Modelling the behaviour of sensitive clays experiencing large deformations using non-local regularisation techniques

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Abstract

This paper presents a methodology to simulate the strain-softening-hardening response of sensitive clays when subjected to extensive remoulding. A non-local strain-softening technique is implemented into the Abaqus Finite Element Analysis (FEA) software through its user defined material subroutine UMAT, in the form of a critical state based strain-softening constitutive model. For the comparison of conventional and non-local formulations, a series of analyses have been performed investigating mesh dependency issues and the validity of a simple softening-scaling rule for practical applications. The non-local model significantly reduces solution mesh-dependency for small-strain analyses of biaxial shear and updated Lagrangian analyses of buried pipe uplift. Finally, the non-local model is applied to simulate undrained cycles of penetration and the consolidated-undrained capacity of a T-bar in kaolin clay. This analysis highlights the limitations of the use of standard element tests (e.g. triaxial tests) to calibrate the model input parameters that control the strain-softening aspect of the constitutive law, and the need for a constitutive law that captures the partial recovery of sensitivity during consolidation.

Keywords: Sensitive clays, Strain localisation, Non-local regularisation, Large deformations, Buried pipe uplift, T-bar penetration tests

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1. Introduction

Fine-grained offshore sediments exhibit strain-softening behaviour with the development of excess pore pressures during the large near-undrained deformations commonly encountered in geotechnical processes such as full-flow penetrometer testing, anchor installation, keying and pullout, pipe laying and buckling. The subsequent dissipation of excess pore pressures induces strain-hardening of the soil and results in partially recovered or even increased strength. It is well established in the literature that numerical modelling of the strain-softening response using classical continuum models such as the finite element method can be challenging due to severe mesh dependent strain localisation [1, 2], which usually appears in the form of shear bands with the thickness of a single element. Due to absence of an internal length scale, these models fail to describe the size effect of strain localisation correctly resulting in mesh dependent solutions [2].

Several regularisation or mitigation techniques have been reported in the literature that can ensure that finite element simulation output remains independent of the mesh discretisation when rate-independent elastoplastic strain-softening models are adopted to describe the material response. The most common mitigation methods involve the implementation of Cosserat continuum methods [3, 4], gradient dependent continuum methods [5, 6, 2] and integral-type non-local strain methods [7, 8, 9]. These techniques all introduce an internal length scale to the constitutive model formulation, which controls the degree of deformation localisation and preserves the well posedness of the governing partial differential equations irrespective of the refinement of the mesh [2].

The simple Tresca-derived elastic-perfectly plastic constitutive model with strain-softening, proposed by Einav and Randolph [10], has been used extensively to simulate ultra large deformation problems such as T-bar penetration in soft clays [11, 12, 13, 14]. In these works, mesh dependency has typically been mitigated by the incorporation of viscous (or rate) effects in the constitutive equations, which enhances the undrained shear strength of the soil by approximately 5-20% per log cycle of rate over the typical laboratory strain rate of 1%/hour [15, 16, 10]. However, the efficacy of this approach in mitigating mesh dependency reduces with reducing strain rate. In other words the transition from undrained to drained behaviour is not

likely to be properly regularised by such an approach. This transition occurs many times in what might be classed as episodic undrained problems, such as T-bar penetration or pipeline buckling, which involve undrained displacement followed by cycles of consolidation and further undrained displacements [17, 18]. These problems require a constitutive model with the capability to model both strain-softening and strain-hardening, as well as a regularisation approach that is effective, irrespective of the operative strain rate.

The main objective of this paper is to demonstrate the application of a regularised large deformation FE based technique for use in the modelling of strain-softening-hardening behaviour. For simplicity, and in order to validate the efficacy of the non-local regularisation approach implemented directly, viscous effects were not considered in this study. The integral-type non-local strain-softening approach [19] was adopted to regularise FE solutions using a simple strain-softening extension of the classical Modified Cam Clay constitutive model [20]. The model was implemented in the commercial finite element software Abaqus, via its user-defined material subroutine UMAT.

The mesh-dependency of the finite element (FE) solutions obtained using the local and non-local models is first demonstrated through a series of small strain biaxial compression simulations. The implementation is then used to simulate a boundary value problem in the form of buried pipe uplift via Large Deformation Finite Element (LDFE) analyses using the Remeshing and Interpolation Technique with Small Strains (RITSS; Hu and Randolph [21]) approach, in order to further explore the mesh dependency when the mesh is periodically updated in an updated-Lagrangian sense [12]. The results obtained using the non-local model are shown to be insensitive to the mesh topology. Perhaps surprisingly, the computational efficiency of the analyses appears largely unaffected, with the additional computational burden of the non-local calculations being offset by an improvement in the rate of convergence due to the preservation of convexity of the governing partial differential equations [2]. Finally, undrained and consolidated-undrained cycles of T-bar penetration in kaolin clay are modelled using the same approach, with the results compared to T-bar penetration experiments. This final analysis illustrates: (i) the difficulty in selecting appropriate constitutive parameters to control the rate of strain-softening using element test data; and (ii) the need for constitutive models that capture the recovery of sensitivity after consolidation induced hardening.

2. A simple constitutive model for strain-softening-hardening

The Structured Modified Cam Clay (SMCC) constitutive model proposed by Mařín [20] has been adopted in this paper. This is a simple elastoplastic critical state model for structured fine-grained soils based on the classical Modified Cam Clay (MCC) [22]. This model was chosen because it is the simplest variant of the Cam Clay genera of constitutive model that can capture both the strain-softening and strain-hardening aspects of soil behaviour and is straightforward to implement in updated-Lagrangian methods such as the RITSS approach.

2.1. Local formulation

The model requires a total of nine parameters, which can be grouped into two distinct groups: (i) the basic parameters M , λ^* , κ^* , N and G (or ν); and (ii) the strain-softening parameters k , A , S_t and S_f . The basic parameters have physical interpretations that are consistent with the original MCC model parameters, except that the so-called Butterfield compression law [23], replacing the conventional e - $\ln p'$ relationship of MCC model by $\ln(1 + e)$ - $\ln p'$, has been adopted. The second group of parameters control the strain softening aspect of the model response. The model has two state variables: void ratio e and sensitivity s_{ep} . The shape of the yield surface (f) is expressed as follows:

$$f = q^2 + M^2 p' (p' - s_{ep} p_c) \quad (1)$$

where p' and q are mean effective stress and deviatoric stress respectively, p_c represents the yield surface size of the reference soil, where it can be seen that the sensitivity state variable s_{ep} acts as a simple scalar multiplier on the size of the yield surface. The initial magnitude of the sensitivity state variable s_{epi} can be derived from the slopes of the normal consolidation line (NCL) λ^* , unload-reload line (URL) κ^* and the initial sensitivity S_t as follows:

$$s_{epi} = S_t^{\left(\frac{\lambda^*}{\lambda^* - \kappa^*}\right)} \quad (2)$$

Similarly, the value of the sensitivity state variable for the reference soil (typically taken as unity) s_{epf} is calculated by:

$$s_{epf} = S_f^{\left(\frac{\lambda^*}{\lambda^* - \kappa^*}\right)} \quad (3)$$

100 The relation in equation 2 represents the ratio of the yield envelope size
 101 for the intact and the reference (i.e. remoulded) soil states. Associated flow
 102 is assumed for simplicity, hence the plastic potential surface is identical to
 103 the yield surface given in Equation 1. The void ratio e is related to the
 104 quantity p_c through:

$$\ln(1 + e) = N - \kappa^* \ln p' - (\lambda^* - \kappa^*) \ln \left(\frac{p_c}{p_r} \right) \quad (4)$$

105 where p_r is the reference pressure 1 kPa.

