Advances in Mechanics: Failure, Deformation, Fatigue, Waves and Monitoring

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PREFACE

This book is a collection of papers submitted to the 11th International Conference on Structural Integrity and Failure (SIF2018), held in Perth, Australia from 3 to 6 December 2018. The Conference aims to provide an opportunity for academics, engineers and postgraduate students to meet, present and discuss the latest research developments, challenges and trends in the area of Structural Integrity, Fracture, Fatigue and Monitoring.

All accepted papers have been peer-reviewed by at least two experts in the relevant field in order to ensure the high standards of the conference and proceedings.

Structural integrity of materials and structures is a key issue in many engineering disciplines. Maintaining structural integrity requires understanding of fracture propagation and damage accumulation under different types of mechanical and physical impacts, fracture monitoring and, when possible, prevention. On the other hand, some applications such as cutting and hydraulic fracturing are based on efficient fracture propagation. In this case monitoring is also needed, especially in hydraulic fracturing to detect dangerous fracture growth to excessively large sizes or in wrong directions. Multiscale nature of fractures ranging from microscopic defects to the Earth’s crust faults makes maintaining structural integrity and controlling fracture propagation and failure quite complex. This collection of papers reflects recent advances in these areas and creates grounds for further development. We hope the presented ground breaking research will attract attention of the research community.

The Editors are grateful to all the authors of the papers included in the book. We are grateful to the Local Organising Committee, as well as the National and International Advisory Committees for their work and guidance. We also thank the colleagues who contributed to the reviewing process. We are grateful to the enormous help of John Pougher and Susan Marie. We are also grateful to our PhD students, Jimmy He and Yuan Xu for enormous effort of formatting and organising the proceedings. The financial and logistic support of the University of Western Australia and the Australian Fracture Group are very much appreciated.

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I FRACTURE AND WEAR
The effect of abrasive particle size on the wear behavior of SS 440

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Keywords: abrasive particle size, abrasive wear

Abstract. The aim of this study is to evaluate the abrasive wear behaviour of an AISI 440 stainless steel (SS 440) when subjected to different abrasive particle sizes under a variety of loading conditions. The abrasive wear tests were designed based on the pin-on-disc model where the stainless steel samples were sliding against a mild steel counterbody but in the presence of silica sand particles. The silica sand particle size ranged from 24-250 mesh. Optical microscopy (OM), scanning electron microscopy (SEM), Vickers hardness tests, nano-indentation tests were used to characterize the SS 440 and the silica sand particles. After each wear test, the wear scars of the SS 440 were analyzed using Digital holographic microscopy (DHM), OM and SEM to relate the wear results to the microstructure of the stainless steel. The general results show that, for the loading conditions studied, the wear rate increases with increasing abrasive particle size, but only up to a certain point. After which, further increases in abrasive particle size resulted in a decrease in the wear rate of the stainless steel.

1. Introduction

Martensitic stainless steels are widely used in engineering applications where high strength and/or excellent wear resistance are required in addition to moderate corrosion resistance, such as pumps, compressor parts, steam and water valves, shafting, cutlery, surgical tools, bearings and plastic moulds [1, 2]. The mechanical strength and corrosion resistance of martensitic stainless steels can be easily altered through heat treatment due to their high carbon content [2].

While martensitic stainless steels are usually received in the annealed state, they are often hardened via heat-treatment because their corrosion resistance is better in the as-hardened condition and weaker in the annealed and tempered (if tempered > 400℃) condition [3, 4]. As SS 440 are frequently used as a wear resistant material, understanding how they behave under abrasive conditions can be immensely useful in future component design. It is known, for instance, that the wear rate of a given alloy subjected to abrasive wear is often affected by the abrasive particle sizes [5-8]. In particular, since white cast irons are often used in many abrasive environments, studies regarding the effects of abrasive particle size on their wear rates have been widely published [6-8]. However, white cast irons are extremely brittle and may not be suitable for many abrasive applications that require some amount of toughness [9, 10]. In these situations, martensitic stainless steels may be a viable option, especially if some amount of corrosion resistance is required. Unfortunately, and as is known to the best knowledge of the authors, there hasn’t been any study done on the role of abrasive particle size on the wear rate of SS 440.

2. Materials and methods

Commercially purchased 6 mm diameter SS 440 balls (the provided nominal composition is shown in Table 1) were tested in a pin-on-disc system that was modified to simulate crushing high stress abrasion
[11]. A figure and a schematic of the set-up are shown in Fig.1. The abrasives are cyclically pressed between the rotating disc (made out of mild steel, 157 HV) and the stainless steel ball, the latter being held rigidly in place (i.e. a “pin”). The abrasive used was silica sand (mass fraction > 99%). The hardness of the sand is approximately $1211 \pm 14$ HV when tested using a Vickers hardness tester under an applied load of 200 g, and approximately 12.32 to 13.08 GPa when tested using nano-indentation (each test was repeated at least three times).

Table 1 Chemical compositions of SS 440

<table>
<thead>
<tr>
<th>Element</th>
<th>C</th>
<th>Cr</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>wt%</td>
<td>0.95-1.2</td>
<td>16-18</td>
<td>1.0 Max</td>
<td>1.0 Max</td>
<td>0.04 Max</td>
<td>0.03 Max</td>
<td>0.75 Max</td>
</tr>
</tbody>
</table>

Fig. 1 The schematic of the set-up (left) [12] and tribometer (right)

In order to study the effects of abrasive particle size on the wear rate of SS 440, the sand was separated into different particle sizes by sieving. The stainless steel balls were thus subjected to four different ranges of abrasive particle sizes: 58-75 μm, 230-300 μm, 300-425 μm, and 425-700 μm. The 58-75 μm sand is achieved by breaking down 230-300 μm sand using the ball mill. The sand particles tend to have a similar morphology, as can be seen in Fig. 2. The set-up was designed such that the sand particles can roll and slide between the two surfaces. Additionally, to investigate how abrasive particle size affects the wear rate of the stainless steel under different loading conditions, an applied load 20 N, 30 N and 40 N was used. Each test was run for 120 minutes using an RPM of 20 but was stopped every 20 minutes for wear scar analysis and wear loss measurements. As feeding direction will be changed because of removing to measure and install again, set a 120 minutes using an RPM of 20 with no stop as control experiment. Wear scar analysis was conducted using optical microscopy (OM) and scanning electron microscopy (SEM).

Fig. 2 Size and morphology of the sand: (a) 230-300 (b) 300-425 (c) 425-700 (d) 58-75 (unit: μm)

Finally, to relate the wear rate of the stainless steel to its microstructure, microstructural characterization of the balls was also conducted via SEM. Prior to SEM analysis, the balls were prepared by being mounted in epoxy resin, ground down to reveal their cross-sections using silicon carbide papers, and polished using diamond suspensions (3 and 1 μm diamond abrasives).
3. Results

3.1 Material characterization

Fig.3 shows the microstructure of the SS 440 balls. As can be observed, this material contains many chromium carbide particles distributed randomly across the martensitic matrix. Alloys with high chromium content naturally contain significant amounts of hard $\mathrm{M}_7\mathrm{C}_3$ carbides that result in excellent wear performance [13-15]. As can be observed in the figure, and as is typical of SS 440, herein primary and secondary chromium carbides are present [1, 16]. The hardness of the stainless steel balls was found to be 770.4±12.3 HV (result were taken from 10 repetitive tests).

![Fig. 3 Microstructure of the SS 440 balls](image)

3.2 Tribological data

Wear rates were computed using mass loss measurements. The Archard Model [17] can be used to calculate the wear rate coefficient $k$ in mm$^3$/N·m (Eq. 1).

$$k = \frac{V}{L \times C} \quad (1)$$

($V =$wear volume in mm$^3$, $L =$sliding length in m, $C =$normal load in N)

Fig.4 shows the wear rate change with the particle size under different load. The wear rate increases with decreasing particle size from 700 to 230 $\mu$m. However, once the particle size drops to 58-75 $\mu$m, a decrease in wear rate was observed. This is contrary to the effect of particle size on regular steels as reported by Halling (Fig.5) [17].

![Fig. 4 Wear Rate of different particle size](image)  ![Fig. 5 The effect of particle size on normal steel](image)
As the Fig.4 shown, applied different load on different load on different particle size test present a similar trend influence. For 58-75 μm particles, the wear rate increase with the load increase. For other particle size, the wear rate is the largest, when applying 30 N.

3.3 Wear surface analysis

After the wear tests, the wear surfaces were characterized by OM and SEM. Fig.6 presents the wear scar evolution for each 20-minute interval of the 300 – 425 silica sand abrasive test.

![Fig. 6 Optical micrograph of new ball surface and wear surface: a: ball surface before wear test, b-g: wear surface after every 20 minutes wear test (particle size: 300-425μm)](image)

4. Discussion

This study has revealed the effects of abrasive particle size on the abrasive wear behavior of SS 440 under two different loading conditions. As was observed in Fig.4, the wear rate of the stainless steel increases with decreasing abrasive particle size but only up to a particular point. After which, the wear rate started to decrease. It is different from a general study when the particle size increase to a certain size the effect of particle size will not play an important role (Fig.5). The selected counterpart is a rather soft mild steel compare to SS440 which will promote two-body abrasion, because the sands tend to be embedded into the disc and scratch the sample[18]. Additionally, silica sand could also adhere to the surface of the stainless steel ball, resulting in a combined two and three-body abrasion wear mode.

The SEM images of the wear scars of the stainless steel balls subjected to an abrasive size of 58 – 75 μm and 230 – 300 μm are shown in Fig.7a, Fig.7b and Fig.7c, respectively. As can be observed, there is a greater degree of wear when the stainless steel was tested against the coarser silica sand. Against 58-75 μm silica sand, the wear surface is dominated by scratches and indents. With the size increase, Fig.7c show the worn surface has higher roughness. However, when the stainless steel was subjected to a much larger abrasive size of 230 – 300 μm, numerous grooves, cracking and large pits can be observed. In Fig.7a the scratch is caused by microcutting, and the groove in Fig.7c is caused by microploughing. From the observation by the SEM, the microcracking and microploughing take domain role in the test when the size bigger than 230 μm. When microploughing occurs, the materials formed on both sides of the groove. After the repeated action of the load, the final formation of wear debris due to fatigue fracture. In the upper part of Fig.7c, lots of pits and cracks can be seen lead to a higher wear loss compare to Fig.7b. In addition, EDS analysis revealed that the wear scar contains a higher amount of Si as compared to the nominal Si value for this stainless steel alloy, at 5.5 wt%. This is indicative of fragmented silica sand wear debris being embedded unto the surface of the stainless steel since the hardness of the former is much higher than the latter [19].
The chromium carbides in SS440 microstructure play a significant role to influence the wear results. As the Fig.3 show, many chromium carbide particles are distributed randomly across the matrix. The hardness of silica sand is around 1211HV. The martensitic matrix of the SS 440 is much softer than the silica sand while the chromium-based carbides are much harder than the silica sand [20]. If the mean free path between the chromium carbide particles is smaller than the indent imposed by the silica sand, the chromium carbides will be able to act as a load bearing element and substantially reduce the penetration depth of the silica sand [18]. However, if the silica sand is much smaller than the mean free path, it will be able to avoid the chromium carbides and preferentially penetrate the much softer martensite. Without the chromium carbides acting as a load bearing element, a smaller silica sand particle has the potential to impose a much deeper indentation than a coarser one. This is illustrated in Fig.8. This could explain why there was a decrease in the wear rate of the stainless steel when the abrasive particle size was increased from 230 – 300 µm to 425 – 700 µm. One possible explanation for the decrease in wear rate when the silica sand particle size is reduced to 58-75 µm could be due to the increasing role that secondary chromium carbides play as load bearing elements. In the tests involving the larger silica sand particle sizes, the secondary chromium carbides are less effective than primary chromium carbides as load bearing elements since the former is much smaller in size and therefore, have a higher probability of being gouged out [21].

In the 58-75 µm silica sand tests, the secondary chromium carbides are not as easily gouged out, which allows them to better resist the 58-75 µm silica sand. Evidence of chromium carbide pull-out can be deduced from presence of pits on the wear surface (Fig.7b). Another possible explanation for the observed wear results for the 58 – 75 µm test is that smaller particles tend to be able to roll in between the spaces across two counterbodies. This is illustrated in Fig. 9. That is consistent with the worn surface images in Fig.7a that shows a significant amount of indents which using 58-75 µm particles. By being able to roll in between the spaces across the stainless steel and the mild steel counterbody, the probability of direct contact between the silica sand particles and the stainless steel surface is reduced, thus decreasing the wear rate of the stainless steel. It was also found that the wear rate show a similar trend under different load. Halling demonstrate that the wear rate will increase with an increase in load [17]. However, in this study, this was only true for 58-75 µm particles. There was a substantial reduction in the wear rate for other particle sizes under 40N (Fig.4). This can be explained by the fracturing of the silica sand particles under high applied loads as the presence of a relatively large hard phase within a metallic matrix (such as the primary chromium carbide particles in the SS 440) has the potential to cause
fracturing of softer abrasive particles [21]. The silica sand particles quickly fracture at 40N, which results in an abrasive wear test against significantly smaller abrasive particles that could resemble that of the 58 – 75 µm test.

5. Conclusion

In the present study the effect of different experimental conditions on the wear behavior. The initial motivation for this study was practical for application. As abrasive wear causes the main industrial wear problems, to understand the wear mechanisms can help avoid the most wear situation. The hard chromium carbides play an important role in the wear test. The free path between the primary chromium carbides is smaller than the particle size, the indentation depth caused by abrasive will substantially reduce by primary chromium carbides. While the particle size drops the specific value, the secondary chromium carbides will become a barrier of increase wear rate. Because the load increases will also crush the abrasive, the wear rate will change because of the changing pressure, changing the working pressure where possible also reduces wear loss.

References

Effect of electrospun cellulose nanocrystals/polysulfone interleaves on the interlaminar fracture toughness of carbon fiber/epoxy composites

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Keywords: Cellulose nanocrystals; Composite laminates; Fracture toughness

Abstract. Cellulose nanocrystals (CNC) were derived from microcrystalline cellulose (MCC) by sulfuric acid hydrolysis. The mechanical properties of CNC/epoxy (EP) composites with different CNC loadings were investigated. A thermoplastic polysulfone (PSF) was added to toughen the epoxy matrix. The synergic effects of CNC and PSF on the mechanical properties of the ternary CNC/PSF/EP composites were also investigated. An electrospinning process was successfully used to fabricate PSF/CNC nanofibrous membranes as an interleaf which was inserted into the mid-plane of the carbon fiber/epoxy (CF/EP) composites to improve the interlaminar fracture toughness. The results showed that after interleaving with the PSF/CNC nanofibrous membrane, both Mode I and Mode II interlaminar fracture toughness values were enhanced compared to the control composites. Also, the flexural properties of the PSF/CNC interleaved composites were increased. The toughening mechanisms for the PSF/CNC nanofibrous membrane were analyzed based on the fracture surface morphologies of the composites.

1. Introduction

Carbon fiber (CF) reinforced polymer composites (CFRP) have been extensively used in aerospace, automotive and marine industries [1-4] owing to their outstanding mechanical properties and physical performance. Epoxy resins (EP) are the most commonly used thermoset polymers because of their relatively high elastic stiffness, good chemical resistance and low cost [5, 6]. However, the poor resistance to crack propagation caused by their brittleness has limited their applications [6, 7]. Thus, many studies were focused on the toughening methods for CFRP.

From previous research studies, several effective methods have been reported to improve the interlaminar fracture toughness of CFRP. A most prevalent method is by incorporating EP with either soft or rigid nanoparticles [7]. Based on the results given in [7, 8], the incorporation of nanoparticles like rubber, silica and carbon nanotubes (CNTs) would increase the fracture toughness of EP. Unlike the high cost of CNTs [9], cellulose nanocrystals (CNCs) are biological materials extracted from plant, tunicate and bacteria [10]. CNCs present many advantages such as biocompatibility, renewability, high tensile strength and elastic modulus [11]. Hence, CNCs are promising nanoparticles to replace existing ones.

Another method to enhance the delamination resistance of laminates is by inserting interleaves [3] between adjacent layers. Various interleaves such as CNTs buckypaper [12], short fiber/EP interleaf [13] and thermoplastic film [14] were proven to improve the delamination toughness of composite laminates. Nanofibrous membranes produced by electrospinning are a kind of burgeoning nanomaterials [15].
Several thermoplastics have been used to produce electrospun nanofibrous membranes to increase the interlaminar fracture toughness of composite laminates [16]. However, the interaction between epoxy and thermoplastics introduces other problems. Nonetheless, thermoplastics like polysulfone (PSF) [17] are compatible with EP and have been widely used as toughening materials.

In this work, mechanical tests were conducted for CNC/EP and PSF/EP samples to identify the optimal concentrations of CNC and PSF in epoxy matrix. Pure PSF and PSF/CNC nanofibrous membranes were produced by electrospinning as interleaves in CFRP laminates. Both mode I and mode II tests were conducted to obtain their interlaminar fracture toughness. The fracture surfaces of tested samples were examined by scanning electron microscopy (SEM) to evaluate the toughening mechanisms of CNC nanoparticles and PSF nanofibrous membranes.

2. Experimental


CNCs were prepared by sulfuric acid hydrolysis of MCC based on the procedures reported in [18]. CNCs were first dispersed in acetone by a high-speed homogenizer and ultra-sonication, followed by mixing with EP or PSF/EP by magnetic stirring. After the solvent was evaporated, the curing agent piperidine was added. The final mixture was poured into the silicon molds and kept in an oven at 120°C for 16 h to obtain samples for mechanical testing.

2.2 Preparation of CF/EP interleaved composite laminates

The nanofibrous membranes were produced by electrospinning based on different PSF/CNC solutions. The obtained nanofibrous membranes (10 wt.% PSF and 10 wt.%PSF/0.5 wt.% CNC) were used as interleaves inserted in the mid-plane of the CF/EP laminates which were fabricated by 16 plies of unidirectional CF fabrics and EP/piperidine by hand layup. These two laminates were referred as CF/EP-PSF and CF/EP-PSF/CNC. A 25 μm thick polytetrafluoroethylene (PTFE) film was also inserted in the mid-plane to serve as an initial crack. The laminates were cured in a hot press at 120°C for 16 h under a pressure of 250 kPa. For comparison, CF/EP (pure epoxy resin) and CF/EP-CNC (epoxy resin modified with 0.5 wt.% CNCs) laminates without interleaves were also prepared.

2.3 Characterization

Scanning electron microscopy (SEM, Zeiss Sigma VP HD) was used to observe the surface morphologies of the fractured composite samples. Tensile, flexural, single-edge-notch bending (SENB), double-cantilever-beam (DCB) and three-point end notched flexure (ENF) samples were all tested on the universal mechanical testing machine (Instron 5567, USA) at a crosshead speed of 1 mm/min, following the ASTM D3039/D3039M, D7264/D7264M, D5045, D5528 and D7905/D7905M standards, respectively.

3. Results and discussion


The tensile, flexural and fracture properties of CNC/EP, PSF/EP and PSF/CNC/EP composites were summarized in Fig. 1. All the mechanical properties reached their maximum values when 0.5 wt.% CNCs were incorporated in EP (Fig. 1 (a-c)). Incorporation of PSF increased the fracture toughness of EP (Fig. 1 (f)), while the tensile and flexural strengths were reduced (Fig. 1 (d, e)). With further CNC addition, all the mechanical properties were only mildly enhanced (Fig. 1 (d-f)).
3.2 Mechanical properties and morphologies of CF/EP composite laminates

The DCB and ENF fracture test results are shown in Fig. 2. As shown in Fig. 3 (a) and (c), after the insertion of the PSF interleaf, phase separation process has occurred during the curing of EP. Many PSF spherical particles are observed which cause crack deflection during delamination growth; and cavities are noted which are due to the debonded and pulled-out PSF particles. These mechanisms lead to increases in both mode I and II fracture toughness. With the addition of 0.5 wt.% CNCs in the PSF nanofibrous membrane, the fracture morphologies did not change much. However, as shown in Fig. 3 (b) and (d), some CNCs pull out from the matrix, leading to further albeit moderate increases of these interlaminar fracture toughness values compared to those CF/EP.
Fig. 2 shows the flexural strengths and flexural moduli of un-interleaved and interleaved CF/EP laminates. The results suggest that phase separation yield poor load transfer between the PSF particles and EP. After further incorporating 0.5 wt.% CNCs in the PSF membranes, the flexural properties are only slightly enhanced but are still much less compared to the CF/EP laminates with CNCs only.

4. Conclusions

In this work, it is demonstrated that the addition of 0.5 wt.% CNCs nanoparticles in epoxy led to the best mechanical properties. PSF was added to toughen epoxy. DCB and ENF tests show that both mode I and mode II interlaminar fracture toughness of the interleaved laminates were enhanced due to the specific toughening mechanisms caused by PSF particles and further addition of CNCs. The flexural properties were also improved with the presence of CNCs.

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References

Boundary smoothing for topologically optimized designs using B-spline

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Keywords: Topology optimization, Boundary smoothing, B-spline

Abstract. Topology optimization has attracted much attention due to its capability of reducing component weight. However, a finite-element-based topology optimization always gives results with a serrate boundary. A detailed procedure for replacing a serrate boundary with smooth B-spline curves is shown in this paper. The end control point can be moved to maintain first order continuity around the connection point between a straight boundary and a spline curve and a minimum distance constraint was developed to maintain relatively smoothness around the connection point. A case study is presented showing a finite element model whose serrate boundary is fitted by a smooth spline curve.

Introduction

Topology optimization is becoming popular in structural designs. Topology optimization endeavors to find an optimal way of distributing material in a given design space which is usually much larger than a traditional design [1]. Such a large design space increases the likelihood of the optimization finding the optimal design. In order to solve the topology optimization problem efficiently in such a large design space, some optimization algorithms divide the design space into finite elements and solve the problem based on these discrete elements, such as SIMP (Solid Isotropic Material with Penalization) method [2, 3] and BESO (Bidirectional Evolutionary Structural Optimization) method [4, 5]. SIMP method solves the optimization problem by applying penalized density to the elements. In this way, the problem becomes continuous and it can be solved based on the gradient calculation. BESO method uses only solid and void elements during the optimization. It continually ranks all elements according to their performances and evolves by modifying the state (solid or void) of top and bottom ranked elements in the mesh. Both methods are based on finite elements and the results always contain a serrate boundary. There are also researchers working on the development of topology optimization with a smooth boundary. This includes the level set method [6, 7], where the structural topology is updated by changing the level set of a higher order function. However, the level set method is still in the developing stage and it has some limitations, including the inability to create new voids. Therefore, boundary smoothing is still a practical solution for postprocessing the designs from finite-element-based topology optimization. A model with a smooth boundary which facilitates further designing and manufacturing can be created using such a solution. This paper presents a detailed procedure of boundary smoothing for designs from finite-element-based topology optimization using B-spline. First order continuity can be achieved along the boundary especially around the connection point between a straight boundary and a spline curve.
Basics of B-spline

Basis spline [8], or B-spline is a parametric function that describes the shape of a curve. Its linear combination can be used to express any kinds of spline functions with same degree. The basic function of B-spline can be expressed as:

$$B(u) = \sum_{i=0}^{n} N_{i,p}(u) C_i.$$  \hspace{1cm} (1)

Where $C_i$ is the $i$th control point, $N_{i,p}(u)$ is the basis function, $p$ is the degree of the spline function and $u$ is the parametric coordinate. The control points have the same degree $p$ as the spline function.

The basis function is created based on a knot vector $U = (u_0, u_1, \ldots, u_m)$, which is a collection of $m+1$ numbers with order $u_0 \leq u_1 \leq \cdots \leq u_m$. These numbers are named as knots and usually $u_0 = 0$ and $u_m = 1$ are used to make the domain in a closed interval $[0, 1]$. Then the basis function can be defined recursively as:

$$N_{i,0}(u) = \begin{cases} 1, & \text{if } u_i \leq u \leq u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u) \hspace{1cm} (2)$$

The basis function can be created using Eq. (2). With control points in a desired degree, the B-spline can be defined accordingly. It should be noted that $n$, $m$ and $p$ must satisfy $m = n + p + 1$.

Fitting the boundary of a finite element model using B-spline

A critical step in boundary smoothing is to fit the original serrate boundary using spline curves. The spline fitting process is the solution of the spline function using given data points. In order to fit the boundary of a finite elements model, the data points are the boundary nodes of elements along the exterior of the model. The first step of fitting is to parameterize the data points into a knot vector. The chord length method is used to parameterize the data points according to their accumulating chord length. This method has been used in other research for spline fitting. Assuming the data points are $D_0, D_1, \ldots, D_m$, the total chord length of the polygon is the sum of all the chord lengths:

$$L = \sum_{i=1}^{n} |D_i - D_{i-1}| \hspace{1cm} (3)$$

The total chord length of the polygon from point $D_0$ to $D_k$ is:

$$L_k = \sum_{i=1}^{k} |D_i - D_{i-1}| \hspace{1cm} (4)$$

Assuming that the domain is $[0, 1]$, the parameters are:

$$t_0 = 0, t_k = \frac{L_k}{L}, t_m = 1 \hspace{1cm} (5)$$

The knot vector can be obtained by averaging all the parameters. This method is suggested by de Boor [14] to avoid singularity in linear equations. The averaging method is:

$$u_0 = u_1 = \cdots = u_p = 0$$

$$u_{j+p} = \frac{1}{p} \sum_{i=j}^{j+p-1} t_i \quad \text{for } j = 1, 2, \ldots, n - p$$

$$u_{m-p} = u_{m-p+1} = \cdots = u_m = 1 \hspace{1cm} (6)$$

where the first $p + 1$ knots are zeros, the last $p + 1$ knots are ones, and the knots in the middle are the average of $p$ consecutive parameters, given that the spline has degree $p$.

There are two ways of fitting a curve to the data points, curve interpolation and curve approximation. For $m + 1$ data points, the interpolation will create $m + 1$ control points for the spline curve, and the
approximation will create fewer number of control points. It should be noted that there are many data points from the finite element model, which means approximation is a better choice as it will reduce the complexity and wiggle of spline curve. In spline approximation, the number of control points is \( h + 1 \), which satisfies \( m > h \geq p \geq 1 \). In this case, the spline function is:

\[
B(u) = \sum_{i=0}^{h} C_i N_{i,p}(u) \tag{7}
\]

The first and last data point are always on the spline curve, which means \( D_0 = B(0) = C_0 \) and \( D_m = B(1) = C_h \).

The approximation is trying to make the spline curve as close to the data points as possible. An error distance is used to measure how close they are. This error distance is calculated by the least square method. For each data point \( D_k \), its corresponding parameter is \( t_k \), hence its corresponding point on the curve is \( B(t_k) \) and the distance between two points is \( |D_k - B(t_k)| \). The summation of all squared distances is:

\[
f(C_1, \ldots, C_{h-1}) = \sum_{k=1}^{m-1} |D_k - B(t_k)|^2 \tag{8}
\]

The control points are obtained by finding the minimum of Eq. (8). Setting the partial derivative of \( f \) with respect to every control point to be zero, the control points can be obtained. It should be noted that the number of control points has a significant effect on the shape of spline curve and this number should be carefully selected. A sample is shown in Fig. 1, where the serrate boundary created by topology optimization is fitted with B-spline.

Figure 2. A serrate boundary of a finite element model replaced by B-spline

**Continuity control during the B-spline fitting**

Increasing the boundary smoothness is the main objective of this work. However, the direct fitting with B-spline doesn’t consider the connection with straight boundary and there are sharp corners at the connection point, as shown in Fig. 1. It is necessary to remove such sharp corners to further improve the boundary smoothness. In this case, first order continuity should be maintained everywhere along the curve. This is achieved by moving the control point adjacent to the point of intersection and making it collinear with the straight boundary. The B-spline with end point always on the spline curve is called clamped B-spline. For clamped B-spline, it is always tangent to the first and last piece of control polygon.
When the piece at the end is collinear with the straight boundary, B-spline curve is now tangent to the straight boundary. In this way, first order continuity can be maintained at the connecting point.

For a given B-spline curve as Eq. (1), if the control point \( C_i \) is to be moved to \( C_i + v \), the B-spline becomes:

\[
B(u) = \sum_{i=0}^{n} N_{i,p}(u)C_i + N_{i,p}(u)v
\]

The moving vector is initially selected to be perpendicular to the straight boundary, one sample movement is shown in Fig. 2(a). However, it was found that sometimes the second ending control point is too close to the first one. When the third point and second point form a blunt angle with the first point in the middle, there can be a sharp turn around the first point and the first order continuity can be difficult to maintain. For example, the B-spline in Fig. 2(b) exhibits such a problem. The first two ending control points are too close to each other and the first order continuity can only be observed when the detail is magnified. This means the radius of curvature of this spline is still relatively small around the first point, which is detrimental in manufacturing.

Figure 3. (a) The second ending control point is moved to maintain first order continuity around the first ending point; (b) The curvature is large around the first ending point when the second control point is directly projected onto the straight boundary; (c) The curvature at the first ending point becomes relatively smaller when the second point is kept at a distance from the first point

In order to resolve this issue, the following strategy was developed and applied during the movement of the control point:
1. Calculate the length between the start control point and the end control point;
2. Obtain an average length via dividing the above length by the number of control points;
3. Project the second end control point onto the straight boundary and make the movement;
4. Measure the distance between the two ending control points. If the distance is less than the average length, make it equal to the average length.

By applying this strategy, the radius of curvature of spline curve near the connection point can be increased, as shown in Fig. 2(c).

Case study

A half bridge created by topology optimization is shown in Fig. 3. B-spline was used to smooth the serrate external boundary created by topology optimization. The internal boundary was not considered in this study and was not smoothed. The B-spline fitting result is shown in Fig. 4, with the points being element nodes along the original boundary.

![Figure 4. Half bridge created by topology optimization.](image)

![Figure 5. Half bridge with boundary smoothed by B-spline](image)

Summary

Finite-element-based topology optimization can lead to designs with reduced weight. But the serrate boundary needs further postprocessing before passing to CAD model or manufacturing. A detailed procedure for fitting a serrate boundary with B-spline is presented in this paper. The end control point was modified to maintain first order continuity around the connection point between the straight boundary and the spline curve. Additionally, a minimum distance constraint was developed to make sure the distance between first two control points was large enough to have relatively smooth turn. Through this procedure, a model based on finite elements was converted to a model with straight boundaries and
spline curves. The serrate boundary was fully removed and first order continuity was maintained everywhere along the entire boundary of the postprocessed model.

References


Stress distribution in osteomorphic blocks with holes
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Keywords: interlocking elements, stress components, Abaqus FEA.

Abstract. The use of topological interlocking based on osteomorphic blocks for mortarless construction requires a one-directional peripheral constraint which can be achieved by running pre-tensioned cables through the blocks. This requires producing holes which produce stress concentrations due to complex stress distribution induced by non-planar interfaces. With the use of special meshes we conducted a finite element analysis and showed that the stress concentration depends upon the hole radius. Thus, a method is developed to analyse the structural integrity of mortarless structures based on osteomorphic blocks.

Introduction
Mortarless construction is a method promising to reduce the cement usage and thus reduce the CO\textsubscript{2} emission that accompanies cement production. One of the ways to built mortarless structures is the principle of topological interlocking [1-5] whereby the blocks or bricks have a special shape engineered in such a way that removal of each block from their assembly/structure is prevented by the neighbouring blocks. This method does not rely on keys or connectors which are strong stress concentratot, but requires peripheral constraint. A promising type of blocks for topological interlocking are osteomorphic blocks, as illustrated in Fig. 1a [2] which require constraint only in one direction. This can be achieved by post-tensioning cables running through the blocks (e.g., [3]), for which the blocks will have to contain holes for the cables. The holes, Fig. 1b, presented in non-uniform stress field created by non-planar contacts between the blocks work as stress concentrators [5]. This research aims at the analysis of this stress concentration and its dependence upon the hole diameter.

Figure 1. Assemblies of osteomorphic blocks: (a) a simple column of osteomorphic blocks and (b) an osteomorphic block mat.
Mathematical modelling

**Geometry.** Each osteomorphic block is symmetrical in two planes: one is parallel to the short plane side of the block and the second one is equidistant from the curvilinear interfaces; therefore, the 3D model of the simple assembly of blocks can be represented by two quarter blocks. The key dimensions of the blocks are presented in Figure 2.

The height function of the non-planar interfaces is defined by a sum of a constant value $h$ and a multiplication of $\sin$-functions in $x$- and $z$-directions together with a magnitude $\Delta h$.

$$f(x, z) = h + \Delta h \sin \left(\frac{\pi x}{a}\right) \sin \left(\frac{\pi z}{a}\right)$$  \hspace{1cm} (1)

Additionally, throughout hole of radius $R$ along $y$-axis is created in each block. Thus, the following geometric parameters are used:

$$h = a = 90 \text{ mm}, \quad \Delta h = \frac{h}{4} = 22.5 \text{ mm}, \quad R \in \left[0; \frac{a}{2}\right]$$  \hspace{1cm} (2)

**Mechanical properties.** A typical material for the blocks is concrete which in tension behaves in a brittle manner, i.e. the loading occurs in the linear elastic part of the stress-strain curve, for which there are two key mechanical properties: the Young’s modulus and the Poisson’s ratio. The Poisson’s ratio can affect the pattern of stress distribution due to nonlinear interaction between the curvilinear interfaces. Another parameter affecting the stress field at the zones of the highest stress concentration is the coefficient of friction. For this analysis, frictionless tangential contact behaviour is chosen because the sliding will induce the highest stress concentrations at the holes and also due to its simplicity for simulation. Thus, in order to understand the influence of the mechanical properties on the stress distribution for frictionless contact behavior between the interfaces, one value of the elasticity modulus and typical physically admissible values of the Poisson’s ratio, $\nu$, are used:

$$E = 10^{10} \text{ Pa}, \quad \nu = [-0.5; 0.5]$$  \hspace{1cm} (3)

![Figure 2. The three-quarter section views of the quarter models of (a) a solid osteomorphic block and (b) a block with throughout holes](image)

**Model and boundary conditions.** As stated previously, the model assembly consisting of a central block and two peripheral half-blocks is analysed, Fig. 3a. Simplification of the analysis can be achieved by taking into account that the block has two planes of symmetry: $XOZ$ and $YOZ$. Thus, only a quarter of
the block is analysed, Fig. 3b. The boundary conditions for the quarter in linear elastic analysis are as follows, Fig. 3c:

- **X-symmetry** \( u_{xx} = u_{xy} = u_{xz} = 0 \) for one of the sides that are parallel to \( YOZ \) surfaces;
- **Y-symmetry** \( u_{yy} = u_{yx} = u_{yz} = 0 \) for the lower horizontal side of the assembly;
- Small elastic displacement controlled loading \( (u_0^y = -1 \, \mu m) \) for the top side of the assembly.

![Symmetry boundaries](image)

**Figure 3.** Simple assembly of osteomorphic blocks: (a) the whole assembly, (b) a quarter model taking advantage of symmetry, and (c) an Abaqus FEA model with applied BCs

**Element type and meshing.** Solid blocks without holes (the benchmark configuration) are analysed using Abaqus FEA, quadratic hexahedron meshing with reduced integration is suitable for contact analysis, Fig. 4a. Mesh convergence is achieved by successive halving all element sizes until the difference between the successive simulations becomes less than 5%.

Blocks with a hole require a more sophisticated meshing in order to correctly analyse the stress concentration that leads to higher maximum stress values. For holes with a small radius, the mesh should be fine close to the hole and the number of elements should be sufficient to reproduce the geometry of the contacts. On the other hand, for larger holes, mesh should not be biased and the number of elements through the walls has to be small. Therefore, in order to accommodate different radii \( R \), an algorithm of two steps for meshing was developed (Fig. 4b). First, the diagonals of the cross-sectioned square divide each quarter into four 8-corner parts, allowing building of a structured mesh of hexagonal elements. Then, biased seeding along the diagonals is generated as the locations of maximum stresses are expected to be close to the hole.

![Meshing](image)

**Figure 4.** Mesh for a solid block (~50000 elements) and a block with a hole (~100000 elements).
Simulation results

Comparing to bricks with planar interfaces, the Poisson’s ratio impacts the stress distribution and the maximum stress value in interlocking elements. The graph of the maximum von Mises stress against the Poisson’s ratio for a solid block assembly under kinematic loading is shown in Fig. 5.

It can be seen that for a simple assembly subjected to displacement controlled loading, the Poisson’s ratio controls non-uniformity of the stress field which can vary by up to 20% depending on the value of the Poisson’s ratio. For instance, the minimum of the highest von Mises stresses is reached when the value of the Poisson’s ratio is equal to -0.1 and the maximum von Mises stresses are the same for the values of the Poisson’s ratio -0.2 and 0.2. This should be taken into account for the accuracy of the determination of the mechanical properties of the material from which osteomorphic blocks are manufactured.

Results for the normal stress distribution from numerical simulations for displacement-controlled loading case for blocks with different radii of the hole, including $R = 0$, are shown in Figure 6.

It is seen that, in the case of solid blocks, Fig. 6a, the maximum normal pressure is observed at the centre of the contact surface decreasing with a radial distance from it. Such a pattern is characterised by the distortion of the blocks due to the Poisson’s ratio and the only point that does not move in the plane perpendicular to the loading is the centre of the interface. As a result, the force propagates mostly through that region.
For relatively small holes, the normal pressure increases due to the redistribution of the force, propagating through the block. It can be seen in Fig. 6b that the maximum values of the normal pressure are at four points around the hole for which the height is equal to $h$, i.e. parallel to $x$- and $z$- axes in Fig. 2.

If the hole has a large radius comparing to the width of the block, see Fig. 6c, then the maximum normal pressure becomes lower than for the small hole. This occurs because the Y-symmetry boundary condition affects the distribution of the normal pressure and the block is turned into a thin wall structure that has higher flexibility than a block with thick walls.

The von Mises stress fields for the corresponding cases are presented in Figure 7.

![Figure 7. von Mises stress [Pa] fields in a block of the simple assembly with (a) $R = 0$, i.e. solid blocks, (b) $R = 0.4 \cdot (a/2)$ and (c) $R = 0.8 \cdot (a/2)$](image)

Here, we can see that the pattern of the stress distribution for the solid blocks, Fig. 7a, replicates that of the conventional pressure, Fig. 6a. In the case of the small hole, Fig. 7b, the zones with the maximum von Mises stress lie on the diagonals to the lowest height of the block which is away from the maximum normal pressure zones in the corresponding normal pressure field, Fig. 6b. Thus, the predominant stress components become hoop stresses instead of axial stresses. This phenomenon can be explained by the Kirsh’s problem since the non-planar contacts cause the emergence of radial stress components that lead to hoop stress concentration.

