THE METAPHYSICS OF IMPENETRABILITY: EULER'S CONCEPTION OF FORCE

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IN this paper I want to examine in some detail one eighteenth-century attempt to restructure the foundations of mechanics, that of Leonhard Euler. It is now generally recognized that the idea, due to Mach, that all that happened in the eighteenth century was the elaboration of a deductive and mathematical mechanics on the basis of Newton's laws is misleading at best. Newton's *Principia* needed much more than a reformulation in analytic terms if it was to provide the basis for the comprehensive mechanics that was developed in the eighteenth century. Book II of the *Principia*, in particular, where the problem of the resistance offered to the motion of a finite body by a fluid medium was raised, was generally (and rightly) thought to be in large part mistaken and confused. There were also a number of areas crucial to the unification of mechanics which Newton did not deal with at all in the *Principia*: particularly the dynamics of rigid, flexible and elastic bodies, and the dynamics of several bodies with mutual interactions. Although a start had been made on some of these topics in the seventeenth century (notably by Galileo, Beeckman, Mersenne, Huygens, Pardies, Hooke, and Leibniz), it was only in the eighteenth century that they were subjected to detailed examination, and Euler's contribution to the development of these topics, and hence to the unification of mechanics, was immense.[1]

Running parallel with this extension and unification of mechanics, there was also a growing concern with foundational questions. These centred upon Newton's conception of force. The general attitude to this conception was one of ambivalence, but many in the eighteenth century thought it to be at best seriously incomplete and at worst an 'obscure and metaphysical being capable of nothing but spreading darkness over a

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[1] For a general discussion of these issues, see R. Dugas, *Histoire de la mécanique*, Neuchatel, 1950, and C. Truesdell, *Essays in the history of mechanics*, Berlin, 1968. For more detailed treatments, see the contributions of Truesdell to *Leonhardi Euleri opera omnia*: 'The rational mechanics of flexible or elastic bodies, 1638–1788', series 2, vol. xi, section 2, Zurich, 1960; 'Rational fluid mechanics, 1687–1765', series 2, vol. xii, Zurich, 1954; 'Editor's introduction [to Euler's treatise on fluid mechanics]', series 2, vol. xiii, Zurich, 1955.

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science clear by itself', as d'Alembert put it. D'Alembert adopted what is generally considered to be a Cartesian view, attempting to reduce force to a kinematic concept, acceleration, and thereby ridding dynamics of the notion altogether.[2] At the opposite pole were Boscovich and Kant, for whom force had to be taken as a physically primitive notion, more primitive even than that of body. Both these positions deviated considerably from Newton's own, but since there were undeniable tensions and contradictions in Newton's view,[3] and since dynamics had to come to terms with force one way or another, some new conception of force had to be formulated. For d'Alembert this new conception was one which ultimately made mention of force redundant. For Boscovich and Kant it was one that conferred upon the idea of repulsive force exactly the same status as Newton had conferred upon attractive force, providing a unified conception of force, a conception which effectively made it a physically primitive notion,[4] and which involved the idea that all forces act at a distance: what many saw as the greatest flaw in Newton's account was thereby made its greatest virtue.

Euler wanted neither to make force redundant nor to make it absolutely primitive. Forces really existed on Euler's account, but they required explanation in terms of something much more intuitive: the impenetrability of matter. For Euler the ultimate foundations of mechanics had to be given in terms of something which could be grasped as being both necessary and self-evident, and it was necessary and self-evident for him that bodies are impenetrable: it was impossible to conceive of a body (i.e. full, solid matter free from vacua) being penetrated since this would require that two bodies be in the same place at the same time, which is impossible. I suggest the procedure here is Cartesian, even if the results are not. In asking why a Cartesian procedure was adopted, at a time when Cartesian mechanics had effectively met its demise, we must remember the great appeal of Cartesian method, with its procedure of building up an apodeictic mechanics on the basis of self-evident and necessary foundations. Newton's foundations for his own dynamics were far from self-evi-

[2] This 'Cartesian' view is not Descartes' own. Descartes himself did not attempt to reduce dynamics to kinematics, although in the eighteenth century he was generally taken to have done so. Part of the reason for this must surely lie in the fact that his system, particularly in its more programmatic aspects, lent itself so easily to mechanism. On Descartes' conception of force, see M. Gueroult, 'The metaphysics and physics of force in Descartes', and A. Gabbey, 'Force and inertia in the seventeenth century: Descartes and Newton', in S. Gaukroger (ed.), *Descartes: philosophy, mathematics and physics*, Brighton, 1980.

[3] A reasonably thorough discussion of the problems in Newton's view is to be found in R. S. Westfall, *Force in Newton's physics* London, 1971, chapters VII, VIII.

[4] I say that force was *effectively* primitive for Boscovich and Kant because Boscovich, at least, although he in fact took force to be primitive as far as mechanics was concerned, did consider that some further explanation of force might be conceivable: 'This propensity is the origin of what we call "force of inertia"; whether this is dependent upon some arbitrary law of the Supreme Architect, or on the nature of points itself, or on some attribute of them, whatever it may be, I do not seek to know; even if I did wish to do so, I see no hope of finding the answer; and truly I think that this also applies to the law of forces . . .'; *A theory of natural philosophy*, tr. by J. M. Child, Cambridge, Mass., 1966.

dent; indeed they were highly questionable. The Cartesian method, with its obvious parallels with the axiomatized structure of Euclidean geometry had been so well assimilated into European culture by this period that it continued to serve as a paradigm for many in the eighteenth century, despite the fact that Cartesian mechanics as such had been largely discarded. While Euler made no explicit commitment to Cartesian method, his foundational work was, as I hope to show, heavily indebted to Descartes nonetheless.

To simplify somewhat, for Euler as for d'Alembert it was a question of squeezing Newtonian mechanics into a Cartesian shape, a shape which, it was hoped, was to render it more certain and more fruitful. The certainty of the new mechanics was to derive from its structure, but more importantly from the nature of the foundations of that structure. These foundations were of a rigorously Cartesian type: they could not be doubted without fear of contradiction. This, I suggest, was the essence of Euler's foundational project. Newtonian dynamics, suitably reformulated, had to be shown not just to be true but to be necessarily true.

Euler provided three extended discussions of his foundational project: the first in chapters II and III of the *Mechanica sive motus scientia analytice exposita* (1736),[5] the second in the short treatise 'Recherches sur l'origine des forces' (1750),[6] and the third in the *Introductio* to his *Theoria motus corporum solidorum seu rigidorum* (1765).[7] These accounts differ very little in content (indicating, I think, Euler's satisfaction with his treatment of the issue) so we shall concentrate on the most mature statement, that of the *Theoria*. Since this treatise is only available in the original Latin, I shall begin with a reasonably detailed paraphrase of the relevant sections before proceeding to a discussion of Euler's account.

