Simulating the effectiveness of wave dissipation by FollowerStopper Autonomous Vehicles

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Abstract

The traffic jam phenomenon known as stop-and-go waves is commonplace on many roadways and is unavoidable due to the imperfect nature of human driving behaviour. Previous research by Stern et al. (2018) has demonstrated experimentally that a single Autonomous Vehicle (AV) can be harnessed to dissipate stop-and-go waves produced by 20 other passenger vehicles driving in a cycle. However, the experiment was conducted in an idealistic situation of a single-lane ring road with one AV; meaning no lane changing was possible and multiple AV interaction was left untested. To address these limitations, the AV driving behaviour used to achieve this, known as FollowerStopper (FS), was modelled for use in the Aimsun traffic simulation software and validated before further numerical experiments were conducted. The microsimulation scenarios were designed to observe how multiple AVs driven by the FS controller (referred to as FS-AVs in this paper) affect their traffic wave dissipation capability. Both single and double-lane ring roads experimental designs were used with the latter capturing the untested effect of lane changes. FS-AV was found to be less effective than originally documented in both cases. Multiple FS-AVs included within one lane were still able to improve traffic flow and dissipate the stop-and-go waves although it is sensitive to the combination of different factors and any deviation away from the ideal condition is likely to produce worse-off traffic operations in terms of traffic flow. For double-lanes, human-driven vehicles (HDVs) would change lanes at heightened rates to occupy the larger gap the FS-AV needs for its dissipating strategy, causing it to further pull back and set off a chain reaction to upstream traffic. Stop-and-go waves had not been dissipated and decreased traffic performance in terms of flow, speed and delay time resulted. Our results also showed that the wave dampening effect of the FS-AV does not translate between the ring road and its linear equivalent. On the other hand, our preliminary simulation results using a real-life freeway model did not suggest multiple FS-AVs worsen traffic conditions. The contradicting results might be due to different traffic conditions such as vehicle density. Further research is required to investigate what factors could adversely affect FS-AV’s ability to dampen shockwaves and explore possible improvement to its driving algorithm.

Keywords: Autonomous vehicle, Stop-and-go waves, Traffic wave dissipation

1. Introduction

1.1 Background and motivation

Autonomous Vehicles (AVs) are set to cause the largest disruption to the transport industry since the invention of the car itself. Due to the vast array of AV technology and research available, it is not currently clear which will be used and how our road networks will be designed to accommodate them.
It is important to decipher which technologies would improve traffic and their suitability for implementation. To do this, thorough examination and testing needs to be conducted so we can better prepare for their introduction.

Many can envision AVs being the prominent transportation mode of the future but there will be an inevitable transition period in which AVs are mixed with human-driven vehicles (HDVs) (Sun et al., 2017). Not only will AVs be able to provide unassisted transport, they may also be capable of improving traffic operations and alleviating a certain traffic jam phenomenon known as stop-and-go waves. These traffic waves are detrimental to traffic performance and cause wasteful increases in fuel consumption and air pollution (Stern et al., 2019).

For the purpose of longitudinal driving, AVs exhibit similar driving behaviour to Adaptive Cruise Control (ACC) models (Sun et al., 2017). Previous research has investigated the ways ACC vehicles affect traffic using microscopic driving models which may be inferred to reveal the impact AVs will have. Bose and Ioannou (2001, 2003) found that when ACC vehicles were mixed in with human driven traffic, they were able to smooth the traffic flow by filtering the response of rapidly accelerating vehicles ahead of them. Advancements from Kesting et al. (2006) went on to harness the intelligent driving model (IDM) (Treiber et al., 2000) to perform microscopic multi-lane simulations for the mixed traffic flow state. They found that a market penetration of 10% ACC vehicles made drastic improvement to both traffic flow and the reduction of travel time (Kesting et al., 2006). Kesting et al. (2007a) extended this model by considering five different traffic states that the ACC vehicle would be in. Using the IDM model, parameters values were changed to match the traffic situation with the intention of steadying the traffic flow (Kesting et al., 2007a). It was found that as little as 5% ACC would improve the traffic flow and reducing the travel times (Kesting et al., 2007a). This was supported by another study from Kesting et al. (2007a) which showed that a market penetration of 30% ACC could even prevent stop-and-go waves from occurring (Kesting et al., 2007b). It is important to note that driving models in these studies did not possess any form of jam dissipating control method. They merely examined their ACC driving models on such traffic states.

More recently, Stern et al. (2018) have experimentally shown that AV driving algorithms can be specifically designed to dampen stop-and-go waves. Stern et al. (2018) created and tested two types of automated vehicle control strategies called FollowerStopper (FS) and the PI with saturation controller. They used the same experimental environment as Sugiyama et al. (2008), who first documented stop-and-go waves emerging from human driving behaviour alone. It consisted of a single-lane ring road with no other infrastructure or geometric considerations present. Experiments entailed one FS or PI with Saturation AV included with 20 or 21 human driven vehicles in the ring road to see if the stop-and-go wave that initially formed from all human driven vehicles would be dissipated. Both AV behaviours were shown to be effective but FS performed better in reducing excessive braking and fuel consumption whilst improving throughput (flow) on the road.

Aside from human drivers’ poor estimation of acceleration or braking needed during car following (Sugiyama et al., 2008; Yeo and Skabardonis, 2009), lane changing is another identified behavioural trigger of stop-and-go waves (Ahn and Cassidy, 2007; Laval and Daganzo, 2006). When lane changes occur, the following vehicle may have to decelerate to generate space between itself and the newly merged vehicle. If the vehicle density is sufficiently high, this can act as the perturbation that instigates the traffic wave (Yeo and Skabardonis, 2009).

Smith’s (1985) empirical findings revealed that when lane changes occur a relaxation phenomenon is observed amongst human drivers. Vehicles initially accept smaller gaps before eventually returning to
a more comfortable spacing 20–30 seconds later (see Fig. 1). In essence, this behaviour absorbs the perturbation and smoothens it across its journey back to its preferred headway. This behaviour may be currently limited to human drivers as they anticipate multiple vehicles ahead and are able to make predictive assumptions about the near future traffic state (Treiber, M. Kesting, 2013).

![Stable flow](image1)

![Vehicle cut-in](image2)

![Pullback](image3)

**Fig. 1.** Relaxation phenomenon after a vehicle cut-in, causing a pullback of the following vehicles to a more comfortable distance.

Ioannou and Stefanovic (2005) have studied the effects of unwanted cut-ins in a mixed flow involving ACC and human-driven vehicles (HDVs). Their experiment involved analysing the effects of cut-ins from HDVs in front of the ACC vehicle. They showed that because of the larger gap the ACC following behaviour exhibits, and the smoother driving response, the perturbations caused from the lane change are dampened (Ioannou and Stefanovic, 2005). Their experiment deemed that ACC vehicles were more adept at handling lane changes, though the congested traffic state was not tested.

The Autonomous Vehicle equipped with a FS controller (FS-AV) initially allows larger gaps to open up as part of its strategy to dissipate the wave. Hence Stern et al. (2018) conceived the possibility of lane changes posing a threat to the controllers effectiveness. They explained the central pitfall in a multi-lane setting being the size of the gaps left by the FS-AV may induce lane changes, though had pre-emptively dismissed this would occur as it did not leave a large gap once the wave had been dissipated. Such claims need to be verified by more rigorous testing and analysis. However, field testing would require introducing multiple lanes to the ring road and unwanted cut-ins in a close range, which is difficult and potentially dangerous to do in a field experiment. We therefore resorted to microsimulation models. Well-calibrated models offer a safe and cost-effective environment to test multiple scenarios that can provide meaningful insights.

