The Role of the Real Interest Rate in US Macroeconomic History

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A negative real interest rate has guaranteed macroeconomic equilibrium during every national emergency in the United States since the early 19th century, except the Great Depression in the 1930s when deflation interfered with the interest rate mechanism. During the Great Depression, the interest rate mechanism failed because the zero bound on the nominal interest rate implies that the real interest rate cannot be negative if there is deflation. This points to a monetary explanation of the Great Depression, and it suggests that central banks should suspend monetary policy rules that target inflation if there is an adverse political or economic shock that creates consumer pessimism.

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One would expect that the severity of the Great Depression in the 1930s made it easy to discern its cause, but far from it! No consensus has emerged on the cause of the Great Depression from the writings of three generations of economists, starting with those who, like John Maynard Keynes, lived through it. Certainly, most economists agree that aggregate demand must have declined, but there is no agreement on the source of the decline in demand and why lower demand set into motion a disastrous downward spiral in economic activity from which there was seemingly no escape. Keynes's view that a fall in investment spending reduced demand is still standing side by side with the hypothesis of Temin (1976) that autonomous consumption spending declined, and the hypothesis of Friedman and Schwartz (1956) who argued that a fall in the money stock lowered aggregate demand. Recently, even the consensus on the deficiency of aggregate demand has been challenged by Prescott (1999), Cole and Ohanian (1999, 2004), and Chari et al. (2002) who maintain that New Deal changes in labor market institutions accounted for the persistence of the Great Depression.

This paper applies the tools of dynamic macroeconomic analysis to the Great Depression in the 1930s. The focus is on the first order optimum condition of consumers, which relates consumption growth to the real interest rate. Using American data, the Great Depression is compared with other severe economic downturns since the early 19th century. The main puzzle is that there was no obvious economic shock to the US economy in the 1930s, whereas there were strong shocks that could plausibly have given rise to economic depressions before and after the 1930s. Why has the Great Depression remained unique? Why did an elusive shock trigger an economic crisis in the 1930s, while easily identifiable shocks during national emergencies affected the US economy much less earlier and later? In this paper it is argued that a negative real interest rate guaranteed macroeconomic equilibrium during every emergency, except the Great Depression when deflation accounted for a positive real interest rate. During the Great Depression, the interest rate mechanism failed to produce a macroeconomic equilibrium because the zero bound on the nominal interest rate

implies that the real interest rate can be negative only if there is inflation. The finding that inflation is needed to prevent a depression if the equilibrium real interest rate is negative has important implications for the conduct of contemporary monetary policy. Central banks should abandon inflation targets if an economic shock induces an expected decline in consumption that requires a negative real interest rate.

Section 1 reviews the theory of consumer behavior in dynamic macroeconomic models. Section 2 provides a historical narrative of the behavior of the real interest rate in the United States since the early 19th century. The historical analysis makes two assumptions: (1) the consumption Euler equation represents a macroeconomic equilibrium relationship between the real interest rate and consumption growth, and (2) expected consumption fell and the volatility of consumption rose during national emergencies. Section 3 shows how the monetary standard conditioned the inflation process. The econometric analysis, which is provided in the next two sections, supports the hypothesis that there exists an equilibrium relationship between the real interest rate and consumption growth. In Section 4 the nonlinear consumption Euler equation is estimated, and in Section 5 the log-linear functional form is considered. The second moments, which enter the log-linear Euler equation, are estimated using the EWMA/ARCH methodology. Section 6 concludes with a word of caution against the use of monetary policy rules in the presence of adverse political and economic shocks that give rise to pessimistic consumer expectations that necessitate a negative real interest rate.

1. Consumer Behavior

Consider the decision problem of a consumer who decides on consumption in two time periods.

The optimum condition, which is known as the consumption Euler equation, is:

$$u'(C_t) = \frac{1+r}{1+\rho} u'(C_{t+1}) \quad (1)$$

$u'(C_t)$ is the marginal utility of consumption in time period t . To illustrate, assume that the real interest rate r equals the subjective discount rate ρ . Then, it is optimal to keep consumption constant because the marginal utility of consumption must be the same in each time period. The gap between r and ρ determines the optimal time path of consumption. When r exceeds ρ , consumption grows because marginal utility in period $t+1$ must be less than marginal utility in period t . This assumes that marginal utility falls when consumption increases. Of course, the consumer postpones consumption if the real return on saving exceeds the subjective discount rate. Similarly, the consumer reduces saving if the real interest rate is less than the subjective discount rate.

Equation (1) can be applied to the representative consumer in a dynamic macroeconomic model. Then, the equation includes two endogenous variables, the real interest rate and the growth rate of per capita consumption, C_{t+1}/C_t . To quantify the relationship between the real interest rate and consumption growth, it is necessary to specify the period utility function. The CRRA utility function, $u(C_t) = C_t^{1-\theta}/(1-\theta)$, is compatible with the long-run behavior of key macroeconomic variables (See Prescott 2006). The parameter θ , which is the coefficient of relative risk aversion, determines the degree of diminishing marginal utility. Substituting marginal utility, $u'(C_t) = C_t^{-\theta}$, into equation (1) yields:

$$1+r = (1+\rho) \left(\frac{C_{t+1}}{C_t} \right)^\theta \quad (3)$$

Using $g = \frac{C_{t+1}}{C_t} - 1$ for the growth rate of consumption, this can be written as:

$$1 + r = (1 + \rho)(1 + g)^\theta \quad (4)$$

Since r , ρ and g are all small, applying logarithms yields the approximation:

$$r \approx \rho + \theta g \quad (5)$$

Optimal consumer behavior implies that the real interest rate equals the sum of the subjective discount rate and the product of the coefficient of relative risk aversion and the growth rate of consumption. Since the parameter θ is positive, there exists a positive relationship between the real interest rate and consumption growth. In the steady state, consumption is constant and the real interest rate is approximately equal to the subjective discount rate. In this paper special attention is paid to situations in which a fall in consumption requires a negative real interest rate. For example, the equilibrium real interest rate is minus 13 percent if ρ is 2 percent per year, θ equals 1.5, and consumption falls by 10 percent per year.

Inflation renders the real interest rate uncertain if interest rates are defined in nominal terms in debt contracts. If both the real interest rate and future consumption are uncertain, the consumption Euler equation is:

$$u'(C_t) = \frac{1}{1 + \rho} E_t[(1 + r) u'(C_{t+1})] \quad (6)$$

Since current consumption is known, this can be written as:

$$1 + \rho = E_t \left[(1 + r) \frac{u'(C_{t+1})}{u'(C_t)} \right] \quad (7)$$

Using the same utility function as before, the optimum condition is:

$$1 + \rho = E_t[(1+r)(1+g)^{-\theta}] \quad (8)$$

The expectation operator should not be written into the nonlinear expression on the right-hand side of equation (8). Instead, Romer (2006, p. 369) computes the expectation of a second order Taylor series approximation of $(1+r)(1+g)^{-\theta}$ around $r = 0$ and $g = 0$. Solving for the expected real interest rate yields:

$$E_t[r] \approx \rho + \theta E_t[g] + \theta \text{cov}_t[r, g] - \frac{\theta(\theta+1)}{2} \text{var}_t[g] \quad (9)$$

This equation shows the equilibrium relationship between the real interest rate and consumption growth if both quantities are uncertain. The expected real interest rate is positively related to the expected rate of consumption growth and the covariance between the real interest rate and consumption growth, while the variance of consumption growth is negatively associated with the real interest rate. Consumers demand a high real interest rate if the covariance between the real interest rate and consumption growth is positive because bonds are an ineffective vehicle for consumption smoothing in this situation. The variance of consumption growth interacts negatively with the real interest rate because an increase in consumption volatility induces precautionary saving, putting downward pressure on the real interest rate.

From 1831 to 2004, the growth rate of American per capita consumption was 2 percent per year, the covariance between the real interest rate and consumption growth was 0.0002, and the variance of consumption growth was 0.002. Plausible parameter values of the utility function are $\rho = 2$ percent and $\theta = 1.5$. With these figures, equation (9) yields a mean real interest rate of 4.28 percent, which is not far from the historical average of 3.57 percent. It seems that the covariance and variance terms do not matter much in equation (9), at least in the long-run. The covariance term increases the mean real interest rate by only 0.03 percent, and the variance term reduces it by 0.75 percent. However, this estimate of the average real interest rate is based upon unconditional moments,

whereas equation (9) really holds for conditional moments. The conditional moments of macro-economic time series, including the real interest rate and consumption growth, depend on economic conditions. Therefore, it is possible that the second moments in equation (9) have a sizeable effect on the expected real interest rate during a national emergency, even if their long-run effect is negligible. In Section 5, the conditional second moments are estimated using the EWMA/ARCH methodology.