I

The first three chapters of the Introduction to the *Theoria* are designed to provide the conceptual foundations for mechanics. The concern of the first chapter is exclusively kinematical and it defines and elaborates upon the ideas of position, distance, shape, space, time, rest, motion, speed, and direction of motion. It provides techniques for resolving motions of up to three directrices and introduces the idea of determining the speed and direction of motions with respect to reference frames (something which Euler is much more confident in handling than his predecessors). The chapter provides all the kinematics that Euler needs for his mechanics: all motions are now characterizable vectorially and procedures for determin-

[5] *Leonhardi Euleri opera omnia*, series 2, vols. i, ii, Leipzig & Berlin, 1912.
[6] Ibid., series 2, vol. v, Lausanne, 1957.
[7] Ibid., series 2, vols, iii, iv, Bern, 1948.

ing and resolving motions are provided. The treatment is a paradigm of compactness and economy. The next two chapters then go on to discuss the foundations of dynamics, chapter II concerning itself with the 'internal' principles of motion and chapter III with its 'external' principles.

Chapter II begins with the distinction between internal and external principles (§ 75). All motion and rest, we are told, must have a *ratio*, and this may be either external or internal. The problem is to decide what action is due to internal factors and what to external ones. A condition for solving the problem, Euler argues, is that we imagine an isolated body, since here the separation of internal and external factors is clearest. (It will subsequently turn out that no external forces can be operative on an isolated body.) The justification offered for this abstraction (§ 77) is that even those (*viz*, Leibnizians) who consider that the totality of bodies interacts so closely that the removal of one would destroy this interaction must be able to say what effects the interaction has on the body and what are due to the body itself. The workings of the 'internal' principles are then investigated in terms of the conditions under which a body would have sufficient reason to deviate from its rest or motion. Inertia is defined (§ 95) in terms of the perseverence of a body in its state of rest or uniform rectilinear motion. Euler notes that inertia is sometimes defined in terms of a force, *vis inertiae*, because the *vis* is what opposes changes of state, but he rejects this characterization as tending to lead to misunderstanding since *vis* is usually restricted to what causes changes of state. To avoid confusion, therefore, he dispenses with the term *vis* in the case of inertia.

Euler considers that when we detect no forces acting on a body then the absolute state of the body can be gauged (§ 96). He then proceeds to show that if a body is in absolute rest or motion the axioms for relative rest and motion also apply. Conversely (§ 101), because of inertia, bodies will persist not only in the same absolute state but also in the same relative state providing the body by which the motion is measured is absolutely at rest or has uniform velocity (i.e., providing the reference frame is inertial). The techniques required for a full analytic characterization of inertial motion are then provided (§ 103 *et seq*) and an inertial state is characterized in terms of the second order differential of distance with respect to time, i.e., $dds/(dt^2) = 0$ in Euler's notation.[8]

Chapter III introduces 'external' principles in the form of force, which is defined as whatever changes the state of a body (§ 117). Correlatively, in the absence of force a body will persist in its state (§ 119),[9] where a state is thereby defined such that a body will persist in its state

[8] I shall use modern notation and shall modernize symbols from here on.

[9] That is to say, in the absence of external forces, $d^2s/(dt^2) = 0$. Euler did not mention the converse of this principle—i.e., that in the presence of external forces $d^2s/(dt^2) \neq 0$, but his argument strongly suggested that it is implied. The converse in fact holds so long as we specifiy *net* forces. This is to get round counterexamples where there are forces acting but where the net force is zero, for example the case of a falling body being acted upon by gravity and by air resistance which, as a result, undergoes a uniform rectilinear motion.

(because of its inertia) unless acted upon by external forces (§ 120). A body cannot change its own state but it can change that of others. It does this by striving to persevere in its state during impact, and it is this perseverance which furnishes the force required to change the state of other bodies (§ 121). The reason why two bodies cannot persevere in their states during impact is because they are impenetrable. Moreover, impenetrability must involve inertia. Since bodies are the only things that are impenetrable, and since all bodies necessarily have inertia, whatever is impenetrable must have inertia. (Note that the converse of this principle does not hold and is explicitly denied below.) We can therefore consider impenetrability as being the origin of all forces (§ 122). Impenetrability itself rests upon the principle that two bodies cannot be in the same place at the same time (§ 123). Since it is a necessary property of bodies no force, however great, can even tend to compact two bodies into one place (§ 125). Moreover, since impenetrability depends on a body having a place, place must be distinct from body (§ 128): this argument is clearly directed against Descartes.

Since impenetrability is the essence of body and since bodies are both extended and inertial, then whatever is impenetrable must be extended and inertial. But the converse does not hold. There are phenomena which are extended and inertial, such as 'shadows and images represented by means of optical machines' which we do not consider to be bodies for the simple reason that they are not impenetrable (§ 129). Impenetrability is, moreover, internal to bodies: it is true that we only know bodies to be impenetrable through their impact with other bodies but impenetrability is a feature of bodies whether they are in impact or not (§ 130). If two bodies come together such that neither can maintain its state without penetration then they act upon one another and mutually exert forces by which one changes the state of the other and vice versa, so as to avoid penetration. Since there is a change of state there must be forces acting and these can only be due to impenetrability (§ 131). This analysis provides us with 'a clear and distinct notion of the action of bodies, something which, in most authors, is usually excessively obscure' (§ 132). The forces by which the state of a body is changed originate, then, in impenetrability, and their effect is the prevention of penetration (§ 133). Impenetrability itself is not quantifiable ('*quae nullius quantitatis est capax*') so the magnitudes of the forces cannot depend upon impenetrability: rather, they depend upon the changes of state required to prevent penetration (§ 134). These forces are exerted only to the extent that penetration is avoided, and impenetrability always provides sufficient force for this (§ 135). It is consequently wrong to ascribe to bodies a striving to change their state since 'these forces aim not directly to change their state but to avert penetration, and unless this were imminent then no forces of this kind would exist in the world' (§ 136). The forces that are due to impenetrability are, moreover, the only forces that can change the states of bodies. We do not need to invoke spiritual action changing the states of bodies and there is no benefit to be gained from so doing since we would not be able to determine how these act. And even if they were present, bodies could only act mutually upon one another in the way already described. Any force which acted otherwise would have to act at a distance. Such action would have to be completely independent of impenetrability: it is therefore impossible to say by what mechanism it could act and how it could affect the states of bodies (§ 137). We must conclude, therefore, that the only mechanically relevant forces are those contact forces due to impenetrability.

In order to determine exactly how forces change the state of a body we must take infinitesimal bodies and infinitesimal periods, and then integrate to find the change of motion in a finite period (§ 138). The effect of a force is the distance through which it moves a body over and above that due to the body's inertia (§ 140) and the force is to be estimated from this distance (§ 142). (Euler subsequently makes it clear that he in fact means distance traversed per unit time rather than distance *per se.*) We draw the procedures for measuring forces from statics, although statics only concerns the measurement of forces acting upon stationary bodies whereas in mechanics we must deal with moving bodies (§ 143). The force acting on a moving body can be assimilated to that acting on a stationary one, inasmuch as the magnitude of a force acting on a moving body is equal to the magnitude of that force which would have the same effect in the same time on that body at rest. The distinction between absolute forces (i.e. those forces such as gravitation which act in such a way that their dynamic effects are independent of whether the body affected is at rest or in motion) and relative forces (i.e. those forces the effect of which depends upon the velocity of the body, such as the hydrodynamic force of a liquid current on an object) is not relevant here because the calculation must always include that force which impels a moving body as if it were at rest (§ 144). Therefore it we calculate the effects of forces upon bodies at rest we can calculate the effects on bodies in motion (§ 145). The effects of forces—distances covered in *dt*—are directly proportional to the forces themselves (§ 148).

