Fitness networks for real world systems via modified preferential attachment

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Abstract

Complex networks are virtually ubiquitous, and the Barabási and Albert model (*BA* model) has become an acknowledged standard for the modelling of these systems. The so-called *BA* model is a kind of preferential attachment growth model based on the intuitive premise that popularity is attractive. However, preferential attachment alone is insufficient to describe the diversity of complex networks observed in the real world. In this paper we first use the accuracy of a link prediction method, as a metric for network fitness. The link prediction method predicts the occurrence of links consistent with preferential attachment, the performance of this link prediction scheme is then a natural measure of the "preferential-attachment-likeness" of a given network. We then propose several modification methods and modified *BA* models to construct networks which more accurately describe the fitness properties of real networks. We find that all features assortativity, degree distribution and rich-club formation can play significant roles for the network construction and eventual structure. Moreover, link sparsity and the size of a network are key factors for network reconstruction. In addition, we find that the structure of the network which is limited by geographic location (nodes are embedded in a Euclidean space and connectivity is correlated with distances)

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differs from other typical networks. In social networks, we observe that the high school contact network has similar structure as the friends network and so we speculate that the contact behaviours can reflect real friendships.

Keywords: Complex network; *BA* model; assortative; degree distribution; rich-club; network sparsity

1. Introduction

The Barabási and Albert preferential attachment growth model (the *BA* model) [1] is the *de facto* standard model in the field of complex networks. The model is both intuitive and highly appealing. Popularity, as characterised by node degree, is a sufficient mechanism to lead to the
5 emergence of scale-free degree distribution. As a consequence of the success of the *BA* model, it is often treated as the generative process for a wide range of real scale-free network systems. However, the *BA* model follows the simple principle that popularity is attractive [2] whereas the real world is often more complicated [2, 3, 4]. Similarity (or affinity) between nodes [2], minimum node degree [3] and the dominance of low degree nodes [4] all play significant supporting roles while constructing
10 fitness networks ¹.

Emerging as the solution to a quite separate problem, link prediction can help us to analyse and predict network structure in real-world networks with characteristic structure [5, 6, 7]. By taking the *BA* model as the underlying generative pattern, we aim to adapt a newly described link prediction method (*PAI*) [8] to predict links based on the degree to which preferential attachment is followed
15 and hence measure the level of preferential attachment in real networks. Hence, we propose using the accuracy of the *PAI* algorithm to measure the fitness between these network construction models and real networks. Obviously, the *BA* model cannot model the degree distribution for many real networks. Conversely, the configuration model [9, 10] can achieve the same degree distribution as real networks, but it is difficult to maintain the connectivity of the networks and
20 the algorithm readily generates self-loops when mimicking real networks. Hence, we also use the degree distribution and the connectivity as metrics of networks fitness.

In our previous studies [3, 4], we observed that the assortativity of a *BA* network differs from

¹In our paper, the fitness network means that a network has the similar structure, degree distribution and connectivity with the original real network.

that of *typical* real networks. Furthermore, previous studies already proposed the assortative preferential attachment to construct the scale-free network with a given assortativity value [11] or can control their model parameter to change the network assortativity [12]. A specific modified preferential attachment also has been proposed to describe the local assortativity distribution for the Internet Autonomous System level networks [13]. In this paper, based on Xulvi-Brunet and Sokolov’s algorithm [14], we use corresponding randomized methods to modify the assortativity of an unbiased *BA* network to construct fitness networks — networks which better match real observed data.

However, we find that these assortativity modification methods are not successful in all cases. Moreover, since the actual degree distribution of *BA* models will usually differ from that of real networks, we further propose two modified methods to modify the degree distribution and the product of nodes degrees of the *BA* networks. Finally we note that in our previous study we found that the rich-club plays a significant role in driving the other properties of networks [15]. On the other side, our modified methods cannot work well for the the large size and the sparse network. Hence, to construct the fitness network simpler and more efficient, we propose one additional modified *BA* model to construct networks with more rich-clubs and one additional modified *BA* model to construct networks with more randomly links. In addition, compare these two additional models, we can study the role of the rich-club. We then propose a further two modified models which more effectively construct fitness networks.

