Separation angle for flow past a circular cylinder in the subcritical regime

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Abstract

Instantaneous and time-averaged flow separation locations for the canonical case of flow past a circular cylinder are investigated through direct numerical simulations. It is found that the instantaneous movement of the upper/lower separation point on the cylinder surface is governed by a dynamic balance between the upper/lower separating shear layer and a shear layer generated at the back of the cylinder due to wake recirculation. It is also found that flow three-dimensionality contributes to an upstream movement of the time-averaged separation point through the effect of the separating shear layer. For the three-dimensional flow, the disordered mode B flow structures developed in the separating shear layer (which is directly responsible for the flow three-dimensionality) would result in turbulent energy dissipation and therefore the separating shear layer is less rolled-up in the cross-flow direction, which weakens the effect of the separating shear layer in pushing the separation point downstream. The time-averaged separation angle versus Reynolds number ($\bar{\theta}_s - Re$) relationship is not monotonic over the three-dimensional wake transition regimes of $Re = 190 - 270$, since there is a transition sequence of different wake flow patterns with different levels of flow three-dimensionality. For $Re \geq 270$, on the physical basis that the increasingly disordered mode B flow becomes the sole wake flow pattern, an empirical formula is proposed for the $\bar{\theta}_s - Re$ relationship, i.e. $\bar{\theta}_s = 78.8 + 505Re^{-1/2}$ ($270 \leq Re \lesssim 10^5$).

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1. Introduction

Flow separation is an important phenomenon in the subject of bluff-body flow, since the separation dictates vortex generation and vortex shedding from the body and affects strongly the wake characteristics and hydrodynamic forces on the body. A phenomenological as well as physical understanding of the flow separation, in particular for the canonical case of a circular cylinder, is not only of fundamental importance, but also of guidance to various practical design/applications involving cylindrical structures in the fields of e.g. civil, hydraulic, mechanical, offshore, and wind engineering (Zdravkovich, 1997).

Among different bluff bodies, flow separation around a circular cylinder has received greatest attention in the literature. This is not only because circular cylinder is the most commonly-used bluff body in practice, but also because the separation point on a circular cylinder may move on the cylinder surface with time and the Reynolds number (Re), which is far more complex than some sharp-cornered bluff bodies where the separations are in favour of the sharp corners. For example, it is generally believed that for flow past a square cylinder the flow would always separate at the leading and/or trailing edges of the cylinder (e.g. Robichaux et al., 1999; Sohankar et al., 1997, 1999; Ozgoren, 2006; Bai and Alam, 2018), and therefore the flow characteristics of a square cylinder are less Re -dependent than those of a circular cylinder (Derakhshandeh and Alam, 2019).

Under the ideal condition of steady incoming flow past a nominally two-dimensional (2D) smooth circular cylinder (simply referred to as “flow past a circular cylinder” hereafter) investigated in this study, the flow is governed by a single dimensionless parameter $Re (= UD/\nu)$, which is defined based on the incoming flow velocity (U), the cylinder diameter (D) and the kinematic viscosity of the fluid (ν). The location of the separation point on the cylinder surface is represented by the separation angle (θ_s), defined as the angle from the front stagnation point of the cylinder to the separation point (see the inset in Fig. 1). If unspecified, the separation angle/point mentioned in the present study refers to the time-averaged result. In the

subcritical regime of $Re \lesssim 2 \times 10^5$ (e.g. Williamson, 1996), because the time-averaged flow is symmetric about the wake centreline, the time-averaged separation angles ($\bar{\theta}_s$) on the upper and lower sides of the cylinder are identical. It is also worth noting that in the unsteady flow regime of $Re \gtrsim 47$ (e.g. Henderson, 1997), the instantaneous separation point moves back and forth over time due to vortex shedding, e.g. over a range of $\pm 0.5^\circ$ at $Re = 50$ and $\pm 5^\circ$ at $Re = 160$ (Wu et al., 2004).

The flow separation around a circular cylinder has been studied extensively in terms of the $\bar{\theta}_s - Re$ relationship. As shown in Sumer and Fredsøe (1997), at sufficiently low Re values of smaller than 5, the creeping flow does not separate from the cylinder surface. The critical Re for the onset of flow separation has also been predicted at 6 – 7 in Wu et al. (2004) and 6.29 in Sen et al. (2009) through numerical simulations. Sen et al. (2009) showed that the flow initially separates at the rear point of the cylinder (i.e. $\bar{\theta}_s = 180^\circ$). In the laminar flow regime, Wu et al. (2004) summarised the separation angles predicted by twelve earlier experimental and numerical studies between the 1930s and 1990s, showing that in general the separation point moves upstream with increasing Re . However, the discrepancies in the separation angles predicted by different studies increase with increasing Re , reaching approximately 10° at $Re \sim 200$. Motivated by this considerable discrepancy, Wu et al. (2004) studied the separation angles for the 2D (laminar) flows up to $Re \sim 280$ through both 2D numerical simulations and 2D soap-film experiments, and proposed an empirical formula for the $\bar{\theta}_s - Re$ relationship:

$$\bar{\theta}_s = 95.7 + 267.1Re^{-1/2} - 625.9Re^{-1} + 1046.6Re^{-3/2} \quad (7 \leq Re \leq 200) \quad (1.1)$$

or alternatively a simpler formula for a slightly smaller range of Re :

$$\bar{\theta}_s = 101.5 + 155.2Re^{-1/2} \quad (10 \leq Re \leq 200) \quad (1.2)$$

Equation (1.1) shows that in the 2D laminar regime the time-averaged separation point moves upstream monotonically with increase in Re (see also Fig. 1).

Fig. 1. The $\bar{\theta}_s - Re$ relationship for flow past a circular cylinder.

For higher Re values from the onset of three-dimensionality at $Re \sim 190$ (e.g. Williamson, 1996; Posdziech and Grundmann, 2001; Jiang et al., 2017b) up to the upper bound of the subcritical regime of $Re \sim 2 \times 10^5$ (e.g. Williamson, 1996), the experimental and numerical data summarised from a number of previous studies suggest that the time-averaged separation point also moves upstream with increase in Re (Fig. 1). However, less data are available in the literature for the early three-dimensional (3D) states. Cao et al. (2010) reported the separation angles for a few Re values between 60 and 1000 (Fig. 1). Nevertheless, based on the scarce data points shown in Fig. 1, it is difficult to see if (and how) the separation characteristics would be affected by the flow transition from 2D (laminar) to 3D (turbulent) states, and if the $\bar{\theta}_s - Re$ relationship over the entire subcritical regime is monotonic and

can be described by a single formula.

