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Manuscript title: Elastic stiffness coefficients for circular footings on clay overlying sand under general loading

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Abstract

Foundations are commonly situated on layered soil deposits, where the elastic stiffnesses may differ from values calculated using solutions for single-layer elastic media. This paper reports elastic stiffness coefficients for a rigid circular footing subjected to vertical, horizontal, and moment loads bearing on a clay layer above a stiffer sand layer. Three-dimensional finite-element analyses considered various shear modulus ratios of top to bottom soil layers, as well as the thickness of the top layer below the footing. The elastic stiffnesses are presented as dimensionless coefficients and are bounded by published solutions for circular footings on a single-layer elastic half-space. Analytical expressions that capture the numerical results well are also proposed. The results can be used in the assessment of the elastic response of circular foundations on clay overlying sand, with applications in soil-structure interaction analyses of offshore structures amongst others.

Keywords: elasticity; footings/foundations; soil/structure interaction

Introduction

Elastic soil-structure response at small strains is a key aspect of foundation design, requiring accurate elastic stiffness coefficients for deformation prediction and to include as a boundary condition in the structural analysis of composite structure-foundation systems. It affects not only the structural load distribution and displacements but also the dynamic properties of the structure, such as the natural period.

There is a large volume of published studies describing the elastic response of rigid circular footings subjected to vertical (V), horizontal (H), and moment (M) loading on an elastic half-space. The combinations of soil type, non-homogeneity, foundation geometry and embedment previously studied are summarised in Table 1. However, all of these solutions have dealt with a single layer of soil, with the shear modulus being constant or varying continuously with depth. Furthermore, when circular foundations resting on layered soils have been considered it has been usual for emphasis to be placed on the vertical load case rather than generalised loading conditions. For instance, solutions have been presented for calculating the elastic vertical settlement of foundations for the layered system by, amongst others, Ueshita and Meyerhof (1968); Mayne and Poulos (1999). Consequently, there is a need to extend the elastic stiffness solutions to the behaviour under VHM loads. This will allow easier assessment of the elastic response of the soil-structure interaction under general loading in engineering practice.

This study presents elastic stiffness coefficients of a rigid circular footing bearing on clay overlying sand under combined VHM loading. Analytical expressions are presented to predict elastic stiffnesses in all three directions of motion, derived from three-dimensional finite-element analyses.

Problem definition

The elastic behaviour of a circular footing of radius R subjected to VHM loads is considered, as shown in Fig. 1. In a finite clay layer underlain by an infinite sand layer, three embedment conditions are studied. They are case 1, representing a circular footing on the surface of the clay layer, case 2 a footing resting at the bottom of the cavity in the clay layer, and case 3 a footing fully embedded in the clay layer. The thickness between the base of the footing and the sand layer is denoted by T , while the embedment depth is z_D from the mudline.

Fig. 2 shows the V , H and M loads applied to the circular footing and their conjugate vertical w and horizontal u displacements and rotation θ . All the loads and displacements are applied at the reference point (RP), which is defined at the centreline of the foundation at the base level (Fig. 2). The soil is regarded as an elastic material with a shear modulus G and a Poisson's ratio ν . Poisson's ratio ν is taken as 0.49 for undrained clay, whereas Poisson's ratio of 0.2 is chosen to represent a wide range of sands (Lade, 1977). The elastic stiffness of the foundation system under general loading is expressed as

$$\begin{bmatrix} V/G_{\text{clay}} R^2 \\ H/G_{\text{clay}} R^2 \\ M/G_{\text{clay}} R^3 \end{bmatrix} = \begin{bmatrix} K_v & 0 & 0 \\ 0 & K_h & K_c \\ 0 & K_c & K_m \end{bmatrix} \begin{bmatrix} w/R \\ u/R \\ \theta \end{bmatrix} \quad (1)$$

where K_v , K_h , K_m , and K_c are dimensionless elastic stiffness coefficients. K_v , K_h , and K_m represent the vertical, horizontal, and moment stiffness, respectively. K_c defines the coupling stiffness that exists between the horizontal and moment degree of freedom. The shear modulus of clay G_{clay} is used for normalisation since the circular footing remains embedded in the clay layer.

Numerical analysis methodology

Geometry and mesh

Three-dimensional finite element (FE) analyses were performed using the commercial software Abaqus 6.14 (Dassault Systèmes, 2014). The circular footing was treated as a rough rigid body and bonded to the soil. The footing thickness was assumed to be $R/50$, which was found to be sufficiently small to simulate the footing with infinitesimal thickness. The soil-foundation interface was modelled using the tie constraint in Abaqus, which was computationally efficient and less inclined to convergence problems often encountered when simulating rough contact behaviour. The soil was assumed to be a weightless elastic material, with properties defined by the shear modulus and Poisson's ratio. The first-order reduced-integration hexahedral elements were used to mesh the soil. Displacement controlled analyses applied prescribed displacements and rotations at the RP. As the required extent of soil domain is much larger for the elastic analyses than that for ultimate bearing capacity analyses (Bell, 1991), a semi-cylindrical soil domain of $200R$ in diameter and $200R$ in depth was adopted. Different soil domain dimensions of $50R \times 50R$, $200R \times 200R$, and $500R \times 500R$ were investigated to achieve a computationally efficient model without compromising the accuracy. For vertically loaded rough circular footings sitting on the surface of the clay layer, the differences in K_v compared with the analytical solution of Spence and Temple (1968) are +5.6%, +1.4%, and +0.6%, respectively. A finer mesh zone extended $20R$ both horizontally from the centreline of the footing and vertically from the mudline. The model consisted of several zones with different mesh densities, depending on the stress distribution. Fig. 3 shows a typical mesh for case 1 of a surface circular footing resting on a clay layer overlying sand. The same meshing principles were followed for embedded cases 2 and 3. The base of the mesh was fixed in all three translational directions, while vertical boundaries were fixed in the lateral directions and free to move vertically. On the plane of symmetry, all the nodes refrained from out-of-plane displacements.

For all the cases shown in Fig. 1, the dimensionless thickness between the footing base and the sand layer of $T/2R = 0, 0.05, 0.125, 0.25, 0.375, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0, \infty$, and the shear modulus ratios of $G_{\text{sand}}/G_{\text{clay}} = 1, 5, 10, 15, 20, 25, 30, 35$ were considered, covering a wide range of practical foundation applications. The soil profile becomes a homogeneous sand layer when $T/2R = 0$, while at the opposite extreme, $T/2R = \infty$ refers to a homogeneous clay layer. For $T/2R$ values larger than 3.0, the effects of soil layering on the elastic stiffness were found to be negligible, thus the results for a single-layer soil can be adopted. For case 2 and case 3, the embedment ratios of $z_D/2R = 0.5, 1.0, 2.0$ were considered. For embedment ratios larger than 2.0, the variation in the elastic stiffness coefficients with embedment is insignificant (Bell, 1991; Ngo-Tran, 1996; Doherty and Deeks, 2003), allowing the results of $z_D/2R = 2.0$ to be used with only a slight conservatism. In total, 1722 analysis cases provided the 2296 elastic stiffness coefficients.

