Hydrodynamics of porous plates and influence of free surface proximity

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Abstract

The present work deals with the hydrodynamic behaviour of structures oscillating in an infinite fluid at varying free surface proximities. This is of interest for the offshore industry for applications including subsea structure installation. For this study a circular plate with negligible thickness was chosen due to its axisymmetric shape. The focus of the study is to understand the hydrodynamic forces when the plate porosity and free surface proximities are varied. The aim is to understand the physics of a hydrodynamic problem involving fluid structure interaction by studying added mass and damping behaviour. The work has also enhanced the hydrodynamic literature archive by providing information about hydrodynamic coefficients obtained using mathematical analysis and experimental techniques.

The study commences with an analysis of variation of added mass and damping coefficients with (i) Keulegan Carpenter ($K_C$) number, (ii) frequency parameter ($\beta$), and proceeds to the hydrodynamic study of (iii) structure porosity ($\tau$) and (iv) submergence level of oscillating plate. Mathematical analysis of hydrodynamic forces over the horizontally submerged plate is simulated using potential theory. The fluid domain is divided into three virtual subregions. The harmonic expressions of the velocity potential in three subregions are obtained in terms of unknown coefficients, Bessel functions and trigonometric functions using suitable boundary conditions. The unknowns in the expressions are obtained using boundary conditions and the method of matching Eigen function expansions at the virtual boundaries. The complex hydrodynamic force is evaluated from the velocity potentials, which gives added mass and damping forces by separating the real and imaginary parts. The same methodology is also applied for the porous plates, where the velocity potentials are redefined to include the pressure drop due the porosity of the plate. A number of porous models for discharge of flow through the porous plate were tested before choosing the one used here.
Qualitative and quantitative experimental techniques were chosen to understand the fluid flow at the plate edges using Particle Image Velocimetry (PIV) and to quantify the hydrodynamic forces using a forced oscillation technique. The PIV technique is used to understand the fluid flow around the plate edge and effect of porosity on the edge vortices. Each image pair illuminated with laser was analysed using VidPIV software to obtain velocity of the fluid flow at a number of spatial points. The velocity and vorticity information of the fluid flow allowed understanding the edge flow and circulation around the edge.

An experiment has been designed and performed to evaluate the hydrodynamic forces by varying the frequency and the amplitude of oscillation. The experimental setup consists of a water tank filled with fresh water and a Vertical Planar Motion Mechanism (VPMM) capable of oscillating the plate in sinusoidal motion at a given amplitude and frequency of oscillation. The total force on the plate is obtained using a load cell that is placed on the plate centre and a potentiometer was used to obtain accurate displacement. The added mass and damping are obtained from a Fourier transform of displacement and force time histories.

The impact of free surface proximity on the hydrodynamic behaviour is studied keeping in mind other varying parameters $KC$, $\beta$ and porosity. It is generally noticed that the hydrodynamic forces acting on the porous plate are lower than the solid structure due to increase in void to surface ratio. The added mass coefficients were found to reduce by 45% and 20% when the plate was made 20% and 10% porous respectively at $KC = 1$ and $\beta = 72000$. At the same time free surface proximity enhances the magnitude of hydrodynamic coefficients. It is found that added mass coefficients of plates oscillating at certain frequencies are three times more when the plate is brought in close proximity to the free surface i.e. at 0.2 times radius of the plate as compared to being far away from the boundaries. On the other hand at a few frequencies the added mass coefficients are found to be negative. The porosity has shown the adverse effect, this is due to the reduction in surface area with increasing porosity in the axial direction of oscillation. Another important observation has shown that frequency plays an important role on hydrodynamic behaviour when the free surface is in close proximity. The added mass and damping coefficients obtained for a solid plate oscillating at various free surface proximity levels have shown very good agreement with mathematically derived coefficients as $KC$ tends to zero.
**Key Words:** - Hydrodynamics, Added mass, Damping, Porosity, Free surface proximity, Amplitude of oscillation, Frequency of oscillation, Velocity potential, Linear potential theory, Separation of variables, Discharge model, Matched Eigen function expansions, Particle Image Velocimetry.
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Publications during PhD Study

Partial contribution in Chapter 3.

Partial contribution in Chapter 3.

Partial contribution in Chapter 4 and Chapter 2.

Partial contribution in Chapter 4.

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Partial contribution in Chapter 2.

Not included in this Thesis.
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<tbody>
<tr>
<td>BBC</td>
<td>Body Boundary condition</td>
</tr>
<tr>
<td>BDC</td>
<td>Bottom Dead Centre</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>FSBC</td>
<td>Free Surface Boundary Condition</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FO</td>
<td>Forced Oscillation</td>
</tr>
<tr>
<td>FPS</td>
<td>Floating Production Systems</td>
</tr>
<tr>
<td>iVOF</td>
<td>Improved Volume of Fluid Method</td>
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<tr>
<td>LABVIEW</td>
<td>Laboratory Virtual Instrumentation Engineering Workbench</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
</tr>
<tr>
<td>ROV</td>
<td>Remotely Operated Vehicles</td>
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<td>SBC</td>
<td>Seabed Boundary Condition</td>
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<tr>
<td>SI</td>
<td>International System of Units</td>
</tr>
<tr>
<td>TDC</td>
<td>Top Dead Centre</td>
</tr>
<tr>
<td>TLP</td>
<td>Tension-Leg Platforms</td>
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<tr>
<td>VPMM</td>
<td>Vertical Planner Motion Mechanism</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \tau )</td>
<td>Porosity</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Frequency Parameter</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Fluid density</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Angular Frequency of oscillation</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Displacement</td>
</tr>
<tr>
<td>( \dot{\zeta} )</td>
<td>Velocity</td>
</tr>
<tr>
<td>( \ddot{\zeta} )</td>
<td>Acceleration</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>( \vec{\xi} )</td>
<td>Vorticity</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Circulation</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Time dependant velocity potential</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Spatial velocity potential</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Free surface elevation.</td>
</tr>
<tr>
<td>( a )</td>
<td>Radius of Plate</td>
</tr>
<tr>
<td>( f )</td>
<td>Frequency of oscillation</td>
</tr>
<tr>
<td>( h )</td>
<td>Vertical distance between the free surface and bottom</td>
</tr>
<tr>
<td>( d )</td>
<td>Vertical distance between mean plate position and bottom</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>( S )</td>
<td>Submergence of plate from the free surface</td>
</tr>
<tr>
<td>( A )</td>
<td>Amplitude of oscillation</td>
</tr>
<tr>
<td>( D )</td>
<td>Diameter of the cylinder</td>
</tr>
<tr>
<td>( KC )</td>
<td>Keulegan-Carpenter Number</td>
</tr>
<tr>
<td>( M )</td>
<td>The Oscillating Mass/Moment Inertia</td>
</tr>
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<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( F_0 )</td>
<td>Force amplitude of first harmonic evaluated by FFT</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>Displacement amplitude of first harmonic evaluated by FFT</td>
</tr>
<tr>
<td>( M_{33} )</td>
<td>Added Mass in heave</td>
</tr>
<tr>
<td>( B_{33} )</td>
<td>Damping in heave</td>
</tr>
<tr>
<td>( F_j )</td>
<td>Exciting force in the ( j^{th} ) direction</td>
</tr>
<tr>
<td>( K_{ij} )</td>
<td>Restoring stiffness in the ( i^{th} ) direction due to unit displacement in the ( j^{th} ) direction</td>
</tr>
<tr>
<td>( M_{ij} )</td>
<td>Added Mass induced in the ( i^{th} ) direction due to unit acceleration in the ( j^{th} ) direction</td>
</tr>
<tr>
<td>( C_a )</td>
<td>Added Mass Coefficient</td>
</tr>
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<td>( C_b )</td>
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<td>Drag Coefficient</td>
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<tr>
<td>( C_M )</td>
<td>Inertia Coefficient</td>
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<td>Slamming Coefficient</td>
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Chapter 1

Introduction

1.1 Motivation

Economic opulence has lead to a continuous and remarkable growth in the energy consumption requirements in recent years. Oil and gas, being major resources of energy have picked up lots of attention by various researchers. An offshore oil rig (e.g. Figure 1-1), a large structure is used to house workers and machinery needed to drill and extract oil and natural gas through wells in the sea bed. These structures may be attached to the sea bed, or be floating depending upon the water depth and other circumstances. The offshore structures are pre-constructed onshore, floated to the location and then installed.

Sea-based oil platforms are some of the largest moveable man-made structures in the world. There are several distinct types of platforms used offshore such as fixed platforms, compliant towers, semi-submersible platforms, jack-up platforms, drill ships, floating production systems (FPS), tension-leg platforms (TLPs), spar platforms etc . But the oil and gas recovery at great water depths could have been a very difficult task without subsea structures used as alternative production systems and other subsea equipments. These include manifolds, heat exchangers and separators, suction anchors and hatch covers etc as shown in Figure 1-2-Figure 1-4. Subsea structures like manifolds are unmanned and are installed on the seabed in deep waters. They are designed to operate remotely and only to be visited occasionally by ROVs for routine maintenance and well work.
A suction anchor can be compared with a large can that is completely open from the side that looks downwards as shown in Figure 1-3. This can is then placed on the bottom of the seabed with completely open side facing downwards. Trapped water inside the suction anchor is pumped out to create underpressure inside the can, which results in the penetration of the can into the seabed. A suction anchor of 12m length can safely secure a platform. A hatch cover as shown in Figure 1-4 is not a permanently fixed part of the subsea structures, but is important for the subsea structures. It helps in securing the subsea structures from foreign objects like ship anchors or other objects that get dropped in the sea accidently.
The increasing availability of hydrocarbon reservoirs in the deeper waters as compared to the shallow waters has made operators and engineers to use subsea structures as compared to fixed and floating offshore structures in deeper waters. The use of very large fixed offshore structures was technically and economically impractical for the operators. This fact has continuously increased the number of subsea structures to be installed for deep
water reservoir operations. The idea of using subsea structures instead of bottom supported structures makes it much more cost efficient due to the reduction in the amount of material to be used to build these structures for installing in deeper waters. Adam et al reported on the Albacora manifold, the first deepest subsea manifold installed in Brazil to minimize the utilization of flow lines and production risers, Ref. [1]. One of the major concerns of the study was to determine the safest and least time consuming way to lower the manifold in 620 m water, using a conventional crane vessel. The end result of the operation only has limited tolerance in terms of positioning the manifold, and the fact that the environment can create sudden unpredictable challenges on the load hanging off the crane wire, the offshore operations are rather full of risk and challenge, Ref. [2]. Thus a very careful in-depth study incorporating numerical and experimental simulations would be a prerequisite to the actual operation, Ref. [3]. The installation of these structures is one of the main concerns. Analysis shows that the ease of installation of subsea structures depends upon the weather conditions, since loads on the structure vary with the fluid-structure interaction. The installation of subsea equipments is performed at sea by purpose built lifting vessels. A schematic of the derrick barge used to conduct this operation is shown in Figure 1-5, Ref. [4].

Figure 1-5 A derrick barge in waves lifting a subsea module
The main issues of a pipeline manifold (PLEM) installation are shown in Figure 1-6 as given by Thiagarajan and Yann, Ref. [2]. The rigging is attached to PLEM and attached to the hydralift crane as shown in Figure 1-6(a). The orientation clump weight is connected to the crane and sling to PLEM. PLEM and clump weight are deployed through the splash zone as shown in Figure 1-6(b). PLEM heading is positioned and aligned. PLEM is lowered to the seabed, tolerance is checked and de-rigged. The lowering operation of an anchor pile by a ‘dynamic positioning (DP) construction’ vessel such as CSO Venturer follows the steps as given by Thiagarajan and Yann, Ref. [2]. The pile lift is attached to rigging. The tag lines are attached to both ends of pile. The tag line on the pile tip is maintained until the pile is below the vessel keel. The crane is maintained at minimum radius while lowering the pile to the seabed. The chain catenary is monitored with ROV. The Pile gets turned during deployment due to bias of chain until it is perpendicular to the...
vessel at depth. Pile nominally landed 20 m from location. The doubled sling at the base of the pile is cut. Snap hook is latched into top masterlink using ROV. Chain is adjusted through gypsy until no more than approximately 1 m of chain is lying on seabed. Pile is upended and pile tip is raised 5 m above seabed. Vessel is moved to the location.

Several offshore operations are much more complex than described above. Ellaithy presented design and installation aspects of the first deep water field in the Mediterranean and concluded that there are a number of areas to be improved in design and installation of the subsea structures including manifold, hatch covers and suction cans etc. Ref. [5]. The installation procedure of the structure becomes important, since it affects the operational life of the structure as well as the successful installation of the structure. The loads during installation are mainly dependent on the hydrodynamics of the structure. The knowledge of hydrodynamic properties of any structure becomes interesting and challenging, for successful installation procedure of subsea modules like manifolds, mudmats and hatch covers etc due to the complex design of the structures.

Improper installation procedures can lead to a fatigued structure or even to loss of the structure. The noticeable thing is that the loss of the structure at a location does not lead only to loss of money spent on that structure, but also to loss of the location of that reservoir. The operations such as lowering and lifting the objects require knowledge of hydrodynamic coefficients to analyse the forces on the structure. Current knowledge of this area is very limited and conservative in the available literature and even in the industry accepted standard rules given in DNV and API codes, Ref. [6, 7]. Therefore the present study seeks to contribute further insight and new knowledge to this area by analytical and experimental study of the hydrodynamic forces.

### 1.2 Fundamentals of Hydrodynamic Coefficients

The study of hydrodynamics attracted the attention of researchers in the early 17th century. The fact that a structure vibrating in water has lower natural frequencies than it has in vacuum has long been recognized. Dubuat began investigation of hydrodynamic phenomena and believed that the effect of fluid on the frequencies of a structure is typically represented by a quantity called ‘added mass’ Ref. [8]. According to the ideal fluid or potential flow theory, a body moving at steady speed through an unbounded fluid
experiences no force (“D’Alembert’s Paradox”); but if the body is accelerating, it experiences an opposing hydrodynamic force proportional to the acceleration. This can be thought of as the force necessary to accelerate the fluid surrounding the body “out of the way”, Ref. [9].

A floating body has six degrees of freedom, three translational (surge, sway and heave) and three rotational (roll, pitch and yaw) as shown in the Figure 1-7 below. These motions are oscillatory and are excited by wave forces. Heave, surge and sway are positive upwards, towards the bow and starboard respectively.

![Figure 1-7 The 6 DOF of a free floating body](http://www.ogj.com)

The governing equations of motion of a floating rigid body are given by

\[
(M_{ij} + M)\ddot{\xi}_j + B_{ij} \dot{\xi}_j + K_{ij} \xi_j = F_i(t)
\]

(1.1)

Where

- **M** = the oscillating mass/ moment of inertia
- **M_{ij}** = the added mass induced in the \(i^{th}\) direction due to unit acceleration in the \(j^{th}\) direction
- **B_{ij}** = the damping induced in the \(i^{th}\) direction due to unit velocity in the \(j^{th}\) direction
- **K_{ij}** = the restoring stiffness in the \(i^{th}\) direction due to unit displacement in the \(j^{th}\) direction
- **F_{j}** = the exciting force in the \(i^{th}\) direction
- **\xi_j, \dot{\xi}_j, \ddot{\xi}_j** = the displacement, velocity and acceleration of the vessel in the \(j^{th}\) direction
\( i,j = 1,2,6 \) denote the six degrees of freedom surge, sway, heave, roll, pitch and yaw respectively as shown in Figure 1-7.

The forces on a structure are influenced by the added mass effects and damping introduced by the motion of the structure in the water, Ref. [10]. The dimensionless hydrodynamic coefficients of added mass and damping are important parameters for the equation of motion of a rigid floating body. The restoring coefficient is the measure of resistance of an elastic body to deform when force is applied. These coefficients have different values in different degrees of motion of a structure.

The added mass is the pressure force per unit acceleration acting on an oscillating body, due to the acceleration field setup in the surrounding fluid. It is also defined as the extra force per unit acceleration required for a body to move in fluid as compared to vacuum, which is purely an inertial effect and is independent of viscosity. A \( 6 \times 6 \) added mass matrix gives added masses of the structure in all six degrees of motion. The added mass coefficient \((C_a)\) is the ratio of the added mass to the mass of the body or mass of water in specific circumscribing volume, and Inertia coefficient \((C_M)\) is given by the ratio of total mass to the mass of the body. An interesting fact that can be seen in added mass matrix is that it is symmetrical, irrespective of the symmetry in the shape of the structure. That means added mass induced in the \( i^{th} \) direction due to unit acceleration in the \( j^{th} \) direction is equivalent to added mass induced in the \( j^{th} \) direction due to unit acceleration in the \( i^{th} \) direction. Added mass opposes a positive acceleration of the structure. The added mass for several simple structural elements like cylinders and solid plates etc. was determined experimentally by researchers and the data is available in the literature, Ref. [6,11,12].

The theoretical added mass (Lamb’s theory) of a circular plate oscillating along its axis is approximately equal to the mass of the sphere of water enclosing the plate and is given by

\[
M_a = \frac{1}{3} \rho (2a)^3
\]  \hspace{1cm} (1.2)

where, \( a \) is the radius of the plate, Ref. [13].
\( \rho \) is the density of fluid [kg/m³].

Mathematically, damping may be defined as a force synchronous with the velocity of the object but opposite in direction of its motion. Damping opposes a positive velocity of the structure. The consequence of that tends to reduce the amplitude of oscillation of any oscillating system. The equation of motion shows that the hydrodynamic coefficients are
the important parameters to be measured to find out the loads on the offshore structures of different body types, designs and geometries. Thiagarajan and Troesch showed that for a fixed oscillation frequency, the heave damping coefficient of a vertical cylinder has two components: a) a friction drag component arising due to viscous shear forces, which is independent of the amplitude of oscillation; and b) a form drag term due to flow separation and vortex shedding at the edges; this term is linear with the amplitude of oscillation, Ref. [14]. An additional another heave damping component, surface wave damping may induced due to waves, if structure is in near proximity to free surface.

The change in infinite frequency added mass coefficient with water depth is defined as the slamming coefficient. Impulse loads with high pressure peaks occur during impact between a body and water and this phenomenon is often called slamming, Ref. [15]. If slam force is truly impulsive, the member may be dynamically excited. The slamming coefficient has been assigned really conservative values, due to the lack of the data available in the literature. Sarpkaya has shown empirically that the coefficient $C_s$ may lie between $0.5\pi$ and $1.7\pi$, depending on the rise time and natural frequency of an elastically mounted cylinder in his tests, Ref. [12]. Sarpkaya and Isaacson recommend that if a dynamic response analysis is performed, the theoretical value of $C_s = \pi$ can be used; otherwise, a value of $C_s = 5.5$ should be used, Ref. [16].

It is important to evaluate the hydrodynamic coefficients of subsea structures during installation. The equations to evaluate the hydrodynamic loads on small diameter cylindrical structure due to waves were found mathematically and empirically and were named as Morison’s equation, Ref. [16]

$$F_i = C_a \rho \Psi \ddot{\xi}_j + \frac{1}{2} \rho C_d \chi |\dot{\xi}_j|$$

where $\Psi = \text{submerged volume of the cylinder}$

and $\chi = \text{submerged area of the cylinder}$

Regulatory bodies like American Petroleum Institute, Det Norske Veritas, and Norwegian Petroleum Directorate have some recommended practices for using added mass ($C_a$), slam ($C_s$) and drag ($C_d$) coefficient values, Ref. [6, 7]. The regulatory body API recommend the inertia coefficient ($C_M$) to be 1.5-2 and drag coefficient ($C_d$) between 0.6-1 for fixed cylindrical structures, Ref. [7]. $C_d$ given in Morrison’s equation may be amended to suit purpose by using suitable reference velocities. Sarpkaya presented the charts for inertia and
drag coefficients for cylindrical structures obtained from the model tests, and are function of Keulegan- Carpenter ($KC$) number, Reynolds number ($Re$), roughness factor and are given in, Ref. [17]

\[ KC = \frac{2\pi A}{D} \]

\[ Re = \frac{DU_0}{v} \]

\[ \beta = \frac{Re}{KC} \]

Roughness factor $= \frac{\varepsilon}{d}$. (1.7)

where, $A = \text{amplitude of oscillation}$, and $v = \text{kinematic viscosity}$, $D = \text{diameter of the cylinder}$.

The hydrodynamic coefficients are well defined for a few standard shapes e.g. cylinders and spheres, obtained from a variety of experiments and simulations. But they are not readily available for complex structures including porous structures used by industry nowadays. The following section gives the details of numerical, experimental and theoretical developments of hydrodynamic coefficients to date.

1.3 Literature Review

1.3.1 Hydrodynamics of subsea structures and equipment

Morison and Cermelli found that the added mass of a mudmat during installation could be 6-7 times the buoyancy of the mudmat, Ref. [18]. That can lead to lot of unexpected loads on the crane hook. During the installation of a structure, after entering through the sea surface, several phenomena like sloshing and slamming occur and thus the added mass of the structure changes continuously. That means that even the slamming coefficient is large and changes rapidly. The huge change in added mass thus becomes self-evident for the importance of correct estimation of these hydrodynamic coefficients for a successful operation, Ref. [19]. A compendium of added mass formula and values for a variety of geometries has been published by numerous researchers in references such as Fritz, Ref [20], but the data available is limited to some standard shapes and has not been really extended to the complex subsea structures used by the industry.

The correct approximation of hydrodynamic coefficients will help in determining the
accurate forces on the structure and that in turn will lead to the correct design and installation procedures for the particular structure. Thiagarajan and Yann revealed shortcomings of accepted practice in key areas such as: 1) Over-prediction of wire tension and snap loads, leading to lesser weight being lifted 2) Under-prediction of underwater heave motion of module while being lowered, Ref. [2]. They have discussed the issues in light of the fact that added mass of the module changes with submergence. Numerical simulations using DNV’s software SESAM were used to evaluate the added mass of cylindrical and rectangular modules and to gain an understanding of heave motions experienced by the modules in moderate wave conditions. Sample calculations are used to show areas where existing regulations are inadequate.

Adam et al. found in their analysis that the ease of installation of a subsea structure also depends upon the weather conditions, since loads on the structure vary with the fluid-structure interaction, Ref. [1]. It becomes a real non-linear problem. Operating limitations on the seastates were also required due to the dynamics involved during installation, since it can lead to a large change in magnitude of hydrodynamic parameters, Ref. [21]. The changed forces on the structure also change the loads on the crane tip used for installation. Considering the fact that the crane deck is supported by the buoyancy of the deck, the unknown forces can result in severe conditions.

Figure 1-8 Manifold in splash zone during installation
(http://www.fmctechnologies.com/upload/440_mainpage.jpg)
Critical stages of the operation can be identified through the assessment of the dynamic forces. Roveri et al. found that the calculated and measured manifold (e.g. Figure 1-8) vertical displacements showed good correlation whereas significant discrepancies were observed in calculated and measured dynamic sling forces while performing installation procedures of a structure, Ref. [22]. The main sources of uncertainty were 1) manifold vertical hydrodynamic added mass and 2) friction effects on the hoisting wires and axial hoist wire dynamics. Fernandes and Mineiro performed experiments and numerical simulations adopting two methods, namely frequency limit method and constant acceleration method, to obtain added mass coefficients, Ref. [23]. The authors found good correlation between the results obtained from both methods for the translational added mass coefficients of a manifold model. Although model tests could be one of the solutions to estimate the hydrodynamic coefficients and hence forces on the structures, they are not economically as well as practically a wise decision to perform for each structure due to time constraints and other resources. Due to the different geometries to be installed, performing model tests for each of them becomes unaffordable, Ref. [24]. So it is really important to have some standard rules to estimate these coefficients accurately as available in literature for some of the non-complex shaped structures.

Based on an experimental study Bunnik and Buchner concluded that the complexity of using added mass concept in the splash zone was due to the strong variation of the added mass, Ref. [25]. The variation in added mass coefficients, with oscillations of the structure was due to the water flow over and inside the structure during installation procedures. Also that it was not possible to simulate the installation of subsea structures through the splash zone, since the wave loads were strongly non-linear due to the intricate effects of varying buoyancy, added mass and water in and outflow etc. Bunnik et al. found that the duration of the slamming pressure at one place on the structure could be of the order of milliseconds, Ref. [26]. It was much localized and the position on the structure where slamming pressure occurs, changes with time. Experiments show that even when under harmonic input, since the magnitude and duration of the pressure vary from cycle to cycle, the pressure variation was neither harmonic nor periodic.

