9052	14

b = Length Factor = 0.25, 0.5, 0.75, 1.0

l_i = Order width

d_{i1} = Demand for order length i for $\tilde{d}_1 = 10$

d_{i2} = Demand for order length i for $\tilde{d}_2 = 50$

$\tilde{d}$ = Average Demand per Order = 10, 50

m = No of Ordered Lengths = 3, 5, 7, 10, 15

APPENDIX B Data Sets for the Joint Problem – Chapters 5 & 6

Data Set 1		Intermediate Product *i*											
		Intermediate Product *i* = 1			Intermediate Product *i* = 2			Intermediate Product *i* = 3			Intermediate Product *i* = 4		
		Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate
Finished Product (*i'*)	***i'* = 1**	9	4.50	7	10	3.00	3	0	4.00	7	10	2.80	3
	***i'* = 2**	12	3.75	4	12	4.00	5	18	3.00	3	10	4.50	4
	***i'* = 3**	5	3.00	6	0	2.80	6	0	3.80	6	0	3.80	3
	***i'* = 4**	0	5.50	7	0	4.50	3	13	2.60	4	13	3.00	7
	***i'* = 1**	10	4.50	4	0	3.00	4	10	4.00	4	0	2.80	4
	***i'* = 2**	10	3.75	3	8	4.00	7	0	3.00	5	0	4.50	7
	***i'* = 3**	0	3.00	7	10	2.80	3	0	3.80	4	0	3.80	5
	***i'* = 4**	6	5.50	4	0	4.50	4	0	2.60	3	0	3.00	4
	***i'* = 1**	10	4.50	5	15	3.00	5	0	4.00	6	10	2.80	6
	***i'* = 2**	5	3.75	6	0	4.00	0	0	3.00	4	0	4.50	5
	***i'* = 3**	12	3.00	5	12	2.80	5	10	3.80	7	10	3.80	4
	***i'* = 4**	0	5.50	7	0	4.50	7	10	2.60	5	10	3.00	6
	***i'* = 1**	0	4.50	6	0	3.00	7	10	4.00	5	10	2.80	5
	***i'* = 2**	8	3.75	7	0	4.00	6	0	3.00	6	0	4.50	6
	***i'* = 3**	0	3.00	3	2	2.80	3	12	3.80	3	0	3.80	7
	***i'* = 4**	0	5.50	6	10	4.50	6	8	2.60	6	8	3.00	5
	***i'* = 1**	12	4.50	3	12	3.00	6	10	4.00	3	11	2.80	7
	***i'* = 2**	5	3.75	6	10	4.00	4	0	3.00	7	8	4.50	3
	***i'* = 3**	0	3.00	4	0	2.80	7	0	3.80	5	0	3.80	6
	***i'* = 4**	0	5.50	5	0	4.50	5	0	2.60	3	0	3.00	3

Notations

i ε Intermediate products (IP) i.e jumbo reel

i' ε Finished products (IP) i.e cut rolls

$d_{i'}$ = Demand for the FP *i´* in period *t*

$l_{i'}$ = Order Width of FP *i´* (m)

L = Jumbo Length (m) = 10 metres

Note: A quantity of $d_{i'}$ = 0 means that there is no demand for product i' .

Data Set 2		Intermediate Product *i*											
		Intermediate Product *i* = 1			Intermediate Product *i* = 2			Intermediate Product *i* = 3			Intermediate Product *i* = 4		
		Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate
Finished Product (*i'*)	*i'* = 1	6	5.50	6	8	3.50	3	4	6.00	3	0	5.25	4
	i' = 2	12	3.50	3	5	5.25	4	3	3.08	4	10	4.60	6
	i' = 3	14	4.35	7	12	4.00	7	5	4.50	6	12	2.80	6
	i' = 4	10	2.25	7	0	6.80	4	0	2.50	4	0	6.50	6
	i' = 1	0	5.50	4	5	3.50	5	3	6.00	6	5	5.25	4
	i' = 2	4	3.50	6	5	5.25	7	6	3.08	7	8	4.60	7
	i' = 3	12	4.35	5	6	4.00	6	0	4.50	5	4	2.80	4
	i' = 4	6	2.25	7	0	6.80	6	8	2.50	5	0	6.50	6
	i' = 1	10	5.50	3	3	3.50	7	6	6.00	3	3	5.25	7
	i' = 2	5	3.50	3	6	5.25	6	2	3.08	6	8	4.60	3
	i' = 3	0	4.35	7	2	4.00	5	9	4.50	6	5	2.80	7
	i' = 4	0	2.25	3	4	6.80	6	10	2.50	5	2	6.50	7
	i' = 1	5	5.50	5	4	3.50	5	2	8.75	6	0	5.25	6
	i' = 2	0	3.50	5	6	5.25	5	0	3.08	4	10	4.60	5
	i' = 3	5	4.35	6	1	4.00	3	7	4.50	4	12	2.80	5
	i' = 4	4	2.25	6	0	6.80	3	8	2.50	3	5	6.50	7
	i' = 1	5	5.50	5	5	3.50	7	0	6.00	7	3	5.25	4
	i' = 2	0	3.50	4	0	5.25	6	4	3.08	7	5	4.60	5
	i' = 3	0	4.35	4	4	4.00	5	10	4.50	6	4	2.80	6
	i' = 4	0	2.25	7	4	6.80	3	26	2.50	7	2	6.50	7