Validation

To validate the numerical model, the elastic stiffness coefficients calculated for circular foundations embedded in single-layer sand (ν of 0.2) are compared with the available solutions. Fig. 4 shows that, although K_v , K_h , and K_m obtained from the present FE analyses are only slightly lower than those reported by Bell (1991), Ngo-Tran (1996) and Doherty and Deeks (2003) and K_c is slightly higher, it is in general agreement, suggesting that the mesh discretization and the model boundaries adopted in this study are valid. It is also observed that the fully embedded soil-foundation system (case 3) is stiffer than the soil-foundation system with an open cavity (case 2), as the displacements are restricted from the soil on top

of the footing. The effect of embedment on the elastic stiffness is evident when the footing embedment ratio $z_D/2R$ is less than 1.0 and diminishes asymptotically with embedment.

Results and discussion

All the computed elastic stiffness coefficients are provided in Table A1-A3 in Appendix A.

Surface Footing solutions

Fig. 5 shows the dimensionless elastic stiffnesses for a surface circular footing resting on a clay layer underlain by sand (case 1) for the shear modulus ratio $G_{\text{sand}}/G_{\text{clay}} = 1$. For $T/2R = 0$, the stiffness coefficients correspond to the solutions for rough rigid circular footings on homogenous sand. For $T/2R = \infty$, the stiffness coefficients converge to those on homogeneous clay. Exact analytical solutions (Spence and Temple, 1968; Poulos and Davis, 1974; Gazetas and Tassoulas, 1987) and FE solutions (Bell, 1991) are compared for those two extremes in Fig. 5.

As shown in Fig. 5, the variation of the stiffness coefficients (K_v , K_h , and K_m) with $T/2R$ for $G_{\text{sand}}/G_{\text{clay}} = 1$ can be fitted by a hyperbolic function

$$K_{i,G_{\text{sand}}/G_{\text{clay}}=1} = \frac{m_i K_{i,v=0.2} + K_{i,v=0.49} (T/2R)}{m_i + (T/2R)} \quad (2)$$

where $i = v, h, m$, corresponding to the V, H , and M loading case, respectively; $K_{i,v=0.2}$ = value of the stiffness coefficient as $T/2R$ approaches 0 (homogeneous sand solution);

$K_{i,v=0.49}$ = value of the stiffness coefficient as $T/2R$ approaches infinity (homogeneous clay solution); and m_i = three fitting parameters for each loading case, which are listed in Table 2.

For $G_{\text{sand}}/G_{\text{clay}}$ values larger than 1.0, the elastic stiffness coefficients for the circular footings on the surface of clay-over-sand are shown in Fig. 6. The stiffness coefficients K_v , K_h , and K_m increase significantly with increasing shear modulus ratio $G_{\text{sand}}/G_{\text{clay}}$, and thinner clay thickness below the footing. This is because the stiffer layer underneath restricts the deformation of the soil as the footings near the layer interface. The influence of the underlying stiffer layer on the elastic stiffness becomes more evident when $T/2R$ is less than 1. The results for K_v , K_h , and K_m can be fitted as a function of $G_{\text{sand}}/G_{\text{clay}}$ and $T/2R$

$$K_i = \left[1 + \left(\frac{G_{\text{sand}}}{G_{\text{clay}}} - 1 \right) \mu_i \right] K_{i,G_{\text{sand}}/G_{\text{clay}}=1} \quad (3)$$

where $K_{i,G_{\text{sand}}/G_{\text{clay}}=1}$ can be calculated from equation (2), and μ_i is fitted as a function of $T/2R$

$$\mu_i = \frac{1}{1 + a_i (T/2R)^{b_i}} \quad (4)$$

where b_i is constant, and a_i is found to be fit well by a linear relationship

$$a_i = I_{a,i} + \frac{G_{\text{sand}}}{G_{\text{clay}}} S_{a,i} \quad (5)$$

where $I_{a,i}$ and $S_{a,i}$ are two fitting parameters. All the fitting parameters following equations (2)-(5) are listed in Table 2. It can be seen from Fig. 6 that there is an excellent agreement between the FE results and the values obtained using equation (3).

The coupling stiffness coefficient shows a more complex trend, as presented in Fig. 6(d). A similar fitting strategy can be utilized to fit K_c , although a different form of the equation has been presented. For $G_{\text{sand}}/G_{\text{clay}} = 1$, the same expression as equation (2) is used, given by

$$K_{c,G_{\text{sand}}/G_{\text{clay}}=1} = \frac{m_c K_{c,v=0.2} + K_{c,v=0.49} (T/2R)}{m_c + (T/2R)} \quad (6)$$

where $m_c = 0.125$, and the fit quality is reasonably good as shown in Fig. 5(d).

For larger values of $G_{\text{sand}}/G_{\text{clay}}$, the relationship is expressed as

$$K_c = \mu_c \left[1 + \left(\frac{G_{\text{sand}}}{G_{\text{clay}}} - 1 \right) \mu_h \right] K_{c,G_{\text{sand}}/G_{\text{clay}}=1} \quad (7)$$

where μ_h can be calculated from equation (4), and μ_c can be expressed as a function of $T/2R$

$$\mu_c = 1 - \frac{c_1 (T/2R)}{1 + c_2 (T/2R) + c_3 (T/2R)^2} \quad (8)$$

The values of c_1 , c_2 , and c_3 are found to be fit well by the following functions

$$c_1 = 3.342 \left(\frac{G_{\text{sand}}}{G_{\text{clay}}} \right)^{0.523} \quad c_2 = 0.0367 \frac{G_{\text{sand}}}{G_{\text{clay}}} - 1.795 \quad c_3 = 1.99 \left(\frac{G_{\text{sand}}}{G_{\text{clay}}} \right)^{0.254} \quad (9)$$

Compared to K_v , K_h , and K_m , the effect of the stiffer layer underneath on K_c is insignificant. Although reasonable curve fitting can be achieved by using more parameters and complex expressions, the values of K_c can be interpolated from Table A1 or represented by the single-layer solutions as a conservative estimate.

Embedded footing solutions

The elastic stiffness coefficients for a circular footing sitting on the bottom of an open cavity (case 2) and embedded fully (case 3) in a clay layer underlain by sand are summarised in Table A2 and Table A3 in Appendix A, respectively. Three embedment ratios of $z_D/2R = 0.5, 1.0, 2.0$ are included for each case. For embedment ratios larger than 2.0, the values of $z_D/2R = 2.0$ can be used as a slightly conservative estimate.

It is convenient to develop analytical expressions to fit the computed results for the embedded footings, using equations from the previous section for the surface footings. New fitting parameters are provided to fit K_v , K_h , and K_m for case 2 and case 3 using equations (2)-(5), which are summarised in Table 2. Fig. 7 compares computed FE results and predicted stiffness coefficients for *VHM* loading cases. Predicted values fall close to the computed stiffnesses, with the differences mostly being less than 15%.

For case 2 and case 3, the curve fitting for the coupling stiffness K_c is more complex than the surface footing. Thus, the analytical expressions are not suggested herein since the influence of the stiffer bottom layer on K_c is insignificant compared to K_v , K_h , and K_m . For the geometry and soil conditions investigated, the changes in K_c are relatively small, ranging from -2 to 3. In contrast, the variations in K_v are two orders of magnitude higher in the range of 6 to 160.

