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Standing spin-wave mode structure and linewidth in partially disordered hexagonal arrays of perpendicularly magnetized sub-micron Permalloy discs

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Standing spin wave mode frequencies and linewidths in partially disordered perpendicular magnetized arrays of sub-micron Permalloy discs are measured using broadband ferromagnetic resonance and compared to analytical results from a single, isolated disc. The measured mode structure qualitatively reproduces the structure expected from the theory. Fitted demagnetizing parameters decrease with increasing array disorder. The frequency difference between the first and second radial modes is found to be higher in the measured array systems than predicted by theory for an isolated disc. The relative frequencies between successive spin wave modes are unaffected by reduction of the long-range ordering of discs in the array. An increase in standing spin wave resonance linewidth at low applied magnetic fields is observed and grows more severe with increased array disorder. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4895984>]

I. INTRODUCTION

Understanding the high-frequency dynamics of sub-micron diameter, nanometer thickness magnetic discs is important for potential applications in data storage^{1–5} and spintronics^{6,7} technologies. There have been a number of recent studies concerned with spin wave mode structure^{8–20} and linewidths^{21–25} in the in-plane magnetized configuration of such dipole coupled arrays. There have also been studies conducted for disc arrays in the perpendicularly magnetized state, with characterization achieved by cavity ferromagnetic resonance^{26,27} (FMR) and magnetic resonance force microscopy.²⁸ More recently, broadband FMR has been used to study highly ordered arrays of micrometer diameter, nanometer thickness discs²⁹ and nanometer diameter antidots.³⁰ The large range of resonant frequencies and fields available by the broadband FMR technique has not yet been utilised to study the mode structure and linewidth in closely packed arrays of dipole-coupled magnetic nanostructures in the perpendicularly saturated state.

In this study, broadband FMR was used to study the perpendicularly magnetized spin wave mode structure and linewidth of a series of four disc array samples with varying degrees of array ordering, over a wide range of excitation frequencies. These dipole-coupled arrays have been previously studied in the tangentially magnetized state.²⁴ Each of the four array samples consisted of a locally hexagonal array of Permalloy discs, with each array distinguished from the others by a different degree of long-range ordering.

This paper is arranged as follows. The established theory for the standing wave mode spectra in dipole-uncoupled magnetic discs is presented for reference in Sec. II. The experimental details, including fabrication details, sample characteristics, and FMR experimental methods, are described in Sec. III. FMR mode frequency and linewidth measurement

results are described in Sec. IV. These experimental results are compared with results expected from theory for isolated discs in Sec. V and interpreted in terms of magnetic static and dynamic coupling between elements within the array. Final remarks are given in Sec. VI.

II. THEORY OF MODE STRUCTURE IN PERPENDICULARLY MAGNETIZED DISCS

The theory for FMR modes in isolated, perpendicularly magnetized discs was first applied to sub-micron discs by Kakazei *et al.* in Ref. 26 and is based on the dipole-exchange theory of spin wave spectra in unrestricted in-plane magnetic films^{31,32} and a method of the calculation of demagnetizing fields in nonellipsoidal bodies.³³ This model does not take into account the interaction between elements in a closely packed array and therefore does not take into account the effects of disorder in that array.

In this theory, the finite radius R of the disc is considered to allow only discrete values of the in-plane wave vector: $k \rightarrow k_m$, $m = 1, 2, 3, \dots$. The nonellipsoidal geometry of the disc means that the demagnetizing field inside the dot is inhomogeneous, with the internal bias field becoming a function of the disc radius ρ . The strong dipolar pinning at the disc edges produces dipolar eigenmodes with zeroth-order Bessel function profiles:

$$\mu_m(\rho) = J_0(k_m \rho), \quad (1)$$

with $k_m = \beta_m/R$, for β_m the roots of the zeroth order Bessel function and R the radius of the disc.