Bunnik et al. found that the existing simulation methods were not able to determine in detail the wave loads on a complex subsea structure when it was passing through the splash zone, simply because of the unpredictable motions of the structure with water in and out flow, Ref. [26]. Bunnik and Buchner showed that the total loads on a manifold structure show a good quantitative comparison between model test and improved volume of fluid
Ireland et al. studied the installation of a suction can in fully submerged condition well below the free surface and above the seabed, such that the effects of the free surface and the seabed proximity can be neglected. It was revealed that open hatches in the top plate of the suction can can significantly increase the hydrodynamic damping, particularly in its linear component, Ref. [28]. This phenomenon could be attributed to the strong increase in pressure drop across the hatch orifice with the decrease in the $KC$ number of the orifice flow. The dynamic effects are much more important in deep and ultra-deep water, consequently the parameters (added mass and damping or drag), which have an impact on the dynamic behaviour, have to be determined more accurately. For example in the case of a suction anchor the trapped mass in the structure and added mass are much more than the mass of the structure itself, Ref. [29]. Hydrodynamic coefficients, which are based on the empirical data for steady flow or heaving half submerged cylinders, underestimate both the heave added mass and the viscous damping for the suction can. This study shows that small changes in the geometry of subsea equipment may alter significantly its hydrodynamic properties. For the subsea lift analysis such changes should be carefully examined and taken into account to provide desired level of accuracy.

The installation of subsea equipment plays a significant role on its future operational performance. It is important to keep in mind that complex geometries and procedures of installation, with all kind of redundancies may input unexpected loads on the structure, Ref. [30]. Sandvik et al. hypothesized that ventilated subsea structures may be represented by porous plates of equivalent porosity ($\tau$), whose hydrodynamic properties may be obtained with confidence, Ref. [31]. Sireta et al. also supposed that more complex structures may be broken down to individual structures whose hydrodynamic properties are known, Ref. [32]. As an example, the model of a suction pile may be assumed to be composed of two parts, a circular cylinder and a top cover, Figure 1-9. The circular cylinder can have a variable roughness in order to stimulate transition to turbulent flow if required. The top cover can be a porous plate with different porosity and hole configurations. Examination of these structures as compound structures may allow methodologies to be developed for estimation of hydrodynamic coefficients using known information for cylinders and plates.
1.3.2 Hydrodynamics of porous plates

Lindholm et al. looked at the characteristics of cantilever plates partially submerged in water, Ref. [33]. They found that the presence of a liquid free surface has a significant effect upon the dynamic plate characteristics (frequencies) only when the surface is less than about one-half span length from the plate. The effect of the liquid free surface on the resonant frequency of a vertical surface piercing plate is highly dependent upon the relationship between the depth of immersion of the plate and the mode of vibration. The overall damping of cantilever plate vibration is increased significantly in water as compared with air. Tao and Cai found experimentally that the diameter of the plate has a significant effect on the hydrodynamic damping, since form damping becomes dominant Ref. [34].

The wake of a porous two-dimensional screen or porous flat plate aligned normal to the flow has been investigated by Castro, Ref. [107] and also by Valensi in Ref. [108]. Castro found two distinct regimes of flow based on the solidity of the plate. For a less porous plate a Karman vortex dominates the near wake and for a plate with higher porosity, it does not shed Karman vortices, but there is a dominant frequency present, which he attributed to far-wake instability. Valensi also examined the latter regime in more detail with a 47 % porous plate and presented power spectra and smoke pictures. Taylor studied the flow through flat porous plate to understand its air resistance. Ref. [109] and also discussed distribution of velocity in turbulent flow of a fluid flowing under pressure through pipe and between parallel planes, Ref. [110]
Particle Image Velocimetry, a flow visualization technique, has been used to measure velocities and related properties in fluids, Ref. [35, 36]. Lake et al. conducted experiments and visualisation of the flow using particle image velocimetry experiments around the edge of a plate and a plate attached to a cylinder, Ref. [37]. It was noticed that the characteristics of the flow tend to be the same irrespective of the configuration. They concluded that the local edge flow is the main cause of damping at low $KC$ number, with an additional smaller component from surface shear. Sireta performed some preliminary experiments on the visualization of the flow (coloured by velocity amplitude in the fluid) for a solid plate and for a porous plate, Ref. [32]. The pictures showed a sequence of snap shots over one cycle of oscillation, the fluid domain being coloured from blue to red with increasing fluid velocity amplitude. It is clear that the figures showed a dominant flow pattern through the pores of the surface, which results in viscous diffusion of the flow around the plate as shown in the Figure 1-10.

![PIV visualization of the flow past (a) a solid plate and (b) 15% porous plate, Ref. [32]](image)

Geoffrey found experimentally using forced oscillation tests, that porous plates provide more damping than solid plates for $KC$ number less than one, Ref. [38]. Also the damping increases with increase in the ratio of the radius to thickness of the plate. More added mass is associated with the thick porous plates as compared to thin porous plates. Wang et al. found that solid plate and the porous plate generate flow resistance in different ways, Ref. [39]. The solid plate generates damping entirely by causing flow separation and vortex shedding from its outer edge. The porous plates have some dependency on the orifice size, since it influences the velocity profile.
Molin approached this problem of evaluation of hydrodynamic coefficients of plates theoretically, based on the assumption of perfect fluid and irrotational flow for solid as well as perforated plates, Ref. [40]. In porous plates there is also separation through the openings, which is taken into account approximately by means of semi-empirical model as compared to the separation of flow around the edges of the plate, which is not taken into account in the theory. It was found that no extra damping can be gained by making plates porous when $KC$ number exceeds one. But the case is different if $KC$ number is less than one and for a $KC$ number around 0.2 there is lot to gain by making plates porous. With porosity slightly below 20%, the heave damping can be increased by a factor of 4 or 5. The remarkable thing obtained is the sensitivity of the added mass coefficient of a porous plate to the porosity ratio and $KC$ number. The added mass becomes nil when the $KC$ number goes to zero for a porous plate. Molin and Legras confirmed this behaviour experimentally for the perforated truncated cylinders. Ref. [41]. The sensitivity of added mass with motion amplitude becomes interesting and very important, on considering the resonance problem.

Molin proposed an analytical model using potential theory and irrotational flow assumption to calculate hydrodynamic properties of circular plates and found a reasonable agreement with hydrodynamic coefficients obtained using finite fluid element method, Ref. [40]. For the perforated plate, the discharge equation is used for the flow through the holes. Analytical model assumed a genuine free surface, Ref. [42, 43]. The fluid domain is virtually divided into three sub-domains. The complex radiation potential for each sub-domain was obtained as a series expansion in terms of Bessel and Hankel functions, Ref. [44]. The porosity of the plate was modelled in terms of a discharge equation which provides an expression for pressure drop across the plate. The boundary value problem together with the discharge equation was solved iteratively to obtain the velocity potentials in the sub domains and hence understand hydrodynamic behaviour of oscillating circular plate.

Molin incorporated the viscous flow separation and vortex shedding at the edges empirically, using the data of Lake et al., Ref. [37, 45]. Sireta also looked at the forces on the plate and used Fourier analysis to measure added mass and damping coefficients, Ref. [32]. They also compared the results with the results obtained from the model developed by Molin, Ref. [43]. Sireta et al. noticed a great amount of discrepancies involved in the added mass and damping coefficients obtained by experiments and theoretical model.
Chua et al. conducted experiments on square heave plates and found a significant dependence of $C_M$ and $C_d$ on $KC$ number, particularly when $KC < 0.5$ and the dependence is far less pronounced for larger $KC$ numbers, Ref. [46]. These parameters vary with the porosity as well as with the size of the openings and also that these are dependent on frequency. $C_M$ becomes higher although $C_d$ value lowers with increase in frequency of oscillation.

Vu et al. performed experiments on solid and porous plates to obtain damping coefficients. They found similar trend but different slopes of the curves, Ref. [47]. Damping was found to vary linearly with increasing $KC$ number. Sireta et al. compared experimental results for circular porous plates with theoretical results obtained using the matched Eigen function expansions, Ref. [48]. They found good agreement for the empirically corrected damping coefficients. Although the values are a bit overestimated for porosity greater than 10%, they still showed the same trend. The added mass coefficient, for $0.2 < KC < 0.6$, produced a fairly good agreement. However, for $KC > 0.6$, a large amount of discrepancies was noticed, especially when the porosity ratios are less than 20%. At the same time $KC > 1$ did not contribute to increase damping with increase in porosity. Tao and Thiagarajan numerically studied the effect of edge radius of a solid plate and later He validated the findings in experiments that maximum flow separation will occur when a sharp edge is presented, Ref. [49, 50].

Tao et al. investigated the effect of spacing between two heave plates, Ref. [51]. It was found that the presence of an additional plate increases the added mass value as the relative spacing between the plates is increased. Further the damping value becomes independent of spacing beyond critical spacing. Tao and Dray performed forced oscillations tests at varying frequencies on porous plates of porosity from 0% to 20%. They found the porosity dependence of hydrodynamic damping of an oscillating plate to be sensitive to both motion amplitude and oscillating frequency. The added mass and damping coefficients were linearly varying with, Ref. [52]. The solid plate produced the highest damping at large $KC$ values. At low $KC$ number the damping value varied depending on frequency of oscillation. The porosity in plates increased damping significantly. The model tests performed by Tao and Dray reveals that the plate with 20% porosity yields approximately 30% increase in damping as compared to the solid plate at particular frequency and $KC$ number, Ref. [52]. In contrast to hydrodynamic damping, porous plates
consistently produce lower added mass coefficient than the solid ones. It was noticed experimentally that the reduction in added mass coefficient is more pronounced as porosity increases.

From the literature the main concerns found are

- To achieve accuracy in estimating hydrodynamic coefficients of the oscillating structures during installation rather than over or underestimation of these coefficients.
- To accurately predict the change in added mass coefficients from the splash zone to the water column.
- Conservative standards available for industry to follow, leading to more expensive installations.

1.4 Objectives

The critical literature review has shown that there is a gap between the industry accepted rules for installation and actual hydrodynamic forces felt by structures during installation. This research project is aimed to fill the gap by concentrating on the following:

1) To understand the influence of porosity on structures oscillating in heave motion, due to alteration in hydrodynamic parameters i. e. $\beta$ and $KC$.

2) To perform flow visualization experiments using the Particle Image Velocimetry (PIV) technique to analyse the flow field around the edges of a solid plate and effect of plate porosity on the flow around the edges of the plate.

3) To develop a theory for hydrodynamic coefficients of a solid plate and to extend the theory to include plate porosity effects using the method of matched Eigen function expansions.

4) To perform experiments to find the behaviour of hydrodynamic coefficients of plates oscillating near the free surface at varying frequencies and amplitudes of oscillation.

5) To gain a better understanding of the flow field around the plate from forced oscillation tests, theory and PIV experiments.

6) To find new empirical corrections of theoretically obtained added mass and damping coefficients at higher $KC$ number such that theoretical results can have a better fit with experimental results.

7) To understand the effect of free surface proximity by performing analytical study on plates oscillating at various frequencies and validating the results with experiments.
1.5 Thesis Plan and Methodology

The project consists of two parts being theoretical as well as experimental. Both avenues are chosen to gain the confidence in theoretical development and to validate results with experimental results as well as for better practical understanding of the flow field around the plate. The main two types of experiments are forced oscillation tests and Particle Image Velocimetry experiments. The forced oscillation methodology and influence of plate porosity on the hydrodynamic behaviour of deeply submerged oscillating plates is discussed in Chapter 2. The flow visualization experiments using Particle Image Velocimetry (PIV) are performed to understand the flow field around the solid and porous plate. The quantitative and qualitative study on a solid plate and a porous plate is studied in Chapter 3.

The theoretical development is taken as the main method of hydrodynamic study for the research. The overall aim of the research is to develop an analytical model for calculating hydrodynamic coefficients of porous plates and empirical corrections are introduced using experimental data. The study focuses on added mass and damping coefficients of porous plates in heave motion with varying porosities, amplitudes, frequencies of oscillation and free surface proximities. The methodology is presented and hydrodynamic results obtained are validated with published literature of deeply submerged plates in Chapter 4. This chapter also extends to understand the behaviour of oscillating plates in close proximity to the free surface using theoretical formulation.

The influence of free surface proximity on hydrodynamic coefficients of a solid plate, oscillating in close proximity to the free surface is studied. The comparison between experimental and analytical study is also shown in Chapter 4. The analytically obtained added mass coefficients of porous plates are compared with experimentally obtained added mass coefficients. The empirical correction to the analytical added mass and damping coefficients is proposed for a better agreement with experimental added mass and damping coefficients of deeply submerged oscillating porous plates. The influence of free surface proximity on the oscillating porous plates is also studied in this chapter.

A closing discussion and concluding remarks are given in Chapter 5. This chapter provides an overall discussion of issues raised in the thesis and points towards new open questions as a result of this thesis for further research.
Chapter 2

Forced Oscillation: Hydrodynamic Performance of Porous Plates

“Everything should be made as simple as possible, but not simpler”
- Albert Einstein

The in-line force coefficients (i.e. added mass coefficient and damping coefficient) of porous plates oscillating in the axial direction are studied in this chapter. The dependence upon $KC$ number, $\beta$ parameter and porosity ($\tau$) is studied for circular plates deeply submerged in a water tank using Forced Oscillation (FO) experimental technique.

2.1 Introduction

Hydrodynamic model tests can be conducted either by oscillating a flow over a stationary model or by oscillating a model in calm water. Bearman concluded that for oscillating the flow over a stationary model, the inertia force is increased by an equivalent Froude-Krylov force, a boundary force caused by the pressure gradient imposed on the fluid to generate the oscillating flow, Ref. [53]. Sarpkaya used a U-tube to produce uniform fluid oscillations and conducted extensive study of circular cylinders under oscillatory flow. On the other hand oscillation of a structure in a still fluid is one of the main methods to understand hydrodynamics around oscillating models in calm water, Ref. [54].

Considering laminar boundary theory the frequency parameter $\beta$ represents the ratio of momentum diffusion through the boundary layer to the rate of momentum diffusion
across the diameter of the plate, Ref. [16]. The parameter $\beta$ and $KC$ for an oscillating flow are given in Chapter 1 in Equation (1.4) and Equation (1.6) and are also related to each other via another non-dimensional parameter, the Reynolds number ($Re$). Rewriting them in terms of plate radius gives,

$$KC = \frac{\pi A}{a} \quad (2.1)$$

$$\beta = \frac{4a^2 f}{v} = \frac{Re}{KC} \quad (2.2)$$

where

$a$ = the radius of the plate (m),

$f$ = the frequency of oscillation (Hz),

$A$ = the amplitude of oscillation (m),

$v$ = the kinematic viscosity of fluid (m$^2$/s).

Experimental techniques used to measure hydrodynamic loads on oscillating structures are forced oscillation technique and decrement test method. Forced oscillation is one of the techniques mainly used for this purpose. In this technique the system is designed in such a way that the structure is forced to oscillate at given amplitude and frequency of oscillation in a quiescent flow. Dalzell studied oscillating solid plates for $KC$ ranging from 0.07-0.63 and $\beta$ ranging from $6 \times 10^3 - 4.8 \times 10^4$ using the decrement test method, in which system is freely oscillated and oscillations die out with time, Ref. [55]. The hydrodynamic coefficients obtained using this technique, are questionable when hydrodynamic forces are highly $KC$ dependent, Ref. [15].

Thiagarajan used the forced oscillation technique to understand the flow around cylinders and a cylinder plate configuration and their hydrodynamic behaviour at low $KC$ and $\beta$ range, Ref. [56]. Tao and Dray studied added mass and damping forces acting on porous plates and found hydrodynamic damping to be sensitive to both the amplitude parameter $KC$ and the frequency parameter $\beta$, Ref. [52]. They also found a linear variation of added mass and damping coefficients for a $KC$ range of $(0.2 - 1.2)$ for 4 plates of varying porosity.
For fixed structures in oscillating flow or an oscillating structure in otherwise quiescent flow, the forces acting on these structures can mainly be decomposed into two parts.

- Inertia forces due to the inertia of the accelerated fluid flow
- Damping forces due to the influence of a viscous boundary layer and due to the separation of the boundary layer leading to the shedding of the vortices at the edges.

Other force components may be

- Surface wave damping due to structure’s proximity to free surface. It only exists if the structure is in close proximity to the free surface.
- Froude Krylov inertia forces contribute towards total hydrodynamic force, when incident wave field is not significantly modified by the structure’s presence.

The damping force is a combination of friction force and form drag force. For an oscillating body whose displacement and velocity are given in Equation (2.3) and Equation (2.4), the damping force can be represented in a quadratic form i.e. Morrison’s equation or in a linearized form as given in equations (2.5) and (2.6) respectively.

\[ \zeta = A_0 \sin(\omega t) \]  
\[ \dot{\zeta} = A_0 \omega \cos(\omega t) \]  
\[ F_D = \frac{1}{2} \rho a C_d \zeta \dot{\zeta} \]  
\[ F_D = B \dot{\zeta} = B A \omega \cos(\omega t) \]

\( C_d \) is the drag coefficient, and \( B \) is the damping coefficient. The damping coefficient and drag coefficient are related to each other by equivalent linearization of the quadratic damping using Fourier decomposition, Ref. [16] and their relation can be written as

\[ B = \frac{2}{3} \mu B a C_d K C. \]  

\( \mu \) = dynamic viscosity

The present research has studied a wider range of \( KC \) and \( \beta \) parameters and has performed in depth investigation on plate porosity and its effects on hydrodynamic behaviour using the forced oscillation technique. This chapter investigates the in-line force coefficients i.e. added mass and damping coefficients experienced by the circular plates that are oscillating in the heave direction. The main parameters studied here are the Keulegan Carpenter number \((KC)\), frequency parameter \((\beta)\) and the effect of plate porosity \((\tau)\).
2.2 Mathematical Formulation

In forced oscillation tests the model is oscillated at a predefined frequency and amplitude. Figure 2-1 shows a schematic of the forces acting on a plate during forced oscillation experiments. From the free body diagram in Figure 2-1 and applying Newton’s third law of motion, we get

\[ M \ddot{\xi} = F_{\text{measured}} + F_{\text{Hydrodynamics}} \quad (2.8) \]

\( F_{\text{Hydrodynamics}} \) is the reactive total hydrodynamic heave force acting on the system.

\( F_{\text{measured}} \) is the total force experienced by the plate and hence acting on the load cell.

\[
\ddot{\xi} = \frac{\partial^2 \xi}{\partial t^2} = -A_0 \omega^2 \sin(\omega t) \quad (2.9)
\]

where \( \xi \) given in Equation (2.9) is the acceleration at which the plate is oscillating in heave.

\[ F_{\text{measured}} = F_0 \sin(\omega t + \theta) \quad (2.10) \]

\( F_{\text{measured}} \) given in Equation (2.10) represents the total force experienced by the plate that is out of phase with plate oscillations by phase difference of \( \theta \).

\( F_0 \) and \( A_0 \) are the amplitude of first harmonic of force and displacement obtained by Fast Fourier Transform on force and displacement time histories. \( \omega \) is angular oscillation frequency, \( \theta \) represents the phase difference between displacement and force time histories. The hydrodynamic force in this case is assumed to have two significant
components: a velocity dependent term; damping \( B_{33} \) and an inertia dependent term; added mass \( M_{33} \). It must be noted here that it is assumed that the oscillating system does not allow any stiffness and or structural resonance. Thus

\[
F_{\text{Hydrodynamics}} = -M_{33} \ddot{x} - B_{33} \dot{x}
\]  

(2.11)

The negative sign indicates that the hydrodynamic force is a reaction force. Here

\( M_{33} = \) added mass in heave

\( B_{33} = \) damping in heave

Using Equation (2.4) and Equation (2.9) - Equation (2.11) in Equation (2.8), we get

\[-MA_0^2 \sin(\omega t)\]

\[= F_0 \sin(\omega t) \cos(\theta) + F_0 \cos(\omega t) \sin(\theta) + M_{33} A_0^2 \sin(\omega t) \]

\[- B_{33} A_0 \omega \cos(\omega t)\]

(2.12)

Rearranging Equation (2.12) gives Equation (2.13).

\[\Rightarrow -(M + M_{33}) A_0 \omega^2 \sin(\omega t) + B_{33} A_0 \omega \cos(\omega t) = F_0 \sin(\omega t) \cos(\theta) + F_0 \cos(\omega t) \sin(\theta)\]

(2.13)

In case of a periodic motion, the primary force component occurs at the excitation frequency i.e. first harmonic. The effect of higher order harmonics is generally less. Comparing coefficients of trigonometric functions \( \sin(\omega t) \) and \( \cos(\omega t) \) on both sides of Equation (2.13), we get

\[\Rightarrow -(M + M_{33}) A_0 \omega^2 = F_0 \cos(\theta)\]

(2.14)

\[\Rightarrow M_{33} = -\frac{F_0 \cos(\theta)}{A_0 \omega^2} - M\]

(2.15)

\[B_{33} A_0 \omega = F_0 \sin(\theta)\]

(2.16)

\[\Rightarrow B_{33} = \frac{F_0 \sin(\theta)}{A_0 \omega}\]

(2.17)

The added mass \( M_{33} \) and damping \( B_{33} \) obtained here are non-dimensionalised using suitable formulation. Lamb applied potential theory assuming an ideal and irrotational perfect fluid to estimate the added mass of a plate oscillating in an infinite fluid domain, Ref. [13], as given in Equation (1.2)

\[M_a = \frac{8}{3} \rho a^3.\]

(2.18)

\( M_a \) given in Equation (2.18) is used to non-dimensionalised the added mass and damping forces as follows

\[C_a = \frac{M_{33}}{\frac{8}{3} \rho a^3}\]

(2.19)
The phase difference between force and displacement measurements is given by $\theta$. The added mass and damping coefficients are evaluated based on $\cos(\theta)$ and $\sin(\theta)$, any small error can contribute significant error in the inertia and damping force measurements. Troesch used a mechanical system with springs to minimize the error but this complicates the experiments by introducing springs into the system and hence the stiffness, Ref. [58]. In this research porous plate models are forced to oscillate in a rigid system without using any springs as discussed in Section 2.3.

2.3 Description of Experiments

2.3.1 Experimental Setup

The experiments reported here were conducted at the Hydrodynamics Laboratory, School of Mechanical Engineering, University of Western Australia. A schematic of the forced oscillation experimental setup is shown in Figure 2-2. The forced oscillation experiments were conducted to investigate the heave response of deeply submerged porous plates.

![Figure 2-2 Schematic of forced oscillation experiments setup](image)

A plate was attached to the Vertical Planar Motion Mechanism (VPMM) and forced to oscillate in heave as shown in Figure 2-3. A purpose built steel frame was used to support VPMM above the tank as shown in Figure 2-3. The VPMM consists of a rotatory motor and converts the electrical energy into mechanical energy for motion. A gear box was used...
to further convert the rotatory motion into the required linear motion of the rod attached to it. A linear potentiometer was placed parallel to the rod to measure output displacement of the rod in the axial direction. The rod was attached to the plate and a high sensitivity load cell was placed in between the rod and plate to measure axial loads acting on the plate as shown in Figure 2-2.

The load cell was positioned above the plate to avoid bending moments and eccentricity loads. The sinusoidal motion of the actuator was controlled using a purpose designed Labview (Laboratory Virtual Instrumentation Engineering Workbench) program. It enables the user to operate the system at a given frequency and amplitude of oscillation. It was found that the calibration of actuator movement is not linear with respect to input volts when the frequency parameter was varied. Therefore the system was calibrated for individual frequencies.
The flow chart for calculating the hydrodynamic coefficients is depicted in Figure 2-4. Both the linear potentiometer and load cell were calibrated to obtain the displacement and force in SI units. The linear potentiometer was calibrated from its lower limit to upper limit and then in reverse. A similar procedure was applied for load cell calibration by initially increasing loads and then by decreasing loads in steps. Any hysteresis effects were found to be negligible and less than 0.01mm for linear potentiometer and about 1% for load cell.

![Flow chart to obtain hydrodynamic coefficients](image)

Figure 2-4 Flow chart to obtain hydrodynamic coefficients

### 2.3.2 Experimental Models

Five plates of varying porosity were used as test models and their hydrodynamic performances were analysed. The porous plates were of radius ‘$a = 100 \text{ mm}$’ and thickness $2 \text{ mm}$. The plates were made porous by drilling holes of $3 \text{ mm}$ diameter on the plate as shown in Figure 2-5. The hole pattern for each porous plate was designed in such a way that the local porosity of a small area on the plate was the same as the global porosity of the plate. There was another hole of diameter $12 \text{ mm}$ in the middle of each plate to attach to the rod. The rod was attached in the middle of the plate to avoid eccentricity loads and hence to avoid any added error. The holes drilled on the plate were of much
smaller radius than the ones used by Chua et al. and are relatively bigger than the ones used in Tao and Dray’s experimental models, Ref. [46, 52]. But the size of the plates they used is also about 4 times bigger than the ones used here. The small holes allow investigation of the averaged porosity effect on the hydrodynamic behaviour of the plates. The plates of porosity 0%, 5%, 10%, 15%, and 20% were made of stainless steel to avoid rust and are designed to have clean and sharp edges. The plates were tested for their hydrodynamic performance in a glass tank of $1m^3$ at varying $KC$, $\beta$ and $\tau$ parameters. The water depth in the tank was 0.92m and the disk was submerged at a level of 0.45m below the water surface. This condition was considered deeply submerged condition for the disk.

