Development of a Force-resultant Model for Spudcan Footings on Loose Sand under Combined Loads

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Ning Cheng (corresponding author)

Research Associate

Centre for Offshore Foundation Systems and ARC CoE for Geotechnical Science and Engineering
University of Western Australia
35 Stirling Highway, Crawley
Perth, WA 6009, Australia
Tel: +61 8 6488 3732 Fax: +61 8 6488 1044
Email: ning.cheng@research.uwa.edu.au

Mark Jason Cassidy

Australian Research Council Laureate Fellow
Lloyd's Register Foundation Chair of Offshore Foundations

Centre for Offshore Foundation Systems and ARC CoE for Geotechnical Science and Engineering
University of Western Australia
35 Stirling Highway, Crawley
Perth, WA 6009, Australia
Tel: +61 8 6488 3732 Fax: +61 8 6488 1044
Email: mark.cassidy@uwa.edu.au

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Ning Cheng and Mark Jason Cassidy

Abstract

Spudcans are typical foundations used in shallow to moderate water oil and gas fields to support jack-up drilling units. Understanding the behaviour of spudcans under combined loadings is crucial to the overall response of the jack-up structure. This paper presents the development of a strain-hardening plasticity model for spudcan footing on loose sand. Most of the model components are developed from direct centrifuge observations. The centrifuge tests were performed at an acceleration of 100 times that of the Earth's gravity on a model spudcan footing subjected to combined vertical, horizontal and moment loads. All the experiments have been designed and conducted to allow the results to be interpreted with a strain-hardening plasticity framework. Combined loads were applied by using a novel apparatus, which enables independent vertical, horizontal and rotational movements of the footing. Test results also revealed the existence of a three dimensional sliding surface that intersects with the conventional yield surface. This additional surface has been defined analytically. Retrospective simulation of the experimental data using the plasticity model confirms the model's capability for the use in predicting the behaviour of larger spudcan applications offshore.

Keywords: Offshore Engineering, Centrifuge test, Force-resultant model

Introduction

Jack-ups are dynamically sensitive structures whose load response is dependent on the stiffness of their large spudcan footings. Spudcans are roughly circular in plan with a sharp protruding spigot to provide extra horizontal stability and to minimise lateral movement in the soil. Jack-ups perform drilling in water depths of up to 120 m. At such depths, the magnitude of wind, wave and current loads acting on the hull and three legs are substantial and cause large horizontal (H) and moment (M) loads on the spudcan as well as changes in the vertical (V). Understanding the bearing capacities of these combined VHM loads is essential during the design of jack-up and during their site specific assessments (see ISO (2012)). Many studies have been conducted in the field of spudcan footings in an effort to understand their behaviour under combined VHM loadings (Noble Denton & Associates 1987; Schotman 1989; Osborne et al. 1991; Byrne and Houlsby 1999; Gottardi et al. 1999; Martin and Houlsby 2000; Byrne and Houlsby 2001; Martin and Houlsby 2001; Cassidy et al. 2002a; Houlsby and Cassidy 2002; Cassidy et al. 2004; Bienen et al. 2006; Zhang et al. 2011).

66 Conventional bearing capacity equations for shallow foundations were initially described by Terzaghi
67 (1943) and subsequently extended by Meyerhof (1953) and Hansen (1970). These account for
68 inclined ($V:H$), eccentric loading ($V:M$) and various footing shapes by using a number of
69 empirical factors applied to the bearing capacity formula for vertical loading. As such, they are not
70 suitable for the direct inclusion in numerical analyses programmes of the complete jack-up structure.
71 Recently, the offshore industry has moved towards the use of plasticity-based force-resultant models
72 to describe the behaviour of shallow foundations, as they express the footing behaviour purely in
73 terms of force resultants and can be directly coupled to typical structural analysis software used in
74 industry.

75 A number of authors have proposed such models for strip footings (Nova and Montrasio 1991;
76 Gottardi and Butterfield 1993, 1995) and circular footings (Byrne and Houlsby 1999; Gottardi et al.
77 1999; Byrne and Houlsby 2001; Cassidy et al. 2002a; Houlsby and Cassidy 2002; Bienen et al. 2006)
78 on sand. The usual approach has been to assume a yield surface, a hardening law, a flow rule and an
79 elastic region within the yield surface. An example force-resultant model for describing drained
80 behaviour of circular footing in sand is known as Model *C*, a plasticity model developed by Houlsby
81 and Cassidy (2002). Model *C* was originally based on a series of 1-g experimental tests performed by
82 Gottardi et al. (1999) but has also been calibrated for calcareous sand (Cassidy et al. 2002a) and
83 extended to six-degrees-of-freedom loading conditions (Bienen et al. (2006). Its use in the pushover
84 analysis of jack-ups is shown in Bienen and Cassidy (2006), Bienen and Cassidy (2009) and Cassidy
85 et al. (2010).