For big holes, Fig. 7c, the distribution of stresses is complex and no predominant stress components can be distinguished, so the locations of the maximum stresses do not coincide neither with the maximum normal pressure zones, nor with the diagonal of the interfaces as in Fig. 7b.

**Conclusion**

The study shows that the stress distribution in osteomorphic block assemblies with holes is affected by the hole radius. It is the presence of non-planar interfaces that creates radial stresses. The Poisson’s ratio of the block material influences the von Mises stress, e.g. for a block assembly without holes, the minimum values of the maximum von Mises stress occurs at the Poisson’s ratio equal to -0.1. In the presence of a hole, the hoop stresses get produced that can exceed the axial stress components.

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References

II FATIGUE
Fatigue Scatter of the tests results related to error of applied stress

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Keywords: Fatigue tests, S-N curve, Steel, Statistical analysis, Scatter of tests results.

Abstract. In fatigue test scatter of results is large and depends on accuracy of test machine, production, material defects, environment etc. The paper presents influence of precision of the applied stress on the scatter. Tests results of rotating bending of steel S355J2+C and 42CrMo4 were used to determine the S-N curves. This data was used to estimate a distribution of fatigue strength for $10^5$ cycles. Additionally, the scatter of the applied stress and compare of the distribution were estimated. Verification showed that careful preparation of specimen and adequate accuracy of the testing machine gave 34% and 46% of standard deviation test results, which can be considered satisfactory.

Introduction

In fatigue test the scatter of test is large and depends on the accuracy of the test machine, type of load, frequency, material defects, production and environment. Statistical methods are used to describe randomness of test results. By using normal distribution standard deviation, which gives information about scatter of fatigue life or strength, is calculated. Similar studies were carried out by other researchers in [1] and [2] and it was claimed that in fatigue tests in order to estimate the parameters which influence the fatigue life it is important to minimalize the scatter [3,4].

This paper presents the influence of production and accuracy of the test machine on the scatter of the test results. However, very careful preparation of the specimen significantly impacts the scatter of the dimension, e.g. diameter of the specimen where the stress level is the highest. That gives scatter of the tests results. Another aspect is the measurement method of the applied force, that produces an accepted error, described by the recommended norms [5]. The accuracy of the test directly influences the threshold of the error of the results and can be a problem for applying variable load conditions [6] and [7]. A detailed calculation of the scatter was presented below.

Calculation of scatter of applied stress

Rotating banding stand is presented as an example of calculating the scatter of the applied stress. This equipment was presented in [8]. The applied stress is calculated as follows:

$$S_a = \frac{32 \cdot m \cdot g \cdot l}{\pi \cdot d^3}. \quad (1)$$

where:
- $m$ – loading mass,
- $g$ – gravitational acceleration,
- $l$ – force arm length,
- $d$ – specimen diameter.
Production precision of specimen is described in detail on the graph and presented in Fig. 1. 0.013 kg is the accuracy value adopted for mass applied in the verified test equipment. The force arm length was measured with 0.05 mm precision. Diameter tolerance is at least 0.02 mm accordingly [5]. However, this value is measured before the tests. Precision of this measurement is at least 0.01 mm.

To calculate the error of the applied stress the following equation was used [9]:

\[
B_p = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \cdot \Delta x_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \cdot \Delta x_2^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \cdot \Delta x_n^2},
\]

where:
\( f \) – equation of the function defining the value of the quantity being determined,
\( \Delta x_n \) – standard deviation of the \( n \)-th measured value,
\( x_n \) – \( n \)-th quantity measured in an intermediate measurement.

Substituting Equation (1) into Equation (2) produced the following results:

\[
B_p = \sqrt{-\frac{96 \cdot m \cdot g \cdot l}{\pi \cdot d^4} \cdot \Delta d^2 + \left(\frac{32 \cdot m \cdot g}{\pi \cdot d^3}\right)^2 \cdot \Delta l^2 + \left(\frac{32 \cdot g \cdot l}{\pi \cdot d^3}\right)^2 \cdot \Delta m^2}.
\]

Standard deviation of all the measured values was equal or less precision of measurement. So it was assumed that standard deviation of measurement is equal to the accuracy of measurement.

For the calculation radial runout (calculated by following formula) was applied ([10]):

\[
S_g = S_a \cdot \frac{e \cdot \omega^2}{g \left[1-(\frac{\omega}{\omega_w})^2\right]^2},
\]

where:
\( e \) – eccentric between the diameter of the specimen in the measurement area and the diameter of the applied force (0.015 mm),
\( \omega \) – load frequency (28.5 Hz),
\( \omega_w \) – natural frequency of the mass – spring – specimen.

The scatter of applied stress was calculated as sum of value from Equations (3) and (4):

\[
S_t = B_p + S_g.
\]
Test results

S355J2+C material was used for the tests. Examinations for evaluating the S-N characteristics are presented in the paper [11] and the S-N curve for this steel is presented in Fig. 2. Additionally, 42CoM4 material was used for the test and the procedures evaluating the S-N characteristics are also presented in the paper [12]. The S-N curve for this steel is presented in Fig. 3. All the tests were performed according to the procedures described [13], with 30 specimen for the determination of the S-N reliability. 32 and 33 samples were used for S355J2+C and 42CrMo4, respectively.

Fig. 2 S-N curves for S355J2+C and scatter of test results and applied stress

Fig. 3 S-N curves for 42CrMo4 and scatter of test results and applied stress
The calculated scatter of the applied stress was presented in Fig. 2 and Fig. 3 for S355J2+C and 42CrMo4, respectively and in Table 1. What is more, normal density function of fatigue strength for $10^5$ cycles is presented in the diagram.

A standard deviation of fatigue strength for S-N curve is 10.15 MPa and 10.08 MPa for S355J2+C and 42CrMo4, respectively. The calculations were made according to [13]. The scatter of the applied stress is 34% and 46% of the standard deviation for S355J2+C and 42CrMo4 steel, respectively. It should be mentioned that presented calculation can be easily made for axial and torsion load and it can be assume that the scatter of applied stress will bring similar results to the afore-presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Material</th>
<th>S355J2+C</th>
<th>42CrMo4</th>
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<tr>
<td>$B_p$ [Mpa]</td>
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<td>3.88</td>
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<tr>
<td>$S_g$ [Mpa]</td>
<td></td>
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<td>0.79</td>
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<td>$S_t$ [Mpa]</td>
<td></td>
<td>3.43</td>
<td>4.67</td>
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</tbody>
</table>

Summary

The scatter of the applied stress is small compared to the standard deviation. However, it case of the stress scatter applied in designing new elements such as hammer mill design presented in [14] it should be considered essential. The diameter of the specimen is affected most in Equation (3) (this value is raised to power 4). It is means that most important for designer should be the accuracy of production. The worst dimensions of the design element should be considered to have bigger reliability of machine.

The calculations presented prove that the scatter of the test results is mainly caused by material structure, nevertheless, laboratory technicians should use measuring device with accuracy of at least 0.01 mm.

References


Cyclic torsion and tension-compression behaviour of aluminium alloy and steels for high-cycle fatigue

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Keywords: shear stress, fatigue crack, aluminium alloy, carbon steel, acid-resistant steel

Abstract. The paper presents high-cycle fatigue tests for three selected structural materials: AW 6063 aluminium alloy, S355J2+C carbon steel and 1.4301 acid-resistant steel. All tests were performed under axial and torsion load conditions. A shear-to-normal stress ratio was analysed. A relationship between the direction of fatigue crack propagation and the load type and shear stress amplitude was observed.

Introduction

The fatigue characteristics are usually determined for the engineering materials experimentally using uniaxial fatigue testing methods. The data are available for a wide range of materials [1]. In some cases, the structural components, e.g. axles and crankshafts, operate under torsion load conditions. A key factor when designing those components is fatigue limit under torsion load, however, it is even more difficult to estimate when using multiaxial load testing methods.

Some multiaxial fatigue criteria [2, 3] require data obtained under torsion load to determine all relevant material constants. However, the data are available for a limited range of materials. It would be advantageous to use a fatigue property estimation model for torsion load using easily accessible data, e.g. for axial load.

The tests comparing the fatigue behaviour of materials under cyclic shear and normal loads were performed for steel [2, 4], aluminium alloy [5] and magnesium alloy [6].

The paper presents the results of author’s tests for three selected structural materials. High-cycle fatigue tests were performed under axial load (tension-compression) and torsion load conditions. The orientation of a fatigue cracking plane was analysed. The materials were selected to allow for different structure and chemical composition.

Experimental results

The analysis of the effects of the type of load was carried out for AW 6063 T6 aluminium alloy, S355J2+C carbon steel and 1.4301 acid-resistant stainless steel. The specimens were made from a 10 mm drawn bar. Constant process parameters were maintained. Fig. 1 shows the geometry of the specimens used in the fatigue tests. Table 2 shows the average values of mechanical properties for tested materials. These values were determined experimentally in a static tensile test in accordance to PN EN ISO 6892-1:2016 [7] (\( A \) - percentage elongation after fracture, \( Z \) - percentage reduction of area).

![Fig. 1. Geometry of the round specimen for fatigue testing](image-url)
Table 1. Mechanical properties for the materials tested

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$, MPa</th>
<th>$S_u$, MPa</th>
<th>$S_y$, MPa</th>
<th>$A$, %</th>
<th>$Z$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>AW 6063</td>
<td>69 556</td>
<td>243</td>
<td>201</td>
<td>75.5</td>
<td>10.5</td>
</tr>
<tr>
<td>S355J2+C</td>
<td>208 429</td>
<td>809</td>
<td>684</td>
<td>12.2</td>
<td>63.8</td>
</tr>
<tr>
<td>1.4301</td>
<td>206 553</td>
<td>803</td>
<td>561</td>
<td>78.3</td>
<td>43.8</td>
</tr>
</tbody>
</table>

To verify the effects of load on the fatigue life, the tests were carried out under axial tension-compression load and torsion load on the Instron 8874 biaxial servo-hydraulic testing machine. A symmetric sinusoidal cycle ($R = -1$) was used. The tests were carried out in accordance to [8, 9]. All fatigue tests were performed at a similar frequency of load changes. The purpose was to determine the $S$-$N$ fatigue characteristic for a load-controlled test. The scope of the tests included high-cycle fatigue. The end of test criterion was a macro-crack of the specimen. The number of test specimen was specified according to [10, 11].

An approximation of the experimental points to the linear regression equation in a bi-logarithmic form was carried out (Table 2). $S$-$N$ fatigue characteristics limiting the confidence level were determined for 95% probability (significance level $\alpha = 0.05$). A very small spread of results was obtained for all characteristics (correlation coefficient $R^2$ above 0.97). There was no correlation between the scatter and the type of load. The fatigue data for torsion cyclic load can be compared with the data for cyclic tension-compression. An equivalent stress amplitude determined using the von Mises hypothesis was used. The lower range of tests was determined from the assumed fatigue limit for steel. The aluminium alloy does not show the fatigue limit [12]. Therefore, the fatigue life has been determined in accordance with the knee point. Fig. 2 shows a graphical representation of the results.

To determine a relative position of the fatigue characteristics, a statistical parallelism test was performed for the regression line slopes ($a$). Despite slight differences in slope $a$, the fatigue characteristics for a specific material are parallel. Shear-to-normal stress amplitude ratio ($\lambda = \tau_a/\sigma_a$) [13] for the same fatigue life was calculated using two experimental equations.

Table 2. Fatigue characteristic $S$-$N$ parameters

<table>
<thead>
<tr>
<th>Type of geometry</th>
<th>Bi-logarithmic regression line $\log N = a \log S + b$</th>
<th>Correlation coefficient, $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td>AW 6063</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension-compression load</td>
<td>-6.617</td>
<td>19.290</td>
</tr>
<tr>
<td>Torsion load</td>
<td>-6.420</td>
<td>19.258</td>
</tr>
<tr>
<td>S355J2+C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension-compression load</td>
<td>-12.842</td>
<td>38.497</td>
</tr>
<tr>
<td>Torsion load</td>
<td>-10.520</td>
<td>33.477</td>
</tr>
<tr>
<td>1.4301</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tension-compression load</td>
<td>-14.568</td>
<td>43.491</td>
</tr>
<tr>
<td>Torsion load</td>
<td>-16.194</td>
<td>51.022</td>
</tr>
</tbody>
</table>
Fig. 2. Comparison of fatigue characteristics $S$-$N$ determined by tension-compression and torsion for: a) AW 6063 aluminium alloy, b) S355J2+C carbon steel, c) 1.4301 acid-resistant steel
Fatigue cracks analysis

Fig. 3 shows the photographs of macro-cracks formed under variable torsion load. Different directions of fatigue crack propagation were observed. For AW 6063 aluminium alloy (Fig. 3a) and S355J2+C carbon steel (Fig. 3b), the crack propagated in the plane of the highest shear stress. A fatigue fracture was perpendicular (AW 6063) and parallel (S355J2+C) to the torsional axis. No significant differences in macro-crack length or direction were observed at different stress amplitudes.

For 1.4301 acid-resistant steel, the crack direction depends on the stress amplitude level. At higher loads, the crack direction corresponds to the direction of the maximum shear stress. At lower load levels, the direction of a micro-crack changed (Fig. 3c) and propagated at 45 degrees to the torsional axis. The fracture in the plane of the highest shear stress was promoted by high stress values. The crack length at 45 degrees was higher for the lowest stress amplitude. A similar material response to cyclic torsion is described in the paper [13].

Fig. 3. Fatigue cracks of specimen made of: a) AW 6063 aluminium alloy, b) S355J2+C carbon steel, c) 1.4301 acid-resistant steel
Summary

The paper analyses a shear-to-normal stress ratio for three different construction materials. The $\lambda$ coefficient is 0.667 for AW 6063, 0.733 for S355J2+C and 0.908 for 1.4301. All characteristics are statistically parallel showing no correlation between the $\lambda$ coefficient and stress amplitude level. Other values for each of the tested materials indicated a different rate of fatigue crack propagation at cyclic shear stress.

For 1.4301 acid-resistant steel, the cracking direction depends on the shear stress amplitude. However, it is not visible in the position of fatigue characteristics. The fatigue characteristic for torsion load shows slightly lower slope than the fatigue characteristic for axial load. Based on the analysis of the regression line slopes, the characteristics are statistically parallel. For aluminium alloy and carbon steel, crack length and angle are not dependent on the shear stress amplitude.

References

On the Simplified Modelling of Front Shapes of Fatigue Cracks

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Keywords: Numerical analysis, Front shape, Crack closure, Plasticity, Steady-state conditions

Abstract. A direct three-dimensional (3D) finite element modelling of fatigue crack growth in structural components still represents a formidable task due to a complex singular behaviour of the stress field along the crack front as well as strong non-linearities associated with material plasticity and the change of contact conditions between crack faces during the loading cycle. The complexity of the 3D numerical modelling of fatigue crack growth largely motivates the development of simplified approaches. This paper describes several possible approaches for the evaluation of front shapes of fatigue cracks. These approaches are based on the (1) elimination of the corner singularity effect, (2) predictions based on the first-order plate theory, (3) the equivalent thickness concept, and (4) the Iso-K criterion. This paper briefly outlines these simplified approaches and presents some theoretical predictions for the case of through-the-thickness cracks propagating in plates under quasi-steady-state conditions. The theoretical predictions are also compared with experimental observations.

Introduction

The evaluation of fatigue failure of structural components is of permanent and primary interest for engineers. Hence, significant research effort has been directed towards the development of fatigue crack growth models over the past four decades. In particular, numerous early publications were dedicated to the study of the fatigue crack closure concept, which was first introduced by Elber [1] to explain the experimentally-observed features of fatigue crack growth in aluminum alloys. The number of publications grew rapidly since his pioneering study, reaching a maximum around 1970. It is now commonly accepted that the contributions of various mechanisms of crack closure, specifically the plasticity-induced closure, are significant, particularly at the near threshold fatigue crack growth, in retardation effects associated with overloads and acceleration of crack growth rates of physically short cracks [2]. According to this approach, the shape evolution is governed by the effective stress intensity factor (ΔKₚₑₓ), which is defined as:

\[ ΔK_{\text{eff}} = K_{\text{max}} - K_{\text{op}} = UΔK = U(K_{\text{max}} - K_{\text{min}}) \]

where \( K_{\text{max}} \) and \( K_{\text{min}} \) are the maximum and minimum values respectively and \( U \) is the normalised load ratio parameter (or the normalised effective stress intensity factors) which is often used to describe the effects of loading and plate geometry on crack closure. \( K_{\text{op}} \) is the crack opening stress intensity factor under cyclic loading conditions.

Prior to 1970, the plasticity and crack closure mechanisms were intensively investigated for two-dimensional (2D) geometries utilising both plane strain and plane stress simplifications. With advances in numerical modelling and the increase in computational power, it became possible to study more realistic three-dimensional (3D) geometries as well as investigate the various near crack front 3D effects. A number of finite element (FE) models have been developed in the past to evaluate the effective stress intensity factor, \( ΔK_{\text{eff}} \), and normalised load ratio parameter, \( U \), for various geometries and loading conditions. However, these methods are difficult to implement in fatigue analysis due to convergence...
and repeatability issues. One of the reasons behind the difficulties in modelling plasticity and contact nonlinearities is the complex 3D singular stress fields, specifically near the vertex (corner) points.

In 3D problems the order of the singularity at the intersection of the crack front with the free surface depends on the Poisson’s ratio and intersection angle. From energy considerations, it follows that shape of the fatigue crack front must evolve to preserve the inverse square root singular behaviour along the entire crack front. Therefore the fatigue crack has to intersect the free surface at a critical angle, $\beta_c$, which is a function of Poisson’s ratio, $\nu$. Several experimental studies, specifically for quasi-brittle materials, have confirmed this prediction for mode I fatigue cracks. Other studies have indicated that the effect of 3D corner singularity might not be very significant in the presence of a sufficiently large crack front process zone. This is because the 3D corner singularity effect is a point effect and is quite localised. The experimental results for surface fatigue cracks in round bars show that the fatigue front preserves a semi-elliptical shape rather than the critical angle [3].

In this paper, we briefly outline four simplified approaches for the prediction of front shapes of fatigue cracks. We also describe the application of these approaches to through-the-thickness cracks as well as a comparison with experimental data.

**Methods for Evaluating the Front Shapes of Fatigue Cracks**

In this Section we briefly describe four simplified approaches for evaluating the front shapes of fatigue cracks propagating in plates under quasi-steady state conditions. These approaches are based on (1) the elimination of the corner singularity effect; (2) predictions based on the first-order plate theory; (3) incorporation of plasticity-induced fatigue crack closure effect using the equivalent thickness concept; and (4) the Iso-K concept.

**Approach Based on the Elimination of Corner Singularity Effect**

This approach is based on the so-called stress singularity matching. In accordance with this assumption, the evolution of the crack front occurs in a manner that all points over the crack front (including the corner points) have the same inverse square root singularity of the stress field. This assumption implies that the angle, $\beta$, is the same during the crack front evaluation and equal to the critical angle, $\beta = \beta_c$, (see Fig. 1) at the condition of the steady-state propagation [4]. The critical angle is a function of Poisson’s ratio only; for example, for $\nu = 0.3$, $\beta_c \approx 100.40$. It is interesting to note that in accordance to the experimental study by Heyder et al. [5], in structures with flat free surfaces, such as beams of rectangular or trapezoidal cross-sections, the fatigue crack front appears to follow the stress singularity matching assumption; however, it is generally not supported by experimental observations for structures with curved surfaces such as round bars [6].

![Fig. 1. Critical angle, $\beta_c$, in the case of a through-the-thickness crack propagating in a plate](image)

An application of this approach to a steady state fatigue crack propagation requires the fulfilment of two conditions: (1) stress singularity matching (or $\beta = \beta_c$ at the intersection with the free boundary) and (2) the same value of the stress intensity factor along the crack front (Iso-K approach). The practical realisation of this approach can be based on a minimisation of the stress intensity factor variation along different front shapes, which can be described by a multi-parametric equation.
First-Order Plate Theory Predictions

Another approach for the front shape evaluation is based on first-order theory predictions. This simplified theory is a natural extension of the classical plane stress/plane strain theories. The first-order plate theory explicitly incorporates the plate thickness and the transverse stress components into the governing equations, which retain the simplicity of 2D models. Based on this theory, and utilising Budiansky-Hutchinson crack closure model [7], Codrington and Kotousov [8] provided the following solution for the normalised load ratio, U, in the case of the small-scale plasticity:

\[ U(R, \eta) = a(\eta) + b(\eta)R + c(\eta)R^2 \]  

where \( R \) is the load ratio; \( a, b \) and \( c \) are fitting functions given by the following equations:

\[ a(\eta) = 0.446 + 0.266 \cdot e^{-0.41\eta}; \quad b(\eta) = 0.373 + 0.354 \cdot e^{-0.235\eta}; \quad c(\eta) = 0.2 - 0.667 \cdot e^{-0.515\eta} \]

where \( \eta = K_{\text{max}}/(h\sqrt{\sigma_f}) \) is a dimensionless parameter, \( K_{\text{max}} \) is the maximum stress intensity factor, \( h \) is the half-plate thickness, and \( \sigma_f \) is the flow stress.

These equations correctly recover the limiting cases of very thin and very thick plates, when \( \eta \to \infty \) or \( \eta \to 0 \), respectively. The details of the derivation of these equations can be found in the original paper [8]. The application of this solution to the evaluation of the front shape of through-the-thickness cracks can be found in He et al. [9], and will not be repeated here due to page restrictions.

Equivalent Thickness Concept

Several researches suggested a concept which simplifies the evaluation of the plastic constraint effect on the plasticity-induced crack closure [10,11]. For example, based on an extensive 3D elasto-plastic FE analysis for through-the-thickness cracks, She et al. [12] proposed to define the equivalent thickness for arbitrary point, \( P \), located on the crack front, see Fig. 2, as follows:

\[ B_{\text{eq}} = h - z^2/h \]

where \( z \) is the distance from the mid-plane and \( h \) is still the half-thickness of the plate.

![Fig. 2. Schematic illustration of the equivalent thickness method in the through-the-thickness cracks](image)

The normalized load ratio is defined as:

\[ U = \frac{3\sqrt{\kappa}}{1 - R} \]  

where \( \kappa \) (see Eq. (6)) is a function of \( R \) and a global constraint factor, \( \alpha_g \):

\[ \kappa = \frac{(1 - R^2)^2(1 + 10.34R^2)}{[1 + 1.67R^{1.61} + \frac{1}{0.15\pi^2\alpha_g}]^{4.6}} \]
The global constraint factor is a thickness \( t(\xi) \), and Poisson’s ratio \( \nu \) dependence parameter:

\[
\alpha_g = \frac{1 + t(\xi)}{1 - 2\nu + t(\xi)}
\]  
(7)

The normalized load ratio increases with an increase in the constraint factor at the constant applied stress ratio. In the last equation, \( t \) can be calculated from the following equation:

\[
t(\xi) = 0.2088\left(\frac{r_0}{B_{eq}}\right)^{0.5} + 1.5046\left(\frac{r_0}{B_{eq}}\right)
\]  
(8)

with the plastic zone size, \( r_0 \), as a function of flow stress, \( \sigma_f \), defined as:

\[
r_0 = \frac{\pi}{16} \left(\frac{K_{max}}{\sigma_f}\right)^2
\]  
(9)

The practical realisation of this approach is normally accomplished by a simple crack advance scheme, in which each point along the crack front moves in accordance with the effective stress intensity factor range, \( \Delta K_{eff} \), see Eq. (1), with \( U \) provided by relationships (5) – (9).

A cracked plate under plane stress undergoes a change to plane strain behaviour near the crack tip. The radial position where the plane stress to plane strain transition takes place strongly depends on the position in the thickness direction. The degree of plane strain is essentially zero at distances from the tip greater than approximately five times of thickness. The effect of the distance from the crack tip on the evaluation of stress intensity factor is carefully considered in order to improve the accuracy of the numerical simulations. To simulate applied mode I loading, the displacement boundary conditions constant through the thickness were applied to the plate surface of the finite element model outside the 3D region per the William’s solution [13].

**Iso-K approach**

In accordance with the Iso-K approach, the steady state fatigue crack propagation requires the uniform distribution of the local stress intensity factor range along the crack front. The stress intensity factor range can be evaluated numerically using 3D linear-elastic FEA.

The practical realisation of this approach for a steady-state propagation of through-the-thickness cracks in plates can involve the evaluation of the stress intensity factor for two characteristic points: at the middle, \( z = 0 \) and at the surface, \( z = h \) for a two-parametric set of equations representing the front shapes, e.g. elliptical shapes. Further, the higher value of the crack closure at the free surface may be incorporated into the theoretical predictions using various empirical equations for crack closure proposed in the past. The steady-state crack growth requires the same fatigue crack growth rate, or

\[
\frac{da}{dN} = C_S(\Delta K_S)^n = \frac{db}{dN} = C(\Delta K_M)^n
\]  
(10)

where \( \Delta K_S \) and \( \Delta K_M \) are the stress intensity factor ranges at the surface and the mid-thickness points of the crack front; \( C \) and \( n \) are Paris constants, which can be obtained experimentally for different materials. Newman and Raju proposed the following relationship between the Paris coefficients at the surface and deepest points for the plate components with a semi-circular crack under pure tension loading [14]:

\[
C_S = 0.9^n C
\]  
(11)

In this study we also utilized a coefficient of 0.8 for both selected materials to get a better agreement with experimental data.

**Comparison of Different Approaches**

The proposed approaches for the evaluation of the steady-state crack front shapes were compared against experimental studies [5,15]. In these studies, the centre-cracked panels were made of 2024-T3 aluminum alloy and Polymethylmethacrylate (PMMA) with a thickness of 6.35mm and 40mm, respectively.
Fig. 3. Comparison between the predicted crack shapes and experimental data for the specimens made of a) 2024-T3 aluminium alloy, and b) Polymethyl methacrylate (PMMA)

The advantage of PMMA material is its transparency which enabled an in-situ evaluation of the crack front shape. For the aluminum alloy specimens such an evaluation was done using benchmarking technique and post-mortem analysis of fracture surfaces. Both specimens were subjected to constant amplitude fatigue loading. The fatigue cracks were grown over a sufficiently large distance from the initial notch to ensure the quasi-steady state conditions of propagation.

As it follows from the analysis of Fig.3, the simplified approaches work a bit better for the quasi-brittle material (PMMA); and the Iso-K approach provides the best correlation with the experimental results. Unfortunately, none of the approaches is capable to accurately describe the front shape of fatigue cracks. This can be explained by the complexity of the crack closure phenomenon, which currently represents one of the major challenges in 3D Fracture Mechanics.

Conclusion

The capability of several simplified approaches for the evaluation of the shape of fatigue crack fronts has been studied using experimental results for a steady-state propagation of fatigue through-the-thickness cracks in different materials. It is demonstrated that none of the approaches is capable to accurately describe the shape of the fatigue cracks. An empirically introduced crack closure equation allows for a better matching of the theoretical and experimental predictions. The outcomes of this work and the comparison justify a need of further research in this area.

References

Developments on Risk-Based Fatigue Failure Prediction for Application to Military Aircraft

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Keywords: risk prediction, probability, fatigue failure, military aircraft, structural integrity assessment.

Abstract. In response to the Australian Defence Force requirements the Defence Science and Technology Group has been engaged in the development of tools and methodologies to conduct probabilistic risk analyses on ADF aircraft fleet. The activity started when DST Group replicated the PRA result of Lockheed Martin for C-130H transport aircraft center wing structure. This led to the development of a fatigue failure risk analysis software tool “FracRisk”. In developing this tool, research was conducted to model the equivalent initial damage size distribution. This research resulted in the evaluation and adoption of new models based on bounded probability distributions such as the beta distribution. This work demonstrated that the DST developed risk analysis tool, FracRisk, was able to replicate capability of the USAF developed tool called PROF. This paper summarizes DST’s development of fatigue failure risk analysis capability since 2010.

Introduction

Fatigue life prediction is a major issue in the field of engineering especially in aerospace and aeronautical structures which requires accurate prediction due to underlying costs involved. In the seventies and eighties there was considerable activity associated with the performance of the Damage Tolerance Analysis (DTA) on older aircraft [1]. Over the years, DTAs and crack growth prediction has improved and modelling has become complicated but still the accuracy of deterministic prediction cannot be accepted without question. Some airframe cracking scenarios are highly complex, requiring sophisticated methods to appropriately manage potential risks including variability of influencing parameters. This parameter variability and its influence on fatigue is managed in the MIL-STD 1530D [2], which mandates the application of probabilistic risk analysis (PRA) in structural integrity assessment. Conducting a probabilistic risk analysis of fatigue failure requires the following data: i) the equivalent initial damage size (EIDS) distribution, ii) the master crack growth curve, iii) the maximum stress distribution per flight and iv) the residual strength corresponding to a given crack size. Of the four parameters, the EIDS distribution and the master crack growth curve have been found to be influential. It should be noted that the EIDS is dependent on the master crack growth curve since EIDS is derived by analytically determining the initial damage size distribution that characterizes the damage size distribution observed during test or in service using the master crack growth curve [2]. Risk-based fatigue failure research at the Defence Science and Technology (DST) Group has focused on three areas: i) methods to improve the accuracy and robustness in estimation of the EIDS distribution, ii) the effect of the variability of the master crack growth curve, and iii) development of tools for the risk-based aircraft structural integrity. The risk-based methods and approaches developed by DST have also been partially validated via experimental data obtained by DST and other sources. By 2020, the DST developed risk-based methodology will be used to augment the structural safety management procedures for the ADF aircraft fleet.
Modelling of the initial flaw size distribution

It has been noted in reference [3] that the assumed distribution model used in probabilistic risk analysis is critical to the fatigue failure risk prediction because the analytical result is highly dependent on the right tail of the distribution. For this reason, DST investigated several different models to approximate the EIDS distribution and found that the use of lognormal distribution may over-estimate the predicted risk as the right tail of this distribution is unbounded. Since cracks grow exponentially with time, unrealistically large maximum cracks in the distribution dominate the risk prediction resulting in the overestimation of risk values. For this reason DST proposed a new method of modelling the EIDS by using a bounded distribution model such as the Beta distribution [4].

Preliminary validation of deterministic and risk-based approaches to determine the safety inspection interval of airframes according to MIL-STD1530D

DST investigated the variability of fatigue crack growth of 7075-T7351 aluminium alloys by testing 85 middle tension specimens under a variable amplitude load spectrum. A fatigue starter was introduced through a center notch, as shown in Fig. 1. The load spectrum consisted of 19032 load turning points which were referred to as a ‘load block’. The maximum tensile load was 60 kN. Each test specimen was subjected to the nominally identical load spectrum applied repeatedly until the specimen failed by fracture. Crack sizes were measured at every 2000 load turning points (i.e., 1000 cycles) using the direct current potential drop (DCPD) method [5]. Test results showed that even under well controlled conditions, cracks grow stochastically as shown in Fig. 2. The figure shows two different plots of the specimen crack growth curves (in black) and the risk curves (in blue) which have both horizontal axis units in load blocks. The crack growth curve’s vertical axis coordinates also coloured black are shown to the left of the graph whereas the risk curves’ vertical axis coordinates is to the right of the graph coloured blue similar to the colour of the risk curves. The crack size \(a\) increases with the increase in the number of load blocks applied to the test articles whereas the probability of failure (POF) also increases with the number of load blocks applied to the test articles. The military standard MIL-STD1530D specifies safety inspection requirements to be used in slow damage growth components. The standard specifies that the initial inspection shall occur at or before one-half the life from the assumed maximum probable initial damage size to the critical damage size. Furthermore, the standard requires that risk analysis shall be used to determine if a reduction in the inspection intervals are required to control the safety risk to an acceptable level or to reduce economic or availability consequences associated with damage repair. This is where the risk-based approach or the PRA plays a role in determining the actual inspection interval. In PRA as specified in the MIL-STD1530D, the acceptable risk or the acceptable Single Flight Probability of Failure (SFPoF), is defined as SFPoF=10^{-7}. When SFPoF exceeds 10^{-7} but is less than 10^{-5}, a safety inspection is required and beyond 10^{-5}, the risk is considered unacceptable. Thus, in the probabilistic method, a risk curve is constructed and the flight hours corresponding to 10^{-7} and 10^{-5} are projected to determine the range of flight hours where inspection is necessary.
Direct validation of the risk assessment approach is challenging with limited appropriate data. The approach adopted here is meant to provide some sense that the numbers are meaningful by producing results in the expected vicinity of the deterministic approach. Note that the natural variation in fatigue performance between aircraft is expected to be larger than what is presented here. The first inspection in the MIL-STD1530D is designed to capture this variation and further supported by a risk analysis that may show that an earlier inspection is required. A comparison (base on five independent trials\(^1\)) of the time of the first inspection (for both deterministic and risk analysis) as per the MIL-STD1530D is presented in Table 1. Risk analysis results also compare results evaluated using the mean (fixed) fracture toughness \( K_c \) and also the fracture toughness represented as a distribution (variable \( K_c \)). The comparison shows that using the variability of fracture toughness \( K_c \) in the risk analysis decreases the inspection time as shown in Table 1. The failure data were modelled using a three parameter Weibull distribution and the resulting model’s failure rates corresponding to the range of inspection times of the three approaches (i.e., deterministic, probabilistic with fixed \( K_c \) and probabilistic with variable \( K_c \)) were compared. In addition to these, the initial inspection interval corresponding to Def-Stan 970 allowable failure rate of 1/1000 was also calculated and shown in Table 2. The results demonstrate that the risk analysis methodology produces results within the expected range.

![Crack growth curves for aluminium 7075-T7351 mid-tension specimens under variable amplitude load from DST experiment as compared to risk analysis predictions](image)

**Fig. 2** Crack growth curves for aluminium 7075-T7351 mid-tension specimens under variable amplitude load from DST experiment as compared to risk analysis predictions

**Table 1** Comparison of initial inspection times evaluated using the deterministic and probabilistic approach as per MIL-STD1530 and the DST test data

<table>
<thead>
<tr>
<th>Trial</th>
<th>Evaluated inspection time (Load blocks)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deterministic</td>
</tr>
<tr>
<td>1</td>
<td>7.7</td>
</tr>
<tr>
<td>2</td>
<td>7.6</td>
</tr>
<tr>
<td>3</td>
<td>7.3</td>
</tr>
<tr>
<td>4</td>
<td>7.8</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

\(^1\) A “trial” here consists of 5 randomly chosen crack growth curves with which to conduct the analysis
Table 2 Comparison of the evaluated inspection times for trial 1 with the UK Defence Standard

<table>
<thead>
<tr>
<th></th>
<th>First inspection (deterministic)</th>
<th>Probabilistic (Fixed KC) P=10^{-7}, 10^{-5}</th>
<th>Probabilistic (Variable KC) P=10^{-7}, 10^{-5}</th>
<th>Def-Stan 970 (Probability of failure of an aircraft during its entire life)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inspection times</td>
<td>7.7</td>
<td>11.5, 11.8</td>
<td>9.9, 10.5</td>
<td>10.3</td>
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<tr>
<td>(Blocks)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Probability of</td>
<td>1/15401</td>
<td>1/155, 1/99</td>
<td>1/1790, 1/706</td>
<td>1/1000</td>
</tr>
<tr>
<td>Failure$^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Development of FracRisk – a risk-based fatigue failure assessment tool

As part of DST’s risk-based structural integrity assessment of C-130 aircraft, a new analysis tool called FracRisk has been developed. The development started with a Defence requirement to replicate the C-130H PRA [6] on center wing lower surface panel location. With the RAAF planning to use FracRisk analysis on other platforms, the Defence Aviation Safety Authority (DASA) contracted QinetiQ Engineering to conduct a preliminary evaluation and validation by comparing FracRisk and the United States Air Force (USAF) developed analysis tool PROF [7]. Comparison of the analysis results between FracRisk and PROF for the analysis of C-130J CW-3A location are shown in Fig. 3 and Fig. 4 [8]. In the analysis the input data between FracRisk and PROF were identical, including the EIDS which was modeled as a beta distribution. The FracRisk analysis results are almost indistinguishable with PROF results especially in the region where the SFPoF is 1x10^{-7} or less. This result shows a verification of FracRisk against PROF. Compared to PROF, FracRisk has more flexible input data format such as the capability to use the residual strength curve as the input parameter instead of using stress intensity factors. Robustness of the graphical user interface was also evaluated by QinetiQ after which a few improvements were proposed. Following the recommendations from QinetiQ, FracRisk user interface was modified to incorporate the suggested improvements. An example of the user interface is presented in Fig. 5.

![Fig. 3 Comparison of FracRisk and PROF results using tabulated stress exceedance data for C-130J CW-3A location](image1)
![Fig. 4 Comparison of FracRisk and PROF results using Gumbel peak stress distribution for C-130J CW-3A location](image2)

$^2$ Based on a three parameter Weibull distribution fitted to the failure data
Summary

With the increasing demand for risk-based approaches to structural integrity assessment of military aircraft, DST has been actively supporting the RAAF by conducting research to improve the methods associated with fatigue failure prediction of structures. Investigations examining the application of different distribution models for the EIDS distribution resulted in a new proposed model using bounded distributions such as Beta distribution. The use of a bounded distribution addresses one of the critical issues in risk analysis, which is the maximum crack size cannot be infinitely large. Even the smallest possibility of a unrealistically large initial crack has the potential to increase the calculated single flight probability of failure risk by several orders of magnitude. Further work is needed to study the goodness-of-fit for the Beta distribution to a larger EIDS data set.

DST investigated the validity of a DST developed risk assessment methodology in conjunction with the MIL-STD1530D guidelines. Preliminary investigations based on DST coupon tests showed that probabilistic approach predicted results in the expected range. In the future, analysis of multi-site fatigue damage (MSD) considering multiple and interacting cracks will be added to capabilities of FracRisk.

Acknowledgement

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References


III  DAMAGE AND CRACK MONITORING
Numerical Study of Applied Continuum Mechanics for Damage Detection on a Cantilever Plate

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Keywords: Non-destructive Testing, Savitzky-Golay Filter, Fibre Optic Sensors, Finite Element Analysis, Plate.

