DEVELOPMENT OF UNIFIED PIPE NETWORK METHOD FOR MULTIPHASE FLUID FLOW IN FRACTURED POROUS MEDIA

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ABSTRACT

The fluid flow and mass transfer in geological rock formations are very important for applications, such as in tunnelling, underground mining, oil and gas exploitation, CO₂ geological sequestration, and nuclear waste geological disposal etc. Due to the intrinsic heterogeneous characteristics of the rock masses, the accurate simulation of fluid flow in the rock masses is a challenging task. The presence of fractures in different scales and types, such as cracks, fissures, joints, faults and fault zones etc. in rocks, further complicates the situation. The rock masses not only behave heterogeneously but also anisotropically.

The numerical simulations of the fluid flow in fractured rock masses are one of the effective ways to investigate the flow processes. For successful simulation, a versatile and robust numerical model is indispensable. However, the current numerical models still have many disadvantages in dealing with the fractured rock masses. Stimulated by the challenging tasks, our aim is to develop a conceptually simple and computationally efficient numerical model called a unified pipe network method (UPNM) to simulate different types of flow processes in the fractured rock masses, including steady flow, transient flow, unconfined flow, single-phase flow, and multi-phase flow.

The concept of unified pipe network method has been proposed. The UPNM conceptually and computationally simplifies the seepage simulations in complex fracture networks and fractured porous media. The flow in both the rock matrix and the fractures has been equivalent to flow in matrix pipes and fracture pipes respectively. Their hydraulic properties have been derived. In addition, the coupling of those two pipe elements has been explained in detail. The conforming mesh method for the hybrid method, which is developed in our group has been presented. Massive 2D and 3D numerical examples have been carried out to validate the method. The case studies demonstrate the flexibility and feasibility of the proposed method.

The steady state pipe network method has been developed with the function of stochastic generation of fracture network and it is applied to study the permeability anisotropy of the fractured rock masses. A refined anisotropy factor has been proposed for quantitatively indicating the hydraulic anisotropy for the fractured rock masses. Permeability anisotropy has been analysed for different fracture patterns. In addition,
the proposed anisotropy factor has been validated by the results of numerical simulation. The anisotropic conductivity index (ACI) has also been proposed for reflecting both the hydraulic connectivity quality and the directionality of the connection. These factors can be derived readily from the scan line surveys.

An unconfined flow simulation method for water flow prediction in the fractured rock masses has been developed based on the unified pipe network method. The method has been validated by examples with different geometries, material properties and draining conditions. Unconfined seepage problems in rock slope with different fracture patterns have been studied. The results show that the presence of the fractures in the rock slope can distort the distribution of the hydraulic head and change the shape as well as the location of the phreatic surface.

The numerical simulation method for the transient saturated-unsaturated flow in fractured rock mass has been developed based on the unified network method. In the interface of the fracture and the rock matrix, a capillary continuity condition is applied, and this considers the fluid interchange in the interface at the unsaturated zone. Both 2D and 3D numerical examples for flow in continuous porous media, in discrete fracture networks and fractured rock masses have been presented to demonstrate the versatility and feasibility of the UPNM for transient flow simulation in the fractured rock mass.

A practical project of petroleum underground storage in the unlined rock caverns has been studied by using the transient variably saturated UPNM. The effects of the water curtain system on the water sealing of the rock caverns in the fractured rock mass has been analysed. The reduction in the pressure of the water curtain, the length of the water curtain boreholes and the increase of the spacing of the water curtain boreholes can reduce the seal effect of the water curtain. It has been found that the failure of the water seal effect can occur locally, which cannot be observed by using the continuum methods.

2D and 3D pipe network methods for the simulation of multiphase flow processes in the fractured rock masses have been developed based on the UPNM. Both 2D and 3D models are set up for the problem of gaseous phase injection processes. A compressible gas phase has been chosen as the non-wetting phase in our study. The extended capillary condition is incorporated to merge the fractures into the porous medium. The solution of the fully coupled fully implicit nonlinear equations is explained in detail.
Numerical examples have been simulated to validate the effectiveness of the UPNM. CO\textsubscript{2} injection processes in the saline aquifers with different pressure and temperature are studied using the proposed model. Equations of state for CO\textsubscript{2} are incorporated in the model to calculate the density and viscosity of CO\textsubscript{2} under different conditions. In addition, the effects of fractures on the injection process and CO\textsubscript{2} plume evolution are studied. It has been found that the injection pressure build-up is related closely to the mobility of CO\textsubscript{2}. The fractures can perform as flow barriers across their planes due to the capillary pressure difference with the rock formation.
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CHAPTER 1. INTRODUCTION

1.1 BACKGROUND

Fluid flow and mass transfer in geological fractured rocks are very common yet important phenomena in a natural environment, which are closely related to the engineering practices of tunnelling, underground mining, oil and gas exploitation, CO$_2$ geological sequestration, and nuclear waste geological disposal, etc. With increasing activities of rock engineering projects in these areas, the mechanism of fluid flow and mass transfer in highly fractured and porous rock mass remains a challenging topic and attracts extensive research in the recent decades.

![Figure 1.1 Geological cross-section of a hard rock aquifer (Maréchal et al. 2003).](image)

Other than artificial or idea porous material, the rock masses in the geological formation are rarely intact. As results of compression, tension processes, erosion and so on in the natural environment, rock masses commonly contain fractures in different scales and types, such as cracks, fissures, joints, faults and fault zones, etc. Figure 1.1 illustrates a typical fractured hard rock aquifer. Fractures provide larger permeability but less pore spaces than the rock matrix that contains them. The permeability of the fractures is orders of magnitude higher than that of the matrix. Therefore, the geometry and distribution of the fractures play significant roles in the hydraulic characters of rock masses. The presence of the fractures makes fractured rock masses heterogeneous and
anisotropic. What is more, it causes the fluid flow in the fractured rock masses very intractable.

Numerical simulation of fluid flow in fractured rock masses is a useful tool to predict the flow process and mass transport phenomena in a geological rock formation. A successful numerical simulation can produce accurate and reliable results, which can be a full or partial replacement of expensive and troublesome experiments and site investigations. There are generally two categories of numerical models for the simulation of fluid flow in fractured rock masses based on the way of treating the fractures. One category is so-called continuum models (or equivalent porous medium models), which treat the fractures implicitly. With the assumption of regularity of the fractures, the domain is homogenised at different scales in the continuum models. A typical continuum model is the double-porosity model proposed by Barenblatt et al. (1960). It divides the domain into two equivalent porous media, i.e. fracture network and rock matrix. Fluid interchange between these two media is allowed. The double-porosity model does not require the data of individual fracture; therefore, it is conceptually simple. However, it is unable to describe exactly the flow processes in individual fracture, which becomes one of its main disadvantages. Furthermore, the derivation of the exchange term between fracture network and rock matrix is troublesome.

The other category is known to be discrete models, which treat each fracture explicitly. Moreover, the interaction between rock matrix and fractures is more straightforward when the matrix permeability is of concern. The discrete models are capable of simulating the processes that are dominated by local fractures. However, some challenges exist and need to be resolved in the discrete models. First, due to the drastic differences of the material properties between fracture and rock matrix, advanced numerical methods are required to cope efficiently with the highly localized discontinuities. Second, the intricate geological configuration produced by the complex fracture structure is difficult to mesh. Third, the large number of fractures makes the simulation very time-consuming therefore simple and efficient numerical methods are important for the hybrid models, especially when the transient processes are of concern.
1.2 OBJECTIVES AND SCOPE OF THE THESIS

This thesis is motivated by the remaining challenges in accurately and efficiently simulating fluid flow in fractured porous rock masses. It aims to develop a conceptual simple and computational efficient numerical method to simulate multiphase flow processes in fractured porous rock masses. Specific objectives are summarised as follows:

- To develop a unified pipe network method for the simulation of fluid flow processes in fractured rock masses.
  
  - The fluid flow in both rock matrix and fractures will be considered in a unified way by implementing equivalent matrix pipe networks and fracture pipe networks respectively.
  
  - The coupling of the matrix pipe networks and the fracture pipe networks will be presented.
  
  - A conforming mesh method will be used in this study to facilitate the coupling of the matrix pipe networks and the fracture pipe networks.
  
  - Validations of fluid flow with 2D and 3D examples will be carried out.

- To evaluate the permeability anisotropy of fractured rock masses by using the pipe network method.
  
  - A 2D pipe network method in cooperating with a stochastic fracture network generation tool is developed.
  
  - A refined anisotropy factor is proposed for quantitatively evaluating the hydraulic anisotropy of the fractured rock masses.
  
  - Permeability anisotropy is analysed for different fracture patterns. In addition, the proposed anisotropy factor is validated by the results of numerical simulation.

- To derive an unconfined flow simulation method for fluid flow prediction in fractured rock masses.
An unconfined seepage simulation method for free surface flow in fractured rock masses is derived based on the developed unified pipe network method.

This method is validated by examples with different geometries, material properties and draining conditions.

The influence of the distribution of fractures on the water table is studied.

- To improve the 2D and 3D unified pipe network methods for the simulation of variably saturated transient flow processes in the fractured rock masses.

The unified pipe network method is extended for the simulation of variably saturated single-phase transient flow process.

2D and 3D flow processes in both fracture network and fractured porous medium are studied.

As an example, the water sealing effects of water curtain system on underground oil storage in unlined rock caverns are studied systematically.

- To further extend the 2D and 3D unified pipe network methods for the simulation of multiphase flow process in fractured porous rock masses.

The pipe network method is further extended for the simulation of multiphase flow in both fracture networks and fractured porous medium.

The drainage process where a compressible non-wetting invading fluid displaces a wetting fluid is deliberately selected to show the feasibility of the unified pipe network method.

Capillary pressure relations with different entry pressures for fracture and rock matrix are incorporated in the numerical scheme.

Fully coupled and fully implicit numerical scheme is applied to avoid the possible small saturation step in the implicit pressure, explicit saturation scheme.

CO₂ injections into geological saline formations with different temperature and depth are studied. The equations of state for CO₂ are incorporated in the
numerical model. The effects of fractures on the CO₂ injection process are also studied.

1.3 STRUCTURE OF THE THESIS

This thesis is organised with eight chapters.

Chapter 1 briefly introduces the rock hydraulics and its wide applications in the fields of engineering. The characteristics of fluid flow in fractured rock masses are analyzed. The existing numerical models that have been applied for fluid flow simulations in fractured rock masses are briefly introduced. The challenges in the numerical modelling are discussed, based on which, the objectives and the scopes of the thesis are introduced. The structure of the thesis is outlined in end of the chapter.

Chapter 2 mainly focuses on literature review. Basic knowledge of fluid flow in fractured porous medium is introduced, which includes fluid flow in porous medium as well as in fractures. Besides, it reviews in more details the numerical models applied in the simulation of fluid flow in fractured rock masses. These models are systematically categorized in this review. The advantages and the disadvantages of different models are analyzed and compared. The two-phase flow models in fractured rock masses are emphasized.

Chapter 3 presents the concept and theoretical derivations of the proposed unified pipe network method. The fracture pipe networks and the matrix pipe networks are then treated in a unified way in the proposed pipe network method. The hydraulic properties of these equivalent 1D pipes are derived in this chapter. In addition, the coupling of these pipe networks is implemented. As an efficient mesh tool for the coupling method, an adaptive mesh technique is applied to generate the pipe network models by using denser pipes at more permeable zones. 2D and 3D numerical examples are carried out to validate the developed unified pipe network method.

Chapter 4 studies the permeability anisotropy of fractured rock masses by using the developed unified pipe network method. For quantitatively evaluating the permeability anisotropy of the fractured rock masses, a refined anisotropy factor is derived, which takes the hydraulic property of each individual fracture into account. The permeability anisotropy caused by different fracture patterns is studied by the proposed anisotropy
factor based on the proposed pipe network method. The effectiveness of the refined anisotropy factor is validated.

Chapter 5 derives an efficient simulation method solving unconfined flow problems encountered in the fractured porous medium. A new model for locating the phreatic surface in the fractured porous medium is proposed based on the pipe network method. The model takes the advantages of both fixed and adaptive mesh algorithms. The effectiveness and robustness of the model are validated by a series of cases, which involve different geometries, material properties and draining conditions, etc. Moreover, the results of those cases are compared with those from other models. The influences of the fractures on the water table in the fractured rock masses are studied.

Chapter 6 improves the 2D and 3D pipe network models for the simulation of transient variably saturated flow in fractured rock masses. Both flows in fractures and in rock matrix are modelled with flows in fracture pipe networks and in the matrix pipe networks. At the interface of the fracture and the rock matrix, capillary continuity condition is applied. Both 2D and 3D numerical examples for fluid flow in a continuous porous medium, discrete fracture networks and fractured porous rock mass are presented to demonstrate the versatility and feasibility of the unified pipe network method. A practical project of petroleum underground storage in unlined rock caverns is studied by using the transient variably saturated unified pipe network method. The discrete fracture network model is adopted to analyse the effects of the water curtain system on the water sealing effect of the rock caverns in the fractured rock mass. Both construction phase and storage phase are considered in this study. The effects of the horizontal water curtain system, the vertical water curtain system, the length of the horizontal water curtain boreholes, the pressure of the water curtain and the spacing of the water curtain boreholes are systematically analysed.

In Chapter 7, the proposed pipe network model is further extended for the simulation of multiphase flow in fractured porous media. Both 2D and 3D pipe network models are set up for the problem of gaseous phase injection process, where the wetting phase fluid in the fractured porous medium is displaced by a non-wetting phase. A compressible gas phase is chosen as the non-wetting phase in this study. Due to the material differences between the fractures and the porous medium, the capillary pressure relations for these
two media are different. Therefore, the extended capillary condition is incorporated to couple the fractures into the porous medium. The solution of the fully coupled fully implicit nonlinear equations is derived in detail. 2D and 3D numerical examples are presented to validate the numerical method. CO\textsubscript{2} injection processes in the saline aquifers with different pressure and temperature are studied using the extended model. Equation of state for the super-critical state CO\textsubscript{2} is incorporated in the numerical model to calculate the density and viscosity of CO\textsubscript{2} under different conditions. In addition, the effects of fractures on the injection process and CO\textsubscript{2} plume evolution are studied.

Chapter 8 summarizes the achievements of the PhD work and the recommended future research works.
CHAPTER 2. LITERATURE REVIEW

This chapter reviews flow models that describe the physical principles governing the fluid flow in porous media, fractures and fractured rock masses. Derivation of the physical parameters from microscopic to macroscopic is introduced. First, single-phase flow in a porous medium is discussed. Then, the concept of representative elementary volume (REV) is introduced, which is adopted to describe the material property of a porous medium on an averaged scale. Flow in fractures can be considered as a special case of flow in a porous medium with different material property. Fluid flow in a fractured porous medium is considered by deriving the material property in combining the flow characteristics of porous medium and fractures. Based on the knowledge of single-phase flow, two-phase flow model in different media is introduced, which is the extension of the single-phase flow model with taking the consideration of relative permeability and capillary pressure. Coupling of the flow in a porous medium and in fractures is presented.

With the macroscopic models of fluid flow in fractured porous media, the derived mathematical governing equations must be resolved numerically. Numerical models for the simulation of fluid flow in fractured porous media are reviewed. There are generally two classes of fluid flow models in fractured porous media, the continuum models and the discrete models. Their development as well as the advantages and limitations are compared.

2.1 SINGLE-PHASE FLUID FLOW IN FRACTURED ROCK MASS

2.1.1 Single-phase fluid flow in a porous medium

Intact rock mass is a typical kind of porous medium, which consists of solid phase (solid matrix) and pores (voids) as shown in Figure 2.1. The solid phase is the skeleton of the medium and is distributed throughout the porous medium, while the pores are the spaces between solid phases and usually are occupied by gaseous phase or aqueous phase. Only the interconnected pores (effective pores) form the path for fluid flow. The ratio of the pore volume to the bulk volume of the porous medium is termed porosity. Because only the interconnected pore space is accessible, it is important to distinguish
two types of porosity: total porosity $\phi_t$ and effective porosity $\phi_e$. The total porosity uses total pore volume to calculate the porosity, while the effective porosity counts the effective pores only. The values of those two porosities can have large differences.

![Micro structure of porous medium](image)

**Figure 2.1 Micro structure of porous medium**

In pore scale (also microscopic scale, at about $10^{-3}$ m), the fluid flow in the porous medium can be described by the Navier-Stokes equations for low Reynolds numbers (Wang, Leung & Chow 2003). However, the real pore network of the porous medium is difficult to investigate. Moreover, the computation cost is prohibitive for a macroscopic problem. To overcome the limitation of the Navier-Stokes equations, the concept of Representative Elementary Volume (REV) (Bear 1972; Brown, Hsieh & Lucero 2000) is adopted, which is a homogenization method to simplify the model. It obtains the macroscopic value of the variables in a point of the porous medium by averaging it over a certain volume and assigns the new value to that point of the porous medium thus building a hypothetic continuum. With the new derived properties, the fluid flow in the porous medium can be described macroscopically.
Figure 2.2 Conceptualisation of Representative Elementary Volume (REV) 
(modified after Bear (1972))

According to the concept of REV, there is a minimum volume, from which a given parameter becomes independent of the size of the sample. However, the existence of such volume is not always guaranteed (Neuman 1988). The size of the REV for different variables may also be different even though the REV exists. Figure 2.2 indicates the conceptual REV curves, which illustrate the existence of the REV. When the porous medium is very heterogeneous, for example, containing large pores or sparse fractures, it may fail to define the REV, or the REV is very large, which is impractical to use.

In a macroscopic domain, fluid flow in porous medium is governed by the famous Darcy’s law. The Darcy’s law is a macroscopic approximation, which is formulated by the French engineer Henry Darcy in 1856, based on the experiments on vertical water filtration through sand beds (Darcy 1856). The Darcy’s law gives the relationship between the averaged flow velocity \( \mathbf{v} \) and the hydraulic head (piezometric head) \( h \), which has the following form,

\[
\mathbf{v} = -K \nabla h. \tag{2.1}
\]

Here, \( K \) is the hydraulic conductivity tensor. It can be evaluated by

\[
K = k \frac{\rho g}{\mu}, \tag{2.2}
\]
in which, \(k\) is the intricate permeability tensor (absolute permeability tensor). \(\mu\) is the dynamic viscosity of the fluid. \(\rho\) and \(g\) are the fluid density and the gravitational acceleration, respectively.

The averaged flow velocity \(\mathbf{v}\) is also called the Darcy flux, or macroscopic apparent velocity. The hydraulic head \(h\) is the summation of the pressure head \(p/\rho g\) and the elevation head \(z\),

\[
h = \frac{p}{\rho g} + z.
\]  

(2.3)

Here, \(p\) is the fluid pressure. The positive sign before the elevation head implies that the upward direction is the positive direction in the model.

With the substitution of \(K\) and \(h\) in the equation (2.1) by equations (2.2) and equation (2.3) respectively, the Darcy’s law yields the form

\[
\mathbf{v} = -\frac{k}{\mu} (\nabla p - \rho \mathbf{g}),
\]  

(2.4)

in which, \(\mathbf{g} = (0, 0, -g)^T\) is the vector of gravity.

Although initially established empirically, the Darcy’s law has been theoretically derived from the Navier-Stokes equations by many researchers via homogenization technique to give an insight into the nature of the Darcian flow (Neuman 1977; Mikelić 1991). However, the validity of Darcy’s law is subject to certain assumptions. The first assumption is that the flow is laminar. It indicates that viscous forces dominate the fluid movement. The Reynolds number \(Re\) should be less than 1 (maximum up to 10) for the validity of the Darcy’s law (Bear 1972; Fetter 2001). The second assumption is that the flow is Newtonian. The third assumption requires the boundary condition to be non-slip.

To complete the Darcy’s law, the fluid mass conservation equation (continuity equation) is required. Considering a domain of the porous medium \(\Omega \subset \mathbb{R}^3\) with a boundary of \(\partial \Omega\), the mass conservation equation is expressed as

\[
\frac{\partial (\rho \phi)}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \rho q,
\]  

(2.5)
where \( q \) and \( \phi \) are source term and porosity, respectively.

By substituting equation (2.4) into equation (2.5), the equation for the single phase fluid flow in the porous medium is obtained:

\[
\frac{\partial (\rho \phi)}{\partial t} - \nabla \cdot \left( \rho \frac{k}{\mu} \left( \nabla p - \rho g \right) \right) = \rho q , \tag{2.6}
\]

in which, the fluid pressure \( p \) is the only independent unknown, while the fluid density \( \rho \) is a dependent variable or a constant depending on the compressibility of the fluid. The equation is of a parabolic type if the fluid is compressible. In contrast, it is elliptic when the fluid is incompressible. To solve the above equation, certain initial conditions and boundary conditions should be applied.

The initial condition set the fluid pressure at an initial time \((t = 0)\),

\[
p(t = 0) = p_0 , \text{ in } \Omega . \tag{2.7}
\]

Normally, there are two boundary conditions used in the fluid flow problem, Dirichlet boundary condition (first-type) and Neumann boundary condition (second-type). The domain boundary is the union set of the Dirichlet boundary \( \Gamma_D \) and the Neumann boundary \( \Gamma_N \), \( \partial \Omega = \Gamma_D \cup \Gamma_N \). For the Dirichlet boundary, the boundary condition is stated as,

\[
p|_{\Gamma_D} = p_D \text{ on } \Gamma_D . \tag{2.8}
\]

For the Neumann boundary, the boundary condition is given as,

\[
\mathbf{v} \cdot \mathbf{n}|_{\Gamma_N} = q_N \text{ on } \Gamma_N , \tag{2.9}
\]

where \( \mathbf{n} \) is the outward unit normal vector on \( \Gamma_N \), and \( q_N \) is the normal flow rate on \( \Gamma_N \). Equations (2.6) ~ (2.9) give the complete mathematical model for the fluid flow in a porous medium.

### 2.1.2 Single-phase fluid flow in a fracture

Fracture is one of the most important geological structures in the rock masses. Most rock masses possess fractures and other discontinuities. Fractures are broken surfaces...
along which rocks lost cohesion. The term fracture used in this thesis includes other discontinuities, such as cracks, joints, faults, etc. Usually, fractures are of planar geometry. The space between two surfaces (walls) of a fracture may be filled by different materials. Some filling materials, such as clay, can reduce the permeability of the fracture. In contrast, the unfilled (open) fracture is much more permeable than the matrix rock.

Individual fracture planes are of definite extent. The shape of fracture depends largely on the rock type and on its structure. Usually, circular, elliptical and polygonal planes are adopted to simulate a fracture within a 3D model. For an individual fracture, features such as orientation, aperture, asperity and size are used for characterisation. The number of sets, spacing, persistence, fracture density and connectivity etc. can be used to characterise the features of a fracture network (Singhal & Gupta 2010).

Fluid flow in a single fracture has been studied by many researchers (Brown 1987; Witherspoon et al. 1980; Oron & Berkowitz 1998). Mathematically, the flow through a single fracture is commonly analogous to a laminar flow between two smooth parallel plates (Tsang & Witherspoon 1981; Schrauf & Evans 1986) and the averaged flow velocity in a fracture is proportional to the hydraulic gradient,

\[ \mathbf{v} = \frac{a^2 \rho g}{12 \mu} \nabla h, \quad (2.10) \]

where \( a \) is the aperture of the parallel surfaces of the fracture. Thus, the volumetric flux \( Q \) of the fracture can be given as,

\[ Q = \frac{a^3 \rho g}{12 \mu} \nabla h. \quad (2.11) \]

This is the famous cubic law, where the volumetric flux \( Q \) is proportional to the cubic power of the hydraulic aperture of the fracture. Comparing equation (2.10) with equation (2.1), it implies that the Darcy’s law is valid for the parallel plate model. The intrinsic permeability for the fracture is \( k = \frac{a^2}{12} \).
The assumption of the parallel plate model implies that the fracture is an open fracture, and the fracture surface is smooth and parallel. However, the real fracture surfaces are rough and contact each other at discrete points (Brown & Scholz 1985). The roughness of the fracture surfaces can reduce the flow rate and makes the fluid flow in tortuous paths. The channelling feature of flow in a single fracture has been reported and studied by many researchers, such as Tsang & Tsang (1987), Nordqvist et al. (1992). For the valid application of the cubic law in a real rock fracture, the concept of hydraulic aperture should be adopted (Zimmerman & Bodvarsson 1996; Brown 1987). Although the cubic law for the description of flow in fractures is not suitable for all cases, it is still a well-accepted model in the numerical simulation.

2.2 MULTI-PHASE FLUID FLOW IN FRACTURED ROCK MASS

The phenomenon of multi-phase fluid flow in fractured rock masses is much more complicated than the single-phase fluid flow. Other than single-phase fluid flow, which is mainly driven by a pressure force, capillary pressure plays an important role in the multi-phase fluid flow process. Multi-phase flow involves interaction between fluids of different phases. In order to capture the accurate model of multi-phase flow, faithful description of capillary pressure and relative permeability is inevitable.

2.2.1 Conceptual model in pore scale for multi-phase flow

When a liquid phase contacts with a solid phase, the adhesive forces between the two phases make the liquid phase spread over the solid phase, while the cohesive forces within the liquid phase tend to go against the spreading. The ability of a liquid to maintain contact with a solid phase is called wetting, the degree to which can be
measured by wettability. When two immiscible-phase liquids are present in the porous medium, it is important to know which liquid is the wetting phase. Typically, the wettability of water is stronger than the other non-aqueous phase liquid (NAPL) (such as oil), which means water is the wetting phase, while NAPL is the non-wetting phase. However, oil wetting is also possible in the reservoir. In a water-gas system, the water is the wetting phase.

Because the intermolecular forces between liquid phase and solid phase for wetting phase and non-wetting phase are different, there is a pressure jump between these two liquids when they contact each other. The pressure jump across the interface of the two fluids is known as the capillary pressure,

\[ p_c = p_n - p_w, \]  

(2.12)

where the subscript \( c \), \( n \) and \( w \) denote “capillary”, “non-wetting phase” and “wetting phase”, respectively.

According to the capillary theory, the capillary pressure in a porous medium increases when the pore size decreases (Doe 2001). A wetting phase fluid preferentially resides in the smaller pore space, due to high capillary suction forces. In an imbibition process, the smaller pore spaces absorb wetting phase fluid first. In contrast, the smaller pore spaces yield wetting phase fluid last in a drainage process. Fractures in the porous medium are regarded as large pores, which contain no wetting phase fluid until the surrounding small pore spaces reach nearly wetting-phase-saturated. The air-water two-phase flow in fractures can be described by the characteristic components of each phase present in the mixture, ratios of volume and mass, and the homogeneity of the mixture (Indraratna et al. 2003). According to the interaction and structure between air and water, the classification of two-phase air-water flow is presented in Figure 2.4 (Indraratna et al. 2003; Triplett et al. 1999).

Based on the capillary theory, fluid flows in continuously connected pore spaces. Therefore, fractures in porous medium act as “barriers” for the wetting phase fluid when the fractured porous medium is unsaturated. Considering that the fracture surface may be partially contact or mineral-filled, the wetting phase fluid can flow across fracture via these areas (Wang & Narasimhan 1985; Peters & Klavetter 1988). However, some
laboratory and field observations find the actual fluid flow can be much faster than predicted by the capillary theory (Pruess, Faybishenko & Bodvarsson 1999). The process and mechanism of liquid-gas two-phase flow in partially saturated fractured rock masses are not fully understood (Berkowitz 2002). For Detailed discussion of different conceptual models for multi-phase fluid flow in fractured rock masses can be referred to Doe (2001).

**Figure 2.4 Different air-water two-phase flow models in fracture**

#### 2.2.2 Macroscopic model for multi-phase flow

In macroscopic, the sharp interfaces between fluids are not modelled explicitly. Fluids occupy different portions of spaces in the medium. The averaged ratio of a fluid volume to the total pore volume in an REV is the saturation of this fluid. The saturation $S$ of phase $a$ can be expressed as,
\[ S_\alpha = \frac{V_\alpha}{V_{por}}, \quad (2.13) \]

where \( V_\alpha \), \( V_{por} \) are the volume of phase \( \alpha \) and pore volume, respectively. Pore spaces are fully filled with fluids,

\[ \sum S_\alpha = 1. \quad (2.14) \]

Multi-phase flow in porous medium can be described with an extended Darcy’s law (Finsterle 2000; Pruess & Tsang 1990) and the governing equation for phase \( \alpha \) is written as follows,

\[ \frac{\partial (S_\alpha \rho_\alpha \phi)}{\partial t} - \nabla \cdot \left( \rho_\alpha \frac{k_\alpha}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha g) \right) = \rho_\alpha q_\alpha, \text{ in domain } \Omega. \quad (2.15) \]

The above equation indicates that the Darcy’s law holds for each phase. Furthermore, each phase fluid meets the mass conservation condition and is driven by the respective phase pressure. In addition, these fluids are immiscible. Permeability for each phase is termed to be a phase permeability \( k_\alpha \), which is calculated as,

\[ k_\alpha = k k_{ra}, \quad (2.16) \]

where \( k_{ra} \) is a relative permeability, which is a reduction factor for each phase. Because only a portion of pore is available for flow, the phase permeability is smaller than the intrinsic permeability except for the saturated state (equal). Typically, a relative permeability is described as a function of the phase saturation, \( k_{ra} = k_{ra}(S_\alpha) \).

There are two mostly used capillary pressure and relative permeability models for fluid flow in porous medium, which are the Brooks-Corey model (BC model) (Brooks & Corey 1966) and the van Genuchten model (VG model) (Nielsen & Biggar 1986; Liu & Bodvarsson 2001; Li 2004; Van Genuchten 1980). Both the BC model and the VG model are functions of effective (or normalized) saturation of wetting phase \( S_w^e \), which is defined as follows,

\[ S_w^e = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{nr}}, \quad S_{wr} \leq S_w \leq 1 - S_{nr}. \quad (2.17) \]
Here, $S_{wr}$ and $S_{nr}$ are residual saturations of the wetting phase and the non-wetting phase, respectively. The normalized saturation is also termed as an effective saturation. Brooks and Corey proposed a capillary pressure model based on a large number of experimental data (Brooks & Corey 1966), which has the form as,

$$p_e = p_e S_w^{-\frac{1}{\lambda}},$$

(2.18)

where $p_e$ is the non-wetting fluid entry pressure, and $\lambda > 0$ is a pore size distribution index. $\lambda$ has a small value for material with a wide range of pore sizes and has a large value for material with a narrow range of pore sizes. Usually, $\lambda$ has the value from 0.2 to 4. The entry pressure, also called bubbling pressure, is a threshold capillary pressure, above which a non-wetting phase fluid can penetrate the wetting phase fluid saturated medium.

Van Genuchten (1980) adopted a three-parameter capillary pressure model to predict the hydraulic conductivity of unsaturated soil. The model has the form as,

$$p_e = \frac{1}{\alpha} \left[ S_w^{-\frac{1}{\alpha}} - 1 \right]^{\frac{1}{\alpha}}.$$  

(2.19)

Here, parameters $\alpha$, $m$ and $n$ are fitted according to available data. Usually, $m$ chooses the value of $1-1/n$. $\alpha$ is related to the entry pressure.

Figure 2.5 shows the typical curves of the BC capillary pressure model and the VG capillary pressure model. One of the most significant differences between these two models is that the BC capillary pressure model has an entry pressure when the non-wetting phase begins to infiltrate. In the VG capillary pressure model the capillary pressure decreases to zero as the effective saturation of wetting phase approaches one. The BC capillary pressure model was originally developed for drainage processes; therefore, it does not suit the imbibition one (Li & Horne 2002).

Both the BC capillary pressure model and the VG capillary pressure model are derived for porous media. Because of the rough-walled nature of fractures, it is expected that fractures also exhibit capillary behaviour (Firoozabadi & Hauge 1990; Pruess & Tsang 1990; Kueper & McWhorter 1991). Conceptualization of fractures that are similar to
porous media has been applied to fractures in numerical simulations (Pruess & Tsang 1990; Kueper & McWhorter 1991). Experimental researches has been carried out by Firoozabadi & Hauge (1990), Myer et al. (1993), Reitsma & Kueper (1994) and Bertels et al. (2001). The capillary pressure curves of fractures measured by Reitsma & Kueper (1994) were found to be well represented by the BC capillary pressure model. However, the capillary pressure curves obtained by Bertels et al. (2001) using computed tomography (CT) scanning technology showed non-monotonic behaviour. The macroscopic description of the capillary pressure in rock fractures is thus still not fully understood.

In addition to the BC model and the VG model, a number of other models have been proposed, such as Parker & Lenhard (1987), Firoozabadl & Hauge (1990), Huang et al. (1997), Jing & Van Wunnik (1998), Li & Horne (2002), Li (2004) and others.
Figure 2.5 Capillary pressure models of (a) Brooks-Corey model; (b) van Genuchten model

For relative permeability models, the famous Brooks-Corey-Burdine relative permeability model (BCB model) and van Genuchten-Mualem relative permeability model (VGM model) are introduced. The BCB model (Brooks & Corey 1966) is based on the BC capillary pressure model and the Burdine theory (1953), and it has the following formulae,
\[ k_{rw} \left( S_w^e \right) = S_w^e \frac{2+\lambda}{\lambda}, \quad (2.20) \]

\[ k_{rn} \left( S_w^e \right) = \left( 1 - S_w^e \right)^2 \left( 1 - S_w^e \frac{2+\lambda}{\lambda} \right). \quad (2.21) \]

The VGM model (Van Genuchten 1980; Parker, Lenhard & Kuppusamy 1987; Lenhard & Parker 1987) is based on the VG capillary pressure model and the Mualem theory (1976). The VGM model is as follows,

\[ k_{rw} \left( S_w^e \right) = \left( S_w^e \right)^a \left( 1 - \left( 1 - S_w^e \frac{1}{m} \right)^m \right)^2, \quad (2.22) \]

\[ k_{rn} \left( S_w^e \right) = \left( 1 - S_w^e \right)^b \left( 1 - S_w^e \frac{1}{2m} \right)^{2m}. \quad (2.23) \]

Here, \( a \) and \( b \) are fitting parameters accounting for the tortuosity and connectivity with respect to the wetting phase and the non-wetting phase, respectively. Usually, values of 0.5 are used for these two parameters (Van Genuchten 1980; Parker, Lenhard & Kuppusamy 1987; Kuang & Jiao 2011), based on the analysis by Mualem (1976) on 45 soils representing a wide range of texture. \( m \) takes the value of \( 1 - 1/n \).

