Dynamics of self-excited thermoacoustic instability in a combustion system: Pseudo-periodic and high-dimensional nature

Yuta Okuno, Michael Small, and Hiroshi Gotoda

Citation: *Chaos* **25**, 043107 (2015); doi: 10.1063/1.4914358

View online: <http://dx.doi.org/10.1063/1.4914358>

View Table of Contents: <http://scitation.aip.org/content/aip/journal/chaos/25/4?ver=pdfcov>

Published by the AIP Publishing

Articles you may be interested in

[Coupling between hydrodynamics, acoustics, and heat release in a self-excited unstable combustor](#)

Phys. Fluids **27**, 045102 (2015); 10.1063/1.4916673

[The development of an optically accessible, high-power combustion test rig](#)

Rev. Sci. Instrum. **85**, 035105 (2014); 10.1063/1.4867084

[Characterization of complexities in combustion instability in a lean premixed gas-turbine model combustor](#)

Chaos **22**, 043128 (2012); 10.1063/1.4766589

[Route to chaos for combustion instability in ducted laminar premixed flames](#)

Chaos **22**, 023129 (2012); 10.1063/1.4718725

[Coherent structures in swirling flows and their role in acoustic combustion control](#)

Phys. Fluids **11**, 2667 (1999); 10.1063/1.870128

Dynamics of self-excited thermoacoustic instability in a combustion system: Pseudo-periodic and high-dimensional nature

Yuta Okuno,^{1,2} Michael Small,² and Hiroshi Gotoda^{1,a)}

¹*Department of Mechanical Engineering, Ritsumeikan University, 1-1-1 Nojihigashi, Kusatsu-shi, Shiga 525-8577, Japan*

²*School of Mathematics and Statistics, The University of Western Australia, Crawley, WA 6009, Australia*

(Received 24 November 2014; accepted 25 February 2015; published online 8 April 2015)

We have examined the dynamics of self-excited thermoacoustic instability in a fundamentally and practically important gas-turbine model combustion system on the basis of complex network approaches. We have incorporated sophisticated complex networks consisting of cycle networks and phase space networks, neither of which has been considered in the areas of combustion physics and science. Pseudo-periodicity and high-dimensionality exist in the dynamics of thermoacoustic instability, including the possible presence of a clear power-law distribution and small-world-like nature. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4914358>]

Self-excited thermoacoustic instability arises as a result of the strong coupling between acoustic waves and flames inside a combustor, resulting in high-amplitude oscillations of all physical quantities such as pressure, temperature, and velocity. It prominently appears during operation under a lean premixed condition and leads to a serious deterioration of the combustion system performance in land-based gas-turbine engines and aircraft propulsion. A better understanding of the nonlinear dynamics will have a significant effect in fundamental combustion science and physics. Complex networks have been used with various schemes to represent the dynamics underpinning time series. In this study, we reveal the hidden dynamics in thermoacoustic instability by examining network topologies reconstructed by temporal variations in a pressure field. The use of sophisticated complex networks consisting of cycle networks and phase space networks allows us to fully extract the signature of the pseudo-periodicity and high-dimensionality in thermoacoustic instability, including the possible presence of a clear power-law distribution and small-world-like nature. The consideration of phase space networks in combination with a pseudo-periodic surrogate method based on statistical null hypothesis testing supports the existence of high-dimensional dynamics.

Thermoacoustic instability generated by the strong interaction between an unsteady pressure field and fluctuations in the heat release ratio in a confined system exhibits a rich variety of nonlinear dynamics and is of fundamental importance in a wide spectrum of fields in thermal fluid and combustion physics.^{1–6} A substantial number of experimental studies^{7–11} have applied the concept of thermoacoustic instability to the development of new engines. Quasi-periodicity,^{4,5} intermittent bursts,^{12,13} and chaos^{4,14} in thermoacoustic combustion

systems have been extensively examined both with and without turbulent flow conditions, demonstrating the suitability of nonlinear time series analysis based on dynamical system theory for dealing with complex combustion dynamics. We have also emphasized the importance of dynamical system theory for several nonlinear phenomena involving magneto-hydrodynamic instability,¹⁵ flame front instability induced by buoyancy-swirl coupling in normal gravity,¹⁶ and radiative heat loss in zero gravity.¹⁷ Our recent work¹⁸ related to thermoacoustic instability has also shown the possible existence of low-dimensional chaos in the intermittent combustion oscillations of a laboratory-scale gas-turbine model combustor, providing a promising online detection method for lean blowout on the basis of dynamical system theory. On another front, complex network approaches reveal the features of large-scale networks represented by complex topologies and heterogeneous structures, and yield a comprehensive understanding of complex connectivity patterns in dynamical phenomena by extracting scale-free properties. The importance of this theory and its approach to time series data are summarized in detail in a recent book¹⁹ and paper.²⁰ How can one understand the nonlinear dynamics in thermoacoustic instability using complex network approaches? This important question has not been addressed in the vast literature of combustion-based studies on thermoacoustic instability.

