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Three-dimensional flow around two circular cylinders of different diameters in a close proximity

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Flow past two cylinders of different diameters in close proximity is simulated numerically for a constant diameter ratio of 0.45, a gap ratio of 0.0625, and a Reynolds number of 1000 (defined using the diameter of the main cylinder). The effect of the position angle α of the small cylinder relative to the large one on force coefficients and wake flow patterns are studied. Depending on the position angle α of the small cylinder, four wake flow modes are identified: the upstream interference mode for $\alpha = 0^\circ$, 22.5° , and 45° , the intermittent attached gap flow mode for $\alpha = 67.5^\circ$ and 90° , the attached gap flow mode for $\alpha = 112.5^\circ$ and 135° , and the wake interference mode for $\alpha = 157.5^\circ$ and 180° . The RMS lift coefficients of both cylinders are reduced significantly compared with that of a single cylinder, regardless of the position angle of the small cylinder. Although the variation trends of the mean drag and lift coefficients with the position angle of the small cylinder obtained from the two-dimensional (2D) and three-dimensional (3D) simulations are similar, the 2D simulations overestimate the mean drag coefficient, the RMS drag and lift coefficients compared with those obtained from the 3D simulations. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4928306>]

I. INTRODUCTION

Piggyback pipelines that comprise of two pipelines of different diameters are often used in offshore oil and gas projects. Laying two pipelines together reduces not only the installation cost, but also the stabilization cost. In practice, the two pipelines are bundled together at certain intervals with a very small gap between them. Due to the lack of specific test data for piggyback pipelines, the common hydrodynamic design practice is to use the equivalent diameter assumption where the two pipelines are simplified into a single pipeline with an equivalent diameter of the sum of the diameters of the two pipelines.¹ It is unclear whether such a practice is too conservative or unsafe, because the influence of the gap between the two pipelines on the fluid dynamics and the forces of the piggyback pipelines could be non-linear.

Remarkably, complex flow patterns have been discovered for flow past two identical cylinders either in side-by-side, tandem, or staggered arrangement.² Previous studies have shown that when two cylinders are in a tandem arrangement, they behave as a single elongated body if the gap between them is very small. The vortex shedding from the upstream cylinder occurs only when the gap is large enough, while a re-attachment flow regime is observed for intermediate gaps.^{3,4} When two identical cylinders are in a side-by-side arrangement, the most prominent phenomenon

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is the biased jet flow from the gap and it was found that the biased direction of the flow changes intermittently when the gap is less than 1.2 times the cylinder diameter.⁵ When two cylinders are in a staggered arrangement, the flow approaching angle has a significant effect on the vortex shedding and the forces on the structures.⁶ Early numerical studies about flow past two identical cylinders were mainly based on two-dimensional (2D) models and focused on low Reynolds numbers.^{7,8} Recently, many three-dimensional (3D) simulations have been conducted for flow past two cylinders in various arrangements and Reynolds number in the turbulent flow regime.^{9–11}

The studies on two cylinders of different diameters in fluid flow are rarer than those of two identical cylinders. Tsutsui *et al.*¹² studied the interactive flow past two circular cylinders of different diameters at close proximity experimentally and numerically. In their experimental study, the separated shear layer from the main cylinder was found to re-attach and adhere to the rear surface of the main cylinder. The intermittent re-attachment and time-averaged fluid forces obtained numerically by Tsutsui *et al.*¹² showed good agreement with the experimental results.