The inertia of a body is proportional to the reluctance with which that body, when at rest, resists motion. To determine inertia we must consider unequal particles of matter (§ 150). If equal forces act upon unequal particles of matter then the effects produced in the same infinitesimal time will be reciprocally proportional to the quantities of inertia of the particles of matter (§ 151). The mass of a body is its quantity of inertia (§ 153). Hence a body's mass is not to be estimated from its volume but from the force required to move that body in a particular way (§§ 154–5). Distance moved is directly proportional to force and inversely proportional to mass (§ 158). If a body of mass M moves at a constant velocity then $d^2s/(dt^2)$ will equal 0.

If the body is accelerated by a force F in the direction of its motion, however, then $d^2s/(dt^2)$ will be as F and reciprocally as M. Therefore, $d^2s/(dt^2)$ will be as F/M, or $F=Ma$, assuming units to be fixed (§ 162). On integration this gives us the extra distance traversed and hence a measure of the force, assuming this force to be constant (§ 167). Moreover, by resolving the motion of a body along the three orthogonal axes x, y, and z, we can complete the treatment of infinitesimal bodies by specifying their motions (§ 176) in terms of the forces acting in the three planes, so that we obtain three general equations:

$$\text{I} \quad \frac{d^2x}{dt^2} = \frac{f_1}{M} \qquad \text{II} \quad \frac{d^2y}{dt^2} = \frac{f_2}{M} \qquad \text{III} \quad \frac{d^2z}{dt^2} = \frac{f_3}{M}$$

Chapter IV of the 'Introduction' is devoted to establishing units for these equations (I have already assumed the units as given for the sake of simplicity) and Euler uses the free fall of bodies as a value to establish units. Chapters V and VI then use the equations and the system of units established to examine the motion of a point mass. This provides the basic tools for the treatment of rigid bodies, with which the treatise is primarily concerned.

II

Euler's aim in the sections I have summarized was to reformulate Newtonian dynamics in such a way that its apodeictic character was established. One of the main aims of the reformulation was the clarification of the idea of bodies acting upon one another and, in particular, to clarify the notions of force and mass invoked to explain these actions. Three levels or stages can be distinguished in Euler's argument. First, there is what can be called the metaphysical level, where the conceptual foundations of mechanics are formulated and the notion of impenetrability introduced. Secondly, there is the qualitative level, where the question of the source of forces is examined, the main conclusion here being that forces derive from impenetrability and inertia. Thirdly, there is the quantitative level, where the actions of forces are compared.

Euler's central metaphysical idea was that bodies are essentially impenetrable and this idea served as the foundation for his mechanics in that he considered that all forces could be accounted for in terms of impenetrability. His treatment of these ideas was quite novel, although neither in itself was without precedent. Newton, in his early *De gravitatione et aequipondio fluidorum* (a paper which Euler would certainly not have known), had considered impenetrability to be a defining characteristic of matter. He had construed body as an impenetrable region of space and had explicitly connected impenetrability with the fact that bodies reflect on impact. No details of this connexion were specified but it was just such a connexion that Euler wanted to develop.

Descartes too had argued that 'impenetrability belongs to the essence of extension, but not to that of anything else' (To More, 15 April 1649) but he neither made, nor so far as I can tell intended, any connexion with force, and (contrary to Euler's view; cf § 128) he identified place and body. Nevertheless, Euler's project was, I would argue, Cartesian, in the sense that there was a clear attempt to derive basic concepts of mechanics from the essence of body. For Descartes, of course, this essence was extension (impenetrability only being the essence in a derived sense; cf. Descartes to More, 5 February 1649), whereas for Euler the essence was impenetrability. It is true that Euler also considered that bodies must be extended and inertial, but he was prepared to say (§ 129) that shadows and optical images are extended and inertial also, yet we do not count these as bodies because they are penetrable. Impenetrability then, and not extension or inertia, was what uniquely characterized bodies, yet it presupposed extension and inertia.

As regards extension, the argument was that whatever is impenetrable must be extended, but whatever is extended need not be impenetrable. This claim was different from that of Descartes, and for a very good reason. Descartes identified body and spatial extension, and therefore whatever was extended would turn out to be impenetrable. Once the idea of a plenum was rejected, however, there was no sense in which whatever was extended had to be impenetrable: and this was what Euler wanted to argue. It must be remembered in this context that although Descartes made extension the essence of body, and impenetrability its derived essence, he could, in principle, have made impenetrability the essence and extension the derived essence since, metaphysically speaking, the one involved the other for him. His reasons for not doing so were epistemological. This is not the place to go into these reasons,[10] but it is perhaps worth noting that for Descartes our cognitive understanding of the natural world is, to the extent that we can call it understanding at all, a physico-mathematical understanding, and what is ultimately understood is something which is paradigmatically quantitative: extended magnitude. If one is concerned to provide epistemological foundations for a quantitative mechanics then this procedure has obvious advantages. Euler's approach was completely different since at least one of his basic foundational concepts, impenetrability, was explicitly not quantifiable.

Before considering whether there were any disadvantages in this procedure it is perhaps worth asking why impenetrability, rather than any of the quantitative notions, was adopted. The answer is, I think, that none of the quantitative notions could serve Euler's purpose. Extension, as we have seen, would not do. Force would not do either since the purpose of the

[10] Cf. my 'Descartes' project for a mathematical physics', in S. Gaukroger (ed.), op. cit. (2).

exercise was precisely to clarify this notion; similarly with the idea of mass or 'quantity of matter'. The ideas of space, time, and motion were clarified in the first chapter of the *Theoria*, which was devoted to kinematics. But these notions could not be used to provide the foundation for dynamics; Euler was fully prepared to accept the reality of force and there could be no question of his providing, as d'Alembert did, a purely kinematic account of force in terms of acceleration. Except for inertia, which we shall look at below, this exhausted the viable quantitative notions available.

Why impenetrability then? One answer would be that Euler thought that impenetrability worked as a foundation for mechanics in the sense that it enabled him to get the required results. But unless we suppose that Euler hit on the idea purely by chance then this answer is unhelpful. The important thing about impenetrability is that it could plausibly be construed as the essence of body, as Euler understood the term 'body'. The idea that impenetrability constituted the essence of body depended upon two claims that Euler made explicitly: that it is unique to body, and that we cannot conceive of a body without it; and one which was only implicit: that impenetrability is irreducible in the sense of being a primitive notion. The last condition was clearly important since if impenetrability had been reducible he would not have been able to claim that it was the essence of body: that to which it was reducible would have been a more likely candidate for this status. Now what one chooses to treat as a primitive notion will, in general, depend upon whether one considers this notion to be in need of elucidation. Both Boscovich (*Theoria philosophiae naturalis*, 1758) and Kant (*Metaphysische Anfangsgründe der Naturwissenschaft*, 1786) thought impenetrability an obscure idea requiring elucidation in terms of a more basic notion, that of repulsive force. For them, repulsive force was the physically primitive notion not requiring further elucidation, just as attractive force was physically primitive. Euler saw things the other way round: force was what required elucidation, *it* was the obscure notion.

