ECONOMICS

Thinking Outside the Box: Edgeworth, Pareto and the Early History of the Box Diagram

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DISCUSSION PAPER 18.03

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Abstract: This study enhances McLure (2017) by considering the significance Edgeworth's reciprocal demand curves to the early history of the Edgeworth box. In short, it clarifies Edgeworth's and Pareto's respective contributions to the development the Edgeworth box diagram by establishing the relationship between the 'trade' representations of Edgeworth's Figures 1 and 5 and the associated 'allocation' representations of those two diagrams. A reconstruction of Edgeworth's Figure 1 is also provided to make the implicit aspects of his treatment of production explicit. A new early history of the Edgeworth box diagram is also presented.

Key Words: Box diagram, Edgeworth, Exchange, Pareto, Production

JEL Code: B13

¹ The authors would like to thank Ken Clements, John Creedy, Greg Moore and Mary Morgan for their comments on, or discussion of, an early draft of this paper. Michael McLure would also like to acknowledge the *LSE* Economic History Department, where he held a visiting appointment when he was making many of his contribution to this paper.

“A picture paints a thousand words”

1) Introduction

Views on the origin of the ‘Edgeworth box’ diagram are polarised. Some economists and historians of economics – such as Luigi Amoroso (1942), John Creedy (1980, 1986 and 2010), Thomas Humphrey (1996), Mary Morgan (2004, 2012) and Alberto Zanni (2014) – recognise F. Y. Edgeworth’s Figure 1 from *Mathematical Psychics* (Edgeworth 1881, p 28.) as the ‘original version’ of the Edgeworth box. Others working in the 1970s – such as Vincent Tarascio (1972, 1980), William Jaffé (1972, 1974), Maurice Allais (1975), James L. Weatherby Jr (1976) – explicitly state that Vilfredo Pareto’s Figure 16, or its companion diagram Figure 50, from the *Manual of Political Economy* (Pareto 1906/1909 [2014, p. 95, p.180]) is the ‘original version’ of the Edgeworth box.

This paper commences with a critical review of the early history of the Edgeworth box with reference to the contributions of Edgeworth and Pareto. As the polemical character of this current history of the box diagram largely turns on different interpretations of Edgeworth’s diagrams, they are the main focus of this study. To provide a clear basis for establishing the respective contributions of these two theorists, we differentiate between Edgeworth’s diagrammatic illustrations of pure exchange and of labour exchange and establish the relationship between both diagrams and the Edgeworth box diagram by drawing on the characterisation of Edgeworth’s indifference curves by W. E. Johnson (1913) and Jaffé (1974), who point out that Edgeworth defined the x axes for his diagrams such that a movement east from the origin indicates an increase in the ‘traded’ quantity x of good X (i.e. either the quantity given up by one dealer or the corresponding quantity acquired by the other dealer); and the y axis such that a movement north from the origin indicates an increase in the ‘traded’ quantity of good Y.

We establish that the simplest direct transformation of Edgeworth’s ‘trade’ oriented illustration of pure exchange (his Figure 5) and of labour exchange

(Figure 1) into a conventional ‘allocation’ representation – in which a horizontal movement away from the origin indicates an increase in an individual’s allocation of X and a vertical movement away from the origin indicates an increase in his or her allocation of Y – does not involve recalibration of the x and y axes in the manner of a box diagram. Rather, allocation is mapped by two separate ‘systems of’ coordinates, one for each dealer, with each dealer’s allocation considered in reference to the same origin. It is possible to impose assumptions on this ‘non-box’ orientation of the allocation plane to transform it into a ‘box’ calibrated allocation plane, that can be undertaken in the case of Edgeworth’s illustration of pure exchange, but he did not undertake such a transformation himself. But when the aspect of production associated with the exchange of labour for remuneration is introduced as Edgeworth did in his famous Figure 1, his insights are concealed or even lost if the diagram is presented in the form of a ‘box’ allocation. In view of this we then illustrate how some of Edgeworth’s insights can be clarified by making the treatment of production within his Figure 1 more explicit.

For the history of the development of the Edgeworth box, however, the fundamental point is that Edgeworth did not develop a box diagram, although he did lay the foundation from which it could be constructed. In view of this, a new history of the relative contributions of Edgeworth and Pareto to the Edgeworth box is presented in this paper. A contribution in that regard is the identification of Pareto’s 1902 graph, from his article ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, which is presented as an important intermediate step between Edgeworth’s 1881 diagrams and the emergence of the box diagram in Pareto’s 1906 *Manual*.

The paper is structured in seven sections. Section 2 presents a critical review of the literature on the history of the Edgeworth box diagram, primarily dating from the 1970s. Section 3 is a systematic investigation of the ‘allocation’ and ‘trade’ representations of pure exchange presented in Edgeworth’s Figure 5. That line of investigation is then extended in Section 4 to the illustration of exchange of labour and remuneration in Edgeworth’s Figure 1. A reconstruction of Edgeworth’s Figure 1 is presented in Section 5, which makes explicit the

implicit aspects of Edgeworth's treatment of production. Section 6 presents a new history of the Edgeworth box. It is concluded, in Section 7, that the box diagram is rightly named in honour of Edgeworth, for his development of indifference curves and the contract curve, but, nonetheless, he did not create an open axes box diagram for allocations.

2) Literature on Edgeworth, Pareto and the Original Box Diagram

Edgeworth's Figure 1 (slightly modified as Diagram A below) is an open axes graph that shows a *contract curve* (CC'); a single indifference curve for one individual that appears convex to the 'northwest' of the graph (the curve from O to $\eta_0 x_0$); and a single indifference curve for the other individual that appears convex to the 'southeast' (the curve from O to $y_0 \xi_0$ and beyond). Two *reciprocal demand curves*,² $O\eta$ and $O\xi$, are also drawn on that diagram that intersect on the contract curve marked at point $\eta\xi$. Significantly, this figure is presented by Edgeworth as an illustration of the famous fictional relationship between Friday and Robinson Crusoe, originally characterised by Daniel Defoe (1718) in his classic novel.³

² Edgeworth refers to these curves as “demand curves”, which are fundamentally a development of Alfred Marshall's trade diagrams (see Creedy 1986, p. 71 and Morgan, 2012, p.126-127). Following Johnson (1913) and Bowley (1924), Edgeworth's demand curves were generally referred to as “offer curves”, although Edgeworth did not like that label and many economists now refer to them as *reciprocal demand curves* (see Creedy 1986, Gram 1987).

³ Fritz Söllner (2016) presents a critical but exaggerated account of the manner in which neo-classical economists have used and abused the Robinson Crusoe story.

Diagram A: Edgeworth's Figure 1 (1881, p. 28)⁴

In contrast, Pareto's Figure 16 (Diagram B below) is a closed box diagram that has two origins, one given by the 'south-west' corner o of the box and the other given in the 'north-east' corner ω of the box, with each origin associated with a different trader. Ordinal systems of indifference curves are mapped for two individuals, each convex to the respective origin for each individual. Relative to the modern stylised Edgeworth box diagram, Edgeworth's contract curve is a notable omission.

Diagram B: Pareto's Figure 16 (1906 [2014, p.95])

The suggestion that the origin of the 'Edgeworth box' goes back to Edgeworth's Figure 1 was implicit by at least as early as 1942, when the Italian Paretian Luigi Amoroso (1942) outlined Pareto's system and then discussed the "Rappresentazione di Edgeworth" (a slight variation of Diagram A) in his *Meccanica Economica* (Amoroso 1942), which was recently discussed by Alberto Zanni (2014). Notably, Amoroso used an open axes diagram in the plane to illustrate a contract curve and two indifference curves and explain the

⁴ To underline the indeterminacy of efficient bilateral exchange, Edgeworth's original figure also indicates points $y_1\xi_1$ and $y_2\xi_2$ as potential exchange points on the contract curve. A label $\frac{x'_1y'_1}{2} \frac{y'_2}{2}$ is also included in the original diagram because Edgeworth (1881, pp. 36-40) subsequently discussed the introduction of two Crusoes and two Fridays and so on with reference to his Figure 1 to explain how a unique point on the contract curve is eventually attained when the number of agents increases and that this point corresponds to the free competition equilibrium. Gerard Debreu and Herbert Scarf (1963) refer to Edgeworth's analysis of this issue in their influential study of the 'core of the economy'.