Notations

i	ε	Intermediate products (IP) i.e jumbo reel
i'	ε	Finished products (IP) i.e cut rolls
$d_{i'}$	=	Demand for the FP *i´* in period *t*
$l_{i'}$	=	Order Width of FP *i´* (m)
L	=	Jumbo Length (m) = 10 metres

Note: A quantity of $d_{i'}$ = 0 means that there is no demand for product i' .

Data Set 3		Intermediate Product *i*											
		Intermediate Product *i* = 1			Intermediate Product *i* = 2			Intermediate Product *i* = 3			Intermediate Product *i* = 4		
		Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate
Finished Product (*i'*)	*i'* = 1	6	2.40	6	8	2.40	3	4	2.75	5	10	3.10	5
	i' = 2	12	4.90	4	5	4.90	5	3	4.50	6	0	5.40	4
	i' = 3	14	3.90	6	12	3.90	3	5	3.70	7	12	4.30	5
	i' = 4	10	5.70	3	0	5.70	7	0	6.20	4	0	6.50	3
	i' = 1	0	2.40	7	5	2.40	6	3	2.75	5	5	3.10	3
	i' = 2	4	4.90	7	5	4.90	4	6	4.50	7	8	5.40	5
	i' = 3	12	3.90	5	6	3.90	4	0	3.70	5	4	4.30	7
	i' = 4	4	5.70	5	0	5.70	5	8	6.20	4	0	6.50	3
	i' = 1	10	2.40	4	3	2.40	4	6	2.75	5	8	3.10	3
	i' = 2	5	4.90	7	6	4.90	3	2	4.50	3	3	5.40	5
	i' = 3	0	3.90	6	2	3.90	4	9	3.70	3	5	4.30	6
	i' = 4	0	5.70	6	4	5.70	4	0	6.20	5	2	6.50	4
	i' = 1	5	2.40	6	4	2.40	3	2	2.75	6	0	3.10	7
	i' = 2	0	4.90	4	6	4.90	4	10	4.50	5	4	5.40	3
	i' = 3	5	3.90	6	1	3.90	3	7	3.70	3	12	4.30	6
	i' = 4	4	5.70	4	0	5.70	3	8	6.20	7	5	6.50	4
	i' = 1	5	2.40	6	5	2.40	6	16	2.75	4	3	3.10	4
	i' = 2	0	4.90	5	0	4.90	3	4	4.50	4	5	5.40	3
	i' = 3	0	3.90	5	4	3.90	6	10	3.70	6	4	4.30	4
	i' = 4	0	5.70	5	4	5.70	7	0	6.20	5	2	6.50	4

Notations

i	ε	Intermediate products (IP) i.e jumbo reel
i'	ε	Finished products (IP) i.e cut rolls
$d_{i'}$	=	Demand for the FP i' in period t
$l_{i'}$	=	Order Width of FP i' (m)
L	=	Jumbo Length (m) = 10 metres

Note: A quantity of $d_{i'}$ = 0 means that there is no demand for product i' .

