Low-loss materials for high Q -factor Bragg reflector resonators

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A Bragg resonator uses dielectric plates within a metallic cavity to confine the energy within a central free space region. The importance of the permittivity is shown with a better Q factor possible using higher permittivity materials of larger intrinsic dielectric losses. This is because the electric energy in the reflectors decreases proportionally to the square root of permittivity and the coupling to the metallic losses decrease linearly. In a sapphire resonator with a single reflector pair a Q factor of 2.34×10^5 is obtained, which may be improved on by up to a factor of 2 using higher permittivity materials. © 2008 American Institute of Physics. [DOI: 10.1063/1.2828025]

Bragg reflector resonators obtain high Q factors by confining most of the energy in a central low-loss region (usually vacuum) using outer-layered dielectric materials loaded in a metallic cavity. The dielectric and resistive losses are decreased due to the reduced electromagnetic energy in the dielectric layers and at the cavity surface. So far, the highest Q factors ($\sim 7 \times 10^5$) have been obtained in Bragg resonators with two pairs of sapphire reflectors due to its extremely low loss at microwave frequencies.^{1,2} In contrast traditional TE_{01δ} modes only obtain a Q factor of order $(4-5) \times 10^4$.^{3,4} In this work, we investigate a range of alternative low-loss crystalline and noncrystalline dielectric materials with the aim to obtain a higher Q factor in a Bragg reflector resonator.

To compare the efficiency of various materials, a standard structure has been analyzed made from a single Bragg reflector pair in the radial and axial regions, as shown in Fig. 1. It has been shown that the near optimum solution may be obtained when we use a simple model to calculate the dimensions of the resonator with a Q factor of order $(2-3) \times 10^5$.^{5,6} The simple model is briefly presented and used in this work to calculate the dimensions then a numerical technique based on the method of lines (MoL) was used to calculate the resonator properties. The MoL is a semianalytical method with a rigorous formulation initially developed by mathematicians for the solution of differential equations.⁷

To design such structures, many methods of solving the electromagnetic boundary value problems have been developed. Previously, we have developed an approximate solution for a Bragg resonator, using a standard non-Maxwellian model procedure.^{5,6} The method divides the resonator into sections then applies separation of variables technique for the E_θ component of field in each of the sections individually, while matching the boundary conditions between them (see Fig. 2). The structure is divided into the main resonance region of thickness t_0 and radius a_0 (region 0), and the Bragg reflector regions 1 and 2. The Bragg reflector region has two

layers in the axial (thickness t_1 and t_2) and radial (thickness a_1 and a_2) directions. From solving the boundary conditions, Eqs. (1)–(3) are derived⁵ and can be used to calculate the dimensions (the derivation is not repeated here) for a specific aspect ratio (AR), ϵ_r , and resonant frequency f_0 [related to the propagation constant k_0 by $k_0 = (2\pi/c)f_0$].

Solving the boundary conditions in the r direction gives (related to the roots of the first order Bessel function, $J_1(\chi)$, and its derivative)

$$a_0 = \frac{\gamma \chi_{1,1}}{k_0 \sqrt{\gamma^2 - 1}}, \quad (1a)$$

$$a_1 = \frac{\gamma}{k_0 \sqrt{\epsilon_r \gamma^2 - 1}} (\chi'_{1,2} - \chi_{1,1}), \quad (1b)$$

$$a_2 = \frac{\gamma}{k_0 \sqrt{\gamma^2 - 1}} (\chi_{1,2} - \chi'_{1,2}). \quad (1c)$$

Solving the boundary conditions in the z direction gives

FIG. 1. (Color online) Right: photo of a single pair sapphire Bragg reflector resonator of frequency 9.7 GHz and Q factor 2.34×10^5 at room temperature. Left: electric field density plot (E_θ) of the fundamental TE mode as calculated using the method of lines, showing the internal resonant region (free space), and outer antiresonant layered region. For 10 GHz, the size ranges between $L=60-100$ mm, with $R \sim 30$ mm for AR ranging between 1.0 and 1.75.