Figure 2.6 shows the typical BCB model and the VGM model. It is shown from the figure that the increase of \( k_{rw} \) is slower as the increase of \( S_w^e \) when \( S_w^e \) is smaller. In comparison, the increase of \( k_{rn} \) is relatively quicker as the increase of \( S_w^e \) when \( S_w^e \) is smaller. This is because the wetting phase tends to fill pores of small size first before it fills the larger ones. The strong adhesive forces in the small pores make the flow harder than flow in large pores.
In addition to the above relative permeability models, models combining the BC capillary pressure model with the Mualem theory and models combining the VG capillary pressure model with the Burdine theory are also available. For more information about these combinations, refer to Kuang & Jiao (2011).
2.2.3 Two-phase flow model for porous media

Based on equation (2.15), the model for flow of a wetting phase $w$ and a non-wetting phase $n$ in a porous medium domain $\Omega \subset \mathbb{R}^3$ is expressed as,

$$\frac{\partial (S_w \rho_w \phi)}{\partial t} - \nabla \cdot \left( \frac{k_w}{\mu_w} (\nabla p_w - \rho_w \mathbf{g}) \right) = \rho_w q_w, \text{ in } \Omega, \quad (2.24)$$

$$\frac{\partial (S_n \rho_n \phi)}{\partial t} - \nabla \cdot \left( \frac{k_n}{\mu_n} (\nabla p_n - \rho_n \mathbf{g}) \right) = \rho_n q_n, \text{ in } \Omega. \quad (2.25)$$

The saturation and pressure of the wetting phase and the non-wetting phase are coupled as,

$$S_w + S_n = 1, \quad (2.26)$$

$$p_n - p_w = p_c. \quad (2.27)$$

The initial conditions for saturations and pressures are

$$S_w (t = 0) = S_{w0}, \text{ in } \Omega, \quad (2.28)$$

$$S_n (t = 0) = S_{n0}, \text{ in } \Omega, \quad (2.29)$$

$$p_w (t = 0) = p_{w0}, \text{ in } \Omega, \quad (2.30)$$

$$p_n (t = 0) = p_{n0}, \text{ in } \Omega. \quad (2.31)$$

Boundary conditions for saturations are

$$S_w \big|_{\Gamma_{wD}} = S_{wD}, \text{ on } \Gamma_{wD}, \quad (2.32)$$

$$S_n \big|_{\Gamma_{nD}} = S_{nD}, \text{ on } \Gamma_{nD}, \quad (2.33)$$

where, $\Gamma_{wD}$ and $\Gamma_{nD}$ are Dirichlet boundaries for saturations of the wetting phase and the non-wetting phase, respectively.

Boundary conditions for phase pressures are
\[ p_w \big|_{\Gamma_{wD}} = p_{wD}, \text{ on } \Gamma_{wD}, \quad (2.34) \]

\[ p_n \big|_{\Gamma_{nD}} = p_{nD}, \text{ on } \Gamma_{nD}, \quad (2.35) \]

where \( \Gamma_{wD} \) and \( \Gamma_{nD} \) are Dirichlet boundaries for phase pressures of the wetting phase and the non-wetting phase, respectively.

Neumann boundary conditions are expressed as,

\[ \rho_n \mathbf{v}_n \cdot \mathbf{n} \big|_{\Gamma_{wN}} = q_w, \text{ on } \Gamma_{wN}, \quad (2.36) \]

\[ \rho_n \mathbf{v}_n \cdot \mathbf{n} \big|_{\Gamma_{nN}} = q_n, \text{ on } \Gamma_{nN}, \quad (2.37) \]

where \( \Gamma_{wN} \) and \( \Gamma_{nN} \) are the Neumann boundaries for the wetting phase and the non-wetting phase, respectively.

Equations (2.24) ~ (2.27) are the governing equations for two-phase flow in porous media, which have four unknowns: \( S_w, S_n, p_w \) and \( p_n \). Because of the coupling nature of saturation and pressure of the two phases, there are actually two independent unknowns out of four. With the boundary conditions and initial conditions from equation (2.28) to equation (2.37), the two-phase governing equations can be solved.

### 2.2.4 Two-phase flow model for fractured porous media and the interface conditions

Fracture can be regarded as a kind of porous medium, which has large porosity and permeability. The Darcy’s law is assumed to be held for two-phase flow in fractures. Therefore, equations (2.24) ~ (2.37) also are valid for two-phase flow in fractures. The flow in fractured porous media \( \Omega \) can be decomposed of flow in the matrix subdomain \( \Omega^m \) and flow in the fracture subdomain \( \Omega^f \) (Monteagudo & Firoozabadi 2004), where \( \Omega = \Omega^m + \Omega^f \).

As noticed, fractures and rock matrix are two kinds of materials with different properties. It is reported that the differences between the absolute permeabilities of rock matrix and fractures have several orders of magnitude (Wu et al. 2002). Due to the
drastic differences of material properties (material discontinuity) between fractures and rock matrix, the interaction between matrix and fractures should be carefully treated.

In order to correctly model physical behaviour between matrix and fractures, appropriate interface conditions are required. The first condition is the mass conservation for both the wetting phase \( w \) and the non-wetting phase \( n \) across the interface, which is expressed as (Van Duijn, Molenaar & De Neef 1995),

\[
\rho_\alpha \mathbf{v}_\alpha \cdot \mathbf{n}|_{\Gamma_+^{mf}} = \rho_\alpha \mathbf{v}_\alpha \cdot \mathbf{n}|_{\Gamma_-^{mf}}, (\alpha = w, n),
\]

where \( \Gamma_+^{mf} \) and \( \Gamma_-^{mf} \) are the upstream side and the downstream side of the fracture-matrix interface, respectively.

The second interface condition is the capillary pressure condition. There are two kinds of capillary pressure conditions depending on the capillary pressure model chosen. For the displacement of a non-wetting phase by a wetting phase, there is no entry pressure. Therefore, the VG style capillary pressure model can be used. The capillary pressure condition assumes that the fracture capillary pressure is equal to the matrix capillary pressure at the interface, which is called the capillary pressure continuity condition (Monteagudo & Firoozabadi 2004; Firoozabadi & Hauge 1990; Van Duijn, Molenaar & De Neef 1995),

\[
p_c^f(\Sigma_w^f) = p_c^m(\Sigma_w^m).
\]

As shown in Figure 2.7a, for a given capillary pressure at the interface, the saturations of wetting phase on both sides of the interface can be calculated. Although the capillary pressure is continuous, the saturations of the wetting phase in fracture and matrix are discontinuous.
For the displacement of the wetting phase by a non-wetting phase, the BC type capillary pressure model can be used, where there is an entry pressure for the non-wetting phase invading to the wetting phase saturated media. According to the Leverett scaling (Leverett 1941), a lower permeability yields a higher entry pressure. For the drainage process, the capillary pressure is not always continuous. As illustrated in Figure 2.7b,
there is a threshold saturation $S_w^*$ for fractures such that the continuous capillary pressure can only be achieved if the wetting phase saturation in the fracture $S_w'$ is less than or equal to $S_w^*$ (Van Duijn, Molenaar & De Neef 1995). The threshold saturation is calculated by

$$S_w^* = \left( p_c' \right) \left( p_c^m \right),$$

(2.40)

where, $p_c^m$ is the entry pressure of the matrix.

When $S_w' > S_w^*$, the wetting phase saturation in the matrix is 1. There is no definition of capillary pressure in the matrix. Therefore, the capillary pressure is discontinuous. This kind of capillary pressure condition is called the extended capillary pressure condition (Bastian 2014; Reichenberger et al. 2006; De Neef & Molenaar 1997; Van Duijn, Molenaar & De Neef 1995), which is expressed as,

$$p_c^m \left( S_w^m \right) = p_c' \left( S_w' \right), \text{ if } S_w' \leq S_w^*,$$

(2.41)

$$S_w^m = 1, \text{ if } S_w' > S_w^*.$$  

(2.42)

### 2.3 UNSATURATED WATER FLOW IN FRACTURED ROCK MASSES

Unsaturated water flow which takes water and air as its fluid pair is a common kind of two-phase flow. The unsaturated water flow assumes that the air phase is continuously distributed in pores. Furthermore, it also assumes that the air phase is connected to the atmospheric air and it is more mobile than the water phase (Szymkiewicz 2012). Therefore, the pressure of the air phase can be considered equal to the atmospheric pressure. The equation of the air phase thus can be eliminated. The original two-phase air-water flow is simplified to consider the water flow only, which can be described by the famous Richards’ equation (Richards 1931).

The geological disposal of high-level nuclear waste has attracted intensive research interest in investigating flow and transport processes in unsaturated fractured rocks. Due to the heterogeneity of fractures, water tends to flow through the path of least resistance. The phenomenon that flow is focused along these preferential paths is known as flow
focusing (Zhang et al. 2004). Flow focusing phenomena and discrete flow characteristics are considered to be of significant importance to potential repository performance (Pruess 1999). Tokunaga and Wan found that the film flow could be an important mechanism at low fracture saturation (Tokunaga & Wan 1997). Glass et al. (1996) demonstrated that gravitational instability and aperture heterogeneities could cause fingering flow, which is the main flow mechanism for vertical unsaturated fracture flow. Su et al. (1999) studied intermittent flow behaviour by laboratory experiments. Liu et al. (1998) proposed an active fracture model to simulate unsaturated flow in fractured rock. Although many research works have been carried out, flow in the unsaturated fractured rock is still poorly understood, which is mainly because of technical difficulties in observing flow phenomena in fractured rock as well as accurately describing the physical processes (Zhang et al. 2004).

2.4 NUMERICAL MODELS FOR FLUID FLOW IN FRACTURED ROCK MASSES

The presence of a large number of irregular fractures in different scales makes the numerical simulation of fluid flow in fractured rock mass a very challenging task. Different numerical models have been proposed to solve the heterogeneous and high nonlinear system. Generally, these numerical models can be categorised into two classes, continuum models and discrete models. In the continuum models, the fractures in the fractured porous media are homogenised into equivalent continuum media based on the regularity assumption of the fractures. In comparison, discrete models explicitly treat fractures as “local” fracture continuum, which are gaining more research attention.

2.4.1 Continuum models

Based on the number of continuum derived from the homogenisation process, the continuum models can generally be categorised into single continuum models and multi-continuum models. The single continuum model is the simplest one, which homogenises the fracture network and the rock matrix into an equivalent porous medium with appropriate effective parameters. The single continuum model is also called the equivalent porous medium model, while the multi-continuum models treat the fracture network and the rock matrix separately. The fracture network alone is homogenised into an equivalent porous medium, which interacts with the rock matrix.
In 1960, Barenblatt et al. (1960) first introduced the concept of the double-porosity model (or dual-porosity model), which is one kind of the multi-continuum model. The fractured rock is modelled as two overlapped porous media, i.e. the rock matrix and the fracture network. Fluid flow takes place in both of the two porous domains, which change fluid via an exchange term. Fluid flow in each domain is modelled by the classical continuum method. Obviously, the material and hydraulic properties of the two porous domains are quite different. The fracture domain has a high permeability and a low storage capacity, while the matrix domain has a low permeability and a relatively high storage capacity. The governing equations of the system are as follows (Barenblatt, Zheltow & Kochina 1960),

\[
\phi^m C^m \frac{\partial p^m}{\partial t} - \nabla \cdot \mathbf{v}^m - q^{mf} = 0
\]  

\[
\phi^f C^f \frac{\partial p^f}{\partial t} - \nabla \cdot \mathbf{v}^f + q^{mf} = 0
\]

where \( C \) is the total compressibility. \( q^{mf} \) is the matrix-fracture interchange term, which is represented by a pseudo-steady state relation,

\[
q^{mf} = \frac{\sigma k^m}{\mu} (p^m - p^f)
\]

Here, \( \sigma \) is a shape factor that controls exchange rate.

Warren & Root (1963) proposed the well-known sugar cube model, in which the matrix domain is separated by the orthorhombic continuum of fractures. In this model, fluid flow between matrix blocks is neglected. The matrix continuum only acts as a tank to store fluid, and it changes fluid with the fracture continuum. The model has simple regular blocks, which helped the understanding of fluid flow in uniform fracture networks. However, the intrinsic complexity of fractures limits the application of the model. Kazemi (1969) and Najurieta (1980) modified the double-porosity model considering the transient fluid exchange between fracture and matrix. Belani (1988), Johns & Jalali (1991) and Spivey & Lee (2000) extended the double-porosity model to include variable matrix block sizes to make the models more realistic. Kazemi et al. (1976) and Rossen (1977) extended the double-porosity model to simulate multi-phase
flow in fractured rock mass. Since then, more double-porosity models are developed and used in the reservoir simulations. Although refined by many researchers, the double-porosity models still contain too many adjustable parameters, some of which have no clear physical meaning and are difficult to estimate (Singhal & Gupta 2010).

With the development of the double-porosity models, many improvements have been made to the models, such as gravity effects on the transfer between matrix and fracture (Gilman & Kazemi 1983; Sonier, Souillard & Blaskovich 1986), matrix block subdivision (Saidi 1983; Gilman 1986) and the multiple interacting continua (Pruess 1985).

The double-porosity models were later extended to triple-porosity models. There are two kinds of triple-porosity models. One kind is to consider one fracture continuum with two matrix systems with different properties (Liu 1981; Abdassah & Ershaghi 1986). The other kind model is to consider two fracture continuum systems with one matrix continuum (Kazemi 1969). More triple-porosity models are discussed by Al-Ahmadi & Wattenbarger (2011).

Another improvement for the double-porosity models is to consider the matrix-to-matrix flow, which is called double-permeability model (Blaskovich et al. 1983; Hill & Thomas 1985; Dean & Lo 1988).

The continuum models suffer some drawbacks and limitations. Firstly, the continuum models are based on the concept of REVs, which are frequently not satisfied in natural rock fracture networks (Neuman 2005). Secondly, the equivalent fracture parameters in the continuum models are derived from the inverse modelling (Bandurraga & Bodvarsson 1999; Van den Daele et al. 2007) and consequently may not be able to represent the characteristics of physical fractures (Reeves et al. 2014). Thirdly, the homogenisation of individual fracture properties into a constant one over-neglects the important geometrical characteristics of the fracture network (Liu, Doughty & Bodvarsson 1998; Renshaw 1999; Dreuzy, Davy & Bour 2001; Reeves et al. 2013; Neuman 2005).
2.4.2 Discrete models

The homogenised fracture continuum in the continuum models makes the numerical simulation of fluid flow in fractured rocks much easier. However, fractures in the natural geological material are highly random and heterogeneous in size, shape, spacing and aperture (Odling et al. 1999; Gillespie et al. 1993). The inherent uniformity assumptions of the double-porosity models have large discrepancies with the natural fractured rocks. Therefore, discrete models are developed to simulate each individual fracture explicitly, thus reducing the non-physical abstractions in continuum models.

Compared to the continuum models, discrete models represent the “exact” geometry of fracture network and account explicitly for the influence of individual fractures. Furthermore, fracture geometry is adaptable and updatable other than the orthogonal fracture geometry in the double-porosity models. More importantly, the fluid exchange term in the double-porosity models is not required in the discrete models. The fluid exchange between matrix and fractures is more straightforward. However, the accurate geological information of the fracture network is difficult to obtain. The computation efficiency also is another issue to be solved.

According to the treatment of the matrix, the discrete models can be categorised into discrete fracture network (DFN) models and hybrid models (also called the discrete fracture method). The DFN models only concern fluid flow in fracture networks, and they neglect the influence of the rock matrix. They assume that the hydraulic behaviour can be represented by the conductivity of individual discontinuities. Therefore, it is only valid for rocks with relatively low permeability. In contrast, the hybrid models consider the fluid flow in the discrete fracture network together with the flow in the rock matrix.

Discrete Fracture Network Models

The DFN models idealize fractures as 2D discrete polygons in 3D space (1D line segments in a 2D domain). Each fracture is assigned with its own individual hydraulic properties. The geometry and hydraulic properties of fracture networks are obtained from site survey. However, the exact geometry and connectivity of the fracture network is hard to investigate. Usually, statistical models are generated with the data collected using the scan-line method, window mapping, borehole logging, image processing and statistical analysis (Mauldon, Dunne & Rohrbaugh 2001; Mauldon 1998; Paillet 1993;
The probabilistic distribution models of the fracture network relate to the parameters of density, spacing, orientation, location, size and aperture (Mauldon 1998; Xu & Dowd 2010). The DFN models adopt the direct representation of the fracture network with the statistical information, therefore the geometrical structure of fracture network can be very complex. Figure 2.8 shows a typical 3D realization of a fracture network in a cubic region using elliptic disk fractures.

**Figure 2.8 Typical 3D realization of fracture network and its Delaunay mesh**

The DFN models are first introduced for homogenising fractured media (Long et al. 1982). Later, the DFN models have been applied to understanding the flow behaviour in complex fracture network and permeability upscaling (Leung & Zimmerman 2010; Davy et al. 2006; de Dreuzy, Davy & Berkowitz 2001; Baghbanan & Jing 2007; de Dreuzy, Méheust & Pichot 2012). Mostly, the DNF models are applied in 2D due to the difficulties in generating meshes for complex 3D fracture networks (Kalbacher et al. 2007; Maryška, Severýn & Vohralík 2005; Vohralík, Maryška & Severýn 2007).

For a 2D fracture flow problem, a fracture network is a system consisting of connected line segments. The numerical solution of the line network is straightforward, which can be treated as a pipe network (Priest 1993; Li, Xu & Ma 2014). The numerical problem is to solve equations describing mass balance at intersections. The numerical simulation of fluid flow in 3D DFN is a more demanding task. The complex flow geometry makes the problem computationally intensive. There are different numerical models proposed to solve the fluid flow in DFN, such as the finite element methods (FEM) (Kolditz 1995; Golder Associates Inc. 2010; Kalbacher et al. 2007), the boundary element methods
(BEM) (Elsworth 1986a; Elsworth 1986b), the discrete element methods (DEM) (Itasca 2004; Jing, Ma & Fang 2001; Ben et al. 2012; Koyama et al. 2011), the extended finite element methods (XFEM) (Berrone, Pieraccini & Scialò 2013), the equivalent pipe models (EPM) (Dershowitz & Fidelibus 1999; Nordqvist et al. 1992; Cacas et al. 1990) and other hybrid methods (Erhel, Dreuzy & Poirriez 2009; Vu, Pouya & Seyed 2013). These methods are incorporated in some software such as FracMan (Golder Associates Inc. 2010), RockFlow (Kalgacher et al. 2007), UDEC (Itasca 2004), 3DEC, ROCMAS (Noorishad 2010) and NAPSAC (Hartley, Holton & Hoch 1998).

Hybrid models

Hybrid models take into account the permeability of the rock matrix and consider the interaction between fracture network and matrix blocks. Wilson & Witherspoon (1974) used 2D elements for simulation fluid flow in both rock matrix and fractures, which is one of the earliest papers of hybrid models. The concept of lower-dimensional elements for the modelling of fractures makes the models easier to construct (Gureghian 1975; Baca, Arnett & Langford 1984; Woodbury & Zhang 2001).

The governing equations of the hybrid method can be solved with different numerical methods. One of the most commonly used methods is the finite element method (FEM). Noorishad & Mehran (1982) used an FEM to simulate transient solute transport by dispersion and convection in 2D fractured porous media. Baca et al. (1984) presented an FEM for single-phase flow with heat and solute transport. The FEM also has been extended to 3D to model single-phase flow in reservoirs (Juanes, Samper & Molinero 2002). Moreover, the FEM has been extended to model two-phase fluid flow in fractured reservoirs considering gravity and capillary effects (Kim & Deo 2000; Karimi-Fard & Firoozabadi 2003). The FEM does not guarantee the local mass conservation, therefore the FEM will cause unphysical results for multi-phase flow. Researchers (Hoteit & Firoozabadi 2006; Rivié et al. 2000) proposed a mixed finite element and discontinuous Galerkin method to solve the mass conservation error for multi-phase flow. However, these methods are computationally expensive.

Another widely used numerical method is the control volume method (CVM) or finite volume method (FVM). The origin of the CVM can be traced back to Varga (1962) who referred to the CVM as the integral method. The integral method first was used to solve
heat transfer problems in heterogeneous isotropic multidimensional regions (Stark 1956). Later, the CVMs were applied in computational fluid dynamics (Narasimhan & Witherspoon 1976; Shahcheraghi & Dwyer 1998; Chan & Anastasiou 1999; Sohankar, Norberg & Davidson 1999). The CVMs apply integration of differential equations over control volumes constructed in a mesh system. Each control volume is associated with a node, where the unknowns of the control volume are evaluated. The CVMs have some distinct advantages over the conventional FEMs. Firstly, they are locally mass conservative. Secondly, the upwind criterion used in the CVMs is based on the flow direction between adjacent control volumes, which has a clear physical interpretation. Due to the advantages, the CVMs have been typically used to simulate multi-phase immiscible fluid flow in porous media (Helmig 1997). The convergence of the CVMs for two-phase immiscible flow in porous media has been proved by Michel (2003). Karimi-Fard et al. (2004) presented a general simulator to model multi-phase flow through 2D and 3D fractured porous media using CVM.

The standard CVM uses a finite difference formulation to evaluate the potential gradient at the interface between adjacent control volumes, therefore it is also called the control-volume finite-difference method. The standard CVM only evaluates the potential in the node. However, the values of potential in the element can also be obtained by interpolation using the same types of shape functions as those in the finite element methods. The potential gradient at the interface of the adjacent control volumes can be evaluated by the interpolated potential. This CVM is called the control-volume finite-element method (CVFEM). Bogdanov et al. (2003b) presented a 3D single-phase discrete fracture simulator to study pressure drawdown well tests in fractured porous media by using the CVFEM. The CVFEMs have also been used to simulate multi-phase flow in discrete fractured porous media (Geiger-Boschung et al. 2009; Monteagudo & Firoozabadi 2004; Reichenberger et al. 2006; Matthäi, Mezentsev & Belayneh 2007; Bastian & Helmig 1999).

Other models

The fracture continuum methods (FCM) (Botros et al. 2008; Reeves et al. 2014; Reeves, Benson & Meerschaert 2008; Parashar & Reeves 2011; Neuman 1988; Svensson 2001; Langevin 2003) map discrete fractures onto a finite difference grid. In this way the
fractures are locally homogenised as a continuum, whose material property is affected by the grid resolution. It is virtually a continuum method, while it preserves the relatively detailed geometry of the fracture network and maintains the simplicity of continuum methods. However, the connectivity of the fracture network depends on mesh size, which may cause unrealistic overestimation.

2.5 CONCLUSION

Fluid flow in fractured rock mass is of critical importance in engineering application. The macroscopic fluid flow theory in fractured rock mass that is highly important in engineering practice, has been developed from macroscopic porous flow theory. It treats fractures in rock as special porous materials. The macroscopic theory mainly concerns about the averaged flow properties and the REV methods are generally used but these are only valid to a limited extent because of the existence of highly irregular fractures. The tremendous difference in material properties between fractures and matrix rock also contributes the heterogeneity of the fractured rock mass. Besides, the size of fractures can be in a great range, which further complicates the situation. The fluid flow in fractured rock mass can behave discontinuously and the influences of fractures should be considered separately.

A wide variety of numerical approaches has been proposed to simulate the complex flow process. Generally, these approaches can be divided into two categories, namely the continuum models and the discrete models, which differ in their representation of the fracture system. The former ones are based on the regularity assumption of the fractures. Therefore, fracture network is homogenised into an equivalent porous medium, which interacts with the rock matrix. It takes the advantages of simplicity in representing fracture network. However, its limitations are also obvious. It oversimplifies the geometrical characteristics of the fracture network and is not able to represent the physical fractures. The later ones explicitly represent fractures in rock to reduce the non-physical abstractions in continuum models. The treating of fractures is much straightforward thus avoiding artificial assumptions. However, the accurate geological information of the fracture network is not always available. It also suffers from the low computation efficiency.
CHAPTER 3. UNIFIED PIPE NETWORK METHOD FOR SIMULATION OF FLUID FLOW IN FRACTURED POROUS MEDIUM

3.1 INTRODUCTION

Fractures are important geological structures that affect both mechanical and hydraulic behaviour of fractured rock mass. Researchers have proposed a number of numerical methods to reflect the influences of fractures on fluid flow in natural rock mass, among which the continuum models (Barenblatt, Zheltow & Kochina 1960; Warren & Root 1963; Blaskovich et al. 1983) have been widely used due to the simplicity of treating the fracture network as a homogeneous continuum. However, the continuum models inherently suffer the drawbacks such as the over homogenization of characteristics of individual fractures (Reeves et al. 2014) and too many nonphysical adjustable parameters caused by the homogenization and the introduction of the fluid interchange term between fractures and the matrix (Singhal & Gupta 2010).

For the accuracy of catching the complex geometry of fractures and their spatial connectivity, the discrete models should be applied. Because of the relative low permeability of the matrix block of hard rock, only the permeability of the intricate fracture network is considered in many cases. For rocks with high porosity and permeability, consideration of the interaction between the fractures and rock matrix is necessary. The difficulties of implementation of the discrete model mainly lie in the complexity of the geometry of fracture networks and the interaction between the two materials, the fractures and the rock matrix, which have drastically different material properties. Furthermore, the computational cost is another important issue when the number of the fractures increases.

Most discrete models implement relatively sophisticated mathematical models to achieve accurate results. The FEM (Kolditz 1995; Noorishad & Mehran 1982; Baca, Arnett & Langford 1984; Kim & Deo 2000; Karimi-Fard & Firoozabadi 2003) is one of the representative methods to solve the governing equation describing the flow in fractures and matrix. To guarantee mass conservation for multi-phase flow, a mixed finite element and discontinuous Galerkin method (Hoteit & Firoozabadi 2006; Riviè et
al. 2000) is required. However, these methods are computationally expensive. It is also not easy for computer programming.

To overcome the difficulties encountered in the discrete models, researchers have proposed some simplified models to conduct simulations. One of the simplified methods is to treat disk-like 2D fractures as mono-dimensional pipes connecting fracture centres with connected fractures (Cacas et al. 1990; Nordqvist et al. 1992; Dershowitz & Fidelibus 1999). This method simplifies the mesh generation of each fracture and reduces the size of the problem. However, the flow pattern in the fracture is unrealistic and the choice of the pipe transmissivities is difficult. It is also impossible to evaluate the uncertainties (Erhel, Dreuzy & Poirriez 2009). What is more, it is unrealistic to couple the flow in fractures with flow in matrix when necessary.

The fracture continuum method is another simplification, which maps fractures onto a finite difference grid (Neuman 1988; Svensson 2001; Reeves, Benson & Meerschaert 2008; Botros et al. 2008). It locally homogenises the fracture property as continuum grids. In this way, the discontinuous problem is converted to the continuous problem, which preserves certain geometrical characteristics of the fracture network. However, the accuracy of the method is highly depended on the resolution of the grids. A coarse grid will overestimate the connectivity of the fracture network, which results in unrealistic prediction.

Li (Li, Xu & Ma 2014) and Xu (Xu, Ma & Li 2014) have proposed the graph-theoretic pipe network method (GPNM) for water flow simulation in discrete fracture networks and porous media, respectively. The features of the method have been demonstrated in some 2D static saturated single-phase flow problems. The GPNM models the line fractures as oriented and weighted pipes. Porous media also can be equivalent to oriented and weighted pipes (Xu, Ma & Li 2014). It is convenient to treat porous media in a similar way to the fracture network, especially when these two materials are coupled. However, the GPNM has some drawbacks. Firstly, the pipes in the GPNM should be oriented, and this becomes time-consuming when labelling all the directions of pipes in a large complex fracture network. Secondly, each boundary node should be connected to a reference node to form boundary pipes for the purpose of establishing the unified governing equation, and this is not so flexible to apply boundary conditions.
Thirdly, the governing equation incorporates the unknowns of the pipe flow rate, which largely increases the problem size. Actually, when the pressures of the two ends of the pipe are obtained, the flow rate of the pipe can easily be calculated. What is more important, the incorporation of pipe flow rate into the governing equation makes the coefficient matrix unsymmetrical and ill-conditioned.

In this chapter, a unified pipe network model (UPNM) is proposed to simulate fluid flow in discrete fracture networks and porous media. The UPNM is conceptually quite simple, which models fractures and porous media as connected pipes in domain space. The equivalences of fracture pipes and matrix pipes are derived, which is the conceptualization of fractures and porous medium. The fractured porous media can be modelled by the systematic assembly of fracture pipes and matrix pipes. The model building process is straightforward. The proposed model well balances the accuracy and the simplicity in flow simulation. Some numerical cases are presented to demonstrate the feasibility and accuracy of the UPNM.

3.2 DERIVATION OF UNIFIED PIPE NETWORK MODEL

3.2.1 Conceptualization of unified pipe network of fractured porous rock

Figure 3.1 Conceptualization of fractured rock mass into pipe network
Natural fractured rock mass contains a large number of fractures in different sizes. The fractures are large voids, which link each other to form main flow conduits in the rock mass. The rock block is regarded as a kind of porous medium, which contains interconnected small pores for fluid flow. These spatial interconnected pores and voids are conceptualised as interconnected pipes with different equivalent hydraulic parameters. In this way, the fractured rock mass is treated as a pipe lattice system or a pipe network system, which is shown in Figure 3.1. This conceptualization can significantly simplify the treatment of the fractured rock mass, which becomes the assembly of the weighted flow pipes. Multi-scale fractures and rock matrix can be uniformly handled. Both 2D and 3D problems can be constructed by the discrete 1D-pipe segments. There are two types of pipes, namely fracture pipes corresponding to fractures and matrix pipes corresponding to rock matrix. The equivalent hydraulic parameters of the pipes can be derived by reconstruction of the locally homogenized macroscopic material, which will be discussed in the following sections.

This conceptualization is based on the exact information of fracture network as well as the permeability of the rock matrix. Admittedly, it is not always possible to obtain the exact information of the fractures in rock under current survey technology. However, some detailed statistical information of the fractures such as the distributions of the dip, dip direction, size and aperture, can be collected without much difficulty, based on which the statistical equivalent UPNM models can be built to conduct Monte Carlo simulation if necessary.

### 3.2.2 Fluid flow in a unified pipe network

In a unified pipe network system that includes fracture pipe network and matrix pipe network, fluid flow occurs through the connected pipes. Fluid flow in each pipe follows certain flow law that is termed pipe flow law. Recent studies have shown that when Reynolds number is less than 1 (maximum to 10) (Bear 1972; Zimmerman et al. 2004), or Forcheimer number is less than 1.3 (Ranjith & Viete 2011), the Darcy’s law is valid for fracture flow. This suggests a relatively low flow velocity. In this study, the Darcy’s law is assumed to be applicable for the flow in both fractures and matrix. Therefore, the pipe flow law for both the fracture pipe and the matrix pipe has the following unified form,
\[ Q = K \Delta \Phi, \]  \hspace{1cm} (3.1)

where \( Q \) is the flow rate in the pipe. \( K \) is a conductance coefficient of the pipe, which will be discussed in the following sections. \( \Phi = p + \rho gh \) is the flow potential at the end node of the pipe. \( p \) and \( \rho \) are pressure and density of the fluid respectively. \( g \) is the gravitational acceleration. \( h \) is the elevation head of the fluid.

In each node, fluid flow obeys a nodal law, which represents the mass conservation condition. For a saturated incompressible fluid flow, the nodal law is expressed as,

\[ \sum_{i=1}^{N_i} Q_{n_i} = Q_S, \]  \hspace{1cm} (3.2)

Here, subscript \( N_i \) denotes the total number of pipes connecting to the node. \( Q_{n_i} \) is the flow rate of pipe \( n_i \). Flow out of the node has the positive sign. \( Q_S \) is the source term of the node \( i \).

By instituting Equation (3.1) into Equation (3.2), the governing equation for the unified pipe network flow can be obtained,

\[ \sum K \Delta \Phi = Q_S, \text{ for all nodes.} \]  \hspace{1cm} (3.3)

Rearrange Equation (3.3) and the governing equation can be written in a matrix equation form,

\[ [K]_{n \times n} \{ \Phi \}_{n \times 1} = \{Q_S\}_{n \times 1} \]  \hspace{1cm} (3.4)

For a \( n \)-node fracture pipe network, the unknown is a \( n \)-element pressure vector. The right hand side is a source vector. The coefficient matrix \([K]_{n \times n}\) is defined as a conductance coefficient matrix, which contains the nodal connection information as well as the hydraulic characteristics of all fracture pipes. The building of the governing equations and the coefficient matrix is illustrated in Figure 3.2. The coefficient matrix of the UPNM is a symmetric and positive-definite matrix. The value of the main diagonal entry in the matrix is the summation of the conductance coefficients of the pipes that link to the node. Taking node \( i \) for example, the entry \( K_{ii} \) can be calculated as
\[ K_{ii} = \sum K_{(i,j)}. \]  

(3.5)

Node \( j \) is the node that connects to node \( i \) through pipe \((i, j)\).

The value of the off-diagonal entry is the negative conductance coefficient of the corresponding pipe. For example, if there exists a pipe \((i, j)\) in the pipe network, then the value of the entry \( K_{ij} \) in \( i \) th column \( j \) th row is \(-K_{(i,j)}\). If there is no such pipe, the value of the entry is 0.

![Figure 3.2 The building of the governing equations for the UPNM](image)

Complemented by boundary conditions, the governing equation can be solved numerically without much effort. The application of the boundary conditions is straightforward. The Dirichlet boundary condition is applied by assigning boundary conditions such that

\[
p \big|_{\Gamma_D} = p_D \quad \text{on} \quad \Gamma_D, \]  

where \( \Gamma_D \) is the Dirichlet boundary.

The Neumann boundary condition is as follows

\[
Q \big|_{\Gamma_N} = Q_N \quad \text{on} \quad \Gamma_N, \]  

where \( \Gamma_N \) is the Neumann boundary.

There are two kinds of boundary conditions for the unified pipe network, the Dirichlet boundary condition and the Neumann boundary condition. The Dirichlet boundary condition is stated as

\[
p \big|_{\Gamma_D} = p_D \quad \text{on} \quad \Gamma_D, \]  

(3.6)

where \( \Gamma_D \) is the Dirichlet boundary.

The Neumann boundary condition is as follows

\[
Q \big|_{\Gamma_N} = Q_N \quad \text{on} \quad \Gamma_N, \]  

(3.7)

where \( \Gamma_N \) is the Neumann boundary.

Complemented by boundary conditions, the governing equation can be solved numerically without much effort. The application of the boundary conditions is straightforward. The Dirichlet boundary condition is applied by assigning boundary
pressures to the boundary nodes. The Neumann boundary condition is applied by assigning the Neumann boundary nodes with source terms.