It has been found that a significant reduction of the fluid forces on a large cylinder can be achieved by properly placing a smaller cylinder close to it. Strykowski and Sreenivasan¹³ discovered experimentally the suppression of vortex shedding from a circular cylinder at low Reynolds numbers ($Re = 40$ – 100) by placing a small circular cylinder close to it. Dipankar *et al.*¹⁴ studied numerically the suppression of vortex shedding behind a circular cylinder by another control cylinder in the near wake region at low Reynolds numbers ($Re = 63, 79$, and 150). Dalton *et al.*¹⁵ studied the effect of installing a small control rod in the wake of a large cylinder on the vortex shedding flow through numerical simulations and flow visualizations, while Lee *et al.*¹⁶ investigated reduction of the lift force of a cylinder by an upstream control rod. In previous studies of flow past two cylinders of different diameters, the small cylinder was either placed upstream or downstream of the large cylinder, while less attention has been paid to the side-by-side and staggered cases. Zhao *et al.*^{1,17} studied flow past two circular cylinders of different diameters using two-dimensional numerical simulations. It was found that the numerical results of the time-averaged force coefficients from the two-dimensional numerical model agree well with the experimental data. In contrast to the time-averaged drag force coefficients, it is well known that the 2D models overestimate the oscillation of the lift coefficient of cylinders. For example, the two-dimensional numerical results of the root mean square (RMS) lift coefficient in the subcritical flow regime in Zhao *et al.*¹⁷ are significantly greater than the measured data. It is therefore preferable to use three-dimensional simulations to investigate the interference between two cylinders in fluid flow.

In this study, flow past two cylinders of different diameters in close proximity is investigated numerically by solving the three-dimensional Navier-Stokes (NS) equations. The position of the small cylinder is determined by the gap G between the two cylinders and the position angle α as shown in Fig. 1. The diameters of the large and the small cylinders are D and d , respectively. The

FIG. 1. Definition sketch of flow past two circular cylinders of different diameters.

diameter ratio between them is $d/D = 0.45$. The study focuses on a very small gap between the two cylinders and the aim is to examine the effect of the position angle α of the small cylinder relative to the large cylinder on the wake flow pattern and the forces on the cylinders. The position angle ranges from 0° to 180° with an interval of 22.5° . The gap between the two cylinders is fixed at $0.0625D$. The diameter and gap ratios are set the same as their counterparts used in the experiments by Tsutsui *et al.*¹² in order to compare the numerical results with the experimental data. Since the vortex shedding characteristics from a circular cylinder are less dependent on the Reynolds number in the subcritical Reynolds number regime than in other regimes, the Reynolds number defined as $Re = UD/\nu$ with ν being the kinematic viscosity of the fluid and U , the free stream velocity, is fixed at 1000. The emphasis has been placed on the effects of the position angle of the small cylinder on the flow around the large cylinder.

II. NUMERICAL METHOD

The two cylinders investigated in this study are shown in Fig. 1. The position of the small cylinder is determined by the gap between the cylinders G and the position angle α . The length, time, velocity, and pressure are non-dimensionalised by $x_i = x'_i/D$, $t = Ut'/D$, $u_i = u'_i/U$, and $p = p'/\rho U^2$, respectively, where the primes stand for the dimensional values, x_1 , x_2 , and x_3 represent the coordinates x , y , and z , respectively, t is the time, u_i is the velocity component in the x_i -direction, p is the pressure, and ρ is the density of the fluid. The non-dimensional three-dimensional incompressible NS equations are written as

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j^2} = 0, \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (2)$$

The NS equations are solved by the Petrov-Galerkin finite element method developed by Zhao *et al.*¹⁸ The details of the numerical method can be found in Ref. 18 and will not be repeated here. The dimensions of the rectangular non-dimensional computational domain are 60 in the x -direction (incoming flow direction), 40 in the y -direction, and 9.6 in the z -direction (axial direction of the cylinders). Fig. 2 shows a typical computational mesh near the cylinders for $\alpha = 22.5^\circ$. The whole computational domain is divided into 192 layers of 8-node hexahedral tri-linear finite elements along the axial direction of the cylinders. On the circumferences of the large and the small cylinders, 130 and 96 elements are distributed, respectively. The total finite element numbers for all the cases are about 3.6×10^6 . The non-dimensional computational time step Δt was set to 0.003. The initial values of the velocity and the pressure are set at zero in the whole domain. The non-dimensional velocity at the inlet boundary is given as

$$(u_1, u_2, u_3) = (1, 0, 0). \quad (3)$$

The free-slip boundary condition is employed at the two side boundaries that are parallel to the x - z plane and the two end boundaries that are perpendicular to the cylinder axes. To employ the free-slip boundary condition, the velocity component and the pressure gradient perpendicular to the boundary are set to zero. On the cylinder surfaces, no-slip condition is applied.