The standing mode profiles of the first five modes of such a disc geometry are shown in Figure 1. These modes are cylindrically symmetric. These profiles have been confirmed experimentally by Mewes *et al.* in square arrays of sub-micron diameter discs using MRFM.²⁸ Assuming that there is no significant out-of-plane film anisotropy, the frequencies of these modes are given by

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FIG. 1. Plot of the theoretical spin wave eigenmodes $\mu_m(\rho)$ vs disc radius ρ for the first five modes for a perfect, isolated disc, as calculated with Eq. (1). The amplitudes of the depicted eigenmodes do not reflect expected experimental amplitudes.

$$\omega_m^2 = \gamma^2 \left(H - 4\pi M_S N_m + \frac{2Ak_m^2}{M_S} \right) \times \left(H - 4\pi M_S N_m + \frac{2Ak_m^2}{M_S} + 4\pi M_S \left(1 - \frac{1 - e^{-k_m L}}{k_m L} \right) \right). \quad (2)$$

In Eq. (2), k_m is the in-plane wave vector, order $m = 1, 2, 3, \dots$, γ the gyromagnetic ratio, H the applied magnetic field, M_S the saturation magnetization, A the exchange stiffness constant, and L the disc thickness. The demagnetizing factor of mode m , N_m , is determined as in Ref. 34:

$$N_m = \frac{1}{A_m} \int_0^R N(\rho) J_0(k_m \rho)^2 \rho d\rho, \quad (3)$$

with:

$$A_m = \frac{R^2 J_1(k_m R)^2}{2}, \quad (4)$$

and the radius-dependent demagnetizing factor defined as in Ref. 33:

$$N(\rho) = \frac{R}{2} \int_0^\infty J_0(t\rho) J_1(tR) (e^{-tz} + e^{-t(L-z)}) dt. \quad (5)$$

or, averaged over the thickness:³³

$$N(\rho) = -\frac{R}{L} \int_0^\infty J_0(t\rho) J_1(tR) \frac{e^{-Lt} - 1}{t} dt. \quad (6)$$

III. EXPERIMENT

The fabrication process for the large area (40 mm²) ordered Permalloy disc array samples used in this study has also been described elsewhere.²⁴ Three continuous Ni₈₁Fe₁₉ films, thickness 27 ± 3 nm were deposited by radio-frequency sputtering. These films were denoted f1, f2, and f3. A hexagonal mask of carboxylate-terminated polystyrene microspheres of diameter 780 nm was deposited on each of the films via water-surface self-assembly technique. One section of each parent film was removed prior to patterning to

FIG. 2. AFM image representative of the microscale character of the disc arrays; the high contrast regions in the centre of the discs indicate that some of the polystyrene mask units remain. This particular image is a $5 \times 5 \mu\text{m}$ image of sample f3b. Reprinted from J. Appl. Phys. **109**, 013906 (2011). Copyright 2011 American Institute of Physics.

provide references of the saturation magnetization for each film: these samples were denoted f1c, f2c, and f3c. The saturation magnetization was measured via FMR. Sections of films to be patterned were denoted f1a (parent film f1), f2a (parent film f2), f3a, and f3b (parent film f3). Films to be patterned were reactively ion etched in an oxygen atmosphere of 100 mTorr, at a power areal density of 0.5 W m^{-2} to reduce the diameter of the masking microspheres. The films were then patterned via argon milling through the gaps between mask units at a pressure 25 mTorr and power areal density of 4.25 W m^{-2} . The resulting patterned samples consisted of large areas of physically isolated, 27 ± 3 nm thick, ~ 700 nm diameter Permalloy discs organised in hexagonal arrays.

The diameter of the discs was verified with atomic force microscopy: an example image is shown in Figure 2. The degree of long-range ordering of the array in each sample was measured via scanning electron microscopy (SEM). An example SEM image is shown in Figure 3. A montage of 4 evenly spaced $3600\times$ magnification SEM images was recorded over each 1 mm section of each sample, running from one edge to the other through the mid-line of the long axis of the sample. The 2-D Fourier transform of this composite image was calculated and used to measure ϕ' , the average variation in array lattice angle per millimeter. Each of the four samples was confirmed to have a different degree of long range ordering.

The result of the fabrication process was three continuous reference films (f1c, f2c, f3c) and four samples (f1a, f2a, f3a, and f3b) consisting of hexagonal arrays of 27 ± 3 nm Permalloy discs, diameter ~ 700 nm, with a disc-to-disc separation of 780 nm, as defined by the diameter of the original polystyrene mask units. The AFM images showed the discs had smooth boundaries, and there was no significant evidence in previously reported SQUID-measured in-plane hysteresis loops of structures which do not support compensated magnetic vortices at remanence.²⁴ With the exception of small differences in saturation magnetizations M_S and parent

FIG. 3. An example SEM image with inset 2-D Fourier transform of a montage of 4 evenly spaced images taken over a one millimeter section of the sample: these particular images of sample f3b. Note that the SEM image shown was not among the images used to produce the composite FFT. Images similar to the inset from each 1 mm section of the sample were used to calculate the average variation in lattice angle per unit length, ϕ' . Reprinted from J. Appl. Phys. **109**, 013906 (2011). Copyright 2011 American Institute of Physics.

film linewidths ΔH , the only parameter known to vary significantly between the arrays was the degree of long-range array ordering, ϕ' . The structural characteristics of each of the samples, including the long range ordering parameter ϕ' are listed in Table I. A diagrammatic representation of the sample geometry is shown in Figure 4 for clarity.