For saturated flow problems, nodal pressure is the only unknown in the model. The flow rate in a pipe can be readily obtained by using the pipe flow law after knowing the pressures of its two end nodes.

### 3.2.3 Pipe network model with 1D fracture pipes

![Figure 3.3 Equivalence of 1D fracture pipe](image)

Actually, discrete fractures in 2D fluid flow problem are commonly treated as 1D segments (Priest 1993; Li, Xu & Ma 2014), due to their negligible thicknesses compared to their lengths. As shown in Figure 3.3, a natural rough fracture can be homogenized into a parallel plate model with an equivalent aperture \( a \). According to the parallel plate model (Tsang & Witherspoon 1981; Schrauf & Evans 1986), the volumetric flow rate \( Q \) in the fracture obeys the cubic law,

\[
Q = \frac{a^3}{12\mu} \frac{\partial p}{\partial l}.
\]  

Here, \( l \) is the length of the fracture. \( \mu \) is the dynamic viscosity of fluid.

The 2D fracture segment can further be simplified as 1D line called fracture pipe, denoted with \( fp(i, j) \), which is associated with two end nodes \( (i, j) \). Each node represents a half volume of the fracture, which is \( al/2 \). If the fluid pressures are known in the end nodes, the flow rate in the pipe can be evaluated by the difference equation,
\[ Q_y = K_{fp(i,j)} (p_i - p_j), \quad (3.9) \]

where \( K_{fp(i,j)} \) is defined as an equivalent conductance coefficient of the pipe, which can be calculated by

\[ K_{fp(i,j)} = \frac{a^3}{12 \mu l_y}. \quad (3.10) \]

Equation (3.9) is the pipe flow law for the fracture segment. If gravitational effect is considered, the pressure \( p \) in Equation (3.9) can be replaced by flow potential \( \Phi \).

Fracture network in a 2D domain is modelled as the assembly of the fracture pipes. The fluid pressures are assumed continuous in the connecting nodes. In addition, the fluid should be conserved in each node. With the governing equation (3.3), the fluid flow in a fracture network can be readily solved. If higher accuracy is required, such as a transient flow problem, each fracture segment can be further divided to more fracture pipes.

### 3.2.4 Matrix pipe network model in 2D porous media

In microscopy, fluid flow in porous medium can be modelled by the network models (Bryant, King & Mellor 1993; Fatt 1956). The structure of the network can be constructed according to the X-ray scanning (Coker, Torquato & Dunsmuir 1996; Spanne et al. 1994) or stochastic algorithm (Adler & Thovert 1998). The pore-scale network model uses a graph of connected nodes to represent the pore space of the porous medium. The nodes in the graph correspond to the pore bodies in the porous medium, and the line segments link the nodes corresponding to pore throat. The model can be used to predict the macro properties of the porous medium according to its microstructure. However, it is not suitable for simulation of fluid flow macroscopically, because of the high computation costs.

For efficient simulation of fluid flow in a macroscopic porous medium, the macroscopic porous medium is reconstructed by using an equivalent pipe network graph, in which the volume of the porous medium is discretised and represented by nodes. Besides, the fluid flow in the porous medium is modelled by equivalent pipe flow, which transports fluid between connected nodes. Because the macroscopic porous medium is a homogenised material, the macroscopic reconstruction of the medium is more flexible...
than the microscopic reconstruction of the porous medium, and this is influenced by the geometry and topology of the connected pores. Figure 3.4 shows reconstructions of a porous medium using structured and unstructured pipe networks. Unit thickness is assumed for the 2D domain. Flow in the porous medium can be discretised into flow in the pipe network graph. All unknowns are assigned on the discrete nodes.

**Figure 3.4 Reconstruction of 2D porous medium using pipe network**

The unstructured grid is capable of modelling domain with complex geometry. Therefore, it is preferable to use the unstructured triangular grid. Xu et al. (2014) have derived the equivalent parameters of pipes of triangular grids. However, volume concentration into separated nodes has not been considered in his work, and this is important for unsaturated flow.
For the derivation of the parameters of pipes constructing a triangular grid, the circumcentre $o$ of the triangle is selected as its division points as shown in Figure 3.5. The line linking the circumcentre and the mid-point of each edge is perpendicular to the edge and these perpendicular bisectors divide the triangle into three parts: polygon $oeif$, polygon $ofjg$, polygon $ogme$. The three polygons form the Voronoi diagram of the triangle with the three nodes working as its generating seeds. The total pore volume of each polygon is assigned to its corresponding node,

$$
\begin{align*}
V_i &= \phi A_{oeif} \\
V_j &= \phi A_{ofjg} \\
V_m &= \phi A_{ogme}
\end{align*}
$$

(3.11)

Here, $\phi$ is the porosity of the porous medium. $A$ represents the area. $V$ is the volume of node.

Any link between two nodes constructs a flow pipe, namely pipes $mp(i, j)$, $mp(i, m)$ and $mp(j, m)$ for the triangle $ijm$. Each pipe conducts fluid between two end nodes. The volume of each pipe node is half the volume of the triangle composed by pipe nodes and the circumcentre. For example, the nodal volume of pipe $mp(i, j)$ is $l_{of} l_q / 4$.

To evaluate the pipe flow conductivity between two nodes, pipe $mp(i, j)$ is taken as an example, which transfers fluid between node $i$ and node $j$. In a porous medium, it represents the fluid transfer between domain $oeif$ and domain $ofjg$ through their common face $of$. Assuming unit thickness for the 2D porous medium, the fluid flow through face $of$ can be evaluated by
\[ Q_{of} = \int_{l_{of}} v \cdot n_{of} dl , \quad (3.12) \]

where, \( n_{of} \) is unit vector normal to face \( of \), which can be calculated as,

\[ n_{of} = \frac{1}{l_{ij}} \left( (x_j - x_i) \hat{x} + (y_j - y_i) \hat{y} \right). \quad (3.13) \]

If the pressures of three nodes are known as \( p_i, p_j \) and \( p_m \), the pressure within the triangular \( p(x,y) \) can be approximated by using the linear shape function as that in FEM,

\[ p(x,y) = N_i p_i + N_j p_j + N_m p_m . \quad (3.14) \]

The linear shape function \( N_k \) has the form as follows,

\[ N_k = \frac{1}{2A_{ijm}} (a_k + b_k x + c_k y) , \quad k = (i,j,m). \quad (3.15) \]

The pressure gradient in the triangular domain is calculated as,

\[ \nabla p = \frac{1}{2A_{ijm}} \left( (b_i p_i + b_j p_j + b_m p_m) \hat{x} + (c_i p_i + c_j p_j + c_m p_m) \hat{y} \right) , \quad (3.16) \]

where \( b_k \) and \( c_k \) are geometry dependent coefficients,

\[ b_i = y_j - y_m , \quad b_j = y_m - y_i , \quad b_m = y_i - y_j , \quad (3.17) \]

\[ c_i = x_m - x_j , \quad c_j = x_i - x_m , \quad c_m = x_j - x_i . \quad (3.18) \]

The fluid flow through the face \( of \) can be approximated as:

\[ Q_{of} = \int_{l_{of}} v \cdot n_{of} dl \approx -l_{of} \frac{k}{\mu} (\nabla p - \rho g) \cdot n_{of} . \quad (3.19) \]

The right term \( \nabla p \cdot n_{of} \) of the above equation can be calculated as,

\[ \nabla p \cdot n_{of} = \frac{1}{l_{ij}} ( -p_i + p_j ) . \quad (3.20) \]
The fluid flow $Q_{of}$ can finally be obtained as follows,

$$Q_{of} = \frac{l_m k}{l_m \mu} (\Phi_i - \Phi_j). \quad (3.21)$$

The fluid flow $Q_j$ in the pipe $mp(i,j)$ is equal to $Q_{of}$. Therefore, the flow law in the pipe $mp(i,j)$ can be written as,

$$Q_j = K_{mp(i,j)} (\Phi_i - \Phi_j). \quad (3.22)$$

Here, $K_{mp(i,j)}$ is the equivalent conductance coefficient of the pipe. Comparing Equation (3.22) with Equation (3.21), $K_{mp(i,j)}$ is found as,

$$K_{mp(i,j)} = \frac{l_m k}{l_m \mu}. \quad (3.23)$$

Similarly, the equivalent conductance coefficients for pipes $mp(i,m)$ and $mp(j,m)$ are derived as,

$$K_{mp(i,m)} = \frac{l_m k}{l_m \mu}, \quad (3.24)$$

$$K_{mp(j,m)} = \frac{l_m k}{l_m \mu}. \quad (3.25)$$

Now, three equivalent pipes for the unstructured triangle grid have been derived. In the domain boundary, one equivalent pipe associates with one triangle only. However, inside the domain, one equivalent pipe associates with two adjacent triangles. Therefore, the equivalent conductance coefficient of this pipe is the summation of contributions of these two triangles.

### 3.2.5 Fracture pipe network equivalence in 3D domain

Fractures in 3D domain are represented by 2D disks (either elliptic or polygonal). Therefore, the equivalence of fracture pipe network in 3D domain is similar as the equivalence of the 2D matrix pipe network. The differences lie in that the 2D fracture has hydraulic aperture $a$ and its permeability $k$ is $a^2/12$. For a triangular discretization
which is similar in Figure 3.5, the conductance coefficient $K_{fp(i,j)}$ for fracture pipe $fp(i,j)$ is derived as

$$K_{fp(i,j)} = \frac{l_{of}a^3}{12l_{ij}\mu}.$$  \hfill (3.26)

Conductance coefficients for fracture pipes $fp(i,m)$ and $fp(j,m)$ are as follows

$$K_{fp(i,m)} = \frac{l_{oe}a^3}{12l_{im}\mu},$$  \hfill (3.27)

$$K_{fp(j,m)} = \frac{l_{og}a^3}{12l_{jm}\mu}.$$  \hfill (3.28)

### 3.2.6 Matrix pipe network equivalence of 3D porous media

The 3D porous medium can also be reconstructed by the pipe network model. The simplest case to reconstruct the 3D domain is to use tetrahedrons, each of which have six edges that correspond to six flow pipes. For deriving the equivalent flow pipes of a tetrahedron, the centre of the circumscribed sphere of the tetrahedron $o$ is chosen as the division point as shown in Figure 3.6. Points $c_n \ (n = 1,...,4)$ are circumcentres of four faces of the tetrahedron. Sequentially linking the point $o$ and circumcentres of adjacent faces and the midpoint of their joint edge encloses a partition face of the tetrahedron. There are totally six partition faces for the tetrahedron, and they are perpendicular to their intersecting edges respectively. The six partition faces divide the tetrahedron into four parts. The total pore volume of each part is assigned to its corresponding node. Fluid transferred by flow pipe between two nodes is equal to fluid flow between two adjacent parts divided by their partition face. Similar to the derivation of the equivalent coefficient of a 2D triangular porous medium, an FEM-like linear shape function is adopted to approximate the distribution of the pressure.
Figure 3.6 Pipe equivalence of tetrahedral porous medium

The pressure \( p(x, y, z) \) within the tetrahedron can be approximated using interpolation of nodal pressure as,

\[
p(x, y, z) = \sum N_k p_k, \quad (k = i, j, m, n)
\]  

\( N_k, (k = i, j, m, n) \) are linear shape functions of the tetrahedron, which have the following form,

\[
N_k = \frac{1}{6V_{jmn}} (a_k + b_k x + c_k y + d_k z), \quad (k = i, j, m, n),
\]

where, \( V_{jmn} \) is the volume of the tetrahedron.

Then the pressure gradient in the tetrahedral domain can be estimated as,

\[
\nabla p(x, y, z) = \frac{1}{6V_{jmn}} \left( \sum b_k p_k x + \sum c_k p_k y + \sum d_k p_k z \right), \quad (k = i, j, m, n).
\]

Here, \( b_k \), \( c_k \) and \( d_k \) are geometry dependent coefficients, the value of which can be obtained by using the coordinates of the four nodes, and they are explicitly expressed as,

\[
b_k = (y_n - y_j)(z_m - z_j) - (y_m - y_j)(z_n - z_j) \]
\[
b_j = (y_m - y_i)(z_n - z_m) - (y_n - y_i)(z_m - z_i) \]
\[
b_m = (y_j - y_n)(z_i - z_m) - (y_n - y_m)(z_j - z_m) \]
\[
b_n = (y_i - y_m)(z_j - z_i) - (y_m - y_i)(z_j - z_m)
\]
\[
\begin{aligned}
    c_i &= (z_n - z_j)(x_m - x_i) - (z_m - z_j)(x_n - x_i) \\
    c_j &= (z_m - z_i)(x_n - x_m) - (z_n - z_m)(x_m - x_j) \\
    c_m &= (z_j - z_m)(x_i - x_m) - (z_i - z_m)(x_j - x_m) \\
    c_n &= (z_i - z_m)(x_j - x_i) - (z_j - z_i)(x_j - x_m)
\end{aligned}
\]  

(3.33)

\[
\begin{aligned}
    d_i &= (x_m - x_i)(y_n - y_i) - (x_n - x_i)(y_m - y_j) \\
    d_j &= (x_n - x_j)(y_m - y_n) - (x_m - x_j)(y_n - y_i) \\
    d_m &= (x_j - x_n)(y_i - y_n) - (x_n - x_m)(y_j - y_n) \\
    d_n &= (x_i - x_m)(y_j - y_i) - (x_m - x_j)(y_j - y_m)
\end{aligned}
\]  

(3.34)

The fluid flow through pipe \( mp(i, j) \), \( Q_{ij} \) can be estimated by the flow through partition face \( oc_{1, fc} \), \( Q_{oc_{1, fc}} \) as follows,

\[
Q_{ij} = Q_{oc_{1, fc}} = \int_{A_{oc_{1, fc}}} \mathbf{v} \cdot \mathbf{n}_{oc_{1, fc}} dA \approx -A_{oc_{1, fc}} \frac{k}{\mu} (\nabla p - \rho g) \cdot \mathbf{n}_{oc_{1, fc}},
\]  

(3.35)

where, \( A_{oc_{1, fc}} \) and \( \mathbf{n}_{oc_{1, fc}} \) are area and unit vector normal of the partition face \( oc_{1, fc} \), respectively. \( \mathbf{n}_{oc_{1, fc}} \) is calculated as,

\[
\mathbf{n}_{oc_{1, fc}} = \frac{1}{l_{ij}} \left( (x_j - x_i) \mathbf{x} + (y_j - y_i) \mathbf{y} + (z_j - z_i) \mathbf{z} \right).
\]  

(3.36)

By substituting Equation (3.31) and (3.36) into Equation (3.35), the \( Q_{ij} \) can be found as follows,

\[
Q_{ij} = K^{3D}_{mp(i, j)} (\Phi_i - \Phi_j),
\]  

(3.37)

where, \( K^{3D}_{mp(i, j)} \) is the equivalent conductance coefficient of pipe \( mp(i, j) \) in the tetrahedron, and it has the value:

\[
K^{3D}_{mp(i, j)} = \frac{A_{oc_{1, fc}} k}{l_{ij} \mu}.
\]  

(3.38)
Similarly, the equivalent conductance coefficients for pipes $mp(i,m)$, $mp(j,m)$ and $mp(m,n)$ can be derived. In a 3D domain, the flow pipes may be shared by several tetrahedrons. Therefore, the overall equivalent conductance coefficient of the pipe is the summation of equivalent conductance coefficients of those tetrahedrons sharing the pipe. As seen from the derivation, the equivalent conductance coefficient of a pipe can be expressed uniformly as

$$K_{(i,j)} = \frac{Ak}{l_j\mu}$$

(3.39)

where $k$ is the intrinsic permeability and it is $a^2/12$ for fractures. $A$ is the influence area of the pipe. In view of the unit thickness of the 2D domain, $A$ equals $a$ for 1D fracture pipe and $A$ is $l_h$ for 2D matrix pipe, which is the distance from the circumcentre to the pipe. In 3D domain, $A$ is $al_h$ for 2D fracture pipe. For 3D matrix pipe, $A$ is the area of the quadrilateral comprising of the middle point of the pipe and the centre of the circumsphere and two circumcentres of the adjacent faces of the tetrahedron.

### 3.2.7 Coupling of fracture pipes and matrix pipes

The 2D and 3D porous media have been reconstructed by using discrete 1D flow pipes, which is the same form as that used in 2D DFN models. Actually, in 3D problems, the fractures are modelled as 2D planes, which also can be reconstructed by using the 1D flow pipes. Thus the fractured porous media in both 2D and 3D can be modelled by the unified pipe network model (UPNM). In this subsection, the coupling of the fractures and the matrix using the UPNM for the saturated steady flow is expressed. The simulation of the unsaturated transient flow using the UPNM will be carried out in later chapters.
Figure 3.7 The coupling of 2D fractured porous medium using fracture pipes and matrix pipes

For a 2D fractured porous medium as shown in Figure 3.7, the fracture and the matrix in the domain are reconstructed by using the fracture pipes and the matrix pipes respectively. It is noticed that the fracture pipe and the matrix pipe share the same nodes in the interface. Therefore, the fracture pipes and the matrix pipes at the interface can be merged together, which are called interface pipes. The nodal volume and the equivalent conductance coefficient of the interface pipe can be obtained by the superposition of the values of the merged pipes. Figure 3.7 shows the merging of the fracture pipes denoted by $fp(a,b)$, $fp(b,c)$, $fp(c,d)$ and $fp(d,e)$ with the matrix pipes denoted by $mp(a,b)$, $mp(b,c)$, $mp(c,d)$ and $mp(d,e)$. After the pipe merging, the 2D fractured porous medium is represented by a pipe network graph consisting of the matrix pipes and the interface pipes.
For a 3D fractured porous medium, the coupling process of the fracture pipes and the matrix pipes is quite the same as that in a 2D problem. As shown in Figure 3.8, at the interface of the fracture plane and the matrix, the fracture pipes share nodes with the matrix pipes. Thus, these two kinds of pipes can be successfully merged into interface pipes using the superposition law.

After the reconstruction and coupling process, the fractured porous medium is uniformly represented by the pipe network graph. The flow, both in the fractures and in the matrix, is modelled by the flow in the pipes. The mass conservation holds at each node. In this derivation, the solute transport has not been considered, for which the diffusion may overtake the advection in the matrix rock, especially in the tight rock, such as shale, where its permeability is negligible comparing to fractures.

3.2.8 Calculating the Darcy velocity from the pipe velocity

The nodal pressure can be obtained by solving the global matrix equation. The direction of the flow velocity at each node can be determined by the direction of the pressure gradient at each node after knowing the pressure distribution. In order to get the magnitude of the Darcy flow velocity of a node, taking node $i$ for example, $v_i$, it is supposed that $\beta_i$ is the directional angle of the flow velocity at node $i$ and $\alpha_{ij}$ is the angle between the pipe $(i,j)$. The flow rate of the pipe $(i,j)$ can be written as,

$$Q_{(i,j)} = v_i \cos(\alpha_{ij} - \beta_i) A,$$

(3.40)
where \( A \) is the area of the partition face relating to pipe \((i, j)\).

Thus the magnitude of the flow velocity at node \(i\) can be evaluated according to the flow rate \(Q_{(i,j)}\) as,

\[
v_i = \frac{Q_{(i,j)}}{A \cos(\alpha_i - \beta_i)}.
\]  

The Darcy flow velocity can be evaluated by any flow rate of the matrix pipe connecting to the node \(i\). Those velocities should be averaged to get a more accurate value.

### 3.3 CONFORMING MESH METHOD FOR UNIFIED PIPE NETWORK MODEL

The coupling of the matrix pipes and the fracture pipes requires the pipe nodes of these two kinds of pipes to be identical in position. Therefore, the domain reconstruction process requires an efficient conforming mesh generator for the fractured porous medium in both 2D and 3D. Currently, the generation of high-quality conforming mesh for fractured porous media with complex fracture geometry is still a challenge. Several public domain tetrahedral mesh generators have been tested by Monteagudo and Firoozabadi (2004), and they cannot generate satisfactory 3D good quality conforming mesh with intricate fracture network.

(a) 7911 nodes and 15539 elements  
(b) 2952 nodes and 5720 elements
A high efficient and high quality mesh generator has been developed to reconstruct the fractured porous medium using the UPNM. For 2D domain, the fractures are discretised firstly by inserting nodes in the fracture lines to form connected pipes. Then the meshed fracture pipes and nodes are input as constraints for the discretization of the porous medium. The generation of mesh nodes of the porous medium is based on the Analogous Force Equilibrium method (Persson & Strang 2004) to improve the mesh quality. A mesh size function is introduced to modify the mesh density and the density-changing gradient around the fractures. Finally, the mesh nodes of the porous medium and the pre-meshed fractures are triangulated using the constrained Delaunay triangulation, which relaxes the circumcircle criterion and nearest-connection rule around the fractures to guarantee that the fractures will not be crossed by any triangular edge. Figure 3.9 shows constrained Delaunay triangulation of a 2D fractured porous medium with different mesh density and density changing gradient.

For a 3D domain, bottom-up conforming mesh processes are adopted for the tetrahedrization, which is shown in Figure 3.10. The first level conforming mesh is applied to the intersected 3D fracture network only. Every 2D fracture plane with intersecting lines is meshed by the constrained Delaunay triangulation, which is the same mesh method used for the 2D fractured porous medium. The discretised intersecting lines between fractures are regarded as the constraining conditions. The
second level confirming mesh is to produce tetrahedrons to conform to the existing 2D fractures by the tetrahedrization algorithm proposed by Si (Si 2015; Si 2008).

![Figure 3.10 Three dimensional constrained Delaunay tetrahedrization process](image)

(a) Level 1: confirming triangulation     (b) Level 2: confirming tetrahedrization

**Figure 3.10 Three dimensional constrained Delaunay tetrahedrization process**

### 3.4 VALIDATIONS AND APPLICATIONS

In this section, several cases are studied to validate the UPNM for saturated steady state fluid flow in discrete fracture network, porous medium and fractured porous medium. 2D and 3D cases are demonstrated to show the flexibility of the method.

#### 3.4.1 Simulation of water flow in 2D porous media

The critical point of the UPNM for the seepage in the porous medium relies on correctly converting the flow parameters from the porous medium to the flow parameters of the connected pipe network. Unless otherwise stated, the default parameters for the following studies are set as follows. The permeability of the porous medium $k_m$ is set to be $1 \times 10^{-11}$ m$^2$. The fluid is water with the viscosity 0.001 Pa·s and the density is 1000 kg/m$^3$. No gravitational effect is considered. In order to examine the correctness of the model, a seepage model (case 1) with different meshes is used to verify the UPNM. The 2D model is a rectangular porous medium whose length and height are 10m and 2m respectively. Figure 3.11 shows the different pipe networks that have been checked. The boundary conditions also are shown in Figure 3.11. Identical results of the distributions
of the pressure have been obtained, regardless of the mesh sizes. Figure 3.12 shows the distribution of the pressure. The results validate the accuracy of the method.

Figure 3.11 Validation of the UPNM with different meshes: case 1

Figure 3.12 The pressure distribution with different meshes: case 1

Figure 3.13 Pressure contour and velocity vector of case 2
Case 2 is seepage in a square porous medium with a circular tunnel in the centre. The size of the medium is 50 m × 50 m. The diameter of the circular tunnel is 10 m. The pressure boundary is applied at the top and the bottom boundaries of the domain with 0.5MPa and 1MPa, respectively. The pressure boundary condition for the tunnel is 0. The left and right boundaries of the domain are non-flow boundaries. Figure 3.13 depicts the pressure distribution as well as the velocity distribution in the medium. The results also are compared with the commercial software FLAC (Itasca 2007). The pressure as well as the velocity in X direction at the position of Y=25m are compared with the results gained from the commercial software FLAC illustrated in Figure 3.14. As noticed, the results readily agree.

3.4.2 Simulation of water flow in 2D fractured porous medium

The presence of the fractures in the host medium can increase the permeability of the whole blocks. The UPNM merges the fracture pipes into the original host pipe system, thereby modelling the seepage both in the fractures and the porous medium. Models (case 3) with both penetrating and non-penetrating fracture in the rock block shown in Figure 3.15, are simulated to verify the applicability of the UPNM. The model parameters and boundary conditions are the same as that in case 1, except that both fractures with the equivalent aperture size of 0.6 mm are embedded. The total outflow rate of the penetrating crack model derived from the simulation is \(3.04 \times 10^{-3} \text{ m}^3/\text{s}\), which is exactly the summation of the flow rates of the crack and the porous material.
Therefore, the equivalent permeability of the fractured porous medium in longitudinal direction is $1.9 \times 10^{-11}$ m$^2$, increased by 90% compared to the original permeability of the porous medium. When the crack is embedded in the middle of the medium, as shown in Figure 3.15(b), the total outflow rate obtained is $1.817 \times 10^{-3}$ m$^3$/s. The embedded fracture can be modelled as a very thin zone in the FLAC software using the equivalent permeability to validate the result derived from our method. The stable flow rate by the simulation of FLAC is $1.818 \times 10^{-3}$ m$^3$/s, which is very close to the result obtained by the UPNM. When the grid size is very small, the stable explicit time step size in FLAC also decreases to a very small value accordingly. Therefore, the FLAC simulation of this case is quite time consuming. Figure 3.16 shows the pressure contour together with the velocity vector and flow trace of the non-penetrating fracture model. In this case, the overall equivalent permeability of the fractured porous medium can be increased by 11.9% compared to the porous matrix, though the crack is tiny and short.

![Impermeable boundary](image)

(a) Porous medium with a penetrating fracture

(b) Porous medium with a non-penetrating fracture

Figure 3.15 Case 3: fractured porous medium with penetrating fracture and non-penetrating fracture
Case 4 demonstrates the application of the UPNM in the simulation of water seepage in a model of the excavation of the underground rock cavern for petroleum storage. The model shown in Figure 3.17 is the layout of Huangdao water sealed underground petroleum storage rock caverns (Shi & Liu 2008; Shi & Liu 2010), which is a real engineering project consisting of three main oil storage tunnels and one water curtain tunnel. Because the water curtain has not been considered in this case, it has not been presented in Figure 3.17. A seepage field analysis without explicitly considering fractures has been carried out by Shi and Liu (Shi & Liu 2010) by using the single continuum model. However, large fractures in rock can significantly affect the seepage field and may lose the important seepage characteristics if they are simply homogenized with the rock matrix. Figure 3.18 shows the hybrid model of the Huangdao project and the mesh with the stochastic fracture network. For the sake of simplicity, the equivalent apertures of the fractures are set to be 1 mm. Both the left and the right boundaries are non-flow boundaries. The top and the bottom boundaries are pressure boundaries with the pore pressures of 2.0586 MPa and 0, respectively. The pressures of all caverns are 0. Gravitational acceleration is 9.8m/s². Figure 3.19 compares the effects of fractures on the pore pressure distributions of the Huangdao project. It is obvious that the presence of fractures changes the distribution of the pore pressure, which is crucial to evaluate the safety of the underground petroleum storage.
Figure 3.17 Layout of Huangdao water sealed underground petroleum storage rock caverns

Figure 3.18 The hybrid model for the Huangdao water sealed underground petroleum storage project
3.4.3 Simulation of water flow in 3D fracture network

In fractured hard rock, the permeability of the rock matrix is relatively much lower than that of the fractures. Water is mainly transferred by the fracture network. Therefore, the permeability of the rock matrix can be ignored. The fracture network in the 3D domain is reconstructed by using the pipe network. The flow is modelled as in the discrete fracture pipes. Case 5 shows a random fracture network in rock mass, the geometry and discretization of which are shown in Figure 3.20. The fractures are in a 10 m × 10 m × 10 m domain. Those out-of-domain parts of the fractures have been cut. Pressure boundaries are set on the planes at x=0 and x=10m with 1MPa and 0.5 MPa,
respectively. For simplicity, the equivalent hydraulic aperture of the fractures are all set to 0.1 mm. Figure 3.21 shows the pressure distribution and the flow velocity in the fracture network. Although the interconnectivity in the fracture network is very intricate, the result clearly illustrates the flow path. The UPNM is capable of simulating DFNs with any complex geometry.

Figure 3.20 Three dimensional discrete fracture network of case 5

Figure 3.21 Pressure distribution and flow velocity of case 5
3.4.4 Simulation of water flow in 3D porous medium

When the fractures in the rock mass are well connected, the fractured rock mass may be approximated as a single porosity continuum. The effects of the fractures on the flow are not explicitly considered. The continuous porous medium can also be reconstructed and simulated by using the UPNM. Figure 3.22 shows two porous media. The difference between these two media is that the model of (b) has a cuboid non-flow obstacle in the centre. Pressure boundaries are applied at the two ends with 1MPa and 0.5MPa respectively. The analytical solution of Darcy velocity in the case 6(a) is $1.67 \times 10^{-4}$. The simulation result is equivalent to the analytical result, which shows the accuracy of the UPNM. Figure 3.23(a) shows the pressure and velocity distribution of case 6(a). Figure 3.23(b) illustrates the longitudinal cross-section view of the result of case 6(b). The Darcy velocities of case 6(b) calculated by the UPNM are compared with results calculated by the commercial software COMSOL (2011) to validate the UPNM. Figure 3.24 illustrates the Darcy velocity at $Y=5$ m, $Z=6.8$ m, and $X$ from 0~30 m. The results are consistent.

Figure 3.22 The geometry of 3D porous media of case 6

(a) Pressure contour and velocity distribution of case 6(a)
When fractures are explicitly considered together with the porous medium, the situation is much more complex. The drastic difference of material properties makes the flow change abruptly at the interface. The irregular distributions of the fractures further strengthen the complexity. The UPNM reconstructs the fractures and porous medium by the assembly of fracture pipes and matrix pipes. The unified treatment makes the medium tractable. A simple model is presented to show the effectiveness of the UPNM, which is demonstrated in Figure 3.25. The size of the model is 10 m × 10 m × 10 m, with two fractures imbedded in the middle. The fracture coordinates are presented in Table
3.1. The permeability of the porous matrix is $1 \times 10^{-13}$ m$^2$ in this case. The fracture aperture is 0.5mm. No gravitational force is applied. Pressure boundaries are applied at surfaces of $x=0$ and $x=10$ m with 1MPa and 0.5MPa, respectively. The simulation results at $Y=5$ m, $Z=5$ m and $X$ from 0–10 m are compared with the results calculated by the commercial software COMSOL in Figure 3.26. COMSOL simulates the fractures as interior boundaries. The two results agree well. A cross-section view at plane of $z=6$ m, which cut the fracture planes, is presented in Figure 3.27 to show the simulation result of the UPNM. The result shows that the high permeable fractures severely distort the pressure distribution in the porous medium. The simulation of the case also demonstrates the feasibility of the UPNM for solving seepage problems in 3D discontinuous medium.

Table 3.1 The coordinates of the fractures

<table>
<thead>
<tr>
<th>Points</th>
<th>A</th>
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<th>C</th>
<th>D</th>
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<td>(9.0, 9.0, 4.0)</td>
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<td>Points</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
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<td>(2.0, 6.8, 4.0)</td>
<td>(2.0, 6.8, 6.0)</td>
</tr>
</tbody>
</table>

Figure 3.25 Fracture-imbedded porous medium of case 7
Figure 3.26 Pressure comparison between results from COMSOL and UPNM

Figure 3.27 Pressure contour of case 7 at z=6m
3.5 CONCLUSION AND DISCUSSION

In this chapter, a domain-reconstruction method by using the unified pipe networks is proposed for the seepage simulations in either two-dimensional or three-dimensional porous medium, discrete fracture network and fractured porous medium. The unified pipe network method treats different flow domains uniformly, which is the assembly of the discrete one-dimensional pipes associated with different properties. The flow equivalences of the fracture pipe and the matrix pipe for the fractures and the porous medium are derived respectively. Mass conservation is guaranteed in the UPNM. The UPNM conceptually and computationally simplifies the seepage simulations in complex fracture networks and fractured porous media. Integration for each element is not required. Confirming mesh methods in two dimension and three dimension are adopted to discretise the flow domain and construct the flow pipes. The case studies demonstrate the flexibility and feasibility of the proposed method.

The current model adopts the cubic law, which indicates that the current model suits for flow with low Reynolds number in fractures. An equivalent hydraulic aperture is used to account for the roughness of the fracture surface. In case of solute transport, the diffusion, sorption and dispersion processes are more sensitive to the fracture geometry, where fracture roughness may require more detailed description. A coefficient of space function may be introduced to the pipe model to represent the variation of the roughness in space. This will be our further research topic.

The concept of material reconstruction is both applicable in microscope and macroscope. Therefore, the UPNM has the potential to combine the seepages in multi-scales. The pipes can model pore scale flow up to tiny fracture flow and finally form to macroscopic flow. The multi-scale computation is the work to be carried out in the future.

Furthermore, the advantage of the UPNM makes it an attractive method for the coupled hydro-mechanical simulation. The lattice model (Zhao, Fang & Zhao 2012; Zhao 2014; Quintana-Alonso & Fleck 2010) used in mechanics is one example that has its advantage in simulating fracturing in brittle material. Therefore, the lattice model suits modelling the mechanical behaviour of rock mass. The lattice model and UPNM are both using one-dimensional elements to model discontinuous material. It is possible to
combine these two methods for the coupled hydraulic and mechanic modelling of rock mass, especially for the hydraulic fracturing problem.
CHAPTER 4. INVESTIGATION OF THE PERMEABILITY ANISOTROPY OF 2D FRACTURED ROCK MASSES

This chapter evaluates the permeability anisotropy of fractured rock masses by using the developed unified pipe network method.

4.1 INTRODUCTION

Effective evaluation of the permeability of fractured rock masses has increasingly attracted researchers’ attentions due to its importance in the fields of geotechnical application, hydrogeology, petroleum reservoir exploitation, civil engineering, mining and environmental waste repositories, etc. (Berkowitz 2002). The intrinsic discontinuities, such as faults, fracture sets and cracks, make rock masses highly heterogeneous media, and render the handling of water flow in the fractured rock masses intractable. Discontinuities in hard rocks are the main flow path and the fracture network system plays a significant role in determining the permeability in rock masses.

The existence of discontinuities makes a high degree of permeability anisotropy in fractured rock masses (Zhang 2013). The parameters of the distributed fractures directly govern the hydraulic behaviour of rock masses. Specifically, the permeability anisotropy of fractured rock masses is greatly affected by the geometric features, such as density, length, orientation, spacing and interconnectivity of the fractures, and it is related to the hydraulic performance of each fracture which is governed by its roughness and aperture (Liu 2005).