Using the Aimsun software package ([https://www.aimsun.com/](https://www.aimsun.com/)), we have successfully developed a microsimulation model for the FS controller (Section 2). It has closely reassembled the field experiment data reported by Stern et al. (2018) (Section 3) and therefore deemed to be fit-for-purpose for testing various scenarios.
Stern et al. (2018) has empirically demonstrated the remarkable effectiveness of one FS-AV’s ability to dissipate the waves generated by HDVs. Nevertheless, before extending the conclusion to “flow control will be possible via a few mobile actuators” (Stern et al., 2018, p.205), there is a need to examine how multiple FS-AVs perform in the traffic stream instead of the assumed interpolation. This paper first extends the original research by comparing the performance of multiple FS-AVs in the single-lane ring road (Section 4.1 & 5.1).

It is followed by testing the impact of lane changing in a two-lane ring (Section 4.2 & 5.2). We hypothesised that the larger gap left by the FS-AV invites lane changing in front of it, especially when density is high. How humans might interact with AVs is currently unclear, although it is conceivable that they might drive aggressively towards them and take advantage of these “law-obeying” machines with no temper, especially if they appear to be slow-driving by leaving a larger gap in front of themselves. This is expected to translate into an increased amount of lane changes by HDVs to fill in the gap left by the FS-AVs. We also hypothesised that when this happens, the FS-AV will immediately react by pulling back further to maintain its preferred gap because it cannot exhibit the relaxation phenomenon known to human drivers (Fig. 1). Its abrupt braking action might consequently trigger a shockwave behind it if the following traffic has a sufficiently high density, which is the opposite of what it is intended to achieve.

Section 4 documents the methodologies used in both single-lane and double-lane numerical experiments, the results of which are discussed in Section 5. The latter also includes discussions of why conclusions drawn from the ring-road cannot be extrapolated to linear roads. Section 6 concludes the paper and proposes further investigations.

If the microsimulation experiments support our hypothesis, human drivers’ aggression might compromise the effectiveness of well-intended FS vehicles. For this technology to be realised commercially, the driving algorithm needs to be extended to cope with more realistic traffic conditions. This research will improve the understanding of incorporating AVs with jam dissipating technology into modern day traffic. From this, future AV shockwave absorption behaviour can account for the effects found and be used to develop an optimum driving behaviour to dampen the stop-and-go waves.

2. Modelling FS-AVs in Aimsun

This section summarises how we modelled FS, which inevitably repeats some part of Stern et al. (2018). It is advised to read it in conjunction with the original paper.

FS driving behaviour was coded in C++ programming using Aimsun’s microSDK module. The FS car-following behaviour governing equations are shown in Appendix A and are reproduction from Stern et. al. (2018). In Aimsun microsimulation, each vehicle is modelled as an agent and its behaviour including interaction with others and the road environment is explicitly simulated. At every time step, each vehicle’s position and velocity are updated as well as any other decisions such as lane changing. The reaction time of vehicles is also accounted for in the form of time delay. A few issues translating the FS driving behaviour into Aimsun had presented themselves with some minor adjustments necessary to overcome them, as described in the following subsections.

2.1 Insufficient distance sampling rate

A constraint unavoidable in Aimsun’s microsimulations is the minimum simulation time step being 0.1 seconds. Therefore, if all vehicles update every 0.1 seconds, the AVs can only calculate the
distance between itself and the lead vehicle $\Delta X$ (1), at a sample rate of 10 Hz. With Stern et al.'s (2018) experiment, their FS-AV was able to calculate $\Delta X$ at a sampling rate of 30 Hz. As the AV estimates the change in velocity $\Delta V$ between itself and the lead vehicle using $\Delta X$, a more frequent sampling rate yields a more accurate $\Delta V$. The sampling rate of 10 Hz in Aimsun for $\Delta X$ proved inadequate for sufficient calculation of $\Delta V$, causing the AV to travel in a jerkier motion.

$$\Delta V = \frac{d}{dt} \Delta X = V^{lead} - V^{AV}$$

(1)

To address this issue, it was assumed that the AVs distance sensor was ideal and would lead to the estimation of $\Delta V$ to be the actual value. This was implementable in Aimsun using the microSDK functions to obtain the lead vehicles velocity $V^{lead}$, as well as the AVs current velocity $V^{AV}$, to then calculate $\Delta V$. Although the sampling rate could not be increased, the value of $\Delta V$ was still able to be calculated accurately and a smoother driving profile was obtained.

2.2 Acceleration & deceleration control heuristics

In essence, the FS controller uses the combination of $\Delta V$ and $\Delta X$ to determine its new commanding speed $V^{cmd}$ (Fig. 2). Three curves each representing a constant decelerating rate divides the decision space into four regions. The goal is to achieve its desired speed $U$ when it is safe to do so, which is what Safe Region covers. Otherwise, it will make a compromise between 0 and $U$ depending on the lead vehicle, as shown in the other three regions.

Additional control heuristics were implemented to mimic the AVs acceleration and deceleration. The real AV uses actuators to control accelerator and brake pedals at a variable level which in turn regulate its movement. In our agent-based model, the driving algorithm to account for this is simplified, though achieving analogous results. The control heuristics included a maximum limit on how quickly the AV could accelerate and use of interpolation techniques to overcome oscillatory commanded vehicle speed when decelerating.

At each simulation time step the AV would update its new velocity and position based on Fig. 2. If the AV’s new commanded velocity, when compared to the last simulation step’s velocity meant an acceleration larger than its maximum value, then its maximum velocity associated with that acceleration is commanded, as shown in (2). The maximum acceleration level of the AV was set to 2.5 m/s², consistent with the maximum value humans are comfortable with as a passenger inside an AV (Dias et al., 2015; Yi and Chung, 2001).
Fig. 2. The regions for determining FS controller’s command speed and its oscillation at the boundary between regions (Reproduction of Fig 2. of Stern et al. (2018) with modifications)

\[
\text{If } \left( V_{\text{cmd new}} \geq V^{AV \text{ previous}} \right) \text{ AND } \left( \frac{V_{\text{cmd new}} - V^{AV}}{\Delta t} > a_{\text{max}} \right) \\
\text{Then } V^{\text{cmd}} = V^{AV} + \left( a_{\text{max}} \cdot \Delta t \right) \\
\text{Else } V^{\text{cmd}} = V^{\text{cmd}}
\]

(2)

The zig-zag line in Fig. 2 illustrates an oscillating $V^{\text{cmd}}$ at the boundary between regions amplified by the low refresh rate limited by Aimsun. At 10HZ it had a visible impact to the vehicle’s driving behaviour. To rectify this, if the AV was decelerating and its commanded velocity crosses the boundary between two regions, the AV would decelerate at an interpolated value between them, as described in (3). This smoothed the deceleration profile of the AV and granted more realistic operation.