Data Set 4		Intermediate Product i											
		Intermediate Product i = 1			Intermediate Product i = 2			Intermediate Product i = 3			Intermediate Product i = 4		
		Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate
Finished Product (i')	i' = 1	4	6.70	6	8	3.50	5	4	2.20	6	10	4.50	6
	i' = 2	12	3.60	6	5	4.20	6	3	5.80	7	0	5.40	6
	i' = 3	14	2.80	6	12	5.20	3	5	3.80	3	12	2.70	7
	i' = 4	10	4.10	3	0	6.50	6	0	5.10	4	0	3.70	6
	i' = 1	0	6.70	4	5	3.50	5	3	2.20	7	5	4.50	3
	i' = 2	4	3.60	4	5	4.20	5	6	5.80	5	8	5.40	6
	i' = 3	12	2.80	5	6	5.20	3	0	3.80	6	4	2.70	4
	i' = 4	4	4.10	3	0	6.50	4	8	5.10	5	0	3.70	7
	i' = 1	10	6.70	5	3	3.50	5	6	2.20	6	8	4.50	4
	i' = 2	5	3.60	5	6	4.20	6	2	5.80	3	3	5.40	4
	i' = 3	0	2.80	3	2	5.20	3	9	3.80	6	5	2.70	4
	i' = 4	0	4.10	5	4	6.50	4	0	5.10	4	2	3.70	6
	i' = 1	5	6.70	4	4	3.50	7	2	2.20	7	0	4.50	4
	i' = 2	0	3.60	4	6	4.20	4	10	5.80	7	4	5.40	7
	i' = 3	5	2.80	3	1	5.20	6	7	3.80	7	12	2.70	3
	i' = 4	4	4.10	7	2	6.50	7	8	5.10	5	5	3.70	6
	i' = 1	5	6.70	5	5	3.50	3	16	2.20	4	3	4.50	6
	i' = 2	0	3.60	6	0	4.20	4	1	5.80	6	4	5.40	7
	i' = 3	0	2.80	4	4	5.20	3	10	3.80	5	4	2.70	5
	i' = 4	0	4.10	7	6	6.50	3	0	5.10	3	12	3.70	7

Notations

- i ε Intermediate products (IP) i.e jumbo reel
- i' ε Finished products (IP) i.e cut rolls
- $d_{i'}$ = Demand for the FP i' in period t
- $l_{i'}$ = Order Width of FP i' (m)
- L = Jumbo Length (m) = 10 metres

Note: A quantity of $d_{i'}$ = 0 means that there is no demand for product i' .

Data Set 5		Intermediate Product i											
		Intermediate Product i = 1			Intermediate Product i = 2			Intermediate Product i = 3			Intermediate Product i = 4		
		Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate	Qty. $d_{i'}$	Width $l_{i'}$	Duedate
Finished Product (i')	i' = 1	6	3.70	3	8	4.20	6	4	5.20	4	10	2.20	3
	i' = 2	12	4.20	6	5	2.50	3	3	6.40	3	0	6.90	5
	i' = 3	14	2.90	6	12	7.30	3	5	4.80	6	12	5.70	5
	i' = 4	10	5.70	3	0	5.80	3	0	3.20	3	0	4.40	5
	i' = 1	0	4.50	7	5	3.00	7	3	4.00	3	5	2.80	7
	i' = 2	4	3.75	4	5	4.00	5	6	3.00	4	8	4.50	3
	i' = 3	12	3.00	5	6	2.80	3	0	3.80	7	4	3.80	6
	i' = 4	4	5.50	3	0	4.50	3	8	2.60	7	0	3.00	4
	i' = 1	10	4.50	6	3	3.00	4	6	4.00	4	8	2.80	6
	i' = 2	5	3.75	5	6	4.00	3	2	3.00	4	3	4.50	3
	i' = 3	0	3.00	7	2	2.80	3	9	3.80	4	0	3.80	7
	i' = 4	0	5.50	5	4	4.50	4	0	2.60	3	2	3.00	6
	i' = 1	5	4.50	3	4	3.00	5	2	4.00	5	0	2.80	7
	i' = 2	0	3.75	5	6	4.00	5	0	3.00	3	4	4.50	5
	i' = 3	5	3.00	5	1	2.80	7	7	3.80	7	6	3.80	5
	i' = 4	4	5.50	4	0	4.50	3	8	2.60	4	5	3.00	6
	i' = 1	8	4.50	7	5	3.00	4	10	4.00	5	3	2.80	5
	i' = 2	0	3.75	7	0	4.00	7	4	3.00	3	0	4.50	6
	i' = 3	0	3.00	3	2	2.80	6	10	3.80	5	0	3.80	6
	i' = 4	5	5.50	7	4	4.50	6	0	2.60	3	2	3.00	7

Notations

i	ε	Intermediate products (IP) i.e jumbo reel
i'	ε	Finished products (IP) i.e cut rolls
$d_{i'}$	=	Demand for the FP i' in period t
$l_{i'}$	=	Order Width of FP i' (m)
L	=	Jumbo Length (m) = 10 metres

Note: A quantity of $d_{i'}$ = 0 means that there is no demand for product i' .