“Rappresentazione di Edgeworth”. The implicit origin for one indifference curve, for one dealer, appears in the ‘northwest’ of the diagram; and the implicit origin for the other indifference curve, for the other dealer, appears in the ‘southeast’. Only a modest transformation is required to transpose the diagram that Amoroso developed, as a representation of Edgeworth’s Figure, into the ‘Box diagram’. On that reckoning, labelling of the box diagram as the Edgeworth box would be entirely reasonable.

As William Jaffé (1974) had noted, a few economists, such as Erich Schneider (1952), G. L. S. Shackle (1967) and Maurice Dobb (1969), subsequently reported on Pareto’s diagram and its relation to the Edgeworth box diagram. It was not until the 1970s, however, following the translation of the French edition of the *Manual* by A. S. Schweir and A. N. Page (Pareto 1909 [1971]), that the relationship between Edgeworth’s Figure 1 and the box diagram was seriously brought into question, starting with Jaffé’s (1972, p. 1190) and his dramatic announcement that the Edgeworth box is not to be found in Edgeworth’s *Mathematical Psychics*. In the same year, Vincent Tarascio reviewed the origins of the Edgeworth box (or the Edgeworth-Bowley box as it has sometimes be called) and reached a similar conclusion, although he observed that two possible diagrams in *Mathematical Psychics* that could be considered as possible origins for the box diagram: Edgeworth’s Figure 1 (our Diagram A) and Figure 5 (our Diagram C). But he concluded that these diagrams cannot be converted into a box diagram by “simple geometric manipulation” (Tarascio 1972, p. 193).

In regard to Edgeworth’s Figure 1, Tarascio reminded us that Edgeworth drew his diagram to illustrate the particular economic relationship between Robinson Crusoe and Friday:

“It might be pointed out that Edgeworth had a box diagram in this sense: the ordinate of Figure 1 (p.28) is the amount of labour (time) given up by Friday; the abscissa is the remuneration received by Friday. Then Y is received by Crusoe, and X is given up. Hence quantities are conserved in the diagram; this mode inherently limits the quadrant to tenable points.” (Tarascio 1972, p. 193).

Any geometrical manipulation of Edgeworth's Figure 1 would result, in Tarascio's assessment, in a 'nonsensical' box diagram in the sense that "the indifference curve for one trader is parallel to the indifference curve of the other trader" (Tarascio 1972, p. 193).

For his Figure 5, Edgeworth's specific objective was to explain how each individual's semi-circular *reciprocal demand curve* may be derived as the "the locus of the point where lines from the origin *touch* the curves of indifference" (Edgeworth 1881, p. 113). Importantly, unlike his Figure 1, Edgeworth's Figure 5 is not a representation of the exchange of labour for remuneration in terms of goods produced by labour. Rather, goods X and Y are both consumer goods and the axes on the graph are such that x indicates the quantity of corn and y indicates the quantity of beef. The initial endowment of goods for the two dealers are not stated, but one indifference curve is concave to point C and the other is concave to point C' . The origin, point O , indicates no trade, with the semicircle $OQTC$ representing person A 's *reciprocal demand curve* and; the semicircle OTC' representing person B 's *reciprocal demand curve*. The single indifference curve for dealer A is tangential to the market price ray at point T and the other indifference curve, for dealer B , passes through point Q .

Diagram C: Edgeworth's Figure 5 (1881, p. 114)

Tarascio rejected any suggestion that Diagram C is an early version of the Edgeworth box because the "results are inconsistent with the theory underlying the analysis" (1972, p. 194). He attempts to clarify his reasoning, noting the following:

“Again, the result is parallel indifference curves. If, on the other hand, one begins with individual diagrams and then superimposes them by reversing the origins and axes, the result is indifference curves along a contract curve, but the direction of the contract curve is from northwest to the southeast and the indifference curves are not concave to/from the origins.” (Tarascio 1972, p. 194)

Tarascio writes nothing more on the issue, so his complete rationale for the abovementioned conclusions is not explained. But it is certainly incorrect because, as demonstrated in Sections 3 of this paper, Edgeworth’s Figure 5 is entirely consistent with theory. Nevertheless, Tarascio’s article is important for introducing Edgeworth’s Figure 5 to the history of Edgeworth’s contribution to the box diagram

Tarascio also observes that Edgeworth’s Figure’s 1 and 5 both show a single indifference curve, rather than a system of indifference curves, that is mapped for each individual. He interprets this as evidence for the proposition that Edgeworth was not thinking in terms of systems of indifference curves. In support of this, he argues that neither the text nor the equations in *Mathematical Psychics* suggest a basis for doing so. Jaffé (1974) a few years later similarly noted the difference between Edgeworth and Pareto over the number of indifference curves shown for each person.

Jaffé’s assessment of Edgeworth’s ‘pristine’ diagram (our Diagram A) failed to account for relationship between the Friday-Crusoe and the associated economic scenario that the diagram was drawn to illustrate. Instead, he read Edgeworth Figure 1 as if it were a representation of two dealers, a Mr A and a Mr B, exchanging consumer goods that, implicitly, have positive marginal utility to both individuals. On that basis, he pointed out that the dealers’ initial endowment poses serious problems for the box interpretation of Edgeworth’s Figure 1.

“Edgeworth’s Contract Curve diagram could only be converted into a box diagram of specific dimensions if the initial endowments, which Edgeworth deliberately chose to leave implicit, were explicitly specified. (Jaffé 1974, pp. 343-344).

To link Edgeworth’s contract curve to trade choices, Jaffé (1974, p. 347) devised Diagram D (below), which treats exchange as incremental variations in

quantities of goods from that given by unspecified initial endowments marked by the no trade situation represented by the origin O.

Diagram D: Jaffé's explanation of Edgeworth's Figure 1 (1974, p. 347)

The fundamental feature of Jaffé's work and diagram D is that it clarifies the distinction between an indifference curve mapped in the 'trade' plane of each dealer and the 'trade' plane that represents exchanges between the two dealers, as Edgeworth did. In diagram E, the 'northwest' and the 'southeast' quadrants map indifference for 'Mr A' and 'Mr B' respectively in their 'trade' planes relative to the no trade point O. That is, the 'northwest' quadrant maps the exchanges for quantities x , of X , (which is being given up by Mr A) and y , of Y , (which is being acquired by Mr A) to which "Mr A" is indifferent. Similarly, the 'southeast' quadrant maps the exchanges of y being given up and x acquired to which 'Mr B' is indifferent to. The 'northeast' quadrant, however, maps both dealers' indifference curves in the common 'trade' plane. For dealer A, the first indifference curve from his origin 'trade' plane in the 'northwest' quadrant is I_A ; and the vertical mirror image of that indifference curve is given in the common 'trade' plane (i.e. I_A in the 'northeast' quadrant shown in bold font). For dealer B, the first indifference curve from his origin 'trade' plane in the 'southeast' quadrant is I_B ; and the vertical mirror image of that indifference curve is given in the common 'trade' plane (i.e. I_B in the 'northeast' quadrant). But, in the light of Tarascio (1972), the critical problem associated with Jaffé's interpretation of Edgeworth's Figure 1 is that he interprets x and y as quantities of consumer

goods, without considering what interpretative adjustments are required for one to consider labour and remuneration on the same basis as consumer goods.

Notwithstanding the difficulties associated with both Tarascio's and Jaffé's explanation of Edgeworth's Figure 1, they each contributed a precious gem to the historical assessment of Edgeworth's diagrams. But their influence was less than might have been expected because Creedy (1980) succeeded in highlighting significant shortcomings in their respective papers. Specifically, Creedy insisted that it is hardly legitimate for Tarascio or Jaffé to suggest that Edgeworth did not appreciate that systems of indifference curves may be modelled for each individual because the very definition of a contract curve "implies a 'system' of indifference curves" (Creedy 1980, p. 274). He also pointed to many instances in the text of *Mathematical Psychics* which envisage more than one indifference curve per dealer.