2. The American Real Interest Rate

The theory of consumer behavior predicts that the real interest rate is negative during a national emergency that affects consumption. A fall in consumption requires a negative real interest rate in equation (5), and a fall in expected consumption reduces the expected real interest rate in equation (9). The expected real interest rate also falls because the conditional variance of consumption growth increases during a crisis. As will be seen in Section 5, the sign of the conditional covariance between the real interest rate and consumption growth depends on the nature of a crisis. In Figure 1, the real interest rate is the short-term interest rate minus the inflation rate in the preceding year. The shaded areas indicate the most severe national emergencies that have affected the United States since the early 19th century: the Civil War (1861-65), World War I (1914-18), the Great Depression (1929-33), World War II (1939-45), the Korean War (1950-53), the oil crises (1973-74 and 1979-80), and the attack on the World Trade Center that led to the US invasions of Afghanistan and Iraq (since 2001). The real interest rate became negative during every major war, it turned negative when deflation ceased in the 1930s, and it was negative during the oil crises. At the same time, the real interest rate became negative without obvious national distress only once, during the recession in 1957-58. The Vietnam War (1964-75) did not lead to a negative real interest rate because the United States was drawn into it gradually over a lengthy period of time. Therefore, the

conditional moments of consumption growth remained unaffected. Table 1 summarizes all national emergencies during which the real interest rate became negative.

Figure 1. Real Interest Rate

Figure 2. Inflation Rate

Sources: See the Appendix.

It is easy to calibrate equation (9) with realistic parameter values so that plausible assumptions on the conditional moments produce the observed real interest rate during the national emergencies in Table 1. During the Civil War, the minimum of the real interest rate occurred in 1864, when it fell to minus 19.0 percent. In the same year, real per capita GNP, which serves as proxy for consumption, dropped by 11.1 percent. If $\rho = 0.02$ and $\theta = 1.5$, equation (5) predicts a real interest rate of minus 14.7 percent. From 1831 to 2004, the standard deviation of consumption growth was 4.5 percent and the covariance between the real interest rate and consumption growth was close to zero. Assuming the conditional standard deviation was twice as high in 1864 and neglecting the covariance, equation (9) predicts a real interest rate of minus 17.7 percent. The equations also work well at the end of the sample period. During the first oil crisis in 1974, real per capita consumption fell by 2.3 percent. Making the same assumptions as before, equation (5) predicts a real interest rate of minus 1.5 percent and equation (9) yields minus 4.5 percent. In fact, the real interest rate was minus 3.1 percent, which lies between these two estimates.

Negative real interest rates are a key feature of American business cycle history. The real interest rate was negative in 35 years during the 174 years covered by Figure 1. Macroeconomic equilibrium requires a negative real interest rate if the economy is hit by a strong shock that affects consumption. During wars and national emergencies, people saved because they expected that consumption would fall and the conditional variance of consumption growth was high. People who face adverse economic prospects save in order to maintain consumption. There is an incentive to save because the expected marginal utility of future consumption is high if expected consumption is low and uncertain. During wars and national emergencies, the incentive to save was so strong that there would have been excess saving and a corresponding excess supply of commodities without a negative real interest rate. For this reason, negative real interest rates prevented the recessions during wars and emergencies from becoming outright depressions.

Table 1. National Emergencies and the Real Interest Rate*

	Duration	Minimum real interest rate	Year
1. Civil War	1861-1865	-19.0%	1864
2. World War I	1914-1918 USA: 1917-1918	-12.5%	1917
3. World War II	1939-1945 USA: 1941-1945	-9.8%	1942
4. Korean War	1950-1953	-6.5%	1951
5. 1 st Oil Crises	1973-74	-3.1%	1974
6. 2 nd Oil Crisis	1979-80	-1.9%	1980
7. Afghanistan/Iraq	since 2001	-1.3%	2004

* All dates are from the US Department of Veterans' Affairs.

The history of the Great Depression in the 1930s confirms that negative real interest rates were instrumental in preventing more depressions in American economic history. Since real per capita consumption dropped by 10.1 percent in 1931 and 11.5 percent in 1932, it is likely that expected consumption fell and the conditional variance of consumption increased. Using equation (9) with the same parameter values and second moments as for 1864, the real interest rate should have been minus 16.2 percent in 1931 and minus 18.2 percent in 1932. Instead, the real interest rate was strongly positive in these years, namely 11.0 and 13.1 percent. This is a gap of 27.2 percent in 1931 and 31.3 percent in 1932! Clearly, the positive real interest rate that prevailed during the Great Depression was not an equilibrium rate. The finding that the real interest rate exceeded the equilibrium rate by a wide margin gives credence to the Keynesian view that saving was excessive during the Great Depression, although Keynes attributed this more to a decline in investment than to an increase in saving. But Temin (1976) has a strong case that the Great Depression was caused by insufficient consumption. Romer (1990) and Greasley, Madsen and Oxley (2001), who consider the consumption hypothesis, use stock market volatility as a measure of consumer expectations after the stock market crash in 1929. Weder and Harrison (2006) compute a consumer confidence index that is based on the spread between high risk and low risk corporate bonds. In this paper, consumer uncertainty is measured directly, using the conditional variance of consumption growth.

During the Great Depression, the interest rate mechanism broke down because a negative real interest rate can be achieved only if there is inflation. It is well known that the nominal interest rate cannot be negative in a monetary economy. The real return on money is the negative of the inflation rate ($-\pi$) and the real return on bonds is the nominal interest rate minus the inflation rate ($R-\pi$). It is not worthwhile to hold bonds if the nominal interest rate is negative because the real return on money would exceed the real return on bonds at any inflation rate. The fact that the nominal interest

rate cannot be negative implies that a negative real interest rate can prevail only if there is inflation.¹ The crucial difference between the national emergencies and the Great Depression is that there was moderate to high inflation during the former, while prices fell during the latter. During the national emergencies, inflation made it possible that the negative equilibrium real interest rate, which was required by pessimistic consumer expectations, was indeed realized. During the Great Depression, the interest rate mechanism failed to achieve macroeconomic equilibrium because the nominal interest rate could not fall further and deflation produced a positive real interest rate.²

3. Monetary Standard

Figure 2 shows that there is a close correspondence between national emergencies and peaks in inflation. The annual inflation rate reached 26.4 percent in 1864, 17.8 percent in 1918, 10.5 percent in 1942, 8.0 percent in 1951, 11.0 percent in 1974, 13.5 percent in 1980, and 3.4 percent in 2005. The inflation process was conditioned by the monetary standard. The gold standard is incompatible with a flexible inflation rate, whereas the monetary authority is free to inflate in a paper standard. Although the United States did not change the official gold price from 1837 to 1933, the gold standard was not always fully operational.³

To finance the Civil War, the Union issued paper money, the so-called greenbacks, and it sold government bonds to national banks, which held them as legal reserves against their bank notes. The expansion of the supply of greenbacks and national bank notes generated inflation during the Civil War. During the first three years of World War I, it was easy for the United States, which was still

¹ Since $R \geq 0$, $r = R - \pi < 0$ requires $\pi > 0$.

² Eggertsson and Woodford (2003, 2004) show that there is the risk of an economic collapse if the zero bound on the nominal interest rate is binding.

³ Friedman and Schwartz (1963) pay close attention to the institutional restrictions that conditioned US monetary policy and inflation.

neutral, to maintain the gold standard because European gold flowed across the Atlantic to pay for armaments and strategic raw materials. But the influx of gold led to an expansion of the American money supply that caused inflation. After entering the war in 1917, the United States ran a budget deficit that was partly monetized by the Federal Reserve. This produced more inflation and a loss of official gold reserves. As a consequence, the United States restricted the export of gold, undercutting the gold standard. During World War II, the gold standard was not operational and, as during the Civil War and in 1917-18, inflation was fueled by the printing press. After World War II, the Bretton Woods international monetary agreement linked all countries indirectly to gold through fixed exchange rates with the US dollar, which was defined in gold. Unlike American residents, foreign governments had the right to exchange dollars for gold at the US Treasury. This arrangement gave the United States some leeway in monetary policy because foreign governments were expected to exercise restraint in the demand for American gold during emergencies. The Bretton Woods system was sufficiently flexible to absorb the spike in inflation during the Korean War, but American inflation went on for too long in the 1960s and the system collapsed in 1971. Since then, national paper standards have given central banks control of the inflation rate.

The restoration of the gold standard after World War I restricted the conduct of monetary policy in the interwar period. The United States abolished the export restriction on gold in 1919, and the international gold standard had been restored by the mid-1920s. Therefore, the world entered the Great Depression with a monetary system that did not allow for inflation when a negative real interest rate was required for macroeconomic equilibrium. Central bankers, who were impervious to the social cost of falling output and high unemployment, embraced deflation in order to bring commodity prices in line with the official gold price. In the United States the deflationary process ended only when Franklin D. Roosevelt abandoned the gold standard after taking office in 1933. Not surprisingly, consumer pessimism persisted for several years and the real interest rate became

belatedly negative in 1934 (Figure 1). This analysis implies that there was a macroeconomic disequilibrium during the Great Depression, whereas the economic contractions during the wartime emergencies and oil crises were equilibrium responses of the economy to exogenous shocks. The inability of the economy to achieve a new macroeconomic equilibrium, which was caused by the failure of the interest rate mechanism to equate saving and investment, explains the unusual severity of the Great Depression.