\[
\text{If } V_{\text{cmd new}} < V^{AV \text{ previous}} \\
\text{Then } \frac{\Delta X - \Delta X_{\text{lb}}}{\Delta X_{\text{ub}} - \Delta X_{\text{lb}}} \cdot \left( d_{\text{ub}} - d_{\text{lb}} \right) + d_{\text{lb}} \\
\therefore V^{\text{cmd}} = V^{AV} - \left( d^{\text{cmd}} \cdot \Delta t \right)
\]

(3)

Where:
- $d^{\text{cmd}} = \text{commanded deceleration}$
- $\text{ub and lb denotes upper bound and lower bound of region respectively}$
3. FS-AV Validation

3.1 Numerical experiment design

Once the Aimsun FS-AV model had been created, it was necessary to validate it in comparison to Stern et al.’s (2018) original results. To do this, the original experiment was replicated in Aimsun, which consisted of the following:

- Single-lane circular ring road with a radius of 41.4 meters to the centre of the lane (260 meter circumference).
- Fleet of 21 human driven vehicles (simulated by Aimsun’s Gipps model (Gipps, 1981; TSS-Transport Simulation Systems, 2017)) within the ring road, with one able to change to a FS-AV.
- The vehicles to be initially evenly spaced within the ring road (Fig. 3).
- The lengths of the vehicles were approximated according to those used (see Stern et al., 2018, Appendix A.1.). This was required to match the distribution of distance gaps between vehicles on the ring road whereby differences may alter stop-and-go wave formation and dynamics.
- The simulation ran for 14 minutes, opposed to Stern et al.’s (2018) of nearly 10 minutes. This was due to the data output intervals needing to be a regular fixed interval in Aimsun and each was set to last for two minutes. Stern et al.’s (2018) intervals were irregular and selectively chosen during the operation of their experiment.

According to the Stern et al. (2018) experiment, the human drivers were told the following instructions: “Drive as if you were in rush hour traffic. Follow the vehicle ahead without falling behind. Do not pass the car ahead. Do not hit the car ahead. Drive safely at all times. Do not tailgate. But put an emphasis on catching up to the vehicle ahead, if a gap starts opening up” (p.208 – 209).

The purpose of these instructions is to leave the wave dissipation behaviour solely due to the AV rather than the HDVs, so the AV’s effectiveness can be studied.

In our numerical experiments, the HDVs were simulated by Aimsun’s default model, which has been well accepted for its ability to approximate human driving behaviour (Hidas, 2005). Some parameters were altered from Aimsun default values in order to achieve closest calibration to the original experiment (See Section 3.3 for choice explanation). The best performing calibrated parameters are shown in Table 1.
Table 1
Calibrated parameters for HDVs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reaction Time</td>
<td>0.6 seconds</td>
<td>This is within reasonable estimates. It has a generally accepted value of about 1 second, with values been shown to range between 0.4 – 1.5 seconds (Mehmood and Easa, 2009). However, in congested traffic such as the stop-and-go waves in the original experiment, HDVs’ reaction time may be shortened due to heightened alertness levels (Olstam and Tapani, 2004). This value allowed for most similar behaviour of the human drivers upon visual inspection of Stern et al.’s (2018) experiment video.</td>
</tr>
<tr>
<td>Sensitivity factor</td>
<td>0.78</td>
<td>As human drivers are poor at estimating the acceleration and deceleration of the vehicle in front, a sensitivity value of less than 1.00 causes the vehicle to underestimate the deceleration of the leader. Therefore, the vehicle decreases the gap between itself and the leader and conforms to the instructions drivers were told to exhibit.</td>
</tr>
<tr>
<td>Clearance</td>
<td>2.60 meters</td>
<td>Based on visual inspection and gave the closest match to the original experiment.</td>
</tr>
</tbody>
</table>

Similar to human drivers, AVs react to the environment around them and also include a reaction time. Literature suggests a maximum value of approximately 0.5 seconds with an attributed delay of 0.1 – 0.2 seconds to sensors and 0.1 – 0.3 seconds to mechanical actuators (Ploeg et al., 2013; Rajamani, 2011; Wang et al., 2018; Xiao and Gao, 2011). The value of 0.5 seconds has been used for FS-AV reaction time.

3.2 Validation results

A video of the Aimsun validation simulation is provided in the supplementary materials and under the following link: https://tinyurl.com/ydektkg8

The model was validated by comparing the results and video produced by Stern et al. (2018) and the Aimsun microsimulation. The metrics compared were flow (veh/hr) and velocity standard deviation (m/s), presented in Fig. 4. The overall trend and values for flow and velocity standard deviation correspond well across the original experiment and the created FS model. It is worth noting that the exact values of the metrics are not required to match precisely as these outputs are based off single experiment runs which can vary. What is important is the trend the values follow across the different intervals. Some notable differences between Stern et al.’s (2018) and the Aimsun FS model may be explained by the interval lengths used and the additional randomness human driving comprises over the agent-based model. For example, the last interval where autonomy is disabled actually produced the highest flow value for Stern et al.’s (2018) experiment. Upon analysis of their associated experiment video, it revealed the stop-and-go wave had not yet fully formed in that time interval before the experiment was ended. This was not the case in the Aimsun FS model, whose interval was longer and returned to form a traffic wave with results akin to the first interval of autonomy at the start of the experiment.
Fig. 4. Comparison of flow and velocity standard deviation between Stern et al. [1] and Aimsun micro simulations for FS-AV. All HDV and All HDV (2) indicate FS-AV autonomy is not activated and all vehicles behave as human driven vehicles.

When visually inspected, the Aimsun model performed consistently with the original field experiment video (2018). The stop-and-go wave had been dissipated from the Autonomy 6.5 m/s interval till the Autonomy 7.5 m/s interval, before reaching the threshold and breaking down at Autonomy 8.0 m/s.

3.3 Sensitivity analysis

Simulation models are often sensitive to certain parameters and the interaction between them so we conducted sensitivity tests to ensure the values used in Table 1 are optimal.

The phenomenon of the stop-and-go wave within a ring-road emerges from insufficient free space for vehicles to drive at their chosen speeds. The ring road has a set length, part of which is physically occupied by vehicles within it, so the total empty space is fixed. A number of factors determine how it is distributed between vehicles and how much slack is available as free space. In our Aimsun model, the most important parameters are clearance, time gap and sensitivity factor. Each vehicle needs to maintain clearance (also referred to as jam gap), the minimum distance to its leader, and the total clearance from all vehicles reduces free space. Time gap imposes a comfortable following distance that a vehicle wishes to drive at and is dependent on its speeds and reaction time. With increased speeds and reaction time, the required time gap is enlarged and once more reduces free space. The sensitivity factor in Aimsun determines how well a vehicle estimates the deceleration of its leader. Once free space is lower than a critical threshold, it contributes to vehicle following instability that can allow the onset of traffic waves. These three interacting factors of clearance, reaction time and sensitivity factor were hence required to be calibrated.
The observed flow (Fig. 5) and speed deviation (Fig. 6) data from Stern et al. (2018) was used as metrics for calibration. From the chosen best performing parameter values presented in Table 1, selected adjustment factors were chosen to illustrate their effects. An adjustment factor of ± 25% for clearance value, ± 16.67% for reaction time (Aimsun requires reaction time be a multiple of simulation step), and ± 5% for sensitivity.