Conclusions

This study has presented a comprehensive series of elastic stiffness coefficients for vertical, horizontal, and moment motions of a rigid circular footing bearing on undrained clay over stiffer sand. The 2296 stiffness coefficients are based on 1722 three-dimensional finite-element analyses. The coefficients cover a wide practical range of foundation embedment ratios, the shear modulus ratios between the layers, and the clay layer thickness below the footing.

It has been shown that soil layering leads to significant changes in the pure vertical, horizontal, and moment elastic stiffnesses, but has an insignificant effect on the coupling stiffness between the loads. Analytical expressions are proposed to derive elastic stiffnesses for the two-layered system, which are bounded by the single-layer solutions.

The results contribute to a better assessment of the elastic response of circular foundations on clay overlying sand under working loads. They can also be applied in soil-structure interaction analyses of structures, with immediate application to those found in offshore engineering.

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Appendix A

See Table A1, Table A2, and Table A3.

Notation

K_v , K_h , K_m , and K_c are dimensionless elastic stiffness coefficients

K_v , K_h , and K_m represent the vertical, horizontal, and moment stiffness, respectively

K_c defines the coupling stiffness

G_{clay} is the shear modulus of clay

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Table 1. Summary of published work on elastic stiffnesses of circular foundations under VHM loading.

Soil ^a			Embedment		Foundation geometry			cross-coupling ^b	Method ^c	Reference
Clay	Sand	Clay-over-sand	Surface	Embedded	Circular	Conical	Caisson			
H	H	—	✓	—	✓	—	—	—	Analytical	Poulos and Davis (1974)
H	H	—	✓	—	✓	—	—	—	Analytical	Dobry and Gazetas (1986) Gazetas and Tassoulas (1987)
H	H	—	✓	✓	✓	—	—	✓	FEM	Bell (1991)
H	H	—	✓	✓	✓	✓	—	✓	FEM	Ngo-Tran (1996)
H/N	H/N	—	—	✓	✓	—	✓	✓	SBFEM	Doherty and Deeks

										(2003)
H/ N	–	–	✓	✓	–	✓ ^d	–	✓	FEM	Zhang et al. (2012)
H	H	H	✓	✓	✓	–	–	✓	FEM	This study

^aH, homogeneous; N, non-homogeneous

^bCross-coupling that exists between the horizontal and moment degree of freedom

^cFEM, finite-element method; SBFEM, scaled boundary finite-element method

^dShape of a generic spudcan foundation

Table 2. Summary of fitting parameters for vertical, horizontal, and moment stiffnesses
[equations (2)-(5)].

Case no.	$z_D/2R$	K_v				K_h				K_m			
		m_v	$I_{a,v}$	$S_{a,v}$	b_v	m_h	$I_{a,h}$	$S_{a,h}$	b_h	m_m	$I_{a,m}$	$S_{a,m}$	b_m
Case 1	0	0.28 2	1.5 2	0.7 1	1. 4	0.20 7	2.9 1	3.3 8	1. 0	0.05 5	4.1 2	4.4 3	1. 6
Case 2	0.5	0.35 9	1.5 3	0.5 2	1. 1	0.10 1	1.5 9	3.7 5	0. 9	0.05 4	1.6 7	3.6 1	1. 3
	1	0.32 2	1.7 6	0.5 3	1. 0	0.07 9	2.1 4	4.1 2	0. 9	0.04 3	2.0 9	3.8 9	1. 3
	2	0.24 8	2.2 4	0.5 6	1. 0	0.06 8	3.0 3	4.4 8	0. 9	0.03 8	2.8 5	4.0 7	1. 3
Case 3	0.5	0.37 9	1.5 6	0.4 7	1. 0	0.14 9	1.1 9	3.2 5	0. 8	0.04 2	1.6 1	3.6 5	1. 2
	1	0.37 5	1.6 7	0.4 7	0. 9	0.11 4	1.4 7	3.6 4	0. 8	0.02 9	2.6 1	4.2 8	1. 1
	2	0.29 8	2.0 5	0.5 2	0. 8	0.08 3	2.2 0	4.0 7	0. 8	0.02 6	3.7 8	4.5 9	1. 1

Table A1. Elastic stiffness coefficients for surface circular footings (case 1).

$T/2R$	$G_{\text{sand}}/G_{\text{clay}}$	K_v	K_h	K_m	K_c
0	1	5.2709	4.5711	3.6046	-0.5715
0.05	1	5.7218	4.7310	4.2392	-0.4644
	5	25.9547	14.2051	16.7091	-1.2737
	10	49.0939	20.1131	29.0153	-1.3621
	15	70.7677	23.4892	39.3889	-1.1650
	20	91.3089	25.6816	48.4349	-0.8870
	25	110.8925	27.2229	56.4640	-0.5912
	30	129.6329	28.3673	63.6720	-0.3004
	35	147.6133	29.2513	70.1973	-0.0232
0.125	1	6.1092	4.8599	4.6518	-0.3412
	5	25.1071	10.5099	13.8088	-0.0036
	10	44.4073	12.9494	20.1388	0.6418
	15	60.7033	14.1155	24.1543	1.1377

	20	74.7781	14.8040	26.9543	1.5090
	25	87.0964	15.2601	29.0236	1.7936
	30	97.9827	15.5851	30.6169	2.0177
	35	107.6800	15.8287	31.8821	2.1983
0.25	1	6.5266	4.9869	4.9518	-0.2148
	5	22.5035	8.3995	9.6518	0.5530
	10	34.6313	9.5425	11.4615	1.0148
	15	42.5867	10.0350	12.2841	1.2493
	20	48.2239	10.3101	12.7554	1.3899
	25	52.4307	10.4859	13.0611	1.4835
	30	55.6912	10.6081	13.2755	1.5501
	35	58.2928	10.6978	13.4342	1.6000
0.375	1	6.7989	5.0643	5.0713	-0.1463
	5	19.4197	7.5081	7.5708	0.5165
	10	26.4393	8.2165	8.2676	0.7777
	15	30.1983	8.5055	8.5483	0.8924
	20	32.5439	8.6628	8.7002	0.9568
	25	34.1476	8.7617	8.7954	0.9979
	30	35.3136	8.8296	8.8606	1.0265
	35	36.1996	8.8792	8.9082	1.0475
0.5	1	6.9871	5.1155	5.1242	-0.1075
	5	16.9123	7.0060	6.5710	0.4053
	10	21.2241	7.5041	6.9132	0.5665
	15	23.2611	7.7011	7.0443	0.6329
	20	24.4489	7.8068	7.1138	0.6690
	25	25.2272	7.8727	7.1568	0.6918
	30	25.7767	7.9178	7.1860	0.7074
	35	26.1853	7.9506	7.2072	0.7188
0.75	1	7.2242	5.1776	5.1640	-0.0696
	5	13.7848	6.4588	5.7529	0.2351
	10	15.8532	6.7604	5.8743	0.3120
	15	16.7140	6.8759	5.9191	0.3419
	20	17.1860	6.9369	5.9425	0.3579
	25	17.4843	6.9747	5.9569	0.3677
	30	17.6897	7.0005	5.9666	0.3745
	35	17.8400	7.0191	5.9735	0.3794
1	1	7.3637	5.2131	5.1769	-0.0530
	5	12.1159	6.1697	5.4639	0.1398
	10	13.3616	6.3815	5.5203	0.1840
	15	13.8505	6.4614	5.5409	0.2008
	20	14.1119	6.5033	5.5516	0.2097
	25	14.2745	6.5292	5.5581	0.2151
	30	14.3856	6.5467	5.5626	0.2188
	35	14.4661	6.5594	5.5658	0.2215
1.5	1	7.5173	5.2468	5.1845	-0.0422
	5	10.5018	5.8734	5.2810	0.0523