4. Econometric Analysis

Many empirical studies have been conducted that yield plausible parameter values for the representative consumer's utility function. This suggests that the consumption Euler equation represents a macroeconomic equilibrium relationship that links the real interest rate with the growth rate of real per capita consumption. In this section, the parameters of the utility function are estimated with annual data on the real interest rate and consumption growth from 1831 to 2004. Before 1920, GDP growth serves as proxy for consumption growth. This sample period is much longer than those of earlier studies, which use monthly and quarterly data from the second half of the 20th century. The advantage of the longer sample period is that it covers the Civil War and both World Wars as well as the financial crises in the second half of the 19th century, which all had a strong impact on the real interest rate and consumption. The estimated subjective discount rate and the coefficient of relative risk aversion are close to those in earlier studies. Thus, the analysis of historical data confirms that the consumption Euler equation has provided an equilibrium relationship between the real interest rate and consumption growth during national emergencies since 1831.

The following arguments pin down the values of the parameters of the representative utility function. Equations (5) and (9) imply that the subjective discount rate equals the real interest rate

when real per capita consumption is constant. Therefore, the low real interest rates that prevail in countries that are close to a steady state – for example Japan and Switzerland in the 1990s – suggest that ρ must be low, perhaps two percent per year or less. A similar argument does not apply to the coefficient of relative risk aversion, for any value of θ is compatible with a steady state. But values between one and four yield a plausible marginal rate of substitution between consumption in two successive years if real per capita consumption grows at two percent per year, which is the average rate of growth from 1831 to 2004. Setting $\rho = 0.02$, $\theta = 1.5$ and $C_{t+1}/C_t = 1.02$, the marginal rate of substitution between current and future consumption is:⁴

$$-\frac{dC_t}{dC_{t+1}} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{1+\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} = \frac{1}{1+0.02} 1.02^{-1.5} = 0.952 \quad (10)$$

A marginal rate of substitution of 0.952 implies that the consumer would agree to trade one unit of current consumption for $1/0.952 = 1.05$ units of future consumption. Similarly, the consumer asks for 1.061 units of future consumption if $\theta = 2$, and for 1.104 units of future consumption if $\theta = 4$. Cochrane (2005, Ch. 1) shows that the inverse of the marginal rate of substitution is the riskfree interest rate (gross return). Since the riskfree interest rate is low, most macroeconomists use values of θ at the lower end of the range from 1 to 4. For example, Prescott (2006) sets $\theta = 1$; Attanasio and Low (2004) work with $\theta = 1.5$, and Walsh (2003, Ch. 2) adopts $\theta = 2$ as benchmark. Analyzing the behavior of the saving rate, Barro and Sala-i-Martin (2004, Ch. 2) conclude that θ must lie between 2 and 4. Therefore, Barro (2006) sets θ equal to 3 and 4 in a model that deals with the equity premium puzzle. These are high values for macroeconomists, but they are close to those used by financial economists. Shiller (2003, p. 86), for example, computes a marginal rate of substitution

⁴ The marginal rate of substitution is derived from the utility function $U(C_t, C_{t+1}) = u(C_t) + \beta u(C_{t+1})$, where $\beta = 1/(1+\rho)$.

(stochastic discount factor) with $\theta = 3$, and the option-implied coefficients of relative risk aversion of Bliss and Panigirtzoglou (2004, p. 429) lie between about 2 and 10.

Hansen and Singleton (1982, 1984) first applied the generalized method of moments (GMM) to the nonlinear consumer Euler equation (4). Using monthly data from 1959 to 1978, they estimated several models with an increasing number of lags on instruments. They found that the annualized ρ lies between 0.6 percent and 9.8 percent and θ is between 0.35 and 1. The annual data that are used in this study provide a similar range for both parameters. Table 2 presents estimates for three time periods: 1831-2004, 1831-1929, and 1934-2004. The depression years from 1930 to 1934 are excluded in all three regressions, including the one covering the entire time period from 1831 to 2004, because during the Great Depression there was a macroeconomic disequilibrium that was incompatible with the consumption Euler equation. The instruments include two lags of the inflation rate and two lags of consumption growth. The estimate of ρ is 6.63 percent before the Great Depression and is insignificantly different from zero afterwards. The decline in ρ accounts for the secular fall in the real interest rate, which can be seen in Figure 1. The estimate of θ is less than one in both subperiods. Using the entire sample period, θ is 2.95 and ρ is insignificantly different from zero.

Table 2. Nonlinear Least Squares - Estimated by GMM

Equation: $(1 + r)(1 + \rho)^{-1}(1 + g)^{-\theta} - 1 = 0$

Parameter	1831-2004	1831-1929	1934-2004
Discount rate (ρ)	-0.0211 [#] (0.0171)	0.0663 (0.0173)	-0.0078 [#] (0.0108)
Coefficient of relative risk aversion (θ)	2.9495 (0.9763)	0.1684 [#] (0.6395)	0.7342 (0.3262)

J(3)	0.6462 (0.8858)	2.9935 (0.3926)	3.2986 (0.3478)

Notes:

The exogenous and predetermined variables include two lags of the inflation rate and two lags of consumption growth.

The standard errors of coefficients are shown in brackets. Coefficients that are not significant at the ten percent level are marked with the superscript #. To correct for serial correlation, the standard errors are based on a Newey-West covariance matrix that was estimated with four lags. The bracket under the J statistic is the level of significance (p-value).

5. Conditional Second Moments

The main advantage of the regression in Table 2 is that it does not require the conditional variance of consumption and the conditional covariance between consumption and the real interest rate. The catch is that the consumption Euler equation (4) is nonlinear, and nonlinear GMM leads to inconsistent estimates in the presence of measurement error. The next regression uses estimates of the second moments as regressors in the log-linear consumption Euler equation (9). The standard theory of measurement error applies to this equation because it is linear in the coefficients.⁵

Two models are used to estimate the second moments: a univariate EWMA model and a bivariate EWMA model, which was adapted from a multivariate GARCH(1,1) model that was proposed by Engle (2002). The univariate model yields estimates of the conditional variances of consumption growth and the real interest rate, and the bivariate model adds the conditional covariance between the two variables. The estimated variances are identical in the univariate and bivariate models. The GARCH(1,1) model assumes that the variance is mean reverting, but the EWMA model does not. As seen in Figure 3, the variances of consumption growth and the real interest rate do not return to a stable long-term value. Since there is no mean reversion, the EWMA model is preferable to the GARCH(1,1) model.⁶ The regression output of the volatility models is included in an Appendix that is attached to the electronic version of this paper.

⁵ Since second moments are not readily available, the simple log-linear equation (5) is often estimated. Then, ρ is not identified because the constant includes both ρ and the omitted second moments. In addition, the estimate of θ is inconsistent if the second moments are correlated with the instruments. See Carroll (2001) and Attanasio and Low (2004).

⁶ GARCH(1,1) models yield negative weights for the long-term variances. Hull (2006, pp. 466-467) suggests the use of the EWMA model if the conditional variance is not mean reverting.

Figure 3. Bivariate EWMA Model

During the Civil War, consumption volatility rose more than fivefold from a prewar level of around 0.001 to above 0.005. Reflecting the deteriorating political and economic situation, consumption volatility started to rise three years before the outbreak of open hostilities. Between the Civil War and World War I, there were three peaks in consumption volatility – 1884, 1896-98 and 1910 – which all coincided with severe financial crises. The economic contraction that started in 1882 culminated in a financial panic in 1884. A growing scarcity of gold caused deflation, debtor insolvency and a financial panic in 1893. In response to falling commodity prices, a populist movement emerged that demanded government intervention in the economy and the monetization of silver to expand the money supply. Although this would have ended deflation, the immediate effect of the silver controversy, which climaxed during the Presidential election in 1896, was to create more financial uncertainty.⁷ The financial crisis in 1907 was followed by rising consumption volatility. Consumption volatility also rose during the recession after World War I and during the Great Depression in the 1930s. World War II had no major impact on consumption volatility, and by the late 1950s consumption volatility had returned to the level that had prevailed before the Civil War. In the second half of the 20th century, consumption volatility remained low, although there was a small increase after the second oil crisis.

The volatility of the real interest rate rose markedly only in exceptional circumstances: the European revolutions in 1848 that affected the transatlantic credit market, the American Civil War, World War I, the political upheaval that led to the rise of totalitarian regimes in Europe in the interwar period, and World War II. The financial crises under the classical gold standard and the collapse of the Bretton Woods international monetary system all left no mark on the volatility of the real interest rate. Thus, financial crises did not destabilize the real interest rate, but credit markets were disrupted by political confrontations that threatened civic society.

⁷ Friedman and Schwartz (1963) investigate the political and economic ramifications of monetary and financial crises in the United States.

