CHARACTERISATION OF MINORITY CARRIER DIFFUSION LENGTH USING THE PHOTOCURRENT TECHNIQUE IN EXTRINSICALLY DOPED GaN

by
Danny Wee
B. Eng (Hons) & B. Com

This thesis is presented for the degree of
Doctor of Philosophy of Electrical and Electronic Engineering of
The University of Western Australia

School of Electrical, Electronic and Computer Engineering
The University of Western Australia
2014
Dear Madam/Sir,

It is with great relief that I present this thesis in fulfilment of the requirements for the degree of Doctor of Philosophy, entitled

*Characterisation of Minority Carrier Diffusion Length Using the Photocurrent Technique in Extrinsically Doped GaN*

This work was performed under the guidance and supervision of Professor Brett Nener and Professor Giacinta Parish of the Microelectronics Research Group within the School of Electrical, Electronic and Computer Engineering.

Yours sincerely

Danny K. W. Wee
DECLARATION

I, Danny Wee, hereby declare that I have the permission from all co-authors of my publications to include the published work in this thesis. A detailed statement of contribution is provided on page iii.

_______________________
Candidate
Danny Wee

_______________________
Co-ordinating Supervisor
Prof. Brett Nener

_______________________
Supervisor
Prof. Giacinta Parish
STATEMENT OF CONTRIBUTION

The contents of this thesis are based mainly on the following published works. The relative contribution made by the author as well as the relevant chapters in this thesis for each publication are provided for each publication. The name of the author of this thesis is emphasised in boldface.


Relative Contribution
1<sup>st</sup> Author: 40% (theory development, analysis, manuscript composition and review)
2<sup>nd</sup> Author: 20% (wafer growth, device fabrication, analysis and manuscript review)
3<sup>rd</sup> Author: 20% (experimental work and manuscript review)
4<sup>th</sup> Author: 20% (theory development, analysis and manuscript review)

Relevant Chapter
Chapter 6


Relative Contribution
1<sup>st</sup> Author: 50% (simulation work, analysis and manuscript composition)
2<sup>nd</sup> Author: 25% (analysis and manuscript review)
3<sup>rd</sup> Author: 25% (analysis and manuscript review)

Relevant Chapter
Chapter 7

Relative Contribution

1st Author: 50% *(experimental work, theory development, data analysis and manuscript composition)*

2nd Author: 25% *(wafer growth, device fabrication, analysis and manuscript review)*

3rd Author: 25% *(theory development, analysis and manuscript review)*

Relevant Chapters

Chapters 3 and 4


Relative Contribution

1st Author: 50% *(experimental work, theory development, data analysis and manuscript composition)*

2nd Author: 25% *(wafer growth, device fabrication, analysis and manuscript review)*

3rd Author: 25% *(theory development, analysis and manuscript review)*

Relevant Chapters

Chapters 4 and 5

__________________________  _____________________  ______________________
Candidate  Co-ordinating Supervisor  Supervisor
Danny Wee       Prof. Brett Nener        Prof. Giacinta Parish
ABSTRACT

Gallium nitride (GaN)-based electronic devices such as light emitting diodes and n-p-n heterojunction bipolar transistors require a good quality p-type GaN layer for high performance and efficiency. Therefore, assessment of material quality for p-type GaN is of critical importance in the advancement of GaN-based technology. One way of quantifying material quality is to measure the minority carrier diffusion length. In this work, the photocurrent technique is applied to determine this parameter. To be confident that the results obtained using this technique are accurate for GaN, which exhibits diffusion length ranging from tens of nanometers to several micrometers, the validity of the standard mathematical model with respect to GaN is investigated.

The validity of the model is assessed by examining four of the assumptions made in the derivation of this model, namely, that there is no excess minority carrier density at the depletion edge plane, the absorption coefficient is uniform throughout the material, including in the depletion region, there is no recombination in the depletion region and the induced photocurrent consists of the minority carrier current only. The validity of these four assumptions, their effects on the accuracy of the diffusion length values obtained and the overall applicability of the photocurrent technique are investigated.

The investigations found that all four assumptions are not always valid for GaN and therefore the standard mathematical model is also not always valid. The invalidity of the assumption of no recombination in the depletion region was found to have the most significant negative impact on the accuracy of the diffusion length value while the invalidity of the remaining three assumptions do not have a significant detrimental effect. Overall, precaution is recommended when applying the standard mathematical model to GaN samples, particularly if the samples have been subjected to energetic post-growth processes, which can give rise to recombination in the depletion region.

Due to the high absorption coefficient of GaN, the photocurrent technique cannot be used when the sample has a wide depletion region. This is because all incident photons will be absorbed in the depletion region. Therefore, the electron-beam induced current (EBIC) technique is investigated as an alternative technique. Like the photocurrent technique, the applicability of this technique needs to be validated for the wide range of diffusion length values. Using numerical simulations, it is shown that the diffusion
length values determined using this technique are fairly accurate estimates of the true values even at very low sub-micrometer diffusion length values. Additionally, it is found that the accuracy of this technique is insensitive to the beam diameter, which suggests that in practice, it is not imperative to know the size of the electron beam.

Application of the EBIC technique to one of the p-type samples studied in this work found the diffusion length to be an order of magnitude greater than the value obtained using the photocurrent technique. There are two possible reasons for this discrepancy. Firstly, the higher value obtained using the EBIC technique could be the result of the overestimation of the diffusion length due to the electron injection-induced increase in diffusion length effect. Secondly, the lower value obtained using the photocurrent technique can be attributed to undesirable recombination caused by a thin, damaged surface layer in the depletion region of the p-type sample. Additionally, because the minority carriers generated by the electron beam also have to traverse the same depletion region that contains the damaged surface layer, it is possible that the same undesirable recombination in the depletion region also has a detrimental effect on the accuracy of the EBIC technique. It is likely that both of these competing effects are present and because their combined effects on the accuracy of the EBIC technique are unknown, it is difficult to make any meaningful comparison between the results obtained using the photocurrent and EBIC techniques.
ACKNOWLEDGEMENTS

I would like to give thanks and acknowledge the various people who have assisted in the production of this thesis during my many years of studies here in the Microelectronics Research Group.

Firstly, I would like to thank my supervisors, Professor Brett Nener and Professor Giacinta Parish, for their guidance, support, patience and consideration as well as for all the reading and editing of draft papers and chapters.

I would also like to thank Professor Adrian Keating, Professor David Pulfrey, Dr. Martin Kocan, Professor Gilberto A. Umana Membreno and Associate Professor Asghar Asgari for the technical discussions and assistance they provided. The discussions I had with and the assistance I received from each one of them were invaluable in the production of this thesis. Additionally, I would like to give special thanks to Peter Duncan for the technical assistance he provided to enable the electron-beam induced current experiments.

Sincere thanks must also be given to Sabina Betts for the administrative support she provided. I am also grateful for the friendship, support and help of other members of the Microelectronics Research Group, in particular, Azlan Baharin, Gino Putrino, Gordon Tsen, Ivan Teo, James Sharp, Jiang Fei, Lai Mei Fang and Zhang Jing.

As the production of this thesis is not possible without funding, I want to thank my supervisors, Professor Brett Nener and Professor Giacinta Parish, as well as Professor Lorenzo Faraone for providing some of the financial support in the form of a scholarship. I also want to thank the University of Western Australia for providing the University Postgraduate Award scholarship and acknowledge the partial funding provided by the Australian Research Council.

In addition to the financial support received, I would also like to acknowledge the facilities, scientific and technical assistance of the Australian Microscopy & Microanalysis Research Facility at the Centre for Microscopy, Characterisation & Analysis, The University of Western Australia, a facility funded by the University of Western Australia, the State Government of Western Australia and Commonwealth
Government of Australia. I am also grateful to Professor Umesh Mishra at the University of California, Santa Barbara, for provision of the samples used in this work.

Finally, I am eternally grateful to my family and Michele for the moral support they provided as well as for the patience, compassion, consideration and grace they showed me over so many years. I am also very appreciative of the guidance, perspectives and opportunities given by Dr. Krystina Haq, Eve, Tara, Linda, Simon and Tim.
# TABLE OF CONTENTS

Declaration ....................................................................................................................... ii

Statement of Contribution ................................................................................................ iii

Abstract ............................................................................................................................ v

Acknowledgements .......................................................................................................... vii

Table of Contents ........................................................................................................... ix

List of Symbols .............................................................................................................. xii

## 1 Introduction

1.1 Introduction ........................................................................................................ ..... 1

1.2 Motivation ............................................................................................................. 4

1.3 Thesis Objectives ............................................................................................... 5

1.4 Thesis Organisation ........................................................................................... 6

## 2 The Photocurrent Technique

2.1 Basic Principles .................................................................................................... 8

2.2 Mathematical Model ............................................................................................ 10

2.2.1 The Derivation of the Photocurrent Equation ............................................. 11

2.3 Application of the Photocurrent Technique ...................................................... 15

2.3.1 Measurement as a Function of Depletion Width ........................................ 15

2.3.2 Measurement as a Function of Wavelength (Spectral Response Method) ................................................................. 19

2.4 Model Assumptions and Limitations ................................................................ 23

2.5 Chapter Summary .............................................................................................. 27

## 3 Validity of the First Boundary Condition

3.1 The First Boundary Condition ............................................................................ 31

3.2 Relationship between Material Parameters and $v_1$ ..................................... 37

3.2.1 Model 1 ........................................................................................................ 38

3.2.2 Model 2 ........................................................................................................ 41

3.2.3 Comparison of Models 1 and 2 ................................................................. 42
3.3 Values of $S_1$ and the Error Due to the $S_1 \gg 1$ Assumption ....................... 49
3.4 Values of $v_1$ determined from Numerical Simulations .................................. 52
  3.4.1 Details of Numerical Simulation ................................................................. 52
  3.4.2 Simulation Results and Discussion ............................................................... 54
  3.4.3 Implications for the Accuracy of the Photocurrent Technique
      when Applied to GaN ..................................................................................... 63
      3.4.3.1 Error Calculations using Synthetic Photocurrent Data
            Produced by Simulations ................................................................. 64
3.5 $L_{est}$ obtained using the Linear Fit Extrapolation Method ............................. 73
3.6 Chapter Summary ................................................................................................ 76

4 Application to Experimental Results
  4.1 Sample Details ................................................................................................. 78
  4.2 Experimental Setup ......................................................................................... 81
  4.3 Experimental Results and Discussion .............................................................. 82
  4.4 Chapter Summary ............................................................................................. 85

5 Influence of the Franz-Keldysh Effect
  5.1 Electric Field-dependent Absorption Coefficient ........................................... 88
  5.2 Application to Experimental Results ............................................................... 93
  5.3 Corrections to Account for the Inherent Error in the Standard
      Mathematical Model ....................................................................................... 97
  5.4 Chapter Summary ............................................................................................. 100

6 Effects of Depletion Region Recombination and the Majority Carrier Current
  6.1 Recombination in the Depletion Region .......................................................... 103
      6.1.1 Defects Introduced by Postgrowth Processing ...................................... 103
      6.1.2 Effects on the Photocurrent Technique ............................................... 105
  6.2 Effects of the Majority Carrier Current .......................................................... 107
      6.2.1 Expression for the Majority Carrier Current .......................................... 108
      6.2.2 Application and Analysis ..................................................................... 111
  6.3 Chapter Summary ............................................................................................. 113

7 The Electron-Beam Induced Current Technique
  7.1 Basic Principles ............................................................................................... 116
  7.2 The EBIC Model ............................................................................................. 117
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>optical absorption coefficient</td>
<td>cm⁻¹</td>
</tr>
<tr>
<td>α</td>
<td>surface recombination dependent coefficient (in Chapter 7 only)</td>
<td>-</td>
</tr>
<tr>
<td>α₀</td>
<td>zero-field optical absorption coefficient</td>
<td>cm⁻¹</td>
</tr>
<tr>
<td>αₑₑ</td>
<td>electric-field dependent optical absorption coefficient</td>
<td>cm⁻¹</td>
</tr>
<tr>
<td>αₑₑₑₑᵉᵉ</td>
<td>effective mean absorption coefficient</td>
<td>cm⁻¹</td>
</tr>
<tr>
<td>ε₀</td>
<td>free space permittivity: (8.854 \times 10^{-14})</td>
<td>Fcm⁻¹</td>
</tr>
<tr>
<td>εᵣ</td>
<td>semiconductor relative permittivity</td>
<td>-</td>
</tr>
<tr>
<td>εₛ</td>
<td>semiconductor permittivity, (εₛ = ε₀εᵣ)</td>
<td>Fcm⁻¹</td>
</tr>
<tr>
<td>Φ₀</td>
<td>incident photon flux density</td>
<td>#cm⁻²s⁻¹</td>
</tr>
<tr>
<td>qφₘₙ</td>
<td>Schottky barrier height for a n-type Schottky barrier diode</td>
<td>eV</td>
</tr>
<tr>
<td>qφₚₚ</td>
<td>Schottky barrier height for a p-type Schottky barrier diode</td>
<td>eV</td>
</tr>
<tr>
<td>qφₙₙ</td>
<td>conduction band energy at metal-semiconductor interface</td>
<td>eV</td>
</tr>
<tr>
<td>γ</td>
<td>surface recombination and depletion depth dependent coefficient (in Chapter 7 only)</td>
<td>-</td>
</tr>
<tr>
<td>ηₙ</td>
<td>quantum yield</td>
<td>-</td>
</tr>
<tr>
<td>φ</td>
<td>collection probability</td>
<td>-</td>
</tr>
<tr>
<td>λ</td>
<td>optical wavelength</td>
<td>nm</td>
</tr>
<tr>
<td>λ</td>
<td>width of transition region arising from the λ-effect (in Chapter 8 only)</td>
<td>nm</td>
</tr>
<tr>
<td>μₙ</td>
<td>electron mobility</td>
<td>cm²V⁻¹s⁻¹</td>
</tr>
<tr>
<td>τₙ</td>
<td>minority carrier lifetime for electrons</td>
<td>s</td>
</tr>
<tr>
<td>τₚ</td>
<td>minority carrier lifetime for holes</td>
<td>s</td>
</tr>
<tr>
<td>A</td>
<td>illuminated area of Schottky contact</td>
<td>cm²</td>
</tr>
<tr>
<td>C</td>
<td>speed of light in a vacuum: (2.997 \times 10^8)</td>
<td>ms⁻¹</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$D_n$</td>
<td>diffusion coefficient for electrons</td>
<td>cm²s⁻¹</td>
</tr>
<tr>
<td>$D_p$</td>
<td>diffusion coefficient for holes</td>
<td>cm²s⁻¹</td>
</tr>
<tr>
<td>$d_b$</td>
<td>electron-beam diameter</td>
<td>nm</td>
</tr>
<tr>
<td>$E(x)$</td>
<td>electric field as a function of position, $x$</td>
<td>Vcm⁻¹</td>
</tr>
<tr>
<td>$E_A$</td>
<td>photon energy</td>
<td>eV</td>
</tr>
<tr>
<td>$E_b$</td>
<td>electron-beam energy</td>
<td>keV</td>
</tr>
<tr>
<td>$E_C$</td>
<td>energy of bottom of the conduction band</td>
<td>eV</td>
</tr>
<tr>
<td>$E_{Fn}$</td>
<td>electron quasi-Fermi level</td>
<td>eV</td>
</tr>
<tr>
<td>$E_{Fp}$</td>
<td>hole quasi-Fermi level</td>
<td>eV</td>
</tr>
<tr>
<td>$E_g$</td>
<td>band-gap</td>
<td>eV</td>
</tr>
<tr>
<td>$E_V$</td>
<td>energy of the top of the valence band</td>
<td>eV</td>
</tr>
<tr>
<td>$G_{op}$</td>
<td>steady-state optical generation rate</td>
<td>cm⁻³s⁻¹</td>
</tr>
<tr>
<td>$G_{op}^{FK}$</td>
<td>steady-state optical generation rate in the presence of the Franz-Keldysh effect</td>
<td>cm⁻³s⁻¹</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant: 6.625×10⁻³⁴</td>
<td>Js</td>
</tr>
<tr>
<td>$I_{EBIC}$</td>
<td>electron-beam induced current</td>
<td>A</td>
</tr>
<tr>
<td>$I_{total}$</td>
<td>total current</td>
<td>A</td>
</tr>
<tr>
<td>$I_{sim}^{total}$</td>
<td>simulated total current</td>
<td>A</td>
</tr>
<tr>
<td>$J_0$</td>
<td>total current density calculated using the zero-field absorption coefficient, $a_0$</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{diff}$</td>
<td>diffusion current density</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{eqn}^{diff}$</td>
<td>calculated diffusion component of $J_{sim}^{total}$</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{sim}^{diff}$</td>
<td>actual diffusion component of $J_{sim}^{total}$</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{drift}$</td>
<td>drift current density</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{eqn}^{drift}$</td>
<td>calculated drift component of $J_{sim}^{total}$</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>$J_{sim}^{drift}$</td>
<td>actual drift component of $J_{sim}^{total}$</td>
<td>Acm⁻²</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$\Delta J_{\text{drift}}$</td>
<td>difference between $J_{\text{drift}}^{\text{sim}}$ and $J_{\text{drift}}^{\text{eqn}}$</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_E$</td>
<td>total current density that takes into consideration the Franz-Keldysh effect</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_{\text{min}}$</td>
<td>minority carrier current density</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_{\text{maj}}$</td>
<td>majority carrier current density</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_n$</td>
<td>electron current density</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_p$</td>
<td>hole current density</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_{\text{total}}$</td>
<td>total current density</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$J_{\text{total}}^{\text{sim}}$</td>
<td>total current density produced by simulation</td>
<td>Acm$^{-2}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann’s constant: $1.38 \times 10^{-23}$ J K$^{-1}$</td>
<td>JK$^{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>coefficient of the EBIC equation (in Chapter 7 only)</td>
<td>-</td>
</tr>
<tr>
<td>$L_D$</td>
<td>extrinsic Debye length</td>
<td>nm</td>
</tr>
<tr>
<td>$L_{\text{est}}$</td>
<td>estimated diffusion length</td>
<td>nm</td>
</tr>
<tr>
<td>$L_{\text{est}}^{\text{exp}}$</td>
<td>diffusion length calculated using experimental data and Eqn. (2.16)</td>
<td>nm</td>
</tr>
<tr>
<td>$L_{\text{est}}^{\text{ave}}$</td>
<td>diffusion length calculated as the average of $L_{\text{est}}^{\text{exp}}$ in the $300 \leq \lambda \leq 325$ nm range</td>
<td>nm</td>
</tr>
<tr>
<td>$L_{\text{est}}^{\text{FK}}$</td>
<td>diffusion length determined from synthetic photocurrent data that are calculated using $L_{\text{est}}^{\text{ave}}$ and Eqn. (5.7)</td>
<td>nm</td>
</tr>
<tr>
<td>$L_n$</td>
<td>electron diffusion length</td>
<td>nm</td>
</tr>
<tr>
<td>$L_p$</td>
<td>hole diffusion length</td>
<td>nm</td>
</tr>
<tr>
<td>$m_c$</td>
<td>effective mass at the edge of the conduction band</td>
<td>kg</td>
</tr>
<tr>
<td>$m_r$</td>
<td>reduced mass, equivalent to $(m_c^{-1} + m_v^{-1})^{-1}$</td>
<td>kg</td>
</tr>
<tr>
<td>$m_v$</td>
<td>effective mass at the edge of the valence band</td>
<td>kg</td>
</tr>
<tr>
<td>$N_A$</td>
<td>acceptor concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$N_C$</td>
<td>conduction band density of states</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>$N_D$</td>
<td>donor concentration</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>(N_V)</td>
<td>valence band density of states</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(n)</td>
<td>electron concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(n_i)</td>
<td>intrinsic electron concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(n_{n0})</td>
<td>equilibrium majority electron concentration in an n-type material</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(n_{p0})</td>
<td>equilibrium minority electron concentration in a p-type material</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(\Delta n)</td>
<td>excess electron concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(p)</td>
<td>hole concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(p_{n0})</td>
<td>equilibrium minority hole concentration in an n-type material</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(p_{p0})</td>
<td>equilibrium majority hole concentration in a p-type material</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(\Delta p)</td>
<td>excess hole concentration</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(q)</td>
<td>electronic charge: 1.602×10(^{-19})</td>
<td>C</td>
</tr>
<tr>
<td>(R)</td>
<td>recombination rates</td>
<td>cm(^{-3})s(^{-1})</td>
</tr>
<tr>
<td>(R^2)</td>
<td>coefficient of determination</td>
<td>-</td>
</tr>
<tr>
<td>(S_1)</td>
<td>normalised charge carrier velocity at the depletion edge plane</td>
<td>-</td>
</tr>
<tr>
<td>(S_{1}^{\text{eqn}})</td>
<td>(S_1) calculated using (v_{1}^{\text{eqn}})</td>
<td>-</td>
</tr>
<tr>
<td>(S_{1}^{\text{sim}})</td>
<td>(S_1) calculated using (v_{1}^{\text{sim}})</td>
<td>-</td>
</tr>
<tr>
<td>(S_W)</td>
<td>normalised recombination velocity at the ohmic contact plane</td>
<td>-</td>
</tr>
<tr>
<td>(T)</td>
<td>temperature</td>
<td>K</td>
</tr>
<tr>
<td>(T_{\lambda})</td>
<td>optical transmission factor</td>
<td>-</td>
</tr>
<tr>
<td>(V_{bi})</td>
<td>built-in voltage</td>
<td>V</td>
</tr>
<tr>
<td>(v_{1})</td>
<td>velocity at which minority carriers are swept away from the depletion edge plane towards the Schottky contact</td>
<td>cm(^{-1})s(^{-1})</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$v_{1}^{\text{eqm}}$</td>
<td>$v_1$ calculated using Model 1 or Model 2</td>
<td>cms$^{-1}$</td>
</tr>
<tr>
<td>$v_{1}^{\text{sim}}$</td>
<td>$v_1$ obtained from least-squares fitting of Eqn. (3.2) to the electron density distribution obtained from the simulation</td>
<td>cms$^{-1}$</td>
</tr>
<tr>
<td>$v_d$</td>
<td>diffusion velocity</td>
<td>cms$^{-1}$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>surface recombination velocity</td>
<td>cms$^{-1}$</td>
</tr>
<tr>
<td>$v_W$</td>
<td>recombination velocity at the ohmic contact plane</td>
<td>cms$^{-1}$</td>
</tr>
<tr>
<td>$W$</td>
<td>thickness of GaN layer</td>
<td>µm</td>
</tr>
<tr>
<td>$w_{gv}$</td>
<td>lateral width of the electron-hole pair generation volume created by an electron beam</td>
<td>nm</td>
</tr>
<tr>
<td>$x$</td>
<td>spatial coordinate</td>
<td>-</td>
</tr>
<tr>
<td>$x_d$</td>
<td>depletion width</td>
<td>nm</td>
</tr>
<tr>
<td>$x_{da}$</td>
<td>depletion width calculated using depletion approximation and does not include the transition region</td>
<td>nm</td>
</tr>
<tr>
<td>$x_{de}$</td>
<td>effective depletion width that includes the transition region</td>
<td>nm</td>
</tr>
<tr>
<td>$y$</td>
<td>spatial coordinate</td>
<td>-</td>
</tr>
<tr>
<td>$y_d$</td>
<td>depletion depth</td>
<td>nm</td>
</tr>
<tr>
<td>$z$</td>
<td>spatial coordinate</td>
<td>-</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Introduction

In recent decades, significant research has been carried out to advance the science and technology associated with gallium nitride or GaN-based semiconductor materials and devices. GaN and its alloys with aluminium nitride (AlN) and indium nitride (InN) are direct and wide band-gap semiconductors. This, together with other attributes such as high breakdown field and high bond strength, makes this group of materials highly suited to optoelectronic applications involving wavelengths in the blue to ultraviolet (UV) spectral range and to power electronics applications in harsh environments.

Examples of GaN-based optoelectronic devices include UV detectors and optical emitters such as high brightness blue light emitting diodes (LED) and UV laser diodes. Compared to silicon-based UV detectors, GaN-based detectors have better wavelength selectivity due to the wide and direct band-gap of the constituent materials and are more suitable for applications in a harsh environment such as space. Visible-blind photodetectors based on GaN and its alloys can be used to detect UV sources against a background that is flooded with visible and infra-red radiation. Using GaN-based lasers that emit light in the blue or UV wavelength range, a smaller laser spot size can be achieved compared to that of the red light emitted by gallium arsenide or GaAs-based laser that are commonly used in compact disc appliances. The smaller spot size results in a greater storage density and hence greater storage capacity for the same disc size.

In power electronic applications, GaN-based power amplifiers are useful in power distribution systems, radar systems and mobile phones base stations. Due to the high breakdown field, good thermal conductivity and high bond strength, GaN-based electronics such as AlGaN/GaN high electron mobility transistors (HEMT) and heterojunction bipolar transistors (HBTs) offer superior performance in terms of power density and heat tolerance compared to GaAs-based electronics. The ability to handle higher voltages and currents means a GaN-based radar system, for example, can produce more power, which translates to greater detection range and volume while the
reduced cooling requirements allows such a system to be more compact.

An essential part of the advancement of any semiconductor technology is the characterisation of minority carrier transport properties. In semiconductor physics, minority carrier transport properties generally refer to the minority carrier recombination lifetime and diffusion length. These two quantities can be considered, respectively, as the average time and distance travelled between the generation of excess minority carriers and their recombination with the majority carriers.

The characterisation of the transport properties of minority carriers is important because the operating characteristics and performance of many semiconductor devices such as photodetectors and bipolar junction transistors depend on them. For example, the responsivity of a photodiode is directly influenced by the diffusion length, which is a measure of the ability of the photogenerated minority carriers to diffuse to the built-in junction and contribute to charge collection. A short diffusion length means that a large proportion of the photogenerated carriers will recombine before they reach the junction. Therefore, a long diffusion length is desirable to achieve high photodiode responsivity.

In bipolar junction transistors, the common emitter current gain is proportional to the square of the ratio of the diffusion length and the width of the base. Therefore, a long diffusion length is desirable to achieve high current gains. In addition to these performance metrics, minority carrier diffusion length and recombination lifetime are also used by engineers as measures of material quality. Either parameter can be used to monitor the defect density and uniformity of wafers after different processing steps [1].

Various experimental techniques have been devised to measure minority carrier transport properties in semiconductor materials. A list of techniques and discussions of their principles, applications and merits can be found in texts by Blood and Orton [2] and Schroder [3]. Typically, the choice of techniques is dependent on many factors. Chief among them is the ability to directly determine the parameter of interest. For example, the Time-Resolved Photoluminescence technique allows the direct determination of lifetime whereas the Electron-Beam Induced Current (EBIC) technique allows the direct determination of diffusion length.

In principle, because the recombination lifetime is proportional to the square of the
diffusion length and provided the proportionality coefficient, that is, the diffusion coefficient is known, then knowledge of one of these parameters will allow the other to be calculated. However, such calculations require knowledge of the diffusion coefficient and by implication, the minority carrier mobility (the diffusion coefficient is the product of the mobility and the thermal voltage). There are many other factors that influence the choice of techniques. For instance, considerations such as whether the sample has a direct band-gap will steer the investigator towards or away from techniques that rely on the detection of luminescence. More detailed discussions of the considerations and the suitability of the various techniques can also be found in [2,3].

Regardless of which of the two parameters are measured and what techniques are employed, the basic principles are the same. Every technique requires the injection of a suitable amount of excess minority carriers into the sample. These charge carriers are typically generated via optical, electron beam or p-n junction injection and the presence of these excess carriers will have an appreciable effect on measurable quantities such as current, voltage or luminescence intensity. Many of the techniques described in [2,3] involve the measurement of one of these quantities and the key to interpreting the measured data and extracting either the recombination lifetime or diffusion length from the data is to have an appropriate mathematical model that relates the measured quantities to the desired minority carrier property.

In this work, the principal focus is the accuracy of the diffusion length values of GaN when determined using a technique that involves the measurement of the current induced in a photodiode via optical means. This technique is known as the photocurrent technique. It is also known as the spectral photocurrent or response technique when measurement is carried out as a function of illumination wavelength. The bulk of the work undertaken in this thesis is concerned with the investigation of the validity of the widely-accepted mathematical model that describes the relationship between the diffusion length and the current produced by a Schottky barrier photodiode under zero bias condition.
1.2 Motivation

The initial motivation for this work stems from the need to assess the material quality of magnesium (Mg)-doped, p-type GaN devices. Devices such as GaN-based LEDs and AlGaN/GaN HBTs contain p-type GaN layers. For example, p-type GaN forms the top (anode) layer of a GaN-based LED. It is also the material used as the base layer of an AlGaN/GaN n-p-n HBT. Both these devices require a good quality p-type layer for high performance and efficiency.

Mg is the dopant of choice to achieve p-type conductivity in GaN [4] and typically, Mg-doped, p-type GaN has poor material quality. This is because the high activation energy of Mg in GaN (~170meV [5]) results in the ionisation of only about one percent of the acceptor dopant. In order to achieve a given hole concentration, a higher nominal acceptor concentration, $N_A$, is required and this leads to a degradation of material quality [6] and consequently, poor device performance [7]. Therefore, assessment of material quality for p-type GaN is of critical importance in the advancement of GaN-based technology.

The purpose of measuring minority carrier diffusion length is thus to provide a quantitative measure of material quality so as to enable the improvement and optimisation of GaN growth and p-type doping technologies as well as postgrowth processing steps for device fabrication.

A survey of the literature found that the diffusion lengths of electrons in p-type GaN range between 21-950nm [8-16]. For n-type GaN, hole diffusion length values of between 10nm-3.4µm have been reported [10,12,14-26]. With reported values as low as 10-20nm [8,17], any technique employed to determine the diffusion length in GaN must be capable of accurately detecting values in the low sub-micrometer range as well as at the upper end of the reported range. The work undertaken here is motivated especially by the need to have confidence in the ability of the photocurrent technique to produce accurate diffusion length values at the lower end of the sub-micrometer range. This technique was chosen because it has been widely used in the past to determine the diffusion length in compound semiconductors such as gallium arsenide (GaAs) [27-32], gallium phosphide (GaP) [33,34], cadmium telluride (CdTe) [35] and indium phosphide (InP) [36] as well as silicon solar cells [37,38]. The diffusion lengths in these materials
are usually much longer than the diffusion lengths reported for GaN. Therefore, the applicability of this technique to a material with very short diffusion lengths such as GaN needs to be validated.

### 1.3 Thesis Objectives

The principal driver for the work undertaken in this thesis is to be able to characterise short minority carrier diffusion lengths in p-type GaN and have confidence in the results obtained. For this thesis, the general aim is to investigate the accuracy of the photocurrent technique in the determination of low sub-micrometre diffusion length in GaN. This is because the diffusion lengths of GaN can be very short, with values as low as 10-20 nm being reported in the literature [8,17]. In this work, the investigation will focus mainly on the standard mathematical model for a test structure in the form of a Schottky barrier diode and its applicability with respect to GaN. The validity of the model will be assessed by examining four of the assumptions made in the derivation of this model. These four assumptions are that:

1. the minority carrier density at the depletion edge retains its equilibrium value;
2. the absorption coefficient, $\alpha$, is uniform throughout the material, including in the depletion region;
3. there is no recombination in the depletion region; and that
4. the photocurrent consists of the minority carrier current only.

The end goal of the investigations is to be able to make a determination of the:

1. validity of each assumption with respect to GaN;
2. the effects of the validity or otherwise of each assumption on the accuracy of the diffusion length values in GaN calculated using the standard mathematical model;
3. overall validity of the standard mathematical model with respect to GaN; and
4. the applicability of the photocurrent technique to GaN.
1.4 Thesis Organisation

This thesis consists of eight chapters. In addition to presenting the motivation and objectives of this thesis, this chapter also provides the introductory information which places the motivation and objectives into context.

Chapter 2 presents a review of the photocurrent technique, which includes a brief qualitative description of the underlying physical mechanisms of the photocurrent technique, the derivation of the standard mathematical model for an ideal Schottky diode, an assessment of the applicability of two common implementations of this technique to GaN and a brief discussion of the assumptions and limitations of the model.

In Chapter 3, the first assumption listed in the previous section, namely, that the photo-generated electrons arriving at the depletion edge are swept away instantaneously by the built-in electric field, thereby leaving no excess electrons at that plane, is investigated in detail. The aims of this chapter are to determine the validity of this assumption and to quantify its effects on the accuracy of the diffusion length values extracted using the standard mathematical model presented in Chapter 2.

In Chapter 4, the findings of Chapter 3 are applied to experimental data acquired from spectral photocurrent measurements of both n-type and p-type GaN Schottky diodes. The results obtained demonstrate that another assumption made in the derivation of the standard mathematical model, that is, the second assumption listed in Section 1.3, needs to be examined in more detail. The investigations of the validity of this assumption are presented in Chapter 5. This is followed by investigations of the third and fourth assumptions mentioned in Section 1.3.

Due to the limitations of the photocurrent technique, which are explained in Chapter 4, an alternative technique, namely, the EBIC technique is applied to both the n-type and p-type GaN samples. Investigations of the accuracy of this technique when it is used to detect low sub-micrometer diffusion length, its application to the GaN samples and discussions of the results are presented in Chapter 7. Finally, Chapter 8 summarises the thesis and discusses some future research directions.
Four appendices follow Chapter 8. Derivations of two models used to calculate the velocity at which minority carriers are swept away from the depletion edge plane towards the Schottky contact are presented in Appendix A and Appendix B. Details of numerical simulations employed to supplement these models are presented in Appendix C. And finally, Appendix D presents the derivations of the modified drift and diffusion currents given in Chapter 5.
CHAPTER 2

THE PHOTOCURRENT TECHNIQUE

In this chapter, a review of the photocurrent technique is presented. The review begins with a brief qualitative description of the underlying physical mechanisms of the photocurrent technique. This is followed by the derivation of the standard mathematical model for an ideal Schottky diode in Section 2.2, an assessment of the applicability of two common implementations of this technique to GaN in Section 2.3 and a brief discussion of the assumptions and limitations of the model in Section 2.4. Extended discussions of the effects of some of the assumptions on the accuracy of the photocurrent technique are presented in Chapters 3 to 6. The chapter finishes with a summary in Section 2.5.

2.1 Basic Principles

The photocurrent technique studied in this thesis is a steady-state, non-scanning, junction method for direct measurement of diffusion length [2]. The term steady-state specifies that the optical excitation source and hence the photocurrent signal is time-invariant. The method is non-scanning because the sample being investigated is illuminated by a stationary, monochromatic optical beam. This is in contrast to a moving light spot, such as a scanning laser-beam, that adds the ability to change the physical separation of the excitation source and the junction that collects the injected excess carriers. This junction can be found in a Schottky barrier or a p-n junction diode for example.

The basic mechanisms of the photocurrent technique can be divided into three sequential parts – carrier generation, diffusion and collection. Consider a semi-transparent metal contact Schottky barrier diode subjected to front-side illumination as shown by the schematic in Figure 2.1; in this case the semiconductor is p-type and the electrons are the minority carriers. The illumination covers and is uniform over the entire Schottky contact. As the incident radiation penetrates into the material, photons,
with energy greater than the band-gap energy of the semiconductor material, are absorbed and excess electrons and holes are created. As the radiation propagates deeper into the structure, the photon flux density decreases. In the absence of recombination in the depletion region, all the electrons that are created in the depletion region as well as those in the neutral bulk region that diffuse to the edge of the depletion region are swept towards the Schottky contact by the built-in electric field and generate a current in the external circuit, the photocurrent. From measurements of this current, material properties such as the minority carrier diffusion length, which is an indicator of material quality and the focus of this work, can be determined. This information is extracted from the measured photocurrent by using a mathematical model that describes the relationship between these quantities. This model is presented in the next section.

Figure 2.1. The schematic of the p-type Schottky diode with the band diagram superimposed showing the drift and diffusion components of the photocurrent. The parameters $\Phi_0$, $\lambda$, $h$, $c$ and $E_g$ are incident photon flux density, wavelength, Planck’s constant, speed of light, and material band-gap respectively. Here, the horizontal and vertical axes are the $x$- and $y$-axis respectively. The depth (not shown) is the $z$-axis. For simplicity, all quantities are assumed to be invariant along the $y$- and $z$-axis so that the problem is reduced to one dimension only.
2.2 Mathematical Model

The derivation of the theoretical expression that relates the minority carrier diffusion length to the photocurrent generated by a Schottky barrier diode under constant monochromatic illumination is based on the drift-diffusion model and has been presented in the literature [2,27,39-41]. The standard treatment involves mathematically differentiating the minority carrier distribution function at the depletion edge plane in the neutral region to obtain an expression for the diffusion current. This is then added to the current generated in the depletion region to obtain the standard mathematical model. The distribution function is obtained by solving the minority carrier continuity equation using two boundary conditions.

Overall, the mathematics involved in the derivation is straightforward. Nevertheless, the essential steps in the derivation and the assumptions made along the way are presented here in order to provide the basis for subsequent chapters that explore some of these assumptions in more detail. Because the electron diffusion length in p-type GaN can be very short [8] and therefore difficult to measure accurately, in this work, significant attention is given to investigating the theoretical aspect of the technique. The investigation involves validating or otherwise, some of the assumptions of the standard mathematical model with respect to GaN, so that there is confidence in applying the model to extract diffusion length values from experimental data.

The derivation presented in the following section is based on an ideal, uniformly doped, non-degenerate p-type Schottky barrier diode with a neutral bulk region that is much greater than the minority carrier diffusion length and the absorption length. The layout of the device structure is shown by the schematic in Figure 2.1. For simplicity, all quantities are assumed to be invariant along the y- and z-axis so that the problem is reduced to one dimension only. Analogous equations and results can be obtained for an ideal n-type diode by using the same treatment.
2.2.1 The Derivation of the Photocurrent Equation

The expression for the current collected in the depletion region is given by

\[ J_{\text{drift}} = q \int_0^{x_d} \varphi(x) G_{\text{op}}(x) \, dx \]  \hspace{1cm} (2.1)

where \( q \) and \( x_d \) are the electronic charge and depletion width respectively. The position-dependent collection probability function, \( \varphi(x) \), specifies the probability that a carrier produced at position \( x \) will be collected and contribute to the photocurrent. \( G_{\text{op}}(x) \) is the steady-state electron-hole pair generation rate for a monochromatic radiation and is given by

\[ G_{\text{op}}(x) = T_\lambda \Phi_0 \eta_\lambda \alpha e^{-\alpha x} \]  \hspace{1cm} (2.2)

where \( T_\lambda \) and \( \Phi_0 \) are the wavelength-dependent optical transmission factor of the semi-transparent Schottky barrier material and incident photon flux density at the \( x = 0 \) plane respectively. The absorption coefficient is denoted by \( \alpha \) and is usually assumed to be a function of wavelength, \( \lambda \), only. The quantum yield at a given wavelength is denoted by \( \eta_\lambda \) and represents the number of electron-hole pairs produced for every photon that is absorbed by the material; it is also known as the photon quantum efficiency.

