Title

Spatial scale invariance of southern Australian forest fires mirrors the scaling behaviour of fire-driving weather events

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Supporting Material (separate file)

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Table S2: Details on weather stations, observations and fire weather events.

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Abstract Power law frequency-size distributions of forest fires have been observed in a range of environments. The scaling behaviour of fires, and more generally of landscape patterns related to recurring disturbance and recovery, have previously been explained in the frameworks of self-organized criticality (SOC) and highly optimized tolerance (HOT). In these frameworks the scaling behaviour of the fires is the global structure that either emerges spontaneously from locally operating processes (SOC) or is the product of a tuning process aimed at optimizing the trade-offs between system yield and tolerance to risks (HOT). Here, we argue that the dominant role of self-organized or optimised fuel patterns in constraining unplanned-fire sizes, implicit in the SOC and HOT frameworks, fails to recognise the strong exogenous controls of fire spread (i.e. by weather, terrain, and suppression) observed in many fire-prone landscapes. Using data from southern Australia we demonstrate that forest fire areas and the magnitudes of corresponding weather events have distributions with closely matching scaling exponents. We conclude that the spatial scale invariance of forest fires may also be shaped by a mapping of the meteorological forcing pattern.

Key words forest fires, fire weather, power law frequency distributions, scale invariance, criticality, self-organization

Introduction

Scale invariant forest fires

Fire is an intrinsic aspect of the ecology and management of flammable vegetation formations, such as grasslands, savannas, mediterranean shrublands/woodlands, and boreal forests, which cover some 40% of the global land mass (Bond et al. 2005; Chapin et al. 2002). Annually, many thousands of fires burn several hundred Mha of these fire-prone landscapes (Giglio et al. 2006). At the regional scale, the total annual area burnt is largely comprised of a small number of relatively large fires (e.g. $\gg 100$ ha). Small (e.g. < 10 ha) and medium-sized (e.g. 10-100 ha) fires occur much more frequently but make only a minor contribution to the total fire-affected area, i.e. the sizes and frequencies of fires are negatively correlated, albeit non-linearly. The size distribution of fires can be accommodated within a region's fire regime (Gill et al. 2003), providing useful information for the design of forest and fire management strategies, including the planning of timber harvest patterns, conservation reserves, and the allocation of fire suppression resources (for a review see e.g. Cui and Perera 2008).

Frequency-area distributions of forest fires commonly follow a scale-free power law of the type $N(> A) \propto A^{-\alpha}$ over several orders of magnitude (Cui and Perera 2008; Gill et al. 2003; Malamud et al. 1998; Malamud et al. 2005; Minnich 1983; Ricotta et al. 2001; Song et al. 2001; Song et al. 2006; Telesca et al. 2005), where $N(>A)$ is the number of fires greater than an area A , and α is a scaling exponent. Power law behaviour of a frequency size distribution implies that the object of study has no particular length scale – it is scale invariant (Brown et al. 2002; Gisiger 2001). The origin of the spatial scale invariance of forest fires has only been explained in terms of

fuel-age effects, and not by other realistic process-pattern relationships operating in fire-prone landscapes.

Scale-free landscape patterns related to recurring disturbance and recovery, such as gaps in forest canopies or mussel beds and forest fires, are commonly (but see e.g. Solow 2005; Sornette 2002) interpreted as a signature of a self-organizing dynamical system (Pascual et al. 2002; Solé and Manrubia 1995; Solé and Bascompte 2006; Wootton 2001). In a self-organizing system the observed (spatial) structure emerges solely from locally operating endogenous processes, independently of global (i.e. exogenous) control or an environmental blueprint (Levin 2005; Pascual et al. 2002; Perry 1995; Peterson 2002; Rohani et al. 1997). Self-organization has been the suggested cause for spatial pattern formation in a range of marine and terrestrial ecosystems (Hallet 1990; Kessler and Werner 2003; Rietkerk et al. 2002; Scanlon et al. 2007; Van de Koppel et al. 2005; Werner 1999) and is thought to generally play a key role in the structuring of ecological systems (Solé and Bascompte 2006).

The spatial scale invariance of forest fires has largely been attributed to self-organization, where fire sizes are controlled by feedbacks between the spatial pattern of fuel in the landscape, and rates of energy/fuel accumulation, fire incidence and propagation (Peterson 2002). The theoretical underpinning for the ‘spontaneous’ formation of scale-free fire size distributions was initially sought in the theory of self-organized criticality (SOC) (Bak et al. 1987; Jensen 1998; Malamud et al. 1998; Ricotta et al. 1999; Ricotta et al. 2001; Song et al. 2001). SOC theorizes that complex spatiotemporal behaviour arises spontaneously out of many-body systems where individual elements are only interacting over a small, local neighbourhood (Bak et al. 1987; Bak 1996; Jensen 1998). The complex system behaviour is characteristic of a

critical state that can be described by general system properties such as scale invariant distributions (Pascual and Guichard 2005). Drossel and Schwabl's (1992) 'Forest Fire Model' has been highly influential in linking the scale invariance of forest fires to SOC (Malamud et al. 1998; Song et al. 2001), to the point that forest fires have become a type example of SOC in ecological systems (Pascual and Guichard 2005; Solé and Bascompte 2006).

Carlson and Doyle (1999, 2000) proposed an alternative conceptual framework for the formation of scale invariant patterns in complex systems without any self-organization or criticality, called highly optimized tolerance (HOT). HOT is sourced in advanced engineering technologies and emphasizes that the power law behaviour of disturbance event sizes results from an optimisation of the tradeoffs between system yield (e.g. sustained vegetation density of a fire-prone landscape) and tolerance to risks (e.g. fires) that drives the system to a specific (spatial) configuration (Carlson and Doyle 1999, 2002). The association of HOT with forest fires was initially mainly metaphoric (Carlson and Doyle 1999, 2002). More recently, Moritz et al. (2005) showed that a simple HOT model could closely reproduce the size distribution of recorded fires in the chaparral-dominated Los Padres National Forest in California. However, like the self-organized critical 'Forest Fire Model' (Drossel and Schwabl 1992) the HOT models are abstractions of fire-prone landscapes; in particular, the optimisation mechanism that forms the core of the HOT framework cannot be readily related to known ecological processes (Moritz et al. 2005). Thus, it is unclear to what degree HOT models are representing differing causal controls on the shape of the fire size distribution, despite their efficacy in reproducing observed fire sizes

Although the SOC and HOT frameworks propose fundamentally different mechanisms for the organization of fire-prone landscapes into a specific spatial patterning of fuels, the simulation models used to explore the validity of these theoretical frameworks for actual forest fires share key underlying assumptions about how model fires propagate across the simulated landscape and about the impact of past fires on future fires via the spatial patterning of fuels. Those key assumptions can be summarised as follows (Carlson and Doyle 2002, p. 2540-2542; for details see e.g. Clar et al. 1996, p. 6804; Malamud et al. 1998, p. 1840): i) model landscapes essentially consist of sites in two contrasting states that are either susceptible or non-susceptible to fire depending on the presence or absence of ‘fuel’, while other factors controlling the susceptibility to fire such as weather or terrain are either ignored or not considered as a source of structure in fire sizes; ii) model fires change the fuel status of a site from ‘present’ to ‘absent’, the equivalent to fuel reduction rates of 100% for all model fires, which will then be non-susceptible to fire until the fuel load is restored; iii) model fires only propagate through the landscape by contagion from burning sites to adjacent flammable sites. Hence, in both frameworks the sizes of forest fires are solely constrained by the connectedness of fuel patches in the landscape. Significantly, the connectedness of the fuel patches is assumed to be a function of their spatial organization only.

