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THE VALLEY OF DEATH FOR NEW ENERGY TECHNOLOGIES

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1 Introduction

More than 90% of the world's primary energy currently is supplied by fossil fuels, while more than 8% comes from nuclear power and hydroelectricity. Thus, despite the recent publicity for energy sources such as wind, solar, geothermal or biofuels, they provide only a tiny fraction of the world's energy, and even then mainly as a result of subsidies. On the positive side, large-scale energy production from non-hydroelectric renewable sources has at least become technologically feasible.

One of the commonly cited reasons why new energy technologies have had difficulty gaining commercial viability is the so-called “valley of death.” According to Markham et al. (2010), the phrase “valley of death” was first used in 1995 to refer to the challenges of transferring agricultural technologies to Third-World countries. It was later applied to describe a paucity of funding for the commercialization of new technologies relative to the funds available for more basic R&D.

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The more academic literature¹ on the valley of death concept, especially as applied to the energy industry, is reviewed in the next section of this paper. Markham et al. provide a simple illustration of the notion, repeated as Figure 1. They claim that while adequate resources are available during the basic research phase, available resources often drop precipitously once the basic research has been completed. If an idea makes it through the valley of death to prove commercial viability, however, once again ample resources are available to take the idea to market.

Figure 1: Illustration of Valley of Death Concept after Markham et al. (2010)

Most of the papers we reviewed assumed, either explicitly or implicitly, that the valley of death reflects a misallocation of resources. Much of their focus is on what types of government policies might be most effective at alleviating the presumed problem. Apart from Beard et al. (2009), the authors were content with a rather intuitive and imprecise description of the phenomenon.

The critical question to answer is why entrepreneurs fail to exploit apparently profitable opportunities in the middle of the new product development cycle while at the same time they are willing to finance R&D at later stages of the process. The analysis by Beard et al. provides the most explicit answer to this question. As we explain in the next section, they show that subsidies at stage 1, but not later stages, of the R&D process are a necessary condition. In effect, relative to the

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¹The concept has also been much discussed by consulting firms and government organizations concerned with funding of industry R&D, especially in the context of renewable energy technologies.

free market outcome, the subsidies provide an excess supply of projects that have completed stage 1. When the government subsidies are no longer available in stage 2, the proportion of projects taken up at that stage reverts to a lower level. However, projects successfully transiting stage 2 are taken up at a normal commercial rate in the commercialization phase. In effect, Beard et al. interpret the vertical axis in Figure 1 as the *proportion* of projects funded at each stage of the process (or the *probability* that any one project at that stage will receive funding).

Beard et al. note that the existence of a valley need not imply an inefficient outcome. For example, the basic research undertaken in stage 1 of the process may yield spillover social benefits that cannot be appropriated by the researcher. In the absence of subsidies, the amount of R&D might be too low from the social point of view. However, the more applied, and patentable, research at later stages might not have the same problems. The efficient policy would then have subsidies for stage 1 R&D only, even though one consequence is a “valley” of reduced probability of receiving funding at stage 2. While the existence of a valley of death need not indicate an inefficient allocation of resources, Beard et al. suggest that it most likely does. Certainly, almost all the equilibrium outcomes they discuss in their paper are not efficient.²

Weyant (2011) argues that companies backed by venture capital do not generally focus on basic R&D since venture capitalists are less willing to assume technological than institutional risks. He notes that while large energy companies could fund some of the R&D, regulations may reduce the ability to recover costs and investors may react negatively to higher perceived risk. The inability to appropriate some of the benefits of R&D will also constrain private investors.

Another issue that Weyant highlights arises from the work of Schumpeter (1942) and especially his concept of “creative destruction.” New technologies will make old ones obsolete and Weyant suggests that existing firms in energy related industries may have strong incentives to delay this process if their current technologies are more profitable than the new ones. Weyant suggests that the likelihood of this happening is greater if the industries are oligopolistic or imperfectly regulated. He says that some of the key articles that have examined the relationship between oligopolistic

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²Weyant (2011) is the only paper we encountered that raises the opposite concern as a possibility. He observes that “technologies that are not likely to be remotely economically competitive (or maybe not even technically feasible) at commercial scale have often nonetheless been pursued at great expense, through a pernicious combination of political self-seeking and technological over optimism.”

competition and the pace of technological innovation show that the relationship could in theory go either way. He claims that in practice, however, imperfect competition is more likely to slow technological progress through either explicit or tacit collusion.

This paper makes several contributions to the literature on the valley of death concept as applied to new energy technologies.

First, we look at the issue in the context of a genuinely dynamic intertemporal model of the displacement of fossil fuel energy technologies by non-fossil alternatives. The model distinguishes between investment in energy industry R&D and investment in the physical capital required to produce the energy services used elsewhere in the economy. Since the physical capital used to supply energy services from fossil fuels has to be displaced by a different set of capital facilities used to supply energy services from non-fossil sources, our model highlights the “creative destruction” element mentioned by Weyant.

A second distinguishing feature of our analysis is that we allow for progress in the fossil fuel technologies as well as the alternatives. The recent turnaround in the trend of oil and natural gas production in the US as a result of the unconventional oil and gas revolution is just the latest example of technological progress in the fossil fuel producing or consuming industries. Earlier examples included the development of 4-D seismic, the ability to use seismic to see beneath salt layers, deepwater drilling, the production of natural gas from tight sands and coal seams, the development of techniques to refine heavy oil, the invention of combined cycle natural gas fired power stations, the use of computer technology to control gasoline engines and raise their fuel efficiency, and many more. These technological changes offset what would otherwise be a rising cost of energy services produced from fossil fuels and make it harder for non-fossil alternatives to compete.

The third novel feature of our analysis in the context of the literature on the valley of death in developing new energy technologies is that we do not assume that reductions in the cost of supplying renewable energy services result from explicit R&D alone. Instead, following much of the empirical literature investigating technological progress in the renewable energy sector, we assume that learning by doing also contributes to cost reductions. In fact, we assume a two-factor learning model, whereby direct R&D expenditure can accelerate the accumulation of knowledge about the renewable technology. Following estimates in a paper by Klaassen et. al. (2005), we assume that

direct expenditure on R&D is roughly twice as productive as is learning-by-doing in lowering the cost of supplying renewable energy services.

Finally, and most importantly, we give a new interpretation to the “valley of death” notion. Following previous authors we associate the early stage of the process as consisting largely of R&D expenditure. However, we associate the “commercialization” phase with the need to build physical capital in order to supply energy services using the alternative technology. The issue that our model highlights is that investment in new energy technologies is required before fossil fuels are abandoned so the supply of energy services can continue uninterrupted. However, capital used to produce energy services from fossil fuels is a sunk cost, so it will continue to be used so long as the price of energy is sufficient to cover short-run operating costs. Thus, from the time that investment in capital used to produce energy services from fossil fuels ceases until the time fossil fuels are abandoned, the operating cost of fossil fuel production sets the energy price. Furthermore, at the switchover point, the price of energy just matches the operating costs of renewable energy production and prior to that time the price of energy is insufficient to cover even the operating costs of renewable energy production, let alone providing a competitive rate of return to the capital employed. In fact, we show that the full long-run costs of renewable energy (including a competitive rate of return) are not covered until some time after fossil fuels are abandoned.

We stress that the paths of investment, technological progress and energy price that we calculate in the model are efficient (they solve the Pareto optimum problem). Whether those paths could be implemented in a competitive equilibrium is an entirely different matter. In particular, a private firm investing in capital to supply renewable energy services would have to accept an energy price that is below the long-run cost of supplying renewable energy for a long time period. Conceptually, the return from lower costs as a result of technological change ought to pay for some of the costs of investing in renewable energy technology and productive capital. In practice, however, some of the benefits to R&D may be external to the firm and not lead to appropriable returns. This could exacerbate the problems of making the new technologies competitive with fossil fuels. Alternatively, if government were to subsidize R&D into alternative energy technologies, it could lead to an overhang of “first stage” research projects as discussed by Beard et al. (2009). Strictly speaking, however, to investigate this matter further one would need to examine equilibrium in an explicit decentralized model with various assumptions about the appropriability of the benefits of R&D

and different amounts of subsidization of research.

Some of the literature on the valley of death in technology industries has noted that the phenomenon does appear to be present in some sectors such as pharmaceuticals or information technology. One reason why these may differ from the energy sector is that patents and copyrights might be more effective at enforcing property rights in the pharmaceutical and IT industries. However, our analysis suggests another reason. Once a new drug has been discovered or invented, it often can be produced at very low marginal cost.³ Similarly, the marginal cost of reproducing, distributing, marketing and supporting software is often much lower than the cost of writing it in the first place. By contrast, very large capital investments are required after the R&D phase in order to supply energy services using new energy technologies. Our model points to potential difficulties in financing these investments in productive capital before fossil fuels are no longer competitive.

Our model abstracts from some real world complicating factors. Imperfect substitutability between energy from fossil and non-fossil sources allows non-fossil sources to be more competitive under special circumstances – for example, solar panels are competitive without subsidies in remote locations. The model also assumes that when we are talking about replacing fossil fuels by renewables, we are talking about a complete displacement, including economic means of electricity storage for renewable production. We also abstract from the fact that a significant part of the current energy supply comes from hydroelectricity and nuclear instead of fossil fuels. To accommodate these sources we would need to add a third type of energy producing capital that is used with both fossil fuels and renewables. From this perspective the model can be seen as representing the replacement of the “fossil fuel only” part of the current energy supply system by alternative renewable sources, while those parts of the current system that do not rely on fossil fuels are retained and used in both regimes.

2 Related literature

In one of the earliest papers to use the phrase “valley of death” to describe a paucity of funding for commercializing new technologies, Frank et al. (1996) applied the concept to investment in environmental remediation technologies. They argued that the valley occurs when “the developer

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³That is why the price often falls dramatically when patents expire and generic alternatives become available.

...has successfully demonstrated the efficacy of the technology but is unable to obtain financing for the scale-up and manufacturing process.” They note that while the initial funding for basic research often comes from government, taking technologies beyond the basic research stage is often not considered to be part of the government’s role. They claim, however, that the private sector is reluctant to invest in technologies that have not yet been implemented.

Frank et al. contrasted environmental remediation technologies to the pharmaceutical industry, noting that the latter does not suffer from a valley of death. They suggest that this might be because “Government has funded medical research across the continuum of technology development, from basic R&D, through human clinical trials, to supporting health care for the needy.” They also point to the ongoing significant involvement of large, well-capitalized private sector firms in pharmaceutical industry R&D at all stages of new product development.

A Report by the US House of Representatives Committee on Science (Sensenbrenner, 1998) also identified a valley of death for new technology developments as “a widening gap between federally-funded basic research and industry-funded applied research and development.” The committee suggested partnerships between universities and firms as a means of spanning the valley, but cautioned against the federal government doing so through direct funding. It argued that the government would not have sufficient resources for such a task and the attempt would draw funds away from basic research and create a void “no other entity could fill.”

One of the earliest authors to apply the valley of death notion to energy technologies, Norberg-Bohm (2000), noted that the phrase is meant to reflect the common experience that many new technologies “die” before being successfully commercialized. She argued that, “For technologies such as power plants, which may be standardized but not mass produced, the initial plant is much more expensive than the 5th or 10th plant” and that early versions may not survive “an extended period of negative net cumulative cash flow.” Furthermore, she suggested that generating technologies may find it difficult to capture a market niche because the homogeneity of electricity as a product meant that it was not possible to use “quality improvements . . . to charge higher prices to the lead adopters.”⁴

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⁴This claim is ironic in light of the fact that many more recent commentators have criticized renewable technologies such as wind for providing a lower quality electricity supply on account of its intermittency, unpredictability and generally weak correlation to peak system demand.

Murphy and Edwards (2003) also applied the concept to renewable energy technologies. They noted that the U.S. Department of Energy, and especially its National Renewable Energy Laboratory, have provided substantial financial support for renewable energy R&D. Furthermore, this support has helped improve technology performance and reduce costs. However, progress toward commercialization has been hindered by barriers to financing resulting from five basic risks:

1. Information asymmetries whereby the entrepreneur or public sector investors know more about the technology than private sector investors, suppliers, or strategic partners.
2. Technology push by public sector efforts paradoxically creates uncertainty about market size, customer benefits and profitability because these have not been critical to success of the venture to date.
3. Uncertainty about whether the research program will succeed, or the new product will have the right characteristics to successfully compete, and how the firm's rivals will respond and what they may be developing in secret.
4. Volatility of both financial and product markets, exacerbated by potential withdrawal of public subsidies or tax credits, leads to high and variable required rates of return.
5. Many of the firm's assets – such as trade secrets, patents and key human resources – are hard to evaluate and cannot be used as collateral for funds.

Bürer and Wüstenhagen (2009) asked a principal or senior fund manager in 60 fund management firms to rate the effectiveness of different policies in stimulating interest in new energy technology investments. They motivated their study by claiming that venture capital and private equity investors are the most important sources of funds for new energy technology firms in the intermediate stage after, primarily government-funded, R&D ceases but before “self-sustaining funding from customers” begins.

Bürer and Wüstenhagen see “technology push” as important for moving firms into the valley of death and “market pull” as important for taking them out of it. They found that, among various technology push policies, government grants for demonstration plants were most favored by the investors. The favorable rating for such policies also exceeded the ratings for most of the market-pull policies, with the notable exception of feed-in tariffs. Other technology push policies favored

included more funding for public and private R&D entities, investment subsidies for manufacturing facilities, grants to install equipment and tax breaks for entrepreneurs.

Market pull policies on average got higher scores than technology push ones. Feed-in tariffs were rated higher than any other policy option including renewable portfolio standards and tradable green certificates. Many investors also rated “reduced fossil fuel subsidies” as important, while technology performance standards and residential and commercial tax credits were also popular.