Assuming that each photon absorbed produces one electron-hole pair, then \( \eta_\lambda \) can be set to unity. Additionally, if, following their separation by the built-in electric field, the electron and hole do not recombine as they drift in opposite directions towards the surface barrier and neutral region respectively, then \( \varphi(x) \) can also be set to unity. Thus, the evaluation of Eqn. (2.1) yields

\[ J_{\text{drift}} = q T_\lambda \Phi_0 (1 - e^{-\alpha x_d}) \]  \hspace{1cm} (2.3)

The equation for the diffusion current is given by

\[ J_{\text{diff}} = q D_n \left. \frac{dn(x)}{dx} \right|_{x = x_d} \]  \hspace{1cm} (2.4)
where $D_n = \mu_n kT/q$ is the electron diffusion coefficient and $\mu_n$, $k$ and $T$ are the electron mobility, Boltzmann’s constant and temperature respectively. $n(x)$ is the electron density distribution function in the region $x \geq x_d$ and is obtained by solving the minority carrier continuity equation

$$\frac{dn(x)}{dt} = D_n \frac{d^2n(x)}{dx^2} + G_{\text{op}}(x) - \frac{n(x) - n_{p0}}{\tau_n} = 0 \quad (2.5)$$

under low-level injection and steady-state conditions. In Eqn. (2.5), $n_{p0}$ and $\tau_n$ denote the electron equilibrium density and recombination lifetime respectively. $\tau_n$ is related to the electron diffusion length by $L_n^2 = D_n \tau_n$. The term involving the electric field, $E(x)$, is eliminated because $E(x)$ is assumed to be approximately zero in the region $x \geq x_d$. To solve for $n(x)$, two boundary conditions are required – one at the depletion edge, $x = x_d$, and the other at the ohmic contact plane, $x = W$. In the literature [2,37,40,42-44], the following Dirichlet boundary condition is commonly specified at the $x = x_d$ plane:

$$n(x_d) = n_{p0} \quad (2.6)$$

This condition represents the assumption that photo-generated electrons arriving at the edge of the depletion region are swept away instantaneously by the built-in electric field. Therefore, there is no excess electron at that plane and $n(x_d)$ maintains its thermal equilibrium density. At the $x = W$ plane, a similar Dirichlet boundary condition can be imposed:

$$n(W) = n_{p0} \quad (2.7)$$

This condition also states that there are no excess electrons at this plane and implies that the ohmic contact is a perfect sink for electrons. Alternatively, a Neumann boundary condition that links the recombination velocity at the ohmic contact, $v_W$, to the excess electron density at that plane can be specified:

$$D_n \left. \frac{dn(x)}{dx} \right|_{x=W} = -v_W [n(W) - n_{p0}] \quad (2.8)$$
Applying these conditions to the general solution of Eqn. (2.5) yields

\[ n(x) = C_0 e^{-a(x - x_d)} + C_1 e^{-(x - x_d)/L_n} + C_2 e^{(x - x_d)/L_n} + n_{p0} \]  

(2.9)

where \( L_n \) is the electron diffusion length and

\[ C_0 = \frac{T_0 \Phi_0 e^{-\alpha x_d} a L_n^2}{D_n (1 - \alpha^2 L_n^2)} \]  

(2.10)

\[ C_1 = -C_0 \left[ \frac{e^{(W - x_d)/L_n} - e^{-a(W - x_d)}}{e^{(W - x_d)/L_n} - e^{-(W - x_d)/L_n}} \right] \]  

(2.11)

\[ C_2 = \frac{C_0 \left[ \frac{(S_w - 1)e^{-(W - x_d)/L_n} - (S_w - aL_n)e^{-a(W - x_d)}}{S_w + 1} \right]}{\left[ \frac{e^{(W - x_d)/L_n} - (S_w - 1)e^{-(W - x_d)/L_n}} \right]} \]

using Eqn. (2.7)

or

\[ C_2 = \frac{C_0 \left[ \frac{(S_w - 1)e^{-(W - x_d)/L_n} - (S_w - aL_n)e^{-a(W - x_d)}}{S_w + 1} \right]}{\left[ \frac{e^{(W - x_d)/L_n} - (S_w - 1)e^{-(W - x_d)/L_n}} \right]} \]

using Eqn. (2.8)

(2.12)

\( S_W = v_W L_n / D_n \) is known as the normalised recombination velocity. The unit of the ratio \( D_n / L_n \) is \( \text{cms}^{-1} \) and this quantity is often referred to as the diffusion velocity, \( v_d \) [2], which means that \( S_W \) is a measure of the relative magnitude of \( v_W \) to the diffusion velocity. Note that the two expressions in Eqn. (2.12) are effectively equivalent when \( S_W \gg 1 \) and \( S_W \gg \alpha L_n \), that is, when the recombination velocity at the ohmic contact is very high and the ohmic contact behaves like a perfect sink for electrons. However, if the neutral region is sufficiently thick such that \( (W - x_d) \gg L_n \) and \( (W - x_d) \gg \alpha^{-1} \), then the value of \( S_W \) becomes irrelevant because \( C_2 = 0 \) irrespective of the value of \( S_W \). For p-type GaN, the short diffusion and absorption lengths mean that these two inequalities are easily satisfied except when the active region is a very thin epitaxial layer. Therefore, the electron density distribution is given by
\[ n(x) = C_0 e^{-\alpha(x-x_d)} - \frac{C_0 e^{-(x-x_d)\alpha L_n}}{1 + \alpha L_n} + n_{p0} \]  

(2.13)

By differentiating Eqn. (2.13) and substituting into Eqn. (2.4), the diffusion current can be expressed as

\[ J_{\text{diff}} = qT_x \Phi_0 e^{-\alpha x_d} \left( \frac{\alpha L_n}{1 + \alpha L_n} \right) \]  

(2.14)

By adding \( J_{\text{drift}} \) and \( J_{\text{diff}} \), the total photocurrent produced is given by

\[ J_{\text{total}} = qT_x \Phi_0 \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right) \]  

(2.15)

This is the standard mathematical model that relates the diffusion length to the photocurrent produced by a Schottky diode under monochromatic illumination. Finally, rearranging Eqn. (2.15) to obtain \( L_n \) yields

\[ L_n = \frac{1}{\alpha} \left( \frac{qT_x \Phi_0 e^{-\alpha x_d}}{qT_x \Phi_0 - J_{\text{total}}} - 1 \right) \]  

(2.16)

Because this equation is the same as Eqn. (2.15) but presented in another form, any mention of the standard mathematical model in the remainder of this thesis shall be taken to mean either equation.

Note that Eqn. (2.15) is similar to the equation derived by Gärtner [39] in 1959. The only difference is the presence of the additional term \( qD_n n_{p0}/L_n \) in the equation derived by Gärtner. In his derivation, Gärtner used \( n(x_d) = 0 \) as the boundary condition rather than \( n(x_d) = n_{p0} \) as was done here and in \([37,40,42-44]\). However, since \( n_{p0} = n_i^2/p_{p0} \), where \( n_i \) and \( p_{p0} \) are the intrinsic carrier and equilibrium hole densities respectively, this additional term is negligible. This is especially true for a wide band-gap material such as GaN because the intrinsic carrier density is very low. Therefore, both equations are effectively equivalent. It should be noted that the Gärtner model also assumes that there is no recombination occurring in the depletion region.
In the following section, two common implementations of the photocurrent technique are described and their applicability to GaN are investigated using synthetic photocurrent data, which are calculated using Eqn. (2.15) and a set of known diffusion length values.

### 2.3 Application of the Photocurrent Technique

The non-scanning, junction-based photocurrent technique has been used to determine the diffusion length of compound semiconductors such as GaAs [27-32], gallium phosphide [33,34], cadmium telluride [35] and indium phosphide [36] as well as silicon solar cells [37,38]. In the literature, this technique is usually implemented in one of two principal ways, namely the measurement of

(i) the photocurrent as a function of depletion width using light of a single wavelength [28-35]; or

(ii) the response as a function of wavelength (spectral response) at a constant depletion width [27,31,34,36-38].

In both methods, the theoretical expression of the measured photocurrent is given by Eqn. (2.15). Provided the remaining parameters are known, the diffusion length, which is assumed to be independent of bias voltage and wavelength, can be calculated directly using Eqn. (2.16). However, depending on the sample and experimental conditions, Eqn. (2.15) can be simplified further so that knowledge of one or more parameters becomes unnecessary. Therefore, the number of additional measurements can be reduced. Each method is described in more detail in Sections 2.3.1 and 2.3.2.

#### 2.3.1 Measurement as a Function of Depletion Width

In this method, the illumination wavelength is kept constant and the induced photocurrent is measured as a function of depletion width, which is modulated electrically by applying a reverse bias. The wavelength is typically chosen such that $\alpha x_d \ll 1$ so that the exponential term in Eqn. (2.15) can be approximated by $1 - \alpha x_d$. With this approximation, Eqn. (2.15) can be expressed as [2]
\[
J_{\text{total}} = qT_0 \Phi_0 \left( \frac{\alpha}{1 + \alpha L_{\text{est}}} \right) (x_d + L_{\text{est}}) \tag{2.17}
\]

where \( L_{\text{est}} \) is the diffusion length estimated as a result of the approximation.

Before proceeding further, a brief remark on the notation for the diffusion length is needed to minimise confusion. The change in the subscript of \( L \) from ‘\( n \)’ to ‘\( \text{est} \)’ in Eqn. (2.17) was intended to distinguish the accurate or actual value from the estimated value. Henceforth, any diffusion length value determined from either synthetic or experimentally measured photocurrent data using the standard model or its variation as a result of additional approximation is regarded as an estimated value and will thus be denoted by \( L_{\text{est}} \). In this work, \( L_n \) is used to denote the electron diffusion length value used in producing any synthetic photocurrent data. In this case, this value is known precisely and is therefore the accurate value. The notation \( L_n \) is also used to represent the actual diffusion length value in a real, physical sample under investigation. This actual value is obtainable only under the most perfect of conditions, meaning that there is no uncertainty or inaccuracy in the value of any of the quantities required by the mathematical model that fully and accurately captures the relationship between \( L_n \) and the measured photocurrent. Therefore, unless such ideal conditions can be met, \( L_{\text{est}} \) represents an estimate of \( L_n \) in an actual sample. Whether it is synthetic or experimental photocurrent data, the difference between \( L_n \) and \( L_{\text{est}} \) is taken to be a measure of the accuracy of the photocurrent technique. The definitions and descriptions of \( L_n \) and \( L_{\text{est}} \) are summarised in Table 2.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Descriptions</th>
</tr>
</thead>
</table>
| \( L_n \) | the value of \( L_n \) is known precisely. It is used in producing photocurrent data  
(i) calculated using equations such Eqn. (2.15); or  
(ii) from numerical simulations. |
| \( L_{\text{est}} \) | a value estimated from either synthetic or experimentally measured photocurrent data using the standard mathematical model or its variation. |

Table 2.1. Definitions and descriptions of \( L_n \) and \( L_{\text{est}} \).
The plot of $J_{\text{total}}$ versus $x_d$ should be a straight line with the slope given by the constant term indicated in Eqn. (2.17). From the plot, $L_{\text{est}}$ can be determined from the zero photocurrent intercept on the $x$-axis. Measurements can be performed at several different wavelengths to verify that the value of $L_{\text{est}}$ is unique [31]. Knowledge of $T_\lambda$, $\Phi_0$ and $\alpha$ are not necessary because their values are constant for a given wavelength. This greatly simplifies the task of determining $L_{\text{est}}$ from photocurrent measurements. Note that, in practice, one always measures the photocurrent, $I_{\text{total}}$, rather than the current density, $J_{\text{total}}$. Even though the illuminated area of the Schottky contact, $A$, remains constant and can be grouped together with the other parameters in the constant term, its value is still required to determine $x_d$ from capacitance-voltage measurements.

To illustrate this method, synthetic photocurrent data for a p-type GaN Schottky diode with an acceptor density of $10^{19}$ cm$^{-3}$ and indium tin oxide (ITO) as the semi-transparent barrier material was calculated using Eqn. (2.15) and the parameters listed in Table 2.2. The ITO work function was taken to be 4.3eV [45] and using the depletion approximation, the zero-bias depletion width was calculated to be approximately 20nm. The $J_{\text{total}}$ data, calculated for a set of assumed $L_n$ values and at $\lambda = 301$nm and $\lambda = 364$nm, are plotted against $x_d$. These plots are shown in Figure 2.2. Each solid line represents the best attempt to fit a straight line to each curve, which, clearly is not the expected straight line. For each curve, only the first few data points ($x_d < 50$nm and $\alpha x_d < 0.7$) are used because these points are the most linear sections of the curves where $\alpha x_d < 1$. The values of $L_{\text{est}}$ are shown in Table 2.3.

The examples presented in Figure 2.2 and Table 2.3 show that because the condition $\alpha x_d \ll 1$ is not met for GaN, the plots of $J_{\text{total}}$ versus $x_d$ are not linear curves and the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength, $\lambda$ [nm]</td>
<td>301, 364 (band edge)</td>
<td></td>
</tr>
<tr>
<td>Absorption coefficient, $\alpha$ [cm$^{-1}$]</td>
<td>$1.44 \times 10^5, 7.87 \times 10^4$</td>
<td>[46]</td>
</tr>
<tr>
<td>Electron diffusion length, $L_n$ [nm]</td>
<td>20, 50, 100, 300</td>
<td></td>
</tr>
<tr>
<td>Depletion width, $x_d$ [nm]</td>
<td>20 – 150</td>
<td></td>
</tr>
<tr>
<td>$\alpha x_d$ (range for $\lambda = 301$nm and $\lambda = 364$nm)</td>
<td>0.29 - 2.16, 0.16 - 1.18</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2. The parameter values used in the calculation of the $J_{\text{total}}$ versus $x_d$ plots shown in Figure 2.2.
Figure 2.2. Plots of synthetic photocurrent density data, $J_{\text{total}}$ versus $x_d$ for a p-type GaN Schottky diode with a zero-bias depletion width of approximately 20nm. The $J_{\text{total}}$ values (solid and open symbols) were calculated using Eqn. (2.15) for a set of assumed electron diffusion length values ($L_n = 20$nm, 50nm, 100nm and 300nm) and at two wavelengths ($\lambda = 301$nm and 364nm). Each solid line represents the fitting of a linear equation to the linear segment ($x_d < 50$nm) of the calculated data points. These lines were extrapolated to the $x$-axis to obtain estimates of the diffusion length values, $L_{\text{est}}$.

<table>
<thead>
<tr>
<th>$L_n$ [nm]</th>
<th>$L_{\text{est}}$ (%)</th>
<th>$\lambda = 301$nm (0.29 ≤ $\alpha x_d$ ≤ 2.16)</th>
<th>$\lambda = 364$nm (0.16 ≤ $\alpha x_d$ ≤ 1.18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>34nm (−70%)</td>
<td>27nm (−35%)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>76nm (−52%)</td>
<td>64nm (−28%)</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>148nm (−48%)</td>
<td>124nm (−24%)</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>435nm (−45%)</td>
<td>367nm (−22%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3. The diffusion length values (and the percentage errors) determined from the zero photocurrent intercepts on the $x$-axis of the linear segments ($x_d < 50$nm) of the $J_{\text{total}}$ versus $x_d$ curves shown in Figure 2.2.
differences between \( L_m \) and \( L_{est} \) can be quite significant. Furthermore, the values of \( L_{est} \) obtained at different wavelengths using this method are not the same. \( L_{est} \) obtained at \( \lambda = 364 \text{nm} \) is more accurate because \( \alpha \) and hence \( \alpha x d \) is lower.

The condition \( \alpha x d \ll 1 \) effectively means that the width of the expanded depletion region must be much less than the absorption length at the chosen wavelength. Wide band-gap materials such as GaN have very large absorption coefficient values, even at wavelengths close to and above the band edge. The band edge absorption length of GaN is approximately 127\text{nm} [46]. Therefore, the applicability of this method is limited to heavily-doped GaN materials only.

Even when the condition \( \alpha x d \ll 1 \) can be satisfied, the expansion of the depletion region increases the transit time of the minority carriers through this region. Due to the short diffusion length and lifetime in GaN, the likelihood of carrier recombination in this region increases and the collection probability, \( \phi(x) \), can no longer be assumed to be unity. This is an example of a scenario where a model assumption does not hold as a result of the short diffusion length of p-type GaN.

Additional discussions about the assumptions of the mathematical model and their limitations on the accuracy of the model are presented in Section 2.4. In the following section, the spectral response method is discussed.

### 2.3.2 Measurement as a Function of Wavelength (Spectral Response Method)

The alternative to the depletion width modulation method is the measurement of the photocurrent as a function of wavelength at a constant depletion width, otherwise known as the spectral response method. In the literature, the procedure for extracting diffusion length from the spectral response data varies according to the equation used. The diffusion length can be calculated directly using Eqn. (2.15) if the values of all the remaining quantities are known (although this does not require wavelength-dependent measurements, the spectral data can be used to verify that the diffusion length value obtained is unique). It can also be determined by fitting Eqn. (2.15) to the experimentally measured \( J_{\text{total}} \) as a function of \( \alpha \), with the diffusion length as one of the
fitting parameters [34]. Alternatively, a simplified form of Eqn. (2.15), obtained as a result of approximations made possible due to the relative magnitudes of the various parameters in the equation, can be used. Depending on the quantities chosen as the independent and dependent variables, a linear curve may be obtained and the diffusion length is then given by either the $x$-intercept [31,38] or the slope of the curve [37].

To determine the hole diffusion length from the spectral response of an n-type GaAs sample, Stagg [31] chose a wavelength range that satisfies the condition $\alpha x_d \ll 1$ and rewrote Eqn. (2.17) as

$$\frac{\Phi_0}{J_{total}} = \frac{1}{\frac{qT_{\lambda}}{x_d + L_{est}}} \left( \alpha^{-1} + L_{est} \right)$$  \hspace{1cm} (2.18)

When the ratio of the photon flux density, $\Phi_0$, to the measured photocurrent, $I_{total}$, was plotted against $\alpha^{-1}$, a straight line was obtained and the hole diffusion length, which was assumed to be a constant over the wavelength range, was determined from the $x$-intercept of the linear curve. Provided the illumination area on the Schottky contact, $A$, is constant, knowledge of $A$ is not required and the use of $I_{total}$ in place of $J_{total}$ will not alter the value of $L_{est}$. In [31], $T_{\lambda}$ was also assumed to be constant over the range of wavelengths used. Therefore, $T_{\lambda}$ was grouped together with the other constants on the right hand side of Eqn. (2.18) and hence, its measurement was unnecessary. In general, $T_{\lambda}$ is a function of $\lambda$ and a separate measurement of $T_{\lambda}$ is usually necessary. Therefore, Eqn. (2.18) should be modified to read

$$\frac{T_{\lambda} \Phi_0}{J_{total}} = \frac{1}{\frac{q}{x_d + L_{est}}} \left( \alpha^{-1} + L_{est} \right)$$  \hspace{1cm} (2.19)

or

$$\frac{T_{\lambda} \Phi_0}{I_{total}} = \frac{1}{\frac{qA T_{\lambda}}{x_d + L_{est}}} \left( \alpha^{-1} + L_{est} \right)$$  \hspace{1cm} (2.20)

and the plot should be $T_{\lambda} \Phi_0/I_{total}$ versus $\alpha^{-1}$ instead. Unlike the method described in the
previous section, the values of $T_\lambda$, $\Phi_0$ and $\alpha$ are required but $x_d$ and $A$ are not. The incident photon flux density, $\Phi_0$, can be determined by measuring the power of the optical beam using a calibrated photodetector and an aperture of known area, which does not necessarily have to be the same as the area of the Schottky contact. Division of the measured power density by the photon energy that corresponds to the wavelength at which the measurement was made gives $\Phi_0$.

Figure 2.3 shows examples of the application of this method to p-type GaN Schottky diodes with zero-bias depletion widths of approximately 20nm and 100nm. The synthetic photocurrent data, $J_{\text{total}}$, were calculated using Eqn. (2.15) and the parameters listed in Table 2.4. Each solid line represents a straight line fit to the linear segment of the calculated data points. These lines were extrapolated to the $x$-axis to obtain the values of $L_{\text{est}}$ presented in Table 2.5.

<table>
<thead>
<tr>
<th>$L_n$</th>
<th>$x_d=20\text{nm}$</th>
<th>$x_d=100\text{nm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20nm</td>
<td>0.0</td>
<td>2.0</td>
</tr>
<tr>
<td>50nm</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>100nm</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>300nm</td>
<td>1.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 2.3. Plots of theoretical $T_\lambda \Phi_0/J_{\text{total}}$ versus $\alpha^{-1}$ for a p-type GaN Schottky diode with a zero bias depletion width of approximately 20nm and 100nm. The $J_{\text{total}}$ values (open symbols) were calculated using Eqn. (2.15) for a set of assumed electron diffusion length values ($L_n = 20\text{nm}, 50\text{nm}, 100\text{nm}$ and $300\text{nm}$). Each solid line represents the fitting of a linear equation to the calculated data points. These lines were extrapolated to the $x$-axis to obtain estimates of $L_n$.  

21
Table 2.4. The parameter values used in the calculation of the $J_{total}$ versus $\alpha^{-1}$ plots shown in Figure 2.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength, $\lambda$ [nm]</td>
<td>301 - 364</td>
<td></td>
</tr>
<tr>
<td>Absorption coefficient, $\alpha$ [cm$^{-1}$]</td>
<td>$1.44 \times 10^5 - 7.87 \times 10^4$</td>
<td>[46]</td>
</tr>
<tr>
<td>Electron diffusion length, $L_n$ [nm]</td>
<td>20, 50, 100, 300</td>
<td></td>
</tr>
<tr>
<td>Depletion width, $x_d$ [nm]</td>
<td>20, 100</td>
<td></td>
</tr>
</tbody>
</table>

The results show that, as with the depletion width modulation method, the error in the values of $L_{est}$ obtained from the $x$-intercept of the $T_\lambda \Phi_0/J_{total}$ versus $\alpha^{-1}$ curve can be quite significant when the condition $\alpha x_d \ll 1$ is not satisfied and that this method is therefore also only applicable to heavily-doped GaN materials.

A variation of this wavelength-dependent measurement was applied by Stokes and Chu [38] to the measurement of $L_n$ in shallow junction n$^+$-p polycrystalline silicon solar cells. In that experiment, $\Phi_0$ was adjusted to maintain a constant $J_{total}$ and the photocurrent was assumed to be from the p-type region only. Then, by plotting $T_\lambda \Phi_0$ versus $\alpha^{-1}$ ($J_{total}$ is now grouped together with the other constant parameters on the right-hand side of the equation), $L_{est}$ is obtained from the $x$-intercept of the linear curve.

In cases where the value of $x_d$ is small and $L_n \gg x_d$, the drift current may be neglected and $J_{total}$ is given by the expression for the diffusion component only. Using this approximation for their n$^+$-p silicon solar cell samples, Arora et al. [37] rearranged Eqn.
(2.14) to read

\[
\alpha^{-1} = L_{\text{est}} \left( \frac{qT_e \Phi_0 e^{-\alpha x_d}}{J_{\text{total}}} - 1 \right)
\]  \hspace{1cm} (2.21)

and plotted \( \alpha^{-1} \) against the non-diffusion length term on the right hand side of the equation. The set of points plotted should be a straight line through the origin and \( L_{\text{est}} \) is given by the slope of the linear curve. The applicability of this approach to GaN depends on the relative magnitude of the diffusion length and \( x_d \). This approach may not be suitable for materials such as p-type GaN because the diffusion length can be as short as 21nm [8] and thus is comparable to \( x_d \). The idealised depletion width for p-type GaN with indium-tin-oxide (ITO) as the Schottky barrier material is calculated to be approximately 20nm (the work function of ITO is taken to be 4.3eV [45]).

In general, diffusion length values calculated directly using Eqn. (2.16) or those obtained by fitting the Eqn. (2.15) to the experimentally measured \( J_{\text{total}} \) as a function of \( \alpha \) are more accurate compared to the values determined from the \( x \)-intercept or slope of a linear curve because no additional approximation (such as \( \alpha x_d \ll 1 \) or that the photocurrent consists only of the diffusion current) is made to simplify the equation. The accuracy is therefore limited only by the reliability of the values of the other parameters in the equation and the validity of the various assumptions made in the derivation of Eqn. (2.15). The latter are explored further in Section 2.4.

A summary of the depletion width modulation and wavelength-dependent photocurrent measurement methods described here and in Section 2.3.1 is given in Table 2.6.

## 2.4 Model Assumptions and Limitations

The reliability of diffusion length values determined from photocurrent measurements using the standard mathematical model of the photocurrent

\[
J_{\text{total}} = qT_e \Phi_0 \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_{n,p}} \right)
\]  \hspace{1cm} (2.15)
<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Approximation(s)</th>
<th>Equation</th>
<th>Plot</th>
<th>Quantities required</th>
<th>Diffusion length is given by</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depletion width modulation</td>
<td>reverse-bias applied to expand the depletion width, ( x_d )</td>
<td>( \alpha x_d \ll 1 ) so that ( e^{-\alpha x_d} \equiv 1 - \alpha x_d )</td>
<td>( J_{\text{total}} = qT_k \phi_0 \left( \frac{\alpha}{1 + \alpha L_{\text{est}}} \right) (x_d + L_{\text{est}}) )</td>
<td>( J_{\text{total}} ) vs. ( x_d )</td>
<td>( J_{\text{total}}, x_d, A )</td>
<td>magnitude of the ( x )-intercept</td>
<td>[31]</td>
</tr>
<tr>
<td></td>
<td>vary the wavelength, ( \lambda ), and measure the photocurrent, ( I_{\text{total}} = J_{\text{total}} )</td>
<td>( L_n = \frac{1}{\alpha} \left( \frac{qT_k \phi_0 e^{-\alpha x_d}}{qT_k \phi_0 - J_{\text{total}}} - 1 \right) )</td>
<td>-</td>
<td>-</td>
<td>( J_{\text{total}}, T_k, \phi_0 )</td>
<td>( \alpha, x_d )</td>
<td>direct calculation</td>
</tr>
<tr>
<td></td>
<td>vary the photon flux density, ( \phi_0 ), to maintain constant photocurrent, ( I_{\text{total}} )</td>
<td>( \alpha x_d \ll 1 ) so that ( e^{-\alpha x_d} \equiv 1 - \alpha x_d )</td>
<td>( J_{\text{total}} = qT_k \phi_0 \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right) )</td>
<td>( J_{\text{total}}/T_k \phi_0 ) vs. ( \alpha )</td>
<td>( J_{\text{total}}, T_k, \phi_0 )</td>
<td>( \alpha )</td>
<td>fitting equation to measured data</td>
</tr>
<tr>
<td></td>
<td>( x_d ) small, ( L_n \gg x_d, J_{\text{total}} \approx J_{\text{diff}} )</td>
<td>( \alpha^{-1} = \frac{L_{\text{est}}}{q} \left( \frac{qT_k \phi_0 e^{-\alpha x_d}}{J_{\text{total}}} - 1 \right) )</td>
<td>( \alpha^{-1} ) vs. ( J_{\text{total}}/T_k \phi_0 )</td>
<td>( \alpha^{-1} ) vs. ( J_{\text{total}}, T_k, \phi_0 )</td>
<td>( \alpha^{-1} )</td>
<td>magnitude of the ( x )-intercept</td>
<td>[31]</td>
</tr>
<tr>
<td></td>
<td>vary the photon flux density, ( \phi_0 ), to maintain constant photocurrent, ( I_{\text{total}} )</td>
<td>( \alpha x_d \ll 1 ) so that ( e^{-\alpha x_d} \equiv 1 - \alpha x_d )</td>
<td>( T_k \phi_0 = \frac{J_{\text{total}}}{q (x_d + L_{\text{est}})} (\alpha^{-1} + L_{\text{est}}) )</td>
<td>( T_k \phi_0 ) vs. ( \alpha^{-1} )</td>
<td>( J_{\text{total}}, T_k, \phi_0 )</td>
<td>( \alpha )</td>
<td>slope of linear curve</td>
</tr>
<tr>
<td>Spectral response</td>
<td>vary the photon flux density, ( \phi_0 ), to maintain constant photocurrent, ( I_{\text{total}} )</td>
<td>( \alpha x_d \ll 1 ) so that ( e^{-\alpha x_d} \equiv 1 - \alpha x_d )</td>
<td>( T_k \phi_0 = \frac{J_{\text{total}}}{q (x_d + L_{\text{est}})} (\alpha^{-1} + L_{\text{est}}) )</td>
<td>( T_k \phi_0 ) vs. ( \alpha^{-1} )</td>
<td>( J_{\text{total}}, T_k, \phi_0 )</td>
<td>( \alpha )</td>
<td>magnitude of the ( x )-intercept</td>
</tr>
</tbody>
</table>

Table 2.6. Summary of various methods and procedures described in Section 2.3 that are used to extract diffusion length from photocurrent measurement of a Schottky barrier diode or shallow p-n junction diodes.
depends on the validity of this equation and the assumptions made in its derivation. If the assumptions can be justified for the sample under investigation, then the accuracy of the measurement of \( L_n \) is limited only by the reliability of the experimental setup and the parameter values used in the calculation.

With the exception of \( L_n \), each of the parameters in Eqn. (2.15), namely, the transmission factor of the Schottky contact, \( T_\lambda \), incident photon flux density, \( \Phi_0 \), material absorption coefficient, \( \alpha \), and depletion width, \( x_d \), can be measured separately.

As outlined in Chapter 1, the objective of this work is to ascertain the validity and accuracy of the standard mathematical model with respect to GaN by examining some of the assumptions made in the derivation of this model. The supplementary experimental methods used to measure these parameters are not addressed. The assumptions to be discussed below do not include any additional assumptions made in the derivation of the drift-diffusion model, which consists of the electron and hole current density and continuity equations as well as Poisson’s equation. This model is a simple formulation of the Boltzmann Transport Equation derived in the relaxation time approximation [47].

One of the assumptions made in the derivation presented in Section 2.2.1 is that low-level injection conditions prevail everywhere and at all times. This allows the potential throughout the semiconductor material to be approximated by the equilibrium potential and Poisson’s equation to be decoupled [48]. When this assumption is valid in a strongly extrinsic material, that is, \( n_{p0} \ll p_{p0} \) (p-type) or \( n_{n0} \gg p_{n0} \) (n-type), the ambipolar diffusion coefficient and mobility as well as excess carrier lifetime are reduced to those of the minority carrier. In effect, analysis of the carrier transport is limited to minority carriers only. In practice, the low-level injection condition is easily satisfied by controlling the intensity of the optical injection source.

Another assumption that facilitates the derivation of Eqn. (2.15) is material uniformity. The doping concentration, \( N_A \), thermal equilibrium densities, \( n_{p0} \) and \( p_{p0} \), mobilities, \( \mu_n \) and \( \mu_p \), minority carrier lifetime and diffusion length, \( \tau_n \) and \( L_n \), are assumed to be spatially invariant over the entire semiconductor bulk material of the Schottky diode. Hence, the electron diffusion coefficient, \( D_n = \mu_n kT/q \), is also a constant. The assumption that the material is uniform and that any spatial variation is limited to just
one dimension may not always reflect reality. In GaN, dislocations may not be uniformly distributed [49] and assuming that these defects are involved in recombination, the diffusion length value will vary spatially. In such cases, the diffusion length value determined using the standard model is an average value. A spatial map of the diffusion length can be obtained by using scanning-based methods that involve either a light-spot or electron-beam that moves away or towards a collecting junction.

The derivation presented in Section 2.2.1 assumes that the neutral region is much thicker than the diffusion length and absorption length. Therefore, Eqn. (2.15) is not suitable when the sample is a thin epitaxial layer. In such cases, the coefficient, \( C_2 \), in the expression for the electron density distribution function, that is, Eqn. (2.12), is not equal to zero. Moreover, reflection at the semiconductor-substrate interface comes into play and must be accounted for in the expression for the optical generation profile, \( G_{\text{op}}(x) \). Modifications to boundary conditions are necessary to accommodate thin epitaxial layers and the resulting expression for the photocurrent is more complicated than Eqn. (2.15). Additional discussions on measurements of epitaxial layers can be found in [2].

The boundary condition imposed at the depletion edge, that is, \( n(x_d) = n_{p0} \), expresses the assumption that the photo-generated electrons arriving at the depletion edge are swept away instantaneously by the built-in electric field, \( E(x) \). However, the electric field is zero at the depletion edge and the instantaneous evacuation of electrons from this edge is unlikely [27]. Therefore, the velocity at which electrons are swept away from the edge will be finite. The value of this velocity and the validity of the boundary condition as well as the consequence on the accuracy of the diffusion length values in GaN determined using the photocurrent technique is discussed in Chapter 3.

Along with \( N_A, \mu_n, \tau_n, D_n \) and \( L_n \), the absorption coefficient, \( \alpha \), is also assumed to be uniform throughout the material, including in the depletion region. Therefore, it should be a function of \( \lambda \) only. This assumption overlooks the fact that if the doping concentration is high enough, the magnitude of the electric field in the depletion region can be quite large and such a field can modify \( \alpha \), resulting in an electric field- and spectral-dependent oscillation of \( \alpha \) in the depletion region [50]. This electric field induced modification of \( \alpha \) is known as the Franz-Keldysh effect and its impact on the diffusion length values determined from measured spectral photocurrent using Eqn.
(2.15) will be discussed in Chapter 5. An additional assumption related to $\alpha$ is that the quantum yield, $\eta_\lambda$, is unity. This implies that an interband transition that leads to the production of an electron-hole pair occurs for every photon absorbed. At $\lambda$ near the band-gap, absorption processes other than direct band-to-band transitions may be present and this can make the unity quantum yield assumption invalid. Therefore, the range of $\lambda$ close to the band-edge is not used either in the analytical and numerical analyses or in the experimental measurements of the photocurrent of GaN Schottky diodes.

Another assumption made in the derivation of the standard model is that there is negligible or no recombination in the depletion region. This assumption limits the ability of the photocurrent technique to measure diffusion length very much shorter than the depletion width [2]. Recombination in the depletion region is discussed in Sections 3.4.3 and 6.1.

Finally, an assumption that is implicit in all the examples cited so far is that the photocurrent is composed entirely of the minority carrier current. Photo-generated majority carriers near the metal-semiconductor interface can diffuse towards the contact in opposition to the electric field and be emitted into the metal or recombine with minority carriers, resulting in a reduced total photocurrent [51]. This effect is expected to be more significant in direct band-gap materials [52] such as GaN because of the high absorption coefficient [51,52]. Analysis of the significance of this effect is presented in Section 6.2. A summary of the assumptions discussed in this section and the associated implications are shown in Table 2.7.

## 2.5 Chapter Summary

In this chapter, a review of the photocurrent technique was presented. Following a brief discussion of the basic principles of the technique and the derivation of the standard mathematical model that relates the photocurrent produced by an ideal Schottky barrier diode under constant monochromatic illumination to the diffusion length in Sections 2.1 and 2.2 respectively, the two principal ways of implementing this technique commonly found in the literature were applied to GaN to determine their accuracy.
<table>
<thead>
<tr>
<th>No.</th>
<th>Assumptions</th>
<th>Implications/Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>strongly extrinsic material and low-level injection</td>
<td>(i) $n_{p0} \ll p_{p0}$ (p-type) or $n_{n0} \gg p_{n0}$ (n-type); (ii) ambipolar diffusion coefficient and mobility are reduced to those of the minority carrier; (iii) electric field term in continuity equation is eliminated due to an essentially field-free, quasi-neutral region</td>
</tr>
<tr>
<td>2</td>
<td>uniform material</td>
<td>(i) $N_A, \mu_n, \tau_n, L_n$ and $D_n$ are spatially invariant; (ii) these parameters can be treated as constant coefficients in the continuity equation.</td>
</tr>
<tr>
<td>3</td>
<td>thick bulk material</td>
<td>(i) the standard mathematical model cannot be applied in its current form to measurements of epitaxial layers. (ii) modifications to the derivation to take into account the back interface will result in a more complicated equation.</td>
</tr>
<tr>
<td>4</td>
<td>minority carrier density at the depletion edge plane is equal to its thermal equilibrium value</td>
<td>(i) this assumption simplifies the resulting photocurrent equation and removes the need to know the velocity at which minority carrier are swept away from the depletion edge plane. (ii) this is considered in detail in Chapter 3.</td>
</tr>
<tr>
<td>5</td>
<td>$\alpha$ is uniform throughout the material, including the depletion region</td>
<td>(i) $\alpha$ is a function of wavelength only and the Franz-Keldysh effect is not considered. (ii) This is explored in Chapter 5.</td>
</tr>
<tr>
<td>6</td>
<td>one electron-hole pair created for every photon absorbed</td>
<td>(i) quantum yield, $\eta_\lambda = 1$ (ii) wavelengths close to the band-edge will not be used either in the analytical and numerical analyses or in the experimental measurements of the photocurrent of GaN Schottky diodes.</td>
</tr>
<tr>
<td>7</td>
<td>no recombination in the depletion region</td>
<td>(i) collection probability, $\varphi(x) = 1$; $0 \leq x \leq x_d$. (ii) this is discussed in Sections 3.4.3 and 6.1.</td>
</tr>
<tr>
<td>8</td>
<td>photocurrent is made up entirely of the minority carrier current only</td>
<td>(i) hole current, $J_p = 0$; (ii) $J_{total} = electron current, J_n$; (iii) this is discussed in Section 6.2.</td>
</tr>
</tbody>
</table>

Table 2.7. List of assumptions made in the derivation of the standard mathematical model and their implications.
The first method involves measurements of the photocurrent as a function of depletion width using a single wavelength while the second method, otherwise known as the spectral response method, relies on measurement of the photocurrent as a function of wavelength at zero bias, that is, at a constant depletion width. Common applications of both methods take advantage of additional simplification of the mathematical model when the condition $\alpha x_d \ll 1$ is satisfied. When this condition is met, knowledge of particular parameters in the model becomes unnecessary, thereby simplifying the measurement process and the extraction of the diffusion length from experimental data.

It was shown in Section 2.3 that, except when the material is heavily-doped, the diffusion length values of GaN determined using these methods were found to be unreliable because the condition $\alpha x_d \ll 1$ was not met. This was due to the high absorption coefficient of GaN. Therefore, it was concluded that in general, the simplifications commonly found in the literature are unsuitable for GaN and the standard mathematical model must be used without additional simplification. A summary of the various methods and procedures described in Section 2.3 that can be used to extract diffusion length from photocurrent measurement of a Schottky barrier diode or shallow p-n junction diodes were given in Table 2.6.

In the last section of this chapter, the various assumptions of the standard mathematical model and their implications on the applicability of this model were discussed and summarised in Table 2.7. The discussions presented in this section as well as the chapter as a whole provide the basis for Chapters 3 to 6 where more detailed investigations of some of the assumptions identified in Section 2.4 are presented.
CHAPTER 3

VALIDITY OF THE FIRST BOUNDARY CONDITION

In this chapter, the first of the two boundary conditions commonly imposed on the minority carrier continuity equation, outlined in Section 2.2.1, is considered in detail. This boundary condition is specified at the depletion edge plane and contains an implicit assumption about the velocity at which minority carriers are swept away from the depletion edge plane towards the Schottky contact. In this work, this velocity is denoted by $v_1$.

The second boundary condition, which is specified at the ohmic contact plane and expresses the assumption that the ohmic contact behaves like a perfect sink for minority carriers, is not considered in this work. It was shown in Section 2.2.1 that this assumption is mathematically irrelevant because the derivation of the standard mathematical model

$$J_{\text{total}} = q T_z \Phi_0 \left( 1 - \frac{e^{-\alpha v_1 z}}{1 + \alpha L_{n,p}} \right)$$ (2.15)

is based on a Schottky diode with a neutral region width that is much greater than the diffusion length and absorption length. For p-type GaN, the short diffusion and absorption lengths mean that this is true except when the active region is a very thin epitaxial layer.

The aim of this chapter is twofold. The first aim is to determine the value of $v_1$ using a combination of analytical and numerical means so that the validity of the assumption with regards to GaN can be investigated. The second aim is to use the results obtained to determine the effects of this assumption on the accuracy of the diffusion length values extracted using the standard mathematical model.

To these ends, the chapter begins with the presentation of a photocurrent equation that
contains the parameter $v_1$. In Section 3.2, two analytical models that describe the relationship between $v_1$ and material parameters are presented. Then, in Section 3.3, the results calculated using these models are used to determine the errors that arise from the assumption of the first boundary condition. This is followed by Section 3.4, in which numerical simulations are employed to overcome the limitations of both analytical models. A discussion of the results and their implications on the accuracy of the photocurrent technique follows a brief description of the simulation setup.

Finally, in Section 3.5, the method described in Section 2.3.2 is applied to the simulated photocurrent data. In this method, the ratio of the transmission-factor-corrected photon flux density to the photocurrent is plotted against the absorption length and a straight line fitted to the plotted data is extrapolated to the $x$-axis where the negative $x$-intercept gives the desired diffusion length value. The chapter finishes with a summary in Section 3.6.