Abstract. Through the use of continuum mechanics, the static governing differential equation was used as an algorithm to detect delamination in composite plates. The algorithm outcome indicated whether or not the structure contained discontinuity through the presence of a residual term. The parameters of the algorithm were defined as the measured in-plane stain values, where a Savitzky-Golay (SG) filter was then used to process the governing differential equation. This paper presented a theoretical study into the use of this algorithm with an array of embedded fibre Bragg grating strain sensors. The measured in-plane strains were obtained using a finite element analysis (FEA) study for a cantilever plate modelled in ANSYS Mechanical 18. This study investigated various delamination sizes ranging from 25 to 75 mm² and the spaces between the fibre optic array consisting of 10, 20 and 40 mm. The measured strain values will be obtained from the FEA simulation through multiple ‘Construction Geometry Paths’ to illustrate the potential strain that the fibres could measure.

Introduction

Since the 1980s, there has been a high demand for real-time monitoring and damage detection within the Naval, Civil and Aerospace industries. A significant interest developed with the multiplexing of fibre Bragg grating and their alignment in array configurations. The development of the fibre Bragg grating has shown to be a feasible solution within the industries, with apparent advantages over more traditional methods, which are as follows [1]–[3]:

- immunity to electromagnetism at a sensor level,
- little to no change to the host material,
- no wires required for the sensors and,
- real-time integration of areas considered to be out of reach, dangerous and or impractical to measure.

Fibre optic arrays were investigated initially as a concept in 1983, with damage detection in composite materials, as complications arose with field inspections of Naval architecture while at sea [3]. This resulted in a promising outcome with the array configurations, as they were able to detect and locate severe damage when impacted [3]. The 1990’s then showed promising results with fibre Bragg grating in Civil Structures, early work investigated embedding fibre Bragg grating in concrete beams [4]. Early field trials were later implemented, with one of the previous projects tested a quarter scale bridge consisting of 60 fibre Bragg gratings [5]. Most recently, the Japanese national project for aerospace structural health monitoring developed an array technology to detect the accumulating water in honeycomb structures by the change in temperature [6]. This later progressed to impact detection of a large-scale aircraft by measuring the change in strain [7]. To isolate fibre optic failures the design was based on the nervous system [7].
This paper will provide a demonstration of a new technique developed by Wildy et al. [8,9] for the detection and localisation of damage within a structure. The technique evaluated non-continuity within a structure, by applying an algorithm that consisted of the static governing differential equation of in-plane strain processed using a SG filter. A plate structure will be used to demonstrate the proposed technique by simulating a model in ANSYS. The FEA result will then be used to obtain the in-plane strain values at the ideal sensor location of the fibre optic arrays.

**Damage Detection Algorithm**

Wildy and Codrington [8] demonstrated a practical solution by applying governing differential equations to a plate for the detection of a crack damage. The same governing differential equation is expressed in Eq. 1, where the final form was derived from the equations of equilibrium and the strain compatibility equations [8]. The $\varepsilon_x$ and $\varepsilon_y$ represents the in-plane strains as referenced in Fig. 1 and $\nabla^2$ represents the Laplacian differential operator.

$$\nabla^2(\varepsilon_x + \varepsilon_y) = 0$$  \hspace{1cm} (1)

The governing differential equation is said to be satisfied if the material structure remains continuous (i.e. undamaged). When a plate structure is deemed to be damaged, either from a crack or delamination the governing differential equation will not be satisfied. By evaluating the governing differential equation into two states damaged and undamaged, the damaged governing differential equation can be expressed as follows,

$$\nabla^2(\varepsilon_x + \varepsilon_y) = R,$$  \hspace{1cm} (2)

where the residual term, $R$, represents the discontinuity within the plate.

Wildy and Codrington [8] processed the governing differential equation using a two-dimensional SG filter, which is a variation of a moving average. The benefits of the SG filter were highlighted, as the filter has the ability to preserve the features of the sampling data [10]. These included the maxima, minima and the widths, while smoothing and/or differentiating by using a polynomial least-square fit [8,10]. The derivation of the SG filter will not be presented in this paper, but can be found in [11,12]. For this paper, the governing differential equation will be evaluated using the SG filter, by performing a second-order differentiation of the in-plane strains. A sampling window of 3x3 data points will be selected alongside a second order polynomial fit.

**Model Prototype (Method)**

The geometry of the plate was divided into four segments as seen in Fig. 1, this was to preserve the midsection of the plate to prevent symmetrical artefacts occurring in the numerical solution. Fig. 1 illustrates the simple plate geometry with Region I showing the undamaged section (i.e. non-delamination) which consists of segments 1 and 2. Additionally, Region II shows the damaged section (delamination) located at points ‘ABCD’ which consists of segments 3 and 4. The physical material properties of the plate were composed of an isotropic material assigned as structural steel, with lengths of 500 mm and a depth of 6 mm.
A contact region was created between segments 3 and 4 at point ‘ABCD’ which allowed the delamination dimensions to vary from 25 to 250 mm². The contact region was created to be frictionless to prevent segments 3 and 4 from passing through one another. The mesh was kept uniform throughout the plate with a global element size of 1 mm as a brick type element. The initial boundary conditions were modelled as represented in Fig. 1 (b), where a ‘Fixed Support’ constraint was applied to the face touching the y-axis of segments 1 and 2. A constant static force of 250 N was then also applied to the edge on the opposite side of segment 1.

The in-plane strain values were extracted from the simulation using multiple ‘Construction Geometry Paths’ to simulate the potential values that the fibre optics will measure. The strain values will later be processed using the SG filter to detect damage. To preserve the fibre optic measurements from any singularities presented in the simulation, a boundary was applied which was 60 mm from each edge of the plate.

Results & Discussion

Fig. 2 shows the in-plane strain fields with the presence and absence of extreme delamination at a size of 250 mm². The strain values presented in Fig. 2 were the surface results from the FEA simulation instead of the fibre optic path locations. Whereas, Fig. 3 presented the new damage detection technique with the applied algorithm, showing the residual term values of the governing differential equation. The residual term results differed as the density of the fibre optic array spacing and delamination changed. The studied fibre optic spacings were 10, 20 and 40 mm, and delamination sizes were 25, 50 and 75 mm². The results in Fig. 3 also showed the total area of the plate in grey and the damage detection of the fibre optic arrays from both Region I and II.
Figure 3: Residual term (R) from SG filter at array sizing (a) 10, (b) 20 and (c) 40 mm spacing, and delamination sizing at (1) 25, (2) 50 and (3) 75 mm².

By evaluating the damage detection results from Fig. 3, both governing differential equation states were noticeable with the residual terms as referenced in Eq. 1 and 2. First, the residual terms were zero for the undamaged parts of the plate (Region I), and secondly, the non-zero residual terms were shown at the delamination zone (Region II). Notably, by changing either the delamination size and or the fibre optic array spacing, the residual terms were shown to be affected. As the delamination dimensions decrease (Region II) the residual term reduces. To see similar results of a small delamination a larger bending force will be required to generate a higher strain field. As the fibre optic spacing was increased the results had a greater distance between the measured peaks. Thus, resulting in a reduction in the accuracy when measuring the delamination size.

Conclusion

This paper demonstrated a successful implementation of an algorithm to detect damage by the means of a static governing differential equation. The technique was examined using a numerical study of a plate under cantilever conditions while in the presence and absence of delamination. The in-plane strain values were obtained using multiple ‘Construction Geometry Paths’ in ANSYS to illustrate the potential values a fibre optic array could measure. The governing differential equation was then investigated to find the discontinuity within the structure. Three variations for both the delamination and fibre optics array spacing were investigated. The results of the processed algorithm indicated that the finer array spacings showed a truer delamination size, a higher residual term presence was solved with a larger
delamination. The results showed an excellent application for the real-world use of fibre Bragg grating arrays and the detection of delamination using continuum mechanics.

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References


The application of piezoelectric strain gauges to enhance fatigue crack closure measurement

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Keywords: Strain measurement, piezoelectric strain gauge, fatigue, compact tension specimen

Abstract. The Defence Science and Technology (DST) Group is responsible to support the Australian Defence Force via research and advice relating to aspects of aircraft structural integrity. One important aim has been to safely manage and improve our understanding of the fatigue life of structures, components and materials that make up an airframe. To quantify and predict fatigue behaviour numerous techniques have been developed (such as the crack closure concept) to predict the influence of load history on the fatigue life of aerospace components. To date, limited experiments have been conducted to assess crack closure in variable amplitude load sequences due to limitations in measurement techniques. Crack closure produces a very small non-linear strain response dependant on location (~1-2με) requiring a measurement device with high sensitivity and accuracy. Piezoelectric strain gauges offer new potential by providing superior signal to noise measurements. These sensors provide the potential to monitor crack closure in variable amplitude load spectrums to an unprecedented level of fidelity. This paper presents recent DST research that applies a remote piezoelectric strain gauge to measure the non-linear strain response in a compact tension specimen. It is anticipated that this work will lead to an enhanced understanding of crack closure and provide a mechanism to improve existing predictive tools.

Introduction

It is a requirement of the Australian Defence Force that all aircraft in the military undergo fatigue testing to certify aircraft designs under representative loads and expected usage. The information that is derived from these tests are instrumental in the development of structural management plans to safely operate the aircraft to their planned withdrawal date. One area of research at DST within the innovative sustainment domain is focused on development of spectrum compression techniques. These techniques are used to radically reduce the size of load sequences (spectrums that are representative of the entire load history an aircraft is expected to see during operation).

DST has developed a new spectrum compression technique based on crack growth to reduce the size of load sequences but relies heavily on the prediction of loads at which fatigue crack closure occurs [1, 2]. To validate the process it is highly desirable to experimentally verify the open load predictions in a variable amplitude (VA) load sequence. The measurement of crack closure has been problematic under VA load sequences due mainly to the resolution of existing measurement techniques [3, 4]. For VA load sequences measurements of crack closure is generally limited to the largest load transitions in a load sequence with limited ability to study the interaction with smaller cycles due to the noise in the system. Measurement of crack closure for constant amplitude (CA) load sequences has been far more successful as reflected by the inclusion of a compliance technique in the ASTM 647 standard. Unlike VA load sequences, CA load sequences can be averaged to improve the signal to noise ratio but nuances in transition from one load amplitude to the next is lost.
In this paper the authors wish to exploit the sensitivity of piezoelectric strain gauges. The piezoelectric PCB 740B02 strain gauge used in this investigation has a quoted broadband resolution of 0.6 nε significantly higher than a typical resistive strain gauge where the best achievable resolution is approximately 10 με. The aim is to achieve an absolute strain measurement on a compact tension specimen including both dynamic and static contributions. More specifically to achieve a level of accuracy to enable the study of crack closure in VA load sequences.

**Experimental Methodology**

Two specimens were manufactured from 7075-T7351 Aluminium; the first, a calibration specimen and the second a compact tension (CT) specimen. The calibration specimen was used to calibrate the voltage response of the piezoelectric strain gauge to a known applied strain as well as evaluating the non-linear components of the test arrangement. Both resistive and piezoelectric strain gauges were bonded to both specimens to allow comparison of the response. The dimensions of the specimens are presented in Fig 1. A resistive strain gauge (CEA-I3-125UW-350) was located both axially and centrally on the calibration specimen with a piezoelectric strain gauge on the opposite side. On the CT specimen the resistive strain gauge was placed centrally on the back face and the piezoelectric strain gauge was located on the back face but indented 11.5 mm from the edge. The maximum operational strain range for the piezoelectric strain gauge is quoted as 100 με. Careful consideration of the position is required to avoid overloading the piezoelectric gauge.

The test sequence applied to both calibration and CT specimens utilised sequence 2 used in White et al [5]. This sequence kept a constant maximum load while alternating groups of CA with stress ratio of 0.5 with steadily decreasing groups of CA loading. Each group consisted of 100 cycles and the entire sequence in the following text will be referred to as a block. The spectrum block was repeatedly applied to the CT coupon until a crack grew to failure. The resistive strain gauge response in Fig. 2a shows an inverted profile of the loading that was applied. The test specimens were loaded in a 100kN MTS test machine to a maximum load of 4kN and were cycled at a frequency of 5.7Hz. This frequency was chosen to match the manufacturers quoted calibration frequency. A data sample rate of 1000 samples per second was used.
Figure 1: Dimensions (in mm) of a) the calibration specimen and b) the compact tension specimen

**Effects of non-linearity**

Discussion of various non-linear issues in this paper refers to deviations from purely linear responses. Crack closure is the non-linear response we wish to measure but decouple from other sources of non-linearity. Crack closure manifests as a tail at the bottom of a load cycle in the force vs reduced strain plot and is due to plastic deformation around the crack tip and physical contact of the crack faces. Other non-linear effects can creep into the results from several sources including the pin connections, adhesive bonds on the strain gauges, and phasing issues in acquisition of data from various sources. In the present analysis the first loading block applied to the CT specimen was used to characterise the nonlinear behaviour of the test system in the absence of crack closure. Hence with this calibration the unwanted non-linear responses are subtracted from subsequent load actions to accentuate the crack closure non-linear response.

**Challenges: resistive vs piezoelectric strain sensing**

While piezoelectric strain gauges offer superior resolution, new challenges to absolute strain measurement are introduced. Piezoelectric strain gauges suffer from signal decay if the applied loading does not have a zero mean. This presents a problem as realistic load sequences used in fatigue crack growth testing generally contains cycles with nonzero mean. To use a piezoelectric strain gauge to measure crack closure a methodology to recover the direct current (DC) corrected signal is required. The piezoelectric strain gauge results are presented in Fig 2b and it is clear from this figure that the amplitude of the CA blocks regresses to a mean of zero. Correcting this phenomenon is critical to recovering the absolute strain response necessary to measure closure levels in a VA load sequence. Piezoelectric devices are also sensitive to temperature highlighting the importance of sensor calibration and a controlled test environment. Frequency dependence is also an important consideration especially at lower frequencies. Three frequencies (10Hz, 5.7Hz and 1Hz) were examined and 5.7Hz was chosen a compromise between limits on data acquisition rate and the response of the gauge.
Genetic Symbolic Regression (GSR)

The strength of GSR [6] lies in the methods ability to trial an enormous number of equations, and through the application of a genetic algorithm, converge on an equation that best fits the data while minimizing the complexity of the equation. This technique has been used two fold in the present investigation; one to characterise the decay in the piezoelectric strain gauge response, and two to adjust the piezoelectric and resistive strain gauge responses to remove test system nonlinearities.

Decay correction. The DC component of each cycle in the sequence in Fig. 2b is extracted (shown in Fig. 2c) and is used as the dependant variable in the GSR where the force and local time (re-zeroed with each change in CA block) are the independent variables. The subsequent equation can then be used to subtract the DC component from the entire sequence. The DC component can be inferred from the mean of the applied forcing cycles scaled by a local scaling factor $\gamma$. The $\gamma$ factor can be calculated in two ways. Firstly, the $\gamma$ factor can be evaluated by taking the ratio of the stabilised piezoelectric response amplitude and equivalent amplitude for the applied force. Secondly it can be evaluated by taking the ratio of the peak rate change in piezoelectric response to the equivalent rate change in force. The former method is simple and approximates the behaviour per full spectrum application. The latter method is more computationally intensive but provides a cycle-by-cycle correction. The corrected sequence is presented in Fig. 2d.

Nonlinear correction. A new dependent variable representing the difference of the actual response to a purely linear response is created. The GSR technique is then applied to determine the functional relationship between this dependent variable and the independent variables; force and force derivative. This functional relationship is constructed with either data from the calibration specimen or the first applied block on the CT specimen and then applied to correct the strain gauge response for all other applied loading blocks.

Figure 2: Block 1 CT specimen gauge results for a) the resistive strain gauge b) piezoelectric strain gauge c) DC drift in piezoelectric strain gauge response and d) DC corrected piezoelectric response
Results and Discussion

The applied force vs reduced strain is plotted in Fig 3 for both piezoelectric and resistive strain gauges. The reduced strain is the total strain with the linear portion subtracted to reveal nonlinear response. The crack length at block 282 reached 4 mm in size where closure was observable on the largest load cycle in the sequence with an opening load of ~0.25 of the maximum load. It is clear in Fig 3 that the signal to noise ratio for the piezoelectric strain gauge is far superior to that of the resistive strain gauges. The piezoelectric strain gauge displayed a measurement error of approximately 0.5 με where the resistive strain gauge displayed a measurement error of approximately 45 με. The results indicate that crack closure is observable on a cycle-by-cycle basis in a variable amplitude load sequence. The GSR proved to be a powerful tool to aid in the correction of DC decay in the piezoelectric signal and in normalising system nonlinearities associated with the experimental system. Although, nonlinearities in the system response were removed this ultimately did not affect the measurement of the crack opening stress which is a dominant asymmetrical nonlinear effect easily recognisable relative to dilation of hysteresis loops. Correction of nonlinearities however does permit an effective comparison between resistive strain gauge performance versus piezoelectric strain gauge performance. As can be seen in Fig 3 the piezoelectric strain gauge generates consistent loops relative to the averaged resistive strain gauge response. Notably individual cycles produced by the piezoelectric strain gauge are consistent with the averaged loops, unlike the resistive strain gauge results where crack closure is only observable after averaging across 100 cycles. The nonlinear response in the strain gauge does appear to be greater but this is related to the strain gauge position which experiences strains five times larger than the location where the piezoelectric strain gauge was mounted.

Conclusion

The present investigation has demonstrated that piezoelectric strain gauges have significant potential to be used to measure absolute strain, but more importantly possess the sensitivity and resolution to allow detailed investigation of second order nonlinear effects like crack closure. However limitations exist, such as DC decay observed when monitoring nonzero mean load cycles, limits on the operational strain range, sensitivity to temperature and sensitivity frequency. A method to correct DC decay and system nonlinearities has been proposed and demonstrated to yield effective results. This investigation has found
that piezoelectric strain gauges are a viable measurement tool to measurement of higher order strain based effects such as crack closure.

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References


Vibration fault detection of fracture in a wind turbine tower foundation

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Keywords: Tower, rotor blade, test-rig, wind turbine, fatigue, crack, vibration.

Abstract. Supporting towers of large wind turbines undergo significant vibrations and dynamic stresses due to the effect of wind gusts and dynamic loads. The use of regular condition monitoring vibration measurements on the tower can facilitate detection of large changes to the stiffness of joints and connections of the structure, including the foundation. This paper reports on experimental work performed on a small scale instrumented wind turbine test rig in the lab that suffered a significant foundation fatigue crack failure after two years of operation.

The experimental work investigates novel techniques of dynamic coupling detection, between the tower and rotor blades by utilizing a comprehensive array of experimental observations of the small wind turbine system. The coupling investigation includes the analysis of tower vibration in the three orthogonal X, Y, and Z directions and the rotating blade vibrations at different speeds. High-frequency dynamic measurements have been obtained from a triaxial accelerometer located at the top of the tower positioned in a plane parallel to the blade rotation plane. Simultaneous tower and blade vibration condition monitoring measurements have been implemented for more than one year, including rotating and non-rotating, contact and non-contact sensors.

The presence of a large fatigue crack around the base of the tower was detected by observation during transient testing of the wind turbine. Subsequent analysis of the vibration measurements confirmed that changes to the tower vibration response were apparent during previous tests. This paper will outline the analysis of the rotating and non-rotating vibration measurements taken in the months prior to the fatigue failure showing how the incipient failure has appreciably been detected.

Introduction

The supporting towers of fixed foundation wind turbines need to withstand transient lateral, axial and reaction torque load situations resulting from fluctuating wind speed conditions. The foundation of the supporting tower is normally able to tolerate the excessive loads resulting from the wind turbine assembly and tower weight; however, long-term dynamic stresses and tower fluctuations are able to produce structural damage in the foundation, which begins with crack initiations. Khatri [1] conducted stress analysis by creating a model of the tower in ANSYS, which shows the coupling effect between yaw eccentricity and tower fluctuation and its pronounced effect on growing longitudinal cracks in the wind turbine foundation. Operation with dynamic vibrations may lead to the creation of transverse cracks in the steel tower body and concrete foundation which may result in turbine failure. Hassanzadeh [2] pointed out that a crack in the wind turbine foundation is not a rare incident. Damage to the tubular steel tower and concrete foundation caused by fatigue cracks result in weakness of the structure, which when left unmonitored, can lead to structural failure. Additionally, connection flanges that contain bolts were found to be likely sites for localised crack initiation. Monitoring in situ of the wind turbine foundation for the strain and crack lateral propagation utilized fiber optic strain gauges positioned along the tower structure.
Cracks that were found to be propagating on the tower bending areas occurred predominantly on the wind side more than the other sides according to strain measurements [3]. Moreover, linear variation was found between the crack displacement and tower strain measurement through the dynamic analysis. This work presents practical outcomes of the crack initiation between the supporting tower and foundation during different loading conditions created by applying artificial faults to a small scale test rig.

**Small-scale wind turbine test rig components**

A small-scale wind turbine test rig has been manufactured and assembled in the vibration lab, for implementing different blade fault conditions and monitoring dynamic vibrations of the rotating and non-rotating components over a range of rotating speeds. A variety of measurement sensors have been utilized for the blade, rotor shaft, and tower vibration condition monitoring as shown in Fig. 1. Three 1 m long slim blades have been bolted to the hub, where two of the blades have been instrumented with in-plane strain gauges, and one has axial and out-of-plane strain gauges for vibration monitoring during rotation. As well, a miniature piezoelectric accelerometer has been chosen to be positioned near the blade tip on one of the blades. Orthogonal laser sensor measurements have been located on the main rotor shaft for shaft vibration monitoring at the free end after bearing 2. Additionally, to achieve comprehensive tower dynamic vibration monitoring, a high sensitivity triaxial accelerometer has been positioned in the upper part of the tower, enabling the measurement of vibrations in the X, Y, and Z directions. The Z direction corresponds to the in-plane rotor direction, the Y direction corresponds to the out-of-plane direction, while the X-axis is the vertical direction.

A low noise slip ring assembly at the end of the rotor shaft has been utilized for connecting all rotating measurement sensors with the data acquisition system. A servo motor with 10N maximum torque has been chosen to the drive the test rig to simulate wind gust behaviour. Various case study experiments have been conducted on the test rig using the ABB servo motor Mint WorkBench software. MATLAB data acquisition software was also used with an NI Compaq daq system for digitising the dynamic sensor measurements during the tests. The vibration condition monitoring comprises the analysis of the rotating and non-rotating dynamic components by analyzing the raw signals of dynamic strain and acceleration, as well as the rotor shaft dynamic displacements during the operations to evaluate the component health for several diverse artificial case studies. The tower has been designed from hollow circular aluminum tubing, to support the drivetrain at a height of 1.75 m. More details about the test rig, sensors and dimensions can be found in reference [4].

![Figure 1 Horizontal axis and small-scale wind turbine test rig.](image-url)
Fig. 2 illustrates the first theoretical bending mode shape of the supported tower indicating the behaviour and bending zones along the length. Higher modes further show the relative bending amplitudes for each mode, as well as the coupling effect of the blade vibrations.

![Figure 2 The dominant mode shape of the supporting tower in the wind turbine test rig.](image)

Significant dynamic bending of the turbine support tower system was observed during the operation, for the various case study conditions, especially in the Z in-plane direction, excited by the servo-motor reaction torque. The reaction torque at the top of the tower was fed to the welded joint at the lower base of the tower during the operation.

Various case studies were developed for vibration testing of the test rig including steady state running speeds from 40 rpm to 150 rpm, addition of mass at the blade tips, reduction of mass at the blade tips, loose bearing support, slow ramp-up of speed from 0 to 120 rpm and transient impulse torque testing.

**Monitoring crack propagation based on acceleration measurements**

Monitoring of the peak tower vibrations during several of the various blade fault tests, over a period of more than 1 year showed significant increases, regardless of the rotor speed or the type of embedded fault. Fig. 3 shows the maximum measured tower acceleration amplitudes for different case studies on the current wind turbine test rig, accumulated over the operational period. The increase in acceleration amplitudes affirms the probable increasing fatigue stresses on the tower foundation due to the persistence of dynamic vibrations. The initial primary data (X=0.494 m/s^2, Y=0.832 m/s^2, and Z=0.536 m/s^2) were the lowest readings in the process. A pronounced increase can be observed in tower vibration amplitudes in the Z-direction, coupling with the blade in-plane vibrations. Additionally, vibrations in the Y-direction were seen to further increase caused by coupling with blade out-of-plane vibrations, and all modes were known to increase with the increase of rotating speeds. After February 2018 (data point 12), a rise in the tower vibrations in the Z-direction was observed, despite the lower rotational speed (100 rpm), which possibly indicated that the crack had initiated in the test rig foundation. A significant 4 cm weld crack was observed visually at the base of the tower during May 2018, (data point 16). The tower was then disassembled and repaired with a stronger reinforced welded joint. After the repair of the tower root connection, a bump test was implemented to monitor the fundamental tower bending frequency, confirming the increased foundation support stiffness post repair.
Monitoring crack propagation based on dynamic frequency

Monitoring the fundamental tower resonance frequency during the experimental work was known to be a helpful technique to detect any change in the health of the overall structure, such as caused by fatigue cracks. As mentioned above, several of the artificial faults were known to trigger high-frequency responses as illustrated in Fig. 4, which shows a spectrogram analysis of tower vibration for operation at 100 rpm and 100 seconds duration with an imbalance effect resulting from adding a localised 200 g mass on blade1 (the rotating blade that has the imbalance mass). It can be seen that this provided excitation of high frequency, especially during the period from 10 to 25 seconds. The pronounced fluctuation of the tower during such tests provides high stresses to the welding joint with the foundation, likely providing the initiation of the cracks in this area. The dark yellow region at the bottom of the spectrogram shows the rotor operating frequencies, measured from Y and Z directions of the tower. This example relates to the measured peak acceleration point (data at 10/08/2017) from Fig. 3.

Additionally, Fig. 5 shows the trend of the first resonance frequency of the tower obtained from several of the tests over the period of more than one year of test rig operation. This representation demonstrates the change in tower resonance frequencies and loading condition on the foundation during the operational period. These results show the resonance close to 2.1 Hz after the initial tower construction, during the year of operation, and the subsequent change in resonance to 1.3 Hz when the 4 cm weld crack was growing and then finally 2.25 Hz after the repair had been performed.

The change to the tower bending resonance frequency provides a further example of how the servo-motor reaction torque provides increasing fatigue loading onto the tower foundation. It is likely that the increasing tower vibrations, coupling with blade vibrational modes (axial, in-plane and out-of-plane) led to the crack initiation on the right side of the tower foundation welded area, which then extended to a 4 cm length around the circumference.

Figure 4 Spectrogram analysis from 10/8/2017 (maximum tower acceleration in figure 3, 100 rpm), with 200 g, added mass on the rotor blade, a) Y-direction, and b) Z-direction.
Figure 5 Fundamental tower bending resonance frequencies from the experiments during the fatigue crack growth period and subsequent repair.

**Conclusion**

A 4 cm weld fatigue crack occurred in the tower foundation of a small-scale wind turbine test rig during the conduct of various experiments that were used for assessing vibration condition monitoring methods. It is likely that the increasing rotor blade vibrations from various seeded fault tests including transient impulsive torque loadings led to excessive vibration towards the Z and Y directions, hence reducing the welded joint fatigue life and initiating a fatigue crack which subsequently grew over a short period of time. Frequency analysis of the tower vibration showed that the fundamental tower resonance changed from 2.1 Hz before the crack and then reduced to 1.3 Hz before the crack was visually identified and repaired. After the repair, the resonance was increased to 2.25 Hz. This shows the value of careful monitoring of the tower foundation welded joints using trends of the tower bending resonance.

**References**


Abstract. This paper presents the outcomes of an experimental study conducted on a short concrete column loaded to failure under uniaxial compression. Piezoceramic transducers bonded to the free surfaces of the specimen were used for ultrasonic inspection during the progressive loading. The evolution of damage was monitored by means of change in ultrasonic wave velocity and scaling subtraction method, with the aim to highlight the capabilities and limitations of these approaches. The results show that the change in the nonlinear parameter derived by the scaling subtraction method is over two orders of magnitude greater than the relative change in the Rayleigh wave speed, thereby indicating the much greater sensitivity of the nonlinear ultrasonics approach for damage detection.

Introduction

Over the past two decades, Non-Destructive Testing (NDT) techniques have gained prominence in the structural integrity management of concrete-based infrastructure assets across the world [1]. The usefulness of NDT techniques is strongly linked to the smallest defect size that can be reliably detected. This is because the time between the formation of a detectable defect and failure largely determines the frequency of safety inspections, and consequently, the maintenance costs [2]. This provides the motivation for the continuous improvement of defect detection techniques.

Linear ultrasonic techniques have been extensively utilised in NDT applications and rely on changes in wave velocity, phase or amplitude to locate and identify structural defects and measure the elastic properties of materials [3]. However, these techniques are only sensitive to defects or microstructural features which are comparable in size to the ultrasonic wavelength [4]. Frequencies less than 100 kHz are generally utilised for ultrasonic testing of concrete, since higher frequency waves experience significant attenuation and distortion in the heterogeneous medium [5].

Nonlinear ultrasonic techniques, such as higher harmonic generation, vibro-acoustic modulation and scaling subtraction method, are promising tools for early detection of damage, i.e. the detection of damage that cannot be resolved using linear ultrasonic techniques at typical testing frequencies. The latter techniques rely on measuring the nonlinear response of the propagating medium, which originates from the localised deformation of compliant features such as micro-cracks, voids and grain boundaries under applied stress [6]. The nonlinear response may manifest itself in a variety of ways, including resonance frequency shift, harmonic generation, frequency mixing, nonlinear attenuation and slow dynamic effects [6, 7]. The nonlinear response of concrete and other cement-based composite materials is strongly enhanced with the accumulation and propagation of micro-cracks (aperture of few microns) in the cement matrix and along the matrix/aggregate interfaces [8]. These features make nonlinear ultrasonic techniques suitable for early detection of distributed damage (micro-cracking), despite the wavelength being much larger than the defect size [7, 8].
In this paper, the sensitivity of some linear and nonlinear ultrasonic techniques to compressive damage in concrete specimens is investigated and compared. A similar comparative assessment was undertaken previously [9] and the key highlights of the present study are: (1) ultrasonic inspection with Rayleigh (surface) waves rather than Longitudinal (bulk) waves, and (2) the demonstration of the inspection technique on a test specimen with dimensions typical of realistic structural members.

**Experimental setup**

The experiments were conducted on a short concrete column with dimensions $300 \times 300 \times 600$ mm. Piezoceramic disc transducers (Ferroperm Pz27) of diameter 10 mm and thickness 2 mm were bonded to the surface of the specimen using conductive epoxy glue and used for both excitation and sensing of the ultrasonic waves. Measurements were made for four actuator and sensor configurations described in Table 1, corresponding to four inspection zones along the front and back surfaces of the specimen. These inspection zones are shown in Fig. 1.

The transducer spacing, excitation frequency and pulse duration were selected empirically to ensure that the primary Rayleigh wave packet can be distinguished from bulk and surface waves reflected from the specimen boundaries. A 7-cycle Hann-windowed tone burst signal at 65 kHz was selected for the excitation of Rayleigh waves in the specimen. This excitation frequency is selected so that the wave reflections from the concrete aggregate are minimised [10]. The excitation signal was varied from 1.0 $V_{pp}$ to 9.0 $V_{pp}$ in 1 V increments and amplified by 20 dB using a Krohn-Hite 7602M amplifier. During the experiment, the transducers labelled F3 and B3 were used for excitation of Rayleigh waves in the front and back surfaces of the specimen, respectively. The sensor outputs were recorded using a National Instruments PXIe-5105 digital oscilloscope at a sampling frequency of 30 MHz and averaged 1000 times.

![Fig. 1: Schematic diagram of the experimental setup showing transducer arrangement on the specimen.](image)

Table 1: Actuator and sensor configurations for the four inspection zones on the specimen surface

<table>
<thead>
<tr>
<th>Zone</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuator</td>
<td>F3</td>
<td>F3</td>
<td>B3</td>
<td>B3</td>
</tr>
<tr>
<td>Sensor at 50 mm distance</td>
<td>F2</td>
<td>F4</td>
<td>B2</td>
<td>B4</td>
</tr>
<tr>
<td>Sensor at 100 mm distance</td>
<td>F1</td>
<td>F5</td>
<td>B1</td>
<td>B5</td>
</tr>
</tbody>
</table>
Data processing

The voltage vs. time signal recorded for the transducers in each of the four inspection zones can be processed to obtain a range of linear and nonlinear ultrasonic parameters. Subsequently, the change in magnitude of these ultrasonic parameters with damage progression can be monitored. In the present work, two ultrasonic parameters are evaluated, namely, the Rayleigh wave velocity, \( c_R \), and the Scaling Subtraction Method (SSM) parameter, \( \theta \). The method for determining each of these parameters is discussed briefly in the following sub-sections.

**Rayleigh wave velocity, \( c_R \).** Each inspection zone has two sensors at a nominal distance of \( d = 50 \) mm. The Rayleigh wave velocity is calculated using the formula

\[
c_R = \frac{d}{\Delta t},
\]

where \( \Delta t \) is the time of flight of the travelling wave, which can be evaluated using the cross-correlation technique [11]. For example, Fig. 2a shows the time-domain signals for sensors F2 and F1 in the undamaged specimen and Fig. 2b shows that variation of the cross-correlation of the two signals with the time shift in the signal of F2. The time of flight is taken as the time shift at which the cross-correlation is maximised. The Rayleigh wave velocities in the four inspection zones of the undamaged specimen are reported in Table 2. The wave velocity on the back surface of the specimen is roughly 14% greater than the front surface, possibly due to non-uniform distribution of aggregate in the specimen or due to the presence of pre-existing defects underneath the front surface. The wavelength of the Rayleigh wave at the excitation frequency (65 kHz), calculated from the velocities in Table 2, lies in the range of 32 mm to 41 mm. The wavelength is comparable to the sensor spacing of 50 mm but approximately 10 times smaller than the specimen width and depth.

![Fig. 2: (a) time domain signals of sensors F2 and F1 in zone 1, (b) cross-correlation of the two signals vs. the time shift of the signal of F2.](image)

**Table 2: Rayleigh and longitudinal wave speed calculations for undamaged specimen**

<table>
<thead>
<tr>
<th>Propagation path</th>
<th>F2 to F1</th>
<th>F4 to F5</th>
<th>B2 to B1</th>
<th>B4 to B5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of flight, ( \Delta t ) [μs]</td>
<td>22.433333</td>
<td>22.766666</td>
<td>19.233333</td>
<td>21.133333</td>
</tr>
<tr>
<td>Distance, ( d ) [mm]</td>
<td>50 ± 2</td>
<td>50 ± 2</td>
<td>50 ± 2</td>
<td>50 ± 2</td>
</tr>
<tr>
<td>Wave velocity, ( c_R ) [m/s]</td>
<td>2229 ± 89</td>
<td>2196 ± 88</td>
<td>2600 ± 104</td>
<td>2366 ± 95</td>
</tr>
</tbody>
</table>

**Scaling-Subtraction Method (SSM) parameter, \( \theta \).** The response of a specimen, \( v_l(t) \), excited by an elastic wave of sufficiently low amplitude, \( A_{o,l} \), is quasi-linear regardless of the damage state. At large excitation amplitudes, \( A_o = kA_l \) (\( k \gg 1 \)), the specimen response, \( v_{nl}(t) \), becomes nonlinear, i.e. not proportional to the excitation voltage. The residual signal, \( v_{res}(t) \), defined according to [9]

\[
v_{res}(t) = v_{nl}(t) - kv_l(t), \quad k = A_o/A_{o,l}
\]

(2)

can provide an indication of the material nonlinearity. In Eq. (2), \( v_{nl}(t) \) and \( v_l(t) \) are the windowed time domain signals from a sensor at high and low excitation voltages, respectively.
Fig. 3: (a) Example of a residual signal obtained using SSM for the undamaged specimen, (b) Baseline dependence of the SSM parameter on the excitation amplitude evaluated from sensor F2 output.

The RMS amplitude $\theta$ of the residual signal obtained from Eq. (2) can serve as a nonlinear parameter, defined as

$$
\theta = \sqrt{\sum v_{res}^2(t_i)} / n, \quad i = 1, 2, ..., n,
$$

where $n$ is the number of samples in the windowed signal.

Fig. 3a shows the residual signal obtained for sensor F2 in the undamaged specimen for $k = 9$ in Eq. (2). The signal $v_{n1}(t)$ was obtained at an excitation voltage of 9 V$_{pp}$ and the signal $v_{f1}(t)$ was obtained at an excitation voltage of 1 V$_{pp}$. Fig. 3b shows the experimentally obtained dependence of the nonlinear parameter $\theta$ on the excitation amplitude $A_o$ as well as the prediction bounds obtained through curve fitting.

**Specimen loading procedure**

Damage was induced in the specimen by applying a uniaxial compressive load along the longitudinal direction of the specimen using a 5000 kN hydraulic press (Fig. 4). The specimen was loaded and unloaded cyclically and the maximum load was incremented by 90 kN per cycle until failure was reached. Ultrasonic measurements were performed on the unloaded specimen at the end of each cycle. The specimen failed at a load of 2730 kN corresponding to a compressive strength of approximately 30 MPa.

Fig. 4: (a) Arrangement of the test specimen in the hydraulic press, (b) side view of the fractured specimen, (c-d) front and back views of the fractured specimen.

**Sensitivity of ultrasonic parameters to compressive damage**

From Eq. (1), the relative change in Rayleigh wave velocity in the damaged specimen can be obtained as

$$
\frac{\Delta c_R}{c_{R0}} = \frac{c_R - c_{R0}}{c_{R0}} = \frac{d / \Delta t - d / \Delta t_0}{d / \Delta t_0} = -\frac{(\Delta t - \Delta t_0)}{\Delta t}
$$

(4)
where $\Delta t_0$ is the baseline value of the time of flight between two sensors in the undamaged state and $\Delta t$ is the time of flight in the unloaded but damaged specimen. Fig. 5a presents the dependence of the relative change in Rayleigh wave velocity upon the normalised compressive load. The results obtained for inspection zones 1 and 2 on the front surface are in close agreement, and a similar trend is observed for inspection zones 3 and 4 on the back surface. However, the wave velocity decreases more rapidly with progressive loading on the front surface than the back surface. The maximum relative change at failure load is greater than 10% for the front surface and approximately 1% for the back surface.

The relative change in the nonlinear SSM parameter, $\theta$ with respect to its baseline value, $\theta_0$, as shown in Fig. 5b, is over two orders of magnitude greater than the relative change in the Rayleigh wave velocity.

Fig. 5: (a) Relative change in Rayleigh wave velocity, and (b) Normalised SSM parameter, vs. the normalised compressive load on specimen.