While the above authors could be read as suggesting that renewable technologies are already competitive but inefficiently excluded from the marketplace by the valley of death phenomenon, Zindler and Locklin (2010) claim that, “Much more work remains to drive down costs so that renewables can truly compete with and beat their fossil rivals on cost.” They describe the task of moving technology from the research to the commercial phase as “arduous” as a result of the “high cash demands and a significant scarcity of capital.” Like Burer and Wüstenhagen (2009) they also based their discussion and analysis partly on interviews. In the case of Zindler and Locklin, the interviews were conducted with “more than 60 thought leaders in 10 countries . . . soliciting all ideas on addressing the commercialization valley of death challenge facing new low-carbon energy technologies.”

Zindler and Locklin report that “participants actually identified two critical locations where a shortfall of capital often comes into play. The first occurs early in a technology’s development, just as it is ready to exit the lab. The second occurs later, when much more substantial levels of capital availability are needed to prove viability at commercial scale.” Zindler and Locklin claimed that “there are clear, well-proven, and fairly low-cost responses available” for the first challenge, citing government support for government energy technology laboratories supplemented by grants to private firms. However, the second financing gap was judged to be more intractable. Many interviewees apparently believed that “fundamental, structural market shortcomings” inhibit progress and, further that these shortcomings “cannot be resolved by the private sector acting on its own.” It was felt that while banks and other financial institutions are “not structurally positioned to back large-scale projects deploying new technologies” venture capital firms “have high technology risk tolerance but relatively limited capital, and they demand short-to-medium returns.”

Azar and Sandén (2011) ascribe the difficulty of transiting the valley of death to the difficulties of capturing the benefits of learning by doing, learning by using, economies of scale and network

externalities. Many of these benefits are external to the firm even if they are internal to the industry. Failure to capture (some of) the external benefits limits the ability of new technology firms to attain the economies of scale that could lead to lower costs. Azar and Sandén also offer some criticism of feed-in tariffs. In particular, they point out that the policies often support only specific identified renewable electricity generating technologies. As a result, “very promising but currently expensive technologies will not gain the support that is needed to come down in cost.”

In an analysis of the commercialization potential for high altitude wind power (HWP), Bronstein (2011) comments: “[I]nvestors are most interested in turning a short or medium term profit from HWP ... [S]ignificant uncertainty remains as to when these small, R&D heavy firms will be able to create a deployable, revenue generating system ... But ... shortening the R&D process can place firms at odds with investors, who may lack an understanding of the technical barriers to commercialization. In addition, investors may have a lower technical standard for a product to be deemed commercializable, which can create a conflict between the investors and researchers/engineers that may have more stringent and technically sound commercialization criteria.”

We could only find one paper, by Beard et al. (2009) and mentioned briefly in the introduction, that presented a formal economic model of the valley of death phenomenon. As the authors say, while “several explanations for the valley of death have been proffered ... none provides a mechanism that clearly explains the non-linear ... valley characteristic of the phenomenon ... [that is] a shortfall of funding at an intermediate stage that is more systematic and profound than the shortfall to either side of the intermediate stage.” Their model is quite insightful and is worth explaining in some detail.

They begin by observing that Arrow suggested that private industry might under-invest in invention and research because many of the returns are non-appropriable. Arrow’s argument implies, however, that the problem should be more severe in very early stage basic research, while the valley of death notion applies to a funding shortage when research has become more applied but not yet commercialized.

In their model, some “non-economic” motivation for R&D, that is, choosing the nature and level of R&D at the first stage on criteria other than prospective return, is a necessary condition for the existence of a “valley” of funding shortfall at the intermediate stage. “So, while pathologies such as risk, uncertainty, appropriation problems and so forth are present at intermediate stages of

an innovation process, a non-economic actor operating at early stages is required for there to be a valley of death.”

The authors consider a project that takes 3 stages of R&D expenditure. At stage i of the project, there is a probability of success P_i , and an opportunity cost of funds tied up in the project I_i . The project pays off a private value V only if it progresses to stage 3.

Assuming that a project has made it to stage 2, a risk neutral entrepreneur will undertake the final stage only if $P_3V > I_3$. Similarly, a project that has progressed through stage 1 will enter stage 2 only if $V > I_3/P_3 + I_2/(P_2P_3)$. Finally, a project will be begun in stage 1 only if $V > I_3/P_3 + I_2/(P_2P_3) + I_1/(P_1P_2P_3)$. Observe that since each of the terms on the right side of the inequalities is positive, a successful project at either stage 1 or stage 2 will automatically be taken to the next stage.

From these inequalities, if an exogenous factor makes stage 2 investment less attractive by raising I_2 or reducing P_2 , it will also make stage 1 investment less attractive in the same way. It cannot result in a “valley” of death in stage 2 *relative to* stage 1. However, a “valley” can occur if decisions made in stage 1 proceed in part independently of what is likely to happen in the later stages. In particular, subsidies in stage 1 can result in more output of stage 1 projects than the private sector is willing to finance in stage 2, where the original decision criterion remains in force.

Beard et al. note that this outcome is not necessarily inefficient. For example, one motivation for subsidizing stage 1 R&D could be that some of the fruits of stage 1 research are not appropriable. For example, the research at stage 1 may include a component v that contributes to general knowledge and is not part of the return V if a technology is commercialized. Stage 2 or 3 development may not be associated with similar contributions to basic scientific knowledge, however, and would not be subsidized. Subsidies at stage 1 alone, however, imply that many more projects will be undertaken at stage 1 than proceed to stage 2, although all projects that succeed at stage 2 will proceed to stage 3. This “valley” would not indicate any market failure.

Alternatively, if the social value component of the R&D, v , also is only realized if the project proceeds to stage 3, the private equilibrium with *optimal* subsidy of only stage 1 research will still be inefficient. Again, there will be a “valley” of stage 2 projects, and some socially beneficial projects at stage 2 will fail to acquire funding, but all stage 2 projects that succeed will be fully funded.

Beard et al. next show how non-economic activity in very early stages of the sequence may not only inflate “the output of basic research above what profit-maximizing behavior is prepared to fund in later stages of the innovation sequence.” It may also “alter the cost of funds at intermediate stages by altering the location of lenders along the sequence.”

The starting point for this analysis is an asymmetry of information between the entrepreneur and potential investors. The entrepreneur knows much more about the details of the technology, but the investors know much more about market prospects and potential competitors. In particular, the less investors know about the details of the technology, the greater the chance that they can lose money as a result of opportunistic behavior by the entrepreneur. Hence, as the information asymmetry increases, the investors will require a higher rate of return before they are willing to finance the investment. The higher cost of funds in turn leads to under-investment by the entrepreneur relative to what would obtain absent the information differential.

Now suppose the entrepreneur and the investors can each choose to specialize in the type of knowledge they invest in. The entrepreneur could, for example, make the technology more like existing technologies and thus easier for outside investors to assess, but perhaps at the expense of making it less effective. On the other side, the investors could employ scientific experts to help them better assess new technologies, but it may be more expensive to do so.

The authors then show that as I_3 increases relative to I_1 and I_2 , the entrepreneur will choose to invest in projects that are closer to the stage 3 investors’ ideal information point. That will, however, tend to drive up the cost of financing the earlier stages of the process. The analysis also implies, conversely, that as the probabilities of success at the earlier stages decline, the entrepreneur will be encouraged to choose projects closer to the ideal investment points of the early stage investments.

The most important conclusion, however, is that non-economic financing of stage 1 R&D may allow the investor to disregard the information preferences of later stage investors. Furthermore, with opportunities to participate in stage 1 investments effectively precluded to them, the private investors have no need to invest in information about stage 1 R&D and thus have information expertise that is even further from that of stage 1 entrepreneurs. The result will be a widening of the valley of death created by the non-economic financing of stage 1 investments.

As we noted in the introduction, we focus on a somewhat different notion of under-funding of investment in the final stages of commercializing a new energy technology. This does not mean

that we reject the alternative notions presented in the literature and discussed above. Indeed, if problems with appropriating some of the returns to early stage R&D were introduced into our model they would exacerbate the difficulty of private firms funding development of new energy technology. Our explanation of the valley of death for new energy technologies is thus meant to complement rather than displace the notions already canvassed in the literature.

The other literature that our paper is related to is intertemporal optimization models of economic growth and energy use. We follow most closely Hartley et al. (2013). The underlying economic growth model in this paper is the same as in Hartley et al. The way technological change affects the cost of supplying fossil fuel energy also is drawn from that paper, as is the way that both learning by doing and explicit investment in R&D affect the cost of renewable energy. However, that paper does not have investment in physical capital needed to supply energy services, and so cannot address the issues examined in this paper. Restricting physical capital accumulation to the capital needed to produce final output simplifies the model in Hartley et al. relative to this paper. It is nevertheless reassuring for the central results of that paper that the more complicated model in this paper also produces “an endogenous energy crisis” around the time of the transition between energy sources.

Our analysis in this paper also is related to a paper by Chakravorty, Roumasset, and Tse (1997). They also consider a model with substitution between energy sources, improvements in extraction, and a declining cost of renewable energy. They find that if historical rates of cost reductions in renewables continue, a transition to renewable energy will occur before over 90% of the world’s coal is used. In contrast to their paper, we generate an endogenous transition to renewable energy and allow for explicit investment in physical capital stocks. Unlike Chakravorty et. al., we do not study the implications of energy use for environmental externalities and we do not conduct policy experiments.

Other papers in the literature have also examined the transition from fossil fuels to renewable technologies in a dynamic intertemporal modeling framework. In particular, Acemoglu et al. (2009) study a growth model that takes into consideration the environmental impact of operating “dirty” technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures or the need to invest in physical capital in order

to supply energy services. They find that subsidizing research in the “clean” sectors can speed up environmentally friendly innovation without resorting to taxes or quantitative controls on carbon dioxide emissions with their negative impact on economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production.

3 The Model

We model economic activity in continuous time, indexed by t . The state variables, the controls, and the technology variables are functions of time. We shall usually simplify notation, however, by omitting time as an explicit argument.

3.1 Goods and services production and consumption

There is a single consumption good in the economy. Letting c denote per capita consumption, we assume that the lifetime utility function is given by:

$$U = \max \int_0^\infty e^{-\beta\tau} \frac{c(\tau)^{1-\gamma}}{1-\gamma} d\tau \quad (1)$$

The term $e^{-\beta\tau}$ acts as a discount factor, capturing the fact that utility from future consumption is less valuable than today’s consumption.

The production function is an Ak model augmented by an explicit accounting for energy input. Specifically, for a per capita capital stock of k producing per capita output of goods and services of $y = Ak$, we assume per capita energy input $e = Fk$ is required where F can be interpreted as the fuel intensity of the capital stock.⁵

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⁵The usual energy identity relates energy input e to delivered energy service u per unit of capital, such as miles per vehicle, times the capital stock k , such as the number of vehicles, divided by energy efficiency E , such as miles per gallon. In the vehicle example this yields total gallons of fuel consumed. The fuel intensity F then is u/E . In the subsequent discussion, we will often refer to F as “end-use energy efficiency” but it should be understood that fuel intensity of final production can change for reasons other than improvements in energy efficiency as usually understood. For example, Medlock (2010) emphasizes that changes in the composition of production, for example the shift to services, reduce energy intensity as an economy grows.

3.2 Energy production

Energy can be provided by two different technologies that also require capital investments to produce useful output. One, with capital stock denoted k_R , mines fossil fuels that are depletable and converts them into useable energy products using, for example, refineries and power stations. The other, with capital stock denoted k_B , is a backstop or renewable technology where the energy source itself is “harvested” from the environment using the capital equipment, so there is no resource depletion, although there are operating and maintenance costs. Once energy-producing capital is in place it cannot be converted from one type to the other. However, energy inputs derived from fossil fuels and renewable sources are perfect substitutes for producing goods output. Total energy input into goods production is given by $e = R + B$ where R equals the energy produced using k_R and B the energy produced using k_B . We also assume linear production functions for the energy producing industries

$$\begin{aligned} R &= Gk_R \\ B &= Hk_B \end{aligned} \tag{2}$$

Each type of capital (the goods-producing capital and the two-energy producing capital stocks) is accumulated via investment i, i_B, i_R and depreciates at the rate δ :⁶

$$\dot{k} = i - \delta k \tag{3}$$

$$\dot{k}_B = i_B - \delta k_B \tag{4}$$

$$\dot{k}_R = i_R - \delta k_R \tag{5}$$

Primary fossil fuel inputs required to produce per capita fossil energy output R need to be extracted and converted at a per unit cost that increases⁷ with the total quantity of resources mined to date, S . We define units of fossil fuel resources so that operating one unit of capital k_R

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⁶Although different types of capital could depreciate at different rates, the data we use to calibrate the model provides only a single rate of depreciation for capital. Having just one rate also simplifies the analysis somewhat.

⁷Heal (1976) introduced the idea of an increasing marginal cost of extraction to show that the optimal price of an exhaustible resource begins above marginal cost, and falls toward it over time. This claim is rigorously proved in Oren and Powell (1985).

requires a fixed input of one unit of fossil fuel resource.⁸ Since k_R is measured in per capita terms, so also will be the fossil fuel input, implying that population growth will also increase the total amount of fossil fuel resources that are mined and thus the increase in mining cost. Letting Q denote the (exogenous) population and labor supply, $0 \leq \rho_R \leq 1$ the utilization variable⁹ for k_R , the total fossil fuel used will be $\rho_R Q k_R$, and S will satisfy:

$$\dot{S} = \rho_R Q k_R \quad (6)$$

We will assume that Q grows at the constant rate π , that is, $\dot{Q} = \pi Q$.

We further modify the resource depletion model to allow for technical change in mining and conversion. Explicitly, we assume that the per unit cost of mining and conversion, $\mu(S, N)$, depends not only on S but also the state of technical knowledge N , which can be augmented through investment:

$$\dot{N} = n \quad (7)$$

Per capita mining and conversion costs will then be given by $\mu(S, N) \rho_R k_R$. We can interpret n as investments that raise the productivity of capital in the mining and conversion industries. This could include new discoveries, improvements in mining technology and improvements in conversion efficiency (for example, refining and electricity production). The latter reduces the primary energy input required to supply a given amount of useful secondary energy.