	10	11.1457	6.0036	5.2993	0.0721
	15	11.3857	6.0519	5.3060	0.0794
	20	11.5112	6.0771	5.3094	0.0833
	25	11.5883	6.0926	5.3115	0.0856
	30	11.6406	6.1031	5.3129	0.0872
	35	11.6783	6.1106	5.3139	0.0883
2	1	7.5984	5.2665	5.1864	-0.0374
	5	9.7491	5.7257	5.2289	0.0170
	10	10.1695	5.8184	5.2369	0.0280
	15	10.3226	5.8526	5.2398	0.0320
	20	10.4019	5.8704	5.2413	0.0341
	25	10.4504	5.8813	5.2422	0.0354
	30	10.4831	5.8887	5.2428	0.0363
	35	10.5067	5.8940	5.2433	0.0369
3	1	7.6815	5.2870	5.1873	-0.0340
	5	9.0493	5.5795	5.2002	-0.0098
	10	9.2916	5.6373	5.2026	-0.0051
	15	9.3780	5.6585	5.2035	-0.0034
	20	9.4224	5.6695	5.2040	-0.0025
	25	9.4494	5.6763	5.2042	-0.0019
	30	9.4676	5.6808	5.2044	-0.0016
	35	9.4807	5.6841	5.2045	-0.0013
∞	1	7.9543	5.3407	5.1864	-0.0303

Table A2. Elastic stiffness coefficients for circular footings embedded in a cavity (case 2).

T/ 2R	$G_{\text{sand}}/$ G_{clay}	$z_D/2R = 0.5$				$z_D/2R = 1$				$z_D/2R = 2$			
		K_v	K_h	K_m	K_c	K_v	K_h	K_m	K_c	K_v	K_h	K_m	K_c
0	1	6.39 22	6.44 11	4.84 59	- 0.8 524	7.04 88	6.76 70	5.14 08	- 0.9 074	7.65 82	7.00 43	5.26 07	- 0.9 473
0.0 5	1	6.83 38	6.93 13	5.51 64	- 0.8 491	7.67 10	7.32 26	5.99 18	- 0.9 926	8.52 12	7.60 93	6.22 94	- 1.0 932
	5	27.4 839	16.9 345	18.2 986	- 1.6 960	28.3 369	17.1 532	18.5 986	- 1.7 997	29.1 704	17.2 973	18.7 534	- 1.8 589
	10	50.9 057	23.2 888	31.0 373	- 1.8 427	51.6 808	23.4 427	31.2 543	- 1.9 343	52.4 547	23.5 345	31.3 705	- 1.9 844
	15	72.8 612	26.9 756	41.8 593	- 1.6 911	73.5 664	27.1 031	42.0 282	- 1.7 753	74.2 916	27.1 731	42.1 211	- 1.8 215
	20	93.6 973	29.3 919	51.3 461	- 1.4 456	94.3 429	29.5 064	51.4 827	- 1.5 234	95.0 288	29.5 652	51.5 586	- 1.5 670

	25	113. 5877	31.1 008	59.8 000	- 1.1 730	114. 1816	31.2 084	59.9 136	- 1.2 451	114. 8348	31.2 609	59.9 760	- 1.2 864
	30	132. 6436	32.3 748	67.4 144	- 0.8 989	133. 1921	32.4 785	67.5 109	- 0.9 656	133. 8173	32.5 271	67.5 621	- 1.0 049
	35	150. 9467	33.3 619	74.3 269	- 0.6 338	151. 4547	33.4 633	74.4 101	- 0.6 954	152. 0560	33.5 093	74.4 517	- 0.7 329
0.1 25	1	7.12 53	7.02 28	5.87 23	- 0.7 326	7.95 35	7.42 52	6.34 97	- 0.8 788	8.82 22	7.72 06	6.59 31	- 0.9 793
	5	26.6 051	13.2 105	15.5 304	- 0.5 279	27.2 932	13.4 228	15.7 332	- 0.6 300	27.9 792	13.5 456	15.8 314	- 0.6 849
	10	46.4 805	15.9 515	22.5 196	0.0 734	47.0 085	16.1 253	22.6 272	- 0.0 062	47.5 448	16.2 132	22.6 759	- 0.0 475
	15	63.4 300	17.2 793	27.0 791	0.5 667	63.8 455	17.4 444	27.1 459	0.5 038	64.2 787	17.5 228	27.1 729	0.4 711
	20	78.1 983	18.0 689	30.3 165	0.9 479	78.5 307	18.2 320	30.3 626	0.8 983	78.8 872	18.3 073	30.3 779	0.8 723
	25	91.2 225	18.5 941	32.7 403	1.2 461	91.4 919	18.7 575	32.7 747	1.2 071	91.7 889	18.8 316	32.7 831	1.1 865
	30	102. 8108	18.9 695	34.6 247	1.4 842	103. 0311	19.1 336	34.6 524	1.4 540	103. 2811	19.2 075	34.6 564	1.4 376
	35	113. 1953	19.2 513	36.1 326	1.6 782	113. 3771	19.4 165	36.1 563	1.6 552	113. 5889	19.4 905	36.1 573	1.6 424
0.2 5	1	7.46 27	7.11 19	6.14 95	- 0.6 142	8.26 01	7.52 22	6.61 41	- 0.7 611	9.13 13	7.82 60	6.85 54	- 0.8 621
	5	24.2 571	10.9 942	11.4 476	0.0 284	24.7 787	11.2 465	11.6 331	- 0.0 721	25.2 813	11.3 883	11.7 029	- 0.1 285
	10	37.5 380	12.3 287	13.6 843	0.5 014	37.8 703	12.5 634	13.8 078	0.4 244	38.1 619	12.6 859	13.8 413	0.3 832
	15	46.5 704	12.9 112	14.7 406	0.7 563	46.8 097	13.1 445	14.8 436	0.6 927	46.9 936	13.2 641	14.8 652	0.6 599
	20	53.1 369	13.2 390	15.3 577	0.9 134	53.3 276	13.4 732	15.4 511	0.8 586	53.4 505	13.5 928	15.4 674	0.8 311
	25	58.1 308	13.4 493	15.7 627	1.0 197	58.2 955	13.6 849	15.8 508	0.9 708	58.3 817	13.8 051	15.8 641	0.9 472
	30	62.0 583	13.5 959	16.0 490	1.0 962	62.2 091	13.8 328	16.1 339	1.0 518	62.2 723	13.9 535	16.1 453	1.0 309
	35	65.2 284	13.7 039	16.2 622	1.1 539	65.3 721	13.9 419	16.3 448	1.1 129	65.4 205	14.0 632	16.3 550	1.0 942
0.3	1	7.69	7.16	6.26	-	8.46	7.57	6.72	-	9.32	7.88	6.95	-