In this paper, we use 14 different real datasets to construct a range of archetypal ”real” networks. We use the *BA* model to construct corresponding networks. Meanwhile, we compute the connectivity, the accuracy of *PAI* and the degree distribution as metrics of network fitness. Our various modified methods (which will be described below) and network generation models all maintain the connectivity of the original real networks. However, compared to real networks, we find that there is only one *BA* network which matches according to the second metric. Hence, we use our methods to modify the assortativity of the *BA* networks, after which, we find that there are now six modified *BA* networks coincide according to the second metric. Finally, we use our new methods to modify the degree distribution and the product of nodes degrees of *BA* networks, following this we find that there 12 networks are able to match the model with the second metric. Based on the rich-club phenomena and random networks, we propose two network models *RCBA* and *RBA* respectively. Moreover, we propose the corresponding modified models of *RCBA*

(*MRCBA*) and *RBA* (*MRBA*). Finally, we can combine our modified methods and models to
 55 construct all fitness networks that fit the second metric. Our models also have degree distributions
 more similar to the original real networks. In addition, we find that the human behaviours can
 reflect underlying real ("true") friendships. The network which is limited by the geographic location
 has the special structure, and the rich-club, the sparsity and the size of network all play the key
 roles for the network reconstruction.

60 2. Data

Different kinds of networks have different structures for the reconstruction, to consider the
 diversity of networks, we employ large open and publically available data sets from 14 different
 undirected networks: from the social network to the engineering network, from the sparse network
 to the dense network, from the small size network to the large size network. (1) *Football* network
 65 [16]: a node represents one country, a link stands for a pair of nodes at least sharing one common
 football player which participated in the World Championship in Paris, 1998. (2) *Dolphin* network
 [17]: a node represents one dolphin which in a community living off Doubtful Sound, a link stands
 for there is associations between a pair of nodes. (3) *PolBooks* network [16]: a node represents
 one book about US politics which is sold by Amazon.com, a link stands for there is a pair of nodes
 70 are purchased by a same buyer. (4) *Friendship* network [18]: a node represents one high school
 student, a link stands for one node reported a friendship with another one. We get link information
 from the publically site SocioPatterns directly [19]. (5) *C.Elegans* network [20]: a node represents
 one neuron, a link indicates a synapse or a gap junction between a pair of nodes. (6) *Contact*₁
 network [18]: a node represents one high school student, a link indicates the face to face contact
 75 of a pair of nodes. The students of the *Friendship* network also are the students of the *Contact*₁
 network. We get link information from the publically site SocioPatterns directly [19]. (7) *USAir*
 network [16]: a node represents one airport, a link indicates there is a flight line between a pair
 of nodes. (8) *SmaGri* network [16]: a node represents one author who cite the papers of Small
 & Griffith and Descendants, a link indicates a pair of nodes who cooperate on a paper. (9) *Pblog*
 80 network [21]: a node represents one blog on US politics, a link indicates a pair of nodes have
 hyperlinks between them. (10) *NetScience* network [22]: a node represents one author working
 on network science, a link indicates there is a co-authorship between a pair of nodes. (11) *Yeast*
 network [23]: a node represents one protein in budding yeast, a link indicates there is interactions

between a pair of nodes. (12) *USPower* network [20]: a node represents one power station of the Western states of US, a link indicates there is electric cables between a pair of nodes. (13) *Contact*₂ network [24]: a node represents one person in the infectious social networks, a link stands for the contact of a pair of nodes. We get link information from the publically site SocioPatterns directly [19]. (14) *Router* network [25]: a node represents one router at the level of autonomous systems, a link indicates the traffic flow between a pair of nodes. Table. 1 provides the number of nodes and links for each network.

Table 1: The number of nodes (N) and links (L), and the ratio of the L to the N for 14 networks. We do not count the number of loop links, the isolated nodes and links, and the duplicate nodes and links. Here, L/N can reflect the sparsity of the network. The networks which have the lower value of L/N are sparse networks, such as the *Netscience* network, the *USPower* network and the *Router* network.