We assume that $\mu(S, N)$ is given by the following function:

$$\mu(S, N) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S - \alpha_2/(\alpha_3 + N)} = \alpha_0 + \frac{\alpha_1(\alpha_3 + N)}{(\bar{S} - S)(\alpha_3 + N) - \alpha_2} \quad (8)$$

illustrated in Figure 2. For a given state of technical knowledge N , mining and conversion costs become unbounded as $S \rightarrow \bar{S} - \alpha_2/(\alpha_3 + N)$. The absolute maximum fossil fuel available, $\bar{S}$, can only be accessed asymptotically as the stock of investment in new fossil fuel technology $N \rightarrow \infty$. The terms $\alpha_0, \alpha_1, \alpha_2$ and α_3 in (8) are parameters. We define N and S so that initially $N(0) = S(0) = 0$.

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⁸As we note below, however, one can interpret investment to slow the increase in mining costs as investment in the efficiency of fossil fuel use that allows the same energy services to be produced using less input of primary fossil fuel resource.

⁹Since capital depreciates exponentially once investment in k_R ceases, a positive amount of k_R will always remain. Mining and conversion costs can then be avoided by choosing $\rho_R = 0$ and ceasing to use k_R to provide energy.

Figure 2: Unit cost of mining fossil fuels

By differentiating $\mu(S, N)$ one can show that $\partial\mu/\partial S > 0$ and $\partial\mu/\partial N < 0$, so depletion raises fossil fuel energy costs while investment in N lowers them. Also, $\partial^2\mu/\partial S^2 > 0$, so depletion increases cost at an increasing rate, while $\partial^2\mu/\partial N^2 > 0$, so investment in N decreases costs at a decreasing rate. Investment in fossil fuel technology also delays the increase in fossil fuel energy costs accompanying increased exploitation, that is, $\partial^2\mu/\partial S\partial N < 0$. However, since $\partial\mu/\partial N \rightarrow 0$ as $N \rightarrow \infty$, it will become uneconomic at some point to invest further in reducing the costs of fossil fuel energy. The costs of depletion will then swamp improvements in mining technology and conversion efficiency. Fossil fuel resources will be abandoned before all known deposits are exhausted as rising costs make renewable energy technologies more attractive. For mining to cease at that time, the utilization ρ_R of k_R has to fall to zero and remain at zero thereafter. Also, once fossil fuel use ceases, S, N and μ will remain constant.

The production of energy from renewable sources requires investment in k_B and incurs operating and maintenance (O&M) costs. Specifically, for renewable energy capital k_B , we assume that renewable energy production is given by $\rho_B H k_B$ and O&M costs by $m\rho_B k_B$, where $0 \leq \rho_B \leq 1$ is the utilization rate of k_B .

We allow technological progress to increase H , and hence reduce the amount of capital k_B required to yield a given level of energy output B . Explicitly, we assume that the accumulation of

knowledge that leads to a change in H follows a two-factor learning process whereby increases the stock of knowledge require the construction of renewable energy production capital k_B in addition to direct R&D expenditure j :

$$\dot{H} = \begin{cases} bk_B^\psi j^{\alpha-\psi} & \text{if } H \leq \bar{H}, \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Here we have assumed that merely constructing k_B allows cost reductions through learning. Using the capital to produce energy output is not required. In addition, we assume that the stock of capital previously constructed influences growth in H , not just new investments i_B .¹⁰ Finally, the learning process also assumes that once H reaches its upper limit, further investment in the technology would be worthless and we should have $j = 0$.

We assume that $\psi < \alpha < 1$, so there are decreasing returns to investing in renewable energy efficiency. The parameter ψ determines the relative contribution from experience versus explicit investment in research to the accumulation of knowledge H . Klaassen et. al. (2005)¹¹ derive robust estimates suggesting that direct R&D is roughly twice as productive for reducing costs in wind turbine farms as is learning-by-doing. Hence, we assume that $\psi = \alpha/3$. The coefficient b relating investment in knowledge to the resulting technological progress is analogous to the coefficient A in the production function for final output.

3.3 The resource constraint

Goods output is consumed, used to produce energy, or invested in k, k_R, k_B, N or H . Let c denote per capita consumption. This leads to a resource constraint (in per capita terms):

$$c + i + i_R + n + i_B + j + \mu(S, N)\rho_R k_R + m\rho_B k_B = Ak \quad (10)$$

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¹⁰This effectively assumes that, prior to $\bar{H}$ being attained, the physical depreciation of capital also determines the depreciation rate on knowledge. If the empirical data were available, different rates could be set.

¹¹Building on an earlier paper by Kouvaritakis et al. (2000), Klaassen et. al. (2005) estimated a two-factor learning curve model that allowed both capacity expansion (learning-by-doing) and direct public R&D to produce cost reducing innovations for wind turbine farms in Denmark, Germany and the UK. They interpret their results as enhancing the validity of the two-factor learning curve formulation.

Also, equilibrium in the energy market requires¹²

$$Fk = \rho_R Gk_R + \rho_B Hk_B \quad (11)$$

4 The optimization problem

The objective function (1) is maximized subject to the differential constraints (3), (4), (5), (6), (7) and (9) with initial conditions $k(0) = k_0, k_R(0) = k_{R0}, k_B(0) = k_{B0}, H(0) = H_0$ and $S(0) = N(0) = 0$, the resource constraint (10), and the energy market equilibrium condition (11). The control variables are $c, \rho_R, \rho_B, i, i_R, i_B, n$ and j , while the state variables are k, k_R, k_B, N, S and H . Denote the corresponding co-state variables by q, q_R, q_B, ν, σ and η . Let λ be the Lagrange multiplier on the resource constraint and p_e the multiplier on the energy market equilibrium constraint. Use θ_{RL} and θ_{RU} for the Lagrange multipliers on the inequality constraints on ρ_R , and θ_{BL} and θ_{BU} the corresponding multipliers on the inequality constraints on ρ_B . Also use ω to denote the multiplier on the constraint $i \geq 0$, ω_R the multiplier on the constraint $i_R \geq 0$, ω_B the multiplier on the constraint $i_B \geq 0$, ω_N the multiplier on the constraint $n \geq 0$, and ω_H the multiplier on the constraint $j \geq 0$. We can then define the current value Hamiltonian and hence Lagrangian by

$$\begin{aligned} \mathcal{H} = & \frac{c^{1-\gamma}}{1-\gamma} + q(i - \delta k) + q_R(i_R - \delta k_R) + q_B(i_B - \delta k_B) + \nu n + \sigma \rho_R Qk_R \\ & + \eta b k_B^\psi j^{\alpha-\psi} + \lambda \left\{ Ak - c - i - i_R - i_B - n - j - \mu(S, N) \rho_R k_R - m \rho_B k_B \right\} \\ & + p_e \{ \rho_R Gk_R + \rho_B Hk_B - Fk \} + \theta_{RL} \rho_R + \theta_{RU} (1 - \rho_R) + \theta_{BL} \rho_B \\ & + \theta_{BU} (1 - \rho_B) + \omega i + \omega_R i_R + \omega_B i_B + \omega_N n + \omega_H j \end{aligned} \quad (12)$$

The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\gamma} - \lambda = 0 \quad (13)$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \rho_R} = & \sigma Qk_R - \lambda k_R \mu(S, N) + p_e Gk_R + \theta_{RL} - \theta_{RU} = 0 \\ \theta_{RL} \rho_R = & 0, \theta_{RL} \geq 0, \rho_R \geq 0, \theta_{RU} (1 - \rho_R) = 0, \theta_{RU} \geq 0, \rho_R \leq 1 \end{aligned} \quad (14)$$

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¹²We can use (11) to write Ak in terms of the *net* contribution to output from the energy sector:

$$c + i + i_R + i_B + n + f + j = \rho_R k_R \left[\frac{AG}{F} - \mu(S, N) \right] + \rho_B k_B \left(\frac{AH}{F} - m \right)$$

We assume A is large enough, and initial fuel intensity F is low enough, that A/F exceeds $\mu(0, 0)/G$ and m/H_0 .

$$\frac{\partial \mathcal{H}}{\partial \rho_B} = -\lambda m k_B + p_e H k_B + \theta_{BL} - \theta_{BU} = 0 \quad (15)$$

$$\theta_{BL} \rho_B = 0, \theta_{BL} \geq 0, \rho_B \geq 0, \theta_{BU}(1 - \rho_B) = 0, \theta_{BU} \geq 0, \rho_B \leq 1$$

$$\frac{\partial \mathcal{H}}{\partial i} = q - \lambda + \omega = 0; \omega i = 0, \omega \geq 0, i \geq 0 \quad (16)$$

$$\frac{\partial \mathcal{H}}{\partial i_R} = q_R - \lambda + \omega_R = 0; \omega_R i_R = 0, \omega_R \geq 0, i_R \geq 0 \quad (17)$$

$$\frac{\partial \mathcal{H}}{\partial i_B} = q_B - \lambda + \omega_B = 0; \omega_B i_B = 0, \omega_B \geq 0, i_B \geq 0 \quad (18)$$

$$\frac{\partial \mathcal{H}}{\partial n} = \nu - \lambda + \omega_N = 0, \omega_N n = 0, \omega_N \geq 0, n \geq 0 \quad (19)$$

$$\frac{\partial \mathcal{H}}{\partial j} = \eta(\alpha - \psi) b k_B^\psi j^{\alpha-\psi-1} - \lambda + \omega_H = 0, \omega_H j = 0, \omega_H \geq 0, j \geq 0 \quad (20)$$

The differential equations for the co-state variables are:

$$\dot{q} = \beta q - \frac{\partial \mathcal{H}}{\partial k} = (\beta + \delta)q - \lambda A + p_e F \quad (21)$$

$$\dot{q}_R = \beta q_R - \frac{\partial \mathcal{H}}{\partial k_R} = (\beta + \delta)q_R - \sigma \rho_R Q + \rho_R \lambda \mu(S, N) - \rho_R p_e G \quad (22)$$

$$\dot{q}_B = \beta q_B - \frac{\partial \mathcal{H}}{\partial k_B} = (\beta + \delta)q_B - \eta \psi b k_B^{\psi-1} j^{\alpha-\psi} + \rho_B \lambda m - \rho_B p_e H \quad (23)$$

$$\dot{\nu} = \beta \nu - \frac{\partial \mathcal{H}}{\partial N} = \beta \nu + \lambda \rho_R k_R \frac{\partial \mu}{\partial N} \quad (24)$$

$$\dot{\sigma} = \beta \sigma - \frac{\partial \mathcal{H}}{\partial S} = \beta \sigma + \lambda \rho_R k_R \frac{\partial \mu}{\partial S} \quad (25)$$

$$\dot{\eta} = \beta \eta - \frac{\partial \mathcal{H}}{\partial H} = \beta \eta - \rho_B p_e k_B \quad (26)$$

We also recover the resource constraint (10), the energy market equilibrium condition (11), and the differential equations for the state variables, (3), (4), (5), (6), (7) and (9).

Let V denote the maximized value of the objective function (1) subject to the constraints. Recalling that $\mathcal{H}$ is the current value and not present value Hamiltonian, from the Hamilton-Jacobi-Bellman equation we have $-V_t = \max e^{-\beta t} \mathcal{H}$. Furthermore, the (current value) co-state variables satisfy $e^{-\beta t} q = \partial V / \partial k \geq 0$, $e^{-\beta t} q_R = \partial V / \partial k_R \geq 0$, $e^{-\beta t} q_B = \partial V / \partial k_B \geq 0$, $e^{-\beta t} \sigma = \partial V / \partial S \leq 0$, $e^{-\beta t} \nu = \partial V / \partial N \geq 0$ and $e^{-\beta t} \eta = \partial V / \partial H \geq 0$.

Observe that (14) and (22) together can be interpreted as a version of the peak load problem. Specifically, we can recognize $p_e G - (\lambda \mu - \sigma Q)$ as a short run marginal profit of energy supply using fossil fuels. Each unit of k_R delivers G units of energy each valued at p_e . Explicit marginal

production costs measured in output units are $\lambda\mu$. In addition there is an implicit depletion cost measured by $-\sigma Q > 0$. If the capacity is non-binding, $\rho_R < 1$ and marginal profits are simply these short run profits. If $\rho_R = 1$, however, $\theta_{RU} = p_e G - (\lambda\mu - \sigma Q) > 0$ measures the “surplus profits” that represent an implicit return to scarce capacity. The differential equation (22) then implies that the shadow value q_R of k_R is equal to the discounted future value of these implicit rents when $\rho_R = 1$, with a discount rate given by the time discount rate β plus the depreciation rate δ of k_R . Equation (17) then implies that when $i_R > 0$ this shadow value equals the marginal costs of investment represented by λ , the marginal utility of consumption and thus the shadow cost of investing.