75		73	64	82	0.5 504	64	93	18	0.6 980	72	76	85	0.7 997
	5	21.4 497	10.0 260	9.24 83	0.0 077	21.9 283	10.3 124	9.48 09	- 0.1 039	22.3 249	10.4 782	9.55 93	- 0.1 688
	10	29.7 143	10.8 784	10.1 692	0.2 784	30.0 673	11.1 533	10.3 617	0.1 833	30.2 751	11.3 057	10.4 126	0.1 305
	15	34.3 668	11.2 306	10.5 525	0.4 039	34.6 808	11.5 045	10.7 319	0.3 171	34.8 148	11.6 553	10.7 738	0.2 701
	20	37.3 573	11.4 234	10.7 631	0.4 760	37.6 592	11.6 978	10.9 361	0.3 941	37.7 588	11.8 487	10.9 738	0.3 507
	25	39.4 427	11.5 451	10.8 962	0.5 228	39.7 423	11.8 202	11.0 656	0.4 441	39.8 240	11.9 715	11.1 007	0.4 030
	30	40.9 803	11.6 290	10.9 881	0.5 555	41.2 815	11.9 046	11.1 550	0.4 792	41.3 531	12.0 564	11.1 885	0.4 397
	35	42.1 608	11.6 903	11.0 553	0.5 798	42.4 652	11.9 665	11.2 205	0.5 052	42.5 311	12.1 187	11.2 529	0.4 669
0.5	1	7.86 83	7.20 28	6.32 44	- 0.5 146	8.61 51	7.61 61	6.77 09	- 0.6 628	9.46 19	7.92 68	7.00 37	- 0.7 654
	5	19.0 620	9.46 80	8.13 23	- 0.0 953	19.5 741	9.77 60	8.41 60	- 0.2 198	19.9 369	9.96 06	8.51 32	- 0.2 940
	10	24.4 345	10.0 825	8.60 50	0.0 678	24.8 867	10.3 807	8.86 18	- 0.0 455	25.0 995	10.5 542	8.93 66	- 0.1 108
	15	27.1 075	10.3 283	8.79 13	0.1 385	27.5 524	10.6 251	9.03 91	0.0 303	27.7 152	10.7 970	9.10 65	- 0.0 308
	20	28.7 099	10.4 609	8.89 10	0.1 779	29.1 583	10.7 575	9.13 46	0.0 727	29.2 993	10.9 292	9.19 83	0.0 140
	25	29.7 780	10.5 439	8.95 33	0.2 030	30.2 321	10.8 407	9.19 42	0.0 998	30.3 624	11.0 125	9.25 57	0.0 426
	30	30.5 409	10.6 008	8.99 58	0.2 204	31.0 009	10.8 977	9.23 50	0.1 185	31.1 252	11.0 698	9.29 51	0.0 625
	35	31.1 133	10.6 422	9.02 67	0.2 331	31.5 785	10.9 393	9.26 47	0.1 323	31.6 992	11.1 116	9.32 38	0.0 770
0.7 5	1	8.09 73	7.24 86	6.37 01	- 0.4 794	8.81 41	7.65 97	6.80 99	- 0.6 288	9.63 42	7.97 20	7.03 77	- 0.7 328
	5	15.8 871	8.84 65	7.16 50	- 0.2 520	16.5 215	9.17 69	7.52 25	- 0.3 952	16.9 192	9.38 54	7.65 79	- 0.4 838
	10	18.6 415	9.23 38	7.34 36	- 0.1 808	19.2 901	9.55 53	7.68 86	- 0.3 193	19.5 961	9.75 39	7.80 89	- 0.4 028
	15	19.8	9.38	7.41	-	20.5	9.70	7.75	-	20.7	9.89	7.86	-

		407	35	10	0.1 517	072	28	18	0.2 880	864	93	69	0.3 693
	20	20.5 127	9.46 30	7.44 65	- 0.1 359	21.1 927	9.78 14	7.78 52	- 0.2 710	21.4 607	9.97 72	7.89 77	- 0.3 510
	25	20.9 428	9.51 24	7.46 83	- 0.1 260	21.6 325	9.83 03	7.80 59	- 0.2 603	21.8 949	10.0 259	7.91 67	- 0.3 395
	30	21.2 416	9.54 60	7.48 31	- 0.1 192	21.9 388	9.86 37	7.81 98	- 0.2 529	22.1 980	10.0 591	7.92 97	- 0.3 316
	35	21.4 613	9.57 04	7.49 38	- 0.1 143	22.1 643	9.88 79	7.83 00	- 0.2 475	22.4 216	10.0 833	7.93 91	- 0.3 258
1	1	8.24 08	7.27 46	6.38 92	- 0.4 681	8.93 97	7.68 39	6.82 45	- 0.6 174	9.73 97	7.99 69	7.05 29	- 0.7 233
	5	14.0 724	8.51 15	6.80 29	- 0.3 368	14.8 052	8.85 34	7.19 81	- 0.4 892	15.2 793	9.07 59	7.36 49	- 0.5 876
	10	15.7 843	8.79 31	6.89 02	- 0.2 999	16.5 612	9.12 64	7.27 97	- 0.4 507	16.9 796	9.33 93	7.43 63	- 0.5 462
	15	16.4 824	8.90 00	6.92 26	- 0.2 852	17.2 841	9.23 08	7.31 02	- 0.4 352	17.6 877	9.44 12	7.46 34	- 0.5 294
	20	16.8 620	8.95 63	6.93 95	- 0.2 774	17.6 789	9.28 60	7.32 62	- 0.4 269	18.0 767	9.49 54	7.47 76	- 0.5 204
	25	17.1 007	8.99 12	6.94 99	- 0.2 725	17.9 279	9.32 02	7.33 61	- 0.4 217	18.3 227	9.52 90	7.48 63	- 0.5 147
	30	17.2 647	9.01 48	6.95 70	- 0.2 691	18.0 992	9.34 34	7.34 28	- 0.4 181	18.4 924	9.55 19	7.49 23	- 0.5 108
	35	17.3 841	9.03 19	6.96 21	- 0.2 667	18.2 243	9.36 03	7.34 76	- 0.4 155	18.6 165	9.56 86	7.49 66	- 0.5 079
1.5	1	8.40 94	7.30 73	6.39 91	- 0.4 561	9.09 45	7.71 16	6.83 31	- 0.6 061	9.86 88	8.02 23	7.05 94	- 0.7 130
	5	12.1 947	8.16 09	6.54 78	- 0.4 045	13.0 343	8.51 59	6.97 35	- 0.5 637	13.6 453	8.75 32	7.17 25	- 0.6 699
	10	13.0 959	8.34 32	6.57 78	- 0.3 915	13.9 949	8.69 11	7.00 27	- 0.5 515	14.5 896	8.91 92	7.19 70	- 0.6 567
	15	13.4	8.41	6.58	-	14.3	8.75	7.01	-	14.9	8.98	7.20	-

