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Jump chaotic behaviour of ultra low loss bulk acoustic wave cavities

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We demonstrate a previously unobserved nonlinear phenomenon in an ultra-low loss quartz bulk acoustic wave cavity ($Q > 3 > 10^3$), which only occurs below 20 mK in temperature and under relatively weak pumping. The phenomenon reveals the emergence of several stable equilibria (at least two foci and two nodes) and jumps between these quasi states at random times. The degree of this randomness as well as separations between levels can be controlled by the frequency of the incident carrier signal. It is demonstrated that the nature of the effect lies beyond the standard Duffing model. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4892926]

Ultra low loss resonant systems are excellent tools to experimentally study nonlinear effects because of the higher probability of interaction between bounded energy quanta over a system nonlinearity. Common systems include photonic devices such as very high-Q Whispering Gallery Mode Resonators, which have drawn considerable attention in both the optical and microwave domains. Another experimental implementation of nonlinear low-loss systems encompass circuits based on Josephson Junctions, where a considerable degree of nonlinearity is apparent even near the quantum ground state and is the basis of circuit Quantum Electrodynamics. All these systems have potential applications in which the high degree of nonlinearity may be exploited, applications include cryptography, nonlinear signal processing, quantum information processing as well as in study of fundamental principles of complex nonlinear systems. On the other hand, better understanding of these phenomena may also be used to avoid them when necessary.

Low-loss phononic devices are capable of demonstrating some degree of nonlinearity. Although this is mostly limited to the well studied Duffing type nonlinearity arising from crystal anharmonicity. Since nonlinear effects are more apparent in high quality factor systems, ultra low loss Bulk Acoustic Wave (BAW) devices at low temperature are good candidates for the study of mechanical nonlinearities beyond this model. These devices have the largest $Q \times f$ product among all the mechanical devices cooled to near their quantum ground state exhibiting quality factors over $1 \times 10^{13}$ at frequencies approaching 1 GHz. As a result, BAW cavities at cryogenic temperatures demonstrate great potential for many physical applications, not only as a mechanical system at the quantum limit but also as a platform for studying complex nonlinear behaviour. In most of the cases, this behaviour is still dominant by the Duffing nonlinearity, which is observed through the appearance of the hysteretic behaviour and third harmonic generation. Despite predictions of the period doubling bifurcation leading to chaotic behaviour, it has been never observed experimentally. Nevertheless, recently some nonlinear effects beyond the Duffing model were discovered in BAW cavities at liquid helium temperatures due to exceptionally high $Q$-factors. The effect was explained by relatively high concentration of light impurities of the crystalline structure. As a result, the effect could not be represented by the Duffing model whose main source is due to higher order terms in the crystal Hamiltonian. This work demonstrates another type of nonlinear behaviour, discovered in a cryogenically cooled ultra low loss BAW cavity made of purified artificially grown crystalline quartz, which only occurs in high-$Q$ higher order overtones (OT) at relatively low drive powers below 20 mK.

The current experiment is made with a quartz SC (Stress Compensated) cut BVA (Boîtier à Vieillissement Amélioré) type plano-convex BAW resonator. The curved plate device is 1 mm thick, 30 mm diameter electrode-separated disk cavities with higher grade surface polishing. Although this BAW resonator is initially designed to sustain slow shear (C-mode) vibration of 5 MHz at room temperature, its longitudinally polarized vibration (A-modes) exhibits extremely high values of $Q$-factor at cryogenic temperatures. Such record high values of $Q$-factor are achieved by the effective phonon trapping due to the curved plate surface. The device is cooled in a Dilution Refrigerator to 17 mK. The resonator electrodes are connected to a single microwave transmission line split by several DC blocks. Frequency stability of the measurements is controlled by a Hydrogen maser that guarantees fractional frequency stability better than $2 \times 10^{-13}$ at 1 s and better than $2 \times 10^{-15}$ at 1000 s of averaging times. For the OTs analysed here, the measurement setup provides frequency fluctuations on the order of 10 MHz for averaging over 1 s. The influence of cryocooling system on instabilities of BAW resonators, in particular, vibration, and thermal fluctuations have been investigated previously and does not play a significant role in the current experiments.