In light of this knowledge gap, the present study will refine the $\overline{\theta_s} - Re$ relationship for a circular cylinder over the early 3D flow states up to $Re = 1000$ through 3D direct numerical simulations (DNS). A primary aim is to investigate the influence of the flow three-dimensionality and turbulence on the variation of the separation angle (both instantaneous and time-averaged) and to explore its underlying physical mechanism.

In addition, each of the previous studies shown in Fig. 1 generally focused on the 3D $\overline{\theta_s} - Re$ relationship over approximately one order of magnitude of Re . Unlike a unified formula proposed by Wu et al. (2004) for the 2D $\overline{\theta_s} - Re$ relationship (i.e. equation (1.1) or (1.2)), there still lacks a formula for the 3D $\overline{\theta_s} - Re$ relationship. Therefore, the second aim of this study is to seek a proper formula for the 3D $\overline{\theta_s} - Re$ relationship.

2. Numerical model

2.1. Numerical method

Direct numerical simulations have been used in this study in solving the continuity and incompressible Navier–Stokes equations:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (2.2)$$

where $(x_1, x_2, x_3) = (x, y, z)$ are Cartesian coordinates, u_i is the velocity component in the direction x_i , t is time, ρ is fluid density, p is pressure, and ν is kinematic viscosity. The DNS are carried out with the finite volume method by using the open-source code OpenFOAM (www.openfoam.org). The convection, diffusion and time derivative terms are discretized, respectively, using a fourth-order cubic scheme, a second-order

linear scheme, and a blended scheme consisting of the second-order Crank–Nicolson scheme and a first-order Euler implicit scheme. The same numerical approach has been adopted in Jiang et al. (2016, 2018a,b) for the simulations of 3D wake transition of a circular and a square cylinder.

2.2. Computational mesh

The computational mesh for a circular cylinder employs the standard mesh used in Jiang et al. (2016) for $Re \leq 300$ and the refined mesh used in Jiang and Cheng (2017) for $300 < Re \leq 1000$. Some key parameters for the standard and refined meshes are listed in Table 1. In particular, while the spanwise domain length L_z does not have to be an integer multiple of the intrinsic wavelength of the wake mode, an L_z of at least three times the intrinsic wavelength of the wake mode is required to avoid inaccurate vortex patterns induced by the restriction of L_z (Jiang et al., 2017c). Therefore, an L_z of $12D$ is used for $Re \leq 300$ to accommodate at least three spanwise periods of mode A (with a spanwise wavelength $< 4D$). Since the mode A structure disappears at $Re \sim 270$ and only the finer-scale mode B structure (with a spanwise wavelength $< 1D$) exists at higher Re values, the L_z value for $300 < Re \leq 1000$ is reduced to $6D$. Fig. 2 shows a schematic model and a close-up view of the refined mesh. Mesh convergence regarding the hydrodynamic forces on the cylinder (including the time-averaged drag coefficient, the root-mean-square lift coefficient and the Strouhal number) have been reported separately in Jiang et al. (2016) for the standard mesh at $Re = 220$ and 300 and in Jiang and Cheng (2017) for the refined mesh at $Re = 400$ and 1000 .

Fig. 2. The refined mesh for a circular cylinder for $300 < Re \leq 1000$: (a) Schematic model of the computational domain, and (b) Close-up view of the mesh near the cylinder.

Table 1. Key parameters for the computational meshes for a circular cylinder.

Parameter	Standard mesh (for $Re \leq 300$)	Refined mesh (for $300 < Re \leq 1000$)
Distance from the cylinder centre to the inlet	$20D$	$30D$
Distance from the cylinder centre to the cross-flow boundaries	$20D$	$30D$
Distance from the cylinder centre to the outlet	$30D$	$30D$
Number of cells around the cylinder surface	132	240
Height of the first layer of mesh next to the cylinder	$1 \times 10^{-3}D$	$2.315 \times 10^{-4}D$
Cell expansion ratio in the domain	≤ 1.1	≤ 1.1
Spanwise domain length for the 3D mesh	$12D$	$6D$
Spanwise cell size for the 3D mesh	$0.1D$	$0.05D$

In addition, since the present study focuses mainly on the flow separation, mesh convergence has been further demonstrated in Appendix A regarding the prediction of the location of separation. Based on Wu et al. (2004), the location of separation is determined at the position on the cylinder surface where the shear stress is zero. In the present study, the instantaneous separation angle θ_s is obtained from an instantaneous flow field, while the time-averaged separation angle $\overline{\theta}_s$ is obtained from a time-averaged flow field. For the time-periodic 2D flows, the time-averaged flow field is averaged over an exact vortex shedding period. For the irregular 3D flows, the time-averaged flow field is obtained by averaging at least 800 non-dimensional time units (defined as $t^* = tU/D$) of the fully developed flow (after discarding an initial period of at least 500 non-dimensional time units). Since the 3D flows may also be irregular along the spanwise direction, the $\overline{\theta}_s$ values reported for the present 3D cases have been averaged over both time and the cylinder span.

2.3. Boundary conditions and time step size

The boundary conditions for the computational domain shown in Fig. 2(a) are as follows. At the inlet, a uniform velocity $(U, 0, 0)$ is specified. At the outlet, the Neumann boundary condition (i.e. zero normal gradient) is applied for the velocity, while the pressure is specified as a reference value of zero. Symmetry boundary conditions are applied at the top and bottom boundaries, while periodic boundary conditions are employed at the front and back boundaries that are perpendicular to the spanwise direction. A no-slip condition is applied on the cylinder surface. The internal flow follows an impulsive start.

The time step size Δt is chosen based on a Courant–Friedrichs–Lewy (CFL) limit, where the CFL number is defined as:

$$\text{CFL} = \frac{|u| \Delta t}{\Delta l} \quad (2.3)$$

where $|u|$ is the magnitude of the velocity through a cell, and Δl is the cell size in the direction of the velocity. Generally speaking, the CFL number cannot exceed 1.0, meaning that the flow cannot travel more than one cell over one time step. In the present study, (after the very initial transients) the CFL number is kept below 0.5 for the standard mesh (for $Re \leq 300$) and below 0.4 for the refined mesh used at higher Re values (for $300 < Re \leq 1000$). For each case, a fixed time step size is used for the entire domain and throughout the time integration, e.g. the time step sizes for $Re = 300, 400$ and 1000 are 6.42×10^{-3} , 3.43×10^{-3} and 2.87×10^{-3} , respectively.