		413	13	87	0.3 864	664	69	34	0.5 466	587	21	61	0.6 514
	20	13.6 241	8.44 69	6.59 45	- 0.3 838	14.5 637	8.79 14	7.01 90	- 0.5 440	15.1 556	9.01 52	7.21 09	- 0.6 486
	25	13.7 373	8.46 89	6.59 79	- 0.3 821	14.6 861	8.81 27	7.02 25	- 0.5 424	15.2 781	9.03 57	7.21 38	- 0.6 468
	30	13.8 144	8.48 37	6.60 03	- 0.3 810	14.7 696	8.82 71	7.02 48	- 0.5 413	15.3 617	9.04 96	7.21 58	- 0.6 455
	35	13.8 701	8.49 45	6.60 20	- 0.3 801	14.8 300	8.83 75	7.02 65	- 0.5 405	15.4 225	9.05 97	7.21 72	- 0.6 447
2	1	8.50 35	7.32 70	6.40 19	- 0.4 516	9.18 49	7.72 77	6.83 55	- 0.6 020	9.94 73	8.03 62	7.06 13	- 0.7 094
	5	11.2 693	7.97 91	6.46 95	- 0.4 285	12.1 429	8.34 30	6.90 30	- 0.5 884	12.8 465	8.58 95	7.11 60	- 0.6 973
	10	11.8 572	8.11 44	6.48 29	- 0.4 230	12.7 878	8.47 28	6.91 67	- 0.5 842	13.4 941	8.71 06	7.12 75	- 0.6 929
	15	12.0 762	8.16 45	6.48 78	- 0.4 208	13.0 299	8.52 11	6.92 17	- 0.5 825	13.7 395	8.75 61	7.13 17	- 0.6 911
	20	12.1 906	8.19 07	6.49 03	- 0.4 197	13.1 569	8.54 63	6.92 43	- 0.5 816	13.8 687	8.77 99	7.13 40	- 0.6 901
	25	12.2 610	8.20 67	6.49 19	- 0.4 190	13.2 351	8.56 18	6.92 60	- 0.5 810	13.9 484	8.79 46	7.13 54	- 0.6 895
	30	12.3 087	8.21 76	6.49 30	- 0.4 185	13.2 882	8.57 23	6.92 70	- 0.5 807	14.0 027	8.80 46	7.13 63	- 0.6 891
	35	12.3 431	8.22 54	6.49 37	- 0.4 182	13.3 265	8.57 99	6.92 78	- 0.5 804	14.0 419	8.81 18	7.13 69	- 0.6 888
3	1	8.60 35	7.35 04	6.40 33	- 0.4 485	9.28 70	7.74 71	6.83 70	- 0.5 993	10.0 354	8.05 13	7.06 24	- 0.7 070
	5	10.3 767	7.79 02	6.42 41	- 0.4 443	11.2 494	8.16 55	6.85 96	- 0.6 024	12.0 419	8.42 39	7.08 20	- 0.7 125
	10	10.7 114	7.87 94	6.42 81	- 0.4 434	11.6 290	8.25 15	6.86 42	- 0.6 027	12.4 396	8.50 31	7.08 61	- 0.7 131
	15	10.8	7.91	6.42	-	11.7	8.28	6.86	-	12.5	8.53	7.08	-

		326	22	96	0.4 430	674	33	58	0.6 028	857	25	76	0.7 132
	20	10.8 953	7.92 93	6.43 03	- 0.4 428	11.8 392	8.29 98	6.86 67	- 0.6 028	12.6 617	8.54 79	7.08 83	- 0.7 133
	25	10.9 336	7.93 98	6.43 08	- 0.4 427	11.8 831	8.31 00	6.86 72	- 0.6 029	12.7 083	8.55 73	7.08 88	- 0.7 133
	30	10.9 594	7.94 68	6.43 11	- 0.4 426	11.9 127	8.31 69	6.86 75	- 0.6 029	12.7 399	8.56 37	7.08 91	- 0.7 134
	35	10.9 780	7.95 19	6.43 14	- 0.4 425	11.9 341	8.32 18	6.86 78	- 0.6 029	12.7 626	8.56 83	7.08 93	- 0.7 134
∞	1	8.94 67	7.42 95	6.39 86	- 0.4 457	9.67 45	7.82 07	6.83 39	- 0.5 979	10.4 537	8.11 30	7.05 67	- 0.7 047

Table A3. Elastic stiffness coefficients for fully embedded circular footings (case 3).

T/ 2R	$G_{\text{sand}}/$ G_{clay}	$z_D/2R = 0.5$				$z_D/2R = 1$				$z_D/2R = 2$			
		K_v	K_h	K_m	K_c	K_v	K_h	K_m	K_c	K_v	K_h	K_m	K_c
0	1	7.10 13	7.78 58	6.83 34	0.0 321	8.41 85	8.61 12	7.73 34	0.1 097	9.73 42	9.17 15	7.97 71	0.1 040
0.0 5	1	7.65 08	8.24 57	7.86 68	0.0 925	9.20 44	9.13 24	9.22 21	- 0.0 338	10.9 313	9.77 80	9.63 82	- 0.1 812
	5	29.1 307	19.3 012	21.2 146	- 0.5 495	31.0 053	20.0 141	22.2 449	- 0.7 665	32.6 369	20.4 245	22.5 108	- 0.9 018
	10	53.0 208	26.2 841	34.2 946	- 0.6 031	54.9 436	26.9 345	35.2 133	- 0.8 646	56.4 825	27.2 717	35.4 285	- 1.0 014
	15	75.3 063	30.3 497	45.3 725	- 0.4 023	77.2 441	30.9 783	46.2 311	- 0.6 858	78.7 237	31.2 875	46.4 185	- 0.8 230
	20	96.4 148	33.0 215	55.0 741	- 0.1 266	98.3 607	33.6 415	55.8 917	- 0.4 220	99.7 965	33.9 375	56.0 597	- 0.5 583
	25	116. 5453	34.9 148	63.7 152	0.1 658	118. 4972	35.5 316	64.5 024	- 0.1 360	119. 8984	35.8 208	64.6 560	- 0.2 707
	30	135. 8213	36.3 280	71.4 976	0.4 537	137. 7787	36.9 441	72.2 603	0.1 484	139. 1507	37.2 296	72.4 027	0.0 157
	35	154. 3280	37.4 241	78.5 616	0.7 286	156. 2907	38.0 405	79.3 051	0.4 218	157. 6373	38.3 240	79.4 373	0.2 913
0.1	1	7.90	8.31	8.19	0.1	9.43	9.20	9.57	0.0	11.1	9.85	9.99	-