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FIG. 2. Schematic of the E_θ field patterns from the simple model, showing boundary conditions in the axial (left) and radial (right) directions. The gray regions 1 represent the dielectric reflector layer, while 2 is the free space reflector region. Region 0 is the central resonant region where the energy is confined, with the origin in the middle.

$$t_0 = \frac{\gamma\pi}{k_0}, \quad (2a)$$

$$t_1 = \frac{\pi}{2k_0\sqrt{\epsilon_r}}, \quad (2b)$$

$$t_2 = \frac{\pi}{2k_0}. \quad (2c)$$

Thus, the AR may be calculated to be

$$\text{AR} = \frac{L/2}{R} = \frac{\pi\sqrt{\gamma^2 - 1}}{2} \left[\frac{\left(1 + \frac{1}{\gamma\sqrt{\epsilon_r}} + \frac{1}{\gamma}\right)}{\chi_{1,1} + \chi_{1,2} - \chi'_{1,2} + \sqrt{\frac{\gamma^2 - 1}{\epsilon_r\gamma^2 - 1}}(\chi'_{1,2} - \chi_{1,1})} \right]. \quad (3)$$

From the chosen AR and ϵ_r , one then calculates the coefficient γ from Eq. (3), then from the selected f_0 all the dimensions from Eqs. (1) and (2).

A dielectric material may be characterized by its complex permittivity $\epsilon = \epsilon_r - j\epsilon''$. The loss tangent can be expressed from the complex permittivity as $\tan \delta = \epsilon''/\epsilon_r$. For the chosen resonant mode in a dielectric resonator, the Q factor and frequency will depend on the above dielectric properties, as well as the dimensions, surface resistance (R_s) of the metallic enclosure, and specific mode properties. The mode properties are characterized by the geometric factor (G) and the electric energy filling factor in the dielectric (Pe).

The dimensions of the cavity (as calculated using the simple model) were entered in the MoL mesh for a range of ϵ_r values between 10 and 200 and the G , Pe , and f_0 calculated for each value of ϵ_r . The calculated f_0 using the MoL always gave the solution of 10 GHz to within 0.02%–0.3% as expected reconfirming the applicability of the simple

TABLE I. Low-loss dielectric materials plotted in Fig. 2, with measured permittivities and loss tangents referred to 10 GHz (Refs. 9–25). The Q factor of the structure in Fig. 1 is calculated using method of lines. For anisotropic materials we only give the perpendicular values, which is the only one relevant for the TE Bragg confined mode.

No.	Material	Permittivity	$\tan \delta(10 \text{ GHz})$	Q factor
1	CaTiO ₃ (see Ref. 9)	162	7.7×10^{-4}	4.7×10^4
2	TiO ₂ +0.05 mol % Fe (see Ref. 9)	104	2.0×10^{-4}	1.3×10^5
3	Pb _{0.7} Ca _{0.3} La _{0.5} (Mg _{1/2} Nb _{1/2})O ₃ (see Ref. 10)	50	1.2×10^{-4}	1.5×10^5
4	Ba[(Zn _{0.6} Co _{0.4}) _{1/3} Nb _{2/3}]O ₃ (see Ref. 11)	35.6	2.8×10^{-5}	3.9×10^5
5	Ba(Mg _{1/2} Ta _{2/3})O ₃ : 0.5 mol % Ba(Mg _{1/2} W _{1/2})O ₃ (see Ref. 12)	24.2	2.5×10^{-5}	3.4×10^5
5	Ba(Mg _{1/3} Ta _{2/3})O ₃ : BaSnO ₃ , BaWO ₄ (see Ref. 12–15)	24	2.3×10^{-5}	3.5×10^5
6	0.92(Mg _{0.95} Co _{0.05})TiO ₃ –0.08CaTiO ₃ (see Ref. 16)	22.1	1.2×10^{-5}	4.7×10^5
7	Al ₂ O ₃ (TiO ₂ doped) (see Ref. 17)	10.5	8.4×10^{-6}	2.9×10^5
8	Al ₂ O ₃ alumina (see Ref. 18)	10.05	1.5×10^{-5}	2.3×10^5
9	TiO ₂ rutile (see Ref. 19)	85.7	1.5×10^{-4}	1.5×10^5
10	LaGaO ₃ (see Ref. 20)	26	1.7×10^{-5}	4.5×10^5
11	LaAlO ₃ (see Ref. 21 and 22)	24	2.0×10^{-5}	3.9×10^5
12	SrLaAlO ₄ (see Ref. 23)	16.85	1.6×10^{-5}	3.3×10^5
13	YAG (see Ref. 23)	10.6	1.2×10^{-5}	2.6×10^5
14	Al ₂ O ₃ sapphire (see Ref. 23–25)	9.935	8.5×10^{-6}	2.8×10^5