### 3.1 The First Boundary Condition

The solution of the continuity equation requires two boundary conditions – one at the depletion edge, that is, the $x = x_d$ plane and the other at the ohmic contact plane, that is, $x = W$. In this work, the first boundary condition refers to the depletion edge and is typically given \[2,37,40,42-44\] by the following Dirichlet boundary condition

\[
n(x_d) = n_{p0}
\]

This condition represents an implicit assumption that photo-generated electrons arriving at the depletion edge are swept away instantaneously by the built-in electric field, thereby leaving no excess electrons at that plane. Therefore, the electron density at the depletion edge retains its equilibrium value, $n_{p0}$. However, because the electric field is zero at $x = x_d$, the instantaneous evacuation of electrons from this plane is unlikely \[27\]. A more conservative approach is to use a Neumann boundary condition that links the excess electron density at that plane to the velocity, $v_1$, at which they are swept away from the depletion edge. This boundary condition is similar in form to
Assuming that the thickness of the neutral region (see Figure 2.1, which is reproduced here for convenience) is well in excess of the diffusion length \((W - x_d \gg L_n)\) and the absorption length \((W - x_d \gg \alpha^{-1})\), then the solution of the continuity equation, which is the electron density distribution, \(n(x)\), is given by

\[
D_n \frac{dn(x)}{dx} \bigg|_{x = W} = -v_n n(W) - n_{p0} 
\]

and is given by

\[
D_n \frac{dn(x)}{dx} \bigg|_{x = x_d} = v_i [n(x_d) - n_{p0}] = v_i \Delta n(x_d)
\]

Figure 2.1. The schematic of the p-type Schottky diode with the band diagram superimposed showing the drift and diffusion components of the photocurrent. The parameters \(\Phi_0, \lambda, h, c\) and \(E_g\) are incident photon flux density, wavelength, Planck’s constant, speed of light, and material band-gap respectively. Here, the horizontal and vertical axes are the \(x\)- and \(y\)-axis respectively. The depth (not shown) is the \(z\)-axis.
\[ n(x) = C_0 e^{-\alpha(x-x_d)} - C_0 \left( \frac{S_1 + \alpha L_n}{S_1 + 1} \right) e^{-(x-x_d)/L_n} + n_{p0} \]  \hspace{1cm} (3.2)\\

where

\[ C_0 = \frac{T_s \Phi_0 e^{-\alpha x_n} \alpha L_n^2}{D_n(1 - \alpha^2 L_n^2)} \]  \hspace{1cm} (2.10)\\

and \( S_1 = v_1 L_n/D_n \). The unit of the ratio \( D_n/L_n \) is cm\textsuperscript{s}\textsuperscript{-1}. This quantity represents the average velocity at which electrons diffuse in the neutral region and it is often referred to as the diffusion velocity, \( v_d \), [2]. Thus, \( S_1 = v_1/v_d \) is a measure of the relative magnitude of \( v_1 \) to \( v_d \). By differentiating Eqn. (3.2) and inserting the resulting expression into

\[ J_{\text{diff}} = qD_n \frac{dn(x)}{dx} \bigg|_{x=x_d} \]  \hspace{1cm} (2.4)\\

the diffusion current, evaluated at the \( x = x_d \) plane, is given by

\[ J_{\text{diff}} = qT_s \Phi_0 e^{-\alpha x_d} \left( \frac{\alpha L_n}{1 + \alpha L_n} \right) \left( \frac{S_1}{1 + S_1} \right) \]  \hspace{1cm} (3.3)\\

The difference between Eqn. (2.14) and Eqn. (3.3) is the presence of the term involving \( S_1 \). In the limit where \( v_1 \) is much greater than \( v_d \), then \( S_1 \gg 1 \) and \( S_1/(S_1 + 1) \approx 1 \). Consequently, Eqn. (3.3) is reduced to Eqn. (2.14). Note that this only affects the diffusion component of the photocurrent and the expression for the drift current remains unchanged.

Using Eqn. (3.3), the total photocurrent is given by

\[ J_{\text{total}} = q T_s \Phi_0 \left\{ 1 - \left[ \frac{\alpha L_n + (S_1 + 1)}{(1 + \alpha L_n)(S_1 + 1)} \right] e^{-\alpha x_d} \right\} \]  \hspace{1cm} (3.4)
Substituting $S_1 = v_1L_n/D_n$ into Eqn. (3.4) and rearranging to get $L_n$ yields

$$L_{n\pm} = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

(3.5)

where

$$a = \frac{\alpha v_1}{D_n} \left( \frac{\gamma}{\gamma - 1} \right);$$

(3.6)

$$b = \alpha + \frac{v_1}{D_n}$$

(3.7)

$$\gamma = 1 - \frac{J_{\text{diff}}}{qT_\lambda \Phi_0 e^{-\alpha x_d}} \text{ or } \frac{qT_\lambda \Phi_0 - J_{\text{total}}}{qT_\lambda \Phi_0 e^{-\alpha x_d}}$$

(3.8)

The second term in the expression for $\gamma$ should always be less than unity because $|J_{\text{diff}}/q|$ cannot exceed the photon flux density at the $x = x_d$ plane, that is, $T_\lambda \Phi_0 e^{-\alpha x_d} > |J_{\text{diff}}/q|$. Therefore, the values of $\gamma$ must lie between zero and unity and the parameter $a$ must be less than zero. Consequently, $L_{n+}$ must also be negative because $(b^2 - 4a)^{1/2} > b$, which means that $L_n$ is given by $L_{n-}$:

$$L_n = L_{n-} = \frac{\left( \alpha + \frac{v_1}{D_n} \right) - \sqrt{\left( \alpha + \frac{v_1}{D_n} \right)^2 - 4 \left( \frac{\alpha v_1}{D_n} \right) \left( \frac{\gamma}{\gamma - 1} \right)}}{2 \left( \frac{\alpha v_1}{D_n} \right) \left( \frac{\gamma}{\gamma - 1} \right)}$$

(3.9)

In practice, the standard mathematical model

$$J_{\text{total}} = qT_\lambda \Phi_0 \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right)$$

(2.15)

or
\[ L_n = \frac{1}{\alpha} \left( \frac{qT_x \Phi_0 e^{-\alpha x_d}}{qT_x \Phi_0 - J_{\text{total}}} - 1 \right) \]  

(2.16)

is more convenient to apply because \( L_n \) is the only unknown. However, the assumption that \( n(x_d) = n_{p0} \) is not necessarily valid because the electric field is, by definition, zero at \( x = x_d \). Therefore, assuming \( S_1 \gg 1 \) may not always be appropriate and hence an error analysis is necessary. By substituting Eqn. (3.4) into Eqn. (2.16) and substituting \( L_{\text{est}} \) for \( L_n \) in Eqn. (2.16) to indicate that the diffusion length given by this equation is an estimate of the true value, \( L_n \), Eqn. (2.16) becomes

\[ L_{\text{est}} = \frac{S_1 L_n}{\alpha L_n + 1 + S_1} \]  

(3.10)

and the percentage error is given by

\[ \frac{L_n - L_{\text{est}}}{L_n} = \frac{\alpha L_n + 1}{\alpha L_n + 1 + S_1} \]  

(3.11)

Because the error determined by Eqn. (3.11) is positive for any combination of \( \alpha, L_n \) and \( S_1 \), using Eqn. (2.16) to determine diffusion length will underestimate \( L_n \). When \( S_1 \ll (\alpha L_n + 1) \), the error approaches but never exceeds 100%. Eqn. (3.11) suggests that for any values of \( L_n \) and \( S_1 \), it is possible to reduce the error by choosing an incident illumination wavelength that is close to the absorption edge so that the absorption coefficient is as low as possible.

Figure 3.1 shows the percentage error in calculated diffusion length as a function of \( \alpha L_n \). The curves shown were calculated using Eqn. (3.11) for \( S_1 \) values of 0.1, 1, 10, 100 and 1000. This figure shows that the error decreases as \( S_1 \) increases, that is, as \( v_1 \) becomes increasingly greater than \( v_d \). Clearly, the accuracy of Eqn. (2.16) depends on \( S_1 \) or in other words, the relative magnitude of \( v_1 \) and \( v_d \). Accordingly, it is beneficial to know the value of \( S_1 \). Since \( v_d \) can be calculated from \( D_n \) and \( L_n \), the task is to determine the values of \( v_1 \) as a function of \( L_n \). This will be achieved through both analytical and numerical means.

In Section 3.2, two analytical expressions describing the relationship between \( v_1 \) and
material parameters will be presented. In addition to the calculation of $v_1$ for an ideal p-type GaN Schottky diode, the same calculations for an n-type Si diode will also be included to aid the analysis of these equations. Next, the values of $v_1$ will be determined from the results of numerical simulations of a p-type GaN Schottky diode only, followed by comparisons with the values calculated from the second analytical equation. Even though numerical simulations may provide more accurate results due to the ability to dispense with some of the assumptions made in the derivation of $v_1$, an analytical expression of $v_1$ in the ideal case can provide insight into the material parameters that affect the value of $v_1$ and hence $S_1$. Following that, the effects of $v_1$ on the accuracy of diffusion length values calculated using Eqn. (2.16) are discussed in Section 3.2.3.

Figure 3.1. Percentage errors in the estimation of electron diffusion length using the assumption $S_1 \gg 1$ as a function of $\alpha L_n$ for various $S_1$ values.
3.2 Relationship between Material Parameters and $v_1$

The analytical expression for $v_1$ is obtained by using the relationship between $v_1$ and the excess electron density at the depletion edge, $\Delta n(x_d)$. This relationship is given by Eqn. (3.1). From Eqn. (3.2),

$$\Delta n(x_d) = C_0 - C_0 \left( \frac{S_1 + \alpha L_n}{S_1 + 1} \right)$$ \hspace{1cm} (3.12)

and

$$D_n \frac{dn(x)}{dx} \bigg|_{x = x_d} = -\frac{D_n C_0}{L_n} \left[ \alpha L_n - \left( \frac{S_1 + \alpha L_n}{S_1 + 1} \right) \right]$$ \hspace{1cm} (3.13)

Rearranging Eqn. (3.12) to obtain $(S_1 + \alpha L_n)/(S_1 + 1)$ and inserting the resulting expression into Eqn. (3.13) leads to

$$D_n \frac{dn(x)}{dx} \bigg|_{x = x_d} = -\frac{D_n C_0}{L_n} \left[ \alpha L_n - 1 + \frac{\Delta n(x_d)}{C_0} \right]$$ \hspace{1cm} (3.14)

Since Eqn. (3.14) is equal to Eqn. (3.1), $v_1$ can be expressed as

$$v_1 = \frac{D_n}{L_n} \left[ \frac{(1 - \alpha L_n) C_0}{\Delta n(x_d)} - 1 \right]$$ \hspace{1cm} (3.15)

In order to determine $v_1$, an alternative expression for $\Delta n(x_d)$ that does not contain $S_1$ is required. In the following, two separate models for $v_1$, derived using two different expressions of $\Delta n(x_d)$ are presented. The first model, which was derived by Card [53], is presented in Section 3.2.1. The second model, derived using the results of Lavagna et al. [52], is shown in Section 3.2.2. An analysis of the differences between these two models as well as the conditions under which the first model represents the asymptotic limit of the second model is discussed in Section 3.2.3.

---

$^1$ See Eqn. (11) in [53].
3.2.1 Model 1

Following the steps described in [53,54], the expression for the excess electron density, $\Delta n(x)$, in the depletion region of a zero-biased, ideal Schottky barrier diode under constant monochromatic illumination is derived using an alternative description of the electron current, $J_n$, which is given by

$$J_n = \mu_n n(x) \frac{dE_{Fn}(x)}{dx}$$  \hspace{1cm} (3.16)$$

where $E_{Fn}(x)$ is the electron quasi-Fermi level. In the formulation described in [53,54], the minority carrier current density was taken to be independent of position $x$ in the depletion region and was equated to the total short-circuit photocurrent density, $J_{total}$. In addition, the majority carrier quasi-Fermi level was assumed to be flat and coincident with the metal-Fermi level, which was taken to be the reference zero energy level. This implies that there is no majority carrier current because the gradient of the quasi-Fermi level is zero. By invoking the depletion approximation and further assuming that $\mu_n$ is independent of position $x$ as well as using

$$n(x) = N_C e^{-[E_C(x) - E_{Fn}(x)]/kT}$$ \hspace{1cm} (3.17)$$

and

$$E_C(x) = q\varphi_n - \frac{q^2 N_A (x^2 - 2x_d x)}{2e_s} ; \quad 0 \leq x \leq x_d$$ \hspace{1cm} (3.18)$$

the electron density distribution in the depletion region is given by

$$n(x) = N_C e^{-E_C(x)/kT} \left\{ 1 + \sqrt{\frac{\pi L_D J_n}{2 \mu_k kT n_0}} \times \left[ \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} \right) - \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} - \frac{x}{\sqrt{2}L_D} \right) \right] \right\}$$ \hspace{1cm} (3.19)$$

where $N_C$ and $E_C(x)$ are the conduction band density of states and energy as a function
of position respectively. The product $q\phi_n$ is the value of the conduction band energy at $x = 0$ and is given by the difference between the band-gap energy, $E_g$, and the Schottky barrier height, $q\phi_{bp}$, while the error function, $erf(\theta)$, is given by

$$erf(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta e^{-t^2} dt$$

(3.20)

The parameter $L_D$ represents the extrinsic Debye length and is given by

$$L_D = \sqrt{\frac{\epsilon_s kT}{N_A q^2}}$$

(3.21)

where $\epsilon_s$ is the semiconductor permittivity. A more detailed derivation of $n(x)$ in the depletion region using the steps described in [53,54] can be found in Appendix A.

By using $n_{p0} = N_C e^{-E_C(x_d)/kT}$ in Eqn. (3.19), the excess electron density distribution in the depletion region can be simplified to

$$\Delta n(x_d) = \sqrt{\frac{\pi}{2} \frac{L_D J_n}{\mu_n kT} e^{\frac{1}{\sqrt{2L_D}}}}$$

(3.22)

The assumption that $J_n$ is constant in the depletion region and equal to $J_{total}$ is equivalent to assuming that either recombination is negligible and there is no generation in the depletion region or that generation is balanced by recombination so that the current in this region is zero [55]. This assumption implies that the main contribution to $J_n$ comes from the neutral region [55] and $J_n$ or $J_{total}$ predominantly results from $J_{diff}$ only. Inserting the expression for $J_{diff}$, Eqn. (3.3), into Eqn. (3.22) and substituting the resulting expression into Eqn. (3.15) yields the equation for $v_1$

$$v_1 = D_n \sqrt{\frac{2 L_n}{\pi L_D} e^{\frac{1}{\sqrt{2L_D}}}} $$

(3.23)

Using $v_d = D_n/L_n$ and substituting the expression for $L_D$, into the above equation leads
to

\[ v_1 = \frac{2N_A kT}{\pi \varepsilon_s} \mu_n \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) \]  

(3.24)

If \( x_d/L_D > 4.5 \), or equivalently, when the built-in voltage, \( V_{bi} \), is greater than \( 10.15kT/q \), the error function term is approximately unity and can be eliminated. The resulting expression is then equivalent to the equation derived by Card [53], which for convenience, is reproduced here for a p-type material:

\[ v_1 = \frac{2N_A kT}{\pi \varepsilon_s} \mu_n \]  

(3.25)

\( v_1 \) is a function of the material permittivity, \( \varepsilon_s \), doping concentration, \( N_A \), and electron mobility, \( \mu_n \), at a given temperature, \( T \). The same expression applies for an n-type material, with the subscripts ‘A’ and ‘n’ replaced by ‘D’ and ‘p’ respectively. From Eqn. (3.23), \( S_1 \) is then given by

\[ S_1 = \sqrt{\frac{2}{\pi} \frac{L_n}{L_D}} \]  

(3.26)

This equation shows that the assumption \( S_1 \gg 1 \) is valid only when the diffusion length is much greater than the extrinsic Debye length. For p-type GaN with an acceptor density of \( 10^{19} \text{cm}^{-3} \), \( L_D \) is approximately 1nm, which means that \( S_1 \gg 1 \) is a good assumption when \( L_n \) is approximately 15nm or longer (here, the condition much greater than one is taken to be satisfied when \( v_1 \) is an order of magnitude greater than \( v_d \)).

Note that the results based on the formulation of [53,54] indicate that \( v_1 \) and, hence, \( S_1 \), do not depend on the absorption coefficient of the material. In materials with direct and wide band-gaps such as GaN, the absorption coefficient, \( \alpha \), is high. Consequently, in the case of a front-illuminated Schottky diode, generation in the depletion region is not negligible and depending on the diffusion length, which can be quite short in GaN [8,17], current generated in this region may be comparable to that originating from the neutral region. The relationship between \( v_1 \) and material parameters obtained on the
basis of the model set out in [53,54] is applicable to materials like GaN only when the
diffusion length is sufficiently long such that the diffusion current dominates the total
current.

In the next section, a second model, which does not assume that the diffusion current
dominates the total photocurrent and is therefore more general than Eqn. (3.23), is
presented.

### 3.2.2 Model 2

An alternative derivation of $v_1$ is based on the solution of the continuity equation in the
depletion region of an ideal diode, which was obtained by assuming unity collection
efficiency in the depletion region and by neglecting any perturbation in the electrostatic
potential due to image force effects [52]. Additional details of the derivation can be
found in Appendix B.

Using the results of Lavagna et al. [52], the excess electron density at the depletion edge
is given by

$$
\Delta n(x_d) = \frac{T_d \Phi_0 \lambda_n}{D_n} \left[ \frac{F_3 (1 + \alpha L_n) - F_3 e^{-\alpha x_d}}{(1 + \alpha L_n) (L_n + F_3)} \right]
$$

(3.27)

where the constants $F_3$ and $F_4$ are given by

$$
F_3 = \sqrt{\frac{\pi}{2}} L_D \text{erf} \left( \frac{x_d}{\sqrt{2} L_D} \right)
$$

(3.28)

and

$$
F_4 = \sqrt{\frac{\pi}{2}} L_D e^{-\alpha x_d - \alpha^2 L_D^2/2} \left[ \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) + \text{erf} \left( \frac{x_d}{\sqrt{2} L_D} - \frac{\alpha L_D}{\sqrt{2}} \right) \right]
$$

(3.29)
respectively. Inserting Eqn. (3.27) into Eqn. (3.15) leads to

\[
v_1 = v_d \left[ \frac{\alpha L_n^2 e^{-\alpha x_d} - (1 + \alpha L_n) \left(F_4 - F_3 e^{-\alpha x_d}\right)}{\alpha L_n F_4 + \left(F_4 - F_3 e^{-\alpha x_d}\right)} \right]
\]  

(3.30)

The expression for \(S_1\) is given by the terms in the brackets in Eqn. (3.30). The same expression applies for an n-type material, with the subscripts ‘n’ replaced by ‘p’. Eqn. (3.30) shows that, in addition to temperature, \(T\), material permittivity, \(\varepsilon_s\), doping concentration, \(N_A\), and mobility, \(\mu_n\), \(v_1\) is also a function of the depletion width, \(x_d\), absorption coefficient, \(\alpha\), and electron diffusion length, \(L_n\).

In the following section, the analysis of the differences between these two equations as well as the conditions under which Model 1 represents the asymptotic limit of Model 2 are discussed in detail.

### 3.2.3 Comparison of Models 1 and 2

The primary difference between the two different approaches described above is the assumption of a constant \(J_n\) in the depletion region, which was not made in the derivation of Eqn. (3.30). As stated earlier, assuming that \(J_n\) is constant in the depletion region implies that \(J_{\text{total}} = J_{\text{diff}}\). Therefore, Model 1 can be considered as a special case of Model 2.

To compare the values of \(v_1\) calculated using both models as well as to illustrate the conditions under which Model 2 can be well-approximated by Model 1, values of \(v_1\) were calculated for n-type Si and p-type GaN Schottky diodes using both models and a set of assumed diffusion length values. The wavelengths chosen for the calculations correspond to energies that are above the band-gap energy of each material. The parameter values used in the calculations are tabulated in Table 3.1 and plots of the calculated values of \(v_1\) versus \(L_{n,p}\) are presented in Figure 3.2 (see pg. 44).

As shown in Figure 3.2 the values of \(v_1\) calculated using Model 1 for the parameter

---

\(^2\) These two expressions are denoted by \(F_3\) and \(F_4\) because the notations \(F_3\) and \(F_2\) were used to denote two similar constants derived for the majority carrier (see Eqns. (B30) and (B31) in Appendix B).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value(s) [source]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature, $T$ [K]</td>
<td>300</td>
</tr>
<tr>
<td>Relative Permittivity, $\varepsilon_r$</td>
<td>11.9 [56]</td>
</tr>
<tr>
<td>Band-gap, $E_g$ [eV]</td>
<td>1.12</td>
</tr>
<tr>
<td>Doping concentration [cm$^{-3}$]</td>
<td>$N_D = 5 \times 10^{14}, 5 \times 10^{15}, 10^{17}$</td>
</tr>
<tr>
<td>Barrier height, $\phi_{bn,p}$ [eV]</td>
<td>0.8 [56]</td>
</tr>
<tr>
<td>Depletion width, $x_d$ [nm]</td>
<td>1155, 386, 92</td>
</tr>
<tr>
<td>Wavelength, $\lambda$ [nm]</td>
<td>900, 1040</td>
</tr>
<tr>
<td>Absorption coefficient, $\alpha$ [cm$^{-1}$]</td>
<td>306, 22.6 [58]</td>
</tr>
<tr>
<td>Extrinsic Debye length, $L_D$ [nm]</td>
<td>184, 58.3, 13.1</td>
</tr>
<tr>
<td>Normalised diffusion length [nm]</td>
<td>$L_n/\lambda &gt; 0.1$</td>
</tr>
</tbody>
</table>

Table 3.1. The parameter values used in the calculation of $v_1$ for p-type GaN and n-type Si Schottky diodes using Eqns. (3.23) and (3.30).

Values listed in Table 3.1 are constant for a given material permittivity and doping concentration. These values do not vary with $L_{n,p}$ or $\alpha$. This is expected because the equation for Model 1

$$v_1 = v_d \left[ \frac{2 L_n}{\sqrt{\pi L_D} \, \text{erf} \left( x_d / \sqrt{2L_D} \right) } \right], \quad (3.23)$$

does not contain these two parameters. On the other hand, for both materials, the values of $v_1$ calculated using the equation for Model 2

$$v_1 = v_d \left[ \frac{\alpha L_n^2 e^{-\alpha x_d} - (1 + \alpha L_n) \left( F_4 - F_3 e^{-\alpha x_d} \right) }{\alpha L_n F_4 + \left( F_4 - F_3 e^{-\alpha x_d} \right) } \right], \quad (3.30)$$

show mixed characteristics. In the case of Si with $N_D = 5 \times 10^{14} \text{cm}^{-3}$ and $5 \times 10^{15} \text{cm}^{-3}$
Figure 3.2. Comparison of the values of $v_1$ calculated using the parameter values tabulated in Table 3.1 and Eqs. (3.23) and (3.30) for (a) n-type Si and (b) p-type GaN Schottky diodes. The values calculated are plotted against the diffusion length, $L_{n,p}$. $x_d$ is the idealised depletion width, which is defined as the depletion width calculated on the basis of the depletion approximation. The wavelengths, $\lambda$, chosen for the calculations have corresponding energies that are above the band-gap of each material. The approximate point at which the $v_1$ curves plateaued are indicated by the arrows.
(see Figure 3.2(a)), the value of $v_1$ calculated at $\lambda = 900\text{nm}$ increases with $L_p$ and reaches a plateau that is almost equal to the value calculated using Model 1. This occurs when $L_p$ is approximately 1.5 times greater than the idealised depletion width, $x_d$, which is calculated on the basis of the depletion approximation. The differences between the values produced by both models when the value of $L_p$ is large are not discernible in Figure 3.2(a) because the differences are very minute. For GaN (see Figure 3.2(b)), similar characteristics can be observed for $\lambda = 301\text{nm}$ and $355\text{nm}$, except that $v_1$ plateaued only when $L_n > 10x_d$ and the differences between the values produced by both models at high $L_n$ values are more apparent. For both materials, the value calculated using Model 1 represents the asymptotic limit of the value calculated using Model 2 when $L_{n,p} \to \infty$.

For $v_1$ calculated for Si with $N_D = 5 \times 10^{14}\text{cm}^{-3}$ and $5 \times 10^{15}\text{cm}^{-3}$, both models produce the same value only for $\lambda = 1040\text{nm}$ but for $N_D = 10^{17}\text{cm}^{-3}$, both models produce the same value at both wavelengths. In the case of GaN, the values of $v_1$ calculated using Model 2 for $\lambda = 355\text{nm}$, which is near the band-edge wavelength, is slightly greater than the values for $\lambda = 301\text{nm}$. All the results shown in Figure 3.2 can be explained by the relative proportion of the drift and diffusion current, which in turn are the results of the relative magnitudes of $L_{n,p}$, $\alpha$, $x_d$ and $L_D$. Note that the last two parameters are functions of $N_{D,A}$.

In the case of Si, for every combination of the $N_D$ ($x_d$ and $L_D$) and $\alpha$ values listed in Table 3.1, $x_d/L_D \gg \alpha L_D$. Consequently, the second $erf$ term in the expression for $F_4$ can be approximated by

$$erf\left(\frac{x_d}{\sqrt{2L_D}} - \frac{\alpha L_D}{\sqrt{2}}\right) \equiv erf\left(\frac{x_d}{\sqrt{2L_D}}\right) \tag{3.31}$$

Furthermore, because $\alpha^2L_D^2/2$ is much less than unity, this means

$$e^{\alpha^2L_D^2/2} \equiv 1 \tag{3.32}$$

By applying these approximations to Eqn. (3.29), the expression for $F_4$ can be approximated by
\[ F_4 = \left( \frac{\pi}{2} L_D e^{-\alpha_D} \right) \left[ \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) + \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} \right) \right] \]  

(3.33)

Using the expression for \( F_3 \), which is given by Eqn. (3.28), this equation can also be expressed as

\[ F_4 = \frac{\pi}{2} L_D e^{-\alpha_D} \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) + F_3 e^{-\alpha_D} \]  

(3.34)

Therefore, inserting Eqn. (3.34) into the equation for Model 2 leads to \(^3\)

\[ v_1 = v_d \frac{\alpha L_p^2 - (1 + \alpha L_p) \sqrt{\pi L_D} \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right)}{\alpha L_p F_3 + (1 + \alpha L_p) \sqrt{\pi L_D} \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right)} \]  

(3.35)

For \( N_D = 5 \times 10^{14} \text{cm}^{-3} \) and \( 5 \times 10^{15} \text{cm}^{-3} \) and \( \lambda = 1040 \text{nm} \), the values of \( v_1 \) calculated using Model 2 are almost equal to the values calculated using Model 1 regardless of the value of \( L_p \) because at \( \lambda = 1040 \text{nm} \), the value of \( \alpha \) is very low such that \( \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) \approx 0 \). Therefore, \( v_1 \) can be approximated by

\[ v_1 = v_d \left( \frac{L_p}{F_3} \right) \]  

(3.36)

Inserting the expression for \( F_3 \) into Eqn. (3.36) leads to the exact equation for Model 1 \(^3\)

\[ v_1 = v_d \left[ \frac{\sqrt{2} L_p}{\sqrt{\pi L_D} \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right)} \right] \]  

(3.23)

Since \( x_d/L_D > 4.5 \) for every value of \( N_D \) listed in Table 3.1, \( \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) = 1 \), thereby allowing the equation for Model 1 to be further reduced to the expression

\(^3\) Note the change in the subscript of \( L \) from ‘n’ to ‘p’. The equations presented in the preceding sections were derived for a p-type material. However, the present analysis pertains to n-type Si.
derived by Card in [53], that is, Eqn. (3.25), with the appropriate subscripts for n-type material.

Note that the value of \( v_1 \) calculated using Eqn. (3.35), and by extension, the exact equation for Model 2, that is, Eqn. (3.30), will always be slightly less than the value calculated using either the exact equation for Model 1 or Eqn. (3.25) because \( \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) \) is very close but not equal to zero. This is why the value produced by Model 1 represents the asymptotic limit of the value produced Eqn. (3.35) or Eqn. (3.30). In the case of Si, the differences between the values calculated for \( N_D = 5 \times 10^{14} \text{cm}^{-3} \) and \( 5 \times 10^{15} \text{cm}^{-3} \) at \( \lambda = 1040\text{nm} \) using Models 1 and 2 exist but are not discernible from a visual inspection of Figure 3.2(a) due to the extremely minute differences.

When the diffusion length is short and \( \lambda = 900\text{nm} \), the value of \( \alpha \) is sufficiently high such that \( \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) > 0 \) and Eqn. (3.35) cannot be simplified further. However, as the diffusion length increases beyond \( 1.5x_d \) to the point where \( \alpha L_p \gg 1 \) and \( L_p \gg D \), Eqn. (3.35) can then be approximated by

\[
L_p \gg \sqrt{\frac{\pi L_D}{2}} \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right),
\]

Eqn. (3.35) can then be approximated by

\[
v_1 = v_d \left[ \sqrt{\frac{2 L_p}{\pi L_D}} \frac{1}{\text{erf} \left( x_d / \sqrt{2L_D} \right) + \text{erf} \left( \alpha L_D / \sqrt{2} \right)} \right]
\]

(3.38)

Since \( \text{erf} \left( x_d / \sqrt{2L_D} \right) = 1 \) and \( 0 < \text{erf} \left( \alpha L_D / \sqrt{2} \right) \ll 1 \), the sum of both error functions is slightly above unity. This explains why the value calculated using Eqn. (3.38) or Eqn. (3.30) approaches but will always be slightly less than the value calculated using Model 1 when the diffusion length is long and \( \lambda = 900\text{nm} \).

In the case of \( N_D = 10^{17} \text{cm}^{-3} \), the values of \( v_1 \) calculated using Model 2 are almost equal to those calculated using Model 1 regardless of wavelength or diffusion length because the value of \( L_D \) is very low such that \( \text{erf} \left( \alpha L_D / \sqrt{2} \right) \equiv 0 \) is true for both \( \lambda = 900\text{nm} \).
and \( \lambda = 1040\text{nm} \). Consequently, the equation for Model 2 can be well approximated by the equation for Model 1.

In terms of the relative proportion of the drift and diffusion currents, when \( x_d \) is narrow, \( \alpha \) is low or \( L_p > 1.5x_d \), the main contribution to the total current comes from the \( x \geq x_d \) region and the assumption implicit in the formulation described in [53,54], that is, the drift current is negligible and the total current is equal to the diffusion current, is valid. As \( x_d \) widens as a result of a decreasing \( N_D \), \( \alpha \) increases or \( L_p \) becomes shorter, the proportion of the drift current relative to the diffusion current increases and the aforementioned assumption is no longer valid, resulting in the observed differences between Eqns. (3.23) and (3.30). The results for GaN can also be explained qualitatively in the same manner.

The more apparent differences between the values of \( v_1 \) calculated using Eqns. (3.23) and (3.30) for GaN (when \( L_n > 10x_d \)) compared to those for Si can be explained by the higher absorption coefficient values of GaN. Unlike for Si, the value of the product \( \alpha L_D \) is sufficiently high such that \( e^{\alpha^2L^2_D/2} > 1 \). Therefore, \( F_4 \) cannot be expressed in the form of Eqn. (3.33) and the original equation for \( F_4 \), Eqn. (3.29), must be used as it stands. For \( L_n > 10x_d \), the equation for Model 2 can be approximated by

\[
v_1 = v_d \left\{ \frac{2}{\pi} \frac{L_n}{L_D} \left[ \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) + \text{erf} \left( \frac{\alpha L_D}{\sqrt{2}} \right) \right] e^{\alpha^2L^2_D/2} \right\}
\]

(3.39)

This equation is similar to Eqn. (3.38), except for the exponential term in the denominator. The presence of this term results in the more obvious difference between the values calculated using Models 1 and 2 when the diffusion length is long.

The calculations presented in Figure 3.2 also show that as the doping concentration increases, the value of \( v_1 \) for a given diffusion length value increases. This is because the values of the depletion width and extrinsic Debye length are inversely proportional to the square root of the doping concentration. As the values of these two quantities are reduced, the values of \( F_3 \) and \( F_4 \) as well as \( F_4 - F_3 e^{-\alpha x_d} \) approach zero. Therefore, the term enclosed in the brackets in Eqn. (3.30) increases, resulting in a higher \( v_1 \) value.
Note that the value of $v_1$ is directly proportional to $v_d$ and is therefore also a function of the minority carrier mobility of the material. Thus, in principle, the functional dependence of mobility on the doping concentration of a particular material can have an effect on the behaviour of $v_1$ with respect to the doping concentration. Consequently, the characteristics of $v_1$ shown in Figure 3.2 for both GaN and Si are not necessarily unique.

However, as mentioned earlier, the accuracy of the standard mathematical model depends on the relative magnitude of $v_1$ and $v_d$ rather than the absolute value of either quantity. The ratio of $v_1$ to $v_d$, which was denoted by $S_1$, is given by the terms enclosed by the brackets in Eqn. (3.30). An inspection of the expression of $S_1$ shows that this parameter is not a function of mobility. Therefore, an analysis of the relationship between carrier mobility and the doping concentration as well as its effect on the behavior of $v_1$ are not necessary and is excluded. The values of $S_1$ and its effect on the accuracy of the diffusion length value determined using the standard mathematical model are presented and discussed in the following section.

### 3.3 Values of $S_1$ and the Error Due to the $S_1 \gg 1$ Assumption

Plots of $S_1$ versus $L_{n,p}$ for both n-Si and p-GaN are presented in Figure 3.3(a). The percentage errors in the diffusion length values calculated using the assumption $S_1 \gg 1$ are presented in Figure 3.3(b). These values were calculated using the values of $S_1$ shown in Figure 3.3(a) in Eqn. (3.11).

Figure 3.3(a) shows that the magnitude of $S_1$ increases with diffusion length for both materials. This is expected because $v_1$ increases while $v_d$ decreases with diffusion length. The results plotted in Figure 3.3(b) show that the percentage error resulting from the use of the standard mathematical model decreases monotonically with diffusion length. For n-Si, the error ranged from as low as 0.4% when $N_D = 10^{17}$ cm$^{-3}$ and $L_p = 4000$nm to almost 100% when $N_D = 5 \times 10^{14}$ cm$^{-3}$ and $L_p = 200$nm. For p-GaN, the error ranged from just over 1% when $N_A = 10^{19}$cm$^{-3}$ and $L_n = 1000$nm to nearly 50% when $N_A = 10^{18}$cm$^{-3}$ and $L_n = 10$nm. In addition, for a given diffusion length, the percentage error increases with decreasing doping concentration.

With the exception of n-Si at diffusion length values that are less than 1.5 times greater
Figure 3.3. (a) Values of $S_1$ calculated using the parameters values for n-Si and p-GaN, which are tabulated in Table 3.1. The equation used is given by the terms in the square brackets in Eqn. (3.30). (b) Percentage errors for diffusion length calculated using the assumption $S_1 \gg 1$. The percentage errors were calculated using the $S_1$ values plotted in Figure 3.3(a) in Eqn. (3.11). The approximate points at which the percentage error for n-Si exhibits weaker dependence on wavelengths are indicated by the arrows.
than the depletion width value, the percentage errors at a given doping concentration for both materials are only weakly dependent on the optical wavelengths. Therefore, the choice of optical wavelength does not have a significant impact on the accuracy of the calculated diffusion length value.

Even though the exact characteristics of the percentage error curves as a function of diffusion length and doping concentration for p-Si and n-GaN may be different from the ones shown in Figure 3.3(b) for n-Si and p-GaN respectively, the same trend and hence conclusions can be expected to hold for either p-Si or n-GaN.

Obviously, the verdict on the accuracy of the photocurrent technique would depend on what constitutes unacceptable error. If the threshold for acceptable error is defined at 50%, then in general, as far as the examples presented here are concerned, the standard mathematical model is fairly accurate. In principle, even when the error is unacceptably high, the necessary corrections can be made to estimate the true diffusion length value by using the equations presented in the preceding sections to create a chart that plots the true diffusion length value versus the value calculated using the standard mathematical model. However, this can be done only if the analytical equations can sufficiently model the Schottky diode.

In addition to the assumption of no recombination in the depletion region, the foregoing analysis assumes a clear delineation between the regions that give rise to the drift and diffusion components of the total current. Furthermore, in the case of p-GaN, the p-type dopant is assumed to be completely ionised. These assumptions afford simplicity that leads to mathematical tractability. However, in practice, the border that separates the depletion and neutral regions is not clearly defined. Furthermore, magnesium (Mg), which is the usual dopant species used for p-type doping of GaN, is not fully ionised. Typically, only about one percent of the Mg atoms are ionised [8].

To take these into consideration, further analysis of $v_1$ and hence $S_1$ was undertaken using the results obtained from numerical simulations of an ideal, p-type GaN Schottky barrier diode under monochromatic illumination. These are presented in the following sections. Further analysis for n-Si will not be undertaken because the objective of this chapter is to investigate the applicability of the photocurrent technique to the measurement of diffusion length in GaN only, particularly p-GaN.
3.4 Values of $v_1$ determined from Numerical Simulations

In order to determine the values of $v_1$ for p-GaN, the theoretical expression of the electron density distribution, $n(x)$, was fitted to those obtained from numerical simulations of the structure shown in Figure 2.1. In the following sub-sections, a brief description of the simulation setup is presented, followed by discussion of the results obtained. To allow comparisons with the results calculated using the analytical equations presented in the preceding sections, the same device structure and material parameters as those listed in Table 3.1 were used in the simulations.

3.4.1 Details of Numerical Simulation

To obtain the steady-state, one-dimensional spatial distribution of the electron density, $n(x)$, in a p-GaN Schottky barrier diode under optical excitation, 2D simulations of the diode structure as shown in Figure 2.1 were undertaken using Synopsys® Sentaurus TCAD. This commercial software is a multi-dimensional device simulator for electrical, thermal and optical characterisation of semiconductor devices.

The values of some of the important material, device and modelling parameters used in the simulations are listed in Table 3.2. For each doping concentration, a series of simulations were conducted for the range of optical wavelength and diffusion length values listed in Table 3.2. In each simulation, the temperature was set to 300K and the device was unbiased. In addition, the resistance of the contacts was taken to be zero.

The simulations of the electrical characteristics of the device were performed on the basis of the drift-diffusion model, which numerically solves the system of five partial differential equations, namely, the current density and continuity equations for both electrons and holes as well as Poisson’s equation. To ensure that the expected spatial variations in charge and carrier density around the $x = x_d$ plane, which was estimated using the depletion approximation, can be resolved, the mesh size in the vicinity of this plane was set to the Debye length. The same mesh size was also applied in the region directly adjacent to the Schottky contact.