The context for Tarascio's point on 'systems' of indifference curves is given by his important observation that Pareto (1902) had provided the mathematical formalism for ordinal systems of indifference curves, in which utility is explicitly a function of the index numbers used to label indifference curves.⁵ But the lack of such an explicit equation is hardly a weakness of *Mathematical Psychics* when one recalls that Edgeworth's utility functions are valued in cardinal terms. As a result, the implicit index number for each individual's indifference curve is simply the total utility that is associated with a curve; and a separate index number function would not need to be specified because an individual's potential total utilities constitute perfectly sound index numbers for his, or her, system of indifference curves.

Creedy also notes that Jaffé was unjustifiably critical of Edgeworth for suggesting that the lack of an initial endowment impedes the conversion of Edgeworth's diagram into a box diagram because Edgeworth (1881, p. 105) had made it clear that he fully appreciated the role of endowment. It is just that his main equations and his diagram reflected a greater interest in exchange rather than allocations. The origin point O of Edgeworth's Figure 1 indicates the initial

⁵ Pareto first introduced the mathematical formalism for ordinal theory in Pareto (1900).

allocation and, as Creedy subsequently noted, he regards the lengths of the x and y axes as being drawn without regard to fixed supplies of goods (Creedy 1986, p.59) because the initial allocation of goods X and Y between the two individuals is not specified. He maintained that it was Bowley who demonstrated that the initial allocation could be introduced by expanding the XY plane beyond the axes drawn by Edgeworth (see Diagram E below).

Diagram E: Bowley's Figure 1 (1924, p. 5)

⁶ The point is, however, emphasised by Morgan (2012), but she did not attribute any significance to it when interpreting Edgeworth's Figure 1 and its relationship to the Edgeworth box.

(1980) in an edited collection (Blaug 1993), reached the conclusion that the now conventional box diagram was introduced by Pareto and not Edgeworth (Blaug 1985, p. 311). Werner Hildenbrand (1993) and Alain and Christiane Alcouffe (1998) have stated similar views. But in his entry on “The Edgeworth box” (Creedy 2010) in *Famous Diagrams and Figures in Economics*, Creedy sets that view aside and completely writes Pareto out of the history of the Edgeworth box diagram. He does so partly because of his earlier stated view that “the use of a box has of course no special analytical value, though it is perhaps a useful pedagogical addition” (Creedy 1986, p.59), but mainly because he continued to characterise the origin of Edgeworth’s Figure 1 as a unique allocation between two individuals on the presumption that Friday’s (or dealer B’s) indifference curves are mapped with respect to the northwest area of Edgeworth’s Figure 1 for one dealer (Friday) and to the Southeast area for the other dealer (Crusoe). The end result is that Creedy’s account of the Edgeworth box is confined to the contributions of Edgeworth (1881) and Bowley (1924).

The detailed and scholarly literature on the history of the Edgeworth box in more recent years has been authored by Thomas Humphrey, in his “The Early History of the Box Diagram” (1996), and Mary Morgan, provisionally in “Imagination and Imaging in Model Building” (2004) and more fully in “The History of the Edgeworth Box Diagram – as Told by Itself” from her *The World in the Model: How Economists Work and Think* (2012). Both Humphrey and Morgan are far more appreciative and nuanced in their treatment of Pareto than Creedy, in that the close resemblance between Pareto’s box diagram and the modern stylized Edgeworth box is fully recognised. But neither Humphrey nor Morgan cite or consider Tarascio (1972) and both treated Edgeworth’s Figure 1 as the ‘original version’ of the Edgeworth box diagram. And they do so on a similar basis as Creedy by treating goods X and Y as allocations of stocks that have positive marginal utilities to both dealers under which: (i) Edgeworth’s Figure 1 implicitly, but not explicitly, incorporates a system of indifference curves for each individual because of the inclusion of the contract curve; (ii) the initial endowments can be treated more generally by moving from the origin point O to

a point inside the box;⁷ and (iii) by creating the box diagram in modern Paretian orientation by rotating Edgeworth's diagram by 90°.

In regard to point (i) in the paragraph above, Creedy, Humphrey and Morgan are on solid ground. But the status of points (ii) and (iii), on which the thesis that “Edgeworth's Figure 1 is the original Edgeworth Box diagram” ultimately depends, rests on two propositions that have not been demonstrated. First, that Edgeworth intended his ‘trade’ oriented diagrams to be also interpreted as open axes ‘allocation’ diagrams in box format for the case of pure exchange of consumer goods. Second, that Edgeworth also intended his illustration of labour exchange and remuneration under the Friday-Crusoe scenario to be transformed into a form that would allow it to be treated analytically as if it were a case of pure exchange.

3) Edgeworth's Figure 5 in ‘Allocation’ and ‘Trade’ Representations

The trade orientation of Edgeworth indifference curves, and their relationship to the allocation orientation of indifference curves, was first made explicit by W. E. Johnson in his account of the axes that define the plane in which they are mapped.

“in Edgeworth's [indifference] curves the coordinates of any point represent two variables, one which is acquired and the other is sacrificed in return for the former. ... [for one party] x measures the amount acquired and z ⁸ the amount sacrificed ... [and] the other party ... sacrifices x in order to acquire z (Johnson 1913, pp. 484-486)

While Johnson did not indicate that indifference curves had been subsequently mapped in the now conventional allocation orientation by Irving Fisher (1892, p. 88, Figure 18)⁹ and Pareto, he nevertheless proposed that Edgeworth's

⁷ Morgan (2012, p. 121), however, notes that Marcel Lenoir (1913), not Bowley, was actually the first economist to bring the initial endowment point inside the box.

⁸ Johnson substitutes ‘ z axis’ for Edgeworth's ‘ y axis’ when discussing Edgeworth.

⁹ The southwest portion of Fisher's Figure 18 shows indifference curves that are convex to the origin and a budget constraint, which is a conventional ‘allocation’ orientation. But his indifference curves are not monotonic; and the indifference curve mapped for the highest level of utility is presented in the shape of an oval.

indifference curves should be ‘modified’ or ‘transformed’ into the allocation orientation.

“The modification of Edgeworth’s diagram, which is here proposed, is in effect equivalent to turning them [indifference curves] upside down. ... The immediate results of this transformation are: - (1) That the utility curves are *convex* to both axes, instead of being convex to the sacrifice axis and concave to the acquisition axis. (2) That the values of the two coordinates vary along a utility curve *in opposite senses*, instead of the same sense”¹⁰ (Johnson 1913, p. 488).

While this section of our study was initiated as a clarifying response to the work of Jaffé on Edgeworth, it should be recognised that the significance of Jaffé (1974) derives, in large part, from his independent rediscovery of the main insights into Edgeworth’s indifference curves that Johnson had identified more than sixty years earlier.¹¹ Consequently, this section may be regarded as an extension and development of ideas initially presented by both Johnson and Jaffé, although they are directed strictly towards the goal of establishing whether or not Edgeworth created the original box diagram. Given that goal, distinguishing between ‘allocation’ and ‘trade’ orientation of Edgeworth’s diagrams must commence with reference to his Figure 5, which illustrates pure exchange between dealers A and B of quantities goods X (corn) and Y (beef), because the modern Edgeworth box diagram typically presents the case of pure consumer exchange. The more well-known case of Edgeworth’s Figure 1, which is not limited to pure exchange, is considered in the next Section of the paper.

The ‘trade’ representation of Figure 5 is predicated on the *OXY* plane only illustrating quantities traded when the two dealers share the same origin, which represents the no-trade situation. Allocations associated with those trades are not indicated in Figure 5, but, if they were, and if point O is taken as the same origin of the axes for each dealer when their allocations are represented, then dealer A’s post-trade allocation of *X* and *Y* would be mapped by different coordinates to that of dealer B (except for the case when both dealers attain identical allocations). When pre-trade allocations are known, it is also logically possible to transpose

¹⁰ Johnson is here indicating that, for any individual who engages in trade, an increase (decrease) in *x* must correspond with a decrease (increase) in *y*.