The drag and lift coefficients on the large cylinder are calculated as $C_{D1} = \frac{2F_{D1}}{\rho DLU^2}$ and $C_{L1} = \frac{2F_{L1}}{\rho DLU^2}$, respectively, and those on the small cylinder are calculated by $C_{D2} = \frac{2F_{D2}}{\rho dLU^2}$ and $C_{L2} = \frac{2F_{L2}}{\rho dLU^2}$, respectively, where F_D and F_L are the drag and lift forces on a cylinder in the x - and y -direction, respectively, the subscripts 1 and 2 represent the large and small cylinders, respectively, and L is the length of the cylinder. The drag and lift forces (F_D and F_L) of a cylinder are calculated by integrating the pressure and shear stress along the cylinder surface.

FIG. 2. Computational mesh near two circular cylinders at $\alpha = 22.5^\circ$; (a) 2D view around the cylinders; (b) zoom in view near the gap, and (c) 3D view around the cylinders.

III. RESULTS AND DISCUSSIONS

While the gap ratio and diameter ratio are fixed at $G/D = 0.0625$ and $d/D = 0.45$, respectively, this study is focused on the influence of the position angle of the small cylinder relative to large (main) cylinder on the vortex shedding and the forces on the two cylinders. The Reynolds numbers based on the diameters of the large and small cylinders are 1000 and 450, respectively. The simulations are conducted for non-dimensional time of at least $Ut/D = 350$ to ensure that the fully developed wake flow has been obtained. The analysis is based on the results after $Ut/D > 110$ when the fully developed vortex shedding has been obtained. The qualities of the meshes for the cases in this study are equivalent to that shown in Fig. 2.

A. Mesh dependency study

The mesh dependency study is conducted to ensure that the meshes used in this study are dense enough to obtain reliable results. Simulations for $\alpha = 45^\circ$ are conducted using four different meshes (referred to be Meshes 1–4, respectively) listed in Table I. In Table I, N_{c1} and N_{c2} are the finite element numbers along the circumferential direction of cylinders 1 and 2, respectively, Δz is the mesh size in the axial direction of the cylinder, and Δr is mesh size in the radial direction on the cylinder surfaces, and $\overline{C}_D$ and $\overline{C}_L$ are the mean (time averaged) drag and lift coefficients, respectively. Because the frequency of the lift coefficients of the two cylinders is found to be the same, only the

TABLE I. Results of the mesh dependency study.

Mesh	L	N_{c1}	N_{c2}	Δz	Δr	$\overline{C}_{D1}$	$\overline{C}_{D2}$	$\overline{C}_{L1}$	$\overline{C}_{L2}$	S_{tl}
1	9.6	130	96	0.05	0.001	0.9846	0.2992	−0.0663	0.3308	0.1549
2	4.8	130	96	0.05	0.001	1.0033	0.3008	−0.0660	0.3314	0.1523
3	9.6	96	72	0.10	0.0018	1.0052	0.3015	−0.0600	0.3318	0.1530
4	9.6	96	72	0.05	0.0018	1.0020	0.3202	−0.0612	0.3312	0.1531
e12 (%)	...	...	...	...	...	1.90	0.55	0.45	0.18	1.68
e13 (%)	...	...	...	...	...	2.09	0.79	9.49	0.29	1.26
e14 (%)	...	...	...	...	...	1.77	7.03	7.68	0.11	1.17

Strouhal number of the large cylinder S_{L1} is shown in Table I. The differences between the results from Mesh k and those from Mesh 1 are defined as $e1k$ and they are listed in Table I. It can be seen that the maximum changes in force coefficient and the Strouhal number as the result of reducing the cylinder length from 9.6 to 4.8 (Mesh 1 to Mesh 2) are 1.90% and 1.68%, respectively. Increasing the finite element sizes in the radial, axial, and circumferential directions on the cylinder surface (comparing between Mesh 1 and Mesh 4) leads to a 9.49% change in $\overline{C}_{L1}$. Because the mean lift coefficient of the large cylinder $\overline{C}_{L1}$ is very small, a 9.49% change in $\overline{C}_{L1}$ corresponds to a very small value (0.0063). The difference of the lift coefficients of cylinder 2 between meshes 1 and 4 is only 0.29%. The comparison between the results from the different meshes demonstrates that the finite element sizes in the radial, circumferential, and axial directions of the cylinders for Mesh 1 has been sufficiently fine to obtain reliable results. Therefore, in the present study, the simulations for all the position angles (α) are conducted using meshes with the same density as Mesh 1.