The determination of the permeability of fractured rock masses has been studied by many researchers. Parsons (1966) studied the overall apparent permeability of fracture-rock systems, including both regular fracture-matrix models and heterogeneous fracture systems. The heterogeneous fracture systems were built by placing fractures in a given pattern, such as a square pattern or a triple hexagonal pattern. Snow (1969) proposed an analytical method to calculate the permeability tensor of fractured rock masses based on statistical information of fractures. The relationship between the fracture geometry and the permeability anisotropy has also been assessed. However, the finite size effect and the spatial variation of the fractures were not considered in the analytical method. Oda
(1985) and Oda et al. (1987) developed a method to evaluate the equivalent permeability tensor for the fractured rock masses involving numerous arbitrarily oriented joints and faults. Highly fractured rock masses are treated as homogeneous anisotropic porous media when the concept of representative elementary volume (REV) is applicable. Lee et al. (1995), Talbot and Sirat (2001), Zhou et al. (2008) and Rong et al. (2013) studied the stress effect on the permeability of fractured rock masses and showed that the in-situ rock stresses and the embedded depth can affect its permeability because of their effects on apertures. Leung and Zimmerman (2012) proposed a method for estimating two-dimensional macroscopic effective hydraulic conductivity by using parameters of fracture network, such as fracture density and aperture distribution. Their work is valid for isotropic networks. Permeability tensors were used to represent the hydraulic features of the REV and thus for the simulation of an equivalent continuum model. The permeability tensor can provide a comprehensive insight into the permeability anisotropy of fracture networks if it is readily available. However, in some cases the information of a fracture network is limited, and difficult to collect. For example, determining of the probability density functions of random variables in the geometries of fracture networks is still a practical challenge (Rong et al. 2013).

Apart from the derivation of the equivalent permeability, a lot of research has also focused on the phenomena of permeability anisotropy of a fracture network system and its origin. Balberg and Binenbaum (1983) used the conducting stick model to study the relationship between the microscopic anisotropy and the percolation threshold of a fracture system. Their study did not emphasise the relationship between the fracture patterns and the anisotropy of the system. Zhang and Sanderson (1995) defined a practical geometric anisotropy factor to describe the effects of orientation and spacing of fractures on the permeability anisotropy. Their results showed that the fracture orientation was a main factor controlling the geometric anisotropy and the permeability anisotropy. However, their model did not consider the hydraulic behaviour of each fracture and interconnectivity effects on the permeability anisotropy. The total permeability of fractured rock masses is governed by the permeability of the fracture system and the permeability of the rock matrix. When simulating the permeability of a fractured hard rock, it is reasonable to ignore the permeability of the matrix rock, and only to consider the permeability of fracture networks. Therefore, the individual
permeability of each fracture in the network can affect the macroscopic permeability of
the fractured rock masses.

Research has also shown that not all the fractures within rock masses contribute to the
flow. In fact only a small portion of the fractures is conductive and contributes to fluid
flow (Long & Billaux 1987; Talbot & Sirat 2001). Dense fracture networks are not
necessarily hydraulically connected (Berkowitz 2002). Therefore, the connectivity also
has important effects on the permeability anisotropy. Percolation theory is an effective
tool to characterize the connectivity of fracture networks. Generally, the issue of
connectivity of a discrete fracture network mainly concerns the percolation of the whole
region (Berkowitz 1995; Robinson 1983; Sahimi 1993). It focuses on fracture networks
required to connect the specific region (percolating state). Different indicators have
been proposed to investigate the permeability of a fractured system when using the
percolation theory. Hestir and Long (1990) and Sahimi (1993) suggested using \( X_f \), the
average number of intersections per fracture; while Dershowitz and Herda (1992)
preferred using a conductive intensity, \( P_{32} \) (\( P_{21} \) in 2D), which is a fracture areal
intensity parameter (average length per unit area in two-dimensional cases). These two
connectivity indicators cannot reflect the directional permeability of a 3D fracture
network. More recently, Xu et. al. (2006) proposed a connectivity index, CI, to quantify
the connectivity probability between two points in space. However, it cannot describe
the directionality for a specific 3D fracture network model.

Factors that affect the permeability features, such as stress (Zhang et al. 2007;
Baghbanan & Jing 2008; Talbot & Sirat 2001; Yang, Billiotte & Su 2010; Zhou et al.
2008) and temperature (Summers, Winkler & Byerlee 1978; Moore, Lockner & Byerlee
1994) can also be studied by the methods of field surveys, laboratory experiments and
numerical simulations. However, they are beyond the scope of this study.

A number of numerical simulation methods have been developed for evaluating the
permeability of a fractured rock mass. These can generally be classified into three
categories, namely, continuum models, discrete fracture network models and hybrid
models (Neuman 2005). Each method has its own distinct advantages and disadvantages
and suits different situations and problems.
A fractured porous medium or a dual porosity medium is always analysed by coupling fracture networks with a permeable matrix. These hybrid models explicitly delineate the fracture system and consider the permeability of the matrix. When the matrix is less permeable, the fluid flow in the discrete fracture network becomes dominant in the fractured rock mass.

In order to consider the highly heterogeneous and directionally dependent nature of the fractured rock masses in numerical simulations, a 2D pipe network model (with a unit thickness in the third direction) has been presented to determine the equivalent permeability of the fractured rock masses (Li, Xu & Ma 2014; Priest 1993).

In this chapter, a hydro-geometric anisotropy factor (HAF) is proposed to estimate the permeability anisotropy in a given direction of a fracture network. Different fracture network patterns with different HAFs are generated, and their permeability anisotropy ratios (PAR) are evaluated by applying the developed pipe network method. The macroscopic permeability anisotropy of the fractured rock masses caused by the geometric anisotropy, including the variation of individual fracture permeability, is studied. A 2D anisotropic conductivity index is also introduced to quantify the directional hydraulic connectivity for a specific fracture network.

### 4.2 A HYDRO-GEOMETRIC ANISOTROPY FACTOR (HAF) FOR FRACTURE NETWORKS

The geometric anisotropy and hydraulic anisotropy of fracture networks are two major causes of the permeability anisotropy of a fractured rock mass. The geometric anisotropy of fracture networks is caused by the non-uniform distribution of geometric properties, such as the orientation, length and spacing etc., while the hydraulic anisotropy arises from hydraulic heterogeneity, which is the non-uniform distribution of the hydraulic properties of fractures, such as aperture and roughness.
In order to investigate the geometric anisotropy and hydraulic anisotropy of a fracture network, a combined hydro-geometric anisotropy factor (HAF) is proposed. In investigating two arbitrary orthogonal directions X and Y with a sampling area $L_x$ and $L_y$ as shown in Figure 4.1, the equivalent permeability in a given direction can be evaluated as,

$$k_{eq} = \frac{Q}{A \nabla p}$$  \hspace{1cm} (4.1)

where $k_{eq}$ is the equivalent permeability of the fractured rock mass in a study direction (direction X, for example). $Q$ is the outflow rate from the boundary area $A$, and $\nabla p$ is the pressure gradient in the direction. $\mu$ is the dynamic viscosity of the fluid.

If the same boundary condition with a constant pressure gradient is assumed in both the X and the Y directions and considering a unit thickness of the model, the ratio of the equivalent permeability of the two orthogonal directions can be expressed as,

$$\frac{k_{eq,x}}{k_{eq,y}} = \frac{Q_x}{Q_y} \frac{L_x}{L_y},$$  \hspace{1cm} (4.2)

where the subscripts $x$ and $y$ denotes the two directions. In the above derivation, linear variation of hydraulic head is assumed, which is supported by Long et al. (1982) and
Oda (1985) if a sufficient number of fractures are contained and well connected. The pressure gradient in the study direction is uniform over the whole flow region.

Due to the randomness of the fractures, $Q_x$ is different at different sections perpendicular to the X direction. In order to obtain the averaged flow rate $\bar{Q}_x$ from the discrete fractures over the volume, a typical effective fracture $i$ that has the dip angle of $\theta_i$ ($0 \leq \theta_i \leq 90^\circ$) with respect to the X direction in the sampling area is considered, as shown in Figure 4.1. An arbitrary cross sectional area $A_x$ that is perpendicular to the X direction is selected to calculate the flow rate. Thus, the flow rate through the area $A_x$ is $Q_x = \sum q_i$, where $q_i$ is the flow rate of the fracture that intersects with $A_x$. If the area $A_x$ scans along the range $L_x$, the overall volumetric averaged flow rate $\bar{Q}_x$ can be obtained as follows,

$$\bar{Q}_x = \frac{\sum_{i=1}^{n_x} \lambda_i q_i}{L_x L_y}, \quad (4.3)$$

where $n_x$ is the number of the effective fractures in the area, and $\lambda_i$ is the length weight factor with orientation effect considered for the fracture $i$ that is scanned by the area $A_x$. It can be obtained as:

$$\lambda_i = \frac{l_i \cos \theta_i}{L_x}. \quad (4.4)$$

Here, $l_i$ is the effective fracture length (excluding the dead tips at each end), and $q_i$ is the flow rate for a single fracture, which contains the information of the characteristic hydraulic behaviour of each fracture in the system. The roughness and fracture width are two key parameters governing the fluid flow in the fracture. Usually, the effects of roughness and mechanical aperture on the fracture permeability can be described by their hydraulic apertures (Renshaw 1995). Therefore, $q_i$ is found to be

$$q_i = \nabla p \cos \theta_i \frac{q_i^3}{12 \mu}, \quad (4.5)$$
where $a_i$ is the equivalent hydraulic aperture, which reflects hydraulic property of each fracture that was influenced by the mechanical aperture and the roughness of the fracture wall. Equation (4.5) also implies that the flow in the fracture is a laminar flow, therefore the cubic law is applicable. The onset of Reynolds number from transition to a turbulent flow varies from 1800 to 4000 for a parallel smooth plate model, which can be lowered by the surface roughness of the fracture (Parsons 1966). More recent research suggests that the onset Reynolds number for flow in real rock fractures within the range of Darcain flow could be much smaller (around 10) (Bear 1972; Zimmerman et al. 2004), or Forcheimer number is less than 1.3, if it is used as an alternative index (Ranjith & Viete 2011).

It is worth mentioning that the fractures involved in the analysis should be effective. An effective fracture means that the fracture should contribute to the fluid flow from one boundary to the opposite boundary in the survey direction. In this way, the fracture connectivity can be taken into consideration. Any isolated fractures will be identified automatically and excluded from the calculation. The orientation and length of the fractures are considered by the length weight factor $\lambda$ and the flow rate $q$ of each effective fracture. The flow rate in the derivation also reflects the hydraulic characteristics of each fracture. Finally, the summation of $q_i$ and the averaging cross section flow rate $\bar{Q}$ over the whole domain reduce the randomness effect of a fracture distribution. In this way, it also reflects the length effect of each fracture, the spacing of fracture sets and the intensity of the fracture system, and makes the results statistically more accurate.

For the Y direction, the overall volumetrically averaged flow rate $\bar{Q}_y$ can be obtained using the same approach. Then, the HAF denoted by $G_f$ is defined by the ratio of the volumetrically averaged equivalent permeability in the two orthogonal directions as,

$$G_f = \lim_{L_x \to \infty} \frac{\bar{k}_{eq,x}}{k_{eq,y}} = \lim_{L_y \to \infty} \frac{\bar{Q}_x}{\bar{Q}_y} \frac{L_y}{L_x}$$

(4.6)

By substituting Equations (4.4) and (4.5) into Equation (4.3), and then substituting Equation (4.3) into Equation (4.6), the factor $G_f$ is reformulated as
When the sampling area tends to be infinity, the scale and randomness effects of the fracture distribution can be eliminated. Therefore, the HAF can be used to estimate the permeability anisotropy together with the hydro-geometric information of the fractured rock masses.

In practice, it is often difficult to collect all information concerning the fractures. In most cases, underground information of fracture geometry is derived from borehole loggings. The number of open fractures that intersect a borehole and their orientations can be determined, whereas the fracture density and the fracture lengths are difficult to estimate from a logging method (Long & Witherspoon 1985). In a practical calculation, the cross sectional flow rate can be added up by counting the fractures that intersect with the scan lines or boreholes and be averaged over the scan lines or borehole lines accordingly. In fact, the length weight factor \( \lambda \) reflects the characteristics of length and orientation distributions of fractures. Together with the position distribution of fractures, the length weight factor \( \lambda \) can reflect the probability of a fracture that is intersected by the scan line. In the case of using the scan line method, the length weight factor \( \lambda \) is not needed because of the certainty of a survey, and the volumetrically averaged cross-section flow rate from Equation (4.3) can be changed to

\[
\bar{Q}_x = \frac{\sum_{i=1}^{n_{ss}} q_i}{L_x L_y},
\]

where \( n_{ss} \) is the number of fractures that intersect with the scan lines along the X direction. The equation in the Y direction can be derived similarly, and the \( G_i \) is simplified to \( D_i \),

\[
D_i = \lim_{L_i \to \infty} \frac{\sum_{i=1}^{n_{ss}} \cos \theta \alpha_i}{L_i}, \quad \theta \in \{x, y\}.
\]
In the above derivation, the scan line of the X direction is perpendicular to the X direction, and it is similar for the scan line of the Y direction. Equally spaced scan lines are assumed in both directions for accuracy.

The simplified HAF $D_f$ in this chapter is neither the same as the one defined by Zhang and Sanderson (1995) nor the one defined by Balberg and Binenbaum (1983). They did not consider the differences of each fracture as in the present definition. The equivalent hydraulic aperture is a critical parameter to affect the permeability of a fracture, which has been incorporated in the proposed HAF. Although the aperture size is difficult to measure, it has a major effect on the permeability anisotropy. In the above definition, the equivalent hydraulic aperture is independent. For a practical reason, it can be further simplified if the equivalent hydraulic aperture is assumed to correlate directly with the fracture length (Klimczak et al. 2010; Bonnet et al. 2001; Oda 1985; Olson 2003).

The current study is based on 2D analyses, which means that the study volume has a unit thickness. Fracture planes are assumed to be perpendicular to the coordinate plane. This is the major limitation of the current model. If fracture planes are not perpendicular to the coordinate plane, errors will arise.

According to the study by Zhou et al. (2008), the permeability tensor of a fracture network $[K]_c$ can be estimated by

$$[K]_c = \frac{\pi}{12 \mu V_p} \sum_{i=1}^{n_i} \sum_{j=1}^{m_i} w_{ij} r_{ij}^2 d_{ij}^3 [M]_{ij}, \quad (4.10)$$

where $V_p$ is the volume of the study region. $n_i$ and $m_i$ are the number of fracture sets and the number of fractures in each set respectively. $w_{ij}$ is a weight coefficient to describe the contribution of each fracture. This is considered as one of the major differences between the discretised form of Oda’s formulation and Equation (4.10) (Zhou et al. 2008). $r_{ij}$ is the radius of fracture (circular disk shape assumption for fractures). $[M]_{ij}$ is a conversion matrix, which is determined by the orientation of a fracture.
\[ [M]_{ij} = \begin{pmatrix} 1-n_1^2 & -n_1n_2 & -n_1n_3 \\ -n_1n_2 & 1-n_2^2 & -n_2n_3 \\ -n_1n_3 & -n_2n_3 & 1-n_3^2 \end{pmatrix}. \] (4.11)

Here, \( n_1, n_2 \) and \( n_3 \) denote the direction cosines of the unit vector normal to the fracture plane in the direction of the axes of X, Y and Z, respectively.

If a 2D problem is concerned, and fractures are assumed to be perpendicular to the X-Y plane, \([M]_{ij}\) is a 2×2 matrix determined only by the fracture inclination angle \( \theta_{ij} \).

\[ [M]_{ij} \text{ is calculated to be,} \]
\[ [M]_{ij} = \begin{pmatrix} \cos^2 \theta_{ij} & \sin \theta_{ij} \cos \theta_{ij} \\ \sin \theta_{ij} \cos \theta_{ij} & \sin^2 \theta_{ij} \end{pmatrix}. \] (4.12)

According to Equation (4.10), the permeability ratio of X and Y directions is obtained as,
\[ \frac{k_x}{k_y} = \frac{\sum_{j=1}^{m_y} \sum_{i=1}^{m_x} w_{ij} \pi r_{ij}^2 d_y^3 \left( \cos^2 \theta_{ij} + \sin \theta_{ij} \cos \theta_{ij} \right)}{\sum_{j=1}^{m_y} \sum_{i=1}^{m_x} w_{ij} \pi r_{ij}^2 d_y^3 \left( \sin^2 \theta_{ij} + \sin \theta_{ij} \cos \theta_{ij} \right)}. \] (4.13)

Comparing Equation (4.13) with Equation (4.7), it is noticed that the term \( \sin \theta_{ij} \cos \theta_{ij} \) is not included in the HAF. This means that the directional permeability does not include the effect of its orthogonal direction. According to the interpretation of \( w_{ij} \) by Rong et al. (2013), \( w_{ij} \pi r_{ij}^2 \) is the effective volume of a fracture. Therefore, it has the same physical meaning as the effective length \( l_i \) in Equation (4.7), which also represents the effective volume of the fracture, acknowledging the unit thickness of the study domain. Recall that the HAF considers two orthogonal directions. For a comprehensive estimate of the permeability anisotropy in a fracture network, the model needs to be transformed to account for the characteristics of the fracture network in different directions. The calculation efficiency can be affected to some extent. The HAF is obviously more applicable in determining the permeability anisotropy in given orthogonal directions.
The aim of the proposed HAF is not for finding the permeability tensor of a REV. We propose a simple way and probably a more practical method (such as the simplified scan line method) for estimating the hydro-geometric anisotropy of fractured rock masses in a given direction at a field scale. With the knowledge of anisotropic features in different directions, it is possible to estimate the principal direction and magnitude of anisotropy (Zhang & Sanderson 1995). Because it is difficult to collect information of natural fractures, it is not always possible to estimate the permeability tensor. The HAF provides an alternative way to estimate the anisotropy characteristics of fractured rock masses.

4.3 PIPE NETWORK MODEL

To obtain the relationship between the hydro-geometric anisotropy and the permeability anisotropy, the unified pipe network model proposed in Chapter 3 is adopted to simulate the fluid flow in fracture networks. For hard fractured rock, the permeability of the matrix will be ignored, providing that the permeability of the matrix is much smaller than that of the fracture networks (Matthäi & Belayneh 2004). Therefore, the fracture system is simply treated as a pipe network to model the bulk permeability of the fractured rock masses.

The pipe network model is a simple and effective flow model that has been studied and applied in many areas. Priest (1993) described the concepts of the pipe network model and used this model to calculate fluid flow in rock fractures. Islam and Chaudhry (1998) used the pipe network model to study the constituent transport in a water supply system. Djordjevic et al. (2004) simulated the transcritical flow in pipe networks. Suchomel et al. (1998) studied the porosity and permeability changes in the porous medium due to the changes in the amount of biomass with the network model. The pipe network model has also been applied to analyse the development of karst aquifers by Liedl et al. (2003) and Clemens et al. (1996). They simulated the conduit network within the pipe network model and coupled it within a continuum model representing the fissured rock.
4.3.1 Theory of the pipe network model

In this subsection, the theory of the pipe network model will be briefly reviewed. The incompressible single phase flow in fracture networks obeys the mass conservation law, which can be described mathematically as,

$$\nabla \cdot Q_f = Q_s. \quad (4.14)$$

$Q_s$ is the source term, while $Q_f$ denotes the flow rate in the fracture, which is assumed to be governed by the well-known “cubic law”,

$$Q_f = \frac{a^3}{12\mu} \nabla p, \quad (4.15)$$

where $\nabla p$ is the pressure gradient along the fracture plane. $\mu$ is the dynamic viscosity of the fluid. The direction of the flow in the fracture is down the pressure gradient. $a$ is the equivalent hydraulic fracture aperture.

In a 2D flow model, each fracture is, conceptually, viewed as a straight pipe, which is discretised into a certain pipe segment (fracture element). If the length of a pipe segment is $l$, the flow rate within this segment is obtained by

$$Q_{fp} = K_{fp} \Delta p, \quad (4.16)$$

where $Q_{fp}$ is the fracture flow rate in the pipe segment, and $\Delta p$ is the pressure decrement. Here, $K_{fp}$ is a conduct coefficient of the fracture flow in the pipe segment, which is evaluated by

$$K_{fp} = \frac{a^3}{12\mu l}. \quad (4.17)$$
Figure 4.2 depicts a typical fracture pipe connection. The discretised form of the nodal mass conservation equation is

$$\sum_{j=1}^{j_{m}} Q_{fp(i,j)} = Q_{S_i}$$ (4.18)

where the subindex $fp(i,j)$ denotes the fracture pipe defined by nodes $i$ and $j$. $i_1, ... i_m$ denote all nodes connecting to node $i$. $Q_{S_i}$ is a fluid source item at node $i$. For each fracture pipe node $i$, the mass conservation law is applied. When the flow Equation (4.16) is substituted into Equation (4.18), it is found

$$\sum_{j=1}^{j_{m}} K_{fp(i,j)} (p_i - p_j) = Q_{S_i}.$$ (4.19)

For a $n$ node problem, Equation (4.19) is applied to each node. Then the global equation can be written as,

$$[K]_{nn} \{p\}_{nn} = \{Q_s\}_{nn}.$$ (4.20)

For problems with given pressure boundary conditions and flow rate boundary conditions, Equation (4.20) can be solved using either direct or iterative methods. After the nodal pressures are calculated, the flow rate in each fracture pipe is obtained according to Equation (4.16).

4.3.2 Validation of the pipe network model

In this subsection, a 2D fracture flow problem is simulated to validate the effectiveness and correctness of the pipe network model. Water is used as the simulation fluid unless otherwise specified.
Figure 4.3 Model 1 - the fractures reside in the impermeable rock block

Figure 4.3 shows a fractured rock formation model with the height of 120m and the length of 200m. Fractures are randomly distributed in the impermeable rock and they all are assumed to have an equivalent hydraulic aperture size of 2mm for simplicity, although it is not necessary for the apertures to be the same in practice. Pressure boundaries are applied at the left and the right sides with 1MPa and 0.5MPa, respectively. Both the top and the bottom boundaries are impermeable. The dynamic viscosity of water $\mu$ is $1\times10^{-3}$ Pa·s. Neither gravity nor surface tension forces are considered.
## Job Title: Steady-State Fluid Flow in Joints for Validation

**UDEC (Version 4.00)**

**Legend**
- 30-Oct-13 12:12
- cycle 3301
- time 6.210E-02 sec
- domain pore pressures
- maximum pressure = 9.05E+05
- each line thick = 1.9900E+03
- boundary plot

---

### Diagram Description:
- **Flow rate distribution**
- **Scale:**
  - 0.00 = 1.00
  - 2.81E-03
  - 2.05E-03
  - 1.83E-03
  - 1.60E-03
  - 1.37E-03
  - 1.14E-03
  - 9.13E-04
  - 6.75E-04
  - 4.57E-04
  - 2.29E-04
  - 0.00
- **Dimensions:**
  - 200 m
  - 120 m

---

### Additional Information:
- **Table**

---

(c)
Figure 4.4 The fracture pressure and flow rate distribution of Model 1

The pressure and the flow rate distributions of the fractures calculated by the pipe network method and the commercial code UDEC (Itasca 2004) are illustrated in Figures 4.4a, 4.4b, 4.4c and 4.4d respectively. Because the intact rock is assumed impermeable, the isolated fractures are identified automatically and are coloured grey in Figures 4.4a and 4.4b. These fractures are not included in the calculation. For the same reason, fractures are trimmed if their ends are not connected to another fracture. The same model is simulated by the UDEC software for verification. UDEC also automatically deleted the fracture tips that do not intersect other fractures. Table 4.1 compares the results between the pipe network method and the UDEC software. As shown, the results agree very well for both the pressures and the flow rates at different points. Although UDEC can be used to simulate the fluid flow in fracture networks, the pipe network method is more efficient and flexible for simulating steady-state fluid flow in discrete fracture networks. The calculation of the HAF can also be conveniently incorporated into the pipe network method.
Table 4.1 Comparison of pressures and flow rates in the same points between UDEC and current model

<table>
<thead>
<tr>
<th>Positions</th>
<th>Pressures (MPa)</th>
<th>Flow rates ($\times 10^{-3}$m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UDEC</td>
<td></td>
<td>A  B     C    D</td>
</tr>
<tr>
<td>1</td>
<td>0.9903</td>
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</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td>0.9212</td>
</tr>
<tr>
<td>Difference</td>
<td>0.98%</td>
<td>-0.04%</td>
</tr>
</tbody>
</table>

4.4 INFLUENCES OF FRACTURE PATTERNS ON THE HYDRO-GEOMETRIC ANISOTROPY AND PERMEABILITY ANISOTROPY

In this section, different fracture patterns are analysed in order to study the effects of different geometric and hydraulic parameters of fracture sets on the bulk permeability anisotropy, and the relation between the hydro-geometric anisotropy and the permeability anisotropy. Multiple fracture sets are very common in natural rock masses. Therefore fracture sets with different mean inclination angles between the fracture sets and the main axis, different mean intersection angle between two fracture sets and different equivalent hydraulic apertures are studied. The effective fractures in the simulation models are searched automatically by the developed pipe network method. Fractures that belong to any route that connects two boundaries are activated. Furthermore, the combined HAF and PAR are used to study the seepage through the fractured rock masses.

The PAR is a quotient of the equivalent permeabilities of two orthogonal directions. The values of the PARs are derived by numerical simulations of the pipe network method. Figure 4.5 demonstrates the boundary conditions for the determination of the equivalent permeability. For each direction, two constants and two linearly varied pressure boundaries are applied. These are the same boundary conditions adopted by Oda (1985) and Zhang et al. (1996). Although a symmetrical hydraulic boundary condition is applied, the outflow rate is not equal to the inflow rate in the same direction. This is due to the unsymmetrical distribution of fracture networks. Therefore, the averaged flow
rate of inflow and outflow is adopted to evaluate the equivalent permeability. The HAF is calculated by Equation (4.7).

**Figure 4.5 Pressure boundaries for the determination of the equivalent permeability of the fractured rock masses**

### 4.4.1 Two orthogonal persistent fracture sets

First, rock masses with two orthogonal (vertical and horizontal) persistent fracture sets are studied. The models’ size in this section is set to be $10\times 10\text{m}$, which has the same size with the models used in the following sections, unless otherwise specified. The parameters of the two fracture sets are the same except for the equivalent hydraulic apertures. The spacing of each fracture set is 0.5m. The equivalent hydraulic aperture for one fracture set is assigned to be 0.05mm as a datum, and the other fracture set has the equivalent hydraulic aperture that is respectively 2 times, 1.7 times and 1.3 times that of the reference size. A model with the same equivalent hydraulic aperture has also been studied for comparison. Rotations of the models are carried out to study the permeability anisotropy in different orthogonal directions. Figure 4.6 shows the correlation of the HAF and PAR with respect to the different equivalent hydraulic aperture ratios of the two fracture sets. The Figure is a polar graph, the radius of which represents the values of the HAF and the PAR. When the HAF is 1, the fracture network pattern is isotropic. The permeability is accordingly also isotropic. The larger the HAF, the stronger is the permeability anisotropy. As shown in Figure 4.6, the HAFs agree well with the PARs in this pattern. When the equivalent hydraulic apertures of the two fracture sets are the same, the distribution of the fractures is isotropic both geometrically
and hydraulically. The anisotropy factors remain a constant of 1.0, regardless of the rotation angle between the fracture set and the reference axis direction. However, the anisotropy of the equivalent hydraulic aperture distribution affects the permeability anisotropy significantly. When the equivalent hydraulic aperture ratio is 2.0, the equivalent permeability in the principal axis direction is 8 times that in the orthogonal direction, following a cubic law of the equivalent hydraulic ratio.

Figure 4.6 HAF and PAR with different hydraulic aperture ratio of the two orthogonal fracture sets

a. Simulation model for rock masses with oblique fracture sets
b. HAF and PAR with hydraulic aperture variation of the oblique fracture sets

**Figure 4.7 HAF and PAR with hydraulic aperture variation of the oblique fracture sets**

### 4.4.2 Two fracture sets crossing obliquely

Rock masses that have two fracture sets crossing obliquely with an intersection angle of 50° are studied. The inclination angle of the main fracture set (set I) is 30° and the trace length, gap and spacing are 5 m, 0.5 m and 0.5 m respectively. The other fracture set (set II) has an inclination angle of 80° and the corresponding parameters are 3m, 0.3m and 0.5m respectively. The simulation model is generated stochastically following normal distribution of the parameters as shown in Figure 4.7a. The standard deviations for the inclination angle, trace length, gap and spacing are 0.01, 0.2, 0.2 and 0.1, respectively. The aperture size of the main fracture set is increased to 1.3, 1.7 and 2.0 times of its original size of 0.05mm. The same value of 0.05mm is also used as the fixed aperture size for the second fracture set. As illustrated in Figure 4.7b, the permeability anisotropy is enlarged with the increase of the differences in the equivalent hydraulic aperture, though other geometric parameters have not been changed. It is also noticed that the principal permeable direction is affected by the equivalent hydraulic aperture. With the increase of the aperture of the main fracture set, the principal permeable direction changes from about 55°, 40° and 35° to about 32° with respect to the 1 time, 1.3 times, 1.7 times and 2.0 times of the original aperture size. Varying the equivalent
hydraulic aperture distribution makes the fracture set with the larger aperture the
dominant flow conduits that control the principal permeable direction. The results by
using the HAF is consistent with that from the PAR according to these simulations. The
HAF not only reflects the common geometric parameters of fracture networks such as
orientation, length and spacing, but also contains information about the equivalent
hydraulic aperture of the fractures, thus it is able to capture the specific permeability
feature of each fracture set.

4.4.3 Two fracture sets with different intersection angles

The main fracture set (set I) in subsection 4.4.2 is fixed at a mean inclination angle of
30° with the reference axis direction. The other fracture set (set II) has an intersection
angle that is from 20° to 140° counter clockwise with respect to set I. Other
parameters of the two fracture sets are the same as those used in section 4.4.2. The
equivalent hydraulic apertures are set to be 0.05 mm. Figure 4.8 illustrates the variation
of the PAR of the two reference directions caused by changing the intersection angles
between the two fracture sets. As depicted in Figure 4.8, the permeability along the two
directions is the same when the intersection angles between the two fracture sets are
near 30° and 90°. Where the intersection angle is 60°, the PAR reaches its lowest
value which is approximately 0.7. This means at the angle, the fracture II contributes
most in the Y direction than in the X direction. From 60° to 140°, the PAR increases,
and reaches up to 6.7. This means that the permeability in the X direction increases and
the direction becomes a relatively more favourable permeable direction as the
intersection angle increases from 60° to 140°. The curve fitting on Figure 4.8 indicates
that the variation of the PAR with the change of the intersection angle between the two
fracture sets follows a power function, which has a form of

\[ y = a(x-x_0)^n + y_0. \] (4.21)

The power \( n \) for the fitting curve of the permeability anisotropy ratio is 3.2 in this
study. The values of \( a, x_0 \) and \( y_0 \) are \( 4.07 \times 10^{-6}, 58.47 \) and 0.85 respectively. This
example only compared the anisotropy in a given direction. In order to obtain a
comprehensive understanding of the anisotropy in different directions, more directions
should be investigated.
Figure 4.8 HAF and PAR with different intersection angles between two fracture sets

a. Fracture generation in the background domain     b. Fractures in the study domain

Figure 4.9 Generation of fracture patterns in the study domain
4.4.4 Two fracture sets with different fracture densities

In order to study the effect of fracture density on the permeability anisotropy, more realistic and complex fracture patterns are generated in an area (or a background domain) that is slightly larger than the study domain, so that the fractures outside the study domain are trimmed, as shown in Figure 4.9. In the present study, the boundary size of the background domain is 50% larger than that of the study domain, which is $10m \times 10m$ in this study. Two types of fracture sets are generated. To minimize the randomness effect, for each specific parameter, different realizations should be generated and their results are then averaged. In this study, 5 groups of realizations are studied. The parameters used in this study are listed in Table 4.2. Different probability distributions are also applied to the two sets deliberately to check the feasibility of handling different fracture patterns. Figure 4.10 shows the fracture patterns and density changes of different realizations. The fracture number of the second set varies to study its effects on the permeability anisotropy. Figure 4.11 illustrates how the permeability anisotropy and hydro-geometric anisotropy change as the number of fractures of the second fracture set increases. It is obvious that as the number increases, both the PAR and the HAF decrease in this case. It indicates that the permeability along the Y direction increases faster than that along the X direction as the fracture number increases. The increasing ratio slows down gradually and becomes stable when the second fracture set starts to dominate the flow. In addition, the influence of the non-effective lengths of fractures (dead fracture tips that contribute no flow to the network) on the calculation of the HAF is studied. As shown in Figure 4.11, the difference of the HAFs between cases using the trimmed and non-trimmed models becomes larger when the fracture density decreases. This is due to the increase of the proportion of the trimmed tips as the fracture density decreases. The maximum difference of the HAFs between cases using the trimmed and non-trimmed models is less than 5%. Figure 4.11 also indicates that the results of the PAR and the HAF are consistent when the fracture network has a complicated fracture pattern. The maximum difference between the PAR and the non-trimmed HAF for the permeability anisotropy is only 5.05%. However, it increases when the fracture density decreases. In this case, the uniform pressure gradient assumption in the domain will not be fully satisfied. When the distribution of fractures becomes sparse, the bulk domain behaves less uniformly and
the fracture randomness effect becomes significant. The flow in fracture networks is thus dominated by a few sparsely connected fractures. As a result, the overall permeability from numerical simulations tends to be more scattered with different random distributions of the fractures. In comparison, the result of the HAF varies more smoothly because of the assumption of the uniform pressure gradient over the flow region. Both the HAF and the PAR decrease with a similar trend with the increase of the fracture number of the second fracture set. When this number is greater than 1500, the fluctuations of the PAR become smaller. The critical fracture density of a fracture pattern, above which the HAF is applicable, can be obtained by comparing the trend of discrepancy between the PAR and the HAF.