¹¹ Jaffé (1974) does not cite Johnson (1913).

post-trade allocations of X and Y between dealers A and B that result from trade illustrated in Edgeworth's Figure 5, such that they appear as a single coordinate within the modern Edgeworth box diagram (i.e. with each coordinate within the diagram indicating the allocation of goods between the two dealers). But, as established below, Edgeworth himself did not undertake such a transformation.

The Interpretation of Edgeworth's Figure 5

An explanation of Edgeworth's Figure 5 must have regard to his statement that "the curves of indifference consist of a series of circles round a point C" (Edgeworth 1981, 113), which is reasonably assumed to be the bliss point for the dealer under examination (dealer A).¹² Although he did not explicitly indicate the initial endowment of the two dealers, Figure 5 nevertheless implies that the initial endowment of dealer A can be inferred as $\hat{O}_A = (x_{0A}, y_{0A})$, such that $x_{0A} = x_{CA}$ i.e. the initial endowment of good X for dealer A is the same quantity of good X given at the bliss point $C_A = (x_{CA}, y_{CA})$; with y_{0A} falling anywhere between zero and the quantity of Y given at the bliss point i.e. $y_{0A} \in [0, y_{CA})$. Analogously for dealer B, with $\hat{O}_B = (x_{0B}, y_{0B})$, $C_B = (x_{CB}, y_{CB})$, $y_{0B} = y_{CB}$ and $x_{0B} \in [0, x_{CB})$. This situation is illustrated in Diagram F, in which the reciprocal demand curves are obtained as the locus of the best points reachable for any given budget constraint (which is a line with slope equal to the price and passing through the initial endowment).

¹² The alternative, although less likely interpretation under which points C and C' are taken, not as the bliss points, but as the initial pre-trade allocations is presented in McLure (2017).

Diagram F: Reciprocal Demand & Indifference in the Allocation Plane

The reciprocal demand curves shown in Diagram F are semicircles ($\hat{O}_A C_A$ for dealer A and $C_B \hat{O}_B$ for dealer B), as demonstrated by Edgeworth.¹³ The points T_A and T_B indicate the situation reached after the exchange in market equilibrium. Dealer A goes from $\hat{O}_A$ to T_A while dealer B goes from $\hat{O}_B$ to T_B . Market equilibrium requires that these two segments are equal, with the same slope and with opposite direction: a dealer sells what the other dealer buys. Given the goals of this paper, it should be emphasised that points T_A and T_B are not the same coordinate (unless both dealers have identical post trade allocations). The allocation plane of Figure F is not the allocation plane of an Edgeworth box.

Diagram G: Reciprocal Demand and Indifference in the 'Trade' Plane

¹³ This is true if $x_{CA} \geq (y_{CA} - y_{0A})/2$ and $y_{CB} \geq (x_{CB} - x_{0B})/2$. Otherwise, the points on the semicircle with $x_A < 0$ (or $y_B < 0$) disappear and are substitute by points with $x_A = 0$ (or $y_B = 0$).

However, Edgeworth does not determine the market equilibrium starting from the ‘allocation’ plane, as presented in Diagram F. Rather, he starts from the ‘trade’ plane, as indicated by Diagram G. While, in Diagram F, a point represents a commodity bundle (*i.e.*, the quantity of goods referred to a dealer), in Diagram G, a point represents the exchanged quantity of goods. For dealer A, a point (x, y) indicates the quantity x of good X given by dealer A in exchange for the quantity y of good Y. For dealer B, a point (x, y) indicates the quantity x of good X obtained by dealer B in exchange for the quantity y of good Y. The connection between Diagram F and Diagram G is straight forward. For dealer A, point O (no trade) in diagram G corresponds to point $\hat{O}_A$ (initial endowment) in Diagram F, and the orientation of axis OX and $O_A X$ are inverted (since dealer A is selling good X, so that an increase in trade is a decrease in the quantity of good X available for dealer A). Analogously for dealer B, point O (no trade) corresponds to point $\hat{O}_B$ and the orientation of axis OY and $O_B Y$ are inverted. Notice also that dealer A’s bliss point, which has coordinates $C_A = (x_{CA}, y_{CA})$ in the ‘allocation’ plane, is coordinate $C = (0, y_{CA} - y_{0A})$ in the ‘trade’ plane. Analogously for dealer B, the bliss point, which has $C_B = (x_{CB}, y_{CB})$ in the ‘allocation’ plane, is $C' = (x_{CB} - x_{0B}, 0)$ in the ‘trade’ plane.

In regard to dealer A, the area east of original allocation $\hat{O}_A$ (with $\hat{O}_A = O$ in Diagram G) and south of bliss point C [quadrant IV] of the first window of Diagram G maps indifference curves as having a positive slope and being concave to point C. An increase in the relative price ratio is given by an anticlockwise rotation of the price ray from its anchor point at $\hat{O}_A$ as the point of dealer A’s original allocation $(x_{0A}, y_{0A}) = (x_{CA}, y_{0A})$, from which the reciprocal demand curve emerges. In other words, quadrant IV of Diagram G illustrates all the information on dealer A in the same form that it is shown in Edgeworth’s Figure 5. The impact of potential trade on dealer A’s allocation is shown in the first window of Diagram F within the area to the west of original allocation $\hat{O}_A$ and south of bliss point C_A . In the ‘allocation’ plane, all information on dealer A is now a vertical mirror image of that which is shown in the ‘trade’ plane. The indifference curves are now in the standard allocation orientation (convex to the

origin), as is the reciprocal demand curve (increased trade increases the allocation of Y and initially decreases, then increases, the allocation of X) and the price ray (i.e. price increase as the price ray rotates clockwise around the anchor point $\hat{O}_A$). All are all mirror images.

An analogous story emerges for the dealer B in the second windows of these two diagrams. The difference is that the mirror images are no longer vertical but horizontal, because dealer B sells a different good to that of dealer A. Consequently, while dealer A's 'trade' quadrant IV of Diagram G has his or her 'allocation' equivalent given in quadrant III of Diagram F; dealer B's 'trade' quadrant I in Diagram G has its 'allocation' equivalent in quadrant III of diagram F. Edgeworth's line of reasoning underlying Diagram G is illustrated in Diagram H, with reciprocal demand curves being derived as "the locus of the point where lines from the origin touch curves of indifference" (Edgeworth 1881, p. 113).

Diagram H: Derivation of Reciprocal Demand Curves Implied by Figure 5

For illustrative purposes, Diagram H includes three discrete price rays so that ray OB indicates a higher price of X relative to Y than ray OD; and ray OA indicates a higher relative price of X than rays OD and OB. More generally though, anticlockwise rotation of the price ray for dealer A means there is a relative price increase in X; or for clockwise rotation there has been a relative increase in the price of Y. The paths of the reciprocal demand curves for dealer A and dealer B

are consistent with theory.¹⁴ When the market determined price ray is given by OB, then there is market equilibrium and the corresponding trade is represented by point T, since the price is such that the desired exchanges (represented by the segment OT for both dealers) are feasible. Efficiency is attained since both dealers' indifference curves are tangential in point T. With the trade represented by point T the dealers reach the allocations indicated by the points T_A and T_B in Diagram F.

Edgeworth's Figure 5 is obtained by overlaying the two sub diagrams in H, thereby generating Diagram I. The relationship between trade and allocation could be further illustrated by taking Diagram I and laying upon it the 'allocation' orientation for dealer A, from quadrant III window one Diagram F, as well as dealer B's information from quadrant III window two Diagram F. That would reveal that, although a single point T in trade orientation is derived that applies to both dealers A and B, there are two 'allocation' points T, one for dealer A, that is T_A , and another for dealer B, that is T_B .