The samples were placed face-down on a 0.2032 mm microstrip waveguide connected to a two-port vector network analyzer and magnetized perpendicular-to-plane with respect to the substrate. The broadband FMR measurement was performed at frequencies between 6 and 17 GHz, in intervals of one GHz. The microwave transmission parameter S_{21} was measured as the applied magnetic field H was swept through the experimentally available range, in analogy to the cavity FMR experiment. Negligible reflections allowed S_{11} to be ignored.³⁵ An example of the spin wave spectra obtained is shown in Figure 5. This spectrum of modes is in qualitative agreement with those measured previously^{26–29} and arises directly from the cylindrical symmetry of the discs in the axially magnetized configuration, as shown in the previous section. The largest peak at highest field corresponds to the $m = 1$ mode, with each successive mode at a lower magnetic field H . The decrease of

TABLE I. Table showing average disc diameter, d , array angular variation per unit length ϕ' , and saturation magnetization $4\pi M_S$ for the samples used in this study. All discs and films had thickness 27 ± 3 nm.

Film	d (nm)	ϕ' [° mm ⁻¹]	$4\pi M_S$ (kOe)
f3c	—	—	8.49
f3b	695 ± 28	6.0 ± 0.8	—
f3a	703 ± 37	9.4 ± 1.1	—
f2c	—	—	8.69
f2a	697 ± 31	11.3 ± 1.7	—
f1c	—	—	8.85
f1a	699 ± 28	19.9 ± 2.1	—

FIG. 4. Diagram of the sample geometry showing the diameter, thickness, pitch, and variation in long range order of the disc arrays ϕ' away from perfect hexagonal order. The applied magnetic field H and the precessing magnetization M are also represented, with the deviation of M from H highly exaggerated. The diagram is not to scale. ϕ' is measured in degrees per millimeter.

amplitudes with increasing m is in good agreement with the simple idea that the FMR absorption amplitude scales as the mean value of the mode amplitude (Figure 1) over the nano-element area.

IV. RESULTS

The range of applied magnetic fields available for the field sweep was defined at its lower end by the minimum field expected to be necessary to saturate the magnetization of the discs out of the substrate film plane, $H \simeq 4\pi M_S$, and at its upper end by the maximum field attainable with the available electromagnet, $H = 14$ kOe. A representative plot of the resonance frequencies f of the first five radial modes against the resonance field H_{res} in this field range is shown in Figure 6. The solid lines are the fits for the data to Eq. (2), having left the demagnetizing factor N_m as the free fitting parameter in each case and otherwise using measured film characteristics. The value of N_m can be considered as a proxy to the strength

FIG. 5. Plot of normalized transmission parameter S_{21} vs applied out-of-plane magnetic field H for sample f2a, at an excitation frequency of 10 GHz, as measured by Broadband FMR. Five radial modes are resolved and labelled as $m = 1$ through $m = 5$.

FIG. 6. Plot of frequency f vs out-of-plane magnetic resonance field H_{res} for the first five radial modes of sample f3b (circles: $m=1$; diamonds: $m=2$; triangles: $m=3$; squares: $m=4$; pentagons: $m=5$), as measured by Broadband FMR. Solid lines are fits to Equation (2). These data are qualitatively representative of those data obtained from all of the samples.

of dipole inter-element coupling on an array of nano-elements.^{36,37}

The fitted values of N_m are tabulated in Table II, along with the values of N_m calculated for a theoretical isolated “disc,” 700 nm diameter, 27 nm thickness. The uncertainties in the N_m values were calculated by propagating the uncertainties in the disc radius R and thickness L through the fit. There was a slight decrease in demagnetising parameter N_m with increasing array disorder and increasing mode number. The changes in N_m across the range of ϕ' available were small enough to be comparable with the uncertainties. However, N_m values for an isolated disc differ from the $N=1$ for a continuous film by less than 0.07. The change in N_m with reduced array ordering cannot be expected to be higher than this difference value.