3. Numerical results

3.1. Separation in the early 3D flow states

The $\overline{\theta_s} - Re$ relationships for flow past a circular cylinder predicted by the present 2D and 3D DNS are shown in Fig. 3. The present 2D results agree well with equation (1.1). Although equation (1.1) is based on a curve fitting of the 2D numerical results in Wu et al. (2004) and is valid for $Re = 7 - 200$ (Wu et al., 2004), it is found

that an extrapolation of equation (1.1) in the extended laminar regime of $Re = 200 - 1000$ is in an excellent agreement with the present 2D results (Fig. 3). This is a surprising yet reasonable agreement because the 2D flow in the extended laminar regime (without the influence of three-dimensionality and turbulence) is expected to follow the same variation trend with Re , such that the $\overline{\theta}_s - Re$ relationship in the extended laminar regime is a smooth extension of that in the laminar regime and can be described by the same formula, i.e. equation (1.1). It is worth noting that although naturally the flow becomes 3D for $Re \gtrsim 200$, the present 2D results for $Re \gtrsim 200$ are necessary for the analysis and discussion later on, because the difference between the 3D and 2D results quantifies the effect of flow three-dimensionality.

The influence of the flow three-dimensionality on the separation angle is revealed by a comparison of the 3D results with the corresponding 2D results in the extended laminar regime (Fig. 3). While the 2D $\overline{\theta}_s - Re$ relationship is a smooth curve up to at least $Re = 1000$, the 3D $\overline{\theta}_s$ values deviate from their 2D counterparts to a certain degree and the variation trend is not monotonic. Hence after the flow transitions to 3D states the $\overline{\theta}_s - Re$ relationship can no longer be represented by a single empirical formula.

Fig. 3. The $\bar{\theta}_s - Re$ relationships for flow past a circular cylinder predicted by the present 2D and 3D DNS.

The deviation between the 2D and 3D $\bar{\theta}_s$ values (denoted as $\Delta\bar{\theta}_s$) in the early 3D flow states is quantified in Fig. 4, where $\Delta\bar{\theta}_s$ is calculated as:

$$\Delta\bar{\theta}_s (\%) = \frac{\text{The 2D } \bar{\theta}_s \text{ value} - \text{The 3D } \bar{\theta}_s \text{ value}}{\text{The 2D } \bar{\theta}_s \text{ value}} \times 100\% \quad (3.1)$$

Meanwhile, for the 3D flows the degree of flow three-dimensionality in the near wake is quantified by the time-averaged spanwise kinetic energy (E_z) and streamwise and transverse enstrophies (ε_x and ε_y), where E_z , ε_x and ε_y are defined as:

$$E_z = \frac{1}{2} \int_V \left(\frac{u_z}{U} \right)^2 dV \quad (3.2)$$

$$\varepsilon_x = \frac{1}{2} \int_V \omega_x^2 dV \quad \text{where } \omega_x = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \right) \frac{D}{U} \quad (3.3)$$

$$\varepsilon_y = \frac{1}{2} \int_V \omega_y^2 dV \quad \text{where } \omega_y = \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \right) \frac{D}{U} \quad (3.4)$$

where V is the volume of the flow field of interest, which is the near-wake region of

$x/D = 0 - 10$ and the entire lengths in the y - and z -directions for the present study. To facilitate comparison of the variation trends, the E_z , ε_x and ε_y results shown in Fig. 4 have been divided by 1.1, 30 and 15, respectively. As shown in Fig. 4, the variation in $\Delta\bar{\theta}_s$ shares similar trends to the variations in E_z , ε_x and ε_y , which suggests that the variation in $\Delta\bar{\theta}_s$ is induced by the variation in the degree of flow three-dimensionality.

Fig. 4. Comparison between the reduction in the 3D $\bar{\theta}_s$ value and the degree of flow three-dimensionality in the near wake.

The variations in $\Delta\bar{\theta}_s$ and the degree of flow three-dimensionality with Re are further correlated with the 3D wake structures. Fig. 5 shows instantaneous vorticity fields for Re in different flow regimes, where the spanwise vorticity (ω_z) is defined as:

$$\omega_z = \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) \frac{D}{U} \quad (3.5)$$

Specifically, the sudden reduction in the 3D $\bar{\theta}_s$ value at the onset of flow three-dimensionality (Fig. 3 and Fig. 4) is due to a sudden increase in the degree of flow three-dimensionality at this point, owing to the subcritical nature (i.e. wake

transition that contains a hysteresis loop) of the mode A wake instability (Henderson and Barkley, 1996). The mode A instability, which accounts for the wake transition from 2D to 3D (Fig. 5b to Fig. 5c), originates in the primary vortex cores (Williamson, 1996; Leweke and Williamson, 1998; Thompson et al., 2001). The relatively large-scale mode A flow structures (Fig. 5c) remain dominant for $Re < 253$ (Jiang et al., 2016), which results in a similar degree of flow three-dimensionality and therefore a similar level of $\Delta\overline{\theta}_s$ over this range of Re . With the subsequent instability of mode B that originates in the braid shear layer region (Williamson, 1996; Leweke and Williamson, 1998; Thompson et al., 2001), the finer-scale mode B flow structures (Fig. 5d) gradually take over the position of the mode A structures to become the dominant flow pattern beyond $Re = 253$ (Jiang et al., 2016), and a sharp drop of the degree of flow three-dimensionality is observed in Fig. 4. Hence the 3D $\overline{\theta}_s$ value shown in Fig. 3 increases towards its 2D counterpart for $Re \sim 250 - 270$, rather than continuing the general trend of a decreasing $\overline{\theta}_s$ with increasing Re . For $Re > 270$, the increasingly disordered mode B flow structures (Fig. 5d–f) (which lead to increasingly turbulent wake flow) produce an increasing degree of flow three-dimensionality with increasing Re , and therefore the 3D $\overline{\theta}_s$ values deviate increasingly from their 2D counterparts (Fig. 3). The deviation at $Re = 1000$ is $\Delta\overline{\theta}_s = 8.7\%$ (i.e. 9.0°), and this deviation is expected to grow further with further increase in Re , as the flow becomes increasingly turbulent.

Fig. 5. Instantaneous vorticity fields in the near wake of a circular cylinder for Re in different flow regimes. The translucent iso-surfaces represent spanwise vortices while the opaque iso-surfaces represent streamwise vortices. Dark grey and light yellow denote positive and negative vorticity values, respectively. The flow is from left to right past the cylinder on the left. (a) At $Re = 45$, the flow is 2D and steady, (b) at $Re = 185$, the flow is 2D and unsteady, (c) at $Re = 194$, 3D mode A flow structures (with vortex dislocations) appear, (d) at $Re = 270$, relatively ordered mode B flow structures appear, (e) at $Re = 400$, the mode B flow structures become disordered, and (f) at $Re =$

1000, the mode B flow structures become increasingly disordered.