Diagram I: Edgeworth's Figure 5 with Tangency Indicated

¹⁴ Each dealer's reciprocal demand is a semicircle. Consequently, 'allocation' calibration of the XY axes (Diagram F) suggests that the increases, in the relative market, of a determined price of the good that each dealer sells from near zero to parity (one-to-one exchange) results in the quantity of the good that the dealer is willing to exchange for the other good increasing at a decreasing rate; but a rise in the market determined relative price from unity to near infinity will result in the dealer being willing to sell relatively less of the good that he/she deals in to obtain an increased allocation of the other good.

But it must also be recognised that the ‘allocation’ representation given in Diagram F can be seen inside the ‘allocation’ representation associated with the current Edgeworth box diagram if the quantities associated with the initial endowments are known for each dealer. That may be obtained from Diagram F by drawing a box with sides of length equal to the total quantity of goods and by reversing dealer B’s plane, so that the origin of the axes for dealer A is the southwest corner of the box and that one of dealer B is the northeast corner. On that basis, the initial endowment of the two dealers would be represented by the same point O, which indicates the situation of no trade. On that basis, Figure 5 appears as the vertical mirror image of what is northwest to point O, as represented in Diagram J.

**Diagram J: Allocation Consequences of Edgeworth's Figure 5
Presented as a Modern Box Diagram**

However, the modern Edgeworth box diagram cannot be seen directly from Figure 5 because it only describes trade and the initial endowment of the two dealers cannot be deduced. But allocations can still be represented in the OXY plane of Figure 5 if each dealer's axes are calibrated in the reverse direction to that of the other dealer by altering the origins for each dealer's indifference curves (*i.e.*, O_A and O_B) such that O_A is located south of C' (southeast if $x_{CB} < x_{0A} + x_{0B}$, southwest if $x_{CB} > x_{0A} + x_{0B}$) and O_B is located west of C (northwest if $y_{CA} < y_{0A} + y_{0B}$, southwest if $y_{CA} > y_{0A} + y_{0B}$). On that basis, movement east along the x axis would indicate an increase in dealer B's allocation of x and a corresponding reduction of dealer A's allocation of x ; and a movement north along the y axis

would indicate an increase in dealer A's allocation of Y and a corresponding reduction in dealer B's allocation of Y. More specifically, the coordinates of O_A , which are (0, 0) in dealer A's 'allocation' plane, would be $(x_{0A}, -y_{0A})$ in the 'trade' plane OXY; and those of O_B , which are (0, 0) in dealer B's 'allocation' plane, would be $(-x_{0B}, y_{0B})$ in the 'trade' plane. Diagram K is the resulting representation of allocations in the trade plane OXY. It appears as a vertical mirror image of the modern Edgeworth's box diagram.

Diagram K: 'Allocations' Shown in Edgeworth's Figure 5 with Each Dealer's Axes Calibrated in the Reverse Direction to that of the Other Dealer

For the very particular case in which dealer A's origin is given at point C' (i.e. C' is coordinate 0,0 for dealer A) and dealer B's origin is given at point C (i.e. C is coordinate 0,0 for dealer B), the representation of allocations in diagram K would be identical to that of Edgeworth's Figure 5. But that does not correspond to Edgeworth's treatment of Figure 5, not just because it requires the that dealer A must initially have all x and dealer B initially have all y , but because it also requires the bliss points for both parties, as shown in Diagram F, to be identical (i.e. it requires $C_A = C_B$ in Diagram F)¹⁵. But once those propositions are imposed, Edgeworth's 'trade' representation of consumer exchange would have exactly the same shape as an open axes 'box' diagram if the origin for the mapping of indifference curves shifts from a single point – from the point O that

¹⁵ That is, dealer A must be able to achieve satiation in good y by obtaining all of quantity y held by dealer B in the pre-trade state; and dealer B must be able to achieve satiation in good x by obtaining all of quantity x held by dealer A in the pre-trade state; such that $x_{CB} = x_{CA}$ and $y_{CB} = y_{CA}$ in Figure F.

Edgeworth used for his mapping ‘trade’ indifference curves for both parties – to two diagonally opposite points, $(x_{CA}, 0)$ in the southeast of the diagram for dealer A; and $(0, y_{CB})$ in the northwest, for dealer B.

4) Interpretations of Edgeworth’s Figure 1

The Friday-Crusoe scenario being illustrated by Edgeworth’s Figure 1 concerns ‘labour given by Friday’ and ‘remuneration given by Crusoe’. More specifically, it concerns contracted exchange of ‘labour’ for the purpose of cultivating abundant land, with Crusoe providing superintendence of that labour as well as the knowledge of cultivation.¹⁶ Friday also contributes knowledge to Crusoe on local conditions.¹⁷

Perhaps the main point of the Friday-Crusoe exchange is that Friday’s exertion of the tiresome effort associated with labouring is an economic bad to Friday (there is a marginal disutility of work) but an economic good to Crusoe (an indirect indicator of the positive marginal utility that Crusoe gains from the production of Friday’s labour). Conversely, payment of remuneration is an economic bad to Crusoe (marginal disutility from giving up the remuneration good) but an economic good to Friday (marginal utility gained from gaining the remuneration good). As a result, Edgeworth’s Figure 1 is not just an illustration of Jevons’s ‘theory of exchange’, it also extends to Jevons’s ‘theory of labour’ (Jevons 1888 [1871]).

The indifference curves in Edgeworth’s figure are, like those of his Figure 5, drawn in trade orientation. Friday’s trade indifference curve $0\eta_0x_0$ is positively sloped, which confirms the trade orientation, and concave to the x axes because of

¹⁶ Crusoe: “...having two mouths to feed, instead of one, I must provide more ground for harvest, and plant larger quantities of corn than I used to do; so I market out a larger piece of land, ...which Friday not only worked very willingly, and very hard; but did it very cheerfully ... [He] let me know that he thought I had much more labour upon me on his account, than I had for myself; and that he would work the harder for me, if I would tell him what to do” (Defoe 1719 [2001, p. 168]).

¹⁷ “I asked Friday a thousand questions about the country, the inhabitants, the sea, the coast, and what nation were near; and he told me all he knew with the greatest openness imaginable. (Defoe 1719 [2001, p. 170])

the increasing disutility of work and diminishing marginal utility of remuneration. Crusoe's indifference curve $0y_0\zeta_0$ is also positively sloped, again confirming trade orientation, but it is concave to the y axis because of the increasing disutility of paying remuneration and diminishing marginal utility of Friday's labour.

But in the case of Edgeworth's Figure 1, we must also distinguish between two potential cases within the Friday-Crusoe scenario. The former case involves Crusoe drawing on his existing stock of corn to remunerate Friday for his labour. This is referred to below as the 'remuneration from previous production' case. The latter case involves Crusoe remunerating with corn actually produced by Friday's work, which is referred to as the 'remuneration from current production' case. The latter case would appear to be the more important of the two to the extent that the Friday-Crusoe scenario is intended to address an ongoing arrangement that is repeated from period to period over time. It is also consistent with Edgeworth's characterisation of the indifference curve for Friday (and implicitly that of Crusoe) as indicating zero utility from exchange.¹⁸

In regard to the 'remuneration from previous production' case, the starting point for transforming the 'trade' representation diagram into the 'allocation' representation is to transform goods with a mix of negative and positive marginal utility into goods that only have positive marginal utility to both dealers. In the case of Friday, that can be achieved simply enough by substituting, on axis OY, his labour with the quantity of Friday's free time and by maintaining, on axis OX, the quantity of corn. In the case of Crusoe, it can be achieved by maintaining, on axis OY, the quantity of Friday's labour and by substituting, on axis OX, the quantity of corn given to Friday with the quantity of corn (produced in previous times) that remain to Crusoe after the payment of Friday's labour. Consequently, in the 'allocation' planes, we have, for Friday, the plane $O_F X_F Y_F$, where x_F indicates the quantity of corn and y_F the quantity of free-time, thus with

¹⁸ "The utility of the contract represented at $\eta_0 x_0$ is for Friday zero, or rather, the same as if there was no contract." (Edgeworth 1881, p. 29). It may be that Edgeworth had in mind an ongoing arrangement in which Crusoe's initial stock of corn is maintained from season to season, but, within the season, remuneration is met from Crusoe's stock of corn but replenished upon harvest. On that basis, the constant ongoing stock of corn between seasons may, as an approximation, be set aside as Friday's is still effectively producing his own remuneration, albeit subject to a one season lag.

indifference curves convex with respect to O_F . For Crusoe, the plane $O_C X_C Y_C$, where x_C indicates the quantity of corn produced in previous times that remains available to Crusoe and y_F the quantity of Friday's labour (which is useful to Crusoe), thus with indifference curves convex with respect to O_C . The initial endowment of Friday is $\hat{O}_F = (0, T_F)$, *i.e.*, nil quantity of corn and T_F quantity of free time (for instance, 24 hours per day). The initial endowment of Crusoe is $\hat{O}_C = (T_C, 0)$, *i.e.*, the quantity of corn T_C produced in previous times and owned initially by Crusoe and nil quantity of Friday's time.