2.3.3 Data Acquisition and post-processing

Agilent DAQ was used to acquire the data from the load cell and the linear potentiometer. U2351A is a 16 channel multifunctional DAQ as shown in Figure 2-6. Only two channels were used to acquire data from the load cell and linear potentiometer. DAQ was connected to a Toshiba laptop equipped with Agilent software via a simple USB connection. The data was acquired at a sampling rate of 1000 Hz in binary format on the laptop. The data was processed using a MATLAB code that automated the mathematical calculations given in Section 2.2. The data was acquired for more than 120 seconds and during the data post processing it was ensured to use the integer number of cycles ($2^n$)
A MATLAB routine was developed to automate the task of post processing of the acquired force and displacement time histories. The routine was capable of reading the acquired data in volts to process. Each data series was truncated to keep only $2^n$ cycles for accurate and fastest FFT calculations. A sensitivity analysis was performed to analyse the influence of using data filters on the value of first harmonic obtained by FFT. The analysis suggested that the application of filters significantly increasing the post processing time but the first harmonic evaluated in each case did not have as much influence. From each time history only 64 cycles long window was analysed and rest of the series was truncated. The truncation was applied on both displacement and force time series while ensuring to keep the properties of phase lag.

### 2.3.4 Experimental Limitations

#### 2.3.4.1 Planer Motion Mechanism Limitations

The actuator was found to have varying limitations on the displacement when the frequency was changed. It was noticed that the limitations are in terms of $Re$ number; hence with increasing $\beta$ parameter the upper limit of $KC$ parameter that could be tested went down and vice versa as shown in Figure 2-6. The region below the green solid line in Figure 2-7 shows the working region of the actuators. This has made limitations on the experimental test matrix given in Table 2-1. For $KC$ and $\beta$ parameters above the green solid line, the amplitude of displacement time series is found to vary at every individual cycle.

<table>
<thead>
<tr>
<th>No.</th>
<th>Plate Submergence</th>
<th>$\beta$</th>
<th>$\tau (%)$</th>
<th>$KC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Deeply submerged</td>
<td>20000-72,000</td>
<td>0</td>
<td>0.05-2.6</td>
</tr>
<tr>
<td>2</td>
<td>Deeply submerged</td>
<td>20000-72,000</td>
<td>5</td>
<td>0.05-2.6</td>
</tr>
<tr>
<td>3</td>
<td>Deeply submerged</td>
<td>20000-72,000</td>
<td>10</td>
<td>0.05-2.6</td>
</tr>
<tr>
<td>4</td>
<td>Deeply submerged</td>
<td>20000-72,000</td>
<td>15</td>
<td>0.05-2.6</td>
</tr>
<tr>
<td>5</td>
<td>Deeply submerged</td>
<td>20000-72,000</td>
<td>20</td>
<td>0.05-2.6</td>
</tr>
</tbody>
</table>
2.3.4.2 Test Tank dimensions and Effectiveness of the damping mats

It was noticed in the experiments that the close proximity of the oscillating disk to the free surface resulted in wave radiation when KC was high. The wave reflection due to the proximity of the wall tanks was possible source of error for the force measurements. In order to reduce the error, the damping mats were installed along four sides of the tank. The damping mats were such that they had approximately 3mm holes to provide it with porosity. The rest of the surface of the mats was covered with bristles of length approximately 6mm. It was observed that the brushes helped to further reduce the wave radiation from tank walls. The disk was oscillated at a constant frequency, different amplitudes and submergences as shown in Table 1. At each submergence, care was taken to limit the amplitude such that the disk did not pierce the free surface at any point in the oscillation cycle. However, at small submergences, considerable free surface disturbance was noticed. Perforated boards of dimensions 0.4 m x 0.3 m were suspended vertically at 20 mm from the tank walls. The boards effectively reduced the reflected waves from reaching the disk, and hence any resultant noise in the load cell data. Figure 2-8 shows a sample measurement of the load cell with and without damping boards. The data obtained with the damping board was cleaner and of comparable quality to data obtained in deeper submergence.
2.4 Results and Discussion

2.4.1 Quality of Experimental data

Figure 2-9 (a) - Figure 2-12 (b) illustrate the force and displacement measurement time histories obtained from potentiometer and load cell at different $KC$ number and $\beta$ parameters. Figure 2-9 (b) - Figure 2-12 (b) shows corresponding amplitude spectra of force and displacement obtained by applying FFT analysis. The displacement time histories were found to be sinusoidal and higher order harmonics are small. The total force measured by the load cell on the other hand has shown non-linear behaviour especially at very low $KC$ and very low $\beta$. The higher order harmonics in force measurements can be due to the higher order harmonics in displacement amplitude spectrum or may be due to the non-linear effects of lower harmonics in force measurements, Ref. [49]. It is found from Figure 2-9 that the higher order harmonics were reduced when $KC$ and $\beta$ parameters were increased, but are relatively prominent in low $KC$ and low $\beta$ cases. This probably could also occur due to the load cell sensitivity for small loads.
Figure 2-9 Force on right hand side axis & displacement on the left hand axis (a) time histories (b) amplitude spectrum for $K_C = 1.9$, $\beta = 40,000$
Figure 2-10 Force on right hand side axis & displacement on the left hand axis (a) time histories (b) amplitude spectrum for $k_c = 0.82$, $\beta = 40,000$.
Figure 2-11 Force on right hand side axis & displacement on the left hand axis (a) time histories (b) amplitude spectrum for $K_C = 0.69$, $\beta = 72,000$
For model tests error analysis is an important step in developing the confidence in obtained results. The force and displacement time histories of a plate, submerged at 0.495 m under the free surface, oscillating at frequency of 1 Hz and various amplitudes were used for this analysis. Various segments of 64 cycles or 32 cycles length were extracted from each time history and were analysed by applying FFT. The first harmonic peak of displacement and force is used in this analysis. The complete set of analysis is performed for 95% confidence interval as well as for 99% confidence interval. The maximum % error in force for up to 0.3 KC number is 12% and for KC higher than 0.3 the maximum percentage error with respect to mean was found to be less than 4%. The maximum error in the displacement is evaluated to be 2.5% for 95% confidence interval.

The error is calculated using the standard statistical formulation as discussed below.
\[ \text{Error} = \text{Mean(Sample)} \pm z\text{value} \times (\text{Std Dev(Sample)}/\sqrt{\text{sample size}}) \]

Number of samples used for the analysis = 6

\( z\text{value} = 1.96 \) for 95% Confidence interval and 2.58 for 99% confidence interval.

Figure 2-13, Figure 2-14, Figure 2-15, Figure 2-16 show the mean value force and displacement for 95% and 99% confidence intervals respectively. The percentage error and the error bars indicate the quantitative error in the added mass and damping calculations. For the relative complexity and limitations of the experimental setup, the error is acceptable and should be accounted for while using the results presented here.
Figure 2-15: Error Bars-Displacement-99% Confidence Interval

Figure 2-16: Error Bars-Force-99% Confidence Interval
2.4.3 Experimental data validation

In order to validate the experimental estimation of hydrodynamic loads, a comparison was made with the published literature, Ref. [47, 52, 59]. This comparison is presented for the case of a solid circular plate submerged at \( S = 2a \) oscillating at a frequency of \( 1 \text{ Hz} \) and at various amplitudes of oscillation. As shown in Figure 2-17 and Figure 2-18, a reasonable agreement was obtained for added mass and damping coefficients. Minor differences with the results published by Tao & Dray (research conducted at Griffith University) may be due to different geometry configuration, i.e. diameter to thickness ratio. The experimental setup used by Tao and Dray was 3 times bigger than what was used to conduct experiment presented here. Similarly, damping also shows the same slope in relation to \( KC \) as given in the published literature, Ref. [47, 52, 59].

![Figure 2-17 Added Mass coefficients of solid plate for \( KC < 2.5 \)](image)

![Figure 2-18 Damping coefficients of solid plate for \( KC < 2.5 \)](image)
2.4.4 Influence of $\beta$ and $KC$ parameters

The data presented here is for a plate oscillating away from the free surface, with water depth $0.9 \, m$ and plate submerged at $4.5 \, m$ below the free surface. Figure 2-19 - Figure 2-23 show the effect of both $KC$ and $\beta$ parameters on added mass coefficients of plates of different porosities. The added mass coefficients were found to be approximately linear with $KC$ parameter for the range of $\beta$ parameter and plate porosities tested. The $\beta$ parameter seems to have a little effect on the added mass coefficients as well, but does not seem to dominate. This indicates the negligible influence of oscillation frequency on deeply submerged oscillating plates. The added mass coefficients of a solid plate approximately tend to unity as $KC \rightarrow 0$ as shown in Figure 2-23. This is consistent with the theoretical value by Lamb, Ref. [13].

Figure 2-11 in Section 2.4.1 shows that the higher order harmonics peak is more than $25\%$ of the first harmonic peak. That suggests a considerable amount of energy spread and hence some error in the hydrodynamic coefficients obtained at $\beta = 20,000$. For other $\beta$ parameters tested, the error in experimental measurements was relatively less and more reliable information was obtained. The added mass coefficients obtained at $\beta = 40,000$ and $\beta = 72,000$ have shown relatively low dependence upon $\beta$ for all the plate porosities tested. The force data obtained at very low $KC$ and $\beta$ for the solid plate was noisy, with a noise to signal ratio greater than 1 and hence these data were discarded.

Figure 2-24 - Figure 2-28 show the damping force coefficients of different porous plates tested for varying $KC$ and $\beta$ parameters. The damping coefficients have shown some ‘systematic displacement’ at $KC$ less than $0.5$ for different $\beta$ parameters tested. The ‘systematic displacement’ begins to diminish for plates of high porosity. The damping coefficients were found to be very weakly dependent upon $\beta$ parameter for $20\%$ porous plate as shown in Figure 2-24, whereas the damping coefficients in Figure 2-27 obtained for plate with $5\%$ porosity have shown the maximum ‘systematic displacement’ at low $KC$ range. Similar trends were noticed by Tao & Dray and they observed that the damping coefficient dependence upon $\beta$ tends to be weaker as porosity of the plate is increased (52).
Figure 2-19 Added mass coefficients variation with $KC$ for 20% porous plate at varying $\beta$

Figure 2-20 Added mass coefficients variation with $KC$ for 15% porous plate at varying $\beta$

Figure 2-21 Added mass coefficients variation with $KC$ for 10% porous plate at varying $\beta$
Figure 2-22 Added mass coefficients variation with $KC$ for 5% porous plate at varying $\beta$

Figure 2-23 Added mass coefficients variation with $KC$ for solid plate at varying $\beta$

Figure 2-24 Damping coefficients variation with $KC$ for 20% porous plate at varying $\beta$
Figure 2-25 Damping coefficients variation with $KC$ for 15% porous plate at varying $\beta$

Figure 2-26 Damping coefficients variation with $KC$ for 10% porous plate at varying $\beta$

Figure 2-27 Damping coefficients variation with $KC$ for 5% porous plate at varying $\beta$
2.4.5 Effect of plate porosity ($\tau$)

The hydrodynamic coefficients presented here are for the plates that are submerged 0.495 m below the free surface, which is 4.95 times the radius of the plate. The free surface disturbance is assumed to be negligible due to plate position being far away from the free surface boundary as well as the bottom of the tank. The behaviour of added mass coefficient as a function of $KC$ is shown in Figure 2-29 - Figure 2-31 for the five plates of porosity 0%, 5%, 10%, 15% and 20%. Each of these figures corresponds to added mass coefficients of porous plates oscillating at frequency parameters $\beta = 72,000$, $\beta = 40,000$ and $\beta = 20,000$ respectively. The added mass coefficient was found to be increasing linearly with $KC$ parameter for all plate porosities tested for their hydrodynamic performance. The added mass coefficient was also found to be decreased in magnitude as plate porosity was increased. This is caused by the pressure drop across the upper and lower side of the plates when the plate porosity was increased.

A linear curve fit approach was used to analyse the data as shown in Figure 2-29 - Figure 2-30 for $\beta = 72,000$ and $\beta = 40,000$. The trend lines were drawn through the data points of added mass coefficients varying with $KC$ parameter for all the plate porosities tested. The information obtained in terms of equations of trend lines is presented in Table 2-2. The added mass coefficient of plates oscillating at $KC = 1$ were found to be dependent upon plate porosity and it was found that the added mass coefficient of a 20% porous plate was approximately 50% of the solid plate added mass coefficient as shown in Table 2-3. The present research indicates that the added mass coefficients are the
maximum for a solid plate and magnitude of added mass coefficients decreases as the plate porosity is increased.

Figure 2-32 - Figure 2-34 show the dimensionless damping coefficients for all the five plates of different porosity and at frequency parameters $\beta = 72,000$, $\beta = 40,000$ and $\beta = 20,000$. For the $KC$ range tested, the damping coefficients were also found to be linearly dependent upon $KC$. This behaviour was noticeable for all the frequency parameters tested and represented in Figure 2-32 - Figure 2-34. The influence of plate porosity on damping coefficients was noticed at lower $KC$ numbers. The influence of plate porosity on the damping coefficient was prominent at low $KC$ as shown in Figure 2-32 - Figure 2-34. This effect diminished as the plates are tested higher $KC$ parameter. The plate porosity influence tended to diminish for $KC > 1.8$ and increasing porosity did not increase the damping coefficients after $KC > 1.8$. The trend of damping coefficients was not directly proportional to plate porosity and the experimental study shows that the damping coefficients of 5% and 10% porous plates were higher than 20% porous plate.

<table>
<thead>
<tr>
<th>$\tau$ (%)</th>
<th>Slope @ $\beta=40,000$</th>
<th>Offset @ $\beta=40,000$</th>
<th>Slope@ $\beta=72,000$</th>
<th>Offset @ $\beta=72,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.45</td>
<td>0.91</td>
<td>0.40</td>
<td>0.92</td>
</tr>
<tr>
<td>5</td>
<td>0.36</td>
<td>0.87</td>
<td>0.38</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>0.44</td>
<td>0.50</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>15</td>
<td>0.41</td>
<td>0.40</td>
<td>0.56</td>
<td>0.36</td>
</tr>
<tr>
<td>20</td>
<td>0.37</td>
<td>0.29</td>
<td>0.54</td>
<td>0.19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tau$ (%)</th>
<th>$Ca$ @ $\beta=72,000$</th>
<th>Relative % decrease in $Ca$ due to $\tau$</th>
<th>$Ca$ @ $\beta=40,000$</th>
<th>Relative % decrease in $Ca$ due to $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.31</td>
<td>0.00</td>
<td>1.35</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>1.20</td>
<td>8.82</td>
<td>1.23</td>
<td>9.05</td>
</tr>
<tr>
<td>10</td>
<td>1.06</td>
<td>19.72</td>
<td>0.94</td>
<td>30.48</td>
</tr>
<tr>
<td>15</td>
<td>0.92</td>
<td>29.92</td>
<td>0.81</td>
<td>40.34</td>
</tr>
<tr>
<td>20</td>
<td>0.73</td>
<td>44.58</td>
<td>0.66</td>
<td>51.15</td>
</tr>
</tbody>
</table>

The $Ca$ curves tend to zero at low $KC$ range for the higher porosities. It indicates significant pressure drop at low $KC$ range due to high porosity. The offsets presented in Table 2-2 for the $Ca$ Vs $KC$ trendlines given in Figure 2-29 - Figure 2-30 can be challenged at the lower $KC$ range due to $Ca$ curve actually bending towards zero. A dedicated set of tests for the low $KC$ range will be useful to acquire detailed knowledge and conclude behaviour of $Ca$ curve at the low $KC$ range for high porosity plates. The definition of low $KC$ range is a function of plate porosity and frequency of oscillation.
Figure 2-29 Variation of added mass coefficients with KC of porous plates at β=72,000

Figure 2-30 Variation of added mass coefficients with KC of porous plates at β=40,000

Figure 2-31 Variation of added mass coefficients with KC of porous plates at β=20,000
Figure 2-32 Variation of damping coefficients with $KC$ of porous plates at $\beta=72,000$

Figure 2-33 Variation of damping coefficients with $KC$ of porous plates at $\beta=40,000$

Figure 2-34 Variation of damping coefficients with $KC$ of porous plates at $\beta=20,000$
2.4.6 Influence of free surface proximity and bottom boundary conditions

A solid plate was also made to oscillate at varying free surface proximities and bottom boundary proximities. The experimental study performed at frequency of 1 Hz has shown influence of the boundary proximities. The added mass and damping coefficients were found to be increasing with increasing $KC$ parameter and plate proximities to the upper and lower fluid boundaries at frequency of 1 Hz. The experimental results were published in conference proceedings and are included in Appendices I and II, Ref. [4, 59]. The detailed discussion on the influence of free surface proximity and frequency parameter on the hydrodynamic coefficients of plates is presented in Chapter 4 using the theoretical development of the method of matched Eigen function expansion.

2.4.7 Summary

The added mass and damping coefficients both have shown dependence upon $KC$ and $\beta$ parameters. The added mass and damping coefficients have shown linear dependence upon the $KC$ parameter and relatively weak dependence upon the $\beta$ parameter. The added mass coefficients are found to decrease with increasing porosity. The increase in plate porosity shows relatively less influence on slope of the added mass coefficients with increasing $KC$ parameter for all plate porosities tested, but the offset is shifted. The added mass coefficient of 20%, 15%, 10%, 5% porous plates are found to be 50%, 40%, 30% and 10% of the added mass coefficient of solid plate oscillating at $KC = 1$ and $\beta = \mathbf{40,000}$. The damping coefficients show dependence upon porosity at small $KC$ parameter. It is found that at higher $KC$ (e.g. > 1.8) the plate porosity does not strongly influence the damping coefficients. The study summarises that the porosity influences the hydrodynamic performance of oscillating plates. Chapter 3 uses a flow visualization technique to understand influence of plate porosity on the fluid flow around the plate edge.
Chapter 3

Particle Image Velocimetry: Effect of plate porosity

“If nature were not beautiful, it would not be worth studying it, 
And life would not be worth living”
–Henry Poincare

The behaviour of hydrodynamic coefficients due to plate porosity enhanced the curiosity to visualize the flow around the plate and hence understand the cause of this behaviour. A flow visualization technique, Particle Image Velocimetry (PIV) was chosen to visualize the flow around a solid and a porous plate. The PIV method was used because it is capable of providing quantitative information along with qualitative information. Quantities including velocity, vorticity and circulation obtained when the plate was oscillated at $KC = 0.63$ and $\beta = 20,000$ are presented in this chapter.

3.1 Introduction

Flow visualization has always played a major role in understanding fluid flows. It is also a common experience that the understanding and interpretation of physical processes are facilitated by visual observations. In fluid mechanics many phenomena have been discovered from observation of flow patterns, recorded in the form of photographs, sketches and more recently in the form of digital images. Many researchers including Leonardo da Vinci, Osborne Reynolds, Ernst Mach, and Ludwig Prandtl have made important discoveries on the basis of flow pattern observations, Ref. [60].
Conventional flow visualization methods involved particle tracking and hence suffered from complications that arise due to tracking individual particles to obtain spatial shift information. An non-intrusive flow visualization technique PIV has been used in numerous fluids experiments i.e. flow around cavitation bubbles, turbulent wake of a cylinder, jets in water, airfoil flows, vortex breakdown in spin-up and Spin down process, Ref. [61, 62, 63, 64, 65].

Many researchers conducted detailed investigation to understand the fluid flow and vortex formation around structures like cylinders and plates, Ref. [66, 67, 68, 56, 15, 16, 49]. For a moving body with high surface curvature like a plate edge, the flow is not able to follow the surface and separates from the plate edge forming a shear layer. For an oscillating plate the shear layer rolls up and forms vortices. These vortices are then convected and diffused with the flow oscillations, Ref. [56] As a result, the oscillating plate experiences complex vortex dynamics that depend upon oscillation amplitude. De Bernardinis noticed that vortices moved from one edge to another and convected at large angles for a body with two edges e.g. cylinders, Ref. [66]. The flow around blunt-shaped structures with no sharp edges will not separate in oscillating fluid motion at low amplitudes of oscillation.

The flow separation does not always develop vortex shedding. The transition from no vortex shedding to vortex shedding should be noticeable as amplitude of oscillation is increased. Thiagarajan studied flow around a plate of uniform thickness and a sharp edge plate that had vanishing thickness at the edges, and showed that the flow was antisymmetric about the mean position of an oscillating plate at high amplitudes of oscillation, Ref. [56]. Vortices were shed from the plate edges and rolled into vortex rings, which were convected and diffused over time. The vortex shedding was observed to be dependent upon the initial start-up position of the plate. The gross features of the flow were periodic and repeatable.

The flow around thin plate edge experienced four vortex formation modes as the $KC$ number increased from 0 to 1.1, Ref. [49]. The $KC$ dependence of the damping coefficients was closely related to the four vortex formation modes and the thickness of the plate affected the vortex flow by changing the transitional $KC$ value. The study also found vortex modes to be independent of frequency in the range of 2Hz to 8Hz. As a plate
moves through its oscillation cycle, say from the bottom dead centre (BDC) to the top dead centre (TDC), the free shear layer on the top surface is shed off the edge, and rolls up into a vortex ring which is propelled initially by the plate movement. When the plate movement is arrested at the TDC, the vortex continues to be propelled by its self-induced velocity, and changes the vortex formation topology of the core vortex formed in the second half of the cycle. Thus only one dominant vortex ring is formed in a cycle, which depends on the starting condition. Such phenomena were observed by others; see for example, Ref. [56]. For an isolated edge in oscillatory flow, the pair of vortices formed during any two half cycles convected away from the edge due to mutual induction. It is found through flow visualization studies that convection was asymmetric for single edge and anti-symmetric for a two edge body, Ref. [67].

In this case, the effects of porosity on vortex roll-up and formation at the edge of a thin plate is of interest. Tao et al. described various contributions to damping of a solid plate. The friction damping is due to the shear forces on the surface of the plate, and the form damping is due to vortex formation and shedding at the outer edge of the plate, Ref. [69]. In the case of a porous plate, porosity will induce an additional contribution to damping. The total damping of a porous plate was then hypothesised by Sireta et al. as the damping of a solid plate plus the additional term i.e. damping due to porosity, Ref. [32]. This however assumes that the friction and form drag for the two structures are similar in magnitude.

For a porous plate oscillating normal to its axis, one may hypothesise that locally the flow would travel from one side of the plate to the other for every half cycle through the orifices. This is on the assumption that hole diameter is large enough to neglect surface tension effects. The local flow firstly neutralizes the pressure differential across the plate and provides the inertial forces that alter the added mass effects. On the contrary, the local flow can cause additional friction and vortex roll-up at scales of orifice diameter. These may contribute to an increase in damping. It is conceivable that these two effects may be of a similar order of magnitude resulting in no significant change in damping due to porosity. Such a conclusion is in accordance with theoretical and experimental force measurements presented in, Ref. [32].

Although flow visualization studies of oscillating structures exist in the literature, relatively less information is available on the porosity of the plates. The present study concentrates
on the effect of plate porosity on the vortex shedding induced by the plate edges. In this chapter, the flow physics of a solid and a 15% porous circular plates oscillating in an infinite fluid domain are studied. These plates have sharp edges and hence vortex formation and shedding are expected to dominate the flow characteristics. Qualitative and quantitative information recorded in the form of images and displacement in a given time interval is presented. The derivative quantities including velocity, vorticity and circulation are presented here for both a solid plate and 15% porous plate to understand the influence of porosity on the flow around the plate.