There are, I suggest, two reasons for Euler's decision not to take force as primitive. First, we must remember that Boscovich, Kant, and Euler were all working within the general framework of Newtonian dynamics. There was a tension in this dynamics between inert bodies occupying an equally inert space on the one hand, and the idea of action at a distance on the other. Boscovich and Kant modified the former conception whereas Euler modified the latter. As a result, the kinds of forces accepted by Newton, Boscovich, and Kant would not necessarily all turn out to be those that Euler wished to invoke, and obviously the central bone of contention here would be the existence of attractive gravitational forces acting at a distance. Strategically, therefore, Euler was in a strong position if he could base his account of force on something accepted by almost everyone: Boscovich and Kant were very exceptional, even in their own period, in treating absolute impenetrability as being an obscure notion.[11] Secondly, Euler had a strong Cartesian desire to base mechanics upon something which was intuitively self-evident. We do have an intuitive conception of force, of course, and this conception may appear self-evident to us, but it may also be mistaken; in particular, it may well conflict with the Newtonian law of inertia that Euler wished to adopt. Our intuitive grasp of force, therefore, cannot be trusted. Impenetrability, on the other hand, is an idea such that our intuitions can be made full use of.

There were, then, good reasons why Euler should have wanted to take impenetrability as being primitive. The relation of inpenetrability to extension and inertia, which Euler also took as being primitive, was not, however, immediately apparent. He made no attempt to deduce these from impenetrability so it is reasonably clear that he did not consider them to be derived essences: indeed, it is very difficult to see how something like inertia could possibly be derived from impenetrability. What is puzzling is his claim that 'without extension impenetrability is inconceivable since in that case bodies would not be moveable, and if mobility is assumed then inertia is assumed' (§ 129). I suggest that we take this claim at the most minimal level, i.e., not as an inference of any kind but simply as an indication of the fact that, since motion is necessary for impact, we must be clear about the motion of bodies, and in particular about their impenetrability, their extension and their inertia.[12] In other words the statement was simply that: a statement, and not a demonstration. It is, I admit, very tempting to take the claim to be much stronger than this since what Euler seems to have been doing was deducing extension and inertia from the fact that bodies move. But there was simply no way in which inertia could be deduced purely from the fact of a body's motion. Other assumptions and arguments were necessary and Euler provided these elsewhere, as we shall see below. As far as extension is concerned, if Euler did intend to derive extension from motion it would be totally unclear why the fact that bodies move should be treated as being especially significant, since there is surely nothing that we can deduce about extension from a body's motion that we cannot deduce from that body at rest. The only extra insight that we can gain from considering bodies in motion lies in the fact that for motion to occur then, by definition, an extended region must be traversed: but this tells us nothing about whether the body itself is extended.

This leaves us with the problem of showing why bodies must necessarily be extended and inertial, as Euler claimed. He provided no explicit arguments in the case of extension but we can gain some idea of the

[11] For sketches of the major schools of thought on this issue in the period, see M. Jammer, *Concepts of force*, Cambridge, Mass., 1957, chapters VII–XI.

[12] What Euler is referring to when he claims that 'if mobility be assumed, then inertia is assumed' is what he earlier defined as that property by which a body persists in a state of rest or uniform rectilinear motion unless acted upon by an external force. He is not referring to inertial mass, a concept which has not yet been introduced.

kind of defence he probably had in mind from looking at how he dealt with impenetrability. Impenetrability, as we have seen, was defended on the grounds that we cannot conceive of an impenetrable body: it is both necessary and self-evident that bodies be impenetrable. The paradigm for arguments of this kind was of course Descartes' argument that bodies are necessarily extended. Euler accepted the conclusion here and there is every reason to think he accepted the argument. If we construe Euler's reasoning in this way then the conclusion is that impenetrability and extension are necessary to body because we cannot conceive of a body being either penetrable or unextended; it is essential to what we mean by 'body' that bodies be impenetrable and extended. It might be objected that this was just an exercise in definition, and that a mere definition was not going to convince anyone, particularly those such as Kant who conceived of body in a very different way. But there was more at stake than mere definition. Euler's argument can be seen as specifying the necessary and sufficient conditions for something to be a body in the normal, generally accepted, sense of the term, and then proceeding to show that, given this unobjectionable, intuitive and self-evident notion of body, we can build up a sophisticated quantitative mechanics without invoking any of the peculiar agencies that Newton had introduced.

Inertia was a problem in this respect, although Euler clearly thought it had the same primitive status as extension and impenetrability. The only justification for the principle of inertia that we are given in the *Theoria* (§ 85), and indeed the only justification that we ever find in Euler,[13] is in terms of the principle of sufficient reason: a body will not change its state without sufficient reason, where the sufficient reason is specified in terms of external forces. The idea that the law of inertia could be justified in these terms was quite common in the eighteenth century, but the proposed justification was clearly question begging. Aristotle, for example, had considered that every motion must have an external cause, so that in the absence of this cause no body will maintain its motion. This view of inertia could just as easily be based on the principle of sufficient reason, but the law of inertia that would result would clearly be different from Euler's. Everything depends on how, and under what conditions, we assign forces. Only given a particular characterization of forces does the law of inertia follow from the principle of sufficient reason. Because of the nature of the relation between a law of inertia and one's characterization of force, any attempt to justify the one in terms of the other must be circular.

Despite the fact that the law of inertia was universally accepted (by those who knew anything about mechanics) by Euler's time, inertia did not and could not have the same status as impenetrability and extension. We may not be able to conceive of a body being penetrable or unextended, but we can surely conceive of it not obeying Newton's law of inertia; indeed,

[13] Cf., for example, § 56 of the *Mechanica*, and § 3 of the 'Recherches sur l'origine des forces'.

people had been doing so for thousands of years prior to the seventeenth century. Having said this, however, what must not be missed is the fact that inertia was introduced in such a way as to make it look as innocuous as extension and impenetrability, by formulating it in terms of an apparently unobjectionable metaphysical principle, that of sufficient reason.