Microwave reflection from cryogenic BAW cavities, $S_{11}$ parameter, is typically measures by the Network Analysis method, which allows compensation for the connecting cable load. During this procedure, the incident probing signal is swept across the resonant frequency with the rate at which all transient effect could be neglected. Then a reflected signal is compared to the incident one at each frequency near the...
resonance. The correction procedure includes measurements of three calibration standards allowing the apparatus to remove the loading parasitic lines from the data. Measurement results are presented in the form of Z-parameters, complex impedance at different frequencies. The device impedance Z is related to the one port scattering parameter $S_{11}$ as $Z = \frac{1+S_{11}}{1-S_{11}} Z_0$, where $Z_0$ is the source impedance.

To ensure linear response of the device, measurements are made at the lowest accessible incident power of the order of $-45$ dBm in order to remove possible nonlinear behaviour.\(^{20}\) Although extremely high Q values in excess of 1 Billion may result in considerable degree of nonlinearity even at such low power. Our experiments demonstrate that this is the case of the 37th and 39th OTs of the longitudinal mode. These OTs at $f_{A37} = 116.16018289$ and $f_{A39} = 134.9927188$ MHz, respectively, exhibit quality factors of $4 \times 10^9$ and $3 \times 10^9$ (within 10% error\(^{20}\)), respectively,\(^{13}\) that are the highest values of Q-factors among all the modes at 17 mK. Since the 39th OT exhibits similar but less pronounced behaviour, we demonstrate the results only for the 37th OT below.

During the frequency sweep in the vicinity of the 37th OT under the conditions described above, the amplitude instability behaviour is observed (Fig. 1). The sweep rate is kept as low as a few $\mu$Hz per second to avoid any additional dynamical effects. As it is seen from the plot, the systems exhibit instabilities near the frequency of $f_A = 116.160182855$ MHz, point A in Fig. 1, of the corresponding softening Duffing oscillator. Although this dynamics may be interpreted as jumps between two stable states of the Duffing oscillator potential, the system also demonstrates intermediate states.

To characterise the observed instability in the time domain, the frequency of the incident signal was kept constant at some point of the instability region, while monitoring both quadratures of the system response. The results are presented in two forms: time-series response for the magnitude of the impedance and histogram of the result with respect to the real and imaginary part of the impedance in the logarithmic colour scale. The time series plots could be understood as demodulation of the resulting (current) signal with an original (voltage) signal related by the device impedance. In this case, $f_X$ plays a role of the carrier frequency. The histogram of the device impedance is presented on a 2D plot with axis ($\Re Z$, $\Im Z$) (Figs. 2–4) with color scale of $\log_{10} N$ where $N$ is the number of measurements at a specific value of Z. The measurements are made with a sampling rate of 1.04 samples/s, for three frequencies $f_A$, $f_B$, and $f_C$ (see Fig. 1) in Figs. 2, 3, and 4, respectively.

In the linear system with a fixed signal frequency, one expects to observe $Z$ independent of time after sufficiently long settling (transient) time. This situation is shown by a constant line on any of time-series plots (1) and a single bright spot on histograms (2) shown in Figs. 2–4. Although in our case due to the observed instability, $Z$ exhibits significant time fluctuations. Fig. 2 reveals existence of at least four values (A)–(D) of the impedance where the resonant impedance $Z$ has a higher chance to be observed. These quasi-levels are depicted as shadowed areas in (1) and appear as bright islands on the histogram (2). The shaded areas in the subplots (1) demonstrate the regions of residence of the impedance that predominate over time, which correspond as brighter areas in the subplots (2). The brighter these islands, the higher the chance of measuring this particular impedance at a given moment. Level (A) corresponds to the expected value of $|Z|$ at the resonance. Fig. 2(2) shows that level (B) has an internal structure consisting of three additional sublevels.

Due to dynamic properties of the device, there are additional ringing (transient) effects leading to fluctuations of $Z$ around these four values as well as transitions between them.

FIG. 1. Reflection coefficient amplitude $|S_{11}|$ instabilities arising during characterisation of the 37th OT of the longitudinal mode in the quartz BAW cavity at 17 mK with $-45$ dBm of the incident power. $\Delta f$ is the offset frequency from the $f_{A37}$.

FIG. 2. (1) Time response of the magnitude of the scattering coefficient at pumping frequency $f_A = 116.160182855$ MHz. (2) Two-dimensional histogram $\log_{10} N$ of the device impedance ($\Re Z$, $\Im Z$). All insets show variation of $|Z|$ on different time scales.
This results in a particular spiral-like appearance of the levels (D) and (C), which can be interpreted as stable foci. Contrary to that, level (A) and all sub-levels of (B) behave as stable nodes of a nonlinear dynamical system. So, the whole behaviour could be characterised as jumping between quasi-levels of stable equilibria.