B. Force coefficients

The time histories of the force coefficients of both cylinders at different position angles are shown in Fig. 3. It is confirmed by the numerical flow visualization that 2D vortex shedding is responsible for the large oscillation amplitudes of the force coefficients observed at the initial stage

FIG. 3. Time histories of force coefficients for $G/D = 0.0625$. (a) $\alpha = 0^\circ$, (b) $\alpha = 22.5^\circ$, (c) $\alpha = 45^\circ$, (d) $\alpha = 67.5^\circ$, (e) $\alpha = 90^\circ$, (f) $\alpha = 112.5^\circ$, (g) $\alpha = 135^\circ$, (h) $\alpha = 157.5^\circ$, (i) $\alpha = 180^\circ$.

of the simulations (except for $\alpha = 0^\circ$ and 180°). The amplitudes of the force coefficients are reduced significantly when the vortex shedding becomes fully 3D. Due to the symmetric configuration at $\alpha = 0^\circ$ and 180° , the flow remains symmetric until $Ut/D = 50$. The drag and lift coefficients are reduced significantly after the flow transits from 2D to 3D because part of the energy in the spanwise vortices, which is responsible for the oscillation of the force coefficients, is transferred to the streamwise vortices¹⁹ and the synchronization of the wake structure in the spanwise direction becomes weak. The sudden reduction of the force coefficients is not observed until $Ut/D = 110$ at $\alpha = 157.5^\circ$, implying that the two-dimensionality of the flow exists for a longer period than any of the other position angles.

The correlation between the significant reduction of the oscillation amplitude of the force coefficients and the transition of the wake flow from 2D to 3D is further demonstrated by the wake flow structures during the transition period shown in Fig. 4. The 3D wake flow patterns are presented by the iso-surface of the second negative eigenvalue e_2 of the tensor $\Psi^2 + \Omega^2$, where Ψ and Ω are the symmetric and the anti-symmetric parts of the velocity-gradient tensor, respectively. The second eigenvalue e_2 can be used to accurately identify the location of the vortex cores.²⁰ The two dimensional vortex shedding characterized by the alignment of vortices in the spanwise direction of the cylinders can be seen clearly at the early stage of simulation ($Ut/D < 55$) in Fig. 4(a), whereas in Fig. 4(b) the development of streamwise vortices from the two ends of the cylinders

FIG. 4. Iso-surface of $e_2 = -0.5$ at $\alpha = 22.5^\circ$ for different non-dimensional time steps.

FIG. 5. Variations of the mean force coefficients with α .

implies the formation of three-dimensional vortex shedding in the wake. The formation of vortex shedding patterns presented in Fig. 4(b) is similar to the 2D to 3D flow transition of an isolated cylinder reported in Williamson.²¹ The energy transfer from the spanwise to the streamwise vortices and the slight phase variation of the vortex shedding along the cylinder span due to the wobbly spanwise vortices lead to the reduction in the oscillation amplitudes of the force coefficients. The fully developed three-dimensional vortex shedding can be observed in Fig. 4(c). The simulation results shown in Figs. 4 and 3(b) clearly show that the reduction of oscillation amplitudes of force coefficients is induced by the flow transition from 2D to 3D. 2D numerical simulations are also performed in this study and the results of the force coefficient are compared with the 3D results later on. No significant reduction of the oscillation amplitudes of the force coefficients is observed in the 2D simulations. Since 3D numerical results are the focus of this study, the time histories of the force coefficients of the 2D simulations are not presented here.