Summary

The measurements for both ultrasonic parameters along the inspection zones 1 and 2 on the front surface exhibit an abrupt change in slope at approximately 60% of the failure load. This trend is not clearly observed for either parameter along the inspection zones 3 and 4 on the back surface. This observation indicates the localisation of damage near the front surface, possibly due to the presence of pre-existing defects.

References


Ultrasonic based crack imaging in concrete

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Keywords: Ultrasonic monitoring, imaging, concrete, cracks, heterogeneity, aggregates

Abstract. This paper describes a numerical technique for visualising wave propagation through structural materials. Structural concrete has been modelled as a mixture of aggregates and binders. A finite difference in time domain technique has been developed to model propagation of ultrasonic waves through the material. The effect of heterogeneity in terms of wave scattering has been studied. Crack has been introduced in the model to understand its effect on wave propagation. Images of the structure have been developed from the ultrasonic signals. The results highlight the reliability of the technology for construction materials. They would serve as a guide for the field applications of the technology.

1. Introduction

A significant amount of money is spent on the maintenance of existing civil infrastructure worldwide. In EU countries these costs account for ~70% of the public investment [1] whereas, within Australia, it is equivalent to $438 billion [2]. These facts highlight the inefficiency of existing structural health monitoring (SHM) practices. Hence, there is a clear need for improved technologies which can facilitate the timely detection and repair of existing damages in the structures, hence improving their sustainability. Concrete is the most used construction material globally. The occurrence of local cracks in concrete structures is a prime reason for the loss in their service lives. The heterogeneity in concrete and bigger size of civil structures makes it a challenging task to effectively monitor concrete structures with existing technologies.

The common practices for SHM involves visual inspection of the structures which are subjective in nature, time inefficient and limited to detection of surface damages only. Other practices includes eddy current [3], radiography [4] and magnetic particle testing (MPT) techniques. These techniques dominate the non-destructive testing (NDT) global market with an estimated equipment cost of around $1 billion [5]. Apart from being cost inefficient, these techniques face limitations of being specific to structural material type; eddy current techniques can only monitor electrically conductive materials whereas MPT is constrained to ferromagnetic materials. Hence, these techniques are unsuitable for monitoring of civil structures which are generally made up from different construction materials such as steel and concrete. Wave based techniques are generic in nature and can be utilized for time efficient monitoring of civil structures.

Wave based inspections are categorized into (a) standing wave approach and (b) travelling wave approach. In the former inspection, the vibrational response of a structure vibrating at a low frequency is analysed [6]. The existence of damage modifies the modal parameters in the vibrational response mapping the global health of the structure. However, local cracks do not interact efficiently with the low frequency waves, hence go unnoticed in these inspections. In the latter approach, a stress wave travels through the structure and interacts with its internal components [7]. The local cracks act as scatterers to the travelling stress wave. Wave interaction with the incipient cracks result in generation of new modes in the structural response [8] whereas full grown cracks cause significant attenuation in the propagating signal. The ultrasonic waves (frequency > 20 kHz) have been established to be effective for the detection of damage in composites [9] and steel-concrete [10] and plate-like structures [11].
Apart from identifying the presence of local cracks, it is important to locate them in the structure to determine the severity of the damage. For this purpose, an efficient structural imaging technique becomes imperative. In past, several imaging techniques like synthetic aperture focusing [12], reverse time migration [13], C-scan [14], B-scan [15] have been used to image damages. However, most of these techniques rely on high sensor density making them difficult to implement on big structures. In this work, we have tried to develop an imaging technique which is suitable to image local cracks in concrete with a limited numbers of sensors.

The main objective of this study is to use the ultrasonic imaging technique for crack imaging in a concrete structure. For investigation purpose, we have numerically modelled a two dimensional concrete specimen with different sizes of aggregates. The ultrasonic wave will be injected into the specimen, which will be allowed to interact with the specimen for the significant time. The reflected waves from the aggregates and the cracks will be recorded and post processed to locate the local crack.

2. Problem of interest

We have chosen to numerically investigate a 450 mm x 450 mm concrete block using ultrasonic wave based technique. For realistic representation of concrete, we have modelled the specimen as a mixture of cement paste, fine aggregates and coarse aggregates as shown in Figure 1. We will implement our imaging technique for in-situ detection of a 150 mm x 3 mm crack located at a depth of 300 mm in the specimen. We have assumed that the crack is filled with air. For the fundamental understanding of the problem, we will carry out this investigation firstly on a specimen with no aggregates and then on the specimen with coarse and fine aggregates.

![Figure 1: Numerical model of the concrete specimen](image)

3. Theoretical background

3.1 Elastic wave theory

The wave propagation in an elastic medium can be described with the force equation and the stress-strain constitutive relation as represented by equations 1 and 2. Here, $\rho$ represents the density of the medium, $v$ represents the particle velocity and $T$ represents the Cauchy stress tensor; $\xi, \gamma$ represents normal and shear strains; $\lambda$ and $\mu$ represent the Lame’s parameters. Since, we are interested in determining
the depth at which the crack is present, we have made plane strain (negligible strain in z-direction) throughout this study for simplifications of the equations.

\[
\frac{\partial \nu}{\partial t} = \nabla \cdot T
\]  \hspace{1cm} (1)

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} =
\begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
\xi_{xx} \\
\xi_{yy} \\
\xi_{zz} \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix}
\]  \hspace{1cm} (2)

Equations 1 and 2 can be combined to form stress-velocity relations 3-4 which are sufficient to simulate the propagation of an elastic wave.

\[
\frac{\partial \sigma_{pp}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_p}{\partial p} + \lambda \frac{\partial v_q}{\partial q} \quad (p, q = x, y; \ p \neq q)
\]  \hspace{1cm} (3)

\[
\frac{\partial \tau_{pq}}{\partial t} = \mu \left( \frac{\partial v_p}{\partial q} + \frac{\partial v_q}{\partial p} \right) \quad (p = x, q = y)
\]  \hspace{1cm} (4)

where \(\sigma\) and \(\tau\) represent the normal and shear stress components.

### 3.2 Numerical modelling

We have used Yee’s grid scheme [16] to discretise the specimen into small grid cells. For simulation of the ultrasonic wave propagation inside the specimen, we have used the finite difference discretization method for solving the equations 3 and 4 into following velocity-stress update equations.

\[
v_x \left( m, n, t + \frac{\Delta t}{2} \right) = v_x \left( m, n, t - \frac{\Delta t}{2} \right) + \frac{\Delta t}{\rho} \left( \frac{\sigma_{xx}(m,n,t) - \sigma_{xx}(m-1,n,t)}{\delta x} + \frac{\tau_{xy}(m,n,t)}{\delta y} \right)
\]  \hspace{1cm} (5)

\[
\sigma_{xx}(m, n, t + \Delta t) = \sigma_{xx}(m, n, t) + \Delta t \left( (\lambda + 2\mu) \frac{v_x(m+1,n,t+\Delta t/2) - v_x(m,n,t+\Delta t/2)}{\delta x} + \lambda \frac{v_y(m,n,t+\Delta t/2) - v_y(m,n-1,t+\Delta t/2)}{\delta y} \right)
\]  \hspace{1cm} (6)

Here, \(m\) and \(n\) in the parenthesis show the co-ordinates of the unit cell and \(t\) represents the time step of the calculations. The update equations for the other field components can be solved similarly. Table 1 shows the medium properties used for the simulations. For numerical stability of the model, we have used the unit grid cell of size less than one-tenth of the minimum wavelength excited in the system. We have used free surface boundary conditions at the crack interfaces. The averaging at all the medium interfaces is done to avoid instability due to different material properties.

<table>
<thead>
<tr>
<th>Table 1: Medium properties used in the numerical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>(\rho) (\text{(kg/m}^3)</td>
</tr>
<tr>
<td>(P) wave velocity (\text{(m/s)})</td>
</tr>
<tr>
<td>(S) wave velocity (\text{(m/s)})</td>
</tr>
</tbody>
</table>
3.3 Imaging technique

Figure 2 shows the schematic for the imaging of the specimen. S and R shows the position of the wave source and the receiver. The wave generating from S travels and interact with the internal components of the specimen. The reflected wave from the crack will be recorded at R. In this way, the whole specimen will be scanned at different S and R locations as shown with S’-R’ and S’’-R’’. We have chosen to move S by 20 mm distance with a fixed distance of 30 mm between the S and R. The final image of the specimen is generated using the time of flight information for each grid pixel recorded from different S-R locations as per equation 7. The recorded intensity $I$ of reflection from each pixel is calculated using equation 8.

$$T_{ToF}(i, j) = \frac{dist(i, j)}{v_{conc}}$$  \hspace{1cm} (7)

$$I(i, j) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(T_{ToF}(i, j))$$  \hspace{1cm} (8)

4. Results

We have used a 200 kHz Gaussian stress wave for the inspection of the specimen. Figure 3 shows the wave propagation in a specimen without and with fine and coarse aggregates. The two figures can be compared to see that the presence of aggregates results in significant scattering of the travelling wave.

![Figure 2: Schematic for imaging of the specimen](image)

Figure 3: Snapshots of wave propagation in a specimen with (a) no aggregate and (b) fine and coarse aggregates

The reflections from the crack are recorded for different source and receiver positions and the image of the specimen is generated as per the discussion in section 3.3. Figure 4 shows the generated images for the specimens with no aggregates and with both aggregates. Figure 4a validates that the efficacy of our developed model by imaging the crack location with a high level of accuracy. Figure 4b, shows that with the inclusion of aggregates resolution of the crack image has significantly reduced. The reason is the significant scattering from the aggregates. However, a crack at reduced depth of 200 mm can still be discerned with higher degree of confidence.
Figure 4: Crack imaging for specimen with (a) no aggregate, crack location: 300 mm (b) both the aggregates, crack location: 300 m (c) both the aggregates, crack location: 200 mm

5. Conclusions and Future Scope of work

The use of wave based imaging technique facilitates the in-situ detection of the local cracks. However, the heterogeneous nature of the concrete reduces the efficacy of the imaging technique to image cracks beyond a certain depth. Overall, the ultrasonic imaging techniques shows promising outcomes to monitor the damages in the civil structures.

The present imaging technique can be further enhanced. The new ideas may involve adding some filtering parameters in the imaging equations, which can recognize the scattering pattern from aggregates and eliminate their effects from the final specimen image. This will facilitate the imaging of cracks up to greater depths from the limited accessible structural surface.

6. References


Ultrasonic Based Detection of Steel-concrete Interfacial Debonding in Reinforced Concrete due to Top-Bar Effect

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\textbf{Keywords:} Ultrasonic, steel-concrete interface, debonding, top-bar, FE simulation

\section*{Abstract}
Formation of steel-concrete (SC) interfacial debonding in reinforced concrete (RC) walls due to top-bar effect is widely recognized. The presence of interfacial defects can lead to higher corrosion rate of steel reinforcement and reduce the time to the initiation of corrosion induced crack. Hence, detecting the corrosion rate before cracking initiation is critical for the accurate assessment of the residual service life of concrete structures. This paper presents a numerical investigation of propagation of ultrasonic waves in RC with pre-existing SC interfacial debonding due to the top bar effect. The effects of interfacial debonding on the propagation of ultrasonic waves and energy transfer between steel rebar and concrete have been studied. The results have shown that when the bar is used as a wave guide, there is a significant increment in the energy content of the received signals due to the presence of interfacial debonding. A noticeable delay in the time of arrival of the signals was also noticed at the transducer placed on the concrete surface. The results demonstrate the capability of guided wave ultrasonics in the detection of top bar effect.

\section{Introduction}
There has been increasing concerns among public and governments worldwide about the degradation of civil infrastructures, such as historic monuments, bridges and buildings, due to the economic and social costs caused by the ageing or failure of these structures. In Australia, most of the critical infrastructures, especially reinforced concrete structures, are located in the marine environment where the salinity is high and therefore in danger of chloride-induced corrosion. Chloride contamination is the primary cause of reinforcing steel corrosion in RC structures in marine area [1]. Chloride ions, presenting in seawater, penetrate into concrete, break down the passive film and induce active corrosion when the concentration of chloride exceeds the threshold [2]. The presence of defect at steel-concrete interface has been reported to be essential for active corrosion in RC as corrosion tends to initiate at the locations where interface quality is poor [2-5]. One of common interface defect is the void/debonding underneath the horizontal steel reinforcements due to segregation, settlement and bleeding during curing process of high concrete members, such as wall. This is known as top-bar effect. The presence of such SC interface defect significantly reduces the bond strength, accelerates the corrosion activity and increases the corrosion rate at corrosion cracking initiation [5]. Hence, detecting the presence of interface defect and the corrosion rate before crack initiation is critical for accurate assessment of corrosion process and evaluation of residual service life of concrete structures.

Guided wave-based technique has been extensively employed for damage detection in RC [6-12]. Guided wave features the ability to propagate over long distance with low energy consumption and high sensitivity to defect, making it a promising and effective structural health monitoring method. When guided waves propagate in steel reinforcement (waveguide), energy keeps leaking into surrounding concrete through SC interface. The irregularities and discontinuities present at SC interface result in scattering and reflection of guided waves. Due to the presence of interface defect, the leakage will be blocked and energy will be retained in waveguide, leading to an increment in signal strength. Therefore, SC interface can be characterized.
In this paper, guided wave is employed to detect the formation of SC interface defect owning to top-bar effect. Finite element study has been conducted to simulate guided wave propagation in RC block subjected to different damages scenarios.

2. Effects of pre-existing debonding owning to top-bar on wave propagation

2.1. Specimen

The concrete compositions are given in Table 1. A 800mm x 100mm x 300mm deep beam concrete wall, shown in Figure 1, is designed including 4 horizontal steel rebar (M12) located at different height (100, 300, 500 and 700mm from concrete bottom with 200mm center-to-center spacing). According to ACI Guide 440, SC interface defect are expected to form underneath steel rebars located at 300mm, 500mm and 700mm from concrete bottom. After 28 days of concrete curing, the deep beam is sawn into 4 identical specimens as shown in Figure 2. Each specimen consists of 1 steel rebar with a minimum concrete cover 20mm.

![Figure 1: 800mm deep concrete member with 4 rebar at c-c spacing 200mm (a) Side view and (b) Front view](image1.png)

![Figure 2: Schematic representation of Small specimen obtained after sawing the deep concrete member (a) side view and (b) front view](image2.png)

<table>
<thead>
<tr>
<th>Mix Component (kg/m³)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse aggregate (10mm basalt)</td>
<td>1190.50</td>
</tr>
<tr>
<td>Fine aggregate</td>
<td>590.30</td>
</tr>
<tr>
<td>OPC (GP cement)</td>
<td>430</td>
</tr>
<tr>
<td>Water/binder ratio</td>
<td>0.45</td>
</tr>
<tr>
<td>Free water</td>
<td>193.5</td>
</tr>
<tr>
<td>Total</td>
<td>2404.3</td>
</tr>
</tbody>
</table>
2.2. Numerical simulation

2.2.1. Propagation of guided wave in steel rebar

Finite element models were employed to investigate the wave propagation characteristics in the embedded steel rebar and the surrounding concrete in RC with/without SC interface defect owning to top-bar effect. The model was created in ABAQUS®/CAE and analysed using ABAQUS®/Explicit (Version 6.14-2) to investigate the wave propagation characteristics and wave-defect interaction [13, 14]. 2-dimentional models were created to investigate the wave propagation in longitudinal (Figure 2(a)) and transverse (Figure 2(b)) section of the specimen. The models were uniformly meshed using Structured technique and four-node plane stress element (CPS4R) type as shown in Figure 3. Each node has 2 degree of freedom (x- and y- axis). The model consists of three part, i.e. the steel bar, the concrete and the changing part which is subjected to different material properties or void to simulate different bonding conditions. A seed size of 1mm was used for all parts, providing 28 elements and 17 elements per wavelength for wave in steel part and concrete part respectively. The total number of elements and nodes generated for different models are given in Table 2.

Table 2 Element number, node number and element type of models

<table>
<thead>
<tr>
<th></th>
<th>Perfect bonding</th>
<th>Debonding in the middle</th>
<th>Debonding at the end</th>
<th>Partial corrosion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of node</td>
<td>63,006</td>
<td>62,604</td>
<td>62,604</td>
<td>63,006</td>
</tr>
<tr>
<td>Number of element</td>
<td>61,600</td>
<td>61,400</td>
<td>61,350</td>
<td>61,600</td>
</tr>
<tr>
<td>Element type</td>
<td>CPS4R</td>
<td>CPS4R</td>
<td>CPS4R</td>
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</tbody>
</table>

The properties of materials used in Abaqus are given in Table 3. Four damage scenarios, namely perfect bonding, partial corrosion, partial debonding in the middle and at the end of embedded rebar, were analysed to investigate the influence of the presence and the location of debonding, as shown in Figure 3. Tie constraint was applied between steel rebar and both bottom and top part of concrete to simulate perfect SC bonding, as shown in Figure 3 (a). One layer of elements of different length were removed from the bottom part of concrete to create SC debonding, as denoted by the white block in Figure 3 (b) and (c). RC corrosion was simulated by adding one layer of elements with material properties of rust. The rust part, denoted by the brown block in Figure 3 (d), was tied with the steel rebar and the concrete. The steel rebar was used as waveguide and one cycle sine wave at 200 kHz was excited by applying time varying concentrated forces at the left surface of the rebar to simulate guided waves. In experiment, the concrete specimen will be placed on a table. To simulate the boundary condition of the specimen, the x- and y-axis displacement were restricted at the bottom of the model. No boundary condition was applied to restrict the displacement of the top, left and right surface of the model and hence allowing free wave-induced vibration at these surfaces. For finite element method, the traction boundary condition can be automatically satisfied after solving the model without any external condition. The normal and shear traction at these surfaces were assumed to be zero as the initial condition, no traction boundary condition was imposed on these surfaces except at the location where the waves were excited. The Dynamic Explicit procedure was used with a fixed time increment of 2x10⁻⁸ second.

Table 3: Properties of materials used in FEM

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Concrete</th>
<th>Steel</th>
<th>Rust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2400</td>
<td>7800</td>
<td>780</td>
</tr>
<tr>
<td>Young's Modulus (GPa)</td>
<td>25</td>
<td>210</td>
<td>21</td>
</tr>
<tr>
<td>Poison Ratio</td>
<td>0.18</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Figure 3: FEM of 4 damage scenarios (a) perfect bonding, (b) debonding in the middle of embedded rebar, (c) debonding at the end of rebar, and (d) corroded rebar

FEM results of wave propagation are illustrated in Figure 4. One cycle of 200 kHz sine wave is applied at left side of the steel rebar. For RC with perfect SC interface bonding (Figure 4 (a) and (b)), once the excited waves reach embedded part of the steel rebar, the transmitted waves start and continue to leak into the top and the bottom part of concrete while propagating along the waveguide. For RC with SC interface debonding in the middle of model (Figure 4 (c) and (d)), the waves leak into the both parts of concrete before the debonding zone. Once the waves reach the debonding zone, due to the discontinuity between the steel rebar and the bottom part of the concrete, the waves can only leak into the top part of the concrete. When the waves travelling in rebar pass the debonding zone, the waves start to leak into the both parts of the concrete again. When the debonding zone is extended to the right end of the concrete (Figure 4 (e) and (f)), the bottom part of the concrete only contains the waves leaked from the steel rebar before they reach the debonding zone. For RC with rebar corrosion, (Figure 4 (g) and (h)), the transmitted waves keep leaking into the top and the bottom part of concrete from the steel rebar. Due to rebar corrosion in the middle of the RC block, the energy of waves leak into the bottom part of concrete is less than that of the top part.

Figure 4: FEM results of guided wave propagation in reinforced concrete with (a) perfect bonding at 20ms, (b) perfect bonding at 60ms, (c) debonding in the middle at 20ms, (d) debonding in the middle at 60ms, (e) debonding at the end at 20ms, (f) debonding at the end at 60ms, (g) rust at 20ms, and (h) rust at 60ms.

The simulated signals of four damage scenarios are summarised in Figure 5. The signal excited at the left end of the rebar is illustrated in Figure 5 (a). Figure 5 (b) shows the signals in time-domain acquired at the right end of steel rebar. Due to the presence of SC interface debonding in the middle of
embedded rebar, a noticeable amplitude increment can be observed and a further increment can be achieved when the debonding extended to the right end of concrete. The debonding leads to SC interface discontinuities, the leakage of waves into the bottom part of the concrete from the steel rebar is blocked, more waves can be retained in the steel rebar and thus received by the sensor at the right end of steel rebar. When debonding zone is filled with rust, the waves can leak into the bottom concrete via rust. However, the stiffness of rust is weaker than that of concrete, only small amount of energy can be transmitted via rust into the bottom part of the concrete.

Figure 5 (c) and (d) show the signals received at the top and the bottom part of the concrete respectively. For RC with perfect bonding, the signals received at the top and the bottom part are identical. Once interface debonding occurs in the middle of the concrete, more waves can be retained in steel rebar and thus more energy can leak into the top part of concrete, resulting in amplitude increment in received signals. However, it is observed that there is also an increment in amplitude received at the bottom part of concrete. As the waves transmit pass the debonding zone, waves leak into the bottom part of concrete again. Comparing to RC with perfect bonding, more waves are retained in the rebar at this point, leading to more waves leaking into concrete. When the debonding zone extends to the end of model, only waves leaked from steel rebar before the debonding zone can propagate in the bottom part of concrete. As waves travel slower and experience stronger attenuation in concrete than in steel, a noticeable delay in time and reduction in amplitude is observed.

Figure 5: (a) excited signal at 200 kHz; simulated signals received at (b) steel rebar, (c) top part of concrete, and (d) bottom part of concrete

2.2.2. Propagation of bulk wave in concrete

2D models, as shown in Figure 6, were created to investigate bulk wave propagation in the cross-sectional face of the concrete specimen. 2 scenarios, the intact scenario with perfect SC bonding, and debonding scenario where void presents underneath the steel rebar, were simulated to investigate the wave propagation in concrete and the interaction between wave and SC interface. The models were
meshed using Structured technique and four-node plane stress element (CPS4R) type. The element size is 1mm. Due to the presence of the circular steel rebar and the void, the entire model cannot be uniformly meshed, therefore the model was meshed in 3 different regions, i.e. the steel rebar (denoted by yellow circle), the surrounding concrete (denoted by yellow dash line) and the major part of the concrete (denoted by blue dash line), as illustrated in Figure 6. The major part of the concrete was uniformly meshed whilst the steel rebar and the surrounding concrete were non-uniformly meshed, as shown in Figure 6. To simulate perfect bonding condition, the circular steel rebar was tied with the surrounding concrete. One layer of elements were removed from the surrounding concrete underneath the steel rebar to simulate the SC debonding. A surface-to-surface contact property was applied between the steel rebar and concrete to restrict node/element penetration into each other. The total number of elements and nodes are 87,215 and 87,922 for RC with perfect bonding and 87,174 and 87,846 for RC with SC interface debonding. 5 cycles of sine wave at 200 kHz were applied at the right surface of the specimen and the reflected signal was received at 20mm centre-to-centre spacing from the exciting point. To simulate the boundary condition of the specimen in experiment, the x- and y-axis displacement of the bottom surface were restricted and no boundary condition was applied on the top, left and right surfaces. The normal and shear traction at these surfaces were assumed to be zero. The Dynamic Explicit procedure was used with a fixed time increment of $2 \times 10^{-8}$ second.

Figure 6: FEM of damage scenarios (cross section): (a) perfect bonding and (b) void underneath rebar (top bar)

Figure 7 shows the FEM results of bulk wave propagation in concrete and the interaction between wave and SC interface. For model with perfect SC bonding, when the incident waves encounter SC interface, part of the waves reflects from the interface and the remaining part transmits via the interface. For model with void underneath the steel rebar, when the incident waves interact with the top half of rebar (perfect bonding), the waves can reflect from and transmit through the interface, whilst for the incident waves encountered the void, the waves can only reflected and diffracted from the void, no wave can transmit via the void.
Typical reflected signals in RC with perfect bonding and SC interface debonding are illustrated in Figure 8 (a). The first wave package arrives at the receiver at time $t_1$, which corresponding to the direct incident wave from signal exciting point. The time $t_2$ denotes the arrival time for the first reflected waves. The signals are identical during the period between $t_1$ and $t_2$, regardless of the presence of void. A noticeable deviation appears at time $t_2$, which indicates the arrival of reflected signal from the void. The amplitude of the reflected waves from the void is greater than that from the perfect bonding, indicating that more waves can be reflected from void, and therefore less waves can be transmitted or diffracted from the void to the left boundary of the model, which explains the reduction in the amplitude of boundary reflection.

The transmitted signals received at the right surface of the RC are shown in Figure 8 (b). A good match can be observed in the first valley of the received signal, because this part of the waves is transmitted directly from the signal actuator, which is not influenced by the rebar or void. The transmitted and diffracted signals from the SC interface reach the signal receiver at the time $t_3$. Due to the interface debonding, more waves are reflected and hence less waves can reach the signal receiver, which explains the amplitude reduction in signal captured in RC with interface debonding after the time $t_3$.
3. Conclusion and future work

FE models were conducted to investigate the wave propagation in RC with and without SC debonding owning to top-bar effect. High frequency guided waves were employed to evaluate the quality of SC bond using steel rebar as the waveguide. Bulk waves were also introduced to assess the influence of SC bonding on the wave propagation. Good correlation between ultrasonic wave features, such as amplitude and time of flight, were observed, indicating the proposed method a promising method for detection of SC debonding.

Future work will focus on the experimental validation of the proposed wave-based method. The proposed technique will also be used to monitor the corrosion of steel rebar and corrosion induced cracking in reinforced concrete.

Reference

IV ROCK FRACTURES AND FAULTS
Interactions of hydraulic fractures

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Keywords: hydraulic fracture, fracture interaction, stress, geomechanics, laboratory experiment, porous medium.

Abstract. The aim of the study was an investigation of the influence of the hydraulic fracture presence in the neighbour boreholes on the fracture propagation. The study was made with the help of laboratory experiments, which were conducted on the artificial porous saturated samples in accordance with the similarity criteria. A set of experiments was conducted, in which the main stress axis orientation was changed after the first hydraulic fracture creation, and the influence of the new stress state on the fracture orientation was examined. It was found that the deviation of the hydraulic fracture from the initial direction of the maximum compressive stress was related to the perturbation of the stress field by fractures created earlier. The obtained results can be used to verify the numerical simulation of the hydraulic fracture propagation and for adequate interpretations of the field data.

Introduction

Hydrofracturing is used widely by hydrocarbon producing companies to enhance the oil and gas production. Meanwhile the hydrofracturing technology was developed several decades ago and was improved and adjusted during its applications, it was found that the existed hydrofracturing theories does not describe the observed parameters of the fractures with necessary accuracy. The problems appeared when oil and gas producing industry start to develop nonconventional hydrocarbon resources (tight sand, shale). For effective design of multi-fracturing in horizontal wells it is necessary to use more complicated fracturing models as before. One of the questions to be resolved is related with interaction between the fractures, both hydraulically produced and the natural ones. The scaled laboratory experiments could be considered as the most reliable method for validation of the various models of the hydraulic fracture formation.

There are a lot of research works (theoretical, numerical and experimental simulations) devoted to the study of the hydraulic fractures [1-7]. Nevertheless, some questions continued to be unresolved and of interest for better understanding the fracturing processes to improve the technology of multir fracturing in unconventional oil & gas reserves.

In the paper, the results of the laboratory experiments for study of the influence of the hydraulic fracture presence in the neighbor boreholes on the fracture propagation are presented. In the experiments, the main stress axis orientation was changed after the first hydraulic fracture creation, and the influence of the new stress state on the orientation of the fracture in the same or in the neighbor borehole was examined.
Experimental Setup and Procedure

The experimental setup was described in detail in [8-10] and is shown in Fig. 1. It consists of two metal disks of 600 mm in diameter and 75 mm in thickness. A metal ring (height 74 mm, thickness 25 mm, the inner diameter 430 mm) is placed between the disks to form the pressure chamber with diameter 430 mm and height 66 mm. A rubber diaphragm is placed between the upper disk and the pressure chamber, four thin chambers are located along the inner side of the ring. The stresses are applied by fluid injection into these side chambers and into the gap between the rubber diaphragm and the upper disk.

The disks and the ring have holes with diameter of 6 mm: 29 holes in the upper disk, 13 - in the bottom disk, and 6 - in the ring. These holes are used for mounting the pore fluid pressure sensors and acoustic emission sensors as well as for pumping fluids into or out of the model reservoir through the boreholes in the sample.

The samples were produced by filling the pressure chamber by gypsum/cement mixture (proportion 9:1) with addition of 45% of water and were dried for 2-3 days. The material for samples was chosen on the basis of the scaling criteria described in [11, 10]. The static and dynamic Young’s modulus, Poison ratio, density, and p-wave velocities are shown in Table 1. The unconfined uniaxial strength of the samples was measured to be 6.4±0.95 MPa. Tensile strength was measured by Brazilian test as 0.8±0.18 MPa. The permeability of the samples was measured as 1.0-1.7 mD. The samples were saturated by gypsum water solution. Vacuum mineral oil was used as the fracturing fluid (the viscosity 112 cP, the density 0.86 g/cm³ for the temperature 22 °C).
Table 1. Mechanical parameters of the sample

<table>
<thead>
<tr>
<th>Measurement type</th>
<th>Bulk density [kg/m³]</th>
<th>P-wave velocity [m/s]</th>
<th>Young’s Modulus [GPa]</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>1770</td>
<td>2310</td>
<td>7.7</td>
<td>0.26</td>
</tr>
<tr>
<td>Static</td>
<td>1770</td>
<td>2260</td>
<td>3.6</td>
<td>0.21</td>
</tr>
</tbody>
</table>

After the setup was assembled, the horizontal stresses were applied up to desired values (0.1 – 4 MPa), and after that the vertical stress was increased to the value 7 – 10 MPa. The fracture was produced by injection of mineral oil through one or another borehole with constant rate 0.22-0.37 cm³/s during 200 s. The fracture forming was controlled by the fracturing fluid pressure change in the borehole. After the first fracturing, the setup was disassembled, the fracture position was detected, the setup was assembled again, the horizontal stresses were changed, and another experiment was conducted by the fracturing fluid injection through the same or through another borehole. The fracturing fluid pressure as well as the pore fluid pressure in the bottom part of the sample were registered during the all experiments.

**Experimental Results**

The sample after the first fracturing in one of the experiments is shown in Fig. 2 (the photo, the pressure changes and experiment parameters). Then, the orientation of the main horizontal stress axis was changed, and another fracturing was made through the same borehole. The results and the parameters are shown in Fig. 3. It can be seen that no new fractures appeared, and the initial fracture continue to grow in spite of the fact that the maximal horizontal stress is perpendicular to the fracture.

Fig. 2. Stage 1 of the fracturing experiment: photo of the sample after fracturing, borehole fluid pressure variation, fracturing conditions.
Fig. 3. Stage 2 of the experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical stress [MPa]</td>
<td>3.0</td>
</tr>
<tr>
<td>Max horizontal stress [MPa]</td>
<td>1.9</td>
</tr>
<tr>
<td>Min horizontal stress [MPa]</td>
<td>0.1</td>
</tr>
<tr>
<td>Fracturing pressure [MPa]</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Fig. 4. Stage 3 of the experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical stress [MPa]</td>
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</tr>
<tr>
<td>Max horizontal stress [MPa]</td>
<td>3.2</td>
</tr>
<tr>
<td>Min horizontal stress [MPa]</td>
<td>0.0</td>
</tr>
<tr>
<td>Fracturing pressure [MPa]</td>
<td>5.5</td>
</tr>
</tbody>
</table>
Then, the fracturing was made through another borehole but under the same stress conditions (Fig. 4). It can be seen, that the new fracture orientation agrees with the stress axis orientation.

In the next stage of the experiment, the stress axis orientation was changed once more, and fracturing was made through the third borehole. The small new fracture (length of one wing 2 cm and of another wing 1 cm) appeared in accordance with the stress axis orientations. Then, the stress orientation was changed, fracturing was produced in the same borehole. The fracture continued to grow but with significant inclination from the initial orientation (Fig. 5).

**Discussion**

Observed growth of the initial fractures in spite of change in the horizontal main stress axis orientations can be expected if we take account of higher permeability of the fractures in comparison with the permeability of the surrounding material, and the stress concentration at the tip of the fracture. As it is seen in the last stage of the considered experiment, the change of the stress axis orientation leads to inclination of the fracture in direction to the previous fracture (Fig. 5). Result of the calculation of the principle horizontal stress axis orientations around the first and the last fractures with account of perturbations of the stresses caused by the first fracture is shown in Fig.6. For the calculation, the first fracture is considered as a gap filled by fluid. The inclination of the last fracture to the first fracture agrees with calculated maximal horizontal stress axis orientations.
Conclusions

The experiments were conducted, in which the main stress axis orientation was changed after the first hydraulic fracture creation in the boreholes, and the influence of the new stress state on the new fracture orientations was examined. It was observed, that in spite of the new stress axis orientation, the initial fracture continues to grow, if we repeat hydraulic fracturing in the same borehole. New fracture in new borehole will be oriented in accordance with the new stress axis orientation. The deviation of the hydraulic fracture from the initial direction of the maximum compressive stress is related to the perturbation of the stresses by the fractures created earlier.

The obtained results can be used to verify the numerical simulation of the hydraulic fracture propagation and for adequate interpretations of the field data.

Acknowledgements

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References

On the coarse-scale residual opening of hydraulic fractures created using the Channel Fracturing technique

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Keywords: Channel fracturing, optimal proppant usage, residual opening, homogenisation procedure.

Abstract. Channel fracturing is a novel technique utilised to achieve discontinuous placement of proppant within a hydraulic fracture and create a network of open channels or voids between the proppant-filled regions (proppant columns), which can significantly increase the conductivity of the fracture. The problem of deformation and fluid flow in a partially-filled fracture involves two length scales: a large scale comparable to the length of the fracture \( \sim O(10^2) \) m and a fine scale comparable to the length of the proppant filled regions or ‘columns’ \( \sim O(1) \) m. In this paper, a homogenisation procedure is developed to obtain the residual opening profile and effective fracture conductivity at the large scale from the solution of a ‘unit-cell’ problem at the fine scale. The application of the model in a practical scenario is demonstrated by performing a mock numerical simulation.

Introduction

The open channels created by channel fracturing technique are very conductive pathways for fluid flow from oil/gas reservoir to the wellbore, so the effective fracture conductivity can be increased up to several folds higher than that using the conventional hydraulic fracturing techniques [1,2]. The effective fracture conductivity can be maximised by selecting the optimal width of the open channels, i.e. the optimal spacing between the proppant-filled regions or ‘columns’.

The optimisation requires solution to the problem of rock deformation and fluid flow in a partially-filled fracture at two length-scales: a coarse length-scale, \( \tilde{X} \) comparable to the half-length of the fracture, \( L \sim O(10^2) \) m, and a fine length-scale, \( \tilde{x} \) comparable to the half-length of the proppant-filled regions or ‘columns’, \( b \sim O(1) \) m. The initial opening of the fracture, \( \delta_0 \), the fluid pressure within the fracture, \( p_f \), as well as the compressive overburden stress normal to the fracture plane, \( \sigma_{yy} \), vary at the coarse scale. However, these variations are expected to be negligible at the fine scale, except close to the wellbore (\( X = 0 \)) or the fracture tips (\(|X| = \pm L \)). Hence, the problem geometry can be treated as periodic at the length-scale of the proppant columns, and the fine scale can be formulated as a “unit cell” problem (see Fig. 1) [2].

Fig. 1: (a) 2D model of a partially-filled hydraulic fracture, (b) detailed view of unit-cell (not-to-scale)
From a practical viewpoint, it is of greater interest to consider the effective conductivity of the entire fracture, rather than a unit-cell. The solution at the coarse scale can be obtained in a computationally efficient manner by adopting a homogenisation procedure. The homogenisation or averaging procedure replaces the system of discrete proppant columns along the fracture length by a continuously-distributed ‘fictitious’ porous medium. The purpose of this paper is to develop the displacement-dependent traction for the fictitious medium, which is a necessary first step towards the solution of homogenised problem.

**Mathematical model for the unit-cell problem**

The present study adopts a simple one-dimensional model for proppant consolidation, i.e. the lateral expansion of the proppant columns is ignored. The compressive stress at a given location in the proppant column, \( \sigma_p(x) \) is related to the change in height of the proppant column, \( \delta_0 - \delta(x) \) using the following power-law relationship [2]:

\[
\sigma_p(x) = \alpha \left( \frac{\delta_0 - \delta(x)}{\delta(x)} \right)^\beta,
\]

where the constants \( \alpha \) and \( \beta \) are the fitting parameters determined from experimental data. The height of the proppant column \( \delta(x) \) lies in the interval \( (0, \delta_0] \) and Eq. (6) implies that \( \sigma_p = 0 \) at \( \delta(x) = \delta_0 \) and \( \sigma_p \to \infty \) as \( \delta(x) \to 0 \).

The relative opening between the crack faces, \( \delta(x) \), is modelled by a continuous distribution of ‘edge dislocations’. The singular integral equation which governs the distribution of the dislocations is derived in [2] and can be written as

\[
\frac{E}{4\pi} \int_0^a B_y(\xi) \left[ \frac{2\xi}{x^2 - \xi^2} + K(x, \xi) \right] d\xi = \sigma_o - \sigma_p(x)H(b - |x|), \quad 0 < x < a,
\]

where \( B_y(\xi) \) is the unknown dislocation density function which represents the continuous distribution of dislocations, \( H(\quad) \) is the Heaviside step function and the kernel \( K(x, \xi) \) is given by

\[
K(x, \xi) = \sum_{n=1}^{\infty} \frac{4\xi(x^2 - \xi^2 + 4a^2n^2)}{((x - \xi)^2 - 4a^2n^2)((x + \xi)^2 - 4a^2n^2)}
\]

The dislocation density is related to the residual opening profile according to

\[
\delta(x) = \delta_{\text{min}} + \int_x^a B_y(\xi) d\xi, \quad B_y(\xi) = \frac{d\delta(\xi)}{d\xi}, \quad 0 < x < a.
\]

The method of solution of Eq. (2) and the residual opening profile is described in [2].