3.2. Formula for the 3D $\bar{\theta}_s$ - Re relationship

For the 2D laminar flows up to $Re \sim 200$, Wu et al. (2004) and Sen et al. (2009) proposed empirical formulae for the $\bar{\theta}_s$ - Re relationship in the form of $\bar{\theta}_s = f(Re^{-1/2})$ (e.g. equations (1.1) and (1.2)). However, due to the influence of the flow three-dimensionality on the separation angle shown in section 3.1, the formulae proposed by Wu et al. (2004) and Sen et al. (2009) for the 2D flows cannot be extrapolated to the 3D flows. Actually, an extrapolation of equation (1.1) to $Re = 200 - 1000$ corresponds to the present 2D DNS results (Fig. 3). To the best knowledge of the author, while there have been different studies focusing on the 3D $\bar{\theta}_s$ - Re relationship over different ranges of Re (Fig. 1), there still lacks a formula for the 3D $\bar{\theta}_s$ - Re relationship.

As shown in Fig. 4, over the 3D wake transition regimes of $Re \sim 190 - 270$ the 3D $\bar{\theta}_s$ values are influenced by different wake flow patterns in a transition sequence of “2D $\rightarrow$ mode A dominance $\rightarrow$ mode B dominance $\rightarrow$ increasingly disordered mode B”. Therefore the 3D $\bar{\theta}_s$ - Re relationship shown in Fig. 3 is not monotonic and cannot be described by a single formula. For Re from approximately 270 up to at least 10000, the increasingly disordered mode B flow becomes the sole wake flow pattern (Williamson, 1996), which forms the physical basis of using a single formula to describe the 3D $\bar{\theta}_s$ - Re relationship. Based on a curve fitting of the 3D $\bar{\theta}_s$ values predicted by the present 3D DNS for $Re = 270 - 1000$ in the form of $\bar{\theta}_s = f(Re^{-1/2})$, the following 3D $\bar{\theta}_s$ - Re relationship is proposed:

$$\bar{\theta}_s = 78.8 + 505Re^{-1/2} \quad (270 \leq Re \leq 1000) \quad (3.6)$$

where the coefficient of determination R^2 is 0.9997. Furthermore, it is found that equation (3.6) can be well extrapolated up to the upper bound of the subcritical

regime of $Re \sim 10^5$, matching very well most of the data points from previous studies (Fig. 6).

Fig. 6. The $\bar{\theta}_s - Re$ relationship for flow past a circular cylinder. The 3D results for Re from 270 up to approximately 10^5 can be well represented by equation (3.6).

It is worth noting that in the range of Re between 270 (the transition to turbulence in the wake) and 2×10^5 (the transition to turbulence in the boundary layer), the relationship between a specific flow property and Re normally includes a changeover in the variation trend (from increasing to decreasing, or vice versa) at $Re \sim 1200$, for example the time-averaged drag coefficient (Zdravkovich, 1997), the time-averaged base pressure coefficient (Norberg, 1994) and the formation length (Noca et al., 1998; Norberg, 1998), owing to the transition to turbulence in the separating shear layer (Williamson, 1996). In contrast, the 3D $\bar{\theta}_s - Re$ relationship shown in Fig. 6 does

not show any change in the variation trend at $Re \sim 1200$ (where the present 3D DNS have been extended up to $Re = 1400$). The reason is that the shear-layer instability is inherently a 2D phenomenon that intensifies with both distance downstream and Re (Prasad and Williamson, 1997), which affects the wake recirculation region (hence contributes to the variations in the time-averaged drag coefficient, the time-averaged base pressure coefficient and the formation length) but may not influence the flow three-dimensionality near the cylinder.

3.3. Instantaneous separation angle of the 2D flows

Having established the $\overline{\theta_s} - Re$ relationship in sections 3.1 and 3.2, sections 3.3 and 3.4 focus on the instantaneous variation of the separation angle. For 2D unsteady flows, the instantaneous θ_s values do not vary along the cylinder span but vary in time. The variation of θ_s over a fully developed vortex shedding period (of $t/T = 0 - 1$, with T being the vortex shedding period) is shown in Fig. 7(a). For each case $t/T = 0$ is the phase when the lift coefficient reaches a maximum. An interesting feature shown in Fig. 7(a) is that for each case the time variations of the upper and lower separation angles are not exactly anti-phase. Specifically, a maximum of the upper/lower separation angle appears slightly earlier than a minimum of the lower/upper separation angle, and the variation of θ_s from a minimum to a maximum is faster than that from a maximum to a minimum. Fig. 7(b) shows the maximum and minimum gradients of variation of θ_s (calculated as $d(\theta_s)/d(t/T)$) for each case shown in Fig. 7(a), which shows more clearly that for each case the maximum gradient is larger than the minimum one.

Fig. 7. (a) Variation of the instantaneous separation angle over a fully developed vortex shedding period for the 2D flows, and (b) maximum and minimum gradients of variation of θ_s for each case shown in panel (a).

The physical mechanism for the above-mentioned feature is investigated. The case $Re = 300$ is chosen to be illustrated here because it will also be used in section 3.4 to investigate the corresponding 3D flow and to make direct comparison between the 3D and 2D results (so as to reveal the effect of flow three-dimensionality on flow separation). While the case $Re = 300$ is used here for the illustration, the general findings have been validated against all the cases shown in Fig. 7 and are deemed universal.

Fig. 7(a) shows that at $Re = 300$ the lower separation point moves upstream over $t/T = 0.33 - 0.94$ and moves downstream over $t/T = 0.94 - 1.33$ (where $t/T > 1$ refers to the next vortex shedding period), while the upper separation point moves downstream over $t/T = 0.44 - 0.83$ and moves upstream over $t/T = 0.83 - 1.44$. Fig. 8 shows the evolution of the instantaneous ω_z field over a vortex shedding period. In each panel of Fig. 8, the positive and negative ω_z contours are plotted by solid and dashed black lines, respectively, while the neutral contour $\omega_z = 0$ is highlighted by a thick blue line. Each intersection between a thick blue line (where $\omega_z = 0$) and the cylinder surface is a point that separates clockwise and anti-clockwise fluid rotations on the cylinder surface. Hence in Fig. 8 the intersection nearest to the front stagnation point of the cylinder marks the upper/lower separation point. It has been validated that the separation points determined in Fig. 8 are consistent with those determined by the method of a zero shear stress location.