Fig. 5 shows the time-averaged force coefficients (defined as the mean force coefficient hereafter) of the two cylinders. The experimental data by Tsutsui *et al.*,¹² the numerical results based on 2D Reynolds-averaged Navier-Stokes (RANS), and 2D NS equations are also included for comparison. The 2D NS results are obtained in this study using the 2D version of the numerical model. The experimental results by Tsutsui *et al.*¹² and the 2D RANS results by Zhao *et al.*¹⁷ were obtained for a higher Reynolds number of 4.1×10^4 , and they are only available for $\alpha \geq 90^\circ$. It is expected that the mean flow feature and the effects of the position angle α on the force coefficients should be the same for Reynolds numbers in the subcritical flow regime ($300 < Re < 10^5$). The mean drag coefficient of the large cylinder is increased significantly when the position of the small cylinder is at the side of the large cylinder. At $\alpha = 90^\circ$, it is increased by about 56.5% compared with that at $\alpha = 0^\circ$. It is decreased when the small cylinder lies either in front ($0^\circ < \alpha < 45^\circ$) or behind the large cylinder ($135^\circ < \alpha < 180^\circ$). The drag coefficient on the large cylinder is reduced because the small cylinder causes a reduction of the pressure on the front surface of the large cylinder when $0^\circ < \alpha < 45^\circ$ and an increase of the base pressure of the large cylinder is observed when $135^\circ < \alpha < 180^\circ$.^{17,22}

FIG. 6. Variations of RMS force coefficients with α for $Re = 1000$.

The mean lift coefficient on the large cylinder is non-zero except for $\alpha = 0^\circ$ and 180° . The variation of the mean drag coefficient of the small cylinder is similar to that of the large cylinder. The maximum mean lift coefficient on the small cylinder occurs at $\alpha = 22.5^\circ$ and 45° , where the flow accelerates due to the presence of the large cylinder. From Fig. 5, it can be seen that the variation trends of the mean force coefficients with the position angle α calculated from different numerical models are similar to each other and close to the measured data. However, quantitatively, the differences between different models are obvious. The difference between the 2D and the 3D results occurs mainly because the variation of the flow in the spanwise direction of the cylinder is not considered in the 2D model. The difference between the 3D results and the experimental data is mainly due to the difference in the Reynolds number.

Fig. 6 shows the variation of the RMS lift coefficient with the position angle α , together with the numerical results based on the 2D NS equations. Although the mean force coefficients based on the 2D NS equations seem to be similar to those based on the 3D NS equations as shown in Fig. 5, the RMS force coefficients are significantly overestimated based on the 2D model. Previous independent investigations also showed that the RMS lift coefficient for $Re = 1000$ obtained from 2D simulations is significantly greater than that from the 3D simulations.⁶ The 3D results of the RMS drag and lift coefficients of both cylinders occur when the small cylinder is located upstream shoulder of the large cylinder ($\alpha = 45^\circ$ or 67.5°). The maximum RMS drag coefficients of the large and the small cylinders are 0.041 and 0.162, respectively, and they both occur at $\alpha = 67.5^\circ$. The maximum RMS lift coefficients of the large and the small cylinders are 0.237 and 0.093, and occur at $\alpha = 45^\circ$ and 67.5° , respectively. The RMS drag coefficient on the large cylinder from the 2D numerical model follows a similar trend to that from the 3D model, but is significantly greater than the latter. In contrast to those obtained from the 3D model, the maximum drag and lift coefficients of the small cylinder from the 2D model occur at about $\alpha = 135^\circ$ and 157.5° when the small cylinder is behind the large cylinder. From Fig. 6, it can be seen that the RMS drag and lift coefficients on both cylinders are reduced significantly compared with their counterparts of a single cylinder in the 3D numerical results, except for the RMS drag coefficient of the small cylinder at $\alpha = 67.5^\circ$, which is very close to that of a single cylinder.

FIG. 7. Amplitude spectra of the lift coefficient for large and small cylinders: (a) $\alpha = 0^\circ$, (b) $\alpha = 22.5^\circ$, (c) $\alpha = 45^\circ$, (d) $\alpha = 67.5^\circ$, (e) $\alpha = 90^\circ$, (f) $\alpha = 112.5^\circ$, (g) $\alpha = 135^\circ$, (h) $\alpha = 157.5^\circ$, and (i) $\alpha = 180^\circ$.