25		24	37	49	877	42	45	01	632	710	78	83	0.0 837
	5	28.3 356	15.5 807	18.5 267	0.5 489	30.2 185	16.3 007	19.5 333	0.3 313	31.8 189	16.7 069	19.7 596	0.1 966
	10	48.8 809	18.7 969	25.9 743	1.1 941	50.8 619	19.4 840	26.8 607	0.9 547	52.3 367	19.8 435	27.0 199	0.8 279
	15	66.3 676	20.3 552	30.8 510	1.7 058	68.4 153	21.0 375	31.6 822	1.4 643	69.8 112	21.3 860	31.8 093	1.3 460
	20	81.6 113	21.2 811	34.3 252	2.0 977	83.7 189	21.9 643	35.1 249	1.8 591	85.0 585	22.3 100	35.2 331	1.7 484
	25	95.0 701	21.8 963	36.9 330	2.4 031	97.2 345	22.5 817	37.7 128	2.1 687	98.5 317	22.9 273	37.8 085	2.0 644
	30	107. 0604	22.3 353	38.9 649	2.6 465	109. 2793	23.0 233	39.7 311	2.4 164	110. 5433	23.3 696	39.8 181	2.3 175
	35	117. 8195	22.6 648	40.5 934	2.8 446	120. 0907	23.3 552	41.3 499	2.6 185	121. 3279	23.7 024	41.4 306	2.5 242
0.2 5	1	8.21 11	8.38 92	8.46 41	0.2 906	9.71 11	9.28 06	9.83 95	0.1 662	11.4 460	9.94 12	10.2 704	0.0 198
	5	26.0 985	13.1 736	14.3 819	1.0 857	28.0 609	13.9 355	15.4 245	0.8 861	29.6 360	14.3 800	15.6 246	0.7 543
	10	40.2 836	14.8 153	16.9 258	1.6 151	42.5 080	15.5 651	17.8 947	1.4 085	43.9 633	15.9 853	18.0 389	1.2 892
	15	50.0 205	15.5 305	18.1 396	1.8 988	52.4 540	16.2 816	19.0 833	1.6 942	53.8 547	16.6 995	19.2 067	1.5 834
	20	57.1 528	15.9 325	18.8 526	2.0 737	59.7 609	16.6 863	19.7 843	1.8 718	61.1 351	17.1 051	19.8 971	1.7 670
	25	62.6 095	16.1 904	19.3 222	2.1 920	65.3 644	16.9 467	20.2 470	1.9 926	66.7 259	17.3 669	20.3 534	1.8 920
	30	66.9 209	16.3 700	19.6 549	2.2 773	69.8 004	17.1 284	20.5 754	2.0 799	71.1 565	17.5 500	20.6 776	1.9 826
	35	70.4 143	16.5 023	19.9 030	2.3 416	73.4 003	17.2 623	20.8 205	2.1 460	74.7 549	17.6 852	20.9 198	2.0 511
0.3 75	1	8.43 47	8.44 06	8.58 65	0.3 508	9.91 17	9.32 95	9.95 45	0.2 247	11.6 370	9.99 37	10.3 820	0.0 777
	5	23.3 507	12.0 590	12.0 409	1.0 883	25.4 427	12.8 518	13.1 494	0.8 929	27.0 367	13.3 322	13.3 604	0.7 535
	10	32.4 861	13.1 407	13.1 348	1.4 208	34.9 827	13.9 276	14.1 986	1.2 183	36.5 204	14.3 921	14.3 668	1.0 875
	15	37.7 095	13.5 877	13.5 948	1.5 743	40.4 839	14.3 761	14.6 442	1.3 714	42.0 175	14.8 396	14.7 975	1.2 460
	20	41.1 000	13.8 325	13.8 485	1.6 623	44.0 757	14.6 228	14.8 911	1.4 600	45.6 188	15.0 872	15.0 370	1.3 380
	25	43.4 804	13.9 871	14.0 095	1.7 194	46.6 075	14.7 789	15.0 482	1.5 177	48.1 631	15.2 444	15.1 896	1.3 981
	30	45.2 439	14.0 936	14.1 206	1.7 594	48.4 889	14.8 868	15.1 569	1.5 582	50.0 569	15.3 532	15.2 954	1.4 404
	35	46.6	14.1	14.2	1.7	49.9	14.9	15.2	1.5	51.5	15.4	15.3	1.4

		029	715	021	890	421	656	366	883	216	329	730	718
0.5	1	8.60 25	8.47 79	8.64 78	0.3 876	10.0 639	9.36 34	10.0 093	0.2 593	11.7 779	10.0 293	10.4 329	0.1 113
	5	20.9 351	11.3 997	10.8 118	0.9 999	23.1 447	12.2 116	11.9 787	0.8 040	24.7 935	12.7 182	12.2 120	0.6 540
	10	27.0 212	12.1 985	11.3 910	1.2 215	29.6 900	13.0 067	12.5 278	1.0 170	31.3 475	13.5 004	12.7 270	0.8 719
	15	30.0 996	12.5 186	11.6 211	1.3 168	33.0 425	13.3 279	12.7 484	1.1 104	34.7 295	13.8 204	12.9 358	0.9 683
	20	31.9 620	12.6 914	11.7 449	1.3 698	35.0 859	13.5 019	12.8 676	1.1 627	36.8 000	13.9 948	13.0 490	1.0 225
	25	33.2 108	12.7 997	11.8 221	1.4 035	36.4 627	13.6 111	12.9 422	1.1 962	38.1 991	14.1 045	13.1 202	1.0 573
	30	34.1 065	12.8 738	11.8 750	1.4 268	37.4 535	13.6 860	12.9 934	1.2 194	39.2 080	14.1 801	13.1 690	1.0 815
	35	34.7 804	12.9 278	11.9 135	1.4 439	38.2 008	13.7 407	13.0 307	1.2 365	39.9 701	14.2 352	13.2 045	1.0 993
0.7 5	1	8.83 42	8.52 90	8.70 19	0.4 279	10.2 791	9.40 75	10.0 556	0.2 958	11.9 727	10.0 734	10.4 731	0.1 453
	5	17.5 963	10.6 472	9.70 98	0.8 451	19.9 323	11.4 794	10.9 569	0.6 495	21.7 167	12.0 177	11.2 367	0.4 821
	10	20.7 939	11.1 675	9.94 01	0.9 650	23.5 645	11.9 977	11.1 719	0.7 596	25.4 365	12.5 247	11.4 283	0.5 920
	15	22.2 051	11.3 692	10.0 274	1.0 133	25.1 935	12.1 999	11.2 543	0.8 048	27.1 220	12.7 249	11.5 026	0.6 376
	20	23.0 012	11.4 766	10.0 734	1.0 394	26.1 200	12.3 078	11.2 980	0.8 294	28.0 860	12.8 323	11.5 421	0.6 626
	25	23.5 125	11.5 432	10.1 019	1.0 557	26.7 181	12.3 749	11.3 250	0.8 449	28.7 105	12.8 993	11.5 666	0.6 784
	30	23.8 689	11.5 886	10.1 212	1.0 669	27.1 364	12.4 207	11.3 434	0.8 555	29.1 481	12.9 450	11.5 834	0.6 892
	35	24.1 313	11.6 216	10.1 351	1.0 750	27.4 452	12.4 539	11.3 566	0.8 633	29.4 717	12.9 783	11.5 955	0.6 972
1	1	8.98 47	8.56 29	8.72 32	0.4 485	10.4 247	9.43 41	10.0 706	0.3 114	12.1 036	10.0 994	10.4 858	0.1 597
	5	15.6 156	10.2 286	9.27 23	0.7 500	17.9 783	11.0 710	10.5 647	0.5 585	19.8 771	11.6 269	10.8 815	0.3 812
	10	17.6 181	10.6 138	9.38 99	0.8 273	20.3 527	11.4 556	10.6 743	0.6 266	22.3 728	12.0 004	10.9 743	0.4 463
	15	18.4 431	10.7 607	9.43 38	0.8 575	21.3 472	11.6 029	10.7 155	0.6 536	23.4 303	12.1 451	11.0 097	0.4 726
	20	18.8 937	10.8 383	9.45 66	0.8 735	21.8 948	11.6 809	10.7 371	0.6 681	24.0 160	12.2 220	11.0 284	0.4 868
	25	19.1 777	10.8 863	9.47 08	0.8 836	22.2 416	11.7 291	10.7 504	0.6 772	24.3 881	12.2 698	11.0 399	0.4 957
	30	19.3 732	10.9 189	9.48 03	0.8 904	22.4 809	11.7 619	10.7 594	0.6 834	24.6 456	12.3 023	11.0 477	0.5 018