Fig. 5. Sensitivity analysis of reaction time, clearance and sensitivity factor, using flow as the metric. The line with sensitivity factor of 0.78 in (e) represents results using values contained in Table 1 and is the same as Fig. 4. Note that flow axis values are shifted accordingly and preserve same scale.

From Fig. 5 and 6 the balancing effect of these three factors towards available space within the ring road are visible. Parameters in graphs a), b) and d) resulted in too much free space. For the desired speed U of 8.0 m/s, the value for flow and speed deviation does not degenerate as it did in Stern et al.’s (2018) experiment (shown as the dashed line). Furthermore, the FS-AV hampers performance as it limits vehicles speeds beyond what is most suitable for given tested desired speed U value range. The opposite is true for graphs f), h) and i) by which the free space was too little, causing recurrent waves being observed in the simulation and led to relative impaired performance. Along the top right to bottom left diagonal of graphs c), e) and g), the three competing parameters appeared more balanced. Their results matched Stern et al.’s (2018) trend more accurately. Both figures suggest that values used in Table 1 produced the best match to the original data. Although they suggest the model is sensitive to some parameters, some degree of numerical stability is preserved along the diagonal, meaning that the same conclusions would have been reached using those alternative values (despite them producing slightly different results). We endeavoured to best match Stern et al.’s experiment findings but it is also important to note that their results were based on a single run and may too have had varying findings with more replications or even a different sample of human drivers.
Fig. 6. Sensitivity analysis of reaction time, clearance and sensitivity (α), using speed deviation as the metric. The line with sensitivity factor = 0.78 in (e) represents results using values contained in Table 1 and is the same as Fig. 4.

4. Numerical experiment design and methodology

After model validation, the FS-AVs in all the following numerical experiments were activated during the entire simulation.

4.1 Single-lane ring road multiple FS-AVs

The same experimental environment used for FS-AV’s validation was used to test whether the stop-and-go wave was still able to be dissipated when the percentage of FS-AV was increased. Ten scenarios (FS 1 – FS 10) were tested. Each succeeding scenario had one more FS-AV added to the mix to replace an HDV so the total number of vehicles was kept constant at 21.

Simulations initially began with FS-AV autonomy disabled with successive ten-minute intervals declining from Autonomy 8.0 m/s to Autonomy 3.5 m/s in 0.5 m/s steps. This design decision allows for the true threshold of desired speed $U$ able to dissipate the traffic wave. Two initial conditions were tested regarding the positioning of the FS-AVs. These were ‘Equal Distance’ in which FS-AVs were as evenly distributed between human vehicles as possible, and ‘Platoon’ where FS-AVs vehicles were all grouped together. The platoon simulations were conducted because our double-lane simulations revealed FS-AVs’ tendency to congregate (Fig. 18). Road agencies often aim to achieve the highest flow through the road as it measures the productivity of the system, hence it was employed as the key metric to gauge the traffic performance.

4.2 Double-lane numerical experiment design

The double-lane ring road numerical experiment was designed to encompass the same characteristics of the original ring road but further introduced the common traffic manoeuvre of lane changes by adding a second lane. A ring road is useful in that a stop-and-go wave can continuously propagate around it, solely formed by the vehicles driving behaviours inside it. It provides a controlled platform to perform simulations that can be compared against each other.
The geometry and design of the double-lane ring road numerical experiment had been altered from the single-lane version (Fig. 7). The ring road had been made larger with a radius of 159.2 meters (500 meter circumference). This was done to lessen the effect of curvature the ring road had on vehicle paths, though was limited to a size that the entire scene was observable on the screen. The total number of vehicles was selected as 80 to match the vehicle density in Stern et al.’s (2018) experiment and served as precursor for stop-and-go waves to form. It also produces integers when translating various FS-AV penetration rates in Table 2 into actual vehicle numbers. Unlike single-lane simulations, all double-lane simulations were ‘Equal Distance’ – FS-AVs were evenly spaced between HDVs at the start of simulations, and equal numbers of FS-AVs occupied each lane.

Five separate numerical experiments have been developed in which varying percentages of AVs have been incorporated into the traffic (Table 2), with the 0% AV incorporation simulation serving as the base case. These simulations were run twice setting the desired speed U to 7.5 and 7.0 m/s for each instance. These values were chosen because they generated the best performance of FS-AV in the single-lane numerical experiment with flow exceeding what all human vehicles achieved and the stop-and-go wave dissipated.

<table>
<thead>
<tr>
<th>Numerical experiment</th>
<th>No. of AVs</th>
<th>No. of HDVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% AV Incorporation</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>2.5% AV Incorporation</td>
<td>2</td>
<td>78</td>
</tr>
<tr>
<td>5% AV Incorporation</td>
<td>4</td>
<td>76</td>
</tr>
<tr>
<td>10% AV Incorporation</td>
<td>8</td>
<td>72</td>
</tr>
<tr>
<td>25% AV Incorporation</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

As AV lane changing models are generally not well documented given it is an emerging technology so the FS-AVs in the microsimulation were not able to change lane. On the other hand, HDVs were able to do so using Aimsun’s default lane-changing model. This is appropriate as the simulations were intended to study the effects of HDV changing lane in front of FS-AVs.
For the purpose of a ring road experiment, vehicle overtaking behaviour will resemble that of some jurisdictions whereby vehicles are not restricted to overtaking in the right lane for a left rule of the road. That is, vehicles in the right lane (the inner lane for ring road in Fig. 7) are allowed to overtake vehicles using the left lane. This allows for more effective analysis of FS’s performance as other vehicles may overtake the AV in this lane if they choose to. Imprudent lane changes which permit vehicles to change lane with a smaller gap and replicate the relaxation phenomenon (Fig. 1) were enabled. This allows for disruptive behaviours to proceed and observe whether the AVs could overcome them. We have also done a sensitivity test by turning off imprudent lane changes with results discussed in Section 5.2.

It is noted that traffic measures of performance (MOP) have been collected during simulations which had not been present in Stern et al.’s (2018) paper. As mentioned previously, for this technology to be implemented it not only needs to be able to dissipate stop-and-go waves, it must also be shown it can improve traffic performance. For this reason, the mean traffic MOP of the following metrics are gathered:

- Speed (km/hr)
- Flow (veh/hr)
- Delay Time (sec/revolution of ring road) – relative to speed of 40 km/h
- Stop Time (sec/revolution of ring road)
- No. of Stops (/revolution of ring road)
- Total Lane Changes

Due to the stochastic nature of microsimulation experiments, numerous replications are required to obtain an estimate of the mean MOP. The number of replications was determined using the student-t table method disclosed in Chu et al. (2003), Toledo and Koutsopoulos (2004) and Truong et al. (2015). Each simulation was repeated until the most varied MOP was within a 95% confidence interval of the mean. Replications across numerical experiments ranged from 10 – 35. The simulation time of each replication lasted 60 minutes.

5. Results and discussion

Tabulated summary of all metrics across all vehicles and individual vehicle classes for double-lane numerical experiment are shown in Appendix B. Select videos of the single-lane and double-lane
Numerical experiments are provided in the supplementary materials and also under the following link:
https://tinyurl.com/ydektkg8
Note that single-lane ring road multiple FS-AVs numerical experiment videos have been altered to two-minute intervals in place of ten for viewing experience.