The main purpose of this study is to conduct a comprehensive investigation to reveal the hidden nature of thermoacoustic instability under turbulent flow conditions, mainly by making use of two sophisticated complex network approaches (cycle networks²¹ and phase space networks²²) in combination with a pseudo-periodic surrogate (PPS) method.^{23,24} We deal with thermoacoustic instability in a laboratory-scale gas-turbine model combustor¹⁸—one of the fundamentally and practically important combustion systems. Details of our experimental system have been reported in a previous study.¹⁸ We measure the pressure fluctuations p' inside the combustion chamber as an important physical quantity that represents the nonlinear dynamics of thermoacoustic instability. The sampling frequency of the time series

^{a)}Author to whom correspondence should be addressed. Electronic mail: gotoda@rs.tus.ac.jp. Present address: Department of Mechanical Engineering, Tokyo University of Science, 6-3-1 Nijuku, Katsushika-ku, 125-8585, Japan.

FIG. 1. Time variation of pressure fluctuations p' in thermoacoustic instability.

data of p' is 5 kHz. The mean axial flow velocity at the inlet of the combustion chamber U and the equivalence ratio ϕ are set to 5.8 m/s and 0.70, respectively, for which self-excited thermoacoustic instability clearly appears (see Fig. 1). We here examine the spatial distribution of the pseudo-Rayleigh index (PRI) shown in Fig. 2 to clarify the flame dynamics of thermoacoustic instability. Note that the PRI, defined by Eq. (1), can identify regions where thermoacoustic driving and damping effects appear and is a useful indicator for identifying the source of thermoacoustic instability

$$R(x, y) = \frac{1}{\tau_c} \int p'(t) \cdot I'_{\text{OH}^*}(t, x, y) dt. \quad (1)$$

Here, R is the PRI, τ_c is the time of one cycle, and I'_{OH^*} is the phase-locked OH* image obtained using a high-speed camera with an image intensifier. Three assumptions²⁵ are adopted to calculate the spatial distribution of the PRI: (I) the spatial distribution of pressure in OH* chemiluminescence intensity images is uniform; (II) the OH* chemiluminescence intensity is proportional to the heat release rate of the flame front; (III) Abel inversion is adopted for nearly axisymmetric OH* chemiluminescence intensity images along the centerline of the axial swirl vane obtained by a high-speed camera. As shown in Fig. 2, a significantly positive PRI is observed in the formation region of the oscillating

flame. According to the results of a study by Tachibana *et al.*,²⁵ the entire flow field in our combustion chamber primarily consists of two recirculation flows: a vortex breakdown bubble in the wake of the centerbody and an outer recirculation region located in the dump plate. Thermoacoustic oscillating flames are sustained between the two recirculation regions, and the vortex structures originating from the convective interaction in the shear layers between the two recirculation regions induce the rolled-up structure of the oscillating flame. The important point to note here is that the formation region above the rolled-up structure generated by the convective vortices corresponds to the regions with a strongly positive PRI, resulting in the sustainment of thermoacoustic instability observed in this study.

A previous study²⁶ using a different type of gas-turbine model combustor demonstrated that the permutation entropy²⁷ based on the coarse-graining concept allows us to quantify the degree of randomness estimated from a sequence of rank order patterns in pressure fluctuations close to lean blowout. We thereby adopt the permutation entropy h_p defined as Eq. (2) in combination with the PPS method^{23,24} to thermoacoustic instability in this study. Note that in our recent study,²⁸ the permutation entropy has been adopted as a useful detector to prevent lean blowout. Given a sequence with embedding dimension D , we index all possible permutations ($D!$ permutations) of order $D > 2$ as π

$$h_p = - \frac{\sum_{\pi} p(\pi) \log_2 p(\pi)}{\log_2 D!}. \quad (2)$$

Here, $p(\pi)$ is the relative frequency for each permutation pattern π .^{26,27} The PPS method,^{23,24} which considers the null hypothesis that the underlying dynamics is dominated by a periodic orbit with uncorrelated noise, enables us to test for the existence of non-periodic inter-cycle dynamics. We first select an initial position vector $\{\mathbf{S}_1 \in \mathbf{x}_i : i = 1, 2, \dots, N - (D - 1)\tau\}$ from the reconstructed phase space by Takens' embedding theorem²⁹ consisting of p' , where N is the data number of scalar time series and τ is the time delay. Let $i = 1$. We then select one of the neighbors of $\mathbf{S}_i$ with probability

$$\text{Prob}(\mathbf{S}_j = \mathbf{x}_i) \propto \exp \frac{-\|\mathbf{x}_i - \mathbf{S}_j\|}{r}, \quad (3)$$