The case of $f_A$ corresponds to the maximum separation between the stable states of the Duffing oscillator (Fig. 1) as well as the quasi-states in the current experiment (Fig. 2). Increasing the offset frequency $\Delta f$ of the incident (carrier) signal, the gap between the two stable states of the Duffing oscillator and separations between the quasi-states of the actual mode decrease as a working point moves leftwards on Fig. 1. This can be observed also in Fig. 3(1) for the frequency $f_B$, where one of the quasi-states becomes indistinguishable from the others. As a result, the life time of the system in each quasi-state reduces leading to higher dispersion shown in the histogram (Fig. 3(2)). Consequently this leads to higher degree of randomness of the jumps between levels. These effects are amplified by further change of the carrier frequency. At frequency $f_c$ the system impedance demonstrate irregular motion with hardly distinguishable quasi-states (Fig. 4(1)). In fact, the histogram (Fig. 4(2)) demonstrates very high dispersion of the quasi-states. Comparison of the plots also demonstrates a considerable reduction of the characteristic time scale of switching between the quasi-states as $\Delta f$ is increased. This may be interpreted as shorter lifetimes associated with each quasi-state.

Figs. 2–4 demonstrate steady state behaviour of the system. Additionally, a system transient response is measured where the resonator exhibits a step-function change of the input carrier signal magnitude. For the carrier signal frequency equal to the effective resonance frequency, the system response in the magnitude-phase domain could be described by the first order filter response function $\frac{1}{\tau s + 1}$, where $\tau = \frac{C_0}{Q} \approx 150$ seconds is the resonance time constant and $s$ is the Laplace variable. Measurements results for different values of the incident power are shown in Fig. 5. Contrary to the expected exponential transient process with the time constant $\tau$, the system exhibits jumps between observed previously quasi-states at random times. The system demonstrates a decrease of the separations between the quasi-states with the increase of power. This could be related

FIG. 3. (1) Time response of the magnitude of the scattering coefficient at pumping frequency $f_B = 116.160182874$ MHz. (2) Two-dimensional histogram $\log_{10}N$ of the device impedance ($\Re Z, \Im Z$). All insets show variation of $|Z|$ on different time scales.

FIG. 4. (1) Time response of the magnitude of the scattering coefficient at pumping frequency $f_C = 116.160182886$ MHz. (2) Two-dimensional histogram $\log_{10}N$ of the device impedance ($\Re Z, \Im Z$). All insets show variation of $|Z|$ on different time scales.

FIG. 5. Transient response of the mode impedance $Z$ for different values of the incident power near $f_c$. Input signal frequency is kept constant for all the measurements.
to the change of the frequency response of the corresponding Duffing oscillator with the applied power.

The described effects are reproducible, they have been observed several times during separate cooldowns. The only requirement is operation at the coldest accessible temperature below 20 mK. Above this temperature only standard Duffing nonlinearity in the form of the hysteresis as shown in Fig. 1 is observed. Based on these results at different temperatures, the influence of the measurement apparatus on the observed chaotic dynamics can be excluded.

Although the behaviour of the ultra-high quality factor BAW cavity at milli-K temperatures under strong pumping is related to the dynamics of the Duffing oscillator, the system possesses some extra features beyond this model. Prior modelling of chaotic behaviour in this type of system predicts unobserved period-doubling phenomenon, but does not predict the quasi states observed here. Indeed, the nature of additional stable quasi-states and random jumps is so far not clear. In addition to this previously unobserved phenomenon, a successful theory has to explain the following peculiarities of the effect: (1) temperature threshold (the effect goes away for \( T > 20 \) mK), (2) loss threshold (the effect goes away for the resonator with the degraded \( Q \), as well as it is not observed for other lower-\( Q \) modes); (3) upper power threshold. In particular, we rule out thermal effects, such as those that occur in high-\( Q \) optical resonators, as they do not result in appearance of extra quasi-states, upper power threshold, and sharp temperature threshold. In contrast, thermal effects usually produce almost periodic (relaxation oscillation type) behaviour and become more pronounced at higher power. A certain degree of similarity can be observed between that described above, in particular, density plots in Figs. 2–4, and Liouville/Husimi densities calculated for the driven quantum oscillator with chaotic parameters.22

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