In accordance with this interpretation, we will call the energy price:

$$p_e = \frac{\lambda\mu - \sigma Q}{G} \quad (27)$$

the short run cost of fossil energy production. Using (21) and (22) when both $i, i_R > 0$, $\rho_R = 1$ and (17) and (16) imply $\dot{q}_R = \dot{q}$, we also define the long run cost of fossil energy production:

$$p_e = \frac{\lambda(A + \mu) - \sigma Q}{F + G} \quad (28)$$

Equations (15) and (23) have a similar interpretation for the renewable energy supply. The marginal costs in this case are just the explicit marginal costs λm . Thus, the short run cost of renewable energy analogous to (27) is

$$p_e = \frac{\lambda m}{H} \quad (29)$$

Similarly, (18) and (16) with $i, i_B > 0$ imply $\dot{q}_B = \dot{q}$, and yield a long run cost of renewable energy. In this case, however, the differential equation (23) includes an additional implicit long run return to investment in capacity $\eta\psi b k_B^{\psi-1} j^{\alpha-\psi}$ so long as $H < \bar{H}$.¹³ This arises because the learning process implies that investing in k_B brings an additional benefit by lowering future production costs and making investment j in explicit R&D more productive. Defining a function $Y \equiv [\eta(\alpha - \psi)b/\lambda]^s$, where¹⁴ $s \equiv 1/(1 + \psi - \alpha) > 1$, the solution for j from (20) can be written $j = Y k_B^{s\psi}$. Using this

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¹³While $H < \bar{H}$, $\eta > 0$ and hence $Y > 0$. Once $H = \bar{H}$, however, $\eta = e^{\beta t} \partial V / \partial H = 0$ and renewable energy R&D investment j will cease even though k_B remains positive.

¹⁴It is useful to note that $(\alpha - \psi)s + 1 = s$ and $(\alpha - 1)s + 1 = \psi s$.

solution for j , we conclude that when $\rho_B = 1$, the long run cost of renewable energy production is:

$$p_e = \frac{\lambda}{F + H} \left[A + m - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y \right] \quad (30)$$

Differential equation (21) has an analogous interpretation to (22) and (23). Specifically, (21) implies that q equals the discounted marginal returns to k arising from selling A units of output at λ and buying F units of energy at marginal cost p_e . Equation (16) then equates this shadow value q to the marginal costs of investment represented by λ whenever $i > 0$.

Similarly, when $n > 0$, (19) equates the marginal cost λ of investing in n to a shadow value ν that, from (24), represents the discounted marginal reduction in fossil fuel production costs accruing at the utilization rate ρ_R of fossil fuel capital. In this case, the discount rate is solely the time discount rate β . Similarly, equations (25) and (26) imply that the marginal costs of depletion σ , and the marginal benefits of increasing renewable energy efficiency η , are discounted at rate β .

5 The evolution of the economy

In this section, we describe the regimes of energy-related investments and production that the economy traverses. The differential equations that determine the evolution of the state and co-state variables in each regime are derived and presented in an appendix.

First, we note that the utility function ensures that $c > 0$ and hence $\lambda > 0$. Focusing next on $t = 0$, we assume that initially fossil fuels alone are used to supply energy input.¹⁵ Specifically, we assume that at $t = 0$ the price of energy p_e is determined by the long run cost of fossil fuel energy supply (28), and that this exceeds the short run cost of fossil energy (27) but is less than the short run cost of renewable energy $\lambda m / H_0$. Then (14) implies $\rho_R = 1$ while (15) implies $\rho_B = 0$ at $t = 0$.

Although only fossil sources are used to supply energy at $t = 0$, renewable production capacity will be non-zero at $t = 0$. The reason is that, as already noted above, the benefits of learning by doing imply there is an additional return $\lambda \psi k_B^{(\alpha-1)s} Y / (\alpha - \psi)$ to investing in k_B (using (20) to

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¹⁵Assuming that the renewable sources do not provide energy services does not imply that there is investment in k_B at $t = 0$. In practice, non-fossil sources other than hydroelectricity and nuclear power have a small share and generally would be uncompetitive without subsidies or mandates. Some are competitive in special situations, such as providing electricity in remote locations, because the energy services they provide are not perfect substitutes for the energy from fossil fuels, a fact that we abstract from in the model.

eliminate $j = Yk_B^{s\psi}$). Then since $\eta > 0$ for $H < \bar{H}$ and we have assumed that $1 > \alpha > \psi > 0$, this additional marginal value of k_B becomes unbounded as $k_B \rightarrow 0$. Thus, k_B , and also i_B , are strictly positive at $t = 0$. From the assumptions that $i_B > 0$ but $\rho_B = 0$ at $t = 0$, together with $i > 0$ and (16), (18), (21) and (23), the price of energy at $t = 0$ also has to satisfy

$$p_e = \frac{\lambda}{F} \left[A - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y \right] \quad (31)$$

From (14) energy production from fossil fuels will continue with $\rho_R = 1$, and k_R fully utilized, so long as p_e at least equals the short run cost of fossil energy production (27). However, depletion (increasing S) must eventually overwhelm investment in N , so μ must rise and fossil fuel use ultimately must cease. Even if investment n in fossil fuel technology could keep μ from rising as a result of depletion, eventually the economy would reach a time when p_e matched the short run cost of renewable energy production (29). The reason is that increases in H must reduce the short run cost of renewable energy production (29). From that time, (15) would imply $\rho_B > 0$, so renewable capital would be used to produce energy.

Suppose fossil fuel production first ceases at T_R , so $\rho_R(t) = 0$ for $t \geq T_R$. Since further changes in S, N or k_R then have no effect on V , $\sigma = \nu = q_R = 0$ for $t \geq T_R$.

Although $\nu = 0 < \lambda$ at T_R , (19) implies $\nu = \lambda$ while $n > 0$. Furthermore, (24) implies $\dot{\nu} \neq 0$ for $\rho_R > 0$.¹⁶ Solving (24) backwards in time from T_R , ν must rise faster than λ . When $\nu = \lambda$, n will become positive. This will occur at some $T_N < T_R$ with $\nu < \lambda$ and $n = 0$ for $t \in [T_N, T_R]$.

Similarly, $0 = q_R < \lambda = q$ at T_R , and (17) implies $q_R = \lambda = q$ while $i_R > 0$. In contrast to ν , however, $\dot{q}_R$ can equal zero if $\rho_R > 0$, specifically when p_e equals the short-run cost of fossil energy supply (27). We return to this point below. Define T_Q as the first time that $i_R = 0$ so that $i_R > 0, \forall t < T_Q$.

Before returning to the issue of investments when $t < T_R$, we first discuss the use of fossil capacity, that is, the value of ρ_R for $t < T_R$. By the definition of T_R as the first time fossil fuels are no longer used, $\rho_R(T_R) = 0$ and $\rho_R(t) > 0$ for $t < T_R$. Since $k_R > 0$, the first order condition (14) implies that $[\sigma Q - \lambda \mu + p_e G] \rho_R \geq 0$ for $t < T_R$ and is zero if $\rho_R < 1$ or p_e equals the short-run fossil fuel cost (27). In particular, for $t < \min(T_N, T_Q)$, $i_R, n > 0$ and from (17) and (19), $q_R = \lambda = \nu$.

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¹⁶Since $\partial \mu / \partial N < 0$, (24) implies that ν starts out positive, but declines to zero at T_R , and since $\partial \mu / \partial S > 0$, (25) implies that σ starts out negative and rises to zero at T_R .

Thus, $\dot{q}_R = \dot{\lambda} = \dot{\nu}$, and since ν is positive and strictly decreasing for $t < T_R$, $q_R > 0$ and $\dot{q}_R < 0$ for $t \leq \min(T_N, T_Q)$ and hence, from (22), $\sigma Q - \lambda\mu + p_e G > (\beta + \delta)q_R/\rho_R > 0$ and thus $\rho_R = 1$.

Next, we consider the transition to renewable energy in more detail. Observe first that the energy market equilibrium condition (11) will require $\rho_B > 0$ and $k_B > 0$ at T_R to ensure that energy input to final production can be maintained as fossil fuel is abandoned. Specifically, if $k_R, k_B > 0$ and $1 \geq \rho_R, \rho_B > 0$, ρ_B must be given by:

$$\rho_B = \frac{Fk - \rho_R G k_R}{H k_B} \quad (32)$$

Thus, if $\rho_R \downarrow 0$ continuously on $(\hat{t}, T_R)$, (32) requires $\rho_B \uparrow 1$ continuously over the same interval. But then $0 < \rho_R, \rho_B < 1$ on $(\hat{t}, T_R)$, and the price of energy would have to equal the two short run costs (27) and (29) over that interval. However, the short-run cost of fossil fuel $\mu - Q\sigma/\lambda \rightarrow \mu$ as $t \rightarrow T_R$ and the absence of investment in N and the effects of continuing depletion cause μ to increase rapidly. On the other hand, increases in H would imply mG/H is decreasing. We thus arrive at a contradiction, implying we must have $\rho_R = 1$ until T_R , when it jumps discontinuously to zero.

Given this behavior of ρ_R , (32) then implies that, for $t < T_R$, $\rho_B < 1$. Since ρ_B jumps from a value less than 1 to equal 1 at T_R , (15) implies that the price of energy at T_R must equal the short-run renewable energy cost (29) at T_R . However, for $t < T_R$, H must be smaller than it is at T_R and thus the short-run real cost of renewable energy must be higher than it is at T_R . On the other hand, the real cost of energy as determined by fossil fuel production must be rising as $t \uparrow T_R$. Hence, (15) implies $\rho_B = 0$ for $t < T_R$. Thus, we have a “bang-bang” solution for energy capital utilization whereby $\rho_R = 1$ and $\rho_B = 0$ for $t < T_R$ and then $\rho_R = 0$ and $\rho_B = 1$ for $t > T_R$.

Once investment in k_R ceases at T_Q , k_R will decline at the rate δ . Then since $\rho_R = 1$ and $\rho_B = 0$, and the depreciation rate on k also is δ , energy market equilibrium (11) will require $i = 0$ and $\dot{k} = -\delta k$ for $t \in [T_Q, T_R]$. Thus, energy market equilibrium will be just exactly satisfied by $\rho_R = 1$. As a result, p_e must equal the short-run cost of fossil energy production (27) for $t \in [T_Q, T_R]$. Then with $q_R = 0$ at T_R and p_e satisfying (27), solving (22) backwards in time from T_R , we conclude that $q_R = 0, \forall t \in (T_Q, T_R]$.¹⁷

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¹⁷Note q_R will be left continuous (and differentiable) as $t \uparrow T_Q$, but will jump discontinuously to zero at T_Q and remain zero $\forall t \geq T_Q$.

With both $i_B > 0$ and $\rho_B = 0$ for $t < T_R$, (18), (20), and (23) imply that $\dot{\lambda} = \dot{q}_B$ satisfies

$$\dot{\lambda} = (\beta + \delta)\lambda - \eta\psi b k_B^{(\alpha-1)s} Y \quad (33)$$

On the other hand, for $t > T_R$, $i > 0$ and (16) imply $q = \lambda$ and thus from (22)

$$\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F \quad (34)$$

In particular, if $i_B > 0$ at T_R , these two values for $\dot{\lambda}$ would be equal and the price of energy at T_R would be given by (31). However, the price of energy at T_R must also equal the short-run cost of renewable energy production (29). Hence, we would have

$$\frac{m}{H} = \frac{A}{F} - \frac{\psi}{(\alpha - \psi)F} k_B^{(\alpha-1)s} Y \quad (35)$$

For the parameter values we will consider later, however, $A/F = 1$ and the second term on the right hand side of (35) is close to zero. Thus, we would have $H \approx m$ at T_R . But we also have $H > m$ at $t = 0$, and H is increasing over time while m is constant. We conclude that we cannot have $i_B > 0$ at T_R and there must be an interval immediately after T_R when $i_B = 0$. In effect, the need to replace k_R at T_R leads to over-investment (from an ex-post perspective) in k_B at T_R and there is a pause from investing in k_B while the price of energy rises from the short-run cost of renewable energy production (29) to the long-run cost (30).

We can summarize the above discussion in Figure 3. The upper part of Figure 3 shows the different investment regimes, while the lower part shows the different regimes of energy production and use. The economy passes through five regimes of investment and energy production before entering the final regime where an analytical solution is possible.

The most striking feature of the solution is that even though investment in new energy technologies is required before fossil fuels are abandoned at T_R , the price of energy is insufficient to cover the operating costs of renewable energy for $t < T_R$. Furthermore, the full long-run costs of renewable energy are not covered until $T_B > T_R$. As noted in the introduction, this outcome provides a different way of formalizing the “the valley of death” notion. The efficient growth path for the economy requires investment in renewable energy sources well before fossil fuels are abandoned. Until fossil fuels are abandoned, however, they set the energy price and not until some time after fossil fuels are abandoned does the energy prices rise to equal the long-run costs of renewable energy.

Figure 3: A schematic representation of the different regimes

The detailed solution of the model is discussed in more detail in an appendix. In particular, the reader is referred to the appendix to see the differential equations that are solved in each regime. The appendix also discusses how initial values are chosen to ensure a unique solution to the optimization problem.

6 Calibration

Before we can solve the model, we need to specify numerical values for the parameters. As far as possible, we choose values consistent with observations from the actual world economy. Unless stated otherwise, all the measured quantities below are equated to their corresponding theoretical variable values at $t = 0$.

By definition, we start the economy with $S = N = 0$ and with $Q = Q_0$ and $H = H_0$. For convenience, we take the current population $Q_0 = 1$ and effectively measure future population as multiples of the current level. We will assume that the population growth rate is 1%.¹⁸

¹⁸This is consistent with a simple extrapolation of recent world growth rates reported by the Food And Agriculture Organization of the United Nations, <http://faostat.fao.org/site/550/default.aspx>

In line with standard assumptions made to calibrate growth models, we assume a time discount factor $\beta = 0.05$. From previous analyses of macroeconomic and financial data, we would expect the coefficient of relative risk aversion γ to lie between 1 and 10, but there is no strong consensus on what the value should be. From the appendix, the long run per capita growth rate of the economy depends inversely on γ . However, larger values of γ also extend the time it takes the economy to enter the terminal regime. We set $\gamma = 5$ as a compromise between these considerations.

To calibrate values for the initial production, capital stocks and energy quantities we used data from the EIA,¹⁹ the *Survey of Energy Resources 2007* produced by the *World Energy Council*,²⁰ and *The GTAP 8 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University.²¹ The last mentioned data source is useful for our purposes because it provides a consistent set of international accounts that also take account of energy flows. We also use the GTAP depreciation rate δ on capital of 4%.