In this paper, we argue that the dominant role of (self-organized or optimised) fuel patterns in constraining forest fire sizes, implicit in the SOC and HOT frameworks (Malamud et al. 1998; Moritz et al. 2005; Ricotta et al. 1999; Ricotta et al. 2001; Song et al. 2001), is inconsistent with a strong exogenous control of fire spread (i.e. by

weather, terrain, and suppression) observed in many fire-prone landscapes (Bessie and Johnson 1995; Johnson and Miyanishi 2001; Moritz et al. 2004).

Forest fire behaviour and landscape connectivity

The behaviour of forest fires, usually characterized in terms like the rate of spread, flame dimensions, fire intensity or heat release, is a function of at least three key factors: i) fuel properties (e.g. mass, density, moisture content), ii) weather conditions (e.g. air temperature, relative humidity, wind speed), and iii) terrain (e.g. slope) (Johnson and Miyanishi 2001). These factors vary in time and/or space, resulting in spatiotemporal variation in fire behaviour. The propagation of individual fires is a non-linear growth process characterized by distinct modes of fire behaviour in which the fire responds to fuel patterns and terrain features at different spatial scales (Peters et al. 2004). Fire behaviour tends to be more variable in tall multi-layered vegetation formations such as woodlands and forests than in low formations such as grasslands and shrublands where any fire is likely to consume most of the short-statured canopy. Here, we focus on woodlands and forests.

Under severe weather conditions (i.e. low relative humidity, high air temperature and strong winds) most of the dead and live fuel on a forest site will be engaged in the fire (i.e. crown fire) thereby driving measures of fire intensity, such as heat release, flame height, spread rate and fuel consumption rate, towards their maxima. High intensity fires can propagate through highly fragmented fuel arrays spanning large gaps (e.g. 10^2 - 10^4 m) by burning brands causing fires downwind (i.e. spotting). These high intensity fires ‘perceive’ the landscape at relatively coarse spatial scales and are relatively insensitive to the finer scale fuel patterning (Peters et al. 2004). Conversely,

only the most readily ignitable fuel on a forest site such as fine surface litter will burn under mild weather conditions, strongly limiting fire intensity, flame dimensions and reducing the recovery period. Thus, low intensity forest fires can only bridge small gaps in the fuel array (e.g. 10^0 - 10^1 m) and do respond to fine-grained variation in the landscape. Hence, though fuel properties largely determine potential fire behaviour, weather conditions experienced during the fire's development control the extent to which that potential is realized and, importantly, the relative connectivity of fuels in the landscape.

Endogenous and exogenous controls of fire size distributions

The basic principles of forest fire behaviour summarised above are key to our discussion about the origin of spatial scale invariance of forest fires. Firstly, they imply that fuel patterns created by past fires may constrain fire propagation under mild weather conditions but usually lose significance under moderate to severe weather conditions (Bessie and Johnson 1995; Johnson and Miyanishi 2001; Moritz et al. 2004). Secondly, as fires burning under mild weather conditions are more likely to remain small than fires burning under severe weather conditions (e.g., Moritz 2003), we may expect some association between prevailing fire behaviour modes and fire sizes, and observe fire size distributions that exhibit multiple scaling regions (e.g., Reed and McKelvey 2002; Ricotta et al. 2001; Telesca et al. 2005) rather than a single power law for all fire sizes. Thirdly, if fire spread is strongly weather-dependent we may expect the structure of variation in fire-driving weather conditions (Fig. 1) to be recorded in the spatial scaling behaviour of forest fires. Considering that power law frequency distributions have been reported for a range of meteorological phenomena (Lovejoy and

Schertzer 1986), including fluctuations of surface wind speeds (Edwards and Hurst 2001; Kavasseri and Nagarajan 2005), we hypothesize that the spatial scaling behaviour of forest fires is a mapping of a meteorological forcing pattern. In our view the temporal variation of fire weather conditions at seasonal, daily and hourly time scales (Fig. 1) sets important constraints for potential fire sizes by controlling the frequency, duration and length scale of connections among ignitable sites in the landscape. If true, the fire size distribution could be a signature of that forcing pattern (i.e. exogenous control) rather than a product of a particular spatial configuration to which the system evolves by the nature of locally operating processes alone (i.e. endogenous control).

The hypothesized mechanism for the formation of scale-free fire sizes differs fundamentally from the existing SOC and HOT frameworks. External drivers have no role in pattern formation in SOC systems (Bak et al. 1987; Bak 1996; Jensen 1998; Pascual and Guichard 2005), while in HOT they may be taken implicitly into account as part of the boundary conditions for the optimisation process (Carlson and Doyle 1999, 2002). In contrast, we hypothesize that the dominant external driver of forest fires (i.e. weather) has scaling behaviour of its own that is mirrored in the size distribution of fires. The spatiotemporal structuring of exogenous variables has previously been suggested as a cause of scale-free patterns in auroral indices (Freeman and Watkins 2002) and fluctuations in population sizes of Peruvian Pacific Ocean pelicans (Milne 1997), but has so far been ignored as a potential mechanism for the formation of spatial scale invariant landscape patterns.

Identification of the origin of scale-free landscape patterns as being top-down (exogenous) or bottom-up (endogenous) control (or both) will impact significantly on how those landscapes are managed. Current management of fire-prone landscapes

commonly relies on bottom-up control being strong, for example, when using fuel reduction measures such as prescribed burning to reduce the risk of large unplanned fires. Such management strategies might be improved or adapted to future climate by incorporating regional knowledge of the frequencies, durations, and magnitudes with which meteorological conditions (i.e. top-down) may override fuel patterns (i.e. bottom-up) as controlling factors of fire spread (Bessie and Johnson 1995).