	<i>Football</i>	<i>Dolphin</i>	<i>PolBooks</i>	<i>Friendship</i>	<i>C.Elegans</i>	<i>Contact</i> ₁	<i>USAir</i>
N	35	63	105	134	296	327	332
L	118	159	441	406	2148	5818	2126
L/N	3.3714	2.5238	4.2000	3.0298	7.5267	17.7920	6.4036
	<i>SmaGri</i>	<i>Pblog</i>	<i>NetScience</i>	<i>Yeast</i>	<i>USPower</i>	<i>Contact</i> ₂	<i>Router</i>
N	1059	1222	1589	2361	4941	10972	22963
L	4916	16714	2742	6646	6594	44517	48436
L/N	4.6421	13.2357	1.7256	2.8149	1.4074	4.0573	2.1093

3. Metrics and methods

In this section we introduce the metrics which we will employ to assess the fitness network generation models and the similarity to real data. The metrics described below measure several aspects of the networks: connectivity; accuracy of *PAI*, or faithfulness of the network to that expected from preferential attachment; and, degree distribution. We then describe the standard link prediction problem: the definition of link prediction; the link prediction method *PAI*; and, how to measure the accuracy of *PAI*. Finally, we will describe the process via which we are able to construct fitness networks. To do this we will propose two modification methodologies to alter the structure of networks generated via existing methods and four distinct network models to attempt to close the gap between the output of the *BA* model, and most real world data.

3.1. Metrics

In previous studies, link prediction methods have been applied to analyse the structure of networks [5, 6, 7]. The *BA* model is the foundational method for the construction of complex networks [3, 4] and has been widely used and widely discussed. The separate preferential attachment index methods (*PAI*) [8] for link prediction are based on the premise that the *BA* model [1] can reflect the level of preferential attachment and the structural properties of the networks. Hence, we take the connectivity, the accuracy of *PAI*, the degree distribution as the metric to measure the similarities of our models with original real networks or the construction of fitness networks.

Meanwhile, the standard link prediction problem, as considered by [26], can be formulated as follows. From the non-observed links construct the set $\bar{E}$, the observed link set E is divided into two parts E^T and E^P where $E^T \cup E^P = E$ and $E^T \cap E^P = \emptyset$. The division into E^P (typically including 10% of the observed links in [26]) and E^T (typically 90% of the observed links) is arbitrary and will be used for scoring purposes. That is, all the links in $E = E^P \cup E^T$ have been observed and are known, however, links in E^T will form a *training set* and are used to implement a link prediction score, the efficacy of which will be evaluated over the *probe set* E^P . In addition, the *PAI* can be simply described as follows: $Score_{xy} = k_x \times k_y$, where k_i is the degree of node i . The so-called receiver operating characteristic curve (*AUC*), originally applied to evaluate communication schemes, has since been widely applied to measure prediction accuracy [27] in a variety of settings. Here, we use *AUC* as a prediction accuracy measure for networks. Only the information of E^T is allowed to be used to compute the performance score $Score_{xy}$, we compare the prediction scores of m pairs of nodes from E^P and $\bar{E}$ randomly, if there are m' times that the score measured from E^P is bigger than the score measured from $\bar{E}$ and m'' times that the two scores are equal, then, $AUC = (m' + 0.5m'')/m$. **Here, the *AUC* value is closer to 1 means that the link prediction method is more efficient. Moreover, the *AUC* values are determined by the relationship between the network structure and the link prediction algorithm.**

As shown in Fig. 1, we describe a simple example to explain the *AUC* schematic. Here, we only compute the *AUC* once ($m = 1$): First, the observed link set (a) can be divided into a *training set* (b) and a *probe set* (c). Next, we can choose a pair of nodes AB from the *probe set* (c) and a pair of nodes from the non-observed links set (d). Only the *training set* (b) can be used to compute the node degree. If we choose the pair of nodes AD from the non-observed links set (d), $Score_{AB} = k_A \times k_B = 0$, $Score_{AD} = k_A \times k_D = 1$, namely $Score_{AB} < Score_{AD}$,

hence $m' = 0, m'' = 0$, then $AUC = (0 + 0.5 \times 0)/1 = 0$. If we choose the pair of nodes BD from the non-observed links set (d), $Score_{AB} = k_A \times k_B = 0, Score_{BD} = k_B \times k_D = 0$, namely $Score_{AB} = Score_{BD}$, hence $m' = 0, m'' = 1$, then $AUC = (0 + 0.5 \times 1)/1 = 0.5$.