As shown in Fig. 8(e–j), the upstream movement of the lower separation point over $t/T = 0.33 - 0.94$ is due to the gradual development and downward movement of shear layer A at the back of the cylinder (which is induced by the instantaneous wake recirculation region attached at the back of the cylinder), pushing the contour $\omega_z = 0$ indicated with an arrow in Fig. 8(h) downward and thus the separation point upstream. On the other hand, the downstream movement of the lower separation point over $t/T = 0.94 - 1.33$ is due to the gradual roll-up of the lower separating shear layer (Fig. 8a–d), pushing the contour $\omega_z = 0$ indicated with an arrow in Fig. 8(c) upward and thus the separation point downstream. The critical condition at $t/T = 0.94$ is a balance between (i) the fact that the strength of shear layer A decreases significantly at $t/T \gtrsim 0.9$ (Fig. 8j) and eventually shear layer A is absorbed by vortex A at $t/T \sim 1.1$ (Fig. 8b), and (ii) the beginning of the roll-up of the lower separating shear layer. The critical condition at $t/T = 1.33$ (or 0.33) is a balance between (i) the further roll-up of the lower separating shear layer, and (ii) the emergence and development of shear layer A (Fig. 8e). In short, the time variation of the lower separation point is governed by a dynamic balance between shear layer A and the lower separating shear layer. Owing to the spatio-temporal symmetry of the flow, the time variation of the upper separation

point is governed by a similar dynamic balance between shear layer B (as indicated in Fig. 8(j)) and the upper separating shear layer. The phase difference between the maximum of the upper separation angle at $t/T = 0.83$ and the minimum of the lower separation angle at $t/T = 0.94$ (Fig. 7a) is due to the phase difference between the upward movement of shear layer B and the roll-up of the lower separating shear layer. The shear layers A and B at the back of the cylinder play an important role in shaping the time variation of the separation point.

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Fig. 8. Evolution of the instantaneous spanwise vorticity field over a vortex shedding period (from panels (a) to (j) with an interval of $0.1T$) for the 2D flow at $Re = 300$. The positive and negative ω_z contours are plotted by solid and dashed black lines, respectively. The neutral contour $\omega_z = 0$ is highlighted by a thick blue line.

3.4. Instantaneous separation angle of the 3D flows

For the 3D flows, the instantaneous variations of the separation angle over time and along the cylinder span are investigated separately. The separation angle averaged over time (but not along the span) is denoted as $\overline{\theta_{s,time}}$, while that averaged along the span (but not over time) is denoted as $\overline{\theta_{s,span}}$.

The instantaneous variation of the separation angle is illustrated with a typical case at $Re = 300$. Fig. 9(a,b) shows the variations of the instantaneous upper and lower separation angles along the span (of $z/D = 0 - 12$) and over a typical vortex shedding cycle of the fully developed flow (of $t/T = 0 - 1$). Different from the 2D flows where the θ_s values are the same along the span, for 3D flows the θ_s values may vary along the span, in accordance with the variation in the degree of flow three-dimensionality along the span. For example, in Fig. 9(a) the largest variation range of the θ_s values along the span (at $t/T = 0.8$) can be up to 8.19° . The variation of θ_s along the span is random, yet over a long statistical time period of 1050 non-dimensional time units, the $\overline{\theta_{s,time}}$ values along the span become quite consistent, being within $\pm 0.1^\circ$ of the $\overline{\theta_s}$ value of 107.92° (Fig. 9c). With a sufficiently long statistical time period, the $\overline{\theta_{s,time}}$ value at each and every spanwise location is expected to converge to the $\overline{\theta_s}$ value.

Fig. 9. Variation of the separation angle along the cylinder span for $Re = 300$: (a) variation of the instantaneous upper separation angle, (b) variation of the instantaneous lower separation angle, and (c) variation of the time-averaged upper and lower separation angles.

Different from the randomness of the variation of θ_s along the span, the variation of θ_s over time is cyclic, in accordance with the vortex shedding cycles. Over the

vortex shedding cycle shown in Fig. 9(a,b), the upper separation angle shown in Fig. 9(a) and the lower separation angle shown in Fig. 9(b) are largely in an anti-phase manner, which is in accordance with the alternate vortex shedding from the upper and lower sides of the cylinder. The largely anti-phase variations of the upper and lower separation angles become more evident when the θ_s values shown in Fig. 9(a,b) have been span-averaged in Fig. 10(a). Nevertheless, similar to the 2D flow discussed in section 3.3, for the 3D flow the time variations of the upper and lower separation angles are also not exactly anti-phase, with the maximum of the upper/lower separation angle being slightly earlier than the minimum of the lower/upper separation angle.

Fig. 10. Variation of the span-averaged separation angle over time for $Re = 300$: (a) for the vortex shedding cycle investigated in Fig. 9(a,b), and (b) for a period of 60 vortex shedding cycles, including the first vortex shedding cycle enlarged in panel (a).

It is noticed in Fig. 10(a) that the cyclic variation trend of the 3D $\overline{\theta_{s,span}}$ values is not inhibited by the spanwise randomness of θ_s . Moreover, the variations of the 3D $\overline{\theta_{s,span}}$ values over time are quite smooth, which is because the evolution of the randomness of θ_s along the span due to the flow three-dimensionality (specifically due to the development of streamwise vortices of mode B at various spanwise locations with varying strengths over time) is a relatively slow process. As shown in Fig. 9(a,b), similar patterns of spanwise randomness may persist for at least $0.5T$ (i.e. the duration of vortex shedding from one side of the cylinder). Fig. 10(b) shows the variation of $\overline{\theta_{s,span}}$ over a time period of 60 vortex shedding cycles (after discarding an initial period of 1400 non-dimensional time units, which corresponds to ~ 285 shedding cycles), including the first cycle shown in Fig. 10(a). Due to the gradual variation of the spanwise randomness of θ_s over the vortex shedding cycles, the variation of the 3D $\overline{\theta_{s,span}}$ values is not exactly time-periodic.