It remains for us to ask why inertia should have been introduced at this early stage; more precisely, what was its foundational status? Dynamics, whether it be Aristotelian, Cartesian, or Newtonian, deals with states, which are characterizable kinematically, and changes in these states, which are characterizable dynamically. The equation characterizing an inertial state in Euler's mechanics is a second order differential of distance with respect to time: $d^2s/(dt^2)=0$, taken vectorially. On such a characterization rest and uniform rectilinear motion are clearly equivalent. When $d^2s/(dt^2)$ has any value other than zero a change of state must be involved, and this requires the action of an external force. Euler's dynamics was primarily concerned with such changes in state, to be explained in terms of force and change of motion. The foundational concepts of extension and impenetrability were not sufficient, in themselves, to allow an adequate characterization of the notions of force and change of motion. For this we also need inertia, which links kinematics and dynamics by linking the notions of motion and force. Indeed when it comes to the quantitative discussion, the value of the measure of inertia, inertial mass, is what directly links the value of the force to the value of the change of motion produced by that force.

Clearly, then, no foundation for Eulerian dynamics could have been adequate without inertia. Insofar as his proposed justification of inertia, at the foundational level, was circular, he failed to achieve his aim, *viz.* to render the system of Newtonian dynamics, suitably reformulated, apodeictic. Its apodeictic appearance arises from the fact that two of its foundational concepts were based upon apparently unobjectionable metaphysical principles—the principle that two bodies cannot be in the same place at the same time, and the principle of sufficient reason—and the third on something that had not only never been questioned but appeared unquestionable—that all bodies are extended. The fact remains, however, that although these foundations may appear apodeictic, they involved at least one crucial assumption—the assumption of inertia—which could not be justified at this level.

III

Inasmuch as it is not quantifiable, impenetrability appears an unlikely *prima facie* contender for the role of a foundational concept. In this respect, Euler's choice was quite unprecedented in the history of quantitative mechanics. We shall be concerned in this section with the question of how something which is not quantifiable could, together with the law of inertia, provide the explanatory basis for Euler's account of force. Although forces are measurable we shall not be concerned with their actual measurement in this section, nor with the measurement of changes of state. This follows Euler's own procedure, and this is why I have called this stage of the argument 'qualitative'.

Elaborating somewhat on the account Euler provides in § 131, the way in which impenetrability is invoked to account for forces is as follows. Imagine two very small, perfectly solid (and presumably spherical[14]) bodies, both of which are initially in inertial states, colliding at a sufficiently large distance from any other bodies for these other bodies not to have any effect upon them. We know that bodies change state in impact and Euler takes the generally accepted view (*contra* Leibniz and his followers) that such changes of state must be instantaneous and hence discontinuous. Finally, we can imagine the situation as being one in which there are no forces acting on the bodies before or after impact, so that the motion of the bodies is inertial both immediately before and immediately after impact. Now since we also know, from the law of inertia, that any change of state must be due to forces acting on the bodies then, since there is a change of state, there must be such forces acting. The question therefore arises as to the source of these forces. Euler approaches this question by considering what would happen if there were no forces acting. In such a situation, the bodies would continue in their inertial motion, but to do so they would have to penetrate one another. Mutual penetration is impossible, however, and it is this very impossibility that results in forces being exercised.

In considering more precisely how bodies resist penetration it may be helpful to look at a formulation of impact that Euler would not accept. Let us say that A, in order to avoid its own penetration, acts to change its own state; similarly, B acts to avoid its own penetration by changing its own state. If we can determine what is wrong with this we can shed more light on why Euler accepts the formulation he does. Consider A. A obeys the law of inertia and it is impenetrable. In impact, it cannot both remain impenetrable and remain in its inertial state. Therefore it changes its own inertial state and for this a force is required. But where does this force derive from? Bodies surely *resist* changes in their states, they do not produce them. The characterization would therefore be completely at odds with New-

[14] One gets the strong impression that Euler's 'very small' particles are spherical, and in fact it helps to avoid misunderstanding to assume that they are. In particular, since the extent to which bodies are impenetrable is not a function of the areas of the parts of the surfaces that are in contact in impact, then bodies with flattened surfaces do not offer more impenetrability, or suffer more risk of penetration, than spheres, whose point of contact is infinitely small. The advantage of imagining spherical bodies lies in the fact that we are not tempted to think in terms of the impenetrability of a body as being determined by the extent of its impenetrable surface area: since the surface area of the point of contact does not matter we may just as well make it infinitely small, to avoid possible confusion.

tonian dynamics: and Euler's foundations are, after all, foundations for *Newtonian* mechanics.

Euler's own characterization of what happens can be formulated as follows: in order to avoid its own penetration, *A* acts to change *B*'s state; similarly, *B* acts to avoid its own penetration by changing *A*'s state. Here, *A* changes the state of the body that would penetrate it. Consequently, the change of state that *B* undergoes is not due to some force which it produces itself, but to a force which acts from outside, from *A*. This force is 'external', but not in the sense in which gravity, conceived as acting at a distance, is external. Euler accepts no forces of the latter kind: he accepts the phenomenon of universal gravitation and indeed has constant recourse to it, but he considers that it must ultimately be accounted for in terms of some contact force mechanism (§ 137).[15] Euler's 'external' forces are not external in the sense that they can act outside the boundaries of bodies, only in the sense that their source is external to the body on which they act. Now as § 137 makes clear, Euler considers that these are the *only* forces that mechanics has to deal with. There are no internal forces as such, *vis inertiae* only being called a *vis* in a misleading sense (§ 95), and spiritual forces, if there be such, are not the business of mechanics and cannot affect conclusions arrived at in mechanics. As far as mechanics is concerned, then, all forces are contact forces deriving from impenetrability and inertia.

There are two features of this account that require closer consideration. The first concerns the claim that the forces acting in impact are external, and the second concerns the legitimacy of using the conception of force derived from the analysis of impact as a model for the action of all forces.

The first issue centres around the question of whether Euler had really been able to dispense with recourse to internal forces in his account of impact. He established that *B*'s state is changed in impact because *A* exerts a force external to *B* which acts to change *B*'s state, and vice versa. The forces were thereby shown to be external in the required sense and this, he clearly considered, was all that was needed for his purposes. But one might entertain doubts about the completeness of this explanation. In particular, the force that changes *B*'s state may be external to *B*, but it might be argued that it is internal to *A* on the grounds that if it were external to both *A* and *B* it would have to act at a distance; moreover, if the source of the force is *A* then surely there must be some sense in which the force is internal to *A*. For Euler's account to have been complete and coherent then this type of objection had to be shown to rest upon a misunderstanding, and the clarification of the misunderstanding had to show that the dichotomy between internal forces and forces acting at a distance was a false one. If

[15] Cf. also letters 52 to 57 of the *Lettres à une Princesse d'Allemagne*, *opera omnia*, series 3, vol. xi, Zurich, 1969.

this could be shown then Euler's conception of force could be clarified considerably. And in fact it stood in need of clarification on more than the question of internal forces, since Euler provided apparently conflicting accounts of the nature of the source of the forces involved in impact. Until this problem is solved the question of whether forces are internal cannot be answered.