	35	19.5 159	10.9 425	9.48 72	0.8 953	22.6 561	11.7 857	10.7 660	0.6 879	24.8 344	12.3 259	11.0 534	0.5 063
1.5	1	9.16 57	8.60 15	8.73 46	0.4 639	10.6 084	9.46 83	10.0 828	0.3 273	12.2 719	10.1 294	10.4 947	0.1 730
	5	13.5 225	9.77 53	8.95 38	0.6 512	15.8 177	10.6 298	10.2 837	0.4 723	17.8 547	11.2 047	10.6 447	0.2 885
	10	14.5 817	10.0 298	8.99 81	0.6 937	17.1 445	10.8 853	10.3 257	0.5 074	19.3 192	11.4 497	10.6 778	0.3 190
	15	14.9 901	10.1 251	9.01 45	0.7 098	17.6 640	10.9 813	10.3 411	0.5 209	19.8 995	11.5 427	10.6 902	0.3 309
	20	15.2 068	10.1 751	9.02 29	0.7 183	17.9 415	11.0 318	10.3 492	0.5 280	20.2 112	11.5 917	10.6 967	0.3 373
	25	15.3 412	10.2 059	9.02 81	0.7 235	18.1 141	11.0 629	10.3 542	0.5 325	20.4 059	11.6 220	10.7 007	0.3 413
	30	15.4 327	10.2 268	9.03 16	0.7 271	18.2 320	11.0 840	10.3 575	0.5 355	20.5 391	11.6 426	10.7 035	0.3 440
	35	15.4 989	10.2 419	9.03 41	0.7 296	18.3 176	11.0 992	10.3 600	0.5 377	20.6 359	11.6 575	10.7 054	0.3 460
2	1	9.26 88	8.62 79	8.73 96	0.4 726	10.7 197	9.48 99	10.0 870	0.3 344	12.3 754	10.1 469	10.4 975	0.1 785
	5	12.4 722	9.53 26	8.84 86	0.6 034	14.6 733	10.3 937	10.1 900	0.4 344	16.7 665	10.9 800	10.5 725	0.2 513
	10	13.1 639	9.72 27	8.87 03	0.6 315	15.5 676	10.5 863	10.2 109	0.4 571	17.7 939	11.1 633	10.5 885	0.2 693
	15	13.4 225	9.79 34	8.87 82	0.6 420	15.9 063	10.6 581	10.2 186	0.4 656	18.1 876	11.2 322	10.5 944	0.2 762
	20	13.5 579	9.83 03	8.88 23	0.6 475	16.0 847	10.6 956	10.2 226	0.4 701	18.3 961	11.2 684	10.5 975	0.2 799
	25	13.6 412	9.85 30	8.88 49	0.6 509	16.1 948	10.7 187	10.2 251	0.4 729	18.5 253	11.2 906	10.5 995	0.2 822
	30	13.6 977	9.86 84	8.88 66	0.6 532	16.2 696	10.7 344	10.2 268	0.4 748	18.6 132	11.3 058	10.6 007	0.2 838
	35	13.7 384	9.87 95	8.88 78	0.6 549	16.3 237	10.7 457	10.2 279	0.4 762	18.6 768	11.3 167	10.6 017	0.2 849
3	1	9.38 05	8.66 01	8.74 29	0.4 806	10.8 458	9.51 73	10.0 900	0.3 410	12.5 056	10.1 687	10.4 996	0.1 833
	5	11.4 491	9.27 43	8.78 21	0.5 579	13.4 928	10.1 417	10.1 285	0.4 008	15.6 021	10.7 433	10.5 277	0.2 227
	10	11.8 433	9.40 01	8.78 98	0.5 737	14.0 188	10.2 714	10.1 362	0.4 135	16.2 393	10.8 661	10.5 335	0.2 317
	15	11.9 865	9.44 66	8.79 26	0.5 796	14.2 117	10.3 194	10.1 390	0.4 182	16.4 756	10.9 119	10.5 356	0.2 352
	20	12.0 605	9.47 08	8.79 41	0.5 827	14.3 120	10.3 444	10.1 404	0.4 207	16.5 989	10.9 358	10.5 367	0.2 370
	25	12.1 057	9.48 56	8.79 49	0.5 845	14.3 733	10.3 598	10.1 413	0.4 222	16.6 747	10.9 505	10.5 374	0.2 381
	30	12.1	9.49	8.79	0.5	14.4	10.3	10.1	0.4	16.7	10.9	10.5	0.2

		363	57	56	858	149	702	419	233	259	604	379	388
	35	12.1 583	9.50 29	8.79 60	0.5 867	14.4 448	10.3 777	10.1 423	0.4 240	16.7 629	10.9 676	10.5 382	0.2 394
∞	1	9.78 13	8.77 07	8.74 43	0.4 943	11.3 661	9.62 78	10.0 900	0.3 526	13.1 453	10.2 656	10.4 985	0.1 915

Figure captions

Fig. 1. Cases of three embedment conditions: (a) case 1; (b) case 2; (c) case 3

Fig. 2. Sign conventions for loads and displacements

Fig. 3. A typical finite element mesh (case 1 surface circular footing)

Fig. 4. Comparison of elastic stiffness coefficients for circular footings embedded in single-layer sand ($\nu = 0.2$): (a) K_v ; (b) K_h ; (c) K_m ; (d) K_c

Fig. 5. Elastic stiffness coefficients for surface circular footings on clay overlying sand ($G_{\text{sand}}/G_{\text{clay}} = 1$): (a) K_v ; (b) K_h ; (c) K_m ; (d) K_c

Fig. 6. Variation of stiffness with $T/2R$ and $G_{\text{sand}}/G_{\text{clay}}$ for surface footings on clay overlying sand: (a) K_v ; (b) K_h ; (c) K_m ; (d) K_c

Fig. 7. Comparison of numerical results and predicted elastic stiffness coefficients: (a) K_v ; (b) K_h ; (c) K_m

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7