One of the first issues we need to address is that national accounts include government spending in GDP, which does not appear in the model.²² In the GTAP database, more than 97.6% of the input cost of government spending is output from other government sectors or the service sector. A substantial amount of government spending, such as that on health care or education, is directly substitutable with private consumption spending. We therefore classified government spending as part of consumption.

Next, we calibrate the initial capital stocks k and k_R . Converting the GTAP data base estimates of the total capital stock to units of GDP, we obtain $k + k_R = 2.6551$. The GTAP data gives firm purchases of the capital resource endowment (and other factors of production) by sector. We identify the “energy sector” as including production of the primary fuels (coal, oil and natural gas), plus the energy commodity transformation sectors of refining, electricity generation and natural

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¹⁹International data is available at <http://www.eia.doe.gov/emeu/international/contents.html>

²⁰This is available at http://www.worldenergy.org/publications/survey_of_energy_resources.2007/default.asp The data are estimates as of the end of 2008.

²¹Information on this can be found at <https://www.gtap.agecon.purdue.edu/databases/v8/default.asp> The GTAP 8 data extracted below pertains to data for 2007.

²²Note that in the GTAP data base, aggregate world exports equal aggregate world imports so world GDP equals consumption plus investment plus government expenditure.

gas distribution, plus the transportation services sector.²³ After doing so, we find that 12.35%, or 0.3278 of the capital would be k_R while $k = 2.3273$. We also note that defining units so that output equals 1 implies that the parameter A would equal the ratio of output to capital, that is, $A = 1/k = 0.4297$. It is convenient to also choose units of energy service inputs into final production so $R = Gk_R = 1$. Then the fossil fuel energy intensity parameter $G = 1/k_R = 3.0506$. The energy market equilibrium condition then requires $Fk = Gk_R = 1$, so we must also have $F = A = 0.4297$.

Similarly, we identify investment in physical capital as defined in the GTAP data as $i + i_R$ in the model. Rescaling units so output equals 1, we conclude that $i + i_R = 0.2299$.

Next we calibrate the initial marginal cost of producing energy. From the resource constraint, the difference between total output and $i + i_R$, namely 0.7701, would equal $c + \mu k_R + n$, which is all classified as *consumption* expenditure in the GTAP data. We then associate $\mu k_R + n$ with the share of consumption falling on the output of the energy sector as defined above and c with the remainder. The result is $c = 0.7140$ and $\mu k_R + n = 0.0561$.

We turn to additional data sources to obtain a value for n . The *International Energy Agency* has statistics on energy sector investment in R&D²⁴ categorized into: energy efficiency (around 18% of the total from 2005–2010), fossil fuels (14%), renewable energy sources (15%), nuclear (33%), hydrogen and fuel cells (6%), other power and storage technologies (5%), and other cross-cutting technologies and basic research (8%). Much of the explicit R&D expenditure on renewable, nuclear, hydrogen and fuel cells, and cross-cutting technologies and basic research would be part of the variable j in the model. Like the use of renewables to produce energy, a substantial amount of this expenditure would not exist without extensive government support. Since we do not have government in the model, we abstract from this activity. The model also abstracts from heterogeneity in levels of economic development and assumes that the “representative country” at the current “average” level of world economic development initially relies solely on fossil fuels. We thus ignore these alternative energy investments in the calibration for $t = 0$.²⁵ Inspection of the items in the

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²³Nuclear, hydroelectric, wind and solar energy, currently supply some electricity, and some transportation is powered by electricity and biofuels, but as an approximation we are assuming that fossil fuels are the only energy source at $t = 0$.

²⁴The data is available at <http://www.iea.org/stats/rd.asp>.

²⁵Although $j > 0$ at $t = 0$ in the solution, it is immeasurably different from zero.

other power and storage technologies category reveals that they relate mainly to electrical system improvements that are not specific to renewable energy sources and thus could be grouped with R&D into fossil fuels as part of n .

The IEA also reports real GDP for the economies in their panel allowing the R&D expenditures to be expressed as a proportion of GDP. Taking the average ratio from 2005–2010 for all countries in the sample, we find that the R&D part of n would be just 0.00644% of GDP. However, the countries in the sample are mostly higher income countries from the OECD. We would expect these countries to invest a higher proportion of GDP in energy sector R&D than the remaining countries. Using the GTAP data, the countries in the IEA sample supplied about 71.7% of total GDP versus 28.3% for the remaining countries. If we assume the proportion of GDP invested in energy sector R&D in the remaining countries is on average one fourth that of the included countries, we would conclude that, for the world as a whole, the R&D part of n would be 0.0051% of GDP.

The variable n in the model should also include expenditures to expand fossil fuel reserves through exploration and investment in new mines. The EIA reports exploration and development expenditures by oil and natural gas firms.²⁶ Over the period 2003–2009, these averaged more than seventy-two times the total fossil fuel R&D expenditure recorded by the IEA. We could not find comparable data for coal mining firms, but we expect it to be much smaller. The Australian Bureau of Statistics reports²⁷ exploration expenditures by Australian coal mining firms, which according to EIA data produce around 20% of the total coal output from those countries included in the IEA R&D expenditure data set. Multiplying the recent annual Australian coal exploration expenditure by five, we arrive at a figure that is only about 3% of the oil and gas exploration and development expenditure reported by the EIA. Adding these exploration expenditures to the previously calculated fossil fuel R&D spending, we arrive at a total value for n of 0.31% of GDP. Expressing this in units defined so output equals 1, we have $n = 0.0031$.

Subtracting n from the previously calculated $\mu k_R + n = 0.0561$, we obtain $\mu k_R = 0.0530$,²⁸ and

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²⁶The data is available at <http://www.eia.gov/finance/performanceprofiles/> as Table 15.

²⁷See publication 8412.0 – “Mineral and Petroleum Exploration, Australia” available at <http://www.abs.gov.au> and searching by catalog number.

²⁸By comparison, the EIA Annual Energy Review gives energy expenditures as amounting to around 5% of gross output in the U.S.

using the previously obtained k_R we then obtain $\mu = 0.1614$.²⁹ After we set the initial values of S and N to zero and R to 1 (by defining energy units), the initial value for μ also would imply

$$0.1614 = \alpha_0 + \frac{\alpha_1}{\bar{S} - \alpha_2/\alpha_3} \quad (36)$$

Next, we evaluate $\bar{S}$ in the same units as R . The EIA web site gives world wide production of oil in 2007 of 178.596 quads (where one quad equals 10^{15} BTU), of natural gas 107.391 quads and of coal 133.367 quads. Summing these gives a total of 419.354 quads, which we will take as our measure of one unit of R .

To obtain an estimate of total fossil fuel resources $\bar{S}$ in the same units, we used data from the World Energy Council and the US Geological Survey (USGS). The millions of tonnes of coal, millions of barrels of oil, extra heavy oil, natural bitumen and oil shale and trillions of cubic feet of conventional and unconventional natural gas were converted to quads using conversion factors available at the EIA. The result is 70.282 quintillion BTU of coal, 72.122 quintillion BTU of conventional and unconventional oil and 13.821 quintillion BTU of conventional and unconventional natural gas. These resources are nevertheless relatively small compared to estimates of the volume of methane hydrates that may be available. Although experiments have been conducted to test methods of exploiting methane hydrates, a commercially viable process is yet to be demonstrated. Partly as a result, resource estimates vary widely. According to the National Energy Technology Laboratory (NETL),³⁰ the United States Geological Survey (USGS) has estimated potential resources of about 200,000 trillion cubic feet in the United States alone. According to Timothy Collett of the USGS,³¹ current estimates of the worldwide resource in place are about 700,000 trillion cubic feet of methane. Using the latter figure, this would be equivalent to 718.900 quintillion BTU. Adding this to the previous total of oil, natural gas and coal resources yields a value for $\bar{S} = 875.125$ quintillion BTU or around 2086.8425 in terms of the energy units defined so that $R = 1$.

Using $\bar{S}$, (36) will give us one equation in the four parameters $\alpha_i, i = 0, \dots, 3$. The quantity $\bar{S} - \alpha_2/\alpha_3$ represents the initial level of fossil fuel extraction S at which marginal costs of extraction

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²⁹Observe that, since we have defined units so $R = 1$, this value of μ implies fossil fuels yield positive net output at $t = 0$, that is, $AG/F_0 - \mu = G - \mu = 2.8892 > 0$.

³⁰<http://www.netl.doe.gov/technologies/oil-gas/FutureSupply/MethaneHydrates/about-hydrates/estimates.htm>

³¹<http://www.netl.doe.gov/kmd/cds/disk10/collett.pdf>

$g(S, 0)$ become unbounded. Thus, we associate $\bar{S} - \alpha_2/\alpha_3$ with current proved and connected reserves of fossil fuel.³² A report from Cambridge Energy Research Associates (CERA, 2009),³³ for example, gives weighted average decline rates for oil production from existing fields of around 4.5% per year. They also note that this figure is dominated by a small number of “giant” fields and that, “the average decline rate for fields that were actually in the decline phase was 7.5%, but this number falls to 6.1% when the numbers are production weighted.” As an approximation, we shall use 6% as a decline rate for oil fields. Using United States natural gas production and reserve figures as a guide, we find that natural gas decline rates are closer to 8% per year. The United States data on coal mine decline rates approximate 6% per year. In accordance with these figures, we assume the ratio of fossil fuel production to proved and connected reserves equals the share weighted average of these figures, namely $(178.60 * 0.06 + 107.391 * 0.08 + 133.367 * 0.06)/419.354 = 0.0651$. Thus, in terms of the energy units defined so that $R = 1$, the initial value of proved and connected reserves $\bar{S} - \alpha_2/\alpha_3$ would equal $1/0.0651 = 15.35586$. Using the previously calculated value for $\bar{S}$, this leads to $\alpha_2/\alpha_3 = 2071.48667$.

We can obtain two more equations by examining the investment in fossil fuel production at $t = 0$. As noted above, we calibrated the initial value of $n = 0.0031$. We assume that this level of investment at $t = 0$ is sufficient to increase proved and connected reserves by a percentage amount equal to the average annual growth over 2004-10 of around 2.43%.³⁴ In other words, we assume that the investment $n = 0.0031$ increases proved and connected reserves to 15.729, that is, $\alpha_2/(\alpha_3 + 0.0031) = 2071.1135$. Then using $\alpha_2/\alpha_3 = 2071.4867$, we find

$$\alpha_3 = \frac{0.0097 \cdot 2071.1135}{2071.4866 - 2071.1135} \approx 17.0411 \quad (37)$$

The previously calculated value for α_2/α_3 then implies $\alpha_2 \approx 35300$. We thus have determined the μ function up to one degree of freedom. Specifically, once we specify either α_0 or α_1 , (36) along with the previously determined $\bar{S}$ and α_2/α_3 will give us the remaining α_i parameter. We chose α_1

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³²Note that current official reserves are not the relevant measure since many of these are not connected and thus are unavailable for production without further investment, denoted n in the model.

³³“The Future of Global Oil Supply: Understanding the Building Blocks,” Special Report by Peter Jackson, Senior Director, IHS Cambridge Energy Research Associates, Cambridge, MA.

³⁴These calculations are again based on data from the EIA.

in an attempt to ensure that $\lambda(0) = c(0)^{-\gamma}$ at $t = 0$.³⁵

Turning next to the learning process on renewable efficiency, (??) and (9), the literature provides a range of estimates for the efficiency gains associated with learning by doing. An online calculator provided by NASA³⁶ gives a range of learning percentages between 5 and 20% depending on the industry. A learning percentage of x , which corresponds to a value of ψ of $-\ln(1-x)/\ln(2)$, has the interpretation that a doubling of the installed capital (our measure of “experience”) will lead to a cost reduction of $x\%$. Thus, $x = 0.2$ is equivalent to an exponent on the installed capital equal to 0.322 while $x = .05$ corresponds to to an exponent of 0.074.

We rely on some empirical estimates based on experience with subsidized installations of wind turbines and solar panels to set ψ and α . In a study of wind turbines, Coulomb and Neuhoﬀ (2006) found values corresponding to the parameter ψ in our model of 0.158 and 0.197. Grbler and Messner (1998) found a value for $\psi = .36$ using data on solar panels, while van Bentham et. al. (2008) report several studies finding a learning percentage of around 20% ($\psi = 0.322$) for solar panels. We will take $\psi = 0.25$. Klaassen et. al. (2005) estimated a model that allowed for both learning-by-doing and direct R&D. Although they assume the capital cost is multiplicative in total R&D and cumulative capacity, while we assume the *change in knowledge* is multiplicative in new R&D and cumulative capacity, we can take their parameter estimates as a guide. They find direct R&D is roughly twice as productive for reducing costs as is learning-by-doing.³⁷ Consequently, we assume that $\alpha = 0.75$.

Finally, we need to establish values for the initial $H(0)$ and final $\bar{H}$ values of the productivity of k_B in producing energy services, and the operating and maintenance costs m for renewable energy production. Although a substantial amount of current primary energy consumption is direct rather than indirect through the consumption of electricity,³⁸ we focus on the relative costs of producing

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³⁵The model is highly non-linear making it difficult and time consuming to solve. Each time α_1 is changed, the model needs to be solved many times over to find solution paths with values at $t = 0$ that approximate initial values of the state variables. Setting $\alpha_1 = 0.15$, we obtained $c(0) = 0.6614$ instead of the required 0.7140.

³⁶Available at <http://cost.jsc.nasa.gov/learn.html>

³⁷Of course, the learning-by-doing has the advantage that it directly contributes to output at the same time it is adding to knowledge.

³⁸Data from the EIA for the US shows that around 28% of primary energy is consumed in transportation, 20% in industry and another 10% in residential and commercial activities. The remainder is used to generate electricity and

electricity using fossil and non-fossil sources. In a world without fossil fuels, vehicles would probably use electric propulsion, although some fossil fuels (especially those used in air transportation) may be replaced by biofuels or other manufactured liquid fuels.