Table 4.2 Statistical parameters for the cases in subsection 4.4.4

<table>
<thead>
<tr>
<th>Fracture set</th>
<th>Inclination angle 0 (degree)</th>
<th>Length (m)</th>
<th>Aperture (mm)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>120 – 10 * (normal)</td>
<td>0.7 – 0.15 (normal)</td>
<td>0.05</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5 – 1.8 (uniform)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>50 – 80 ** (uniform)</td>
<td></td>
<td>0.05</td>
<td>600–8000</td>
</tr>
</tbody>
</table>

* For the normal distribution, the parameters are presented in the form of: mean value-standard deviation. ** For the uniform distribution, the parameters are two limits.
Figure 4.10 Fracture networks of different fracture numbers of the second fracture set

Figure 4.11 PAR and HAF change with fracture number variation
4.4.5 Two fracture sets with different fracture lengths

Fracture length is also an important parameter that affects the bulk fluid flow. Therefore, the effects of the distribution of fracture length on the hydro-geometric anisotropy as well as the permeability anisotropy need to be quantitatively studied. Two sets of fractures with different length distributions are generated and studied. Each fracture set has 300 fractures. The mean length of the second fracture set varies according to a normal distribution. The parameters used in these cases are listed in Table 4.3. The standard deviations for all the normal distributions in these cases are 1. For each set of fracture parameters, 4 groups of realizations are generated and their results are averaged. Figure 4.12a shows one of the fracture patterns used in the simulation. Figure 4.12b depicts how the HAF and PAR change with the variation of the fracture length of the second fracture set. When the mean length of the second fracture set increases, the permeability in the X direction increases much faster than that in the Y direction. Therefore, both the HAF and the PAR increase as the fracture length increases. Moreover, the increase is linear at the first stage when the mean length of the second fracture set is less than the domain boundary size. However the increasing rate of PAR with the fracture length becomes nonlinear at the second stage when the mean fracture length is larger than the domain boundary size. The rate of increase in PAR reduces after the mean length exceeds the boundary size of the study domain. The effect of the fracture length to affect the permeability anisotropy is limited by the size of the study domain.
Table 4.3 Statistical parameters for the cases in subsection 4.4.5

<table>
<thead>
<tr>
<th>Fracture set</th>
<th>Inclination angle $\theta$ (degree)</th>
<th>Length (m)</th>
<th>Aperture (mm)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>80 (normal)*</td>
<td>6 (normal)</td>
<td>0.05</td>
<td>300</td>
</tr>
<tr>
<td>II</td>
<td>160 (normal)</td>
<td>1~25 (normal)</td>
<td>0.05</td>
<td>300</td>
</tr>
</tbody>
</table>

* The values in the table are mean values for the normal distribution.

Figure 4.12 PAR and HAF change with fracture length variation of the second fracture set

Figure 4.12 PAR and HAF change with fracture length variation
The distribution of fractures in the rock masses follows certain stochastic laws. Therefore, in order to estimate the anisotropic features of the fracture pattern, theoretically, an infinite domain is theoretically required to dampen the fluctuation caused by the uncertainty that arises from the scale effect. The above result demonstrates how permeability anisotropy is affected by the study scale. From the results of Figure 4.12b, it is suggested that the size of the study domain should be larger than the mean fracture length to capture the characteristics of permeability anisotropy of fracture networks.

4.4.6 The influence of anisotropy distribution of the equivalent hydraulic aperture on the permeability anisotropy

Both statistically and geometrically isotropic fracture patterns have been generated for the sake of investigating the effects of anisotropy of hydraulic parameters on the permeability anisotropy of fracture systems. Two orthogonal fracture sets with the same geometric distribution parameters are adopted in this case. Both fracture sets have 300 fractures, and their lengths obey a normal distribution with the mean length of 3m. Set I has the mean inclination angle of 10° and set II has the mean inclination angle of 100°. Both inclination angles follow a normal distribution. Equivalent hydraulic apertures are assumed to be independent and follow normal distributions. The mean equivalent hydraulic aperture for set II is 0.02mm. Only the mean aperture of set I changes from 0.01mm to 0.05mm. These fracture patterns are geometrically isotropic and are hydraulically anisotropic when the mean hydraulic aperture of the two sets is not equal. Figure 4.13 shows how the permeability anisotropy changes as the equivalent hydraulic aperture varies. The geometric anisotropy factor $A_f$ defined by Zhang and Sanderson (1995) and the HAF as well as the simplified HAF ($D_f$) in Equation (4.9), are plotted together in Figure 4.13. Factors $A_f$ and $D_f$ in Figure 4.13 are derived by using three equally spaced scan lines in both X and Y directions. Because factor $A_f$ only considers the geometric information, it is not sensitive to the hydraulic anisotropy of the fractures. It only shows the isotropic feature of the geometry distribution. Both HAF and $D_f$ reflect the hydraulic anisotropy and agree well with the PAR. The HAF is able to give a proper measurement of the anisotropy of the hydraulic distribution of fracture networks, which cannot be represented by the geometric parameters alone.
Figure 4.13 The impact of the equivalent hydraulic aperture variation on the PAR and the HAF

4.5 ANISOTROPIC CONDUCTIVITY INDEX (ACI)

The conventional indexes for the connectivity of the discrete fracture network, such as $X_f$ (Hestir & Long 1990; Sahimi 1993) and P21 (Dershowitz & Herda 1992), cannot reflect the directional differences of the connectivity, which limits their applications in different fracture network settings. An appropriate connectivity index should reflect the connection quality and also be able to indicate the directionality of the fracture system. Based on the above requirements, an anisotropic conductivity index (ACI) is proposed based on Equation (4.9), which has the following definition when the X direction is considered,

$$ACI = \frac{\sum \cos \theta \alpha_i^3}{n_y L_y}$$  \hspace{1cm} (4.22)

Here, $\alpha_i$ and $\theta_i$ are the equivalent hydraulic aperture and inclination angle between a fracture and the X direction respectively. $n_y$ is the number of scan line. $L_y$ is the length of the domain in the Y direction, which is also the length of each scan line.
By using the scan line method, only the fractures intersected with the scan lines are counted. Figure 4.14 shows the scan line method used for evaluating ACI in the X direction. The scan lines should be perpendicular to the X direction. The position of the scan line is arbitrary. However, three to five equally spaced scan lines may be adopted to calculate ACI for the accuracy of the information being collected. The number of scan lines for each direction should be equal. With the scan line method, a probabilistic relationship for fracture frequency $\lambda_{x}$ with areal density $\lambda_{d}$, mean fracture length $\bar{L}$ and dip angle $\theta$ in a two-dimensional domain can be expressed as (Baecher, Lanney & Einstein 1977; Long & Witherspoon 1985),

$$\lambda_{x} = \lambda_{d} \bar{L} \cos \theta.$$  \hspace{1cm} (4.23)

The fracture frequency $\lambda_{x}$ is the number of fractures intersected with a scan line per unit length. Fracture length together with the inclination angle and density will affect the probability of a fracture being scanned or penetrated by the scan lines. With the scan line method, the key geometric properties such as direction, length, spacing and density can be well expressed. ACI also takes the hydraulic features into account. Therefore, ACI can reflect the directional hydro-geometric features of fracture networks effectively.

Figure 4.14 The schematic diagram of scan line method

Figure 4.15 shows the relationship between ACI and the corresponding equivalent permeability in the same direction according to the simulation of the cases in subsection 4.4.4. ACI has a linear relationship with the directional equivalent permeability. Both
a. Anisotropic conductivity index (ACI) in X direction with equivalent permeability in X direction

b. Anisotropic conductivity index (ACI) in Y direction with equivalent permeability in Y direction

Figure 4.15 Relations between the anisotropic conductivity index (ACI) and the equivalent permeability

regression correlation coefficients $R^2$ for the two directions are greater than 0.97. It is noticed that ACI is defined based on the derivation of HAF. ACI is an index containing directional hydro-geometric information of a fracture network. Thus, it is able to indicate the permeability in a specific direction according to the present study. It is reasonable to assume that the permeability of a fracture network is related to the quality of its connectivity at a given condition (stress, thermal etc.). Therefore for a given fracture network, ACI can readily reflect the connectivity quality in a certain direction.
Furthermore, the ratio of the ACIs in two orthogonal directions reflects the anisotropy of the fracture network.

**4.6 CONCLUSIONS**

A hydro-geometric anisotropy factor (HAF) has been derived to describe the macroscopic geometric anisotropy and the hydraulic anisotropy of fracture networks to predict the permeability anisotropy of a given domain. A discrete fracture method for seepage simulation based on a pipe network method is developed to simulate the permeability anisotropy in fracture networks.

Six patterns of fracture sets have been generated and studied to show the effects of fracture distribution on the hydro-geometric anisotropy and the permeability anisotropy. It is shown that the change of the fracture equivalent hydraulic aperture can cause the permeability to become anisotropic even if the orientation, length and spacing of the fracture sets are isotropically distributed. The HAF can reflect the variation of the permeability anisotropy effectively. When the two fracture sets are orthogonal, the change of the equivalent hydraulic aperture does not change the principal permeable direction. Where the two fracture sets are non-orthogonal, the variation of aperture of one fracture set can increase the permeability anisotropy and also tune the principal permeable direction. Varying the intersection angle between two fracture sets affects the characteristics of permeability anisotropy. A power relationship has been found between the PAR and the intersection angle. The variations of both the density and the mean fracture length of two fracture sets cause significant changes of the permeability anisotropy. However, the variation rate varies at different stages. The range to change the anisotropy by increasing the fracture length is limited by the domain size. The permeability anisotropy changes linearly with the increase in the fracture length in one fracture set when the mean fracture length is less than the domain boundary size. Both the HAF and the simplified $D_f$ are capable of indicating the hydraulic anisotropy of fracture networks and agree well with the PAR.

The HAF is consistent with the PAR according to the present simulation results. It is an effective parameter to describe the geometric anisotropy as well as the hydraulic anisotropy of fracture networks and to reflect the permeability anisotropy in two arbitrary directions of the fractured rock masses.
The proposed anisotropic conductivity index (ACI) is an excellent connectivity indicator that can reflect both the hydraulic connectivity quality and directionality of the connection. It also has a linear relationship with directional equivalent permeability. In addition, it is relatively more convenient to derive the ACI from scan line surveys.
CHAPTER 5. UNCONFINED SEEPAGE ANALYSIS IN FRACTURED POROUS MEDIA USING THE UNIFIED PIPE NETWORK METHOD

This chapter derives an unconfined flow simulation method based on the unified pipe network method for fluid flow prediction in fractured porous media.

5.1 INTRODUCTION

A seepage problem is one of the most important issues in the design and safety analyses of civil and hydraulic structures, such as earth dams, rock slopes, tailing dams etc. It can be categorized into two types, the confined seepage and the unconfined seepage. The former type is a problem with known boundaries while the boundary of the later one is not known as a priori, which brings difficulties in the unconfined seepage analysis. The determination of the location of the phreatic surface in the unconfined seepage analysis is one of the major issues towards a successful seepage assessment at an engineering site.

The unconfined seepage problem in a porous medium has been studied by many researchers. Analytical solutions (Luther & Haitjema 1999; Kacimov & Yakimov 2001) are always preferred. However, they can only deal with some simple and particular cases. It is difficult to address problems with complex geometries, and anisotropic and inhomogeneous materials. Therefore, numerical methods are generally adopted.

The finite element method (FEM) has been widely used in analysing the unconfined seepage problem due to its advantages in simulating complicated geometries, and anisotropic and heterogeneous materials (Neuman & Witherspoon 1970; Desai & Li 1983; Bathe & Khoshgoftaar 1979; Gioda & Gentile 1987). There are two main types of mesh algorithms used in FEM in simulating unconfined seepage problems, the fixed mesh algorithm and the variant mesh (adaptive mesh) algorithm. In the fixed mesh algorithm, the mesh of the geometry domain is fixed while the hydraulic conductivity is iteratively adjusted within the fixed mesh (Zheng, Dai & Liu 2009). The unsaturated zone is incorporated into the whole simulation domain by reducing the hydraulic conductivity. A Heaviside function or a penalized quasi-Heaviside function is
commonly selected to achieve the hydraulic conductivity reduction and make the flow in the dry domain approximately zero (Zheng et al. 2005). An inappropriate choosing of the Heaviside function may lead to inaccuracy or numerical instability. Alternatively a suitable pressure-permeability law is used to adjust the parameters of the material (Zhang, Xu & Chen 2001). However, the relationship between the permeability and the pore pressure is difficult to obtain by tests. In comparison, the variant mesh algorithm holds the hydraulic conductivity as a constant while the mesh is modified during the successive iterations to match the variable flow domain (Neuman & Witherspoon 1970; Ouria & Toufigh 2009). The mesh modification and remeshing process is time-consuming, especially in cases of large change in geometry. It is also hard to treat inhomogeneous material near the phreatic surface.

To overcome the mesh deformation problem, some meshless methods have been proposed by researchers, such as the natural element method (NEM) (Jie et al. 2013) and the element free method (EFM) (Li, Ge & Jie 2003). However, the disadvantages of the meshless methods are mainly in their expensive computational time and difficulties in satisfying essential boundary conditions.

The boundary element method (BEM) has also been applied in the unconfined seepage analyses (Chugh & Falvey 1984; Wu, Lin & Chiou 1996). For a variable domain problem, BEM has an intrinsic advantage in treating the change of the geometry boundary by relocating the boundary elements. However, its drawbacks are also obvious in that it is difficult to handle inhomogeneous materials as well as nonlinear problems. There are many other methods that have been applied in the solution of the unconfined flow problem, such as the finite difference method (FDM) (Bardet & Tobita 2002), the finite volume method (FVM) (Darbandi et al. 2007), the numerical manifold method (NMM) (Jiang et al. 2010) and the boundary-fitted coordinate transformation method (Jie et al. 2004).

Although a number of research results have been published, all the above mentioned methods were developed for unconfined seepage in porous media. The continuum-based unconfined seepage methods make it difficult to analyse problems in porous media with discontinuities, such as rock slopes and nuclear repositories. Because an equivalent continuum method cannot take the geological and hydraulic characteristics of each
fracture into account explicitly, the spatially distributed flow features are difficult to be modelled. Only a few works have been published on the study of unconfined seepage analysis in fracture networks (Jing, Ma & Fang 2001; Jiang et al. 2013), they mainly considered the flow in the fracture network while neglecting the flow in the porous medium.

For seepage flow in a fractured porous medium, there are some available numerical methods, such as the dual-permeability method (Vogel et al. 2000) and hybrid methods (Kim & Deo 2000; Bogdanov et al. 2003a). The dual-permeability method couples the discontinuities with the continuous medium to form two continuous media with different permeabilities. Hybrid methods model the discontinuities explicitly and discretise the continuous medium using different methods such as FEM and FVM.

The traditional pipe network method is one that models flow in the discontinuous networks such as flow in karst aquifers (Clemens et al. 1996; Liedl et al. 2003) as well as in fracture networks (Li, Xu & Ma 2014). Recently the pipe network method has been developed to simulate the continuous media (Xu, Ma & Li 2014). It is effective to treat flow in a fractured porous medium as well as flow in fracture networks.

In this chapter, a novel unconfined seepage analysis method is derived based on our newly developed unified pipe network method. It treats continuous medium and discontinuous medium both as virtually assembled pipe networks. The unconfined seepage problems in both the continuous medium, and the continuous medium coupled with discontinuities, are solved. Unconfined seepage problems with homogeneous and inhomogeneous materials, different geometrical boundaries, different drain conditions and arbitrary fracture distributions are studied to verify the flexibility and versatility of the developed method.

5.2 UNCONFINED SEEPAGE FLOW

Figure 5.1 illustrates an example of unconfined seepage flow in an earth dam. The phreatic surface $AE$ divides the whole domain into two parts, the dry region $\Omega_d$ and the wet region $\Omega_w$. The determinations of the phreatic surface are the main objectives of the unconfined seepage problem.
The total head or hydraulic head \( h \) of a point in the wet region is the summation of the pressure head \( p/\rho g \) and the elevation head \( y \), and it satisfies,

\[
h = \frac{p}{\rho g} + y \quad \text{(in } \Omega_w),
\]

where \( p \) is the fluid pressure, \( \rho \) is the fluid density, \( g \) is the gravitational acceleration and \( y \) is the ordinate of the point. Assuming Darcy’s law is satisfied in \( \Omega_w \),

\[
v = -K \nabla h, \quad \text{(in } \Omega_w). \tag{5.2}
\]

Here, \( v \), \( K \) and \( \mu \) are Darcy velocity, hydraulic conductivity and fluid dynamic viscosity, respectively. \( \nabla h \) is the total head gradient.

The fluid flow in the wet region obeys the continuity equation,

\[
\nabla \cdot v = 0, \quad \text{(in } \Omega_w). \tag{5.3}
\]

It also satisfies the following boundary conditions:

1) the water head boundary conditions on the upstream face AB and the downstream face CD,

\[
\begin{align*}
h &= h_u, \\
h &= h_d.
\end{align*} \tag{5.4}
\]

2) the impervious boundary condition on BC,

\[
q_n = v \cdot n = 0, \tag{5.5}
\]

where \( n \) is the unit vector normal to the boundary surface.
3) the boundary condition on the seepage surface DE,

\[ h = y . \]  

(5.6)

4) the boundary condition on the phreatic surface AE,

\[ \begin{cases} \mathbf{v} \cdot \mathbf{n} = 0 \\ h = y \end{cases} . \]  

(5.7)

The two conditions for the phreatic surface should be satisfied simultaneously. The flux normal to the phreatic boundary should be zero and the pore pressure on the phreatic surface should also be zero. Because of the nonlinear boundary conditions as well as the unknown phreatic surface, the solution of the problem is an iteration process.

## 5.3 THE UNIFIED PIPE NETWORK METHOD (UPNM)

As derived in Chapter 3, either the porous medium or the fracture network can be reconstructed by discrete one-dimensional pipes. In order to simulate the unconfined fluid flow in fractured porous medium, the UPNM is adopted. In this section, the theory of the UPNM is briefly reviewed.

In a 2D domain, the fractures can be regarded as pipes that transfer water between their two ends. According to the “cubic law”, water flow in the fracture pipe is governed by,

\[ Q_{ij}^f = \rho g K_{fp(i,j)} (h_i - h_j) , \]  

(5.8)

where, \( K_{fp(i,j)} \) is a conduct coefficient of the fracture pipe segment \( fp(i,j) \).

The 2D porous medium can also be reconstructed by using matrix pipes,

\[ Q_{ij}^m = \rho g K_{mp(i,j)} (h_i - h_j) . \]  

(5.9)

Here, \( K_{mp(i,j)} \) is an equivalent conduct coefficient of the matrix pipe segment \( mp(i,j) \).

The derivation of equivalent conduct coefficients can refer to chapter 3.

After the reconstruction of the porous medium, the continuous medium is regarded as an unstructured pipe network. The fractured porous medium is treated as the assembly of the matrix pipes and the fracture pipes. When the fracture system is embedded in the porous matrix, the equations for the fractures and the porous medium are coupled.
together by the pipe merging or pipe superposition at the interface of fracture and porous medium to form interface pipes. For an interface pipe $ij$, which shares the same nodes with the fracture pipe $fp(i,j)$ and the matrix pipe $mp(i,j)$, the combined equivalent conduct coefficient $K_{ij}$ can be obtained by a superposition principle as follows,

$$K_{ij} = K_{mp(i,j)} + K_{fp(i,j)}.$$  \hspace{1cm} (5.10)

Therefore the global equation of the UPNM is,

$$\rho g [K][h] = [Q_s].$$ \hspace{1cm} (5.11)

The assembly of the coefficient matrix is according to the connection of pipes, which can refer to subsection 3.2.2.

The nodal hydraulic head is obtained by solving the global matrix equations. The direction of the flow velocity at each node is determined by the direction of the head gradient at each node after knowing the head distribution. The Darcy velocity of a node is calculated by Equation (3.41).

In such a simple yet effective manner, the fluid flow in discontinuities and continuous medium is combined together under the unified pipe network frame. No capillary force is considered in this model. The treatment of the flow becomes efficient and flexible.

### 5.4 Unified Pipe Network Method for Unconfined Seepage

As stated beforehand, there are two algorithms to determine the shape and location of the phreatic surface for the unconfined flow problem if a mesh based method is applied. Both the fixed mesh algorithm and the adaptive mesh algorithm have their own advantages and drawbacks. The Non-Boundary-Fitted Meshes (NBFM) algorithm (Bathe & Khoshgoftaar 1979; Desai & Li 1983; Cividini & Gioda 1984) used in the finite element method is an attractive approach that decreases the mesh dependency. The variable domain problems are solved without mesh iteration. However, for the boundary elements cut by the phreatic surface, the integration of the irregular parts is rather complex and difficult. Some works have been carried out to eliminate the
difficulty of the domain integration (Kazemzadeh-Parsi & Daneshmand 2013; Kazemzadeh-Parsi & Daneshmand 2012). With regard to the UPNM, the integration of the elements is unnecessary. It would be much easier to deal with the boundary intersecting meshes in the UPNM. Therefore, an auxiliary boundary pipe method is proposed to conduct the searching of the phreatic surface in unconfined seepage problems using the UPNM. In this approach, both the advantages of the fixed mesh algorithm and the adaptive mesh algorithm are integrated in the UPNM.

5.4.1 Auxiliary boundary pipe method

Considering an already meshed possible flow domain as illustrated in Figure 5.2, the mesh is not necessary to coincide with the model boundary that is confined by the phreatic surface. The possible flow domain usually adopts the entire geometric area of the problem. The mesh is intersected by the phreatic surface. Thus the initial mesh is divided into three parts: the Internal Meshes (IM), the External Meshes (EM) and the Boundary Intersecting Meshes (BIM). The original nodes are separated by the phreatic surface into Internal Nodes (IN) and External Nodes (EN). As shown in Figure 5.2, the nodes of the IM meshes are all IN nodes, while the EM meshes are all formed by the EN nodes. The BIM meshes are enveloped by both the IN nodes and the EN nodes. In a 2D problem, the phreatic surface (dashed line) is approximated by a polygonal curve linking a serial of vertexes. The vertexes of the polygonal curve as well as the points intersected by the phreatic surface and the edges of the BIM meshes are the Boundary Intersecting Nodes (BIN).

![Figure 5.2 Schematic of non-boundary fitted mesh in UPNM](image)
For the current UPNM, a triangular element mesh is used. The BIM meshes are cut by the phreatic surface into two parts: the internal parts and the external parts. The internal parts may not be the triangular shape. Therefore auxiliary pipes are needed to maintain the internal parts to be all triangles. There are typically four types of possible patterns that BIM meshes are cut by the phreatic surface as illustrated in Figure 5.3, when it is assumed that any two vertexes of the polygonal curve is not in the same mesh. Other patterns are the variations of these four patterns. For example, there may be the situation that the phreatic surface may go through the nodes of the mesh. The hatched part of a mesh, which is enveloped by IN nodes and BIN nodes, is the internal part of the BIM mesh. For patterns (a) and (b), the vertex of the phreatic polygonal curve is not in the mesh, while for patterns (c) and (d), the vertex is inside of the BIM mesh. In patterns (a) and (c), the internal part is a quadrilateral, therefore the auxiliary pipe (in red) is the diagonal line of the quadrilateral. In this way, two new boundary meshes are formed. In pattern (b), the internal part is a triangle. Therefore, no treatment is required. For pattern (d), the internal part is a pentagon. Two auxiliary pipes are required, and this is obtained by linking the vertex of the polygonal curve with two IN nodes respectively. Three new boundary meshes are formed. With the reconstruction of the auxiliary pipes, all of the internal parts are triangular meshes. As such, new boundary meshes as well as new boundary pipes are formed and incorporated into the simulation domain. The IM meshes are fixed, and only the information of new boundary pipes, meshes and nodes need to be updated.

![Figure 5.3 Four types of typical cutting patterns](image)

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5.4.2 Solution algorithm

The phreatic surface is approximated by a polygonal curve. The solution process is trying to find the location of the vertexes at the polygonal curve. Figure 5.4 illustrates the flow chart of the solution algorithm. Before the numerical simulation, either the whole domain or the domain bounded by the phreatic surface with the initial guess, is assumed to be the flow domain. In this chapter, the boundary parameterization algorithm is adopted as shown in Figure 5.5, which suits for both porous media and fractured porous media. With the given base point $B_i$ as well as the unit direction vector $e_i$, the location of the vertex $x_i$ is determined by (Kazemzadeh-Parsi & Daneshmand 2012),

$$x_i = B_i + r_ie_i,$$

where, $i$ represents the $i$th vertex of the polygonal curve. $r_i$ is the distance between the base point and the corresponding vertex.

Figure 5.4 Flow chart of solution algorithm
At the beginning of the simulation, the whole domain is discretised by triangular meshes, which are fixed throughout the simulation process. Theoretically, \( B_i \) and \( e_i \) can be chosen arbitrarily except that the seepage point should be on the seepage surface. What is more, the direction vector for the seepage point should be along the direction of the possible seepage surface. With the initial guess of the distance \( r_0 \), the flow in the model is simulated by the UPNM with the boundary condition in Equations (5.4), (5.5), (5.6) and zero normal flux condition (first equation) of Equation (5.7). Thereafter, the elevations and the distance vector of the vertexes are updated iteratively according to the new hydraulic head of each step, by

\[
y_i^{n+1} = (1 - \omega) y_i^n + \omega h_i^n, \quad i \in \{ \text{vertex points, excluding the seepage point } s \}, \quad (5.13)
\]

and

\[
r_i^{n+1} = \left( y_i^{n+1} - y_i^B \right) / e_{s’, i}, \quad i \in \{ \text{vertex points, excluding the seepage point } s \}. \quad (5.14)
\]

Here, \( n \) denotes the iteration step; \( y_i \) is the elevation of the vertex; \( \omega \) is the relaxation factor. \( y_i^B \) and \( e_{s’, i} \) are the elevation of the base points and the vertical component of the direction vector \( \cos \theta_i \) for the \( i \) th vertex, respectively.

As for the last seepage point, which is located on the seepage surface, the zero pore pressure boundary condition is satisfied in advance. Therefore the zero normal flux
condition is applied to update the location of the seepage point, the distance parameter $r_n$ of which can be adjusted by

$$r_{s}^{n+1} = r_{s}^{n} + \eta (\hat{v} \cdot n),$$  

(5.15)

where, subscript $s$ represents the seepage point; $\hat{v}$ is the unit flux vector at the seepage point and $n$ is the unit outward normal vector to the phreatic surface at the seepage point. $\eta$ is a parameter that adjusts the updating speed of each step.

Normally the iteration stops after convergence is reached. For a given tolerance $\varepsilon_0$, the simulation stops, if the following condition is reached:

$$\|r^{n+1} - r^n\| \leq \varepsilon_0.$$  

(5.16)

Alternatively, it stops after the maximum iteration step is exceeded.

### 5.4.3 Convergence acceleration

Fast convergence is essential for the solution process. Although, theoretically, the iteration process will converge after steps of calculation; with the increase of the complexity of the problems or the choices of inappropriate calculation parameters or both of them, it may fail to converge or it converges very slowly. For the sake of fast and stable convergence, an average acceleration method (AAM) is proposed.

In the iteration process of solving a nonlinear problem, a relaxation factor is often used to achieve a fast or stable result. When the under-relaxation factor ($\omega < 1$) is used, it takes part of the previous iteration result to dampen the solution and cut out the steep oscillations. However, the convergence speed is slowed down. While the over-relaxation factor ($\omega > 1$) is used, it gives a higher weight on the current iteration result to accelerate the convergence. Nevertheless, it may lead to numerical instabilities. The results sometimes will oscillate around the true result within a relative small range and difficult to converge. An effective way to accelerate the convergence is to average the historical results when the oscillation happens. For a sequential iteration result series $r^1$, $r^2 \ldots r^n$, if the new step result $r^{n+1}$ is the recurrence of one former result $r^k$ (where $k$ is the step number and satisfies $0 < k \leq n$), then the new step result $r^{n+1}$ is modified as,
\[ r^{n+1} = \frac{1}{n-k+1} \sum_{j=k}^{n} r^j. \]  
\[ (5.17) \]

In order to check whether the result \( r^{n+1} \) is the recurrence of the former result, a tolerance \( \varepsilon_1 \) is used to estimate. The result \( r^{n+1} \) is the recurrence of the result \( r^k \), if

\[ \| r^{n+1} - r^k \| \leq \varepsilon_1. \]  
\[ (5.18) \]

5.5 **NUMERICAL EXAMPLES**

In this section, some typical numerical examples with various geometry and boundary conditions are solved to evaluate the application of UPNM in simulating the unconfined seepage problem. The results are compared with those reported in the literature to validate the effectiveness and versatility of the proposed method. A tolerance of \( \varepsilon_0 = 1.0 \times 10^{-4} \) is used for the following examples.

5.5.1 **Example 1. Homogeneous rectangular dam**

The unconfined seepage through a rectangular cross-section dam as shown in Figure 5.6 (a) has been studied by many researchers. The dimension and water elevations of the upstream and the downstream have been shown in the figure. The unit direction vector of the seepage point has been set along the seepage face as illustrated in Figure 5.6 (a). The convergent phreatic surface obtained by the present method is compared with the results obtained by other methods.
The model of the example 1; (b) The phreatic surface obtained by UPNM compared with the results from Bardet-Tobita (FDM) (2002), Bazyar-Graili (SBFEM) (2012), Lacy-Prevost (FEM) (1987), Borja-Kishnani (FEM) (1991) and Jie et.al. (NEM) (2013); (c) The flow net

Figure 5.6 (a) Rectangular dam with tailing water (example 1)

Table 5.1 Comparison of parameters of different methods for Example 1

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of degrees of freedom</th>
<th>Number of iterations</th>
<th>Pressure head on phreatic surface (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bardet-Tobita (Bardet &amp; Tobita 2002)</td>
<td>44</td>
<td>4</td>
<td>-0.03 to 0.04</td>
</tr>
<tr>
<td>Bazyar-Graili (Bazyar &amp; Graili 2012)</td>
<td>231</td>
<td>11</td>
<td>-0.604</td>
</tr>
<tr>
<td>Lacy-Prevost (Lacy &amp; Prevost 1987)</td>
<td>231</td>
<td>4</td>
<td>0 or 0.13</td>
</tr>
<tr>
<td>Borja-Kishnani (Borja &amp; Kishnani 1991)</td>
<td>32</td>
<td>7</td>
<td>0.057</td>
</tr>
<tr>
<td>UPNM</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1 lists the number of degrees of freedom, the number of iterations, the pressure head on the phreatic surface from literatures and the current method. Figure 5.6 (b) shows the phreatic surfaces determined by different methods. It is found that the
phreatic surface calculated by the present UPNM agrees well with those reported by other researchers. Figure 5.6 (c) shows the flow net obtained by the UPNM. Models of different mesh sizes as well as arbitrary initial guesses are also studied in Figure 5.7 and Figure 5.8 respectively. It shows that the UPNM is neither mesh sensitive nor initial guess sensitive. All models can converge to the same final position with reasonable errors.

Figure 5.7 Phreatic surface of the homogeneous rectangular dam with different meshes

Figure 5.8 Shape variation history in successive iterations with different initial guesses
The effectiveness of the AAM is studied in this example. With different relaxation factors $\omega$, total iteration steps with and without the AAM are both listed in Table 5.2. It is found that when the relaxation factors $\omega$ equals 1.0, most less iteration steps are required for both methods with and without the AAM in this example. When the AAM is not used, both under-relaxation factors and over-relaxation factors will increase the iteration steps significantly. In contrast, with the help of AAM, the iteration steps can be reduced obviously compared to the cases without AAM. What is more important is that the using of AAM can reduce the dependency of the choice of the relaxation factor to the convergence. The iteration steps can be stabilized in an acceptable range with the different choices of $\omega$.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
<th>1.2</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without AAM</td>
<td>17</td>
<td>14</td>
<td>9</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>With AAM</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

5.5.2 Example 2. Homogeneous trapezoidal dam with slanted downstream face

Figure 5.9 shows the geometry of a homogeneous trapezoidal dam with a slanted downstream face. The size of the problem as well as the unit directional vector of the seepage point is also illustrated in the figure. The direction of the directional vector should be along the downstream face. The number of degrees of freedom of the original fixed mesh is 324. The iteration step is 9 to fulfill the tolerance $\varepsilon_0$. As demonstrated in Figure 5.10 (a), the phreatic surface computed by the proposed method is compared with those found by other researchers. It is obvious that the result is consistent with other results. Figure 5.10 (b) shows the flow net obtained by the present simulation.
Figure 5.9 Trapezoidal dam with tailing water

(a) The phreatic surface obtained by UPNM compared with the results from Bardet-Tobita (FDM) (2002), Bazyar-Graili (SBFEM) (2012), Lacy-Prevost (FEM) (1987) and Borja-Kishnani (FEM) (1991); (b) The flow net

Figure 5.10 (a) Simulation results of the homogeneous trapezoidal dam (example 2)
5.5.3 Example 3. Homogeneous rectangular dam with sheet pile

(a) The Phreatic surface obtained by UPNM compared with the results from Bardet-Tobita (FDM) (2002), Bazyar-Graili (SBFEM) (2012), Lacy-Prevost (FEM) (1987) and Borja-Kishnani (FEM) (1991); (b) The flow net

Figure 5.12 The simulation results of the homogeneous rectangular dam with sheet pile (example 3)
Figure 5.11 illustrates the geometry of a homogeneous rectangular dam with an impermeable sheet pile on the upstream. The unit directional vector is vertical along the downstream face. The position of the phreatic surface on the upstream face is not fixed in this problem. For the present model, the number of degrees of freedom for the initial fixed mesh is 681. The iteration step is 8 to fulfil the tolerance $\varepsilon_0$. Sound agreement is achieved as demonstrated in Figure 5.12 (a) compared with other results obtained from many different methods. The flow net of the problem is shown in Figure 5.12 (b).

5.5.4 Example 4. Homogeneous rectangular dam with horizontal toe drain

(а) The phreatic surface obtained by UPNM compared with the results from Bardet-Combina (FDM) (2002), Bazyar-Graili (SBFEM) (2012), Lacy-Prevost (FEM) (1987) and Borja-Kishnani (FEM) (1991); (b) The flow net

Figure 5.14 The simulation results of the homogeneous rectangular dam with horizontal toe drain (example 4)
Figure 5.13 shows the geometry of a homogeneous rectangular dam with the horizontal toe drain. As illustrated in the figure, the possible seepage point is on the toe drain; therefore the unit directional vector for the seepage point should be horizontal and along the toe drain. The number of degrees of freedom of the initial fixed mesh is 412. Convergence is achieved in 22 iterations to fulfil the tolerance $\varepsilon_0$. Sound agreement can be found in Figure 5.14(a) by the comparison of the current result with those results by other investigators. Figure 5.14(b) shows the flow net of the seepage in example 4.