To investigate how the instantaneous separation angle is influenced by flow three-dimensionality, the instantaneous separation angles over a vortex shedding cycle calculated through 2D DNS are also plotted in Fig. 10(a), and the variation range of the 2D θ_s values is marked in Fig. 10(a,b) with horizontal dashed lines. To facilitate comparison of the 2D and 3D cases shown in Fig. 10(a), for both cases $t/T = 0$ is the phase when the lift coefficient reaches a maximum. A comparison of the 2D and 3D results in Fig. 10(a,b) shows that the minima of the 3D $\overline{\theta_{s,span}}$ values (with an average of 102.92° over the 60 cycles) are relatively close to their 2D counterpart (of 102.26°), while the maxima of the 3D $\overline{\theta_{s,span}}$ values (with an average of 113.19° over the 60 cycles) decrease considerably from their 2D counterpart (of 119.33°). This uneven reduction of the 3D variation range from its 2D counterpart is the reason for the reduction of the 3D $\overline{\theta_s}$ value from its 2D counterpart shown in Fig. 3.

The physical mechanism for the above-mentioned uneven reduction is explained

with the time evolution of the span-averaged upper separation angle over the vortex shedding cycle shown in Fig. 10(a). As shown in Fig. 10(a), the deviation of the span-averaged upper separation angles calculated in 3D and 2D is built up by a difference in the gradient of variation in $\overline{\theta_{s,span}}$ between the 3D and 2D cases over $t/T \sim 0.5 - 0.8$ (while those over $t/T \sim 0.45 - 0.5$ are still very similar). As discussed in section 3.3, the time variation of the upper separation angle is governed by a dynamic balance between shear layer B (as indicated in Fig. 8(j)) and the upper separating shear layer. For the time range of $t/T \sim 0.5 - 0.8$, shear layer B has not yet developed (Fig. 8f-i), such that the deviation of the 3D and 2D results is primarily affected by the upper separating shear layer. For the 3D flow, the disordered mode B flow structures developed in the separating shear layer would result in turbulent energy dissipation and therefore the separating shear layer is weaker and less rolled-up in the cross-flow direction (i.e. the y -direction) (Fig. 11(b) versus Fig. 11(a)). Norberg (2001) also suggested that “the increase in secondary (essentially streamwise) circulation occurs probably at the expense of the primary (essentially spanwise) circulation associated with the roll-up of the von Kármán vortices”. Since the roll-up of the separating shear layer has an effect of pushing the contour $\omega_z = 0$ indicated with an arrow in Fig. 11 downward and thus the separation point downstream (as discussed in section 3.3), the less roll-up of the separating shear layer for the 3D flow results in a separation point less downstream (i.e. a smaller $\overline{\theta_{s,span}}$) than its 2D flow counterpart.

Fig. 11. Instantaneous spanwise vorticity field at $t/T = 0.8$ for $Re = 300$: (a) the 2D flow, and (b) the span-averaged 3D flow. The positive and negative ω_z contours are plotted by solid and dashed black lines, respectively. The neutral contour $\omega_z = 0$ is highlighted by a thick blue line.

Fig. 12 shows the variation ranges of the instantaneous separation angles for the 2D and 3D flows at various Re values, together with the $\overline{\theta_s}$ values reproduced from Fig. 3. The results of the 2D flows agree well with those reported by Wu et al. (2004). For the irregular 3D flows, the maximum/minimum of the instantaneous separation angles shown in Fig. 12 is an average of the maxima/minima of $\overline{\theta_{s,span}}$ over 60 vortex shedding cycles (see e.g. Fig. 10b). Similar to the 3D flow at $Re = 300$ discussed in the previous paragraph, for $Re = 400 - 1000$ the increasingly disordered mode B flow structures developed in the separating shear layer result in increasingly less roll-up of the separating shear layer in the cross-flow direction, such that the separation point is increasingly less pushed downstream (compared with its 2D flow counterpart), as shown by the average of the maxima of $\overline{\theta_{s,span}}$ in Fig. 12. On the other hand, the average of the minima of $\overline{\theta_{s,span}}$ shown in Fig. 12 is only slightly influenced by the 3D effect. As discussed earlier in this section with the example of $Re = 300$ and further shown in Fig. 12 with the results of Re up to 1000, such an uneven reduction of the 3D variation range from its 2D counterpart is the reason for the reduction of the 3D $\overline{\theta_s}$ value from the corresponding 2D one.

Fig. 12. Variation ranges of the instantaneous separation angles for the 2D and 3D flows.

Another interesting feature shown in Fig. 12 is that for the increasingly turbulent 3D flows the variation range of the instantaneous separation angle (shaded in dark grey in Fig. 12) reduces with increasing Re , which is in contrast to that of the 2D (laminar) flows (shaded in light grey in Fig. 12). For the 2D/3D flows, the increase/decrease in the variation range with increasing Re is mainly induced by the increase/decrease in strength of the separating shear layer and hence the cross-flow roll-up of the separating shear layer with increasing Re . While the present study focuses primarily on the early 3D flow states up to $Re = 1000$, time histories of the instantaneous separation angle at a much higher Re of 7.4×10^4 are available from the experimental study by Capone et al. (2015), which enables a similar analysis at this much higher Re . Capone et al. (2015) plotted the time histories of the instantaneous separation angle at $Re = 7.4 \times 10^4$ at two spanwise locations over approximately 45 vortex shedding cycles. The averages of the maxima and minima of θ_s for the first time history are 78.1° and 75.0° , respectively, while those for the second time history are 78.2° and 75.1° , respectively, which suggest that the variation range of the instantaneous separation angle remains small (of approximately 3.1°) at $Re = 7.4 \times 10^4$. In addition, Capone et al. (2015) showed the probability density functions of θ_s for Re

$= 4.45 \times 10^4 - 1.87 \times 10^5$ and also found a tendency toward reduced level of variation of θ_s with increasing Re .

Having examined the instantaneous separation angles of the 3D flows, it is worthwhile to note that the influence of flow three-dimensionality on the separation angle may be stronger than that indicated by the $\Delta \overline{\theta}_s$ values shown in Fig. 4. For example, at $Re = 300$ the flow three-dimensionality contributes to a reduction in $\overline{\theta}_s$ of only 1.39° (Fig. 4), which seems to suggest that the separation angle is insensitive to the 3D effect, as was commented by Wu et al. (2004). However, the 3D effect may be strong at some particular phases. For example, at $t/T = 0.8$ the 3D $\overline{\theta}_{s,span}$ value of the upper separation shown in Fig. 10(a) is 6.62° smaller than its 2D counterpart. In addition, at $t/T = 0.8$ the instantaneous upper separation angles along the span have a variation range of 8.19° (Fig. 9a), with the smallest value being 11.59° smaller than the 2D counterpart.