We focus on natural gas and coal generated electricity for the fossil fuel cost and nuclear,³⁹ wind and pumped storage for non-fossil generation. Not all locations have suitable geography for pumped storage, however, so a certain fraction of capacity needed to provide ancillary services will have to take more expensive forms such as batteries, flywheels, or compressed air. In the *Annual Energy Outlook, 2010* the EIA gives indicative costs for different types of generation capacity as outlined in the first four rows of Table 1 and in the heat rates for the natural gas and coal plants. The heat rate for nuclear plants comes from the average realized heat rate in the US in 2010 as reported in Table 5.3 of the EIA publication *Electric Power Annual*.

The natural gas and coal fuel prices for 2009 were obtained from Table 3.5 of the *Electric Power Annual*. The uranium price is the 2009-2011 average monthly price of U_3O_8 per pound obtained from the IndexMundi web site⁴⁰ divided by the average energy content of U_3O_8 , namely 180 MMBTU per pound.⁴¹

The load factors for the coal and nuclear plants were obtained by dividing net generation from Table 1.1 of *Electric Power Annual* by net summer capacity from Table 1.1.A of the same publication and then averaging the result for 2007-2010. Performing the same calculation for natural gas fired plants produces an average capacity factor of 0.2396. However, this would cover combined cycle and conventional steam plants fired by natural gas, which are operated as base or intermediate load, and combustion turbines, which are operated as peaking plants at a very low

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allocated as primary energy to users based on their consumption of electricity.

³⁹Although nuclear power is strictly speaking non-renewable, there is a huge supply not only of uranium but also thorium and other fissionable material. The available fuel supply can also be extended using breeder reactors, while if nuclear fusion is ever perfected as a means of generating electricity, the fuel supply would, for all practical purposes, be inexhaustible. We can also use nuclear power costs as a proxy for the costs of unconventional geothermal energy based on “hot rocks”.

⁴⁰Available at <http://www.indexmundi.com/commodities/?commodity=uranium&months=60>

⁴¹This figure was obtained from the TradeTech web site http://www.uranium.info/unit_conversion_table.php, but similar values are given elsewhere.

Table 1: Indicative Electricity Generation Costs

	Fossil Fuels			Non-fossil Energy		
	Natural Gas	Natural Gas		Electricity	Onshore	
	Combustion	Combined	Pulverized			
	Turbine	Cycle	Coal	Storage	Wind	Nuclear
Capital Cost (\$/MW)	0.665	1.003	2.84	8.393	2.438	5.34
Size (MW)	210	400	1300	250	100	2236
Fixed O&M (\$m/MW)	0.0067	0.01462	0.02967	0.01955	0.02807	0.08875
Variable O&M (\$/MWh)	9.87	3.11	4.25	0	0	2.04
Fuel (\$/MWh)	46.22	30.48	19.45	0	0	2.88
Heat Rate (MMBTU/MWh)	9.75	6.43	8.80	–	–	10.452
Fuel (\$/MMBTU)	4.74	4.74	2.21	–	–	0.28
Load Factor	0.1	0.6	0.727	0.1	0.2	0.9008
Plant Life	30	30	40	40	25	40
Capital EAC (\$m/MW)	0.5631	0.1415	0.3106	10.1477	1.0936	0.4702
Annual O&M Cost (\$m/MWyr)	0.0491	0.1765	0.1509	0	0	0.0388
Capacity Shares	0.1	0.45	0.45	0.2	0.3	0.5
Implied Output Shares	0.0165	0.4447	0.5388	0.0377	0.1131	0.8492

load factor. A briefing paper from the *National Energy Technology Laboratory* (NETL)⁴² assumes that combined cycle plants would be operated in the US as base load plants at 85% load factor, but they may sometimes be used to supply intermediate load. A technology brief from the IEA⁴³ claims that typical international values for load factors of combined cycle plants range from 0.2–0.6, while corresponding values for combustion turbines range from 0.1–0.2. For the calculations in the table, we have assumed that combined cycle plants are operated at the top of the IEA range (0.6) and combustion turbines at the low end of the IEA range (0.1). We also assumed that the load factor for pumped storage, which we assumed would provide half the peaking services in a non-fossil fuel world, would equal the load factor (0.1) of natural gas combustion turbines in the fossil fuel world. We assume that the cost of other forms of storage would be double the cost of pumped storage, so we inflate the pumped storage capital and fixed O&M costs by 50%. We also assume an average plant life for backup capacity of 25 years.

The data on wind energy from the *Electric Power Annual* implies an average load factor for 2007–2010 of only 0.1306 for US wind generators. The EIA web site also provides data on annual electricity generation by country and type and installed generation capacity by country and type. For the world as a whole in 2009 the average load factor for wind generators was 0.2025.⁴⁴ Since one might expect the better sites for wind generation to be used first, we would not expect load factors to rise much over time.⁴⁵ Nevertheless, we assumed a load factor of 0.20 for wind.

Calculations of the levelized cost of electricity generation often assume a plant life of 30 years. However, the Environmental Protection Agency has produced a database (the National Electric

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⁴²“Cost and Performance Baseline for Fossil Energy Plants,” Vol. 1, DOE/NETL-2007/1281, May 2007 available at http://www.netl.doe.gov/energy-analyses/pubs/deskreference/B_NGCC_051507.pdf

⁴³Available at http://www.iea-etsap.org/web/E-TechDS/PDF/E02-gas_fired_power-GS-AD-gct.pdf

⁴⁴The same source produces an average worldwide load factor for nuclear plants of only 0.7739, but for the calculations we assumed the higher US factor. Unfortunately, the coal and natural gas generators were lumped together as “conventional thermal,” so we could not use this data to separately estimate the load factor for coal and natural gas plants. However, the overall load factor of 0.4506 for conventional thermal plants again suggests that combined cycle natural gas plants are operated at lower load factors than the 0.85 assumed by NETL.

⁴⁵Some sites with good wind conditions might not be connected in early years of development because they are remote from markets and need to await construction of suitable transmission infrastructure.

Energy Data System (NEEDS) database)⁴⁶ of characteristics of all electric generating plant in the US, including their year of commissioning. The average age of coal-fired generators in the database is 38 years, but many plants will be far from the end of their useful life. We therefore assumed an operating life of 40 years for the coal and nuclear plants. The average age of the latter in the NEEDS data was 24 years, but many more of these plants would still be a long way from their full life span. The conventional oil or gas-fired steam plants had an average age of 44 years. The average age of the combustion turbines in the NEEDS database was 27 years, while the combined cycle plants averaged just 13 years, but only because combined cycle is a relatively new technology. Nevertheless, the single and combined cycle turbines might be expected to last a shorter period of time than the steam plants. Hence, we assumed the conventional 30 year life span for the single and combined cycle turbines. The average age of the pumped storage plants in the NEEDS database was also 30 years, so we also assumed a 40 year life span for these. Finally, the wind generators in the NEEDS database were also constructed recently, so we do not have a good indication of how long they may last. However, several sources on the internet gave a design life of 20 years for wind turbines, while the maximum estimated lifespan we found was 30 years. For our calculations we assumed a 25 year life.

The lower half of Table 1 shows various calculated quantities based on the data in the top half of the Table. The costs are divided into costs that would be included in investment in the GTAP data (capital costs) versus those that would be part of “consumption” (operating and maintenance expenditures).

The equivalent annual capital cost (EAC) of capital per MW of capacity was calculated based on an assumed annual real required rate of return of 7.5%. The EAC essentially assumes a continuous cycle as old plants are replaced when they attain the end of their useful life. To obtain an overall annual capital cost, we need to weight the EAC/MW for each technology by the share of that technology in the overall generating capacity.

Based partly on the NEEDS data, but also allowing for the share of combined cycle plants to be somewhat higher in the future to reflect the higher efficiency of those plants relative to older single cycle plants, we assumed that peaking plant (combustion turbines) would constitute 10% of

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⁴⁶ Available at <http://www.epa.gov/airmarkt/progsregs/epa-ipm/past-modeling.html#needs>.

total capacity (and be used only for 10% of total hours). We then divided the remaining required capacity equally between coal and natural gas combined cycle plants.

For the non-fossil world, we increased the share of backup storage to 20% of capacity to account for the fact that the intermittency of wind (and also solar) power will require additional peaking capacity which nevertheless will not be as fully utilized as in the fossil fuel world (so we retain the load factor of 0.1). The intermittency of wind generation also makes it unlikely that network stability could be maintained with such sources constituting more than 30% of system wide capacity. We therefore set the wind capacity at 30% and nuclear at 50% in the non-fossil world.

The final row of Table 1 then gives the implied shares of each type of capacity in annual electricity production. These result from the assumed capacity shares and the load factors of each type of plant.

The annual capital costs *for the system* are calculated as a capacity-weighted average of the EAC for each type of plant. Similarly, the annual O&M costs for the system are calculated as a capacity-weighted share of the fixed O&M costs (from row 3) plus an output-weighted sum of the combined variable O&M and fuel costs (rows 4 plus 5). The latter were converted to an annual basis by multiplying by the number of hours in a year that each type of plant would be operated, namely the load factor times 8760.

We next obtained the ratio of the annual capital costs for the non-fossil system to the annual capital costs for the fossil-based system and the ratio of the annual O&M costs for the two generating systems. The resulting numbers were 9.9807 for the capital costs and 0.4948 for the O&M costs. We already calculated above that the energy output to capital ratio for fossil fuels G was 3.0506. We therefore assumed that the corresponding initial energy output to capital ratio for non-fossil fuels was $H = 3.0506/9.9807 \approx 0.3056$. Similarly, since we calculated above that the initial value for $\mu = 0.1614$, we take the value of $m = 0.1614 * 0.4948 \approx 0.0799$.

Finally, we need to specify a final limiting value for H and the coefficient b in the renewable energy technological change function. We arbitrarily assumed that H could be increased by a factor of 4 to $\bar{H} \approx 1.2224$. Then the final ratio $m/\bar{H} \approx 0.0653$ would be not much above the current ratio for fossil fuels $\mu/G \approx 0.0529$. Once the values of $\bar{H}$ and m have been set, the long run per capita growth rate can also be calculated as 4.14%. We could not find a suitable data source to calibrate b , so we simply set it to 0.006 to obtain an approximate 100 years until the renewable

sources attain their final long-run efficiency level.

7 Results

Figure 4: Calculated paths of the state variables

Figure 4 graphs the solution paths for the state variables.⁴⁷ The critical times for transition between the various regimes are $T_Q = 73.5248$, $T_N = 79.0248$, $T_R = 80.0424$, $T_B = 85.7931$ and

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⁴⁷The solution discussed here has $k(0) = 2.3285$, $H(0) = 0.3070$, $S(0) = 0.00004$ and $N(0) = -0.0924$ compared to target values of 2.3273, 0.3056, 0 and 0. The initial value of S is closest to its target because S responds most sensitively to changes in the values used to determine the solution to the model's differential equations. The initial value of N is least sensitive, making it hardest to match. The highly non-linear nature of the solutions prevented us using an automatic solution procedure to find the best "initial values" to hit the required target values. Small changes in the "initial values" easily move the differential equations to regions where they cannot be solved.

$T_H = 99$. Thus, fossil fuels are abandoned after about 80 years, but investment in k_R ceases after about 73.5 years. Investment in fossil fuel technology continues for another 5.5 years, ceasing after slightly more than 79 years.

Figure 5: Calculated paths of the control variables

The need to replace fossil fuel energy production at T_Q means that there has to be considerable investment in k_B prior to T_Q . In addition, the rapid increase in H after T_Q means that less k_B is needed to supply a given level of energy services. The result is a “hang over” of additional k_B at T_R . As illustrated in the third graph in Figure 5, investment i_B in k_B is thus zero for about five years (from T_R until T_B) and k_B declines over this period. Similarly, k declines along with k_R between T_Q and T_R as both i and i_R are zero over this interval (see the second and fourth graphs in Figure 5). The fossil fuel capital stock continues to decline, but remain positive, for $t > T_R$,

although it is not used to supply energy services after T_R since renewable sources are cheaper.

The solution for S implies that slightly over 77% of the initial stock of fossil fuel resources are exploited. Fossil fuels are abandoned not because they are exhausted but because they ultimately become too costly relative to the alternative. Investment in fossil fuel technology N does not play a large role until the end of the fossil fuel era. Indeed, there is a large spike in investment n as investment in k_R ceases, with a smaller jump in n occurring right before n drops of quickly to zero. Similarly, investment i_B in k_B and j in H do not take off until investment in k_R ceases at T_Q . The efficient path in this economy thus has investment largely restricted to k and k_R between $t = 0$ and $t = T_Q$, with the focus on exploiting fossil fuel to supply energy services with minor investments in technology N until the end of the fossil fuel era looms.

A striking feature of the optimal paths of the control variables graphed in Figure 5 is their non-linearity and somewhat volatile character – especially investment in final capital k and renewable energy producing capital k_B . Some of these fluctuations appear to be to facilitate investment n and j in the two R&D variables. The economy also is severely disrupted by the switch from using fossil fuels to renewables to supply energy services. Part of the explanation is that the real price of energy rises to a peak at T_R and the need to spend on energy takes resources away from investment in k and consumption. Around the same time, the growth in energy R&D in the form of increases in both n and j also reduces the resources available for consumption. It is quite striking that per capita consumption actually declines for nearly 7 years (between T_Q and T_R).

Figure 6 graphs the solution paths for the main⁴⁸ co-state variables. The shadow price ν of investing in N is equal to λ while $n > 0$ (that is, between $t = 0$ and T_N) after which point it declines quickly to zero and remains there.

The shadow price σ of the fossil fuel resource mined to date S is negative until fossil fuels are abandoned at T_R , at which point σ becomes zero and remains at zero thereafter. The negative shadow price reflects that assumption that increased mining raises future costs. The real (or utility) value of the shadow price σ/λ declines continuously (increases in absolute value) until T_N , after which the increase to zero is swift.