**Homogenisation procedure**

The aim of the homogenisation procedure is to replace the proppant column, which partially occupies the unit-cell, by an effective medium which fills the entire unit-cell. The nonlinear response of the effective medium is also described by Eq. (6), except for a multiplicative constant \( C \), which varies with the geometrical parameters \( a, b \) and \( \delta_0 \) and the remote stress, \( \sigma_o \). The constant \( C \) must be found in such a manner that the potential energy of the unit-cell, defined below, remains conserved [3].
\[ \Pi = U_1 + U_2 + W. \tag{5} \]

In Eq. (5), \( U_1 \) is the strain energy of the rock in the deformed configuration over the region \( x \in [-a, a], \ y \in (-\infty, \infty) \) and can be written as:

\[ U_1 = 4 \int_0^\infty \int_0^a \left( \frac{\sigma_{xx}^2 + \sigma_{yy}^2 - 2v \sigma_{xx} \sigma_{yy} + \frac{\sigma_{xy}^2}{2G}}{2 \bar{E}} \right) \, dx \, dy, \tag{6} \]

where \( \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \) are elastic stress components in the rock formation, \( \bar{E} \) is the generalised Young’s modulus, and \( G \) is the shear modulus.

The term \( U_2 \) in Eq. (5) corresponds to the strain energy stored in the deformed proppant column and can be obtained as:

\[ U_2 = 2 \int_0^b \int_0^\delta_0 - \delta(x) \alpha \left( \frac{u}{\delta_0 - u} \right)^\beta \, du \, dx, \tag{7} \]

where \( u = \delta_0 - \delta \) denotes the change in height of the proppant column.

Finally, the term \( W \) in Eq. (5) represents the work done due to the displacement of the remote boundary upon which the compressive traction \( \sigma_{yy}(x, y \to \pm \infty) = -\sigma_o \) is applied. Since the displacement field due to the dislocation density tends to zero at the remote boundary \( y \to \infty \), the work done can be written as

\[ W = \lim_{y \to \infty} \left( -4a \frac{\sigma_o^2}{E} y \right) - 2a(\delta_0 - \delta_{\text{min}})\sigma_o. \tag{8} \]

A unit-cell filled entirely with the effective medium undergoes uniaxial compression and the strain energy stored in the rock over the region \( x \in [-a, a], \ y \in (-\infty, \infty) \) is simply given by

\[ U_1^\ast = 4 \int_0^\infty \int_0^a \left( \frac{\sigma_o^2}{2 \bar{E}} \right) \, dx \, dy. \tag{9} \]

The strain energy stored in the effective medium can be written as

\[ U_2^\ast = 2 \int_0^{+a} \int_0^{\delta_0 - \delta^\ast} \alpha \left( \frac{u}{\delta_0 - u} \right)^\beta \, du \, dx, \tag{10} \]

where \( \delta^\ast \) is the constant opening of the fracture filled with the effective medium, and can be obtained as:

\[ \delta^\ast = \delta_0 \left( 1 + \left( \frac{\sigma_o}{\bar{\alpha}} \right)^\beta \right)^{-1}. \tag{11} \]

Analogous to (8), the work done at the remote boundary is

\[ W^\ast = \lim_{y \to \infty} \left( -4a \frac{\sigma_o^2}{E} y \right) - 2a(\delta_0 - \delta^\ast)\sigma_o. \tag{12} \]

The equivalence of potential energy requires that \( \Pi = \Pi^\ast \), i.e. \( U_1 + U_2 + W = U_1^\ast + U_2^\ast + W^\ast \). Utilising Eqs. (5)-(12), the potential energy equivalence requirement can be stated as
Eq. (13) is satisfied by a unique value of the constant $C$ which can be obtained using a suitable root finding algorithm.

**Numerical results**

In this section, some numerical results are presented for the effective properties of the homogenised medium. In these numerical calculations, the initial opening $\delta_0$ is fixed at 5 mm and the width of the proppant filled region, $2b$ is fixed at 1 m. The Young’s modulus and Poisson’s ratio of the rock are selected to be $E = 10$ GPa and $\nu = 0.3$ and the fitting parameters in Eq. (6) are selected to be $\alpha = 5.543$ MPa and $\beta = 3.873$.

The first step of the analysis is to determine the critical spacing between the proppant columns, $2a$, at which the minimum residual opening of the unit-cell, $\delta_{\text{min}} = \delta(|x| = a)$ equals to zero, i.e. the fracture walls come in contact (see Fig. 1b). This critical value of proppant column spacing, $2a_{\text{cr}}$, corresponds to a drastic reduction in the fluid conductivity of the open channels. The selection of proppant column spacing greater than this critical value will result in sub-optimal increase in the effective fracture conductivity, hence represents a case of little practical interest. The dependence of $a_{\text{cr}}$ on the remotely applied compressive stress $\sigma_0$ was obtained through an extensive parametric study and the results are presented in Fig. 2. The best fit equation recovers the limiting cases, i.e. $2a_{\text{cr}} \to \infty$ as $\sigma_0 \to 0$ and $2a_{\text{cr}} \to 2b = 1$ m as $\sigma_0 \to \infty$.

![Fig. 2: Envelope showing combinations of proppant column spacing, $2a$ and remotely applied compressive stress, $\sigma_0$ which ensure that the fracture faces do not come in contact.](image-url)

Numerical results for the constant $C$ are obtained for the remotely applied stress in the range $10 < \sigma_0 < 50$ MPa with increments of 1 MPa and the proppant column spacing in the range $1.0 < 2a < 2.0$ m, with increments of 0.05 m. A spline function was fitted through the discrete data points to obtain the
interpolated value of the constant $C$ for any combination of parameters $2a$ and $\sigma_o$ which yields $\delta_{\text{min}} > 0$ as shown in Fig. 3.

The conductivity of the unit cell fully-filled with the effective medium is equivalent with the effective conductivity of the partially-filled unit cell, $K_{\text{eff}}$, which is obtained from [4]. Hence, the permeability of the effective medium, $\kappa_p$, can be gained as:

$$\kappa_p = \frac{K_{\text{eff}}}{\delta^*}$$

where $\delta^*$ is the constant opening of the unit cell when it is fully-filled with the effective medium, see Eq. (11). $\kappa_p$ is the key to compute the conductivity of each unit cell along the fracture length, and the conductivity of unit cells will be utilised to calculate the conductivity of the entire fracture.

Fig. 3: Contour plot showing the variation of the effective medium stiffness constant $C$ upon proppant column spacing and remotely applied stress.

**Conclusion**

In this paper, the periodic system of proppant columns within a hydraulic fracture is replaced by a continuous distribution of springs along the fracture length using a homogeneous procedure. The energy conservation principle and the solution for “unit-cell” developed in [2] are utilised to define the power-law for the nonlinear springs. The numerical results present the effective medium stiffness constant $C$ according to any combination of proppant column spacing and the confining stress. The application of the effective medium stiffness concept allows a significant reduction of the complexity of the problem and an application of well-developed methods of Fracture Mechanics to evaluate the residual opening of a periodically supported fracture. The outcomes of this work provide the first necessary step to analyse the hydraulic channel fracturing technique, which is of great interest to the gas and oil industries.
References


Field Scale Case Studies of Blue Shift Damage Monitoring Method

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Keywords: hydraulic fracture, slope instability, microseismic monitoring technique, fracture geometry, fractal

Abstract. We applied the Blue Shift method to analyse microseismic events generated from large scale rock damage processes, i.e. hydraulic fracturing of Marcellus Shale in Pennsylvania, USA and rock wall instability of Century mine in Queensland, Australia. It is shown that the Blue Shift method is able to detect the onset of localisation of rock damage from the non-zero value of the localisation exponent and can reveal the fractal nature of the geometry of the localised damage zone.

Introduction

Since the development of the damage zone is accompanied by microseismic events at the field scale, recording of the microseismic events can be useful to detect and monitor the damage localisation. The microseismic monitoring technique relies on the accurate recovery of the spatial coordinates of the events based on the P-wave arrival time and the full-field P-wave velocity measurements. It usually requires a multi-channel seismic acquisition system, theoretically at least four channels and more to reduce errors in source location determination, to obtain accurate locations of the microseismic events. In addition, large hard disk volume is required to store the data because the number of events can be large if the recording time is long.

Pasternak and Dyskin [1-5] proposed a damage localisation monitoring method, called Blue Shift Indicator, solely based on the recording of the arrival times of the microseismic events. The distinct temporal pattern of the microseismic events generated from the localisation zone should stem from the characteristic spatial development of the localisation zone and hence be related to the Euclidean dimension of the damage localisation zone. The spectral analysis of the simulated time series of the microseismic events generated during the evolution of the localisation zone shows that the higher the dimension of the damage localisation zone leads to the shift of the spectrum of the arrival times of microseismic events to a higher frequency, i.e. ‘blue shift’. The merits of the Blue Shift Indicator are that it could potentially reduce the required number of channels to ideally a single channel as only arrival times of microseismic events are needed and it mitigates the overwhelming hard disk storage capacity as before. Therefore, more cost-effective microseismic acquisition system can be built based on this method.

Two case studies of the application of the Blue Shift method in analysing different rock failure mechanisms and fracture geometries will be presented. The first case study is concerned with the hydraulic fracturing of Marcellus Shale in Pennsylvania, USA. The analysis result of the first case study is compared with the analysis by Zorn et al. [6]. The second case study is related to the rock wall instability of Century mine in Queensland, Australia. The analysis result of the second case study is compared with the analysis by Salvoni and Dight [7].
Blue Shift Method

The basis of the Blue Shift method [1-5] is to find the ‘energy’ content of the arrival times of the microseismic events generated during fracture propagation. The ‘energy’ content of the time series can be obtained from the Fourier transform of the time sequences and is given by

\[ S(M, \theta, \Omega) = \Omega \ast M + 2 \ast \sum_{m=2}^{M} \sum_{l=1}^{m-1} \frac{\sin[\Omega \ast (t_m - t_l)]}{(t_m - t_l)} \]  

(1)

where \( \Omega \) is the frequency; \( M \) is the total number of the microseismic events; \( t_m, t_l \) represent individual arrival times of the microseismic events and \( \theta \) is the set of arrival times of the microseismic events.

We introduce a benchmark case assuming that the event-generating fracture or a localisation zone propagates in a step-wise manner and that the microseismic events are only generated from the contour of the fracture or zone. It is assumed that in the \( k^{\text{th}} \) step of fracture or localisation zone propagation, there are \( k^\alpha \) events that would be generated. The parameter \( \alpha \), called the localisation exponent, varies with the fracture geometry. Subsequently, the full dimension of the cloud of events is \( \alpha+1 \). The localisation exponent can be fractional in which case it would correspond to the geometry fractal of the fracture or localisation zone [8]. The localisation exponent is determined by back analysis by comparing the ‘energy’ content of the actual arrival times with the benchmark case.

Hydraulic Fracturing of Marcellus Shale

In 2012, hydraulic fracturing with the “Zipper-frac” configuration was applied to the Marcellus Shale over the six horizontal wells in the southwest of Pennsylvania. Fig. 1 provides a map view of the configuration of the six horizontal wells and the spatial distribution of the microseismic events associated with the six stages. Zorn et al. [6] used the fractal dimension, D-value, to quantify the shape of the spatial distribution of hypocentres. When the D-value is equal to 0, 1, 2 or 3, it indicates that the spatial distribution of the microseismic events is a point, line, plane or cloud respectively. In this paper, only Stage 3 in Well 5 will be discussed.

![Figure 1](image)

Figure 1 Map view of the six horizontal wells. The microseismic events are coloured according to different stages of hydraulic fracturing. (Courtesy of Zorn and Bunger).

In stage 3 of well 5, there were 449 microseismic events indicated as dark green dots in Fig. 1. Events with the same arrival times are considered as the same event for the purpose of Blue Shift analysis. Therefore, there were only 433 events with different arrival times. The microseismic data are divided into four groups based on the pressure profile as shown in Fig. 2. The average localisation exponents for each group and the whole stage are summarised in Table 1. The time evolution of the simulated
localisation exponent is plotted with blue solid lines and the localisation exponent for the whole stage is indicated as a black dashed line in Fig. 2.

**Table 1 Average Localisation Exponents for Well 5, Stage 3**

<table>
<thead>
<tr>
<th>Group</th>
<th>Time Interval (second)</th>
<th>Event Interval</th>
<th>Average Localisation Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-3409</td>
<td>1-80</td>
<td>1.77</td>
</tr>
<tr>
<td>2</td>
<td>3409-6980</td>
<td>80-260</td>
<td>1.67</td>
</tr>
<tr>
<td>3</td>
<td>6980-9248</td>
<td>260-413</td>
<td>1.02</td>
</tr>
<tr>
<td>4</td>
<td>9248-9857</td>
<td>413-433</td>
<td>0</td>
</tr>
<tr>
<td>Whole stage</td>
<td>0-9248</td>
<td>1-413</td>
<td>1.64</td>
</tr>
</tbody>
</table>

As the pressure gradually increased to 8000 psi, the simulated localisation exponent of the first group of events reached 1.77. This indicates that the microseismic events of the initial pumping stage were distributed in multiple fracture planes and formed a 3D fractal with fractal dimension (the localisation index + 1) of 2.77. From approximately 3400 seconds to 7000 seconds, the pressure started to slowly decrease from 8700 psi to 8000 psi. The localisation exponent of the second group of events is slightly decreased to 1.67. It implies that the spatial distribution of microseismic events started to shrink towards a planar distribution but the microseismic cloud still occupied multiple fracture planes at this stage with fractal dimension of 2.67. After 7000 seconds, the pressure stabilised at 8500 psi till shut-in was applied. The simulated localisation exponent of the third group of microseismic events is further decreased to 1.02. It indicates the microseismic events were localised in a main fracture plane with fractal dimension of 2.02. The last group of microseismic events was recorded after shut-in and cannot be fitted with the dependencies provided by the *Blue Shift* method. It therefore implies that the microseismic events did not localise after shut-in.

![Figure 2](image_url)  
**Figure 2** Plot of pumping pressure (red curve) and localisation exponent (blue solid lines) varying with time of well 5, stage 3. Localisation exponent for the whole stage is 1.64 (black dashed line) and corresponds to the time interval from the start of receiving microseismic signals to shut-in.
The overall localisation exponent for the whole fracturing stage before shut-in is equal to 1.64. This suggests that multiple fractures were stimulated and the microseismic cloud formed is a 3D fractal with fractal dimension of 2.64. Zorn et al. [6] reported that the D-value for the whole stage before shut-in is 2.59 indicating spatial uniform distribution of the microseismic events with a strong planar component. They also pointed out that there were two distinct clusters of microseismic events developed. The first cluster was distributed within 150 meter around the perforation. This mainly corresponds to the first and second groups of microseismic events with localisation exponents equal to 1.77 and 1.67 respectively, indicating the development of multiple fracture planes. The second cluster showed a strong planar geometry between 280 and 840 meters from the perforation in the southwest direction after 6000 seconds. This mainly corresponds to the third group of microseismic events with localisation exponent equal to 1.02, indicating a strong planar fracture. The observed fracture development of this fracturing stage is aligned with the localisation exponent determined by the Blue Shift method. It can be seen that the Blue Shift method can detect the changes in the spatial distribution of microseismic events and indicates development of large multiple fractures.

**Rock Wall Instability of Century Mine**

This case study is focused on instability of the southwest (SW) wall of the Century mine and fractal characteristics of the associated microseismic events. The general geological setting of the SW wall is shown in Fig. 3. The SW wall has five main geological structures: i) Lower Footwall (LFW) composed of black layered carbonaceous shales, ii) a block of high strength Carbonate Breccia (CBX), iii) Upper Footwall (UFW) composed of black laminated shales, iv) Page Creek Fault and v) Pandora’s Fault. Four microseismic arrays with four geophones per array were installed along four drilled boreholes behind the SW wall. More detailed geological settings and the configuration of geophones can be found in [8].

![Figure 3](image-url)

**Figure 3** General geological structures of the SW wall of Century mine. (A) Front view of the SW wall showing main geotechnical units: Lower Footwall (LFW), Carbonate Breccia (CBX), Upper Footwall (UFW), Page Creek Fault and Pandora’s Fault; (B) Top view showing complex bedding planes of the area ([3]).

From 18th November 2013 to 30th April 2014, deformation of the SW wall and the associated seismicity increased significantly, especially between February and April. The majority of the microseismic events were located behind the SW wall with only a small amount of events recorded below the floor of the open pit because the mining activities were limited [7]. The observed seismicity has a strong correlation with rainfall during this time period [7]. The total number of microseismic events recorded with different arrival times is 1852. The microseismic events are divided between five time intervals for ease of comparison with the finding by [7]. The five time intervals are: i) 18th November 2013 to 14th February 2014, ii) 15th to 22nd February 2014, iii) 23rd February to 14th March 2014, iv) 15th March to 14th April 2014 and v) 15th April to 30th April 2014.

The Blue Shift method was applied to analyse the fractal characteristics of the recorded microseismic events for the five time intervals. The average localisation exponents for each time interval and for the whole stage are computed over ten realisations. The simulation results based on the Blue Shift method
are summarised in Table 2. In Fig. 4 the average localisation exponents are plotted as blue solid lines for the five time intervals and black dashed line for the whole stage (15\textsuperscript{th} February to 14\textsuperscript{th} April 2014). The spatial distribution of the microseismic events corresponding to all five time intervals with a best fit plane by Least Absolute Residual (LAR) robust regression is shown in Fig. 5.

It can be observed that the microseismic events were mainly located in the lower section of the Page Creek Fault, the wedge failure zone, in the upper wall sector, around the CBX block and along the Pandora’s Fault. The residual plot, Fig. 5 (B), shows a cross-section view of the microseismic events from the best fit plane. It can be seen that the majority of events are within 50 meter bounds from the fitting plane which is also indicated in [7]. The result of the LAR fitting shows the adjusted R-square is 0.9876 and the root mean square error is 5.375. It indicates that the fitting model of a 2D plane can adequately represent the actual spatial distribution of the microseismic events. From the Blue Shift method analysis, the average localisation exponent for this time interval is found to be 1.64, indicating the microseismic events were localised and possibly occupied multiple fracture planes. The events formed a fractal dimension of 2.64. Salvoni and Dight [7] also concluded the development of multi-batter failure behind the SW wall.

### Table 2 Average Localisation Exponents for Century Mine

<table>
<thead>
<tr>
<th>Group</th>
<th>Time Interval</th>
<th>Event Interval</th>
<th>Average Localisation Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18 Nov 2013 ~ 14 Feb 2014</td>
<td>1-68</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>15 Feb 2014 ~ 22 Feb 2014</td>
<td>69-154</td>
<td>1.65</td>
</tr>
<tr>
<td>3</td>
<td>23 Feb 2014 ~ 14 Mar 2014</td>
<td>155-721</td>
<td>1.67</td>
</tr>
<tr>
<td>4</td>
<td>15 Mar 2014 ~ 14 Apr 2014</td>
<td>722-1697</td>
<td>1.63</td>
</tr>
<tr>
<td>5</td>
<td>15 Apr 2014 ~ 30 Apr 2014</td>
<td>1698-1852</td>
<td>-</td>
</tr>
<tr>
<td>Whole stage</td>
<td>15 Feb 2014 ~ 14 Apr 2014</td>
<td>69-1697</td>
<td>1.64</td>
</tr>
</tbody>
</table>

**Figure 4** Plot of cumulative seismic events (red curve) and localisation exponent (blue solid lines) varying with time from November 2013 to April 2014. Localisation exponent for the whole stage =1.64 (black dashed line) corresponds to for the time interval from 15\textsuperscript{th} February to 30\textsuperscript{th} April 2014.
Figure 5 (A) Spatial distribution of microseismic events recorded from 15th February to 14th April 2014. The microseismic events are fitted with a plane using LAR robust regression method. The adjusted R-square is 0.9876 and the RMSE is 5.375. The Northing is along the SW wall. (B) Residual plot of microseismic events recorded from 15th February to 14th April 2014.

Conclusions
The Blue Shift method has been applied to analyse the arrival times of microseismic events recorded from large scale rock damage processes. From the first case study, it is interesting to note that the localisation exponent is generally greater than 1 for the initial pumping stages when the pressure is still rising. This possibly indicates that the microseismic events are localised around the major plane of rock damage and occupy multiple fracture planes. Furthermore, the localisation exponent generally reduces to around 1 after the pressure stabilised. This indicates the propagation of a planar fracture. From the second case study, it is worth pointing out that the localisation exponent remains almost constant and above 1 for the whole rapidly growing instability stage.

Literature References

Influence of the shape of 3-D cracks on their growth in biaxial compression

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Keywords: Mining; civil engineering; high stress; wing crack; transparent brittle sample; biaxial compression; penny-shape crack; square-shape crack.

Abstract: When excavating tunnels or shafts in civil or mining engineering the rock at depth may be subjected to high stress resulting in fracture and rockburst. The cause of the fracturing has been the subject of many studies but few have identified the role of the intermediate principal stress in the fracture growth. An experimental study of 3-D crack growth in biaxial compression is presented. The biaxial compression tests with different biaxial load ratios ($\sigma_x/\sigma_y$) were conducted on transparent casting resin samples each with a single internal crack of different shapes. Unlike 3-D crack growth in uniaxial compression, where there are intrinsic limits of growth of wing cracks imposed by wing wrapping, in biaxial compression the wing crack can grow extensively, parallel to the free face of the sample. The biaxial load ratio influences the morphology of the induced wing surface, whereas the effect of the shape of the initial crack is minor.

Introduction

When excavating tunnels or shafts in civil or mining engineering the rock at depth may be subjected to high stress resulting in fracture and rockburst [1]. The cause of the fracturing has been the subject of many studies but few have identified the influence and role of the intermediate principal stress in the fracture growth. Heterogeneous materials like rock and concrete contain abundant morphologies of the structural imperfections or weaknesses at all scales. These imperfections include grain boundaries, pores, and cracks on the small scale, and joints, faults, shears, foliation, schistosity and bedding planes on the larger scale [2]. In compression, the elements of material heterogeneity act as stress concentrators producing local stress redistribution, generating local tension and thus initiating crack growth and local failure.

The internal pre-existing cracks are often modelled as penny-shape cracks. Direct experiments conducted by Dyskin et al [3, 4] show that in 3-D samples with the initial crack located in the centre of the sample, the wing cracks are restricted to the sizes comparable to the initial crack and therefore cannot substantially grow to cause the macroscopic failure of the sample, see Fig. 1. This is opposite to the 2-D situations, where there is no room for the wrapping and thus the secondary crack can grow extensively and finally causes the sample splitting [5-13].
Fig. 1. The crack growth in resin sample under uniaxial compression loading condition[4]: (a) front view (b) side view.

The crack growth behaviour can be radically changed when the intermediate principal stress is applied. Sahouryeh et al. [14] conducted biaxial compression tests transparent casting resin with a single penny-shape crack situated in the centre of the sample. It was found that in biaxial compression with approximately equal loads, the intermediate principal compression stress prevented the wing cracks from wrapping, and therefore wing cracks were able to grow extensively forming a crack surface parallel to both loading directions, which ultimately gave rise to the splitting failure of the tested samples. It should be noted that in that study, the applied biaxial load ratio ($\sigma_2/\sigma_1$) was 1 and the initial crack was of penny shape.

In the field, generation of surface parallel fractures (slabbing or spalling type failure) are often observed at the excavation boundary [1, 15, 16], where rocks are under biaxial compression. Under these conditions, extensive growth of wing cracks parallel to the excavation boundary could be induced leading to separation of thin rock plates from the rock mass followed by their buckling, formation of a new free surface. Then this process of rock surface spalling repeats itself eventually changing the shape of the opening [17]. In a more severe case, a skin rockburst may occur [18].

In the light of the abovementioned observation, a question should be asked as to whether the extensive crack growth is dependent on the special biaxial load ratio or the shape of the initial crack. In order to answer this question we conducted experiments with (i) 3-D initial crack of different shapes (disc-like and square) and (ii) different biaxial load ratios of 0.5, 0.25, 0.1 and 0.06.

Sample Preparation and Experimental Set-up

Sample Preparation. Cubic samples of 100 mm × 100 mm × 100 mm were made from transparent casting polyester resin “CRYSTIC 406 COS PA FC”. To model the initial penny or square-shape cracks, two greased aluminium foil discs or squares, 10 mm in diameter or in side, inclined at 30° to one of the loading axes (y-axis) were hanged in the centre of a cubic aluminium mould by a pair of copper wires prior to resin casting. Then the thoroughly mixed resin and the catalyst was poured into the mould.

After being completely cured (around 7 days according to the resin casting manual), the samples were cut and polished to the required dimensions. In addition, to ensure a brittle failure regime, samples were kept in a freezer for at least 48 h (at $\approx -16^\circ C$) before testing.

Experimental Set-up. The resin samples were tested by using the True Triaxial Stress Cell (TTSC) at Curtin University (WA, Australia). The sample surfaces and the loading platens were in direct contact; the contact dimensions were 98mm × 98mm (the size of loading platen was 1 mm smaller on each side
than resin samples) to prevent loading platens from touching and interlocking. The biaxial loading condition can be achieved by setting the load on two opposite sample surfaces to zero and removing the corresponding loading platens.

The biaxial compression tests were conducted in load-control (a limitation of the loading system); the ratio of loading rates in two directions was fixed during each test, which means that the predefined biaxial load ratio \( \sigma_x / \sigma_y \) can be maintained within the accuracy of the control module of hydraulic pumps during the whole testing process.

**Experimental Results**

Progressively decreasing biaxial load ratios \( \sigma_x / \sigma_y \) of 0.5, 0.25, 0.1 and 0.06 were applied in a series of biaxial compression tests for both initial crack types. In total, 8 resin samples were tested. Table 1 and Table 2 report the testing parameters including the loading rate, the final stress reached, the biaxial load ratio and the error between final and pre-defined biaxial load ratio in biaxial compression tests on two types of the initial cracks, respectively.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>( \sigma_y ) [MPa]</th>
<th>( \sigma_x ) [MPa]</th>
<th>( \sigma_z ) [MPa]</th>
<th>( \sigma_x / \sigma_y )</th>
<th>( \dot{\sigma}_y ) [MPa/min]</th>
<th>( \dot{\sigma}_x ) [MPa/min]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-1</td>
<td>83.6</td>
<td>41.5</td>
<td>0</td>
<td>0.495</td>
<td>12</td>
<td>6</td>
<td>0.50</td>
</tr>
<tr>
<td>P-2</td>
<td>84.1</td>
<td>20.5</td>
<td>0</td>
<td>0.244</td>
<td>12</td>
<td>3</td>
<td>0.25</td>
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<tr>
<td>P-3</td>
<td>72.7</td>
<td>8.0</td>
<td>0</td>
<td>0.11</td>
<td>12</td>
<td>1.2</td>
<td>0.1</td>
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<tr>
<td>P-4</td>
<td>71.7</td>
<td>4.3</td>
<td>0</td>
<td>0.06</td>
<td>12</td>
<td>0.72</td>
<td>0.06</td>
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<table>
<thead>
<tr>
<th>Test No.</th>
<th>( \sigma_y ) [MPa]</th>
<th>( \sigma_x ) [MPa]</th>
<th>( \sigma_z ) [MPa]</th>
<th>( \sigma_x / \sigma_y )</th>
<th>( \dot{\sigma}_y ) [MPa/min]</th>
<th>( \dot{\sigma}_x ) [MPa/min]</th>
<th>Error [%]</th>
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<tr>
<td>S-1</td>
<td>84.2</td>
<td>43.4</td>
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<td>0.516</td>
<td>12</td>
<td>6</td>
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<tr>
<td>S-2</td>
<td>83.5</td>
<td>20.6</td>
<td>0</td>
<td>0.245</td>
<td>12</td>
<td>3</td>
<td>0.25</td>
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<tr>
<td>S-3</td>
<td>73.4</td>
<td>7.0</td>
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<td>0.095</td>
<td>12</td>
<td>1.2</td>
<td>0.1</td>
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<tr>
<td>S-4</td>
<td>75.7</td>
<td>4.7</td>
<td>0</td>
<td>0.062</td>
<td>12</td>
<td>0.72</td>
<td>0.06</td>
</tr>
</tbody>
</table>

**Biaxial Compression Tests on Penny-shape Crack.** When a resin sample was tested with \( \sigma_x / \sigma_y = 0.5 \) (test No. P-1), the wing crack grew extensively towards both loading axes. At the end of the test, the wing produced from the upper part of the initial crack contour extended to the sample boundaries and produced splitting failure of the sample. As for the wing sprouted from the lower part of the initial crack contour, it assumed the form of “moth-shape”. The “moth-shape” wing was completely parallel to the free surfaces [19].

Two wings produced from the upper and lower contour of the initial crack grew extensively and approached the top and bottom boundaries of the sample with \( \sigma_x / \sigma_y = 0.25 \) (test No. P-2). An overlapping area was formed between two individual wings presented in the form of “moustache-shape” at the middle of the sample as shown. Unlike the splitting failure observed in test No. P-1, the whole
sample remained intact even though both wing cracks propagated nearly to the sample boundaries. The wing crack surfaces were completely parallel to the load-free surfaces.

When $\sigma_x/\sigma_y$ ratio was reduced to 0.1 (test No. P-3), the indications of a transition pattern of crack growth modes could be observed. The crack growth with this value of $\sigma_x/\sigma_y$ has the following features: (1) the wings sprouted from the upper and lower contour of the initial crack merged together (by overlapping); (2) the total area of overlapping was narrower as compared with that when the higher $\sigma_x/\sigma_y$ ratios were applied; and (3) in spite of the fact that the overall wing surfaces were almost parallel to the sample free surface, waving could be observed in local area near the initial crack. Fig. 2 shows the pattern of the produced wing crack growth when $\sigma_x/\sigma_y = 0.06$ (test No. P-4). It is seen that the abovementioned features can still be observed with further reduction in the $\sigma_x/\sigma_y$ ratio.

![The wavy shape near the initial crack](image)

**Fig. 2.** The crack growth in biaxial compression with $\sigma_x/\sigma_y = 0.06$ (test No. P-4) when the initial penny-shape crack is inclined at 30° to $\sigma_y$ (a) lateral view; (b) front view.

**Biaxial Compression Tests on Square-shape Crack.** A resin sample was tested with $\sigma_x/\sigma_y = 0.5$ (test No. S-1). At the end of the test, the extensive wing crack growth was produced. The wing produced from the upper part of the initial crack contour nearly extended to the sample boundaries and was completely parallel to the free face of the sample. The wing sprouted from the lower part of the initial crack contour, it had the form of “moth-shape”, which is similar to test No. P-1. On the other hand, the wing surface produced from the upper part of the initial crack contour has obvious shrinkage compared with that sprouted from the penny-shape crack when the same $\sigma_x/\sigma_y$ was applied. Together with results obtained in test No. S-2, within 0.25 to 0.5 of the range of $\sigma_x/\sigma_y$ ratio, the growth of the lower wing crack was suppressed to some extent, which might be attributed to the interaction between two individual wings.

When $\sigma_x/\sigma_y = 0.1$, the waving shape near the initial crack and appreciable undulation can be identified on the wing crack surface, resembling that sprouting from the penny-shape crack with the same ratio of $\sigma_x/\sigma_y$.

When $\sigma_x/\sigma_y$ was reduced to 0.06, Fig. 3, the continuity of the wing crack near the initial crack was affected. Most importantly, the wing crack at this part started to show discreteness instead of forming a continuous plane.
It is worth noting that the extensive crack growth occurred, and the near arc wing crack surfaces were induced despite the square shape of the initial crack. This phenomenon suggests that in biaxial compression the influence of the shape of the initial cracks on their growth is minor.

**Fig. 3.** The crack growth in biaxial compression with $\sigma_x/\sigma_y = 0.06$ (test No. S-4) when the initial square-shape crack is inclined at $30^\circ$ to $\sigma_y$ (a) lateral view; (b) top view.

**Influence of the Intermediate Principal Stress on Crack Growth.** In uniaxial compression, the direction of wing crack growth is only controlled by the direction of the maximum principal stress ($\sigma_y$ in this case). A curly crack surface due to wrapping of the wings is formed. This results in a limited wing crack growth in the direction of $\sigma_y$, as shown in Fig. 4(a). When the intermediate principal stress ($\sigma_x$ in this case) is applied, wing wrapping in the direction perpendicular to $\sigma_x$ can be suppressed, depending on the stress ratio. When $\sigma_x/\sigma_y$ ratio is high enough, the wrapping can be completely suppressed, which gives rise to extensive crack growth, as shown in Fig. 4(b). Then the wing cracks propagate parallel to both $\sigma_y$ and $\sigma_x$ directions.

**Fig. 4.** Wing crack growth in compression from the top view [14]: (a) wing crack wrapping in uniaxial compression (the loading axis is perpendicular to the drawing plane) (b) extensive crack growth in biaxial compression.
Conclusion

The cause of spontaneous growth of fractures leading to strain burst and buckling failure in underground development has been identified. The material heterogeneities generate tensile stresses in a compressive stress field. The penny-shape and square-shape cracks are two extreme cases of modelling the pre-existing cracks in heterogeneous materials. In uniaxial compression, for both cases, the wrapping restricts the induced wing cracks to grow sufficiently to produce failure. In biaxial compression, the wrapping gets suppressed and the wing crack is capable of growing extensively parallel to the free face of the sample. The extensive crack growth pattern does not depend on the biaxial load ratio as long as it is sufficient to suppress the wrapping. Some geometrical features of wing cracks depend upon the magnitude of the intermediate principal stress: when the relatively high intermediate principal stress is applied, the wing crack surface is larger and flatter; reducing the biaxial load ratio leads to the fluctuation and even discreteness of the wing crack surfaces. On the other hand, the effect from the shape of the initial crack on wing growth is minor.

Thus in compression the extensive crack growth is highly sensitive to the intermediate principal stress which is capable of changing the mechanics of brittle fracturing in compression.

References


The dynamic fracturing patterns of multiple types of rock with Brazilian tests investigated by high-speed 3D-DIC

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Keywords: Rock fracturing, SHPB, Brazilian Disc test, High-speed imaging, 3D-DIC

Abstract. Dynamic Brazilian Disc (BD) test was performed on Carrara marble, Hawksbury sandstone and impala black gabbro with split Hopkinson pressure bar (SHPB). The full-field and real-time fracturing processes were captured by the high-speed 3D-DIC technique with the resolution of 256 × 256 pixels and 200,000 frame-per-second (fps) in frame rate. The development of horizontal and vertical displacement field of the BD disc showed different patterns for all three types of rock. The locations for the initiation of strain localization (as well as crack) depends on shear strength of the rock. The ratio of propagation velocity of tensile strain localization between rocks is dependent on the wave impedance.

Introduction

In the last two decades, the development in the dynamic BD test mainly focused on the improvement of loading method, introduction of innovative measurement and taking external treatment or internal effect into the dynamic tensile properties. In terms of the improvement of loading method, four typical loading configurations of the loading platens were proposed successively. They are flat loading platens, flat loading platens with two small-diameter steel rods, flat loading platens with cushion, curved loading jaws and the flattened Brazilian[1]. The improvement all came from the validity of the Brazilian test which is the location of the failure initiation. According to the theoretical analysis based on the Griffith criteria, the failure should start from the center of the disc. However, Hudson et al. (1972) found that failure always initiated directly under the loading points in the Brazilian test if only flat steel platens were used to load the specimens in a servo-controlled testing machine. With regards to the measurement development in dynamic BD test, the milestone is introduction of high-speed imaging with digital optical measurements monitoring the deformation of the specimen. Wong et al. [2] observed the cracking processes in static and dynamic BD test of Carrara marble by high-speed video footage captured at a frame rate of 100,000 frames per second. Rock-like material were investigated by dynamic BD test with high-speed photography as well, Zhou and Zhu studied the tensile behavior of five types of 3D printing material by BD test with the aid of [3] a high-speed camera of 100,000 frames per second to monitor the fracturing process. Colback applied high-speed photography to capture the photoelastic patterns induced in birefringent layers to determine the fracture initiation point and study its subsequent propagation [4]. The digital image correlation (DIC) method was firstly employed by Zhang and Zhao to calculate the strain fields of specimen to observe the failure initiation and propagation [5]. Microscopic observation were applied as a post failure analysis means to investigate the mechanism of the fracture. Li, Hui, et al. used X-ray diffraction (XRD) [6] to estimate the mineralogy and total organic content (TOC) after the sample failed. Zhang and Zhao [7] applied thin section to study the relationship between crack path and grain boundary. Scanning electron microscopy (SEM) was also applied to investigate the topography of the fractured surface. The external treatment includes the saturation of water, carbon dioxide, brine, and the heat, microwave treatment and the pre-stress. Both static and dynamic tensile strength of sandstone
under nominally dry and full saturated conditions were quantified by Brazilian tests [8]. The effect of high temperature on the fracture toughness was investigated by Flattened Brazilian Disc Specimen[9]. Tavallali et.al investigated the effect of layer orientation on the failure of layered sandstone under Brazilian test conditions [10]. Zhou et.al. applied [11] coupled static and dynamic SHPB to conduct the Brazilian test to investigate the coupled-load properties of rock.

As can be seen from above studies, the Brazilian test mostly investigate the single type of rock and the high speed photography in the previous studies has the low resolution and frame rate. In study, rock type effect on the tensile loading behavior was investigated with three types of rock representing igneous, metamorphic and sedimentary rock. The initiation and propagation of crack were discussed by the DIC results.

**Experiment Setup**

**Rock Materials**

Three types of rocks are investigated which are Hawkesbury sandstone, Cararra marble and impala black gabbro. The diameter of specimens is 50 mm with a thickness of 25 mm. The mechanical properties for the sandstone are as following, density= 2.21 g/cm³, P-wave velocity= 1957 m/s, elastic modulus = 8.39 GPa, and UCS = 41 MPa. The mechanical properties for the gabbro are as following, density= 2.90 g/cm³, P-wave velocity = 6821 m/s, elastic modulus = 63.6 GPa, and UCS = 284 MPa. The mechanical properties for the marble are as following, density= 2.68 g/cm³, P-wave velocity = 5340 m/s, elastic modulus = 40.0 GPa, and UCS = 112 MPa.