106 The incremental strain rate drives the evolution of the state variables e
 107 and s_{ep} according to:

$$\dot{e} = -(1 + e) \dot{\epsilon}_v \quad (5)$$

$$\dot{s}_{ep} = -\frac{k}{(\lambda^* - \kappa^*)} (s_{ep} - s_{epf}) \dot{\epsilon}_s \quad (6)$$

108 where k controls sensitivity degradation rate, $\dot{\epsilon}_v$ is the volumetric strain
 109 rate and $\dot{\epsilon}_s$ is the equivalent plastic strain rate obtained from the plastic
 110 volumetric and the deviatoric strain rates (i.e., $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_q^p$, respectively) by:

$$\dot{\epsilon}_s = \sqrt{(\dot{\epsilon}_v^p)^2 + \frac{A}{1 - A} (\dot{\epsilon}_q^p)^2} \quad (7)$$

111 The parameter A governs the relative contribution of $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_q^p$ driving
 112 the strength degradation. In the present work, a slightly different form of
 113 the equation 7 (previously used in several strain-softening models including
 114 Taiebat et al. [24]) has been adopted to facilitate greater flexibility as it
 115 allows the value of A to be set to unity in order to prescribe softening driven
 116 purely by plastic shear strain, writing the plastic strain rate as:

$$\dot{\epsilon}_s = \sqrt{(1 - A) (\dot{\epsilon}_v^p)^2 + A (\dot{\epsilon}_q^p)^2} \quad (8)$$

117 In this model formulation, the local incremental plastic strains drive the
 118 strain-softening aspect of the model response, which is why it is referred to
 119 herein as a ‘local’ formulation.

120 2.2. Regularisation method: non-local softening

121 The regularised variant of the model simply uses a non-locally weighted
 122 measure of the volumetric and deviatoric strains, $\dot{\varepsilon}_v^p$ and $\dot{\varepsilon}_q^p$, to drive the
 123 softening response, similar to Galavi and Schweiger [25], Summersgill et al.
 124 [26], Mánica et al. [27], and Monforte et al. [28]. In this method, the strain-
 125 softening at a material point is linked to the state of plastic deformation in
 126 the material points surrounding the current integration point, rather than
 127 being solely a function of the strain at the current integration point. The
 128 basic constitutive relations remain unchanged and it is therefore relatively
 129 straightforward to implement the required modifications into LDFE sim-
 130 ulations compared to other regularisation methods. Technically, since only
 131 the variable controlling strain-softening response is considered non-local, this
 132 technique is referred to as a partially non-local approach; fully non-local
 133 variants of this approach utilise non-local stresses as well as strains in the
 134 constitutive response [25].

135 In the local SMCC model, the equivalent plastic strain expressed by Equa-
 136 tion 8 governs the strain-softening response at a FE integration point (IP).
 137 In the non-local SMCC model, this value is replaced by a spatially averaged
 138 value of the same quantity, $\bar{\varepsilon}_s$, calculated as:

$$\bar{\varepsilon}_s(X_i) = \int_V \alpha(X_i, X_j) \dot{\varepsilon}_s(X_j) dV \quad (9)$$

139 where X_i and X_j are positions of any IP under consideration and its
 140 neighbouring IPs, respectively, dV is the volume of the element represented
 141 by each integration point and $\alpha(X_i, X_j)$ is a normalised weight function used
 142 in spatial averaging of local variable $\dot{\varepsilon}_s$ at all integration points around the
 143 IP at which the stress state is being updated. The normalised form of the
 144 function preserves the uniform non-local distribution of strains when the local
 145 distribution is uniform and is expressed by:

$$\alpha(X_i, X_j) = \frac{\alpha_0(\|X_i - X_j\|)}{\int_V \alpha_0(\|X_i - X_j\|) dV} \quad (10)$$

146 where $\alpha_0(\|X_i - X_j\|)$ is the weight function value that controls relative
 147 contribution of any IP in strain softening. The value of α_0 depends on
 148 assumed shape of the weight function and distance between two IPs (i.e.
 149 $\|X_i - X_j\|$). Various weighting schemes have been proposed to determine
 150 the non-local strains from the local strain, including Gaussian [29], over

151 non-local [29] and bi-modal functions [25]. In this study, both Gaussian and
 152 bi-modal functions were implemented, given respectively by:

$$\alpha_0 = \frac{1}{l_c \sqrt{\pi}} e^{-\left(\frac{\|X_i - X_j\|}{l_c}\right)^2} \quad (11)$$

$$\alpha_0 = \frac{\|X_i - X_j\|}{l_c^2} e^{-\left(\frac{\|X_i - X_j\|}{l_c}\right)^2} \quad (12)$$

153 in which the parameter l_c - typically referred to as the ‘characteristic
 154 length’ - introduces a length scale dependency into the constitutive response
 155 in order to indirectly control the size of the strain localisation features (or
 156 shear bands). Figure 1 plots both weight functions against distance between
 157 two IPs for an arbitrary characteristic length of one metre. It can be seen
 158 that the Gaussian function assumes a maximal contribution from the IP
 159 under consideration, whereas the bi-modal function suggested by Galavi and
 160 Schweiger [25] (hereafter referred to as the ‘G&S method’ following Summers-
 161 gill et al. [26]) ignores the local equivalent strain at the current integration
 162 point. The non-local averaging calculations can only be guaranteed to be
 163 effective if the characteristic length, l_c , takes a value greater than the largest
 164 element size in the FE mesh discretisation.

Figure 1: Comparison of non-local weighting functions for $l_c = 1.0$ m.

165 2.3. Finite element implementation

166 The local SMCC model has been implemented in the commercial FE code
 167 Abaqus [30] via its user specified-material subroutine or UMAT. It was writ-
 168 ten in Fortran, based on the implementation of Gudehus et al. [31], which
 169 uses an adaptive Runge-Kutta-Fehlberg method (RKF23) for the stress inte-
 170 gration, with yield surface drift correction following the procedure described
 171 by Sloan et al. [32]. The non-local aspect of the model has been imple-
 172 mented by extending the UMAT subroutine for the local model, following
 173 the methodology suggested by Chow et al. [33]. To implement the weighted
 174 spatial averaging in the numerical algorithm, equivalent discrete integrals of
 175 equations 9 and 10 are adopted which can be expressed as:

$$\bar{\varepsilon}_s(X_i) = \sum_{j=1}^{NIP} \alpha_{ij} \dot{\varepsilon}_s(X_j) \quad (13)$$

$$\alpha_{ij} = \frac{\alpha_0(\|X_i - X_j\|)}{\sum_{k=1}^{NIP} \alpha_0(\|X_i - X_k\|)} \quad (14)$$

176 where NIP is the total number of integration points in the adopted do-
 177 main discretisation. When performing non-local spatial averaging of the
 178 plastic strains at any IP, access to information of the current state of all IPs
 179 in the simulation is required. To perform constitutive calculations, Abaqus
 180 successively calls the UMAT subroutine at all of the integration points of
 181 the elements for which the material definition includes a user-specified con-
 182 stitutive response. The UMAT interface allows access to state information
 183 (e.g., stresses, strain increment, step time, position coordinates and other
 184 state-dependent variables) of the IP currently under consideration but not
 185 all IPs in the model. This is the most critical issue with regards to the imple-
 186 mentation of the non-local calculations in a UMAT subroutine. Previously,
 187 the access to global state information has been gained by combining two or
 188 more subroutines, specifically UMAT (for constitutive response calculations)
 189 and UEL (a user-element subroutine to implement a user-defined element)
 190 [34].