Finally, the shadow price of H increases to a peak at T_R and then declines to zero at T_H when

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⁴⁸A graph of the small interval over which q_B differs from λ has been omitted. Also, $q_R = \lambda$ until T_Q , at which point it declines to zero and remains at zero thereafter.

Figure 6: Calculated paths of the co-state variables

additional gains in H are no longer possible. This implies that the incentive to invest in H follows the same pattern. The ratio η/λ looks much more similar to the graph of investment j in H . However, it does start to increase earlier than j (after 60 years) and the “bump” immediately prior to T_R is longer lasting and more noticeable in η/λ than in j .

8 Concluding remarks

We have presented an intertemporal optimizing model of economic growth where energy inputs are essential to final production. The energy services can be supplied by either fossil fuels or renewable sources. Initially, the fossil fuels sources are much cheaper than the alternative and supply all the energy used in final production. However, depletion eventually raises the cost of fossil fuels, while

R&D allows the cost of renewable sources to decline gradually over time. The result is that fossil fuels are replaced by renewable sources, which eventually supply all energy used by the economy. Fossil fuels will cease to provide energy services not because they are exhausted physically but because they eventually become too costly. In the numerical example presented in the paper, only slightly over 77% of the initial stock of fossil fuel resources ends up being exploited.

A feature of our model relative to many alternative models of the energy and growth process is that we allow technical progress in fossil energy production to offset the cost increases from depletion. This fossil fuel R&D delays the time that fossil fuels are displaced by renewables but cannot prevent the transition indefinitely. Once the renewable sources displace fossil fuels in supplying energy for final production, the rate of technological progress in renewables accelerates as a result of a substantially increased rate of learning by doing.

Another distinguishing feature of our model is that physical capital is needed to turn fossil fuel resources into useful energy services or to “harvest” the fundamental non-fossil energy sources (sunlight, wind, geothermal resources, fissionable nuclear material etc) and make them available as energy that can be used in final production. We assume that these capital stocks are specific in the sense that capital used to provide energy services from fossil fuels cannot be re-purposed to be used with renewable energy. An important consequence is that the economy has to invest in a stock of the latter type of capital so it is ready to take over from fossil fuels in supplying energy services at the transition point.

Prior to the transition, the price of energy is set by the cost of fossil fuels, and this is rising over time. Also, the cost of renewable energy is falling over time as a result of R&D investment and learning by doing. At the moment of transition, the short-run cost of producing energy from the two sources will be equal. Thus, prior to the transition, the short-run cost of renewable energy will exceed the price of energy as set by fossil fuels. The result is a “valley death” type of scenario for renewable energy in the sense that investment in renewable productive capacity is required well before the energy price is sufficient to cover the cost of that capacity. In the numerical example presented in the paper, renewable sources take over from fossil fuels after about 80 years and investment in renewable energy production capital begins in earnest after about 74 years. However, energy prices are not sufficient to cover the full costs of producing energy (including a competitive return on capital) until after almost 86 years.

Investment in renewable energy R&D and productive capacity prior to the transition point delivers value to the economy. It means not only that renewable energy is ready to take over from fossil fuels in supplying energy at the transition, but also these investments will lower the cost of renewable energy through direct R&D and learning by doing. The latter effects will not only deliver benefits after the transition by allowing the economy to have a lower cost of energy. They will also hasten the transition and reduce the cost of energy in the fossil fuel era by lowering the opportunity cost of exploiting fossil fuels more rapidly. These myriad benefits, of course, explain why such investments in renewable energy prior to the transition point appear as part of the efficient path for the economy. Whether those benefits would be fully appropriable to private entrepreneurs and allow a competitive equilibrium to support the Pareto optimum is, of course, an entirely different matter. Our model may in this sense complement much of the literature explaining the origin of the “valley of death” phenomenon, which has focused on the inability of entrepreneurs to appropriate all of the benefits of early research into alternative energy technologies. On the other side of this argument, however, we note that in the numerical example we presented in the paper, the efficient level of investments in both renewable energy R&D and productive capacity are very low for more than 70 years. In the absence of other reasons, such as environmental externalities, for hastening the transition to alternative energy technologies, it may be questioned presently weak support for investments in alternative energy technologies truly reflects an inefficient allocation of resources.

9 Appendix: The differential equations applying in each regime

In this appendix, we outline the differential equations that apply in each regime and discuss the numerical solution procedure. Following the numerical solution procedure, we work through the regimes backwards in time. This discussion will be easier to follow if the reader keeps Figure 3, showing the time line and paths of investment and energy production, available for reference.

9.1 The long run endogenous growth economy

Beyond T_H , H is constant at $\bar{H}$. The control variables are c , i and i_B , while the state variables are k and k_B . In this regime, the resource constraint simplifies to

$$c + i + i_B + mk_B = Ak \quad (38)$$

while the energy market equilibrium condition becomes

$$Fk = \bar{H}k_B \quad (39)$$

To maintain the ratio (39), the two investments cannot be chosen separately. The economy must evolve along a balanced growth path. Intuitively, since the marginal costs of investing in the two types of capital are identical, the marginal benefits also must be kept the same. Differentiating (39) and using (39) and the assumption that the depreciation rates are identical, we also obtain

$$Fi = \bar{H}i_B \quad (40)$$

With both $i, i_B > 0$, (16) and (18) imply $q = \lambda = q_B$. Noting also that $j = 0$ and $\rho_B = 1$, the co-state equations for q and q_B in this regime then imply

$$\dot{\lambda} = (\beta + \delta)\lambda - \lambda A + p_e F = (\beta + \delta)\lambda + \lambda m - p_e \bar{H} \quad (41)$$

In particular, the price of energy is constant at

$$p_e = \frac{A + m}{\bar{H} + F} \lambda \quad (42)$$

while λ satisfies the differential equation

$$\frac{\dot{\lambda}}{\lambda} = \beta + \delta - \frac{A\bar{H} - mF}{\bar{H} + F} \equiv -\bar{A} \quad (43)$$

where $\bar{A}$ is a constant. To get perpetual growth, we must have $c \rightarrow \infty$ as $t \rightarrow \infty$, which from (13) will require $\lambda \rightarrow 0$ and hence $\bar{A} > 0$, that is⁴⁹

$$A > \beta + \delta + \frac{F}{\bar{H}}(\beta + \delta + m) \quad (44)$$

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⁴⁹High productivity A of k in the production of goods, a low terminal fuel intensity F of goods production, a high terminal efficiency $\bar{H}$ of renewable energy production, and a low discount rate β , depreciation rate δ and operating and maintenance cost of renewable energy production m all make it more likely that (44) will hold.

The solution to (43) can be written

$$\lambda = \bar{K}e^{-\bar{A}t} \quad (45)$$

for some constant $\bar{K}$ yet to be determined. Now use the differential equation for k , (45) and the first order condition (13) for c , the resource constraint (38), the constraint on investment (40) and the definition of $\bar{A}$ in (43) to obtain

$$\dot{k} = (\bar{A} + \beta)k - \frac{\bar{H}\bar{K}^{-1/\gamma}}{\bar{H} + F}e^{\bar{A}t/\gamma} \quad (46)$$

The integrating factor for the differential equation (46) is $e^{-(\bar{A}+\beta)t}$, so the solution can be written

$$k = C_0e^{(\bar{A}+\beta)t} + \frac{\bar{H}\gamma\bar{K}^{-1/\gamma}}{(\bar{H} + F)[\beta\gamma + \bar{A}(\gamma - 1)]}e^{\bar{A}t/\gamma} \quad (47)$$

for another constant C_0 . However, the transversality condition requires

$$\lim_{t \rightarrow \infty} e^{-\beta t} \lambda k = C_0\bar{K} + \lim_{t \rightarrow \infty} \frac{\bar{H}\gamma\bar{K}^{(1-1/\gamma)}}{(\bar{H} + F)[\beta\gamma + \bar{A}(\gamma - 1)]}e^{(\bar{A}/\gamma - \bar{A} - \beta)t} = 0 \quad (48)$$

Equation (48) in turn requires

$$\lim_{t \rightarrow \infty} e^{(\bar{A}/\gamma - \bar{A} - \beta)t} = 0, \text{ that is, } \bar{A}(1 - \gamma) < \beta\gamma \quad (49)$$

and also $C_0 = 0$.⁵⁰ Thus, the value of k in the final endogenous growth economy will be given by

$$k = \frac{\bar{H}\gamma\bar{K}^{-1/\gamma}e^{\bar{A}t/\gamma}}{(\bar{H} + F)[\beta\gamma + \bar{A}(\gamma - 1)]} \quad (50)$$

with λ (and also q and q_B) given by (45) and where $\bar{K}$ is a constant yet to be determined.

From (39) and (50), the capital stock allocated to renewable energy production in the final endogenous growth economy will be

$$k_B = \frac{F\gamma\bar{K}^{-1/\gamma}e^{\bar{A}t/\gamma}}{(\bar{H} + F)[\beta\gamma + \bar{A}(\gamma - 1)]} \quad (51)$$

---

⁵⁰Since $\bar{A} > 0$, the inequality in (49) will be satisfied if $\gamma > 1$, as is the case in our calibration. If $0 < \gamma < 1$, the inequality in (49) would require $\bar{A} < \frac{\beta}{1-\gamma} + \delta + \frac{F}{\bar{H}}(\frac{\beta}{1-\gamma} + \delta + m)$ which would further limit the range of acceptable parameter values relative to (44).

The growth rate of the final regime will be⁵¹

$$\frac{\bar{A}}{\gamma} = \frac{1}{\gamma} \left[\frac{A\bar{H} - mF}{\bar{H} + F} - (\beta + \delta) \right] \quad (52)$$

Working backwards in time, the beginning of the final analytical regime at T_H occurs when H attains $\bar{H}$ and $\eta = 0$ (which in turn implies $Y = 0$). The values k, k_B, λ and φ at T_H must match the values at the end of regime 5 since these variables must be continuous across the boundary.

9.2 Regime 5: Fully dynamic renewable regime

Regime 5 will have direct investment in renewable energy production efficiency ($j > 0$) in addition to investment in end-use capital ($i > 0$) and renewable energy production capital ($i_B > 0$). The solutions for c and j are given by (13) and (20) respectively. In particular, technological progress in renewable energy production will satisfy

$$\dot{H} = bk_B^{s\psi} Y^{\alpha-\psi} \quad (53)$$

where the function Y was defined previously.

The differential equations for the remaining state variables involve i and i_B . Noting that we still have $\rho_B = 1$, the resource constraint now implies

$$i + i_B = Ak - Yk_B^{s\psi} - mk_B - \lambda^{-1/\gamma} \quad (54)$$

and the energy market equilibrium condition is

$$Fk = Hk_B \quad (55)$$

Differentiating (55), and using (53), (55), and (20), and the differential equations (3) and (4), we obtain a second equation linking i and i_B

$$Fi - Hi_B = bk_B^{s\psi+1} Y^{\alpha-\psi} \quad (56)$$

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⁵¹In an endogenous growth model with a single capital stock k , linear production function Ak , depreciation rate of capital δ , and representative consumer with time discount rate β and constant relative risk aversion γ , the growth rate of the economy is $[A - (\beta + \delta)]/\gamma$. Thus, the need for energy input to production reduces the *gross* productivity of capital A by an amount that depends on the marginal cost of producing renewable energy m . Furthermore, the weights on A and m in *net* productivity depend on the relative values of the productivity of capital in producing renewable energy $\bar{H}$ and the energy intensity of final production F .

Equations (56) and (54) then give us two equations to solve for i and i_B . The differential equations (3) and (4) then yield $\dot{k}$ and $\dot{k}_B$.

In this regime, we will again have $q = \lambda = q_B$ and hence we obtain distinct co-state differential equations only for λ and η . Noting that the price of energy p_e will be given by the long run cost of renewable supply (30) and $\rho_B = 1$, the co-state differential equations are $\dot{\eta} = \beta\eta - p_e k_B$ and $\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F$.

Figure 7: Solving for investments in regime 5

The solution for investments in regime 5 can be illustrated as in Figure 7, where both i and i_B are positive. However, we expect that the incentive to invest in H will tend to decrease over time as $H \uparrow \bar{H}$. Solving backwards in time in regime 5, we therefore expect $\dot{H}/H$ to increase, which will shift the upward sloping line to the right. On the other hand, as we move backwards through time, the resources available to support both investments will tend to decrease, shifting the downward sloping line to the left. Thus, as we solve backwards in time, i_B is likely to decline rapidly and we would find the constraint $i_B \geq 0$ binding at the lower boundary T_B of regime 5.

9.3 Regime 4: No investment in renewable capacity

Regime 4 involves full use of available renewable capacity ($\rho_B = 1$) but no investment in additional capacity ($i_B = 0$ and hence $\dot{k}_B = -\delta k_B$). The lower boundary of regime 4 will be T_R where energy production shifts out of fossil fuels into renewables.