The data produced by the simulations were for a two-dimensional structure. However, since the illumination is invariant along the $y$-axis and the minority carrier lifetime is set
<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Value(s) [source]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Relative Permittivity, $\varepsilon_r$</td>
<td>9.5 [57]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Electron Affinity [eV]</td>
<td>4.1 [59]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Band-gap, $E_g$ [eV]</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dopant</td>
<td>Mg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Doping concentration, $N_A$ [cm$^{-3}$]</td>
<td>$10^{18}$, $10^{19}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equilibrium hole concentration, $p_0$ [cm$^{-3}$]</td>
<td>$9 \times 10^{16}$, $3 \times 10^{17}$</td>
<td>$p_0 &lt; N_A$ due to incomplete ionisation of Mg, which has an activation energy of 170meV [5]</td>
</tr>
<tr>
<td></td>
<td>Absorption coefficient, $\alpha$ [cm$^{-1}$]</td>
<td>$1.44 \times 10^5$, $7.87 \times 10^4$ [46]</td>
<td>values correspond to $\lambda = 301$nm and 355nm respectively.</td>
</tr>
<tr>
<td>Device</td>
<td>Dimensions of p-GaN bulk material, widthxthickness [µm]</td>
<td>5x100</td>
<td>width of neutral region, which is given by $100\mu$m - $x_d$, is much greater than both the diffusion and absorption lengths so that the normalised recombination velocity at the ohmic contact plane becomes irrelevant (see Section 2.2.1).</td>
</tr>
<tr>
<td></td>
<td>Barrier height, $\phi_B$ [eV]</td>
<td>3.2 [45]</td>
<td>ITO as Schottky barrier material with 4.3eV work function.</td>
</tr>
<tr>
<td></td>
<td>Contact Resistance [Ω]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>Temperature, $T$ [K]</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Optical excitation wavelength, $\lambda$ [nm]</td>
<td>301, 355</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Electron diffusion length, $L_n$ [nm]</td>
<td>10 - 1000</td>
<td>set by specifying a spatially uniform minority carrier lifetime profile, which is calculated using $L_n^2 = D_n \tau_n$.</td>
</tr>
</tbody>
</table>

Table 3.2. A list of some of the important GaN material, device and modelling parameters used in the simulations.
to be uniform over the entire device, any spatial variation of the output quantities is limited to the \( x \)-axis only. Thus, the desired one-dimensional electron density distribution is obtained by taking a cross-section of the 2D data along the \( x \)-axis.

Additional details about the simulation setup can be found in Appendix C. In the next section, the results of the simulations, that is, the values of \( v_1 \) and the method by which they are extracted from the simulation data, are presented and discussed. Then, the implication of these results for the accuracy of the photocurrent technique is presented in Section 3.4.3.

### 3.4.2 Simulation Results and Discussion

Figure 3.4 shows an example of the least-squares fitting of

\[
    n(x) = C_0 e^{-\alpha(x-x_d)} - C_0 \left( \frac{S_1 + \alpha L_n}{S_1 + 1} \right) e^{-(x-x_d)/L_n} + n_{p0}
\]  

(3.2)

to the electron density distribution obtained from the simulation of a p-type GaN Schottky diode with a doping concentration of \( 10^{19} \) cm\(^{-3} \) and diffusion length of 100nm. The depletion width was calculated to be approximately 20nm. This is the idealised depletion width calculated on the basis of the depletion approximation and it will, henceforth, be denoted by \( x_{da} \). The data and the fit shown in this figure are representative of those obtained for other values of doping concentration, diffusion length and wavelength.

From Figure 3.4, it can be seen that the excess electron density at \( x = x_{da} \) is not zero as is usually assumed and Eqn. (3.2) does not fit well to the simulation data. To understand why this is so and to improve the fit, it is important to consider that \( x_{da} \) was calculated according to the depletion approximation but, in practice, the electric field is not actually zero at \( x = x_{da} \), as shown in Figure 3.5(a) (see pg. 56). In fact, it decays to zero gradually within a transition region of finite width, beyond \( x = x_{da} \) (see Figure 3.5(b)). This gradual decay can be regarded as complete in approximately 4-6 extrinsic Debye lengths, \( L_D \) [60,61]. This is supported by Figure 3.5(b), which shows that the transition from almost full depletion to positions of quasi-neutrality takes place over a region of
Figure 3.4. Eqn. (3.2) fitted to the simulation generated electron density distribution (solid circle) to extract $v_1^{\text{sim}}$, using the idealised depletion width calculated on the basis of the depletion approximation. $v_1^{\text{sim}}$ represents the value of $v_1$ obtained from simulation data as opposed to $v_1^{\text{eqn}}$, which denotes the value of $v_1$ calculated from Eqn. (3.30). The inset shows the section enclosed by the dashed rectangle, plotted on a semi-log scale to highlight the non-zero electron density at $x = x_{\text{da}}$.

about 4-6$L_D$ in width, at which point the hole density becomes approximately equal to the bulk equilibrium hole density. In addition, the magnitude of the electric field decreased by 3 orders of magnitude to just under 100Vcm$^{-1}$ within this region.

Before proceeding further, a clarification of notations is required. The extension of $x_{\text{da}}$ by the width of this transition region results in an effective depletion width which is, henceforth, denoted by $x_{\text{de}}$. For the sake of generality, the notation $x_d$, which appeared numerous times in the preceding sections, is used to represent any depletion width value, be it $x_{\text{da}}$ or $x_{\text{de}}$. Additionally, the values of $v_1$ determined using Eqn. (3.30) and from simulation data will be denoted by $v_1^{\text{eqn}}$ and $v_1^{\text{sim}}$ respectively. Likewise, $S_1^{\text{eqn}}$ and $S_1^{\text{sim}}$ denote the values of $S_1$ calculated using $v_1^{\text{eqn}}$ and $v_1^{\text{sim}}$ respectively. These
Figure 3.5. (a). The solid line represents a straight line fit to the linear section of the simulated electric field data (solid squares) for a p-type GaN Schottky diode with $N_A = 10^{19}$ cm$^{-3}$. The inset shows an enlarged view of the region around the zero-field intercept on the x-axis. (b) The simulated electric field and hole density data are super-imposed on the same graph to highlight the extend of the transition region.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depletion Width</td>
<td>$x_d$</td>
</tr>
<tr>
<td></td>
<td>$x_{da}$</td>
</tr>
<tr>
<td>Velocity at which electrons are swept away from the depletion edge towards the Schottky contact</td>
<td>$v_1$</td>
</tr>
<tr>
<td></td>
<td>$v_1^{\text{sim}}$</td>
</tr>
<tr>
<td>Ratio of $v_1$ and diffusion velocity, $v_d$</td>
<td>$S_1$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3. Summary of the definitions of the notations introduced in this section.

In the analysis presented in Sections 2.2 and 3.1, $n(x)$ was found by solving the electron continuity equation for the neutral region only, where there is only a very small electric field. Therefore, instead of $x_d = x_{da}$, using $x_d = x_{de}$ should yield a better fit because the electric field should be closer to zero as $x$ increases. The fitting procedure implemented to extract $v_1^{\text{sim}}$ from the data produced by the simulations involves an iterative process where the depletion edge, with a starting value of $x_{da}$, is extended in 0.1$L_D$ increments until the value of the coefficient of determination of the fit, $R^2$, reaches or exceeds 0.99. The best fits for all the diffusion length values simulated were obtained using values of between 3-6$L_D$. Choo et al. [41], who also considered the transition region in an unrelated study of the steady-state surface photovoltage method, used a different criterion to determine the width of this region. By imposing the condition that the effective depletion edge be the position where the diffusion current is at least two orders of magnitude greater than the drift current, they determined the width of the transition region to be approximately 10$L_D$. In fitting Eqn. (3.2) to the simulation data, extending $x_{da}$ by 10$L_D$ was found to be excessive. In fact, the fits were very poor for $L_n < 75$nm.

As shown in Figure 3.5(b), the position of almost complete neutrality is attained when $x \approx 6$L_D.
Figure 3.6. Eqn. (3.2) fitted to the simulation data shown previously in Figure 3.4 using the effective depletion width $x_{de} = x_{da} + 6L_D$. $x_{da}$ is the depletion width calculated using the depletion approximation and $L_D$ is the extrinsic Debye length. The width of the transition region is bounded by the two dotted vertical lines.

Figure 3.6 shows the fit for the same data as Figure 3.4, using $x_{de} = x_{da} + 6L_D$. The effective extrinsic Debye length, $L_D$, was calculated to be approximately 6.8nm using the equilibrium hole density instead of the nominal doping concentration (since the acceptors are not fully ionized). This fit is clearly an improvement over the fit shown in Figure 3.4. From this fit, $v_1^{\text{sim}}$ was determined to be approximately $3.3 \times 10^5 \text{ cm s}^{-1}$, which is on the order of the diffusion velocity, $v_d$.

Figure 3.7(a) shows the values of $v_1^{\text{sim}}$ extracted from the simulation data for $N_A = 10^{18} \text{ cm}^{-3}$ and $10^{19} \text{ cm}^{-3}$. Also shown for comparisons are the values of $v_d$ and $v_1^{\text{eqn}}$. The latter were presented earlier in Figure 3.2(b).
Figure 3.7. (a) Plots of $v_1$ as a function of $L_n$ for $\lambda = 301\text{nm}$ and $\lambda = 355\text{nm}$. The top four curves represent the values of $v_1^{\text{eqn}}$, which were calculated using Eqn. (3.30) while the curves with open symbols represent the diffusion velocity, $v_d$. $v_1^{\text{sim}}$, which were extracted from the simulation data are represented by the remaining curves. Parts of the $v_1^{\text{sim}}$ curves ($L_n \geq 100\text{nm}$), which are obscured due to the log scale, are shown in the inset on a linear scale. (b) Plots of $S_1 = v_1/v_d$ as a function of $L_n$. These values were calculated using the $v_1$ and $v_d$ values shown in Figure 3.7(a).
The results for $N_A = 10^{18}\text{ cm}^{-3}$ show that the differences between $v_i^{eqn}$ and $v_i^{sim}$ can be quite substantial, ranging from just over an order of magnitude at $L_n = 1000\text{nm}$ to slightly over seven orders of magnitude at $L_n = 10\text{nm}$. For $N_A = 10^{19}\text{ cm}^{-3}$, the differences between $v_i^{eqn}$ and $v_i^{sim}$, which range from just over one order of magnitude at $L_n = 1000\text{nm}$ to slightly over two orders of magnitude at $L_n = 10\text{nm}$, are not as significant by comparison. The differences between $S_i^{eqn}$ and $S_i^{sim}$ mirror that between $v_i^{eqn}$ and $v_i^{sim}$, as shown in Figure 3.7(b). This is because the same values of $v_d$ are used to calculate both $S_i^{eqn}$ and $S_i^{sim}$ at each $L_n$ value for a given $N_A$ value.

The differences in magnitude between the values obtained from calculations and simulations can be attributed to the position of the depletion edge. The effects of this position on the value of $v_i^{sim}$ can be seen by examining Eqn. (3.15) as well as plots of the electron density distributions obtained from calculation and simulation (see Figure 3.8). Eqn. (3.15), which is reproduced here in a slightly different form, is obtained following the substitution of the expression for $C_0$, which is given by Eqn. (2.10).

$$v_i = v_d \left[ \frac{T_d \Phi_0 \alpha L_n}{v_d (1 + \alpha L_n) \Delta n(x_d)} e^{-\alpha x_d} - 1 \right]$$ (3.40)

In Figure 3.8, the curve with solid circles represents the electron density distribution, $n(x)$, produced by the simulation of a p-type GaN Schottky diode with $N_A = 10^{19}\text{ cm}^{-3}$ and $L_n = 100\text{nm}$. This curve has been presented earlier in Figure 3.4 and Figure 3.6. The curve with open circles is the electron density distribution calculated using the same material parameters, $x_d = x_{da}$ and two separate equations – one for the depletion region and another for the neutral region. The equation for the latter is given by Eqn. (3.2) and is valid for $x \geq x_{da}$. The value of $S_1$ used is given by the terms in the brackets Eqn. (3.30). In the depletion region, $n(x)$ was calculated using the expression obtained from the results of Lavagna et al. [52], which is given by Eqn. (B7) in Appendix B.

Since the material parameters used to obtain both curves are the same, the obvious discrepancies between these two curves can be attributed to the differences between the analytical and numerical treatments of the Schottky junction. The idealisation incorporated in the former, which was necessary to maintain mathematical tractability,
Figure 3.8. The electron density distributions obtained from simulation and calculation. The curve with solid circles represents \( n(x) \) produced by the simulation of a p-type GaN Schottky diode with \( N_A = 10^{19}\text{cm}^{-3} \) and \( L_n = 100\text{nm} \). The curve with open circles is \( n(x) \) calculated using the same material parameters, \( x_d = x_{da} \) and Eqns. (3.2) and (B7).

includes the assumptions embodied by the depletion approximation that, in turn, enabled the explicit calculation of a clearly defined depletion edge. The latter allows for the consideration of the incomplete ionisation of the acceptor and the numerical solution of the electrostatics within the device gives rise to the aforementioned transition region.

As indicated in Figure 3.8, at their respective depletion edge positions, the electron density values of both curves are different, with \( n(x_{da}) < n(x_{de}) \). In addition, the shifting of the position of the depletion edge in the positive \( x \) direction results in \( e^{-\alpha x_{da}} > e^{-\alpha x_{de}} \). Hence, for a given combination of \( T, \Phi_0, \alpha, N_A \) and \( L_n \),

\[
\frac{e^{-\alpha x_{da}}}{\Delta n(x_{da})} > \frac{e^{-\alpha x_{de}}}{\Delta n(x_{de})} \quad (3.41)
\]
and consequently, $v_1^{\text{eqn}} > v_1^{\text{sim}}$. The differences between $v_1^{\text{eqn}}$ and $v_1^{\text{sim}}$ are accentuated by the incomplete ionisation of Mg in GaN, which leads to a longer extrinsic Debye length that gives rise to a wider transition region.

Note that the similarities in shape between the curves representing the results obtained from calculation and simulation for both doping densities are misleading because of the different type of scale used in Figure 3.7(a). As shown in Figure 3.7(b), the shape of the $S_1$ curves are different at low diffusion length values ($L_n < 100\text{nm}$), especially for $N_A = 10^{18}\text{cm}^{-3}$. The break in the y-axis of Figure 3.7(a) and the use of a logarithmic and linear scale in the bottom and top halves of this axis respectively were necessary to allow all data points to be presented in one figure. Therefore, another point of difference between the calculated and simulated results is the strong dependence of $v_1^{\text{sim}}$ on $L_n$ when $L_n < 100\text{nm}$ versus the weaker dependence of $v_1^{\text{eqn}}$ on $L_n$. The strong dependence of both $S_1^{\text{eqn}}$ and $S_1^{\text{sim}}$ on $L_n$, even when $L_n > 100\text{nm}$ can be attributed to the values of $v_d$, which decreased by two orders of magnitude between $L_n$ values of 10nm and 1000nm.

As shown in Figure 3.7(a), except when $N_A = 10^{18}\text{cm}^{-3}$ and $L_n < 100\text{nm}$, the values of $v_1^{\text{sim}}$ obtained for both $\lambda = 301\text{nm}$ and $\lambda = 355\text{nm}$ are comparable, with the values of the latter being slightly greater over the entire $L_n$ range simulated. These results imply that for the most part, $v_1^{\text{sim}}$ is, like $v_1^{\text{eqn}}$, only weakly dependent on the optical wavelength. Not surprisingly, this characteristic is also mirrored in plots of $S_1$ versus $L_n$, as shown in Figure 3.7(b). In addition, as with the calculated results, the values of $v_1^{\text{sim}}$ and $S_1^{\text{sim}}$ obtained for $N_A = 10^{19}\text{cm}^{-3}$ are greater than the values found for $N_A = 10^{18}\text{cm}^{-3}$.

Having obtained the values of $v_1^{\text{sim}}$ and hence $S_1^{\text{sim}}$, the next step is to determine and compare the errors in the diffusion length values calculated from synthetic photocurrent data produced by the simulations. These diffusion length values are calculated using Eqns. (2.16) and (3.9). In addition to these, the errors calculated using Eqn. (3.11) are also presented for comparison. In the following section, the implications of the results presented in this section for the accuracy and suitability of the photocurrent technique when applied to GaN is discussed.
3.4.3 Implications for the Accuracy of the Photocurrent Technique when Applied to GaN

The simulation-derived results presented in Figure 3.7(b) show that for almost the entire range of the simulated $L_n$ values, the condition $S_1^{\text{sim}} \gg 1$ is not satisfied.\(^4\) The minimum values of $S_1^{\text{sim}}$ for $N_A = 10^{18} \text{cm}^{-3}$ and $10^{19} \text{cm}^{-3}$ are on the order of $10^{-7}$-$10^{-8}$ and $10^{-1}$-$10^{-2}$ respectively. Maximum values of around 10-50 are obtained only at high $L_n$ values. Using these results, the error in the estimation of $L_n$ as a result of the $S_1 \gg 1$ assumption is found to be very significant. As shown in Figure 3.9, errors of nearly 100% are predicted for both $N_A = 10^{18} \text{cm}^{-3}$ and $10^{19} \text{cm}^{-3}$ when $L_n = 10$nm. From there, the error drops to just below 50% for $N_A = 10^{19} \text{cm}^{-3}$ at $L_n = 1000$nm while the errors for $N_A = 10^{18} \text{cm}^{-3}$ at the same $L_n$ value remains above 50%.

\(^4\) The condition much greater than unity is taken to be satisfied when $S_1 \geq 10$. 

---

![Figure 3.9](image-url) 

**Figure 3.9.** Plots of the percentage errors in the calculation of $L_n$ due to the assumption that $S_1 \gg 1$. Each data point was calculated using Eqn. (3.11). Eqn. (3.30) refers to the analytical equation used to calculate $v_1^{\text{sim}}$. The values of $S_1$ used in the calculations were plotted in Figure 3.7(b). The top four curves represent the errors calculated using $S_1^{\text{sim}}$ values obtained from the simulations while the bottom four curves represent the error calculated using $S_1^{\text{sim}}$ values.
The significant difference between the percentage errors calculated using $S_1^{\text{sim}}$ and $S_1^{\text{eqn}}$ reflects the substantial differences in the magnitudes of these two parameters. Overall, the error at longer optical wavelengths is slightly lower, which is expected according to Eqn. (3.11)

$$\frac{L_n - L_{\text{est}}}{L_n} = \frac{\alpha L_n + 1}{\alpha L_n + 1 + S_1}$$

(3.11)

because the absorption coefficient is lower. The percentage error curves for wavelengths between 301nm and 355nm should lie between these two curves since the absorption coefficient decreases monotonically with wavelength.

Based on the simulation results shown in Figure 3.9 alone, at this point, one may easily conclude that the diffusion length values derived from the photocurrent method are quite inaccurate and therefore, this technique should not be applied to determine minority carrier diffusion length in GaN without making the necessary corrections. However, it is premature to make this conclusion because the analysis is incomplete. As is shown in the following, further analysis of these results that take into consideration recombination in the depletion and transition regions yields a different conclusion.

### 3.4.3.1 Error Calculations using Synthetic Photocurrent Data Produced by Simulations

Strictly speaking, the use of $S_1^{\text{sim}}$ in Eqn. (3.11) to calculate the percentage error is incorrect because $S_1^{\text{sim}}$ was determined from simulation results whereas Eqn. (3.11) was derived using an analytical model, which, unlike the numerical simulation, is not capable of accounting for the transition region. Instead, the error analysis should be extended to the simulation results by using the synthetic photocurrent density value, $J_{\text{total}}^{\text{sim}}$, which can be obtained from the same simulation that produced the electron density distribution data.

A set of percentage error curves similar to the ones shown in Figure 3.9 can be calculated using $1 - L_{\text{est}}/L_n$, where $L_n$ is the diffusion length value specified in each simulation run and is therefore considered to be the known, true value. $L_{\text{est}}$ is the
diffusion length estimated from the total current density produced by the simulation, $J_{\text{total}}^{\text{sim}}$. It is calculated using\(^5\)

$$L_{\text{est}} = \left( \alpha + \frac{v_1^{\text{sim}}}{D_n} \right) - \sqrt{ \left( \alpha + \frac{v_1^{\text{sim}}}{D_n} \right)^2 - 4 \left( \frac{\alpha v_1^{\text{sim}}}{D_n} \right) \left( \frac{\gamma}{\gamma - 1} \right) }$$  \hspace{1cm} (3.9)

where\(^5\)

$$\gamma = \frac{q T_k \Phi_0 - J_{\text{total}}^{\text{sim}}}{q T_k \Phi_0 e^{-\alpha x_d}}$$  \hspace{1cm} (3.8)

Note that $v_1^{\text{sim}}$ is defined at $x = x_{\text{de}}$, which means that the depletion edge, insofar as one can be defined, is effectively located at $x = x_{\text{de}}$. Therefore, in Eqn. (3.8), $x_d = x_{\text{de}}$ rather than $x_{\text{da}}$. The results of this calculation are presented and labelled (ii) in Figure 3.10.

In addition to these, another set of percentage error curves can also be calculated. By using\(^5\)

$$L_{\text{est}} = \frac{1}{\alpha} \left( \frac{q T_2 \Phi_0 e^{-\alpha x_d}}{q T_k \Phi_0 - J_{\text{total}}^{\text{sim}}} - 1 \right)$$  \hspace{1cm} (2.16)

and $x_d = x_{\text{da}}$, the results obtained are arguably more relevant because it represents how $L_{\text{est}}$ will be calculated from experimental data, that is, calculating $L_{\text{est}}$ using the standard mathematical model and without consideration of $v_1$ or the transition region. In Figure 3.10, the results of these calculations are labelled (iii). Also shown for comparison with (ii) and (iii) are the percentage error curves calculated using $S_1^{\text{eqn}}$ and Eqn. (3.11). These curves are labelled (i). A summary of the differences between these three sets of curves are presented in Table 3.4 (see pg. 67).

---

\(^5\) $v_1$ and $J_{\text{total}}$ are replaced by $v_1^{\text{sim}}$ and $J_{\text{total}}^{\text{sim}}$, respectively.
Figure 3.10. Comparison of percentage errors in the diffusion length values calculated using Eqns. (3.9) and (2.16) as well as Eqn. (3.11) using the values of $v_i^{\text{eq}}$. The depletion widths used in the calculations of these curves are also indicated. Percentage error is defined as $(L_n - L_{\text{est}})/L_n$, where $L_{\text{est}}$ is the value calculated using equations and $L_n$ is the diffusion length value specified in each simulation run, which means that it is also the true value. (a) Results for $N_A = 10^{18}\text{ cm}^{-3}$. (b) Results for $N_A = 10^{19}\text{ cm}^{-3}$. 

<table>
<thead>
<tr>
<th>Equation</th>
<th>$\lambda = 301\text{ nm}$</th>
<th>$\lambda = 355\text{ nm}$</th>
<th>$\lambda = 301\text{ nm}$</th>
<th>$\lambda = 355\text{ nm}$</th>
<th>$\lambda = 301\text{ nm}$</th>
<th>$\lambda = 355\text{ nm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eqn. (3.11)</td>
<td>$\Delta$</td>
<td>$\triangle$</td>
<td>$\Delta$</td>
<td>$\triangle$</td>
<td>$\Delta$</td>
<td>$\triangle$</td>
</tr>
<tr>
<td>Eqn. (3.9)</td>
<td>$\triangle$</td>
<td>$\square$</td>
<td>$\triangle$</td>
<td>$\square$</td>
<td>$\triangle$</td>
<td>$\square$</td>
</tr>
<tr>
<td>Eqn. (2.16)</td>
<td>$\square$</td>
<td>$\square$</td>
<td>$\square$</td>
<td>$\square$</td>
<td>$\square$</td>
<td>$\square$</td>
</tr>
</tbody>
</table>
Table 3.4. Summary of the differences between the three sets of curves shown in Figure 3.10. Percentage error is defined as $(L_n - L_{est})/L_n$. In Eqn. (3.9), $L_n$ and $v_1$ have been replaced by $L_{est}$ and $v_1\text{sim}$ respectively. In Eqn. (2.16), $L_n$ and $J_{total}$ have been replaced by $L_{est}$ and $J_{\text{sim total}}$ respectively. The latter denotes the total current density produced by the simulation.

The results presented in Figure 3.10 show that for a given doping concentration, the degree to which the calculated diffusion length, $L_{est}$, deviates from the true value, $L_n$, depends not only on $L_n$ itself but also on the equation and the depletion width value used in the calculation, and to a lesser extent, the wavelengths at which these calculations are made. Overall, $L_{est}$ calculated using Eqns. (3.11) and (3.9) underestimates $L_n$. In contrast, $L_{est}$ calculated using Eqn. (2.16) generally overestimates $L_n$. These general trends are, however, not universal. As shown in Figure 3.10(a), exceptions to these characteristics occur in the case of $L_{est}$ calculated using Eqn. (2.16) for $N_A = 10^{18} \text{cm}^{-3}$. The results show that $L_{est}$ underestimates $L_n$ initially and as the value of $L_n$ increases above approximately 40nm, $L_{est}$ overestimates $L_n$. In the case of $\lambda = 301\text{nm}$, beyond $L_n = 40\text{nm}$, the accuracy of $L_{est}$ deteriorates with increasing $L_n$ values before stabilizing when $L_n$ exceeds 150nm. For $\lambda = 355\text{nm}$, the accuracy improves beyond $L_n = 150\text{nm}$.

Except for the results obtained using Eqn. (3.11), the described characteristics of the results presented in Figure 3.10 can be explained by the rate at which $L_n - L_{est}$ increases with respect to $L_n$ and the presence of recombination in the effective depletion region, which has thus far been excluded from consideration.
Recall that in deriving the standard mathematical model, it was assumed that recombination does not occur in the depletion region so that a simple analytical equation can be obtained. However, numerical simulation is not constrained by this requirement and therefore, no such assumption was made. Even though this assumption may be reasonable for the greater part of the region $0 \leq x \leq x_{da}$, it is not valid in the transition region, $x_{da} \leq x \leq x_{de}$, in which the current is made up of both drift and diffusion currents. As shown in Figure 3.11, there is considerably more recombination in the transition region than in the depletion region calculated according to the depletion approximation. Note that in deriving Eqn. (3.9), only the expression of the diffusion component of the total current was modified to include the parameter $S_1$ and the expression for the drift current, which is given by

$$J_{\text{drift}} = qT \Phi_0 (1 - e^{-\alpha x_d})$$,  

remained unchanged. Consequently, Eqn. (3.9) inherited the assumption of no recombination in the depletion region. Because $\nu_1^\text{sim}$ and hence $S_1^\text{sim}$ were defined at $x = x_{de}$ instead of $x_{da}$, this assumption was implicitly extended to the transition region.

To facilitate discussions of Figure 3.10, the following notations will be used. The total current density produced by the simulation is denoted by $J_{\text{total}}^\text{sim}$, as before. The actual drift and diffusion components of $J_{\text{total}}^\text{sim}$ are denoted by $J_{\text{drift}}^\text{sim}$ and $J_{\text{diff}}^\text{sim}$ respectively. Their calculated counterparts are denoted by $J_{\text{drift}}^\text{eqn}$ and $J_{\text{diff}}^\text{eqn}$ respectively. Such differentiation is necessary because $J_{\text{drift}}^\text{eqn}$, which is calculated using Eqn. (2.3) and $x_d = x_{da}$, may differ from $J_{\text{drift}}^\text{sim}$. Note that while the relative proportions of $J_{\text{drift}}^\text{sim}$ and $J_{\text{diff}}^\text{sim}$ may differ from that of $J_{\text{drift}}^\text{eqn}$ and $J_{\text{diff}}^\text{eqn}$ respectively, the sum of the actual components must equal that of the calculated components. In other words, the following relation must hold

$$J_{\text{total}}^\text{sim} = J_{\text{drift}}^\text{sim} + J_{\text{diff}}^\text{sim} = J_{\text{drift}}^\text{eqn} + J_{\text{diff}}^\text{eqn}$$  

Therefore, $J_{\text{diff}}^\text{eqn}$ can be expressed as

$$J_{\text{diff}}^\text{eqn} = \Delta J_{\text{drift}} + J_{\text{diff}}^\text{sim}$$  

(3.43)
Figure 3.11. Plots of selected recombination rates as a function of position. These are the recombination rate curves that correspond to the electron density distribution data used in the least-squares fitting to obtain $v_1^{\text{sim}}$, an example of which was shown in Figure 3.6. (a) Recombination rates for $N_A = 10^{18}\text{cm}^{-3}$ and $L_n = 10\text{nm}, 100\text{nm}$ and $1000\text{nm}$ at $\lambda = 301\text{nm}$ and $\lambda = 355\text{nm}$. (b) Recombination rates for $N_A = 10^{19}\text{cm}^{-3}$ and $L_n = 10\text{nm}, 100\text{nm}$ and $1000\text{nm}$ at $\lambda = 301\text{nm}$ and $\lambda = 355\text{nm}$. 
where \( \Delta J_{\text{drift}} = J_{\text{drift}}^{\text{eqn}} - J_{\text{drift}}^{\text{sim}} \). A summary of these notations and their definitions is presented in Table 3.5.

A direct consequence of extending the depletion region to include a zero recombination transition region is that \( J_{\text{drift}}^{\text{eqn}} > J_{\text{drift}}^{\text{sim}} \). Therefore, \( J_{\text{diff}}^{\text{eqn}} < J_{\text{diff}}^{\text{sim}} \) and the diffusion length value calculated using Eqn. (3.9) must underestimate \( L_n \), as shown in Figure 3.10. This effect is more pronounced as \( L_n \) decreases because the increased recombination in the depletion region exacerbates the discrepancy between \( J_{\text{drift}}^{\text{eqn}} \) and \( J_{\text{drift}}^{\text{sim}} \). Note that Eqn. (3.9) cannot be evaluated below a certain \( L_n \) value. For \( N_A = 10^{18} \text{cm}^{-3} \), the lower limit is 200nm while for \( N_A = 10^{19} \text{cm}^{-3} \) the lower limit is 50nm. Below these limits, there is sufficient recombination in the depletion region such that the calculated drift current, \( J_{\text{drift}}^{\text{eqn}} \), becomes greater than the total current density, \( J_{\text{total}}^{\text{sim}} \). Hence, \( J_{\text{diff}}^{\text{eqn}} \) is negative and consequently the term in the square root is also negative.

The error curves calculated using Eqn. (2.16) and \( x_d = x_{da} \) for \( N_A = 10^{18} \text{cm}^{-3} \), which were presented earlier in Figure 3.10(a), shows non-monotonic characteristics. Even though the depletion region does not include the transition region and \( J_{\text{drift}}^{\text{eqn}} \) should be less than \( J_{\text{drift}}^{\text{sim}} \), there is sufficient recombination in the depletion region when \( L_n \) is less than approximately 30-40nm such that \( J_{\text{drift}}^{\text{eqn}} > J_{\text{drift}}^{\text{sim}} \). This means that \( \Delta J_{\text{drift}} \) is negative and \( J_{\text{diff}}^{\text{eqn}} < J_{\text{drift}}^{\text{sim}} \). Thus, \( L_{\text{est}} \) underestimates \( L_n \). As \( L_n \) increases towards 30-40nm, \( L_{\text{est}} \) calculated using Eqn. (2.16) becomes more accurate because the reduced recombination

<table>
<thead>
<tr>
<th>Notation</th>
<th>Source</th>
<th>Definition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_{\text{total}}^{\text{sim}} )</td>
<td>Simulation</td>
<td>\text{Total} photocurrent density produced by the simulation</td>
<td>-</td>
</tr>
<tr>
<td>( J_{\text{drift}}^{\text{sim}} )</td>
<td></td>
<td>\text{Actual} drift component of ( J_{\text{total}}^{\text{sim}} )</td>
<td>-</td>
</tr>
<tr>
<td>( J_{\text{diff}}^{\text{sim}} )</td>
<td></td>
<td>\text{Actual} diffusion component of ( J_{\text{total}}^{\text{sim}} )</td>
<td>-</td>
</tr>
<tr>
<td>( J_{\text{drift}}^{\text{eqn}} )</td>
<td>Calculation</td>
<td>\text{Calculated drift} component of ( J_{\text{total}}^{\text{sim}} ) (Eqn. (2.3)).</td>
<td>( qT_s \Phi_0 (1 - e^{-\alpha x}) )</td>
</tr>
<tr>
<td>( J_{\text{diff}}^{\text{eqn}} )</td>
<td></td>
<td>\text{Calculated diffusion} component of ( J_{\text{total}}^{\text{sim}} )</td>
<td>( \Delta J_{\text{drift}} + J_{\text{drift}}^{\text{sim}} )</td>
</tr>
<tr>
<td>( \Delta J_{\text{drift}} )</td>
<td></td>
<td>\text{Difference} between ( J_{\text{drift}}^{\text{sim}} ) and ( J_{\text{drift}}^{\text{eqn}} )</td>
<td>( J_{\text{drift}}^{\text{sim}} - J_{\text{drift}}^{\text{eqn}} )</td>
</tr>
</tbody>
</table>

Table 3.5. Summary of the notations used in explaining the results shown in Figure 3.10.
in the depletion region results in an increasing \( J_{\text{drift}}^{\text{sim}} \) that gradually approaches \( J_{\text{drift}}^{\text{eqn}} \). When \( J_{\text{drift}}^{\text{eqn}} = J_{\text{drift}}^{\text{sim}} \), \( L_{\text{est}} \) becomes equal to \( L_n \) and the percentage error is zero. As \( L_n \) increases further, recombination in the depletion region continues to decrease as shown in Figure 3.11, which means that \( J_{\text{drift}}^{\text{eqn}} < J_{\text{drift}}^{\text{sim}} \) and \( \Delta J_{\text{drift}} \) is positive. Therefore, \( J_{\text{diff}}^{\text{eqn}} > J_{\text{diff}}^{\text{sim}} \) and \( L_{\text{est}} \) switches from underestimating to overestimating \( L_n \). Beyond approximately 40nm, the non-monotonicity of the error curves is simply due to \( L_n \) and \( |L_n - L_{\text{est}}| \) increasing at different rates rather than because \( |L_n - L_{\text{est}}| \) is decreasing with \( L_n \). In all cases, \( |L_n - L_{\text{est}}| \) increases non-linearly with \( L_n \).

In the case of \( N_A = 10^{19} \text{cm}^{-3} \), using Eqn. (2.16) and a depletion width value of \( x_{da} \) to calculate \( L_{\text{est}} \) also leads to an overestimation of \( L_n \) as expected and the same qualitative explanation can be applied. Underestimation of \( L_n \) is not observed only because the minimum \( L_n \) simulated is not short enough and depletion width, \( x_{da} \), is sufficiently narrow such that even at \( L_n = 10\text{nm} \), recombination in this region is not significant enough to result in \( J_{\text{drift}}^{\text{eqn}} > J_{\text{drift}}^{\text{sim}} \) and \( J_{\text{diff}}^{\text{eqn}} < J_{\text{diff}}^{\text{sim}} \).

The percentage error curves calculated using Eqn. (3.11), which were presented earlier in Figure 3.10(b), show that \( L_{\text{est}} \) always underestimates \( L_n \). This result has nothing to do with recombination in the depletion region or the transition region because these are not considered in the analytical model at all. Instead, this result simply reflects the fact that \( L_{\text{est}} \) must be less than \( L_n \) so that

\[
\left( \frac{\alpha L_n}{1 + \alpha L_n} \right) \left( \frac{S_{1}^{\text{eqn}}}{1 + S_{1}^{\text{eqn}}} \right) = \left( \frac{\alpha L_{\text{est}}}{1 + \alpha L_{\text{est}}} \right)
\]

This relation is obtained when the diffusion current calculated using Eqn. (3.3) is equated to the diffusion current calculated using Eqn. (2.14), that is,

\[
\frac{qT \Phi_0 e^{-\alpha x_{da}} \left( \frac{\alpha L_n}{1 + \alpha L_n} \right) \left( \frac{S_{1}^{\text{eqn}}}{1 + S_{1}^{\text{eqn}}} \right)}{\text{Eqn. 3.3}} = \frac{qT \Phi_0 e^{-\alpha x_{da}} \left( \frac{\alpha L_{\text{est}}}{1 + \alpha L_{\text{est}}} \right)}{\text{Eqn. 2.14}}
\]

Note that for Eqn. (3.3), \( S_{1}^{\text{eqn}} \) is defined at \( x = x_{da} \), which means that the depletion width used in Eqn. (3.3) is the same as that used in Eqn. (2.14). This allows the \( qT \Phi_0 e^{-\alpha x_{da}} \)
term to be eliminated from both sides of Eqn. (3.45).

The purpose of the analysis presented in this section is to determine which equation is more appropriate to apply to experimental data. It has been shown that the assumption which differentiates

$$L_n = \frac{1}{\alpha} \left( \frac{qT_\alpha \Phi_0 e^{-\alpha x_d}}{qT_\alpha \Phi_0 - J_{\text{total}} - 1} \right)$$  \hspace{1cm} (2.16)

and

$$L_n = \frac{\alpha + \frac{\nu_1}{D_n}}{\sqrt{\left( \alpha + \frac{\nu_1}{D_n} \right)^2 - 4 \left( \frac{\alpha \nu_1}{D_n} \right) \left( \frac{\gamma}{\gamma - 1} \right)}}$$  \hspace{1cm} (3.9)

that is, $\nu_1 \gg \nu_d$, is technically incorrect, especially when the diffusion length is short. However, an obvious disadvantage of Eqn. (3.9) is the need to know $\nu_1$. As shown in Figure 3.7, $\nu_{1,\text{eqn}}$ can differ greatly from $\nu_{1,\text{sim}}$ because the analytical model cannot fully capture the characteristics of the Schottky diode. Even when the value of $\nu_1$ is known, the results presented in Figure 3.10 show that using Eqn. (3.9) does not necessarily lead to more accurate results. In fact, in some cases, Eqn. (2.16) can produce more accurate results. In addition, it was shown that $L_{\text{est}}$ cannot be determined below a certain $L_n$ value when Eqn. (3.9) is used. Even when Eqn. (3.9) is more accurate, such as when $N_A = 10^{19} \text{cm}^{-3}$ and $L_n > 75\text{nm}$, the error due to Eqn. (2.16) is less than 20%. Therefore, in practice and as far as GaN is concerned, Eqn. (2.16) is recommended simply because it is more convenient to apply and the error that results from using it is not very severe.

Having determined that, in general, it is acceptable to use Eqn. (2.16) to calculate diffusion lengths in GaN from spectral photocurrent data, the method described in Section 2.3.2 is revisited in the next section using the same simulated photocurrent data because it is more convenient to apply in practice. In this method, the ratio of the transmission-factor-corrected photon flux density to the photocurrent is plotted against
the absorption length and a straight line fitted to the plotted data is extrapolated to the \( x \)-axis where the negative \( x \)-intercept gives the desired diffusion length value.

### 3.5 \( L_{est} \) obtained using the Linear Fit Extrapolation Method

In this section, the method described in Section 2.3.2 will be applied to the simulated photocurrent, \( I_{total}^{\text{sim}} \). In this method, the ratio of the transmission-factor-corrected photon flux density to the photocurrent, \( \frac{T_{x} \Phi_{0}}{I_{total}^{\text{sim}}} \), is plotted against the absorption length, \( \alpha^{-1} \). By fitting a straight line to the plotted data points and extrapolating to the \( x \)-axis, \( L_{est} \) can be obtained as the negative \( x \)-intercept. This relationship is described by the following equation

\[
\frac{T_{x} \Phi_{0}}{I_{total}^{\text{sim}}} = \frac{1}{qAT_{x}(x_{d} + L_{est})} (\alpha^{-1} + L_{est})
\]

The above is an approximation of

\[
J_{total}^{\text{sim}} = qT_{x} \Phi_{0} \left( 1 - \frac{e^{-\alpha x_{d}}}{1 + \alpha L_{est}} \right)
\]

made possible when the condition \( \alpha x_{d} \ll 1 \) is satisfied. It is easier to apply compared to

\[
L_{est} = \frac{1}{\alpha} \left( \frac{qT_{x} \Phi_{0} e^{-\alpha x_{d}}}{J_{total}^{\text{sim}}} - 1 \right)
\]

because knowledge of the depletion width and area of the Schottky contact are not required. \( I_{total}^{\text{sim}} \) is used instead of \( J_{total}^{\text{sim}} \) because in any measurement, the output current is recorded rather than its density. Because this method is more convenient in practice compared to using Eqn. (2.16), an assessment of its accuracy will be discussed.

Figure 3.12(a) shows a selection of \( \frac{T_{x} \Phi_{0}}{I_{total}^{\text{sim}}} \) versus \( \alpha^{-1} \) plots for \( N_{A} = 10^{18} \text{cm}^{-3} \) and \( N_{A} = 10^{19} \text{cm}^{-3} \). The wavelength range of the simulated data points is between 301nm to
Figure 3.12. (a) A selection of $T_{\lambda_0} \Phi_0/\Phi_{\text{total}}$ versus $\alpha^{-1}$ plots for $N_A = 10^{18}$ cm$^{-3}$ and $10^{19}$ cm$^{-3}$. Each straight line represents the fitting of a straight line to the simulated data points. These lines were extrapolated to the $x$-axis to obtain estimates of $L_n$. The inset shows the section enclosed by the dashed rectangle. (b) Comparison of percentage errors in the diffusion length values obtained from (a) and those calculated using Eqn. (2.16). Percentage error is defined as $(L_n - L_{\text{est}})/L_n$. 