Having transformed both goods (labour and remuneration) into goods with positive marginal utility for both individuals (leisure and remuneration for Friday; and Friday's labour and his existing stock of corn, for Crusoe), almost all that is indicated above with respect to Figure 5 can be extended Figure 1. The only differences are that the assumption that the curves of indifference being circles round a bliss point no longer applies; and that the initial allocations for Friday and Crusoe are the allocations indicated above. With reference to Diagrams F and H, the situation of Friday is comparable with that of dealer B, since they are both selling good Y, and the situation of Crusoe is comparable with that of dealer A, since they are both selling good X. Consequently, all information on Friday is now, in the allocation plane, a horizontal mirror image of what is shown in the 'trade' plane and a vertical mirror image for Crusoe.

By the same reasoning, the conversion of remuneration and labour into two goods with positive marginal utility for both Friday and Crusoe in this case also means that the diagrams equivalent to Diagrams F and H for Edgeworth's Figure 1 can be transformed into a box type diagram. That is, Edgeworth's 'trade' oriented Figure 1 can be transformed into an 'allocation' oriented box diagram without any change to the structure of the figure itself if the implied origin for the mapping of the indifference curves were shifted from a single point, from point O in Edgeworth's trade oriented diagram, to two diagonally points such that Friday's indifference curves are mapped to an implicit point in the northwest and Crusoe's curves are mapped to an implicit point in the southeast. That would lead to a reversing of the calibration of the X and Y axes for Friday and Crusoe

respectively and coordinates within the OXY allocation plane would then take on the meaning of coordinates within a box diagram (i.e. each coordinate would signify the allocation of goods X and Y between Friday and Crusoe).

In regard to the case of ‘remuneration from current production’, however, there are two reasons why it is impossible to interpret Figure 1 as a representation of a pure exchange economy, as done in the former case analogously to Figure 5. The first reason is that in this economy there is only one good for Crusoe, which is the current production of corn that remains to him after the payment of Friday’s labour. In the previous case, where there is an ‘initial stock of corn’ that is depleted over time, two goods were available to Crusoe: the corn produced in previous times and the corn produced presently, of which Friday’s labour-time is an indicator (as represented by the production function). But in the situation where there is only corn produced presently, we are unable to draw indifference curves in a plane for Crusoe as consumer. The second reason is that if the initial endowment for Crusoe is a nil quantity of corn, what he initially owns is the possibility of production (we can say that he owns a production function). Thus, we are outside the representation of a pure exchange economy: the total quantity of corn to be allocated between Friday and Crusoe depends on current production, which has to be determined and it is not a given quantity as in a pure exchange economy. Therefore, the current Edgeworth’s box diagram does not apply in the ‘no initial stock of corn’ case. In that case, Edgeworth’s Figure 1 needs another explanation, which must involve the choice in relation to production.

5) A Reconstruction of Edgeworth’s Figure 1

A problematic feature of Edgeworth’s Figure 1 is that quantity y serves as a direct indicator of Friday’s labour and an indirect indicator of Crusoe’s consumption. Consequently, the production of x is only given implicitly and not explicitly. In this reconstruction, that is redressed by defining Crusoe’s utility function and replacing his indifference curves with, what Pareto called, ‘indifference lines of obstacles’ (1906 and 1909 [2014, p. 88]) that incorporates a production function.

For the purpose of this reconstruction, x_F is the quantity of basic remuneration good X (e.g. food) that Crusoe exchanges with Friday, in the form of remuneration for his labour services supplied; and x_C , the quantity of x that is allocated to Crusoe as profit, *i.e.*, the residual of total quantity of good X after Friday has been paid, such that $x_F + x_C = x$. The quantity of ‘labour’ that Friday provides direct to Crusoe is defined as y , which is equal to or greater than 0 hours per day but less than the maximum of 24 hours per day. On that basis, the relationships of indifference in trade are given as follows: Friday’s relationship of indifference is given between x_F and y (curve $O\eta_0x_0$ is one of his indifference curves); and Crusoe’s indifference between x_C (profit obtained from Friday’s labour) and y (Friday’s labour), which is represented by ‘indifference lines of obstacles’, which are now perhaps more commonly known as ‘iso-profit’ lines (the zero profit line is represented by $O\eta_0x_0$ in Edgeworth’s Figure 1).

Friday’s utility Π is represented by the function $\Pi = \Phi(x_F, y)$, with utility increasing with respect to x_F and decreasing with respect to y . Therefore, when his utility function is mapped into Edgeworth’s Figure 1, Friday’s indifference curves are convex to the y axis and the indifference curve relative to the initial situation is the curve $O\eta_0x_0$. That is, it is a ‘trade’ flow indifference curve relative to the origin point O. For readers who have a strong preference to use the standard ‘allocation’ orientation of indifference curves, they may view the indifference curves as if they were mapped with respect to the initial endowment in the northwest region (for Friday) and the south east region (for Crusoe).

While Crusoe’s utility P is given by the single variable function $P = F(x_C)$, his ‘indifference line of obstacles’ that corresponds to zero profit is mapped to the XY ‘trade’ plane as the curve from O to $y_0\zeta_0$, but the welfare implications from consumption are confined to the x axis. However, as that curve passes through the origin, the curve from O to $y_0\zeta_0$ also indicates total production of X , from which Crusoe receives none *i.e.* $x_C = 0$. In other words, this indifference line of obstacles corresponds to the production function $x = f(y)$, for which

production is increasing ($f'(y) > 0$) at a decreasing rate ($f''(y) < 0$) and is therefore convex to the (x axis) abscissa. It equates with the post-production allocation all the possible production of x for y on the basis that $x_C = 0 = f(y) - x_F$. The other indifference lines of obstacles, or iso-profit lines, may then be represented by the function $x_F = f(y) - x_C$, where x_C indicates Crusoe's profit given as a quantity of the produced good. That is, Crusoe's welfare increases as the amount of Friday's product which he retains increases.

The contract curve CC' is the locus of points for which Friday's marginal rate of substitution of y for x_F equates with Crusoe's marginal rate of substitution of output for wages, the latter being given by the marginal product of labour:

$$-\frac{\frac{\partial \Pi(x_F, y)}{\partial y}}{\frac{\partial \Pi(x_F, y)}{\partial x_F}} = \frac{df(y)}{dy}$$

If a market were to exist, efficient exchange/production in Edgeworth's Figure 1 would be given by the intersection between *reciprocal demand curves* and the contract curves at point η^ζ . That would require an equilibrium price ratio $x_e y_e$, as given in a market under free exchange. As indicated in Diagram L, when Friday provides y labour to Crusoe, total output is x ; with quantity x_F being used to pay remuneration to Friday; and with the residual product $x - x_F$ being retained by Crusoe as x_C .