4. Conclusions

This study examines the instantaneous and time-averaged flow separation angles for the canonical case of flow past a circular cylinder in the subcritical regime. It is found that the instantaneous variation of the upper/lower separation angle is governed by a dynamic balance between the upper/lower separating shear layer and a shear layer generated at the back of the cylinder due to wake recirculation. Due to the phase difference between the development of the shear layer at the back of the cylinder (slightly earlier) and the roll-up of the separating shear layer of the same sign of vorticity (slightly later), the instantaneous variations of the upper and lower separation angles are not exactly anti-phase.

Based on a comparison of the present 2D and 3D DNS results for $Re = 190 - 1000$, it is found that the reduction in the 3D $\overline{\theta}_s$ value from its 2D counterpart is correlated with the degree of flow three-dimensionality. Fundamentally, the reduction in the 3D $\overline{\theta}_s$ value comes from a reduction in the local maxima of θ_s , induced by the

separating shear layer. For the 3D flow, the disordered mode B flow structures developed in the separating shear layer (which is directly responsible for the flow three-dimensionality) would result in turbulent energy dissipation and therefore the separating shear layer is weaker and less rolled-up in the cross-flow direction, which weakens the effect of the separating shear layer in pushing the separation point downstream (i.e. a larger θ_s). As the mode B flow structures become increasingly disordered and the wake flow becomes increasingly turbulent with increasing Re , this 3D influence becomes stronger. Hence for the 3D flows the variation range of θ_s decreases with increasing Re , which is in contrast to that of the 2D (laminar) flows.

Over the 3D wake transition regimes of $Re = 190 - 270$, there is a transition sequence of different wake flow patterns with different degrees of flow three-dimensionality, such that the 3D $\bar{\theta}_s - Re$ relationship is not monotonic and cannot be described by a single empirical formula. For $Re \geq 270$, on the physical basis that the increasingly disordered mode B flow becomes the sole wake flow pattern, an empirical formula is proposed for the 3D $\bar{\theta}_s - Re$ relationship, i.e.

$$\bar{\theta}_s = 78.8 + 505Re^{-1/2} \quad (270 \leq Re \lesssim 10^5).$$

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Appendix A. Mesh convergence

The mesh convergence regarding the location of separation is examined as follows, where the numerical accuracy for the prediction of $\bar{\theta}_s$ is demonstrated to be in the order of magnitude of 0.1° . First, the 2D standard and refined meshes in the x - y plane (i.e. the plane perpendicular to the spanwise direction) are examined through 2D

DNS, and the $\overline{\theta}_s$ values predicted by different meshes are summarised below:

- (i) At $Re = 1000$, the refined mesh produces $\overline{\theta}_s = 103.64^\circ$.
- (ii) Based on the reference case in (i), two variation cases are examined. The first case with doubled numbers of cells in both the x - and y -directions (and a total number of cells four times that of the refined mesh) produces $\overline{\theta}_s = 103.77^\circ$, while the second case with a doubled computational domain size (from $60D \times 60D$ to $120D \times 120D$) produces $\overline{\theta}_s = 103.72^\circ$. The $\overline{\theta}_s$ values predicted by the two variation cases are within 0.2° of that predicted by the reference case in (i), which suggests that the refined mesh is adequate for $Re \leq 1000$.
- (iii) At $Re = 300$, the $\overline{\theta}_s$ values predicted by the refined mesh and the standard mesh are 109.31° and 109.24° , respectively. This close agreement suggests that the standard mesh (a mesh coarser than the refined mesh so as to reduce the computational cost) is adequate for $Re \leq 300$.

Having established the adequacy of the 2D meshes in the x - y plane, the 3D meshes will be examined regarding the adequacy of the mesh resolution and domain size in the spanwise direction. The following 3D cases are examined:

- (i) At $Re = 1000$, the refined mesh produces $\overline{\theta}_s = 94.89^\circ$.
- (ii) Based on the reference case in (i), two variation cases are examined. The first case with an increased spanwise domain size from $6D$ to $12D$ produces $\overline{\theta}_s = 94.79^\circ$, while the second case with a refined spanwise cell size from $0.05D$ to $0.03D$ produces $\overline{\theta}_s = 94.62^\circ$. The $\overline{\theta}_s$ values predicted by the two variation cases are within 0.3° of that predicted by the reference case in (i), which suggests that the refined mesh is adequate for $300 < Re \leq 1000$. In addition, for $Re = 1000$ this study uses the results calculated with the spanwise cell size of $0.03D$ so as to achieve the best available accuracy, since data are available through the mesh dependence

study.

- (iii) For $Re \leq 300$, due to the appearance of the relatively large-scale mode A flow structures with a spanwise wavelength of approximately $3 - 4D$ (Williamson, 1996), a spanwise domain size of $12D$ is adopted for the standard mesh to accommodate at least three spanwise periods of mode A in the domain (Jiang et al., 2017c). At $Re = 300$, the standard mesh produces $\bar{\theta}_s = 107.92^\circ$.
- (iv) Based on the reference case in (iii), two variation cases are examined. The first case with an increased spanwise domain size from $12D$ to $24D$ produces $\bar{\theta}_s = 108.01^\circ$, while the second case with a refined spanwise cell size from $0.1D$ to $0.05D$ produces $\bar{\theta}_s = 107.84^\circ$. The $\bar{\theta}_s$ values predicted by the two variation cases are within 0.1° of that predicted by the reference case in (iii), which suggests that the standard mesh is adequate for $Re \leq 300$.

References

- Achenbach, E., 1968. Distribution of local pressure and skin friction around a circular cylinder in cross-flow up to $Re = 5 \times 10^6$. *Journal of Fluid Mechanics* 34, 625–639.
- Bai, H., Alam, M.M., 2018. Dependence of square cylinder wake on Reynolds number. *Physics of Fluids* 30, 015102.
- Ballengee, D. & Chen, C. F. 1971 Experimental determination of the separation point of flow around a circular cylinder. *Flow, its Measurement and Control in Science and Industry*, vol. 1, Proc. ASME Fluids Engineering Conference on Flow Characteristics (ed. R. B. Dowell), pp. 419–427. Instrument Society of America, Pittsburgh.
- Breuer, M., 1998. Large eddy simulation of the subcritical flow past a circular cylinder: numerical and modeling aspects. *International Journal for Numerical Methods in Fluids*, 28, 1281–1302.
- Cao, S., Ozono, S., Tamura, Y., Ge, Y., Kikugawa, H., 2010. Numerical simulation of Reynolds