In this paper we investigate to what extent the frequency size distributions of fires in four large forest regions of southeast and southwest Australia can, as hypothesised above, be explained by regional fire weather statistics. We introduce a simple metric for the quantification of weather events in terms of the potential fire area they may produce and compare its scaling behaviour with that of recorded forest fires in our study regions. Fire occurrence and fire behaviour in the study areas is relatively well-documented (Abbott and Burrows 2003; Bradstock et al. 2002; Luke and McArthur 1978). Our study included fire records from the Australian Capital Territory that have previously been discussed in the context of both SOC and HOT (Malamud et al. 1998; Moritz et al. 2005).

Material and methods

Study areas

Fire and weather records from four forest regions with a total area of 3.6 Mha were analysed: southwest Western Australia (SW-WA), the Australian Capital Territory (ACT), northeast Victoria (NE-VIC), and the greater Blue Mountains west of Sydney, southeast New South Wales (SE-NSW). Our data sets hold 3725 wildfires (Table S1)

and are highly representative of southern Australian eucalypt forests (Abbott and Burrows 2003; Bradstock et al. 2002; Luke and McArthur 1978). The fire data has been analysed as received from the fire management agencies. The completeness and accuracy of the fire data may vary to some extent between regions, and in the case of long-term data sets (e.g. ACT) also over time, due to variation in fire recording and measurement procedures. The databases include fires that primarily occurred on land managed by government agencies, but may also include some records of fires on private land. Our study focused on unplanned fires (i.e. wildfires); hence all prescribed fires were excluded from the analyses. The majority of the fires can be considered ‘forest fires’ since eucalypt forest predominates in all four study areas.

Quantifying the magnitudes of fire-driving weather events

We used the McArthur’s Forest Fire Danger Index (FFDI) (McArthur 1967) to quantify the magnitudes of fire weather events. The FFDI is used throughout southern Australia as an index of potential forest fire danger, rate of fire spread and difficulty of suppression. Half-hourly or hourly observations of air temperature, relative humidity and wind speed over 5-7 year periods were obtained from automatic weather stations in each of the forest regions and used to compute FFDI time-series (Fig. 1) according to equations proposed by Noble et al. (1980). The computed FFDI values were not capped to fit the traditional operational range from 0 to 100, producing occasional peaks of FFDI greater than 100 (Fig. 1). The Keetch and Byram drought index (KBDI) (Keetch and Byram 1968), one of the inputs to the FFDI, was calculated from daily observations of precipitation and maximum daily air temperature at the same weather stations. The initial KBDI value for the start of the FFDI time series was computed from daily

observations of precipitation and maximum air temperature recorded at the same station over the preceding 5 years. The FFDI is highly sensitive to wind speed values, and therefore to the time over which wind speed recordings are averaged. McArthur (1967) suggested using average wind speeds “over a period of at least five minutes” for the calculation of the FFDI. In our application 10-minute wind speed averages were used.

We designed a simple metric to quantify the magnitudes, M , of fire-driving weather events from time-integrated FFDI such that M is proportional to the potential area burnt, A , during that event. This condition of proportionality is critical to demonstrating a one-to-one relationship between scaling exponents when the scaling behaviour of M is compared to that of A for each forest region.

The FFDI is a predictor of the linear rate of forward spread of a fire in eucalypt forests with a given litter fuel load on flat terrain burning with the wind (McArthur 1967). In its usual context of operational fire management the FFDI is combined with information on litter-fuel loads and topographic slope to predict the forward rate of spread of the head fire (Luke and McArthur 1978). In this study, we focus on the meteorological drivers of fire spread, and therefore applied the index under the assumption of uniform fuels and terrain. For further simplicity we also assumed the FFDI’s proportionality with forward rate of fire spread to hold for the full range of fire weather conditions and fire behaviour models. As the FFDI was designed for surface fire, not crown fire, this approach likely underestimates fire spread rates under severe weather conditions.

If the (half-)hourly FFDI is proportional to the forward rate of spread of a fire, the total length of an isotropic fire run would be proportional to the FFDI integrated over the duration of the fire, and the total fire area, A , proportional to the square of the time-

integrated FFDI. Using this relationship the magnitude, M , of fire-driving weather events was quantified as:

$$M^{(i)} = \left(\sum_{t=t_1^{(i)}}^{t_2^{(i)}} 1[F(t) > F_0] F(t) \Delta t \right)^2$$

$$\{t_2^{(i)} = \min(t) : F(t^*) \leq F_0 \ \forall t^* \text{ such that } (t - \beta) < t^* \leq t\} \quad (1)$$

where $F(t)$ is the regional time series of the FFDI with time step Δt , and $t_1^{(i)}$ and $t_2^{(i)}$ are the start and end times of weather event i . F_0 defines the FFDI value below which there is a zero rate of fire spread, and β sets the maximum length of time (hours) a fire could continue to burn at a given location when weather conditions would not allow the fire to spread to other locations.

We focused our investigation on the behaviour of the FFDI at the time of actual fires. We therefore used the actually recorded fire start dates plus the criterion $\text{FFDI} > F_0$ to define, respectively, the starting date and starting time of the fire weather events. Our data bases do not hold end date/time of recorded fires. Fire weather events were therefore assumed to end when $\text{FFDI} \leq F_0$ for a time exceeding β hours. Although the physical interpretation of the parameters F_0 and β is rather straightforward their exact values cannot be readily derived from the literature. Moreover, our four forest regions will likely have similar albeit different F_0 and β values as their fuel and terrain characteristics vary. Consequently, a precise choice of F_0 and β from the available data cannot be arrived at, with different combinations of F_0 and β values effectively constituting different metrics of fire weather behaviour. Our experience with fires in southern Australian eucalypt forests suggests that β has values in the order of half a day to several days, while results of experimental fires in SW Western Australia suggests an approximate value of $F_0 \sim 6$ (Burrows 1999). We computed distributions of M for each

of the study regions using F_0 values of 6 ± 4 , combined with β values of 12, 24, 36, and 48 hours, and compared them with the corresponding distribution of recorded wildfire areas.

Re-scaling of data and model fitting

The log-log cumulative frequency-area distributions of the fires and cumulative-magnitude distributions of the fire weather events were collapsed onto the same scaled axes using the transformation:

$$y^t = \frac{y - \min(y)}{\max(y) - \min(y)} \quad (2)$$

where y is either the log transform of the observed fire area, A , or the calculated magnitude, M , of the corresponding fire weather event (Eq. 1). A further transformation of $y^t = y + 1$ was applied to the M values before the log transformation as there were fire weather events of magnitude zero, corresponding to periods when a fire did occur but the FFDI did not exceed the threshold FFDI of F_0 within a time length β .