C. Vortex shedding frequency

The vortex shedding frequencies around both cylinders are determined by conducting Fast Fourier Transform (FFT) of the lift coefficients and the FFT spectra are shown in Fig. 7. The dominant frequencies of the lift coefficient of the two cylinders are found to be the same because there is only one vortex street in the wake. Tsutsui *et al.*¹² found two frequencies in the lift coefficients for position angles α between 90° and 130° and the two frequencies are very close to each other (less than about 15% difference). In Fig. 7, the secondary peak can also be seen for $67.5^\circ \leq \alpha \leq 112.5^\circ$. However, the amplitudes of these secondary peaks are much smaller than that of the primary peaks. In this study, the Strouhal number is determined based on the frequencies corresponding to the primary peak.

The Strouhal number is defined based on the lift coefficient of the cylinders as $S_t = fD/U$, where f is the frequency of the oscillation of the lift coefficient. Fig. 8 shows the variation of the Strouhal number with the position angle α of the small cylinder, together with the experimental data in Tsutsui *et al.*¹² and two-dimensional numerical results in Zhao *et al.*¹⁷ The two values for each angle α in the range of 90° – 130° correspond to the so-called reattachment state as in Tsutsui *et al.*¹² and Zhao *et al.*¹⁷ The Strouhal number corresponding to the secondary peak in the power spectra of the lift coefficient is not included in Fig. 8 because it is much weaker than the primary peak.

The Strouhal number decreases with increasing α as the small cylinder is in the upstream position ($0^\circ \leq \alpha \leq 90^\circ$) and increases with increasing α as the small cylinder is located at the downstream position ($90^\circ \leq \alpha \leq 180^\circ$), as shown in Fig. 8. The minimum Strouhal number occurs when the small cylinder is positioned nearly about side-by-side ($67.5^\circ < \alpha < 112.5^\circ$) with the main cylinder. The reason for this is that when the two cylinders are in side-by-side arrangement, increased dimension of the system in the cross-flow direction increases the vortex shedding period. The vortex shedding period is reduced also because the jet flow through the gap weakens the interaction of the vortices in the wake of the cylinder which will be discussed in detail in Sec. III D. The maximum Strouhal number is observed when the small cylinder is located either in front ($\alpha = 0^\circ$) or behind ($\alpha \geq 157.5^\circ$) the main cylinder. This is because the streamlined configuration reduces the dimension of the system in the cross-flow direction and makes the interaction between the shear

FIG. 8. Variation of the Strouhal number with α .

layers from the two sides of the large cylinder easier. The calculated Strouhal numbers agree well with experimental results in Ref. 12.

D. Wake flow patterns

Fig. 9 shows the iso-surface of e_2 at instants when the lift coefficient of the large cylinder reaches its maximum value for all the position angles. The distortion of the spanwise vortices and the development of the streamwise vortices can be clearly seen in Fig. 9. When $Re = 1000$, the wake structure of a single cylinder is in Mode B, which is featured by the generation of strong streamwise vortices and the dissipation of the spanwise vortices.²³ The wake flow patterns in Fig. 9 are very similar to the wake flow of Mode B with strong streamwise vortices. At a few diameters downstream of the cylinders, the spanwise vortices are hardly identifiable using the $e_2 = -0.5$ iso-surfaces.

The cross-sectional flow features are visualized using the contours of the vorticities on planes perpendicular to the cylinders. The vorticity components in the x -, y -, and z -directions are defined as $\omega_x = \partial w / \partial y - \partial v / \partial z$, $\omega_y = \partial u / \partial z - \partial w / \partial x$, and $\omega_z = \partial v / \partial x - \partial u / \partial y$, respectively. Fig. 10 shows the contours of the axial vorticity (ω_z) at the mid-section ($z = 4.8$) for small position angles of $\alpha = 0^\circ$, 22.5° , and 45° at the instants when the lift coefficient on the large cylinder is at its maximum value. For $\alpha = 0^\circ$, no flow exists between the gap, and the vortex shedding from the large cylinder is similar to that of a single body. The oscillation of the lift coefficient due to the vortex shedding is weak because the upstream cylinder makes the whole structure more streamlined. As the angle α is increased from 0° to 45° , the velocity through the gap also increases. The jet flow from the gap merges into the shear layer from the top of the small cylinder. This enhances the shear layer from the top side of the large cylinder, and as a result, increases the oscillation amplitude of the lift coefficient of the large cylinder as shown in Fig. 6 compared with other position angles. The vortex shedding from the cylinders for $\alpha \leq 45^\circ$ is very similar to that of a single cylinder and the vortex shedding mode in Fig. 10 is called the upstream interference mode in this study.