86 The four major components associated with Model *C* are:

- 87 a) An empirical expression for the yield surface which separates the elastic and plastic boundary,
- 88 b) A model for the elastic load-displacement behaviour within the yield surface,
- 89 c) An empirical strain-hardening expression to define the expansion and contraction of the yield
- 90 surface with the plastic component of vertical displacement,
- 91 d) A suitable flow rule to allow for the prediction of the footing plastic displacements during
- 92 yield.

93 Further reference to Model *C* can be found in Houlsby and Cassidy (2002), Cassidy et al. (2002a),
94 Byrne and Houlsby (1999, 2001) and Bienen et al. (2006). Although there is considerable
95 experimental evidence to support those strain-hardening plasticity models, most of the components
96 were developed from small-scale model 1-g experiments. This paper addresses if these accurately
97 replicate the soil-stress strain relationship of the much larger prototype spudcan footing.

In an attempt to ensure confidence when employing Model *C* in larger footings applications, Cassidy (2007) verified components of Model *C* with centrifuge experiments on a flat circular footing. Although, a similar yield surface shape was broadly observed, a complete *VHM* yield surface could not be tested because only a fixed arm (pure horizontal translation) or a hinged arm (pure rotation) were used (see details of the apparatus in Cassidy et al. (2004)). Subsequently, Bienen et al. (2007) conducted a series of centrifuge tests, also on a shallow circular foundation on sand, but in this case only in the vertical-torsional loading plane. Following the same theoretical framework of strain-hardening plasticity, Govoni et al. (2010, 2011) investigated circular footings resting on medium dense silica sand subjected to combined vertical, moment and horizontal loading and compared results to existing data obtained at 1- *g*. Special emphasis was given to the embedment effect. However, only two specific load paths were investigated, as the same fixed and hinged arms were used in this experiment as in Cassidy (2007). Cocjin and Kusakabe (2012) conducted a series of displacement controlled vertical loading and swipe tests at various vertical load levels on a flat strip footing. The study confirms that the deviatoric load capacity is highly dependent on the ratio of the vertical load and vertical peak bearing capacity. However, only horizontal swipe tests were performed in their study.

Although there is considerable experimental evidence from the small scale 1- *g* environment to support the well-established strain-hardening plasticity models, no comprehensive centrifuge evidence, to date, has been provided. This is because with only single or at the best two load paths, it is difficult to establish any complete yield surface or flow rule in three dimensional space. Another restriction of the previous studies is that the small-scale model being tested was a representative flat circular plate; no model with an inverted conical spudcan shape has been studied. A summary of the major studies in the framework of plasticity based force-resultant models is provided in Table 1.

Table 1 Comparison of the major studies publicly available concerning plasticity based force-resultant models

	Footing type		Sand type	g level	<i>VHM</i> load paths	Yield Surface	Flow rule	Force-resultant model
	Flat circular	Spudcan						
Gottardi (1999)	✓		Very dense	1	Full	✓	✓	
Byre & Houlsby (1999)	✓		Very dense	1	Full	✓		
Byrne & Houlsby (2001)	✓		Carbonate	1	Full	✓		
Houlsby & Cassidy (2002)	✓		Very dense	1	Full	✓	✓	✓
Cassidy (2002)	✓		Carbonate	1	Full	✓	✓	✓
Bienen (2006)	✓		Loose	1	Full	✓	✓	✓
Cassidy (2007)	✓		Loose	100	2			
Govani (2010, 2011)	✓		Medium	100	2			
Cocjin & Kusakabe (2012)	✓		Dense	50	1			

This study	✓	Loose	100	Full	✓	✓	✓
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122 This paper details such a plasticity-based force-resultant model for spudcan footing on loose sand
123 with most of the model components developed from direct centrifuge observations. The experiments
124 were conducted at an acceleration of 100 times that of the Earth's gravity in the drum centrifuge at the
125 University of Western Australia (UWA) and is the first comprehensive centrifuge programme
126 investigating spudcan behaviours on sand in all *VHM* load directions.

127 **Experimental setup**

128 The sign convention for the loads and displacements adopted in this study follow the standardised
129 convention system proposed by Butterfield et al. (1997) for combined loadings. Foundational loading
130 was applied by means of displacement-control to a load reference point (LRP). The LRP is located at
131 the centreline of the foundation at the mud line level. The load was described by horizontal translation
132 (u), vertical translation (w) and rotation (θ); positive sign conventions together with size dimensions
133 are shown in Fig. 1.

134

135 **Fig. 1 Model dimensions and sign conventions for load and displacement**

136 The experiments were conducted at the UWA drum centrifuge, which has a swinging platform at a
137 radius of 0.6 m and a maximum rotational speed of 850 rev/min. The drum centrifuge consists of a
138 central tool table, driven by a Dynaserv servo motor, and a channel for sample placement, which are
139 able to rotate differentially. This creates relative motion between the sample and any tool connected to
140 the actuator, situated within the tool table. The soil channel containment area has width of 300 mm
141 and a depth of 200 mm. Details of this drum centrifuge are provided by Stewart et al. (1998).