Finally, we fitted a broken-stick model with two objectively identified breakpoints to describe the double log-transformed data (Appendix S3). Other attempts at modelling the scaling behaviour of forest fires have arbitrarily chosen linear scaling regions (Malamud et al. 1998; Malamud et al. 2005; Ricotta et al. 1999; Ricotta et al. 2001; Song et al. 2001). However, it is obvious that the fire and weather distributions from our study areas are not linear unless truncated in some manner. A linear model does not provide the repeatability across studies that an automatic method of scaling region selection would. A broken-stick model (also piece-wise linear or segmented) can be fitted by maximum likelihood using the “Segmented” package available for the open source statistical graphics and computing environment R (Muggeo 2003; R

Development Core Team 2008) Our study concentrated on fitting a broken-stick model with just two break-points. This broken-stick model is in general appropriate for the data, except for fire areas of less than ca. 0.1 ha (Appendix S3).

Results

Observed forest fires in our study areas cover a size range of six to eight orders of magnitude (i.e. 10^{-2} - 10^6 ha). In each of the study areas, fire size distributions have multiple spatial scaling regions (Fig. 2), which is consistent with the existence of distinct modes of fire behaviour, cross-scale shifts in the factors that control fire spread and extinguishment (Peters et al. 2004), and corresponding changes in fire fighting tactics (e.g. from direct to indirect attack). Frequency-area distributions with several scaling exponents have also been reported for unplanned fires in the Mediterranean (Ricotta et al. 2001; Telesca et al. 2005) and North America (Reed and McKelvey 2002) and contrast with the single scaling exponent obtained for SOC- and HOT-based models (Malamud et al. 1998; Moritz et al. 2005; Song et al. 2001). The fire area distributions from our southern Australian study areas have a central scaling region that represents three to four orders of magnitude in fire area and just over half the number of fires (Fig. 2). The central scaling region of the cumulative frequency distribution is well-described by the power law:

$$N(> A) \propto A^{-\alpha_A} \quad (3)$$

where $N(>A)$ is the number of fires with area greater than A (ha) and α_A is the scaling exponent. Values for α_A vary from 0.32 for the data set from SW Western Australia to 0.62 for the fires in NE Victoria (Fig. 2). The relatively small scaling exponent for the

SW-WA data may reflect the more subdued relief of this study area compared to the more rugged terrain of the other three study areas.

For many F_0 - β combinations the cumulative frequency distribution of M has a similar shape as the frequency-area distributions of recorded forest fires; that is, manifesting two to three scaling regions and a central scaling region that is well-described by the power law: $N(>M) \propto M^{-\alpha_M}$. To explore the similarity of the frequency distributions of fire areas and the distribution of weather event magnitudes we rescaled both data sets (eq. 2), giving A' and M' , and compared the slopes ($\alpha_{A'}$ and $\alpha_{M'}$) and proportions of data represented by the central scaling region of the rescaled distributions ($prop_{A'}$ and $prop_{M'}$) for each parameter combination (F_0, β). For the data sets from SW Western Australia and NE Victoria the closest match between $\alpha_{A'}$ and $\alpha_{M'}$ and between $prop_{A'}$ and $prop_{M'}$ was obtained for $F_0=4$ and $\beta=24$ hours, while $F_0=6$ and $\beta=24$ produced the closest match for the data from The Australian Capital Territory and $F_0=4$ and $\beta=12$ for SE New South Wales. The corresponding distributions of A' and M' are shown in Figure 3. In all cases the selected values of F_0 and β are reasonable from knowledge of fire behaviour in southern Australian eucalypt forests (Burrows 1999; Gould et al. 2007; Luke and McArthur 1978). The sensitivity of $\alpha_{M'}$ to unit changes in F_0 as a percentage of $\alpha_{M'}$ was $\partial\alpha_{M'}/\partial F_0 = 11.1\%$ on average across all regions. The sensitivity of $\alpha_{M'}$ to β was $\partial\alpha_{M'}/\partial\beta = 1.6\%$, or 19.4% for every 12 hours. The resemblance of fire and weather distributions is slightly stronger for the data from SW Western Australia (Fig. 3a) and SE New South Wales (Fig. 3c) than for those from The Australian Capital Territory (Fig. 3b) and NE Victoria (Fig. 3d) where the rescaled exponents took on higher values. The causes of this disparity are unclear, but we suspect an as yet unknown correlation with terrain.

Discussion

Matching scaling behaviour of fires and weather events

Our findings strongly suggest that a mapping of the meteorological forcing pattern is a legitimate causal mechanism for explaining the spatial scale invariance of fire sizes. This explanation for the origin of the power law distribution of forest fire sizes poses an alternative to existing SOC and HOT theories (Malamud et al. 1998; Moritz et al. 2005; Ricotta et al. 1999; Ricotta et al. 2001; Song et al. 2001) by showing that scale invariance of fires may be imposed on the forest system by regional weather conditions. Like the SOC- and HOT-based models our weather-driven model ignores much of the complexity of forest fire behaviour in order to explore the role of a particular mechanism in producing the observed scaling behaviour of fire sizes. For the interpretation of the results of such modelling exercises, the rationale for disregarding one set of processes and emphasizing another is as important as demonstrating the statistical similarities of simulated and observed patterns. In the introduction we argued that a focus on temporal variation in weather conditions, rather than spatial variation in fuels, is warranted by the strong meteorological impact on forest fire behaviour generally and on landscape connectivity in particular.

Despite the simplicity of our weather-driven model there are several reasons to have confidence in the observed matching between the scaling behaviours of fire sizes and weather events. Firstly, a matching scaling behaviour of fire sizes and weather events was found for all four study regions, which represent much of the diversity in southern Australian forest landscapes.

Secondly, the finding that only one of several scaling regions observed in the fire size distributions matches the frequency-magnitude distribution of fire weather events is consistent with current understanding of fire behaviour and its drivers/constraints. Among fire scientists it is well-accepted that fuel, terrain and weather conditions are the three key determinants of forest fire behaviour in general and of fire propagation rates in particular (Gould et al. 2007; Johnson and Miyanishi 2001; McArthur 1967). Though fire behaviour may always be a function of multiple factors, theory and field experience suggest that those factors do not contribute equally in all fire-prone landscapes, in every fire, or throughout the course of a single fire (Falk et al. 2007; Peters et al. 2004). More precisely, the main fire-controlling factors in fuel, ignition, terrain and weather, have different ranges of variation and affect fire behaviour at different spatial scales (Bessie and Johnson 1995; Falk et al. 2007). Consequently, the predominance of one factor over another may occur over a limited fire size interval but no single factor explains the entire fire size distribution. Our findings are in accordance with this scale-dependence of fire-controlling factors: fire sizes and fire weather events were found to have matching scaling behaviour over a considerable, yet restricted, range of fire sizes, corresponding to roughly 50-60% of the recorded fires. Thus, other fire-controlling factors than weather including fuel patterns may still determine the distribution of a significant proportion of the (smaller) fires but, as our findings suggest, they do not explain the spatial scale invariance of the fires in our study areas. Previous studies (e.g. Carlson and Doyle 2002; Malamud et al. 1998; Song et al. 2001) have often ignored the existence of multiple scaling regions, focusing narrowly on constructing an explanation for the power law behaviour of fire size distributions from simulations with models designed to study complex system behaviour in general but lacking essential ingredients

of the fire propagation process (e.g. the non-stationary forcing by variability and pattern in weather events).