The time histories of the drag and lift coefficients on both cylinders vary with time in a beating pattern at $\alpha = 67.5^\circ$ (Fig. 3(d)). This beating phenomenon in forces is related to the changes of flow between one pattern and another alternatively, leading to the increase and decrease of the amplitude of the force coefficients. To explain this further, Fig. 11 shows the contours of the instantaneous spanwise vorticity at the mid-section of the cylinders at $\alpha = 67.5^\circ$. Figs. 11(a)–11(d) correspond to the instants when the amplitudes of the forces on both cylinders are large, while Figs. 11(e)–11(h) correspond to the instants when the amplitudes are small (see also the time histories of the force coefficients in Fig. 3(d)). From Fig. 11, it can be seen how the jet flow through the gap affects the wake flow structure and hence the forces. The forces of the two cylinders oscillate almost

FIG. 9. Vortex structures at the instants when the lift coefficient reaches its maximum value represented by the isosurfaces of $e_2 = -0.5$.

FIG. 10. Contours of axial vortices at the mid-section ($z = 4.8$) of the two cylinders: (a) $\alpha = 0^\circ$, (b) $\alpha = 22.5^\circ$, and (c) $\alpha = 45^\circ$.

regularly with time because of the vortex shedding generated by the interaction between the two shear layers from the top of the small cylinder and the bottom of the large cylinder. It has been found that the formation length of the wake vortices affects the velocity fluctuation in the wake and the force oscillation. The formation length is defined as “that point downstream of the body where the velocity fluctuation level has grown to a maximum, and thereafter decays downstream.”²¹ An increase in the formation length leads to a decrease in the level of velocity fluctuation and a decrease in the base suction.^{21,24,25} In Figs. 11(a)–11(d), the jet flow through the gap bends downward and attaches to the rear surface of the large cylinder. This allows the shear layer from the top of the small cylinder to form vortices near the cylinder. The biased gap flow toward the large cylinder weakens the oscillation of the forces on the large cylinder compared to that of a single isolated cylinder. The reduction of the force due to biased gap flow for flow past two side-by-side identical cylinders has been observed in Refs. 10 and 26. The gap flow detaches from the rear surface of the large cylinder intermittently. It can be seen in Figs. 11(e)–11(h) that the flow from the gap is nearly in the horizontal direction but slightly biases toward the small cylinder. This delays the interaction between the shear layers from the two sides of the system and further reduces the amplitude of the force coefficients of the cylinders.

By conducting experimental investigation of flow through two cylinders at a nearly side-by-side arrangement with a diameter ratio of 0.5, Ko *et al.*²⁷ found that the characteristics of the gap flow through two cylinders with a diameter ratio of 0.5 are very similar to that of two side-by-side cylinders of an equal diameter. Two flow modes were observed by Ko *et al.*,²⁷ the gap flow biases toward the large cylinder in Mode 1 and toward the small cylinder in Mode 2. The two flow modes are also clearly observed in Fig. 11. When the gap flow is biased towards the large cylinder, it attaches to the rear surface of the large cylinder, forming a wall jet. Tsutsui *et al.*¹² define this phenomenon as “reattachment.” However, when the gap flow is biased towards the small cylinder, instead of attaching to the back surface of the small cylinder, the flow through the gap forms a free-jet that biases toward the small cylinder as shown in Figs. 11(e)–11(h). This is because the small cylinder cannot generate strong enough wake to suck the gap flow towards its back surface. The intermittent change of the flow pattern for $\alpha = 90^\circ$ (Fig. 12) is similar to that for $\alpha = 67.5^\circ$ (Fig. 11). The gap

FIG. 11. Instantaneous contours of vorticity in the z -direction at the mid-section ($z = 4.8$) of the two cylinders for $\alpha = 67.5^\circ$.

flow attaches to the rear surface of the large cylinder in Figs. 12(a)–12(c) and detaches in Fig. 12(d). The flow mode observed at $\alpha = 67.5^\circ$ and 90° is defined as the intermittent attached gap flow mode.