In § 121 of the *Theoria*, the internal principles of a body are said to be the source of the forces acting in impact: 'it is the very faculty of individual bodies each to persist in its own state that furnishes the forces by which the state of other bodies is changed'. On the other hand, in § 131 we are told just as clearly that it is impenetrability that is responsible: 'as soon as bodies are unable to persist in their state without penetrating one another, impenetrability supplies forces by which their state is changed so that penetration is avoided'. These statements must somehow be reconciled with one another, and the only clue to this reconciliation is provided in § 122:

> The cause of those forces by which the state of a body is changed may be agreed to lie not in inertia alone but in inertia coupled with impenetrability. Indeed, seeing that only bodies can be said to be impenetrable, and since bodies are necessarily endowed with inertia, impenetrability as such involves inertia, so that impenetrability alone is rightly considered the source of all forces by which the state of bodies is changed. It will therefore be proper to consider this property more exactly as being the origin of all forces.

This suggests that when Euler subsequently talked about impenetrability being responsible for force—as in § 131—what he meant was impenetrability *and* inertia. I shall take it that this is the case, and that all subsequent propositions had to be compatible with § 121. Other interpretations are possible and this is a question that we shall look at below. For the moment, however, let us see where the present interpretation takes us.

The first problem is to determine the kinds of contribution made by inertia and impenetrability to the forces arising in impact. Impenetrability is absolute and a body's inertia clearly cannot affect its impenetrability in any way. But the fact of a body's impenetrability does have an effect on its inertial behaviour since if, as § 121 indicates, inertia furnishes the forces to change other bodies' states, then impenetrability would have to be presupposed here since these forces could not arise unless the body were impenetrable. Shadows are inertial, for example, but because they are not impenetrable they furnish no forces to change the states of other shadows, or of bodies for that matter. Consequently, inertia with impenetrability is very different in its effects from inertia without impenetrability. On the interpretation that I am proposing, impenetrability is a condition of inertia having any dynamic effect, but it is inertia, and not impenetrability, that actually has the dynamic effect.

In asking what this dynamic effect is, we must remember that bodies are maintained in their states by their internal principles. Although Euler did not draw the conclusion explicitly, this implies that the state maintained is proportional to the internal principle maintaining that state. That internal principles are quantifiable is indicated by the way in which they were compared with external principles in § 76:

> For whether a body be at rest or in motion, whether it remain at rest or acquire motion and continue in it in any way, it is necessary that these phenomena originate from particular causes. Certainly no matter what occurs in a body in respect of rest or motion, this must in no wise be set down as happening by chance and without a reasonable cause. Moreover, whatever this cause may be, it is necessary that it be sought either in the body itself which is being investigated, or outside it. Hence two classes of principles, by which the motion of bodies may be defined, must be set up, the former of which I shall call *internal* and the latter *external*. I naturally classify among the *internal* principles whatever is present in the bodies themselves, containing the reasonable cause of their motion or rest.

Now since bodies resist changes to their states, and since their states are due to their internal principles, they must resist changes to their internal principles, and this resistance must take the form of a force. Bearing this in mind, and bearing in mind that we do not have to take account of mass since we are, *ex hypothesi*, dealing with bodies of the same mass, we can construe impact as follows. When *B* comes into contact with *A* in impact, we can say that it experiences *A*'s internal principles as a force, a force which we would normally term *A*'s force of resistance to change of state. Note that the force is not in any sense *in A*: what is in *A* is its internal principle, which is not a force because it only maintains *A*'s state. But this internal principle is experienced by *B* as a force. There is, therefore, an external force acting on *B* and this force is not internal to *A*. Nor does it act at a distance because it is a prior condition of there actually being a force that *A* and *B* be impenetrable and that they be in contact. Impenetrability and contact are therefore necessary conditions for this force, but they cannot be sufficient conditions since there would be no force, for example, acting on two stationary impenetrable bodies in contact. For the sufficient conditions to be realized there must be 'fear of penetration' and this only occurs when the bodies cannot continue in their present states, i.e., when one of the bodies is moving with respect to the other such that the two bodies come into contact. Impenetrability and contact as such cannot, therefore, give rise to any forces, nor can inertia as such. All three are required if there is to be a force: this, incidentally, provides an interesting vindication of the inclusion of extension in the foundational concepts, for if bodies were not extended they could never be said to be in contact.

This interpretation of impact obviates the need for any internal forces and shows how impenetrability underpins the changes of state that result from impact. The interpretation depends, however, on the idea that contact and impenetrability provide the conditions under which inertia is dynamically effective, and this needs defending since, despite what he says in § 122, Euler occasionally wrote in a way which suggests that it is impenetrability alone that is dynamically effective, or even perhaps that impenetrability and inertia may both be dynamically effective. § 131 suggests this, as does § 133: 'the forces by which the state of bodies is changed [in impact] originate from their impenetrability and they produce so great an effect that penetration is prevented; and these forces are always so great that they suffice for this'. § 134 elaborates on the point: 'the magnitude of these forces is not determined by impenetrability, which is not quantifiable, but by the change of state that it is obliged to bring about so that the bodies do not mutually penetrate one another'. If the only forces that acted were due to impenetrability, however, and if these acted solely to avoid penetration, then there would be no reason why the bodies should not simply stop, since this would be quite sufficient to avoid penetration, as well as being very economical. Inertia must therefore be dynamically effective, and if it is, then recourse to extra forces deriving from impenetrability is otiose at best. Indeed, it is wholly obscure how impenetrability, which is unquantifiable, could give rise to a quantifiable force: moreover, we would have to imagine each body having a potentially infinite reservoir of repulsive force so that it would have sufficient force to repulse any body in impact. So far as I can see, any coherent conception of a dynamically effective impenetrability would have to be along the lines of Boscovich's and Kant's accounts, where impenetrability is a function of repulsive force, and not vice versa, and this is a conception that Euler could not have accepted.

Assuming, then, that the interpretation that I have suggested is correct, how does it compare in clarity with those other accounts that Euler considered to be 'excessively obscure' (§ 132) and 'shrouded in profound obscurity' (§ 137)? Its strength, for Euler, lay in the fact that it invokes neither internal forces nor action at a distance. The former was only really an issue, at least as far as impact is concerned, for the Leibnizians. Although Newton's terms *vis inertiae* and *vis insita* suggest that forces were being designated,[16] the context of the *Principia* made it clear that they were not internal *forces* and that they could easily be accounted for along the lines of the interpretation suggested above, i.e., as internal principles experienced by other bodies as forces. But the account would have had to be slightly different from Euler's because impenetrability did not play the role in Newton's work that it played in Euler's. It is clear from the *Principia* that Newton considered that bodies must be impenetrable, but he did not

[16] The terms did originally designate forces for Newton. They do so in the *De motu* (1684) and in the revisions to this treatise; cf. in particular definition 12 of the third version (the manuscript *De motu sphaericorum in fluidorum*). But by the immediately subsequent drafts of the definitions and the laws of motion, in the *De motu corporum in medijs regulariter* (Law 1), and the second draft of the definitions, in the *De motu corporum* (definition 3), *vis insita* could no longer be regarded as a force maintaining a body in its motion, and this is true of all Newton's subsequent works. For the texts see J. W. Herivel, *The background to Newton's* Principia, Oxford, 1965.

use this idea to account for force. On Euler's view, the full consequences of impenetrability had to be drawn out, and one rather important consequence was that forces could only arise because of 'fear of penetration', which occurred only in impact. Hence there could be no forces acting at a distance (§ 137). If Euler's conclusion was correct, then his account of force would be considerably clearer than Newton's since he would have been able to explain the action of all forces on the one clear model, which is far more than Newton was able to do.