Thirdly, the similarity between the slopes of the central scaling region for the distributions of the weather metric, M and the fire areas, A , is in accordance with the way we quantified the magnitude of the weather events: M should scale to A by definition of the FFDI as a fire weather index that is proportional to the linear rate of fire spread (McArthur 1967). The total linear spread of a fire is then proportional to the sum of FFDI over the time length of the fire, and the total planar size of the fire (i.e. area) is proportional to the sum of FFDI all squared (assuming fuel and terrain conditions are uniform and fire propagation is isotropic on average over a large region and many fires):

$$A = \gamma (\sum 1[FFDI > F_0] FFDI)^2 \quad (4)$$

Since we plot both distributions on a log-log scale:

$$\log A = \log \gamma + \log (\sum 1[FFDI > F_0] FFDI)^2 \quad (5)$$

Importantly, the proportionality constant disappears when the log-log axes are rescaled (Eq. 2). We therefore use the kernel of the expression on the right hand side

$M = (\sum 1[FFDI > F_0] FFDI)^2$ when considering $\log A$, where M quantifies fire weather event magnitudes in terms of potential fire areas. Actual fire areas, A , recorded in our study areas are not necessarily equal to the potential areas, A_{pot} , predicted from the value of M due to impacts of additional fire constraining factors, of which suppression is perhaps the most significant. Although suppression will generally cause A to be smaller than A_{pot} , a closer look at how suppression may work statistically suggests that the power law distributions of A retains the same scaling exponent as A_{pot} .

In southern Australia, fire management agencies will in principle attempt to extinguish any unplanned forest fire as soon as possible after detection. Thus, many fires are contained at an early stage and will only burn a small area (e.g. < 10 ha). For fires that are not contained at an early stage (e.g. due to weather conditions), extinguishment by direct attack usually becomes impossible once the size of the fire exceeds a threshold (e.g. > 10 ha), forcing suppression efforts to change to indirect attacks (e.g. creating containment lines) until weather conditions provide new opportunities to stop the fire. In both scenarios, suppression reduces the area actually burnt by the fire (i.e. the recorded value of A) to some fraction, l , of A_{pot} . Under the assumption that l has a uniform distribution, and considering $A = lA_{pot}$ and $A_{pot} \propto M$, A may correlate weakly to M (Fig. 4) but the power law distributions of A , A_{pot} , and M should have the same scaling exponent (for details, see Appendix 1).

Applicability of results to other fire-prone landscapes

The four areas studied here are highly representative of southern Australian eucalypt forests in terms of stand structure, fuels, relief, climate, fire behaviour and management (Abbott and Burrows 2003; Bradstock et al. 2002; Luke and McArthur 1978) and we may therefore expect our results to apply generally to other areas of similar forest. It is likely that fire size distributions from other fire-prone environments also hold a signature of a meteorological forcing pattern as the weather is a key controlling factor of most vegetation fires. This control may be exerted either directly with wind as a driving force of fire spread or indirectly, for example, by creating particular spatiotemporal patterns of extremely dry fuels (Beverly and Martell 2005). Where the scaling of weather events is expressed in the distribution of fire sizes our results suggest this

applies to only one of the several scaling regions that comprise the entire distribution. Breakpoints in the fire size distribution have been attributed to shifts in fire behaviour modes and corresponding sets of drivers/constraints (e.g. Ricotta et al. 2001). They are more likely observed in fire size distributions from complex, multilayered vegetation types such as the eucalypt forests of this study than in more homogenous or continuous fuel types (e.g. grasslands, boreal forest) characterised by stand-replacing fires. On the other hand, we believe fire size distributions with multiple scaling regions could be more common than suggested by the relatively low frequency with which they are reported in the literature. Multiple scaling regions are commonly reported for fire size distributions from relatively small or environmentally uniform districts (e.g. Ricotta et al. 2001; Telesca et al. 2005) while they are apparently less common for distributions from very large and environmentally heterogeneous regions (Malamud et al. 2005; Song et al. 2001). This is not surprising as the position of breakpoints in the fire size distributions varies, even among the distributions from fairly similar forest regions (Fig. 2). Well defined break points are likely to disappear in the mixing of distributions when fire records from vast and heterogeneous regions such as the ‘U.S. Fish and Wildlife Service lands’, ‘western United States’ (Malamud et al. 1998, p. 1841) or ‘China’ (Song et al. 2001) are pooled and plotted as one data set.

Potential management implications

Fire management agencies in Australia rely heavily on fuel reduction treatments (i.e. mainly prescribed burning) to reduce potentially adverse effects of unplanned fires in forests and other fire-prone vegetation types such as shrublands and woodlands (Abbott and Burrows 2003; Bradstock et al. 2002; Cary et al. 2003). This management strategy

is based on well-established relationships between fuel loads and potential fire intensity, and the possibilities/success of fire suppression measures (Gould et al. 2007; Luke and McArthur 1978). Forest managers are obviously aware of the fact that climate and weather conditions set important boundary conditions for this strategy, and that future climate change may force current fire management strategies to be revised and/or adapted. Our results may provide some guidance in that process. So far, potential impacts of climate change on unplanned fire occurrence in Australia has mostly been discussed in terms of possible changes in seasonal fuel moisture conditions or the frequencies of days of severe fire weather (Beer and Williams 1995; Hennessy et al. 2005; Pitman et al. 2007; Williams et al. 2001). Though such predictions may indicate whether the fire potential is likely to increase or decrease under future climate, they provide little information about possible changes in the probabilities of occurrence of fires of a given size. Our analysis showed that fire sizes are related to the magnitudes of fire weather events and consequently that the structure of temporal variation in weather conditions is important (Fig. 1). This may imply that changes in night-time conditions or more generally in the frequency/duration of conditions that temporarily reduce fire intensity and spread rate (e.g. dew or rain) are as important for predicting probabilities of future fire sizes as changes in the frequencies of extreme fire weather days.