<table>
<thead>
<tr>
<th>$L_n$</th>
<th>$10^{18}$ cm$^{-3}$</th>
<th>$10^{19}$ cm$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10nm</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>20nm</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>50nm</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>100nm</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>200nm</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>300nm</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Straight Line Fit
solid line $10^{18}$ cm$^{-3}$
dash-dot line $10^{19}$ cm$^{-3}$
355nm. Each straight line, which was extrapolated to the x-axis to obtain \( L_{est} \), represents the fitting of a straight line to the simulated data points. The percentage error curves of the diffusion length determined in this way are shown in Figure 3.12(b) alongside those obtained using Eqn. (2.16). Note that the latter was presented earlier in Figure 3.10.

The results show that the diffusion length determined using this method generally overestimates the true value. In the case of \( N_A = 10^{18} \text{cm}^{-3} \), the error ranges from approximately \(-400\%\) at \( L_n = 10\text{nm} \) to approximately \(-50\%\) at \( L_n = 1000\text{nm} \). For \( N_A = 10^{19} \text{cm}^{-3} \), the percentage error is approximately \(-100\%\) at \( L_n = 10\text{nm} \) but as \( L_n \) increases, the overestimation error improves rapidly to better than \(-25\%\). Between approximately 50nm and 400nm, the error becomes comparable to the errors that result from using Eqn. (2.16). Beyond 400nm, the accuracy deteriorates again.

Note that, as with the results presented in Figure 3.10, the non-monotonicity of the percentage error curves obtained using this method simply reflects the differences in the rates at which \( |L_n - L_{est}| \) and \( L_n \) increases. In all cases, the former increases non-linearly with the latter.

That the error is much worse for \( N_A = 10^{18} \text{cm}^{-3} \) than it is for \( N_A = 10^{19} \text{cm}^{-3} \) is not surprising because the condition \( \alpha x_d \ll 1 \) is not satisfied. In the case of \( N_A = 10^{18} \text{cm}^{-3} \), the minimum value of \( \alpha x_d \) exceeds 0.5 when the depletion width is taken to be \( x_{da} \). If the depletion width is extended to include the transition region, then \( \alpha x_d \) exceeds unity. Regardless of the value of \( x_{da} \), clearly, this condition is not satisfied. For \( N_A = 10^{19} \text{cm}^{-3} \), the minimum value of \( \alpha x_d \) varies between 0.2 and 0.7, depending on the value of \( x_d \). The lower \( \alpha x_d \) value translates into a lower percentage error. Although a value of 0.2 barely qualifies as being much less than unity, it is sufficiently low such that the resulting error approaches that which arises from using Eqn. (2.16) when \( L_n > 50\text{nm} \).

The requirement that \( \alpha x_d \ll 1 \) essentially limits this method to heavily-doped GaN only. Even when the material is heavily-doped, there may be additional complications when this method is applied to real experimental data. In heavily-doped material, the optical absorption coefficient in the depletion region directly adjacent to the Schottky contact can be modified by the high, built-in electric field. This is known as the Franz-Keldysh
effect. A more detailed discussion of this effect and its impact on the accuracy of this method is discussed in Chapter 5.

3.6 Chapter Summary

In this chapter, the assumption that the velocity at which minority carriers are swept away from the depletion edge plane towards the Schottky contact, $v_1$, is much greater than the diffusion velocity, $v_d$, was considered in detail. It was shown at the end of Section 3.1 that when this is not the case, the error that results from this assumption can be quite severe. Therefore, it was necessary to determine the values of the velocity at which the carriers are swept to the surface contact. This was achieved using both analytical and numerical means.

In Section 3.2, two analytical expressions that describe the relationship between $v_1$ and material parameters were presented. Using results calculated for Si and GaN, it was shown that the first model, which assumed that the main contribution to the photocurrent came from the neutral region only, represents the asymptotic limit of the second model. In addition, the second model showed that $v_1$ is a function of temperature, material permittivity, doping concentration, mobility, depletion width, absorption coefficient and diffusion length. Using the second model, the results obtained for GaN showed that the assumption $v_1 \gg v_d$ is not always appropriate, especially when the diffusion length is short.

Because these models were derived by assuming that there is no recombination in the depletion region and that the boundary between this region and the neutral region is clearly defined so that a closed-form equation can be obtained, the resulting equations cannot fully model the electrostatic and carrier density profile inside the Schottky diode. Therefore, numerical simulations using Synopsys® Sentaurus TCAD were performed so that a more accurate value of $v_1$ can be obtained. The simulation results, which were presented in Section 3.4.2, showed that in general, the value of $v_1$ obtained is much less than the values calculated using the analytical expression, especially when the diffusion length is short. The significant discrepancies were attributed to the differences between the analytical and numerical treatments of the Schottky junction, with the former being less accurate because it is limited by certain assumptions that are invalid in actual
devices so as to allow a closed-form equation to be obtained.

Overall, the discussions in this chapter showed that the assumption that $v_1 \gg v_d$, is technically incorrect, especially when the diffusion length is short. Nevertheless, it was demonstrated that in practice, knowledge of $v_1$ and the application of the equation that relates both $v_1$ and the diffusion length to the measured photocurrent do not necessarily lead to a more accurate diffusion length value. In fact, in some cases, the standard mathematical model can produce more accurate results. Even when that is not the case, this equation is still recommended for GaN simply because it is more convenient to use in practice.

In the next chapter, the findings of this chapter are applied to experimental data acquired from spectral photocurrent measurements of both n-type and p-type GaN Schottky diodes. The results obtained will show that another assumption made in the derivation of the standard mathematical model needs to be examined in more detail. This investigation is presented in Chapter 5.
CHAPTER 4

APPLICATION TO EXPERIMENTAL RESULTS

In the previous chapter, the main conclusion was that the standard mathematical model

\[ L_n = \frac{1}{\alpha} \left( \frac{qT_d \Phi e^{-\alpha T_d}}{qT_d \Phi_0 - J_{\text{total}}} - 1 \right) \]  

(2.16)

is recommended over the more exact equation, which is given by

\[ L_n = \frac{\left( \alpha + \frac{v_l}{D_n} \right)}{2} - \sqrt{\left( \alpha + \frac{v_l}{D_n} \right)^2 - 4 \left( \frac{\alpha v_l}{D_n} \right) \left( \frac{\gamma}{\gamma - 1} \right)} \]  

(3.9)

because the former is more convenient to apply and the error that results from its use is acceptable. In this chapter, Eqn. (2.16) is applied to experimental data acquired from spectral photocurrent measurements of both n-type and p-type GaN Schottky diodes. Details of the samples measured and the experimental setup will be provided in Sections 4.1 and 4.2 respectively. This is followed by a discussion of the results obtained in Section 4.3. The chapter finishes with a summary in Section 4.4.

4.1 Sample Details

Spectral photocurrent measurements were conducted on three samples of p-type GaN and one sample of n-type GaN. Details of each sample are shown in Table 4.1. The test structures are coplanar Schottky diodes with circular-geometry. The plan and section views of the diode structures are shown in Figure 4.1(a). Note that this figure only shows the top-most layer. Additional layers beneath this, including the sapphire substrate, are present but not shown. Also shown is an example of the circuit connection
| Sample | Type | Dopant Species | Growth | \(|N_D - N_A| [cm^{-3}] | n or p [cm^{-3}] | \(x_{de} [nm] | Layer Thickness [µm] | Ohmic Contact | Schottky Contact | Comment |
|--------|------|----------------|--------|----------------|----------------|------------|----------------|---------------|----------------|---------|
| 1      | p    | Mg             | MOCVD  | \(~10^{19a}\) | \(~3 \times 10^{17b}\) | \(~20^a\)  | 0.75          | Pd/Au        | Indium Tin Oxide (ITO)\(^c\) (50nm) | 200  
|        |      |                |        |                |                |            |               |              | \(a\) C-V measurements \(b\) Hall data \(c\) Sputter-deposited |
| 2      | p    | Mg             | MOCVD  | \(~2.0 \times 10^{19d}\) | Unknown\(^e\) | \(~13.7^d\) | 0.75          | Pd/Au        | ITO\(^f\) (50nm) | 400  
|        |      |                |        |                |                |            |               |              | \(d\) C-V measurements \(e\) No Hall data because material is too resistive \(f\) Sputter-deposited |
| 3      | p    | Mg             | MOCVD  | \(~2.2 \times 10^{19g}\) | Unknown\(^h\) | \(~12.5^g\) | 0.75          | Pd/Au        | ITO\(^i\) (50nm) | 400  
|        |      |                |        |                |                |            |               |              | \(g\) C-V measurements \(h\) No Hall data because material is too resistive \(i\) Sputter-deposited |
| 4      | n    | Si             | MOCVD  | \(~3.6 \times 10^{16j}\) | \(~3.6 \times 10^{16j}\) | \(~189^j\) | 1.5           | Al/Cr/Au     | Ni/Au\(^k\) (1nm/5nm) | 600  
|        |      |                |        |                |                |            |               |              | \(j\) C-V measurements \(k\) deposited via thermal evaporation |

Table 4.1. Details of samples used in the photocurrent measurements.
Figure 4.1. (a) Schematic showing two different views of the n-type and p-type GaN Schottky diodes used in the photocurrent measurements (dimensions are not to scale). The Schottky contact diameter is given in Table 4.1 for each of the samples. The axis labelling and dimension indicators follow that of Figure 2.1. The circuit connections and the direction of the short-circuit photocurrent, $J_{\text{total}}$, are appropriate for an n-GaN sample. For p-GaN, current flows in the reverse direction. 

(b) Experimental setup used to conduct spectral photocurrent measurements (see below).
for measurement of an n-type material and the direction of the short-circuit photocurrent, $J_{\text{total}}$.

### 4.2 Experimental Setup

The experimental setup used to perform the spectral photocurrent measurements is shown in Figure 4.1(b). Wavelength selection was accomplished using an Oriel Cornerstone 260 1/4 m monochromator. A 150W xenon arc lamp, placed in an enclosed lamp housing designed specifically for the monochromator, was used as the broadband light source. The UV spectral range used in the measurements was 301-355nm. Because wavelengths below 180nm are absorbed by air, an order sorting filter to remove higher diffraction orders was unnecessary.

Carrier injection into the device was achieved by front-side illumination through the semi-transparent Schottky contact. Because the illuminated area is larger than that of the Schottky contact, the gaps between the two contacts were also exposed to the optical radiation. Hence, there is photo-generation around the periphery of the Schottky contact. However, because the diameter of the Schottky contacts (200-600µm) are significantly greater than the width of the depletion region that extends beyond the edge of the circular contacts (5-10nm for the p-type samples and about 100nm for the n-type sample), the peripheral currents are expected to be negligible. As the ohmic contact is opaque, there was no photo-generation under this contact. The power density of the optical excitation was measured using a factory-calibrated, Hamamatsu S1227 series silicon photodiode and a circular aperture with known diameter.

To reduce signal noise, standard phase-sensitive detection technique was used. Signal detection was achieved using the Stanford Research Systems SR570 low-noise current preamplifier and SR830 digital lock-in amplifier. The exiting light was chopped by an SR540 optical chopper at approximately 180Hz.

The devices being measured were connected to the measurement electronics via two probes. The probe for the Schottky contact was positioned in such a way as to minimise blocking of the incident light as well as the resulting shadow. A custom program written using National Instruments LabVIEW was used to automate instrument control and data
acquisition.

4.3 Experimental Results and Discussion

Figure 4.2(a) shows plots of the measured responsivities versus wavelengths, $\lambda$, for Samples 1 to 4. Plots of the estimated diffusion lengths, $L_{est}$, versus wavelength are shown in Figure 4.2(b). $L_{est}$ was calculated from the measured photocurrent data using Eqn. (2.16) and the idealised depletion widths, $x_{da}$.

The plot of diffusion length versus wavelength for the n-GaN sample is not shown in Figure 4.2(b) because the values calculated for all wavelengths are negative. This is because the depletion width of approximately 165nm exceeds the absorption lengths of the optical wavelengths in GaN. The absorption lengths at $\lambda = 301$nm and 355nm are approximately 69nm and 127nm respectively. Therefore, the measured photocurrent consisted of the drift component only. As shown in Figure 4.3, responsivity calculated using

$$\text{Responsivity} = \frac{J_{\text{drift}}}{\text{Power Density}} = \frac{qT_{e}\Phi_{0}(1 - e^{-\alpha x_{da}})}{\text{Power Density}}$$

(4.1)

fits the experimental data fairly well for this sample. These results highlight another limitation of the spectral photocurrent method and the standard mathematical model, namely that if the depletion region is sufficiently wide such that the measured current consists of drift current only, then determination of the diffusion length will not be possible. To determine the hole diffusion length of this sample, the Electron Beam Induced Current, (EBIC) method was used. A discussion of this method and its application to the n-GaN sample as well as one of the p-GaN samples is presented in Chapter 7.

The results presented in Figure 4.2(b) for the p-GaN samples show that the relative values of the calculated diffusion lengths are consistent with the relative doping levels. Note that the diffusion length values plotted in Figure 4.2(b) have not been corrected for the overestimation error which results from using Eqn. (2.16) and the idealised depletion width. Before applying the appropriate corrections to these diffusion length
Figure 4.2. (a) Plots of responsivity versus wavelengths for Samples 1 to 4. (b) Plots of minority carrier diffusion lengths versus wavelengths for Samples 1 to 3 (p-GaN samples). Values of $L_{est}$ were calculated using Eqn. (2.16). The results for Sample 4 are not shown because all values calculated using Eqn. (2.16) are negative.
values, it is necessary to consider the origins of the minor but perceptible spectral variations in the calculated diffusion lengths. As shown in Figure 4.2(b), the diffusion length appears to oscillate slightly with wavelength and its dependence on wavelength is more pronounced when the wavelength is above 340nm. It is unlikely that the spectral variation can be explained by the slightly different overestimation errors at different wavelengths because the differences between the error curves at \( \lambda = 301 \text{nm} \) and \( \lambda = 355 \text{nm} \) (see Figure 3.10) are not significant enough to cause the more pronounced wavelength dependence near \( \lambda = 355 \text{nm} \). In addition, the fact that the same features can be observed in the results of three different samples measured on repeated occasions suggests that it is also unlikely that these are the consequence of experimental errors or noise in the data. Further investigation of these observations is warranted. It is possible that the observed
spectral variation simply reflects a spatially-varying diffusion length. An increase in the wavelength of the excitation source effectively increases the depth (in the $x$ direction, see Figure 4.1(a)) at which the material is being probed because the absorption length increases with wavelength. If the diffusion length somehow degrades with increasing depth, then the diffusion length calculated from photocurrent measured at longer wavelengths can be used to detect this degradation. Note that the calculated diffusion length represents the average or effective diffusion length of the material between $x = 0$ and probing depth.

This scenario is by no means the only possible explanation. In fact, it can be shown that the observed spectral variation in the diffusion length is an artefact of the calculations and that the diffusion length of the material is constant with respect to the optical wavelength. The explanations for these observations can be found by considering the Franz-Keldysh effect. In the next chapter, this effect and its consequence are discussed. Thereafter, the appropriate corrections are made to the calculated diffusion length values shown in Figure 4.2(b) to obtain a better estimate of the true values.

### 4.4 Chapter Summary

In this chapter, the results obtained from spectral photocurrent measurements of four GaN sample – three p-type and one n-type Schottky diodes – were presented. The results revealed one of the limitations of this technique as well as an unexpected effect of another assumption made in the derivation of the standard mathematical model.

It was found that the photocurrent technique cannot be applied to the n-type sample because the high absorption coefficient of GaN meant that all the photons are absorbed in the wide depletion region and the photocurrent is made up of the drift current only. Therefore, this technique is limited to GaN samples (both p- and n-type) with sufficiently high doping concentration such that enough photons can reach the neutral region and induce a diffusion current. Note that this limitation is true for all semiconductor materials with wide depletion width and high absorption coefficients and not just for GaN or the samples investigated in this work.

The unexpected effect was that the diffusion lengths calculated for the three p-type
samples exhibited minor but perceptible spectral variation. The source of this spectral anomaly could be the assumption that the absorption coefficient is uniform throughout the sample. In the next chapter, the proposed source of the observed spectral variation in the calculated diffusion length, namely the Franz-Keldysh effect, is discussed in detail.
CHAPTER 5

INFLUENCE OF THE FRANZ-KELDYSH EFFECT

In the previous chapter, the diffusion length values calculated from spectral photocurrent measurements of three p-type GaN Schottky diodes were found to exhibit a minor but perceptible spectral variation. The fact that the same spectral variations are observed from data obtained on repeated occasions and on different samples suggests that it is unlikely that these observations are the consequence of experimental errors or noise in the data.

The objective of this chapter is to show that, by reconsidering one of the assumptions in the derivation of the standard mathematical model, which is that the absorption coefficient, $\alpha$, is uniform throughout the sample, these observations can be explained and a constant diffusion length value with varying incident wavelength can be obtained for each of the three p-type samples investigated.

The assumption of uniform absorption coefficient throughout the sample requires that the absorption coefficient is only a function of the optical wavelength, $\lambda$. However, it is shown in Section 5.1 that for the p-type GaN samples studied in this work, the absorption coefficient can be modified by the built-in electric field in the depletion region. This phenomenon is known as the Franz-Keldysh effect and it results in electric-field- and wavelength-dependent oscillations of the absorption coefficient about its zero-field value [50]. In addition, because the electric field varies linearly with $x$, the absorption coefficient is also expected to vary spatially in the depletion region. For the three p-GaN samples, the nominal acceptor densities are approximately $10^{19}\text{cm}^{-3}$, $2\times10^{19}\text{cm}^{-3}$ and $2.2\times10^{19}\text{cm}^{-3}$ respectively and the zero-bias maximum field strength in the depletion region of each sample, which is proportional to the square root of the doping concentration, exceeds $10^{6}\text{Vcm}^{-1}$. Therefore, an appreciable change in the absorption coefficient in the depletion regions of these samples can be expected.

In Section 5.2, the analysis presented in the Section 5.1 is applied to the experimental
results obtained for Samples 1 to 3 to show that a constant diffusion length value with
wavelength can be obtained for each sample. In addition, the results obtained are also
corrected for the inherent errors which result from using the standard mathematical
model and the idealised depletion width, $x_{da}$ (these errors were discussed earlier in
Section 3.4.3.1). The chapter then finishes with a summary in Section 5.4.

To facilitate discussions in the following sections, the standard mathematical model,
Eqn. (2.15), is modified to read

$$J_0 = qT_0 \Phi_0 \left(1 - e^{-\alpha_0 x_{da}} \right) \left(1 + \alpha_0 L_a \right) \tag{5.1}$$

The only differences are the substitutions of $J_0$ and $\alpha_0$ for $J_{\text{total}}$ and $\alpha$ respectively. The
addition of zero as a subscript to $\alpha$ serves to indicate that $\alpha_0$ is the zero-field absorption
coefficient. In other words, $\alpha_0$ represents the bulk absorption coefficient value. $J_0$ is
therefore the photocurrent which results from using $\alpha_0$ in any calculation. Note that the
photon flux density, $\Phi_0$, is not affected by the electric field. The subscript of $\Phi_0$ refers to
the incident photon flux density at the $x = 0$ plane.

### 5.1 Electric Field-dependent Absorption Coefficient

Callaway [62] presented an expression for the electric field-dependent absorption
coefficient, $\alpha_E$, in terms of the square of the Airy function. When the incident photon
energy, $E_{\lambda}$, is above the band-gap energy, $E_g$, the asymptotic expansion of the Airy
function can be used instead and this enables $\alpha_E$ to be expressed as [30]

$$\alpha_E(x) = \alpha_0 \left\{1 - \frac{qhE(x)}{8\pi \sqrt{2m_r \left(E_g - E_{\lambda}\right)^3}} \cos \left[\frac{8\pi \sqrt{2m_r \left(E_g - E_{\lambda}\right)^{3/2}}}{3qhE(x)}\right]\right\} \tag{5.2}$$

where $h$ is Planck’s constant, $m_r = \left(m_c^{-1} + m_v^{-1}\right)^{-1}$ is the reduced mass and $m_c$ and $m_v$
are the effective masses at the edge of the conduction and valence bands respectively.
The electric field strength in the depletion region is given by $E(x) = qN_A(x_d - x)/\varepsilon_s$, 
where \( \varepsilon_s = \varepsilon_r \varepsilon_0 \) is the material permittivity and \( \varepsilon_0 \) and \( \varepsilon_r \) are the free space and GaN relative permittivity respectively.

Since the spatially varying absorption coefficient only occurs in the depletion region, the expressions for the drift and diffusion currents

\[
J_{\text{drift}} = q T_\lambda \Phi_0 \left( 1 - e^{-\alpha x_d} \right) \tag{2.3}
\]

and

\[
J_{\text{diff}} = q T_\lambda \Phi_0 e^{-\alpha x_d} \left( \frac{a L_n}{1 + a L_n} \right) \tag{2.14}
\]

are modified to read\(^6\)

\[
J_{\text{drift}} = q T_\lambda \Phi_0 \left[ 1 - e^{-\int_0^x a_s(x) dx} \right] \tag{5.3}
\]

and\(^6\)

\[
J_{\text{diff}} = q T_\lambda \Phi_0 e^{-\int_0^x a_s(x) dx} \left( \frac{a_0 L_n}{1 + a_0 L_n} \right) \tag{5.4}
\]

respectively (see Appendix D for derivation). Summing these two components leads to

\[
J_E = q T_\lambda \Phi_0 \left[ 1 - \frac{e^{-\int_0^x a_s(x) dx}}{1 + a_0 L_n} \right] \tag{5.5}
\]

Note that the total photocurrent is now denoted by \( J_E \) to differentiate it from \( J_0 \) in Eqn. (5.1).

\(^6\) \( e^{-\alpha x_d} \) in Eqn. (2.3) is replaced by \( e^{-\int_0^x a_s(x) dx} \)
For comparison with the zero-field absorption coefficient, the definite integral can be divided by $x_d$ to obtain an effective mean absorption coefficient

$$a_{\text{eff}} = \frac{\int_0^{x_d} a_{E}(x)dx}{x_d}$$

(5.6)

so that Eqn. (5.5) can be written as

$$J_{E} = qT_{x} \Phi_{0} \left( 1 - \frac{e^{-a_{\text{eff}}x_{d}}}{1 + \alpha_{0}L_{n}} \right)$$

(5.7)

Eqn. (5.7) is similar in form to Eqn. (5.1), but the diffusion length values calculated from the same experimental photocurrent data using these equations will differ due to the presence of $\alpha_{\text{eff}}$ in the exponential term in Eqn. (5.7). To illustrate the influence of the Franz-Keldysh effect on the absorption coefficient, calculations of $\alpha_{E}(x)$ for one of the p-GaN samples is presented below.

Figure 5.1(a) shows plots of the electric field-dependent absorption coefficient, $\alpha_{E}(x)$, versus the electric field strength, $E(x)$, in the depletion region of Sample 1, which has nominal acceptor density and depletion width of approximately $10^{19}$ cm$^{-3}$ and 20nm respectively. The wavelengths for which $\alpha_{E}(x)$ was calculated are indicated in the figure. $E(x)$ was calculated using the idealised depletion width, $x_{da} \cong 20$nm. The bandgap energy, $E_{g}$, was taken to be 3.4eV and the zero-field absorption coefficient, $\alpha_{0}$, was obtained from Muth et al. [46]. The bottom axis shows the electric field strength, $E(x)$.

In order to correlate physically with the sample, the electric field axis was reversed, with the maximum value corresponding to the left-hand metal-semiconductor interface. The top axis shows the corresponding depth $x$ in the depletion region.

The non-monotonic, field- and spectral-dependent oscillations of the absorption coefficient are due to the corresponding behaviours of the electron and hole wave functions. In the presence of an electric field and when $E_{k} > E_{g}$, these wave functions are composed of an incident wave and a reflected wave that have different phases [63]. The resulting interference of these overlapping waves causes the observed oscillatory
Figure 5.1. Plots of the electric-field dependent absorption coefficient, $\alpha_E$, versus electric field strength, $E(x)$, in the depletion region of Sample 1 ($N_A = 10^{19}$ cm$^{-3}$): (a) for seven wavelength values from 301nm to 355nm, and (b) for 301nm and 355nm, plotted against the optical generation rates calculated with and without consideration of the Franz-Keldysh (FK) effect. The left axes coincide with the metal-semiconductor interface.
behavior of the absorption coefficient [63]. The amplitude of the oscillation decreases with the magnitude of the electric field and the point at which the absorption coefficient returns to its zero-field value, \( \alpha_0 \), depends on the wavelength, as shown in Figure 5.1(a). The oscillations also become less pronounced as the wavelength decreases.

Because the optical generation rate is a function of the absorption coefficient, the characteristics of \( \alpha_E(x) \) will be reflected in the generation profile calculated using

\[
G_{op}^{FK}(x) = T_x \Phi_0 \alpha_E(x) e^{-\int_0^x \alpha_E(u) du}
\]  

(5.8)

where the superscript ‘FK’ is the acronym for Franz-Keldysh and \( u \) is a dummy variable (see Appendix D for derivation). These characteristics can be seen in Figure 5.1(b), where \( G_{op}^{FK} \) was plotted against \( E(x) \). Also shown for comparisons are the optical generation profiles calculated using \( \alpha_0 \) and Eqn. (2.2), which is reproduced below for convenience

\[
G_{op}(x) = T_x \Phi_0 \alpha_0 e^{-\alpha_0 x}
\]  

(2.2)

Figure 5.2 shows the spectral-dependence of \( \alpha_E \) at three values of depth \( x \) that correspond to the following three electric field strengths: \( E_{max} \approx 3.8 \times 10^6 \text{Vcm}^{-1} \), \( E_{max}/2 \approx 1.9 \times 10^6 \text{Vcm}^{-1} \) and \( 10^4 \text{Vcm}^{-1} \). Also shown is the spectral-dependence of the effective mean absorption coefficient, \( \alpha_{eff} \), calculated using Eqn. (5.6).

These plots show that the amplitudes of the oscillations of \( \alpha_E \) decrease with decreasing wavelengths. Most importantly, \( \alpha_{eff} \) is approximately equal to \( \alpha_0 \) when the wavelength is short (the curve for \( 10^4 \text{Vcm}^{-1} \) can be taken to represent \( \alpha_0 \) because the oscillations at \( E = 10^4 \text{Vcm}^{-1} \) are negligible for all the wavelengths shown in Figure 5.1(a)).

These results suggest that photocurrent measurements should be carried out using photons with energies that are well above \( E_g \) to minimise the effect of the spatial oscillations of the absorption coefficient in the depletion region. Note that the Franz-Keldysh effect is still present at short wavelengths but its net effect is negligible.

\footnote{To be consistent with the notations used in this chapter, \( \alpha \) has been replaced by \( \alpha_0 \).}
Absorption Coefficient, $\alpha_E(x) \times 10^5 \text{ cm}^{-1}$

<table>
<thead>
<tr>
<th>Wavelength, $\lambda$ [nm]</th>
<th>295</th>
<th>300</th>
<th>310</th>
<th>320</th>
<th>330</th>
<th>340</th>
<th>350</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon Energy, $E_\lambda$ [eV]</td>
<td>4.20</td>
<td>4.13</td>
<td>4.00</td>
<td>3.87</td>
<td>3.76</td>
<td>3.65</td>
<td>3.54</td>
<td>3.44</td>
</tr>
</tbody>
</table>

$\alpha_{\text{eff}} \approx \alpha_0$

Electric Field
- $3.8 \times 10^6 \text{ Vcm}^{-1}$
- $1.9 \times 10^6 \text{ Vcm}^{-1}$
- $10^4 \text{ Vcm}^{-1}$

Figure 5.2. Plots of $\alpha_E$ versus wavelength, $\lambda$, at three $x$ positions that correspond to the electric field strength, $E_{\text{max}} = 3.8 \times 10^6 \text{ Vcm}^{-1}$, $E_{\text{max}}/2 = 1.9 \times 10^6 \text{ Vcm}^{-1}$ and $10^4 \text{ Vcm}^{-1}$. Also shown is the curve for the effective mean absorption coefficient, $\alpha_{\text{eff}}$ versus $\lambda$, calculated using Eqn. (5.6).

### 5.2 Application to Experimental Results

In this section, the results presented in Section 5.1 are applied to the experimental results obtained for Samples 1 to 3 to show that the minor but perceptible spectral variation in the diffusion length values calculated using $\alpha_0$ and the standard mathematical model is an artefact, and that the electron diffusion length in each sample is in fact constant with respect to the optical wavelength.

The wavelength-dependent diffusion lengths shown in Figure 4.2(b), which were calculated using $\alpha_0$ and Eqn. (2.16), are reproduced in Figure 5.3 using the same solid symbols and colours. Also shown are two additional sets of curves (dashed and solid lines) for each sample. To facilitate discussions of these curves in the remainder of this section, the notations used are discussed briefly below and summarised in Table 5.1.
Figure 5.3. The solid symbols are the electron diffusion lengths of Samples 1 to 3, which were calculated using Eqn. (2.16) and $\alpha_0$. They are denoted by $L_{\text{est}}^{\text{exp}}$. The dashed lines are the average of the $L_{\text{est}}^{\text{exp}}$ values in the $300\text{nm} \leq \lambda \leq 325\text{nm}$ range where $\alpha_{\text{eff}} \cong \alpha_0$. These average values are denoted by $L_{\text{est}}^{\text{ave}}$. The solid lines represent $L_{\text{est}}^{\text{FK}}$, which are the wavelength-dependent diffusion length values reproduced from synthetic photocurrent data calculated using Eqn. (5.7) and $L_{\text{est}}^{\text{ave}}$.

<table>
<thead>
<tr>
<th>Variants of $L_{\text{est}}$</th>
<th>Description</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\text{est}}^{\text{exp}}$</td>
<td>Calculated using experimental data and Eqn. (2.16)</td>
<td>Previously denoted by $L_{\text{est}}$ in Figure 4.2(b). ‘exp’ indicates experimental data.</td>
</tr>
<tr>
<td>$L_{\text{est}}^{\text{ave}}$</td>
<td>Calculated as the average of $L_{\text{est}}^{\text{exp}}$ in the $300\text{nm} \leq \lambda \leq 325\text{nm}$ range</td>
<td>At these wavelengths, the Franz-Keldysh effect, while present, has negligible net effect because $\alpha_{\text{eff}} \cong \alpha_0$ (see Figure 5.2). The superscript ‘ave’ is the abbreviation for average.</td>
</tr>
<tr>
<td>$L_{\text{est}}^{\text{FK}}$</td>
<td>Calculated using synthetic photocurrent data and Eqn. (2.16). It reproduces the observed wavelength dependence of the diffusion length.</td>
<td>The synthetic photocurrent data is calculated using Eqn. (5.7) and $L_{\text{est}}^{\text{ave}}$ in place of $L_0$. The superscript ‘FK’ is the acronym for Franz-Keldysh. Additional description is given below in the main text.</td>
</tr>
</tbody>
</table>

Table 5.1. Summary of the notations used to discuss the results presented in Figure 5.3.
Throughout Section 5.1, the parameter $L_n$, which appears in Eqns. (5.1) to (5.7), functioned mainly as a placeholder in the equations. However, as described in Section 2.3.1, it is also used to represent the actual value of the electron diffusion length in any p-type Schottky diode sample, so that the discrepancy between $L_n$ and $L_{est}$, which is used to denote the diffusion length value calculated from either synthetic or experimental data, can be taken to be a measure of the accuracy of the photocurrent technique.

To aid discussions, superscripts are added to the notation $L_{est}$. The data points (solid symbols) plotted in Figure 5.3, which were calculated from the measured photocurrent using $\alpha_0$ and Eqn. (2.16), previously denoted by $L_{est}$ in Figure 4.2(b), are now denoted by $L_{est}^{exp}$. The superscript ‘exp’ indicates experimental data and serves to differentiate $L_{est}^{exp}$ from the other two variants of $L_{est}$, namely $L_{est}^{ave}$ and $L_{est}^{FK}$. These are also plotted in Figure 5.3.

$L_{est}^{ave}$ denotes the constant diffusion length in each sample while $L_{est}^{FK}$ represents the diffusion length values determined from synthetic photocurrent data that are calculated using Eqn. (5.7) and $L_{est}^{ave}$. $L_{est}^{FK}$ reproduces the observed wavelength dependence of the diffusion length caused by the Franz-Keldysh effect. The relationship between $L_{est}^{FK}$ and $L_{est}^{ave}$ is explained below.

The value of $L_{est}^{ave}$ can be determined by taking the average of $L_{est}^{exp}$ in the 300-325nm wavelength range. As shown in Section 5.1, at these wavelengths, the Franz-Keldysh effect, while present, has negligible net effect because the effective absorption coefficient, $\alpha_{eff}$, is approximately equal to its zero-field value (see Figure 5.2).

Under low-level injection conditions, which prevail in each measurement, the electrostatics should not be affected by this effect or for that matter the presence of a non-zero generation rate. Therefore, the Franz-Keldysh effect should not affect the underlying principles of the photocurrent method or the standard mathematical model and the values of $L_{est}^{exp}$ calculated for the 300-325nm wavelength range can be taken to represent the constant diffusion length in each sample. The average diffusion length value, $L_{est}^{ave}$, for the data points in this wavelength range was found to be approximately

\[8\] ‘exp’ = experimental data; ‘ave’ = average; ‘FK’ = Franz-Keldysh.
14nm, 5.5nm and 4.5nm for Samples 1, 2 and 3 respectively (dashed-lines in Figure 5.3).

The observed spectral variation in the diffusion length values can be reproduced by calculating \( L_{est}^{FK} \) using the values of \( J_E \) in place of \( J_0 \) in Eqn. (5.1). The values of \( J_E \) are synthetic photocurrent data (not shown in Figure 5.3) calculated using Eqn. (5.7) and \( L_{est}^{ave} \) in place of \( L_n \). Doing so is equivalent to equating Eqns. (5.1) and (5.7)\(^9\)

\[
qT_x \phi_0 \left( 1 - \frac{e^{-\alpha_E x_d}}{1 + \alpha_0 L_{est}^{FK}} \right) = qT_x \phi_0 \left[ 1 - \frac{e^{-\alpha_E x_d}}{1 + \alpha_0 L_{est}^{ave}} \right] 
\]

Eqn. (5.1) Eqn. (5.7)

Therefore, the observed spectral variation in the diffusion length values can be reproduced by using

\[
L_{est}^{FK} = \left( \alpha_0^{-1} + L_{est}^{ave} \right) e^{-\left( \alpha_0 - \alpha_{est} \right)x_d} - \alpha_0^{-1} \tag{5.10}
\]

The results calculated using Eqn. (5.10), which are presented in Figure 5.3 as the solid lines, show good agreement between \( L_{est}^{FK} \) and \( L_{est}^{exp} \) (solid symbols).

These results show that the Franz-Keldysh effect, through the spatial-variation of the absorption coefficient in the depletion region, can affect the diffusion length values calculated from spectral photocurrent measurements and that Eqn. (5.2), which had been used by Dmitruk et al. [50] and Dorantes-Dávila et al. [30] to explain the diffusion lengths of GaAs determined from similar photocurrent measurements, can also be used to describe this effect in GaN.

The causal relationship between the spatially varying absorption coefficient, \( \alpha_E(x) \) and the observed spectral variation in the diffusion length, \( L_{est}^{exp} \), can be understood by considering the deviation of \( \alpha_{est} \) from \( \alpha_0 \) and the consequent modification of the relative proportion of the drift and diffusion components of the photocurrent. When \( \alpha_{est} < \alpha_0 \),

\(^9\) \( L_n \) has been replaced by \( L_{est}^{FK} \) in Eqn. (5.1)
the drift component calculated using $\alpha_0$ is greater than its actual value and since the total photocurrent must remain constant, the calculated diffusion current must be commensurately lower. Hence, the diffusion length calculated will also be lower than its actual value. The converse is true when $\alpha_{\text{eff}} > \alpha_0$. The magnitude of the over- and underestimation errors are greater for $\lambda > 325\text{nm}$ because of the more pronounced spectral variation in $\alpha_{\text{eff}}$ relative to $\alpha_0$, as shown in Figure 5.2. At the longest wavelength used in the measurement of Samples 1 to 3, namely, $\lambda = 355\text{nm}$, ignoring the Franz-Keldysh effect leads to underestimation errors of approximately 3nm (20%), 4nm (72%) and 4.1nm (91%) for Samples 1, 2 and 3 respectively. It should also be noted that, if the true diffusion length of a sample is sufficiently short, measurements at wavelengths near the band-gap can produce erroneous negative diffusion length values.

In conclusion, the results presented in this section show that the spectral variation observed is not due simply to measurement error or actual variation in diffusion length but rather it is an artefact that can be explained by the Franz-Keldysh effect. In addition, the results also show that in the presence of this effect, the expected constant diffusion length can be determined from data points obtained at short wavelength, which for the three p-GaN samples investigated in this work is approximately 325nm and below.

Before concluding this chapter, a correction due to the inherent error which results from using the standard mathematical model will be applied to the diffusion length values obtained in this section. The inherent error, which was discussed earlier in Chapter 3, is the subject of the next section.

### 5.3 Corrections to Account for the Inherent Error in the Standard Mathematical Model

In addition to corrections for the Franz-Keldysh effect, the experimentally determined diffusion lengths, that is, $L_{\text{est}}^{\text{AVE}}$ also need to be corrected for the inherent errors which result from using the standard mathematical model

$$L_n = \frac{1}{\alpha} \left( \frac{qT_k \Phi_0 e^{-\alpha x_n}}{qT_k \Phi_0 - J_{\text{total}}} - 1 \right)$$

(2.16)
and the idealised depletion width, $x_{\text{da}}$. Such errors, which have been discussed in Section 3.4.3.1, are due to the presence of a transition region situated between the fully depleted and neutral regions. Within this transition region, the current is made up of both drift and diffusion currents and there is recombination that is unaccounted for. The standard mathematical model does not consider this region and therefore, it also ignores the recombination, leading to errors in the diffusion length values calculated using this model.

The necessary corrections to the diffusion length values determined for the three p-GaN samples investigated in this work can be made by using error curves such as the ones shown in Figure 3.10, obtained using numerical simulations. The error refers to the difference between the known diffusion length value, $L_n$, used to produce synthetic photocurrent data and the diffusion length value calculated from these data using Eqn. (2.16), $L_{\text{est}}$.

Figure 5.4 shows error curves obtained from numerical simulations of p-GaN Schottky diodes with nominal acceptor density of $10^{19}$ cm$^{-3}$, $2 \times 10^{19}$ cm$^{-3}$ and $2.2 \times 10^{19}$ cm$^{-3}$, that is, the doping densities of Sample 1, 2 and 3 respectively. Only the results obtained for $\lambda = 301$nm are shown because they are almost indistinguishable from the results obtained for $\lambda = 355$nm. The results obtained for $N_A = 10^{19}$ cm$^{-3}$ were presented earlier in Figure 3.10(b). Note, however, that Figure 3.10 and Figure 5.4 differ from one another in two respects.

Firstly, the results in Figure 3.10 are presented in the form of percentage error versus simulated diffusion length values whereas the results in Figure 5.4 are presented in the form of simulated versus estimated diffusion length values. Using notations of the preceding chapters, the percentage error is calculated as $(L_n - L_{\text{est}})/L_n$, where $L_n$ is the simulated value and $L_{\text{est}}$ is the estimated value.