5.1 Single-lane ring road with multiple FS vehicles

The simulation results confirmed that multiple FS vehicles were able to dissipate stop-and-go waves in a single-lane ring road, but with a few caveats. Traffic flow degenerated in all scenarios with high U values (Fig. 8). The trend was reversed by a decreasing U at some point, which is different for each scenario. For example, in the Platoon FS 2 scenario in Fig. 8(a), U = 8.0 m/s produced a reduced flow relative to All HDV’s which was improved by U = 7.5 m/s before peaking at U = 7.0 m/s. By comparison, in the FS 5 Equal Distance scenario in Fig. 8(b), U values between 8.0 m/s to 6.0 m/s produced similar flows around 1200 veh/hr which was significantly lower than what was achieved by HDVs (slightly below 2000 veh/hr). It had a sudden improvement at U = 5.5 m/s with additional flow of about 400 veh/hr but setting smaller U will see flow decrease again. Generally, there is a threshold for U at each FS-AV penetration level, above which FS vehicles cannot dissipate stop-and-go waves, and below which wave dissipation continues but with linearly declining flow. Therefore, these thresholds also mark the desired speed U that can maximise flow for each scenario.

Fig. 8 also reveals the trend that when more FS vehicles were included, their desired speed U needed to be progressively smaller in order to dampen the traffic wave, which had negative consequences of lowering the system flow and therefore offsetting the benefit of wave dissipation. This causes a tight boundary to exist in which the inclusion of FS vehicles is actually beneficial to traffic performance. The only scenarios that were able to produce higher flow rate than 100% HDVs were FS 1 – 4 (includes FS 1, 2, 3 and 4) Platoon simulations at U = 7.5 m/s (Fig. 8(a)) and FS 1 – 3 in Equal Distance simulations at U ≥ 7.0 m/s (Fig. 8(b)). The best of all scenarios was FS 2 with U = 7.5 m/s in Equal Distance with flow approaching 2200 veh/hr.

An explanation to decreased flow with more FS-AVs added before the wave is dissipated can be attributed to the slower return to speed FS-AVs exhibit. With more of these vehicles present while the wave is not able to be dissipated, the flow will decrease accordingly.

The question arising from this numerical experiment is why wave dissipating performance is negatively affected with increasing FS incorporation. Intuitively, each extra FS vehicle introduces additional stop-and-go wave dissipating effects. However, it appears that the system itself contains a threshold of how much of this behaviour is actually needed to alleviate the instabilities and dampen the wave before too much of it might have contributed towards their formation. The reverse is also true in whereby there is not enough wave dissipation behaviour included, although there was only one instance of it (Equal Distance simulation for U = 7.5 m/s) in all the numerical experiments conducted. In this case, FS 2 performs better than FS 1 indicating that one FS vehicle did not provide sufficient dissipation effects for the system whereas two did.

Evident from the simulations is that FS 1 at desired speed U = 7.5 m/s could not dissipate the wave, although it could in Stern et al.’s (2018) original and validation numerical experiment in Section 4. This is explained by the differing experimental designs. In the original experiment, the stop-and-go wave was already dissipated before the desired speed U was changed to 7.5 m/s. This cleared the instability of the system allowing the transition to U = 7.5 m/s smoother without the traffic wave forming. By contrast, the multiple FS numerical experiment presented here was preceded with U =
and the traffic wave present before $U = 7.5 \text{ m/s}$ was commanded. The same reasoning of these different initial conditions also explains the reduction in flow performance FS 1 has for $U = 8.0 \text{ m/s}$ relative to the original (2018) and validation numerical experiment.

These experiments highlight the difficulties in improving traffic flow using this kind of technology since it shows the performance is sensitive to the combination of different factors. Any deviation away from the ideal condition is likely to produce worse-off traffic operations in terms of traffic flow. The incorporation percentage of FS vehicles, their set desired speed, their position amongst human vehicles and the initial conditions of traffic all affect how effective FS-AVs are. This poses questions of FS-AVs’ effectiveness in real life traffic since it is appreciably more chaotic with higher probabilities of deviating from the ideal operating conditions.

**Fig. 8.** Single-lane numerical experiments with multiple FS vehicles (FS 1 indicates 1 FS vehicle within ring). Each MOP data point has been extracted from 10 minutes worth of simulation time by which steady-state is relatively quick to achieve.

### 5.2 Double-lane numerical experiment

Visual inspection of the simulation has revealed that the stop-and-go waves have not been dissipated in any of the double-lane numerical experiments. Table 3 contains the numerical results to further prove this point. The average speed and standard deviation presented were calculated following Stern et al.’s (2018) original definition and calculation method (Section 4.1 in p. 212), with the time interval $m$ set to 1 second and the total duration set to the entire hour of the simulation. Stern et al. (2018) had put forward the threshold of an instantaneous speed standard deviation of $2.5 \text{ m/s}$ being the indicator of an onset of a traffic wave. It can be seen that all FS-AV numerical experiments have lower average speed than the 100% HDV base case and it declines with increasing FS-AV percentage. Some FS-AV numerical experiments have standard deviation of speed lower than the $2.5 \text{ m/s}$ threshold but given the reduced average speed, Coefficient of Variation (CoV = standard deviation / mean) is probably a better measure of relative variability of speed among vehicles of the same numerical experiment. Although all FS-AV numerical experiments have achieved lower CoV values, they are at the cost of lower average speed and not enough to dissipate the waves.
Table 3
Speed average, standard deviation and coefficient of variation data for a single replication of each numerical experiment

<table>
<thead>
<tr>
<th>Numerical experiment</th>
<th>Average speed (m/s)</th>
<th>Standard deviation of Speed (m/s)</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% FS-AV (100% HDV)</td>
<td>6.75</td>
<td>4.53</td>
<td>0.67</td>
</tr>
<tr>
<td>2.5% FS-AV U = 7.0 m/s</td>
<td>6.36</td>
<td>2.60</td>
<td>0.41</td>
</tr>
<tr>
<td>2.5% FS-AV U = 7.5 m/s</td>
<td>6.40</td>
<td>3.03</td>
<td>0.47</td>
</tr>
<tr>
<td>5% FS-AV U = 7.0 m/s</td>
<td>6.14</td>
<td>2.31</td>
<td>0.38</td>
</tr>
<tr>
<td>5% FS-AV U = 7.5 m/s</td>
<td>6.16</td>
<td>2.55</td>
<td>0.41</td>
</tr>
<tr>
<td>10% FS-AV U = 7.0 m/s</td>
<td>5.70</td>
<td>2.20</td>
<td>0.39</td>
</tr>
<tr>
<td>10% FS-AV U = 7.5 m/s</td>
<td>5.70</td>
<td>2.48</td>
<td>0.44</td>
</tr>
<tr>
<td>25% FS-AV U = 7.0 m/s</td>
<td>4.46</td>
<td>2.60</td>
<td>0.58</td>
</tr>
<tr>
<td>25% FS-AV U = 7.5 m/s</td>
<td>4.45</td>
<td>3.02</td>
<td>0.68</td>
</tr>
</tbody>
</table>

The reason for this is as hypothesized as the larger gap left in front of the FS-AVs invited more lane changes by HDVs. Up to 25% penetration rate, with a greater FS-AV proportion, the number of total lane changes across the simulation increased (Fig. 9).