3.2 Problem formulation

The flow physics was studied using a flow visualization technique called Particle Image Velocimetry (PIV). PIV is capable of producing quantitative and qualitative information of a flow field. In this method the flow is seeded with particles and particles are illuminated by laser flashes. Two images are recorded on two frames at times $t_1$ and $t_1 + \Delta t$ of a particular region of the flow. The images are divided into small sub-domains called interrogation windows. Typical interrogation windows are of size 32 pixels $\times$ 32 pixels. The flow displacement throughout one interrogation window is assumed to be uniform. An averaged spatial shift of particles is observed from one sample image to its counterpart in the other image when the flow is illuminated. The resultant output image is a combination of spatial shift as well as the added noise. The added noise can be due to background noise in the images or due to a few particles leaving the window before the second image was recorded after time $\Delta t$. The presence of noise can complicate the main task of computing spatial shift i.e. displacement in 2-D. A cross-correlation technique is used to determine the average displacement of the particles in the interrogation window. The average displacement and time interval $\Delta t$ are used to evaluate the velocity of the flow in that interrogation region. A schematic is shown in Figure 3-1.
The flow displacement in time $\Delta t$ is obtained by using intensity functions of both images of the interrogation window. Let us assume that the intensity function of the two consecutive images are $I_1(x, y)$ and $I_2(x, y)$. The intensity functions are assumed to be comprised of contributions from the seed particles. If the particles in the second image are the same as those of the first, but only displaced linearly in $x$ and $y$ directions by $\Delta x$ and $\Delta y$ (70), then

$$I_2(x, y) = I_1(x, y) \ast \delta(x - \Delta x, y - \Delta y)$$

(3.1)

where the delta function is defined as

$$\delta(x, y) = 0, x \neq 0, y \neq 0$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta(x, y) \, dx \, dy = 1$$

and the cross-convolution of two functions $I_1(x, y)$ and $I_2(x, y)$ is given by

$$C(X, Y) = I_1(x, y) \ast I_2(x, y)$$

(3.2 - A)

Using Equation (3.1), Equation (3.2-A) can be rewritten as

$$C(X, Y) = I_1(x, y) \ast I_1(x, y) \ast \delta(x - \Delta x, y - \Delta y)$$

(3.2)
Where * denotes the spatial convolution and Equation (3.2) is written as

\[ C(X, Y) = A(x, y) * \delta(x - \Delta x, y - \Delta y) \quad (3.3) \]

\[ C(X, Y) = A(X - \Delta x, Y - \Delta y) \quad (3.4) \]

The autocorrelation function \( A(x, y) \) is defined as the convolution of \( I_1(x, y) \) on itself. Since the maximum of an autocorrelation function occurs at the origin, Ref. [71], the above equation states that the maximum of the cross correlation function occurs at

\[ X = \Delta x, \quad Y = \Delta y \quad (3.5) \]

Thus the displacement of the particles within the interrogation region is given by the location of the cross correlation peak. A high cross correlation is observed when many of the particle images match up with their corresponding spatially shifted partners and small cross correlation peaks occur when individual particle images match up with the other particle images. The flow velocity is evaluated in each window using spatial shift information of each sub-sampled image and time interval as given in the equations

\[ u = \frac{\Delta x}{\Delta t}, \quad (3.6) \]

\[ v = \frac{\Delta y}{\Delta t}. \quad (3.7) \]

where \( u, v \) represent the velocities in \( x \) and \( y \) directions respectively. The overall flow velocity at a point in space is given by \( \vec{V} \). Further detailed information is available in the reference, Ref. [72]. The local flow velocity is obtained over the entire image using the same methodology. To understand the flow characteristics this problem is formulated and discussed in terms of vorticity. The vorticity transportation equation is obtained by taking the curl of the unsteady Navier Stokes equation, Ref. [73, 74, 75, 76].

\[ \frac{\partial \vec{\xi}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{\xi} = (\vec{\xi} \cdot \vec{\nabla})\vec{V} + \nu \nabla^2 \vec{\xi} \quad (3.8) \]

where \( \vec{\xi} \) represents the vorticity.

This equation states that the local rate of change of vorticity is governed by the net difference between convection and generation of vorticity. The vortex stretching is not expected to occur in two-dimensional flows, Ref. [73]. The vorticity strength of a 2-D flow in \( xy \) plane is defined in terms of the velocity by
\[
\hat{\xi} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.
\] (3.9)

The vorticity contours are obtained using this formulation given in Equation (3.9). The strength of the dominant vortex core can also be quantified by its circulation, Ref. [76], defined as

\[
\Gamma = \oint \nabla \cdot \vec{V} \cdot dl
\] (3.10)

### 3.3 Experimental setup:

The schematic of PIV experimental setup is shown in Figure 3-2. Experiments were conducted on solid and porous plates oscillating perpendicular to the plane of the plates. The measuring system consisted of a New Wave Solo 15 Nd:YAG PIV laser, a PCO pixel fly camera with a 1390 x 1024 CCD pixel array, and a 50 mm Nikon lens for image capture Figure 3-4. The exposure time between consecutive images was 1ms. A Labview program was designed in house to control the oscillating motion of the plate. The plate was made 15% porous with 3mm diameter holes uniformly spaced such that the local porosity was equivalent to the global porosity as shown in Figure 3-3. The noise due to any kind of added reflections was minimised by painting both plates in black and covering the overall setup in black to avoid any interference from external source of light.

After a few trials and calculations the interrogation window was chosen, such that it captures the complete vortex formation on the plate and also vertically long enough to capture the lower and upper end vortex formations. The seeding particle size was chosen such that the small particle displacement in a given time duration is on average sufficient to move to next grid. The particles used for PIV imaging were Polyamid 2157 of 57μm diameter. The cross-correlation or auto correlation applications can have direct influence from the interrogation window size, grid size. Small scale motions cannot be discernible if the interrogation window is too large. For interrogation very small interrogation window, it cannot resolve large displacements. For this set of experiments the field of view was set to 97 mm x 72 mm and interrogation grid was chosen to be 32X32 pixels.

The images captured are in 8-bit grey scale (0-255) for each pixel. The camera and laser were synchronised using PIV Sync software such that an image pair was captured for each pair of laser flashes at a given time interval. The flow patterns around both plates were captured at 8 different plate positions within the oscillating cycle. The digital dual images obtained using PCO pixel fly camera were post processed using VidPIV software.
The dual images were imported in the software window and an adaptive cross correlation scheme was used. A linear mapping scheme was used to calibrate the images from pixels to real units in millimetres. The velocity vectors obtained from the dual images and timing information as shown in Figure 3-5 and Figure 3-6 were filtered using a global filter and a local median velocity filter to determine the global or ghost vectors. The ghost vectors shown in blue in Figure 3-6 come into existence when the cross-correlation peaks are very small and were removed using the filters as shown in Figure 3-7.

Figure 3-2 Schematic of Particle Image Velocimetry Experimental Set up
Figure 3-3 A 15% Porous plate

Figure 3-4 Camera and lens used to capture PIV image pairs
A global velocity filter was performed by plotting a scatter plot of the velocity vectors in terms of their magnitude and direction. A scatter plot of the velocity data was found to be a very intuitive method of looking at the distribution of velocities in a vector map. Outliers that had occurred due to identification of noise peaks rather than signal peaks were randomly distributed about the velocity domain. The valid vectors only existed in a limited region of the velocity domain where vectors varied smoothly from one value to other.

The local median filter allowed further determination of non-valid vectors. Empirical evidence has suggested that the median filter is most reliable when there is a possibility of a significant number of non-valid vectors around the test vector. In this local filter only 24 neighbouring vectors were used to calculate median and standard deviation values rather than using all the velocity domain information from the vector map as was the case in global filters.

Vectors in Figure 3-7 were used to interpolate vectors in the filtered sites using the interpolation node in PIVSync. The method used for the interpolation was based on multiple passes of a weighted mean interpolation algorithm. This scheme scanned the
vector map to be interpolated and determined all of the filtered vector sites that had a maximum number of valid neighbour vectors and then performed interpolation of these sites. A number of successive passes were performed using the same analysis till the vector map was completed and are shown in green in Figure 3-8.

Figure 3-8 shows that even though about 8% of the vectors were interpolated in the overall vector field, less than 5% vectors were obtained by interpolation in the main region of interest i.e. vortex core.

![Figure 3-6 Determination of bad or ghost vectors](image)
Figure 3-7 Vector plot after deleting the outliers

Figure 3-8 Vector plot with interpolated vectors
### 3.4 Risk and Safety

For the purpose of managing risk, emergency stop safety buttons were located on the tank. These safety buttons were able to isolate the actuator motion immediately. Since the laser is classified Class 4, there is a high risk of eye injury and burns from the beam and reflections of the beam. The hazards identified in the Table 3-1 have safety control measures in place to mitigate the risk and satisfy the recommendations of UWA Safety and Health Office in achieving laser safety compliance. The author has also undergone a laser safety course and exam before conducting experiments.

**Table 3-1 Laser safety control measures**

<table>
<thead>
<tr>
<th>TASK HAZARD IDENTIFICATION</th>
<th>HAZARD</th>
<th>SAFETY CONTROL MEASURES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure to laser beam</td>
<td>Severe eye damage or skin burns</td>
<td>1. Wear correct laser safety goggles.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Beam path fixed to prevent operator exposure.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. The prescribed black cardboard sheets must always be in position around the tank to</td>
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<tr>
<td></td>
<td></td>
<td>prevent reflections.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Warning sign and flashing light on laboratory door whenever the laser is in use.</td>
</tr>
<tr>
<td>Unsafe or inappropriate use of laser</td>
<td>Injury to operators or equipment failure</td>
<td>1. Operators complete half day UWA Laser Safety Training</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Operators are trained and experienced in the operation and maintenance of the laser</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Laser Safe Working Rules are displayed and followed.</td>
</tr>
<tr>
<td>Motion of actuators</td>
<td>Trapping or impact injuries</td>
<td>1. Access to actuators is restricted by their location above the middle of the tank and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>the positioning of cardboard sheets at the top and sides of the tank as shown in</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Figure 3-9.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Emergency Stop button to isolate actuator motion.</td>
</tr>
</tbody>
</table>
3.5 Results and Discussion

The results shown here correspond to an oscillation amplitude of $20 \text{ mm}$, and frequency of $0.5 \text{ Hz}$. The maximum velocity of the plate ($\omega A$) was approximately $63 \text{ mm/s}$ in both the cases of solid plate as well as porous plate. The maximum velocity of the fluid due to the motion of the plate was found to be approximately $105 \text{ mm/s}$ for the solid plate and $99 \text{ mm/s}$ for the porous plate. Similar findings were also observed by Thiagarajan Ref. [56]. Figure 3-11 shows that the maximum fluid velocity occurs around the mean position of the plate. This may be influenced by the initial position of the plate and depending upon where the strong wake is formed.

As discussed earlier, the free shear layer rolls up into the vortex rings which then convect and diffuse as the flow reverses when the plate completes its oscillatory cycle of motion.
At the end of an oscillation cycle, the maximum roll up results in a large vortex ring, whose cross-section is visualized by PIV. The results showing velocity and vorticity profiles of flow around the solid and porous plates are shown in Figure 3-11 and Figure 3-12. The x-axis and y-axis in Figure 3-11 and Figure 3-12 represent the radial and heave axial directions respectively and are in millimetres and both are presented at same scale for easy comparison. The area of interest consists of 41 mm of the 200 mm plate diameter. The coloured legend on the side of each figure represents the vorticity scale. It is noticeable in these figures that the vortex ring grew and became stronger close to the upper or lower end of the cycle. When the plate changed direction of motion in the oscillatory cycle a pair of vortices was observed below and above the plate. Figure 3-12 differs from Figure 3-11 due to capability of the flow to also pass through the holes in the porous plate during oscillations, Ref. [77]. The porosity leads to the formation of secondary vortices below and above the plate depending upon direction of plate motion and also reduces the strength of the main vortex at the edge due to diffusion Figure 3-12.

Figure 3-10 Position of plate oscillating at A=20mm and f=0.5Hz.

Figure 3-13 and Figure 3-14 show zoomed in vortex regions for the two plates. The variation in vortex strength due to the presence of holes in the porous plate compared to the solid plate was noticeable as shown in Figure 3-13 and Figure 3-14. Using Equation (3.7) and trapezoidal rule of integration, the circulation in the vortex of the solid and porous plates was calculated for varying rectangular contours centred at the vortex core.
The circulation plots in Figure 3-15 show the circulation strength versus the perimeter of the contour which is linearly related to the enclosed area. Once the perimeter of the vortex core is reached, the circulation reaches a mesa region. Beyond this region, the vortex strength may diminish, due to local diffusion. However, the maximum circulation occurring at the mesa region is an indication of the strength of vortex roll-up and directly relates to the convection damping induced by the plate. Figure 3-15 shows that the effect of porosity is to lower the circulation strength by 50% of that of the solid plate, Ref. [78]. This may be used to visualize and explain the effects found in Chapter 2 of this thesis during hydrodynamic force analysis on porous plates. In the case of porous plate apart from the decrease in the amount of circulation and low velocities, the vortex shape appears stretched, which may further contribute to a reduction in convection.

It can be concluded here that the plate porosity affects the fluid flow around the plate and hence gives some insight to the hydrodynamic behaviour of porous plates noticed in Chapter 2. The pressure drop due to plate porosity contributes to decrease in added mass coefficients. The reduced vortex strength around the plate edge and presence of small vortices below and above the plate contribute towards the damping behaviour noticed in Chapter 2. The hydrodynamic problem of oscillating solid and porous plates is formulated analytically using the method of Eigen function expansions in Chapter 4.
Figure 3-11 Velocity (mm/s) and vorticity (s⁻¹) plots for the oscillatory flow due to solid plate motion: A=20 mm, f=0.5 Hz with x and y axis in mms. 8 Images correspond to the eight plate positions as shown in Figure 3-10.
Figure 3-12 Velocity (mm/s) and vorticity (s⁻¹) plots for the oscillatory flow due to porous plate motion: A=20 mm, \( f=0.5 \) Hz with x and y axis in mms. 8 Images correspond to the eight plate positions as shown in Figure 3-10.
Figure 3-13 Zoomed in Vector plots and vorticity contours of vortex around the edge of the solid plate (both axis are in millimetres and vorticity is in s⁻¹).

Figure 3-14 Zoomed in Vector plots and vorticity contours of the vortex around the edge of the 15% porous plate (both axis are in millimetres and vorticity is in s⁻¹).
Figure 3-15 Circulation in the vortex contours formed around the edges of solid and a 15% porous plates
Chapter 4

Flow Physics: Using Potential Flow Theory

“As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality”
- Albert Einstein

The forced oscillation study in Chapter 2 has shown the influence of $K_C$ and $\beta$ parameters on oscillating solid and porous plates. The flow visualization study in Chapter 3 has also shown the influence of plate porosity on the edge vortices and velocity profile around the plate. In this chapter the hydrodynamic performance of the solid and porous plates oscillating in close proximity to the free surface is studied analytically. The case of an oscillating circular plate in an otherwise quiescent fluid is considered. The main parameters studied here are the non-dimensional frequency parameter ($\omega^2 a/g$), Keulegan-Carpenter number ($K_C$), plate porosity ($\tau$) and submergence of the plate ($S$). A theoretical hydrodynamics approach using the method of Eigen function expansion is used to understand the behaviour of the plates with varying porosity, oscillating at different frequencies, amplitudes and submergence depths.

Introduction

For a plate that lies in the free surface, the problem is known as a dock problem and has been studied by researchers, Ref. [79, 80, 81, and 82] and various other researchers. This problem is relatively simple to solve because it can be easily reduced to a Fredholm integral equation of second kind, Ref. [83]. Most of the theoretical and experimental research
work has concentrated on solid plates, Ref. [4, 13, 34, 37, 42, 52, 56, 59, 84, and 85]. Cho and Kim have studied flexible solid plates by decomposing the solution into angular Eigen functions, Ref. [86]. Hassan et al. studied submerged elastic plates using the method of matched Eigen function expansions and discussed the solution of a semi-finite plate, a finite plate using symmetry and a circular plate, Ref. [87]. Iafrati and Korobkin studied hydrodynamic loads on a two-dimensional plate using asymptotic estimates, Ref. [87]. Wadhwa and Thiagarajan extended Lamb’s approach to an oscillating plate by perturbing the solution in terms of small $KC$ parameter. The added mass coefficients obtained here showed variations with $KC$ parameter but were underestimated as compared to experimental results, Ref. [85]. Recently researchers have also expanded theoretical developments to porous plates using theoretical and experimental studies, Ref. [40, 45, 52, 89]. The study presented here uses the method of matched Eigen function expansions to estimate the hydrodynamic coefficients of a solid plate and extends to include plate porosity effects by using various flow discharge models including the ones also published in the literature. The aim is to choose the best suitable discharge model that provides a better agreement with experimental results. Empirical corrections are proposed and analytical development is used to understand the behaviour of the plates with varying porosity, oscillating at different frequencies, amplitudes and submergence depths.

### 4.1 Boundary Value Problem: Solid Plate

The physical problem showing the position of a plate and the fluid domain is sketched in Figure 4-1. A cylindrical co-ordinate system is employed for the mathematical description. The origin of the co-ordinate system is at the bottom of the seabed and the centre of the plate is at $r = 0$. The free surface is at a vertical distance '$h$' and the mean position of the plate is at a distance '$d$' from the sea bottom. The radius of the circular plate is '$a$'. The time dependent velocity potential $\Phi(r, \theta, z, t)$ is used to describe the fluid flow by assuming that the fluid is incompressible and inviscid, as well as irrotational. The velocity potential $\Phi$ satisfies the Laplace’s equation. The oscillating plate problem is linearized in terms of the first harmonic by expressing $\Phi$ as

$$\Phi = Re[\varphi e^{-i\omega t}]$$  \hspace{1cm} (4.1)

where

$\varphi = \varphi(r, \theta, z)$ is a spatial function
$\omega =$ The angular frequency of plate oscillation

Velocity in the heave (z) direction is obtained by taking the derivative of $\varphi$ with respect to $z$ in Equation (4.1) is given by

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \varphi}{\partial z} e^{-i\omega t}$$  \hspace{1cm} (4.2)

The plate motion as given in Equation (2.1) in Chapter 2 can be written as Equation (4.3) below

$$\dot{z}_{plate} = iA e^{-i\omega t} = A e^{-i(\omega t - \pi/2)}$$  \hspace{1cm} (4.3)

The fluid velocity in the normal direction at the plate position is given by

$$\dot{z} = V_n = A \omega e^{-i\omega t}$$  \hspace{1cm} (4.5)

From Equation (4.2) and Equation (4.5), we get

$$\frac{\partial \Phi}{\partial z}_{z=d} \bigg|_{t} = \frac{\partial \varphi}{\partial z}_{z=d} \bigg|_{t} e^{-i\omega t} = A \omega e^{-i\omega t}$$  \hspace{1cm} (4.6)

Therefore the velocity potential can be non-dimensionalised to obtain a unit velocity

$$\frac{\partial \varphi}{\partial z}_{z=d} \bigg|_{t} = 1,$$  \hspace{1cm} (4.8)

by redefining the velocity potential $\Phi$ in terms of $\varphi$ as

$$\Phi = \varphi A \omega e^{-i\omega t}.$$  \hspace{1cm} (4.9)

The Laplace's equation is given by

$$\nabla^2 \Phi = 0.$$  \hspace{1cm} (4.10)

Therefore the spatial function $\varphi$ can also be written as

$$\Rightarrow \nabla^2 \varphi = 0.$$  \hspace{1cm} (4.11)

**Bernoulli’s equation:** For inviscid flows the Navier-Stokes equation is simplified to obtain the Euler's equation. Integrating Euler’s equation yields Bernoulli’s equation, Ref. [76]

$$p = -\rho \frac{\partial \Phi}{\partial t} - \frac{1}{2}(\nabla \Phi)^2 - \rho gz + p_0(t)$$  \hspace{1cm} (4.12)

Equation (4.12) is used to obtain the linearized Bernoulli’s equation, Ref. [90] and is given by

$$p = -\rho \frac{\partial \Phi}{\partial t} - \rho gz + p_0(t)$$  \hspace{1cm} (4.13)

where $p_0(t)$ is the constant and usually taken as gage pressure i.e. zero $p_0(t) = 0$. 

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4.1.1 Assumptions

- The fluid motion is assumed to be irrotational.
- There is no or negligible flow through the seabed porosity.
- Seawater is incompressible and inviscid.
- The plate shape is assumed to be axisymmetric.
- The plate thickness is negligible as compared to the radius of the plate.

4.1.2 Boundary Conditions

Theoretical hydrodynamics is based on the mathematical theories that describe the flow of fluids in prescribed confinements. These confinements are called boundaries in fluid dynamics and the mathematical expressions used to define these confinements are called boundary conditions. The following boundary conditions define the confinements of the given fluid flow:

**Seabed Boundary Condition (SBC):** - The lower boundary of our region of interest is the seabed. The seabed floor is assumed to be impermeable at \( z = 0 \). This implies that the normal component of a fluid particle on a motionless solid surface must be equal to zero and is given mathematically as

\[
\frac{\partial \Phi}{\partial n} = 0 \text{ at } z = 0
\]  

(4.14)

'\( n \)' denotes the unit vector normal to the seabed.

\[
\Rightarrow \frac{\partial \Phi}{\partial z} = 0 \text{ at } z = 0
\]  

(4.15)

Using Equation (4.9) in Equation (4.15), we get

\[
\frac{\partial \Phi}{\partial z} = 0 \text{ at } z = 0
\]  

(4.16)

**Free Surface Boundary Condition (FSBC):** - The free surface boundary condition is defined by the kinematic and the dynamic boundary conditions, Ref. [91].

**Kinematic free surface boundary condition**

The kinematic free surface boundary condition on the free surface explains that a fluid particle initially lying on the free surface will remain on the free surface. That means

\[
z - \eta(r, \theta, t) = \text{constant at } z = h
\]  

(4.17)

\( \eta \) represents the free surface elevation.

\[
\Rightarrow \frac{D(z - \eta(r, \theta, t))}{Dt} = 0 \text{ at } z = h
\]  

(4.18)
Dynamic free surface boundary condition

The dynamic free surface condition on the free surface relies on the assumption that the pressure outside the fluid is constant. This can be expressed mathematically as

\[ \frac{Dp}{Dt} = 0 \quad \text{at } z = h. \]  

(4.19)

Using Equation (4.5) in Equation (4.12), we get

\[ D \left( -\rho \frac{\partial \Phi}{\partial t} - \rho gz + p_0(t) \right) = 0 \quad \text{at } z = h. \]  

(4.20)

A linearized free surface boundary condition given below is obtained by simplifying the kinematic free surface boundary condition and the dynamic free surface boundary condition, Ref. [91].

\[ g \frac{\partial \Phi}{\partial z} + \frac{g^2 \Phi}{2} = 0 \quad \text{at } z = h \]  

(4.21)

Using Equation (4.9) in Equation (4.21), we get

\[ g \frac{\partial \Phi}{\partial z} - \omega^2 \Phi = 0 \quad \text{at } z = h \]  

(4.22)

\textbf{Sommerfeld Radiation Boundary Condition (RBC): -} The fluid oscillations in the radial outward direction diminish at an infinite distance from the source of oscillation and are represented by Sommerfield as the radiation boundary condition, Ref. [92].

\[ \lim_{r \to \infty} (r)^{1/2} \left( \frac{\partial \Phi}{\partial r} - ik \Phi \right) = 0 \]  

(4.23)

Using Equation (4.9) in Equation (4.23), we get

\[ \lim_{r \to \infty} (r)^{1/2} \left( \frac{\partial \Phi}{\partial r} - ik \Phi \right) = 0 \]  

(4.24)

where \( k = \) progressive wave number and is a function of wavelength.

\textbf{Body Boundary Condition (BBC): -} The body boundary condition represents fluid flow velocity on the surface of the body. For a body moving with a normal velocity component \( V_n \) at its surface at the given point, the boundary condition on the surface is given by

\[ \frac{\partial \Phi}{\partial n} = V_n = V_n, \quad z = d \text{ and } r \leq a \]  

(4.25)
Using Equation (4.5), Equation (4.6) and Equation (4.7) in Equation (4.25) we obtain Equation (4.8) that represents the body boundary condition at the plate as

\[ \frac{\partial \varphi}{\partial z} = 1 \text{ at } z = d \text{ and } r \leq a. \]  

(4.8)

### 4.2 Solution: Boundary Value Problem

The plate problem described in Section 4.1 is solved using separation of variables method and the method of matched Eigen function expansions.

#### 4.2.1 General Solution: Laplace Equation

A convenient method of solving linear partial differential equations, called separation of variables is used to obtain the velocity potential expression. In this method it is assumed that the solution can be expressed as a product of terms, each of which is a function of only one independent variable, Ref. [91]. For this fluid problem it can be given as

\[ \varphi(r, z, \theta) = R(r) \times Z(z) \times \Theta(\theta) \]  

(4.26)

The plate is circular and hence axisymmetric. Thus without loss of generality one may assume

\[ \Theta(\theta) = 1 \]  

(4.27)

\[ \Rightarrow \varphi(r, z) = R(r) \times Z(z) \]  

(4.28)

Applying it to Laplace’s equation given in Equation (4.3), we get

\[ \nabla^2 \varphi = \frac{1}{r} \left( Z(z) R'(r) + Z(z) R''(r) + R(r) Z'(z) \right) = 0 \]  

(4.29)

\[ \Rightarrow \frac{1}{r} Z(z) R'(r) + Z(z) R''(r) + R(r) Z'(z) = 0 \]  

(4.30)

\[ \frac{1}{r} \frac{R'(r)}{R(r)} + \frac{R''(r)}{R(r)} = -\frac{Z''(z)}{Z(z)} \]  

(4.31)

Let us assume both sides of the Equation (4.31) are equal to \( \kappa^2 \), where \( \kappa \) can be zero, real or complex number. Thus three possible general solutions of \( \varphi \) obtained from Equation (4.31) depending upon the value of \( \kappa \) being real, complex or zero are

\[ \varphi^{(1)} = (A' J_0(\kappa r) + B Y_0(\kappa r)) \times (D \cosh(\kappa z) + C \sinh(\kappa z)) \]  

(4.32)

\[ \varphi^{(2)} = (E K_0(\kappa r) + F I_0(\kappa r)) \times (G \sin(\kappa z) + H \cos(\kappa z)) \]  

(4.33)

\[ \varphi^{(3)} = (I \ln(r) + J) \times (Mz + L). \]  

(4.34)

where \( A', B, C, D, E, F, G, H, I, J, M, L \) are unknown constants,

\( J_0, Y_0 \) = Bessel functions of the first and second kind,  
\( I_0, K_0 \) = modified Bessel functions of first and second kind and zero order,
\( k \) = wave number.