**Loading and high-speed imaging system**

The dynamic loading was applied by the SHPB system including a striker of 0.4 m, an incident and transmitted bars of 2.4 m and 1.4 m, respectively. The striker and bars made with high strength 40Cr steel and share the diameter of 50mm, and have a nominal yield strength of 800 MPa, the P-wave velocity of 6100 m/s, and elastic modulus of 208 GPa. The principle of the SHPB refers to [12]. The pressure of the gas gun was kept same for each test to achieve the same velocity of projectile which is around 8.5 m/s. Static uniaxial compression test was also conducted for a reference. High-speed cameras were employed to capture the deformation and fracturing process of the specimens. Two cameras were mounted up and down by tripods to establish a stereovision for the 3D-DIC. The resolution was set as 256×256 pixels and the frame rate was 200,000 fps with 4 μs exposure time. DIC belongs to a class of non-contact methods that acquire images of an object, store images in digital form and perform image analysis to extract full-field shape, deformation or motion measurements [13]. The principle of DIC refers to [12]. When there is an out-of-plane displacement, 3D-DIC is necessary to be applied to eliminate the error in 2D-DIC caused by out-of-plane displacement.
Results

The specimen ID and its corresponding loading condition are listed together with the tensile strength in Table 1. It could be seen from Table 1 that even the projectile velocity has small deviation, the loading rate generated in three rock types could be dramatically different. Therefore, the loading rate does not practically depend on the incident wave or the size of the specimen and bar but also depends on the intrinsic properties of rock. More commissioning tests have to be done if similar loading rates are required between different rocks for comparison. It could be seen that the strengthening effect on tensile strength caused by the increasing loading rate exists on each type of rock. The dynamic increase factor is 1.34, 1.9 and 2.0 for gabbro, marble and sandstone, respectively, which has the opposite order with the tensile strength.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Impedance (MPa·s)</th>
<th>Loading type</th>
<th>Loading condition*</th>
<th>ID</th>
<th>Tensile strength (MPa)</th>
<th>Loading rate (Gpa/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gabbro</td>
<td>2.02</td>
<td>static</td>
<td>0.2mm/min</td>
<td>G1</td>
<td>13.5</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>G2</td>
<td>12.7</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>G3</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>dynamic</td>
<td>8.4 ±0.4 m/s</td>
<td>G4</td>
<td>17.7</td>
<td>430</td>
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<td></td>
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<td></td>
<td>G5</td>
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<td>G6</td>
<td>18.2</td>
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<tr>
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<td>0.2mm/min</td>
<td>M1</td>
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<td>M6</td>
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<td>0.2mm/min</td>
<td>S1</td>
<td>2.8</td>
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<td>9.0±0.1 m/s</td>
<td>S4</td>
<td>5.5</td>
<td>119</td>
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<td></td>
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<td>S5</td>
<td>6.1</td>
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<td></td>
<td></td>
<td>S6</td>
<td>6.5</td>
<td>170</td>
</tr>
</tbody>
</table>

*Condition refers to loading speed for static tests and projectile velocity for dynamic tests.

Fig. 2 shows the horizontal displacement within the 60 µs in all three types of rock which can indicate the stress wave propagation within the disc. It can be seen from the contour that the horizontal displacements of three types of rock firstly concentrate at the interface between the bar and the disc. The horizontal displacement gradually propagated though the specimen with an arc front (indicated by red dash) symmetrized to the axis. When the wave propagation across the center of the specimen, the arc front become vertical then turned back to arc again but with the opposite curvity. Finally, most part of the specimen share the similar displacement while the edges have the largest and least displacements. The characteristics of the displacement is the interaction between the spherical wave (caused by point loading source) and the boundary of the disc. Because the gabbro has the largest wave velocity, the horizontal displacement propagated and reached the end the of the specimen with the shortest time followed with marble and sandstone.
The out-of-plane displacement which is explained as the shear failure is shown in Fig. 3. The displacement firstly appeared at 40 µs at the end of the disc for gabbro then the loading edge which is opposite to the case of marble and sandstone. The out-of-plane for sandstone showed earliest at 20 µs with the most significant degree due to the lowest shear strength. The rest part of the specimen had less out-of-plane but not symmetric for upper and lower semi-circles for gabbro and marble which indicating that the deformation was not consistent through the z direction. This may caused by the heterogeneity or manufacturing error in z-direction. We used the images from single camera to calculate the results in 2D-DIC and found that, the maximum error is below 5% in strain and displacement. Therefore, it is acceptable to use 2D-DIC to study the BD test.

Fig. 4 shows the vertical strain localisation development in three types of rock, the first photo represents the initiation of the strain localisation and the conection throughly in the last one. Only the strain localisation in gabbro initiated from the end of the specimen, marble and sandtone both initiated from the loading edge. This trend is as same as the one in out-of-plane displacement (W) and the time for the initiation of out-of-plane displacement and tensile strain localization. Therefore, the location of the strain localization in BD test with plane platen depends on the shear failure place which will form a fractured
opening, the tensile stress will dramatically concentrate and pull two semi-circles apart along the center axis. The duration for the strain localization is 20 µs, 50 µs and 90 µs for gabbro, marble and sandstone which has the same scale with the wave impedance in Table 1.

Therefore, the propagation velocity of the strain localization (as well as crack) is dependent on the wave impedance of the rock which represents the resistance of rock to momentum transfer. Because the strain localization did not appear from the center which is not constant with the Griffith theory, the tensile strength cannot represent the maximum tensile strength of the rock. It confirms that the plane platen can hardly obtain a good stress distribution in dynamic tests to find out the tensile strength of the rock.

References


Transitional Negative Stiffness and its Effect on Material Instability and Failure under Compression

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\textbf{Keywords:} Transitional Negative Stiffness, Instability, DEM modelling.

\textbf{Abstract.} In this paper we demonstrate that \textit{transitional negative stiffness} effect exists when spring breakage happens. This transitional negative stiffness effect can momentarily increase the effective Poisson’s ratio of the material. Subsequently, the transitional negative stiffness effect is capable of bringing the Poisson’s ratio of dilating materials to the critical value of 0.5. It can be readily seen from equilibrium equations that when Poisson’s ratio equals to 0.5 non-trivial solution exits which induces the loss of stability and finally shear failure. This new mechanism of instability is further verified through Distinct Element Method modelling. It is found that the critical Poisson’s value is reached in near peak stress region which confirms the proposed mechanism.

\textbf{Introduction}

Stiffness - a factor that controls how an elastic body deforms when subjected to positive loading. Yet there are cases when the mechanical behavior of the system can be interpreted in terms of negative stiffness. For instance negative stiffness was observed in human bodies for instance hair bundles in the ear and joints [1-3]. Negative stiffness can also be observed in compressive rock tests manifesting itself through the post peak softening in the stress and strain curve [4, 5]. (The post peak softening stage when exists is only obtainable when the loading frame has high stiffness.) In numerical modelling post peak softening stage is also observed in compression test simulations [6-11]. In the literature, mechanisms of instability involving negative stiffness effect induced by rotations of non-spherical particle were proposed [12-17]. Here in this paper we discuss and verify a new mechanism of instability of geomaterial based on the \textit{transitional negative stiffness} effect.

\textbf{Concept of Transitional Negative Stiffness}

The concept of transitional negative stiffness can be illustrated using the following simple system Fig. 1 [18].

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{simple-springs-model.png}
\caption{Simple three springs model (a) before spring $k_2$ breaks (b) after spring $k_2$ breaks [18]}
\end{figure}
Here $F_1$, $F_2$ and $F$ are the forces in the three springs, $k_0$, $k_1$ and $k_2$ are the stiffnesses of the springs, $u_0$ and $u$ are the displacements of the springs. In Fig. 1 (b) $F_1'$ and $F'$ are the forces of the remaining springs after spring $k_2$ breaks, $\Delta u$ is the change in displacement. The latter can be expressed as

$$\Delta u = u_0 \frac{k_0 k_2}{(k_0 + k_1)(k_0 + k_1 + k_2)}$$

(1)

It can be seen from Eq. (1) that the change of the displacement of spring $k_1$ is positive when the force of $F_2$ reduces to zero. The reduction of the force $F_2$ can be modelled as application of force $-F_2$ to spring $k_2$. Assuming that this negative force is applied gradually from 0 to $-F_2$ one obtains that during this process work done this force is negative. It can be expressed as

$$A = -0.5F_2\Delta u = \frac{1}{2} u_0^2 k_{eff} k_{eff} = -\frac{k_0^2 k_2^2}{(k_0 + k_1 + k_2)^2(k_0 + k_1)}$$

(2)

Thus this simple three-spring model shows that negative stiffness effect exists during the breakage of the spring. Since this negative stiffness element only exists in very short time it is termed transitional negative stiffness.

**The Effect of Transitional Negative Stiffness on Effective Poisson’s Ratio**

Consider an isotropic elastic space subjected to uniform load $\sigma_{ij0}$. Suppose a disc-like crack with radius $a$ is being formed. Introduce a coordinate system $(x'_1, x'_2, x'_3)$ where $x'_3$ direction is normal to the crack plane and the applied stress has components $\sigma_{ij0}'$. Assume that normal stress component $\sigma_{330}'$ is negative, such that the crack does not get closed. Under such load the crack develops displacement discontinuity $\Delta u_i'$ distributed over the crack plane. The average deformability of the material is controlled by displacement discontinuity integrated over the crack plane [19].

$$V_i' = \iint_A \Delta u_i' dS = \frac{8\pi}{3} \frac{1 - v_0^2}{E_0} a^3 c_i(\sigma_{ij0}')\sigma_{ij3}' \quad c_1 = c_2 = \frac{1}{2} \frac{1}{1 - v_0}, \quad c_3 = \frac{2}{\pi}$$

(3)

where $E_0$ and $v_0$ are the Young’s modulus and Poisson’s ratio of the material. For pure normal crack $c_1 = c_2 = 0$.

The energy change caused by the crack opening is.

$$U_{opening} = \frac{1}{2} V_i' \sigma_{ij0}' = \frac{4\pi}{3} \frac{1 - v_0^2}{E_0} a^3 c_i(\sigma_{ij3}' \sigma_{ij3}')$$

(4)

Now let’s consider the moment of the crack formation. At this moment the corresponding stress components acting on the future crack plane $\sigma_{ij3}'$ must be equal to those in the material. After the crack has formed these stress components vanish. Thus when the crack is being formed these stresses can be considered acting on the crack faces in the direction opposite to the displacement of the crack faces. Hence these stresses do negative work during the process of crack formation. If the reduction of stress $\sigma_{ij3}'$ is linear then the negative energy associated with crack generation is

$$U_{formation} = -\frac{1}{2} V_i' \sigma_{ij0}' = -\frac{4\pi}{3} \frac{1 - v_0^2}{E_0} a^3 c_i(\sigma_{ij3}' \sigma_{ij3}')$$

(5)

Comparing Eqs. (4) and (5) one can see that the fact that the energy associated with crack formation and crack opening only differ by sign makes it easy to determine the effect of transitional negative stiffness on the effective Poisson’s ratio of the materials.

For isotropic material with randomly oriented disc-like cracks the effective Poisson’s ratio reads.

$$v = v_0 \left[ 1 - \frac{4\pi}{45} (1 - v_0^2) [2(1 + 3v_0)c_3 - (1 - 2v_0)(c_1 + c_2)]v \right]$$

(6)
where \( v \) is the crack concentration, \( v = N\langle a^3 \rangle \ll 1 \), where \( N \) the number of cracks per unit volume and symbol \( \langle . \rangle \) denotes averaging over all cracks.

Since the energies are only differ in sign by simple reversing the sign in Eq. 6 the effective Poisson’s ratio associated with the transitional negative stiffness can be expressed as follows

\[
v = v_0 \left[ 1 + \frac{4\pi}{45} (1 - v_0^2) [2(1 + 3v_0)c_3 - (1 - 2v_0)(c_1 + c_2)]v \right]
\]  

(7)

It should be noted here that this simple reversing sign method is only valid when the crack concentration is low and crack interaction is neglected. This assumption is valid since the transitional negative stiffness is short lived so not many cracks can be formed under this short time.

Assuming that only normal cracks are formed \( [20] \) \( c_1 = c_2 = 0 \) and Eq. (7) becomes

\[
v = v_0 \left[ 1 + \frac{16}{45} (3 + v_0)(1 - v_0^2) \right]v
\]  

(8)

Thus the transitional negative stiffness momentarily increases the effective Poisson’s ratio.

**Critical Value of Poisson’s Ratio and Instability of Isotropic Material**

Consider following \([18]\) Lame’s equation for isotropic material.

\[
(1 - 2v)\Delta \mathbf{u} + \text{grad} \text{ div} \mathbf{u} = 0
\]  

(9)

where \( \mathbf{u} \) displacement vector and \( \mathbf{u}=(u_1,u_2,u_3) \) in a coordinate system \((x_1,x_2,x_3)\).

We look for non-trivial solutions of Lame’s equation in the form

\[
\mathbf{u} = \mathbf{u}_0 e^{i\lambda (kx)}
\]  

(10)

where \( \mathbf{u}_0 \) and \( \lambda \) are constants, \( k=(k_1,k_2,k_3) \) and \( |k| = 1 \).

Substituting Eq. (10) into Eq. (9)

\[
\begin{pmatrix}
1 - 2v + k_1^2 & k_1k_2 & k_1k_3 \\
k_1k_2 & 1 - 2v + k_2^2 & k_2k_3 \\
k_1k_3 & k_2k_3 & 1 - 2v + k_3^2
\end{pmatrix}
\begin{pmatrix}
u_1 \\
u_2 \\
u_3
\end{pmatrix} = \begin{pmatrix}0 \\
0 \\
0
\end{pmatrix}
\]  

(11)

Non-trivial solutions of Eq. (11) exist when the determinant of the matrix vanishes, which gives:

\[
2(1 - 2v)^2(1 - v) = 0
\]  

(12)

Eq. (12) has two roots 0.5 and 1. Therefore at \( v=0.5 \) a non-trivial solution exists that is the incompressible material loses stability.

Suppose during late loading stage a material has Poisson’s ratio of \( 0.5 - \varepsilon \) where \( \varepsilon \ll 1 \) (e.g., due to dilantancy). Then from Eq (8) the concentration of forming cracks needed to bring the effective Poisson’s ratio to 0.5 is quite low \( v=(15/7)\varepsilon \). Therefore, even a low forming crack concentration can bring the almost incompressible material to full incompressibility and induce the instability.

This is the new mechanism of instability of material associated with transitional negative stiffness. It is based on the increasing of Poisson’s ratio of an almost incompressible material due to the transitional negative stiffness associated with fracture formation. This will bring the material to full incompressibility, instability and subsequently failure of the material.

**Verification of the proposed instability mechanism by Distinct Element Method (DEM)**

In order to verify the proposed mechanism of instability a 3D virtual sample is created and uniaxial compressive test is simulated by the Particle Flow Code 3D (PFC3D). The 3D sample consists around
15,000 particles. PFC$^{3D}$ built in Linear contact model and Linear Parallel Bond model are used. Parameters of the virtual sample in 3D have been listed in Table 2 and Table 3.

<table>
<thead>
<tr>
<th>Parameters of the sample</th>
<th>H (m)</th>
<th>W (m)</th>
<th>$D_{\text{min}}$ (m)</th>
<th>$D_r$</th>
<th>Porosity</th>
<th>$\rho$(kg/m$^3$)</th>
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</thead>
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<td>0.0634</td>
<td>0.0317</td>
<td>0.0019</td>
<td>1.66</td>
<td>0.35</td>
<td>2630</td>
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</tbody>
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<table>
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<tr>
<th>Parameters of parallel bond model</th>
<th>$E_p$ (Pa)</th>
<th>$\bar{\sigma}$ (Pa)</th>
<th>$\sigma_{\text{std}}$ (Pa)</th>
<th>$\tau$ (Pa)</th>
<th>$E$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$73 \times 10^9$</td>
<td>$105 \times 10^6$</td>
<td>$20 \times 10^6$</td>
<td>$10^{10}$</td>
<td>$73 \times 10^9$</td>
</tr>
</tbody>
</table>

Fig. 2 shows the incremental Poisson’s ratio and the axial stress vs. loading time. It can be seen that the critical value of 0.5 of incremental Poisson’s ratio is reached in the peak region.

![Figure 2 Incremental Poisson's ratio and axial stress vs. time step](image)

**Conclusions**

We demonstrate the existence of *transitional negative stiffness* during the process of local failure and that the transitional negative stiffness can momentarily increase effective Poisson’s ratio. In particular, when the Poisson’s ratio is close to 0.5, as in compressive tests of dilatant materials, the formation of even low concentration of cracks can momentarily bring the materials to complete incompressible stage (Poisson’s ratio of 0.5) and thus induce global instability.

While during crack formation the material is no longer elastic, when fracture growth is stable (as in compression) and lasts very short time the material can still be considered elastic.

DEM 3D simulations of uniaxial compressive test show that the incremental Poisson’s ratio reaches the critical value at the stress that is close to the peak stress.
References

Development of Numerical Rock Model Using Discrete Element Method

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Keywords: triaxial compression testing, discrete element method, constitutive relationship.

Abstract. In petroleum-related geomechanics engineering applications, triaxial compression testing is widely used to characterize stress-strain relationships of underground rocks. However, the test requires multiple core plugs with similar mechanical properties. Also coring reservoir rocks requires the suspension of drilling operation and often becomes very expensive. Thus, the availability of real reservoir rock materials is often limited. In this study, a commercially available software, DEMPack is used to develop a DEM (Discrete Element Method) model predicting triaxial stress-strain curves at various confining pressures. The model is based on basic rock properties such as grain size distribution, porosity, grain-scale constitutive relationships. From these fundamental rock properties, the DEM model predicts a core-scale constitutive relationship. Numerical stress-strain curves predicted by the DEM model are compared with those measured from laboratory triaxial compression test data. Through extensive calibration of mechanical parameters, DEM-predicted stress-strain curves match those measured from laboratory test data. The example presented in this work shows good matches for linear elastic stress-strain relationships, yield points, and maximum compressive stresses. In this study, we also present three empirical correlations for grain-scale Young’s modulus, internal friction angles, maximum compressive stresses as a function of confining pressures, which can be used for developing numerical rock models.

Introduction

Background. Sedimentary rocks are formed by a number of small solid particles that are compacted and bonded to each other by cementation. Particles have their own mechanical properties such as Young’s modulus, Poisson’s ratio, yield point, and so on. Also, the strength of the bonds between the particles significantly varies depending on the degree of cementation. In rock mechanics applications, the structural stability and deformations are analyzed while rocks are treated as continuum in many situations. Such continuum approaches require the estimation of bulk rock properties. Recently, the discrete element method has widely been applied in rock mechanics problems because of advancements in computer technology and availability of 3D pore structure characterization. Li [1] and Li et al. [2] presented a DEM model to simulate the constitutive behavior of the sandstone by developing a contact law that considers non-linear elastic and plastic deformations. They suggested that it is necessary to link the upscaled modeling to the grain and pore-scale modeling using the micro-scale model to derive the relation between the strain and the rock mechanical and petrophysical parameters. In this study, we present three empirical correlations for grain-scale Young’s modulus, internal friction angles, and maximum compressive stresses as a function of confining pressures. The proposed correlations allow a DEM model to compute triaxial stress-strain curves under a range of confining pressures.

Method. In this study, a numerical rock model is developed for a high porosity sandstone core using a fully parallelized DEM code, DEMPack [3]. In the DEM simulation, the model consists of a number of spheres (i.e., elements) to construct a sandstone core sample numerically. It should be noted that the size of the element is much larger than actual grain sizes in the model. Therefore, a single element
represents a number of strongly bonded grains. In the DEM model, there are a number of parameters such as inter-element Young’s modulus, Poisson’s ratio, shear and tensile strengths, etc. Stress-strain data of five triaxial tests were used for calibrating the numerical rock model. In the later section, we discuss which parameters should be changed to match the laboratory triaxial stress-strain data.

Theory

Laws of motion. In this study, a commercial software, DEMPack is used to develop a DEM model simulating triaxial stress-strain curves. The model consists of a collection of rigid spherical elements having various diameters. The translational and rotational motion of particles is described by Newton’s law of motion. For the \( i \)-th particle with mass \( m_i \) and the moment of inertia \( I_i \) we have

\[
m_i \frac{d^2 u_i}{dt^2} = F_i^{ext} + \sum_{j=1}^{n_i^c} F_{ij}.
\]

\[
I_i \frac{d^2 \theta_i}{dt^2} = T_i^{ext} + \sum_{j=1}^{n_i^c} r_{ij} \times F_{ij}.
\]

where \( u_i \) is the particle centroid displacement, \( \theta_i \) – the particle rotational displacement, \( F_{ij} \) – the contact force to act between \( i \)-th and \( j \)-th particles, \( n_i^c \) – the number of particles being in contact with the \( i \)-th particle, \( r_{ij} \) – the vector connecting the centroid of the \( i \)-th particle with the contact point \( c \) at the interface between particles \( i \) and \( j \). All forces and moment applied to the \( i \)-th particle due to external loads, \( F_i^{ext} \) and \( T_i^{ext} \), respectively. The displacement and rotational displacement of the next time step can be found by integrating Eqs. 1 and 2 with time.

Contact model. In the two-dimensional case, the contact theory between spherical elements is given by a dynamic model as shown the Fig. 1 [3]. The contact force can be divided into two components in the normal and tangential directions, respectively. The normal force is represented by a parallel system of a linear spring and a dashpot while the tangential force is represented by a series of a linear spring and a slider. This contact model shows a connection between elements where tensile and shear failures of the springs are caused by tensile and shear forces, respectively. The sum of the two forces is expressed by the following equation:

\[
F_{ij} = \left(K_n u_n + C_n \frac{du_n}{dt} \right) n_{ij} + K_s u_s.
\]

Figure 1  Contact model used in DEMPack

Figure 2  Elastoplastic stress-strain curve defined in DEMPack

Elasto-plastic model. The compressive stress-strain behavior at the contact surface follows the elasticity law under low stress conditions while it follows nonlinear stress-strain relations under high
stress conditions. In the DEM model, once the compressive stress reaches the yield point at the contact surface, stress-strain behavior in the normal direction becomes nonlinear and is computed by changing Young’s moduli where a piecewise linear function is used as shown in Fig. 2. LCS1, LCS2, and LCS3 defines normal stresses where the Young’s modulus changes after yielding where $E_1$, $E_2$ and $E_3$ represents the changing Young’s moduli. In the DEM model, $E_1$, $E_2$, and $E_3$ are defined by the following equation.

$$E_i = \frac{E}{YRC_i_i},$$

where YRC$_i$ is a reduction parameter for the initial Young’s modulus, $E$.

**Example DEM Simulation for Triaxial Compression Test**

**Creation of numerical rock model.** Table 1 shows basic rock properties and mechanical parameters of the sandstone core. UCS is a value of the maximum compressive stress measured in a uniaxial compression test. Based on this data, we developed a DEM model consisting of 19,000 spherical elements. The created DEM model consists of spherical elements with the mean diameter of 0.565 mm providing a porosity of 0.28. It should be noted that the model porosity is slightly higher than one measured from the actual core sample. This indicates that the size of the element used in the DEM model is approximately 12.6 times larger than the actual grain size.

<table>
<thead>
<tr>
<th>Density (g/cm$^3$)</th>
<th>Porosity</th>
<th>UCS (MPa)</th>
<th>$\nu$</th>
<th>Mean diameter of grains (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.24</td>
<td>0.21</td>
<td>2.66</td>
<td>0.20</td>
<td>0.045</td>
</tr>
</tbody>
</table>

**Calibration Results of DEM Model.** Using the numerical rock model developed in the previous section, curve fittings for triaxial stress-strain data are performed by changing various DEM parameters. In Fig. 3 the dots show a stress-strain curve predicted by the DEM model for UCS test. The result shows that the model can predict the maximum compressive strength with a reasonable accuracy. However, the initial non-linearity of the rock is not captured by the DEM model.

![Figure 3 Comparison of stress-strain curves for DEM calculation and laboratory measurements for UCS test.](image)

Fig. 4 shows stress-strain curves predicted by the DEM model for low confining pressures of 3.45, 6.90, 10.34 MPa. Similar to the UCS test, the maximum compressive strengths can be predicted reasonably well. Also, Young’s moduli and Poisson’s ratios are matched very well for the three tests.
After the maximum compressive strength of the rock, the load supported by the rock decreases as the strain increases (i.e., brittle state). As shown in the figure, the brittle rock behavior is not properly simulated by the current model.

![Figure 4 Comparison of stress-strain curves for DEM calculation and laboratory measurements under low confining pressures.](image)

Fig. 5 shows stress-strain curves predicted by the DEM model at high confining pressures of 17.24 and 24.13 MPa. The results show that Young’s modulus, Poisson’s ratio and yield stress are predicted reasonably well by the DEM model. However, the triaxial test data at the highest confining pressure shows the axial stress continues to increase with strain after the yield point has been passed. This hardening behavior is not captured by the current DEM model.

![Figure 5 Comparison of stress-strain curves for DEM calculation and laboratory measurements under high confining pressures.](image)

Table 2 summarizes the constitutive parameters required for the calibration in the DEM model.

<table>
<thead>
<tr>
<th>Confining pressure (MPa)</th>
<th>Initial Young’s modulus (MPa)</th>
<th>LCS1 (MPa)</th>
<th>LCS2 (MPa)</th>
<th>LCS3 (MPa)</th>
<th>YRC1</th>
<th>YRC2</th>
<th>YRC3</th>
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<td>0</td>
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<td>1993</td>
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<td>11</td>
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<tr>
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<td>24.13</td>
<td>4998</td>
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<td>24</td>
<td>3</td>
<td>9</td>
<td>24</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Derivation of empirical correlations. In this numerical experiment, as shown in Table 2, initial Young’s modulus, internal friction coefficient and critical compressive stresses (LCS1, LCS2, LCS3) between elements are changed to match the laboratory test data. It was determined that these parameters may be functions of the confining pressure. Based on these observations, the following three correlations were obtained.

\[ E = 17E_0 \{1 - (e^{-0.09\sigma_c})\} + E_0. \]  
\[ \mu = 0.8908(e^{-0.044\sigma_c}). \]  
\[ LCS_i = 0.003\sigma_c^3 - 0.1484 \sigma_c^2 + 2.6187\sigma_c + 1.7063, \quad i = 1, 2, 3. \]

where \(E_0\) is the Young’s modulus of the rock measured at UCS test and \(\sigma_c\) is the confining pressure.

Discussion

In this study, in order to match laboratory triaxial stress-strain curves, Young’s modulus, internal friction angle, and critical compressive stresses are changed as functions of confining pressures. As discussed previously, elements used in the DEM model consists of a number of grains. Since the element is considered perfectly rigid, no deformation is allowed between grains within the element. Inter-element Young’s modulus, coefficient of internal friction, critical compressive stresses need to be adjusted. It should be noted that Eqs. 5, 6, and 7 are derived for a specific numerical rock model in this particular example. It is believed that these equations can be changed depending on the size of elements (or the number of elements) used in the DEM model. To improve our understanding of the relationship between the bulk rock properties and inter-element (inter-grain) mechanical properties defined in the DEM model, the size effect should be further investigated in future work.

Conclusions

Following conclusions can be drawn from the numerical work performed in this study:

- A numerical rock model developed by the discrete element method can simulate triaxial stress-strain curves of the sandstone core sample at confining pressures of 0 to 24 MPa.
- The DEM model can predict Young’s moduli, Poisson’s ratios, and yield stresses with reasonable accuracy.
- On the other hand, the initial nonlinearity of the rock, hardening and softening behaviors can’t be properly simulated by the current model. Further investigation and model calibration are required for improved prediction on complex stress-strain relations of highly nonlinear rock samples.
- Three empirical correlations for inter-element Young’s modulus, coefficient of internal friction, and critical compressive stresses are developed in this work. These correlations can be used as a curve fitting tool for triaxial stress-strain data in the DEM model.

Acknowledgement

The present work was performed as a part of activities of Research Institute of Sustainable Future Society, Waseda Research Institute for Science and Engineering, Waseda University. Also, the support of CIMNE for making the software code, DEMPack available for this study is gratefully acknowledged.

References

On the stress and constitutive relations determination from fault-slip data in the Earth's crust

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Keywords: principal stress axes, strain rate, constitutive equations, equilibrium conditions.

Abstract. According to widespread concepts of tectonophysics, the problem of retrieving the stress tensor $T$ from the fault-slip data is routine and solved locally for a block of the Earth's crust without taking into account its interaction with adjacent blocks. This contradicts mechanics where stress from kinematics in micro- and/or meso-scale can be determined locally only for an isolated volume. If the block is an open system, exchanging momentum and angular momentum with the environment, then we can talk about the construction of constitutive relations, rather than about the stress determination. The stress is determined by integrating the equilibrium equations. On top of that, in tectonophysics stress is thought to be the only cause of the observed kinematics. The paper presented discusses these and other fundamental misconceptions of the tectonophysical approach to stress reconstruction.

Introduction

The study of the stress fields in the Earth's crust becomes increasingly important. It is widely accepted that one of the suitable sources of information about stresses is the data on $N$ slips on pre-existing faults occurring during the observation time $0<t<\Delta t$. These data can be either geological data on discontinuous displacements along joint planes in rocks, or seismological data about earthquake mechanisms. According to the tectonophysical method of local kinematic reconstruction (MLKR) [1] the data on the orientation tensors $O_{(J)}=n_{(J)}\otimes h_{(J)}$ (in geology) and $O_{(J)}=(n_{(J)}\otimes h_{(J)}+h_{(J)}\otimes n_{(J)})/2$ (in seismology), $J=1,\ldots,N$ ($N\geq4$) are per se sufficient for local retrieving a reduced stress tensor $T_R$ in a crustal block (macrovolume $x$). Here the mutually orthogonal unit vectors $n_{(J)}$ and $h_{(J)}$ are the normal to the plane of the $J^{th}$ discontinuity and the direction of movement along it respectively. Tensor $T_R$ is defined by 4 parameters: 3 angles $\phi_1, \phi_2, \phi_3$, and the stress ratio $R=(T_2-T_3)/(T_1-T_3)$. The angles specify the orientation of the trihedron composed by mutually orthogonal unit vectors $m_1, m_2, m_3$, which correspond to axes of the principal stresses $T_1, T_2, T_3$ ($T_1\geq T_2\geq T_3$). The tensor $T_R$ can be written as $T_R=(1-R)m_1\otimes m_1-Rm_3\otimes m_3$. In MLKR, stresses are considered as the only cause of deformations, and the reconstruction procedure itself is performed locally without taking into account the interaction of $x$ with the surrounding blocks and ignoring deformation for $t<0$. The purpose of this article is to demonstrate the false goals of MLKR and the possibility of a correct approach to solving the problem.

Determination of macroscopic stresses and constitutive relations in physics and MDS

Let us consider how the stresses are determined and constitutive equations are constructed in physics and mechanics of a deformable solid (MDS) by the kinematics at micro- and meso- scales. Fig. 1 illustrates some methods for a volume $\Delta V$ which is accepted as the representative material element $x$ at macroscale. In Fig. 1 $a, b$ the macrovolume $x$ is isolated, $x=x_{\text{closed}}$, it does not exchange linear and angular momentum with the environment. In Fig. 1 $c$ $x$ is not isolated, $x=x_{\text{open}}$. 

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Let us suppose that \( x^{\text{closed}} \) has rigid walls and contains \( N \) non-interacting microparticles of mass \( m \) moving with velocities \( v \) (Fig. 1 a). As known from the theory of ideal gases, when conservation of momentum is taken into account, the macroscopic pressure \( P \) in \( x^{\text{closed}} \) is expressed as \( P = n m < v \cdot v > / 3 \), where \( n = N / \Delta V \) is the particle concentration, \( < > \) mean volume averaging. The method of determining \( P \) at \( \Delta V = \text{const} \) excludes the simultaneous determination of the constitutive equation. The latter for a constant gas mass in isothermal conditions (in this case, it is the Boyle-Mariott law \( P \Delta V = \text{const} \) at \( \Delta V \neq \text{const} \)) is established in fundamentally independent way.

For an ensemble of interacting particles performing finite motions in \( x^{\text{closed}} \) (Fig. 1 b), the stress tensor \( T \) is usually determined from the Clausius virial theorem. The potential part of tensor \( T \) depends on the dyads \( (r_j - r_i) \otimes f_{ij} / 2 \Delta V \), which are summed over all pairs of particles with radius vectors \( r_i \) and \( r_j \) in the actual configuration, \( f_{ij} = -f_{ji} \) is the force with which the particle \( j \) affects the particle \( i \). Internal forces \( f_{ij} \) are long-range (for separated particles) or contact (e.g., for a granular medium) forces. External forces on the faces of volume \( \Delta V \) are absent. If this volume is not isolated, then the particles are additionally acted upon by external forces that affect \( T \) which is now not determined by local kinematics.

According to the physical plasticity, edge dislocations become activated over a certain crystallographic plane if the resolved shear stress along the direction \( \mathbf{h} \) of possible slip exceeds the critical threshold \( \tau_c \) (Fig. 1 c). If the stress in \( x \) is assumed to be homogeneous (the Reiss hypothesis), the macrostrain is determined by averaging. On the contrary, if a homogeneous deformation is assumed (Voigt's hypothesis), a macro-stress is obtained by averaging. It is important that the above procedures are aimed at constructing the constitutive relationships of the material at the macrolevel rather than at reconstruction of stresses. Stresses are subsequently determined by integrating the equilibrium equations.

On the basis of the above examples and results obtained in [2], we can formulate some general rules for correct local reconstruction of macroscopic stresses or for determination of macroscopic constitutive relations from information on the local kinematics at micro- and/or meso-level:

- data on local kinematics allow us to determine either the stresses or the constitutive relations, but not both at the same time by the same techniques;
- macrostresses are locally retrievable in \( x^{\text{closed}} \) by using laws of dynamics at micro- and/or meso-levels;
- external forces are not uniquely determined by local kinematics, therefore for \( x^{\text{open}} \) the constitutive relations are determinable rather than stresses; the latter are to be retrieved by subsequent integration of the equilibrium equations;
- in general, strain rate \( \dot{E} \) in \( x^{\text{open}} \) is determined not only by stresses; in particular, if \( x^{\text{open}} \) is capable of changing the elastic energy, \( \dot{E} \) necessarily depends on the rate of the stress change.
Tectonophysical approach to the problem of stress reconstruction and its falsification

What does tectonophysics offer for solving the problem of the stress reconstruction? The laws of conservation of momentum and angular momentum, expressed in quasistatic equations of equilibrium, are absolutely ignored. At the same time, it is asserted that the stresses "can only be estimated through deformations caused by them" or that "orientation of the stress axes is always established on the basis of deformation analysis" [3]. Such a tectonophysical approach has been implemented in MLKR for 40 years (e.g., [1]).

The concept of tectonophysics creates an illusion that stresses can be estimated from local observations of deformation only. Deformations mean increments $\Delta E$ of strain or strain rates $\dot{E}$ that were observed for a period of time $\Delta t$ following the moment of action of the sought for tensor $T=T_0$ or simultaneously with it.

The key elements of conceptual deficiency of tectonophysics are:
- the sought for stresses are thought to be the only cause of the observed kinematic phenomena;
- as a result, the chronological relations "simultaneously" or "subsequently" (between $T_0$, on the one hand, and $\dot{E}$ or $\Delta E$, on the other) are illegally replaced by the cause-effect relation "due";
- the postulated existence of a unique relationship between $T_0$ and $\Delta E$ (or $\dot{E}$ ) is considered sufficient to locally determine the principal stress axes from $\Delta E$ (or $\dot{E}$) without using the equilibrium conditions;
- the ratio of the observation time $\Delta t$ to the stress relaxation time is not taken into account, while this ratio can be of fundamental importance in the reconstruction of stresses in the Earth’s interior.

The concept of tectonophysics is refuted by both theoretical analysis and thought experiments [2], one of which is presented in Fig. 2. The requisites for the experiment are two identical weightless rods I and II, two identical loads $p$ and a horizontal crossbar $EF$, to which the rods are suspended. The material of the rods has the property of aftereffect (i.e., delay of deformation in comparison with the change in the load) and is described, in particular, by the viscoelastic Kelvin-Voigt model.

![Fig. 2. Schematics of a thought experiment for the stress retrieving in rods I and II. Left – the preparatory phase; centre – the initial stage; right – the observed deformations of the rods (1 - elongation, 2 - shortening).](image)

For $t<0$, rod I is free from the load, and both loads act on rod II. At instant $t=0$ one of the loads is removed from rod II and applied to rod I. As a result, for $t>0$, under the action of the same load, rod I is elongated, and unloaded rod II is shortened. Thanks to the aftereffect, deformation processes are visually detected during the time interval $\Delta t$. Suppose that an experiment is observed by a tectonophysicist and a physicist who are asked to reconstruct the (uniaxial) stresses in the rods during time $0<t<\Delta t$ by the methods of their craft. The loads are hidden from the observers by table $ABCD$ (Fig. 2).
The tectonophysicist, seeing the elongation of the rod I and the shortening of the rod II, will come to a wrong conclusion that the axis of maximum extension is vertical in rod I, and the axis of maximum compression is vertical in rod II. Opposite to that, the physicist will use the conservation laws. To this end, he will disconnect the rods from the EF crossbar and weigh them. He will conclude that the weight of both rods is \( p \), and, consequently, an identical tension is realized along them. He will then investigate the dependence of the rates of elongation (in rod I) and shortening (in rod II) on time. This determines the delay function both under loading and unloading.

Thus, opposite deformations are realized in the equally loaded rods, which contradicts the concept of tectonophysics. Locally, the deformations do not determine neither the magnitude of the stresses, nor the orientation of the principal axes. Contrary, at the stage of stress reconstruction by the physicist, the most important is utilizing the equilibrium equations and boundary conditions. At this first stage, the problem reduces to a statically determinate one, while the deformation features are ignored. They are used in the second stage, when the physicist, on top of the stresses, retrieves additional information about the rheology of the material at nonzero stresses. Other experiments have been considered in [2], showing that it is always possible to select such individual properties of the medium and features of the dynamic process that are poorly known to the Earth's interior, so that in the macrovolume \( \mathbf{x} \) the principal axes of the tensor \( \Delta \mathbf{E} \), on the one hand, and the principal stress axes on the other, were disoriented by any predefined way. Especially, in the processes accompanied by the release of elastic energy \( U \), it is possible to conclude that the maximum compression is directed along the maximum elongation rate, and the maximum tension is oriented along the maximum rate of shortening, as in Fig. 2. In these processes, an inherited stress regime can be realized, which is meant independence of the axes of principal stresses \( T_1, T_2, T_3 \) from the increment of deformation \( \Delta \mathbf{E} \).

**Peculiarities of the tectonophysical approach in MLKR**

In MLKR, the tectonophysical approach is complicated by the fact that the initial data are usually not sufficient to determine total strain tensor \( \Delta \mathbf{E} \) or, which is the same, tensor \( \mathbf{\dot{E}} \). This is partly because the data on the amplitudes of motions and the areas of discontinuity are not used in the analysis. The reconstruction of tensor \( \mathbf{T}_R \) in MLKR is schematically shown in Fig. 3.