FIG. 3. (Color online) Left: loss tangent versus permittivity showing the locus of $\tan \delta|_{Q_0=3 \times 10^5}$ at 10 GHz as calculated using the MoL for the resonator design in Fig. 1 for a range of AR values. Fourteen low loss materials of varying permittivity are also plotted (see Table I). The bold circles (1–8) represent noncrystalline material, while the white squares represent crystalline material. The materials below the locus have a Q factor greater than 3×10^5 , while the ones above have a lower Q factor. Right: electrical energy filling factor and mode G factor of the as a function of permittivity as calculated using the MoL for a range of AR values. Results show an inverse square root and linear dependence, respectively.

model. For a dielectric resonator the standard relationships between Q_0 , G , and Pe is given by

$$Q_0^{-1} = Pe \tan \delta + \frac{R_s}{G}. \quad (4)$$

Thus, the required $\tan \delta$ of the material necessary to give an unloaded Q factor (Q_0) of 3×10^5 (assuming the R_s to be 26 m Ω the value of good silver plated copper⁸ at 10 GHz) allowed the numeric calculation of a locus in the permittivity versus loss tangent graph shown in Fig. 3 and can be calculated from Eq. (4) to be

$$\tan \delta|_{Q_0=3 \times 10^5} = \frac{3.333 \times 10^{-6} - 0.026/G_{BM}}{Pe_{BM}}. \quad (5)$$

Here, Pe_{BM} and G_{BM} are the electrical energy filling factor and G factor numerically calculated for the Bragg mode (BM) at the specific value of ϵ_r .

A range of materials at 10 GHz from the database given in Refs. 9–26 was compared with the calculated $\tan \delta|_{Q_0=3 \times 10^5}$ locus from Eq. (5) and is shown in Fig. 3 and tabulated in Table I. As expected, crystalline sapphire (No. 14) and rutile-doped alumina (No. 7) are the lowest loss tangent materials and lie close to the curve (Q factor calculated to be $\sim 2.8 \times 10^5$ and measured to be 2.34×10^5).

There is a range of crystalline and noncrystalline materials with higher loss and permittivity, which give higher Q factors above 3×10^5 , with only a small reduction in the thickness of the reflectors when compared to sapphire (thickness of the reflector is inversely proportional to the square root of the permittivity). This includes dielectric materials 4, 5, and 6 and the crystalline materials 10, 11, and 12 (see Table I). The machining of such structures should be no more difficult than sapphire or alumina,²⁶ and in particular the dielectric materials can offer a cheaper alternative and a higher Q factor. However, the loss tangent of materials of very high

permittivity (greater than 50 labeled 1–3 and 9), are not low enough to give a Q factor greater than sapphire.

In conclusion, we have shown that an improved Q factor may be obtained with a Bragg resonator by constructing the resonator from higher permittivity materials even if they have higher losses than sapphire. This occurs because of the smaller electric filling factor in the dielectric reflector and the higher G factor of the mode for larger permittivity materials. This work was also extended to multilayered Bragg reflector resonators, but is not presented here as the results give the same trends, except that Q factors of better than 7×10^5 may be obtained.

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