In addition to changes with m and ϕ' , there was a dramatic difference in the contrast between analytical and measured N_1 and N_2 , the demagnetizing parameters for the first and second radial modes. Displayed across the all four samples was the phenomenon that the difference between N_1 and the higher order demagnetizing factors $N_{2,3,4,5}$ was larger than for the isolated analytical disc. This difference is most easily observed when framed as an average frequency difference between the fit lines of Figure 6. The average frequency differences between successive nodes are tabulated in Table III and show that the frequency difference for the first two modes, $\bar{f}_2 - \bar{f}_1$ is larger in all cases than for the theoretical, isolated disc.

The very small amplitudes of modes beyond $m=2$ precluded the meaningful extraction of linewidths from those modes. Plots of the field linewidths ΔH extracted from

TABLE III. Table of average frequency differences in GHz between fits to Eq. (2) of successive modes, $\bar{f}_{m+1} - \bar{f}_m = 1/(H_{\text{max}} - H_{\text{min}}) \int_{H_{\text{min}}}^{H_{\text{max}}} (\omega_{m+1} - \omega_m)/(2\pi) dH$, on the field interval [10000–12000] Oe for a disc of 700 nm diameter, 27 nm thickness (denoted “disc”), with a saturation magnetization M_S and spectroscopic splitting factor g identical to film f3c, and for disc array samples f3b, f3a, f2a, and f1a.

Film	$\bar{f}_2 - \bar{f}_1$	$\bar{f}_3 - \bar{f}_2$	$\bar{f}_4 - \bar{f}_3$	$\bar{f}_5 - \bar{f}_4$
“disc”	1.59	1.42	1.42	1.48
f3b	1.93 ± 0.09	1.44 ± 0.06	1.36 ± 0.05	1.40 ± 0.08
f3a	2.00 ± 0.17	1.44 ± 0.09	1.49 ± 0.09	1.39 ± 0.07
f2a	2.01 ± 0.09	1.44 ± 0.06	1.47 ± 0.06	1.26 ± 0.04
f1a	2.03 ± 0.09	1.55 ± 0.06	1.47 ± 0.06	1.46 ± 0.05

Lorentzian fits to the first two radial modes of all four samples are shown in Figure 7, alongside the linewidth data from the corresponding parent continuous films.

In previous studies of spin wave mode broadening in thin films³⁸ and patterned structures,^{21,24,29} the effects of intrinsic and extrinsic damping have been separated by fitting the data with the equation:

$$\Delta H = \Delta H_0 + \frac{4\pi\alpha}{\gamma\mu_0} f \quad (7)$$

α is the intrinsic damping parameter in the Landau-Lifshitz-Gilbert equation,^{39,40} the “viscous” damping of energy to the lattice,⁴¹ and ΔH_0 is a term representing inhomogeneous broadening. However, ΔH does not increase with frequency in the affine fashion expected from Eq. (7), even in the case of the parent continuous films. Instead, at low frequency values the linewidths are very broad, decreasing to some minimum, then increasing with increasing frequency in an approximately linear fashion from some onset frequency, or equivalently from the resonance field corresponding to that frequency. The linewidth in the $m=2$ mode is always larger than in the $m=1$ mode. This effect is more severe for films with higher disorder parameter ϕ' .

V. DISCUSSION

As the disc radii, film thickness, and processing conditions were very similar between all of the array samples, the slight decrease in demagnetizing parameter N_m with increasing array disorder ϕ' may be attributable to a reduction in average dipole coupling strength due to lowered symmetry and/or a slight reduction in neighbour density associated with increased array disorder. The change in N_m across the range of films was between 0.018 for $m=1$ and 0.032 for

TABLE II. Table of demagnetizing factors N_m as calculated from Eq. (3) for a disc 700 nm diameter, 27 nm thickness (denoted “disc”), and for the disc array samples f3b, f3a, f2a, and f1a, as extracted by fitting Eq. (2) to the data in Figure 6.

Film	ϕ' [° mm ⁻¹]	$N_1 \times 10$	$N_2 \times 10$	$N_3 \times 10$	$N_4 \times 10$	$N_5 \times 10$
“disc”	—	9.51	9.40	9.35	9.33	9.31
f3b	6.0 ± 0.8	9.67 ± 0.05	9.44 ± 0.08	9.38 ± 0.10	9.37 ± 0.12	9.39 ± 0.15
f3a	9.4 ± 1.1	9.69 ± 0.08	9.43 ± 0.14	9.37 ± 0.18	9.32 ± 0.21	9.37 ± 0.23
f2a	11.3 ± 1.7	9.65 ± 0.04	9.38 ± 0.08	9.32 ± 0.10	9.28 ± 0.12	9.34 ± 0.14
f1a	19.9 ± 2.1	9.49 ± 0.04	9.21 ± 0.08	9.12 ± 0.10	9.08 ± 0.12	9.07 ± 0.14

FIG. 7. Plot of FMR linewidth ΔH vs frequency f for samples (a) f3b, (b) f3a, (c) f2a, and (d) f1a, for applied field in the direction of the film perpendicular, as measured by Broadband FMR. The first (circles) and second (diamonds) radial modes are shown alongside the data from the corresponding parent continuous films (inverted triangles).