Fig. 9. Average total lane changes occurred during double-lane ring road numerical experiments

Fig. 10 demonstrates this problem using captured images from experiment simulations. When HDV1 detects that its lane is now moving at a slower rate than adjacent lane, it considers to change lane. As the gap left in front of the FS-AV is sufficient enough to perform a lane change, it cuts in front of the FS-AV causing it to decelerate. Because the line of vehicles following the FS-AV are densely positioned, each successive vehicle is forced to brake to avoid collision, initiating a stop-and-go wave behind it. In this instance, the FS-AV becomes part of the problem that it is designed to negate.
On the other hand, imprudent lane changes do have the benefit of HDVs not getting delayed by slower moving FS-AVs. A lane change not only benefits the vehicle that initiated the manoeuvre, but also empties the space it occupied and allows successive vehicles to advance and possibly change lane themselves. This will result in less delay time and improved flow. Our results in Appendix Table B1 and B4 have not shown significant difference in the MOP values or trends except for the number of lane changes being decreased with imprudent lane changes disabled. This suggests the benefit and disbenefit of imprudent lane changes appears to cancel each other out.

With increasing incorporation percentage of FS-AVs (up to 25%), the collective systems metrics of average speed and flow both worsened (Fig. 11).
The only improvement the FS-AVs achieved was in the metrics of stop time and number of stops (Fig. 12). It was found that the 5% penetration rate reached a minimum for these MOPs, indicating a smoother speed profile for individual vehicles. This indicates that FS-AV’s strategy had some potential minor improvements with regards to fuel consumption. With less stops and reduced stop time, there are fewer occasions in which vehicles need to waste fuel accelerating back to full speed. However, this is at the expense of system productivity indicated by lower flow and speed.

Fig. 11. Speed and flow across all vehicles in double-lane ring road numerical experiments
The double-lane numerical experiments tested the FS penetration rate up to 25%. It might be possible that the trend will be reversed with further increase, which should be the subject of future research.

From Fig. 13 the speed of FS-AVs is slower, delay and stop time are longer and the number of stops are more frequent, which is not suitable as a consumer technology. Occupants would not enjoy having vehicles cut in front of them more often and experiencing deceleration motions. The longer travel times would also contribute to this technology not being sought after. In fairness, the situation would be different if FS vehicles are deployed by the government for congestion easing, which is presumably what the original author envisaged. In this case the metric would be their effectiveness vs. capital and operational costs.
5.3 Ring-road and linear road equivalence

Stern et.al. (2018) inferred that the experimental results they obtained in a ring-road can be extrapolated to long sections of roads in the real world. They suggested that one FS-AV dissipating the stop-and-go waves for 20 other HDVs in a circle is equivalent to 5% evenly distributed FS-AVs dampening the waves on real-life linear roads. However, our results do not support this equivalence. The FS-AV needs a large enough gap in front of it to act as a buffer to dampen the waves but its driving algorithm does not guarantee such a sufficient headway when it cruises (Fig. 14(a)). Consequently, it is not able to dissipate the wave when it first hits (b). Its headway only increases as the reaction to the first pass, i.e. after the event but not before or during it. The increase is also due to the proceeding HDVs running away from it. The large headway also featured in the field experiment video that Stern et.al. (2018) published. As Fig. 14(c) shows that wave dissipation can only start from the second pass. On the ring road, the same FS-AV keeps on circling back and takes consecutive passes until the wave dissipates. However, on a linear road with multiple FS-AVs, each of them will only encounter the same wave once so there is only one pass, which means they cannot dissipate the waves in the absence of multiple passes.
We created a linear equivalent of the ring road to test the ring-road and linear road equivalence. Five platoons of vehicles were released continuously to match five rounds on the ring road (Fig. 15). Each platoon had the same composition as the ring road experiment (Fig. 14), with one FS-AV and 20 HDVs. In both cases, one leading HDV was set to trigger a perturbation by slowing down for a brief period before returning to its original speed. Fig. 15(b) shows that the first FS-AV could not dissipate the wave when it hit, just as the wave not being dampened after the first pass on the ring road Fig. 14(b). Throughout the process, the second FS-AV maintained the same ‘cruising’ headway until it encountered the wave so it was just as unprepared as its predecessor and therefore could not dampen the wave and pass it onto its followers (Fig. 15(c)). The same was repeated by the third, fourth and fifth FS-AVs, which is totally different to the wave being dampened by the second pass in the ring road experiment shown in Fig. 14(c).
The string stability analysis further illustrates the difference between the ring road and linear road. Although the initial disturbance decayed with consecutive passes in the ring road numerical experiment, as shown in Fig. 16(d, e & f), the linear road numerical experiment exhibited string instability with never settling oscillations along the platoon Fig. 16(a, b & c). The results of setting the desired speed to 6.5 m/s (Appendix C) displayed similar patterns and led to the same conclusion.

![Fig. 16](image)

**Fig. 16** String instability comparison of linear and ring road environment. The perturbation was trigged by a lead vehicle decelerating from a speed of 7.0 m/s (25.2 km/h) to 2.78 m/s (10 km/hr) over 10 second period before returning to 7.0 m/s. (a), (b) & (c) show the 1st, 22nd, 43rd, 64th, 85th follower on the linear road. In the ring road numerical experiment, the same vehicle behind the lead vehicle represents all these vehicles with subsequent passes of the wave, e.g. it represents the (1st follower in the first pass, and 22nd follower in the second pass. X on ring road graphs indicate approximate crossover point from 1st, 22nd, 43rd, 64th, 85th following vehicle progressively with time. Appendix C contains the results of setting lead vehicle speed to 6.5m/s.

While the FS-AV’s control regime could be further optimised, its current form does not explicitly guarantee a sufficiently large headway so it is unlikely to be reliable in wave dissipation on real roads. Any adjustment to its controller should also consider potential implications to traffic performance measures such as flow, as well as the detrimental effect of unwanted cut-ins if a larger headway is pursued. It is also worth noting that real road traffic is an open system with many uncontrollable external variables that could not been included in either Stern et al.’s (2018) original experiment or our simulation so the real traffic performance of FS-AVs might deviate further and their optimal speed could be different to 7.0 m/s.

The lack of proactively maintained headway could also explain why more FS-AVs tend to produce worse results on ring road. As previously mentioned, in the original experiment the large headway was not only because of FS-AV’s reaction to the first pass but also the result of proceeding HDVs running away from it. When there are more FS-AVs in the ring, they all react to the first pass by slowing down. Subsequently, they constrain the freedom of HDVs between them and their ability to run away from the next FS-AV. This in turn limits the gap that appears in front of each FS-AV and its capacity to dampen the second and following passes.
5.4 Other observations

To counter the adverse effect of lane-changing by HDVs, FS-AVs might be aligned across all lanes to form a line to block any cars from overtaking or lane changing immediately in front of them. The initial larger gap buffer left in front of the FS-AV will therefore stay intact and be used to dissipate the stop-and-go wave. This occurred for brief instances in the simulations (Fig. 17) though did not last as the inside lane was shorter in distance, causing that FS-AV to pull ahead and break the line. However, implementing such strategy in real-life would face many possible challenges. For example, what would happen if there was a long queue protruding from an off-ramp onto the outer lane of the freeway with normal free flow traffic continuing otherwise? If the aligned AVs came to this incident would they stay aligned and disrupt the flow of traffic or separate?