![Fig. 3. Determination of macroscopic reduced tensor \( \mathbf{T}_R \) in the framework of MLKR from \( N \) directions \( \mathbf{h}_{(j)} \) of slips along planes with unit normals \( \mathbf{n}_{(j)} \) (a). \( \mathbf{T}_R \) is represented by the trihedron of the indexed axes and the parameter \( R (1 \geq R \geq 0) \) (b). Tensor \( \mathbf{T}_R \) generates unit vectors \( \mathbf{p}_{(j)} \) on planes \( \mathbf{n}_{(j)} \) (c), which are treated in MLKR as directions of the "shear stress". Based on some subjective assumptions, a relationship is constructed between vectors \( \mathbf{h}_{(j)} \) and \( \mathbf{p}_{(j)} \), which leads to the "constitutive relation" (Eq. 1), from which reduced tensor \( \mathbf{T}_R \) is calculated.](image-url)
The central point of MLKR is the postulation of a certain connection between directions $h_{(j)}$ of motions and directions $p_{(j)}$ of the "shear stress" on the set of rupture planes with normals $n_{(j)}$, $j=1,...,N$. Vectors $p_{(j)}$ and consequently tensor $T_R$ generating these vectors are determined on the basis of various speculative hypotheses. In fact, this procedure is a certain algorithmic assignment of some "constitutive relations", expressed by the tensor functional

$$T_R=O_R(O_{(1)},...,O_{(N)}),$$

which is defined on the set of tensors $O_{(j)}$, $j=1,...,N$, and has the range of values $\phi_1$, $\phi_2$, $\phi_3$, and $R$. Simultaneously (Eq. 1) serves as a source of information about $T_R$. The methods of introducing and reconstructing the stresses in MLKR and MDS contradict each other in all principal points.

- In MLKR, the tensor $T_R$ is reconstructed as if the macrovolume $x$ were isolated, $x=x_{closed}$. In fact, $x=x_{open}$, because when $T_R\neq0$ the tractions (ignored in MLKR) act at the $x$ surface. Therefore, (Eq. 1) should be treated as a "constitutive relation", and not as a source of information about stresses.
- The assumption $x=x_{closed}$ leads to the violation of equilibrium equations in the quasistatic system of interacting macrovolumes. However, only these conditions guarantee the existence of stresses as a symmetric tensor of the second rank. Therefore, $T_R$ obtained in MLKR is not related to stresses.
- (Eq. 1) connects a macroscopic object ($T_R$) with the mesolevel values, which does not satisfy the principle of macroscopic definability of MDS. Besides, (Eq. 1) while different from one author to another, is considered in MLKR universal, applicable to a massif of rocks of any nature. Opposite to it in MDS the constitutive relationships characterize only a specific material.
- Simultaneous reconstruction of "stresses" and formulation of "constitutive relations" means combining the laws of dynamics and properties of a particular medium into a single feature, which is a return to the pre-Cauchy views.

Summary

From the above, it is clear that in MLKR a reconstructed object is different from what in classical physics and MDS is understood as stress. It is possible to outline those types of deformation processes for which the results of MLKR can be radically different from the true axes of tectonic stresses. The above goals and representations of the tectonophysical approach are incorrect, since neither the physical meaning of the stress tensor, nor the way in which it is introduced has any relation to deformations. The tectonophysical problem of local determination of the stress axes from the strain rate data is analogous to the archaic problem of local determination of the direction of force from data on the instantaneous velocity of a body. The possibility of reconstructing any pre-assigned orientation of the principal stress axes indicates that the results obtained within the framework of tectonophysics and MLKR are subjective, depending on the accepted system of assumptions. An increase in the reliability of the reconstruction of tectonic stresses by kinematic data and, in particular, by discontinuous shear displacements is seen in the rejection of the tectonophysical concept of locality and in the return to the notions of classical physics, namely, the use of the conservation laws. When using the equilibrium equations (at least for an ideally-dissipative medium), it is possible to separately determine the stresses and constitutive relationships just as the physicist did in the experiment depicted in Fig. 2. Such an approach using the example of an ideal plastic material was first proposed in [4], where the stresses were reconstructed on the basis of a statically determinate problem, and the plastic potential was determined using kinematic data.
References


Modelling Fault Instability Caused by Asymmetric Friction

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Abstract. In this study sliding over a geological fault with a section of anisotropic rocks that create asymmetric friction is considered. We analyse the simplest model of such a sliding – longitudinal oscillations of two blocks connected by a spring: one block with asymmetric friction and the other block with symmetric friction. These blocks are set in a constrained environment to which harmonic vibrations are applied. Results show that the presence of asymmetric friction can cause fault instability (sliding) at amplitudes of external excitation lower that in the case of standard symmetric friction.

Introduction

Unidirectional asymmetric friction created in a constraint environment by material anisotropy with a principal axis inclined to the line of contact has been discovered by Bafekrpour and colleagues [1]. This effect is created when a block with inclined principal axis of anisotropy is placed in the constraint environment (in the direction normal to sliding) such that the normal stress becomes different when the block moves in different directions. One method of creating the effect is through the use of internal architecture – namely inclined ribs, which has been conducted in the previous study. This resulted in coupling shear stress with normal stress; increasing normal stress when shear stress is applied in the direction opposite to the inclination of the ribs, and decreasing normal stress when the shear stress is applied in the opposite direction, i.e. aligned with the projection of the ribs on the contact surface. This inclined rib structure can occur in geological faults, which can be an issue as one characteristic of asymmetric friction is the ability to produce unidirectional locomotion in a vibrating environment, thus, potentially producing catastrophic fault instability during earthquakes.

In the first section of the paper, an analytical model is presented simulating faults that involve an asymmetric friction part connected to a part with symmetric friction. This assembly will be set in a constrained environment to which vibrations are applied. The second section of the paper presents the numerical solution of the model. Finally, the third section discusses the frequency response of the system.

Dynamic Model

Mathematical formulation

Consider a fault that consists of a section with asymmetric friction and another section that has symmetric friction. These two sections are connected together by a spring, slide in a constrained environment and subjected to longitudinal vibrations. Fig. 1 (a) shows a fault with a part exhibiting asymmetric friction. Fig. 1 (b) shows the assembly that models such a fault.
Fig. 1 Model of a fault with an asymmetric friction part: (a) Schematics of the fault. The rock is layered (anisotropic) with the layering inclined to the contacts forms the left-hand section. The right-hand section contains rock that display conventional symmetric friction; (b) A two-mass model of the fault; M₁ is the mass with asymmetric friction whereby friction force \( f_1 \) depends upon the sign of the sliding velocity; M₂ is the mass with symmetric friction, friction force \( f_2 \) is constant. The masses are connected by a spring of stiffness, \( k \) and are vibrated in horizontal direction with displacement \( u_g \). Displacements of sliding of masses M₁ and M₂ are defined as \( v_1 \) and \( v_2 \) respectively.

For the sake of simplicity it is assumed that friction force \( f_1 \) is zero for positive sliding and infinitely large otherwise.

To formulate the equations of motion for the system the event-driven scheme [2,3] will be used. Events involving static and dynamic frictions are considered for both M₁ and M₂. Since M₁ involves asymmetric friction, events involving the forward and backwards motion are also considered.

First assume both masses are frictionless. Using Lagrangian mechanics, the resulting equations of motion for mass M₁ and M₂ are provided as follows:

\[
M_1 \ddot{v}_1 = -M_1 \ddot{u}_g + k(v_2 - v_1),
\]
\[
M_2 \ddot{v}_2 = -M_2 \ddot{u}_g + k(v_1 - v_2).
\]  

### Asymmetric friction

Consider mass M₁. This mass will experience no friction force when the system has net force or velocity in the positive direction, but will experience infinite friction when having net force or velocity in the negative direction. In essence, the mass will have the net force as governed by the Lagrangian equation when it experiences force or is moving in the positive (easy) direction. However, static and kinetic friction in the opposite (hard) direction is infinitely high such that overcoming friction in this direction is impossible, implying that no acceleration will occur in this direction. From this we see that velocity of mass M₁ in the negative direction is zero since the motion is prohibited. Table 1 encapsulates the constraints faced by M₁ due to asymmetric friction.
### Table 1 Summary of the constraints faced by M₁ due to asymmetric friction

<table>
<thead>
<tr>
<th>(\dot{v}_1)</th>
<th>(\geq 0)</th>
<th>(&lt; 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{v}_1)</td>
<td>(M_1\ddot{v}_1 = -M_1\ddot{u}_g + k(v_2 - v_1))</td>
<td>(M_1\ddot{v}_1 = -M_1\ddot{u}_g + k(v_2 - v_1))</td>
</tr>
<tr>
<td>(= 0)</td>
<td>(M_1\ddot{v}_1 = -M_1\ddot{u}_g + k(v_2 - v_1))</td>
<td>(M_1\ddot{v}_1 = 0)</td>
</tr>
<tr>
<td>(&lt; 0) ((\text{cannot occur}))</td>
<td>(M_1\ddot{v}_1 = 0)</td>
<td>(M_1\ddot{v}_1 = 0)</td>
</tr>
</tbody>
</table>

The governing equation of motion for M₁ can be rewritten with the use of the Heaviside step-function, \(\theta(x)\), which equals one for \(x\) larger than or equal to zero and equals zero for \(x\) less than zero. With constraint that velocity is non-negative, the resulting equation for M₁ is shown below:

\[
M_1\ddot{v}_1 = \begin{cases} 
1 - \theta(-\dot{v}_1)\theta\left(M_1\ddot{u}_g - k(v_2 - v_1)\right) & \left(-M_1\ddot{u}_g + k(v_2 - v_1)\right) \\
|M| - f_2 & \text{sgn}(\dot{v}_2) = \text{sgn}(F) \\
-[|F| + f_2] & \text{sgn}(\dot{v}_2) = -\text{sgn}(F) \\
(|F| - f_2)\theta(|F| - f_2) & \text{sgn}(\dot{v}_2) = 0 
\end{cases}
\]

(3)

Numerical analysis reproduces this behavior (during each time step, velocity was taken as \(\dot{v}_1 = \dot{v}_1\theta(\dot{v}_1)\)).

### Symmetric friction

Now we consider mass M₂. This mass experiences equal friction in both positive and negative directions. For the sake of simplicity, we assume the static and kinetic friction are equal in magnitude and let this magnitude be \(f_2\). Eq. 2 was derived for the case when both masses are frictionless. The forces applied to mass M₂ require an additional friction term. However, it is not as simple as taking the direct difference between the frictionless formulation and formulation accounting for the presence of friction force \(f_2\). In the case of friction, it is possible for forces to be directed against velocity of M₂. Since kinetic friction is always opposing the direction of velocity, the net force on M₂ will be an addition to the frictionless forces and the friction force. Thus, we can define function \(F_m(F, \dot{v}_2, f_2)\), which takes into account all possibilities:

\[
F_m(F, \dot{v}_2, f_2) = \begin{cases} 
(|F| - f_2)\theta(|F| - f_2)\text{sgn}(\dot{v}_2) & \text{sgn}(\dot{v}_2) = \text{sgn}(F) \\
-[|F| + f_2] & \text{sgn}(\dot{v}_2) = -\text{sgn}(F) \\
(|F| - f_2)\theta(|F| - f_2)\text{sgn}(F) & \text{sgn}(\dot{v}_2) = 0 
\end{cases}
\]

(4)

where \(F\) is the force applied to M₂ when M₂ is frictionless (Eq. 2), \(f_2\) is the friction force applied to M₂ and \(\dot{v}_2\) is the velocity of M₂. Using function \(F_m\), the force applied to M₂ is as follows:

\[
M_2\ddot{v}_2 = F_m\left([-M_2\ddot{u}_g + k(v_1 - v_2)], \dot{v}_2, f_2\right). 
\]

(5)

Finally, we assume that the external excitation is harmonic. It reads

\[
\ddot{u}_g = G\sin(\omega t). 
\]

(6)
Comparison of sliding of the masses with symmetric and asymmetric friction, and the masses both with symmetric friction

We use the following normalised variables: $\mu = M_2/M_1$, $\beta = \omega/\Omega$ and $\alpha = (M_1 + M_2)G/f_2$, where $\omega$ is the driving frequency of the constrained environment and $\Omega^2 = k/(M_1 + M_2)$. Fig. 2 (a) shows the case when mass $M_1$ (with asymmetric friction) is connected to mass $M_2$ (with symmetric friction), with $M_1 + M_2 = 1$, $k = 1$, $f_2 = 1$, $\mu = 1$, $\beta = 1$, $\alpha = 1$, and zero initial conditions for displacements and velocities of both blocks. Fig. 2 (b), shows the case when both $M_1$ and $M_2$ are subjected to symmetric friction, with the same parameters as in Fig. 2 (a), except for $f_1 = f_2 = 0.5$ and $\alpha = (M_1 + M_2)G/(f_1 + f_2) = 1$.

Fig. 2 Displacement of masses $M_1$ and $M_2$; $M_1 + M_2 = 1$, $k = 1$, $\mu = 1$, $\beta = 1$, $\alpha = 1$: (a) mass $M_1$ moves with asymmetric friction, $f_2 = 1$; (b) Conventional symmetric friction, $f_1 = f_2 = 0.5$.

Fig. 2 (a) shows that in the presence of asymmetric friction blocks $M_1$ and $M_2$ can move, albeit the movement is restricted while in the absence of asymmetric friction, Fig. 2 (b) the blocks do not move at all. This result is physically plausible, since $\alpha = 1$ is the transition point at which external vibration overcomes total frictional force. Further parametric tests show that the situation when $M_1$ having asymmetric friction can initiate displacement of $M_2$ can be observed even if $\alpha$ is as low as 0.73. One reason for this is that friction in $M_1$ in the easy direction is lower than that on $M_2$ (in fact it is zero in our simulations), allowing $M_1$ to start displacement in the easy direction, causing compression of the spring between $M_1$ and $M_2$. Thus, to keep $M_2$ stationary the friction have to resist both the force of external excitation and the force in the compressed spring. Due to the additional force in the spring, the sliding can occur at lower magnitudes of vibration. This supports the earlier established observation that both masses can start moving for the lower values of $\alpha$ when mass $M_1$ is under asymmetric friction.

We note that the displacements in the case when $M_1$ has asymmetric friction are transient. The displacements are found to be transient when $\alpha < 1$ as well as when $\beta \geq 1/\sqrt{2}$ for $\alpha = 1$. In these scenarios, the amount of displacement of $M_1$ relative to $M_2$ is not sufficient to produce compression of the spring needed for further displacement of $M_2$. When $\alpha > 1$ the masses, in the case when $M_1$ has asymmetric friction properties, will steadily move in a cyclic manner in the easy direction. On the other hand, for the case when both masses $M_1$ and $M_2$ have symmetrical friction, displacement of the masses in both positive and negative directions during the vibration is physically plausible.

Parametric analysis of frequency for the case of masses under symmetric and asymmetric friction conditions

Now explore the (driving) frequency dependence of the system. As a characteristic frequency we use eigenfrequency of the system when block $M_1$ is stationary. This is reduced to an eigenfrequency of a
simple pendulum: $\omega_n^2 = k/M_2$. For the parameters used in the previous section ($M_1 = M_2 = M$), $\beta$ can be found as $\beta = \omega_n / \Omega = \sqrt{2}$. A sequence of plots for varied $\beta$ as multiples of $\sqrt{2}$ for different values of $\alpha$ were produced for the model.

Block 1 always moves in the easy direction when subjected to any frequencies for any non-zero magnitude of vibrations, which is physically plausible with the block’s frictionless property in the easy direction. However, block 2 only moves for certain band of frequencies of the confining environment. These band of frequencies are plotted in Fig. 3 as a function of normalised driving amplitude $\alpha$.

For $\alpha < 0.73$ block 2 is stationary for any values of $\beta$. When $\alpha \geq 0.73$ and within the frequency band for $\beta$ shown in Fig. 3, block 2 has positive displacement that trails the displacement of block 1, restoring the difference in separation between the two blocks when compared to the original separation between them. Frequency band expands at a higher rate when $\alpha > 1.01$. The profile of displacements for both blocks in this band include both transient (finite) and steady increasing displacements in the easy direction. A similar parametric test has been done for the case with both masses are with symmetric friction. In this case, the system’s motion is only dependant on $\alpha$; the masses are stationary when $\alpha \leq 1$ and in motion when $\alpha > 1$ at any frequency of excitation $\beta$.

**Conclusion**

The presence of asymmetric friction in a fault creates instability (fault sliding) at lower magnitudes of external excitation. At lower magnitudes of vibrations ($\alpha \leq 1.01$), the faults which are only under symmetric friction will not slide. However faults with a section with asymmetric friction may slide if the frequency of vibration of the environment lies within a certain frequency band that depends on the magnitude of vibration. At higher magnitudes of vibrations ($\alpha > 1.01$), the frequency band at which motion occurs in faults with sections of symmetric and asymmetric friction expands. In this frequency band, the faults with sections of symmetric and asymmetric friction will have piece-wise increasing sliding in easy direction. On the other hand, faults with only symmetric friction can have displacement in both forward and backward directions during the vibrations at any frequency only when the magnitude of vibrations is high ($\alpha > 1$).

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References


V WAVE PROPAGATION
Transient dissipative solitary waves during oedometric compaction of a highly porous carbonate

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Abstract. We present a dissipative solitary wave phenomenon for an oedometric compaction experiment of a highly porous (45% porosity) carbonate rock with pore collapse. We show that the complexity of the experimental system and the emergence of localized structures, their mechanisms of propagation, their interaction and their uncertainty can be captured by reaction-diffusion equations, which predict migrating and standing solitary waves for a given set of boundary conditions. Although the dissipative solitary waves appear as continuous waves in the reaction-diffusion partial differential equations they also show particle-like behaviour much like the quantum wave-particle analogue. Transient waves groups start at the undrained bottom boundary of the oedometric experiment and propagate with a constant group velocity of the order of 10 mm/s towards the top drained boundary. The waves interact with each other and lead to the formation of stable localized structures. In our experiment the observed solitary waves are dissipative P-waves, which in the long-time scale limit form stationary (standing) solitary waves better known as compaction bands. This observation offers a possible closure to the classical geomechanics and provides a new approach for describing the dynamic properties (damage mechanics) and failure of matter.

Introduction

In this paper we present the hypothesis that the “wave mechanics” approach of quantum statistical mechanics holds the key for understanding the universality of physical phenomena that can be observed over many orders of scale [Sethna et al., 2001]. We propose a new mathematical approach, which we present here for the first time for use of analysis of failure and fluid flow instabilities of materials. The approach gives physics-based insights into the processes that are commonly described by damage mechanics.

We can upscale quantum processes and may consider quantization of energy levels as a key element of Multiphysics processes. Evidence for quantization of energy 12 orders of magnitude larger than Planck constant $\hbar$ has already been found in plasmas [Livadiotis and McComas, 2013]. These mega-quant particles give rise to wave phenomena in solar flares where plasmons exhibit a wave phenomenon equivalent to photons. However, the energy level is not defined by Planck quant but by the diffusional Debye length. In analogy, here we propose a quantization by the hydro-mechanical diffusional length scale of the porous limestone $L_D = \sqrt{2D_H t_c}$ where $t_c$ is the characteristic time scale of the microprocess of pore collapse and $D_H$ is the Terzaghi consolidation coefficient describing the diffusion of the fluid pressure away from the collapsing pore.
The reaction-diffusion equation of the problem

We propose an extended Terzaghi equation considering a cross diffusion of solid and fluid processes:

\[ \begin{align*}
\frac{\partial p_f}{\partial t} &= D_H \frac{\partial^2 p_f}{\partial x^2} + f(p_f, p') \\
\frac{\partial p'}{\partial t} &= D_S \frac{\partial^2 p'}{\partial z^2} + g(p_f, p')
\end{align*} \tag{1} \]

Here the mean effective stress is \( p' = p - p_f \), where \( p_f \) is the fluid pressure in the pore space and \( p \) the bulk pressure. Function \( f(p_f, p') \) is the fluid pressure source term incorporating the effective stress equilibrium condition and any density changes in the filtrating fluid phase due to incorporation of crushed grains. Equivalently, \( g(p_f, p') \) is the effective pressure source term related to the bulk volumetric strain rate and effective modulus of consolidation. \( D_H \) is the fluid diffusivity of the pore fluid pressure, and \( D_S \) the diffusivity of the effective stress. There is no direct method to measure the above implied cross diffusion between solid and fluid flow processes, however, their interaction can lead to a number of characteristic patterns which be stationary or propagate as solitary waves [Vanag and Epstein, 2009]. The observation of these waves can be used to obtain a phenomenological understanding of the complexity of the processes based on a thermodynamical averaging of microprocesses over the waves.

The Physics of the Wave Phenomenon

The physics of the wave phenomenon relies on the formation of correlation clusters of matter defined by their diffusional length scales. Correlation clusters arise from the connections of kinetic and potential energy in long-range interactions. Long range electron oscillations have first been discovered in plasma and are known as plasmons; their cluster size is characterized by the Debye diffusion length in plasma which forms Debye spheres [Livadiotis and McComas, 2013]. In analogy to the generalisation of de Broglie’s electron waves to other matter waves we propose here that the correlation clusters can encompass a representative volume of pores and matrix material with an ensemble mass \( m^* \) encapsulated by the volume described by the characteristic diffusion length.

Material within these clusters form correlated motions and contribute to the macro-scale quantization such as observed in plasmon waves [Livadiotis and McComas, 2013]. We thus translate the simple mechanistic interpretation of single moving particles into the motions of an upscaled material ensemble with mass \( m^* \) defining quantized energy state \( \hbar^* \). We show that for this upscaled material ensemble wave phenomena are a natural solution. In order to show this, we formulate the problem using Schrödinger’s equation.

The corresponding generalised reaction-diffusion equation using the upscaled quant \( \hbar^* \) and upscaled mass \( m^* \) is:

\[ i \hbar^* \frac{\partial}{\partial t} \Psi(r, t) = -\frac{\hbar^*^2}{2m^*} \nabla^2 \Psi(r, t) + V(r) \Psi(r, t), \tag{2} \]

where \( i \) is the imaginary unit and \( \Psi(r, t) \) is the wave function of the position vector \( r \) and time \( t \). In the first successful application Schrödinger proposed a diffusion wave equation for the hydrogen atom by restricting the equation to a time invariant potential energy \( V(r) \) attractor in quasi-steady state (infinite time scale) and a time-dependent wave.
For generality the simplest solution to the Schrödinger equation is a linear combination of plane wave functions. Any wave can be recovered based on the fact that the linear partial differential equation (pde) supports superposition principle of the waves:

\[
\Psi(r, t) = \sum_{n=1}^{\infty} A_n e^{i(k_n r - \omega_n t)},
\]

where \(|k_n| = \frac{2\pi}{\lambda}\) is the magnitude of the wavevector, \(\lambda\) is the wavelength and \(\omega_n\) the angular frequency of the \(n^{th}\) wave. In our application we are also interested in standing waves that can emerge at long time scales and follow Schrödinger’s approach in assuming that for long time/length scales time and space can be uncoupled solutions:

\[
\Psi(r, t) = \sum \psi(r) e^{-i\frac{Et}{h^*}}.
\]

The stationary solutions (standing waves) correspond to the eigenvalues of the Hamiltonian energy \(\hat{H}\) operator characterising the total energy (i.e. the sum of the kinetic and potential energy) of \(\Psi\) of the system. Energy eigenstates are independent of time and correspond to the equation \(\hat{H} = E\). In the simple linear theory, eigenstates are separable and extended eigenstates can be expressed as a linear combination of basis functions.

We now come back to equation (2) where we can identify the first term on the right hand side as the usual diffusion term. The second term is a source term. Thus, the first diffusion term destroys the wave function \(\Psi\), while the second term supports the wave around the potential \(V(r)\). Their interplay is the same as that between kinetic and potential energies in quantum mechanics but in the present form it is the interaction between diffusion and a local energy source or sink. The result is known as the lowest energy eigenstate, or ground state wave function used in diffusion Monte-Carlo calculations [Skinner et al., 1985]. The ground state for a plane wave function in a one-dimensional potential energy well is for instance half a sinus wave.

**Reaction-diffusion equations in Fourier space**

We have postulated above that energy eigenstates exist for the macroscale pore pressure diffusion problem with pore collapse. We have proposed that macroscopic correlated clusters exist which are defined by the diffusion length. In order to show the correspondence to Schrödinger’s wave mechanics we need to prove that the clusters can perform correlated motions described by a ground state wave function and that they can be superposed linearly. In order to test this hypothesis, we need to verify whether the assumption of a linear pde holds, i.e. we need to find out whether the free energy can be decomposed into independent terms. This decomposition of energy eigenstates is known as translational invariance. Under translational invariance energy eigenstates must be retained if the wave moves through the medium. If we use plane waves to describe the energy eigenstates they must become the eigenfunction of the translation operator.

In order to test this invariance in the upscaled version, we plug in the plane waves into the reaction-diffusion equations (1 and 2). This transformation does not change the equation as the Fourier space decomposes a function \(f\) into a family of plane wave solutions with the property that if \(f(z, t)\) is a solution then another solution is obtained from \(f(z - \delta, t)\) where \(\delta\) is any translation into the positive \(z\) direction. In Fourier space the function \(f(z, t)\) is transformed from space and time into a function of the wavenumber \(\hat{f}(k) = f(z, t)e^{-ikz}\) where \(k = \frac{2\pi}{\lambda}\) is the wavenumber and \(\lambda\) the wavelength. We will
illustrate the transformation in the example of Terzaghi’s consolidation of a cohesive material with a dissipative pressure source term from the collapse of pore space.

Superposition of diffusion of microprocesses

We can now come back to equation (3) where we assume long time-scales and separate the wave function into a time independent and a time dependent phase factors. If we assume that the mechanism of pore collapse and fluid expulsion is based on a series of microprocesses with multiple energy ground states we can obtain a solution by exploiting the fact that the Schrödinger equation is a linear pde and superpose equations:

\[ \Psi(r, t) = \sum_n A_n \psi_{E_n}(r) e^{-iE_n t/\hbar^*}, \]

where \( A_n \) is the real wave amplitude of the \( n^{th} \) – microprocess of pore collapse. The superposition principle can also be exploited to provide a link to the classical mechanics by extending it into a non-linear pde [Shi and Hearst, 1994]. This consideration closes the proposition of the new theoretical approach for simplifying the complexity of the experimental system. This model allows us to develop a new perspective on the dynamics and longevity of localized structures. It also provides an avenue for a data analysis method that enables us to extract the coefficients of the pde’s from a Fourier transform of the observed waves if they appear. In order to test the new theory, we consider the simple pressure diffusion equation in 1-D.

First application of the wave mechanics theory

The above approach is conceived as an abstract approach to the solid and fluid mechanics of materials (here geomechanics) based on fundamental physics and thermodynamics. The hypothesis needs to be tested in the laboratory. For this we need to evaluate whether the diffusional waves may be detected in controlled laboratory experiments. The key to the detectability of the wave phenomenon is proposed to be the diffusive length scale \( L_D = \sqrt{2D_H t_c} \) where \( t_c = T \frac{2\pi}{2\pi} \) is the time of the period of a wave \( T \), which defines the time scale of observation. Length scale \( L_D \) defines the correlation length for the macroscale ensemble representing the minimum upscaled quantized energy state \( \hbar^* \). For much larger length scales the wave mechanics phenomenon is lost and the experiments are best described by classical solid and fluid mechanics approaches.

In the laboratory we can readily detect waves with a time resolution of \( t_c \sim 0.1 \text{s} \). Taking representative values for geomaterials of \( D_T \sim 10^{-6} m^2 s^{-1} \), \( D_H \sim 10^{-16} m^2 s^{-1} \) and \( D_C \sim 10^{-16} m^2 s^{-1} \) we obtain the corresponding diffusional wavelength scales of \( L_T \sim 10^{-4} m \), \( L_H \sim 10^{-1} m \) and \( L_C \sim 10^{-9} m \), respectively. From this dimensional analysis the hydromechanical wave is the easiest to detect in the laboratory followed by the thermo-mechanical wave, while the chemo-mechanical wave is the most difficult to detect.

The observation windows must be carefully selected in controlled experiments posing strong technological challenges for the expected chemical and thermal wave phenomena. The hydromechanical wave in porous media also provides some complexity as the diffusivity can incorporate the rheology of the solid matrix in addition to the fluid as well as local chemical interactions. The combination of these processes may give rise to dissipative shear (S) and pressure (P) waves in addition to the familiar elastic (P) and (S) waves. The wave mechanics theory allows us to superpose (P) and (S) waves linearly as long as we can use a linear Schrödinger equation. This may be used to derive the classical Arthur Vardoulakis angles of geomechanics from a simple superposition of dissipative S- and P-waves [Regenauer-Lieb et
Here, we want to simplify further and just look at the possibility of a propagation of a diffusive $P$-wave by selecting an oedometric experimental setup where we compress the sample in 1-D.

**Pressure diffusion in an oedometric experiment**

We consider a simple case of possible pressure waves emitted in a porous medium such as a carbonate under load. As a further simplification the medium is considered to be composed of just two phases, the matrix and the fluid (or air) filling the pore space and the calcite matrix.

We recap Terzaghi’s consolidation experiment where the gradient of the effective stress is in equilibrium with the fluid stress gradient:

$$\frac{\partial p'}{\partial z} + \frac{\partial p_f}{\partial z} = 0 .$$  \hfill (6)

The simplest resulting diffusion equation for the pore pressure (or the effective stress) is given by equation (1) without the source terms

$$\frac{\partial p_f}{\partial t} = D_H \frac{\partial^2 p_f}{\partial z^2}$$  \hfill (7)

where $D_H$ is the Terzaghi consolidation coefficient or the pressure diffusivity. We can use different rheologies for the solid matrix. This classical Terzaghi consolidation problem for compaction of elastic granular media can also be applied for purely viscous media such as magma, where the diffusive length scale describes a compaction length [McKenzie, 1985]. The diffusivity underpinning the correlation length of both problems incorporates the compressibility (elastic or viscous), the fluid viscosity and the permeability of the porous medium.

The next step is to consider the Fourier transform solution of equation (7) with the Fourier transform (identified by the circumflex) defined by:

$$\hat{f}(k) = \int_{-\infty}^{\infty} f(z, t) e^{-2\pi ikz} \, dz .$$  \hfill (8)

Accordingly, taking the Fourier transform of equation (7) we obtain:

$$\frac{d\hat{p}_f(k, t)}{dt} = -D_H k^2 \hat{p}_f(k, t) ,$$  \hfill (9)

Thus, a wave with the wavenumber $k$ decays with an exponential law $e^{-D_H k^2 t}$. With the initial condition that for $t=0$ we have $\hat{p}_f(k, 0)$ the solution is:

$$d\hat{p}_f(k, t) = e^{-D_H k^2 t} \hat{p}_f(k, 0) .$$  \hfill (10)

The general solution to this equation in space-time is obtained from the inverse Fourier transform:

$$p_f(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{p}_f(k, 0) e^{ikz} e^{-D_H k^2 t} dk .$$  \hfill (11)

The general solution predicts that diffusional waves can be emitted based on the correlation length of pressure diffusion. The general solution yields diffusive and dispersive waves with a phase velocity of:
\[ v_i = \frac{\omega_i}{k_i} \] (12)

If we assume a step function of pressure (instead of a plane wave function) applied on one boundary of the 1-D model and assume that the ambient pressure of the medium prior to the application of the step function is recovered infinitely far away (in the direction of the other boundary) we only obtain a single wavefront that diffuses into the medium with the classical error function which defines the diffusional length scale:

\[ p_f = p_0 \text{erf} \left( \frac{z}{2\sqrt{DHt}} \right), \] (13)

where \( p_0 \) is the Heaviside pressure function applied at \( t=0 \).

**First qualitative description of expected results**

We can now come back to the role of the source term and its interplay with the propagating diffusional wave front. We can use the spectral decomposition of the experimental results as a means to test assumptions on the source terms in equation (1) and the diffusivities and quantify the coefficients of the assumed \( \text{pde's} \) based on the observed spectra, the phase velocities and group velocities. We have not yet performed such a quantitative analysis via a spectral decomposition of the observations. We firstly need to quantify the effect of the shear deformation and the interaction with the volumetric strains.

Therefore, in this paper we present the wave phenomenon for the first time and give a qualitative explanation of the observation based on the wave mechanics theory. The separation of the solution \( \Psi = \Psi_1 + i\Psi_2 \) allows us to assess whether the dissipation phenomenon of pore collapse can support a wave propagation. If the real and imaginary wavenumbers \( k_{1,2} \) are equal, waves can be triggered at any frequency of a forcing function applied to the sample. This case is encountered for a poro-elastic matrix. A number of acoustic wave-phenomena (e.g. slow Biot waves) are possible if we deal with an elastic matrix [Vardoulakis and Sulem, 1995].

If the real and imaginary parts of the wavenumber are, however, unequal the imaginary wave cannot pick up enough strength to appear until high frequencies are reached [Mandelis, 2000]. Therefore, at low frequencies there are no waves. If we neglect, for instance, the elastic wave phenomenon and are interested at waves that appear from the slow pore collapse of the matrix material, by adding a dissipative source term, then we should expect a slow propagation of dissipative waves above a critical frequency/wavelength but faster than the applied loading rate. The superposition of diffusion and dissipative wave sources can lead to self-reinforcing wave packets (solitary waves) that maintain their shape and propagate according to a group velocity.

Another important prediction can be drawn from equation (2) which separates the wave into two parts: The time independent (stationary) energy well \( V(z) \) which defines the long-time scale attractor of the dynamic system and the diffusive wave phenomenon which defines the shorter time scale solution discussed above. We expect that short time scale waves ultimately feed into longer time scale standing waves, which are defined by the stationary energy attractor (standing waves). This implies that the standing wave grows by absorbing dissipation carried into the stationary structures through short time scale dissipative waves.
Dissipative solitary waves

The pressure diffusion-reaction equation (1) describes the behaviour of a porous medium when the effect of pressure on fluid migration can be described by Darcy’s law. We are interested in establishing whether the reactive source term and cross diffusion generates solitary waves as expected from the wave-mechanics theory.

As a first test we perform a simple experiment using the theoretical criteria for appearance of solitary waves in an oedometric test. The fluid phase in the porous medium is subject to a compression and the diffusion of the fluid is filter pressed out of the pore space towards the top thus allowing compaction of the medium. The rate of compaction for this problem is characterised by Terzaghi’s consolidation coefficient $D_H$ which is a function of permeability, the compressibility of the matrix and fluid viscosity [Vardoulakis and Sulem, 1995]. A possible solution without the emergence of solitary waves is compaction characterised by the decay of the pressure following equation (13) as described in Terzaghi’s consolidation theory. Terzaghi’s theory predicts a diffusive wave front without solitary waves. However, for specific conditions where the imaginary part of the dissipative source term of pore collapse picks up enough strength a solitary waves phenomena can be expected [Mandelis, 2000].

![Figure 1 Four snapshots of axial compressive strain recorded at the surface of the limestone interpreted by DIC image analysis. The maximum compressive axial strain is shown in blue. Low values of strain are red. The total strain is 10%.

We performed a simple test of the above discussed dissipative wave phenomenon by using a highly porous carbonate (Mt Gambier limestone) in a compression test using an oedometric setup. The pore ‘fluid’ in our experiment was air and crushed grains. A constant axial displacement rate of 0.42 mm/min was applied. We recorded the deformation via two cameras mounted at right angles to observe the propagation of the deformation in the oedometric cell by digital image correlation (DIC). In this first experiment we recorded transient waves close to the noise level of the DIC image analysis and therefore only interpreted the growth of the standing waves. The first stationary localization feature of axial strain appears at the top of the first frame of Figure 1. This band is interpreted here as the standing wave solution of equation (1). In subsequent phases new bands propagate towards the bottom of the sample (three snapshots shown in Figure 1) until after 10% strain the large pores have collapsed, and the sample was compacted homogeneously.
Discussion and Conclusions

We have presented a new approach to investigating failure of materials inspired by Schrödinger’s formulation of wave mechanics. Wave mechanics in its classical definition is a theory of analysis of the behaviour of atomic phenomena with particles represented by wave equations. The basic theory relies on four fundamental principles (1) quantization of energy, (2) duality of particles and waves, (3) Heisenberg’s uncertainty principle of momentum and position of a particle, and (4) the correspondence principle, which defines that at large time/length scale classical mechanics is recovered.

The mathematical approach is based on predicting the criteria for dissipative pattern formation in materials by investigating reaction-diffusion partial differential equations. The equations are subject to a wave test function and the criteria for the onset of solitary waves are investigated.

An alternative particle-physics based approach - defined by discovering the smallest building blocks of matter and identifying their interaction laws - is already well established in geomechanics [Cundall, 2001]. The alternative wave mechanics macroscopic (condensed-matter) approach tries to understand and explain the new laws that emerge when many particles interact [Sethna, 1992]. This approach has not yet been tested in geomechanics and is currently tested in our laboratory.

Two scenarios exist: (1) If there is no delay in the energy conversion to the test function then heavily damped diffusional waves propagate through the material at the triggering wavelength. An example is the thermal-elastic wave triggered by a modulated laser beam hitting a material [Mandelis, 2000]. (2) In most cases relevant for geomaterials the physics controlling the reaction term is not fast enough. There is a phase delay of the reaction and the imaginary wave vector is not equal to the real part. In this case diffusion waves only appear if the imaginary wave vector picks up enough strength [Mandelis, 2000]. In other words, a dispersive, damped diffusional dissipation wave can only propagate through the material above a critical frequency of the force function. At low frequencies or long wavelengths there are no waves. Similarly, at high frequency we expect a cut-off of the wave phenomenon as the wavelength becomes smaller than the size of the ensemble characterised by the ground-state correlation length of the smallest energy eigenstate.

References:


Wave Propagation in Materials with Closing Micro-Cracks Modelled by Discrete Chain of Bilinear Oscillators with Damping

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Keywords: Bilinear oscillators with damping, wave propagation, shock waves

Abstract. This work focuses on wave propagation in materials with micro-cracks or fractures. The presence of micro-cracks makes the material behaviour dependent on the sign of the loading, i.e. materials exhibit what is called bimodularity – different moduli in compression and in tension. The simplest model of such a material is a chain of bilinear springs (bilinear oscillators) having different stiffnesses for compressive and tensile states and dampers. It is assumed that damping is the same for compression and tension. The discrete chain of bilinear oscillators is subjected to external harmonic excitation at one end and fixed at the other end. It was found that for the tension-compression harmonic excitation, only tensile deformations remain, while the compressive deformations get damped out over time.