Figure 1: (Color online) The simple examples of the observed links set E , the non-observed links set $\bar{E}$, the *training set* E^T and the *probe set* E^P .

135 3.2. Methods

Not all real networks, built via growth process and exhibiting scale-free characteristics, share the statistical features of the BA model. We have previously observed that both assortativity [3, 4] and the degree distribution of BA networks differ from that observed in real networks. Moreover, the rich-club plays a significant roles for other properties of networks [15], we propose two modification

140 methods and four network models to modify networks generated via the *BA* process and construct fitness networks.

3.2.1. Modification methods

First, we can use randomized methods to modify the structure of *BA* networks. As shown in Fig. 2, we use the randomized edges method (*RE*) [28] to change the structure, the assortativity and the rich-club properties of *BA* networks [29], but the degree distribution of every *BA* network is maintained.

Figure 2: (Color online) Nodes *A* and *B*, and nodes *C* and *D* are connected, while nodes *A* and *D*, and nodes *B* and *C* are not connected. Then cut the links *AB* and *CD*, and connect the nodes *A* and *D*, the nodes *B* and *C* respectively.

In our previous study, we found that the assortativity of real networks differs from that of *BA* networks [3, 4] and we also observed that we can change the assortativity of these original networks by controlled application of *RE* [30]. Based on Xulvi-Brunet and Sokolov’s algorithm [14], as shown in Fig. 3, we use the disassortative *RE* method (*DARE*) and the assortative *RE* method (*ARE*) to change the structure and the assortativity of the original *BA* networks. In each network, there is a limited number of motifs which are described in Fig. 3, but we simply continue the randomization process until convergence.

In addition we can aim to directly improve the degree product of every pair of nodes which have a link between them to improve the accuracy of *PAI*. Conversely, we could decrease the degree product of every pair of nodes which have a link between each other to decrease the performance

Figure 3: (Color online) (a) Nodes A and B , and nodes C and D are connected, while nodes A and D , and nodes B and C are not connected. In addition, $|k_A - k_B| < |k_A - k_D|$ and $|k_C - k_D| \leq |k_C - k_B|$, where k_i is the degree of node i . Then cut the links AB and CD , and connect the nodes A and D , the nodes B and C respectively. (b) Nodes A and B , and nodes C and D are connected, while nodes A and D , and nodes B and C are not connected. In addition, $|k_A - k_B| > |k_A - k_D|$ and $|k_C - k_D| \geq |k_C - k_B|$, where k_i is the degree of node i . Then cut the links AB and CD , and connect the nodes A and D , the nodes B and C respectively.

of PAI ². Doing so, we obtain similar performance ³, similar degree distribution and the same connectivity as the original networks. The process is depicted in Fig. 4. We propose two new randomization methods to change the structure and the degree distribution of BA networks. These two methods not only can change the degree distribution of BA networks, but also can change the

²The score of PAI is determined by the degree product of every pair of nodes. On the other side, AUC compare the score of the pair of nodes which have a link between each other and that have no link between each other. Hence, if we decrease or increase the degree product of every pair of nodes which have a link, we can decrease or increase the performance of PAI respectively.

³Here, we define the similar performance as the difference between the AUC values of our models and that of original networks are not larger than 0.02.

PAI performance of networks. Moreover, because the pair of nodes which links will be cut have the common neighbours, hence our randomized methods can maintain the connectivity of networks. We denote these methods as increased *RE* method (*IRE*) and decreased *RE* method (*DRE*). As before, while in each network, there is a limited number of motifs which are described in Fig. 4, we will randomize until convergence.

Figure 4: (Color online) (a) Nodes *A* and *B* are connected, and should have a common neighbour *E*, while nodes *C* and *D* are not connected. In addition, $k_A \times k_B < k_C \times k_D$, where k_i is the degree of node *i*. Then cut the link *AB* and connect the nodes *C* and *D*. (b) Nodes *A* and *B* are connected, and should have a common neighbour *E*, while nodes *C* and *D* are not connected. In addition, $k_A \times k_B > k_C \times k_D$, where k_i is the degree of node *i*. Then cut the link *AB* and connect the nodes *C* and *D*.