Conclusion

We have reviewed existing explanations for the formation of spatial scale invariance of forest fires in the context of current knowledge of fire behaviour and its drivers/constraints. The reliance on fire sizes being solely controlled by (self-organized

or optimised) fuel patterns was identified as a key difference between existing theoretical frameworks such as SOC and HOT and the observed behaviour of actual forest fires. To reconcile the field experience of actual forest fires as often being strongly weather-driven with an equal or more plausible mechanism for the observed scale invariance of fires we hypothesized that the fire size distribution is a mapping of the scaling behaviour of fire weather events. Extensive data sets of forest fires and weather observations from four large forest regions in southern Australia were analysed. Distributions of fire sizes and weather event magnitudes displayed multiple scaling regions, with matching scaling exponents observed for power laws fitted to the central scaling regions that represented 53-56% of the fires and 49-64% of weather events. Our findings support the hypothesis that the spatial scale invariance of southern Australian forest fires mirrors the frequencies and magnitudes of fire weather events and poses an alternative for previously proposed explanations based only on self-organizing or optimising fuel patterns (Carlson and Doyle 2002; Malamud et al. 1998; Moritz et al. 2005; Ricotta et al. 1999; Ricotta et al. 2001; Song et al. 2001).

Fire behaviour in southern Australian eucalypt forests may differ in some details from that in forests and woodlands elsewhere (Luke and McArthur 1978), but the key controlling factors (i.e. fuel, terrain, weather) apply to unplanned fires in general. We therefore expect that scale-free power law distributions of fire areas in other fire-prone environments may also mirror a meteorological forcing pattern. Understanding of the functional linkage between the scaling behaviour of regional weather conditions and unplanned fire size distributions contributes to improved assessments of fire potential and identification of management options under changing climate and land use conditions.

Our case study of southern Australian forest fires shows that the functioning of ecological systems may be better understood by examining its spatiotemporal configuration in conjunction with the complexity of its drivers. The relevance of our findings may go beyond forest fire complexity by providing a new perspective on the formation of scale-free landscape patterns, which have so far been commonly attributed to self-organization. We expect that more examples of exogenously controlled scale invariant patterns will be discovered.

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Figure captions

Figure 1. Time series of hourly values of the McArthur Forest Fire Danger Index (FFDI) at Canberra Airport (July 2002-June 2004). The inset shows the index's strong diurnal pattern for January 2003. Extreme index values occurred on 18 January 2003 when bushfires that started in the Australian Capital Territory on 8 January 2003 reached Canberra. These fires finally burnt ~250,000 ha across the ACT.

Figure 2 Cumulative frequency-area distributions of recorded unplanned fires in four forest regions of southern Australia: (A) southwest Western Australia (1995-2004; 397 fires), (B) Australian Capital Territory (1921-2004; 897 fires), (C) southeast New South Wales (1962-2002; 557 fires), (D) northeast Victoria (1972-2004; 1874 fires). $N>A$ is the number of fires, N , exceeding an area A (hectares). Symbols: (•) data points; (—) fitted broken stick model; (---) corresponding breakpoints. Scaling exponents for the central scaling region are given by α_A (Eq. 3).

Figure 3 Inverse cumulative frequency distributions of rescaled fire areas (black circles) and weather event magnitudes (red triangles) in four forest regions of southern Australia. (A) southwest Western Australia, (B) the Australian Capital Territory, (C) southeast New South Wales, (D) northeast Victoria. Fitted power-laws (Eq. 3) for the central scaling regions of rescaled fire areas and weather events have similar scaling exponents, α_A and α_M , and represent similar proportions, $prop_A$ and $prop_M$, of the data. Distributions of weather events are for given values of F_0 and β (Eq. 1).

Figure 4 The recorded areas of unplanned fires, A , as a function of the predicted magnitude, M , of the corresponding fire weather events (eq. 1) in four forest regions of southern Australia: (A) southwest Western Australia, (B) Australian Capital Territory, (C) southeast New South Wales, (D) northeast Victoria. The continuous black curves show smooth kernel regressions of the data, with 95% confidence regions shown in grey. See Appendix 1 for details.

Figures

Figure 1

Figure 2

Figure 3

Figure 4

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Appendix 1

Power law behaviour of fire sizes preserved under suppression

Bivariate plots of observed fire areas, A , as a function of the weather metric M show no or very weak correlation between the two variables (Fig. 4). Here, we show that fire suppression, which is not accounted for in the weather metric M , can explain the apparent contradiction of A and M having power law distributions with matching scaling exponents while not being strongly correlated.

By assuming a close relationship between M and the maximum potential area burnt we will demonstrate through a simple model of suppression that:

- (i) the power law size distributions of suppressed fires and of M retain the same scaling exponent (analytically);
- (ii) correlation between fire areas and M is low when suppression is present (by Monte Carlo simulation).

Suppression Model

Our suppression model possesses three parameters, p_s , k and l_U , where:

p_s is the probability of a fire being effectively suppressed such that the area of the fire is bounded by a dimensionless threshold k ; and

l_U is the lower bound of a uniform distribution which marks what proportion of the maximum potential area burnt A_{pot} is actually burnt. It represents that there may be differing levels of suppression, but may include extra stochasticity based on factors such as topography and any fuel age effects. Note that the maximum potential area burnt is either $A_{pot} = M$ (i.e. the TIFFDI metric) or $A_{pot} = k$ depending on the outcome of the binomial trial given by p_s .

Applying our assertion that the area burnt A_i in the absence of suppression is predicted by M_i for any fire i , $i \in \{1, \dots, n\}$ then the probability distributions of A and M follow a power law, at least in the central region of the support for those distributions:

$$N(A > a) = \lambda a^{-\alpha}; N(M > m) = \gamma m^{-\alpha} \quad (A1)$$

$$\Rightarrow N(A > a) \propto N(M > m) \quad (A2)$$

where α is the scale invariant exponent and λ, γ are constant scalars. Eq. A2 is the conclusion drawn when assuming FFDI is proportional to the linear rate of fire spread (McArthur 1967). Given $X_i \sim \text{Bin}(1, p_s)$ is an indicator variable where $X_i = 1$ represents successful suppression, and $Y_i \sim U(l_U, 1)$ is the proportion of the potential actually burnt, then the suppressed fire size distribution is given by:

$$Z_i = \begin{cases} kY_i, & X_i = 1 \\ M_i Y_i, & X_i = 0 \end{cases} \quad (\text{A3})$$