142 A recently developed *VHM* apparatus (Zhang et al. 2013) was used in the experiments. The
143 conventional drum centrifuge configuration only allowed for a setup direction perpendicular to the

vertical and horizontal plane on the footing (w and u , respectively), thus only two degree-of-freedom movement was available, as shown in Fig. 2. A third degree-of-freedom movement was achieved, by adding another actuator on top of the existing actuator and connected through linkage arms and hinges (Fig. 3(c)). This enables the footing to also rotate in-plane.

Fig. 2 Schematic diagram for conventional actuators in drum centrifuge

The primary advantage of using this displacement controlled apparatus is the ability to apply any combination of the displacement paths to the model footing using computer controlled actuators. This series of movements were calculated in real time by a build-in-house programme developed with NI LabVIEW interface (National Instrument Corporation 2003). More details of the apparatus used in this experiment can be found in Zhang et al. (2013), Cheng et al. (2014) and Cheng and Cassidy (2016). Photos showing the *VHM* apparatus incorporated in the drum centrifuge are presented in Fig. 3(a) and (b).

Fig. 3 Picture and schematic drawing of the *VHM* loading arm

Important features of the new *VHM* apparatus are:

- a) The new *VHM* apparatus can conduct any specified combination of vertical, horizontal and rotational movement with speeds ranging from 0.1 mm/s to 3 mm/s.
- b) The new *VHM* apparatus has been equipped with modern inflight computers to ensure the highest possible accuracy along with the minimum systematic lag. This is particularly important for tests involving sequential movements, such as the swipe tests described in this paper.
- c) The new *VHM* apparatus has been retrofitted with enhanced mechanical systems. This ensures the apparatus can be safely used in high stress testing environment so it is adaptable to any soil.

Soil characterisation

The soil used in this study was a superfine silica sand used in numerous UWA studies. The physical properties of the sand are listed in Table 2 (Cassidy and Cheong 2005).

Table 2. Soil properties of sand used in test

Dry unit weight (kN/m^3)	
Particle size, D_{50} , D_{10} , D_{20} : mm	0.19, 0.099, 0.135
Minimum dry density, $\rho_{\min}$: kg/m^3	1517
Maximum dry density, $\rho_{\max}$: kg/m^3	1829
Maximum $e_{\max}$	0.75
Minimum $e_{\min}$	0.53
Specific gravity, G_s	2.65
Critical-state frictional angle: degrees	31

The sand sample was prepared using a rotating actuator to rain the silica sand into the rotating outer channel of the drum centrifuge. The sample height was 140 mm and remained saturated with 20 mm water above the surface throughout the test. A consistent relative density of 30-35% was interpreted from 8 cone penetrometer tests (Fig. 4) using the procedure of Schneider and Lehane (2006) where

$$R_d = 100 \sqrt{\frac{Q}{250}} \quad (1),$$

and where $Q = (q_c - \sigma_v) / \sigma'_v$, q_c is the cone tip resistance, σ_v and σ'_v are the current total and effective stress levels respectively.

Fig. 4 Interpretation of cone penetrometer test results

Experimental plan

Five distinct types of footing tests were conducted in the experimental programme: vertical penetration, vertical load-unload loops, swipe, monotonic radial displacement test and combined radial displacement test. The vertical penetration tests were used to study the variation of the size of the yield surface, V_0 , with spudcan penetration and the load-unload tests the relevant elastic behaviour.

Tan (1990) introduced the swipe test for a study of conical footings on sand under combined loadings. In the swipe tests of this study, the footing was vertically penetrated to a prescribed depth where the targeted stress level was achieved (≈ 350 kPa). Then, the footing was subjected to a series of combined horizontal displacement and rotational excursions. Radial displacement tests were used to investigate the flow rule, in which the footing was subjected to different combinations of vertical and horizontal displacements or vertical and rotational displacements. It has to be noted that due to the limited testing space, in the paper no yield surface variation with V_0 has been investigated. The detailed experimental plan is listed in Table 3. Vertical loading tests and swipe tests were already presented in Cheng and Cassidy (2016), where a comprehensive interpretation is also given in terms of the determination of the yield surface and additional sliding surface. This paper is mainly relevant to the development of force-resultant model, presentation of radial displacement tests, calibration of the flow rule and model assessment through retrospective analysis of selected experimental data.

200 **Table 3 Details of the experimental plan**

Test type	Test name	V (N)	V/A (kPa)	$u/2R\theta$	w (mm)	u (mm)	θ (Radians)	Figures
Vertical penetration	SVP1	4434	1568	-	25.8	-	-	Fig.5
Load-unload loops	SVP2	3810	1348	-	19.8	-	-	Fig.5
Swipe tests ¹	SW1	1287	455	∞		9	0	Fig. 6-11
	SW2	966	342	0.531		5	0.157	
	SW3	1150	407	0.095		0.9	0.157	
	SW4	903	319	-0.095		-0.9	0.157	
	SW5	1047	370	-0.286		-2.7	0.157	
	SW6	1035	366	-0.572		-5.4	0.157	
	SW7	1197	423	-1.145		-9	0.131	
Monotonic Radial displacement tests	RD1	1822	644	-	15	15	-	Fig. 14,15
	RD2	2385	843	-	15	4.5	-	
	RD3	2152	761	-	15	9	-	
	RD4	1654	585	-	8.5	-	0.148	
	RD5	748	265	-	4.5	-	0.157	
	RD6	3240	1146	-	17.7	-	0.154	
Combined radial displacement tests	RD7	1128	399	0.102	9	9	0.157	Fig. 14,15
	RD8	1793	634	0.971	12	6	0.105	

201 1. Swipe test results also reported in Cheng and Cassidy (2016)

202 Results and discussion

203 In the following section, the results from the experiments are described with respect to the four major
 204 components required for the strain-hardening plasticity model.