This brings us to the second question. Let us agree that Euler provided clarification of the notion of force in his analysis of impact. The problem remains whether the analysis of impact in itself can provide us with a sufficiently exhaustive conception of force to cover all those phenomena in which we detect the action of forces. The external forces invoked by Euler in accounting for impact have two distinctive features: they are contact forces, and they are repulsive (in the passive sense of 'resistive' rather than in an active sense). Now there are at least *prima facie* classes of forces which are not of this type. There are apparent contact forces which are attractive, such as the force of cohesion; apparent non-contact forces which are repulsive, such as the magnetic force existing between like poles; apparent non-contact forces which are attractive, such as gravitation and the magnetic force existing between unlike poles; and finally there are phenomena such as electricity where it is not immediately clear what kinds of force are acting. These were all phenomena of which Euler was aware, as were his contemporaries. Magnetic forces were generally taken to act mechanistically at this time, and several mechanistic models of gravitation were proposed, the most important being that put forward by LeSage in 1747, a model which provided the basis for much speculation and some research up to the end of the century. Euler's attitude to these forces was often instrumentalist—in the sense that he was quite prepared to invoke forces for which he could not discover the causal mechanism—although he made it clear that he thought that, ultimately, no sense could be made of action at a distance. In letter 53 (5 September, 1760) of the *Lettres à une Princesse d'Allemagne*, for example, he explained that magnetism 'presents a somewhat similar phenomenon' to gravity and that 'it is now certain [*sic*] that it is produced by an extremely subtle fluid'. It was left mainly to Aepinus to develop the theory of electricity and magnetism, but by 1759, with the publication of his *Tentamen theoriae electricitatis et magnetismi*, Aepinus had forcefully rejected the idea that electricity and magnetism could be derived from a single aether, as postulated by Euler, and Euler charged Aepinus with introducing arbitrary forces acting at a distance.[17] The postulation of aetherial mechanisms turns out, then, to have been no

[17] See R. W. Home and P. J. Connor, *Aepinus' essay on the theory of electricity and magnetism*, Princeton, 1979, for the relevant section of the *Tentamen* (p. 243), a general discussion of the issues (pp. 68 f), and a statement of Euler's reaction (p. 15).

more than a promissory note on Euler's part, and clearly those who did not share his optimism on this score were unlikely to be convinced. It was, in the final analysis, simply not enough to provide a clear account of repulsive contact forces, maintaining that these are the only kinds of force, and then to speculate on how such phenomena as gravity and magnetism might be accommodated to this view; particularly when Euler had to make constant recourse to forces such as gravitation. The clarity of Euler's discussion of the forces involved in impact must surely be seen in a different light if it rendered the phenomena of gravitation, magnetism, electricity, cohesion, etc, more difficult to comprehend. In this respect, Euler's foundational treatment of force can justly be accused of leaving more unexplained than it explained.

IV

The final stage of Euler's foundational argument (§ 136 *et seq.* of Chapter III of the *Theoria*) consisted in the elaboration of the notions of force and inertia in quantitative terms. It was only here that the quantitative foundations of dynamics were laid, and the transition from the qualitative conceptions to the quantitative ones was a *tour de force.* Newtonian dynamics was provided with the basic algebraic form that we are familiar with today. Even more importantly, mass, which up to this point had been formulated rather vaguely and intuitively in terms of quantity of matter, density, volume, and in a host of other ways, came to be defined operationally as a numerical coefficient dependent upon the ratio between the force required to change a body's state in a particular way and the acceleration of the body produced by that force.

The concepts of measurement that were invoked were introduced from two areas. The measurement of motions, involving infinitesimal calculus and the representation of accelerations (treated vectorially from the beginning) in terms of second order differential equations, had already been treated in the first part of the *Theoria.* The measurement of forces, on the other hand, was derived from statics, and the procedure employed deserves mention. The use of statical principles in mechanics had become reasonably common by Euler's time. It is often maintained that it was Archimedian statics that provided the impetus for the development of mathematical physics in the seventeenth century but, while there is some truth in this claim, it must be remembered that statics often acted as an obstacle to the conceptualization of dynamical problems, at least in the early part of that century. This was particularly true of Galileo's work, for example, where the attempt to pose dynamical problems geometrically took the form of an illegitimate extension of statics, which he could handle geometrically, into dynamics, which he hoped to be able to handle geometrically as a result, so that statics and dynamics were simply conflated, with disastrous consequences.[18] By Euler's time this problem had been overcome: Newton had elaborated a set of genuinely dynamical concepts in the *Principia* and the tendency to derive from statics dynamical notions of force, in particular, declined rapidly thereafter.

In the present context, Euler relied on the statical principle of equilibrium. D'Alembert had also used this principle in his *Traité de Dynamique* (1743) but he had attempted to found the laws of motion upon it and thereby to reduce force to acceleration. Euler's use of the principle was quite different: since he fully accepted the reality of forces, he used it simply as a source of procedures for the measurement of forces. The principle stated, in essence, that force f_1 corresponds to force f_2 as the numbers m to n if f_1, applied n times in a certain direction on a point, and f_2, applied m times on the same point in the opposite direction, leave the point in equilibrium. Euler transferred this equivalence into a dynamical context (§ 151), arguing that the distance that a force F will move two bodies in dt over and above the distance that they would have moved inertially in dt is directly proportional to the force and inversely proportional to the quantities of inertia, or masses, of the bodies. In consequence, the acceleration would be directly as the force and inversely as the mass, thus providing an operational definition relating mass, force, and acceleration: mass was measured by the force necessary to impart to a body a given acceleration. Acceleration and force had been defined in Euler's foundational kinematics and dynamics and both could now be measured. Because force and acceleration were now related to mass in the equation $F \propto Ma$, an operational definition and measurement of the third was possible, once units had been fixed. The definition of mass provided here was free from the operational deficiencies—most notably the apparent circularity—in Newton's definition of mass in terms of density. A very major clarification had therefore been achieved. Moreover, because the quantitative formulations had been given algebraically and in terms of infinitesimal calculus, Euler had at his disposal analytic techniques far superior to the geometry of Newton's *Principia*, and this allowed the treatment of an extended range of mechanical problems—the dynamics of points, finite rigid bodies, flexible bodies, elastic bodies, several bodies with mutual interactions, and finally fluids—on a secure and fruitful basis. Indeed, Euler's treatment of the dynamics of points, rigid bodies, and fluids went a large way towards providing the basis for our modern understanding of these phenomena.

Conclusion

Euler's foundational project, as I have tried to show, was not entirely successful. It could not genuinely claim to have provided the apodeictic foundations for mechanics since, at the metaphysical level of the argument,

[18] R. S. Westfall, op. cit. (3), chapter I.

the law of inertia was smuggled in as an apodeictic principle, whereas in fact it could not be justified at this level. Moreover, at the qualitative level, it is far from clear that the conception of repulsive contact forces that Euler offered could viably be considered the only conception of force that was needed in physics generally.