The covariance between consumption growth and the real interest rate measures the consumption risk of government bonds.⁸ Figure 3-C shows that the covariance between consumption growth and the real interest rate averages zero in the long-run. Therefore, government bonds yield the riskfree interest rate in the long-run. Although low consumption risk is an inherent quality of government bonds, in exceptional circumstances it may change, either positively or negatively. During the Civil War, investors faced the possibility that a bad outcome of the War would reduce consumption and, at the same time, government bonds would become worthless, providing no hedge against the fall in consumption. The positive conditional covariance between consumption growth and the real interest rate confirms that US government bonds were being perceived as a risky investment during the Civil War. In contrast, investors never lost confidence in the United States during both World Wars. Then, the conditional covariance between consumption growth and the real return on government bonds turned negative, indicating that US government bonds were viewed as a hedge against a fall in consumption. The same holds during the Great Depression, when deflation increased the real value of government bonds and bank failures and corporate bankruptcies affected the credit rating of private securities.

Table 3 shows the estimated log-linear consumption Euler equation, using the same time periods as before. For the full sample period, two models are estimated: one including a dummy variable for the time after World War II and another including a trend instead of the dummy variable. The instruments include two lags of the inflation rate and two lags of each independent variable. The estimates confirm that the subjective discount rate, ρ , has declined since the 19th century. Using all data from 1831 to 2004, the constant is 9.9 percent and the coefficient of the postwar dummy is minus 9.1 percent. Thus, ρ was 9.9 percent from 1831 until the end of World War II, and it has been

⁸ For an introduction to consumption based asset pricing see Cochrane (2005) and LeRoy and Werner (2001, Chapter 14).

Table 3. Linear Regression - Estimated by Instrumental Variables

Dependent Variable: Real Interest Rate

Independent Variable	1831-2004		1831-1929	1934-2004
	(1)	(2)	(3)	(4)
Consumption growth	1.2839 (0.5395)	1.2150 (0.4990)	0.8591 (0.5011)	0.7094 (0.2569)
Covariance consumption growth/interest rate	43.9793 (8.7177)	25.5651 (7.2433)	36.5497 (8.3032)	11.9728 [#] (20.6437)
Variance of consumption growth	-37.1246 (9.4168)	-23.1281 (6.9710)	-36.6051 (9.9289)	-14.8933 (5.1299)
Constant	0.0991 (0.0231)	0.1156 (0.0182)	0.1181 (0.0194)	0.0036 [#] (0.0087)
Postwar dummy	-0.0905 (0.0195)			
Trend		-0.0008 (0.0001)		

J(4), J(5)	0.0304 (0.9999)	0.0271 (0.9999)	0.0267 (1.0000)	0.0084 (1.0000)

Notes:

The exogenous and predetermined variables include two lags of the inflation rate and two lags of each independent variable.

The standard errors of coefficients are shown in brackets. Most coefficients are significant at the one percent level. Coefficients that are not significant at the ten percent level are marked with the superscript #. To correct for serial correlation, the standard errors are based on a Newey-West covariance matrix that was estimated with four lags. The bracket under the *J* statistic is the level of significance (*p*-value).

9.9-9.1 = 0.8 percent since then. Replacing the postwar dummy with a trend shows that ρ fell at an average rate of 0.08 percent per year between 1831 and 2004 (column 2). Splitting the sample period confirms this result (columns 3 and 4). Considering the impact of expected consumption growth, the coefficient of relative risk aversion, θ , lies between 0.71 and 1.28 in the four regressions. The value of 1, which is used by Prescott (2006) and others, lies right in the middle of this range of estimates. The estimated coefficients of the second moments all carry the correct sign, but their absolute values are too large. If θ equals 1, the coefficient of the covariance between the real interest rate and consumption growth should also be 1, and the coefficient of the variance of consumption growth should be $-1(1+1)/2 = -1$. Instead, the estimated coefficients of the covariance lie between 12.0 and 44.0 and the coefficients of the variance are between -14.9 and -37.1 . Thus, shocks to the second moments affected the real interest rate much more than predicted by the second order approximation of the consumption Euler equation.

The likely explanation for the strong effect of the second moments on the real interest rate is that these coefficients are biased because of some omitted variable that depressed the real interest rate during wars and emergencies. Adopting an idea of Rietz (1988), Barro (2006) proposed that rare disasters played a role in determining the real interest rate in the United States during the 20th century.⁹ The possibility of a disaster reduces the real interest rate because people prepare for disasters by saving more. Barro (2006, Section 6) conjectures that an increase in the perceived disaster probability accounted for the fall in the real interest rate during wars. Thus, the absolute values of the coefficients of the covariance and variance terms may be too large because the

⁹ Barro (1987) argued that military spending increased the real interest rate in Great Britain during wars, but Barro (1993, pp. 321-322) presented conflicting evidence on the United States, which, according to Romer (2006, p. 76), shows that “real interest rates appear to have been, if anything, generally lower during wars than in other periods.” Romer concludes that the “reasons for this anomalous behavior [of American interest rates] are not well understood.” Both this study and Barro (2006) leave no doubt that the real interest rate falls during wars.

omission of the perceived disaster probability from the regression equation produces biased estimates.

Another explanation for the strong effect of the second moments on the real interest rate is that the log-linear Euler equation does not consider shocks to the marginal utility of consumption. A war, however, causes a temporary proportional downward shift of the period utility function. The utility function shifts downward because the horrors of war affect the welfare of consumers at every level of consumption. In addition, the composition of aggregate consumption changes, as some goods become unavailable and consumers shift away from goods whose prices skyrocket on the black market. During the oil crises, price controls and rationing reduced the availability of petrol for weekend outings and other enjoyable activities that require a car. Consumers save when marginal utility is temporarily low during a war or emergency, and they catch up by spending more when marginal utility recovers afterwards. This extra saving reduces the real interest rate during the war or emergency. In future research, it would be interesting to incorporate both ideas, that the perceived disaster probability increases and that marginal utility falls during wars and emergencies, into the log-linear consumption Euler equation.

6. Conclusion

The consumption Euler equation provides an equilibrium relationship between the real interest rate and the rate of consumption growth in dynamic macroeconomic models with microeconomic foundations. The real interest rate interacts positively with both the rate of consumption growth and the covariance between the interest rate and consumption growth, and it is negatively related to the variance of consumption growth. Therefore, the real interest rate is high during economic expansions and it is low or negative in periods of political and economic distress. A negative real interest rate was a common occurrence in US macroeconomic history. It became negative during every major war

and it was negative during the oil crises. Except for the Vietnam War, whose course was more drawn out than that of other conflicts, wars and national emergencies gave rise to consumer pessimism. People saved because they expected that per capita consumption would fall and because the volatility of consumption was high. Consumption was also postponed because the hardship of wars directly reduced marginal utility, and shortages and rationing affected the composite aggregate consumption good. It is also likely that the perceived probability of a disaster increased. For all these reasons, macroeconomic equilibrium required a negative real interest rate during wars and national emergencies. Without a negative real interest rate, there would have been excess saving and a corresponding excess supply of commodities.

Alas, the interest rate mechanism does not work automatically. The zero bound on the nominal interest rate implies that there must be inflation when the equilibrium real interest rate is negative. The United States interfered with the gold standard during the Civil War and World War I and the gold standard was not operational during World War II, while the Bretton Woods system was sufficiently flexible to accommodate short-run inflation during the Korean War. By chance, the collapse of the Bretton Woods system gave the Federal Reserve the power to inflate shortly before the first oil crisis. Inflation accommodated a negative equilibrium interest rate during every national emergency except the Great Depression. There is no doubt that the economic history of the 1930s would have been different if there had been a period of deliberate inflation and a negative real interest rate after the stock market crash of 1929. Instead, deflation put the negative equilibrium real interest rate out of reach until the rise in the official gold price led to inflation in 1934. For this reason, wars and national emergencies, which are easily identifiable economic shocks, produced normal business fluctuations, whereas the Great Depression became a national emergency without an obvious economic shock.

A growing literature, motivated by the recent occurrence of deflation and zero interest rates in Japan, deals with the conduct of monetary policy when the interest rate constraint is binding.¹⁰ The problem is that standard open market purchases of government bonds by the central bank are ineffective because money and government bonds are perfect substitutes if the nominal interest rate is zero. American economic history shows how a ‘liquidity trap’ can be avoided when the equilibrium real interest rate is negative. The Federal Reserve (or national banks) contributed to the financing of wars and emergencies by buying Treasury bonds. During the Civil War, the Treasury also directly issued paper money, the so-called greenbacks. The monetization of government expenditures guaranteed the normal operation of financial markets because, even with a negative equilibrium real interest rate, the nominal interest rate did not fall to zero if there was sufficient inflation. However, the inflationary financing of rising government expenditure, which required interfering with the gold standard, was considered acceptable only in exceptional circumstances, during wars and national emergencies. During the Great Depression, President Herbert Hoover, who insisted on ‘sound’ budget principles, implemented one of the largest tax increases in American history to pay for Depression related public projects. In this, he had the support of the financial community, which, according to Friedman and Schwartz (1963, p. 322), rejected increases in government spending and monetary expansion as “greenbackism” and as being “inflationary”. Thus, the authorities knew that an expansion in government spending that is financed by the printing press would cause inflation, but, accepting the advice of financial circles, they let deflation run its course in order to safeguard the official gold price. The lesson from American economic history is that monetary policy targets, whether a fixed price of gold or a direct inflation target, should be abandoned when an adverse political or economic shock causes a decline in expected consumption and an increase in consumer uncertainty. The United States adopted a combination of increasing

¹⁰ See Krugman (1998), Auerbach and Obstfeld (2005), Jung et al. (2005), Wolman (2005), Adam and Billi (2006), Eggertsson (2006), Iwata and Wu (2006), and Kam (2006).

government spending and monetary expansion during every episode of negative equilibrium real interest rates except once – during the Great Depression.