FIG. 2. Spatial distribution of pseudo-Rayleigh index in thermoacoustic instability.

where r is the noise radius. After setting $\mathbf{S}_{i+1} = \mathbf{x}_{j+1}$ and incrementing i , we repeat the above procedure until $i = N$. The surrogate time series $\{(\mathbf{S}_i)_1: i = 1, 2, \dots, N\}$ is thus obtained, where $(\mathbf{S}_i)_1$ denotes the first component of the embedding vector. Absolute difference $||h_{p,o} - h_{p,s}||$ between the permutation entropies estimated for the original pressure fluctuations and the PPS data are shown in Fig. 3. There is no significant change in $||h_{p,o} - h_{p,s}||$ for the low dimension of $D = 3$. When D exceeds 5, a notable difference can appear for all times, indicating the existence of high-dimensional dynamics in thermoacoustic instability. Additionally, the surrogate data show higher permutation entropies than the original data when considering the absolute difference as $h_{p,o} - h_{p,s}$. Note that in our preliminary test using a scale-dependent correlation dimension^{23,30} in combination with the PPS method, a clear difference between the original pressure fluctuations and the PPS data is obtained, which supports the existence of high-dimensional dynamics.

We introduce the cycle network approach²¹ proposed for treating strongly periodic-like time series data. In this approach, p' is divided at each local minimum into n cycles $\{C_1, C_2, \dots, C_n\}$. Each cycle C_i represents nodes in a network. The edges, which are the connections between nodes, are determined by the correlation between the cycles ρ_{ij} defined as Eq. (4) or by the phase space distance D_{ij} defined as Eq. (5)

$$\rho_{ij} = \max_{l=0,1,\dots,l_j-l_i} \frac{\text{Cov}[C_i(1:l_i), C_j(1+l:l_i+l)]}{\sqrt{V[C_i(1:l_i)]} \sqrt{V[C_j(1+l:l_i+l)]}}, \quad (4)$$

$$D_{ij} = \min_{l=0,1,\dots,l_j-l_i} \frac{1}{l_i} \sum_{k=1}^{l_i} ||X_k - Y_{k+l}||. \quad (5)$$

Here, $C_i(a:b)$ denotes the segment between the a -th and b -th elements in C_i , $V(\cdot)$ is the variance, l_i and l_j are the lengths of C_i and C_j , and X_k and Y_k are the k -th points of C_i and C_j ($i \neq j$, $l_i < l_j$), respectively. We estimate three basic network properties: the average path length, the clustering coefficient, and the vertex strength in the cycle network. The number of network nodes n is 11330 for p' with 60 s duration. The threshold of ρ for the transformation into a binary network,

FIG. 3. Absolute differences $||h_{p,o} - h_{p,s}||$ between permutation entropies estimated for original and PPS data. The blue, red, green, and black lines correspond to embedding dimensions D of 3, 4, 5, and 6, respectively. We consider a running window width of $N_w = 10000$ (i.e., 2 s of p') and a half window shift (i.e., 1 s of p') in this study, which satisfy the criteria reported by Amigó *et al.*³¹

which stands for just above the critical value to break the network's single giant component,²¹ is set to 0.9943 in this study. Figure 4 shows the average shortest path L and average clustering coefficient C_a in terms of ρ . L at $\rho = 0.9943$ is 1.43 and is sufficiently small compared with n (average degree $\langle k \rangle = 6535$), while C_a has a high value of 0.71 at $\rho = 0.9943$. The sufficiently small L ($= 3.22$) and large C_a ($= 0.45$) can be seen even at a larger ρ of 0.9991 ($\langle k \rangle = 300$), indicating the existence of the small-world-like nature in the network³² between cycles of pressure fluctuations in thermoacoustic instability. The vertex strength $S_i (= \sum_{j \in G} D_{ij})$ distribution is shown in Fig. 5. The distribution exhibits a clear power-law distribution, and the scaling exponent is inferred to be -5.627 , which is estimated by the straight-line fit based on the maximum likelihood method.