$m=5$, or up to half the N_m value difference between a continuous thin film and an isolated disc. Given that the same samples showed no correlation between the demagnetising parameter and ϕ' in the in-plane magnetized configuration,²⁴ the argument from array packing density is less persuasive than the conjecture that the reduced long-range ordering impacted on the static demagnetizing field.

The difference $N_1 - N_2$, or equivalently the average frequency separation between first and second modes $\bar{f}_2 - \bar{f}_1$, was larger than expected from Eq. (2) for an uncoupled disc. Such an effect was not observed in studies of resonance frequency of either square arrays of lower magnetization nickel discs²⁶ or Permalloy discs with large disc-to-disc spacings.²⁹ Furthermore, differences between subsequent modes after the first, for example, $\bar{f}_3 - \bar{f}_2$ *et cetera*, were unremarkable in comparison to the analytical treatment. For a given sample, all of the radial modes were the result of the same out-of-plane static magnetization configuration, effectively ruling out static dipole coupling between the equilibrium magnetic moments of the discs as the cause of the difference in demagnetizing factors. On the other hand, the dynamic stray field of the first radial mode must be stronger than for the other modes, since it alone has no nodes across the diameter of the disc (see Figure 1). Tacchi *et al.* have observed dynamic dipole coupling in travelling Bloch waves in closely packed square

element arrays of comparable array element separation,²⁰ but only in the fundamental and 1DE modes. The higher order modes did not display the same coupling because of their reduced stray fields. The difference between $N_1 - N_2$ and successive mode differences $N_2 - N_3$, etc., in the disc arrays studied here is therefore interpreted as a dynamic dipole coupling between elements in the array which is larger for the $m=1$ mode than for subsequent modes.

The non-linear behaviour of the linewidth with decreasing frequency and applied magnetic field seen in Figure 7 was either not present or present in a more subtle manner in the largely dipole-uncoupled sparse square array system of Castel *et al.*²⁹ In square arrays of anti-dots, applied fields near the bulk saturation magnetization of the film allow canting of static magnetization vectors, leading to non-vanishing ellipticity of moment precession.³⁰ In the exchange coupled antidot arrays, this manifests as a measured decrease in resonance frequency below 10 GHz. For the study presented here, no significant deviation of the resonance frequency is observable in the frequency range in which the linewidth broadens anomalously. However, the linewidth broadening is more severe for less ordered arrays and also is stronger in the $m=2$ mode than the $m=1$ mode. Slight non-uniformity of the disc shapes may contribute to this effect, but the disc shapes and diameter distributions are very close between the samples. Variation in

disc morphology is unlikely to account for the increase in the linewidth broadening with increasing array disorder. The linewidth broadening is therefore likely the result of two static dipole effects: both the canting of the magnetization of a single dot away from the film perpendicular near saturation, and the distribution of this canting due to static dipole coupling of dots across the imperfectly ordered array.

VI. CONCLUSION

The standing spin wave mode structure and linewidth broadening in a series of closely packed hexagonal sub-micron diameter disc arrays in the perpendicularly magnetized state was investigated using broadband ferromagnetic resonance. Comparison of measurements to theory over a wide range of frequencies allowed deviations of the mode structure from that of an isolated disc to be identified. These deviations revealed the importance of array ordering to the absolute size of the demagnetizing factors. Deviations from the expected frequency differences between the first and second modes suggested that dynamic dipole coupling between discs was important to the mode structure. The relative values of demagnetizing fields for successive modes were essentially unaffected by degradation of the array symmetry.

Anomalously large linewidth broadenings were observed close to the out-of-plane demagnetizing fields of the arrays. The increased severity of this broadening in the second radial mode compared to the first suggests that the broadening is the result of the static magnetization configuration of the discs. The effect was more severe in less ordered arrays, suggesting that the increase in linewidth was the result of both the canting of local magnetic moments and of the distribution of the canting across the partially disordered array.

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