(i) Insensitivity of the scaling exponent to suppression

The probability density of the power law distribution for $M = m$, assuming for sake of simplicity a minimum value for m of 1, can be described by:

$$p_M(m) = (\alpha - 1)m^{-\alpha} \quad (\text{A4})$$

then,

$$\begin{aligned} f_Z(z) &= f_{MYX}(x, y, m) \\ &= f_{M|YX}(m | y, x) f_{Y|X}(y | x) f_X(x) \\ &= f_{M|YX}(m | y, X = 1) f_Y(y) p_X(X = 1) + f_{M|YX}(m | y, X = 0) f_Y(y) p_X(X = 0) \quad (\text{A5}) \\ &= \frac{1}{1 - l_U} [f_Y(Y = z/k) p_s + f_{MY}(MY = z)(1 - p_s)] \end{aligned}$$

The first part of Eq. A5 on the RHS may be ignored as it dominates only the lower tail of the distribution of Z for k small enough. The central scaling region is given by the second part on the RHS. Since products of distributions are analytically intractable, we assume that $l_U = 0$ to ensure symmetry of Y , allowing $f_{MY}(MY = z) = f_{MY}(M/Y = z)$ so that the quotient of two random variables may be considered without loss of generality. Using $Z = M/Y$ and M, Y independent then the distribution of the quotient Z is dominated by:

$$\begin{aligned}
f_Z(z) &= f_{MY}(MY = z)(1 - p_s) \\
&= (1 - p_s) \int_0^1 y f_Y(y) f_M(yz) dy \\
&= (1 - p_s) \int_0^1 y (\alpha - 1) (yz)^{-\alpha} dy \\
&= (1 - p_s) \frac{\alpha - 1}{2 - \alpha} z^{-\alpha}
\end{aligned} \tag{A6}$$

which is positive only for $1 < \alpha < 2$. The key result here is that both the distribution with suppression and the distribution without suppression are proportional to each other and share the same scaling exponent α .

(ii) Weak correlation between M and A

For the study area in SW Western Australia the correlation between M and A (i.e. actual area of recorded fires) was 0.3426. The mean correlation between M and Z for 500 Monte Carlo simulations was low (0.237), where Z mimics A in the presence of a high suppression success $p_s = 0.9$. In this example, variable suppression effects had been removed from the model ($l_U = 1$). Inclusion of a high level of variability in suppression effects (e.g. $l_U = 0$) reduced the M, Z correlation by only 0.042 at most for $p_s = 0.9$, meaning suppression success p_s was the more influential parameter in determining the overall correlation between M and Z . Decreasing suppression success meant that the M, Z correlation increased above the observed 0.3426. Parameter k had little influence. As $A \propto Z$ in distribution then low correlations between fire sizes and the metric M are expected in the presence of high rates of successful fire suppression.

Conclusion

The conclusion we draw is that in the presence of strong fire suppression, other factors such as fuel age effects are of less importance in determining the distribution of observed fire sizes. Regardless of the level of suppression, the distribution of fire sizes bounded by the central region of the distributional support will continue to possess a scale invariant exponent similar to that for the distribution of the fire weather magnitudes M corresponding to those fires.

Supporting Material

Corresponding manuscript:

Boer, Sadler, Bradstock, Gill & Grierson: Endogenous and exogenous controls of wildfire complexity

Table S1. Details on study areas and fire data sets. Data sources: 1) The Department of Environment and Conservation - Western Australia (SW-WA), 2) Environment ACT - Australian Capital Territory (ACT), 3) The Department of Environment and Conservation - New South Wales (SE-NSW), 4) The Department of Sustainability and Environment - Victoria (NE-VIC).

Study area	Observation	No. fires	Area of study	Fire area	Data source
	period		region (ha)	range (ha)	
SW-WA	1995-2004	397	9.28×10^5	0.01-28863	1
ACT	1921-2004	897	2.36×10^5	0.13-265864	2
SE-NSW	1962-2002	557	1.04×10^6	0.01-90372	3
NE-VIC	1972-2004	1874	1.40×10^6	0.0001-1092421	4

Table S2. Details on weather stations, observations and fire weather events. The number of events corresponds to the number of days on which one or more fires started during the observation period.

Study	Station name	Station	Observation	Frequency of	No.
area		code	period	observation	events
SW-WA	Manjimup	009573	Oct. 2000 –Aug. 2004	Hourly	113
ACT	Canberra Airport	070014	Jan. 1998 – May 2005	Hourly	64
SE-NSW	Sydney-Obs.Hill	066062	Jan. 1998 - Sep. 2005	Hourly	117
	& Fort Denison	066022			
NE-VIC	Wangaratta	082053	Sep. 2000 - Nov. 2004	Half-hourly	150
	Airport				

Appendix S3. *Choice of distributional model*

In Figure 2 the number of fires of area greater than each fire area bin value, $N > A$ ($\log_{10}$), is plotted against the fire area bin values, A ($\log_{10}$). Before selecting a broken-stick model with two breakpoints two further distributional models were considered; the disjoint broken-stick model and the upper-truncated power (or Pareto) law. A multiple linear regression model which fits disjoint line segments as opposed to joined segments is achieved by minimising residual sum of squares. Disjoint line segments were not used since they introduce further degrees of freedom and were found by the authors to be more sensitive to non-linear features in the data.

The upper-truncated power law distribution is predicated on there being a maximum event magnitude due to the finite size of the system (Tebbens and Burroughs 2005). Parameters can be easily estimated using a maximum likelihood procedure (Aban et al. 2006). However, in practice not all our data sets reflect an upper-truncated distribution. For example, the largest fires in the data sets from the Australian Capital Territory and NE Victoria are an order of magnitude greater than the next largest fires (Fig. 2). A similar distribution was found for the lengths of normal faults on the plains of Venus (Tebbens and Burroughs 2005). While the upper-truncated Pareto function fits some of our data sets reasonably well (e.g. SW-WA) it performs poorly for the data sets with extremely large fires (e.g. NE-VIC) (Figure S3-1). Although the argument for truncated power laws is sound the data do not always behave accordingly.

Supporting Material

Unlike the model fitting procedure for the upper truncated Pareto law the broken-stick model is not fitted directly to the underlying data but instead to the log-log distribution as no algorithm exists for such a procedure. Hence, in having chosen a broken-stick model it is important to use evenly spaced bins in constructing the fitted cumulative distribution function rather than evaluating the cumulative distribution at the fire sizes present in the data. All points on the distribution therefore have equal weight as the cumulative distribution is defined to be continuous (whereas a histogram of the data is not). The rarity of extreme values will mean that there are non-linearities in the upper tail of the distribution rather than sparse data, leading to lower reliability in estimation of scaling exponents for the upper tail. The approach is deemed appropriate as these exponents are not of immediate interest to the argument of this paper and the broken-stick model fits the data reasonably well in this region (Figure S3-1).