Xu *et al.*²² proposed a phase space network approach. Their approach represented the phase space points as nodes and the k -nearest neighbors connecting each point as edges, with k set to 4, while also taking into account temporal separation greater than the mean period of pressure fluctuations in order to choose appropriate neighbors. They reported that the distinction among periodic, chaotic, and noisy-periodic dynamics can be expressed with reference to the relative frequencies of two particular motifs, D and F (see Fig. 6). The

FIG. 4. (a) Average shortest path L vs threshold ρ . (b) Average clustering coefficient C_a vs threshold ρ .

FIG. 5. Vertex strength distribution for pressure fluctuations in thermoacoustic instability.

rank of motif D has a higher frequency and motif F occurs less frequently from periodic (e.g., periodic Rossler oscillations) to chaotic (e.g., a chaotic Lorenz system) to noisy-periodic dynamics (e.g., a sine wave with white Gaussian noise). This is essentially due to the heterogeneity of the attractor and is also related to the intrinsic dimension of the system. Motif D is more likely to appear only in higher dimensions or when the distribution of points is heterogeneous; whereas motif F occurs more often when the points are evenly distributed in a low-dimensional (i.e., linear or planar) attractor.²² As shown in Fig. 7, the motif rank is ABDCEF and motif B has the same order of magnitude as motif D, which indicates that the dynamics of thermoacoustic instability is governed by high-dimensionality. The frequency of the various motifs can be understood as a measure of the local embedding dimension of the underlying attractor—the probability of getting points that are and are not neighbors (motif D vs F, for example) depends on the dimension in which the points are distributed (motif D is virtually impossible in one dimension and fairly uncommon in two dimensions, for example). Moreover, there are significant differences between the approach we take here and the recurrence networks described in previous studies.^{33,34} We incorporate the PPS method into the motif ranks of phase space networks. If the dynamics is governed by noisy periodic time series (i.e., periodic signal plus additive noise), there is no difference between each motif for the original and PPS data. All motifs in Fig. 7 vary by an amount that exceeds two standard deviations except for motif E—the null hypothesis can thus be rejected for all motifs except for motif E. The consideration of phase space networks in combination with the PPS method supports the existence of high-dimensional

FIG. 7. Motif ranks of pressure fluctuation in thermoacoustic instability. The blue dots (no error bars) correspond to the original data. The thin lines with error bars (red) represent the mean and standard deviation from 30 PPS data. The wider error bars (green) represent the range from the minimum to the maximum.

dynamics that cannot be represented by a low-dimensional model. On the basis of the results obtained in this study, it is concluded that pseudo-periodicity and high-dimensionality exist in the dynamics of thermoacoustic instability, including the possible presence of a clear power-law distribution and small-world-like nature in the network.

The detailed elucidation of the combustion dynamics in our model combustor sustained by the physical mechanism discussed above is of great importance in the fields of combustion science and physics, and we focused on revealing the hidden dynamics of thermoacoustic instability from the viewpoint of complex network approaches. Quasi-periodicity,³⁵ low-dimensional chaos,²⁶ the synchronization of a self-excited mode with a forced mode,³⁵ and noise-induced transitions³⁶ have been extensively explored for self-excited thermoacoustic combustion systems involving a turbulent swirl-stabilized or bluff body-type combustor, and, in particular, the limit cycle structure and the regular pattern in the recurrence plots have been discussed for pressure fluctuations in well-developed thermoacoustic instability. Two complex network approaches, cycle networks²¹ and phase space networks, in combination with the PPS method^{23,24} allow us to find hidden new structures in thermoacoustic instability. These include high-dimensionality, and a small-world-like and clear power-law distribution nature, which have not been extracted in previous studies.^{13,14,18,26,35}

FIG. 6. All motifs of size four in undirected networks.

H.G. was partially supported by a Grant-in-Aid for Young Scientists (A) from the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT), and by the JGC-S Scholarship Foundation. M.S. was supported by an Australian Research Council Future Fellowship (FT110100876) and Discovery Project (DP140100203). The authors thank Taku Kuriyama and Shuya Sawada for helping in the implementation of our experiments, and Shohei Domen for estimating the pseudo-Rayleigh index.