191 Chow et al. [33] suggested a simpler approach which does not require
 192 the use of the UEL subroutine to access the state details of neighbour-
 193 hood IPs. In this method, a three dimensional FORTRAN storage array
 194 called a COMMON BLOCK was employed to store the information re-
 195 quired for the non-local implementation (coordinates of IPs and the local

196 strain variables). These details were stored at the beginning of the anal-
 197 ysis step and then updated at the end of every increment. In this paper,
 198 a method similar to this approach has been used. However, instead of
 199 using a COMMON BLOCK array, which limits the storage size to max-
 200 imum of 2GB, Abaqus utility routines are used to create global allocat-
 201 able arrays. At the time of the array creation in a subroutine, an identi-
 202 fier number (i.e a user defined arbitrary integer) is assigned to each array
 203 which can then be used to access the array data in any other subrou-
 204 tine. Four such arrays (EIXYS[3,NIPP,NUEL], IPID[NEGBR,NIPP,NUEL],
 205 ELMID[NEGBR,NIPP,NUEL], and WEGHT[NEGBR,NIPP,NUEL]) were cre-
 206 ated to store and transfer the required information for the non-local strain-
 207 softening calculations. In these arrays, NIPP is the number of total IPs per
 208 element, NUEL is the total number of elements in the mesh and NEGBR
 209 is the maximum number of K nearest neighbouring IPs for any IP in the
 210 model. In the first Abaqus call to the UMAT subroutine, the coordinates of
 211 all of the IPs are stored in the first and second dimensions of EIXYS. The
 212 third dimension is used to store the local strain-softening variable which is
 213 updated at the end of every increment. The non-local averaging is a com-
 214 putationally expensive process and therefore, for computational efficiency, K
 215 nearest neighbours (KNN) of all IPs are identified in the first step increment
 216 based on the the relative distance between them, and their identifier num-
 217 bers (IP and element number in FE mesh) are stored in the arrays IPID and
 218 ELMID. Then, the averaging weights are calculated based on the character-
 219 istic length and are stored into the array WEGHT[NEGBR,NIPP,NUEL] for
 220 all integration points. In this way, the identification of KNNs and calculation
 221 of their respective weights are performed once for each small strain step of
 222 the analysis and simply accessed from the storage array to perform non-local
 223 averaging during each increment.

224 *2.4. Material parameters and model validation*

225 SMCC model parameters for Pisa clay and UWA kaolin clay were adopted
 226 from Mašin [20] and Ragni et al. [35], respectively. These were previously cal-
 227 ibrated using standard geotechnical element tests performed on natural Pisa
 228 and reconstituted kaolin clay samples and the various parameters adopted are
 229 presented in Table 1. For the Pisa clay, the identification of the parameters
 230 ϕ_c or M , N^* , λ^* and κ^* was done by simulating isotropic compression tests
 231 on reconstituted clay, and the softening parameters S_t , k and A were found
 232 by the direct evaluation of the results of experiments on natural Pisa clay,

233 using the structure degradation law of the model[20]. The Poisson's ratio
 234 of the soil was assumed to be 0.3, rather than prescribing a constant shear
 235 modulus as required in the original model equations. In Ragni et al. [35], the
 236 basic parameters of the hypoplastic clay model (of which the SMCC model
 237 is an elastoplastic equivalent) for the UWA kaolin clay were adopted as the
 238 MCC model parameters defined by Stewart [36], with appropriate conversion
 239 from $e - \ln p'$ to $\ln(1 + e) - \ln p'$ space. The initial sensitivity S_t was adopted
 240 as T-bar based sensitivity S_{T-bar} , obtained from the cyclic T-bar tests in
 241 kaolin clay. The softening parameter k was back calculated from the triaxial
 242 compression tests presented in Figure 2, and the value of the parameter A
 243 was assumed as 0.2.

244 The triaxial compression tests on UWA kaolin clay [35] were simulated
 245 using the SMCC model to demonstrate the model capabilities and validate
 246 the model implementation in Abaqus. Figure 2 shows that the slope of
 247 the critical state line M was approximately 0.80 ($\phi_c = 20.6^\circ$) for the tests
 248 with mean effective stresses at the end of isotropic consolidation of 100 kPa
 249 and 400 kPa, and 0.90 ($\phi_c = 23.0^\circ$) for the test with mean effective stress
 250 of 50 kPa. The stress paths deviate from the CSL at approximately 10%
 251 axial strain, which is likely due to the onset of strain localisation so the
 252 data beyond this point has been faded out. The upper bound of $M =$
 253 0.9 derived from the test with lowest initial p' is close to the value of 0.92
 254 ($\phi_c = 23.5^\circ$) reported by Stewart [36], which has been adopted extensively in
 255 past numerical studies([37],[38],[39],[40],[41]). Considering the effective stress
 256 levels important for the boundary value problem simulation presented later
 257 in this study (i.e. ≈ 50 kPa), the critical state friction angle of 23.0° , also
 258 used by Ragni et al. [35], was adopted in this study.

Table 1: Parameters for Pisa and UWA Kaolin clay, after Mašin [20] and Ragni et al. [35], respectively

Parameter	Pisa clay	Kaolin clay
Slope of NCL, λ^*	0.14	0.0811
Slope of URL, κ^*	0.02	0.019
Poisson's ratio, ν	0.3	0.3
Frictional constant, M	0.85	0.90
Intercept of the NCL at $p' = 1$ kPa, N^*	1.56	1.195
Initial sensitivity, S_t	3.45	2.2
Final sensitivity, S_f	1.0	1.0
Degradation rate, k	0.4	0.2
Degradation ratio, A	0.1	0.2

Figure 2: Comparison of experimental and numerical data of Triaxial undrained compression tests on kaolin clay: (a) effective stress path in $q/p'_0 - p'/p'_0$ space, and (b) stress ratio variation with axial strain in $q/p' - \epsilon_a$ space.

259 3. Simulation of a classical element test: biaxial compression

260 3.1. Analyses details

261 Classical biaxial compression simulations have previously been used to
 262 demonstrate the mesh dependency of local strain-softening models and effi-
 263 cacy of the non-local techniques in regularising FE solutions involving strain-
 264 softening [25, 42, 27, 28]. In this work, a series of undrained and drained
 265 small strain FE analyses have been performed for biaxial compression using

266 the local and the non-local strain-softening methods. All the analyses were
267 performed using plane strain models generated in Abaqus. The geometry,
268 typical undeformed mesh and boundary conditions of the test are illustrated
269 in Figure 3. The soil specimen was considered to be a square domain of
270 2m x 2m size. Only 1/4 of the area of the domain was modelled due to
271 the symmetrical boundary conditions. The left hand side boundary of the
272 model (i.e. the vertical axis of symmetry) was allowed to move in the vertical
273 direction but was restrained in the horizontal direction. Along the bottom
274 boundary (i.e. the horizontal axis of symmetry) horizontal displacement was
275 allowed with vertical movement restrained. The top boundary of the mesh
276 was restrained against any horizontal movement and vertical displacement
277 was prescribed to apply the compression to the specimen, which is sufficient
278 to create strain localisation [42].