### 5.5.5 Example 5. Nonhomogeneous rectangular dam

![Diagram of nonhomogeneous rectangular dam](image)

(a) The model of example 5; (b) The phreatic surface obtained by UPNM compared with the results from Bardet-Tobita (FDM) (2002), Bazyar-Graili (SBFEM) (2012), Lacy-Prevost (FEM) (1987) and Borja-Kishnani (FEM) (1991); (c) The flow net

**Figure 5.15 Nonhomogeneous rectangular dam with permeability varying horizontally (example 5)**

The feasibility of using the UPNM to solve the free seepage problem in nonhomogeneous dam is demonstrated in this example. As illustrated in Figure 5.15 (a), a nonhomogeneous rectangular dam with the permeability of the downstream half, which is ten times the permeability of the upstream half, is simulated. Due to the material property changing across the interface, the phreatic surface will drop down...
rapidly near the interface. For the unstructured triangular mesh of the UPNM, it is easy to reduce the mesh size near the interface to model the rapid variation precisely. The number of degrees of freedom of the initial fixed mesh is 885 for current study. The iteration step is 29 to make the error less than the tolerance $\varepsilon_0$. Figure 5.15 (b) shows the results obtained by the proposed UPNM as well as the results from other investigators. The results show ready agreement. Figure 5.15 (c) is the flow net of the seepage problem.

5.5.6 Example 6. Fractured rock slope

In addition to the capability of simulating the free seepage problem in continuous domain, the UPNM can also solve the free seepage problem in the fractured porous media efficiently. Figure 5.16 illustrates the geometry of a rock slope that contains fractures. The influences of fractures in different patterns on the phreatic surface are studied, which include the different number of fracture sets, fracture directions and fracture hydraulic apertures. The ideal homogeneous rock slope without any fractures is studied for comparison. One non-persistent parallel fracture set with the trace length of 30m, gap of 12m and spacing of 20m is embedded in the rock mass with the direction angle of 110° as shown in Figure 5.17 (a). The direction angle of the fracture set is the angle between the fracture and the horizontal direction. To study the influences of the direction of the fracture set on the phreatic surface, non-persistent fracture sets with the same parameters but with the direction angle of 70° and 150° also are studied. Another model is the one that contains two fracture sets with the combination of the fracture set of 70° and 150°. A model that contains random fractures shown in Figure 5.17 (b) is investigated as well. For the above models, the hydraulic apertures of 0.2mm and 0.5 mm are used.
Figure 5.16 Fractured rock slope

Figure 5.17 (a) Mesh for rock slope with one fracture set (110°); (b) Mesh for rock slope with random fractures

Figure 5.18 depicts the hydraulic head of different models with the hydraulic aperture of 0.5mm after the convergence. It is found that the fractures in the rock mass can remarkably distort the distribution of the hydraulic head. Thus, the shapes as well as the positions of the phreatic surface are altered accordingly. The hydraulic head distributions of the two fracture sets case and the random fracture case are affected more severely than that of one fracture set case.
Figure 5.18 Contour of hydraulic head for different models: (a) homogeneous rock slope; (b) rock slope with one fracture (110°); (c) rock slope with two fracture sets (70°+150°); (d) rock slope with random fractures

Figure 5.19 shows the comparisons of the phreatic surface obtained from different models. It is found that the hydraulic apertures of the fractures affect the phreatic surface obviously. Compared to the result of the homogeneous rock mass model, the average elevation decreases of the 110° fracture set models are 1.36m and 4.11m for the 0.2mm and 0.5mm hydraulic aperture cases respectively. It is also noticed that the direction of the fracture set will affect the position of the phreatic surface. When the direction of the fracture set is close to the direction of the gradient of the hydraulic head, the phreatic surface is lowered significantly. While the direction of the fracture set is nearly perpendicular to the direction of the gradient of the hydraulic head, the variation of the phreatic surface will be slight. In the present study, the fracture sets with the direction angles of 110° and 150° affect the phreatic surface more significantly. In
contrast, the influence of the fracture set with the direction angle of 70° on the phreatic surface is weak, no matter what the size is of the hydraulic aperture. The irregular fracture pattern make the phreatic surface much rougher than those affected by the regular pattern.

![Figure 5.19](image)

**Figure 5.19** Comparisons of phreatic surfaces with different fracture patterns (a) fracture apertures are 0.2mm; (b) fracture apertures are 0.5mm

### 5.6 CONCLUSION

The unified pipe network method (UPNM) has been applied to analyse the unconfined seepage problem in both continuous domain and continuous domain with discontinuities. The method deals with both continuous domain and discontinuities as the connected pipe networks. Therefore, it is convenient to simulate flow in continuous domain embedded with discontinuities. An auxiliary boundary pipe method is developed to conduct the searching of the phreatic surface, which takes both the advantages of the fixed mesh algorithm and the adaptive mesh algorithm. Due to the pipe reassembly on the phreatic boundary, it is very effective to reconstruct the global coefficient matrix. The proposed average acceleration method can accelerate the iteration process significantly and make the solution process more stable. The validity and versatility of the UPNM method have been verified by analysing unconfined seepage problems with homogeneous and nonhomogeneous materials, different geometry boundaries, different drain conditions and fractured porous medium. The results computed by the present method are compared with those investigated by other researchers. Significant agreements are achieved.
Unconfined seepage problems in rock slope with different fracture patterns have been studied. The results show that the presence of the fractures in the rock slope can distort the distribution of the hydraulic head and change the shape as well as the location of the phreatic surface. The direction, hydraulic aperture, fracture set number and fracture distribution can affect the phreatic surface significantly. With the increase of the hydraulic aperture of the fractures, the influences on the location of the phreatic surface become stronger. When the direction of the fracture set is close to the direction of the gradient of the hydraulic head, the phreatic surface is lowered more significantly. While the direction of the fracture set is nearly perpendicular to the direction of the gradient of the hydraulic head, the influence on the phreatic surface is weak.
CHAPTER 6. TRANSIENT (SATURATED & UNSATURATED) FLOW ANALYSIS IN FRACTURED ROCK MASS TOWARDS AN INSIGHT INTO UNDERGROUND PETROLEUM STORAGE

This chapter improves the 2D and 3D unified pipe network methods for the simulation of variably saturated transient flow processes in the fractured rock masses and studies the seal effects of the water curtain system in underground petroleum storage.

6.1 INTRODUCTION

The underground storage of petroleum in unlined rock caverns is one of the most important strategic storage methods in many countries. The implementation of the large capacity underground storage is economically competitive compared to other storages. Furthermore, there are some well-recognized advantages of underground storage, such as less evaporation losses, low environmental impact, low maintenance, improved safety during operation, security against terrorism, war and natural calamities, space saving etc. (Naithani 2012).

The concept of petroleum storage in an unlined rock cavern below groundwater was first proposed by a Swedish inventor H. Jansson in 1939 (Lindblom 1994). Similar ideas were patented by another Swede H. Edholm in 1949 and a test plant was built in 1951. The first commercial project of unlined rock cavern underground storage was constructed in 1952 in Göteborg (IVA 1988). From then on, a number of large scale unlined rock cavern storage projects have been constructed around the world (Yu et al. 2013).

The fundamental principles for successful unlined rock cavern storage is mainly based on the principle that the fuel oil is confined in the rock cavern by the hydrostatic pressure of surrounding water and the inward pressure gradient (Goodall, Åberg & Brekke 1988; Åberg 1977). The two prerequisite conditions are that the petroleum is lighter than water and it is insoluble in water. These two conditions are automatically satisfied for petroleum. Therefore, it is theoretically safe when the rock cavern is located well below the surrounding ground water level. However, the natural ground
water recharge may not be sufficient for sealing the rock cavern during both the construction phase and the operation phase. An artificial water curtain is required to make sure that the rock mass around the rock cavern is fully saturated and the pore water pressure is always higher than the static pressure exerted by the petroleum (Froise 1987). Thus, the water in the surrounding rock keeps flowing into the cavern to prevent the inflow of the air and the outflow of the oil and its vapour. In other hand, if too much water leaks in the cavern, it not only wastes fresh water but also increases the difficulty during the construction period, and the running cost during the operational phase.

The accurate seepage analysis and proper design of the water curtain system play key roles in the application of the unlined rock cavern storage. Most of the previous research focuses on the overall performance of the storage facility. There are few detailed studies (Rehbinder, Karlsson & Dahlkild 1988; Li et al. 2014) on the quality of water curtain systems in literature, especially on their performance in the fractured rock mass. However, due to the complexities in rock mass properties, fracture geometry, and hydrogeological conditions for storage caverns, the behaviour of the water curtain can be quite different from the assumption of the continuous rock mass. The seal effect of the water curtain in the fractured rock mass has not been fully investigated.

To analyse the hydraulic behaviour of the ground water in fractured rock mass during the construction and operational stages, numerical simulation is required in most cases. Currently, the most commonly used simulation methods for seepage analysis of tunnel inflows in fractured rock mass are equivalent continuum methods, such as the commercial software FLAC (Sun & Zhao 2010) and FLAC3D (Liu 2009; Sun, Zhao & Zhang 2011; Yu et al. 2013) and the FEM (Coli et al. 2008). The equivalent continuum methods are relatively easy to apply. However the fractured rock mass is a highly heterogeneous material. Individual fractures, especially the large fractures, may dominate the flow. The continuum methods may be incapable of capturing the detailed characteristics of the seepage field. Therefore, the discrete fracture network method (DFN), and the hybrid method which couples the fracture flow with the porous flow, are more competent to simulate the water flow in fractured rock mass.

In this chapter two-dimensional and three-dimensional transient variably saturated flow methods for seepage analysis in fractured rock mass, are derived based on the unified
pipe network method. Water flow can be modelled solely in the discrete fracture network and in the rock matrix, or be modelled in both the rock matrix and the fractures. The water interchange between the matrix and the fracture at the interface is considered in the proposed method. With the developed transient UPNM, numerical simulations are carried out to analyse the seepage field of the underground petroleum storage project in Jingzhou, China. The effects of the water curtain on the seal effect are studied.

6.2 TRANSIENT SATURATED-UNSATURATED FLOW IN FRACTURED ROCK MASS

The variably saturated water flow in the fractured rock mass satisfies the mass conservation law (Richards 1931), which can be expressed as follows,

$$\frac{\partial (\rho S \phi^\tau)}{\partial t} - \nabla \cdot \left( \rho \frac{k^\tau}{\mu} (\nabla p - \rho g) \right) = \rho q,$$

where $\rho$ and $\mu$ are the water density and the water viscosity respectively. $S$ is the water saturation. $\phi$ and $k$ are the medium porosity and permeability respectively. $p$ is the water pressure. $q$ is the source term. $g$ is the gravitational acceleration. The superscript $\tau = (m, f)$ represents the medium type ($\tau = m$ for the rock matrix and $\tau = f$ for the fractures).

The permeability is calculated by

$$k = k_s k_r(S),$$

where $k_s$ is the intrinsic permeability of the medium depending on the pore geometry of the medium. $k_r(S)$ is the relative permeability, which is a function of the saturation. For the fracture, the smooth parallel plates model is adopted (Tsang & Witherspoon 1981). Therefore, the intrinsic permeability of fracture with equivalent hydraulic aperture $a$ can be calculated by

$$k_s = \frac{a^2}{12}.$$
The water pressure is related to the atmospheric pressure $p_g$, which is 0 at the ground water table. In the unsaturated zone, the water pressure has a relationship to the capillary pressure as,

$$p = p_g - p_c.$$  \hspace{1cm} (6.4)

Here, $p_c$ is the capillary pressure. In view of the zero atmospheric pressure, water pressure $p = -p_c < 0$ in the unsaturated zone.

The material properties of the fracture and the rock matrix have large differences. The porosity and void of the fracture are larger than those of the rock matrix. The capillary pressure and relative permeability relationships also are different for the two media. Therefore, the interface conditions should be introduced to model the interaction between the fracture and the matrix. The first condition is that water is conserved across the interface. The second condition is the capillary continuity condition (Monteagudo & Firoozabadi 2004). The capillary pressure in matrix is equal to the capillary pressure in the fracture at the interface:

$$p_c^f (S^f) = p_c^m (S^m).$$ \hspace{1cm} (6.5)

Because the capillary pressure curves of the fracture and the matrix are different, the saturations in the fracture and the matrix are different at the interface in the unsaturated zone, which is illustrated in Figure 2.7. The saturation of the medium can be obtained according to the capillary pressure models adopted.

### 6.3 DERIVATION OF UNIFIED PIPE NETWORK MODEL FOR TRANSIENT SATURATED-UNSATURATED FLOW

As derived in chapter 3, saturated flow in the fractured rock mass can be modelled using the unified pipe network method. The 2D and 3D rock matrix can be reconstructed by the corresponding matrix pipes. The 1D and 2D fractures can be represented by the corresponding fracture pipes. The problem domain is reconstructed by the assembly of different pipes. The fluid transfer in both fractures and rock matrix is equivalent to fluid transfer through pipes. The pipe flow law for any pipe $(i, j)$ can be uniformly expressed as,
\[ Q_{ij}^r = K_{(i,j)}^r \left( \Phi_i - \Phi_j \right). \]  \hfill (6.6)

Here \( Q_{ij}^r \) is the volumetric flow rate from node \( i \) to node \( j \). \( \Phi = p + \rho g \) is the flow potential. \( K_{(i,j)}^r \) is the equivalent conductance coefficient of the pipe, and this has been derived in chapter 3. Generally the form of \( K_{(i,j)}^r \) is found as

\[ K_{(i,j)}^r = \frac{Ak}{l_y \mu}. \]  \hfill (6.7)

where \( k \) is the intrinsic permeability of the medium. \( l_y \) is the length of the pipe. \( \mu \) is the viscosity of water. \( A \) is the influence area of the pipe.

### 6.3.1 Unsaturated pipe flow

In an unsaturated zone, the unsaturated pipe flow need to consider the influence of the saturation by multiplying the relative permeability:

\[ Q_{ij}^r = k_r^r (S) K_{(i,j)}^r \left( \Phi_i - \Phi_j \right). \]  \hfill (6.8)

Because the saturation in the fracture pipe may be different from the saturation in the matrix pipe at the interface, the fracture pipe and the matrix pipe at the interface cannot be merged together by the superposition principle as used in the saturated flow. The two flow pipes should be treated separately as shown in Figure 6.1. For the stability of the numerical calculation, an upwind scheme is applied to calculate the relative permeability,

\[ k_r^r = \begin{cases} k_r^r (S_i) & \text{if } \left( \Phi_i - \Phi_j \right) > 0 \\ k_r^r (S_j) & \text{otherwise} \end{cases}. \]  \hfill (6.9)

![Figure 6.1 Fracture pipes and matrix pipes at the interface](image)
6.3.2 Capillary pressure continuity condition for unsaturated pipe flow

At the interface of fracture and matrix, the capillary continuity condition is applied. If water pressure is $p_i$ at the interface in the unsaturated zone, capillary pressure is $-p_i$. The saturation can be obtained according to the capillary pressure curves adopted,

$$S_i^* = \left[ p_i^c \right]^{-1} (-p_i).$$  \hspace{1cm} (6.10)

Figure 6.2 shows the different capillary pressure curves for the fracture and the matrix, and the calculation of saturation from the capillary pressure.

![Figure 6.2 Typical capillary pressure curves for fracture and matrix, and saturation calculation through capillary pressure continuity condition](image)

6.3.3 Governing equation of saturated-unsaturated pipe flow in fractured rock mass

At every pipe node, mass conservation is applied,

$$\frac{\partial M_i}{\partial t} + \rho \sum_{j=1}^{n_i} Q_{ij} = \rho Q_{S_i}, i \in \{n \mid n \text{ is pipe node}\},$$  \hspace{1cm} (6.11)

where $M_i$ is the mass of water accumulated in the node $i$, $Q_{S_i}$ is the source in the node, $n_i$ is the number of nodes that connect to the node $i$. Equation (6.11) is the governing equation of the saturated-unsaturated pipe flow in fractured rock mass.

The water mass in a node is obtained by
\[ M_i = \rho \sum_{j=1}^{n_i} \phi_{(i,j)}^r V_{(i,j)}^r S_{(i,j)}^r, \]  

(6.12)

where \( \phi_{(i,j)}^r \) and \( V_{(i,j)}^r \) are the porosity and the node volume of pipe \((i,j)\) respectively. \( S_{(i,j)}^r \) is the water saturation in the pipe. The saturation \( S_{(i,j)}^r \) can also be approximated by the upwind scheme,

\[ S_{(i,j)}^r = \begin{cases} S_i & \text{if } (\Phi_i - \Phi_j) > 0 \\ S_j & \text{otherwise} \end{cases}. \]  

(6.13)

### 6.3.4 Time discretization and solution

From equation (6.12), the mass accumulation \( M_i \) is the function of saturation, which can be determined by the water pressure according to equation (6.10). The capillary pressure curve is a nonlinear function. Therefore the governing equation (6.11) is the nonlinear equation of pressure,

\[ \frac{\partial M_i(p(t))}{\partial t} + \rho \sum_{j=1}^{n_i} Q_{ij}^r(p(t)) - \rho Q_{ij}^r(t) = 0, \quad i \in \{n \mid n \text{ is pipe node}\}. \]  

(6.14)

To solve the time-dependent nonlinear equations, different time differencing schemes can be used (Hairer & Wanner 1991). For brevity, \( M_i(p(t)) \) is denoted as \( M_i^t \) and \( Q_{ij}^r(p(t)) \), \( Q_{ij}^r(t) \) are denoted as \( Q_{ij}^{r,t} \) and \( Q_{ij}^{r,t} \) respectively. If the water pressure at time \( t \) is known, for a given time step \( \Delta t \) the time step scheme can be written as

\[ \frac{M_{i}^{t+\Delta t} - M_{i}^{t}}{\Delta t} + \theta \left( \rho \sum_{j=1}^{n_i} Q_{ij}^{r,t+\Delta t} - \rho Q_{ij}^{r,t} \right) + (1-\theta) \left( \rho \sum_{j=1}^{n_i} Q_{ij}^{r,t} - \rho Q_{ij}^r \right) = 0. \]

(6.15)

For \( \theta = 0 \), it yields the explicit Euler scheme. The unknowns at step \( t+\Delta t \) can be directly obtained from the state at step \( t \). However the time step \( \Delta t \) should be small enough to guarantee the numerical stable of the scheme. For \( \theta = 1/2 \), it yields the Crank-Nicholson scheme. For \( \theta = 1 \), it yields the implicit Euler scheme, which is adopted in this study. The implicit Euler scheme is first-order accurate and very stable.
With the initial conditions $p_0 = p(t = 0)$, $S_0 = S(t = 0)$ and the boundary conditions $p|_{\Gamma_D} = p_D$ on Dirichlet boundary $\Gamma_D$ and $Q|_{\Gamma_N} = Q_N$ on Neumann boundary $\Gamma_N$, equation (6.15) can be solved. To find the state at step $t + \Delta t$, the nonlinear system equations can be solved by the Newton-Raphson method. The Jacobian matrix is calculated by the numerical derivatives. The linear system of each Newton-Raphson iteration step is solved by the direct sparse solver GSS (GRUSOFT 2014).

### 6.4 VALIDATIONS AND EXAMPLES

#### 6.4.1 Transient flow in porous medium

In order to evaluate the performance of the transient saturated-unsaturated UPNM, 2D and 3D numerical tests have been carried out. The 2D and 3D examples are shown in Figure 6.3, the analytical solutions of which were developed by Tracy (2006). The domain of the 2D example is a square with the dimensions of $1\text{m} \times 1\text{m}$. The domain of the 3D example is a cube with the dimensions of $1\text{m} \times 1\text{m} \times 1\text{m}$. The parameters of the tests are the same with the 2D example used by BurzyŃski and Szymkiewicz (2011). The porosity and the intrinsic permeability of the two examples are 0.45 and $1.0194 \times 10^{-12}$ respectively. The density and viscosity of water is $10^3 \text{kg/m}^3$ and $10^{-3} \text{Pa}\cdot\text{s}$ respectively. The gravitational acceleration is $-9.81 \text{m/s}^2$. The capillary function and relative permeability function in the examples are as follows,

$$S = S_r + (1 - S_r) e^{\alpha p}, \quad (6.16)$$

$$k_r = e^{\alpha p}, \quad (6.17)$$

where $S_r$ is the residual saturation, which is $1/3$ in the examples. $\alpha$ is a coefficient, which is $2.5484 \times 10^{-5} \text{Pa}^{-1}$. 


Figure 6.3 Numerical examples for 2D and 3D transient flow in porous medium

The initial conditions of the 2D and 3D domains are the uniformly distributed water pressure $p_0 = -9.81 \times 10^4$ Pa. The boundary conditions for both the 2D and 3D domains are the constant uniform pressures at the bottom and all vertical faces with the value $p_b = p_0 = -9.81 \times 10^4$ Pa. The top boundary of the 2D case is a sinusoidal distribution of pressure, which has the form,

$$p(x, y = 1) = \frac{1}{\alpha} \ln \left( e^{x p_0} + \left(1 - e^{x p_0}\right) \sin \left(\pi x\right) \right).$$  \hspace{1cm} (6.18)

The top boundary of the 3D case is as follows,

$$p(x, y = 1, z) = \frac{1}{\alpha} \ln \left( e^{x p_0} + \left(1 - e^{x p_0}\right) \sin \left(\pi x\right) \sin \left(\pi z\right) \right).$$  \hspace{1cm} (6.19)

The analytical solution of water pressure evolution at a specific point $(x, y)$ of the 2D problem is calculated as (Tracy 2006),

$$p(x, y, t) = \frac{1}{\alpha} \left[ e^{x p_0} + \bar{p} \sin(\pi x) e^{2 \gamma (t-y)} \left[ \frac{\sinh(\beta_2 y)}{\sinh(\beta_2)} + \frac{2}{c} \sum_{n=1}^{\infty} (-1)^n \frac{\lambda_n^2}{\gamma} \sin(\lambda_n y) e^{(-\gamma t)} \right] \right].$$  \hspace{1cm} (6.20)
The analytical solution of water pressure evolution at a specific point \((x, y, z)\) of the 3D problem is calculated as (Tracy 2006),

\[
p(x, y, z, t) = \frac{1}{\alpha} \left\{ e^{\alpha p_0} + \bar{p} \sin(\pi x) \sin(\pi z) e^{\alpha (1-y)} \left[ \frac{\sinh(\beta_3 y)}{\sinh(\beta_2)} + \frac{2}{c} \sum_{n=1}^{\infty} (-1)^n \frac{\lambda_n}{\gamma} \sin(\lambda_n y) e^{(-\gamma t)} \right] \right\},
\]

where, \(c = \frac{\alpha \phi (1-S_r)}{k_s}\), \(\beta_2 = \sqrt{\frac{\alpha^2}{4} + \pi^2}\), \(\beta_3 = \sqrt{\frac{\alpha^2}{4} + 2\pi^2}\), \(\lambda_n = n\pi\), \(\gamma = \frac{\beta_2^2 + \lambda_n^2}{c}\) and \(\bar{p} = 1 - e^{\alpha p_0}\).

Figure 6.4 compares the water pressure contour of the numerical result with the water pressure contour from the analytical result of the 2D example at the time of 180s. The simulation model consists of 322 nodes and 899 flow pipes. Figure 6.4 shows that the UPNM result is consistent with the analytical result. For a detailed comparison, the water pressure along the middle line at \(x=0.5m\) is plotted in Figure 6.5.

(a) Analytical result    (b) Numerical simulation result

**Figure 6.4** Comparison of the pressure contour of the 2D example with the analytical result
Figure 6.5 Comparison of the water pressure profile of the 2D example at $x=0.5m$ with the analytical result

The numerical simulation result of the 3D example at the time of 180s and the corresponding analytical solution is shown in Figure 6.6. The simulation model consists of 11430 nodes and 74770 flow pipes. The water pressure profiles of the numerical simulation and the analytical result at the centre line are plotted in Figure 6.7. It also indicates that the result of the UPNM is consistent with the analytical result.

(a) The analytical result
(b) The numerical simulation result

Figure 6.6 Comparison of the pressure contours of the 3D example with the analytical result

Figure 6.7 Comparison of the water pressure profile of the 3D example at x=0.5m and z=0.5m with the analytical result

6.4.2 Transient flow in three-dimensional discrete fracture network

The UPNM can also simulate 3D transient fracture flow efficiently. Fifty elliptic fractures have been randomly generated in a cubic domain with the size 1m × 1m × 1m.
The out-of-domain parts of the fractures have been trimmed. The fracture apertures are set to be 0.1mm. The porosity and intrinsic permeability of the fractures is 1.0 and 8.33×10^{-10} \text{ m}^2 respectively. The model is meshed by using the conforming mesh and it consists of 26200 nodes and 78331 flow pipes. The initial conditions, the boundary conditions, relative permeability function, and capillary function are the same as that used in the example of 3D transient flow in porous medium. The residual saturation of the fractures is 0.33 in this example. The coefficient $\alpha$ in this example is $4 \times 10^{-5} \text{ Pa}^{-1}$. The water pressure distribution at 180s is shown in Figure 6.8.

![Figure 6.8 Distribution of water pressure of the 3D DFN example](image)

### 6.4.3 Transient flow in fractured porous medium

The above examples demonstrate the capability of the UPNM in simulating transient variably saturated flow in the 2D and 3D porous media and fracture network. The UPNM can also simulate transient variably saturated flow in fractured porous media efficiently. In a 2D problem, fracture is simulated as 1D fracture pipe. In a 3D problem, fracture is 2D and it is reconstructed using 1D fracture pipes. To validate the transient variably saturated flow in fractured porous medium, a vertical fracture is imbedded in the 2D example in subsection 6.4.1. The coordinates of fracture location are from (0.5,
1.0) to (0.5, 0.2). In order to compare the results with traditional 2D fracture model, the fracture aperture is set to be 5mm. The porosity and intrinsic permeability of the fractures is 1.0 and $2.083 \times 10^{-6}$ m$^2$ respectively. The relative permeability function and capillary function are the same as those used in the example of 2D transient flow in porous medium. The residual saturation of the fractures also is 0.33 in this example. The initial conditions and boundary conditions are the same as those used in the 2D transient porous flow example. The distribution of water pressure at 10s of the fractured porous is shown in Figure 6.9(a). The fracture also is modelled as a 2D porous medium as in the traditional way. The material properties of the 2D fracture are the same as the 1D fracture. The 2D fracture has been discretised using very small triangular meshes. The 2D fracture imbedded porous medium is simulated using the UPNM as a combined porous medium with different material properties for different parts. The simulation result at 10s is shown in Figure 6.9(b) for comparison. Although the fracture is modelled in different way, the results are very close. However, the 1D fracture is much convenient in the UPNM model. For a rigorous validation, the results of pore pressure at 10s and y=0.5m are compared in Figure 6.10. The results from UPNM also are compared with the results calculated by commercial software COMSOL (Comsol 2011), which implements the FEM methods and simulates fractures as interior boundaries. As illustrated in Figure 6.10, the results are consistent. This numerical test demonstrates the effectiveness of 1D fracture pipe model and the capillary pressure continuity condition adopted at the interface, which is essential to the simulation of fracture flow using 1D fracture pipe. Otherwise, the results could not be such accurate, even if the mesh size near the fracture is significantly improved.
Figure 6.9 Distribution of water pressure at 10s of the transient 2D fractured porous flow
Figure 6.10 Comparison between different methods and models for the 2D transient flow problems in fractured porous medium

To test the variably saturated transient flow in 3D fractured porous medium, two fractures have been imbedded in the 3D porous model in subsection 6.4.1. The position and the size of the fractures are shown in Figure 6.11. Fracture 1 is horizontally and centrally located 0.2m below the top. Fracture 2 is vertically located at x=0.5m and it crosses with the top face of the model. The two vertical edges of fracture 2 are both 0.2m from the model’s boundaries. The aperture and porosity of the fractures are 0.5mm and 1.0, respectively. The coefficient for calculating the saturation of the fracture is $8.0 \times 10^{-5}$; and it is $3.55 \times 10^{-5}$ for calculating the relative permeability. All other conditions and coefficients are the same as those in the 3D example in subsection 6.4.1. The simulation result is shown in Figure 6.12. The fractured porous medium is effectively reconstructed by the connection of the fracture pipes and the matrix pipes. These pipes are assigned with different material properties. With the uniform structure, the building of the coefficient matrix and the solving process is straightforward.
Figure 6.11 Geometry of 3D fractured porous model with two fractures

(a) Result at 36s    (b) Result at 72s

(c) Result at 108s    (d) Result at 180s

Figure 6.12 Slices of pressure distribution of 3D transient fractured porous flow
6.5 SEEPAGE ANALYSIS OF JINGZHOU PETROLEUM UNDERGROUND STORAGE IN UNLINED ROCK CAVERN

Natural rock mass is a discontinuous material. The fractures in the rock are the main conduits of the water flow. For accurate analysis of water flow and mass transfer in the rock mass, it is necessary to consider the influence of the imbedded fractures. In this section, an underground petroleum storage project in Jingzhou China is studied using the transient UPNM.

6.5.1 Site description

The Jingzhou project is located in Liaoning province, in the northeast of China. The rock in the study area consists primarily of micro-weathered granites. Less than 10% of the rock mass is diorite and amphibolite. According to the evaluation of the Q-values system, 80% of rock is classified as fair to very good rock mass quality. Generally competent and unweathered rock provides excellent rock mass conditions for cavern construction. Based on field test data, the dominant strikes of the joint system of the field area are NNW and NEE. There are five well-developed joint sets, which are 270°±30°, 150°±75°, 85°±60°, 330°±30° and approximately horizontal joints.

Groundwater in the site is mainly in the form of pore water in the incompact rock mass and fracture water in the bedrock. According to the water pressure test, the overall permeability of the rock mass is very low, ranging from $1.55 \times 10^{-16} \text{ m}^2$ to $3.57 \times 10^{-14} \text{ m}^2$ in the rock mass where joints are relatively well developed. Flow mainly occurs in the fracture network. The complement of the groundwater is mainly from the precipitation. The average annual precipitation in the area over the past 20 years has been 532.7 mm, and heavy rains tend to occur between June and September when more than 76% of the annual precipitation occurs.
6.5.2 Description of the project

The storage capacity of the project is $3 \times 10^6$ m$^3$. The project is comprised of 8 east-to-west storage caverns, 8 shafts, 2 construction tunnels and 5 water curtain tunnels, which are shown in Figure 6.13. The water curtain tunnels are parallel to the storage caverns. Every two storage caverns form a storage group, which includes one oil inlet shaft and one oil outlet shaft. The two storage caverns in the same storage group are connected by a connection tunnel. Every storage group is sealed by a set of water curtain systems consisting of a set of horizontal water curtain boreholes and two sets of vertical water curtain boreholes drilled from the water curtain galleries. Figure 6.14 is a cross-section view of the project. The length of each storage cavern is 934m, and the designed attitude of the bottom of the storage cavern is at -80m. The shape of the cross section of the storage caverns is vertical wall and arched roof, with 19m width and 24m height. The water curtain galleries are installed 24m above the roof of the storage cavern. The shape of the cross section of the water curtain galleries also is arched roof with 6.5m width and 6m height.
6.5.3 The simulation model

In this study one boundary storage group, which is bounded by the dash lines in Figure 6.14 is selected to conduct the numerical analysis. The simulation model chooses one segment of the storage group, the dimension of which is 141.75m × 120m × 100m, shown in Figure 6.15. The spaces of the horizontal and the vertical water curtain boreholes are 10m and 20m respectively. Two-dimensional elliptic discrete fractures are stochastically generated in the three dimensional space according to the investigated geological data of the joint sets. Figure 6.16 illustrates the generated discrete fracture network model and the spatial flow pipes that reconstruct the fractures. These fractures are cut by the storage caverns and the water curtain tunnels.
Figure 6.16 Three-dimensional discrete fracture network model
In the numerical simulation, different boundary conditions are applied to model work conditions in different phases, namely the construction phase and the storage phase. In the construction phase, the rock caverns are empty. Atmospheric pressure is applied to the surface of the caverns. In the storage phase, the rock caverns are full of oil with the density of 900 kg/m$^3$. Oil pressure will apply to the surface of the caverns at the storage phase. The water curtain system operates before the construction of the main caverns applying fixed water pressure at the water curtain boreholes. The bottom of the model is applied with the static water pressure to consider the effect of the ground water. Other boundaries are non-flow boundaries.

Each fracture can be assigned with different hydraulic properties. However, the fracture apertures are all set to be 0.08mm in this study. The capillary pressure function and relative permeability function use the exponential form functions in Equations (6.16) and (6.17). The residual saturation is 0.25 and the coefficient $\alpha$ is $5 \times 10^{-5}$ Pa$^{-1}$.

### 6.5.4 Results and discussion

The water curtain system plays an important role in oil storage in the underground rock caverns. The oil is confined in the rock caverns by the higher water pressure around the caverns. On the one hand, low water pressure can cause leakage of the oil, and this will contaminate the ground water. On the other hand, high water pressure can reduce the strength of the surrounding rock, deteriorating the stability of the rock cavern. In addition, it increases the operational cost and the water usage.

*The effects of the pressure of the water curtain system*

![Images showing the effects of water curtain pressure](image1.png)  
(a) No water curtain  
(b) 0.1 MPa water curtain pressure
The influences of the water curtain pressure on the pore pressure field are analysed in the study. Figure 6.17 shows the flow velocity and pore pressure contours under cases of different water curtain pressure. For comparative purposes, the case without the water curtain system is presented as well. In Figure 6.17, the unsaturated parts, the pore pressures of which are less than zero, are uncoloured. From the results, it is obvious that the pressure of the water curtain affects the position of the water table of the ground water. The high water curtain pressure can increase the local ground water table and pressure gradient around the caverns. Therefore, the high water curtain pressure improves the water-seal effects. However, the high water curtain pressure increases the inflow rate. The increase of the inflow rate is linear with the increase of the water curtain pressure as shown in Figure 6.18. The inflow rate of the two whole rock caverns

**Figure 6.17 The front views of the pore pressure contours at the construction phase**

(c) 0.2 MPa water curtain pressure  (d) 0.3 MPa water curtain pressure

(e) 0.4 MPa water curtain pressure  (f) 0.5 MPa water curtain pressure
is calculated from the simulation results. Figure 6.18 also plots the increase of the inflow rate with the increase of the water curtain pressure at the storage phase. The increase trend is nearly parallel to the trend at the construction phase. However, the increasing rates of flow into the caverns are different for different water curtain system as shown by deviation from the black square line and the blue triangle line. The difference of the increasing rate of these two lines is solely contributed by the vertical water curtain.