- ¹T. C. Lieuwen, *Unsteady Combustor Physics* (Cambridge University Press, 2012).
- ²S. Candel, *Proc. Combust. Inst.* **29**, 1 (2002).
- ³K. Balasubramanian and R. I. Sujith, *Phys. Fluids* **20**, 044103 (2008).
- ⁴L. Kabiraj, A. Saurabh, P. Wahi, and R. I. Sujith, *Chaos* **22**, 023129 (2012).
- ⁵K. Kashinath, Ph.D. thesis, University of Cambridge, 2013.
- ⁶K. Kashinath, I. C. Waugh, and M. P. Juniper, *J. Fluid Mech.* **761**, 399 (2014).
- ⁷T. Biwa, Y. Tashiro, U. Mizutani, M. Kozuka, and T. Yazaki, *Phys. Rev. E* **69**, 066304 (2004).
- ⁸T. Biwa, Y. Tashiro, M. Ishigaki, Y. Ueda, and T. Yazaki, *J. Appl. Phys.* **101**, 064914 (2007).
- ⁹T. Yoshida, T. Yazaki, H. Futaki, K. Hamaguchi, and T. Biwa, *Appl. Phys. Lett.* **95**, 044101 (2009).
- ¹⁰T. Biwa, D. Hasegawa, and T. Yazaki, *Appl. Phys. Lett.* **97**, 034102 (2010).
- ¹¹H. Hatori, T. Biwa, and T. Yazaki, *J. Appl. Phys.* **111**, 074905 (2012).
- ¹²V. Nair and R. I. Sujith, *Chaos* **23**, 033136 (2013).
- ¹³V. Nair and R. I. Sujith, *J. Fluid Mech.* **747**, 635 (2014).
- ¹⁴V. Nair, G. Thampi, and R. I. Sujith, *J. Fluid Mech.* **756**, 470 (2014).
- ¹⁵H. Gotoda, R. Takeuchi, Y. Okuno, and T. Miyano, *J. Appl. Phys.* **113**, 124902 (2013).
- ¹⁶H. Gotoda, T. Miyano, and I. G. Shepherd, *Phys. Rev. E* **81**, 026211 (2010).
- ¹⁷H. Gotoda, T. Ikawa, K. Maki, and T. Miyano, *Chaos* **22**, 033106 (2012).
- ¹⁸H. Gotoda, Y. Shinoda, M. Kobayashi, Y. Okuno, and S. Tachibana, *Phys. Rev. E* **89**, 022910 (2014).
- ¹⁹M. E. J. Newman, *Networks: An Introduction* (Oxford University Press, 2010).
- ²⁰R. V. Donner, M. Small, J. F. Donges, N. Marwan, Y. Zou, R. Xiang, and J. Kurths, *Int. J. Bifurcation Chaos* **21**, 1019 (2011).
- ²¹J. Zhang and M. Small, *Phys. Rev. Lett.* **96**, 238701 (2006).
- ²²X. Xu, J. Zhang, and M. Small, *Proc. Natl. Acad. Sci.* **105**, 19601 (2008).
- ²³M. Small, *Applied Nonlinear Time Series Analysis: Applications in Physics, Physiology and Finance* (World Scientific Publishing Co. Pte. Ltd., Singapore, 2005).
- ²⁴M. Small, D. Yu, and R. G. Harrison, *Phys. Rev. Lett.* **87**, 188101 (2001).
- ²⁵S. Tachibana, J. Yamashita, L. Zimmer, K. Suzuki, and A. K. Hayashi, *Proc. Combust. Inst.* **32**, 1795 (2009).
- ²⁶H. Gotoda, M. Amano, T. Miyano, T. Ikawa, K. Maki, and S. Tachibana, *Chaos* **22**, 043128 (2012).
- ²⁷C. Bandt and B. Pompe, *Phys. Rev. Lett.* **88**, 174102 (2002).
- ²⁸S. Domen, H. Gotoda, T. Kuriyama, Y. Okuno, and S. Tachibana, *Proc. Combust. Inst.* **35**, 3295 (2015).
- ²⁹F. Takens, *Lecture Notes in Mathematics* (Springer-Verlag, Berlin, 1981), Vol. 898, pp. 366–381.
- ³⁰K. Judd, *Physica D* **56**, 216 (1992).
- ³¹J. M. Amigó, S. Zambrano, and M. A. F. Sanjuán, *Europhys. Lett.* **79**, 50001 (2007).
- ³²D. J. Watts and S. H. Strogatz, *Nature* **393**, 440 (1998).
- ³³R. V. Donner, J. Heitzig, J. F. Donges, Y. Zou, N. Marwan, and J. Kurths, *Eur. Phys. J. B* **84**, 653 (2011).
- ³⁴J. F. Donges, J. Heitzig, R. V. Donner, and J. Kurths, *Phys. Rev. E* **85**, 046105 (2012).
- ³⁵S. Balusamy, L. K. B. Li, Z. Han, M. P. Juniper, and S. Hochgreb, *Proc. Combust. Inst.* **35**, 3229 (2015).
- ³⁶V. Jegadeesan and R. I. Sujith, *Proc. Combust. Inst.* **34**, 3175 (2013).