205 Hardening law

206 The strain-hardening expression, known as the hardening law, is used to define the variation of the
 207 size of the yield surface. For most shallow foundation models, the size of the yield surface is
 208 reasonably deemed as a function of the plastic component of the vertical displacement (Gottardi et al.
 209 1999; Byrne and Houlsby 2001; Cassidy et al. 2002a; Houlsby and Cassidy 2002). One of the key
 210 assumption of the strain-hardening plasticity theory is that when the footing is pushed into soil, the
 211 shape of the yield surface is assumed constant; however, the size expands with the amount of plastic
 212 vertical displacement (w_p). Shown in Fig. 5 are the vertical loading response of two vertical loading
 213 tests SVP1 and SVP2 which show consistency in the experimental programme. From the unload-
 214 reload loops of SVP2 an average elastic stiffness K_v was established as 2.4 kN/mm. The stiffness of
 215 the loading system was taken into consideration through a correction process. In this process,
 216 additional plastic penetration can be calculated for a given change in vertical load and measured

217 actual vertical displacement. If the form of the hardening law is known (such as Equation (2)), then a
 218 correction can be applied to update V_0 from its value at the beginning of the test.

219

220 **Fig. 5 Vertical penetration and load-unload loops**

221 An empirical strain-hardening expression is used to define the expansion and contraction of the yield
 222 surface with the plastic component of vertical displacement. With the elastic vertical displacements
 223 removed, a function previously used to fit carbonate sand experimental data (Cassidy et al. 2002a)
 224 was again used here to fit a hardening law:

$$225 \quad V_0 = \frac{cw_p + k_2w_p^2}{k_1 + w_p} \quad (2).$$

226 The best fit model parameters are found to be $c = 278 \text{ N}$, $k_1 = 0.15 \text{ mm}$, $k_2 = 0.04 \text{ N/mm}$. No special
 227 significance is attached to this equation except that it represents a good fit to the vertical load-
 228 penetration response. Alternative expressions could also be appropriate.

229 Elastic stiffness

230 Increments within the yield surface are assumed to be elastic and follow:

$$231 \quad \begin{Bmatrix} dV \\ dM / 2R \\ dH \end{Bmatrix} = 2GR \begin{bmatrix} k_V & 0 & 0 \\ 0 & k_M & k_C \\ 0 & k_C & k_H \end{bmatrix} \begin{Bmatrix} dw \\ 2Rd\theta \\ du \end{Bmatrix} \quad (3),$$

232 where G is a representative elastic shear modulus and k_V , k_M , k_H , k_C are dimensionless elastic
 233 constants for elastic behaviour within the yield surface, and (dV, dM, dH) and $(dw, d\theta, du)$

vectors of incremental load and displacement respectively. The elastic coefficients have been defined using finite element methods by Bell (1991), Ngo-Tran (1996) and Doherty and Deeks (2003), and appropriate values for circular foundations, as listed in Doherty and Deeks (2003) are:

$$k_V = 2.904; k_M = 0.548; k_H = 2.901; k_C = -0.208.$$

The shear modulus G was estimated by the following formula, suggested by Houlsby and Cassidy (2002):

$$\frac{G}{p_a} = g \sqrt{\frac{V}{Ap_a}} \quad (4),$$

where p_a is atmospheric pressure, V is a representative vertical load on the foundation, A is the plan area of the foundation and g is a dimensionless constant. This latter value of g can be back calculated from the unload-reload test SVP2 as 44.2 (representing an average G of 13.8 N/mm² and $k_V = 2.904$). Without measured data of the stiffness Cassidy et al. (2002b) recommend a g value of approximately 400 for practical offshore conditions.

Yield surface

Following Tan (1990) and Martin and Houlsby (2000), swipe tests were used to explore the yield surface shape. Fig. 6 shows the recorded load path of swipe test SW2 with $M/2R:H \approx 1.5$ in three-dimensional space and its projected view in corresponding the $V:H$, $V:M/2R$ and $H:M/2R$ planes. The colour bar in Fig. 6(a) represents the $M/2R:H$ ratio on each test point; as shown in the figure, despite some changes occurring at the initial stage, the $M/2RH$ ratio is consistent throughout the test. Another interesting trend that can be observed from Fig. 6 (d) is the sliding failure mechanism occurred at the $V/V_0 \approx 0.25$ where the horizontal force continued to increase while moment load kept constant. This behaviour was not observed from previous centrifuge studies when only the horizontal displacement path was applied.