When the position angle α is increased to 112.5° and 135° , the gap flow is always biased towards the large cylinder and attaches to the back surface of the large cylinder as shown in Fig. 13.

FIG. 12. Instantaneous contours of vorticity in the z -direction at mid-section ($z = 4.8$) of the two cylinders for $\alpha = 90^\circ$.

FIG. 13. Contours of axial vorticity at the mid-section ($z = 4.8$) of the two cylinders: (a) $\alpha = 112.5^\circ$ and (b) $\alpha = 135^\circ$.

The alignment of the two cylinders forces the gap jet flow to be in the diagonally downward direction and to attach to the back surface of the large cylinder. Free jet flow (separation of the gap flow from the back of the large cylinder) as observed when $\alpha = 67.5^\circ$ and 90° is not observed. The wake flow for $\alpha = 112.5^\circ$ and 135° is defined as the attached gap flow mode. When the gap flow attaches to the rear surface of the large cylinder, the separation point on the large cylinder becomes very close to its most downstream point, leading to a significant reduction in the oscillation amplitude of the lift coefficient.

When the position angle is 157.5° and 180° , the gap flow disappears and the small cylinder affects the interaction between the two shear layers from the two sides of the large cylinder as shown in Fig. 14. The vortex shedding in the wake of the cylinder is similar to that of a single cylinder. However, due to the existence of the small cylinder, the vortex formation length of the large cylinder is longer than that of a single cylinder, leading to a reduction of the amplitude of the lift force oscillation. Because the interaction between the top and bottom shear layers occurs behind the small cylinder, the vortex shedding does not lead to strong oscillation of the lift force of the small cylinder. The vortex shedding mode for $\alpha = 157.5^\circ$ and 180° is called wake interference mode because the small cylinder affects the interference between the two shear layers from the large cylinder in the wake.

FIG. 14. Contours of axial vorticity for large position angles at the mid-section ($z = 4.8$) of the two cylinders: (a) $\alpha = 157.5^\circ$ and (b) $\alpha = 180^\circ$.

IV. CONCLUSIONS

Three-dimensional numerical simulations of flow past two circular cylinders of different diameters in close proximity for $Re = 1000$ (based on the large cylinder's diameter) at a diameter ratio of 0.45 and a gap ratio of 0.0625 are conducted. The effect of the position angle of the small cylinder (α) relative to the large cylinder is studied for α ranging from 0° to 180° with an interval of 22.5° . The results are summarised as follows.

Depending on the position angle of the small cylinder, the wake flow can be divided into four modes: the upstream interference mode for $\alpha = 0^\circ$, 22.5° , and 45° , the intermittent attached gap flow mode for $\alpha = 67.5^\circ$ and 90° , the attached gap flow mode for $\alpha = 112.5^\circ$ and 135° , and the wake interference mode for $\alpha = 157.5^\circ$ and 180° . The flow through the gap attaches and separates from the back surface of the large cylinder in the intermittent attached gap flow mode, and always attaches to the back surface of the large cylinder in the attached gap flow modes.

The calculated RMS force coefficients using the 3D model on both cylinders are found to be significantly smaller than that of a single cylinder. While the 2D numerical model predicts the similar mean force coefficients to those by the 3D model, it overestimates the RMS force coefficients significantly. The maximum RMS drag and lift coefficients from 3D simulations occur at either $\alpha = 45^\circ$ or 67.5° for the large and small cylinders, respectively. When the two cylinders are in a near side-by-side arrangement ($67.5^\circ \leq \alpha \leq 112.5^\circ$), the RMS lift coefficients on both cylinders are reduced significantly, despite the increase in the mean drag coefficient. When $67.5^\circ \leq \alpha \leq 112.5^\circ$, the jet flow from the gap either attaches to the back surface of the large cylinder or is biased towards the small cylinder, forming a free jet flow. An attached gap flow to the rear surface of the large cylinder reduces the RMS lift coefficient by moving the flow separation position point towards downstream part of the cylinder surface, while a free jet flow reduces the RMS lift coefficient by weakening the interferences between the two shear layers from the two sides of the cylinder system.

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