For a heaving plate only their expansions in terms of zeroth order Bessel functions are adopted, due the fact that the axisymmetric heave motion is independent of \( \Theta \). A linear equation that has a number of solutions, the sum of those solutions is also a general solution of the equation, thus

\[
\Rightarrow \varphi(r, z) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)}
\]

(4.35)

The velocity potential solutions given in Equation (4.32), Equation (4.33), Equation (4.34) and Equation (4.35) may be represented in different forms to satisfy fluid boundary conditions. Detailed derivations are given in Appendix III and are also available in published literature, Ref. [91]. Laplace’s equation is a partial differential equation that has infinitely many solutions. The solutions are chosen with careful analysis to satisfy the boundary conditions and physics of the problem.

4.2.2 Definition of Flow Regions

The flow around the plate is divided into three virtual regions as shown in Figure 4-1. A similar approach was taken by Yu and Chwang for a wave scattering problem, Ref. [42, 43].

Region 1 represents the fluid outside an imaginary cylinder that has a radius equal to that of the plate, and height equivalent to the water depth, i.e.

\( 0 \leq z \leq h \) and \( r > a \)

Region 2 represents the fluid below the plate and contained within the imaginary cylinder of radius \( 'a' \) up to height \( 'd' \), i.e.

\( 0 \leq z \leq d \) and \( r \leq a. \)

Region 3 represents the fluid flow above the plate and contained within the imaginary cylinder of radius \( 'a' \), with the base being at the top of the plate and the top of this region is at the free surface, i.e.

\( d \leq z \leq h \) and \( r \leq a. \)
These three regions around the plate are bounded by different boundary conditions

**Region 1**: Free Surface Boundary Condition (FSBC) at $z = h$

- Summerfield Radiation Boundary Condition (RBC) at $r \to \infty$
- Seabed Boundary Condition (SBC) at $z = 0$

**Region 2**: Seabed Boundary Condition (SBC) at $z = 0$

- Body Boundary Condition (BBC) at $z = d$ and $r \leq a$

**Region 3**: Body Boundary Condition (BBC) at $z = d$ and $r \leq a$

- Free Surface Boundary Condition (FSBC) at $z = h$. 
### 4.2.3 Solid Plate: Flow Regions Solution

The general expressions of velocity potential \( \varphi \), which satisfy the entire boundary conditions on the free surface, at the seabed, on the plate and radial boundary condition in three virtual regions are given below.

In region 1 the general solution is obtained using separation of variables and was chosen such that it satisfies boundary conditions on the free surface, seabed boundary as well as the radiation boundary condition. The velocity potential in region 1 is given by

\[
\varphi_1(r, z) = \left[ P_1 H_0^{(1)}(k_0 r) + Q_1 H_0^{(2)}(k_0 r) \right] \left[ D_1 \cosh(k_0 z) + C_1 \sinh(k_0 z) \right] \\
+ \sum_{n=1}^{\infty} \left[ E_{1,n} K_0(k_n r) + F_{1,n} I_0(k_n r) \right] \left[ G_{1,n} \sin(k_n z) + H_{1,n} \cos(k_n z) \right] \\
+ \left[ I_1 \ln(r) + f_1 \right] \left[ M_1 z + L_1 \right]
\]

(4.36)

where \( P_1, Q_1, D_1, C_1, E_{1,n}, F_{1,n}, G_{1,n}, H_{1,n}, I_1, J_1, M_1, L_1 \) are unknown constants,

\( k_0, k_n = \) wave numbers,

\( I_0, K_0 = \) modified Bessel functions of first and second kind and zero order,

\( H_0^{(1)}, H_0^{(2)} = \) Hankel functions of first and second kind and zero order.

In region 2 the general solution was chosen in such a way that it satisfies the boundary condition on plate surface and seabed boundary. The solution in region 2 is given by

\[
\varphi_2(r, z) = \left[ A_2 J_0(\lambda_0 r) + B_2 Y_0(\lambda_0 r) \right] \left[ D_2 \cosh(\lambda_0 z) + C_2 \sinh(\lambda_0 z) \right] + \frac{T_2}{d} \left( z^2 - \frac{r^2}{2} \right) \\
+ \sum_{n=1}^{\infty} \left[ E_{2,n} K_0(\lambda_n r) + F_{2,n} I_0(\lambda_n r) \right] \left[ G_{2,n} \sin(\lambda_n z) + H_{2,n} \cos(\lambda_n z) \right].
\]

(4.37)

where \( A_2, B_2, D_2, C_2, E_{2,n}, F_{2,n}, G_{2,n}, H_{2,n}, T_2 \) are unknown constants,

\( J_0, Y_0 = \) Bessel functions of the first and second kind,

\( I_0, K_0 = \) modified Bessel functions of first and second kind and zero order,

\( \lambda_0, \lambda_n = \) wave numbers.

The second term in Equation (4.37) has been chosen because it satisfies the boundary conditions in region 2.

In region 3 the general solution of velocity potential was chosen such that it can satisfy the boundary conditions at the free surface and plate surface. The velocity potential in region 3 is given by
\[ \varphi_3(r,z) = [A_3J_0(\mu_0 r) + B_3Y_0(\mu_0 r)][D_3\cosh(\mu_0(z - d)) + C_3\sinh(\mu_0(z - d))] \\
+ \sum_{n=1}^{\infty} [E_{3,n}K_0(\mu_n r) + F_{3,n}I_0(\mu_n r)][G_{3,n}\sin(\mu_n(z - d)) + H_{3,n}\cos(\mu_n(z - d))] \\
+ [I_3\ln(r) + J_3][M_3(z - d) + L_3] \]  
(4.38)

where \( A_3, B_3, D_3, C_3, E_{3,n}, F_{3,n}, G_{3,n}, H_{3,n}, I_3, J_3, M_3, L_3 \) are unknowns,
\( J_0, Y_0 = \) Bessel functions of the first and second kind,
\( I_0, K_0 = \) modified Bessel functions of first and second kind and zero order,
\( \mu_0, \mu_n = \) wave numbers.

The last term in Equation (4.38) has been chosen because it satisfies the boundary conditions in region 3.

**Region 1:** Applying the bottom boundary condition (BBC) on \( \varphi_1 \) given by Equation (4.36) we get
\[ C_1 = 0, \ C_{1,n} = 0 \text{ and } M_1 = 0 \]  
(4.39)

Using Equation (4.39) in Equation (4.36), reduces the equation to
\[ \varphi_1(r,z) = \left[ P_1H_0^{(1)}(k_0 r) + Q_1H_0^{(2)}(k_0 r) \right][D_1\cosh(k_0 z)] + L_1[I_1\ln(r) + J_1] \\
+ \sum_{n=1}^{\infty} [E_{1,n}K_0(k_n r) + F_{1,n}I_0(k_n r)][H_{1,n}\cos(k_n z)]. \]  
(4.40)

The free surface boundary condition in Equation (4.22) solves Equation (4.40) and results in dispersion relations given by the following two equations
\[ gk_0 \tanh(k_0 h) = \omega^2 \]  
(4.41)
\[ -gk_n \tan(k_n h) = \omega^2 \]  
(4.42)

and also evaluates the constants
\[ I_1 = 0 \text{ and } J_1 = 0. \]  
(4.43)

Equation (4.40) is further reduced due to the Sommerfield radiation boundary condition and is given by
\[ \varphi_1(r,z) = (P_1 \ast D_1) H_0^{(1)}(k_0 r)\cosh(k_0 z) + \sum_{n=1}^{\infty} \{(E_{1,n} \ast H_{1,n})K_0(k_n r)\cos(k_n z). \} \]  
(4.44)

To simplify further analysis and make sure the unknown constants are dimensionless, the unknown variables in region 1 are redefined as
\[ P_1 * D_1 = \frac{a R_{1,0}}{\cosh(k_0 h)H_0(k_0 a)} \] (4.45)

\[ E_{1,n} * H_{1,n} = \frac{a R_{1,n}}{K_0(k_n a)} \] (4.46)

where \( a \) is the radius of the plate.

Hence the expression for \( \varphi_1 \) in region 1 can be written as

\[ \varphi_1(r, z) = \frac{R_{1,0} a H_0^{(1)}(k_0 r) \cosh(k_0 z)}{\cosh(k_0 h)H_0(k_0 a)} + \sum_{n=1}^{\infty} \left\{ \frac{R_{1,n} a K_0(k_n r) \cos(k_n z)}{K_0(k_n a)} \right\} \] (4.47)

where \( R_{1,0}, R_{1,1}, R_{1,2, \ldots} R_{1,n} \) are the constants and the set of these constants can be represented in the vectorial form as \( \bar{R}_1 \).

**Region 2:** The behaviour of the Bessel functions \( Y_0 \) and \( K_0 \) tend towards infinity at the origin, Ref. [44]. Hence the constants \( B_2 \) and \( E_{2,n} \) in Equation (4.31) must be zero for the solution to be finite at \( r = 0 \), i.e.

\[ B_2 = 0 \quad \text{and} \quad E_{2,n} = 0 \] (4.48)

\[ \Rightarrow \varphi_2(r, z) \]

\[ = [A_2 J_0(\lambda_0 r)][D_2 \cosh(\lambda_0 z) + C_2 \sinh(\lambda_0 z)] + \frac{T_2}{d} \left( z^2 - \frac{r^2}{2} \right) \]

\[ + \sum_{n=1}^{\infty} [F_{2,n} I_0(\lambda_n r)][G_{2,n} \sin(\lambda_n z) + H_{2,n} \cos(\lambda_n z)] \] (4.49)

The seabed boundary condition (SBC) further reduces the equation to

\[ \varphi_2(r, z) \]

\[ = [A_2 J_0(\lambda_0 r)][D_2 \cosh(\lambda_0 z)] + \frac{T_2}{d} \left( z^2 - \frac{r^2}{2} \right) \]

\[ + \sum_{n=1}^{\infty} [F_{2,n} I_0(\lambda_n r)][G_{2,n} \sin(\lambda_n z)] \] (4.50)

\[ \therefore C_2 = 0 \quad \text{and} \quad G_{2,n} = 0 \] (4.51)

Applying the body boundary condition (BBC) on Equation (4.50) gives us the values of unknown wave numbers and constant as

\[ \lambda_n = \frac{n \pi}{d} \quad \text{where} \quad n = 1, 2, 3 \ldots \] (4.52)

\[ T_2 = \frac{1}{2} \] (4.53)
To simplify further analysis and make sure the unknown constants are dimensionless, the unknown variables in region 2 are redefined as

\[ A_2 \ast D_2 = aR_{2,0} \quad (4.54) \]

\[ F_{2,n} \ast H_{2,n} = \frac{a R_{2,n}}{I_0(\lambda_{n}a)} \quad (4.55) \]

where \( a \) is the radius of the plate.

\( R_{2,0}, R_{2,1}, R_{2,2}, \ldots, R_{2,n} \) are the constants and the set of these constants can be represented in the vectorial form as \( \mathbf{R}_2 \).

Hence the expression for velocity potential of virtual region 2 (\( \phi_2 \)) can be written as

\[ \phi_2(r,z) = aR_{2,0} + \frac{1}{2d} \left(z^2 - \frac{r^2}{2}\right) + \sum_{n=1}^{\infty} \left\{ aR_{2,n} \frac{I_0(\lambda_{n}r)}{I_0(\lambda_{n}a)} \cos(\lambda_{n}z) \right\} \quad (4.56) \]

**Region 3:** Similar to region 2, the behaviour of the Bessel functions \( Y_0 \) and \( K_0 \) is infinite at the origin, Ref. [44]. Hence the constants \( B_3 \) and \( E_{3,n} \) must vanish for the solution to be finite at \( r = 0 \), i.e.

\[ B_3 = 0 \text{ and } E_{3,n} = 0 \quad (4.57) \]

\[ \phi_3(r,z) = A_3I_0(\mu_0r)D_3 \cosh(\mu_0(z - d)) + C_3 \sinh(\mu_0(z - d))] + [I_3J_3 + J_3][M_3(z - d) + L_3] + \sum_{n=1}^{\infty} [F_{3,n}I_0(\mu_nr)] [G_{3,n}\sin(\mu_n(z - d)) + H_{3,n}\cos(\mu_n(z - d))] \quad (4.58) \]

Further applying body boundary condition (BBC) gives

\[ C_3 = 0, G_{3,n} = 0, I_{3,n} = 0 \text{ and } M_3 \times J_3 = 1, \quad (4.59) \]

and velocity potential for region 3 becomes

\[ \phi_3(r,z) = [A_3I_0(\mu_0r)][D_3 \cosh(\mu_0(z - d))] + J_3[M_3(z - d) + L_3] + \sum_{n=1}^{\infty} [F_{3,n}I_0(\mu_nr)][H_{3,n}\cos(\mu_n(z - d))] \quad (4.60) \]

The free surface boundary condition in region 3 gives dispersion relations for this region and hence the wave numbers are determined by

\[ g\mu_0 \tanh(\mu_0(h - d)) = \omega^2 \quad (4.61) \]

\[ -g\mu_n \tan(\mu_n(h - d)) = \omega^2 \quad (4.62) \]

Hence the velocity potential equation for region 3 reduces to
\[ \varphi_3(r,z) = z - h + \frac{g}{\omega^2} + A_3 \cdot D_3 J_0(\mu_0 r) \cosh(\mu_0(z - d)) \]
\[ + \sum_{n=1}^{\infty} \left\{ F_{3,n} \cdot H_{3,n} I_0(\mu_n r) \cos(\mu_n(z - d)) \right\} \quad (4.63) \]

To simplify further analysis and make sure the unknown constants are dimensionless, the unknown variables in region 3 are redefined as

\[ A_3 \cdot D_3 = \frac{a R_{3,0}}{\text{cosh}(\mu_0(z - d))} \quad (4.64) \]
\[ F_{3,n} \cdot H_{3,n} = \frac{a R_{3,n}}{I_0(\mu_n a)} \quad (4.65) \]

where \( a \) is the radius of the plate.

where 'a' is the radius of the plate and \( R_{3,0}, R_{3,1}, R_{3,2}, \ldots, R_{3,n} \) are the constants and can be represented in the vectorial form as \( \vec{R}_3 \).

Thus the velocity potential for virtual region 3 (\( \varphi_3 \)) can be written as

\[ \varphi_3(r,z) = z - h + \frac{g}{\omega^2} + \frac{a R_{3,0}}{\text{cosh}(\mu_0(h - d))} J_0(\mu_0 r) \cosh(\mu_0(z - d)) \]
\[ + \sum_{n=1}^{\infty} \left\{ \frac{a R_{3,n}}{I_0(\mu_n a)} I_0(\mu_n r) \cos(\mu_n(z - d)) \right\} \quad (4.66) \]

### 4.2.4 Matching Velocity Potentials at the Virtual Boundaries:

The expressions of velocity potential in three different regions must match at \( r = a \) to ensure the continuity of fluid flow. This also enables the unknown constants in Equation (4.47), Equation (4.56) and Equation (4.66) to be found. Matching the velocity potentials at \( r = a \), we get

\[ \varphi_1 = \varphi_3 \quad \text{at} \quad r = a \quad \text{and} \quad 0 \leq z \leq d \quad (4.67) \]
\[ \varphi_2 = \varphi_3 \quad \text{at} \quad r = a \quad \text{and} \quad d \leq z \leq h \quad (4.68) \]

Equation (4.61) gives

\[ R_{1,0} \frac{\cosh(k_0 z)}{\cosh(k_0 h)} + \sum_{n=1}^{n_1} R_{1,n} \cos k_n z = \frac{1}{2ad} \left( z^2 - \frac{a^2}{2} \right) + R_{2,0} + \sum_{n=1}^{n_2} R_{2,n} \cos \lambda_n z \quad (4.69) \]

Integrating Equation (4.69) with respect to \( z \) from 0 to \( d \), we get the following
Similarly matching potentials from Equation (4.74) and Equation (4.72) can be written together in the vectorial form as

\[
\bar{R}_{2,n} = \bar{C}_{21} + R_{2,0} \bar{R}_1.
\]  

(4.73)

Equation (4.70) and Equation (4.72) can be written together in the vectorial form as

\[
\bar{R}_2 = \bar{C}_{21} + R_{2,0} \bar{R}_1.
\]

(4.74)

Similarly matching potentials from region 3 and region 1 and using Equation (4.68) at \( r = a \) gives

\[
\frac{1}{a} \left( z - h + \frac{g}{\omega z} \right) + R_{3,0} \frac{\cosh(\mu_0(z - d))}{\cosh(\mu_0(h - d))} J_0(\mu_0 a) + \sum_{n=1}^{n_3} \{R_{3,n} \cos(\mu_n(z - d))\}
\]

\[
= R_{1,0} \frac{\cosh(k_0 z)}{\cosh(k_0 h)} + \sum_{n=1}^{n_1} R_{1,n} \cos k_n z.
\]

(4.74)

Multiplying each side of Equation (4.74) with \( \cosh(\mu_0(z - d)) \) and integrating with respect to \( z \) from 0 to \( d \), we get
\[ R_{3,0} \frac{I_0(\mu_0 a)}{\cosh(\mu_0 (h - d))} \left( \frac{h - d}{2} + \frac{\sinh(2\mu_0 (h - d))}{4\mu_0} \right) \]

\[ = R_{1,0} \int_0^d \cosh(k_0 z) \cosh(\mu_0 (z - d)) \, dz \]

\[ + \sum_{n=1}^{n_1} R_{1,n} \int_0^d \cos(k_n z) \cosh(\mu_0 (z - d)) \, dz \]

(4.75)

Now using the orthogonality of Cosine functions, we multiply Equation (4.68) with

\[ \{P_j\} = \{\cos\mu_1(z - d), \cos\mu_2(z - d), \cos\mu_3(z - d), \ldots \} \]

(4.76)

and integrating with respect to \( z \) from 0 to \( d \), we get

\[ R_{3,n} \left( \frac{h - d}{2} + \frac{\sinh(2\mu_n (h - d))}{4\mu_n} \right) \]

\[ = \frac{R_{1,0}}{\cosh(k_0 h)} \int_0^d \cosh(k_0 z) \cosh(\mu_n (z - d)) \, dz \]

\[ + \sum_{n=1}^{n_1} R_{1,n} \int_0^d \cos(k_n z) \cosh(\mu_n (z - d)) \, dz + \frac{1}{\mu_n^2} \]

(4.77)

Equations (4.76) and Equation (4.78) can be written in the vectorial form as

\[ \vec{R}_3 = \vec{C}_1 + \vec{R}_1 \vec{R}_3 \vec{R}_1 \]

(4.78)

where

\[ \vec{R}_3 = \vec{R}_{3,0}, \vec{R}_{3,1}, \vec{R}_{3,2}, \ldots, \vec{R}_{3,n} \]

\[ \vec{R}_1 = \vec{R}_{1,0}, \vec{R}_{1,1}, \vec{R}_{1,2}, \ldots, \vec{R}_{1,n} \]

\[ \vec{C}_1 \] = constant terms in Equation (4.76) and Equation (4.78).

\[ \vec{R}_1 \vec{R}_3 \] = constant terms in Equation (4.76) and Equation (4.78) and are coefficients of vector \( \vec{R}_1 \).

4.2.5 **Matching Radial Derivatives of Velocity Potentials at \( r = a \)**

Radial velocities are matched at \( r = a \) to ensure the continuity of flow at \( r = a \) as follows:

\[ \frac{\partial \varphi_1}{\partial r} \bigg|_0^h = \frac{\partial \varphi_2}{\partial r} \bigg|_0^d + \frac{\partial \varphi_3}{\partial r} \bigg|_0^h \text{ at } r = a \]

(4.79)

The set

\[ \{U_j\} = \{\cosh(k_0 z), \cos(k_1 z), \cos(k_2 z), \cos(k_3 z), \cos(k_4 z), \ldots\} \]

(4.80)

forms a complete set of orthogonal functions.

Multiplying Equation (4.79) with \( \{U_j\} \) gives the following equations:
\[ R_{1,0} \left( \frac{h}{2} + \frac{\sin(2hk_0)}{4k_0} \right) \left( \frac{k_0}{\cosh(k_0h)} \right) \]

\[ = \sum_{n=1}^{n^2} \lambda_n R_{2,n} \int_0^d \cos(\lambda_n z) \cosh(k_0z) \, dz \]

\[ + \mu_0 R_{3,0} \frac{J_0(\mu_0 r)}{\cosh(\mu_0(h - d))} \int_d^h \cos(\mu_0(z - d)) \cosh(k_0z) \, dz \]

\[ + \sum_{n=1}^{n^3} \mu_n R_{3,n} \int_0^d \cos(\mu_n(z - d)) \cosh(k_0z) \, dz - \frac{\sinh(dk_0)}{2dk_0} \] (4.81)

and

\[ \sum_{n=1}^{n^1} R_{1,n}^* k_n \left( \frac{h}{2} + \frac{\sin(2hk_n)}{4k_n} \right) \]

\[ = \sum_{n=1}^{n^2} \lambda_n R_{2,n}^* \int_0^d \cos(\lambda_n z) \cos(k_nz) \, dz \]

\[ + \mu_0 R_{3,0} \frac{J_0'(\mu_0 r)}{\cosh(\mu_0(h - d))} \int_d^h \cos(\mu_0(z - d)) \cos(k_nz) \, dz \]

\[ + \sum_{n=1}^{n^3} \mu_n R_{3,n}^* \int_0^d \cos(\mu_n(z - d)) \cos(k_nz) \, dz - \frac{\sin(2dk_n)}{2dk_n} \] (4.82)

Equations (4.81) and Equation (4.82) can also be put together in a vectorial form as

\[ \overline{R}_1 = \overline{C} + R_1 \overline{R}_2 + R_1 R_3 \overline{R}_3 \] (4.83)

where

\[ \overline{R}_3 = \overline{R}_{3,0}, \overline{R}_{3,1}, \overline{R}_{3,2}, \overline{R}_{3,3}, \ldots, \overline{R}_{3,n}. \]

\[ \overline{R}_2 = \overline{R}_{2,0}, \overline{R}_{2,1}, \overline{R}_{2,2}, \overline{R}_{2,3}, \ldots, \overline{R}_{2,n}. \]

\[ \overline{R}_1 = \overline{R}_{1,0}, \overline{R}_{1,1}, \overline{R}_{1,2}, \overline{R}_{1,3}, \ldots, \overline{R}_{1,n}. \]

\[ \overline{C}_{31} \] = constant terms in Equation (4.81) and Equation (4.82)

\[ \overline{R}_1 R_3 \] = constant terms in Equation (4.81) and Equation (4.82) and are coefficients of vector \( \overline{R}_1 \).

\[ \overline{R}_1 R_2 \] = constant terms in Equations (4.81) and Equation (4.82) and are coefficients of vector \( \overline{R}_1 \).

Using vectors \( \overline{R}_2 \) and \( \overline{R}_3 \) from Equation (4.73) and Equation (4.78) in Equation (4.83), gives us a set of linear equations to be solved to obtain the constants in vector \( \overline{R}_1 \) and hence \( \overline{R}_2 \) and \( \overline{R}_3 \).
The system of linear Equations (4.73), (4.78) and (4.83) in the vectorial form are solved using the Gaussian elimination scheme and hence the unknown coefficients are obtained for all the three velocity potential expressions. The velocity potentials above and below the plate i.e. $\Phi_2$ and $\Phi_3$ are used to obtain the hydrodynamic behaviour of the plate.