279 For domain discretisation, four meshes with 100 (10 x 10), 400 (20 x
280 20), 900 (30 x 30) and 1600 (40 x 40) eight node quadratic plain strain
281 quadrilateral elements (CPE8RP in the Abaqus Standard library) were used.
282 Figure 3 shows the mesh created for the case with 100 (10 x 10) elements
283 numbered from 1 to 10 along horizontal and vertical sides. To establish the
284 initial stress state in the model, vertical and horizontal stresses of 50 kPa
285 and 100 kPa were applied at the top and the right side boundaries of the
286 mesh respectively. The specimen compression response was obtained using
287 displacement controlled analyses which were performed by applying a 0.1 m
288 vertical displacement at the top boundary of the mesh. The input SMCC
289 model parameters for the Pisa clay were used. The total step time was
290 prescribed such that no dissipation of generated excess pore pressure occurs
291 in the undrained analyses, while full drained conditions were maintained
292 during drained analyses. The characteristic length was assumed to be 0.1 m
293 (i.e. equal to the element size in the coarsest mesh) in all of the non-local
294 analyses.

Figure 3: Illustration of analysis mesh used for biaxial simulations.

3.2. Results: mesh dependency

The vertical reaction force at the top of the analysis mesh is plotted against applied vertical displacement (δ_v) in Figure 4 for all of the considered cases. Figure 5 plots contours of the degree of soil remoulding (i.e. the ratio of current and initial values of the sensitivity state variable s_{ep}/s_{epi}) at the maximum compression of the mesh. The responses obtained from the local undrained simulations is shown in Figure 4a. The force-displacement curves are identical until the peak resistance is reached, after which an increasing rate of strain-softening is clearly observed with an increase in the number of elements. This is due to mesh dependent strain localisation, where the shear band thickness is proportional to the size of a single element (compare sensitivity distribution contours in Figures 5a, d, g, and j). Unlike the local undrained analyses, the local drained analyses do not suffer from the mesh dependency problem (see Figure 4b). In drained conditions, the dissipation of generated excess pore pressure causes strain-hardening of the soil, which is sufficient to override the strain-softening effects. The results for the undrained analyses using the non-local methods (Gaussian and G&S methods) are presented in Figures 4c and d. The strain-softening rate remains almost identical for all mesh densities. This demonstrates that the non-local strain-softening methods alleviate the mesh dependency. The size of the strain-softened zone and degree of remoulding is similar for the different discretisations considered in this work (compare Figures 5b, e, h and

317 k and Figures 5c, f, i and l). For the non-local Gaussian method, a small
 318 mesh dependency persists and the size of the strain-softening zone appears
 319 to reduce with decreasing element size. The non-local G&S method appears
 320 to regularise the simulations more effectively, which is consistent with the
 321 conclusion reached by previous research [43], and so was adopted for all the
 322 boundary value problem analyses performed in this study hereafter.

Figure 4: Small-strain simulation response for varying mesh density: (a) local undrained;
 (b) local drained; (c) non-local (Gaussian) undrained; and (d) non-local (G&S) undrained.

Figure 5: Sensitivity distribution at maximum compression for (a,b,c) 10 x 10, (d,e,f) 20 x 20, (g,h,i) 30 x 30 and (j,k,l) 40 x 40 element discretisations for (a,d,g,j) local undrained (b,e,h,k) non-local (Gaussian) undrained; and (c,f,i,l) non-local (G&S) undrained small-strain analyses.

3.3. Influence of characteristic length and softening scaling

In the non-local strain-softening method, the characteristic length l_c controls the extent of the strain localisation and thus, indirectly, the strain-softening rate. Figure 6 presents the responses obtained from the analyses with 20x20 mesh (i.e. 400 elements) using Gaussian and G&S methods with three different values of l_c . Figure 7 depicts the degree of the soil remoulding at the maximum compression of the mesh. In Figures 6a and b, as l_c increases the effective area over which the non-local averaging is performed increases, and therefore, the strain softening rate decreases (compare Fig 7a with b and 7c with d). For the same l_c value, the size of strain-softened area is slightly larger in the case of the G&S method in comparison to the Gaussian method, due to the wider distribution of G&S weighting function (see Figure 1). Therefore, a non-local softening model based on a critical state model has three principle parameters controlling the strain-softening rate: k , A and l_c .

The simulation of strain-softening in practical geotechnical engineering boundary value problems necessitates the use of a very small value of l_c if the scale of shear band is to be modelled at true scale (i.e. of the order of, at most, a few millimetres for the fine-grained soils; Gylland et al. [44], and 10-20 D_{50} for sands; Mühlhaus and Vardoulakis [3] and Muir Wood [45]). However, the non-local averaging is only effective when l_c is larger than the size of the smallest element in the FE mesh, which necessitates ultra fine domain discretisations rendering such analyses computationally impractical. In order to overcome this problem, a softening scaling rule was suggested in the literature [46, 29, 47, 25], which postulates that a much larger thickness of shear band can be adopted to describe the correct load-displacement response in a numerical model, provided that: (i) the softening rate is adjusted accordingly to compensate; and (ii) the mechanism is not likely to be significantly altered by such scaling. Advanced soil deformation measurement techniques, such as Particle Image Velocimetry, must be applied in geotechnical physical model tests to identify these constraints on mechanism. A linear relationship between the strain-softening rate and the characteristic length is generally shown to be effective [25, 27]. To investigate the effect of the softening scaling, two sets of analyses were performed with the non-local G&S method considering a scaling ratio (i.e. k/l_c) of four. In the first set, the same analysis mesh was used whilst the parameters k and l_c were varied, while both mesh and parameters k and l_c were varied in the second set of analyses. The load-displacement responses obtained from these analyses are

presented in Figure 8. The force-displacement responses are almost identical for both sets of analyses, demonstrating that the linear softening scaling rule is also generally valid for the non-local SMCC constitutive model. Figure 9 plots the contours of the degree of soil remoulding for selected cases from both sets of analyses and illustrates the following points:

1. Identical meshes with different l_c (c.f. Figure 9a and 9c where $l_c=0.05$ m and 0.10 m, respectively) demonstrate that the size of strain-softened zone is different and proportional to the value of l_c , yet practically identical load-displacement responses are obtained (see Figure 8a).
2. Different meshes with different l_c (c.f. Figure 9b and Figure 9d where $l_c=0.05$ m and 0.025 m, respectively) reveal that the size of strain-softened zone is proportional to the value of l_c and yet practically identical load-displacement responses are obtained because the ratio k/l_c is constant (see Figure 8b).
3. Different mesh with identical l_c (c.f. Figure 9a and Figure 9b where $l_c=0.05$ m) show, as demonstrated earlier in Figure 5, that the size of strain-softened zone has been regularised by the addition of the characteristic length in the strain averaging procedure.