3.2.2. Network models

In our previous studies, we found that the rich-club plays a significant role on the other structural properties of networks [15]. Hence, we also propose two modified *BA* models targetting these rich-clubs.

First, we propose a new model which follows the rich-club phenomena and has the same number

of nodes and links as the original network. If the original real network has $m + t$ nodes and N links: (1) Given m ($m \geq 1$) nodes, construct a fully-connected network. (2) Next, introduce one new node to only connect to one existing node every time, and the connected probability of existing node i (degree k_i) is $P_i = \frac{k_i}{\sum_{j=1}^{m+t} k_j}$. After t times, we could construct the network which have $m + t$ nodes and $m(m-1)/2 + t$ links. (3) Finally, we add $N - m(m-1)/2 - t$ links to the pairs of nodes which have no link between them, and the connected probabilities of existing nodes both are equal to that of step (2), which means both the two connected nodes are more like rich nodes. We named this model as the rich-club *BA* model (*RCBA*).

Second, we propose a new model which (as above) we first build a tree via preferential attachment and then (different from *RCBA*) adds additional links randomly — until one has the same number of nodes and links as the original network. If the original real network has $m + t$ nodes and N links. (1) Given m ($m \geq 1$) nodes, construct a fully-connected network. (2) Next, introduce one new node to only connect to 1 existing node every time, and the connected probability of existing node i (degree k_i) is $P_i = \frac{k_i}{\sum_{j=1}^{m+t} k_j}$. After t times, we could construct the network which have $m + t$ nodes and $m(m-1)/2 + t$ links. (3) Finally, we add $N - m(m-1)/2 - t$ links randomly. We named this model as the random *BA* model (*RBA*).

In addition, for the step (2) of the *RCBA* and the *RBA*, we can use the method as shown in Fig. 5 to exchange the new added link. For step (3) of the *RCBA* and the *RBA*, we can use the method as shown in Fig. 6 to exchange the newly added links. We named the modified models as *MRCBA* and *MRBA*. We summarise the various algorithms describe above in Table. 2.

Figure 5: (Color online) Link AB is the new added link, node A is the new node, then cut the link AB and connect the nodes A and C . If the *PAI* performance of our networks are lower than that of original real networks, then $k_C > k_B$. If the *PAI* performance of our networks are better than that of original real networks, then $k_C < k_B$.

Figure 6: (Color online) Link AB is the new added link, nodes C and D are not connected, then cut the link AB and connect the nodes C and D . If the PAI performance of our networks are lower than that of original real networks, then $k_C \times k_D > k_A \times k_B$. If the PAI performance of our networks are better than that of original real networks, then $k_C \times k_D < k_A \times k_B$.

Table 2: All our methods which are described in Section. 3.2. Here, we describe all the purposes of our methods. For all our methods, the connectivity of original real networks is maintained.

Methods	Description
<i>RE</i>	Change the structure, the assortativity and the rich-club properties of networks (see Fig. 2).
<i>DARE</i>	Change the structure and increase the disassortativity of networks (see Fig. 3 (a)).
<i>ARE</i>	Change the structure and increase the assortativity of networks (see Fig. 3 (b)).
<i>IRE</i>	Change the structure and increase the PAI performance of networks (see Fig. 4 (a)).
<i>DRE</i>	Change the structure and decrease the PAI performance of networks (see Fig. 4 (b)).
<i>RCBA</i> Model	The modified <i>BA</i> model which can construct more rich-clubs.
<i>RBA</i> Model	The modified <i>BA</i> model which is more like random networks.
<i>MRCBA</i> Model	The modified <i>RCBA</i> model which either can increase or decrease the PAI performance.
<i>MRBA</i> Model	The modified <i>RBA</i> model which either can increase or decrease the PAI performance.

4. Results

We now present the results of computations. We examine the similarity between the various *BA* models (*BA* model and modified *BA* models) and the real networks by our methods. We find that the *BA* model only matches one of our real world networks. The modified *BA* model matches

195 12. Modifications applied to our generative models can match all 14 fitness networks.