### 4.2.6 Added Mass and damping

The pressure difference above and below the plate is then obtained using Bernoulli’s Equation (4.13) at $z = d$ as

$$p_2 - p_3 = -\rho \left( \frac{\partial \Phi_2}{\partial t} - \frac{\partial \Phi_3}{\partial t} \right) - \rho g (d - d + t')$$  \hspace{1cm} (4.84)

where $t'$ is the thickness of the plate and is assumed to be negligible.

$$p_2 - p_3 = -\rho \left( \frac{\partial \Phi_2}{\partial t} - \frac{\partial \Phi_3}{\partial t} \right)$$  \hspace{1cm} (4.85)

The hydrodynamic forces given in Equation (2.10) in Chapter 2 are determined here by integration of the pressure on the submerged plate surface.

$$F_{\text{heave}} = \iint p \mathbf{n} \, dS$$  \hspace{1cm} (4.86)

Heave added mass $M_{33}$ and damping $B_{33}$ of the plate located at $z = d$ in the complex form is given by the integral of difference in potentials at $z = d$ with respect to $r$ from $0$ to $a$ and then integrated with respect to $\theta$ from $0$ to $2\pi$ as given below.

$$M_{33} = \text{Real} \left( \rho \int_0^a \int_0^\theta (\varphi_2(r,d) - \varphi_3(r,d)) \, r \, dr \right)$$  \hspace{1cm} (4.87)

$$B_{33} = \text{Imaginary} \left( \rho \omega \int_0^a \int_0^\theta (\varphi_2(r,d) - \varphi_3(r,d)) \, r \, dr \right)$$  \hspace{1cm} (4.88)

The added mass ($C_a$) and damping ($C_b$) coefficients are obtained from Equation (4.87) and Equation (4.88) and are non-dimensionalised as given in Equation (2.18) and Equation (2.19) in Chapter 2

$$C_a = \frac{M_{33}}{\frac{8}{3} \rho a^3}$$  \hspace{1cm} (4.89)

$$C_b = \frac{B_{33}}{\frac{8}{3} \omega \rho a^3}$$  \hspace{1cm} (4.90)
4.2.7 Convergence Study

The infinite series in the velocity potentials were truncated at \( n = 100 \) for all the three regions. A convergence study was performed to optimize the length of the series required to obtain hydrodynamic coefficients. It was found that the solution converges asymptotically for increasing \( n \) as shown in Figure 4-2. The percentage difference in added mass coefficients was less than 1\% once the number of terms in the series was truncated at 50. Further increasing the number of terms in the series up to 100 reduced the error to less than 0.14\%. Increasing the number of terms any further did not contribute significantly towards error reduction, but contributed towards the more computation. Hence the total number of terms in the series was truncated at 100 for the results shown here.

![Figure 4-2](image_url)

Figure 4-2 The percentage difference in added mass coefficients for a deeply submerged plate with increasing number of terms in the series.

4.2.8 Validation

A MATLAB routine was written to automate the calculations given in the above sections. The method and the MATLAB computer program were validated here. The input parameters \( h, d, a \), were chosen such that \( a \) was much smaller than \( h \) and \( h - d \). This was done to ensure that a comparison of added mass coefficients obtained here can be made with published literature for plate oscillating in an infinite fluid domain. Figure 4-3 shows only slight difference of the present results with added mass coefficients obtained by Lamb theoretically, Ref. [13]. For a plate deeply submerged in water, the damping
coefficients obtained using potential theory should be negligible. It is because potential theory does not account for edge vortices and assumes the flow to be irrotational. Therefore the damping coefficients of a deeply submerged solid plate were found negligible.

![Graph showing comparison of results obtained with added mass coefficients obtained by (Lamb, 1932)](image)

Figure 4-3 Comparison of results obtained with added mass coefficients obtained by (Lamb, 1932)

### 4.3 Boundary Value Problem: Porous Plate

The hydrodynamic problem of an oscillating plate was made further complex by introducing porosity ($\tau$) to the plate. The plate porosity allows the fluid to discharge through the plate surface in the axial direction and results in a pressure drop that needs to be modelled.

#### 4.3.1 Flow Discharge Models

The discharge of fluid through a porous plate is initially studied by applying three methods as given below:

- Conservation of momentum model,
- locally averaged flow,
- Jet velocity profile through an orifice,

The aim is to choose the best suitable discharge model representing the physics of the porous plate problem and matches with experimental data for added mass and damping coefficients.
Conservation of Momentum Model

A control volume can be finite or infinitesimal volume that is fixed in space in an inertial frame. We consider a small control volume that is assumed to represent the porous plate. The velocity of the oscillating plate is given by \( V = \omega A \cos(\omega t) \). We can consider the flow as moving with relative velocity \( V = -\mathbf{v}_1 \) entering a control volume around the plate as shown in Figure 4-4. The flow coming out of the plate of porosity \( \tau \) is diffused into the ambient. Thus the flow through the plate is modified in a diffuser form. The orifice losses are neglected here and all the losses are due to momentum diffusion in the diffuser.

![Figure 4-4 Schematics of a control volume of plate](image)

\( A_1 \) is the area on top the plate from which the flow can pass through with a velocity \( \mathbf{v}_1 \) and \( A_2 \) is the area from which flow comes out with a velocity \( \mathbf{v}_2 \), then

\[
A_1 = \tau A_2. \tag{4.91}
\]

Now the area of a plate of radius ‘a’ is given by

\[
A_2 = \pi a^2 \tag{4.92}
\]

and hence

\[
A_1 = \tau \pi a^2. \tag{4.93}
\]

Now by the principle of continuity (93)

\[
v_1 A_1 = v_2 A_2 \tag{4.94}
\]

\[
\Rightarrow v_2 = \tau v_1 \tag{4.95}
\]

Momentum is defined as the product of mass of the fluid and velocity of the fluid. The conservation of momentum is a fundamental concept of physics and states that within a problem domain the momentum is neither created nor destroyed, but only changes.
through action of forces. Momentum is a vector quantity and preserves directional information.

Momentum considerations across the volume are shown in Figure 4-5.

\[ M_{in} = \text{momentum flux in} = \rho v_1 |v_1| A_1 \]

\[ M_{out} = \text{momentum flux out} \rho v_2 |v_2| A_2 \]

\[ F = \text{force on the plate} = (P_1 - P_2)(A_2 - A_1) \]

where \( \rho \) is the density of fluid flow

Now applying conservation of momentum we get

\[ P_1A_1 + P_1(A_2 - A_1) - P_2A_2 - (P_1 - P_2)(A_2 - A_1) = \rho v_1 |v_1| A_1 - \rho v_2 |v_2| A_2 \]

\[ \Rightarrow \Delta P (A_2 - A_2 + A_1) = \rho (v_1 |v_1| A_1 - v_2 |v_2| A_2) \]

\[ \Delta PA_1 = \rho (v_1 |v_1| A_1 - v_2 |v_2| A_2) \] (4.97)

Using Equation (4.92), Equation (4.93) and Equation (4.95) in Equation (4.97) we get

\[ \tau \pi a^2 \Delta P = \rho v_1 |v_1| a^2 \pi (r - r^2) \] (4.98)

\[ \Delta P = \rho v_1 |v_1| (1 - r) \] (4.99)

\[ \frac{\Delta P}{\frac{1}{2} \rho v_1 |v_1|} = 2(1 - r) = f_{CM}(r) \] (4.100)

Using Equation (4.97), the pressure differential below and above the plate can be written as

\[ \Delta P = \left( \frac{1}{2} \rho v_1 |v_1| \right) f_{CM}(r) \] (4.101)
**Locally averaged flow**

Following the porous boundary condition due to pressure drop across the plate, Ref. [40, 94], a porous surface is assumed to consist of small openings with sharp edges, so that flow separates and passes through the openings. The small openings on the plate result in pressure drop across the horizontal plate. A number of theoretical and numerical studies have been performed on sloshing problem of tanks and truncated liquid dampers with porous screens, Ref. [95]. A quadratic pressure loss condition is used on porous screens, Ref. [96, 97, 98]. The resultant pressure drop is proportional to the square of flow velocity though the holes on the plate is given below

\[
\Delta P = \frac{1}{2\kappa} \rho v_2 |v_2| \quad (4.102)
\]

The square of the velocity is written as \(v_2 |v_2|\) to preserve the directional information from the velocity vector. \(\kappa\) is the discharge coefficient and is dependent upon the hole shape and the Reynolds number. Reynolds number is given in Equation (1.6) in Chapter 1 as \(Re = \beta * K\).

In the case of steady flows, a typical value of \(\kappa\) can be taken as \(0.5 < \kappa < 1\) , Ref. [40, 89, 96]. It is assumed that the relation between pressure drop and velocity can be written in an averaged sense. In this case the flow will be assumed to pass through a number of openings locally over the plate i.e. the size of each opening and spacing between surrounded openings is much smaller than the size of the plate

\[d_h \ll 2a\]
\[S' \ll 2a\]

where \(d_h\) is the diameter of the opening, \(a\) is the radius of the plate and \(S'\) is the spacing between two consecutive openings on the plate.

Now due to the averaged porosity introduced on the plate, the velocity of the flow coming out of the plate is given by

\[\tau v_2 = v_1 \quad (4.103)\]

The pressure drop \(\Delta P\) is only applied on the solid parts of the plate and quadratic pressure drop \(\Delta p\) due to plate porosity can be written as
\[ \Delta p = \Delta P (1 - \tau) \]  
(4.104)

Hence we can write the pressure drop given in Equation (4.102) as

\[ \frac{\Delta p}{1 - \tau} = \frac{1}{2 \kappa} \rho \frac{v_1}{1} \left| \frac{v_1}{\tau} \right| \]  
(4.105)

\[ \Delta p = \frac{1 - \tau}{2 \kappa T^2} \rho v_1 |v_1| \]  
(4.106)

Molin estimated discharge coefficient \( \kappa \) to be generally taken between 0.5 and 1, Ref. [40]. Faltinsen et al estimated is based on the solidity ratio (which was reported to be between 0.328 and 0.963) of the flat slat screen and was estimated using the formulation given below in, Ref. [96]. The discharge-coefficient is noticed to be monotonically dependent upon increasing solidity ratio, Ref. [96]. The discharge coefficient of 0.51 was found to be most suitable on comparing the analytical results obtained here with experimental measures.

Further detailed discussion on the quadratic pressure loss is also given in reference, Ref. [96]. The pressure drop obtained in Equation (4.106) can be written as

\[ \frac{\Delta p}{1 - \tau} = \frac{1 - \tau}{\kappa T^2} = f_{LAF}(\tau) \]  
(4.107)

Hence the pressure drop obtained assuming locally averaged flow is given by

\[ \Delta p = \left\{ \frac{1}{2} \rho v_1 |v_1| \right\} f_{LAF}(\tau) \]  
(4.108)

**Jet Velocity Profile**

Consider the flow from the orifice at the plate that shoots out like a free jet with a flow rate as given in, Ref. [99]. The \( f_{JV}(\tau) \) is obtained as

\[ f_{JV}(\tau) = \frac{\Delta p}{\frac{1}{2} \rho v_1 |v_1|} = 4(\tau g(\tau))^2 - 1 \]  
(4.109)

Where

\[ g(\tau) = \frac{\sqrt{\tau}}{20.8 \tan(\theta)(1 - \sqrt{\tau})} \tanh \left( \frac{20.8 \tan(\theta)}{\sqrt{\tau} - \tau} \right) - \frac{(1 - \sqrt{\tau})^2}{23 \tau} \ln \left[ \cosh \left( \frac{20.8 \tan(\theta)}{\sqrt{\tau} - \tau} \right) \right] \]

Hence the pressure drop obtained assuming jet velocity profile is given by

\[ \Delta p = \left\{ \frac{1}{2} \rho v_1 |v_1| \right\} f_{JV}(\tau) \]  
(4.110)
The expression in Equation (4.107) shows that the discharge model due to plate porosity cannot be used for the case where plate porosity is zero.

The detailed mathematical expressions are given in Appendix IV.

Equation (4.101), Equation (4.108) and Equation (4.110) can be represented as \( f(\tau) \) as shown in Table 4-1 and given in Equation (4.111) below.

\[
\Delta p = \left( \frac{1}{2} \rho v|v| \right) f(\tau) \tag{4.111}
\]

<table>
<thead>
<tr>
<th>S. No</th>
<th>Method</th>
<th>( f(\tau) )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conservation of momentum model</td>
<td>( f_{CM}(\tau) )</td>
<td>2(1 - ( \tau ))</td>
</tr>
<tr>
<td>2</td>
<td>Locally averaged flow</td>
<td>( f_{LAF}(\tau) )</td>
<td>( \frac{1 - \tau}{\kappa\tau^2} )</td>
</tr>
<tr>
<td>3</td>
<td>Jet velocity profile through an orifice</td>
<td>( f_{JVP}(\tau) )</td>
<td>((4(\tau g(\tau))^2 - 1))</td>
</tr>
</tbody>
</table>

It is noticeable from Figure 4-6 that the \( f(\tau) \) has shown major variation with porosity in the case where plate porosity is modelled assuming locally averaged flow given by Equation (4.101). The \( f(\tau) \) is used in further calculation to represent porosity dependence and to ensure the hydrodynamic forces were evaluated in terms of \( f(\tau) \), which is defined above in Table 4-1. Figure 4-6 indicates that \( f_{LAF}(\tau) \) results in higher pressure drop with increasing plate porosity as compared with other two functions of porosity. The Equation (4.107) shows that \( f_{LAF}(\tau) \) tends to infinity if the plate porosity is reduced to zero. Therefore this model cannot be used for predicting hydrodynamic forces on plates of porosity zero.
4.3.2 Velocity Potentials in three Virtual Regions

From Figure 4-1 with a porous plate, it is clear that only velocity potentials in virtual region 2 and region 3 can be affected due to plate porosity.

**Region 1** - In the case of a porous plate, the velocity potential in region 1 remains the same as the one obtained for solid plate in Equation (4.47) because it is independent of the body boundary condition. Rewriting the velocity potential in dimensional form as $\Phi = \varphi e^{-i\omega t}$ to account for pressure drop and variation in velocity due to porosity and hence is given by

$$\varphi_1(r, z) = A\omega \left( \frac{R_{1,0} H_0^{(1)}(k_0 r) \cosh(k_0 z)}{\cosh(k_0 h) H_0(k_0 a)} + \sum_{n=1}^{\infty} \left\{ \frac{R_{1,n} K_0(k_n r) \cos(k_n z)}{K_0(k_n a)} \right\} \right) \tag{4.112}$$

**Body Boundary condition (BBC) on porous plate**: Let us apply the principle of conservation of mass i.e. the mass in a system can neither be created nor be destroyed. Therefore the velocities below and above the plate of negligible thickness can be given by

$$\frac{\partial \varphi_2(r, d)}{\partial z} = \frac{\partial \varphi_3(r, d)}{\partial z} = \frac{\partial \varphi(r, d)}{\partial z}. \tag{4.113}$$

The pressure differential and the difference between potentials below and above the plate diminish at the plate edge i.e. $r = a$ and $z = d$

$$\varphi_2(a, d) = \varphi_3(a, d) \tag{4.114}$$

and heave velocity is zero at the plate edge i.e. $r = a$ and $z = d$. 

Figure 4-6 Behaviour different porosity models with varying $\tau$. 

[Graph showing behavior of different porosity models with varying $\tau$.]
The velocity potential for a porous plate can be written as sum of velocity potential due to solid plate \( \varphi_s \) and velocity potential due to plate porosity \( \varphi_p \). Hence the dimensional velocity can also be obtained by taking the derivatives.

\[
\frac{\partial \varphi}{\partial z} = \frac{\partial \varphi_p}{\partial z} + \frac{\partial \varphi_s}{\partial z} \tag{4.115}
\]

\[
\frac{\partial \varphi(r, d)}{\partial z} = A\omega + \frac{\partial \varphi_p(r, d)}{\partial z} \tag{4.116}
\]

Using the information in the above equations, heave velocity at \( \mathbf{r} < a \) can be expanded in terms of summations of infinite series of zeroth order Bessel functions due to the axisymmetry of plate, Ref. [40], which have the property of orthogonality and velocity potential due to plate porosity is given by

\[
\varphi_p(r, z) = \frac{A\omega}{v_i} \sum_{l=1}^{\infty} \left[ A_{4,l} J_0(v_i r) * B_{4,l} \cosh(v_i (z - d)) + C_{4,l} \sinh(v_i (z - d)) \right] \tag{4.118}
\]

\[
\frac{\partial \varphi_p(r, d)}{\partial z} = A\omega \sum_{l=1}^{\infty} A_{4,l} * B_{4,l} J_0(v_i r). \tag{4.119}
\]

Redefining the constant \( A_{4,l} * B_{4,l} = R_{4,l} \), we get

\[
\frac{\partial \varphi_p(r, d)}{\partial z} = A\omega \sum_{l=1}^{\infty} R_{4,l} J_0(v_i r). \tag{4.120}
\]

Using Equation (4.116) and Equation (4.120), we get

\[
\Rightarrow \frac{\partial \varphi(r, d)}{\partial z} = A\omega + A\omega \sum_{l=1}^{\infty} R_{4,l} J_0(v_i r). \tag{4.121}
\]

where

\( R_{4,l} \) = represents a set of constants,

\( A \) = amplitude of oscillation of plate,

\( \omega \) = angular oscillation frequency,

\( J_0 \) = Bessel function of first kind,

\( v_i \) = the waves numbers and given by zeroes of \( J_0(v_i r) = 0 \).

This makes orthogonal basis for the plate.
Region 2: The flow below the porous plate is also affected by the flow velocity on the body of the porous plate and is obtained by applying the body boundary condition for porous plate, the seabed boundary condition. Now applying these boundary conditions similarly as is done to obtain Equation (4.56) we get
\[
\varphi_2(r, z) = A\omega R_{2,0} + \frac{A\omega}{2d} \left(z^2 - \frac{r^2}{2}\right) + \sum_{n=1}^{\infty} \left\{ A\omega R_{2,n} \frac{I_0(\lambda_n r)}{I_0(\lambda_n a)} \cos(\lambda_n z) \right\}
\]
\[
+ A\omega \sum_{l=1}^{n^4} \frac{R_{2,0} \cosh(v_l z)}{v_l \sinh(v_l d)} J_0(v_l r)
\]
\[
\text{(4.122)}
\]
and body boundary condition also yields \( J_0(v_l r) = 0 \)
where
\[
R_{2,0}, R_{2,n}, R_{4,l} \text{ represent unknown constants,}
\]
\( J_0, I_0 = \text{Bessel function and modified Bessel function of first kind} \)
\( v_l = \text{the zeroes of } J_0(v_l r) = 0. \)

The velocity potential for flow above the porous plate i.e. in region 3 is a combination of velocity potential of solid plate in Region 3 (i.e. Equation (4.66)) and contribution from velocity on the body of the porous plate as given in Equation (4.122).
\[ \varphi_2(\text{porous}) = \varphi_2(\text{solid}) + \varphi_2 (\text{due to plate porosity}) \]

Region 3: Due to the plate porosity introduced here Equation (4.120) becomes the Body Boundary Condition (BBC) for the porous plate instead of the one used in Equation (4.8) for a solid plate. Now applying this boundary condition for a porous plate and free surface boundary condition, we get the velocity potential in Region 3 written as
\[
\varphi_3(r, z) = A\omega \left(z - h + \frac{g}{\omega^2}\right) + \frac{A\omega R_{3,0}}{\cosh(\mu_0(h - d))} J_0(\mu_0 r) \cosh(\mu_0(z - d))
\]
\[
+ \sum_{n=1}^{\infty} \left\{ \frac{A\omega a R_{3,n}}{I_0(\mu_n a)} \frac{J_0(\mu_n r)}{\cos(\mu_n(z - d))} \right\}
\]
\[
- A\omega \sum_{l=1}^{n^4} \frac{R_{5,l}}{v_l} \left[ K_l \cosh(v_l (z - d)) + \sinh(v_l (z - d)) \right] J_0(v_l r)
\]
\[
\text{(4.123)}
\]
\( I_0 = \text{modified Bessel function of first kind,} \)
\( R_{3,0}, R_{3,n}, R_{5,l} \text{ represent unknown constants,} \)
\( \nu_i \) are the zeroes of \( J_0(\nu_i r) = 0 \),

and \( K_1 \) has come into existence due to the free surface boundary condition and is given by

\[
K_1 = \frac{\omega^2 \sinh \nu_i (h - d) - g \nu_i \cosh \nu_i (h - d)}{g \nu_i \sinh \nu_i (h - d) - \omega^2 \cosh \nu_i (h - d)}.
\] (4.124)

The velocity potential for flow above the porous plate i.e. in region 3 is a combination of velocity potential of solid plate in Region 3 (i.e. Equation (4.66)) and contribution from velocity on the body of the porous plate as given in Equation (4.124).

\[ \phi_3(\text{porous}) = \phi_3(\text{solid}) + \phi_3(\text{due to plate porosity}) \]

### 4.3.3 Pressure Drop: Bernoulli’s Equation

Pressure difference due to the plate porosity in terms of velocity potential can be written using linearized Bernoulli’s equation in Equation (4.13)

\[
p_2 - p_3 = -\rho \frac{\partial (\Phi_2 - \Phi_3)}{\partial t}
\] (4.125)

Using Equation (4.111) in Equation (4.125), we get

\[
\rho \frac{1}{2} \nu |\nu| f(\tau) = -\rho \frac{\partial (\Phi_2 - \Phi_3)}{\partial t}
\] (4.126)

where \( \nu \) represent the velocity due to plate porosity at the plate i.e. \( z = d \).

\[
\nu = \frac{\partial \Phi}{\partial z} - A \omega \Re(e^{-i\omega t})
\] (4.127)

From the dimensional velocity potential, we get

\[
\frac{\partial \Phi}{\partial z} = \frac{\partial \varphi}{\partial z} \Re(e^{-i\omega t})
\] (4.9)

Now using Equation (4.9) and Equation (4.126) in Equation (4.127), we get

\[
\rho \frac{1}{2} \left( \frac{\partial \Phi}{\partial z} - A \omega \Re(e^{-i\omega t}) \right) \frac{\partial \Phi}{\partial z} - A \omega \Re(e^{-i\omega t}) f(\tau) = p_2 - p_3
\] (4.128)

now

\[
\phi_2 - \phi_3 = \frac{-1}{\rho} \int (p_2 - p_3) \, dt
\] (4.129)

Using Equation (4.128) in Equation (4.120), we get

\[
\Rightarrow \phi_2 - \phi_3 = -\frac{i}{2\omega \pi} \int f(\tau) \left( \frac{\partial \varphi}{\partial z} - A \omega \right) \left| \frac{\partial \varphi}{\partial z} - A \omega \right|
\] (4.130)
The problem that arises from the non-linear terms in the equation is tackled by using an iterative procedure where the expressions of velocity potential in three different regions must match at \( (4.113) \) and \( (4.114) \) are found. The velocity potentials and their radial derivatives are matched at

Now using Equations \((4.122)\) and \((4.123)\) in Equation \((4.130)\), we get

\[
aR_{2,0} + \frac{1}{2a} \left( z^2 - \frac{r^2}{2} \right) + \sum_{n=1}^{\infty} \left\{ aR_{2,n} \frac{l_0(\lambda_n r)}{l_0(\lambda_n a)} \cos(\lambda_n z) \right\} + \sum_{i=1}^{n_4} \frac{D_i}{v_i} \coth(v_i d) J_0(v_i r)
- z + h - \frac{g}{\omega^2} + \frac{aR_{3,0}}{\cosh(\mu_0 (h - d))} J_0(\mu_0 r) + \sum_{n=1}^{\infty} \left\{ \frac{aR_{3,n}}{l_0(\mu_n a)} \right\} \sum_{n=1}^{\infty} \frac{aR_{3,n}}{l_0(\mu_n a)} \left. l_0(\mu_n r) \right| \\
- \sum_{i=1}^{n_4} \frac{R_{4,i}}{v_i} J_0(v_i r) = -\frac{8}{23\pi} f(r) \left( \frac{\partial \varphi}{\partial z} - A\omega \right) \left| \frac{\partial \varphi}{\partial z} - A\omega \right| \quad \text{(4.131)}
\]

Using Equation \((4.121)\) in Equation \((4.131)\), we get

\[
aR_{2,0} + \frac{1}{2a} \left( z^2 - \frac{r^2}{2} \right) + \sum_{n=1}^{\infty} \left\{ aR_{2,n} \frac{l_0(\lambda_n r)}{l_0(\lambda_n a)} \cos(\lambda_n z) \right\} + \sum_{i=1}^{n_4} \frac{R_{4,i}}{v_i} \coth(v_i d) J_0(v_i r)
- z + h - \frac{g}{\omega^2} + \frac{aR_{3,0}}{\cosh(\mu_0 (h - d))} J_0(\mu_0 r) + \sum_{n=1}^{\infty} \left\{ \frac{aR_{3,n}}{l_0(\mu_n a)} \right\} \sum_{n=1}^{\infty} \frac{aR_{3,n}}{l_0(\mu_n a)} \left. l_0(\mu_n r) \right| \\
- \sum_{i=1}^{n_4} \frac{R_{4,i}}{v_i} J_0(v_i r) = -\frac{8}{23\pi} A f(r) (R_{4,i} J_0(v_i r)) |R_{4,i} J_0(v_i r)| \quad \text{(4.132)}
\]

The problem that arises from the non-linear terms in the equation is tackled by using an iterative procedure where \( |R_{4,i} J_0(v_i r)| \) has to be evaluated first from the iteration that was done before. This ensures the equation can be solved linearly without loss of the physics.