Secondly, to be consistent with the notations used in this chapter, the $x$-axis is labelled as $L_{\text{ave}}$ instead of $L_{\text{est}}$ because in this chapter, $L_{\text{ave}}$ is the notation used to represent the constant diffusion length calculated from experimental data. For the three p-GaN samples investigated in this work, $L_{\text{ave}}$, was found to be approximately 14nm, 5.5nm and 4.5nm for Samples 1, 2 and 3 respectively (dashed-lines in Figure 5.3).
Figure 5.4. Plots of $L_n$ versus $L_{\text{est}}$ obtained from numerical simulations of Samples 1 to 3 (see Section 3.4.1 for details of simulations). $L_n$ denotes the diffusion lengths set in the simulations and $L_{\text{est}}$ represents the constant diffusion lengths calculated from experimental data (discussed earlier in Section 5.2). The black dashed-line represents the $L_n = L_{\text{est}}$ curve, which is added to show the deviations of the other curves from the ideal relationship. The dotted lines are simply visual guides to aid the reader to identify $L_{\text{est}}$ for each sample and the corresponding $L_n$ value.

So, effectively, Figure 5.4 presents a calibration plot that can be used to make corrections to $L_{\text{est}}$ to account for the inherent overestimation errors that results from using Eqn. (2.16).

When corrected for these errors using the results presented in Figure 5.4, the actual diffusion length in Sample 1 is estimated to be approximately 11.5nm instead of 14nm. For Samples 2 and 3, the actual values for both samples are estimated to be about 4.3nm instead of 5.5nm and 4.5nm respectively.

Note that, although the zero-field absorption coefficient, $\alpha_0$, was used throughout the
numerical simulations presented in Chapter 3 and the Franz-Keldysh effect was not considered, the error estimates from that chapter should remain valid even in the presence of this effect because the low-level injection condition was satisfied in every simulation.

In addition, even though the simulations presented in Chapter 3 were for the simple structure shown in Figure 2.1, which is effectively a one-dimensional structure as far as the equations are concerned and therefore does not take into account the periphery of the Schottky contact of the actual samples measured, given the high doping densities of Samples 1 to 3, the extension of the depletion region beyond the edge of the circular contact is not significant. Coupled with the low diffusion length values and large contact diameter, the peripheral currents are negligible and thus have little to no impact on the measured photocurrent.

5.4 Chapter Summary

In this chapter, it was shown that the minor but perceptible spectral variation in the diffusion lengths calculated for the three p-type samples is not due to measurement error or actual variation in diffusion length. Rather it is an artefact that can be explained by reconsidering one of the assumptions of the standard mathematical model, namely that the absorption coefficient is uniform throughout the sample.

In Section 5.1, it was shown that because of the high doping density typically required to achieve high majority hole concentration in p-type GaN, the built-in electric-field in the depletion region is sufficiently high such that there is an appreciable change in the absorption coefficient in this region. The modification of the absorption coefficient by the built-in electric field, which is known as the Franz-Keldysh effect, resulted in electric-field and wavelength-dependent oscillations of the absorption coefficient about its zero-field value.

In Section 5.2, it was shown that by taking this effect into consideration, the observed spectral variation in the calculated diffusion length can be explained and the value of this parameter is in fact constant with respect to wavelength. Using these results and the percentage error curves obtained in Chapter 3 to correct for the inherent errors that arise
from the application of the standard mathematical model, the estimated diffusion length values for the three p-type GaN samples were determined.

Overall, comparisons of the diffusion length values obtained with and without consideration of the Franz-Keldysh effect showed that this effect does not have a significant impact on the accuracy of the diffusion length values obtained from spectral photocurrent measurements of heavily-doped p-type GaN samples. An exception to this statement is when the diffusion length is sufficiently short such that measurements at wavelengths near the band-gap produce erroneous negative diffusion length values. When this occurs, then the measurements should be repeated at shorter wavelengths to verify the results.

In the next chapter, the possibility of undesirable recombination in the depletion region, which can alter the relative proportion of drift and diffusion currents with respect to their actual values and therefore affect the diffusion length values calculated using the standard mathematical model are discussed. In addition, the assumption that the measured photocurrent is composed entirely of minority carrier current is also explored.
CHAPTER 6

EFFECTS OF DEPLETION REGION RECOMBINATION AND THE MAJORITY CARRIER CURRENT

In this chapter, the validity of two additional assumptions (see assumptions 7 and 8 in Table 2.7) of the standard mathematical model are explored. These assumptions are that:

(1) there is no recombination in the fully depleted part of the depletion region of the Schottky diode in addition to the recombination in the transition region; and

(2) the photocurrent which results from optical excitation of this diode under short-circuit condition is composed entirely of minority carrier current.

The assumption of no recombination in the depletion region is discussed first in Section 6.1. Evidence from the literature that supports the presence of defects introduced into the sample during post-growth processing and that can lead to such undesirable recombination is presented in 6.1.1. This is followed by discussions of the effects these defects can have on the accuracy of the diffusion length values obtained using this technique.

In Section 6.2, the assumption that the photocurrent does not contain contributions from the majority carrier current is investigated using a combination of analytical and numerical means. An analytical expression of the majority carrier current for a Schottky diode is presented in Section 6.2.1. This is followed by the application of this equation to estimate the magnitude of the majority carrier current for the GaN samples investigated in this work and comparisons of the results calculated with those obtained from numerical simulation as well as the analysis of the consequence of the majority carrier current on the accuracy of the photocurrent technique in Section 6.2.2.

The chapter finishes with a summary in Section 6.3.
6.1 Recombination in the Depletion Region

In Section 3.4.3, recombination in the depletion region, which includes a transition region that is approximately 4-6 extrinsic Debye lengths in width, was used to account for the discrepancies between the simulated and calculated diffusion length. Within this transition region, the current is made up of both drift and diffusion currents and there is recombination that is unaccounted for.

As explained in that section, such recombination alters the relative proportion of the calculated drift and diffusion currents with respect to their actual values. Consequently, the diffusion length value calculated using the standard mathematical model can deviate from the true value. Examples of the discrepancies between the true and estimated values for p-GaN samples with acceptor concentration of $10^{18}$ cm$^{-3}$ and $10^{19}$ cm$^{-3}$ were given in Figure 3.10.

Therefore, it follows that any additional recombination that is not accounted for by either the numerical simulations or the standard mathematical model can also be expected to have a detrimental effect on the accuracy of the calculated diffusion length.

6.1.1 Defects Introduced by Postgrowth Processing

In addition to pre-existing defects in GaN such as threading dislocations, energetic postgrowth processing steps such as reactive-ion etching and metallisation using thermal evaporation or sputter deposition can induce surface damage and defects that degrade the performance of GaN-based devices. For the p-type GaN samples investigated in this work, the transparent ITO Schottky contacts were sputter-deposited.

The surface of GaN is relatively sensitive to energetic ion bombardment or thermal degradation and it is known that surface bombardment causes reduction of the surface nitrogen concentration [64]. Nitrogen atoms are lost preferentially because they are lighter compared to gallium atoms and nitrogen vacancies, V$_N$, have a low formation energy in p-type GaN [65]. These vacancies are donor-like and lead to compensation or worse, type conversion of the near surface region of p-type GaN [66].

Cao et al. [67] reported that the near-surface (40-50nm) of p-type GaN exposed to high-
density plasmas is found to become more compensated through the introduction of shallow donors and at high ion fluxes or energies, there can be type conversion of this surface region. They measured the diode breakdown voltage of p-type GaN exposed to inductively coupled plasmas (ICP) of Cl₂/Ar, H₂ or Ar and observed an increase in reverse breakdown voltage on the p-GaN, the magnitude of which was dependent on both ion energy and flux. The increase in breakdown voltage was attributed to the reduction in hole concentration in the near-surface region through the creation of shallow donor states. These shallow donor states are thought to be nitrogen vacancies [67]. Pearton et al. [66] found that at high ICP source powers, the concentration of plasma-induced shallow donors exceeds the hole concentration and there is type conversion on the initially p-type surface, that is, the metal-p GaN diode turned into a metal-n GaN-p GaN double junction diode.

Yan et al. [68] studied the effects of sputtering on the surface of GaN. They compared Ni/Au contacts deposited on p-type GaN through evaporation and sputtering and found that the as-deposited contact resistance is lower when evaporated. After annealing, both evaporated and sputtered contacts exhibit similar characteristics. They concluded that any surface damage on GaN by an energetic process would create nitrogen vacancies. It is suggested that nitrogen vacancies will form preferentially to gallium vacancies, V\text{Ga}, which act as acceptors, due to a lower formation energy. Upon annealing, the damage created will begin to recover.

Hashizume et al. [69] proposed the formation of a thin surface barrier region of nitrogen vacancies or its complex due to thermal evaporation, the least energetic of the three processes mentioned above. Their simulation reproduced the experimental current-voltage-temperature or I-V-T characteristics of their GaN Schottky diodes and gave excellent fitting results to the reported Schottky I-V curves in GaN for both forward and reverse bias at different temperatures.

With this evidence in mind, Pulfrey et al. [59] investigated the effects of these defects on the diffusion length values determined from spectral photocurrent measurement of one of the p-type GaN samples studied in this work. The results of this collaborative effort are presented in the next section.
6.1.2 Effects on the Photocurrent Technique

Pulfrey et al. [59] postulated that the sputter-deposition of ITO onto Mg-doped p-type GaN resulted in the creation of a constant concentration of donor-like vacancies at the surface, \( V_N(0) \). It was suggested that these point defects then diffuse into the bulk, forming a thin, damaged surface layer with a \( V_N \) profile characterised by a complementary error function

\[
V_N(x) = V_N(0) \text{erfc} \left( \frac{x}{L_c} \right)
\]

(6.1)

where \( L_c \) is the characteristic length of the function. Within this damaged layer, the minority carrier lifetime and mobility are assumed to be drastically reduced, leading to significant recombination and therefore, underestimation of the diffusion length calculated from the photocurrent using the standard mathematical model

\[
L_n = \frac{1}{\alpha} \left( \frac{qT_A \Phi_0 e^{-\alpha x}}{qT_A \Phi_0 - J_{\text{total}}} - 1 \right)
\]

(2.16)

The uniform and exponential distributions used in [69,70] to describe the distribution of \( V_N \) can be viewed as approximations to such an \( \text{erfc} \) profile [59].

To investigate this hypothesis, Pulfrey et al. [59] performed numerical simulations of a Mg-doped, p-type GaN Schottky diode with a 50nm-thick ITO as the semi-transparent Schottky barrier material and a doping density of \( N_A = 10^{19} \text{cm}^{-3} \). The simulations were performed using MEDICI, which is a commercial semiconductor device simulation software created by Technology Modeling Associates, Inc. and is similar to Synopsys® Sentaurus TCAD. These material parameters were chosen because the aim of the simulations was to fit the simulated responsivity data to the experimental data obtained for Sample 1. The fit was achieved by varying \( V_N(0) \) and \( L_c \) only. Values of all the other simulation parameters remain unchanged.

In the simulations, the damage was characterised by a \( V_N \)-dependent mobility and recombination lifetime. Therefore, these properties are also functions of position \( x \). The relationships between these properties and \( V_N \) were modelled using empirical equations...
that can be found in the numerical simulation package MEDICI. The parameter values required by the empirical equations were chosen such that these properties are strongly degraded in the damaged layer but assume their bulk values at the expiry of this layer [59]. The bulk mobility and lifetime values used in the simulations were $\mu_n = 132\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ [71] and $\tau_n = 0.32\text{ns}$ [72] respectively. Using $L_n = \sqrt{\frac{\mu_n \tau_n kT}{q}}$ and these bulk values, the simulated diffusion length value, $L_n$, was calculated to be approximately 325nm.

Pulfrey et al. [59] found that the best fit to the experimental data for Sample 1 can be obtained by setting $V_N(0) = 3.7 \times 10^{18}\text{cm}^{-3}$ and $L_c = 23\text{nm}$. Plots of recombination rate versus position $x$ showed that the damage induced by $V_N$ resulted in massive recombination in the damaged layer, which extended beyond the depletion region. The increased recombination, which was not accounted for by standard mathematical model, resulted in a calculated diffusion length value of approximately 10nm instead of 325nm. In view of these results, it was concluded that the postulated damaged layer can affect the measured responsivity and that conventional calculation of the diffusion length using standard mathematical model, which assumed that mobility and lifetime are uniform throughout the material, will produce erroneous results.

While the approach taken by Pulfrey et al. [59] is sound and the causal relationship between the significant underestimation of the diffusion length and substantial recombination in a damaged layer adjacent to the Schottky contact is not in dispute, the uniqueness of the results obtained is not assured. One of the advantages of numerical simulations is the ability to incorporate complex models or phenomena that cannot be easily accommodated by analytical models. However, this advantage also happens to be a disadvantage when using numerical simulations to match experimental data. When many parameters are involved and the values chosen for some of these parameters are neither the results of subsidiary experiments nor are in fact measureable, and are therefore somewhat arbitrary, as was the case here, then in principle, the experimental results could be matched by using a different set of parameter values. Indeed, Pulfrey et al. [59] conceded that the values chosen for the parameters of the empirical equations are not likely to be unique.

In fact, one might also argue that there is also uncertainty as to whether the empirical equations used in the simulations can adequately describe the relationships between $V_N$
and mobility and lifetime, and unless the exact relationships are known, the purpose of such simulations is limited to demonstrating that such a damaged surface layer can have an effect on the diffusion length obtained from traditional analysis. As a matter of fact, this is the primary purpose of this work – to investigate the extent of the degradation in the accuracy of the photocurrent technique when one or more of the assumptions made in the derivation of the standard mathematical model are violated. In this case, the assumptions of no recombination in the depletion region as well as uniform mobility and lifetime, are violated. It is necessary to take account of this effect when applying this technique to GaN that has been subjected to energetic post-growth processes such as reactive-ion etching and sputtering.

Additional sensitivity analysis to determine which parameter has the least or the greatest impact on the simulation output is beyond the scope of this thesis. In the next section, the assumption that the photocurrent consists of the minority carrier current only is investigated.

### 6.2 Effects of the Majority Carrier Current

In using the standard mathematical model, which is derived from the minority carrier continuity equation, the assumption is that the short-circuit photocurrent detected by the external circuit is driven entirely by the minority carrier current. In other words, for a p-type material, the majority carrier current at the Schottky interface, \( J_p(x = 0) \), is zero and \( J_{\text{total}} = J_n(x = 0) \).

In the depletion region directly adjacent to the Schottky interface, it was assumed that the photo-generated minority and majority carriers are swept towards and away from the interface respectively by the built-in electric field. However, this is not always the case. The majority carriers generated near the Schottky interface can diffuse towards the contact in opposition to the electric field and be emitted into the metal or recombine with minority carriers, resulting in a reduced total photocurrent [51]. This effect is expected to be more significant in direct band-gap materials [52] because of the high absorption coefficient [51,52]. As GaN is a direct gap semiconductor material and the absorption coefficient is quite high, the assumption of zero majority carrier current warrants further investigations.
Lavagna et al. [52] solved the electron and hole continuity equations in the depletion region of a zero-biased, n-type Schottky diode, subjected to a uniform, monochromatic illumination, and produced expressions that described the electron and hole currents as a function of depth $x$ in the depletion region. The derivation of these expressions can be found in Appendix B. Note that the results of Lavagna et al. [52] were also used earlier in Section 3.2.2 to derive the relationship between material parameters and $v_1$, which is defined as the velocity at which minority carriers are swept away from the depletion edge towards the Schottky contact.

Using the expressions they derived to calculate the response of an n-type Au-CdTe Schottky diode to strongly absorbed light, Lavagna et al. [52] found that, initially, the quantum efficiency increases with increasing absorption coefficient. At some point, instead of reaching a plateau, as predicted by the standard mathematical model [51], the quantum efficiency decreases with increasing absorption coefficient. They attributed this result to the diffusion of majority carriers towards the Schottky contact and the subsequent emission of these carriers into the metal [52]. Recombination at the Schottky interface was deemed negligible because the emission velocity was judged to be much greater than the recombination velocity.

The emission of the majority carriers into the metal constituted a majority carrier current that opposes the minority carrier current. The reduced total photocurrent will therefore affect the accuracy of the diffusion length value calculated from such photocurrent data. The extent of the inaccuracy depends on the magnitude of the majority carrier current. Hence, it will be beneficial to estimate the magnitude of this current for the GaN samples discussed in Chapter 4. In the next section, analytical expressions for the majority carrier current for both p-type and n-type material will be presented. The application of these equations to GaN and a discussion of the results is presented in the subsequent section.

### 6.2.1 Expression for the Majority Carrier Current

For a p-type material, the hole current density is given by (see Appendix B for derivation)
\[ J_p = -qT_p \Phi_0 \left[ 1 + \left( \frac{1}{D_p + F_1 H_p} \right) \left( \frac{\Delta p(x_d) H_p}{T_p \Phi_0} - \frac{F_2 H_p}{D_p} - 1 \right) D_p \right] \] (6.2)

where \( D_p \) is the hole diffusion coefficient. The constants \( F_1 \) and \( F_2 \) are given by

\[ F_1 = \sqrt{\frac{\pi}{2}} L_D \text{erfi} \left( \frac{x_d}{\sqrt{2}L_D} \right) \] (6.3)

and

\[ F_2 = -\sqrt{\frac{\pi}{2}} L_D e^{-\left(\alpha x_d + \alpha^2 L_n/2\right)} \left[ \text{erfi} \left( \frac{aL_D}{\sqrt{2}} \right) - \text{erfi} \left( \frac{x_d}{\sqrt{2}L_D} + \frac{aL_D}{\sqrt{2}} \right) \right] \] (6.4)

where the imaginary error function, \( \text{erfi}(\theta) \), is a function defined by \( \text{erfi}(\theta) \equiv -i\text{erf}(i\theta) \) [73].\(^{10}\) \( \Delta p(x_d) \) is the excess hole density at the depletion edge plane, \( x = x_d \). Assuming that the region \( x \geq x_d \) is electrically neutral, \( \Delta p(x_d) \) can be equated to \( \Delta n(x_d) \), the expression of which is given by

\[
\Delta p(x_d) = \Delta n(x_d) = T_p \Phi_0 \left[ \frac{1}{H_n} \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right) + \frac{1}{D_n} \left( F_4 - \frac{F_3 e^{-\alpha x_d}}{1 + \alpha L_n} \right) \right] \frac{F_3}{L_n} + \frac{D_n}{L_n H_n} + 1 \] (6.5)

The expressions for the constants \( F_3 \) and \( F_4 \) were given earlier in Section 3.2.2 but are repeated here for convenience

\[ F_3 = \sqrt{\frac{\pi}{2}} L_D \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} \right) \] (3.28)

\[ F_4 = \sqrt{\frac{\pi}{2}} L_D e^{-\left(\alpha x_d - \alpha^2 L_n^2/2\right)} \left[ \text{erf} \left( \frac{aL_D}{\sqrt{2}} \right) + \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} - \frac{aL_D}{\sqrt{2}} \right) \right] \] (3.29)

\(^{10}\) Note that for real \( \theta \), this function is also real.
The constants $H_p$ and $H_n$, which appear in Eqns. (6.2) and (6.5) respectively, are defined as

$$H_p = \frac{A_p^* T^2}{q N_v} e^{-\frac{x_d}{2L_D}}$$  \hspace{1cm} (6.6)$$

and

$$H_n = \frac{A_n^* T^2}{q N_C} e^{-\frac{x_d}{2L_D}}$$ \hspace{1cm} (6.7)$$

where $A_p^*$ and $A_n^*$ are the effective Richardson constants for electrons and holes respectively, while $N_v$ and $N_C$ are the valence and conduction band effective density of states, respectively. Using hole and electron effective mass values of $m_0$ and 0.22$m_0$ [57] respectively, the values of $A_p^*$, $A_n^*$, $N_v$ and $N_C$ for GaN were calculated to be approximately $120 \text{Acm}^{-2}\text{K}^{-1}$, $26.7 \text{Acm}^{-2}\text{K}^{-1}$, $2.51 \times 10^{19} \text{cm}^{-3}$ and $2.63 \times 10^{18} \text{cm}^{-3}$ respectively.

Note that provided that

$$\frac{x_d^2}{2L_D^2} = \left( \frac{2\varepsilon_n V_{bi}}{q N_A} \right) \left( \frac{q^2 N_A}{2\varepsilon_n kT} \right) = \frac{V_{bi}}{kT/q} \gg 1$$ \hspace{1cm} (6.8)$$

then the magnitude of $H_n$ is extremely large such that $1/H_n$ is effectively zero and Eqn. (6.5) can be reduced to

$$\Delta p(x_d) = \Delta n(x_d) = \frac{T_2 \Phi_0 L_n}{D_n} \left[ \frac{F_4 (1 + \alpha L_n) - F_3 e^{-\alpha x_d}}{(1 + \alpha L_n)(L_n + F_3)} \right]$$ \hspace{1cm} (3.27)$$

which was produced earlier in Section 3.2.2.

For an n-type material, the majority electron current density is given by
where

\[ H_n = \frac{A_n^* T^2}{qN_C} e^{-\left(\frac{\Delta \sigma}{\sqrt{2L_n}}\right)^2} \quad (6.10) \]

### 6.2.2 Application and Analysis

Using the equations described in the previous section, the ratio of the minority carrier current, \( J_{\text{min}} \), to the majority carrier current, \( J_{\text{maj}} \), as a function of diffusion length were calculated for Samples 1 to 4 at two wavelengths: \( \lambda = 301\text{nm} \) and \( 355\text{nm} \). The results, shown in terms of the order of magnitude, are presented in Figure 6.1. Also shown in

![Figure 6.1](image)

**Figure 6.1.** Plots showing the order of magnitude for the ratio of the minority carrier current, \( J_{\text{min}} \), to the majority carrier current, \( J_{\text{maj}} \), as a function of diffusion length for Samples 1 to 4. These plots are obtained from calculated and simulated \( J_{\text{min}} \) and \( J_{\text{maj}} \).
the same figure are the $J_{\text{min}}/J_{\text{maj}}$ ratios obtained from numerical simulations using Synopsys® Sentaurus TCAD. These simulations were undertaken using the same material parameters (except for doping concentrations) and simulation settings used in Chapter 3 (see Table 3.2 on pg. 53 for material parameters and Appendix C for additional details of simulation setup).

The plots for the ratio obtained from calculations show that for the three p-type samples, $J_{\text{min}}$ is at least three orders of magnitude greater than $J_{\text{maj}}$. As the diffusion length increases, these ratios also increase. For the n-type sample, the ratio is slightly lower than those of the p-type samples and stays relatively constant with respect to the diffusion length. Nevertheless, $J_{\text{min}}$ is still more than two orders of magnitude greater than $J_{\text{maj}}$.

For the p-type samples, the ionised dopant concentration is higher compared to the n-type samples. Consequently, the magnitude of the built-in electric field in the p-type samples is greater and therefore, the diffusion of holes towards the Schottky contact in opposition of this field is minimal. Hence, the $J_{\text{min}}/J_{\text{maj}}$ ratios for the p-type samples are greater than those of the n-type sample.

Because the majority current is independent of the diffusion length for a given material, the observed increase in the $J_{\text{min}}/J_{\text{maj}}$ ratios is due to $J_{\text{min}}$ increasing with a rising diffusion length value. For the n-type sample, this increase is barely noticeable because minority current density is dominated by the drift current due to the much wider depletion region.

The ratios calculated from $J_{\text{min}}$ and $J_{\text{maj}}$ produced by numerical simulations for all four samples also show that, except for the magnitude of the ratios, the characteristics of the plots are similar to those obtained from equations. These results are deemed to be more accurate compared to the results obtained from equations because the numerical simulations can model the electrostatics inside the Schottky diode more accurately than an analytical model.

Overall, the results presented in Figure 6.1 show that in all cases, $J_{\text{min}}$ is much greater than $J_{\text{maj}}$. Therefore, as far as Samples 1 to 4 are concerned, it is safe to assume that the minority carrier current equals the measured photocurrent.
The reduction of the total photocurrent due to a non-zero majority carrier current is expected to be more significant in direct band-gap materials [52] such as GaN. When applying the photocurrent technique to determine diffusion length in lightly-doped GaN, the presence of non-zero majority carrier current should be investigated especially if the calculated diffusion length is very short or is negative.

### 6.3 Chapter Summary

In this chapter the validity and effects of two additional assumptions made in deriving the standard mathematical model were investigated.

In Section 6.1, the possibility of undesirable recombination in the depletion region due to donor-like nitrogen vacancies which are introduced into the sample in the form of a thin, surface damaged layer as a result of post-growth processing steps such as sputter-deposition of the Schottky contact was discussed. Evidence obtained from the literature supporting the presence of such defects was also presented. It was found that if such recombination is present but was unaccounted for, as is the case in the derivation of the standard mathematical model, calculations of the diffusion length from photocurrent data can significantly underestimate the true value. Therefore, it is worth keeping this in mind when the photocurrent technique is applied to GaN samples that have been subjected to energetic post-growth processes. When the value of diffusion length calculated is very low, the results should be treated with caution and confirmation with an alternative technique should be obtained.

In Section 6.2, the assumption that the photocurrent is made up entirely of the minority carrier current was investigated. Through calculations using an analytical equation and numerical simulations, it was shown that for the all the samples measured in this work, the minority carrier current is much greater than the majority carrier current and therefore it is safe to equate the measured photocurrent to the minority carrier current.

The results presented up to this point show that there are cases in which either the photocurrent technique cannot be used to determine the diffusion length of a sample or the values obtained need to be treated with caution because of concerns that an assumption of the standard mathematical model may be invalid. In Section 4.3, it was
shown that the photocurrent technique cannot be applied to the n-type GaN sample studied in this work because the depletion width is sufficiently wide such that the measured current consisted of the drift current only. Note that this does not mean the technique is not applicable to n-type GaN but rather, this is a general limitation of this technique. It applies to both p- and n-type samples of any materials with wide depletion widths and high absorption coefficients and not just for GaN or the samples investigated in this work. And in Section 6.1, evidence from the literature showed that due to defects introduced during post-growth processing steps, there is a real possibility that significant recombination occurs in the depletion region near the metal-semiconductor interface and consequently, this results in a significant underestimation of the diffusion length calculated using a model that assumes no such recombination. These cases suggest the need for an alternative characterisation technique that serves as either a substitute for or a complement to the photocurrent technique.

In the next chapter, an alternative technique – the Electron-Beam Induced Current technique – is investigated and applied to the GaN samples studied in this work.
CHAPTER 7

THE ELECTRON-BEAM INDUCED CURRENT TECHNIQUE

In this chapter, the Electron-Beam Induced Current or EBIC technique for determining minority carrier diffusion length is investigated and applied to the GaN samples studied in this work. The purpose of investigating and applying this technique is threefold. The first and main reason is that an alternative technique is required when the photocurrent technique is unsuitable. An example of when the photocurrent technique is unsuitable is when the depletion width is too wide relative to the optical absorption length of the material such that all photons are absorbed in the depletion region and the induced photocurrent is made up of drift current only (see Section 4.3). It was found that this is the case for the n-type sample investigated in this work. The second reason is that the results obtained using the EBIC technique can be used for comparisons with the results produced by the photocurrent technique (when it is applicable, for example, for the p-type GaN samples studied in this work). The third reason is that since this technique is widely used to determine diffusion length, like the photocurrent technique, it needs to be carefully validated for GaN to determine its ability to produce accurate diffusion length values at the lower end of the sub-micrometer range.

This chapter begins with a brief introduction to the EBIC technique, and the mathematical model used to determine the minority carrier diffusion length from EBIC data, in Sections 7.1 and 7.2 respectively. Then, in Section 7.3, the accuracy of the model when it is applied to GaN is investigated. The method by which the accuracy of the model is determined is the same as that used earlier in this thesis to ascertain the accuracy of the photocurrent technique, that is, by using numerical simulations to produce synthetic data for a set of known diffusion length values and then comparing these values to those obtained using the model. The discrepancy between the known and extracted values is thus a measure of the accuracy of the model. In Section 7.4, the application of the EBIC technique to the GaN samples studied in this work and the results obtained will be presented and discussed. The chapter then finishes with a summary in Section 7.5.
Note that, the investigation presented here into variants in the EBIC technique implementation and mathematical modelling is not as comprehensive as for the photocurrent technique; the latter being the focus of this thesis. A more detailed history and in-depth investigation of the EBIC technique can be found in texts by Holt and Joy [74], and Blood and Orton [2], a review paper by Leamy [75] as well as the doctoral thesis of Kurniawan [76].

### 7.1 Basic Principles

The EBIC technique is a characterisation tool commonly used to determine minority carrier diffusion length in semiconductors [8-10,12-16,23-26,77]. In this technique, a current is induced in a semiconductor device, such as a Schottky or p-n junction diode that contains a built-in junction, when a beam of electrons with kilo-electron-volt energies produced by an electron microscope is focused on the surface of the sample. This current is made up of a proportion of the excess carriers that are created by the impinging primary electrons and subsequently collected by the built-in junction. The excess carriers are created by collisions between the energetic primary electrons and the valence electrons of the atoms that make up the material [75]. The transfer of energy during each collision produces an electron in the conduction band and hole in the valence band; in other words, an electron-hole pair.

The EBIC technique can be implemented in several ways [10,78-81]. For example, EBIC can be measured as the beam is scanned away from or towards a collecting junction [8,10,24,26,77,80] or as a function of beam energy, $E_b$ [78,82]. The former method is known as an EBIC line-scan measurement and is the method that will be applied to the GaN Schottky diodes studied in this work. Typically, in an EBIC line-scan measurement, an electron beam with constant energy and a spot size that is much smaller than the dimensions of the sample is positioned near the junction (see Figure 7.1) and the beam is then moved away from the contact in a straight line so that the distance between the electron beam and the collecting junction is increased. The beam stays at each point long enough to ensure that each recorded current signal is a steady-state measurement. The diffusion length is then extracted from the line-scan data using an appropriate mathematical model that describes the relationship between the measured current and the diffusion length. In the next section, the model used in this work is
The EBIC Model

Chan et al. [79] proposed a direct method based on the modified asymptotic models of Ioannou and Dimitriadis [83] and Boersma et al. [84] to extract diffusion length from EBIC line-scan measurements of a planar junction device configuration. This method is attractive because of its simplicity and it has been used by researchers in the past to determine diffusion length of GaN and zinc oxide [24,26,77].

The equation proposed by Chan et al. [79] to extract diffusion lengths from EBIC line-scans of a planar junction device with negligible depletion depth is given by

\[ I_{EBIC} = k x^\alpha e^{-x/L_n} \]  

(7.1)

where \( x \) is the beam-to-junction distance while the coefficients \( k \) and \( \alpha \) and electron
diffusion length, $L_n$, are fitting parameters. The coefficient $\alpha$ is strictly a function of the surface recombination velocity along the scanning path. A more detailed discussion of $\alpha$ can be found in [79]. Note that in the preceding chapters, the parameter $\alpha$ denotes the optical absorption coefficient. However, to be consistent with the symbols used by Chan et al. [79], $\alpha$ is also used here to represent one of the coefficients of the EBIC model.

Further development of this model by Kurniawan and Ong [81] for cases where the depletion depth, $y_d$, is non-negligible showed that the same equation can be applied. However, they changed the symbol for the exponent of $x$ from $\alpha$ to $\gamma$ so that Eqn. (7.1) now becomes

$$I_{EBIC} = kx \gamma e^{-x/L_n}$$

(7.2)

This was implemented to highlight the fact that the exponent of $x$ is now a function of both $\alpha$ and $y_d$ [81].

Using $k$, $\alpha$ and $L_n$ as fitting parameters, diffusion length is extracted from EBIC data simply by fitting the logarithm of $I_{EBIC}$ as a function of $x$ to

$$\log(k) + \gamma \log(x) = \frac{x}{L_n}$$

(7.3)

In general, both Eqns (7.1) and (7.2) produce reasonably accurate results provided $x$ is greater than two diffusion lengths away from the collecting junction [79]. Numerical simulations performed by Chan et al. [79] and Kurniawan and Ong [81] showed that the accuracy of this method is within 6% for diffusion length values between 3 $\mu$m and 100$\mu$m.

Even though Eqns. (7.1) and (7.2) were derived using a point source, Chan et al. [79] showed that these equations can also be used for an extended generation source, provided that the lateral width of the electron-hole pair generation volume, $w_{gv}$, is much less than $x$. The accuracy of this method decreases as the difference between these quantities decreases. In summary, to produce accurate results, the conditions that must be satisfied are $x > 2L_n$ and $x \gg w_{gv}$.
In the next section, the accuracy of the method proposed by Chan et al. [79] in determining sub-micrometer diffusion lengths will be investigated via numerical simulations.

### 7.3 Model Accuracy

The investigation of the accuracy of the EBIC model followed the same principle as that of the investigation of the accuracy of the photocurrent technique. Using Synopsys® Sentaurus TCAD, 2D simulations of the structure shown in Figure 7.1 were undertaken to produce synthetic EBIC line-scan data for a set of known diffusion length values. These values were then compared to those obtained using the mathematical model. The discrepancies between the known and extracted values were taken to be a measure of the accuracy of the model.

The use of numerical simulation software to investigate the accuracy of the method described in the previous section has been reported in the literature [79,81]. To determine the accuracy of their method, Chan et al. [79] used the DaVinci 3D device simulation software to produce EBIC data for planar Schottky diodes with diffusion length values of 3µm and 100µm. A point generation source was simulated using a cubic generation volume with sides of 200nm and uniform density.

Kurniawan and Ong [81] used the same approach and performed 2D simulations of a diffused p-n junction diode with planar configuration using the MEDICI semiconductor device simulation software. In their simulations, they set the diffusion length to 3µm and used an extended electron-hole pair generation volume rather than a point source. The generation volume was calculated using a model where the lateral-dose function is based on a Gaussian model proposed by Donolato [85] and the depth-dose function is approximated by a cubic polynomial suggested by Everhart and Hoff [86]. In both examples, the effects of surface recombination velocity on the accuracy of the diffusion length values extracted were also investigated.

The simulations conducted in this work differ from the above examples mainly in the following respects:
(a) diffusion length range
The values used in the simulations conducted in this work are set to between 10nm to 1µm. This is because GaN can have very short diffusion lengths in the low sub-micrometer range and the range of values considered in the examples cited above as well as in several other reports found in literature [87,88] are longer than 1µm.

(b) electron beam characteristics
The simulations conducted in this work considered the effects of beam energy and diameter on the accuracy of the EBIC model in determining sub-micrometer diffusion lengths. These beam characteristics were not considered in the examples cited.

Although the effects of beam diameter on EBIC line-scan results were examined by Luke [89], his study did not investigate the model of Chan et al. [79] and more importantly, did not produce a clear assessment of the effects of beam diameter on the error in the diffusion length values determined from EBIC line-scan data. The results reported included a model to calculate EBIC data that takes beam diameter into account but his investigations were aimed at the methods and specific experimental results of other investigators rather than the accuracy of the diffusion length values extracted for a given range of known diffusion length and beam diameter values.

Therefore, the key difference between this work and that of Luke [89] is the systematic investigation of the effects of beam characteristics on the accuracy of diffusion length values determined from EBIC line-scan data.

(c) generation source
The simulations performed in this work use a generation volume created using Monte Carlo simulations of the electron-solid interactions. This is in contrast to the examples cited above which used a cubic volume with uniform electron-hole pair generation density [79] and a Gaussian-based analytical model that produces non-uniform generation density [81]. This approach was taken for reasons as outlined below.

A model that has the best agreement with the line-scan profiles obtained using the
Monte Carlo technique is a Gaussian-based analytical model proposed by Bonard and Ganière [80]. Kurniawan [76] showed that the EBIC line-scan profile calculated using the Bonard and Ganière model provides the closest approximation to the results calculated using Monte Carlo simulation data. However, there was good agreement in the results primarily because the parameters in the Bonard and Ganière model are obtained through the fitting of this model to the Monte Carlo data [76]. To use the Bonard and Ganière model in this work, the same number of Monte Carlo simulations plus an extra step of fitting the model to the Monte Carlo data would be required. Using the results of the Monte Carlo simulation directly requires one less step. Furthermore, the procedure and programming work required to insert an extended generation volume with non-uniform electron hole pair density into Synopsys® Sentaurus TCAD are the same regardless of the how the generation volume is obtained.

In the following section, the details of how the numerical simulations are performed are discussed. This is followed by discussions of the results in Section 7.3.2.

### 7.3.1 Details of Numerical Simulations

The simulation of the line-scan data involved the application of two different simulation software packages. First, a realistic extended generation volume that contains a spatially varying electron-hole pair generation rate (see Figure 7.2) was simulated using CASINO\(^{11}\) v2.42. CASINO is a software that performs Monte Carlo simulations of electron-solid interactions [90]. After the generation data was obtained, it was imported into Synopsys® Sentaurus TCAD, which was then used to simulate the steady-state electrical responses of a GaN Schottky diode under electron-beam bombardment.

The values of some of the important material, device and modelling parameters used in the simulations are listed in Table 7.1. The doping type and concentration chosen for the simulations corresponds to Sample 3 (p-type with \(N_A \approx 2.2 \times 10^{19} \text{cm}^{-3}\), see Table 4.1 on page 79) and Sample 4 (n-type with \(N_D \approx 3.6 \times 10^{16} \text{cm}^{-3}\), see Table 4.1 on pg. 79). For each doping concentration, a series of simulations were conducted for the range of diffusion lengths, beam voltage and diameter as well as surface recombination velocity.

---

\(^{11}\)CASINO is the acronym for monte CArlo SImulation of electronN trajectory in sOlid.
Figure 7.2. An example of the extended generation source generated by CASINO as viewed in Sentaurus Tecplot. The ohmic contact, which is positioned at the top-right hand corner of the structure, is not shown. The colour scheme of the generation volume indicates the different generation rates inside the volume, with red signifying the highest carrier density and the darkest shade of blue indicating the majority carrier thermal equilibrium density.

<table>
<thead>
<tr>
<th>Software</th>
<th>Parameter/Setting</th>
<th>Description</th>
<th>Value(s) [source]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASINO v2.42</td>
<td>Material</td>
<td>Density [gcm$^{-3}$]</td>
<td>n-GaN</td>
</tr>
<tr>
<td></td>
<td>Beam Characteristics</td>
<td>Beam Energy, $E_b$ [keV]</td>
<td>5 &amp; 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Beam Diameter, $d_b$ [nm]</td>
<td>20, 100, 200</td>
</tr>
<tr>
<td>Synopsys® Sentaurus</td>
<td>Material</td>
<td>Relative Permittivity, $\varepsilon_r$</td>
<td>9.5 [57]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electron Affinity [eV]</td>
<td>4.1 [59]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Band-gap, $E_g$ [eV]</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dopant</td>
<td>Si</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Doping concentration [cm$^{-3}$]</td>
<td>3.6×10$^{16}$</td>
</tr>
<tr>
<td>Device</td>
<td>Dimensions of GaN bulk material, width×thickness [µm]</td>
<td>30×100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Surface recombination velocity, $v_s$ [cm$^{-s}$]</td>
<td>0, 10$^5$, 10$^9$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Contact Resistance [Ω]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>Contact bias [V]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minority carrier diffusion length [nm]</td>
<td>10 - 1000</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1. A list of some of the important GaN material, device and modelling parameters used in the simulations.
values listed in Table 7.1. Many combinations of these parameters were used in the simulations of the electrical responses at various beam-to-junction distances to produce the EBIC line-scan data.

Similar to the simulations undertaken for the photocurrent technique, each simulation of the electrical characteristics of the device was performed on the basis of the drift-diffusion model. In addition, the device was unbiased and the resistance of the contacts was taken to be zero. Additional details relevant to the EBIC simulation setup can be found in Appendix C. Even though Appendix C contains information about the photocurrent simulations, most of the details described there (except for the parts associated with optical generation) are also applicable to the EBIC simulations.

### 7.3.2 Simulation Results and Discussions

Figure 7.3 shows examples of the fitting of Eqn. (7.3) to two sets of synthetic EBIC

![Graph](image)

**Figure 7.3.** Two examples of Eqn. (7.3) being fitted to normalised synthetic EBIC line-scan data obtained from the numerical simulations described in the previous section. Only those data points that were within the range of model validity were used (solid symbols).
line-scan data obtained from numerical simulations of a planar, n-type GaN Schottky diode (see Figure 7.1 for the structure used in the simulation).