4.1. *BA* model

To build the corresponding complex networks which have the same number of nodes and the similar number of links as the original networks, we use the modified *BA* model to construct the fully-connected networks [1]: Given m ($m \geq 2$) nodes, construct a fully-connected network. Next,

200 introduce one new node to connect to n or $n+1$ ($1 \leq n < n+1 \leq m$) existing nodes every time, and

the connected probability of existing node i (degree k_i) is $P_i = \frac{k_i}{\sum_{j=1}^{m+t} k_j}$. After t times, we could construct our destination network which have $m + t$ nodes, the floating number of links is from $m(m-1)/2 + nt$ to $m(m-1)/2 + (n+1)t$. Finally, the probability of the degree (k) distribution also follows a power-law distribution, $P_k \propto k^{-\gamma}$.

As shown in Fig. 7, we find that the *PAI* performance of *BA* networks differ with that of original real networks except the *Pbook* network⁴. To get the similar results of original networks, we try to use randomized methods to modify *BA* networks. Hence, we use *RE* to change the structure, the assortativity and the rich-club properties of *BA* networks. However, as shown in Fig. 7, we see that the *PAI* performance of modified *BA* networks which are randomized by *RE* have no big differences with that of *BA* networks. Here, the number of *RE* randomizations for each network is equal to the 10 times of the number of links. Interestingly, we find that the *PAI* performance of the original contact network and the original friendship network which are from same high school are similar to each other. We suggest that human behaviours can reflect their real friendships to some extent.

We find that the assortativity of real network differ with that of *BA* networks [3, 4] and we are able to change the assortativity of original networks by controlled application of *RE* [30]. Here, we use our new methods *DARE* and *ARE* to change the structure and the assortativity of original *BA* networks. As shown in Fig. 7, we see that the *PAI* performance of *BA* networks which are randomized by *DARE* are better than that of *BA* networks, and not lower than that of *BA* networks which are randomized by *ARE* except for the *USPower* network. However, we can find that the *PAI* performance of four original networks are located between that of the modified *BA* networks which are randomized by *DARE* and that of the modified *BA* networks which are randomized by *ARE*. The modified *NetScience* and *Contact₁* *BA* networks can get the similar *PAI* performance as the original real network. These phenomena suggest that only six different *BA* networks which are randomized by *DARE* or *ARE* can achieve the similar performance as corresponding original networks. Specially, differ from other networks, we find that the performance of modified *USpower* *BA* network by *DARE* are lower than that of modified *USpower* *BA* networks by *ARE*, due to only the power network is limited to the geographic location. However, we can not find a similar

⁴Only by computing the *AUC* accuracy of *PAI* about 10,000 times, we can consistently achieve the stable results. Hence, we compute the *AUC* accuracy of *PAI* more than 10,000 times independently.

Figure 7: (Color online) Under the metric of the AUC accuracy of PAI , the performance of original real networks, BA networks and modified BA networks. **The circles with dashed lines of the original real networks mean that we can use our methods to archive the similar PAI performance of the original networks.**

phenomena in contact networks, due to the fact that the contact networks actually are offline social networks which are not limited by the geographic location in a traffic and communication developed society. On the other side, these phenomena suggest that we can not use $DARE$ and ARE to modify the $USPower$ BA network — interestingly this is a spatially engineered network, it is deliberately designed rather than the evolved network. Table. 3 provides the number of randomizations for each network.

Table 3: The number of randomizations for ($DARE$ and ARE) of each network for the results which shown in Fig. 7.

	<i>Football</i>	<i>Dolphin</i>	<i>PolBooks</i>	<i>Friendship</i>	<i>C.Elegans</i>	<i>Contact₁</i>	<i>USAir</i>
<i>DARE</i>	42	49	178	131	1045	3243	1125
<i>ARE</i>	61	96	341	372	2410	6689	2878
	<i>SmaGri</i>	<i>Pblog</i>	<i>NetScience</i>	<i>Yeast</i>	<i>USPower</i>	<i>Contact₂</i>	<i>Router</i>
<i>DARE</i>	2376	9092	1033	2614	1125	18555	16312
<i>ARE</i>	7314	26493	3891	9296	2878	66416	67383