![Fig. 17. AVs aligning seen to improve traffic upstream](image)

An interesting behaviour also emerged in the advent of multiple FS-AVs in a double-lane environment. The FS-AVs in the same lane would tend to converge to a point in which they form a convoy (Fig. 18). It was witnessed amongst the 5%, 10% and 25% AV numerical experiments and may be seen as an unintended consequence of FS’s strategy. This would transpire due to HDVs overtaking the FS-AVs, bringing the FS-AVs within the same lane closer together. As AVs could not change lane this would eventually lead to them grouping together. They would more commonly stay in this platoon position as it was undesirable for a car to change lane in-between AVs due to them moving at a slower and steadier pace. This observation triggered our platoon simulations in the single-lane setup. This might not be necessarily bad since Fig. 8 shows that the Platoon simulations perform better or no worse than the Equal Distance simulations for the same penetration rate, with the exception of the FS2 scenarios. Congregation of FS-AVs might in fact improve their effectiveness but it does contradict Stern et al.’s (2018) vision of deploying FS vehicles dispersedly amongst HDVs.
6. Conclusions and further research

In this research we have successfully modelled the FollowerStopper driving algorithm in Aimsun microsimulation model and produced similar results to the original Stern et al. (2018) experiment. Using this model, we have reached the following key conclusions:

- Adding multiple FS vehicles in a single-lane ring road environment are not beneficial. When more FS vehicles were included, their desired speed $U$ needed to be progressively smaller in order to dampen the traffic wave, which had negative consequences of lowering the system flow and therefore offsetting the benefit of wave dissipation. Only a few scenarios have achieved equal or slightly better traffic flow (Fig. 8).

- Contrary to single-lane numerical experiments, none of the double-lane numerical experiments displayed the dissipation of stop-and-go waves. The likely reason is the larger gap left in front of the FS-AVs invited more lane changes by HDVs. Up to 25% penetration rate, greater FS-AV proportion was associated with larger number of total lane changes (Fig. 9). Fig. 10 shows that FS-AV reacts to unwanted HDV cut-ins by quickly decelerating in order to restore its desired gap, which sets off a chain reaction behind it since the HDVs closely following behind all need brake.

- For FS-AVs to be useful, they need to show improvements on wider traffic metrics in addition to traffic waves. With increasing incorporation percentage of FS-AVs (up to 25%), the collective systems metrics of average flow, speed and delay time worsened (Fig. 11). Although they were seen to decrease stop time and the number of stops with a low level (e.g. 5%) of incorporation, it is not enough to justify their usage because they caused slower speeds, decreased flow and longer delay times across the system of vehicles.

- Our simulation results do not support the ring road and linear road equivalence suggested by Stern et.al. (2018). Contrary to the ring road results, the introduced local perturbation did not exhibit the same magnitude of decay in the linear road numerical experiment.

- FS-AVs may be more suitable as a traffic control device rather than a consumer technology. However, more questions need to be answered prior to its deployment.

Before dismissing the effectiveness of FS vehicles, one must be careful in generalising the results of the ring road to real life situations. Although we have endeavoured to choose the best parameters to replicate the original field experiment, simulations can never be entirely accurate. On the other hand,
one must also be careful in extrapolating benefits observed in an idealised ring-road experiment to the real road traffic.

The exact reason why adding more FS-AVs worsens the traffic performance deserves more research. It is possible that traffic circulating in a ring road forms a feedback loop and therefore amplifies the shockwaves and their negative consequences. It might be possible that the trend will be reversed with further increase beyond the penetration rates that we tested, which should be the subject of future research. Better understanding of how multiple FS-AVs interact (directly or indirectly) in the traffic flow and how to improve their collective performance also requires further investigation.

Questions also arise with regards to how the added complexity of real traffic and road environment will affect FS vehicle performance. Our preliminary results using real-world traffic models did not reproduce the same adverse impacts reported in this paper. On the contrary, FS-AVs exhibited benefits to the traffic flow (Cummins et al., 2020). That is not to say that results reported here have no real-life implications since many factors could affect the results, as stated above. Rather, we advocate caution in either praising or dismissing FS vehicle effectiveness in dissipating traffic waves. More in-depth analysis is certainly required, for which our Aimsun model can play an important role.

Improvements of the FS driving algorithm should focus on how to proactively maintain a sufficient headway for shock absorption while counteracting the problems created by unwanted cut-ins. Having a large gap will reduce string instability observed in Fig. 16. It might be beneficial to introduce the relaxation phenomenon displayed by human drivers (Fig. 1) to make it less reactive and avoid counterproductive abrupt braking. Of course, such a temporary tolerance of deviation from the desired gap might raise safety concerns, which will need to be adequately addressed.

Acknowledgements
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7. References


Appendix A: The FollowerStopper Controller
The model algorithm presented is accredited to and developed by Stern et al. [1]. The AV measures its own velocity $V^{AV}$ continuously and the gap $\Delta X$ (at a sampling rate of 30 Hz), which is the distance between its front bumper and the rear bumper of the vehicle in front. The velocity difference $\Delta V$ between the two vehicles is then estimated:

$$\Delta V = \frac{d}{dt} \Delta X = V^{lead} - V^{AV} \quad \text{(A 1)}$$

A set of regions is created in a phase space plot of $\Delta X - \Delta V$ using the following formula as the boundaries. They are based on the intercept parameter $\Delta X_k^0$, the deceleration rate $d_k$, and the negative velocity difference $\Delta V_-:

$$\Delta X_k = \Delta X_k^0 + \frac{1}{2d_k} (\Delta V_-)^2, \quad \text{for } k = 1, 2, 3 \quad \text{(A 2)}$$

Where:

$$\Delta V_- = \min (\Delta V, 0) \quad \text{(A 3)}$$

The regions used are a stopping region, adaption region I, adaption region II and a safe region (see Fig. 2). A commanded velocity $V^{cmd}$, is determined based on $\Delta X$ and $\Delta V$ and is sent to the AV to achieve. The AV aims to move at its desired velocity, $U$, when safe to do so. The AV does not determine $U$ itself and is required as an external input. If this is not the attainable, a velocity less than the desired velocity is chosen with the AV falling into either the stopping region or one of the adaption regions.

The commanded velocity $V^{cmd}$ is chosen:

$$V^{cmd} = \begin{cases} 
0 & \text{if } \Delta X \leq \Delta X_1 \\
\left(\frac{\Delta X - \Delta X_1}{\Delta X_2 - \Delta X_1}\right) & \text{if } \Delta X_1 < \Delta X \leq \Delta X_2 \\
\left(\frac{\Delta X - \Delta X_2}{\Delta X_3 - \Delta X_2}\right) + U & \text{if } \Delta X_2 < \Delta X \leq \Delta X_3 \\
U & \text{if } \Delta X_3 < \Delta X 
\end{cases} \quad \text{(A 4)}$$

Where:

$$v = \min (\max (V^{lead}, 0), U) \quad \text{(A 5)}$$

The parameter values used in the experiment by Stern et al. is shown below:

- $\Delta X_1^0 = 4.5 \text{ m}$
- $\Delta X_2^0 = 5.25 \text{ m}$
- $\Delta X_3^0 = 6.0 \text{ m}$
- $d_1 = 1.5 \text{ m/s}^2$
- $d_2 = 1 \text{ m/s}^2$
- $d_3 = 0.5 \text{ m/s}^2$