For both datasets, the electron beam energy and diameter were 5keV and 100nm respectively. In addition, the surface recombination velocity was set to zero. The only difference between these datasets is the hole diffusion lengths, which were 200nm and 1000nm. Note that some of the data points were excluded in each fit because only those that satisfy the conditions $x > 2L_p$ and $x \gg w_g, v_s$ can be included. These data points are indicated by solid symbols in the figure.

In keeping with the notation used in the preceding chapters, the diffusion length values used to produce synthetic data or the actual values of a real physical sample are denoted by $L_p$ (or $L_n$ in the case of a p-type diode) while the diffusion length values estimated from fitting and analysis of either synthetic or experimental data are denoted by $L_{est}$. A summary of the notations used is given in Table 7.2. Note that the $x$-axis of Figure 7.3 represents the position of the centre of the electron beam relative to the edge of the depletion region (see the top view in Figure 7.1).

Figure 7.4 shows the discrepancies (in percentage) between $L_{est}$ and $L_{n,p}$ for both n-type and p-type GaN for various experimental conditions (doping concentration and beam energy) over a wide range of $L_{n,p}$ (10nm-1µm). The results presented are for $d_b = 100nm$ and $v_s = 0cms^{-1}$ (equivalent to no surface recombination). This figure shows that neither the doping type nor densities have a significant effect on the accuracy of $L_{est}$. Additionally, the results show that for $L_{n,p} \geq 75nm$, the percentage errors are more or

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{n,p}$</td>
<td>synthetic EBIC data the value of $L_{n,p}$ is known precisely. It is used in producing EBIC data from numerical simulations.</td>
</tr>
<tr>
<td></td>
<td>experimentally measured EBIC data the notation $L_{n,p}$ also represents the actual diffusion length value of a real, physical sample.</td>
</tr>
<tr>
<td>$L_{est}$</td>
<td>a value estimated from either synthetic or experimentally measured EBIC data using Eqn. (7.2).</td>
</tr>
</tbody>
</table>

Table 7.2. Definitions and descriptions of $L_{n,p}$ and $L_{est}$. 124
Figure 7.4. Percentage error in the estimated diffusion length values, $L_{\text{est}}$, as a function of the known diffusion length values set in the simulations, $L_{n,p}$. The estimated values were extracted from the simulated EBIC line-scan data using the method of Chan et al. [79]. The results shown are for both n-type and p-type GaN at beam voltages of 5keV and 20keV.

less constant and within 1% for both n-type and p-type GaN and both beam energies.

For the $E_b = 5$keV results, the accuracy of $L_{\text{est}}$ for $L_{n,p} < 50$nm deteriorates gradually with decreasing $L_{n,p}$ but the errors are not significantly different from the errors at longer $L_{n,p}$. At the shortest $L_{n,p}$ simulated, $L_{\text{est}}$ is within 2% of $L_{n,p}$. For the $E_b = 20$keV results, the accuracy of $L_{\text{est}}$ is only slightly worse than that for $E_b = 5$keV and it deteriorates gradually with decreasing $L_{n,p}$, from just under $-2\%$ when $L_{n,p} = 50$nm to between $-6\%$ and $-8\%$ when $L_{n,p} = 10$nm.

Note that, for all the results shown in Figure 7.4 as well as the ones to be presented in the next figure, the condition $x \gg w_{gv}$ was found to be unnecessary. It was found that the differences between the results produced by using data points that satisfy either $x \gg w_{gv}$ or $x > w_{gv}$, were negligible.
To determine the effects of beam diameter and surface recombination on the accuracy of $L_{\text{est}}$, the percentage error curves for various combinations of all the beam parameters and surface recombination values listed in Table 7.1 are plotted in Figure 7.5.

The results show that, for the ranges investigated, beam diameter and surface recombination velocity do not have a significant effect on the accuracy of $L_{\text{est}}$ obtained using the method of Chan et al. [79]. For each beam voltage, all the error curves shown have broadly similar shapes and trends as the corresponding curves shown in Figure 7.4.

Overall, these results show that for every combination of doping type and densities, beam characteristics and surface recombination velocities listed in Table 7.1, the diffusion length values extracted using the method of Chan et al. [79] are fairly accurate estimates of the true values even at very low sub-micrometer diffusion length values. The accuracy of $L_{\text{est}}$ is within 2% of the true value for $E_b = 5\text{keV}$ and 8% for $E_b = 20\text{keV}$.

That the beam diameter does not affect the results in a significant way is not surprising because the effect of the wider beam is to increase the dimensions of the electron-hole pair generation volume but the current decay depends mainly on the diffusion length.

The shape of the EBIC line-scan data also depends on the surface recombination velocity to a certain degree. However, this parameter also does not have any significant effect on the accuracy of $L_{\text{est}}$ because the model and method proposed by Chan et al. [79] accounts for it through the $\gamma$ parameter. Similarly, the doping density does not affect the accuracy significantly because the $\gamma$ parameter in the EBIC model also takes into consideration non-negligible depletion depth.

With these results, it can be concluded that the EBIC model and method for extracting diffusion length from EBIC line-scan data proposed by Chan et al. [79] can be applied to obtain reasonably accurate results even when the diffusion length is on the order of 10nm.

In terms of practical application, these results imply that it is not imperative to use narrow and low-voltage beams to obtain EBIC line-scan data. This is beneficial because such conditions may result in poor signal-to-noise ratio in cases where the diffusion
Figure 7.5. Percentage error in the estimated diffusion length values, $L_{\text{est}}$, as a function of the known diffusion length values set in the simulations, $L_{n,p}$. The estimated values were extracted from the simulated EBIC line-scan data using the method of Chan et al. [79]. The results shown are for both n-type and p-type GaN and for combinations of the beam parameters and surface recombination velocity values listed in Table 7.1. (a) Results for $E_b = 5$ keV. (b) Results for $E_b = 20$ keV.
length is very short, as is often the case for p-type GaN. Even though the results show that using $E_b = 5\text{keV}$ can produce more accurate results compared to using $E_b = 20\text{keV}$ when the diffusion length is very short, the need for accuracy must be balanced against the need for sufficient signal-to-noise ratio.

Therefore, in practice, an EBIC line-scan may be conducted at a low beam voltage initially. If the signal-to-noise ratio is found to be poor, the beam voltage is to be increased gradually until a signal of satisfactory strength is achieved. Note that by increasing the beam voltage, the sample will be probed at a greater depth from the surface. This does not present a problem if the diffusion length is uniform throughout the sample. In fact, probing at a greater depth from the surface helps to minimise any effects of surface recombination on signal strength.

However, if the diffusion length varies with depth, then an effective diffusion length that is an average of the volume being probed by the electron beam is obtained. A worse scenario would be if the diffusion length varies not just with depth but also along the scanning path. In such cases, the EBIC model investigated in this work may not be applicable since it assumes a uniform diffusion length.

Finally, because the beam diameter has no significant effect on the accuracy, this means that in practice, it is not necessary to determine its value. In the next section, the application of the model and results described in the preceding sections to actual samples will be presented and discussed.

### 7.4 Application to Experimental Results

The EBIC technique described in Section 7.2 was applied to the samples investigated using the photocurrent technique (see Chapter 4). Of these samples, two yielded useful line-scan data for extracting diffusion length. In the following sections, the experimental setup will be presented first, followed by the experimental results in Section 7.4.2 and discussion of the results in Section 7.4.3.
### 7.4.1 Experimental Setup

The setup used to perform EBIC measurements is shown in Figure 7.6(a). To obtain line-scan data, the Philips XL30 environmental scanning electron microscope was set to line-scan mode after a focused image of the area to be scanned was acquired. In the line-scan mode, the beam scans from left to right (see Figure 7.6(b)). Each line-scan contains 720 points and the electron beam dwells at each point for 66ms before moving to the next point.

A digital oscilloscope is used to record the EBIC signals that have been amplified by a Stanford Research Systems SR570 low-noise current preamplifier. The electron microscope is equipped with a BNC connector, which allows the amplifier input to be connected to the samples in the microscope chamber. Inside the chamber, wires bonded to the metal contacts of the samples are attached to leads soldered to the BNC connector. The circuit connections shown in Figure 7.6(b) follow the examples of Ioannou and Dimitriadis [83].

### 7.4.2 Results

Figure 7.7 (see pg. 131) shows the measured EBIC line-scan data for Sample 3 (p-type with $N_A \approx 2.2 \times 10^{19} \text{cm}^{-3}$, see Table 4.1 on pg. 79) and Sample 4 (n-type with $N_D \approx 3.6 \times 10^{16} \text{cm}^{-3}$, see Table 4.1 on page 79). Both data sets were obtained using a 5keV electron beam. Also shown in the figure are curves obtained by fitting the equation presented earlier

$$\log(k) + \gamma \log(x) - \frac{x}{L_{n,p}}$$

(7.3)

to the data points shown by the solid symbols. The selection of these data points are discussed below.

In applying the method proposed by Chan et al. [79], accurate results can be obtained provided that the data points used satisfy the conditions $x > 2L_{n,p}$ and $x > w_{ge}$.\(^{12}\) In the

\(^{12}\) In the previous sub-section, the condition $x \gg w_{ge}$ was found to be excessive and $x > w_{ge}$ is sufficient to obtain accurate results.
Figure 7.6. (a) Experimental setup used to conduct EBIC line-scan measurements. (b) Schematics showing the top view and cross-section views of an n-type and a p-type planar Schottky diodes (the shapes and sizes of important features such as the depletion region and the pear-shaped generation volume are exaggerated for illustration purposes). A separate cross-section view for each doping type is shown because the circuit connections are different. For the n-type diode, the Schottky contact is grounded. For the p-type diode, the ohmic contact is grounded. These connections follow the examples shown in [83]. The scanning path and direction are shown in both views.
Figure 7.7. Plots of experimental EBIC line-scan data for Samples 3 and 4, which were obtained using a beam energy of 5keV. The solid lines are obtained by fitting Eqn. (7.3) to sections of the experimental data that appear to be in a straight line; these points are indicated by the solid symbols.

In general, however, these will not be known. Therefore, the need to satisfy the first condition, $x > 2L_{n,p}$, requires knowledge of the diffusion length, which is precisely the quantity that the EBIC technique is being applied to determine. Hence, it appears that this method is not applicable in practice. In addition, because the simulation results show that beam diameter does not have a significant effect on the accuracy of the method of Chan et al. [79] thus making it unnecessary to measure the beam diameter, the need to know the dimensions of the electron-hole pair generation volume so that the condition $x > w_{gv}$ can be met would appear to negate this advantage.

However, as will be shown here, in practice, provided that the diffusion length is
uniform throughout the sample, or at least along the scanning path, then parts of the line-scan profile that Eqn. (7.3) should be fitted to can be readily identified from the shape of the line-scan profile and neither knowledge of diffusion length nor the size of the generation volume is necessary.

Typically, a line-scan profile has two distinct parts. The first part starts at \( x = 0 \) and is concave downwards with increasing \( x \). The second part (the remainder), usually appears to be a straight line from visual inspection. In Figure 7.7, the data points that form the second part of each line-scan profile are indicated by the solid symbol. This part of the line-scan profile is not necessarily linear. The concavity of this section of the profile is influenced by surface recombination and is accounted for in the model by the parameter \( \gamma \)[81].

In the first part, the induced current decays more slowly compared to the second part because at these \( x \) positions, a portion of the generation volume overlaps with the depletion region where all the carriers generated are collected as the induced current. As the beam moves further to the right and away from the edge of the Schottky contact, the proportion of the generation volume that remains in the depletion region decreases, resulting in a commensurate decrease in the induced current. Until the generation volume is completely outside the depletion region, the decay of the induced current is a function of both the amount of carriers generated in the depletion region and the diffusion length of the material. Once the generation volume is completely outside the depletion region, the rate of current decay becomes a function of the diffusion length and surface recombination. This decay with increasing distance away from the edge of the Schottky contact forms the second part of the line-scan profile.

Using the data points indicated by the solid symbols, the diffusion lengths of Samples 3 and 4 are found to be approximately 69nm and 139nm respectively. Note that because the electron beam diameter for each line-scan was not measured, the exact error curve (see Figure 7.5) to use to make corrections to these values also cannot be determined. However, as shown in Figure 7.5(a), the errors in the estimated diffusion length are expected to be within \( \pm 2\% \) of the true value for \( E_b = 5\text{keV} \). Therefore, the diffusion lengths of the samples are estimated to be between 68nm and 70nm for Sample 3 and 136nm and 142nm for Sample 4.
Using the EBIC technique, the electron diffusion length value for Sample 3 (p-type with $N_A \approx 2.2 \times 10^{19} \text{cm}^{-3}$) is estimated to be approximately 69nm. In contrast, the electron diffusion length value estimated using the photocurrent technique is approximately 4.3nm. This value is an order of magnitude less than the value determined using the EBIC technique.

One possible explanation for the difference in the diffusion length values estimated using both techniques is the electron injection-induced increase in the diffusion length. Chernyak et al. [91] reported a 3 to 6-fold increase in the electron diffusion length of their p-type GaN samples after they subjected the sample to multiple line-scans lasting a total of approximately 1500s. In their experiment, the duration of each line-scan is 16s, which means they subjected their sample to about 90 scans. They attributed this observation to the charging of deep levels associated with magnesium doping. Once a majority of these levels are charged, they are unable to capture electrons, thereby resulting in an increase in the electron diffusion length [91].

However, this explanation cannot fully account for the difference in the electron diffusion lengths obtained in this work. This is because care was taken to ensure that both samples were not subjected to prolonged scans. Both line-scan data shown in Figure 7.7 were obtained only after several scans and the duration of each scan was 47.5s.

To avoid exposing the area of interest on the sample to prolonged scans, at the beginning of every EBIC measurement conducted in this work, an area on the sample outside the area of interest was used to adjust the magnification and focus of the microscope. After a focused image at the desired magnification was obtained, the $x$-$y$ position of the sample stage was adjusted to move the area of interest into view. After an image of this area was obtained, the line-scan mode was activated.

The measurement was stopped after several line-scans confirmed that the same line-scan profile was obtained in each scan. Each sample was subjected to no more than three line-scans, which means the total scanning duration was less than 150s. Given these experimental conditions and the fact that it took approximately 1500s for the
diffusion length in the p-type MOCVD-GaN sample studied by Chernyak et al. [91] to increase 4-fold, it is unlikely that the p-type sample investigated in this work experienced the same level of electron injection-induced increase in diffusion length reported by Chernyak et al. [91]. And even if the same level of injection-induced increase was occurring, the diffusion length value obtained using the EBIC technique is about 16 times greater than the value obtained using the photocurrent technique as opposed to the 3 to 6-fold increase reported by Chernyak et al. [91].

Another explanation for the discrepancy between the photocurrent and EBIC results is the underestimation of diffusion length using the photocurrent technique due to recombination in the depletion region. As discussed in Section 6.1, Pulfrey et al. [59] showed that it is possible to vastly underestimate the electron diffusion length in p-type GaN using the photocurrent technique when a thin, damaged surface layer adjacent to the Schottky contact is present. It was postulated that this layer is created during the sputter-deposition of the Schottky barrier material onto the surface of GaN. Energetic postgrowth processes such as reactive-ion etching and metallisation using thermal evaporation or sputter deposition have been found to induce surface damage and defects that degrade the performance of GaN-based devices [66-69]. Therefore, the electron diffusion length value of 4.3nm is likely to be lower than the true value.

In the EBIC technique, the minority carriers generated by the electron beam also have to traverse the same depletion region that contains the damaged surface layer. Therefore, it is possible that the same undesirable recombination in the depletion region also has a detrimental effect on the accuracy of this technique. The electron injection-induced increase in diffusion length and recombination in the depletion region have opposite effects on the estimation of the diffusion length. In the absence of the undesirable recombination in the depletion region, the electron injection-induced increase in diffusion length will result in the overestimation of the diffusion length. And the opposite is true when the situation is reversed. In reality, it is likely that both of these competing effects are present and it is unknown which has a more significant impact.

Given the uncertainties surrounding the EBIC results, the impact of both effects on the accuracy of this technique warrants further investigation. However, because the primary focus of this work is the photocurrent technique and the EBIC technique is employed as
a supplementary technique, such an investigation is beyond the scope of this work and therefore, it will not be pursued further.

For Sample 4 (n-type with $N_D \cong 3.6 \times 10^{16} \text{cm}^{-3}$), the hole diffusion length estimated using the EBIC technique is approximately 139nm. Note that unlike the p-type sample, there is no photocurrent technique-derived diffusion length value for comparison. In Chapter 4, it was found that the photocurrent technique is unsuitable for the n-type sample because the depletion depth is sufficiently wide such that all the photons are absorbed in the depletion region and the photocurrent is made up of the drift current only (see Section 4.3). The hole diffusion length value obtained here is expected to be a good estimate of the true value for the following reasons.

Firstly, recombination in the depletion region is not expected to be significant compared to the p-type samples. For the n-type sample, the Schottky barrier metal was deposited onto the sample via thermal evaporation as opposed to sputtering for the p-type samples. As thermal evaporation is less energetic compared to sputtering, less damage to the surface of GaN is expected.

The experimental and calculated responsivities of Sample 4 presented in Figure 4.3, which is reproduced on the next page for convenience, show that the experimental data and the calculated curve match fairly well. In addition to the assumption that there is no diffusion current, these calculations also assumed that all the carriers generated in the depletion region are collected and contribute to the drift current. Therefore, the good agreement between the experimental and calculated responsivities suggests that any recombination in the depletion region due to a thin, damaged surface layer under the Schottky contact caused by thermal evaporation of the contacts is negligible.

Secondly, unlike the p-type GaN, Chernyak et al. [91] found that n-type GaN do not experience the same electron injection-induced increase in hole diffusion length. In addition, during the measurement of Sample 4, the same precaution was taken to avoid repetitive and prolonged scans. Therefore, the hole diffusion length of Sample 4 is expected to be accurate to within 2% of the true value (see Figure 7.5(a) for the simulation-derived percentage error curves for 5keV beams).

In conclusion, the first and primary objective of investigating and applying the EBIC
Figure 4.3. Plots of responsivity versus wavelength for the n-GaN sample (Sample 4). The square symbol represents data obtained from measurements. The solid line was obtained from calculation that assumed the measured current consisted of drift current only.

As for the secondary objective, which is to provide comparisons for the electron diffusion length values obtained using the photocurrent technique, uncertainties about the extent to which two competing effects – electron injection-induced increase in diffusion length and undesirable recombination in the depletion region due to a thin, damaged surface layer under the Schottky contact – affect the diffusion length value obtained means that it is not possible to make any meaningful comparison between the results obtained using both techniques. Therefore, additional investigations into how these effects affect the accuracy of the EBIC is required. However, as the principal focus of this work is the application of the photocurrent technique to the determination...
of diffusion length in GaN, such investigations are outside the scope of this work.

Despite these limitations, the following conclusions can be made about the EBIC technique. Firstly, the simulation results presented in this chapter showed that the EBIC technique described in this work can be used to determine short diffusion lengths to within 2% of the true value, well below the lower limit of 3µm investigated by Chan et al. [79] and Kurniawan and Ong [81]. Secondly, if the undesirable recombination in the depletion region is excluded from consideration, it can also be concluded that the EBIC technique described in this chapter is better suited to determining diffusion length in n-type GaN because the electron injection induced increase in diffusion length affects p-type GaN only.

### 7.5 Chapter Summary

In this chapter, the Electron-Beam Induced Current or EBIC technique for determining minority carrier diffusion length was investigated as an alternative to the photocurrent technique. Specifically, the EBIC technique considered is the line-scan method and diffusion length was extracted from line-scan data using the model and method proposed by Chan et al. [79]. This method involved fitting the line-scan data to an equation that contains three fitting parameters, one of which is diffusion length. Discussions of this model were presented in Section 7.2.

In Section 7.3, the accuracy of the model when it is applied to determine short diffusion lengths in the sub-micrometer range, which is often the case for GaN, was investigated via numerical simulations. This investigation involved the variation of the size of the depletion region (by changing the doping concentration rather than applying a biasing voltage), beam voltage, beam diameter, surface recombination velocity and diffusion length to determine the effects of each parameter on the accuracy of this technique.

The results showed that for every combination of the parameters mentioned above, the diffusion length values extracted are fairly accurate estimates of the true values even at very low sub-micrometer diffusion length values. The accuracy of the estimated diffusion length is within 2% and 8% of the true value when the beam voltage is 5keV and 20keV respectively. Based on these results alone, it can be concluded that the EBIC
model and method for extracting diffusion length from EBIC line-scan data proposed by Chan et al. [79] can be applied to obtain reasonably accurate results even when the diffusion length is on the order of 10nm, which is well below the lower limit of 3µm investigated by Chan et al. [79] and Kurniawan and Ong [81]. Additionally, given that the accuracy is insensitive to the beam diameter, the results also suggest that in practice, it is not imperative to know the size of the electron beam.

Following these investigations, the EBIC technique was applied to the samples examined in this work. However, only Samples 3 (p-type with \(N_A \cong 2.2 \times 10^{19} \text{cm}^{-3}\)) and 4 (n-type with \(N_D \cong 3.6 \times 10^{16} \text{cm}^{-3}\)) yielded useful data. The diffusion length estimated using the EBIC technique for the p-type sample was found to be an order of magnitude greater than the value obtained using the photocurrent technique. Two competing effects were discussed as possible explanations for the discrepancy.

The first is the overestimation of the diffusion length due to the electron injection-induced increase in diffusion length effect reported by Chernyak et al. [91]. This effect was attributed to the charging of deep levels associated with magnesium doping which caused these levels to be unable to capture electrons, thereby resulting in an increase in the diffusion length. The second effect, which was discussed earlier in Section 6.1, is the underestimation of the true diffusion length value due to the presence of undesirable recombination caused by a thin, damaged surface layer in the depletion region.

Because the photocurrent technique was inapplicable to the n-type sample, there is no comparison for the diffusion length value determined using the EBIC technique. However, the estimated value is expected to be accurate because n-type GaN does not experience an electron injection-induced increase in diffusion length and the undesirable recombination in the depletion region was found to be insignificant.

Overall, the first and primary objective of applying the EBIC technique, which is to determine the diffusion length of the n-type sample investigated in this work using an alternative technique to the optical method, has been met. As for the secondary objective, which is to provide comparisons for the electron diffusion length values obtained using the photocurrent technique, uncertainties about the extent to which the two competing effects affect the diffusion length value obtained means that it is not possible to make any meaningful comparison between the results obtained using both
techniques. The third objective, which is to validate this technique for application to GaN, has also been met.
CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Summary of Thesis Objectives

The objective of this thesis was to investigate the accuracy of the photocurrent technique used in the estimation of diffusion length where the diffusion length is expected to be low sub-micrometer in length as found in, for example, GaN semiconductor layers. The bulk of this thesis was concerned with the standard mathematical model of this technique for a Schottky barrier diode test structure

\[
J_{\text{total}} = qT_0 \Phi_0 \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_{n,p}} \right)
\]  

(2.15)

and its accuracy and applicability with respect to GaN. Specifically, the objectives of this thesis were to determine:

1. the validity of the following four assumptions made in the derivation of the standard mathematical model with respect to GaN:
   (a) the minority carrier density at the depletion edge retains its equilibrium value;
   (b) the absorption coefficient, \( \alpha \), is uniform throughout the material, including in the depletion region;
   (c) there is no recombination in the depletion region; and
   (d) the photocurrent consists of the minority carrier current only.
2. the effects of the validity or otherwise of each assumption on the accuracy of the diffusion length values in GaN calculated using the standard mathematical model.
3. overall validity of the standard mathematical model with respect to GaN.
4. the applicability of the photocurrent technique to GaN.

In addition to the photocurrent technique, the Electron-Beam Induced Current or EBIC technique was also investigated and applied to GaN in Chapter 7. The objectives of investigating and applying this technique were to:
(1) determine the diffusion length of the n-type GaN sample studied in this work due to the inapplicability of the photocurrent technique to this sample.
(2) provide comparisons for the electron diffusion length values obtained using the photocurrent technique.
(3) determine the ability of this technique to produce accurate diffusion length values at the lower end of the sub-micrometer range.

8.2 Summary of Results – The Photocurrent Technique

The key outcomes of the work undertaken in this thesis on the photocurrent technique are listed below.

- Objectives 1 and 2
  The validity of the four assumptions and their effects on the accuracy of the photocurrent technique with respect to GaN.

  (a) Assumption 1
  The minority carrier density at the depletion edge retains its equilibrium value. In effect, this assumption means that the photo-generated minority carriers arriving at the depletion edge are swept away instantaneously by the built-in electric field or that the velocity at which minority carriers are swept away from the depletion edge plane towards the Schottky contact, \( v_1 \), is much greater than the diffusion velocity, \( v_d \).

Results

In Chapter 3, an in-depth analysis of this assumption, which has not been performed on GaN or any material for that matter prior to this work, showed that this assumption and therefore, the common practice in solving the continuity equation whereby the minority carrier density at the depletion edge plane is set to its thermal equilibrium value, is technically incorrect, especially when the diffusion length is short. Nevertheless, it was demonstrated that in practice, knowledge of \( v_1 \) and the application of a more complete equation that relates both \( v_1 \) and the diffusion length to the measured photocurrent do not necessarily lead to a more accurate diffusion length value. In fact, in some cases, the
standard mathematical model can produce more accurate results. Even when that is not the case, the standard mathematical model is still recommended for GaN because it is more convenient to apply in practice and it was found that the error that results from using it for diffusion lengths in the 10nm-1µm range is not very severe. Because the simulation results produced in this work are for GaN, this conclusion is applicable to this material only. For other materials with short diffusion lengths in the sub-micrometer range, it is recommended that similar simulations be performed at these diffusion lengths to estimate the errors that result from using the standard mathematical model.

In the course of the investigations that led to the findings summarised above, an analytical model that provides a better description of the relationship between \( v_1 \) and material parameters was also developed. This model showed that \( v_1 \) is a function of temperature, material permittivity, doping concentration, mobility, depletion width, absorption coefficient and diffusion length. Using results calculated for Si and GaN, it was shown that the model for calculating \( v_1 \) proposed by Card [53], which assumed that the main contribution to the photocurrent came from the neutral region only, represents the asymptotic limit of the model developed in this work. In other words, the model developed here is a more general model. Although it was shown subsequently in Section 3.4 that this model overestimates the value of \( v_1 \) by at least an order of magnitude (see Figure 3.7), the significance of this model is that it confirms the dependence of \( v_1 \) on diffusion length and absorption coefficient observed in the simulation results. The discrepancies between the results produced using this model and the numerical simulations can be attributed to the differences between the analytical and numerical treatments of the Schottky junction. The latter is more accurate because it can fully capture the characteristics of the Schottky diode whereas the former is limited by certain assumptions, such as a clearly defined boundary between the depletion and quasi-neutral regions, that are invalid in actual devices but are necessary for mathematical tractability.

(b) Assumption 2

The optical absorption coefficient, \( \alpha \), is uniform throughout the material, including in the depletion region.
Results

It was shown in Chapter 5 that this assumption is not always valid with respect to GaN. For the three p-type GaN samples studied in this work, the absorption coefficient values in the depletion region can differ from the zero-field value found in the quasi-neutral bulk region of the material. In fact, the absorption coefficient in the depletion region exhibits electric-field and wavelength-dependent oscillations about its zero-field value. Because the electric field decreases with distance away from the Schottky contact plane, the absorption coefficient also varies spatially in the depletion region. This non-uniformity of the absorption coefficient is a result of the modification of the absorption coefficient in the depletion region by the built-in electric field. Due to the high doping density of each sample, which is on the order of $10^{19}$ cm$^{-3}$, the magnitude of the electric field is sufficiently large to give rise to an appreciable difference between the absorption coefficient values in the depletion and quasi-neutral bulk regions. The modification of the absorption coefficient by an electric field is known as the Franz-Keldysh effect.

Due to this effect, the diffusion length values calculated from experimentally-measured spectral photocurrent data for the three p-type GaN samples exhibited a minor but perceptible wavelength-dependence. It was shown in Section 5.2 that this apparent spectral variation is an artefact that can be explained by the Franz-Keldysh effect and that by using a modified version of the standard mathematical model, that is, Eqn. (5.7), a wavelength-independent diffusion length value can be produced. Overall, comparisons of the diffusion length values obtained with and without consideration of the Franz-Keldysh effect showed that this effect does not have a significant impact on the accuracy of the diffusion length values obtained from spectral photocurrent measurements of heavily-doped p-type GaN samples. An exception to this is when the diffusion length is sufficiently short such that measurements using photons of energy close to the band-gap energy produce erroneous negative diffusion length values. When this occurs, measurements should be repeated at shorter wavelengths to verify the results.

(c) Assumption 3

There is no recombination in the depletion region. This assumption refers to
recombination in the depletion region that is in addition to those in the transition layer that is approximately 4-6 extrinsic Debye lengths in width.

**Results**

As discussed in Section 6.1, this assumption is also not always valid with respect to GaN. Recombination in the depletion region in addition to those in the transition layer may be present as a result of damage created on the top surface layer of GaN during energetic post-growth processing such as the sputter deposition of Schottky contact. This thin, damaged surface layer can result in a significant underestimation of the diffusion length when the standard mathematical model is used because the recombination in the depletion region alters the relative proportion of the calculated drift and diffusion currents with respect to their actual values. Therefore, it is worth keeping this in mind when the photocurrent technique is applied to GaN samples that have been subjected to energetic post-growth processes. When the value of diffusion length calculated is very low, the results should be treated with caution and confirmation with an alternative technique should be obtained.

(d) Assumption 4

The induced photocurrent consists of the minority carrier current only.

**Results**

In Section 6.2, it was shown, through both analytical and numerical means, that this assumption is also technically incorrect. However, it was also found that, as far as the GaN samples studied in this work are concerned, the minority carrier current is significantly greater than the majority carrier current and therefore, it is safe to assume that the measured photocurrent is made up of minority carrier current only. Nevertheless, because the reduction of the total photocurrent due to a non-zero majority carrier current is expected to be more significant in direct band-gap materials [52] such as GaN, when applying the standard mathematical model to determine diffusion length in lightly-doped GaN, the presence of non-zero majority carrier current should be investigated especially if the calculated diffusion length is very short or is negative.
Objectives 3 and 4
The overall validity of the standard mathematical model and the applicability of the photocurrent technique to GaN

Results
Because all four assumptions were not always valid for GaN, the standard mathematical model is also not always valid. The invalidity of the assumption of no recombination in the depletion region was found to have the most significant negative impact on the accuracy of the diffusion length value calculated using the model while the invalidity of the other three assumptions did not have a significant detrimental effect. Overall, caution is recommended when applying the standard mathematical model to GaN samples, particularly if the samples have been subjected to energetic post-growth processes. When the value of diffusion length calculated is very low or negative, the results should be suspected and confirmation with an alternative technique should be obtained.

In addition, the photocurrent technique cannot be applied to a GaN sample with a wide depletion region because the high absorption coefficient of the material means that all the photons will be absorbed in the depletion region and the photocurrent is then made up of the drift current only. Therefore, this technique is limited to GaN samples (both p- and n-type) with sufficiently high doping concentration that enough photons can reach the neutral region and induce a diffusion current. To determine the diffusion lengths of lightly-doped samples that do not meet this requirement, an alternative characterisation technique such as the EBIC technique is required.

It was also found that the simplification of the standard mathematical model commonly found in the literature, which requires that the condition $\alpha x_d \ll 1$ is satisfied, thereby making knowledge of some of the material or sample parameters unnecessary, is inapplicable to GaN because of the high absorption coefficient of this material. Consequently, it was concluded that when applying the photocurrent technique to GaN, the standard mathematical model must be used without additional simplification. It should be noted that the limitations mentioned in this and the previous paragraph are true for any semiconductor material with wide depletion widths and high absorption coefficients and not just for GaN or the
8.3 Summary of Results – The EBIC Technique

The key outcomes of the work undertaken in this thesis on the EBIC technique are listed below.

- **Objective 1**
  To determine the diffusion length of the n-type GaN sample studied in this work due to the inapplicability of the photocurrent technique to this sample.

  **Results**
  Application of this technique to the n-type sample yielded a hole diffusion length value that is expected to be a good estimate of the true value. This is because, unlike p-type GaN, n-type GaN does not experience an electron injection-induced increase in diffusion length and the undesirable recombination in the depletion region was found to be insignificant.

- **Objective 2**
  To provide comparisons for the electron diffusion length values obtained using the photocurrent technique.

  **Results**
  Application of the EBIC technique to one of the p-type samples studied in this work found the diffusion length to be an order of magnitude greater than the value obtained using the photocurrent technique. There are two possible reasons for this discrepancy. Firstly, the higher value obtained using the EBIC technique could be the result of the overestimation of the diffusion length due to the electron injection-induced increase in diffusion length effect reported by Chernyak et al. [91]. Secondly, the lower value obtained using the photocurrent technique can be attributed to undesirable recombination caused by a thin, damaged surface layer in the depletion region of the p-type sample. Additionally, because the minority carriers generated by the electron beam also have to traverse the same depletion region that contains the damaged surface layer, it is possible that the same
undesirable recombination in the depletion region also has a detrimental effect on the accuracy of the EBIC technique. It is likely that both of these competing effects are present and because their combined effects on the accuracy of the EBIC technique are unknown, it is difficult to make any meaningful comparison between the results obtained using the photocurrent and EBIC techniques. Hence, when applying the EBIC technique to p-type GaN that has been subjected to energetic post-growth processes to create electrical contacts, such as in the case of the p-type samples studied in this work, considerations need to be given to the aforementioned effects and an alternative, contactless technique may be considered.

**Objective 3**
To determine the ability of this technique to produce accurate diffusion length values at the lower end of the sub-micrometer range.

**Results**
The accuracy of this technique at the lower end of the sub-micrometer range, which had not been investigated prior to this work, showed that the diffusion length values extracted are fairly accurate estimates of the true values even at very low sub-micrometer diffusion length values. The accuracy of the estimated diffusion length is within 2% and 8% of the true value when the beam voltage is 5keV and 20keV respectively. Based on these results alone, it can be concluded that the EBIC model and method for extracting diffusion length from EBIC line-scan data proposed by Chan et al. [79] can be applied to obtain reasonably accurate results even when the diffusion length is on the order of 10nm. Additionally, given that the accuracy is insensitive to the beam diameter, the results also suggest that in practice, it is not imperative to know the size of the electron beam.

**8.4 Future Work**

Further work is required in the following areas:

- In this thesis, the Schottky barrier diodes in the simulations and the real, physical samples measured have neutral regions that are much thicker than the diffusion and absorption lengths. In cases where the active region is a very thin epitaxial layer, the
standard mathematical model is not suitable because the coefficient, $C_2$, in the expression for the electron density distribution function, that is, Eqn. (2.9), is not equal to zero. Moreover, reflection at the semiconductor-substrate interface comes into play and must be accounted for in the expression for the optical generation profile. Therefore, modifications to boundary conditions are necessary to accommodate thin epitaxial layers and the resulting expression for the photocurrent is more complicated than Eqn. (2.15). In order to determine the accuracy of the photocurrent technique at the lower end of the sub-micrometer range, the investigations and analysis carried out in this thesis should be extended to samples with a very thin epitaxial layer.

- The additional recombination in the depletion region due to surface layer damage, which was discussed in Section 6.1, requires further investigation. In particular, the relationship between the nitrogen vacancies created during post-growth processing and the mobility and lifetime needs to be investigated so that the effects of this recombination on the diffusion length values calculated can be quantified.

- Due to the high activation energy of Mg in GaN, the energy level of this dopant can be considered a deep level. This results in the $\lambda$-effect, which in essence, is the extension of the depletion region by a transition region, the width of which is known in literature as $\lambda$. The right-hand plane of this transition region is defined by the edge of the neutral region while the left hand plane is defined by the point at which the deep level intersects with the thermal equilibrium Fermi level in the depletion region. The width of this transition is given by $\lambda = \sqrt{2\varepsilon_s (E_T - E_F) / q^2 N_A^-}$, where $\varepsilon_s$, $E_T$, $E_F$, $q$ and $N_A^-$ are the semiconductor material permittivity, acceptor energy level, thermal equilibrium Fermi level, electronic charge and ionized acceptor concentration respectively. As an example, using a Mg level in GaN of approximately 170meV above the valence band, the value of $\lambda$ was calculated to be approximately 20nm for one of the p-type GaN samples investigated in this work, namely, Sample 1, which has an equilibrium hole concentration of approximately $3 \times 10^{17} \text{cm}^{-3}$. The width of this transition region is roughly equal to the idealised depletion width, $x_{da} \approx 20\text{nm}$ and therefore represents a doubling of the depletion region. As shown in Chapter 3, the presence of a transition region means that the actual relative proportion of the drift and diffusion currents differ from the
proportion calculated when it is assumed that the boundary of the depletion and neutral regions are clearly defined. Therefore, the $\lambda$-effect can degrade the accuracy of the photocurrent technique, especially if diffusion is very short. Therefore, this effect should be considered when applying the photocurrent technique to Mg-doped p-type GaN or any semiconductor material with deep level traps. Note that the $\lambda$-effect was not considered in the simulations conducted in this work. Hence, this effect should also be included in future simulation work.

- The two competing effects discussed in Section 7.4.3, namely, the over-estimation of diffusion length in p-type GaN due to the electron injection-induced increase in diffusion length and the under-estimation of the true diffusion length value due to the presence of undesirable recombination caused by a thin, damaged surface layer in the depletion region, also require further investigation. Specifically, simulations that combine these two effects should be carried out to determine which effect has a more significant impact on the diffusion length value of p-type GaN.

- Because the EBIC model used in this thesis assumes that the diffusion length is uniform over the entire material, or at least in the volume being probed along the scanning path, it would be beneficial to perform EBIC simulations where the diffusion length varies with depth as well as along the scanning path to determine the effects of such variation in diffusion lengths on the accuracy of the EBIC model used in this thesis.

### 8.5 Concluding Remarks

In this thesis, it has been shown that the validity of the mathematical model used in any technique is important. However, it is not the sole determinant of the accuracy of the quantitative results obtained using a particular technique. It also depends on the accuracy of the various parameters that are not measured by the technique. In the case of the photocurrent technique, the absorption coefficient, Schottky barrier contact transmission factor, photon flux density and depletion width are also required in the calculation of diffusion lengths using the standard mathematical model. Therefore, in addition to ensuring that the conditions for which the mathematical model is valid are met, attention to the accuracies of these parameters is also required. Ideally, these
parameters should be measured for each sample being investigated as the values of these parameters can vary between samples. However, in this work, not all the parameters were measured. For expedience, the absorption coefficient values were obtained from the literature [46] and the transmission factors of the Schottky barrier contact of the n-type sample was calculated using published optical constants of nickel and gold.

This thesis has investigated in detail, for the first time, the validity of the standard mathematical model of the photocurrent technique in a material where the diffusion length can be very short, specifically GaN. It was shown that, in general, with respect to GaN, the model is not always valid because four of the assumptions used in the derivation are not always valid. To assist the application of this technique to GaN, the general recommendations that result from the investigations conducted in this work are:

- In general, the standard mathematical model is recommended over the more exact one for GaN because it is more convenient to apply. For other materials with short diffusion lengths in the sub-micrometer range, similar simulations should be performed to estimate the errors that result from using the standard mathematical model.

- Avoid photons with energy close to the band-gap as this may result in erroneous negative diffusion length values due to the Franz-Keldysh effect. Using photons with energy well-above the band-gap also avoids absorption processes other than direct band-to-band transitions, which can invalidate the unity quantum yield assumption.

- For GaN samples that have been subjected to energetic post-growth processes and therefore have a damaged surface layer that degrades the accuracy of this technique, the results obtained using the standard mathematical model should be treated with caution and an alternative technique that is not affected by or does not require the sample processing steps that create the damaged, surface layer should be considered. The same caution and consideration should be made if this technique is applied to other materials that are also expected to be damaged by any sample preparation steps.
For direct band-gap materials such as GaN, the presence of non-zero majority carrier current should be investigated for lightly-doped samples, especially if the calculated diffusion length is very short or is negative.