### 4.3.4 Matching Velocity Potentials at the Virtual Boundaries:

The expressions of velocity potential in three different regions must match at \( r = a \) to ensure the continuity of fluid flow and so that the unknown constants in Equations \((4.110)\), \((4.113)\) and \((4.114)\) are found. The velocity potentials and their radial derivatives are matched at \( r = a \) as given in Equation \((4.67)\) and Equation \((4.68)\).

\[
\varphi_1 = \varphi_3 \quad \text{at} \ r = a \ \text{and} \ 0 \leq z \leq d \quad \text{(4.67)}
\]

\[
\varphi_2 = \varphi_3 \quad \text{at} \ r = a \ \text{and} \ d \leq z \leq h \quad \text{(4.68)}
\]

The pressure drop term due to plate porosity becomes nil at \( r = a \) and hence the matching of velocity potentials remain the same as discussed in Section 4.2.4 and from Equation \((4.67)\) we get

\[
\overline{R}_3 = \overline{C}_{31} + R_1 R_3 \overline{R}_1 \quad \text{(4.73)}
\]

and from Equation \((4.68)\) we get

\[
\overline{R}_2 = \overline{C}_{21} + R_1 R_2 \overline{R}_1 \quad \text{(4.78)}
\]
4.3.5 **Matching Radial Derivatives of Velocity Potentials at \( r = a \)**

Radial velocities are matched at \( r = a \) to ensure the continuity of flow at \( r = a \) as given in Equation (4.79):

\[
\frac{\partial \varphi_1}{\partial r} \bigg|_0^h = \frac{\partial \varphi_2}{\partial r} \bigg|_0^d + \frac{\partial \varphi_3}{\partial r} \bigg|_0^h \quad \text{at } r = a
\]  

(4.79)

Even though the terms due to porosity become nil at \( r = a \), their derivatives do not necessarily go to zero and hence contribute towards the radial derivatives of potential in region 2 and region 3.

The set

\[
\{U_j\} = \{\cosh(k_0 z), \cos(k_1 z), \cos(k_2 z), \cos(k_3 z), \cos(k_4 z), \ldots\}
\]

(4.80)

forms a complete set of orthogonal functions.

Multiplying equation (4.73) with \( \{U_j\} \) and integrating gives the following equations (100):

\[
\begin{align*}
R_{1,0} & \left( \frac{h}{2} + \frac{\sinh(2k_0)}{4k_0} \right) \left( \frac{k_0}{\cosh(k_0 h)} \right) \\
& = \sum_{n=1}^{n^2} \lambda_n R_{2,n} \int_0^d \cos(\lambda_n z) \cosh(k_0 z) \, dz \\
& + \mu_0 R_{3,0} \frac{J_0(\mu_0 r)}{\cosh(\mu_0 (h - d))} \int_0^h \cosh(\mu_0 (z - d)) \cosh(k_0 z) \, dz \\
& + \sum_{n=1}^{n^3} \mu_n R_{3,n} \int_0^d \cos(\mu_n (z - d)) \cosh(k_0 z) \, dz - \frac{\sinh(\mu_0 h)}{2d_0 k_0} \\
& - \sum_{l=1}^{n^4} \frac{R_{4,l}}{v_l} J_0(v_l r) \int_0^h \left( K_l \cosh(v_l (z - d) + \sinh(v_l (z - d)) \cosh(k_0 z) \, dz \\
& + \sum_{l=1}^{n^4} \frac{R_{4,l}}{av_l \sinh(v_l d)} \int_0^h \cosh(v_l z) \cosh(k_0 z) \, dz
\end{align*}
\]

(4.133)
\[
\sum_{n=1}^{n_1} R_{1,n} k_n \left( \frac{h}{2} + \sin\left(\frac{2h k_n}{4k_n}\right) \right)
\]
\[
= \sum_{n=1}^{n_2} \lambda_n R_{2,n} \int_0^d \cos(\lambda_n z) \cos(k_n z) \, dz
\]
\[
+ \mu_0 R_{3,0} \frac{f'_0(\mu_0 r)}{\cosh(\mu_0 (h - d))} \int_d^h \cosh(\mu_0 (z - d)) \cos(k_n z) \, dz
\]
\[
+ \sum_{n=1}^{n_3} \mu_n R_{3,n} \int_0^d \cos(\mu_n (z - d)) \cos(k_n z) \, dz - \frac{\sin(d k_n)}{2 d k_n}
\]
\[
- \sum_{l=1}^{n_4} \frac{R_{4,l}}{\nu_l} f_0(\nu_l r) \int_d^h \left\{ K_l \cosh(\nu_l (z - d)) + \sinh(\nu_l (z - d)) \right\} \cosh(k_n z) \, dz
\]
\[
+ \sum_{l=1}^{n_4} \frac{R_{4,l}}{\nu_l} \int_0^d f(\nu_l r) \sinh(\nu_l d) \cosh(k_n z) \, dz
\]
(4.134)

Equations (4.133) and Equation (4.134) can also be put together in a vectorial form as

\[
\bar{R}_1 = \bar{C}_1 + R1R2. \bar{R}_2 + R1R3. \bar{R}_3 + R1R4. \bar{R}_4
\]
(4.135)

Using vectors \( \bar{R}_2 \) and \( \bar{R}_3 \) from Equations (4.73) and Equation (4.78) in Equation (4.135) and that gives us a set of linear equations to be solved to obtain the constants in vector \( \bar{R}_1 \) and hence \( \bar{R}_2 \) and \( \bar{R}_3 \).

The system of linear Equations (4.73), Equation (4.78) and Equation (4.135) in the vectorial form are solved using the Gaussian elimination scheme and hence the unknown coefficients are obtained for all three velocity potential expressions. The velocity potentials above and below the plate i.e. \( \varphi \) and \( \varphi \) are used to obtain the hydrodynamic behaviour of the plate as given in Section 4.2.6.

The discharge coefficient \( \kappa \) is chosen as 0.5 in \( f_{LAF}(\tau) \) because of its better agreement with experimental hydrodynamic findings. It is found that hydrodynamic coefficients obtained using the \( f_{LAF}(\tau) \) fitted better with experimental hydrodynamics results. Figure 4-6 also indicates that this model performs better as compared to other porosity models tested and hence agreement with experimental results is only expected in case when \( f(\tau) = f_{LAF}(\tau) \).

Hence the hydrodynamic coefficients presented in the Section 4.4 are all obtained using Equation (4.108) by incorporating \( f(\tau) = f_{LAF}(\tau) \).

### 4.3.6 Convergence study

The infinite number of terms in the series of velocity potentials in 3 virtual regions was tested for a range of number of terms as shown in Figure 4-7. A convergence study was
performed to optimize the length of the series required to obtain hydrodynamic coefficients. It was found that the solution converged at about ‘\( n = 100 \)’ and ‘\( l = 20 \)’. The percentage difference in added mass coefficients is less than 0.5% once the number of terms in the series is truncated at 50 or more.

4.3.7 Matching velocity potentials and radial velocity potentials

The matching of velocity potentials and their radial derivatives performed in Section 4.3.4 and Section 4.3.5 is verified here by using the evaluated constants back into the potentials and their radial derivatives. Figure 4-8 - Figure 4-11 represent the matching of real and imaginary velocity potentials, and their radial derivatives for a porous plate of ‘\( \tau = 20\% \)’ submerged at ‘\( h = 0.9 \, m \)’, and ‘\( d = 0.45 \, m \)’ and radius ‘\( a = 0.1 \, m \)’. These figures also represent the achieved level of accuracy. The small amplitude vibrations in matching of radial derivatives can be due to Gibb’s effect, Ref. [101].
Figure 4-8 Matching of real velocity potentials for a plate oscillating at KC = 1.6 and frequency = 0.2 Hz.

Figure 4-9 Matching of imaginary velocity potentials for a plate oscillating at KC = 1.6 and frequency = 0.2 Hz.

Figure 4-10 Matching of real radial velocities for a plate oscillating at KC = 1.6 and frequency = 0.2 Hz.
Figure 4-11 Matching of imaginary radial velocities for a plate oscillating at KC= 1.6 and frequency = 0.2 Hz.

4.4 Comparison of Experimental and Analytical Results

This section concentrates on comparing experimental results obtained using forced oscillation method in Chapter 2 and analytical results obtained using the method of matched Eigen function expansions in this chapter. The comparison includes two main subsections

- Solid Plate-Free surface Proximity
- Deeply Submerged Plate-Plate Porosity

4.4.1 Solid Plate-Free surface Proximity

Martin and Farina studied radiation of water waves due to an oscillating submerged plate using Fredholm integral equations and found that the added mass coefficients can vary widely depending upon the submergence level of the oscillating plate and frequency of oscillation, Ref. [83]. The added mass coefficients were also found to be negative at a range of $\omega^2a/g = 0.4 - 1.6$. The added mass coefficient was the maximum at $\omega^2a/g = 0.35$ approximately, when plate was in very close proximity to free surface.

The added mass coefficient obtained for a solid plate as $KC \to 0$ are presented in Figure 4-12 and Figure 4-13. These are obtained using experimental and analytical techniques respectively. The added mass coefficients have shown a non-linear dependence upon non-dimensional frequency parameter. From the experimental data available here it is found that the magnitude of the added mass coefficient is higher at $\omega^2a/g = 0.4$ for each submergence level and lower at $\omega^2a/g = 1.3$. The nonlinear trends due to the frequency of oscillation suggest that the added mass coefficients cannot be interpolated for
other values of $\omega^2a/g$ parameter. The data available here and from the published literature, show that the maximum of added mass coefficient may lie close to $\omega^2a/g = 0.4$. Similar trend were obtained in the analytical study shown in Figure 4-13. The differences in added mass coefficients obtained analytically and experimentally were relatively small for small $\omega^2a/g$ parameter and were very high for $\omega^2a/g = 1.3$. The comparison in added mass coefficients of solid plate deeply submerged in water as $KC \to 0$ are given in Figure 4-12 and Figure 4-13.

![Figure 4-12](image1)

**Figure 4-12** Experimentally obtained added mass coefficients of Solid plate as $KC \to 0$

![Figure 4-13](image2)

**Figure 4-13** Analytically obtained added mass coefficients of Solid plate as $KC \to 0$

The differences in added mass coefficients obtained using experimental technique and the matched Eigen function expansion method are presented in Table 4-2. The small
differences in added mass coefficients at $\omega^2a/g = 0.1$ and $0.4$ can be due to different radiation boundary conditions in the analytical and the experimental study. The analytical study assumes infinite fluid domain in radial direction of oscillating plate, whereas in experimental setup the plates were oscillated in a relatively confined fluid domain. The added mass coefficients presented in Figure 4-12 and Figure 4-13 are for plates oscillating at very low amplitude i.e. $KC \to 0$, hence negligible contribution from the tank boundaries is expected. The tank walls in the experimental study were also made porous by installing the damping mats with uniformly distributed circular holes on the mat surface. The orifice diameter is about 3mm and damping mats were placed at a distance of 20mm from the vertical tank walls. The difference between experimental and analytically obtained added mass coefficients was found to be very large at $\omega^2a/g = 1.3$ and vary in cases where the plate was in very close proximity to the free surface. These differences may be due to more error at high frequency of oscillation resulting from confinements of the experimental tank. The added mass coefficients were found to non-linearly depend upon $\omega^2a/g$ parameter when the plates were submerged at a close proximity to free surface.

Wadhwa and Thiagarajan conducted experiments on a solid plate oscillating at 1Hz frequency and various free surface proximities and have found that added mass and damping coefficients increase with plate proximity to the free surface at the oscillation frequency of 1Hz, Ref. [4]. Wadhwa et al. found from experimental data that proximity to the bottom also influences the added mass and damping coefficients, Ref. [59]. The coefficients were found to be varying with proximity to free surface or bottom boundaries. Sireta et al. empirically estimated damping coefficients and were given by approximately $0.5(KC)$ using Lake et al. experimental data, Ref. [32, 37]. This empirical correction predicts higher values of damping coefficients as compared to the one given in Equation (4.138). This may be due to the bottom proximity of the plate, because the empirical correction to the damping coefficients proposed by Sireta was for a plate oscillating in relatively shallow water and in close proximity to bottom boundary, Ref. [48].
4.4.2 Porous Plate - Deeply Submerged

The experimental study in Chapter 2, Chapter 3 and the analytical study in the previous sections have shown that the porosity affects the flow around the plate and the overall hydrodynamic performance of the structure. For a deeply submerged plate, the hydrodynamic forces are weakly dependent upon frequency of oscillation i.e. $\beta$ parameter and are linearly dependent upon the amplitude of oscillation i.e. $KC$ parameter, when deeply submerged in water. Figure 4-14 shows the $KC$ dependence of added mass coefficients of a solid plate obtained using experimental technique, whereas analytically obtained added mass is independent of $KC$ parameter. A trend line was drawn through experimental added mass coefficients and based on the equation of trend line, it is proposed that the analytical added mass coefficient for a solid plate can also be written as

$$C_a(\text{corrected}) \approx C_a(\text{analytical}) + \frac{4}{8} \frac{\rho a^2}{\rho a^3} (KC)$$

$$\Rightarrow C_a(\text{corrected}) \approx C_a(\text{analytical}) + 0.5(KC)$$

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<th>Test Set #</th>
<th>$s/a$</th>
<th>$f$</th>
<th>$\omega^2 a/g$</th>
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<th>Relative Difference (%)</th>
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<td>-0.28</td>
<td>-0.22</td>
<td>19.52</td>
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Table 4-2: Comparison of added mass coefficients obtained experimentally and analytically for a solid plate
The experimentally obtained added mass coefficients of a solid plate were slightly less than 1 as \( C \rightarrow 0 \), a similar trend is found in published experimental studies, Ref. [4, 52]. The \( KC \) correction data points in Figure 4-14 shows the dependence of empirically corrected added mass coefficient of a solid plate upon \( KC \) parameter.

Figure 4-14 Added Mass coefficients of a solid plate oscillating in deep water

The analytical study of porous plates has shown the dependence of added mass coefficients on \( f(\tau) \) and \( KC \) parameter. Among the porosity models tested, the model assuming locally averaged flow through porous media appears to provide a reasonably good agreement with experimental data at low \( KC \) numbers, but greater than 0.1. For higher \( KC \) numbers, the experimental data deviate from the analytical model. The deviation from experimental data increase with increasing plate porosity and hence added mass coefficients were underestimated at \( KC > 1 \). An empirical correction function dependent on the porosity \( (\tau) \) and \( KC \) number is proposed in this section. The trend line drawn through experimental data suggested different empirical correction factors for each plate porosity. The proposed empirical correction equation for the added mass coefficients of porous plates is given by

\[
C_a(\text{corrected}) \approx C_a(\text{analytical}) + \frac{\tau(KC)f(\tau)}{10\pi} \quad (4.137)
\]

Figure 4-15 - Figure 4-17 show experimental, analytical and empirically corrected added mass coefficients as function of \( KC \) for the plates of porosity 10\% 15\% and 20\%. It is noticeable that the agreement between experimental and the empirically corrected analytical added mass coefficient gets even better as the plate porosity is increased. Analytically obtained and corrected added mass coefficients tended to zero as \( KC \rightarrow 0 \). This trend
was not observed in the experimental study. Therefore the empirical correction proposed in Equation (4.137) can be used to obtain added mass coefficients for plates oscillating at $KC > 0.1$.

The discharge equation or the pressure drop due the porosity allows one to account for viscous effects due to the porosity of the structure, while still assuming the ideal fluid which does not account for amplitude of oscillation of the plate. This limits the accuracy of added mass for higher $KC$ numbers and porosities. Figure 4-15, Figure 4-16 and Figure 4-17 shows that as the porosity is increased and for higher porosity, the lower added mass is predicted analytically as compared experimental analysis.

![Figure 4-15 Added Mass coefficients of a 10% porous plate oscillating in deep water](image1)

![Figure 4-16 Added Mass coefficients of a 15% porous plate oscillating in deep water](image2)
The damping coefficients of a solid plate obtained using the method of matched Eigen functions were found to be of small magnitude as compared to the experimental study. This is due to the fact that the analytical study does not account for the flow around the edges. Wang et al. found that the solid plate generates eddy resistance entirely by causing a flow separation and vortex shedding from the edge. Ref. [39]. Therefore an empirical formulation is proposed here and is given in Equation (4.138). Figure 4-18 represents experimental and empirical damping coefficients and showed reasonable agreement with each other.

\[ C_a (corrected) \approx \frac{\rho a^3 \omega (KC)}{8 \frac{3}{2} \alpha a^3} \approx 0.375(KC) \] (4.138)

The experimental results presented in Chapter 2 indicate that the plate porosity affects the behaviour of damping coefficients. Figure 4-19 - Figure 4-21 represents the damping coefficients of deeply submerged porous plates obtained using experimental technique and analytically corrected using proposed Equation (4.139-a). Along with vortex shedding from the edge, the porous plate also has dependence on orifice size and hence influences the velocity profile as compared to solid plate. In the analytical study performed here using the method of matched Eigen function expansions, the flow through the orifice was taken into account in an averaged sense but the edge effects were not included due to potential theory assumptions. Error and trial method was applied on analytically obtained damping coefficients to get a better fit with experimentally obtained damping coefficients. The empirical correction dependent upon \( KC \) parameter and porosity is proposed to account.
for edge effects. In Figure 4-19 - Figure 4-21 the corrected damping coefficients showed reasonable agreement with the experimental damping coefficients obtained for plates of porosity 10% - 20% using forced oscillation technique.

\[
C_b(\text{corrected}) \approx \frac{C_b(\text{analytical})}{\pi} + (0.45 - \tau) \cdot (KC) \quad (4.139 - a)
\]

There is no physical justification behind the large divider \(\pi\) of small coefficient \(C_b\) first term in Equation (4.139-a). The large divider makes value of first term negligible as compared to the second term. Therefore one may write Equation (4.139-a) purely as an empirical function of porosity \(\tau\) and \(KC\), given by Equation (4.139-b)

\[
C_b(\text{corrected}) \approx (0.45 - \tau) \cdot (KC) \quad (4.139 - b)
\]
4.5 Analytical Results and Discussion

The results presented in Section 4.4.1 indicate that the analytically developed model provides a reasonable agreement with experimental added mass coefficients for solid plate oscillating in close free surface proximity. The theoretical added mass coefficients of deeply submerged solid and porous plates provide a better agreement with experimental data using empirical corrections. Therefore the analytical study mainly concentrates on the added mass coefficients of plates oscillating in close proximity to free surface, even though the damping coefficients are presented here. The damping coefficients are not predicted.
accurately because of potential theory assumption that assumes the flow to be irrotational and hence does not account for edge vortices.

4.5.1 Influence of free surface proximity: non-dimensional frequency parameter

An important non-dimensional frequency parameter given by $\omega^2a/g$ was studied analytically to include gravity effects due to free surface proximity for plates of porosity 10% and 20% and discussed in Figure 4-22 - Figure 4-25 at $KC = 0.47$ and $KC = 1.57$. The dominant nonlinear frequency parameter effects on hydrodynamic performance of solid plate oscillating in close free surface proximity were noticed in Section 4.4.1. A comparison of added mass coefficients of a 5% porous plate oscillating in close proximity to free surface, obtained using analytical and experimental study was found to be within 12% for submergence up to $0.5a$ at $KC = 0.2$. The detailed comparison and percentage differences are given in Table A3-1 in Appendix V. Figure 4-22 and Figure 4-23 represent the added mass and damping coefficients for plates of porosity 10% and 20% at $KC = 0.47$. The dominance of non-dimensional frequency parameter on the hydrodynamic behaviour of oscillating porous plates was shown, when plates were in close proximity to the free surface. At certain frequencies the added mass coefficients also became negative in the range of $\omega^2a/g = 0.8 - 1.4$. Similar trends for solid plates have been noticed and published in the literature, Ref. [83].

The hydrodynamic performance of plates became relatively weakly dependent on $\omega^2a/g$ when plates were submerged at $h - d > a$. Another important observation showed that free surface proximity has stronger influence on added mass and damping coefficients of a 10% porous plate as compared to 20% porous plate. The damping coefficients were found to remain positive for the range of non-dimensional frequency parameters presented here. The damping coefficients of a 10% porous plate has maximum value at $\omega^2a/g = 0.7$ and at $\omega^2a/g = 0.7$ in the case of a 20% porous plate oscillating at submergence '$S = 0.2'$. The magnitude of the damping coefficients decreased approximately by a factor of five and three, as the 10% and 20% porous plates were oscillated at a submergence of '$S = 2'$. Figure 4-24 and Figure 4-25 represents the added mass and damping coefficient for plates oscillating at $KC = 1.6$. Only results obtained for plates submerged at $2a$ and $1a$ submergence are shown due to the fact that a plate submerged at $0.2a$ will pass through the free surface if oscillated at $KC = 1.6$. 

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Figure 4-22 Added mass coefficients variation with $\omega^2 a / g$ - $KC=0.47$, $\tau=10$, 20%-varying submergences

Figure 4-23 Damping coefficients variation with $\omega^2 a / g$ - $KC=0.47$, $\tau=10$, 20%-varying submergences
4.5.2 Influence of frequency-Deeply submerged porous plates

Figure 4-26 - Figure 4-27 show the variation of added mass and damping coefficients with frequency of oscillation for various plate porosities tested analytically. The plate of \( a = 0.1m, h = 0.9m, d = 0.45m \) was oscillated at \( KC = 0.3 \). Figure 4-26 shows the added mass coefficients of deeply submerged plates of varying porosity with respect to frequency of oscillation. The variation of damping coefficients with respect to
plate frequency of oscillation is shown in Figure 4-27. The results presented here are for plates of porosity 2%-20%. The added mass and damping coefficients showed negligible influence of oscillation frequency for all the deeply submerged porous plate cases presented here.

The magnitude of the added mass coefficients was found to be decreased by 50% for a plate of porosity 20% as compared to a plate of porosity 2%. Similar trends were noticed with increasing plate porosity in the experimental study given in Chapter 2. This indicates the major influence of structural porosity on hydrodynamic performance of a structure. The porous plates oscillating in nearly deep water were also studied experimentally by Tao and Dray who found that frequency has a relatively small influence on variation of hydrodynamic coefficients of oscillating plates, Ref. [52]. The damping coefficients given in Figure 4-27 obtained using linear potential theory did not account for the edge vortices that contribute to damping and hence the damping coefficients presented here are not predicted accurately. The magnitude of damping coefficients was small for a 2% porous plate oscillating in deep water and increased as the plates were made further porous. This increase is due to the damping introduced by the porosity of the plate. But the increase in damping coefficients with increasing plate porosity is not found to be linear. The non-dimensional damping coefficients of plates were found to be 0.45, 0.4, 0.3 and 0.1 for plates of porosity 20%, 15%, 10% and 2% at all frequencies of oscillation. The flow visualization study in Chapter 3 also showed presence of small vortices and diffusion around the plate due to plate porosity.
4.5.3 Effect of free surface proximity—Varying plate porosities

This section concentrates on the effect of free surface proximity. In the previous subsection the added mass coefficients obtained for deeply submerged oscillating plates were discussed. This section aims at understanding the effect of free surface proximity on porous plates using analytical method. The parameters are chosen such that plates of porosity 2%, 10% and 20% are oscillating at $S = 2a, 1a, 0.5a, 0.4a, 0.2a$ at frequency of 1Hz. The range of $KC$ numbers for the test cases shown in Figure 4-28 -
Figure 4-30 was chosen such that the plate remained submerged under the free surface at all times including during oscillation. The frequency plays a dominant role for plates oscillating in close proximity to free surface. Therefore the results obtained here cannot be generalised for plates oscillating at other frequencies. The added mass coefficients were up to three times more when the plate of porosity 2% was oscillated in very close proximity to the free surface i.e. $0.2a$ submergence as shown in Figure 4-28. However these magnitudes seemed to be decreasing if the plate was made further porous as shown in Figure 4-29 and Figure 4-30. This behaviour was noticed for a solid plate in experimental studies and is reported in, Ref. [4]. As discussed in the previous sections this analytical study does not predict added mass coefficients accurately for large $KC$ number and this can be noticed in Figure 4-28 for a plate of porosity 2%. With increasing porosity it is found that the added mass coefficients of a 10% and 20% porous plate tended to zero, but this behaviour is not found in the experimental study in Chapter 2 and Tao & Dray, Ref. [52]. The damping coefficients given in Figure 4-31 - Figure 4-33 show the behaviour of damping coefficients with increasing $KC$ parameter. The damping increased with close proximity to the free surface. The damping coefficients increased up to six times when the 2% porous plate was submerged at $S = 0.2a$ as compared with plate a plate submerged at $S = 2a$. But the variation with $KC$ parameter is not consistent with experimental findings for deeply submerged porous plates given in Chapter 2. The experimental study indicates that damping coefficients increase with increasing $KC$ paramter, where as the analytical damping coefficients did not predict this trend. The damping coefficients are predicted using linear potential theory that does not take into account the damping induced by edge vortices i.e. $KC$ dependence due to oscillation. The empirical corrections introduced in the previous section for the damping coefficients of solid and porous plates oscillating in deep water cannot be directly applied here to correct damping coefficients. This is because of the non-linear behaviour of hydrodynamic coefficients with varying frequencies of oscillation for plates oscillating in close proximity to free surface.
Figure 4-28 Added mass coefficients variation with KC - 2% porous plate-frequency 1 Hz - Varying free surface proximities

Figure 4-29 Added mass coefficients variation with KC - 10% porous plate-frequency 1 Hz - Varying free surface proximities
Figure 4-30 Added mass coefficients variation with KC - 20% porous plate-frequency 1 Hz - Varying free surface proximities

Figure 4-31 Damping coefficients variation with KC - 2% porous plate-frequency 1 Hz - Varying free surface proximities
4.5.4  Effect of frequency-Close proximity to free surface

The frequency of oscillation played a negligible role when the plates were oscillated in deep water. Equation (4.22) suggests that frequency contributes towards the hydrodynamic coefficients of oscillating plates when the free surface is in close proximity. Figure 4-34 - Figure 4-36 represent the variation of added mass coefficients with respect to \( KC \) for a 2% porous plate. These show that the frequency contributed towards the added mass coefficients but the trend was not linear. This sort of non-linear behaviour was also
The added mass coefficients had a maximum value when the plate was oscillated at 1 Hz frequency and had a minimum value when the plate was oscillated at 2 Hz frequency from the range of data shown here. Figure 4-37 - Figure 4-39 represent the damping coefficients as a function of $KC$ parameter at varying frequencies of oscillation. The damping coefficients did not vary linearly with increasing $KC$ parameter. The non-linear variation with increasing $KC$ parameter indicates the limitation of the theoretical model for estimating damping coefficients of plates. However the damping coefficients varied widely with increasing frequency of oscillation when the plates are in close proximity to free surface as shown in Figure 4-37 - Figure 4-39. The magnitude of the damping coefficients decreased by up to 5 times when the frequency of oscillation was lowered to 0.1 Hz. The nonlinear damping behaviour with frequency parameter is noticed in Figure 4-39.