Figure 6: Non-local small-strain simulation response for varying characteristic length (20 x 20 elements): (a) non-local (Gaussian) undrained; and (b) non-local (G&S) undrained.

Figure 7: Influence of varying characteristic length on Sensitivity distribution at maximum compression: (a) non-local (Gaussian) undrained with $l_c=0.05\text{m}$; (b) non-local (Gaussian) undrained with $l_c=0.20\text{m}$; (c) non-local (G&S) undrained with $l_c=0.05\text{m}$; and (d) non-local (G&S) undrained with $l_c=0.20\text{m}$.

Figure 8: Influence of softening scaling with $k/l_c=4$ for : (a) same mesh with varying k and l_c ; and (b) different mesh with varying k and l_c .

Figure 9: Influence of softening scaling on the size of softening zone: (a) 900 elements, $k=0.20$, $l_c=0.05$ m; (b) 400 elements, $k=0.20$, $l_c=0.05$ m; (c) 900 elements, $k=0.40$, $l_c=0.10$ m; and (d) 1600 elements, $k=0.10$, $l_c=0.025$ m.

379 4. Simulation of a classical large deformation boundary value prob- 380 lem: buried pipe uplift

381 Buried pipelines transport hydrocarbons at elevated temperatures and
382 pressures. The elevated temperatures and pressures can cause upheaval buck-
383 ling of the pipe in the direction of the least soil resistance. The process of
384 pipe uplift buckling is a large deformation problem in which significant pipe
385 movements and soil deformations are involved. In this section, the mesh de-
386 pendency of local and non-local strain-softening models when implemented
387 in FE simulation of the pipe uplift process is investigated.

388 4.1. Modelling details

389 Large deformation fully-coupled effective stress finite element analyses
390 were performed using plane strain boundary conditions in Abaqus. The
391 LDFF approach adopted in this paper is based on the Remeshing and Inter-
392 polation Technique with Small Strain (RITSS) strategy [21], where a large
393 deformation finite element analysis is divided into several small strain finite
394 element analyses in order to alleviate issues related to mesh distortion. A
395 new mesh is generated at the beginning of every small strain step based on
396 the deformed geometry of the previous step, and the stresses, field variables
397 and state variables are then mapped to the new mesh from the old deformed
398 mesh. The implementation of the RITSS technique into the Abaqus soft-
399 ware package has been described in detail previously by Wang et al. [48],
400 Chatterjee [37], and Tian et al. [49].

401 A two dimensional plane strain model was developed assuming a ‘wished-
402 in-place’ pipe with a diameter D of 0.5 m and an initial embedment H of
403 $4D$. The pipe was modelled as a solid rigid part and the soil was modelled
404 as a solid deformable part. The interaction between the pipe surface and the
405 soil was assumed to be rough and fully bonded. Four mesh densities (coarse,
406 medium, fine and very fine) of eight-noded plane strain quadrilateral, bi-
407 quadratic displacement, bi-linear pore pressure, reduced integration elements
408 (CPE8RP in the Abaqus Standard library) were selected to discretise the soil.
409 A square partition zone with a size of $3D$ by $3D$ containing smaller elements
410 was created around the pipe to optimise the analysis mesh. The adopted
411 mesh discretisations are illustrated in Figure 10. Displacement controlled
412 uplift analyses were performed using the local SMCC model and non-local
413 G&S method. To perform the non-local spatial averaging, a characteristic
414 length of 0.1 m (equal to the mesh element size at the partition edge of

the coarse mesh) is assumed in all the non-local softening calculations. To establish initial normally consolidated conditions, a surcharge pressure of 10 kPa was applied at the top boundary of the mesh. The pipe was present when the surcharge pressure was applied, and the ‘correct’ initial vertical and horizontal stresses were imposed directly throughout the soil domain. The value of lateral coefficient of earth pressure at rest, K_o , was considered equal to $1 - \sin(\phi_c)$, where ϕ_c is the critical state effective friction angle of the soil. The SMCC model parameters for the Pisa clay were once again adopted, the permeability of the soil was assumed as 10^{-9} m/sec, and the pipe was displaced vertically by $0.5D$ in 50 RITSS steps.

4.2. Results: mesh dependency

Figure 11 shows the numerical results for the undrained uplift of the pipe using local and non-local G&S methods for different meshes. The uplift resistance normalised by the diameter and intact shear strength at the level of the pipe centre, $V_u/s_{ui}D$, (calculated from equation 15) is plotted against normalised pipe displacement (d_{pipe}/D). Assuming negligible softening occurs until the intact undrained shear strength is mobilised, the shear strength of NC clay predicted by the SMCC model under the plane strain conditions is calculated as:

$$s_{ui} = \frac{2}{\sqrt{3}} \frac{M}{2} \exp \left[\frac{(\lambda^* - \kappa^*) \ln(p_{ci} s_{epi}/2) + \kappa^* \ln(p'_i)}{\lambda^*} \right] \quad (15)$$

where p_{ci} is the initial value of the quantity p_c and p'_i is the initial mean effective stress.

The upper bound derived by Randolph and Houlsby [50] for uniform Tresca soil and a fully bonded contact condition of 11.94, and responses from elastic-perfectly plastic FE analyses by Maitra et al. [51], are also plotted alongside for comparison purposes. The curve for the case with no strain-softening matches with the published results well. The mesh dependency of the uplift response is clearly observed for the local undrained analyses in Figure 11a. The refinement of the mesh from coarse to very fine causes gradual increase in the strain-softening rate and leads to a decrease in the peak resistance. The peak uplift capacities for the simulations with fine and very fine mesh density are also slightly lower than the corresponding values for cases with the coarse and medium mesh density. As expected, the non-local analyses demonstrates significantly reduced mesh dependency (see Figure 11b) with all the curves falling in a narrow band.

4.3. Computational costs

Non-local averaging is often considered to be a computationally expensive process in the FE simulations; however, this is potentially a misconception. Figure 12 presents a bar chart in which the relative computational time of the local and non-local G&S undrained analyses ($t_a/t_{a,ref}$) is plotted for the analyses with the four different meshes. The computational time for the non-local analysis with very fine mesh is taken as the reference time ($t_{a,ref}$). In only half of the cases was the non-local analysis slower than the corresponding local analysis. The maximum increase in computational time ($\approx 16\%$) between the local and non-local analyses was for the case with the very fine mesh. The non-local analysis with the fine mesh density was even $\approx 30\%$ faster than the corresponding local analysis. This shows that the application of non-local methodologies to ultra large deformation geotechnical analyses is possible with negligible additional computational burden.

Figure 10: Illustration of the initial analysis mesh used for the buried pipe simulations: (a) coarse; (b) medium; (c) fine; and (d) very fine.

Figure 11: Large deformation simulation of buried pipe uplift response for varying mesh density: (a) local undrained; and (b) non-local (G&S) undrained.

Figure 12: Comparison of total analysis time for large deformation simulation of buried pipe undrained uplift response with local and non-local G&S softening models for varying mesh density.