Appendix: Data

Sources

Dick, T.J.O. *Macroeconomic Data: 1790-1935* (www.economics.utoronto.ca/floyd/dick.html). A list of sources and methods of this data set is contained in the Data Appendix to the working paper: Dick, T.J.O. and J.E. Floyd. 2001. Capital Imports and the Jacksonian Economy: A New View of the Balance of Payments, Department of Economics and Institute for Policy Analysis, University of Toronto, Working Paper Number UT-ECIPA-Floyd-01-01.

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Mitchell, B.R. 1998. *International Historical Statistics. The Americas 1750-1993*. 4th ed., London: Macmillan.

National Bureau of Economic Research. *NBER Macrohistory Database* (www.nber.org/macroeconomy).

The data sources of Figures 1 and 2 are:

Figure 1

1831-1856: Commercial paper rate (Dick); 1858-1947: Commercial paper rate (NBER); 1948-2004: 90-day US Treasury bill rate (IMF). No adjustments were made for the breaks in the series in 1857 and 1948. In 1857 the commercial paper rate was 11.00 percent (NBER) and 11.56 percent (Dick). In 1948 the 90-day Treasury bill rate and the commercial paper rate were 1.04 percent and 1.44 percent. The real interest rate is the nominal interest rate minus the annual inflation rate (Figure 2).

Figure 2

Consumer price index (CPI): Mitchell (1831-1947) and IMF (1948-2004).

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Econometric Work

1. Nonlinear Consumption Euler Equation
2. Volatility Models
3. Log-Linear Consumption Euler Equation
4. Figures

The econometric work uses RATS 5.1.

* 1. Consumption Euler Equation

* _____

* See RATS User's Guide, Example 7.2, and the RATS program file GIV.PRG.

```
cal 1831 1 1
all 0 2004:1
```

```
open data
data(format=RATS) / USRINT DUSPCRCN USINFL
* REALINTEREST.RAT
```

```
set USREALRET = USRINT+1
set USCONGROW = DUSPCRCN+1
```

* Exclude the Great Depression.

```
set TREND = t
set DEP = %IF(TREND<=103.AND.TREND>=100,%na,1.0)
```

```
print 1928:1 1935:1 TREND DEP
```

ENTRY	TREND	DEP
1928:01	98	1
1929:01	99	1
1930:01	100	NA
1931:01	101	NA
1932:01	102	NA
1933:01	103	NA
1934:01	104	1
1935:01	105	1

```
smpl 1831:1 2004:1
```

```
nonlin discount riskaver
frml h = dep(t)*(1+discount)**(-1)*usrealret(t)*uscongrow(t)**(-riskaver)-1
compute discount = 0.02,riskaver = 1.1
```

```
instruments constant uscongrow{1 to 2} usinfl{1 to 2}
nlls(inst,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * / resid
```

Nonlinear Instrumental Variables - Estimation by Gauss-Newton
 Convergence in 7 Iterations. Final criterion was 0.0000017 < 0.0000100
 Annual Data From 1833:01 To 2004:01

Usable Observations 168 Degrees of Freedom 166
 Total Observations 172 Skipped/Missing 4
 Sum of Squared Residuals 3.0485154229
 J-Specification(3) 0.646206
 Significance Level of J 0.88577770
 Durbin-Watson Statistic 1.811302

Variable	Coeff	Std Error	T-Stat	Signif
1. DISCOUNT	-0.021050725	0.017084780	-1.23213	0.21789932
2. RISKAVR	2.949499163	0.976255903	3.02124	0.00251745

* Do specification tests on lag lengths 1, 2, 3 and 4. These are
 * done over a common interval (the range supported by the four lag
 * estimation - %regstart() returns the start entry from the last
 * regression). The NLLS output is suppressed and the tests results
 * are displayed using cdf.

```
compute start=%regstart()
report(action=define,hlabel=||'Lags','J-Stat','P-Value'||)
dofor nlag = 1 2 3 4
  instruments constant uscongrow{1 to nlag} usinfl{1 to nlag}
  nlls(inst,noprint,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * start *
  report(atcol=1,row=new) nlag %jstat %jsignif
end dofor
```

```
report(action=show)
Lags      J-Stat      P-Value
1          0.012137    0.912275
2          0.646219    0.885775
3          3.949901    0.556651
4          4.684017    0.698460
```

smpl 1831:1 1929:1

```
nonlin discount riskaver
frml h = dep(t)*(1+discount)**(-1)*usrealret(t)*uscongrow(t)**(-riskaver)-1
compute discount = 0.02,riskaver = 1.1
```

```
instruments constant uscongrow{1 to 2} usinfl{1 to 2}
nlls(inst,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * 1833:1 1929:1 resid
```

Nonlinear Instrumental Variables - Estimation by Gauss-Newton
 Convergence in 15 Iterations. Final criterion was 0.0000038 < 0.0000100
 Annual Data From 1833:01 To 1929:01

Usable Observations 97 Degrees of Freedom 95
 Sum of Squared Residuals 0.3680579095
 J-Specification(3) 2.993501
 Significance Level of J 0.39262823
 Durbin-Watson Statistic 0.827511

Variable	Coeff	Std Error	T-Stat	Signif
1. DISCOUNT	0.0663226629	0.0173463101	3.82345	0.00013160
2. RISKAVR	0.1683842314	0.6394549966	0.26332	0.79230040

```

compute start=%regstart()
report(action=define,hlabel=||'Lags','J-Stat','P-Value'||)
do for nlag = 1 2 3 4
    instruments constant uscongrow{1 to nlag} usinfl{1 to nlag}
    nlls(inst,noprint,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * start *
    report(atcol=1,row=new) nlag %jstat %jsignif
end do for
report(action=show)

```

Lags	J-Stat	P-Value
1	0.595290	0.440381
2	2.993500	0.392628
3	3.261671	0.659715
4	3.550664	0.829830

```

smpl 1934:1 2004:1

nonlin discount riskaver
frml h = (1+discount)**(-1)*usrealret(t)*uscongrow(t)**(-riskaver)-1
compute discount = 0.02,riskaver = 1.1

instruments constant uscongrow{1 to 2} usinfl{1 to 2}
nlls(inst,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * / resid

Nonlinear Instrumental Variables - Estimation by Gauss-Newton
Convergence in 21 Iterations. Final criterion was 0.0000073 < 0.0000100
Annual Data From 1934:01 To 2004:01
Usable Observations 71 Degrees of Freedom 69
Sum of Squared Residuals 0.0874250374
J-Specification(3) 3.298605
Significance Level of J 0.34783679
Durbin-Watson Statistic 0.593544

Variable Coeff Std Error T-Stat Signif
*****
1. DISCOUNT -0.007792058 0.010786261 -0.72241 0.47004500
2. RISKAVAR 0.734218129 0.326219733 2.25069 0.02440544

compute start=%regstart()
report(action=define,hlabel=||'Lags','J-Stat','P-Value'||)
do for nlag = 1 2 3 4
    instruments constant uscongrow{1 to nlag} usinfl{1 to nlag}
    nlls(inst,noprint,frml=h,optimal,robusterrors,DAMP=1.0,LAGS=4) * start *
    report(atcol=1,row=new) nlag %jstat %jsignif
end do for
report(action=show)

```

Lags	J-Stat	P-Value
1	1.722130	0.189419
2	3.298606	0.347837
3	3.614145	0.606191
4	3.817064	0.800591

* 2. Volatility Models

* _____

* 2.1. Consumption Volatility

* _____

* See RATS User's Guide, Chapter 12.

cal 1831 1 1

all 0 2004:1

open data

data(format=RATS) / DUSPCRCON

* REALINTEREST.RAT

set y = duspcon

declare series u

declare series h

nonlin(parmset=meanparms) b0

nonlin(parmset=garchparms) vc va vb

frml resid = y - b0

frml hf = vc + va*h{1} + vb*u{1}**2

frml logl = (h(t)=hf(t)),(u(t)=resid(t)),-.5*(log(h(t))+u(t)**2/h(t))

linreg(noprint) y / u

constant

compute b0=%beta(1)

compute vc=%seesq,va=.05,vb=.05

set h = %seesq

maximize(notrace,parmset=meanparms+garchparms,method=simplex,itors=15) logl 2 *

MAXIMIZE - Estimation by Simplex

Annual Data From 1832:01 To 2004:01

Usable Observations 173

Function Value 468.00514387

Variable	Coeff
1. B0	0.0208
2. VC	-8.8959e-06
3. VA	0.7656
4. VB	0.3157

maximize(notrace,parmset=meanparms+garchparms,method=bfgs,robusterrors,\$
itors=100) logl 2 *