Fig. 6 Swipe test SW2 in three-dimensional and corresponding projected views

Each swipe test follows one particular $M/2RH$ ratio corresponding to one specific load path on the yield surface. Fig. 7 presents all swipe tests collapsed into one three-dimensional space. It can be observed that as moment increases (from SW1 to SW7), the sliding failure mechanism becomes more pronounced. Cassidy (2007) reported that a critical state occurred at $V/V_0 \approx 0.3$. Clearly, this is not supported in the current study, as the critical state point occurs at different V/V_0 ratios for different moment to horizontal load ratios. This is demonstrated in Fig. 7. Due to the three-dimensional nature of the sliding surface that intersects with the yield surface the location of the transition point cannot be determined solely on the V/V_0 ratio but also depending on the ratio of moment to horizontal load.

Fig. 7 All swipec tests in $V/V_0:M/2RV_0$ space

Fig. 8 Representative swipec tests projected on the $V:H$ and $H:u$ plane

Fig. 8 shows three representative swipec tests projected on the $V:H$ plane and their corresponding load-displacement curves. SW1 is a conventional, pure translation test, which can be observed from most previous studies. It should be noted that the $M/2R:H$ ratio increases from test SW1 to test SW3. In SW2 and SW3, after a certain point, V starts to increase rather than decrease as in test SW1. This transition point varies depending on the $M/2R:H$ ratios which means both the horizontal load and the moment load play an important role in determining this point. Gottardi et al. (1999) and Byrne and Houlsby (2001) derived a three-dimensional surface from the experimental swipec tests in a 1 g environment. This surface could be defined as

$$f = \left(\frac{H}{h_0 V_0} \right)^2 + \left(\frac{M/2R}{m_0 V_0} \right)^2 - \frac{2\alpha HM/2R}{h_0 m_0 V_0^2} - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1 + \beta_2)}}{\beta_1^{\beta_1} \beta_2^{\beta_2}} \right]^2 \left(\frac{V}{V_0} \right)^{2\beta_1} \left(1 - \frac{V}{V_0} \right)^{2\beta_2} = 0 \quad (5),$$

where V_0 denoted reference vertical load and representing the apex of the yield surface determined by the hardening law such as Equation (1). Parameters h_0 and m_0 define the extent of the yield surface in the moment and horizontal directions respectively. The shape factors α effects the eccentricity of the elliptical cross-section of the yield envelope in the $H : M/2R$ plane. The factors β_1 and β_2 determine the position of the maximum size of the elliptical section between $V = \beta_2 V_0 / (\beta_1 + \beta_2)$ and $V = V_0 / 2$ (Nova and Montrasio 1991). Previous studies have established various sets of parameters for flat circular footing on sand. For comparison, those parameters are listed in Table 4 along with the parameters identified in the current study (through linear least squares fitting).

Table 4. Comparison of yield surface shape parameters

	Byrne and Houlsby (2001)	Houlsby and Cassidy (2002)	Bienen et al. (2006)	This study
h_0	0.154	0.116	0.122	0.113
m_0	0.094	0.086	0.094	0.096
α	-0.25	-0.2	-0.25	-0.248
β_1	0.82	0.90	0.76	0.71
β_2	0.82	0.99	0.76	0.99
Sand type	Carbonate	Very dense	Very loose	Loose
g level	1	1	1	100

Fig. 9 shows a great consistency of these parameters when compared to the experimental data and the proposed yield surface normalised in the $H : M/2R$ plane.

Fig. 9 Comparison between the analytical VHM yield surface and the experimental data for swipe tests

The overall quality of the fitting was defined by introducing the quantity

$$Q^2 = \left(\frac{H}{h_0} \right)^2 + \left(\frac{M / 2R}{m_0} \right)^2 - \frac{2\alpha HM / 2R}{h_0 m_0} \quad (6).$$

Fig. 10 shows the load paths of all swipe tests projected onto the normalised $V : Q$ plane, validating the best-fit parameters previously mentioned. It has to be noted that all the load paths that deviate from the yield surface has been removed in this figure for data fitting purpose as Equation (6) was not intended for describing sliding phenomena.

Fig. 10 Swipe load path against Equation (6) in the normalised $H:M$ plane

When all swipe tests are plot in three-dimensional space, it becomes clear that the load paths that diverge from yield surface lie on one surface. This surface could be described in the form of

$$h = \frac{1}{\tan \phi} \left(\frac{H}{V_0} \right) - 4 \left(\frac{M / 2R}{V_0} \right) + \frac{V}{V_0} = 0 \quad (7),$$

by assuming a simple Coulomb failure criterion for pure shearing occurred beneath the footing. More details of the formulation are provided in Cheng and Cassidy (2016). The mobilised internal friction angle ϕ during the centrifuge tests was determined from a iterative procedure developed by White et al. (2008). In this study, ϕ was estimated to be 34.3° (see, Cheng and Cassidy (2016)). With ϕ determined, the theoretical sliding surface is defined. Fig. 11 demonstrates the proposed complete form of the yield surface supplemented with the sliding surface in three-dimensional space. This three-dimensional sliding surface intersects the conventional yield surface with certain angles: in $V:H$ plane this angle is determined by mobilised internal friction angles $\tan \phi$ while in $V:M$ plane this angle is of a fixed value. Further, the intersection lines between the yield surface and sliding surface clearly defines all the transition points.