463 5. Simulation of cyclic T-bar penetration

464 The T-bar penetrometer is a popular offshore geotechnical site charac-
 465 terisation tool for profiling soft clay soils. The T-bar is passed through the
 466 soil at an undrained rate ($vD/c_V > 10$ where v is penetration rate and c_V
 467 is coefficient of consolidation; e.g., Lehane et al. [52] and Colreavy et al.
 468 [53]) and the measured penetration resistance is converted to an interpreted
 469 soil strength using plasticity solutions [50]. A cyclic T-bar penetration test,
 470 where the T-bar is moved up and down repeatedly over a distance of a few
 471 diameters, allows evaluation of the intact and remoulded soil strengths as

well as the rate of strength degradation. This genera of test is potentially far better suited to deriving the soil sensitivity S_t and the strain-softening rate parameter k because a fully remoulded state is reached. This allows the sensitivity S_t to be unambiguously defined, which in turn reduces the uncertainty associated with determining an appropriate value of k . Further to that, if the T-bar is left in place for some time after the end of this cycling (allowing dissipation of the excess pore pressures previously generated), the consolidated-undrained soil strength can also be assessed by performing another episode of undrained cycles [54].

The penetration resistance in soft clay soils depends on both the strain rate and the consolidation characteristics of the soil as shown by Lehane et al. [52]. In this paper, the non-local SMCC model, which is a rate-independent model and thus ignores the strain-rate effects on the T-bar resistance, is applied to simulate a cyclic T-bar penetration test using the RITSS technique. It is assumed that during an undrained analysis, no dissipation of the generated excess pore pressure occurs. Firstly, an undrained episode comprising five penetration cycles was performed using the local and non-local model variants, demonstrating important characteristics of the non-local approach in LDFE analyses. Secondly, analyses including an episode of undrained penetration, subsequent consolidation analysis, followed by another episode involving one cycle of the consolidated-undrained penetration were simulated.

5.1. LDFE modelling details

A $D=0.04$ m diameter T-bar was modelled under plane strain boundary conditions, with the FE analysis mesh shown in Figure 13. Due to the symmetrical geometry of the problem, only half of the domain was modelled for computational efficiency. The T-bar was ‘wished-in-place’ with an initial embedment of $5.5D$ and the width and height of the soil domain were taken as $10D$ and $15D$, respectively. Following Zhou and Randolph [14], in order to minimise the propagation of interpolation errors in the stress state and state variables caused by repeated remeshing, a partial remeshing scheme was used where only the region close to the T-bar was periodically remeshed. The remeshing region comprises very fine elements near to the T-bar and then gradually transitions to a coarse mesh in the non-remeshed far field (see Figure 13). The T-bar-soil surface interaction was considered to be smooth with normal separation not allowed (i.e. no breakaway).

To obtain the cyclic penetration response, displacement controlled analyses were performed. The T-bar was penetrated by $4D$ from the initial

509 position in the first penetration cycle to partially remould the soil, estab-
 510 lishing an approximately steady value of normalised resistance, $V_u/s_{ui}D$, fol-
 511 lowing the initial peak. Penetration cycles with $3D$ amplitude were then
 512 carried out until the fully remoulded asymptotic state was achieved. The
 513 mesh discretisation was performed using eight-noded biquadratic plane strain
 514 quadrilateral, reduced integration elements (CPE8R in the Abaqus Standard
 515 library) for the undrained penetration cycles. To perform consolidation anal-
 516 yses, eight-noded plane strain quadrilateral, biquadratic displacement, bilin-
 517 ear pore pressure, reduced integration elements (CPE8RP in the Abaqus
 518 Standard library) were chosen to discretise the soil domain. The undrained
 519 stress-strain soil response was described by adding the bulk modulus of the
 520 pore water to the effective stress-strain matrix in an Abaqus ‘static’ step.
 521 The soil state at the end of undrained cycles was mapped to a coupled pore-
 522 fluid effective stress analysis (an Abaqus ‘soils’ step) to allow the dissipation
 523 of the excess pore pressures. The undrained cycles following the consolida-
 524 tion step were again considered as a ‘static’ step by augmenting the effective
 525 stress-strain matrix once again with the bulk modulus of water. This ap-
 526 proach resulted in improved convergence compared to a purely fully-coupled
 527 analysis approach.

528 The SMCC model parameters for the UWA kaolin clay are provided in
 529 Table 1. For non-local spatial averaging calculations, a characteristic length
 530 l_c value of 0.01m was selected to ensure the non-local calculations are effective
 531 in the very fine mesh region, which captures the failure mechanism mobilised
 532 by T-bar penetration. Matching of the simulation response to the T-bar
 533 experiment on a cycle-by-cycle basis will result in a set of strain-softening
 534 parameters relevant only to this characteristic length. However, parameter
 535 scaling as described earlier [25] could be used to scale the parameters for
 536 a different characteristic length for use in a problem with a different scale
 537 (whether larger or smaller). Similar to the pipe analyses, the initial normally
 538 consolidated state in the soil domain was achieved by applying a surcharge
 539 pressure of 50 kPa at the top boundary of the mesh and the ‘correct’ vertical
 540 and horizontal stresses were imposed as an initial stress condition throughout
 541 the soil domain. The value of lateral coefficient of earth pressure at rest, K_o ,
 542 was considered equal to $1 - \sin(\phi_c)$, where ϕ_c is the critical state effective
 543 friction angle of the soil. The permeability of the soil was taken as 10^{-9} m/sec.

Figure 13: Illustration of the FE analysis mesh used for the cyclic T-bar simulations.

5.2. Results: undrained and consolidated-undrained penetration

The results obtained from numerical simulations of the undrained T-bar cycles using the local and non-local G&S softening methods are plotted in Figure 14. Undrained T-bar penetration resistance normalised by the diameter and intact shear strength at the level of the T-bar centre, $V_u/s_{ui}D$, (calculated from equation 15) is plotted against normalised displacement (d_{T-bar}/D) in Figures 14a and b. Figures 14c and d show numerically obtained penetration resistance degradation factor (V_u/V_{ui}) at the mid-depth of each T-bar pass (i.e. d_{T-bar}/D of 2.5) plotted against the penetration cycle number (N), following the convention suggested by Randolph et al. [17] and Zhou and Randolph [14] that the first pass is referred to as cycle number $N = 0.25$. The results extracted from five cyclic undrained T-bar penetration tests in UWA kaolin clay by Ragni et al. [35] and O’Loughlin et al. [55] are also presented in the figure with error bars representing the

range and the symbols representing the mean value for the five tests. The contour plots of the local incremental equivalent plastic strain variable, $\dot{\epsilon}_s$, are presented in Figure 15 for the points A-H marked in Figure 14.