MAXIMIZE - Estimation by BFGS

Convergence in 15 Iterations. Final criterion was 0.0000000 < 0.0000100

Annual Data From 1832:01 To 2004:01

Usable Observations 173

Function Value 471.86775184

Variable	Coeff	Std Error	T-Stat	Signif
1. B0	0.0196	2.3477e-03	8.33615	0.00000000
2. VC	-2.0883e-05	6.3567e-06	-3.28516	0.00101923
3. VA	0.9242	0.0579	15.95897	0.00000000
4. VB	0.0837	0.0652	1.28378	0.19921814

* The GARCH(1,1) model yields a negative weight for the long-term variance.
 * Therefore, it is best to switch to an EWMA model. Delete the vc terms in the
 * preceding program and replace vb with 1-vb.

```
declare series u
declare series h
nonlin(parmset=meanparms) b0
nonlin(parmset=garchparms) va
```

```
frml resid = y - b0
frml hf = va*h{1} + (1-vb)*u{1}**2
frml logl = (h(t)=hf(t)),(u(t)=resid(t)),-.5*(log(h(t))+u(t)**2/h(t))
```

```
linreg(noprint) y / u
# constant
```

```
compute b0=%beta(1)
compute va=.5
set h = %seesq
```

```
maximize(notrace,parmset=meanparms+garchparms,method=simplex,itors=5) logl 2 *
```

```
MAXIMIZE - Estimation by Simplex
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              470.50130404
Variable                   Coeff
*****
1. B0                      0.0192796234
2. VA                      0.8736962210
```

```
maximize(notrace,parmset=meanparms+garchparms,method=bfgs,robusterrors, $
itors=100) logl 2 *
```

```
MAXIMIZE - Estimation by BFGS
Convergence in      3 Iterations. Final criterion was  0.0000053 <  0.0000100
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              470.50130485
Variable                   Coeff      Std Error      T-Stat      Signif
*****
1. B0                      0.0192805093  0.0019943607      9.66751     0.00000000
2. VA                      0.8736524367  0.0419249821     20.83847     0.00000000
```

```
set VARCONRAW_A = u**2
set VARCON_A = h
```

```
dedit
Editing data file "C:\Documents and Settings\eweber.BIZ\My Documents\Real
Interest Rate\TEMPSTORAGE.RAT", 1 series
include VARCON_A
include VARCONRAW_A ; save ; quit
```

* 2.2 Volatility of Real Interest Rate

*

* See RATS User's Guide, Chapter 12.

```
cal 1831 1 1
all 0 2004:1
open data
data(format=RATS) / USRINT
* REALINTEREST.RAT

set y = usrint

declare series u
declare series h
nonlin(parmset=meanparms) b0
nonlin(parmset=garchparms) vc va vb

frml resid = y - b0
frml hf = vc + va*h{1} + vb*u{1}**2
frml logl = (h(t)=hf(t)),(u(t)=resid(t)),-.5*(log(h(t))+u(t)**2/h(t))

linreg(noprint) y / u
# constant

compute b0=%beta(1)
compute vc=%seesq,va=.05,vb=.05
set h = %seesq

maximize(notrace,parmset=meanparms+garchparms,method=simplex,itors=5) logl 2 *
```

```
MAXIMIZE - Estimation by Simplex
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              420.07222171
```

Variable	Coeff
1. B0	0.0362656475
2. VC	0.0016848322
3. VA	0.0110055017
4. VB	0.2823469178

```
maximize(notrace,parmset=meanparms+garchparms,method=bfgs,robusterrors, $
itors=100) logl 2 *
```

```
MAXIMIZE - Estimation by BFGS
Convergence in      20 Iterations. Final criterion was  0.0000050 <  0.0000100
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              443.57012250
```

Variable	Coeff	Std Error	T-Stat	Signif
1. B0	0.0192413646	0.0025763615	7.46843	0.00000000
2. VC	0.0000282777	0.0000646321	0.43752	0.66173605
3. VA	0.4337129868	0.0898364385	4.82781	0.00000138
4. VB	0.8100967300	0.2660590979	3.04480	0.00232835

* The sum of va and vb exceeds one, implying a negative weight for the long-term variance. Therefore, it is best to switch to an EWMA model. Delete the * vc terms in the preceding program and replace vb with 1-vb.

```
declare series u
declare series h
nonlin(parmset=meanparms) b0
nonlin(parmset=garchparms) va
```

```
frml resid = y - b0
frml hf = va*h{1} + (1-vb)*u{1}**2
frml logl = (h(t)=hf(t)),(u(t)=resid(t)),-.5*(log(h(t))+u(t)**2/h(t))
```

```
linreg(noprint) y / u
# constant
```

```
compute b0=%beta(1)
compute vc=%seesq,va=.5
set h = %seesq
```

```
maximize(notrace,parmset=meanparms+garchparms,method=simplex,itors=5) logl 2 *
```

```
MAXIMIZE - Estimation by Simplex
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              434.25079658
```

Variable	Coeff

1. B0	0.0191157821
2. VA	0.5898507622

```
maximize(notrace,parmset=meanparms+garchparms,method=bfgs,robusterrors, $
itors=100) logl 2 *
```

```
MAXIMIZE - Estimation by BFGS
Convergence in      3 Iterations. Final criterion was  0.0000033 <  0.0000100
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              434.25079844
```

Variable	Coeff	Std Error	T-Stat	Signif

1. B0	0.0191169521	0.0011076665	17.25876	0.00000000
2. VA	0.5899596328	0.0800734845	7.36773	0.00000000

```
set VARINTRAW_A = u**2
set VARINT_A = h
```

```
dedit
Editing data file "C:\Documents and Settings\eweber.BIZ\My Documents\Real
Interest Rate\TEMPSTORAGE.RAT", 3 series
include VARINT_A
include VARINTRAW_A
save
quit
```

* 2.3. Bivariate Volatility Model

*

* This program estimates the multivariate GARCH(1,1) model developed by
 * Engle, JBES 2002, pp 339-350. The program is part of the RATS program file
 * GARCHMV.PRG, updated, March 2003.

cal 1831 1 1
 all 0 2004:1

open data
 data(format=RATS) / USRINT DUSPCRCRCON
 * REALINTEREST.RAT

compute GSTART=1832:1 , GEND=2004:1

* Parameters for the regression function

dec vect[series] y(2) u(2)
 dec vect[frml] resid(2)
 set y(1) = duspccrcon
 set y(2) = usrnt

nonlin(parmset=MEANPARMS) B11 B21
 frml RESID(1) = (Y(1)-B11)
 frml RESID(2) = (Y(2)-B21)

* Do initial regression. Copy initial values for regression parameters

linreg(noprint) Y(1) / u(1)
 # constant

compute B11 = %BETA(1)
 linreg(noprint) Y(2) / u(2)
 # constant

compute B21 = %BETA(1)

* Get the covariance matrix of the residuals.

vcv(MATRIX=RR,NOPRINT)
 # U

* h will have the sequence of variance estimates
 * uu will have the sequence of uu' matrices
 *

declare symm[series] h(2,2)
 declare symm[series] uu(2,2)

* hx and uux are used when extracting elements from h and uu.
 * ux is used when extracting a u vector

declare symm hx(2,2) uux(2,2)
 declare vect ux(2)

* This is used to initialize pre-sample variances.
 * If you want the pre-sample uu' to be the unconditional variance,
 * change the right side of the set uu(i,j) to rr(i,j) (same as h).


```

do i=1,2
  do j=1,i
    set h(i,j) = rr(i,j)
    set uu(i,j) = 0.0
  end do j
end do i

* This is a standard log likelihood formula for any bivariate
* ARCH, GARCH, ARCH-M,... The difference among these will be in
* the definitions of HF and RESID. The function %XT pulls information
* out of a matrix of SERIES, while %PT puts information into one.

declare frml[symm] hf

frml LOGL = $
  U(1) = RESID(1) , U(2) = RESID(2) , $
  HX = HF(T) , $
  UX = %XT(U,T) , UUX = %OUTERXX(UX), $
  %PT(H,T,HX), %PT(UU,T,%OUTERXX(UX)), $
  %LOGDENSITY(HX,UX)

* From here, the code is specific to the model of Engle.

declare symm[series] q(2,2)
declare symm qx(2,2)

declare frml[symm] qf

* Initialize the q sequence

do i=1,2
  do j=1,i
    set q(i,j) = rr(i,j)
  end do j
end do i

dec vect vbv(2) vav(2) vcv(2)
dec real a b

* a and b are the parameters governing the "GARCH" process of the Q sequence

nonlin(parmset=garchparms) vcv vbv vav a b

frml qf = (qx=(1-a-b)*rr+a*%xt(uu,t-1)+b*%xt(q,t-1)), %pt(q,t,qx), qx
frml hf = qf(t), rho=%if(a<1.and.b<1, qx(1,2)/sqrt(qx(1,1)*qx(2,2)), %na), $
  (h11=vcv(1)+vav(1)*h(1,1){1}+vbv(1)*uu(1,1){1}), $
  (h22=vcv(2)+vav(2)*h(2,2){1}+vbv(2)*uu(2,2){1}), $
  ||h11|rho*sqrt(h11*h22), h22||

* Initialize the c's to the diagonal elements of rr, others with typical values

compute vcv=%xdiag(rr), vbv=%const(0.05), vav=%const(0.05), a=0.05, b=0.05
maximize(notrace, parmset=meanparms+garchparms, method=SIMPLEX, iters=10) LOGL $
START GEND