The other assumptions of the model such as a neutral region that is much thicker than the diffusion length and absorption length, low-level injection condition and unity quantum yield are not investigated. This is because the validity of these assumptions can be achieved through appropriate sample selection, for example, a sample with a thick neutral region, and by applying the correct experimental conditions, for example, using an optical injection source with the right intensity level to achieve low-level injection and not using wavelengths close to the band-edge to avoid absorption processes other than direct band-to-band transition that leads to non-unity quantum yield.

In addition to the work carried out on the photocurrent technique, a systematic investigation of the effects of beam characteristics on the ability of the EBIC technique to accurately detect short diffusion lengths at the lower end of the sub-micrometer range was also conducted. The investigations reported in this work differ from those conducted in the past by considering diffusion length values that are well below 1µm and by directly using a realistic generation source produced using Monte Carlo simulations rather than analytical equations that approximate the shape of the generation volume. The results of this investigation showed that the accuracy of the estimated diffusion length is within 10% of the true value. However, in practice, the accuracy of this technique is degraded in p-type GaN due to an electron-injection induced increase in the diffusion length and the underestimation of the true diffusion length value due to the presence of undesirable recombination caused by a thin, damaged surface layer which is commonly present in the depletion region. Because of the uncertainties about the extent to which these two competing effects affect the diffusion length value obtained using the EBIC technique, it is not possible to determine if the electron diffusion length value obtained is a good estimate of the true value and therefore it is not possible to make any meaningful comparisons between the photocurrent and EBIC techniques. Nevertheless, overall, the results presented in this thesis should assist the application of both the photocurrent and EBIC techniques to GaN by highlighting the precautions and considerations required in interpreting the diffusion length results obtained.
APPENDIX A

Derivation of the Equation for the Excess Electron Density Distribution, \( \Delta n(x) \), in the Depletion Region – Eqn. (3.22)

In this section, the derivation of the electron density distribution, \( n(x) \), in the depletion region of a zero-biased, p-type Schottky diode under constant, monochromatic illumination is presented. The expression for \( n(x) \), which is given by

\[
n(x) = N_c e^{E_C(x)/kT} \left\{ 1 + \frac{\pi}{2} \frac{L_D J_n}{\mu_n kT n_p} \right\} \times \left[ \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} \right) - \text{erf} \left( \frac{x_d}{\sqrt{2}L_D} - \frac{x}{\sqrt{2}L_D} \right) \right]
\]

was derived on the basis of the model set out in [53,54]. In this model, recombination in the depletion region is assumed to be negligible and the electron current, \( J_n \), is taken to be equal to the total current, \( J_{\text{total}} \), as well as independent of position in this region [53,54].

The electron current density, \( J_n \), generated by a p-type Schottky diode under constant, monochromatic illumination is the sum of the drift and diffusion currents

\[
J_n = J_{\text{drift}} + J_{\text{diff}}
\]

\( J_n \) can also be expressed in terms of the electron quasi-Fermi level

\[
J_n = \mu_n n(x) \frac{dE_{F_n}(x)}{dx}
\]

where \( \mu_n \) and \( E_{F_n}(x) \) are the electron mobility and quasi-Fermi level respectively. \( E_{F_n}(x) \) is defined by
\[
n(x) = N_C e^{-\left[E_C(x) - E_{fn}(x)\right]/kT}
\]  \(\text{(A3)}\)

where \(n(x)\), \(N_C\), \(k\) and \(T\) are the electron density distribution, conduction band density of states, Boltzmann’s constant and temperature respectively. \(E_C(x)\) is the conduction band energy as a function of position, \(x\), and on the basis of the depletion approximation, it is given by

\[
E_C(x) = q\phi_n - \frac{q^2 N_A (x^2 - 2x_d x)}{2\varepsilon_s}; \quad 0 \leq x < x_d
\]  \(\text{(A4)}\)

\[
E_C(x) = q\phi_n + \frac{q^2 N_A x_d^2}{2\varepsilon_s}; \quad x \geq x_d
\]  \(\text{(A5)}\)

where \(q\), \(x_d\), \(N_A\), and \(\varepsilon_s\) are the electronic charge, depletion width, doping concentration and permittivity respectively. The product \(q\phi_n\) is the value of the conduction band energy at \(x = 0\) and is given by the difference between the band-gap energy, \(E_g\), and the Schottky barrier height, \(q\phi_{\text{bp}}\).

Substituting Eqn. (A3) into Eqn. (A2) yields

\[
J_n = \mu_n N_C e^{-\left[E_C(x) - E_{fn}(x)\right]/kT} \frac{dE_{fn}(x)}{dx}
\]  \(\text{(A6)}\)

Rearranging so that only the terms containing \(E_{fn}(x)\) remain on the right hand side, Eqn. (A6) becomes

\[
\frac{J_n}{\mu_n N_C e^{E_C(x)/kT}} = e^{E_{fn}(x)/kT} \frac{dE_{fn}(x)}{dx}
\]  \(\text{(A7)}\)

By taking \(\mu_n\) to be independent of \(x\) and using

\[
e^{E_{fn}(x)/kT} \frac{dE_{fn}(x)}{dx} = kT \frac{d}{dx} e^{E_{fn}(x)/kT}
\]  \(\text{(A8)}\)

in Eqn. (A7) followed by integration from 0 to \(x\), the resulting expression is
\[ e^{E_{Fp}(x)kT} = 1 + \frac{J_n}{\mu_n kTN_C} \int_0^x e^{E_C(u)kT} du \]  
(A9)

where \( u \) is a dummy variable. Using Eqn. (A4), the integral in Eqn. (A9) becomes

\[
H(x) = \int_0^x e^{E_C(u)kT} du = \int_0^x e^{q\phi_e/kT - q^2N_{\lambda}(u^2 - 2x_\lambda u)/2e_kT} du
\]  
(A10)

Completing the squares and factoring,

\[
H(x) = -e^{q\phi_e/kT + q^2N_{\lambda}x_\lambda^2/2e_kT} \int_0^x e^{-q^2N_{\lambda}(u - x_\lambda)^2/2e_kT} du
\]  
(A11)

In Eqn. (A11), the exponent in the exponential term outside the integral is equivalent to \( E_C(x_d)/kT \), therefore

\[
H(x) = -e^{E_C(x_d)/kT} \int_0^x e^{-q^2N_{\lambda}(u - x_\lambda)^2/2e_kT} du
\]  
(A12)

Setting

\[
s = x_d - u
\]  
(A13)

and

\[
g = \sqrt{\frac{q^2N_{\lambda}}{2e_kT}}
\]  
(A14)

Eqn. (A12) becomes

\[
H(x) = e^{E_C(x_d)/kT} \int_{x_d}^{x_d - s} e^{-g^2s^2} ds
\]  
(A15)

By making another change of variables
\[ t = gs \] 

\[ H(x) \text{ can be expressed as} \]

\[
H(x) = \frac{1}{g} e^{E_c(x)/kT} \int_{g_{x_d}}^{s(x_d-x)} e^{-t^2} dt
\]

(A17)

Using

\[
\int_{g_{x_d}}^{s(x_d-x)} e^{-t^2} dt = \int_{0}^{s(x_d-x)} e^{-t^2} dt - \int_{0}^{g_{x_d}} e^{-t^2} dt
\]

(A18)

and the error function

\[
\text{erf}(\theta) = \frac{2}{\sqrt{\pi}} \int_{0}^{\theta} e^{-t^2} dt
\]

(A19)

Eqn. (A17) becomes

\[
H(x) = \frac{\sqrt{\pi}}{2g} e^{E_c(x)/kT} \left[ \text{erf}(g_{x_d}) - \text{erf}(g_{x_d} - gx) \right]
\]

(A20)

Inserting Eqn. (A20) into Eqn. (A9) leads to

\[
e^{E_{p_m}(x)/kT} = 1 + \frac{\sqrt{\pi} J_n [\text{erf}(g_{x_d}) - \text{erf}(g_{x_d} - gx)] e^{E_c(x)/kT}}{2g\mu_n kTN_C}
\]

(A21)

Therefore, \( n(x) \) can be expressed as

\[
N_C e^{-E_c(x)/kT} \left\{ 1 - \frac{\sqrt{\pi} J_n [\text{erf}(g_{x_d}) - \text{erf}(g_{x_d} - gx)] e^{E_c(x)/kT}}{2g\mu_n kT} \right\}
\]

(A22)

Setting the metal Fermi level as the zero level of energy, \( n(x) \) can be simplified to
\[ n(x) = N_C e^{-E_C(x)/kT} \left\{ 1 + \sqrt{\pi} J_n \frac{[\text{erf}(gx_d) - \text{erf}(gx_d - gx)]}{2g\mu_n kT n_\rho} \right\} \]  \hspace{1cm} (A23)

where \( n_\rho \) is the thermal equilibrium electron density. Note that \( g \) can be expressed in terms of the extrinsic Debye length, \( L_D \), as

\[ g = \sqrt{\frac{q^2 N_A}{2e_s kT}} = \frac{1}{\sqrt{2L_D}} \]  \hspace{1cm} (A24)

Therefore,

\[ n(x) = N_C e^{-E_C(x)/kT} \left\{ 1 + \sqrt{\frac{\pi}{2}} \frac{L_D J_n}{\mu_n kT n_\rho} \right\} \times \left[ \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) - \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} - \frac{x}{\sqrt{2L_D}} \right) \right] \]  \hspace{1cm} (3.19)

Evaluation of Eqn. (3.19) at the \( x = x_d \) plane and substituting \( n_\rho = N_C e^{-E_C(x_d)/kT} \) from the result leads to Eqn. (3.22)

\[ \Delta n(x_d) = \frac{\pi}{2} \frac{L_D J_n}{\mu_n kT} \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) \]  \hspace{1cm} (3.22)

By using

\[ J_p = \mu_p p(x) \frac{dE_{F_p}(x)}{dx} ; \]  \hspace{1cm} (A25)

\[ p(x) = N_v e^{-[E_{F_p}(x) + E_V(x)]/kT} ; \]  \hspace{1cm} (A26)

\[ E_V(x) = -q\phi_p + \frac{q^2 N_D (x^2 - 2x_d x)}{2e_s} ; \hspace{1cm} 0 \leq x < x_d \]  \hspace{1cm} (A27)

\[ E_V(x) = -q\phi_p - \frac{q^2 N_D x_d^2}{2e_s} ; \hspace{1cm} x \geq x_d \]  \hspace{1cm} (A28)
and following the same steps, the hole density function, \( p(x) \), in the depletion region of a zero-biased, n-type Schottky diode under constant, monochromatic illumination can be expressed as

\[
p(x) = N_V e^{E_V(x)/kT} \left\{ 1 - \frac{\sqrt{\pi} J_p [\text{erf}(g x_d) - \text{erf}(g x_d - g x)]}{2 g \mu_p k T p_{n0}} \right\}
\]  

(A29)

Inserting Eqn. (A24) into Eqn. (A29) leads to

\[
p(x) = N_V e^{E_V(x)/kT} \left\{ 1 - \frac{\pi L_D J_p}{2 \mu_p k T p_{n0}} \times \left[ \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right) - \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} - \frac{x}{\sqrt{2L_D}} \right) \right] \right\}
\]  

(A30)

where \( N_V \) and \( E_V(x) \) are the valence band density of states and energy as a function of position respectively. \( \mu_p, E_{Fp}, \phi_p, J_p \) and \( p_{n0} \) have the same meaning as their n-subscripted counterparts. Evaluation of Eqn. (A30) at the \( x = x_d \) plane and subtracting \( p_{n0} \) from the result yields

\[
\Delta p(x_d) = -\frac{\pi L_D J_p}{2 \mu_p k T} \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} \right)
\]  

(A31)

Even though there is a the negative sign in Eqn. (A31), the value of \( \Delta p(x_d) \) will be positive because \( J_p \) is negative (current flowing from right to left).
APPENDIX B

Derivation of the Equation for the Excess Electron Density Distribution, $\Delta n(x)$, in the Depletion Region – Eqn. (3.27)

In this section, an alternative expression to the excess electron density distribution derived in Appendix A will be presented. This alternative expression

$$\Delta n(x_d) = \frac{T_0 \Phi_0 L_n}{D_n} \left[ \frac{F_3 (1 + \alpha L_n) - F_3 e^{-\alpha x_d}}{(1 + \alpha L_n)(L_n + F_3)} \right]$$  \hspace{1cm} (3.27)

was derived by solving the continuity equation in the depletion region of a zero-biased, p-type Schottky diode under constant, monochromatic illumination. The expression was obtained by assuming unity collection efficiency in the depletion region and by neglecting any perturbation in the electrostatic potential due to image force effects [52]. In the following, the essential elements of the derivation, which is based mostly on the results of Lavagna et al. [52], will be presented.

The steady-state electron and hole continuity equations are given by

$$\frac{1}{q} \frac{dJ_n(x)}{dx} + G_{op}(x) - R(x) = 0$$ \hspace{1cm} (B1)

and

$$-\frac{1}{q} \frac{dJ_p(x)}{dx} + G_{op}(x) - R(x) = 0$$ \hspace{1cm} (B2)

where $q$, $G_{op}(x)$ and $R(x)$ are the electronic charge, steady-state electron-hole pair generation rate for a monochromatic radiation and recombination rate respectively. $J_n(x)$ and $J_p(x)$ are the electron and hole current densities respectively and are given by
\[ J_n(x) = q\mu_n n(x) E(x) + qD_n \frac{dn(x)}{dx} \] \hspace{1cm} (B3)

and

\[ J_p(x) = q\mu_p p(x) E(x) - qD_p \frac{dp(x)}{dx} \] \hspace{1cm} (B4)

where \( \mu_{n,p}, n(x), p(x), E(x), \) and \( D_{n,p} \) are the carrier mobility, electron density distribution, hole density distribution, electric field and diffusion coefficient respectively. Similar to the derivation presented in Appendix A, carrier mobility is assumed to be independent of position \( x \). The electric field in the depletion region is given by

\[ E(x) = \frac{qN_A (x_d - x)}{\varepsilon_s} \] \hspace{1cm} (B5)

where \( N_A, \varepsilon_s \) and \( x_d \) are the acceptor doping density, material permittivity and the depletion width respectively. The solutions of Eqns. (B1)-(B4) in a depletion region with unity collection efficiency are given by [52]

\[
\begin{align*}
\int_{-\infty}^{x} e^{\frac{(t-x_d)/\sqrt{2L_D}}{\alpha}} \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt = \frac{1}{D_p} \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt
\end{align*}
\] \hspace{1cm} (B6)

\[
\begin{align*}
\int_{-\infty}^{x} e^{\frac{(t-x_d)/\sqrt{2L_D}}{\alpha}} \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt = \frac{1}{D_n} \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt \int_{0}^{x} e^{\frac{(t-x)/\sqrt{2L_D}}{\alpha}} dt
\end{align*}
\] \hspace{1cm} (B7)

where \( B_0 - B_3 \) are constants while \( n_{p0}(x) \) and \( p_{p0}(x) \) are the thermal equilibrium electron and hole density distribution respectively. \( L_D \) is the extrinsic Debye length, which is given by
\[ L_D = \frac{\varepsilon_s kT}{\sqrt{q^2 N_A}} \]  

(B8)

Using Eqns. (B6) and (B7) in Eqns. (B4) and (B3) respectively leads to

\[ J_n(x) = qT_x \Phi_0 \left( B_3 D_n + e^{-\alpha x} \right) \]  

(B9)

and

\[ J_p(x) = -qT_x \Phi_0 \left( B_1 D_p + e^{-\alpha x} \right) \]  

(B10)

The terms involving \( n_{p0}(x) \) and \( p_{p0}(x) \) are eliminated because

\[ q\mu_n n_{p0}(x)E(x) + qD_n \frac{dn_{p0}(x)}{dx} = 0 \]  

(B11)

and

\[ q\mu_p p_{p0}(x)E(x) - qD_p \frac{dp_{p0}(x)}{dx} = 0 \]  

(B12)

To determine the constants \( B_0 - B_3 \), five boundary conditions are required – two at the Schottky contact plane and three at the \( x = x_d \) plane. The first condition is the result of the requirement that the electron current density is a continuous function of \( x \):

\[ J_n(x_d-) = J_n(x_d+) \]  

(B13)

\( J_n(x_d-) \) is obtained by evaluating Eqn. (B9) at the \( x = x_d \) plane:

\[ J_n(x_d) = qT_x \Phi_0 \left( B_3 D_n + e^{-\alpha x_d} \right) \]  

(B14)

\( J_n(x_d+) \) is the electron current density immediately to the right of the \( x = x_d \) plane. Therefore, the expression of this current is given by
\[ J_n(x_{d+}) = qT_x \Phi_0 e^{-\alpha x_d} \frac{\alpha L_n}{1 + \alpha L_n} - \frac{qD_n \Delta n_d}{L_n} \]  \hspace{1cm} (B15)

where \( \Delta n_d \) is defined as the electron density at the \( x = x_d \) plane. This equation was obtained in the same manner as

\[ J_{\text{diff}} = qT_x \Phi_0 e^{-\alpha x_d} \left( \frac{\alpha L_n}{1 + \alpha L_n} \right) \left( \frac{S_i}{1 + S_i} \right). \]  \hspace{1cm} (3.3)

which was presented earlier in Section 3.1, that is, by solving the electron continuity equation in the region \( x \geq x_d \) under steady-state and low-level injection conditions. Eqns. (B15) and (3.3) are equivalent but different in form as a consequent of the application of the Dirichlet boundary condition

\[ \Delta n(x_d) = \Delta n_d \]  \hspace{1cm} (B16)

instead of the Neumann boundary condition

\[ D_n \frac{dn(x)}{dx} \bigg|_{x = x_i} = v_1 \Delta n(x_d) = v_1 \Delta n_d \]  \hspace{1cm} (3.1)

The difference between both equations is the presence of the parameter \( v_1 \) in Eqn. (3.3). The relationship between \( v_1 \) and \( \Delta n_d \) is given by

\[ v_1 = \frac{D_n}{L_n} \left[ \frac{(1 - \alpha L_n) C_0}{\Delta n(x_d)} - 1 \right] \]  \hspace{1cm} (3.15)

Thus, by equating Eqns. (B14) and (B15), the first boundary condition required to determine the constants \( B_0 - B_3 \) can be expressed as

\[ B_3 D_n e^{-\alpha x_d} = \frac{\alpha L_n e^{-\alpha x_d}}{1 + \alpha L_n} - \frac{D_n \Delta n_d}{T_x \Phi_0 L_n} \]  \hspace{1cm} (B17)

By evaluating \( p(x) \) and \( n(x) \), that is, Eqns. (B6) and (B7) respectively at the Schottky
contact plane and using the resulting expressions in the following definition of the
electron and hole current densities at the same plane [92]:

\[ J_n(x = 0) = qA_n^* T^2 \Delta n(x = 0)/qN_c \]  \hspace{1cm} (B18)

and

\[ J_p(x = 0) = -qA_p^* T^2 \Delta p(x = 0)/qN_v \]  \hspace{1cm} (B19)

where \( A_p^* \) and \( A_n^* \) are the effective Richardson constants for electrons and holes respectively while \( N_v \) and \( N_c \) are the valence and conduction band density of states respectively, the second and third boundary conditions can be written as

\[ B_0 = \frac{D_p}{H_p} B_1 + \frac{1}{H_p} \]  \hspace{1cm} (B20)

and

\[ B_2 = \frac{D_n}{H_n} B_3 + \frac{1}{H_n} \]  \hspace{1cm} (B21)

respectively, where

\[ H_p = \frac{A_p^* T^2}{qN_v} e^{-\left(\frac{x_d}{\sqrt{2}L_p}\right)^2} \]  \hspace{1cm} (B22)

and

\[ H_n = \frac{A_n^* T^2}{qN_c} e^\left(\frac{x_d}{\sqrt{2}L_n}\right)^2 \]  \hspace{1cm} (B23)

The fourth and fifth boundary conditions, which result from the evaluation of Eqns. (B6) and (B7) at the \( x = x_d \) plane, can be expressed as
\[
\Delta p_d = T_\lambda \Phi_0 \left( B_0 + B_1 F_1 + \frac{F_2}{D_0} \right) \tag{B24}
\]

and

\[
\Delta n_d = T_\lambda \Phi_0 \left( B_2 + B_3 F_3 + \frac{F_4}{D_0} \right) \tag{B25}
\]

where the constants \( F_1 - F_4 \) are given by

\[
F_1 = \int_0^{x_d} e^{\left[(t-x_d)/\sqrt{2L_D}\right]^2} dt \tag{B26}
\]

\[
F_2 = \int_0^{x_d} e^{-\alpha t + \left[(t-x_d)/\sqrt{2L_D}\right]^2} dt \tag{B27}
\]

\[
F_3 = \int_0^{x_d} e^{-\alpha t - \left[(t-x_d)/\sqrt{2L_D}\right]^2} dt \tag{B28}
\]

\[
F_4 = \int_0^{x_d} e^{-\alpha t - \left[(t-x_d)/\sqrt{2L_D}\right]^2} dt \tag{B29}
\]

Evaluation of Eqns. (B26) to (B29) lead to

\[
F_1 = \sqrt{\frac{\pi}{2}} L_D e^{erfi}(\frac{x_d}{\sqrt{2L_D}}) \tag{B30}
\]

\[
F_2 = -\sqrt{\frac{\pi}{2}} L_D e^{\left(\alpha x_d + \alpha^2 L_D/2\right)} \left[ erfi(\frac{\alpha L_D}{\sqrt{2}}) - erfi(\frac{x_d}{\sqrt{2L_D}} + \frac{\alpha L_D}{\sqrt{2}}) \right] \tag{B31}
\]

\[
F_3 = \sqrt{\frac{\pi}{2}} L_D e^{erf}(\frac{x_d}{\sqrt{2L_D}}) \tag{3.28}
\]
\[ F_4 = \sqrt{\frac{\pi}{2}} L_D e^{-\left(x_d - a^2 L_D^2 / 2\right)} \left[ \text{erf} \left( \frac{aL_D}{\sqrt{2}} \right) + \text{erf} \left( \frac{x_d}{\sqrt{2L_D}} - \frac{aL_D}{\sqrt{2}} \right) \right] \] (3.29)

where the error function \( \text{erf}(\theta) \) is given by

\[ \text{erf}(\theta) = \frac{2}{\sqrt{\pi}} \int_0^\theta e^{-t^2} \, dt \] (B32)

and the imaginary error function, \( \text{erfi}(\theta) \), is an entire function defined by [73]

\[ \text{erfi}(\theta) \equiv -i \text{erf}(i\theta) \] (B33)

Inserting Eqns. (B20) and (B21) into Eqns. (B24) and (B25) respectively leads to the expressions for \( B_1 \) and \( B_3 \), which are given by

\[ B_1 = \left( \frac{1}{D_p + F_1 H_p} \right) \left( \frac{\Delta p \Phi}{T_\lambda} \left( \frac{F_3 H_p}{D_p} - 1 \right) - \frac{F_3 H_p}{D_p} - 1 \right) \] (B34)

and

\[ B_3 = \left( \frac{1}{D_n + F_3 H_n} \right) \left( \frac{\Delta n \Phi}{T_\lambda} \left( \frac{F_4 H_n}{D_n} - 1 \right) - \frac{F_4 H_n}{D_n} - 1 \right) \] (B35)

Therefore,

\[ B_0 = \left( \frac{D_p}{D_p H_p + F_1 H_p^2} \right) \left( \frac{\Delta p \Phi}{T_\lambda} \left( \frac{F_2 H_p}{D_p} - 1 \right) - \frac{F_2 H_p}{D_p} - 1 \right) + \frac{1}{H_p} \] (B36)

and

\[ B_2 = \left( \frac{D_n}{D_n H_n + F_3 H_n^2} \right) \left( \frac{\Delta n \Phi}{T_\lambda} \left( \frac{F_4 H_n}{D_n} - 1 \right) - \frac{F_4 H_n}{D_n} - 1 \right) + \frac{1}{H_n} \] (B37)
Finally, by inserting the equation for $B_3$ into the first boundary condition, that is, Eqn. (B17), and rearranging the resulting equation to obtain $\Delta n_d$, the excess electron density at the $x = x_d$ plane is given by

$$
\Delta n_d = T \Phi_0 \left[ \frac{1}{\frac{H_n}{\Phi_n}} \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right) + \frac{1}{\frac{D_n}{\Phi_n}} \left( F_4 - \frac{F_3 e^{-\alpha x_d}}{1 + \alpha L_n} \right) \right] \left( \frac{F_3}{L_n} + \frac{D_n}{L_n H_n} + 1 \right)
$$

(B38)

If electrical neutrality in the region $x \geq x_d$ is assumed, then $\Delta p_d$ can be equated with $\Delta n_d$. Therefore, the excess hole density, which appears in the expressions for both $B_0$ and $B_1$, is also given by Eqn. (B38).

Except when $H_n$ is zero, Eqn. (B38) can be reduced to Eqn. (3.27)

$$
\Delta n(x_d) = \Delta n_d = \frac{T \Phi_0 L_n}{D_n} \left[ \frac{F_4 (1 + \alpha L_n) - F_3 e^{-\alpha x_d}}{(1 + \alpha L_n)(L_n + F_3)} \right]
$$

(3.27)

provided that

$$
\frac{x_d^2}{2L_D^2} = \left( \frac{2e_s V_{bi}}{qN_A} \right) \left( \frac{q^2 N_A}{2e_s kT} \right) = \frac{V_{bi}}{kT/q} \gg 1
$$

(B39)

When this inequality is satisfied, $e^{x_d^2/2L_D^2}$ becomes very large such that $1/H_n$ is approximately zero. Therefore, in the numerator of Eqn. (B38)

$$
\frac{1}{H_n} \left( 1 - \frac{e^{-\alpha x_d}}{1 + \alpha L_n} \right) \ll \frac{1}{D_n} \left( F_4 - \frac{F_3 e^{-\alpha x_d}}{1 + \alpha L_n} \right)
$$

(B40)

and in the denominator,

$$
\frac{D_n}{L_n H_n} \ll \frac{F_3}{L_n} + 1
$$

(B41)
For an n-type material, the same equations apply except that the subscripts $p$ are replaced by $n$ and vice versa.
APPENDIX C

Additional Details of Numerical Simulations of the Photocurrent Technique

In this section, the numerical simulations of the photocurrent technique using Synopsys® Sentaurus TCAD, which were described briefly in Section 3.4.1, are discussed in more detail. Additional details such as the method by which optical generation and electron-hole pair generation are implemented and settings for improving the numeric accuracy of the simulation are presented.

To perform the simulation, a two-dimensional representation of the structure shown in

![Diagram of a p-type Schottky diode with band diagram superimposed showing the drift and diffusion components of the photocurrent. The parameters $\Phi_0$, $\lambda$, $h$, $c$, and $E_g$ are incident photon flux density, wavelength, Planck’s constant, speed of light, and material band-gap respectively. Here, the horizontal and vertical axes are the x- and y-axis respectively. The depth (not shown) is the z-axis.](image)
Figure 2.1 was created using Sentaurus Structure Editor. The Schottky and ohmic contact widths of the simulated device were set to 5µm while the p-GaN layer thickness was set to 100µm. This is so that the width of the neutral region is much greater than the diffusion length, $L_n$, and absorption length, $\alpha^{-1}$. Therefore, the normalised recombination velocity at the ohmic contact plane, $S_w = \nu_W L_n / D_n$, becomes irrelevant and the coefficient $C_2 = 0$ (see Section 2.2.1). Consequently, the electron density distribution along the $x$-axis can described by

$$n(x) = C_0 e^{-a(x - x_d)} - C_0 \left( \frac{S_l + aL_n}{S_l + 1} \right) e^{-(x - x_d)/L_n} + n_p$$

(3.2)

The mesh for the simulated device structure was generated by Mesh, which was called from within Sentaurus Structure Editor. The mesh size in the vicinity of the $x = x_d$ plane, which was estimated using the depletion approximation, was set to the Debye length so that the expected spatial variations in charge and carrier density around this plane could be resolved. The same mesh size was also applied in the region directly adjacent to the Schottky contact.

The simulations of the electrical characteristics in the device were performed using Sentaurus Device on the basis of the drift-diffusion model, which numerically solves the system of five partial differential equations, namely, the current density and continuity equations for both electrons and holes as well as Poisson’s equation.

Simulations were performed for p-GaN Schottky diodes with nominal doping concentrations of $10^{18}$ cm$^{-3}$ and $10^{19}$ cm$^{-3}$. The acceptor species used in the simulations is Mg, which has an activation energy of approximately 170meV [5]. The incomplete ionisation of Mg was accounted for by specifying the IncompleteIonization keyword in the Physics section of the Sentaurus Device command file. Activation of the incomplete ionisation model leads to equilibrium hole concentrations of approximately $9 \times 10^{16}$ cm$^{-3}$ and $3 \times 10^{17}$ cm$^{-3}$ for $N_A = 10^{18}$ cm$^{-3}$ and $10^{19}$ cm$^{-3}$ respectively.

The work function of the semi-transparent Schottky barrier material, ITO, was set by assigning a value of 4.3eV [45] to the WorkFunction parameter in the Electrode section of the command file. Due to the thick p-GaN layer, electron and hole concentrations at the ohmic contact plane assume the default values of $n_0$ and $p_0$ respectively. In all
Carrier mobility values were calculated from the mobility model of Mnatsakanov et al. [93] while the absorption coefficient data were taken from the experimental results of Muth et al. [46]. The values of both of these parameters, along with other material parameters such as the relative permittivity and electron affinity (see Table 3.2 on pg. 53), were specified in the parameter file for the simulations.

For each doping concentration, a series of simulations were conducted for the range of optical wavelength and diffusion length values listed in Table 3.2. At the beginning of each simulation run, the desired diffusion length is specified by an instruction in the command file which directs Sentaurus Device to load an external MATLAB-generated file that contains a spatially uniform lifetime profile. The required lifetime value was calculated from $L_n = \sqrt{D_n \tau_n}$. In all simulations, the temperature was set to 300K and the device was unbiased. Additionally, the Fermi keyword was specified in the Physics section of the command file to activate Fermi-Dirac statistics.

Due to the simple geometrical structure of the device, the simulation of optical absorption and electron-hole pair generation in the p-GaN layer as a result of a constant and uniform monochromatic illumination through the semi-transparent ITO was achieved by using the OptBeam statement in the Physics section of the command file. The optical generation rate along the $x$-axis, $G_{op}(x)$, is computed according to

$$G_{op}(x) = T_\lambda \Phi_0 \alpha e^{-\alpha x} \quad (2.2)$$

where $T_\lambda$, $\Phi_0$ and $\alpha$ are the wavelength-dependent optical transmission factor of the semi-transparent Schottky barrier material, incident photon flux density at the $x = 0$ plane and absorption coefficient respectively. The quantum yield, $\eta_\lambda$, is unity for all wavelengths used in the simulations.

Unlike RayTrace or TMM (Transfer Matrix Method), the effects of reflection, transmission and refraction are not calculated when photo-generation are simulated using OptBeam. Calculations of these effects are unnecessary for several reasons. Firstly, the optical beam impinges the device at zero angle of incidence with respect to
the normal of the y-z plane. Therefore, the beam will continue to propagate inside the
device at the same angle.

Secondly, as a result of the thick p-GaN layer coupled with the strong optical absorption
of the material, the reflection from the back surface can be ignored. In the event that the
optical beam is sufficiently intense such that reflection from the back surface occurs, the
reflected photons will be fully absorbed before they are able to reach the incident plane.
With the exception of a thin epitaxial sample, this scenario is unlikely considering that
the optical power of the light beam is usually chosen such that the low-level injection
condition is fulfilled.

Thirdly, calculations and considerations of the effects of a thin-film of ITO on the p-
GaN layer can be avoided because the plane at which the intensity of the optical beam is
maximum can be specified by assigning the coordinate of the metal-semiconductor
interface, that is, the $x = 0$ plane, to the $Semsurf$ parameter in the $OptBeam$ statement. In
the simulations, the optical beam intensity is spatially uniform at every $y-z$ plane along
the $x$-axis.

Being a wide band-gap semiconductor material, the intrinsic carrier density of GaN is
extremely low ($n_i \sim 10^{-10}$ cm$^{-3}$). Therefore, some of the settings for numeric accuracy in
the global $Math$ section of the command file must be modified [94]. Table C1 shows the
changes made to some of the default parameter values as well as the function of each
parameter in relation to the numeric accuracy of the simulations.

The data produced by the simulations were for a two-dimensional structure. However,
since the illumination is invariant along the $y$-axis and the electron lifetime is set to be
uniform over the entire device, any spatial variation of the output quantities is limited to
the $x$-axis only. Thus, the desired one-dimensional electron density distribution is
obtained by taking a cross-section of the 2D data along the $x$-axis. This was achieved
using a custom-written MATLAB program that extracts a cross-section of the electron
density data from the *_des.dat file produced by the simulation and also automatically
performs the least-squares fitting of Eqn. (3.2) to the extracted data to obtain $v_1$. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default Value</th>
<th>Value for GaN</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digits</td>
<td>5</td>
<td>6</td>
<td>defines the relative error convergence criterion and approximates the number of digits of accuracy to which an equation must be solved before it is considered to have converged.</td>
</tr>
<tr>
<td>ErrRef(electron)</td>
<td>$10^{10}$ cm$^{-3}$</td>
<td>$\leq 10^5$ cm$^{-3}$</td>
<td>defines to which carrier concentration level the accuracy of the solution variable is controlled.</td>
</tr>
<tr>
<td>ErrRef(hole)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RHSmin</td>
<td>$10^{-5}$</td>
<td>$10^{-10}$</td>
<td>a solution is accepted as having reached convergence if either the weighted error estimate becomes less than unity or the value of the right-hand-side (RHS) becomes less than the value of this parameter.</td>
</tr>
</tbody>
</table>

Table C1. Settings changes in the global Math section of the command file due to the extremely low intrinsic carrier concentration of GaN. The new values and the description of each parameter’s function in relation to the numeric accuracy of the simulations were taken from an application template published by Synopsys [94].
APPENDIX D

Derivation of expressions for $J_{\text{drift}}$ and $J_{\text{diff}}$ in the presence the Franz-Keldysh effect

In this section, derivations of the modified drift and diffusion currents given in Chapter 5 are presented. These modifications result from consideration of the Franz-Keldysh effect and the consequent spatial variation of the absorption coefficient in the depletion region. The expression for this absorption coefficient, which is denoted by $\alpha_{E}(x)$, is given by

$$\alpha_{E}(x) = \alpha_{0} \left[ 1 - \frac{qhE(x)}{8\pi\sqrt{2m_e(E_g - E_{\lambda})}^{3/2}} \cos \left( \frac{8\pi\sqrt{2m_e(E_g - E_{\lambda})}^{3/2}}{3qhE(x)} \right) \right]$$

The diagram shown in Figure D1 represents the depletion region in a Schottky diode, with the $x = x_0$ and $x = x_n$ planes corresponding to the metal-semiconductor interface and depletion edge respectively. This region is divided into $n$ partitions of equal width, $\Delta x$. In each partition, the absorption coefficient is given by $\alpha_j$, where $j$ is an integer with
values ranging from 0 to n. The incident photon flux density, $\Phi_0$ is defined at the $x = x_0$ plane. At the $j^{th}$ plane, the photon flux density is denoted by $\Phi_j$.

Using Figure D1, the following expressions for the photon flux densities can be obtained:

$$
\begin{align*}
\Phi_1 &= T_\lambda \Phi_0 e^{-\alpha_1 \Delta x} \\
\Phi_2 &= \Phi_1 e^{-\alpha_2 \Delta x} = T_\lambda \Phi_0 e^{-(\alpha_1 + \alpha_2) \Delta x} \\
&\quad \vdots \\
\Phi_{n-1} &= \Phi_{n-2} e^{-\alpha_{n-1} \Delta x} = T_\lambda \Phi_0 e^{-(\alpha_1 + \alpha_2 + \cdots + \alpha_{n-2} + \alpha_{n-1}) \Delta x} \\
\Phi_n &= \Phi_{n-1} e^{-\alpha_n \Delta x} = T_\lambda \Phi_0 e^{-(\alpha_1 + \alpha_2 + \cdots + \alpha_{n-1} + \alpha_n) \Delta x}
\end{align*}
$$

(D1)

Note that $T_\lambda$ is included to account for the losses due to reflection at the air-material interface. The photon flux density at the $j^{th}$ plane is therefore given by

$$
\Phi_j = T_\lambda \Phi_0 e^{-\sum_{i=1}^{j} \alpha_i \Delta x}
$$

(D2)

As the number of partitions, $n$, approaches $\infty$ and $\Delta x$ becomes infinitesimally small,

$$
\lim_{{n \to \infty}} \sum_{i=0}^{j} \alpha_i \Delta x = \int_{x_0}^{x_j} \alpha(x) \, dx
$$

(D3)

Therefore, the photon flux density at any position $x$ in the depletion region, in which the absorption coefficient is known to vary spatially according to $\alpha_E(x)$, is given by

$$
\Phi(x) = T_\lambda \Phi_0 e^{-\int_{0}^{x} \alpha_E(u) \, du}
$$

(D4)

where $u$ is a dummy variable. The relationship between the optical generation rate and photon flux density is given by
\[ G_{\text{op}}(x) = -\frac{d}{dx} \Phi(x) \quad (D5) \]

Therefore, the expression for the optical generation rate in the presence of the Franz-Keldysh effect is

\[ G_{\text{op}}^{\text{FK}}(x) = -\frac{d}{dx} \Phi_0 e^{-\int_0^x a_E(u) du} \]

\[ = -T_k \Phi_0 e^{-\int_0^x a_E(u) du} \frac{d}{dx} \left[ -\int_0^x a_E(u) du \right] \]

\[ = T_k \Phi_0 a_E(x) e^{-\int_0^x a_E(u) du} \quad (D6) \]

Here, the quantum yield is assumed to be unity. Assuming that all carriers generated in the depletion region are collected, the drift current can be expressed as

\[ J_{\text{drift}} = q \int_0^{x_d} G_{\text{op}}^{\text{FK}}(x) \, dx \]

\[ = q \int_0^{x_d} \left[ -\frac{d}{dx} \Phi(x) \right] \, dx \]

\[ = qT_k \Phi_0 \left[ 1 - e^{-\int_0^{x_d} a_E(x) dx} \right] \quad (D7) \]

where \( q \) is the electronic charge and \( x_d \) is the position of the depletion edge. The superscript ‘FK’ corresponds to Franz-Keldysh. Because the photon flux density at \( x = x_d \) is given by

\[ \Phi(x_d) = T_k \Phi_0 e^{-\int_0^{x_d} a_E(x) dx} \quad (D8) \]

and beyond \( x = x_d \), the absorption coefficient returns to its zero-field value, \( a_0 \), the diffusion current originating from the neutral region of a p-type material with an electron diffusion length value of \( L_n \) can be expressed as
\[ J_{\text{diff}} = qT_e \Phi_0 e^{-\int_0^d a_e(x)dx} \left( \frac{\alpha_0 L_n}{1 + \alpha_0 L_n} \right) \]  \hspace{1cm} (D9)

If the absorption coefficient in the depletion region is constant and equal to its zero-field value, Eqns. (D4), (D6), (D7) and (D9) are reduced to

\[ \Phi(x) = T_e \Phi_0 e^{-\alpha_0 x} \]  \hspace{1cm} (D10)

\[ G_{\text{op}}(x) = T_e \Phi_0 \alpha_0 e^{-\alpha_0 x} \]  \hspace{1cm} (2.2)

\[ J_{\text{drift}} = qT_e \Phi_0 (1 - e^{-\alpha_0 x_d}) \]  \hspace{1cm} (2.3)

and

\[ J_{\text{diff}} = qT_e \Phi_0 e^{-\alpha_0 x_d} \left( \frac{\alpha_0 L_n}{1 + \alpha_0 L_n} \right) \]  \hspace{1cm} (2.14)

respectively.
REFERENCES