Noting that c and j are still given by (13) and (20) respectively, we conclude that i will be given by the resource constraint:

$$i = Ak - Yk_B^{s\psi} - mk_B - \lambda^{-1/\gamma} \quad (57)$$

However, as in regime 5, the energy market equilibrium condition will also determine a value for i :

$$Fi = bk_B^{s\psi+1}Y^{\alpha-\psi} \quad (58)$$

To ensure that these two values are equal, the price of energy p_e needs to adjust to determine appropriate values for the co-state variables λ and η . Equating the two expressions for i from (57) and (58) we obtain

$$bk_B^{s\psi+1}Y^{\alpha-\psi} + FYk_B^{s\psi} - FAk + mFk_B + \lambda^{-1/\gamma}F = 0 \quad (59)$$

Before differentiating (59), we use (21) and (26) to obtain the derivative of η/λ :

$$\frac{d}{dt}\left(\frac{\eta}{\lambda}\right) = -\frac{p_e}{\lambda}\left[k_B + F\frac{\eta}{\lambda}\right] + (A - \delta)\frac{\eta}{\lambda} \quad (60)$$

Also, using the definition of Y , we obtain:

$$\dot{Y} = (s - 1)bY^{\alpha-\psi}\frac{d}{dt}\left(\frac{\eta}{\lambda}\right) \quad (61)$$

while we can write the derivative of λ in terms of p_e as

$$\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F \quad (62)$$

The derivative of (59) can then be written in terms of these expressions as:

$$k_B^{s\psi}\left[F\dot{Y} - s\psi\delta FY - b\delta(s\psi + 1)k_BY^{\alpha-\psi} + k_B\frac{\lambda}{\eta}\dot{Y}\right] + F\left[A\delta k - Ai - m\delta k_B - \frac{1}{\gamma}\lambda^{-\frac{1+\gamma}{\gamma}}\dot{\lambda}\right] = 0 \quad (63)$$

Equation (63) can then be solved for an energy price p_e .

Both the resource constraint (57) and the energy market equilibrium condition (58) are satisfied at T_B where p_e equals the long-run cost (30) as it does for all $t > T_B$. The energy price p_e solving

(63) will ensure (57) and (58) both remain satisfied throughout regime 4. Investment i will be given by either (57) or (58). The co-state variable η will evolve according to (24), and q_B will evolve according to (23), with $\rho_B = 1$ in both cases. The co-state q_B will have initial value $q_B = \lambda$ at T_B .

At the lower boundary T_R of regime 4, $q = \lambda$ and $\dot{q} = \dot{\lambda}$ will be given by (21) as also is true for all $t > T_R$. At the same time, $i_B > 0$ in regime 3 and (18) implies $q_B = \lambda$ and $\dot{q}_B = \dot{\lambda}$. Also, $\rho_B = 0$ in regime 3, as it does for all $t < T_R$. Then from (23), we conclude that in regime 3, $\dot{\lambda}$ will evolve according to

$$\frac{\dot{\lambda}}{\lambda} = \beta + \delta - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y \quad (64)$$

The two expressions (62) and (64) for $\dot{\lambda}$ at T_R imply that at the lower boundary of regime 4 (upper boundary of regime 3) p_e will equal the alternative long-run cost of renewable energy (31).

Also at the lower boundary of regime 4, ρ_R jumps from one to zero, while ρ_B jumps from zero to one. Then (15) implies $\theta_{BL} = \theta_{BU} = 0$ while (14) implies $\theta_{RL} = \theta_{RU} = 0$ at T_R . Thus, p_e must equal the two short-run costs of energy production, (29) and (27), at T_R . Using the fact that σ converges to zero at T_R , we therefore must have

$$\frac{p_e}{\lambda} = \frac{\mu}{G} = \frac{m}{H} \quad (65)$$

at T_R . Equation (65) then gives a condition that can be used to determine not only T_R but also the value of S at T_R once $N(T_R) = \bar{N}$ has been specified. Also, given that k_B and H are known at T_R when solving backwards in time, energy market equilibrium will determine a limiting value for k_R at T_R :

$$k_R(T_R) = \frac{H(T_R)}{G} k_B(T_R) \quad (66)$$

9.4 Regime 3: Only fossil fuels used, $i_R = n = 0$

In this regime, only fossil fuels are used to produce energy ($\rho_R = 1, \rho_B = 0$). However, we have $i_R = 0$, so k_R declines according to $\dot{k}_R = -\delta k_R$. We also have $n = 0$, so N remains fixed at $\bar{N}$ throughout the regime.

Using the solutions for c and j , the resource constraint can be written

$$i + i_B = Ak - Y k_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma} \quad (67)$$

A second equation, involving i alone, can again be obtained from the energy market equilibrium condition, which now becomes $Fk = Gk_R$. However, since $i_R = 0$ and F and G are constant, differentiation implies we now must also have $i = 0$. Then (67) implies $i_B = Ak - Yk_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma}$. In addition, since $i = 0$ we cannot conclude that $\lambda = q$ in this regime. However, from (18) and $i_B > 0$, we will have $q_B = \lambda$ and hence from (23), $\rho_B = 0$ and (20), λ will evolve according to (64).

With only one capital stock investment i_B positive, the price of energy p_e cannot be determined by either of the long-run costs of energy production. Instead, throughout this regime (and also in regime 2) the demand for energy Fk exactly matches the available capacity for fossil fuel energy production Gk_R and p_e will be given by the short-run cost of fossil fuel (27). The co-state variables q_R, ν, σ and η will evolve according to (22), (24), (25) and (26) with p_e given by (27), $\rho_R = 1$ and $\rho_B = 0$. In particular, throughout regime 3 (and also regime 2), $\dot{q}_R = (\beta + \delta)q_R$, and since $q_R = 0$ at T_R we must have $q_R = 0$ throughout regime 3 (and also regime 2). In addition, for all $t < T_R$, η will evolve according to

$$\dot{\eta} = \beta\eta \quad (68)$$

Thus, η will be strictly greater than zero and increasing exponentially for $t < T_R$, while for $t > T_R$ it must ultimately decrease to zero at T_H . In other words, the incentive to invest in renewable energy efficiency improvements increases as $t \rightarrow T_R$, but ultimately must decline to zero as $t \rightarrow T_H$. Similarly, (25) with $\rho_R = 1$ implies

$$\dot{\sigma} = \beta\sigma + \lambda k_R \frac{\partial \mu}{\partial S} \quad (69)$$

while (24) with $\rho_R = 1$ implies

$$\dot{\nu} = \beta\nu + \lambda k_R \frac{\partial \mu}{\partial N} \quad (70)$$

The lower boundary T_N of regime 3 will be where $\nu = \lambda$.

9.5 Regime 2: Only fossil fuels used, investment in N but not k_R

For $T \in [T_Q, T_N]$, we again have $\rho_R = 1$ and $\rho_B = 0$. We also still have $i_R = 0$, so k_R again declines according to $\dot{k}_R = -\delta k_R$. As in regime 3, energy market equilibrium will then imply that $i = 0$ and $\dot{k} = -\delta k$. However, we have positive investments i_B, j and n .

Using the solutions for c and j , and the conclusion that $i = 0$ along with $i_R = 0$, the resource

constraint can be written

$$i_B + n = Ak - Yk_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma} \quad (71)$$

Since $n > 0$ for all $t \leq T_N$, we also have $\nu = \lambda$ and hence $\dot{\nu} = \dot{\lambda}$. Noting that $i_B > 0$ and (18) imply $\lambda = q_B$, and using $\rho_B = 0$ and (23), we again deduce that λ evolves according to (64). Thus, using also (24) and $\rho_R = 1$, we obtain an equation:

$$k_R \frac{\partial \mu}{\partial N} = \delta - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y \quad (72)$$

Before differentiating (72), we use (26), $\rho_B = 0$, (64) and (72) to obtain the derivative of η/λ in this regime:

$$\frac{d}{dt} \left(\frac{\eta}{\lambda} \right) = -\frac{\eta}{\lambda} \frac{\partial \mu}{\partial N} k_R \quad (73)$$

Then (recalling also that $(\alpha - 1)s = \psi s - 1$ and $i_R = 0$) the derivative of (72) can be written as:

$$\begin{aligned} & -\delta \frac{\partial \mu}{\partial N} k_R + k_R \frac{\partial^2 \mu}{\partial N^2} n + Q k_R^2 \frac{\partial^2 \mu}{\partial S \partial N} + \\ & \frac{\psi s k_B^{\psi s - 2}}{\alpha - \psi} \left[(\alpha - 1)Y(i_B - \delta k_B) + (\alpha - \psi) b k_B Y^{\alpha - \psi} \frac{d}{dt} \left(\frac{\eta}{\lambda} \right) \right] = 0 \end{aligned} \quad (74)$$

The two equations (71) and (74) can then be solved for the two investments i_B and n .

Using $\nu = \lambda$ we find that $\dot{\lambda}/\lambda$ will now satisfy a much simpler equation

$$\frac{\dot{\lambda}}{\lambda} = \beta + k_R \frac{\partial \mu}{\partial N} \quad (75)$$

The differential equations for the co-state variables η and σ will remain as in regime 3, namely (68) and (69).

Finally, the fact that p_e is given by (27) as in regime 3 implies that $\dot{q}R = (\beta + \delta)q_R$ and hence q_R again remains zero throughout the regime. However, at the lower boundary of regime 2, we must have $q_R = \lambda > 0$ since $i_R > 0$ throughout regime 1 and falls to zero only at the upper boundary of regime 1 (lower boundary of regime 2). The co-state variable q_R must therefore be left continuous and differentiable for all $t < T_Q$, but jump discontinuously to zero at T_Q at the moment that investment in k_R ceases. The lower boundary T_Q of regime 2 cannot be calculated endogenously and thus becomes an additional parameter that has to be set exogenously.

9.6 Regime 1: Investment in both k_R and k_B but only fossil fuel is used

Regime 1 is meant to approximate the current situation. We currently see investment in renewable sources, but primarily to accumulate knowledge about their use and improve their efficiency. Such sources are uncompetitive with fossil fuels for supplying energy so the latter would supply virtually all energy services were it not for subsidies.⁵² In our (somewhat simplified) representation of the outcome we would expect in an equilibrium *without* subsidies, we assume that renewable capital is not used to produce any energy at $t = 0$ ($\rho_B = 0$) and, as argued above, will in fact not be used in the solution until fossil fuel is abandoned at T_R . All energy investments i_R, i_B, n and j are, however, positive in this regime.

Using the solutions for c and j , the resource constraint can now be written

$$i + i_B + i_R + n = Ak - \mu k_R - Yk_B^{s\psi} - \lambda^{-1/\gamma} \quad (76)$$

Once again, the energy market equilibrium condition can be differentiated (noting now that $i, i_R > 0$) to yield

$$Fi - Gi_R = 0 \quad (77)$$

and hence $i = Gi_R/F$. A third equation involving the investments can again be obtained from (72). Since $i_R > 0$, however, (74) is modified to an equation involving i_R, i_B and n :

$$\begin{aligned} & \frac{\partial \mu}{\partial N} i_R - \delta \frac{\partial \mu}{\partial N} k_R + k_R \frac{\partial^2 \mu}{\partial N^2} n + Qk_R^2 \frac{\partial^2 \mu}{\partial S \partial N} + \\ & \frac{\psi s k_B^{\psi s - 2}}{\alpha - \psi} \left[(\alpha - 1)Y(i_B - \delta k_B) + (\alpha - \psi)bk_B Y^{\alpha - \psi} \frac{d}{dt} \left(\frac{\eta}{\lambda} \right) \right] = 0 \end{aligned} \quad (78)$$

where the derivative of η/λ is once again given by (73). The fourth equation involving investments arises from the fact that with both $i_R, i_B > 0$ the price of energy has to equal both long run costs (28) and (31), yielding:

$$\mu - \frac{\sigma}{\lambda} Q - \frac{AG}{F} + \frac{\psi k_B^{(\alpha - 1)s} Y(F + G)}{(\alpha - \psi)F} = 0 \quad (79)$$

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⁵²As noted in the introduction, we do not include hydroelectric and nuclear electricity generation as renewable energy. Renewable energy therefore applies to other sources such as wind or solar energy. A more complete model would include a third type of energy producing capital that is accumulated and used in the fossil regime while also continuing to be used in the renewable regime.

Before differentiating (79), it is useful to note that we can use $\nu = \lambda$ and thus (24) and (25) with $\rho_R = 1$ to obtain a simple expression for the derivative of σ/λ :

$$\frac{d}{dt}\left(\frac{\sigma}{\lambda}\right) = \left[\frac{\partial\mu}{\partial S} - \frac{\sigma}{\lambda}\frac{\partial\mu}{\partial N}\right]k_R \quad (80)$$

Then differentiating (79) we obtain:

$$\begin{aligned} & \frac{\partial\mu}{\partial N}n + \frac{\sigma Q}{\lambda} \left[\frac{\partial\mu}{\partial N}k_R - \pi \right] + \\ & \frac{\psi s(F+G)k_B^{\psi s-2}}{(\alpha-\psi)F} \left[(\alpha-1)Y(i_B - \delta k_B) + (\alpha-\psi)bk_BY^{\alpha-\psi} \frac{d}{dt}\left(\frac{\eta}{\lambda}\right) \right] = 0 \end{aligned} \quad (81)$$

The four equations (76), (77), (78) and (81) can then be solved for i, i_R, i_B and n .

The differential equations governing the evolution of the co-state variables will again have $\rho_R = 1$, $\rho_B = 0$ and p_e given by (31). In particular, $\dot{\lambda}/\lambda$ (with $q_R = q_B = \lambda$) will satisfy the simpler equation (75), while $\dot{\sigma}$ and $\dot{\eta}$ will again satisfy (69) and (68) respectively.

9.7 Initial and terminal conditions

At $t = 0$, there are three initial conditions for the physical capital stocks $k(0)$, $k_R(0)$ and $k_B(0)$, and an initial value for renewable energy capital productivity $H(0)$. In addition, by definition, the initial values of S and N should be zero. However, the initial values of these six variables cannot be calibrated separately since equations (72) and (79) must hold at $t = 0$. These two equations imply two more constraints on possible initial values for the state variables. In effect, active investment in N, k_R and k_B at $t = 0$ requires marginal benefits of investing to be equal (since they all have the same marginal costs) and this in turn constrains the relative magnitudes of these state variables. Altogether, therefore, we only have four independent targets for the state variables. We will take these to be $k(0), H(0)$ and $N(0) = S(0) = 0$. Thus, we need to set four initial values for the differential equations that can then be varied to ensure that we hit these targets. The solution in the final analytical regime depends on an unknown constant $\bar{K}$. We also need to specify the value of T_H , the value of N at T_R and the time T_Q when investment in fossil fuel capital k_R ceases.

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