```

```

MAXIMIZE - Estimation by Simplex
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              894.05583614

```

Variable	Coeff
1. B11	0.024199293
2. B21	0.022571748
3. VCV(1)	0.000913983
4. VCV(2)	0.000356264
5. VBV(1)	0.660348185
6. VBV(2)	0.681747674
7. VAV(1)	0.107437098
8. VAV(2)	0.258144977
9. A	0.186828042
10. B	-0.059647890

```

maximize(parmset=meanparms+garchparms,method=Bfgs,itors=100) LOGL GSTART GEND

```

```

MAXIMIZE - Estimation by BFGS
Convergence in      48 Iterations. Final criterion was  0.0000076 <  0.0000100
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              916.73842735

```

Variable	Coeff	Std Error	T-Stat	Signif
1. B11	0.0193	2.1883e-03	8.80363	0.000000000
2. B21	0.0192	1.6508e-03	11.65783	0.000000000
3. VCV(1)	-2.0059e-05	6.3543e-06	-3.15674	0.00159542
4. VCV(2)	2.5558e-05	3.5270e-05	0.72464	0.46867322
5. VBV(1)	0.1051	0.0542	1.93948	0.05244289
6. VBV(2)	0.7760	0.1865	4.16115	0.00003166
7. VAV(1)	0.9079	0.0441	20.56908	0.000000000
8. VAV(2)	0.4513	0.0752	6.00391	0.000000000
9. A	0.0570	0.0521	1.09481	0.27359923
10. B	0.8458	0.1421	5.95225	0.000000000

```

* The GARCH(1,1) model yields a negative weight for the long-term variance of
* consumption. Therefore, the model is reestimated as an EWMA model.

```

```

declare symm[series] q(2,2)
declare symm qx(2,2)
*

```

```

declare frml[symm] qf

```

```

* Initialize the q sequence

```

```

do i=1,2
  do j=1,i
    set q(i,j) = rr(i,j)
  end do j
end do i

```

```

dec vect vav(2)
dec real a b

```

```

* a and b are the parameters governing the "GARCH" process of the Q sequence

```

```

nonlin(parmset=garchparms) vav a b

frml qf = (qx=(1-a-b)*rr+a*%xt(uu,t-1)+b*%xt(q,t-1)),%pt(q,t,qx),qx
frml hf = qf(t),rho=%if(a<1.and.b<1,qx(1,2)/sqrt(qx(1,1)*qx(2,2)),%na),$
          (h11=vav(1)*h(1,1){1}+(1-vav(1))*uu(1,1){1}),$
          (h22=vav(2)*h(2,2){1}+(1-vav(2))*uu(2,2){1}),$
          ||h11|rho*sqrt(h11*h22),h22||
* Initialize the c's to the diagonal elements of rr, others with typical values

compute vav=%const(0.5),a=0.05,b=0.05
maximize(notrace,parmset=meanparms+garchparms,method=SIMPLEX,itors=10) LOGL $
GSTART GEND

MAXIMIZE - Estimation by Simplex
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              905.41315267

Variable                  Coeff
*****
1.  B11                    0.0192788522
2.  B21                    0.0189713256
3.  VAV(1)                 0.8685713155
4.  VAV(2)                 0.5918197101
5.  A                      0.0532379515
6.  B                      0.0164683129

maximize(parmset=meanparms+garchparms,method=Bfgs,itors=100) LOGL GSTART GEND

MAXIMIZE - Estimation by BFGS
Convergence in      14 Iterations. Final criterion was  0.0000091 <  0.0000100
Annual Data From 1832:01 To 2004:01
Usable Observations      173
Function Value              906.47610423

Variable                  Coeff      Std Error      T-Stat      Signif
*****
1.  B11                    0.0190434927  0.0023084547      8.24945  0.00000000
2.  B21                    0.0189947453  0.0013322550     14.25759  0.00000000
3.  VAV(1)                 0.8703428021  0.0313307612     27.77918  0.00000000
4.  VAV(2)                 0.5879949672  0.0560446332     10.49155  0.00000000
5.  A                      0.0452195167  0.0427792692      1.05704  0.29049207
6.  B                      0.8469252926  0.1478434537      5.72853  0.00000001

set VARCON_B = h(1,1)
set VARINT_B = h(2,2)
set COVAR_B = h(2,1)

set VARCONRAW_B = UU(1,1)
set VARINTRAW_B = UU(2,2)
set COVARRAW_B = UU(2,1)

dedit
Editing data file "C:\Documents and Settings\eweber.BIZ\My Documents\Real
Interest Rate\TEMPSTORAGE.RAT", 5 series
include VARCONRAW_B
include VARCON_B
include VARINTRAW_B

```

```

include VARINT_B
include COVARRAW_B
include COVAR_B
save
quit

```

```

* 3. Log-Linear Euler Equation
* _____

```

```

cal 1831 1 1
all 0 2004:1

```

```

open data
data(format=RATS) / USRINT DUSPCRCON USINFL COVAR_B VARCON_B
* REALINTEREST.RAT
* Exclude Great Depression.

```

```

set TREND = t
set DEP = %if(TREND<=103.and.TREND>=100,%na,1.0)
set DUSPCRCONDEP = DUSPCRCON*DEP
set POSTWAR = %if(TREND>=120,1.0,0)

```

```

* 3.1. OLS Estimation
* _____

```

```

linreg(robusterrors,damp=1.0,lags=4) USRINT 1831:1 2004:1
# CONSTANT POSTWAR DUSPCRCONDEP COVAR_B VARCON_B

```

Linear Regression - Estimation by Least Squares

Dependent Variable USRINT

Annual Data From 1831:01 To 2004:01

Usable Observations	170	Degrees of Freedom	165
Total Observations	174	Skipped/Missing	4
Centered R**2	0.385403	R Bar **2	0.370503
Uncentered R**2	0.533350	T x R**2	90.670
Mean of Dependent Variable	0.0343334448		
Std Error of Dependent Variable	0.0611561802		
Standard Error of Estimate	0.0485218214		
Sum of Squared Residuals	0.3884705808		
Durbin-Watson Statistic	0.996681		

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.10419824	0.01524908	6.83308	0.00000000
2. POSTWAR	-0.07873592	0.01293349	-6.08775	0.00000000
3. DUSPCRCONDEP	0.18964016	0.12568681	1.50883	0.13134198
4. COVAR_B	42.71032275	6.68796120	6.38615	0.00000000
5. VARCON_B	-29.28436767	6.46814586	-4.52747	0.00000597

```
linreg(robusterrors,damp=1.0,lags=4) USRINT 1831:1 2004:1
# CONSTANT TREND DUSPCRCONDEP COVAR_B VARCON_B
```

Linear Regression - Estimation by Least Squares

Dependent Variable USRINT

Annual Data From 1831:01 To 2004:01

```
Usable Observations      170      Degrees of Freedom      165
Total Observations      174      Skipped/Missing        4
Centered R**2           0.433829      R Bar **2           0.420103
Uncentered R**2         0.570119      T x R**2           96.920
Mean of Dependent Variable      0.0343334448
Std Error of Dependent Variable 0.0611561802
Standard Error of Estimate      0.0465710060
Sum of Squared Residuals      0.3578616693
Durbin-Watson Statistic      1.048942
```

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.11654680	0.01048051	11.12034	0.00000000
2. TREND	-0.00066277	0.00007586	-8.73642	0.00000000
3. DUSPCRCONDEP	0.22516529	0.13505025	1.66727	0.09546065
4. COVAR_B	28.47450132	5.47502751	5.20080	0.00000020
5. VARCON_B	-18.04691279	5.00322772	-3.60705	0.00030969

```
linreg(robusterrors,damp=1.0,lags=4) USRINT 1831:1 1929:1
# CONSTANT DUSPCRCON COVAR_B VARCON_B
```

Linear Regression - Estimation by Least Squares

Dependent Variable USRINT

Annual Data From 1831:01 To 1929:01

```
Usable Observations      99      Degrees of Freedom      95
Centered R**2           0.354375      R Bar **2           0.333987
Uncentered R**2         0.638938      T x R**2           63.255
Mean of Dependent Variable      0.0579686679
Std Error of Dependent Variable 0.0656295695
Standard Error of Estimate      0.0535600401
Sum of Squared Residuals      0.2725244006
Durbin-Watson Statistic      1.236058
```