Figure 4-34 Added mass coefficients variation with KC - 2% porous plate- 'sub=0.5a' - Varying frequencies

Figure 4-35 Added mass coefficients variation with KC - 2% porous plate- 'sub=0.4a' - Varying frequencies
Figure 4-36 Added mass coefficients variation with KC - 2% porous plate - 'sub=0.2a' - Varying frequencies

Figure 4-37 Damping coefficients variation with KC - 2% porous plate - 'S=0.5a' - Varying frequencies

Figure 4-38 Damping coefficients variation with KC - 2% porous plate - 'sub=0.4a' - Varying frequencies
4.5.5 Summary

The method of matched Eigen functions predicts the added mass coefficients and a range of results were comparable with experimental findings for plates that were heaving in close proximity to free surface. Empirical corrections are proposed for the added mass and the damping coefficients of solid and porous plates oscillating in deep water for a better fit with experimental findings with increasing $\frac{\tau}{g}$ parameter. The influence of porosity was better predicted using the model that assumes locally averaged flow on plate, but the formulation and empirical corrections show that added mass coefficients tend to zero for $\frac{\tau}{g} < 0.15$ and this trend was not observed in experimental study. The damping coefficients using potential theory are not predicted accurately due to its assumption of irrotational flow and hence the influence of edge vortices is not taken account. The added mass and damping coefficients are found to be nonlinearly dependent upon non-dimensional frequency parameter as plates were oscillated in close proximity to free surface. The experimental and analytical findings suggest that the hydrodynamic coefficients of plates submerged at a distance of two times the radius of a plate have small influence of the boundary proximities.

Figure 4-39 Damping coefficients variation with KC - 2% porous plate- 'sub=0.2a' - Varying frequencies
Chapter 5

Conclusions and Future Work

5.1 Summary and conclusions

This research has investigated the analytical method of Eigen function expansions and partially validated the methodology with model test measurements of oscillating solid and porous plates. The plate porosity influences its overall hydrodynamic performance. Conservation of momentum, assuming locally averaged flow and jet profile are the porosity models developed to analyse porous plate hydrodynamics. By understanding the behaviour of non-dimensionalised pressure drop with varying plate porosity the performance of each porosity model is tested. The conclusion drawn is that the porosity model obtained by assuming locally averaged flow better serves the purpose for high porosity plates when compared to other flow discharge models tested. Even though this model is relatively better than others, but still some discrepancies are noticed when compared with experimental findings and hence empirical corrections are proposed. The methodology developed here can be used for plates of varying sizes and for any finite plate submergence level.

The hydrodynamic performance of the plates is also investigated experimentally using both force measurements to estimate the hydrodynamic forces, and flow visualization to understand the reason behind the differences in hydrodynamic performance of solid and porous plates. The force measurements experimental study includes parametric dependence upon $\tau, KC, \beta$ and $\omega^2 a/g$. This was extended to study the free surface effects by oscillating the plates in close proximity to free surface. The flow visualization experimental study provides further insight as to why added mass and damping vary with plate porosity.
The contributions of the work are as follows:

• **Plate Porosity (τ):**
  Plate porosity influences both the added mass and damping coefficients. The added mass coefficients continuously decrease as plate porosity increases. Both the experimental and analytical studies demonstrate a decrease in added mass coefficients with increasing plate porosity. It is noticed in the experimental study that the added mass coefficients of deeply submerged porous plates decreased by 9% for 5% porous plate; 19% for a 10% porous plate; and 30% for a 15% porous plate when compared with a solid plate. The added mass coefficient of a 20% porous plate is about 50% of the solid plate added mass coefficients. The damping coefficients have shown influence of plate porosity for \( KC < 1.8 \). The trend was not directly proportional to plate porosity and the experimental study shows that the damping coefficients of 5% and 10% porous plates were higher than 20% porous plate. The experimentally obtained damping coefficients have shown influence of porosity with increasing \( KC \). It is noticed that damping cannot be increased by increasing plate porosity for plates oscillating at \( KC > 1.8 \).

• **Keulegan-Carpenter Parameter:**
  \( KC \) number plays an important role in relation to the hydrodynamic behaviour of the solid and porous plates. For deeply submerged plates, the added mass coefficients increase linearly with \( KC \) parameter. A similar trend was found in the case of solid and porous plates oscillating in close proximity to free surface. However it is noticed in the experimental study that the added mass coefficients of deeply submerged porous plates decreased by 9% for 5% porous plate; 19% for a 10% porous plate; and 30% for a 15% porous plate when compared with a solid plate.

• **β Parameter study:**
  Both the added mass and damping coefficients demonstrate a dependence upon \( β \) parameter. Frequency plays a minor role for deeply submerged plates, but frequency plays a significant role when the plates are oscillating in close proximity to free surface.

• **Non-dimensionalised frequency parameter:**
An important non-dimensional frequency parameter $\omega^2 a/g$ is studied to understand the behaviour of the added mass and the damping coefficients due to free surface proximity of plates. The study is primarily conducted using an analytical method. It is concluded that the added mass coefficients can be 3 times greater than the added mass coefficients of plate oscillating in deep water. Simultaneously, the added mass coefficients were found to be negative for $\omega^2 a/g = 0.8 – 1.4$ for a 10% porous plate and $0.85 – 1.3$ in case of 20% porous plate. The damping coefficients also have shown dependence upon $\omega^2 a/g$ parameter. The damping coefficients vary non-linearly with increasing $\omega^2 a/g$ parameter. The influence of the non-dimensional frequency parameter is more as the plates were oscillated in close proximity to free surface. Another important observation is that the dominance of non-dimensional frequency parameter varies with increasing plate porosity. In the case of solid plate it is found both experimentally and analytically that the added mass coefficients become negative for a plate oscillating at $\omega^2 a/g$ of 1.3.

- **Flow visualization findings:**
  In relation to solid and 15% porous plate it was found that as the solid plate moves from one direction to another, say from lower end to upper end, the free shear layer on the upper surface is shed off the edge before rolling into a vortex ring. In case of porous plate, the vortex ring is less dominant. The flow through the orifices in the porous plates influences the velocity field above and below the oscillating plate, affecting the flow pattern around the edge of the plate. It is noticed that the circulation of vortices around the porous plate is weaker than that obtained in the case of solid plate. Moving away from the core in the vortex ring the circulation around porous plate is found to be less than 50%. The velocity profile around porous plate contributes towards pressure drop and hence the added mass coefficients decrease with increasing porosity. The small vortices formed due to plate porosity on either side of the plate contribute towards overall damping behaviour of porous plate.

- **Empirical Corrections:**
  The comparison of experimental and analytical studies suggests that the added mass coefficients are in agreement for a solid plate oscillating in close proximity to free
surface. But the variation of the added mass coefficients is only predicted for a limited range of $KC$ parameter. A reasonable agreement is obtained for $KC$ range of $0.15 - 1'$ for a plate of 20\% porosity, and $0.15 - 0.7'$ for a plate of 15\% porosity. Empirical corrections are proposed that provide a good agreement between experimental and empirically corrected analytical added mass coefficients. The empirical correction predicts added mass coefficients of plates oscillating at deep submergence for $KC$ up to 2. The damping coefficient of a solid plate are obtained using an empirical formula and empirical corrections are proposed for the damping coefficients of a porous plates oscillating at deep submergence for $KC$ up to 1.4.

5.2 Future Work

The current research has covered range of aspects relating to the hydrodynamic performance of oscillating solid and porous plates. However certain limitations do exist, especially in regard to the experimental setup. The following improvements could be made to the experimental setup and theoretical model for future research

- **Experimental tank dimensions**
  Whilst not presented in this thesis, experiments could also be performed at higher $KC$ parameter and free surface proximity of solid and porous plates. It is noticed that the hydrodynamic coefficients are contaminated by the radial proximity of walls at higher $KC$ and $\beta$ parameter. As a consequence the results obtained can only be trusted for low $KC$ and deeply submerged oscillating plates. A further set of experiments could be validated at low $KC$ with the current results and could be further extended for higher $KC$ and $\beta$ parameters.

- **Flow visualization**
  The flow visualization study could be performed on porous plates that have different orifice size and oscillating at close proximities to free surface. This will assist in better understanding the fluid flow around plates of varying porosity and oscillating at submergence levels.

- **Modelling porous tank walls**
  The analytical study could possibly be modified to account for tank wall proximity by changing the radial boundary condition and accounting for porous tank walls at
a distance from the plate edge. This will allow for a direct comparison with the experimental results at higher $K_C$ parameters.

- **Further investigation into plate porosity models:**
The analytical of flow through oscillating porous plates could be further investigated and compared with the models presented here, while incorporating edge effects analytically as compared to empirical corrections presented here.
Appendices

Appendix I

NOTE: removed due to copyright restrictions.

Variation of Heave Added Mass and Damping Near Seabed
Hemlata Wadhwa, Balaji Krishnamoorthy and Krish P. Thiagarajan
Paper No. OMAE2010-20456, pp. 271-277; 7 pages
doi:10.1115/OMAE2010-20456
ASME 2010 29th International Conference on Ocean, Offshore and Arctic Engineering
29th International Conference on Ocean, Offshore and Arctic Engineering: Volume 1
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Conference Sponsors: Ocean, Offshore and Arctic Engineering Division
Appendix II

NOTE: removed due to copyright restrictions.

Experimental Assessment of Hydrodynamic Coefficients of Disks Oscillating Near a Free Surface

Hemlata Wadhwa and Krish P. Thiagarajan

ASME 2009 28th International Conference on Ocean, Offshore and Arctic Engineering
Volume 4: Ocean Engineering; Ocean Renewable Energy; Ocean Space Utilization, Parts A and B
Honolulu, Hawaii, USA, May 31–June 5, 2009
Conference Sponsors: Ocean, Offshore and Arctic Engineering Division
Appendix III

Separation of Variables: - Separation of variables is a convenient method to solve some linear partial differential equations. The method is based on the assumption that the solution of a partial differential equation can be expressed as a product of terms, each of which is a function of only one independent variable. In the case of problem in chapter 4 the velocity potential in general can be written as

\[ \phi(r, z) = R(r)Z(z) \quad (A1 - 1) \]

Where \( R(r) \) is a function of \( r \) only in the radial direction in Cartesian co-ordinate system and \( Z(z) \) depends only upon variable \( z \) in the vertical direction.

Substituting (A1-1) in Laplace’s equation, we get

\[ \frac{R'(r)Z(z)}{r} + Z(z)R''(r) + R(r)Z'(z) = 0 \quad (A1 - 2) \]

\[ \frac{R'(r) + rR''(r)}{rR(r)} = \frac{Z''(z)}{Z(z)} \quad (A1 - 3) \]

In equation (A1-3), the left hand side term is clearly only dependent on variable \( r \) and right hand side only on variable \( z \).

If we consider variation in \( z \) holding \( r \) constant, the right hand side term could conceivably vary, whereas the left hand side term could not. This would not give a satisfactory solution to equation (A1-3).

The only way this equation can be true is by putting each side of the equation equal to some constant.

\[ \frac{R'(r) + rR''(r)}{rR(r)} = k^2 \quad (A1 - 4) \]

\[ \frac{Z''(z)}{Z(z)} = k^2 \quad (A1 - 5) \]
where \( k^2 \) can be positive, negative or zero.

The equation (A1-4) and (A1-5) are ordinary differential equations and can be solved independently. There are three possible solutions depending upon the nature of the ‘\( k \)’ value.

**Case 1:** \( k^2 > 0 \) i.e. \( k \) is real

\[
\frac{R'(r)}{r} + R''(r) + k^2 R(r) = 0
\]

(A1 - 6)

\[ Z''(z) - k^2 Z(z) = 0 \]

(A1 - 7)

\[ R(r) = A' J_0(kr) + B Y_0(kr) \]

(A1 - 8)

\[ Z(z) = C \sinh(kz) + D \cosh(kz) \]

(A1 - 9)

where \( A, B, C, D \) are the constants in the solution

**Case 2:** \( k^2 < 0 \) i.e. \( k \) is complex

\[
\frac{R'(r)}{r} + R''(r) - k^2 R(r) = 0
\]

(A1 - 10)

\[ Z''(z) + k^2 Z(z) = 0 \]

(A1 - 11)

\[ R(r) = E K_0(kr) + F I_0(kr) \]

(A1 - 12)

\[ Z(z) = G \sin(kz) + H \cos(kz) \]

(A1 - 13)

where \( E, F, G, H \) are the constants in the solution

**Case 3:** \( k^2 = 0 \) i.e. \( k \) is zero

\[
\frac{R'(r)}{r} + R''(r) = 0
\]

(A1 - 14)

\[ Z''(z) = 0 \]

(A1 - 15)

\[ R(r) = I \ln(r) + j \]

(A1 - 16)

\[ Z(z) = Mz + L \]

(A1 - 17)

where \( I, J, M, L \) are the constants in the solution

Thus the solution to Laplace’s equation can be written as the sum of the number of solutions.
\[ \varphi(r, z) = (E K_0(kr) + F I_0(kr))(G \sin(kz) + H \cos(kz)) \\
+ (A J_0(kr) + B Y_0(kr))(C \sinh(kz) + D \cosh(kz)) \\
+ (I \ln(r) + J)(Mz + L) \quad (A1 - 18) \]
Appendix IV

Jet Velocity Profile: Consider the flow from the orifice at the plate and assuming that it shoots out at an angle $\Theta$ like a free jet due to the availability of more area to spread as the flow reaches on the other side of the orifice as shown in Figure A2.1. Assuming the flow profile as jet velocity profile given in (99).

![Figure A2.1 Schematic of flow through the plate orifice as jet flow](image)

$v_1$ is the flow velocity when fluid enters the orifice. Let $v_2$ be the flow velocity when fluid reaches on the other side of the orifice. The diameter at $z$ is given by

$$d_z = d_h + 2ztan(\Theta). \tag{A2 - 1}$$

Therefore the area covered by the fluid flow is given by

$$A_z = \frac{\pi d_z^2}{4} = \frac{\pi (d_h + 2ztan(\Theta))^2}{4} \tag{A2 - 2}$$

where $d_h$ = diameter of the hole

Now in the case of a porous plate, we want

$$A_z = \frac{A_d}{\tau} = \frac{\pi d_h^2}{4\tau} \tag{A2 - 3}$$

Using Equation (A2 - 1) in Equation (A2 - 2), we get

$$\frac{\pi d_h^2}{4\tau} = \frac{\pi (d_h + 2ztan(\Theta))^2}{4} \tag{A2 - 4}$$

$$\Rightarrow \frac{1}{\sqrt{\tau}} = 1 + \frac{2ztan(\Theta)}{d_h}$$
\[
\frac{z}{d_h} = \frac{1}{\sqrt{\tau}} - 1 \quad (A2 - 5)
\]

The velocity profile above and below the orifice can be related as given in (99)

\[
\frac{v_2}{v_1} = \text{sech}^2 \left( 10.4 \frac{r}{Z'} \right) \quad (A2 - 6)
\]

Using Equation (A2 - 4) and Equation (A2 - 5), Equation (A2 - 6) becomes

\[
\frac{v_2}{v_1} = \text{sech}^2 \left( 10.4 \frac{r}{d_h} \right) \quad (A2 - 7)
\]

Equation (A2 - 7) representing the ratio of velocities below and above the plate orifice can be written as

\[
\frac{v_2}{v_1} = \text{sech}^2 (N r) \quad (A2 - 8)
\]

where

\[
N = \frac{20.8 \tan(\Theta)}{d_h \left( \frac{1}{\sqrt{\tau}} - 1 \right)} \quad (A2 - 9)
\]

Now flow rate of the fluid flow is given by

\[
dQ = 2\pi v_2 r dr \quad (A2 - 10)
\]

\[
Q = \int_0^{d_h/\tau} 2\pi v_2 r dr \quad (A2 - 11)
\]

Using Equation (A2 - 8) in Equation (A2 - 11), we get

\[
Q = \int_0^{d_h/\tau} 2\pi \text{sech}^2 (N r) v_1 r dr \quad (A2 - 12)
\]

Integrating by parts we get

\[
Q = 2\pi v_1 \left[ \frac{d_h}{N \tau} \tanh \left( \frac{N d_h}{\tau} \right) - \frac{1}{N^2} \ln \left[ \cosh \left( \frac{N d_h}{\tau} \right) \right] \right] \quad (A2 - 13)
\]

Now

\[
\frac{N d_h}{\tau} = \frac{20.8 \tan(\Theta)}{d_h \left( \frac{1}{\sqrt{\tau}} - 1 \right)} \left( \frac{1}{\sqrt{\tau} - 1} \right) \quad (A2 - 14)
\]

and

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\[ \frac{d_h}{N_\tau} = \frac{d_h^2}{20.8 \tan(\Theta) \left( \frac{1}{\sqrt{\tau}} - 1 \right)} \]

\[ \frac{d_h}{N_\tau} = \frac{d_h^2}{20.8 \tan(\Theta) \left( \frac{1}{\sqrt{\tau}} - 1 \right)} \quad (A2 - 15) \]

Using Equation (A2 - 14) and Equation (A2 - 15) in Equation (A2 - 13), we get

\[ Q = 2\pi v_1 d_h^2 \left[ \frac{\sqrt{\tau}}{20.8 \tan(\Theta) (1 - \sqrt{\tau})} \tanh \left( \frac{20.8 \tan(\Theta)}{\sqrt{\tau} - \tau} \right) \right. \]

\[ - \left. \frac{(1 - \sqrt{\tau})^2}{23\tau} \ln \left[ \cosh \left( \frac{20.8 \tan(\Theta)}{\sqrt{\tau} - \tau} \right) \right] \right] \quad (A2 - 16) \]

The average velocity is obtained from flow rate as given below

\[ \bar{v} = \frac{Q}{\pi d_h^2 / \tau} = 2\tau g(\tau) v_1 \quad (A2 - 17) \]

Thus the pressure drop can be written as

\[ \Delta p = \frac{1}{2} \rho (\bar{v}|\bar{v}| - v_1 |v_1|) \quad (A2 - 18) \]

Rearranging Equation (A2 - 18), we get

\[ \Delta p = \frac{1}{2} \rho v_1 |v_1| \left( \frac{\bar{v}|\bar{v}|}{v_1 |v_1|} - 1 \right) \quad (A2 - 19) \]

From Equation (A2 - 19) and Equation (A2 - 17), we get

\[ \frac{\Delta p}{\frac{1}{2} \rho v_1 |v_1|} = (4(\tau g(\tau))^2 - 1) \quad (A2 - 20) \]

where

\[ g(\tau) = \frac{\sqrt{\tau}}{20.8 \tan(\Theta) (1 - \sqrt{\tau})} \tanh \left( \frac{20.8 \tan(\Theta)}{\sqrt{\tau} - \tau} \right) - \left( \frac{1 - \sqrt{\tau}}{23\tau} \right) \ln \left[ \cosh \left( \frac{20.8 \tan(\Theta)}{\sqrt{\tau} - \tau} \right) \right] \]

The right hand side of Equation (A2 - 20) is a function of \( \tau \) and thus Equation (4.126) can be rewritten as

\[ f_{\text{IVP}}(\tau) = \frac{\Delta p}{\frac{1}{2} \rho v_1 |v_1|} = (4(\tau g(\tau))^2 - 1) \quad (A2 - 21) \]

The problem assumes relatively small orifice sizes and hence the flow is expected to jet at small angles. At small angles of flow spread through the orifice, \( f_{\text{IVP}}(\tau) \) has shown
relatively better performance with varying $\tau$. The spread angle of slightly greater than 0.2 radians is chosen in evaluating $f_{\text{VP}}(\tau)$. The function $f_{\text{VP}}(\tau)$ tends to negative as the plate porosity was reduced to about 2%. The pressure drop obtained assuming jet velocity profile is given by

$$\Delta p = \left(\frac{1}{2} \rho v_1 |v_1| \right) f_{\text{VP}}(\tau)$$  \hspace{1cm} (A2 - 22)
Porous plate and free surface proximity: - Experimental and analytical studies were performed on the hydrodynamic behaviour of porous plates of porosity 5%. The added mass coefficients and percentage difference in the results from both studies are given in Table A3-1. The $KC$ parameter was chosen carefully to ensure that the plate remains under the free surface at all times. The percentage difference in the added mass coefficients was found to be reasonable and increases as the plate submergence was increased. This also indicates that analytical study better predicts the added mass coefficients when the porous plates are in close proximity to free surface. Table A3-2 presents the damping coefficients of 5% porous plate. Even though the percentage difference at $KC = 0.2$ and $S = 0.4a$ was less than 13%, the trend of the damping coefficients with increasing $KC$ was not in accordance with experimental data. The damping coefficients obtained from experimental data have shown a linear increment with increasing $KC$, whereas analytical study did not predict the same behaviour. The added mass and the damping coefficients obtained using analytical analysis are presented at a range of $KC$ in the next sections, but the reliability of the damping coefficients is questionable.

Table A3-1: Experimental and analytical added mass coefficients of a 5% porous plate

<table>
<thead>
<tr>
<th>s/a</th>
<th>KC</th>
<th>$F$ (Hz)</th>
<th>$\tau$ (%)</th>
<th>$C_a$ (Experimental)</th>
<th>$C_a$ (Analytical)</th>
<th>% Difference</th>
</tr>
</thead>
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<tr>
<td>0.2</td>
<td>0.1</td>
<td>1</td>
<td>5</td>
<td>1.93</td>
<td>1.74</td>
<td>10.7</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
<td>1.54</td>
<td>1.36</td>
<td>12.9</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
<td>1.41</td>
<td>1.26</td>
<td>11.9</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
<td>1.29</td>
<td>1.03</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Table A3-2: Experimental and analytical added mass coefficients of a 5% porous plate

<table>
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<tr>
<th>s/a</th>
<th>KC</th>
<th>$F$ (Hz)</th>
<th>$\tau$ (%)</th>
<th>$C_b$ (Exp)</th>
<th>$C_b$ (Analytical)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1</td>
<td>1</td>
<td>5</td>
<td>0.75</td>
<td>1.47</td>
<td>-49.0</td>
</tr>
<tr>
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<td>0.2</td>
<td>1</td>
<td>5</td>
<td>0.45</td>
<td>0.45</td>
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<td>5</td>
<td>0.35</td>
<td>0.36</td>
<td>-1.4</td>
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<td>1</td>
<td>5</td>
<td>0.25</td>
<td>0.22</td>
<td>12.5</td>
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