Similar to the pipe uplift simulations, the normalised resistance first mobilises a peak value before decreasing rapidly to a steady value with further advancement for both local and non-local analyses. The initial resistance profile in the local simulation shows oscillations, but these are significantly reduced in the non-local analysis (c.f. points A, B and C in Figure 14a with points E, F and G in Figure 14b). Figure 14c and d demonstrate that the numerically obtained V_u/V_{ui} values at the residual state are higher than those derived from the experiments. In the analysis this is caused by: (i) partial remoulding of the soil caused by the first pass of the T-bar, first reported by Zhou and Randolph [14]; and (ii) over-estimation of the strain-softening rate parameter k resulting in a lower penetration force (V_{ui}) during the first pass, by which all other measurements are normalised. The oscillations observed in the resistance profile of the local analyses are expected in non-regularised strain-softening materials including clays and are caused by periodic development and diffusion of localised shear bands (c.f. Figures 15a, b and c), which was also highlighted by Zhou and Randolph [56]. When the soil is partially remoulded the oscillations disappear because the mechanisms stabilises, with a unique pattern of plastic deformation formed as observed in Figure 15d at the point D, which is chosen at the same level as point A in the third pass of the T-bar penetration. Such artefacts are less evident in the non-local strain-softening simulations because the strain-localisation is adequately regularised and a practically identical zone of plastic deformation is formed after any level of further deformation (at points E, F and G in the first T-bar pass and point H in the third T-bar pass) (c.f Figures 15e, f, g and h).

Several additional analyses were carried out to back-calculate the values of the softening parameters (i.e. k and S_t) that lead to a response comparable with that observed in the experiment. The values yielding the ‘best fit’ presented in Figure 16 (excess pore pressure contours) and Figure 17 (penetration resistance response) are $k=0.06$ and $S_t = 2.78$, which is a decrease in strain-softening rate by a factor of 3.33 and an increase in sensitivity by a factor of 1.26. The question is: could these parameters have been assessed directly from the T-bar penetration test data?

[†]Ragni et al. 2015; O'Loughlin et al. 2020

Figure 14: Large deformation local and non-local simulations of cyclic undrained T-bar penetration with $k=0.2$ and $S_t = 2.2$.

Figure 15: Evolution of the distribution of the incremental local equivalent plastic strains ($\dot{\epsilon}_s$) in the first and third T-bar pass: (a, b, c and d) local analyses, and (e, f, g and h) non-local analyses.

594 Rather than use element test data to define strain-softening parameters,
 595 Einav and Randolph [10] suggested a method of extracting the degradation
 596 parameters $\xi_{s,95}$, which is an equivalent general shear strain required to cause
 597 95% strength degradation, and the sensitivity S_t from cyclic full-flow pen-
 598 etrometer tests. Zhou and Randolph [14] proposed the following equation to
 599 estimate S_t from T-bar test based sensitivity S_{T-bar} :

$$S_t = [1 + 0.3(1 - \alpha)^{1.5}] [\delta''_{rem} + (1 - \delta''_{rem}) e^{1.5\xi/\xi_{s,95}}] S_{T-bar} \quad (16)$$

600 in which the parameter ξ is the current accumulated general shear strain, δ''_{rem}
 601 represents the disturbance during initial penetration and α is the inverse of
 602 the initial sensitivity (i.e. $1/S_t$). According to Einav and Randolph [10], $\xi_{s,95}$
 603 can be obtained from:

$$\xi_{s,95} = 2\xi_P N_{95} \quad (17)$$

604 where ξ_P was obtained from the Upper Bound Strain Path Method (UB-
 605 SPM) for a smooth T-bar as 4.42, and N_{95} is the number of cycles required
 606 for 95% strength degradation to occur in a full-flow penetrometer test. Based
 607 on their numerical analyses, Zhou and Randolph [14] obtained values for N_{95}
 608 in the range of 3.0-4.0, depending upon the sensitivity of the soil assumed.
 609 From the cyclic T-bar penetration test data (see Figure 17b), the N_{95} value
 610 appears to be around 2.0 for kaolin clay. This results in a value of $\xi_{s,95}$ of
 611 17.68.

612 To relate k in the SMCC constitutive model to this value of $\xi_{s,95}$, we begin
 613 by integrating Equation 6, which yields:

$$s_{ep} = s_{epf} + (s_{epi} - s_{epf}) \exp \left[-\frac{k}{\lambda^* - \kappa^*} \varepsilon_s \right] \quad (18)$$

614 in which the parameter ε_s represents the average magnitude of plastic
 615 shear strain operative in the full flow mechanism during one T-bar penetra-
 616 tion/extraction stroke.

617 For the undrained T-bar penetration, if it is assumed that $\dot{\varepsilon}_v^p \ll \dot{\varepsilon}_q^p$,
 618 then Equation 18 reduces to:

$$s_{ep} = s_{epf} + (s_{epi} - s_{epf}) \exp \left[-\frac{k\sqrt{A}}{\lambda^* - \kappa^*} (\varepsilon_q^p) \right] \quad (19)$$

By combining Equation 19 with the generalised form of the Einav-Randolph strength degradation law [10], we generate the following relation between k and $\xi_{s,95}$:

$$k = \frac{3(\lambda^* - \kappa^*)}{\xi_{s,95}\sqrt{A}} \quad (20)$$

The values of the parameters λ^* and κ^* used in the T-bar penetration analyses were 0.0811 and 0.019 respectively. For the softening rate, k derived from the element tests, i.e. $k=0.2$, the $\xi_{s,95}$ value from equation 20 is 2.08 which is much lower than suggested range of around 10 to 50 [10]. This is at least partly because the strain-softening rate parameter k was calibrated from standard triaxial tests, which are ill suited because: (i) they do not impart sufficient deformation for the soil to reach a residual state [45]; and (ii) the strain-localisation that typically occurs within a triaxial sample leads to erroneous over-estimation of the strain-softening rate because the strain-rate applied at the boundaries of the sample is far lower than the operative strain-rate mobilised within the strain-localisation within the sample. Therefore, $\xi_{s,95}$ of 17.68 from equation 17 would imply that a value of k of 0.01 ought to yield the correct rate of strain-softening. However, this is six times lower than the ‘best-fit’ value of 0.06 derived by trial and error. In other words, a much faster than expected rate of strain-softening was required for the simulation to fit the experimental measurements. One reason for this is that the non-local softening method attenuates the strain-softening rate with increasing characteristic length. For this numerical simulation, the attenuation was around 11% for the l_c value selected in this paper (compare Figures 14a and b). For a larger characteristic length this attenuation would be even more significant, therefore, as illustrated also for the pipe uplift problem, the strain-softening parameters are intrinsically linked to the choice of characteristic length. This demonstrates that the UBSPM method proposed by Einav and Randolph [10] is - like element tests - similarly unable to determine appropriate strain-softening parameters, such as $\xi_{s,95}$ and hence S_t (or k and hence S_t for a critical state based constitutive model) directly from cyclic T-bar penetration test data for simulations that utilise non-local regularisation techniques to mitigate mesh dependency. This limitation of the UBSPM method was also highlighted by Einav and Randolph [10], and is due to existence of a velocity discontinuity in the kinematic mechanism for the cylinder-based problems - such as the T-bar penetration - leading to

653 a finite jump in the cumulative shear strain when a soil element traverses
654 a discontinuity. New methods of calibration are required if any non-locally
655 regularised strain-softening models are to be of practical use.