Furthermore, as shown in Fig. 11 (see Appendix), we find that the degree distributions of the BA networks are different from those of the original networks. To get similar PAI performance, more similar degree distribution and the same connectivity as the original networks, we use our

new methods *IRE* and *DRE* to modify the *BA* networks. As shown in Fig. 8 , we can see that the *PAI* performance of the 10 different original networks are located between that of the modified *BA* networks when randomized by *IRE* and that of the modified *BA* networks when randomized by *DRE*. These phenomena suggest that based on *BA* model, we can use *IRE* and *DRE* to get the similar *PAI* performance as 10 smaller size real networks. In addition, we see that the modified *BA* network when randomized by *IRE* also can get the similar performance as the real *contact₂* network. However, compared to the *Yeast BA* network and the *contact₂* *BA* network, we find that the smaller size *BA* network (*NetScience*) does not get the similar result as the original real network. Furthermore, we find that the *NetScience* network is more sparse than the *Yeast* network and the *contact₂* network. Hence, we suggest that the sparsity of networks is one of the keys for the reconstruction of original networks. On the other side, compared to the *contact₂* *BA* network, we find that *IRE* and *DRE* are more effective for the smaller networks which have the similar sparsity as it. Hence, we suggest that the sparsity and the size both are keys for the reconstruction of original networks.

Figure 8: (Color online) Under the metric of the *AUC* accuracy of *PAI*, the performance of original real networks, *BA* networks and modified *BA* networks.

Next, as shown in Fig. 8, we attempt to use *DARE* to modify the modified *BA* networks (by *IRE*) which do not achieve similar performances as the original networks, we can see that only the smaller size *BA* network (*NetScience*) can get the similar performance as the original real network.

255 This phenomenon also shows that the network size is one of the keys for the reconstruction of the original networks. Finally, we suggest that we can combine the product modified methods (*IRE* and *DRE*) and the assortativity modification methods (*DARE* and *ARE*) to get more similar structures to the real networks. Table. 4 provides the number of randomizations for each network.

Table 4: The number of randomizations (*IRE*, *DRE* and *DARE*) for each network for the results which shown in Fig. 8.

	<i>Football</i>	<i>Dolphin</i>	<i>PolBooks</i>	<i>Friendship</i>	<i>C.Elegans</i>	<i>Contact₁</i>	<i>USAir</i>
<i>IRE</i>	129	227	780	784	4178	11305	4592
<i>DRE</i>	407	50	241	98	1339	11306	1260
<i>DARE</i>	0	0	0	0	0	0	0
	<i>SmaGri</i>	<i>Pblog</i>	<i>NetScience</i>	<i>Yeast</i>	<i>USPower</i>	<i>Contact₂</i>	<i>Router</i>
<i>IRE</i>	10560	33048	65	28654	10	1084	94
<i>DRE</i>	741	9706	52	281	44	1063	188
<i>DARE</i>	0	0	1075	0	0	0	16356

4.2. New models and the role of rich-club

260 Based on the *BA* model, the modified methods cannot build all fitness networks for real data, and it is difficult for the configuration model to keep the connectivity of real networks. Especially for the large size network (*Router* network) and the sparse network (*USPower* network), the modified methods are not effective. Hence, to seek better models and study the role of the rich-club, we use our new models *RCBA* and *RBA* to construct the full-connected networks fitting real networks, then compare the performance of the two methods. As shown in Fig. 9, compared to *RBA*, we can see that the *PAI* performance of the networks which are constructed by *RCBA* are more similar to that of all original real networks except the *USPower* network. We can conclude that the rich-club plays a significant role for the construction of fitness networks.

270 Next, to obtain similar performance as the original networks, we try to combine the product modified methods (*IRE* and *DRE*) and the assortativity modification method (*DARE*) to modify our model. As shown in Fig. 10, we can see that the *PAI* performance of about 12 different original networks are located between that of the modified *RCBA* networks which are randomized by *IRE* (or combined with *DARE* ⁵) and that of the modified *RCBA* networks which are randomized by

⁵Here, for the *NetScience* network, the number of *DARE* randomizations for *RCBA* model is 1303.

Figure 9: (Color online) Under the metric of the AUC accuracy of PAI , the performance of original real networks, $RCBA$ networks and RBA networks.