Fig. 11 Yield surface intersected with sliding surface in three dimensional space

In order to evaluate the performance of this integrated yield-sliding surface model, the proposed yield surface along with the sliding surface were sliced cut with the $M/2RH$ ratio taken from the experiments then projected to $V:H$ and $V:M$ planes respectively in two dimensions.

Fig. 12 shows a great consistency when compare the experimental measured value of V/V_0 at the parallel and transition point and the corresponding $M/2RH$ ratio against those values predicted from the theoretical yield surface (Equation (5)) and sliding surface (Equation (7)). The solid blackline represents the model predictions with values greater than 0 while the redline represents those values less than 0. Both lines were derived by solving Equation (5) and (7) simultaneously.

Fig. 13 shows the comparison between experimental measured swipe tests SW3 with the theoretical yield surfaces proposed in previous studies along with the sliding surface derived above. Equation (5) with the parameters derived in this study provides the best fit. Bienen et al. (2012) parameters slightly under estimate both the horizontal and moment load capacities, while the parameters of Byrne and Houlsby (2001) failed to exactly locate the transition point between surfaces of Equation (5) and (7). It is interesting to note that after the transition point, the consecutive load path tracks along the sliding surface resulting in an increase in vertical load. Cassidy (2007) also reported the existence of such a transition point occurred at $V/V_0 \approx 0.3$ for flat circular foundation. However, it has to be emphasised that, this transition point can vary from $V/V_0 \approx 0.45$ to $V/V_0 \approx 0.05$, depending on the ratio of moment to horizontal load due to the three-dimensional nature of the sliding surface. This is explained with the data of this paper and the fit of Equation (5).

Fig. 12 Comparison between the experimental measured V/V_0 and $M/2RH$ values and model predictions

Fig. 13 Comparison between the experimental and theoretical yield surface and sliding surface

Flow rule

The prediction of footing displacement during yield requires a suitable flow rule. If the plastic flow vectors are perpendicular to the yield surface, then associated flow is assumed. However, Gottardi et al. (1999) found that for flat circular footings on sand the associated flow rule only models the ratios between plastic displacements in the $(H : M/2R)$ plane but not in the $(V : M)$ or $(V : H)$ planes. Associated flow rule under-predicts the level of the vertical displacements in these deviatoric planes, and this has been a consistent finding since, including in the experiments of Gottardi et al. (1999); Houlsby and Cassidy (2002), and Bienen et al. (2006). Houlsby and Cassidy (2002) and Cassidy et al. (2002a) defined a non-associated plastic potential based on the yield surface shape, but defined as

$$g = \left(\frac{H}{\alpha_h h_0 V_0'} \right)^2 + \left(\frac{M/2R}{\alpha_m m_0 V_0'} \right)^2 - \frac{2\alpha H M / 2R}{\alpha_h \alpha_m h_0 m_0 V_0'} - \beta_{34}^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} = 0 \quad (8),$$

where

$$\beta_{34} = \frac{(\beta_3 + \beta_4)^{(\beta_3 + \beta_4)}}{\beta_3^{\beta_3} \beta_4^{\beta_4}}$$

where α_h and α_m are non-dimensional horizontal and moment “association” factors, and V_0' is a dummy parameter specifying the intersection of the plastic potential with the V -axis. The parameters β_3 and β_4 are chosen to allow for different variations in the curvature. If $\alpha_h, \alpha_m = 1, \beta_3 = \beta_4 = 1$, the plastic potential coincides with the yield surface and an associated flow is implied. If α_h and α_m are constant and equal, Equation (12) can be rewritten as

$$g = \left(\frac{H}{h_0 V_0'} \right)^2 + \left(\frac{M / 2R}{m_0 V_0'} \right)^2 - \frac{2\alpha HM / 2R}{h_0 m_0 V_0'} - \alpha_v^2 \beta_{34}^2 \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} = 0 \quad (9).$$

where $\alpha_v = \alpha_h = \alpha_m$

As two different governing equations exist for describing yield surface (Equation (5)) and sliding surface (Equation (7)), the load paths that were tracing two surfaces are divided into two sections and were therefore analysed separately. If the associated assumption applies, according to plasticity theory the ratio of plastic displacement to rotation on yield surface and sliding surface can be evaluated as

$$\frac{\delta u_p^{(yield)}}{\delta \theta_p 2R} = \frac{\partial f / \partial H}{\partial f / \partial (M / 2R)} = \frac{H 2R \frac{m_0}{h_0} - \alpha M}{M \frac{h_0}{m_0} - \alpha H 2R} \quad (10),$$

and

$$\frac{\delta u_p^{(sliding)}}{\delta \theta_p 2R} = \frac{\partial h / \partial H}{\partial h / \partial (M / 2R)} = \frac{1}{-4 \tan \phi} \quad (11),$$

(a)

(b)