Diagram L: Edgeworth's Figure 1 with Production

Friday's *reciprocal demand curve* derives from utility maximisation subject to the constraint that his consumption must be equal to the product of his hourly wage rate (w) and his total hours of work. That is, $\max_{x_F, y_c} \Pi(x_F, y)$ subject to $x_F = wy$, the first order conditions of which are $\frac{\partial \Pi(x_F, y)}{\partial x_F} w + \frac{\partial \Pi(x_F, y)}{\partial y} = 0$ and $x_F = wy$; and they result in the following condition for the derivation of Friday's *reciprocal demand curve*:

$$-\frac{\frac{\partial \Pi(x_F, y)}{\partial y}}{\frac{\partial \Pi(x_F, y)}{\partial x_F}} = \frac{x_F}{y}$$

Crusoe's *reciprocal demand curve* is, however, derived from the conditions of profit maximisation (his utility is maximised when profits are maximised). That is, $\max_y f(y) - wy$, which has the first order condition that the wage rate corresponds with the marginal product of labour, $\frac{df(y)}{dy} = w$, and, when the relation $x_F = wy$ is taken into account, the conditions for Crusoe's *reciprocal demand curve* are given by the equation:

$$\frac{df(y)}{dy} = \frac{x_F}{y}$$

As in the case of Edgeworth's Figure 5, and the associated diagrams F and G presented earlier, 'trade' and 'allocation' orientations can also be presented for the reconstructed Figure 1 although they have not been included in the paper due to a lack of space. But the essential point is that the allocation orientation for the reconstruction of Edgeworth's Figure 1 proposed in this Section cannot be transformed into a box diagram because the total production of corn depends on the amount of labour employed and this amount is not constant when we pass from one given point to another point in the diagram. On the contrary, in the pure exchange box diagram the total quantity of goods is given and the box is not modified if we pass from a point to another in the diagram. In other words, a

given ‘box’ is not definable in the case of production. Therefore, if production is involved and Friday is paid with corn currently produced, then Edgeworth’s Figure 1 cannot accommodate a box representation.

6) A New History of the Edgeworth Box

A new early history of the Edgeworth box is required as a result of the account of Edgeworth’s Figures presented in this paper. Of course, Edgeworth’s Figures 1 and 5 must both feature prominently in the history of the creation of the Edgeworth box as a diagram, so there can be no serious suggestion that it is incorrect to name the diagram in honour of Edgeworth. But, just as unequivocally, the trade orientation of his diagrams and his account of those shows that he did not present box representations in an allocation sense or explain what is required to transform his trade representations into box diagrams in ‘allocation’ representation. Furthermore, the labour production aspect of Edgeworth’s Figure 1 is not amendable to incorporation in a ‘box’ diagram (and impossible to incorporate in a ‘box’ diagram when some or all of Friday’s remuneration is paid from current production).

As Pareto was an admirer of Edgeworth’s *Mathematical Psychics*¹⁹, it is certain that he was influenced by Edgeworth in some contextual way when the box diagram was being devised. That contextual influence is evident from Pareto’s polemic with Gaetano Scorza over the first law of welfare economics, which has been discussed by John Chipman (1976) and McLure (2000). Specifically, in ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, Pareto (1902 [2008]) presented the first proof of the first order conditions for the first law of welfare economics in a world with exchange and production. He then introduced an algebraic and numeric example to illustrate the concepts and, in the process, he provided, for the first time, an

¹⁹ Pareto’s entry on “Economie Mathématique” for the *Encyclopédie des Sciences Mathématiques* appears to follow the characterisation of endowment and exchange set out in Edgeworth’s work where he introduces the case of two commodities, X and Y. Pareto then makes Edgeworth’s influence explicit: “Some problems of this type are treated by F. Y. Edgeworth, *Mathematical Psychics*, London 1881. This work is important and remarkable in many respects, especially for the period when it was written.” Pareto (1911 [1955, p61]).

illustrative image that explicitly shows the association between efficiency and the point of tangency between two indifference curves for two individuals in an allocation plane.

Diagram M: Modified Figure from Pareto 1902 [2008, p. 532]

Pareto’s original graph only includes axes for individual 1 from origin O_1 , with u_{1x} indicating the quantity of x and u_{1y} indicating the quantity of y . But he states: “Let us draw on a single diagram the indifference curves of individual 1 and those of individual 2, assuming of course that the coordinate axes are parallel” (Pareto 1902,[2008, p. 531]). Diagram M modifies Pareto’s original graph by adding the implicit axes for individual 2, with u_{2x} indicating the quantity of x and u_{2y} indicating the quantity of y for that individual, which are parallel to those of individual 1.²⁰ The initial allocation for individual 1 is indicated by $\hat{O}_1$ and the initial allocation for individual 2 is indicated by $\hat{O}_2$. The final allocations for individuals 1 and 2 are represented by point m in their respective allocation maps. Individual 1 passes from $\hat{O}_1$ to m and individual 2 does from $\hat{O}_2$ to m , with segments $\hat{O}_1m$ and $\hat{O}_2m$ having the same length and slope but the opposite direction.

²⁰ In addition, the initial allocations are indicated by $\hat{O}_A$ and $\hat{O}_B$ on Diagram M; and the dotted ray tms is not been drawn beyond point s , in accordance with Pareto’s discussion of the diagram, though his original figure presents that ray as extending northeast of point s until it touches the u_{1x} axis.

Pareto's main purpose for introducing Diagram M was to illustrate that, with the price ray under free competition given by the ray between $\hat{O}_1$ and $\hat{O}_2$ economic efficiency, in terms of the Pareto criterion, is attained when the indifference curves for the two individuals are tangential to each other. If the two individuals were to then undertake exchange under a different price ratio, both individuals will move away from point m . If individual 1 passes from m to s , resulting in an increased allocation of x and a decreased allocation in y for that individual (indicated by $+x$ and $-y$ in Diagram M), and individual 2 passes from m to t , necessarily resulting in a reduced allocation of x and an increased allocation in y because segments ms and mt must have the same length and slope in the opposite direction. In that case, individual 2 is made worse off in welfare terms and the welfare of individual 1 is enhanced. Alternatively, if individual 1 were to pass from point m to t and individual 2 passes from m to s , both individuals would be made worse off in welfare terms.

But Pareto's 1902 diagram also represents an intermediate image in the history of the Edgeworth box because it is fundamentally an extension of Edgeworth's Figure 5. While the indifference curves are set in an allocation plane when viewed with respect to the origin for each individual, the diagram is drawn upside down and the origin is in the northwest corner. Consequently, it would be viewed in a trade plane if it were considered with respect to the southwest corner of the graph, where the origin of a graph is typically located). Importantly, from Diagram P it was possible for Pareto to subsequently notice that the indifference map and allocation plane for individual 1 can be rotated 90° anticlockwise, while those for individual 2 can be rotated 90° clockwise; and they can then be superimposed over each other to create a box diagram with each individual's initial endowment points $\hat{O}_1$ and $\hat{O}_2$ coalesced into a single point. Such a diagram shows that, for a price ray given under free competition, the efficient allocation is the one given by the point of tangency between the two individuals' indifference curves. And that is what Pareto illustrated in the box diagram presented in the 1906 *Manual*.

On balance, then, the diagrammatic history of Edgeworth's and Pareto's contributions to the development of the Edgeworth box starts from Edgeworth's Figures 1 and 5, because they illustrate the concepts of contract curve and 'trade' oriented indifference curve on an open XY diagram with single origin given by point O. It progresses with Pareto's 1902 Figure from 'On a New Error in the Interpretation of the Theories of Mathematical Economics', which highlights efficient allocation between two individuals with indifference curves presented in 'allocation' orientation. The story culminates in Pareto's Figure 16 (as well as his figure 50) from his *Manual of Political Economy*, which is a development of his 1902 graph. The 1906 figure is a box diagram with one individual's curves mapped with respect to the southwest corner and the other's mapped with respect to the northeast corner, and every coordinate within the box indicating a unique allocation of two goods between two individuals.

7) Conclusion

Edgeworth and Pareto must both feature prominently in the history of the Edgeworth box diagram. Edgeworth introduced the key economic concepts of indifference and contract curves and his trade representations were suggestive of how allocation issues may be dealt with. He did not develop the box diagram *per se*. That is, he did not indicate how the calibration of the x and y axes must be redefined, or explain where the 'origins' for indifference mapping must be relocated to different points for each individual, for his 'trade' orientated Figure 5 to also acquire the meaning and significance of open axes 'box' diagram in allocation representation. But he provided a platform from which Pareto could move, tentatively in 1902, and then definitively in 1906, to introduce the box diagram.