![Figure 6.18 Water flow rate into the two rock caverns](image)

The fracture network is highly irregular in the flow domain, which greatly influences the seepage field. It is found that the unsaturated flow path can be locally formed when the water curtain pressure is low. Figure 6.19(a) shows the local unsaturated flow path in the case of 0.1 MPa water curtain pressure, which links the rock cavern to the top unsaturated zone. The local unsaturated path can cause either the leakage of the oil vapour or the invasion of the air. This phenomenon cannot be observed if the continuous model is adopted. The increase of the water curtain pressure can seal the unsaturated path effectively as shown in Figure 6.19(b).
(a) Oblique view of the case applying 0.1MPa water curtain pressure

(b) Oblique view of the case applying 0.4MPa water curtain pressure

Figure 6.19 Illustration of the locally connected unsaturated flow path
The effects of the vertical water curtain

To study the effects of the placement of the water curtain on the seepage field, the cases with only the top horizontal water curtain boreholes are simulated at the construction stage. Without the recharge of the two vertical water curtain boreholes, the water inflow rate into the rock cavern is smaller than the cases with both the horizontal and vertical water pressure boreholes. The increasing rate of the inflow with the pressure also is smaller as shown in Figure 6.18. According to the simulation results, the vertical water curtains increase the pressure gradient on the vertical sides of the caverns. The water table does not change significantly. The differences between the cases with the vertical water curtain boreholes and the cases without the vertical water curtain boreholes are not obvious. The importance of the vertical water curtain is less than that of the horizontal water curtain.

The effects of the lengths of the horizontal water curtain boreholes

The horizontal water curtain boreholes in the above cases are continuous between the two water curtain galleries. The distance between the two water curtain galleries is 105.25m. Therefore, each borehole is 52.625m from the water curtain gallery. The cases of different horizontal borehole lengths are studied. The vertical water curtain boreholes on the two sides are unchanged. Figure 6.21 shows the pressure distributions of the models at the construction phase. It is found that the vertical water curtain system alone cannot seal the rock caverns. Large portions on the top of the caverns are unsaturated. With the increase of the length of the horizontal water curtain boreholes, the pressure and the pressure gradient around the rock caverns increase obviously. The inflow rate of water into the rock caverns is shown in Figure 6.20. It is observed that the difference of the inflow rate between the cases with and without the horizontal water curtain system is very large. The placement of the horizontal water curtain system greatly increases the inflow of water. However, a further increase in the length of the horizontal boreholes does not increase the inflow rate significantly. Because the fractures in the space are two-dimensional plates, the increase in the length of the boreholes cannot guarantee intersecting with other fractures. Therefore, it is better to connect the two horizontal water curtain boreholes from the two water curtain galleries to reduce the chance of missing the vertical fractures in the gap. If the vertical fracture goes through to the top
and it is unconnected with other fractures, it may produce the unsaturated path to the top when it is not recharged by the water curtain system.

Figure 6.20 Variation of inflow rate with the length of the horizontal water curtain boreholes
(a) Only vertical water curtains  (b) 10m horizontal boreholes
(c) 20m horizontal boreholes  (d) 30m horizontal boreholes
(e) 40m horizontal boreholes  (f) 52.625m horizontal boreholes

Figure 6.21 The front views of the pore pressure contours at the construction phase with different lengths of the horizontal water curtain boreholes
The effects of the spacing of the water curtain boreholes

The proper choosing of spacing of the water curtain boreholes is important for the design of the water curtain system. Numerical simulations of cases of different spacing of water curtain boreholes are carried out. The horizontal water curtain borehole is continuous between the two water curtain galleries. The pressure of the water curtain system is 0.3 MPa in this study. The spacing of the vertical water curtain boreholes and the horizontal water curtain boreholes is the same in each case. Figure 6.22 shows the pore pressure contours of the simulations at construction phase. As seen from the results, the average water table decreases when the spacing of the water curtain boreholes increases. More importantly, the pore pressure and pressure gradient around the storage cavern decrease significantly, which deteriorate the seal effect of the water curtain. The irregularity and the discontinuity of the fracture network also make the flow field more complex. Unlike the continuous media, the seepage variables of which change smoothly in space, the seepage variables of the fracture network change abruptly in space. It is found that when the spacing of the water curtain increases, the connected unsaturated flow paths from the storage cavern to the top boundary can appear in some local fractures. An example of the unsaturated flow path can be seen in Figure 6.23 when the spacing reaches to 30m. Therefore, the design of the water curtain system should take into account the discontinuity of the fracture network. Any local unsaturated flow path can cause leakage of the oil vapour, which is very dangerous.

(a) The spacing of 10m (b) The spacing of 20m
In fact, the fracture network in the project site is random and irregular. Therefore, for a more detailed design, more fracture network models are required to conduct the Mont Carlo simulation.
6.6 CONCLUSION

In this chapter, the numerical simulation method for the transient saturated-unsaturated flow in fractured rock mass is developed based on the unified network method. Both flows in the fractures and in the rock matrix are modelled with flows in fracture pipes and in the matrix pipes. At the interface of the fracture and the rock matrix, a capillary continuity condition is applied, and this considers the fluid interchange in the interface at the unsaturated zone. However, the saturations in the fracture and in the rock matrix can be different at the interface. Two-dimensional and three-dimensional numerical examples for flow in continuous porous medium, in discrete fracture network and fractured rock mass have been presented to demonstrate the versatility and feasibility of the unified pipe network method for transient flow simulation in the fractured rock mass.

The Jingzhou project of petroleum underground storage in the unlined rock caverns is studied by using the transient variably saturated unified pipe network method. The discrete fracture network model is adopted to analyse the effects of the water curtain system on the water sealing of the rock caverns in the fractured rock mass. Both the construction phase and the storage phase have been considered in the study. The effects of the horizontal water curtain system, the vertical water curtain system, the length of the horizontal water curtain boreholes, the pressure of the water curtain and the spacing of the water curtain boreholes are systematically analysed. It is found that the effect of the horizontal water curtain is more important than the vertical water curtain. The vertical water curtain system alone is incapable of sealing the rock cavern in this study. The reduction in the pressure of the water curtain, the length of the water curtain boreholes and the increase of the spacing of the water curtain borehole can reduce the seal effect of the water curtain. It is worth mentioning that the failure of the water seal effect can occur locally. The locally connected unsaturated flow paths have been observed in the study, which cannot be found by using the continuous simulation model. This is because of the discontinuity and irregularity of the discrete fracture network. The water flow in the fracture network can be very complex. A conventional continuous model may cause inaccurate results. Although an increase in the pressure of the water curtain can enhance the seal effect, it also leads to some disadvantages, such as the drastic increase in the inflow rate and the detriment to the stability of the caverns. In view of the discontinuity of the fracture network, the continuous horizontal water
curtain boreholes and the relatively more closely spaced water curtain boreholes are recommended, and these are more effective methods in enhancing the seal effect of the water curtain system.
CHAPTER 7. TWO-PHASE TRANSIENT FLOW IN FRACTURED ROCK MASS AND GEOLOGICAL SEQUESTRATION OF CARBON DIOXIDE

This chapter further extends the 2D and 3D unified pipe network methods for the simulation of multiphase flow process in fractured porous rock masses and studies the injection process of CO\(_2\) into saline aquifer.

7.1 INTRODUCTION

The emission rate of carbon dioxide (CO\(_2\)) produced by human activity into the atmosphere has exceeded the natural equilibrium rate by many orders of magnitude (Haszeldine 2006). Most scientists consider that the noticeable increase of CO\(_2\) will cause global warming based on the investigation of geological history. The Intergovernmental Panel on Climate Change (IPCC) has projected that the atmospheric concentration of CO\(_2\) may double by the middle of the 21st century and may continue to rise at increasing rates (Houghton et al. 1996). Concerning the global warming and climate change, CO\(_2\) capture and geological storage (CCS) was proposed to mitigate the anthropogenic CO\(_2\) emission and balance the carbon cycle (Benson et al. 2005; Pacala & Socolow 2004).

The geological storage of CO\(_2\) requires certain geological structures to confine the CO\(_2\) in geological formation, which is known as structural and stratigraphic trapping (Class et al. 2009). It is likely to be the most important trap mechanism in the short term (Pruess 2004). Hydrodynamic trapping (Zhao et al. 2010; Finley, Gustison & Leetaru 2004) also is an important trapping mechanism. Other long-term sequestration mechanisms are solubility trapping, residual trapping and mineral trapping (De Silva, Ranjith & Perera 2015). There are mainly three kinds of possible sites that can be used to store CO\(_2\) in a large amount, namely the depleted oil and gas reservoir, the coal beds and the deep saline aquifer (Bachu 2000). The deep saline aquifer is a promising storage site due to its potential large storage capacity and broad distribution around the world (Zhao et al. 2010; Izgec et al. 2008a).
To gain an efficient storage capacity, usually CO$_2$ is expected to be injected into saline aquifers in a supercritical state (Nordbotten, Celia & Bachu 2005). The supercritical state CO$_2$ has a large density while maintaining a relatively low viscosity. For reaching such a state, the pressures should be over 7.38 MPa and the temperatures should be greater than 31.1 °C (critical point). Figure 7.1 illustrates the phase change of CO$_2$ with the variation of temperature and pressure. Therefore, it generally requires the depth of the saline aquifer to be greater than 740m according to the estimation based on the hydrostatic pressure (Bachu 2008).

![Phase changes in carbon dioxide](https://commons.wikimedia.org/wiki/File:Phase_diagram_of_carbon_dioxide.png)

**Figure 7.1 phase change of CO$_2$ (Wikimedia Commons 2005)**

The storage of CO$_2$ in deep geological formations triggers complex physical processes, which involve the multiphase multicomponent flow in the fractured rock formation, the phase transition, the CO$_2$ dissolution and mineral precipitation, and other chemical reactions etc. During the injection phase, advective multiphase processes dominate the CO$_2$ spreading, and these are driven by viscous forces and buoyancy. Analytical and numerical studies have been carried out to obtain insight into the injection scenarios (Nordbotten & Celia 2006; Riaz et al. 2006; Saripalli & McGrail 2002; Dentz & Tartakovsky 2009; Pruess et al. 2003; Doughty & Pruess 2004; Rutqvist & Tsang 2002; Doughty 2007; Bigi et al. 2013). Most of the numerical methods reported for the simulation of CO$_2$ geological sequestration are continuum based methods, such as the commercial reservoir simulator ECLIPSE (Zhao et al. 2010), the black oil simulator Eclipse 100 (Eigestad et al. 2009), the TOUGH serials (Oldenburg & Benson 2002; Pruess et al. 2003; Alkan, Cinar & Ülker 2010), the CMG’s STARS (Izgec et al. 2008b), the MFLOW3D (Gonzalez-Nicolas, Cody & Baü 2011) and others. Pruess et al. (2004)
and Class et al. (2009) evaluated the efficiency and accuracy of different numerical codes in their ability to model the processes of CO$_2$ deep injection into a saline formation. Very few models are based on discrete fracture network models or hybrid models. Actually, the aquifer formation is a fractured porous medium. The fractures in the aquifer have dramatic differences in properties compared with that of the host rock. They can significantly affect the CO$_2$ spreading. They will also affect the pressure build-up and the CO$_2$ distribution. The process of the CO$_2$ injection can be simulated in much detail by the hybrid models.

However, the numerical simulation of multiphase flow in discrete-fractured media can be extremely challenging (Monteagudo & Firoozabadi 2004). The difficulties of a successful numerical simulation arise from, firstly, the large contrast between the permeability in fracture and rock matrix. Secondly, the complex geometry of fracture network and the intense dimension contrast between fracture apertures with the domain size, complicate the modelling process. Thirdly, the capillary pressure and the relative permeability are highly nonlinear, and these cause numerical challenge in solving the governing equations. Fourthly, the compressibility and phase change of the CO$_2$ also complicate the simulation.

In this chapter, 2D and 3D two-phase flow hybrid models are developed using the unified pipe network method (UPNM). Fractures are represented explicitly by fracture pipes. Rock matrix is equivalent to matrix pipes. The CO$_2$ is modelled as a compressible gaseous phase or supercritical phase. Equation of state (EOS) for CO$_2$ is incorporated into the model to simulate the change of the density and viscosity. Capillary pressure is considered in this method. The Brooks-Corey capillary model and relative permeability model are adopted to consider the entry pressure and permeability variation. The extended capillary pressure condition is applied to consider the fluid interchange at the interface between the fracture and the rock matrix. A fully coupled numerical scheme is applied for solving the system equations, thus achieving large and stable time steps.

Numerical examples are demonstrated to validate the UPNM two-phase flow model. Problems of CO$_2$ injection into deep saline aquifer concerning different field temperatures and field pressures are studied. Their influences on the injection pressure build-up and the CO$_2$ plume distribution is systematically analysed. The importance of
the selection of boundary conditions on the injection pressure is studied. Besides, the influences of the fractures on the injection processes and CO$_2$ plume evolution also are studied.

### 7.2 TWO-PHASE TRANSIENT FLOW IN FRACTURED ROCK MASS

Although the geological storage of CO$_2$ is a complex process, the process of CO$_2$ injection into a deep saline aquifer can be simplified as a two-phase immiscible flow problem if the injection phase is of interest, when advective multiphase processes dominate the CO$_2$ spreading. Thus, the long-term sequestration mechanisms such as the solubility trapping, the residual trapping and the mineral trapping are neglected in this study. The CO$_2$ (in either a gaseous phase or critical phase) is the non-wetting phase, while the brine is the wetting phase. The brine is repelled by the invasion of the CO$_2$ when CO$_2$ is injected.

#### 7.2.1 Governing equations

The governing equations for two-phase flow in domain $\Omega \subset \mathbb{R}^n$ ($n = 2, 3$) are,

$$
\frac{\partial (S_w \rho_w \phi)}{\partial t} - \nabla \cdot \left( \rho_w \frac{k_w \rho_w}{\mu_w} \left( \nabla p_w - \rho_w g \right) \right) = \rho_w q_w, \text{ in } \Omega, \quad (7.1)
$$

$$
\frac{\partial (S_n \rho_n \phi)}{\partial t} - \nabla \cdot \left( \rho_n \frac{k_n \rho_n}{\mu_n} \left( \nabla p_n - \rho_n g \right) \right) = \rho_n q_n, \text{ in } \Omega, \quad (7.2)
$$

where the subscripts $w$ and $n$ represent the wetting phase (brine) and the non-wetting phase (CO$_2$) respectively. $S$ is the saturation of fluid. $\phi$ is the porosity of the medium. $\rho$ and $p$ are the density and the phase pressure of fluids. $q$ is the source term. $g$ is the gravitational acceleration, which is 9.81 m/s$^2$ downwards. $k$, and $\mu$ are relative permeability and dynamic viscosity for each phase respectively. $k$ is the intrinsic permeability vector of the media. The intrinsic permeability of a fracture is $k = a^2/12$. $a$ is the hydraulic aperture of the fracture. The relative permeability can be expressed as a function of the saturation of the wetting phase. The Brooks-Corey (1966) relative permeability function is used to describe the relative permeability curve, which is...
\[ k_{rw} \left( S_w^e \right) = S_w^{2+\lambda} , \]  
(7.3)

\[ k_{rn} \left( S_n^e \right) = \left( 1 - S_n^e \right)^2 \left( 1 - S_n^{2+\lambda} \right) . \]  
(7.4)

Here, \( \lambda \) is a pore-size distribution index. \( S_w^e \) is the effective (or normalized) saturation. It has the form as

\[ S_w^e = \frac{S_w - S_{wr}}{1 - S_{nr} - S_{wr}} , \]  
(7.5)

where \( S_{wr} \) and \( S_{nr} \) are residual saturations of wetting phase and non-wetting phase respectively.

### 7.2.2 Coupling equations

Equations (7.1) and (7.2) are mass conservation equations for both the wetting phase and the non-wetting phase. The flow of each phase follows the Darcy’s law. The coupling of the two phases is based on conditions that the saturation of the two phases is one (the pore spaces are fully occupied by the CO\(_2\) and brine) and capillary pressure between the two phases. They are expressed as

\[ S_w + S_n = 1 , \]  
(7.6)

\[ p_n - p_w = p_c . \]  
(7.7)

Here, \( p_c \) is the capillary pressure between the CO\(_2\) and the brine. Equation (7.7) indicates that the phase pressures are discontinuous at the interface. The capillary pressure is a function of the saturation of the wetting phase. According to the Brooks-Corey capillary model, the capillary pressure can be calculated by

\[ p_c = p_c S_w^{\frac{1}{\lambda}} . \]  
(7.8)

Here \( p_c \) is the entry pressure for the non-wetting phase to penetrate the wetting phase. Only when the pressure of the non-wetting phase exceeds the pressure of the wetting phase with the entry pressure, can the non-wetting phase can enter the medium in which
the wetting phase fluid resides. This is important for understanding the mechanisms of the geological sequestration.

7.2.3 Initial conditions and boundary conditions

To solve a transient problem, the initial conditions are necessary. The initial conditions describe the initial states from which the process evolves. They include the saturation distributions and pressure distributions of each phase,

\[
\begin{align*}
S_w(t = 0) &= S_{w0}(x), & \text{in } \Omega, \\
S_n(t = 0) &= S_{n0}(x),
\end{align*}
\]

(7.9)

\[
\begin{align*}
p_w(t = 0) &= p_{w0}(x), & \text{in } \Omega, \\
p_n(t = 0) &= p_{n0}(x).
\end{align*}
\]

(7.10)

The boundary conditions for two-phase flow include the Dirichlet boundary conditions and the Neumann boundary conditions. The Dirichlet boundary conditions for fluid saturations are

\[
\begin{align*}
S_w|_{\Gamma^S_{wD}} &= S_{wD}, & \text{on } \Gamma^S_{wD}, \\
S_n|_{\Gamma^S_{nD}} &= S_{nD}, & \text{on } \Gamma^S_{nD},
\end{align*}
\]

(7.11)

where \(\Gamma^S_{wD}\) and \(\Gamma^S_{nD}\) are Dirichlet boundaries for saturations of the wetting phase and the non-wetting phase respectively.

The Dirichlet boundary conditions for phase pressures are given as

\[
\begin{align*}
p_w|_{\Gamma^p_{wD}} &= p_{wD}, & \text{on } \Gamma^p_{wD}, \\
p_n|_{\Gamma^p_{nD}} &= p_{nD}, & \text{on } \Gamma^p_{nD},
\end{align*}
\]

(7.12)

where \(\Gamma^p_{wD}\) and \(\Gamma^p_{nD}\) are Dirichlet boundaries for phase pressures of the wetting phase and the non-wetting phase respectively.

The Neumann boundary conditions describe the flow across the boundary,
where $\Gamma_{wN}$ and $\Gamma_{nN}$ are Neumann boundaries for the wetting phase and the non-wetting phase respectively. $\mathbf{v}$ is the Darcy velocity across the boundaries, and $\mathbf{n}$ is the unit normal of the boundaries.

### 7.2.4 Interface conditions

Governing equations (7.1) and (7.2) are valid for both the rock matrix domain and the fractures domain. However, at the discontinuous interface between the rock matrix and the fractures, proper interface conditions must be introduced to describe the physical behaviour correctly. Experiment studies indicate that capillary pressure is responsible for trapping and pooling effect at the media discontinuities (Dawe, Wheat & Bidner 1992; Kueper, Abbott & Farquhar 1989). Physical trapping of CO2 by the capillary force is one of the crucial trapping mechanisms (Alkan, Cinar & Ülker 2010). Therefore, it is important to model the capillary force accurately at the discontinuities.

The first condition for two-phase flow at the interface is the mass conservation of each phase,

$$
\rho_a \mathbf{v}_a \cdot \mathbf{n} \bigg|_{\Gamma_{+}^{mf}} = \rho_a \mathbf{v}_a \cdot \mathbf{n} \bigg|_{\Gamma_{-}^{mf}}, \quad (\alpha = w, n),
$$

where $\Gamma_{+}^{mf}$ and $\Gamma_{-}^{mf}$ are the upstream side and the downstream side of the fracture-matrix interface respectively.

The second condition is the so called the extended capillary pressure condition (Bastian 2014; Reichenberger et al. 2006; De Neef & Molenaar 1997; Van Duijn, Molenaar & De Neef 1995). It is assumed that the capillary pressure in the rock matrix is larger than the capillary pressure in the fractures for the same saturation. According to the Leverett scaling (Leverett 1941), a lower permeability yields a higher entry pressure. The entry pressure for the rock matrix also is higher than that of the fractures. Figure 7.2 illustrates the typical Brooks-Corey capillary pressure curves for the rock matrix and the fractures. The extended capillary condition can be expressed as
The superscripts $m$ and $f$ denote the rock matrix and fracture respectively. For the CO$_2$ injection process, Equation (7.15) indicates that the capillary pressure is not always continuous at the interface. There is a threshold saturation $S^*_w$ for fractures such that the continuous capillary pressure can only be achieved if the wetting phase saturation in the fracture $S^f_w$ is less or equal to $S^*_w$ (Van Duijn, Molenaar & De Neef 1995). The threshold saturation can be calculated from the entry pressure of the rock matrix and the capillary pressure curve of the fractures as shown in Figure 7.2.

![Figure 7.2 Brooks-Corey capillary pressure curves and the indication of the extended capillary condition (after Van Duijn et al. (1995))]
Based on the Darcy’s law, the pipe flow law for any pipe \((i, j)\) can be expressed uniformly as

\[
Q_{ij}^r = K_{(i,j)}^r \left( \Phi_i - \Phi_j \right),
\]

(7.16)

where \(Q_{ij}^r\) is the volumetric flow rate from node \(i\) to node \(j\). \(\Phi = p + \rho g\) is the flow potential. \(K_{(i,j)}^r\) is the equivalent conductance coefficient of the pipe, and this has been derived in chapter 3 for different pipes. The superscript \(\tau = (m, f)\) represents the medium type \((\tau = m\) for the rock matrix and \(\tau = f\) for the fractures). Generally the form of \(K_{(i,j)}^r\) is found as

\[
K_{(i,j)}^r = \frac{Ak}{l_{ij} \mu},
\]

(7.17)

where \(k\) is the intrinsic permeability of the medium. \(l_{ij}\) is the length of the pipe. \(A\) is the influence area of the pipe.

### 7.3.1 Two-phase pipe flow

The two-phase pipe flow should consider the influence of the saturation of each phase on the permeability. The pipe flow for a specific phase \(\alpha\) is expressed as

\[
Q_{ij,\alpha}^r = k_{ra}^{\alpha} \left( S_{\alpha} \right) K_{(i,j)}^r \left( \Phi_{i,\alpha}^r - \Phi_{j,\alpha}^r \right),
\]

(7.18)

where \(k_{ra}^{\alpha} \left( S_{\alpha} \right)\) is the relative permeability for the phase \(\alpha\). It is a function of the phase saturation. According to the extended capillary pressure condition, the phase saturation as well as the phase pressure at the interface can be different. Therefore, the fracture pipes and the matrix pipes cannot be merged directly. They should be treated separately as illustrated in Figure 7.3.
For the numerical stability, the evaluation of the relative permeability is based on the upwind scheme (Monteagudo & Firoozabadi 2004; Forsyth 1991). The upwind scheme in two-phase pipe flow is as follows,

\[ k_{ra}^\tau = \begin{cases} 
  k_{ra}^\tau (S_{i,w}) & \text{if } \Phi_{i,a}^\tau - \Phi_{j,a}^\tau > 0 \\
  k_{ra}^\tau (S_{j,w}) & \text{otherwise} 
\end{cases} \]  

(7.19)

The value of the relative permeability of each phase in a pipe is calculated from the saturation of the wetting phase at the upstream node. The judgement criteria of the flow direction accord to the comparison of the phase potential at the two nodes.

### 7.3.2 Governing equations of two-phase pipe flow in fractured rock mass

The partial differential equations (7.1) and (7.2) describe the mass conservation of the two phases. In the unified pipe network model, the mass conservation is applied on each node,

\[ \frac{\partial M_{i,a}}{\partial t} + \rho_a \sum_{j=1}^{n_i} Q_{j,a}^\tau = \rho_a Q_{S,a}, \quad i \in \{n_m \mid n_m \text{ is pipe nodes}\}, \]  

(7.20)

where \( M_{i,a} \) is the mass of the phase \( \alpha (\alpha = w, n) \) accumulated in the node \( i \). \( Q_{S,a} \) is the source of the phase \( \alpha \) in the node. \( n_i \) is the number of nodes that connect to the node \( i \).

The nodal mass accumulation is calculated by

\[ M_{i,a} = M_{i,a}^m + M_{i,a}^f = \rho_a \sum_{j=1}^{n_i} \phi_{i,j}^\tau V_{i,j}^\tau S_{i,j,a}^\tau, \]  

(7.21)
where the superscript \( m \) and \( f \) represent the matrix pipe and the fracture pipe respectively. \( \phi_{(i,j)} \) and \( V_{(i,j)} \) are the porosity and the node volume of pipe \((i,j)\) respectively. \( S_{(i,j),\alpha}^f \) is the phase saturation in the pipe, and this can also be approximated by the upwind scheme,

\[
S_{(i,j),\alpha}^f = \begin{cases} 
S_{j,\alpha} & \text{if } \left( \Phi_{i,\alpha} - \Phi_{j,\alpha} \right) > 0 \\
S_{j,\alpha} & \text{otherwise}
\end{cases}.
\] (7.22)

### 7.3.3 Interface conditions for the unified pipe network model

Equation (7.20) is the discrete governing equation for the two-phase flow in the fractured porous media. It guarantees the mass conservation of the system. Fluid transfer from one node to another is through the connected pipe. In the porous media, the two adjacent nodes are connected by only one pipe, the matrix pipe. However, at the interface, every two adjacent interface nodes are connected by three pipes, two matrix pipes and one fracture pipe, as shown in Figure 7.3. At the interface, the pressure and saturation of each phase in the fracture pipe can be different from the values in the matrix pipe. With the extended capillary pressure conditions, the exchange of fluids at the interface can be achieved with using the transfer function.

Based on the extended capillary pressure conditions, if the saturation of the wetting phase in the fracture pipe \( S_{w}^f \) is not less than the threshold saturation \( S_{w}^* \), the saturation of the wetting phase in the adjacent matrix pipe \( S_{w}^m \) is 1 (fully saturated state). In this case, there is neither the capillary pressure nor the non-wetting phase pressure in the matrix pipe (Reichenberger et al. 2006). The capillary pressure in the fracture pipe \( p_c^f \) can be evaluated by the Brooks-Corey capillary pressure function.

When the wetting phase in the fracture pipe \( S_{w}^f \) is less than the threshold saturation \( S_{w}^* \), the saturation of the wetting phase in the adjacent matrix pipe \( S_{w}^m \) should be calculated by the capillary continuity condition,

\[
S_{w}^m = \left( p_c^m \right)^{-1} \left( p_c^f \left( S_{w}^f \right) \right).
\] (7.23)

If the Brooks-Corey capillary model is used, the \( S_{w}^m \) is found as
where \( p^m_e \) is the entry pressure of the rock matrix. The capillary pressure in the fracture pipe \( p^f_c \) is evaluated still by the Brooks-Corey capillary pressure function. The wetting phase is assumed to be at the whole domain, therefore the wetting phase pressure is continuous across the interface.

### 7.3.4 Time discretization and solution

The governing Equation (7.20) can be reformed as

\[
\begin{align*}
F_{i,w}(S_w, p_w) &= \frac{\partial M_{i,w}(S_w, p_w)}{\partial t} + \rho_w(p_w) \sum_{j=1}^{n_{ij}} Q_{ij,w}(S_w, p_w) - \rho_n(p_w) Q_{s,w} = 0 \\
F_{i,n}(S_n, p_n) &= \frac{\partial M_{i,n}(S_n, p_n)}{\partial t} + \rho_n(p_n) \sum_{j=1}^{n_{ij}} Q_{ij,n}(S_n, p_n) - \rho_n(p_n) Q_{s,n} = 0
\end{align*}
\]

\( i \in \{ n_m \mid n_m \text{ is pipe nodes} \} \).

The phase saturations \( S_\alpha \) and the phase pressures \( p_\alpha \) are unknowns. In view of the coupling of the phase saturation and the phase pressure, there are totally two independent unknowns.

One of the commonly used schemes to solve the coupled equation is the IMPES (implicit pressure, explicit saturation) scheme (Preux & McKee 2011; Markovinović & Jansen 2006), which decouples the phase pressure and saturation equations. For every time step, the pressure equation is solved implicitly, and then the saturation is advanced explicitly.

The other scheme is to solve the fully coupled equations using the implicit time scheme. The fully coupled fully implicit scheme has the advantages of being more robust and stable (Reichenberger et al. 2006). Other than the IMPES, relatively larger time steps can be used for this scheme. Therefore, the fully coupled fully implicit scheme is adopted in the unified pipe network method.

In our model, the wetting phase pressure \( p_w \) and the non-wetting phase saturation \( S_n \) are chosen as the independent unknowns. Based on the coupling equations (7.6) and
(7.7), the non-wetting phase pressure and the wetting phase saturation can be obtained by
\[
\begin{align*}
\begin{cases}
 p_n = p_w + p_c \\
 S_w = 1 - S_n 
\end{cases}
\end{align*}
\] (7.26)

By instituting the Equations (7.26) into the Equations (7.25), we get the fully coupled \((p_w, S_n)\) formulations as,
\[
\begin{align*}
F_{i,w}(S_n, p_w) &= \frac{\partial M_{i,w}}{\partial t} (1 - S_n, p_w) + \rho_w (p_w) \sum_{j=1}^{n} Q_{i,j,w} (1 - S_n, p_w) - \rho_w (p_w) Q_{S,w} = 0 \\
F_{i,n}(S_n, p_w) &= \frac{\partial M_{i,n}}{\partial t} (S_n, p_w + p_c) + \rho_n (p_w + p_c) \sum_{j=1}^{n} Q_{i,j,n} (S_n, p_w + p_c) - \rho_n (p_w + p_c) Q_{S,n} = 0 \\
&\quad i \in \{n_m \mid n_m \text{ is pipe nodes}\}.
\end{align*}
\] (7.27)

In order to solve the coupled equations above, the initial conditions and the boundary conditions should be given. The initial conditions are \(p_{w0} = p_w (t = 0)\) and \(S_{n0} = S_n (t = 0)\). The Dirichlet boundary conditions can be assigned directly to each boundary node, \(p_{w,\Gamma_{wD}} = p_{wD}\) on \(\Gamma_{wD}\) and \(S_{n,\Gamma_{nD}} = S_{nD}\) on \(\Gamma_{nD}\). The Neumann boundary conditions can be treated as the source term at each boundary node, \(Q_{w,\Gamma_{wN}} = Q_{wN}\) on \(\Gamma_{wN}\) and \(Q_{n,\Gamma_{nN}} = Q_{nN}\) on \(\Gamma_{nN}\).

Equations (7.27) is a time-dependent partial differential equations. To numerically solve the problem, the backward Euler scheme is applied. For brevity, the superscript \(t\) is used to denote the state in time \(t\). For a time step \(\Delta t\), the time differencing form of the backward Euler scheme for the governing equations is as follows,
\[
\begin{align*}
\begin{cases}
 F^{t+\Delta t}_{i,w}(S_n, p_w) &= \frac{M^{t+\Delta t}_{i,w} - M^{t}_{i,w}}{\Delta t} + \rho_w^{t+\Delta t} \sum_{j=1}^{n} Q^{t+\Delta t}_{i,j,w} - \rho_w^{t+\Delta t} Q^{t+\Delta t}_{S,w} = 0 \\
 F^{t+\Delta t}_{i,n}(S_n, p_w) &= \frac{M^{t+\Delta t}_{i,n} - M^{t}_{i,n}}{\Delta t} + \rho_n^{t+\Delta t} \sum_{j=1}^{n} Q^{t+\Delta t}_{i,j,n} - \rho_n^{t+\Delta t} Q^{t+\Delta t}_{S,n} = 0 \\
&\quad i \in \{n_m \mid n_m \text{ is pipe nodes}\}
\end{cases}
\end{align*}
\] (7.28)
Equations (7.28) are highly nonlinear. The nonlinear system equations can be solved by the Newton-Raphson method. The iterative linear equations for the Newton-Raphson method are shown in Equations (7.29). Each entry of the Jacobian matrix is calculated by the numerical derivatives. The right hand side is the source term of each phase. If there is no source term in the node, the value will be 0. The linear system of each Newton-Raphson iteration step is solved by the direct sparse solver GSS (GRUSOF 2014).

\[
\begin{pmatrix}
\frac{\partial F_{1,w}}{\partial p_{1,w}} & \cdots & \frac{\partial F_{1,w}}{\partial p_{n_w,w}} \\
\vdots & \ddots & \vdots \\
\frac{\partial F_{n_w,w}}{\partial p_{1,w}} & \cdots & \frac{\partial F_{n_w,w}}{\partial p_{n_w,w}}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial F_{1,n}}{\partial S_{1,n}} & \cdots & \frac{\partial F_{1,n}}{\partial S_{n_n,n}} \\
\vdots & \ddots & \vdots \\
\frac{\partial F_{n_n,n}}{\partial S_{1,n}} & \cdots & \frac{\partial F_{n_n,n}}{\partial S_{n_n,n}}
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
p_{1,w} \\
p_{n_w,w}
\end{pmatrix}
\begin{pmatrix}
Q_{1,w} \\
Q_{n_w,w}
\end{pmatrix}
= \\
\begin{pmatrix}
\begin{pmatrix}
S_{1,n} \\
S_{n_n,n}
\end{pmatrix}
\begin{pmatrix}
Q_{1,n} \\
Q_{n_n,n}
\end{pmatrix}
\end{pmatrix}
\end{pmatrix}
\text{ (7.29)}
\]

7.4 VALIDATIONS AND EXAMPLES

In this section, 2D and 3D numerical examples are presented to demonstrate the capability of the UPNM in modelling the two-phase compressible fluid flow in fractured porous media.
7.4.1 Single fracture 2D two-phase flow example

Figure 7.4 The geometry and boundaries of the two-phase flow example 1

The first example is a gas injection problem in the water saturated fractured porous media, the geometry of which is illustrated in Figure 7.4. The size of the domain is 1m × 1m. One vertical fracture resides in the middle. Its equivalent hydraulic aperture is 5mm. For comparison purposes, the parameters of the simulation are the same with those used by Reichenberger et al. (2006). The parameters of the simulation are listed as follows,

Parameters for the wetting phase:

- \( \rho_w = 1000 \text{ kg/m}^3 \)
- \( \mu_w = 10^{-3} \text{ Pa s} \)

Parameters for the fracture:

- \( \phi^f = 0.3 \)
- \( k^f = 10^{-8} \text{ m}^2 \)
- \( S_{wr}^f = 0 \)
- \( S_{sw}^f = 0 \)
- \( \lambda^f = 2 \)
- \( p_e^f = 1000 \text{ Pa} \)

Parameters for the non-wetting phase:

- \( \rho_n = \frac{p_n}{84149.6} \text{ kg/m}^3 \)
- \( \mu_n = 1.65 \times 10^{-5} \text{ Pa s} \)

Parameters for the porous medium:

- \( \phi^m = 0.1 \)
- \( k^m = 10^{-12} \text{ m}^2 \)
- \( S_{wr}^m = 0 \)
- \( S_{sw}^m = 0 \)
- \( \lambda^m = 2 \)
- \( p_e^m = 2000 \text{ Pa} \)

The initial conditions for the whole domain are \( S_n = 0, p_n = 0 \). The boundary conditions are Dirichlet boundary on the top with \( S_n = 0, p_n = 0 \) and Neumann boundary on the
bottom with gas injection rate at $2.5 \times 10^{-5}$ kg/m$^2$/s. The other two boundaries are non-flow Neumann boundaries.