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.12151879	0.01497127	8.11680	0.00000000
2. DUSPCRCON	0.16052947	0.14413444	1.11375	0.26538725
3. COVAR_B	40.81479185	7.91612717	5.15590	0.00000025
4. VARCON_B	-33.13730446	8.22700441	-4.02787	0.00005628

```
linreg(robusterrors,damp=1.0,lags=4) USRINT 1934:1 2004:1
# CONSTANT DUSPCRCON COVAR_B VARCON_B
```

Linear Regression - Estimation by Least Squares

Dependent Variable USRINT

Annual Data From 1934:01 To 2004:01

```
Usable Observations      71      Degrees of Freedom      67
Centered R**2           0.301285      R Bar **2           0.269999
Uncentered R**2         0.302497      T x R**2           21.477
Mean of Dependent Variable      0.0013772886
Std Error of Dependent Variable 0.0332836828
Standard Error of Estimate      0.0284376026
Sum of Squared Residuals      0.0541827150
Durbin-Watson Statistic      0.769640
```

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.00865582	0.00646068	1.33977	0.18032054
2. DUSPCRCON	0.42289856	0.14627175	2.89118	0.00383793
3. COVAR_B	8.93452955	17.34118655	0.51522	0.60639916
4. VARCON_B	-13.56108142	4.41225758	-3.07350	0.00211562

* 3.2. Estimation with Instrumental Variables
*

```
instruments CONSTANT POSTWAR USINFL{1 to 2} DUSPCRCONDEP{1 to 2} $
VARCON_B{1 to 2} COVAR_B{1 to 2}
```

```
linreg(inst,robusterrors,damp=1.0,lags=4) USRINT 1831:1 2004:1
# CONSTANT POSTWAR DUSPCRCONDEP COVAR_B VARCON_B
```

Linear Regression - Estimation by Instrumental Variables

Dependent Variable USRINT

Annual Data From 1831:01 To 2004:01

Usable Observations	166	Degrees of Freedom	161
Total Observations	174	Skipped/Missing	8
Centered R**2	-0.190566	R Bar **2	-0.220145
Uncentered R**2	0.094917	T x R**2	15.756
Mean of Dependent Variable	0.0342444420		
Std Error of Dependent Variable	0.0611585189		
Standard Error of Estimate	0.0675558053		
Sum of Squared Residuals	0.7347696804		
J-Specification(5)	0.030420		
Significance Level of J	0.99999151		
Durbin-Watson Statistic	1.567679		

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.09909716	0.02307490	4.29459	0.00001750
2. POSTWAR	-0.09048953	0.01954751	-4.62921	0.00000367
3. DUSPCRCONDEP	1.28388967	0.53945008	2.38000	0.01731277
4. COVAR_B	43.97934168	8.71770707	5.04483	0.00000045
5. VARCON_B	-37.12456776	9.41678162	-3.94238	0.00008068

```
instruments CONSTANT TREND USINFL{1 to 2} DUSPCRCONDEP{1 to 2} $
VARCON_B{1 to 2} COVAR_B{1 to 2}
```

```
linreg(inst,robusterrors,damp=1.0,lags=4) USRINT 1831:1 2004:1
# CONSTANT TREND DUSPCRCONDEP COVAR_B VARCON_B
```

Linear Regression - Estimation by Instrumental Variables

Dependent Variable USRINT

Annual Data From 1831:01 To 2004:01

Usable Observations	166	Degrees of Freedom	161
Total Observations	174	Skipped/Missing	8
Centered R**2	-0.021584	R Bar **2	-0.046965
Uncentered R**2	0.223379	T x R**2	37.081
Mean of Dependent Variable	0.0342444420		
Std Error of Dependent Variable	0.0611585189		

Standard Error of Estimate 0.0625781961
Sum of Squared Residuals 0.6304809315
J-Specification(5) 0.027053
Significance Level of J 0.99999366
Durbin-Watson Statistic 1.679707

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.11564949	0.01823828	6.34103	0.00000000
2. TREND	-0.00077342	0.00013395	-5.77373	0.00000001
3. DUSPCRCONDEP	1.21499793	0.49895462	2.43509	0.01488821
4. COVAR_B	25.56509746	7.24327219	3.52950	0.00041635
5. VARCON_B	-23.12806165	6.97100943	-3.31775	0.00090746

instruments CONSTANT USINFL{1 to 2} DUSPCRCON{1 to 2} VARCON_B{1 to 2} \$
COVAR_B{1 to 2}

linreg(inst,robusterrors,damp=1.0,lags=4) USRINT 1831:1 1929:1
CONSTANT DUSPCRCON COVAR_B VARCON_B

Linear Regression - Estimation by Instrumental Variables

Dependent Variable USRINT

Annual Data From 1831:01 To 1929:01

Usable Observations	97	Degrees of Freedom	93
Total Observations	99	Skipped/Missing	2
Centered R**2	0.074227	R Bar **2	0.044363
Uncentered R**2	0.474348	T x R**2	46.012
Mean of Dependent Variable	0.0572204465		
Std Error of Dependent Variable	0.0659257730		
Standard Error of Estimate	0.0644468485		
Sum of Squared Residuals	0.3862658542		
J-Specification(5)	0.026736		
Significance Level of J	0.99999384		
Durbin-Watson Statistic	1.599709		

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.11807147	0.01944620	6.07170	0.00000000
2. DUSPCRCON	0.85911397	0.50108980	1.71449	0.08643859
3. COVAR_B	36.54969109	8.30322612	4.40187	0.00001073
4. VARCON_B	-36.60505462	9.92890170	-3.68672	0.00022717

instruments CONSTANT USINFL{1 to 2} DUSPCRCON{1 to 2} VARCON_B{1 to 2} \$
COVAR_B{1 to 2}

linreg(inst,robusterrors,damp=1.0,lags=4) USRINT 1934:1 2004:1
CONSTANT DUSPCRCON COVAR_B VARCON_B

Linear Regression - Estimation by Instrumental Variables

Dependent Variable USRINT

Annual Data From 1934:01 To 2004:01

Usable Observations	71	Degrees of Freedom	67
Centered R**2	0.248235	R Bar **2	0.214574
Uncentered R**2	0.249538	T x R**2	17.717
Mean of Dependent Variable	0.0013772886		
Std Error of Dependent Variable	0.0332836828		
Standard Error of Estimate	0.0294974193		
Sum of Squared Residuals	0.0582965488		
J-Specification(5)	0.008440		

Significance Level of J 0.99999965
Durbin-Watson Statistic 0.942065

Variable	Coeff	Std Error	T-Stat	Signif
1. Constant	0.00357732	0.00868356	0.41196	0.68036586
2. DUSPCRCO	0.70943756	0.25692837	2.76123	0.00575846
3. COVAR_B	11.97276687	20.64369496	0.57997	0.56193344
4. VARCON_B	-14.89325530	5.12986668	-2.90324	0.00369319

* 4. Figures

* _____

cal 1830 1 1
all 0 2004:1

open data
data(format=RATS) / USRINT DUSPCRCO USINFL VARCON_A VARINT_A VARCON_B \$
VARINT_B COVAR_B
* REALINTEREST.RAT

set USRINT = USRINT*100
set DUSPCRCO = DUSPCRCO*100
set USINFL = USINFL*100
set TREND = t

set EMERGENCIES =
%if (TREND>=1861:1.and.TREND<=1865:1.or.TREND>=1917:1.and.TREND<=1918:1 \$
.or.TREND>=1929:1.and.TREND<=1933:1.or.TREND>=1942:1.and.TREND<=1945:1 \$
.or.TREND>=1950:1.and.TREND<=1953:1.or.TREND>=1973:1.and.TREND<=1980:1 \$
.or.TREND>=2001:1,1,0)

spgraph(vfields=2)
graph(shading=EMERGENCIES,header='Figure 1. Real Interest Rate', \$
subheader='1831-2004') 1
USRINT 1830:1 2004:1
graph(shading=EMERGENCIES,header='Figure 2. Inflation Rate', \$
SUBHEADER='1831 - 2004') 1
USINFL 1830:1 2004:1
spgraph(done)

grparm(bold) header 15
spgraph(samesize,header='Figure 3. Bivariate EWMA Model',vfields=3)
grparm(bold) header 20
graph(max=0.006,vticks=6,shading=EMERGENCIES, \$
header='A. Consumption Volatility') 1
VARCON_B 1830:1 2004:1
set UNIVARIATE = VARINT_A
set BIVARIATE = VARINT_B
graph(shading=EMERGENCIES,header='B. Interest Rate Volatility') 1
VARINT_B 1830:1 2004:1
graph(MAX=0.005,shading=EMERGENCIES, \$
header='C. Covariance Consumption Growth/Interest Rate') 1
COVAR_B 1830:1 2004:1
spgraph(done)