But the historical importance of Edgeworth's Figure 1 does not depend entirely on its status as the original box diagram or as a precursor to that diagram. The insights of Edgeworth's Figure 1 for the study of labour exchange were profound – and that is now explicit in the reconstruction presented in this paper – and are lost if that diagram is viewed as if it were a box diagram.

References

- Alcouffe, Alain and Alcouffe, Christiane 1998. "Edgeworth, l'Utilitarisme et l'Egalité", in 1998. *Mélanges en l'honneur du professeur Jean Vincens*, Toulouse: Presses de l'Univ. des Sciences Sociales de Toulouse, <https://www.archives-ouvertes.fr/hal-01312389/document> (accessed 27 October 2017).
- Allais, Maurice 1975. "The General Theory of Surplus and Pareto's Fundamental Contribution", *Convegno Internazionale Vilfredo Pareto (Roma 25-27 ottobre 1973)*, *Atti dei Convegni Lincei* 9, Roma: Accademia Nazionale Dei Lincei, 109-163; Reprinted in J. C. Wood and M. McLure (eds), 1999, *Vilfredo Pareto: Critical Assessments*, London: Routledge, Vol. 1, 428-473.
- Amoroso, Luigi 1942. *Meccanica Economica*, Citta di Castello: Macri.
- Bowley, A. L. 1924. *The Mathematical Groundwork of Economics*, Oxford: Clarendon.
- Blaug, Mark 1984. *Economic Theory in Retrospect*, 4th Edition, Cambridge, Cambridge University Press
- Blaug, Mark (ed.) 1993. *Alfred Marshall (1842-1924) and Francis Edgeworth (1845-1926)*, Aldershot: Edward Elgar.
- Chipman, John S. 1976. 'The Paretian Heritage', *Revue Européenne des Sciences Sociales*, no.37, 65-173; reprinted in J. C. Wood and M. McLure (eds) (1999), *Vilfredo Pareto: Critical Assessments*, London: Routledge, Vol. 2, 157-257.
- Creedy, John 1980. "Some Recent Interpretations of Mathematical Psychics", *History of Political Economy*, 12(2), pp 267-276.
- Creedy, John 1986. *Edgeworth and the Development of Neoclassical Economics*, Oxford: Basil Blackwell.
- Creedy, John 2010. "The Edgeworth box", in Mark Blaug and Peter Lloyd (eds) *Famous Figures and Diagrams in Economics*, Cheltenham: Edward Elgar.
- Debreu, G. and H. Scarf, H. 1963. "A Limit Theorem on the Core of an Economy", *International Economic Review*, 4(3), Sept., pp. 235-246.

- Defoe, Daniel 1719 [2001] *The Life and Strange Surprising Adventures of Robinson Crusoe of York, Mariner*, Penguin Classics: London.
- Dobb, Maurice 1969, *Welfare Economics and the Economics of Socialism*, Cambridge: Cambridge University Press.
- Edgeworth, F. Y. 1881. *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences*, London: C. Kegan Paul and Co.
- Fisher, Irving 1892 [1925]. *Mathematical Investigations in the Theory of Value and Prices*, New Haven: Yale University Press.
- Gram, Harvey 1987. "Offer Curve/Reciprocal Demand Curve", in J. Eatwell, M. Milgate, and P. Newman (eds.), *The New Palgrave: A Dictionary of Economic Theory and Doctrine*, Macmillan: London, 1987, pp. 694-697.
- Hildenbrand, W. 1993 . "Francis Ysidro Edgeworth: Perfect Competition and the Core", *European Economic Review*, 37, pp. 477-490
- Humphrey, Thomas M. 1996. "The Early History of the Box Diagram". Federal Reserve Bank of Richmond, *Economic Quarterly*, 82(1), pp. 37-75.
- Jaffé, William 1972. "Pareto Translated: A Review Article", *Journal of Economic Literature*, 10(4), pp. 1190-1201.
- Jaffé, William 1974. "Edgeworth's Contract Curve: Part 1. A Propaedeutic Essay in Clarification" *History of Political Economy*, 6(3), pp. 343-359.
- Jevons, W. S. 1888 [1871] *Theory of Political Economy*, 3rd edition, London; Macmillan.
- Johnson, W. E. 1913. "The Pure Theory of Utility Curves", *Economic Journal*, 23(92), pp. 483-513.
- Lenoir, Marcel 1913. *Etudes sur la Formation et le Mouvement des Prix*, M. Giard et E. Brière.
- McLure, Michael 2000. 'The Pareto-Scorza Polemic on Collective Economic Welfare', *Australian Economic Papers*, 39(3), 345-369.
- McLure, Michael 2017 'Thinking Outside the Box: A New History of Edgeworth's and Pareto's Development of the Box Diagram" *University of Western Australia, Economics Program, Discussion Paper Series DP 17.12*.
- Morgan, Mary S. 2004. "Imagination and Imaging in Model Building", *Philosophy of Science*, Vol. 71, No. 5, pp. 753-766.

- Morgan, Mary 2012. “The History of the Edgeworth Box Diagram – as Told by Itself”, in *The World in the Model: How Economists Work and Think*, Cambridge: Cambridge University Press, pp. 106-117.
- Pareto, Vilfredo 1902. ‘Di un Nuovo Errore nello Interpretare le Teorie dell’Economia Matematica’ *Giornale degli Economisti*, 12(15), 401-33; translated into English by John Cairncross (and edited by John Chipman) as ‘On a New Error in the Interpretation of the Theories of Mathematical Economics’, *Giornale degli Economisti e Annali di Economia*, 67(3), December (2008), 515-544.
- Pareto, Vilfredo 1906. *Manuale di Economia Politica*, Milan: Società Editrice Libreria; Revised and translated in French as *Manuel d’Économie Politique* (1909), Paris: Giard et Brière; translated from the French edition by A. Schwier and A. Page as, *Manual of Political Economy*, (1971), New York: Augustus M. Kelly; with both the Italian and French editions translated into English as, *Manual of Political Economy: A Critical and Variorum Edition*, edited by A. Montesano, A. Zanni, L. Bruni, J. S. Chipman and M. McLure, (2014), Oxford: Oxford University Press.
- Pareto, Vilfredo, 1911. ‘Economie Mathématique’, in the *Encyclopédie des Sciences Mathématiques*, Gauthier-Villars, Paris, pp. 591-641; Translated by J.I. Griffin as “Mathematical Economics”, 1955, in *International Economic Papers*, 5, London: Macmillan, pp. 58-102.
- Schneider, Erich 1952. *Pricing and Equilibrium*, London: William Hodge, 1952.
- Shackle, G. L. S. 1967. *Years of High Theory: Invention and Tradition in Economic Thought 1926–1939*, Cambridge: Cambridge University Press.
- Söllner, Fritz 2016. ‘The Use (and Abuse) of Robison Crusoe in Neoclassical Economics’, *History of Political Economy*, 48(1), pp. 35-64.
- Tarascio, Vincent J. 1972. “A Correction: On the Genealogy of the So-Called Edgeworth-Bowley Diagram.” *Western Economic Journal* 10, p. 193-97.
- Tarascio, Vincent J. 1980. Some Recent Interpretations of Mathematical Psychics: a Reply”, *History of Political Economy*, 12(2), pp. 277-281.
- Weatherby, James L. 1976. “Why was it Called an Edgeworth-Bowley Box? A Possible Explanation”, *Economic Inquiry*, 14, June, pp. 294-296.

Zanni, Alberto 2014. “E. N. 19 Edgeworth Box and Pareto Box” in Vilfredo Pareto *Manual of Political Economy: A Critical and Variorum Edition*, edited by A. Montesano, A. Zanni, L. Bruni, J. S. Chipman and M. McLure, (2014), Oxford: Oxford University Press, pp 541-545.

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