The number of bins also impacts upon the smoothness of the cumulative frequency-area distribution, particularly in the upper tail, and thereby affects any estimates of scaling behaviour. However, there is a trade-off between the number of bins used (i.e., computation time) and the accuracy of slope estimates of scaling regions. As a simple experiment the slope estimates α_A of the central scaling region for each data set differed by 0.004 at most when 200 as opposed to 100 bin values were used. Hence, 100 bin values evenly spaced over the range of observed fire areas on the logarithmic scale were chosen for all frequency-size distributions (Figures 2, 3), reducing computation time of any bootstrap sampling without any great loss in accuracy.

Supporting Material

The causes of non-truncated behaviour in the region of extremely large fires may be two-fold, despite the forest systems studied being spatially finite. First, variability of the empirical cumulative frequency-area distribution is greater for extremely large fires due to a sparse number of observations of these events (as demonstrated by the greater bootstrap variability for large fire sizes in Figure S3-1). Second, it may be that the rarely observed extremely large fires have a distinct mode of behaviour driven by a different set of processes. Very large fires burning at extreme intensity produce massive convective rises of heated gases that create low air pressure and draw air towards the fire front, which may then contribute to a further increase in intensity and rate of spread leading to so-called fire blow-ups or fire storms (Clark et al. 1996; Peters et al. 2004). Certainly, extreme fire weather and extreme fire behaviour has contributed to the exceptionally large fire events in the ACT (Fig. 2) and high country of Victoria in 2003 (Ellis et al. 2004). However, as the extremely large fires are rarely observed we can not know their frequency distribution. We have therefore focused this study on the central section of the fire-area distributions corresponding to the commonly observed fire sizes. The variability in the fire area data is well explained by the broken-stick model regardless of which of the two types of empirical distributions (with and without extremely large fires) are being observed (Figure S3-1).

Another form of truncation commonly applied to cumulative frequency-size distributions is the censoring of small fires from the data set. The common justification for this censoring is that as fire size decreases then an increasing proportion of fires are either not reported due to their minimal impact on the studied system or not observed (Ricotta et al. 1999). Fire sizes of < 1 ha are commonly excluded while fitting a typically linear function to

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frequency-area distributions (Malamud et al. 1998; Ricotta et al. 1999). A simple calculation demonstrates that such censoring is poorly justified. We extended the slope of the central region to all fire sizes to quantify the proportions of fires that would need to be censored if it was assumed that there was but a single underlying scaling region. The proportion of fires that are not observed ($P_i^{censored}$) at each binning of size i is therefore given by the difference between the empirical and the underlying scaling distributions:

$$P_i^{censored} = \frac{(F_i^s - F_{i-1}^s) - (F_i^e - F_{i-1}^e)}{F_i^s - F_{i-1}^s} \quad (6)$$

where F^s is the cumulative frequency of the assumed underlying scaling law distribution and F^e is the cumulative frequency of the empirical distribution. For the data set from SW Western Australia more than 80% of fires of 0.1 ha or less, or more than 30% of fires of 1 ha or less, would have to be unobserved to reproduce the empirical cumulative frequency distribution from the assumed underlying distribution (Figure S3-2). Such figures are unrealistic for our study areas according to the expert opinions of people involved in the collection of the data. Instead, the multi-scaling behaviour of the cumulative distribution may be attributed to either the system being spatially finite (e.g., rice pile systems (Aegerter et al. 2004) or shifts in mode of fire behaviour and corresponding fire fighting tactics (Peters et al. 2004), or both. In all cases a broken-stick model seems justified.

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Figure S3-1. Cumulative fire frequency-area distributions for SW Western Australia (A & C) and NE Victoria (B & D). Two models have been fitted to each of the regions' fire area data: the broken-stick (A & B) and the upper-truncated Pareto (C & D). The shaded region is a measure of model applicability, generated by the model being fitted to 1000 bootstrap samples of the data. The broken-stick model fits the data well for both regions, except at small fire sizes (< 0.1 ha), whereas the truncated power law does not.

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Figure S3-2. The proportion of unobserved fires if the underlying distribution of fires is assumed to be described by a single exponent taken from the central scaling region. At each bin on the x-axis two quantities are calculated: (a) the difference between the cumulative frequency-magnitude distribution for that bin and the bin of the next largest size, and (b) the difference between the cumulative frequency-area distributions of the bins when the cumulative frequency-area distribution is the central scaling region extended over the entire domain of fire areas. The proportion censored then becomes (a) divided by (b). The data shown are from SW Western Australia.

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Appendix S4. *Calculating the proportion of observations in the central scaling region.*

The proportion of events observed in the central scaling region ($P^{central}$) is given by:

$$P^{central} = F_{bp_2} - F_{bp_1} \quad (7)$$

where F_{bp1} and F_{bp2} are the cumulative frequencies at the first and second breakpoints respectively.

Appendix S5. *Bootstrap confidence regions for difference in slope.*

Direct comparison of slopes between the rescaled A and M distributions is not possible using standard linear regression tests as the assumption of a normally distributed error does not hold. Ordinary bootstrap confidence regions may be generated by sampling randomly from the fire data and their corresponding fire weather events for the period of weather observation. For each of the bootstrap samples the broken-stick model was fitted to both the fire and weather distributions and a difference in slopes and proportions for the central scaling region then calculated. However, the bootstrap confidence intervals could only be applied in a semi-automatic fashion to the SW Western Australia data due to the sensitivity of the broken-stick model to starting values in the estimation procedure, and perhaps due to either of the underlying distributions of fire or weather events not being sufficiently piecewise linear (although this last concern was not investigated). Further, these insufficiently piecewise linear distributions were excluded from the bootstrap procedure and thus introduced an unknown bias into the estimation of the confidence region. For the other datasets the outlined bootstrap procedure failed.

For the Western Australian data a wide 95% confidence region was returned for the mean difference in slope parameters $\alpha_{A'}$ and $\alpha_{M'}$ (C. R. = $\{-0.241, 0.612\}$; 500 bootstrap samples). Likewise, no difference in the proportions $prop_{A'}$ and $prop_{M'}$ was detected (C. R. = $\{-0.319, 0.014\}$).

Appendix S6. *The reporting of scaling behaviour*

Some researchers report R^2 values for fitted distributions pursuant to a linear regression (Malamud et al. 2005). Reporting R^2 values is meaningless when the number of bins is chosen arbitrarily, as R^2 values increase with increasing bin number for the same empirical cumulative frequency distribution. R^2 values are also dependent on sample size, as this will determine the “smoothness” of the cumulative frequency-magnitude distribution. This is unlike a simple regression setting (Aban et al. 2006) where additional observations in principle do not alter the proportion of the variation in the data that is explained by the model.

Appendix S7: *References for Supporting Material*

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