It is important to note that these equations are functions of time although they have been excluded for simplicity.
## Appendix B: Summary Metrics Double-lane Experiment

### Table B1
All vehicles, $U = 7.5$ m/s

<table>
<thead>
<tr>
<th>Experiment</th>
<th>0% AV</th>
<th>2.5% AV</th>
<th>5% AV</th>
<th>10% AV</th>
<th>25% AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of replications</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Flow (veh/hr)</td>
<td>3895.33</td>
<td>3744.13</td>
<td>3663.23</td>
<td>3412.51</td>
<td>2530.30</td>
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<tr>
<td>Speed (km/hr)</td>
<td>25.74</td>
<td>23.76</td>
<td>23.24</td>
<td>21.66</td>
<td>16.03</td>
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<tr>
<td>Delay time (s/rev)</td>
<td>32.13</td>
<td>35.03</td>
<td>36.69</td>
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<td>70.90</td>
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<tr>
<td>Stop time (s/rev)</td>
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<td>7.94</td>
<td>6.33</td>
<td>9.79</td>
<td>41.47</td>
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<tr>
<td>No. of stops (s/rev)</td>
<td>2.42</td>
<td>2.28</td>
<td>1.80</td>
<td>2.58</td>
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<tr>
<td>Total Lane Changes</td>
<td>658.20</td>
<td>889.20</td>
<td>978.10</td>
<td>1118.54</td>
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### Table B2
HDVs, $U = 7.5$ m/s

<table>
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<tr>
<th>Experiment</th>
<th>2.5% AV</th>
<th>5% AV</th>
<th>10% AV</th>
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<tr>
<td>No. of replications</td>
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<td>10</td>
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<td>Simulation time (min)</td>
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<tr>
<td>Speed (km/hr)</td>
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<td>Stop time (s/rev)</td>
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### Table B3
AVs, $U = 7.5$ m/s

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<td>Speed (km/hr)</td>
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<td>Stop time (s/rev)</td>
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<td>No. of stops (s/rev)</td>
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<td>3.38</td>
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### Table B4
All vehicles, $U = 7.5$ m/s, Imprudent lane changes disabled

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<th>Experiment</th>
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<th>5% AV</th>
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<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Flow (veh/hr)</td>
<td>3932.80</td>
<td>3786.80</td>
<td>3696.00</td>
<td>3367.20</td>
<td>2553.00</td>
</tr>
<tr>
<td>Speed (km/hr)</td>
<td>26.86</td>
<td>24.01</td>
<td>23.56</td>
<td>21.17</td>
<td>16.29</td>
</tr>
<tr>
<td>Delay time (s/rev)</td>
<td>31.46</td>
<td>33.57</td>
<td>34.48</td>
<td>41.08</td>
<td>63.66</td>
</tr>
<tr>
<td>Stop time (s/rev)</td>
<td>22.97</td>
<td>6.96</td>
<td>6.37</td>
<td>11.71</td>
<td>40.80</td>
</tr>
<tr>
<td>No. of stops (s/rev)</td>
<td>2.13</td>
<td>2.10</td>
<td>1.70</td>
<td>2.91</td>
<td>3.41</td>
</tr>
<tr>
<td>Total Lane Changes</td>
<td>392.40</td>
<td>638.60</td>
<td>705.60</td>
<td>781.80</td>
<td>1019.20</td>
</tr>
</tbody>
</table>
### Table B5
All vehicles, $U = 7.0 \text{ m/s}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>0% AV</th>
<th>2.5% AV</th>
<th>5% AV</th>
<th>10% AV</th>
<th>25% AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of replications</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Flow (veh/hr)</td>
<td>3895.33</td>
<td>3722.67</td>
<td>3650.47</td>
<td>3353.69</td>
<td>2550.56</td>
</tr>
<tr>
<td>Speed (km/hr)</td>
<td>25.74</td>
<td>23.57</td>
<td>23.12</td>
<td>21.16</td>
<td>16.22</td>
</tr>
<tr>
<td>Delay time (s/rev)</td>
<td>32.13</td>
<td>34.79</td>
<td>35.58</td>
<td>40.90</td>
<td>62.70</td>
</tr>
<tr>
<td>Stop time (s/rev)</td>
<td>18.66</td>
<td>5.11</td>
<td>3.91</td>
<td>7.90</td>
<td>35.29</td>
</tr>
<tr>
<td>No. of stops (s/rev)</td>
<td>2.42</td>
<td>1.78</td>
<td>1.27</td>
<td>2.33</td>
<td>4.11</td>
</tr>
<tr>
<td>Total Lane Changes</td>
<td>658.20</td>
<td>842.22</td>
<td>983.20</td>
<td>1088.17</td>
<td>1327.22</td>
</tr>
</tbody>
</table>

### Table B6
HDVs, $U = 7.0 \text{ m/s}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>2.5% AV</th>
<th>5% AV</th>
<th>10% AV</th>
<th>25% AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of replications</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Speed (km/hr)</td>
<td>23.63</td>
<td>23.21</td>
<td>21.25</td>
<td>16.31</td>
</tr>
<tr>
<td>Delay time (s/rev)</td>
<td>35.28</td>
<td>36.65</td>
<td>43.39</td>
<td>69.51</td>
</tr>
<tr>
<td>Stop time (s/rev)</td>
<td>5.08</td>
<td>3.76</td>
<td>7.52</td>
<td>34.84</td>
</tr>
<tr>
<td>No. of stops (s/rev)</td>
<td>1.77</td>
<td>1.22</td>
<td>2.24</td>
<td>4.07</td>
</tr>
</tbody>
</table>

### Table B7
AVs, $U = 7.0 \text{ m/s}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>2.5% AV</th>
<th>5% AV</th>
<th>10% AV</th>
<th>25% AV</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of replications</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>Simulation time (min)</td>
<td>60</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Speed (km/hr)</td>
<td>21.31</td>
<td>21.33</td>
<td>20.32</td>
<td>15.98</td>
</tr>
<tr>
<td>Delay time (s/rev)</td>
<td>13.61</td>
<td>13.59</td>
<td>17.49</td>
<td>41.90</td>
</tr>
<tr>
<td>Stop time (s/rev)</td>
<td>6.62</td>
<td>7.18</td>
<td>11.50</td>
<td>36.69</td>
</tr>
<tr>
<td>No. of stops (s/rev)</td>
<td>2.25</td>
<td>2.36</td>
<td>3.23</td>
<td>4.26</td>
</tr>
</tbody>
</table>
Appendix C: Comparison of string stability between the ring-road and its linear equivalent with FS-AV’s desired speed set at 6.5m/s

String instability comparison of linear and ring road environment. Results accompanying Fig. 16 but the perturbation was trigged by a lead vehicle decelerating from a speed of 6.5 m/s (instead of 7.0m/s) to 2.78 m/s (10 km/hr) over 10 second period before returning to 6.5 m/s. (a), (b) & (c) show the 1st, 22nd, 43rd, 64th, 85th follower on the linear road. In the ring road experiment, the same vehicle behind the lead vehicle represents all these vehicles with subsequent passes of the wave, e.g. it represents the 1st follower in the first pass, and 22nd follower in the second pass. X on ring road graphs indicate approximate crossover point from 1st, 22nd, 43rd, 64th, 85th following vehicle progressively with time.