Fig. 14 Plastic flow on the $H:M$ plane for (a) yield surface and (b) sliding surface

Fig. 14(a) proves that associated flow is valid on yield surface in $H:M/2R$ plane by plotting the experimental measured plastic displacement ratios against the analytical flow rule predictions. However a non-associated behaviour was observed on sliding surface as showing in Fig. 14(b). For the benefit of future modelling requirements, a plastic potential characterising the flow rule on the sliding surface was proposed through a modification made to Equation (7) as

$$j = \gamma \frac{1}{\tan \phi} \left(\frac{H}{V_0'} \right)^2 - 4\lambda \left(\frac{M/2R}{V_0'} \right)^2 + \frac{V}{V_0'} = 0 \quad (12).$$

Similarly, λ and γ are the non-dimensional “association” factor, and V_0' is a dummy parameter. In this study, the best fit non-dimensional association factors γ and λ were found to be $\tan \phi$ and -0.5 respectively. This is shown in Fig. 14(b).

On the $V:H$ and $V:M/2R$ planes, monotonic radial displacement tests were performed to evaluate the flow rule. Further analysis of the experimental indicates that the associated assumption was not supported in the deviatoric planes, therefore, Equation (9) can be further derived that

$$\frac{\partial g}{\partial V} = (\alpha_v \beta_{34})^{\frac{1}{\beta_{34}}} \left(\frac{V}{V_0'} \right)^{\left(\frac{\beta_3}{\beta_4} - 1 \right)} \left(\frac{V}{V_0'} - \frac{\beta_3}{\beta_4} \left(1 - \frac{V}{V_0'} \right) \right) / V_0' \quad (13),$$

$$\frac{\partial g}{\partial H} = \frac{1}{\beta_4} \left(\left(\frac{Q'}{V_0'} \right)^2 \right)^{\left(\frac{1}{2\beta_4} - 1 \right)} \left(\frac{H}{h_0 V_0'} - \frac{eM/2R}{\alpha_m m_0 V_0'} \right) / h_0 V_0' \quad (14),$$

$$\frac{\partial g}{\partial M/2R} = \frac{1}{\beta_4} \left(\left(\frac{Q'}{V_0'} \right)^2 \right)^{\left(\frac{1}{2\beta_4} - 1 \right)} \left(\frac{M/2R}{\alpha_m m_0 V_0'} - \frac{eH}{h_0 V_0'} \right) / \alpha_m m_0 V_0' \quad (15),$$

$$\frac{\delta q_p}{\delta w_p} = \frac{\partial g / \partial Q}{\partial g / \partial V} = (Q' / V_0') / (\alpha_v \beta_{34}^2) \left(1 - \frac{V}{V_0'} \right)^{2\beta_4} \left(\frac{V}{V_0'} \right)^{2\beta_3} \left(\frac{\beta_4}{1 - \frac{V}{V_0'}} - \frac{\beta_3}{\frac{V}{V_0'}} \right) \quad (16),$$

$$\beta_{34} = \frac{(\beta_3 + \beta_4)^{(\beta_3 + \beta_4)}}{\beta_3^{\beta_3} \beta_4^{\beta_4}}$$

$$\text{where } Q' = \sqrt{\left(\frac{H}{\alpha_h h_0} \right)^2 + \left(\frac{M / 2R}{\alpha_m m_0} \right)^2} - \frac{2\alpha H M / 2R}{\alpha_h \alpha_m h_0 m_0} \quad (17).$$

Therefore, $\delta q_p / \delta w_p$ can be calculated. δq_p is the plastic displacement in the general deviator plane, as defined in Equation (6). An optimization scheme was adopted here to numerically adjust the values of α_v , β_3 and β_4 so that the difference between the predicted $\delta q_p / \delta w_p$ values and the experimental measured $\delta q_p / \delta w_p$ values at a given Q/V ratio was minimised. This is demonstrated in Fig. 15 as the proposed non-associated flow rule represents most of the experimental data. The following values were regarded as the best fit in this study: $\alpha_v = 2.7$, $\beta_3 = 0.65$, $\beta_4 = 0.65$.

Fig. 15 Associated and best fit non-associated flow against the measured experimental data

390 Table 5 Refined model parameters for Model C taking into account spudcan shape and high-g experiments of this study

Constant	Dimension	Explanation	Typical values	Notes
G	F/L ²	Representative shear modulus		$G = 13.8 \text{ MN/m}^2$ used in simulation
k_V	-	Elastic stiffness factor (vertical)	2.904	As defined by (Doherty and Deeks 2003)
k_H	-	Elastic stiffness factor (horizontal)	2.901	
k_M	-	Elastic stiffness factor (moment)	0.548	
k_C	-	Elastic stiffness factor (horizontal/moment coupling)	-0.208	
h_0	-	Dimension of yield surface (horizontal)	0.113	Maximum value of H / V_0 on $M = 0$
m_0	-	Dimension of yield surface (moment)	0.096	Maximum value of $M / 2RV_0$ on $H = 0$
α	-	Eccentricity of yield surface	-0.248	
β_1	-	Curvature factor for yield surface (low stress)	0.71	$\beta_1 \leq 1.0, \beta_2 \leq 1.0$
β_2	-	Curvature factor for yield surface (high stress)	1	
β_3	-	Curvature factor for plastic potential (low stress)	0.65	$\beta_3 \leq 1.0, \beta_4 \leq 1.0$
β_4	-	Curvature factor for plastic potential (high stress)	0.65	
α_v	-	Association factor (vertical)	2.7	
c	F	Hardening law parameters	278	Fitted from experimental data
k_1	L	Hardening law parameters	0.15	
k_2	F/L	Hardening law parameters	0.04	