- number effects on velocity shear flow around a circular cylinder. *Journal of Fluids and Structures* 26, 685–702.
- Capone, A., Klein, C., Di Felice, F., Beifuss, U., Miozzi, M., 2015. Fast-response underwater TSP investigation of subcritical instabilities of a cylinder in crossflow. *Experiments in Fluids* 56:196.
- Cheng, W., Pullin, D. I., Samtaney, R., Zhang, W., Gao, W. 2017. Large-eddy simulation of flow over a cylinder with Re_D from 3.9×10^3 to 8.5×10^5 : a skin-friction perspective. *Journal of Fluid Mechanics* 820, 121–158.
- Derakhshandeh, J.F., Alam, M.M., 2019. A review of bluff body wakes. *Ocean Engineering* 182, 475–488.
- Franke, J., Frank, W., 2002. Large eddy simulation of the flow past a circular cylinder at $Re_D = 3900$. *Journal of Wind Engineering and Industrial Aerodynamics* 90, 1191–1206.
- Henderson, R.D., Barkley, D., 1996. Secondary instability in the wake of a circular cylinder. *Physics of Fluids* 8, 1683–1685.
- Henderson, R.D., 1997. Nonlinear dynamics and pattern formation in turbulent wake transition. *Journal of Fluid Mechanics* 352, 65–112.
- Jiang, H., Cheng, L., Draper, S., An, H., Tong, F., 2016. Three-dimensional direct numerical simulation of wake transitions of a circular cylinder. *Journal of Fluid Mechanics* 801, 353–391.
- Jiang, H., Cheng, L., 2017. Strouhal–Reynolds number relationship for flow past a circular cylinder. *Journal of Fluid Mechanics* 832, 170–188.
- Jiang, H., Cheng, L., Draper, S., An, H., 2017a. Two- and three-dimensional instabilities in the wake of a circular cylinder near a moving wall. *Journal of Fluid Mechanics* 812, 435–462.
- Jiang, H., Cheng, L., Draper, S., An, H., 2017b. Prediction of the secondary wake instability of a circular cylinder with direct numerical simulation. *Computers and Fluids* 149, 172–180.
- Jiang, H., Cheng, L., An, H., 2017c. On numerical aspects of simulating flow past a circular cylinder. *International Journal for Numerical Methods in Fluids*, 85, 113–132.
- Jiang, H., Cheng, L., An, H., 2018a. Three-dimensional wake transition of a square cylinder. *Journal of Fluid Mechanics* 842, 102–127.
- Jiang, H., Cheng, L., 2018b. Hydrodynamic characteristics of flow past a square cylinder at

- moderate Reynolds numbers. *Physics of Fluids* 30, 104107.
- Leweke, T., Williamson, C.H.K., 1998. Three-dimensional instabilities in wake transition. *European Journal of Mechanics - B/Fluids* 17, 571–586.
- Noca, F., Park, H., Gharib, M., 1998. Vortex formation length of a circular cylinder ($300 < Re < 4000$) using DPIV. *Proceedings on Bluff Body Wakes and Vortex-Induced Vibration*, ASME Fluids Engineering Division, Washington, DC.
- Norberg, C., 1994. An experimental investigation of the flow around a circular cylinder: influence of aspect ratio. *Journal of Fluid Mechanics* 258, 287–316.
- Norberg, C., 1998. LDV-measurements in the near wake of a circular cylinder. *Advances in Understanding of Bluff Body Wakes and Vortex-Induced Vibration: Proceedings of the 1998 Conference*, Washington, DC.
- Norberg, C., 2001. Flow around a circular cylinder: Aspect of fluctuating lift. *Journal of Fluids and Structures* 15, 459–469.
- Ozgoren, M., 2006. Flow structure in the downstream of square and circular cylinders. *Flow Measurement and Instrumentation* 17, 225–235.
- Parnaudeau, P., Carlier, J., Heitz, D., Lamballais, E., 2008. Experimental and numerical studies of the flow over a circular cylinder at Reynolds number 3900. *Physics of Fluids* 20, 085101.
- Posdziech, O., Grundmann, R., 2001. Numerical simulation of the flow around an infinitely long circular cylinder in the transition regime. *Theoretical and Computational Fluid Dynamics* 15, 121–141.
- Prasad, A., Williamson, C.H.K., 1997. The instability of the shear layer separating from a bluff body. *Journal of Fluid Mechanics* 333, 375–402.
- Robichaux, J., Balachandar, S., Vanka, S.P., 1999. Three-dimensional Floquet instability of the wake of square cylinder. *Physics of Fluids* 11, 560–578.
- Sadeh, W.Z., Saharon, D.B., 1982. Turbulence effect on crossflow around a circular cylinder at subcritical Reynolds numbers. *NASA Contractor Report 3622*, National Aeronautics and Space Administration (NASA).
- Sen, S., Mittal, S., Biswas, G., 2009. Steady separated flow past a circular cylinder at low Reynolds numbers. *Journal of Fluid Mechanics* 620, 89–119.
- Sohankar, A., Norberg, C., Davidson, L., 1997. Numerical simulation of unsteady low-Reynolds

- number flow around rectangular cylinders at incidence. *Journal of Wind Engineering and Industrial Aerodynamics* 69–71, 189–201.
- Sohankar, A., Norberg, C., Davidson, L., 1999. Simulation of three-dimensional flow around a square cylinder at moderate Reynolds numbers. *Physics of Fluids* 11, 288–306.
- Son, J.S., Hanratty, T.J., 1969. Velocity gradients at the wall for flow around a cylinder at Reynolds numbers from 5×10^3 to 10^5 . *Journal of Fluid Mechanics* 35, 353–368.
- Sumer, B.M., Fredsøe, J., 1997. *Hydrodynamics around cylindrical structures*. Singapore: World Scientific. (page 2)
- Thompson, M.C., Hourigan, K., 2005. The shear-layer instability of a circular cylinder wake. *Physics of Fluids* 17, 021702.
- Thompson, M.C., Leweke, T., Williamson, C.H.K., 2001. The physical mechanism of transition in bluff body wakes. *Journal of Fluids and Structures* 15, 607–616.
- Wu, M.H., Wen, C.Y., Yen, R.H., Weng, M.C., Wang, A.B., 2004. Experimental and numerical study of the separation angle for flow around a circular cylinder at low Reynolds number. *Journal of Fluid Mechanics* 515, 233–260.
- Williamson, C.H.K., 1996. Vortex dynamics in the cylinder wake. *Annual Review of Fluid Mechanics* 28, 477–539.
- Wissink, J.G., Rodi, W., 2008. Numerical study of the near wake of a circular cylinder. *International Journal of Heat and Fluid Flow* 29, 1060–1070.
- Zdravkovich, M. M., 1997. *Flow around circular cylinders, vol 1: Fundamentals*. Oxford University Press. (page 17)