The simulation results at different times are illustrated in Figure 7.5 and Figure 7.6. The results clearly demonstrate the influence of the fracture to the gas flow. At $t=70s$, the gas front reaches the fracture. After only 9s at $t=70s$, the gas arrives at the top tip of the fracture. Then the gas is accumulated at the top tip of the fracture until it reaches the threshold saturation $1-S_w^* = 0.75$ at $t=84s$. After $t=84s$, the gas penetrates the matrix.

Different mesh grids are employed to test the sensitivity of the UPNM to the mesh size. The saturations of the gas phase at $t=100s$ along $X=0.5m$, which is obtained from models of different mesh sizes are compared with the result by Reichenberger et al. (2006) in Figure 7.7. Results of different mesh sizes are very close to each other and are in good agreement with the result by Reichenberger et al. (2006). The saturation discontinuities can be observed at $Y=0.2m$ and $0.8m$.

![Figure 7.5: Simulation results at different times](image)

- (a) 70s
- (b) 79s
Figure 7.5 The saturation of the gas phase at different times

Figure 7.6 The water pressure at different times
7.4.2 Multiple fracture 2D two-phase flow example

A 2D multiple-fracture medium, which is illustrated in Figure 7.8 is adopted to test the performance of the UPNM. The fracture apertures are assigned to be 0.5mm. Other parameters are the same as those used in example 1. The conforming mesh method is used to reconstruct the flow domain. There are 5152 nodes, 15271 matrix pipes and 411 fracture pipes. In this example, the effects of gravity and capillary pressure are studied. The simulation results are demonstrated in Figure 7.9.
As seen in Figure 7.9, three cases are studied. In case 1 gravitational force is applied at the vertical downward direction, and different entry pressures are assigned for the matrix and the fractures: \( p_e^m = 2000 \text{Pa}, p_e^f = 1000 \text{Pa} \). In case 2, gravitational force is not applied and entry pressures are the same as those in case 1. In case 3, gravitational force is not applied and the same entry pressures are assigned for both the matrix and the fractures: \( p_e^m = p_e^f = 2000 \text{Pa} \). From the simulation results, it is found that the gravitational force and the capillary force have significant effects on the two-phase flow. The fracture network can enhance the uplift effect of the buoyancy. The difference between the capillary pressures of the matrix and the fractures, confines the gas in the fractures before the gas saturation reaches the threshold gas saturation. If the capillary
pressure is modelled improperly, for example, using the same entry pressure, the capillary confinement effect cannot be simulated correctly. The gas phase will flow from fractures into matrix without any confinement, thus significantly affecting the gas distribution and transportation route.

7.4.3 Multiple fracture 3D two-phase flow example

A two-phase flow problem in 3D multiple-fracture porous medium is presented to verify the 3D UPNM two-phase flow model. The configuration of the model is illustrated in Figure 7.10. The dimension of the domain is $10\times10\times10$ m. There are three inter-crossed fractures residing in the block. The hydraulic apertures of the fractures are 0.5mm. A compressible gas is injected through the whole bottom boundary. The injection rate is $5\times10^{-5}$ kg/m$^2$/s. For simplicity, other parameters are the same as those in example 1. The top boundary is a Dirichlet boundary with $S_n = 0$, $p_w = 0$. The other boundaries are non-flow boundaries. Figure 7.11 depicts the development of the gaseous phase in the fractured medium. The UPNM can successfully capture the influences of the fractures on the two-phase flow.
Figure 7.10 The configuration of the 3D multiple-fracture porous medium

(example 3)

(a) 1000s

(b) 1250s
Figure 7.11 Cross-section view of the advance of the gaseous phase at different times

7.5 NUMERICAL STUDY OF CARBON DIOXIDE INJECTION INTO A DEEP SALINE AQUIFER FOR THE GEOLOGICAL SEQUESTRATION

The saline aquifer is a typical geological material, and this consists of fractures and pores that transfer and store fluids. Initially, high-pressure brine occupies the aquifer. The injection of CO\(_2\) into the deep aquifer is a two-phase flow problem. The depth and temperature of the aquifer can affect the injection process. In addition, the discontinuity structures of the aquifer also have significant effects on the injection. In this study, the relatively short-term injection is considered, therefore the chemical reaction is neglected.

7.5.1 Incorporation of equations of state of CO\(_2\) into the two-phase flow model

Normally the CO\(_2\) is in a gaseous phase. However, it may become a supercritical phase or an aqueous phase when it is injected into a deep saline aquifer, depending on the temperature and pressure. In addition, the viscosity of CO\(_2\) also varies with the temperature and the pressure. Therefore precise EOS (equation of state) for CO\(_2\) is required to predict its state change.

For the density-pressure-temperature relationship, the Span-Wagner EOS (Span & Wagner 1996) is adopted. It is one of the most accurate reference equations for CO\(_2\). It
is fitted from an extensive body of experimental data and covers a wide range of
temperature and pressure (up to temperature of 1100k, and up to pressure of 800MPa).
For calculating the viscosity of CO$_2$, the viscosity equation from Fenghour et al. (1998)
is adopted. It also covers a wide range in terms of temperature and pressure and features
high accuracy. A typical range of density and viscosity of CO$_2$ for deep saline injection
is illustrated in Figure 7.12.

(a) density of CO$_2$                   (b) viscosity of CO$_2$

Figure 7.12 Density and viscosity of CO$_2$

7.5.2 The ranges of temperature and pressure for the study

From an efficiency perspective, CO$_2$ is expected to be injected in a sufficient depth to
maintain it in a supercritical state. Assuming the hydrostatic pressure gradient of a
typical saline aquifer is 10.5 MPa/m, the proper depth for a potential “shallow” aquifer
is around 1000m, and the depth for a potential “deep” aquifer may be approximately
3000m (Nordbotten, Celia & Bachu 2005). According to the average land surface
temperature and geothermal gradient, the “warm” and “cold” basins can be identified,
the surface temperature and the gradient of which are about 20 $^\circ$C, 45 $^\circ$C/km and
10 $^\circ$C, 25 $^\circ$C/km respectively (Nordbotten, Celia & Bachu 2005). Therefore, the
typical range of temperature for a storage aquifer is from about 308K (35 $^\circ$C ) to 428K
(155 $^\circ$C ). As illustrated in Figure 7.12, within the typical pressure and temperature
range mentioned above, the density and viscosity of CO$_2$ can change dramatically.
7.5.3 Description of the model

A rectangular saline aquifer with the size of 1000m × 1000m × 100m that is illustrated in Figure 7.13, is used to study the CO$_2$ injection process. The top and the bottom boundaries are non-flow boundaries. CO$_2$ is supposed to be injected in the middle of the aquifer. The injection rate is 5 kg/s (432 t/day). The injection process is assumed isothermal. The parameters for the study are listed as follows,

Parameters for the brine:
- $\rho_w = 1050$ kg/m$^3$
- $\mu_w = 0.28$ mPa · s

Parameters for the aquifer:
- $\phi^m = 0.3$
- $k^m = 10^{-13}$ m$^2$
- $S^m_{sw} = 0.05$
- $S^m_{sr} = 0$
- $\lambda^m = 2$
- $p_e^m = 5000$ Pa

Parameters for the fracture:
- $\phi^f = 0.8$
- $k^f = 2 \times 10^{-8}$ m$^2$
- $S^f_{sw} = 0.05$
- $S^f_{sr} = 0$
- $\lambda^f = 2$
- $p_e^f = 1500$ Pa

![Figure 7.13 Simulation model for CO$_2$ injection](image)

7.5.4 The boundary conditions

Boundary conditions are very important for the numerical simulation. The proper choice of the boundary conditions should be based on the knowledge of the aquifer. Usually, the top and the bottom of the aquifer are treated as a non-flow boundary, thus confining CO$_2$ in the formation. In comparison, the choice of the lateral boundary conditions is...
more complicated. According to the size of the aquifer and the communication with other formations, Dirichlet-type boundary or Neumann-type boundary can be assigned to the lateral boundaries. Normally the constant pressure or flow rate boundaries are used to represent the fluid exchanging with the surrounding aquifer. The non-flow boundary is adopted if the aquifer is a closed one. Alternatively, the non-flow boundary with amplified boundary pore volumes (NAPV) is used to consider the effect of the far field (Eigestad et al. 2009). Based on the rule of thumb, the multiplier for those pore volumes is set to 1000 (Eigestad et al. 2009; Juanes et al. 2006). However, the choice of the multiplier has not been examined carefully. The boundary conditions have significant effects on the pressure build-up of the aquifer, which is important for the CO₂ injection. If the injection pressure is too high, it may cause fracturing in the aquifer.

![Figure 7.14 Effects of boundary conditions on the injection pressure](image)

**Figure 7.14 Effects of boundary conditions on the injection pressure**

The effects of different boundary conditions of the lateral boundaries on the injection pressure have been studied. The injection pressure is defined as the pressure difference between the CO₂ pressure at the injection point and the initial hydrostatic pressure of the injection point (Sasaki et al. 2008). In this study, the initial hydrostatic pressure and the temperature of the aquifer are 25MPa and 350K respectively. The initial saturation of CO₂ in the aquifer is 0.1. The constant injection rate is assumed during the injection. Five different boundary conditions, namely the constant pressure boundary condition, the non-flow boundary condition, and the NAPV with different multipliers, are simulated. Figure 7.14 demonstrates the development of the injection pressure for
different boundary conditions. For a short period (around 4 days) at the beginning, the injection pressures change similarly for different boundary conditions. They surge to a local peak value and then decrease gradually. After that, their developments are very different. For the constant pressure boundary condition, the injection pressure continues decreasing with a very slow but steady rate (about 0.3MPa/year). In comparison, the injection pressure dramatically increases for the non-flow boundary condition (with a rate about 4.7MPa/year). The changes of injection pressures for the NAPVs are among the above two special conditions. Generally, the injection pressures of the NAPVs all increase during the injection process. The larger multiplier, the slower is the increasing rate of the injection pressure.

The constant pressure boundary condition indicates that the brine can easily flow out of the boundary. The aquifer has a sufficient communication with the external aquifers. In this case, the injection will become easier with the increase of the CO$_2$. The non-flow boundary represents a closed aquifer. Compared to CO$_2$, the brine is almost incompressible. Therefore, the injection will become more and more difficult for the closed the aquifer. The NAPV simulates the limited outflow of the brine. The multiplier indicates the connection level of the system with the surrounding aquifers. A larger multiplier represents a higher connection degree. In the case of multiplier 1000, the injection pressure increases at a moderate rate of about 0.1MPa/year.

7.5.5 The influences of the temperature on the injection processes

The temperature of the aquifer has significant effects on the density and viscosity of CO$_2$. It also influences the density and viscosity of the brine. However, in comparison to CO$_2$, the influences of temperature on brine are relatively small. In this study, the influences of temperature on brine are neglected to emphasize its effects on CO$_2$ solely. The initial hydrostatic pressure of the aquifer is 20MPa. The NAPV boundary condition with a multiplier 1000 is adopted in this study and the following studies.
The developments of the injection pressure and the CO₂ plume are studied. Figure 7.15 illustrates the injection pressure change with time. As found in Figure 7.15, the injection pressure surges to a higher value for the “hotter” aquifer at the first stage. The peak value of the injection pressure for the case with temperature of 400K is 46.6% higher than that of case with temperature of 300K. After the peak, the injection pressures decrease gradually. In hotter aquifer, the decreasing rate is a bit slower than that of the colder aquifer. In the time scale of one year, the injection pressures keep decreasing for the cases of 350K, 380K and 400K. In comparison, the injection pressures increase after a period of decreasing for cases of 300K and 320K. The increasing rate for the case of 300K is higher than that of the case of 320K.

The influence of the temperature of the aquifer on the development of the CO₂ plume also is significant. The CO₂ saturations at the injection point (Mid) and the top point at the centre (Top) with the coordinates x=500m, y=100m, z=500m, are observed during the injection. Figure 7.16 illustrates the development of the CO₂ saturations these two points. For different temperature, the trends of the development of CO₂ saturation in the injection point are similar. They are close to each other. In the first 25 days, the saturation in the injection point soars to 0.6. Then the increasing rate gradually slows down. In the hot aquifer, the CO₂ reaches the top earlier than that in the cold aquifer. The saturation at the top point for the high temperature case is higher than that in the
cold aquifer. That is due to the low viscosity and the low density at the high temperature, and these improve the mobility and the buoyancy of CO$_2$.

![Graph showing the effects of temperature on the evolution of the saturation of CO$_2$](image)

**Figure 7.16 The effects of temperature on the evolution of the saturation of CO$_2$**

### 7.5.6 The influences of the aquifer pressure on the injection processes

The aquifer pressure is another important factor affecting the CO$_2$ injection process. The increase of pressure will increase the density and the viscosity of CO$_2$, thus influencing the evolution pattern of the CO$_2$ plume. In this study, the temperature is 350K. Figure 7.17 compares the development of the CO$_2$ plume under different aquifer pressures after a 1-year injection. For a shallow aquifer (low-pressure aquifer), the speed of the spreading of CO$_2$ is much quicker. It floats to the top more easily and tends to spread to a larger area along the top of the formation. For a deep aquifer, the CO$_2$ is concentrated in a relatively smaller space. From this point of view, it is better to choose a deep aquifer as the storage site.
(a) Aquifer pressure: 10MPa

(b) Aquifer pressure: 15MPa

(c) Aquifer pressure: 20MPa
Figure 7.17 The shapes of CO₂ plumes after 1-year injection

Figure 7.18 depicts the development of CO₂ saturations at the injection point (Mid) and the top centre point under different aquifer pressures. It is found that the variation of the saturation at the injection point follows the same rate. In contrast, the speeds of saturation increasing at the top are of significant difference. The CO₂ plume reaches the top earlier under the low aquifer pressure. The saturation also is higher with the lower aquifer pressure at the same injection time.
The influences of aquifer pressures on the development of the injection pressures are depicted in Figure 7.19. It is observed that the injection pressure soars to a higher peak at the beginning for the lower pressure aquifer. Then the injection pressure decreases gradually. Under a higher aquifer pressure, the peak value is lower. However, after decreasing for a short period, the injection pressure starts increasing again. The higher the aquifer pressure, the larger the increasing rate is. Actually, the injection pressure under a low pressure also is expected to rise after a period of decreasing.

Together with the effects of temperature on the injection pressure, it is found that the injection pressure is related closely to the mobility of \( \text{CO}_2 \). When the \( \text{CO}_2 \) is in a better mobility state (such as in a state of higher temperature and lower pressure), it tends to produce a higher injection pressure peak at the beginning of the injection process. Later, the injection pressure will come down to a low level gradually, and it tends not to rise again at a high rate as those in the worse mobility cases. To reduce the peak value of the injection pressure at the beginning, the injection rate can be slowed.

**Figure 7.18 The effects of aquifer pressure on the development of \( \text{CO}_2 \) saturation**
figure 7.19 the effects of aquifer pressure on the injection pressure

7.5.7 The influences of fractures in the aquifer on the injection processes

The heterogeneity is one of the most important characteristics of the rock formation of the aquifer (Tsang, Birkholzer & Rutqvist 2008). The heterogeneity can cause fingering and channelized flow (Tsang et al. 2001), which increases the spread of the injected liquid. Some important heterogeneities for CO₂ storage have been described by Flett et al. (Flett, Gurton & Taggart 2005), and they are stratigraphic layering, faults, depositional mixing, compartmentalization, and channel systems. Fractures in the aquifer can cause high heterogeneity to the rock formation. The permeability within the fracture plane is much larger than that of the formation rock, and this can significantly affect the development of the CO₂ plume. The developed UPNM is very effective in simulating fractures or other thin high permeable layers using one-dimensional pipe model.

Barrier effect of a fracture

Generally, the presence of fractures increases the permeability of the aquifer and extends the range of CO₂ distribution. Fractures perform as high permeable conduits within their planes. However, due to the capillary differences between the fractures (or thin high permeable zone) and the rock matrix, in the direction across their planes, they can also act as flow barriers. To demonstrate the barrier effect of a fracture, a numerical model is built as shown in Figure 7.20. In this model, a square fracture zone happens to
be 10m above the injection point. The size and thickness of the zone are 200m×200m and 0.5mm respectively. The permeability and the porosity of the fracture are $2.0 \times 10^{-8}$ m$^2$ and 0.8 respectively. The aquifer pressure and temperature used in this simulation are 20MPa and 350K respectively. The CO$_2$ plume evolutions are depicted in Figure 7.21. As observed from the simulation results, when the CO$_2$ plume meets the zone, it cannot penetrate the zone plane and migrates directly to the top of the formation. Instead, it flows within the zone and accumulates. When the saturation of CO$_2$ exceeds a threshold, the CO$_2$ penetrates into the upper formation. The saturation at the boundaries of the zone reaches the threshold first. Therefore, the zone in this case acts as a barrier, and the migration of the CO$_2$ plume will be obstructed. The presence of the zone can extend the plume to a larger range, thus enabling the CO$_2$ to mix more thoroughly with the brine. It also prolongs the upward migration time. This helps the mixing of the CO$_2$ with the brine and benefits the solubility trapping and mineral trapping in the long term. The barrier effect is caused by the material discontinuity. This is important to the CO$_2$ storage.

![Figure 7.20 The numerical model to demonstrate the barrier effect of a fracture](image)

Figure 7.20 The numerical model to demonstrate the barrier effect of a fracture
Figure 7.21 The CO$_2$ plume evolution under the blockage of a horizontal fracture

The effects of joint sets—a study of a 3D case

The effects of large joint sets on the evolution of a CO$_2$ plume are studied. In this case, two large joint sets exist in the aquifer. One joint set has the dip angle of 61°, and dip
direction of $40^\circ$. The aperture size is 1mm. The permeability and porosity are $1.0 \times 10^{-8} \text{ m}^2$ and 0.8 respectively. The other set has the dip angle of $56^\circ$, and dip direction of $285^\circ$. The permeability and porosity are $0.8 \times 10^{-8} \text{ m}^2$ and 0.5 respectively. The boundary conditions of the lateral boundaries are constant hydrostatic pressures of 20MPa. The CO$_2$ distribution is illustrated in Figure 7.22. As found from the results, the CO$_2$ migrates along the fracture sets when it meets the fractures and spreads around fractures. The fracture sets cause the permeability anisotropy, which significantly influences the path of migration. They also concentrate the CO$_2$ around the fractures. The shape and the distribution of the CO$_2$ plume are altered by the heterogeneity caused by the joint sets.

(a) CO$_2$ plume in the aquifer and the joints after 50 days

(b) CO$_2$ plume in the aquifer and the joints after 100 days

(c) CO$_2$ plume in the aquifer and the joints after 300 days
The effects of joint sets—a study of 2D cases

In chapter 4, the HAF (hydro-geometric anisotropy factor) has been proposed to evaluate the permeability anisotropy of a fracture network. In this study, the HAF is employed to estimate the permeability anisotropy of a fractured rock mass caused by the presence of fracture networks thus quantifying the influence of permeability anisotropy on the development of CO₂ plume.

A 2D 300m×100m model is built for the study. The injection point is at the coordinates of x=0, y=20m. The injection rate is 0.02kg/s. Most material properties are the same as
those stated in subsection 7.5.3 except that the aperture of fractures in this study is 0.5mm and their permeability and porosity are $1.0 \times 10^{-8}$ m$^2$ and 0.4 respectively. The temperature and initial pressure for the aquifer are 350K and 20MPa respectively. The right vertical boundary is applied with a constant hydrostatic pressure of 20MPa. Other boundaries are non-flow boundaries. The fracture patterns used are illustrated in Figure 7.23. Their HAFs (directions of horizontal to vertical) and intensity p21s are calculated. For comparison purposes, their average intensity is 0.102 m/m$^2$.

![Fracture patterns](image)

Figure 7.23 Fracture patterns used in the study
Figure 7.24 demonstrates the influences of permeability anisotropy caused by the fracture distribution on the evolution of the CO$_2$ plume. When the HAF is less than 1, the horizontal permeability is less than the vertical permeability. It is relatively easier
for the CO_2 to migrate upward to the top of the formation. When the HAF is larger than 1, the horizontal permeability is greater than the vertical one. It is relatively easier for the CO_2 plume to advance horizontally. The advance speed of the CO_2 is closely related to the permeability anisotropy when the intensity of the fractures is the same. Figure 7.25 illustrates the relationship between the horizontal advance distances of CO_2 and the HAFs according to the cases studied. At a specific time, the horizontal advance distance $d$ (m) has a linear relationship with the logarithm of the corresponding HAF,

$$d = a + b \ln HAF.$$  \hspace{1cm} (7.30)

In this study, $a=185.52$, $b=22.06$ for the 231 days’ injection.

![Figure 7.25 The relationship between the horizontal advance distances of CO_2 and the HAFs at different injection times](image)

Observations from the results show that the distribution of CO_2 also is related to the HAF value. When the HAF becomes smaller, CO_2 tends to be more concentrated at the top of the formation. In contrast, CO_2 is more dispersive when the HAF becomes larger. This accelerates the sufficient mixture of CO_2 and the brine, and it also benefits the solubility trapping and mineral trapping.

### 7.6 CONCLUSION

In this chapter, 2D and 3D two-phase transient UPNM are developed for the simulation of immiscible and compressible gaseous phase flow in the fractured rock masses. The
UPNM uses fracture pipes and matrix pipes to simulate fluid flow in fractures and rock matrix respectively. The Brooks-Corey capillary function and relative permeability function are incorporated in the model. The extended capillary pressure conditions are incorporated to consider the fluid exchange at the interface, and this avoids using the transfer functions. The upwind scheme is adopted for the evaluation of flow in the pipes, and this enhances the stability of the numerical solution. In addition, the fully coupled fully implicit scheme is derived to solve the discrete nonlinear equations. Therefore, a larger time step can be achieved for a stable and robust solving process.

2D and 3D examples have been presented to demonstrate the effectiveness of the UPNM for two-phase flow in fractured porous media. The discontinuity characteristics at the interface between the fracture and matrix rock can be simulated successfully. The effects of gravity and capillary pressure can also be modelled readily.

To precisely simulate the state change of CO$_2$ during the injection phase, the Span-Wagner density-pressure-temperature EOS and viscosity equation from Fenghour et al. (1998) are incorporated in the UPNM. The effects of boundary conditions on the injection pressure for the CO$_2$ injection process are studied. The boundary conditions can have significant effects on the injection pressure. The non-flow boundary mimics the closed aquifer, which can cause dramatic injection pressure to rise during the injection. In comparison, the constant pressure boundary represents an effective connection with the surrounding aquifers. After a peak value at the initial stage, the injection pressure will dissipate gradually during the injection. The non-flow boundary with amplified boundary cell volume condition mimics the partial outflow ability of the brine into the surrounding aquifer. The choice of the multiplier is based on its connection quality to the surrounding aquifer.

The influences of the temperature and pressure of an aquifer on the injection pressure build-up and CO$_2$ plume evolution have been studied systematically. It is found that CO$_2$ spreads more readily in a relatively shallow and hot aquifer than in a deep and cold aquifer. The buoyancy effect in a relatively shallow and hot aquifer also is stronger. The injection pressure build-up is related closely to the mobility of CO$_2$. When the CO$_2$ is injected into a relatively shallow and hot aquifer, the CO$_2$ has a higher mobility, and the injection pressure tends to produce a higher injection pressure peak at the beginning of
the injection process. Later, the injection pressure will reduce to a low level gradually. It will not rise again at a high rate. In contrast, when the CO$_2$ is injected into a relatively deep and cold aquifer, the injection pressure surges to a lower peak value and after a short period of decreasing, it increases at a greater rate. The insight into the characteristic of the injection of CO$_2$ into different aquifers can help to make a proper injection scheme to prevent the over build of the injection pressure.

The fracture is one kind of heterogeneity in the rock formation. It has significant effects on the CO$_2$ injection process, especially the CO$_2$ plume migration. On the one hand, the fracture and thin high permeable layer act as a flow conduit within their planes. On the other hand, they also perform as a flow barrier across their planes due to the capillary pressure difference with the rock formation. The barrier effect can extend the range of the CO$_2$ plume and prolong the time of migrating to the top of the aquifer. Therefore, the barrier effect helps the mixing of CO$_2$ with the brine, thus benefiting the solubility trapping and mineral trapping in the long term. The joint sets can cause permeability anisotropy, which influences the evolution of the CO$_2$ plume including its shape and saturation distribution. With the increase of the HAF, the distribution of CO$_2$ plume is more dispersive. It is found that for a specific fracture intensity, the horizontal advance distance of CO$_2$ has a linear relationship with the logarithm of the HAF of the fracture network.
CHAPTER 8. CONCLUSIONS AND FUTURE WORK

8.1 CONCLUSIONS

Accurate and efficient numerical simulation for fluid flow in fractured porous rock requires the numerical method to be capable of considering the influences of individual fracture and to be sufficiently versatile to treat fracture networks with complex geometries. Towards the aims of developing a conceptual simple and computational efficient numerical method to simulate multiphase flow processes in fractured porous rock masses, a unified pipe network method (UPNM) has been proposed and systematically developed to study different fluid flow problems in fractured porous rock. Detailed works are concluded as follows.

1. The UPNM is proposed by using a domain-reconstruction method, in which fractures and porous matrix are equivalent to fracture pipes and matrix pipes respectively. The UPNM treats and combines different flow domains uniformly by the assembly of the discrete one-dimensional pipes associated with different properties. The UPNM is developed for the seepage simulations in either a 2D or 3D porous media, a discrete fracture network and a fractured porous media. The flow equivalences of the fracture pipe and the matrix pipe for the fractures and the porous medium are derived respectively. Mass conservation is guaranteed in the UPNM. The UPNM conceptually and computationally simplifies the seepage simulations in complex fracture networks and fractured porous media. Integration for each element is not required. Confirming mesh methods in two dimension and three dimension are adopted to discretise the flow domain and construct the flow pipes. A number of numerical examples have been tested to demonstrate the flexibility and feasibility of the proposed method.

2. The UPNM is employed to study the permeability anisotropy of 2D fracture networks. A hydro-geometric anisotropy factor (HAF) and the simplified $D_f$ have been derived to describe the macroscopic geometric anisotropy and the hydraulic anisotropy of fracture networks to predict the permeability anisotropy of a given domain. Six patterns of fracture sets have been generated and studied to show the effects of fracture distribution on the hydro-geometric anisotropy and the permeability anisotropy. It is shown that the change of the fracture equivalent hydraulic aperture can cause the permeability to
become anisotropic even if the orientation, length and spacing of the fracture sets are isotropically distributed. Numerical results indicate that HAF can reflect the variation of the permeability anisotropy effectively. The proposed anisotropic conductivity index (ACI) is an excellent connectivity indicator that can reflect both the hydraulic connectivity quality and directionality of the connection. It also has a linear relationship with directional equivalent permeability. In addition, it is relatively more convenient to derive the ACI from scan line surveys.

3. The UPNM has been applied to analyse the unconfined seepage problem in both continuous domain and continuous domain with discontinuities. The method deals with both continuous domain and discontinuities as the connected pipe networks. Therefore, it is convenient to simulate flow in continuous domain embedded with discontinuities. An auxiliary boundary pipe method is developed to conduct the searching of the phreatic surface, which takes both the advantages of the fixed mesh algorithm and the adaptive mesh algorithm. Due to the pipe reassembly on the phreatic boundary, it is very effective to reconstruct the global coefficient matrix. The proposed average acceleration method can accelerate the iteration process significantly and make the solution process more stable. The validity and versatility of the UPNM method have been verified by analysing unconfined seepage problems with homogeneous and nonhomogeneous materials, different geometry boundaries, different drain conditions and fractured porous medium. The results computed by the present method are compared with those investigated by other researchers. Significant agreements are achieved.

Unconfined seepage problems in rock slope with different fracture patterns have been studied. The results show that the presence of the fractures in the rock slope can distort the distribution of the hydraulic head and change the shape as well as the location of the phreatic surface. The direction, hydraulic aperture, fracture set number and fracture distribution can affect the phreatic surface significantly. With the increase of the hydraulic aperture of the fractures, the influences on the location of the phreatic surface become stronger. When the direction of the fracture set is close to the direction of the gradient of the hydraulic head, the phreatic surface is lowered more significantly. While the direction of the fracture set is nearly perpendicular to the direction of the gradient of the hydraulic head, the influence on the phreatic surface is weak.
4. The 2D and 3D UPNMs are improved for the simulation of variably saturated transient flow processes in the fractured rock masses. At the interface of the fracture and the rock matrix, a capillary continuity condition is applied, and this considers the fluid interchange in the interface at the unsaturated zone. However, the saturations in the fracture and in the rock matrix can be different at the interface. Two-dimensional and three-dimensional numerical examples for flow in continuous porous medium, in discrete fracture network and fractured rock mass have been presented to demonstrate the versatility and feasibility of the unified pipe network method for transient flow simulation in the fractured rock mass.

The Jingzhou project of petroleum underground storage in the unlined rock caverns is studied by using the transient variably saturated UPNM. The discrete fracture network model is adopted to analyse the effects of the water curtain system on the water sealing of the rock caverns in the fractured rock mass. Both the construction phase and the storage phase have been considered in the study. The effects of the horizontal water curtain system, the vertical water curtain system, the length of the horizontal water curtain boreholes, the pressure of the water curtain and the spacing of the water curtain boreholes are systematically analysed. It is found that the effect of the horizontal water curtain is more important than the vertical water curtain. The vertical water curtain system alone is incapable of sealing the rock cavern in this study. The reduction in the pressure of the water curtain, the length of the water curtain boreholes and the increase of the spacing of the water curtain borehole can reduce the seal effect of the water curtain. It is worth mentioning that the failure of the water seal effect can occur locally. The locally connected unsaturated flow paths have been observed in the study, which cannot be found by using the continuous simulation model. This is because of the discontinuity and irregularity of the discrete fracture network. The water flow in the fracture network can be very complex. A conventional continuous model may cause inaccurate results. Although an increase in the pressure of the water curtain can enhance the seal effect, it also leads to some disadvantages, such as the drastic increase in the inflow rate and the detriment to the stability of the caverns. In view of the discontinuity of the fracture network, the continuous horizontal water curtain boreholes and the relatively more closely spaced water curtain boreholes are recommended, and these are more effective methods in enhancing the seal effect of the water curtain system.
5. The 2D and 3D UPNMs are extended for the simulation of two-phase transient flow processes in fractured porous rock masses. A compressible gaseous phase is adopted as the non-wetting phase. The Brooks-Corey capillary function and relative permeability function are incorporated in the model. The extended capillary pressure conditions are incorporated to consider the fluid exchange at the interface, and this avoids using the transfer functions. The upwind scheme is adopted for the evaluation of flow in the pipes, and this enhances the stability of the numerical solution. In addition, the fully coupled fully implicit scheme is derived to solve the discrete nonlinear equations. Therefore, a larger time step can be achieved for a stable and robust solving process. 2D and 3D examples have been presented to demonstrate the effectiveness of the UPNM for two-phase flow in fractured porous media. The discontinuity characteristics at the interface between the fracture and matrix rock can be simulated successfully. The effects of gravity and capillary pressure also can be modelled readily.

To precisely simulate the state change of CO$_2$ during the injection phase, the Span-Wagner density-pressure-temperature EOS and viscosity equation from Fenghour et al. (1998) are incorporated in the UPNM. The effects of boundary conditions on the injection pressure for the CO$_2$ injection process are studied. The influences of the temperature and pressure of an aquifer on the injection pressure build-up and CO$_2$ plume evolution have been studied systematically as well. It is found that CO$_2$ spreads more readily in a relatively shallow and hot aquifer than in a deep and cold aquifer. The buoyancy effect in a relatively shallow and hot aquifer also is stronger. The injection pressure build-up is related closely to the mobility of CO$_2$. When the CO$_2$ is injected into a relatively shallow and hot aquifer, the CO$_2$ has a higher mobility, and the injection pressure tends to produce a higher injection pressure peak at the beginning of the injection process. Later, the injection pressure will reduce to a low level gradually.

The barrier effects of fracture in a CO$_2$ injection process are studied, and this is caused by the capillary pressure difference between fracture and the rock formation. The barrier effect can extend the range of the CO$_2$ plume and prolong the time of migrating to the top of the aquifer. Therefore, the barrier effect helps in mixing CO$_2$ with the brine, thus benefiting the solubility trapping and mineral trapping in the long term.
The joint sets can cause permeability anisotropy, and this influences the evolution of the CO$_2$ plume including its shape and saturation distribution. With the increase of the HAF, the distribution of the CO$_2$ plume is more dispersive. It is found that for a specific fracture intensity, the horizontal advance distance of CO$_2$ has a linear relationship with the logarithm of the HAF of the fracture network.

### 8.2 FUTURE WORKS

Some research works are recommended for future studies:

1. Including the dispersion term in the UPNM to study the contaminant migration in naturally fractured rock formation, such as the storage of nuclear waste.

2. Coupling the thermal and mechanical effects into the UPNM, which can extend the application of the UPNM to the geothermal energy exploitation.

3. Further developing the hydraulic fracturing process for the study of the unconventional oil and gas exploitation.

4. Extending the UPNM to multi-scale fluid flow simulation from pore scale fluid flow to large field fluid flow.

5. Further boosting the efficiency by implementing the parallel computation technology.
APPENDIX A. FLOW CHART OF TWO-PHASE FLUID FLOW PROGRAM

The computer program of the UPNM is developed with C/C++ code. The structure and flow chart of the computer program are listed in Figure A.1.

![Flow Chart of Two-Phase Fluid Flow Program](image-url)
Figure A.1 Flow chart of two-phase fluid flow program
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