Retrospective analyses

The plasticity model of this study has been implemented into a FORTRAN programme to demonstrate that the refined Model *C* can be numerically implemented and used to simulate footing behaviours. To investigate the capacity of this programme to predict footing behaviour, numerical back analysis was carried out for a number of representative experiments.

Fig. 16 Comparison between experimental data from test SVP2 and programme prediction

Fig. 16 shows the experimental results for a vertical load-unload loop test, SVP2, compared with a simulation of this test in which the measured displacement is taken as input, and the vertical load is calculated. Model *C* with refined parameters produces a load that accurately represents the original test, indicating that the chosen strain-hardening law and elastic stiffness are suitable.

Fig. 17 Comparison between the experimental test, SW3, and the programme prediction

Fig. 17 represents a combined swipe test with $u/2R\theta = 0.095$ starting at $V \approx 1100N$ (SW3). The results are plotted in both the $V:H$ and the $V:M$ planes. In the numerical simulation, Model C is load-controlled to $V \approx 1100N$ and then displacement-controlled for the swipe. Model C simulates the magnitude of the peak horizontal load adequately, reaching a value $H = 28N$, but the peak moment is slightly overestimated. Model C locates the ‘parallel point’ perfectly in the $V:H$ plane; however, it again over- estimates in the $V:M$ plane. The difference between the experimental tests and simulation results in the early stage of the swipe test is due to a significant component of deviatoric load developed during the vertical loading phase of the swipe tests. This deviatoric load is due to the nature of displacement controlled test. Similar results can also be found in other studies, such as Zhang et al. (2013).

Fig. 18 Comparison between the experimental test, SW2, and the programme prediction

Fig. 18 represents the swipe test, SW2 with $u/2R\theta = 0.531$ starting at $V \approx 950N$. Model C overestimates the horizontal load but the simulation stops tracking at approximately the same horizontal and vertical load, indicating an accurate prediction of the ‘parallel point’ in the horizontal plane. For the equivalent test in the $V : M$ plane, the refined Model C slightly underestimates the peak moment but locates the ‘parallel point’ perfectly well. Further evidence that supports the model’s performance are also provided in Fig. 19 and Fig. 20 for tests SW1 and SW7 respectively.

Fig. 19 Comparison between the experimental test, SW1, and the programme prediction

Fig. 20 Comparison between the experimental test, SW7, and the programme prediction

Constant gradients of the horizontal to vertical and the moment to vertical displacements were used as inputs to simulate the horizontal and moment radial displacement tests. The refined Model C predictions and the corresponding experimental horizontal and vertical loads are shown in Fig. 21. The simulation is of a similar gradient, implying that the flow rule of modified Model C is performing well.

Fig. 21 Comparison between the experimental test RD1, RD2, RD3 and the programme prediction

Conclusions

In this paper, experimental results for an extensive research programme carried out in a drum centrifuge at 100- g are presented and interpreted within the strain-hardening plasticity framework. A three-dimensional sliding surface was observed in the experiments, and an analytical equation was subsequently proposed to describe this sliding surface. Comparison between this new yield-sliding analytical model and the experimental data demonstrates good agreement. The well-established three-dimensional strain-hardening plasticity model (Model C) that can be used to describe flat circular footings has been experimentally recalibrated and refined for a spudcan footing resting on loose sand. Plasticity analysis shows that in the $H : M/2R$ plane, associated flow is valid for the yield surface, but non-associated flows were observed on the sliding surface. Consequently, a non-associated equation was proposed for the flow rule to capture this behaviour. Back-analysis of the experimental observations showed that the force-resultant model adequately captures the main features of the spudcan footing's behaviour although it remains conservative. The outcome from this study reveals the behaviour of a spudcan footing subject to combined loadings on loose sand under stress levels relevant to the offshore industry. The development of a complete strain-hardening plasticity model based on centrifuge observations has been achieved, adding further evidence to a well-established and widely used framework. This numerical model can be coupled with existing structural model thus an integrated analysis of structure-soil interactions can be performed. Further, a new sliding surface was observed in the tests and therefore added into the framework which completes the theory in describing footing's behaviour on frictional material under combined loadings. The need to model the load/displacement path of footing beyond the transition point reveals footing's true behaviour on

454 frictional material under combined loadings. Without recognition of this sliding failure mechanism
455 leads to unconservative assumptions in practice. In response, this study provides greater confidence in
456 the sued of strain-hardening plasticity models in the assessment of jack-up platforms.

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