656 The first two penetration and extraction strokes of the undrained episode
657 with parameters $k = 0.06$ and $S_t = 2.78$ were also repeated using the effective
658 stress approach coupled with pore pressure in order to compare the predicted
659 excess pore pressures with those obtained using a fully undrained approach.
660 Figure 16 presents contours of excess pore pressures u_{excess} using: (i) nor-
661 mal elements (CPE8R in the Abaqus standard library) with bulk modulus
662 equal to that of the pore water (Figures 16a and c); and (ii) pore pressures
663 elements (CPE8RP in the Abaqus standard library) with pore pressures cal-
664 culated using Biot's equations (Figures 16b and d), after T-bar advancement
665 of $0.3D$ in the first penetration pass computed (Figures 16a and b) and in the
666 first extraction pass at the same advancement from the bottom of the pene-
667 tration pass (Figures 16c and d). The analysis with pore pressure elements
668 was performed with a normalised penetration velocity, vD/c_v , of 100. The
669 distributions of the excess pore pressures obtained using the two approaches
670 are practically identical for the first penetration and extraction passes. In
671 the T-bar penetration pass, the suction pressures develop behind the T-bar
672 because of the extension loading, and the positive excess pore pressures gen-
673 erate in front of the T-bar at the invert level due to compression loading
674 (see Figure 16a and b). On reversal of the displacement in the T-bar ex-
675 traction pass, the suction pressures develop at the T-bar invert level because
676 of the extension loading (see Figure 16c and d). Therefore, using a fully
677 undrained approach - which is computationally more efficient in such a large
678 deformation problem because there is one less nodal degree of freedom - is
679 an appropriate choice for investigation of the undrained penetration response
680 being simulated in this instance. However, the use of pore pressure elements
681 is inevitable if partially drained conditions dominate the response.

Figure 16: Distribution of the excess pore pressures (u_{excess}) in T-bar cycling analyses using (a, c) normal elements (CPE8R in the Abaqus Standard library) with bulk modulus and (b, d) coupled pore pressure elements (CPE8RP in the Abaqus Standard library) after T-bar advances by: (a, b) $0.3D$ from its initial position in the first penetration pass; and (c, d) $0.3D$ from the bottom of the penetration pass in the first extraction pass.

682 Following the undrained episode, complete dissipation of the excess pore
 683 pressures was allowed, parking the T-bar at mid-depth of the cycling zone;
 684 the resulting consolidation of the soil leads to regain of the shear strength.
 685 After the re-consolidation of the soil, only a single cycle of undrained pene-
 686 tration (i.e total T-bar displacement of $6D$) was performed. This is because
 687 the resulting highly nonlinear distribution of void ratio around the T-bar fol-
 688 lowing the re-consolidation increases the propensity for interpolation errors,
 689 which results in eventual convergence problems. However, the interpolation
 690 error was straightforward to quantify and apply the drift correction in the
 691 first packet of the undrained penetration due to the void ratio being a state
 692 variable that decreases linearly with depth. The response obtained is shown
 693 in the Figure 17 as a red dashed line with triangular markers. The penetra-
 694 tion resistance increased by about 50% compared to the remoulded resistance
 695 observed at the end of the first packet of cycles. The results derived from
 696 the second packet of cyclic T-bar penetration test [55] are also plotted in the
 697 figure as blue square markers. The actual test contained three packets of
 698 20 cycles of undrained T-bar penetration. However, $\approx 95\%$ strain softening
 699 occurred in the first five cycles, and therefore only the first five cycles are
 700 shown in this comparison.

701 Contrary to the observations made from the T-bar penetration tests at
 702 UWA [54, 55], the subsequent cycle generates a practically constant resis-
 703 tance with no reduction in resistance during the subsequent cycle. This is
 704 because the sensitivity state variable has already reached a minimum and
 705 re-consolidation did nothing to change that. Most existing constitutive mod-
 706 els in their current form, including that applied in this study, do not allow
 707 recovery of the sedimentation or post-sedimentation inter-particle structure
 708 of the soil that occurs due to primary and secondary consolidation, creep and
 709 thixotropy processes following the initial undrained remoulding process. As
 710 these effects cannot be observed in a standard element test, such as a tri-
 711 axial test, the existing strain-softening-hardening constitutive models were
 712 not developed to take into account the aspects of behaviour that these large
 713 deformation simulations have highlighted. If this genera of problem is to
 714 be simulated accurately, new constitutive models are required that facilitate
 715 the recovery of the sensitivity state variable during dissipation and allow
 716 subsequent packets of cycles of softening to be captured.

[†]Ragni et al. 2015; O'Loughlin et al. 2020

Figure 17: Large deformation non-local simulations of cyclic undrained and consolidated-undrained T-bar penetration with $k=0.06$ and $S_t = 2.78$.

6. Conclusions

This paper presented a methodology to simulate large deformation behaviour of sensitive clays in finite element analyses based on a non-local strain-softening approach. The Structured Modified Cam Clay (SMCC) constitutive model and its non-local variants have been implemented into Abaqus via a user material subroutine, or UMAT. Non-local spatial averaging was applied to the plastic strains that control the strain-softening response of the model. The mesh dependency issues of the local strain-softening model have been highlighted and the efficacy of the non-local approaches in mitigating mesh-dependency are demonstrated through a systematic series of small strain biaxial compression and large strain buried pipe uplift analyses under undrained and drained conditions. Finally, an ultra large deformation problem in the form of cyclic undrained T-bar penetration was simulated using the techniques developed. A number of new observations have been

731 made through these analyses:

- 732 1. The non-local regularisation technique is not strictly necessary for the
733 simulation of a drained biaxial compression using coupled effective
734 stress analyses techniques. This is because the degree of strain-hardening
735 in the response is - for the constitutive model parameters adopted -
736 greater than that of the strain-softening that is simultaneously occur-
737 ring during the biaxial simulation.
- 738 2. The validity of the softening-scaling rule proposed by Galavi and Schweiger
739 [25] for the multi-laminate constitutive model, has been confirmed as
740 valid for the critical state based elastoplastic model adopted in this
741 study. This finding is critical because otherwise the simulation of large
742 scale problems involving soft clay would be practically impossible with
743 currently available computational power.
- 744 3. Non-local regularisation techniques have been shown to mitigate mesh
745 dependency in large deformation updated Lagrangian RITSS analyses
746 by simulating classical problems such as pipe uplift and cyclic T-bar
747 penetration. These analyses indicated that the strain-softening rate
748 parameter is likely to be significantly over-estimated if element tests
749 are used as the sole means of model calibration.
- 750 4. Existing analytical methods for determining strain-softening param-
751 eters directly from cyclic T-bar experiments are unable to predict the
752 correct parameters for strain-softening models implemented with non-
753 local regularisation. This is because these methods such as UBSPM
754 method overestimate the accumulated shear strains, therefore a lower
755 value of the the strain-softening rate parameter k is predicted. In ad-
756 dition, the non-local regularisation effectively attenuates the rate of
757 strain-softening, therefore a larger value of k is required to achieve a
758 match between the experimental data and the results of non-locally
759 regularised FE simulations. This necessitates the development of new
760 techniques for strain-softening parameter calibration if non-locally reg-
761 ularised models are to be of practical use.
- 762 5. The model developed was unable to capture the second packet of strength
763 degradation as well as the magnitude observed in T-bar experiments by
764 Hodder et al. [54] and O'Loughlin et al. [55]. This is because the sen-
765 sitivity state variable does not recover during the dissipation process,
766 which results in a monotonic response after the soil is fully remoulded
767 for the first time. Accurately simulating cyclic undrained problems such

768 as T-bar penetration with multiple packets of alternating remoulding
769 cycles and dissipation requires the development of new constitutive
770 model features that allow some recovery of sensitivity.

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