The latter problem should not be overestimated, however, since what is at issue is not so much whether Euler's view turns out to be unsatisfactory by modern criteria, but whether it gave rise to any fruitful lines of research, particularly into problematic forces such as electricity and magnetism. And in fact it did, although it was left primarily to Aepinus rather than to Euler himself to develop and revise the latter's approach to these matters. It is true that Aepinus deviated from Euler on a number of issues, some of them foundational, but it is also clear that his work relied upon Euler's fully analytic treatment of mechanical problems, upon his clear separation of internal and external principles and the consequences this had for the understanding of force and inertia, and upon his willingness to invoke apparently long range forces, the contact-force mechanisms for which he did not discover. Aepinus' work was fruitful—and fruitful in a way derived from the fact that he made use of an essentially Eulerian approach to the problem—because his treatment of electricity and magnetism was mathematical, and some attempt was made to incorporate this treatment within the body of rational mechanics. In this respect, he differed from the mainstream of those engaged in electrical and magnetic studies in the eighteenth century, because this mainstream study was experimental and non-mathematical, taking its cue in the main not from Newton's *Principia* but from his *Opticks*.[19] Aepinus went a long way towards unifying the experimental stream and the mathematical stream (which did not generally concern itself with electricity and magnetism), and the importance of his reliance on Euler's work in his attempt to effect this unification cannot be overestimated. Even though Euler had serious qualms about Aepinus's very liberal use of the idea of action at a distance, there can be no doubt that much of Aepinus's basic conceptual and mathematical equipment was due to Euler.

As far as the question of providing the apodeictic foundations for mechanics is concerned, the situation is a little more complicated. In the first place, Euler clearly wanted these foundations to be self-evident, and this is what I have called his Cartesianism. The self-evidence he sought seems to appeal on the one hand to our intuitions about bodies—we cannot conceive of bodies which are penetrable, unextended or non-inertial—and on the other hand to what Euler clearly regarded as inviolable metaphysical principles—the principle that two bodies cannot be in the same place at

[19] Cf. I. B. Cohen, *Newton and Franklin*, Philadelphia, 1956, and Home's introductory monograph in Home and Connor, op. cit. (18), as well as his 'Out of the Newtonian straightjacket: alternative approaches to eighteenth century physical science', in R. F. Brissenden and J. C. Eade (eds.), *Studies in the eighteenth century*, vol. iv, Canberra, 1979.

the same time, and the principle of sufficient reason. As far as the appeal to our intuitions is concerned, Euler was highly selective, as we have seen. He did not appeal to intuitions of force or mass, for example, for the good reason that intuitions of these turned out to be wrong. One might also argue that his conception of inertia conflicted with intuitions, in which case they were wrong again; hence the importance of bolstering the intuitions that he chose with metaphysical principles. But are these metaphysical principles absolutely self-evident; could they reasonably be denied? The principle of sufficient reason, in the context in which it was used, had to presuppose the universe to be deterministic, and this was something that could be disputed. Hence the principle itself, in this context, could be disputed. The principle that two bodies cannot be in the same place at the same time derived its self-evidence from the dominant conception of matter as being simply full space. If matter was conceived of in this way then the principle was self-evident, since if something is full it cannot be made fuller. But of course it is not self-evident that this is the only way that matter could be conceived of, and a different conception might allow violations of the principle. Nevertheless, Euler was not concerned with providing the foundations for any conceivable physics whatsoever, and considerations of indeterminacy, or of whether matter as it is currently conceived could be totally compacted at the centre of a black hole, are not relevant considerations. He was concerned with providing the foundations for Newtonian mechanics and there can be no doubt that the principle of sufficient reason—interpreted as the principle that no rest or motion of any kind could occur 'by chance and without a reasonable cause' (§ 76)—and the principle that two or more bodies cannot be in the same place at the same time were both principles which were essential to the truth of Newtonian mechanics. That is to say, unless these principles were true, Newtonian mechanics could not be true. Hence the truth of these principles could be derived from the truth (if it be such) of Newtonian mechanics. But, and this is the important point, the truth of Newtonian mechanics could not be derived from the truth of these principles alone, since the principles could hold true in a number of different physical theories; they both held true in Aristotelian physics, for example.

Now what Euler derived from these principles did not hold true in every theory and, in particular, the law of inertia did not hold in Aristotelian physics. Our conclusion that the law of inertia could not in fact be derived from the principle of sufficient reason should therefore come as no surprise. But if it could not be so derived, what precisely was its status? I have indicated that it appears to have had a status different from the foundational concepts of impenetrability and extension. Euler attempted to put the three on the same level, but there was a very important difference between them. His argument comes down to saying that extension, impenetrability, and inertia must be essential properties of bodies if

Newtonian physics is to be true. As far as the argument relates to impenetrability and extension, it was at least plausible. But it was not even plausible as far as inertia was concerned. The law of inertia, as formulated by Euler (following Newton), does not need to hold if Newtonian dynamics is to be true. This is not to deny that if some other law of inertia were adopted then certain revisions might have to be made to the system, but so long as all the results of Newtonian dynamics hold we may say that the alternative law is a genuine alternative within Newtonian dynamics. Ellis, for example, has provided a law of inertia which states that every body has a component of relative acceleration towards every other body directly proportional to the sum of their masses and inversely proportional to the square of the distance between them, unless it is acted upon by a force.[20] The adoption of this law would require us to conceive of stress and momentum differently from Euler, and it would require us to distinguish between inertial and gravitational mass in a way which is not the usual one. But a system based on this law would provide the same results as those provided by Euler's. There is, therefore, nothing necessary about Euler's law of inertia, or any law of inertia for that matter.

This is not to deny that there is some sense in which the law of inertia must nevertheless be foundational for Newtonian dynamics; moreover, it will matter what formulation of the law we adopt. But the foundational value of these formulations will depend upon what areas are rendered problematic or fruitful by the different versions. The idea that in providing the foundations of a physical system we must automatically be concerned with questions of self-evidence, indeed intuitive self-evidence, and necessity, is a Cartesian myth. It cannot be denied that Euler's attempt to realize this myth is remarkable, but it is a myth nevertheless.

[20] B. D. Ellis, 'Universal and differential forces', *British journal for the philosophy of science*, 1963, *14*, 177–94; 'The origin and nature of Newton's laws of motion', in R. G. Colodny (ed.), *Beyond the edge of certainty*, Englewood Cliffs, NJ, 1965; 'The existence of forces', *Studies in history and philosophy of science*, 1976, *7*, 171–85. Related reformulations are discussed in L. Sklar, 'Inertia, gravitation and metaphysics', *Philosophy of science*, 1976, *43*, 1–23. I have discussed some of the philosophical problems arising from the choice of different laws of inertia in my *Explanatory structures*, Brighton, 1978, pp. 22–9.