DRE . Hence, we suggest that we can modify the $RCBA$ networks to get the similar performance of all original real networks except the $USPower$ network and the $Router$ network. Table. 5 provides the number of randomizations for each network.

Table 5: The number of randomizations for each network for the results which shown in Fig. 10.

	<i>Football</i>	<i>Dolphin</i>	<i>PolBooks</i>	<i>Friendship</i>	<i>C.Elegans</i>	<i>Contact₁</i>	<i>USAir</i>
<i>IRE</i>	107	176	738	756	4296	11194	4252
<i>DRE</i>	236	83	315	162	1856	11636	1273
	<i>SmaGri</i>	<i>Pblog</i>	<i>NetScience</i>	<i>Yeast</i>	<i>USPower</i>	<i>Contact₂</i>	<i>Router</i>
<i>IRE</i>	9832	33428	141	24321	5	1791	95
<i>DRE</i>	1166	14182	80	517	43	2410	77

Furthermore, we suggest that we can modify the BA networks at every growth step. Hence, we use our new models $MRCBA$ and $MRBA$ to construct the full-connected fitness networks for the real world. As shown in Fig. 10, we use the $MRCBA$ model to construct the router network, and modified it by $DARE$ (19436 times), then we can get the similar performance as the original router network. However, the $MRCBA$ model does not work for the $USPower$ network.

Moreover, under the metric of AUC , the PAI performance of the original real network, RBA

Figure 10: (Color online) Under the metric of the AUC accuracy of PAI , the performance of original real networks, $RCBA$ networks, modified $RCBA$ networks and one modified $MRCBA$ network.

network and $MRBA$ network are 0.4487, 0.4704 and 0.3857 respectively. The PAI performance of original real network is located between that of RBA network and that of $MRBA$ network. We suggest that only the random model can get the similar structure of original $USPower$ network. In addition, combine the special phenomena of $USpower$ network as shown in Fig. 7 to Fig. 10, we can conclude that the structure of the network which is limited by the geographic location differs from that of other networks which are not limited by the geographic location, and is more like that of random networks.

In addition, as shown in Fig. 11 (see Appendix), compared to BA networks, we find that the degree distributions of $RCBA$ networks are generally more similar to original real networks. Moreover, as shown in Fig. 12 (see Appendix) we use the $USAir$ network as an example, describe the structure of original real network, BA network and modified BA network by IRE . We can see that the structure of the modified BA network is more like the original real network. These phenomena also suggest that we can construct networks more similar to original real networks by our methods.

5. Discussion and conclusion

In summary, we can use our new modified methods and models to construct the fitness networks for real world under our metrics. In detail, based on *BA* model, we can combine our modified methods (*DARE* and *IRE*, or *ARE* and *DRE*) easily to construct the smaller size (the number of nodes is lesser than about 1500) or the dense (L/N is bigger than about 2) fitness networks. Furthermore, based on our rich-club model (*RCBA* and *MRCBA*), we can combine our modified methods to construct all fitness networks except the power network. In particular, we only can use the random network model to construct the fitness network for *USPower* network.

On the other hand, we find that the rich-club, the network sparsity and the network size all play important roles for the construction of fitness networks. Specially, we find that the power network is different with other networks and more like the random network, due to the role of the geographic location. We also find that the connect network actually is a offline social network and not limited by the geographic location in a traffic and communication developed society. In addition, the human behaviours can reflect their real friendships to some extent. We find that the distinct characteristics of engineered versus natural growth processes is evident. Engineered networks are more difficult to mimic through random generative processes and are hence rarer among the space of all networks.

Furthermore, based on the higher-order connectivity patterns, the recently research has developed a nice generalized model for directed networks, especially for clustering networks [31]. More assortativity-related concepts also have been proposed for directed networks [32], using the assortativity modify or construct directed network models should be accepted more attentions in future.

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6. Appendix

Figure 11: (Color online) The degree distribution of the original real networks, the BA networks and the $RCBA$ networks.

(A) USAir (Original)

(B) USAir (BA)

(C) USAir (Modified BA)

Figure 12: (Color online) The structure of the original *USAir* network, the *BA* network and the modified *BA* network by *IRE*.