Introduction

Laboratory experiments on many natural materials with structural singularities (micro-cracks, cleavage) such as rocks [1], graphite [2], concrete [3] revealed they respond differently to tension and compression. The phenomenon was called the bimodularity, which is the dependence of elastic modulus on the sign of the stress state.

It appears that the reason of such bimodularity has not been fully investigated. Antonets et al. [4] suggested the following simple model of realisation of bimodularity: the elastic specimen has a defect in the shape of a crack that opens under tension which, in turn, decreases stiffness of the stretched medium. This makes the stiffness in tension lower than in compression. Brittle materials may contain many of these micro-cracks which produce decrease in the elastic moduli (e.g., [5]). Under compression however these micro-cracks get closed and thus stop affecting the elastic moduli. As a result the material effective moduli in tension are lower than those in compression.

As a simple representation of materials with micro-cracks, we propose to consider a chain of bilinear oscillators with damping (i.e. discrete chains of masses connected by dampers and bilinear springs) and study the dynamical response of systems that exhibit different properties in tension and compression. Bilinear oscillators possess multiple resonances (e.g., [6-9]) and they were applied to a wide range of problems related to non-linear vibrations of mechanical systems, such as vibrations in mooring lines, suspension bridges, parts with clearances or gap-activated springs, seismic isolation systems and topological interlocking assemblies [8]. Chain of bilinear oscillators and their resonances have been considered in [10, 11].

In this paper, we study wave propagation in a very long discrete chain of bilinear oscillators with dampers that could represent fractured (i.e. brittle and bimodal) material with viscous properties. To detect peculiarities of the bilinear behaviour, we consider two external harmonic impulses: a simple harmonic impulse with a faster compressive part followed by a slower tensile part and vice versa.
Formulation of bimodular elasticity with damping properties

Consider a 1D problem of a discrete chain of identical masses $M$ connected by dampers and bilinear springs (see Fig. 1), where the dampers have identical damping $C$, springs have the same length $L$, and the common stiffness is described in the following formula

$$K(U) = \begin{cases} 
K_0(1-a) & \text{for } \Delta U \geq 0 \\
K_0(1+a) & \text{for } \Delta U < 0 
\end{cases}$$

(1)

Here, $U$ is the horizontal displacement, $K_0$ is the average stiffness of the bilinear spring, $a$ is the stiffness ratio, and $\Delta U$ is the difference of displacements of two adjacent spheres, that is the displacement of each spring. The chain is assumed to be fixed at the right end, Figure 1 and loaded by a force $F(T)$ from the other end.

**Fig. 1.** Mass-damper-spring chain.

The governing equation of the longitudinal motion of $i$-th mass is derived from the Lagrange equation in case of non-conservative forces

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tilde{F}_q$$

(2)

where $L$ is the Lagrangian function, $q$ is a generalised coordinate, and $\tilde{F}_q$ is the non-conservative force.

For Lagrangian function $L = E_k - V$ where $E_k = \frac{1}{2}MU_i^2$ is the kinetic energy and $V = \frac{1}{2}K_{i,i-1}(U_{i-1} - U_i)^2 + \frac{1}{2}K_i(U_i - U_{i,i+1})^2$ is the potential energy of the chain and non-conservative force $\tilde{F}_q = F(T) + C_{i,i-1}(U_{i-1} - U_i) - C_i(U_i - U_{i,i+1})$, the equation of motion reads

$$\ddot{U}_i + K_{i,i-1}(U_{i-1} - U_i) + K_i(U_i - U_{i,i+1}) + C_{i,i-1}(U_{i-1} - U_i) + C_i(U_i - U_{i,i+1}) = F_i$$

(3)

Since the force $F(T)$ is applied only to the first mass, it follows that

$$F_i = \begin{cases} 
F(T) & \text{for } i = 1 \\
0 & \text{otherwise} 
\end{cases}$$

(4)

We now rewrite Eq. (3) in non-dimensional form. By introducing the non-dimensional displacement $u = \frac{U}{L}$, time $t = \Omega_0 T$, damping $c = \frac{C_0}{M}$, stiffness $k = \frac{K}{M\Omega_0^2}$, force $f = \frac{F_0}{M\Omega_0^2}\sin(\omega t)$, excitation frequency $\omega = \frac{\Omega}{\Omega_0}$ wherein $\Omega_0$ is the basic frequency of the bilinear oscillator $\Omega_0 = \sqrt{\frac{K_0}{M}}$, one gets the dimensionless equation of motion

$$\ddot{u}_i + k_{i,i-1}(u_{i-1} - u_i) + k_i(u_i - u_{i,i+1}) + c_i(\dot{u}_i - \dot{u}_{i,i+1}) + c_{i,i-1}(\dot{u}_{i-1} - \dot{u}_i) = f_i$$

(5)

Herein, it is assumed that each resulting damping force is proportional to the relative velocity of its connecting points. Also the springs are assumed much stiffer in compression than in tension with
dimensionless stiffnesses \( k_c = 1 + a, \ k_t = 1 - a \) for compression and tension respectively, where \( a \) is a positive stiffness ratio.

We assume that at \( t = 0 \) the system is at rest, that is the displacement and velocity of each mass are zero.

**Dimensionless parameters of the mass-damper-bilinear spring chain**

All the numerical results presented in the paper are obtained for the following dimensionless parameters listed in the table below:

**Table 1. Dimensionless parameters.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of masses, springs and dampers</td>
<td>( N )</td>
</tr>
<tr>
<td>Length of the spring</td>
<td>( l = \frac{\Omega_o}{c} l_s )</td>
</tr>
<tr>
<td>Stiffness ratio</td>
<td>( a )</td>
</tr>
<tr>
<td>Damping</td>
<td>( c )</td>
</tr>
<tr>
<td>Amplitude of the applied force</td>
<td>( f_0 = \frac{F_0}{M\Omega_o c} )</td>
</tr>
<tr>
<td>Frequency of the applied force</td>
<td>( \omega )</td>
</tr>
</tbody>
</table>

An explicit Runge-Kutta algorithm with the time step \( \Delta t = 10^{-3} \) is used for solving the system of \( N \) ODEs (5) in the following section.

**Impulse harmonic excitation**

Let us analyse the effect of damping on the behaviour of the chain subjected to the external loading described by a simple harmonic impulse (Eq. 6).

\[
f(t) = \pm f_0 H(t) H\left(\frac{2\pi}{\omega} - t\right) \sin(\omega t)
\]

where \( H(t) \) is the Heaviside function.

Since there are two critical damping values corresponding to stiffnesses in compression and tension, \( c_{cr} = 2\sqrt{k_c} \) and \( c_{cr} = 2\sqrt{k_t} \) respectively, it becomes interesting to analyse the behaviour of the system while varying the actual damping parameter. Therefore, using Table 1, we consider three different cases \( c = 1 < c_{cr} = 1.63, \ c_{cr} = 1.63 < c = 2 < c_{cr} = 2.31, \) and \( c = 3 > c_{cr} = 2.31. \)

**Compression-Tension.** First of all, we analyse the case of a positive sign in Eq. 6 when a compression impulse is followed by a tensile one. Figs. 2, 3 show displacement and deformation against the mass number (that is the integer coordinate along the chain) at different time moments. As expected, increased damping leads to more prominent attenuation of the wave with time.
Fig. 2. Displacement at different time moments versus the horizontal coordinate for $c = 1$ (solid line), $c = 2$ (dashed line) and $c = 3$ (dash-dot line).

Fig. 3. Deformation at different time moments versus the horizontal coordinate for $c = 1$ (solid line), $c = 2$ (dashed line) and $c = 3$ (dash-dot line).

Tension-compression. Let us now consider a negative sign in Eq. 6 for which a slower tensile impulse is followed by a faster compressive impulse. Displacement and deformation for this case are depicted in Figs. 4, 5 respectively. As in the previous case, attenuation of the wave is greater for higher damping. However, one can also observe in Fig. 5 that a negative (compressive) part of deformation dissipates with time leaving only positive (tensile) deformation. This phenomenon has never been observed before and contradicts the expectation that a higher damping ratio corresponding to the tensile stiffness $\xi_t = \frac{c}{c_{cr}}$ would result in the attenuation of tensile deformation.

Fig. 4. Displacement at different time moments versus the horizontal coordinate for $c = 1$ (solid line), $c = 2$ (dashed line) and $c = 3$ (dash-dot line).
Conclusions

Assuming bilinearity springs (stiffness in compression is assumed to be higher than in tension) of the structure and identical damping for compression and tension, the discrete chain of bilinear oscillators with dampers demonstrated different responses depending on the sequence of the tensile and compressive components of the external loading. The results reveal that for the tension-compression loading compressive deformations get damped over time and only tensile deformations remain. This effect cannot be predicted by the linear theory and it may be used for identification of the micro-cracks or fractures in geomaterials.

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References


Measurement of residual stresses in rails using Rayleigh waves

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Keywords: Acoustoelastic effect, Residual stress, Rail, Rayleigh waves.

Abstract. This paper presents a new nondestructive approach for evaluating the residual longitudinal stresses in rails. The developed approach utilises the acoustoelastic effect to infer the longitudinal stress from the measured speed of Rayleigh waves propagating along the longitudinal direction. The measured Rayleigh wave speed along the longitudinal direction is shown to vary significantly across the height of the rail section, which can be directly correlated to the residual stress profile in the rail section. Unlike existing residual stress measurement techniques, such as hole-drilling or sectioning, the developed approach can be potentially applied for the in-situ residual stress measurement, without taking the rail out of service.

Introduction

Residual stress determination in rails is paramount for accurate life prognosis and assessment of fatigue damage [1]. Traditional methods for stress evaluation include the sectioning method, hole-drilling, and X-ray diffraction. However, these methods require the rail to be taken out of service during evaluation. Additionally, cutting and hole-drilling are destructive techniques, which limits their usefulness for field measurements, while X-ray diffraction is expensive [2] and unable to differentiate between micro- and macro-residual stresses, making measurements potentially unreliable [1]. Ultrasonic methods, which utilise the acoustoelastic effect, can potentially provide accurate, nondestructive residual stress measurement and can be applied in-situ.

Use of the acoustoelastic effect to measure stresses has been well documented in literature. Previous studies have utilised longitudinal waves for through-the-thickness evaluation, and critically refracted longitudinal waves for sub-surface stress interrogation [3]. The major drawback of such methods is the limited interrogation distance achievable with bulk (longitudinal and shear) waves. Guided waves are a type of ultrasonic wave capable of propagating long distances without significant attenuation [4], and therefore may be more suitable for stress monitoring applications. In particular, Rayleigh (surface) waves have previously been used to determine applied stresses in aluminium structures [5].

Rayleigh waves can be generated via mode conversion of longitudinal waves using a wedge. The difference in material properties between the wedge and rail specimen causes the longitudinal wave generated by an ordinary ultrasonic transducer to refract into a Rayleigh wave, which travels along the surface of the rail. The wedge angle required for Rayleigh wave generation is determined using Snell’s law (Eq. 1, where $c_{lw}$ is the longitudinal wave speed in the wedge and $c_{R}$ is the Rayleigh wave speed in the rail). The Rayleigh wave can be detected either using another wedge transducer, or using non-contact instruments, such as an air-coupled transducer or Laser Doppler vibrometer. Due to the non-contact
nature of the measurements, the latter two methods avoid the experimental scatter associated with inconsistent contact conditions [6].

\[ \theta_w = \sin^{-1}\left( \frac{c_{rw}}{c_R} \right) \]  

(1)

**Theory**

The theory of acoustoelasticity relates the speed of a wave to the stress state of the material through which it propagates. This phenomenon was first reported in bulk waves by Hughes and Kelly [7] in the 1950’s, and was later expanded to include Rayleigh waves by Dowaikh [8] in 1990. For a Rayleigh wave, the dependence between stress and wave speed can be expressed as [8]:

\[
\alpha_{22}(\alpha_{11} - \rho_0 c^2)[\gamma_2(\gamma_1 - \rho_0 c^2) - (\gamma_2 - \tau_2)^2] \\
= [\alpha_{12}^2 + \alpha_{22}(\rho_0 c^2 - \alpha_{11})][\alpha_{22}\gamma_2(\alpha_{11} - \rho_0 c^2)(\gamma_1 - \rho_0 c^2)]^{1/2}
\]

(2)

where \( \alpha_{11} = J \mathcal{A}_{0111}, \alpha_{22} = J \mathcal{A}_{0222}, \alpha_{12} = J \mathcal{A}_{0112}, \gamma_1 = J \mathcal{A}_{0121}, \gamma_2 = J \mathcal{A}_{0212}, \tau_2 \) is the stress in the \( x_2 \) direction, \( c \) is the wave speed, and \( \rho_0 \) is the stress-free density. For small values changes in stress, this equation can be represented by:

\[ \Delta c_R = k \Delta \sigma \]  

(3)

where \( k \) is a constant that relates the change in wave speed to a change in stress and has units m/s/MPa. The constant, \( k \), is typically negative for aluminium and mild steel, implying that an increase in wave speed corresponds to an increase in compressive stress. However, it has been documented that for grades of rail steel, \( k \) is positive, such that an increase in wave speed corresponds to an increase in tensile stress [9]. Thus, measurement of the Rayleigh wave speed in the longitudinal direction at various heights on the rail can be used to determine the distribution of tensile and compressive residual stresses.

**Experimental Methodology**

Generation of the Rayleigh wave was achieved using a wedge-transducer assembly. A high frequency wave was used to ensure that the Rayleigh wavelength was significantly smaller than the minimum rail thickness (16 mm in the web) and therefore satisfied the approximate half-space conditions required for the generation of a Rayleigh wave. A 2.25 MHz (\( \lambda_R \approx 1.3 \text{ mm} \)), 20 cycle rectangular windowed sine wave was generated using a Tektronix arbitrary function generator and amplified to 320 Vpp through a RITEC GA-2500A gated amplifier. A piezoelectric transducer was used to generate a longitudinal wave, which was converted into a Rayleigh wave using a 52° polyethylene wedge. The surface of a 1 m sample of new 60 kg/m rail was prepared using sandpaper to remove the surface rust and allow for both a) transmission of the wave from the wedge to the rail and b) a reflective surface for the laser-based measurements. The wedge was coupled to the surface of the rail using light motor oil and secured in place using a clamp. Measurement of the Rayleigh wave was achieved using a Polytec PSV-400-M2-20 scanning laser vibrometer, which measures the out-of-plane displacement of the wave. An initial alignment scan was performed to ensure that the Rayleigh wave propagation occurred in the longitudinal direction. The time of arrival of the wave was then measured at nine locations along a 100 mm segment of the line of propagation. The process of aligning the wedge and measuring the time of arrival was repeated for seven different heights along the rail, as shown in Fig. 1a. The experimental setup can be seen in Fig. 1b, and the wedge-transducer assembly can be seen in Fig. 1c.
The time of flight between measurement locations was determined using the cross correlation algorithm. The Rayleigh wave speed was obtained by plotting the time lag, corresponding to the maximum correlation between the first measurement point and subsequent points, against the propagation distance. A linear regression line was then used to determine the speed of the wave. Example waveforms recorded at locations 1 and 9 are shown in Fig. 2, and determination of wave speed using a regression slope is shown in Fig. 3.

Results

Figure 4 shows the out-of-plane displacement of the Rayleigh wave recorded by the vibrometer during an alignment scan. It can be seen that the wave travels with no angular deviation longitudinally along the rail, which is necessary to ensure that changes in wave speed are caused purely due to longitudinal residual stresses. It can also be seen that the wedge transducer generates a narrow Rayleigh wave beam,
approximately 10 mm in width, which travels along the longitudinal direction. The width of the Rayleigh wave is small in comparison to the height of the rail cross-section, which enables stress measurements to be made at several positions along the head, web and foot of the rail. This feature provides a significant advantage over some other stress measurement techniques, which lack the resolution offered by a narrow-beam Rayleigh wave along the height of the rail. Particular care was taken when aligning the wedge-transducer assembly, as small angles of misalignment can cause an artificial change in wave speed due to the error in measured propagation distance. Similarly, care was taken to ensure that the stand-off distance between the vibrometer head and line of propagation on the rail specimen was constant for different measurement heights. Due to the low number of averages taken during the alignment scan, noise errors are evident, as identified in Fig. 4. Such errors were avoided during time of flight scans by increasing the number of signal averages to 2000.

Fig. 4: a) 2D view of typical alignment scan showing out-of-plane displacement, beam width, and line of propagation; b) Isometric view of alignment scan. An obvious noise error has been identified.

The effect of residual stress on the Rayleigh wave speed can be seen in Fig. 5a. There is a clear increase in wave speed in the region of the rail head and rail foot, implying regions of tensile stress. Conversely, the web of the rail appears to feature mostly compressive stresses, as indicated by the slower wave speed. These results are in qualitative agreement with the results of Webster et. al. [10], also presented in Fig. 5b, which were obtained using neutron scanning techniques for new and used rail specimens.

Fig. 5: a) Results of the current work, showing the variation in wave speed along the height of the rail; b) Results of the Webster et. al. [10] showing the residual stress measured using neutron scanning techniques.
Further work is required to determine the residual longitudinal stress profile from the experimental results for the Rayleigh wave speed presented in Fig. 5b. The conversion constant \( k \) in Eq. (3) could be determined by inducing a known applied stress in the specimen and measuring the change in wave speed. The preliminary experimental results demonstrate the feasibility of using Rayleigh wave acoustoelasticity for longitudinal residual stress measurements in steel rails. The potential benefits of the developed approach include fast, cost-effective, nondestructive stress measurements that can be applied in-situ.

**Conclusion**

This paper demonstrates the application of the acoustoelastic effect towards a new approach for residual stress determination in steel rails. An experimental technique is developed for the measurement of Rayleigh wave speed along the longitudinal direction at various positions along the rail cross-section. These measurements can be used, in principle, to determine the residual longitudinal stress distribution in the rail cross-section. Further experimental work is required to calibrate the Rayleigh wave speed to the longitudinal stress according to the theory of acoustoelasticity. The preliminary experimental results of the present study indicate that the shape of the residual stress distribution as measured using Rayleigh wave acoustoelasticity is consistent with previous findings, which use neutron scattering.

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VI MECHANICS OF STRUCTURAL ELEMENTS
Flexure Behaviour of Rubberised Concrete-Filled Single-Skin Circular Tubes

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Keywords: rubberised concrete; concrete filled steel tubes; bending; energy absorption; ductility.

Abstract. Concrete filled steel tubes (CFST) are increasingly used in engineering construction as columns and beam-columns. Recycled rubber can be used as aggregate supplement in concrete and rubberised concrete is known to be more ductile than conventional concrete however has a lower compressive strength. Rubberised concrete filled steel tubes (RuCFST) is considered to have improved load capacity, ductility and energy absorption, and reduces resource extraction and waste disposal at the same time. This study aims to investigate the flexural behaviours of RuCFST. Steel tubes with four different geometries were tested in four-point bending tests to determine the behaviours of RuCFST under pure bending. Each geometry was tested as a circular hollow section, filled with conventional concrete and two rubberised concrete mixes with different rubber replacement ratios, for a total of 16 specimens. The experiment results were compared with the experimental results along with existing models for CFST. It was found that as the rubber content increased, the ultimate moment capacity of RuCFST decreased. However, the ductility increased overall, resulting in greater energy absorption. The composite material had improved toughness than the sum of the two individual components.

Introduction

The purpose of flexible and rigid roadside barriers is to reduce injuries and fatalities during hazardous events such as car collisions and rollover [1]. In recent years, concrete roadside barriers have been widely used in areas where potentially serious accidents may occur, such as highways, because of its benefits including high durability, variability and maintainability [2]. However, the current, overly solid profile of concrete barriers causes unexpected deformation to vehicles and severe injury to drivers because it cannot fully absorb external shocks and thus cannot reduce damage to colliding vehicles [3].

It has been widely accepted that concrete-filled single-skin tubes (CFST) have increased flexural strength, ductility and energy absorption [4]. Rubberised CFST (RuCFST) has strong potential to contribute to overcoming the problems of RuC as a possible alternative for current concrete roadside barriers. However, experimental use of RuCFST has been relatively limited compared with CFST, which has been implemented and actively researched over past decades [5]. In particular, knowledge of the critical characteristics of RuCFST, such as strength, ductility and energy absorption remains insufficient. This study analysed the flexural behaviours of the RuCFST by experimental and theoretical investigations. The outcome is beneficial for the recycle of rubber and building durable and safe roadside barriers.
Experimental Program

Three mixes were prepared, as shown in Table 1. One mix was a conventional mix of concrete RuC0. The other two mixes contained chip and crumb rubber replacing aggregate used in the conventional mix. In one mix (RuC15), 15% of the aggregate by mass was replaced by rubber and in the third mix (RuC30), 30% of the aggregate by mass was replaced by rubber. Prior to mixing, the rubber was treated with 10% sodium hydroxide solution for one day to improve bond performances in the concrete matrix. Compression and slump tests were performed on the cylinders made of three different concrete mixes.

Table 1 The mix compositions of the three mixes

<table>
<thead>
<tr>
<th></th>
<th>Water (kg/m³)</th>
<th>Cement (kg/m³)</th>
<th>10 mm Aggregate (kg/m³)</th>
<th>7 mm Aggregate (kg/m³)</th>
<th>&lt;4 mm Aggregate (kg/m³)</th>
<th>Sand (kg/m³)</th>
<th>Rubber Chip (7–10 mm) (kg/m³)</th>
<th>Rubber Crumb (1–5 mm) (kg/m³)</th>
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<tbody>
<tr>
<td>RuC0</td>
<td>205</td>
<td>426</td>
<td>444</td>
<td>306</td>
<td>130</td>
<td>843</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RuC15</td>
<td>205</td>
<td>426</td>
<td>311</td>
<td>214</td>
<td>91</td>
<td>648</td>
<td>45</td>
<td>58</td>
</tr>
<tr>
<td>RuC30</td>
<td>205</td>
<td>426</td>
<td>178</td>
<td>122</td>
<td>52</td>
<td>453</td>
<td>90</td>
<td>117</td>
</tr>
</tbody>
</table>

Four-point bending tests were conducted on a total of 16 CFST specimens. Fig. 1 shows the test setup used in this study. The specimens were 1 metre in length and had four different cross-sections, namely 88.9-3.2, 88.9-5, 114.3-3.2, 114.3-3.6 (outside diameter-thickness). Each steel circular hollow section (CHS) was filled with the three types of concrete. An additional CHS tube was tested without concrete infill.

Results and Discussions

The hardening graphs of RuC15 and RuC30 are shown in Fig. 2. It was observed that by 7 days of curing both mixes had attained at least 75% of their overall compressive strength and by 28 days they had obtained 85% of their compressive strength. After 28 days of curing, RuC15 specimens had reached peak compressive strength of 12.9 MPa. The strength of RuC30 was stable at approximately 6.6 MPa from 7 to 28 days. The control group RuC0 with no rubber content reached 41.79 MPa at 56 days. Therefore, as the rubber content increased, the compressive strength of the concrete decreased significantly.
The rotation-moment curves for all the 16 specimens obtained from the four-point bending tests are shown in Fig. 3. Note, the CFST and RuCFST specimens are represented by steel section-rubber replacement ratio. For example, 88.9-3.2-15 means CHS 88.9-3.2 filled with RuC15.

Fig. 3 The moment-rotation graphs for all the 16 specimens

Generally ultimate moment decreased as rubber content increased. Additionally, as rubber content increased ductility increased. All filled specimens were observed to have greater ultimate moment and ductility than their respective hollow specimen. For all specimens, no buckles were observed until the ultimate moment was reached. In the case of multiple peak moments occurring (such as the specimen 88.9-5.0-15), buckling occurred when the first peak was reached. When ultimate moment was reached multiple buckles developed. The three most common locations of buckles forming were on the inside
edges of central loads and at the centre of the beam. As the test continued after the ultimate moment was reached one buckle became dominant and developed at a greater rate than the other buckles. For the filled specimens, the steel cracked on the tension side opposite the dominate buckle. At the point of steel fracture, the test is terminated. The recorded load-displacement curves were converted to moment-rotation curves using eq. 1 and 2.

\[ M = \frac{PL}{6} \]

\[ \theta = \frac{-18L}{L^2 - 27x_0 + 27x_0^2} \]

Where \( P \) represents the load, \( x_0 \) represents the location on the beam, that is 400 mm (mid-span). The energy absorption of the specimens was then calculated based on the moment-rotation curves, by integrating the area underneath the curve using trapezoidal rules. It could be seen that RuCFST showed improved energy absorption compared to normal concrete filled specimens. This is especially beneficial for increasing the safety of the motorists.

Three analytical models for CFST from literature were compared with the experimental ultimate moment. Ultimate moment was predicted according to models in Eurocode 4 [6], CIDECT [7] and research done by Elchalakani et al. [8]. A summary of the theoretical ultimate moments is found in Table 2, along with \( D/t \) and slenderness ratios (\( \lambda_s \)) for the specimens. All models underestimated ultimate moment by similar amounts. This has shown that RuC could be effectively used as green alternative to normal concrete as strong roadside barriers. The accuracy in the predictions also ensured easier adoption of this new composite section as flexural members.

### Conclusions

The behaviour of the RuCFST beams was investigated experimentally through four-point bending tests and theoretically through analytical models. The previous models were found to underestimate the experimentally determined ultimate moment. The models were more accurate for more compact specimens. Filling CHS with conventional or rubberised concrete was found to increase the ultimate moment capacity, ductility and energy absorption of the specimens. As the rubber content increased, the ultimate moment decreased slightly and the ductility increased substantially, which also resulted in greater energy absorption. Due to the high moment capacity, ductility and energy absorption of the RuCFST, they are suitable for use in construction as beam-columns or roadside barriers. The utilisation of waste rubber also has a positive effect on the environment.

<table>
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<td></td>
<td></td>
<td>1.30</td>
<td>7.1</td>
<td>7.9</td>
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<td>27.8</td>
<td>37.6</td>
<td>5.84</td>
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<td>14.5</td>
<td>8.5</td>
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<td>6.54</td>
<td>19.8</td>
<td>8.3</td>
<td>8.6</td>
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<td></td>
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</table>

Table 2 Experiment results and comparisons between the past studies
References


Flexure Behaviour of Rubberised Concrete-Filled Single-Skin Square Tubes

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Keywords: rubberised concrete; concrete filled steel tubes; bending; energy absorption; ductility.

Abstract. Rubberised concrete has the benefit of utilising waste material, preventing resource extraction and improving concrete ductility, however at the cost of reduced strength. This paper explored the option to effectively confine the rubberised concrete with steel tubes to obtain enhanced strength and flexibility, which allows it to be used as roadside barriers. The flexure performances of twelve rubberised concrete-filled single-skin tubes were investigated to predict the relationship between rubber content and ductility of the composite. The experimental program involved three rubber contents, 0\%, 15\% and 30\% and four tube sections. The flexural performances of the rubberised concrete filled steel tubes were analysed. The experimental results were then compared with the theoretical results from the models proposed by various researches. The test results indicate that rubberised concrete filled tubes offers a higher degree of ductility and energy absorption than normal concrete. This study showed that it is effectively used as structural elements or safe roadside barriers.

Introduction

Concrete is a brittle material and generally as concrete strength increases the toughness decreases. As such there is interest in increasing the ductility of concrete to achieve concrete with high strength and toughness. Addition of rubber to concrete increases the ductility of the concrete \cite{1,2}. Consequently, the field of research into rubberised concrete has been continually growing over the past three decades \cite{3}. Despite the benefits of rubberised concrete, compressive and tensile strengths have been shown to decrease as the rubber content increases \cite{1}.

The research into rubberised concrete filled steel tube (RuCFST) has been continued by Elchalakani, et al. \cite{4} by looking at short single and double skin RuCFST. It was found that using rubberised concrete led to increased ductility compared to standard concrete by multiples of 2.5 and 3.5 respectively for axially loaded short single and double skin CFST. The high moment capacity, ductility and energy absorption of CFST have been suggested on multiple occasions to be desirable for design to resist seismic loads \cite{5,6}. Preliminary research indicates that the properties of rubberised concrete in particular for use in RuCFST will be even more desirable for use in designing members to resist seismic loads than concrete filled steel tubes (CFST) \cite{1,2}. However, there is a lack of research into flexural strength of RuCFST. This study contributes to addressing this gap in understanding.
Experimental Program

General purpose ordinary Portland cement was used as the binder material. Three mixes were compared in this study, namely RuC0, RuC15, RuC30 to denote the replacement ratio of rubber to aggregates. The control mix RuC0 has 426 kg cement, 205 kg water, 843 kg fine aggregates (sand) and 880 kg of coarse aggregates in one cubic metre. The coarse aggregates consisted of <4, 7, and 10 mm gravels. A water to cement ratio of 0.48 was adopted to ensure satisfactory workability of 150-190 mm slump. Superplasticiser was added accordingly to reach the target slump. RuC15 and RuC30 had the same water and cement proportions, with varying rubber replacement ratios of 15% and 30%, respectively. Four types of steel tubes were used for this project. The square hollow tubes were grade C350L0 cold-formed steel. Three sizes of the square hollow section (SHS) tubes, 100 x 100 x 3 mm, 89 x 89 x 5 mm and 89 x 89 x 3.5 mm, were painted and the 100 x 100 x 2 mm SHS tubes were galvanised. Four-point bending tests as shown in Fig. 1 were adopted for flexure strength measure of the SHS tubes, CFST and RuCFST. Each specimen was setup on the Baldwin compression machine with 345 mm ground clearance to allow sufficient deformation, 100 mm overhanging segments from each end of the beam and 267 mm distance between each loading/support points.

![Fig. 1 Four-point bending test setup](image)

Results and Discussions

Axial compression tests were performed on RuC0, RuC15 and RuC30 concrete specimens at 28 days. The compressive strength for RuC0, RuC15 and RuC30 are 41.8, 18.7 and 9.5 MPa respectively. The rubber content had a substantial impact on the compressive strength of the concrete.

The load-deflection curves for CFST and RuCFST are presented in Fig. 2. It can be seen that in general, the load capacity of CFST beams with RuC0 (normal concrete) is the greatest while those filled with RuC30 showed greatest deflection. For concrete filled 89 x 3.5 mm SHS tubes (Fig. 2a), the difference in strength between 89-3.5-RuC15 (outer diameter – thickness – concrete) and 89-3.5-RuC30 is relatively small but the deflection of 89-3.5-RuC30 is much greater than that of 89-3.5-RuC15. This means that rubber aggregate can have a positive effect on the flexibility of a beam. In terms of buckling, all three specimens showed local buckling on the compression face.
The 89 x 5 mm SHS specimens with RuC0, RuC15 and RuC30 also were subjected to four-point bending tests. The load-deflection curves for 89 x 5 mm SHS tubes are shown in Fig. 2b. Unlike 89 x 3.5 mm SHS tubes, the specimens filled with RuC15 and RuC30 showed poor results as they slipped during the test. The RuCFST maintained large load and underwent large deformation, and the setup became unstable and the friction was insufficient to resist the relative movement. One side of the beam moved slightly and global deformation increased significantly due to the slip. The specimen fractured shortly after this slipping failure. Thus, the deflections of 89-5-RuC15 and 89-5-RuC30 were lower than that of the control 89-5-RuC0. The buckling between the loading points occurred for all the three specimens.

In regard to the 100 x 2 mm SHS specimens, the load-deflection curve is shown in Figure 2c. These specimens showed satisfactory results for the load-deflection responses. The strength of 100-2-RuC0 was the highest while the 100-2-RuC30 specimens exhibited the highest deflection. The addition of rubber significantly improved energy absorption and ductility of the beams.

Finally, the results for 100 x 3 mm SHS specimens with RuC0, RuC15 and RuC30 are shown in Fig. 2d. These specimens showed similar results to the 100 x 2 mm SHS specimens in terms of load and deflection. The 100-3-RuC0 had the highest capacity, while the 100-3-RuC15 had the highest deflection. The energy absorption and ductility indices of the 12 specimens are shown in Table 1. The ductility index is given in eq. 1. It is measured between the axial displacements at 10% reduction of the failure load before \( \delta_1 \) and after the peak \( \delta_2 \).

\[
DI = \frac{\delta_2 - \delta_1}{\delta_1}
\]

Most specimens show satisfactory results for the energy absorption and ductility indices, except for the 89 x 5 mm and 100 x 3 mm SHS tubes. The discrepancies were due to that specimens slipped under loading. However, Table 1 still shows the improved energy absorption and ductility for the RuCFST, making them suitable for roadside barriers or flexural members. Although huge differences in compressive strengths existed among the 3 types of concrete, the differences reduced significantly to approximately 10-20% through steel confinement. The energy absorption of RuC was on average greater than that of normal concrete. The improvement in absorbed energy increased as the steel contribution (area of steel) decreased. The ductility of RuC also showed comparable or superior ductility when compared to normal concrete. However, further experiment programs with repeated specimens will be carried out to quantify this improvement.
### Table 1 Energy absorption and ductility indices of the RuCFST specimens

<table>
<thead>
<tr>
<th>Steel section</th>
<th>Rubber ratio (%)</th>
<th>Experimental load (kN)</th>
<th>Energy absorption (kJ)</th>
<th>Ductility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHS-89-3.5 mm</td>
<td>0</td>
<td>163.71</td>
<td>3.61</td>
<td>3.41</td>
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<td></td>
<td>15</td>
<td>154.85</td>
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<td>150.88</td>
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<td>30</td>
<td>151.81</td>
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### Conclusions

Twelve RuCFST specimens under four-point bending were tested to determine the behaviour of a confined RuC beam in terms of strength, ductility and energy absorption. The following results were obtained:

Rubber content can affect the strength of a confined beam: the more rubber is added, the less the strength of the beam. However, the addition of rubber increases the ductility index compared with that seen for normal concrete. In the case of RuCFST, SHS tubes in compact sections are stronger and have greater strain values than those in non-compact and slender sections. RuCFST specimens with higher rubber replacement ratio in general had higher energy absorption capabilities than those with no rubber content. Therefore, RuCFST can be effectively used as roadside barriers.

### References


Analytical solution for thermally induced axial stress in an end bearing heat exchanger pile embedded in a homogeneous soil

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Keywords: analytical solutions, energy pile, heat exchanger pile, thermal load

Abstract. Heat exchanger piles (HEP), which are also known as energy piles, are deep foundations with dual role. They transfer loads from a superstructure to a subsurface while simultaneously enabling the exchange of thermal energy between the subsurface and superstructure. The latter is accomplished by embedding plastic pipes into the piles. Usually a mix of propylene glycol and water circulates inside the pipes, thus enabling the transfer of thermal energy. The installation of pipes incurs only a small additional costs while facilitating harvesting of naturally available thermal energy at a shallow subsurface. The energy provides supplemental heating or cooling while decreasing the carbon emission to the atmosphere, thus making the HEP sustainable foundations.

The surrounding soil imposes a restraint to thermally induced deformations of HEP. This leads to generation of additional compressive or tensile axial stresses, the knowledge of which is required for design. Specifically, generation of tensile stresses in concrete HEP is critical and especially in the case that the operation starts with a cooling cycle. To this end, this paper presents analytical solutions for thermally induced axial tensile and compressive stress in HEPs, and for corresponding axial strain and displacement. The solution is derived for end bearing HEP embedded in a homogeneous soil.

Introduction

In addition to safely transferring loads to deeper soil layers, Heat Exchanger Piles (HEP) also need to be designed for a safe exchange of thermal energy between a superstructure and subsurface. Thus, it is of interest to obtain displacement, strain and stress in the HEP. To accomplish this prior relevant research efforts have used finite element method and finite difference method.

The main objective of this study was to derive analytical solutions for axial displacement, axial strain and stress in end bearing HEP subjected to thermal load.

Analytical Solution for Thermal Load

It was assumed in Peric et al. [1] that soil surrounding an end bearing HEP behaves as elastic material under working stresses. The assumption was successfully validated by comparing predictions of a computational model with the corresponding results of field tests that were conducted on an end bearing HEP emebeded in a layered soil profile. Consequently, it is also assumed herein that both, soil and HEP behave as linear elastic materials.

As axial displacements are significantly larger than radial displacements analytical solutions are derived by assuming one dimensional state of stress, strain and displacement. Axial stress and strain are negative in compression, while positive axial displacement is directed upwards. Compressive axial force is negative and heating of the pile induces positive temperature change. Furthermore, it is assumed that
axial displacement at the pile tip is equal to zero, thus representing an end bearing HEP that is embedded into a stiff bedrock. Finally, it is also assumed that HEP is embedded into a homogeneous soil profile whereby only a single soil layer is present above the bedrock.

Free body diagrams for the entire HEP subjected to the temperature change $\Delta T$ only and for its infinitesimal segment are shown in Fig. 1. The positive x-axis is directed upwards as indicated in Fig. 1.

![Free body diagrams](image)

Fig. 1. FBDs of the HEP subjected to a positive temperature change a) the entire HEP, b) an infinitesimal segment of HEP.

For the purpose of obtaining analytical solution the soil is replaced by continuous linear elastic shear springs that are attached along the pile shaft. Their stiffness is denoted by $k_s$. Thus, the shear stress $\tau = \tau(x)$, which acts along the soil pile interface, is given by

$$\tau = -k_s u.$$  \hspace{1cm} (1)

$u = u(x)$ is axial displacement. Eq. 1 is combined with the kinematic relationship for infinitesimal strain and thermo-elastic constitutive relationship that are given by Eq. 2 and Eq. 3 respectively. This leads to the governing equation for displacement of HEP, which is given by Eq. 4.

$$\varepsilon = \frac{du}{dx}.$$ \hspace{1cm} (2)

$$\sigma = E(\varepsilon - \alpha \Delta T).$$ \hspace{1cm} (3)

$$\frac{d^2u}{dx^2} - \psi^2 u = 0.$$ \hspace{1cm} (4)

$\varepsilon = \varepsilon(x)$ is axial strain and $\sigma = \sigma(x)$ is axial stress, $E$ is the Young’s modulus of a pile, $\alpha$ is the coefficient of thermal expansion of a pile, and $\Delta T$ is a temperature change.

The parameter $\psi$ is given by
\[ \psi^2 = \left( \frac{p}{A} \right) \left( \frac{k_s}{E} \right) = \xi_g \xi_s. \]  

Parameters \( p \) and \( A \) are the perimeter and cross sectional area of a pile respectively. Thus, the parameter \( \psi \) contains the pile geometry in the form of \( \xi_g \) and ratio of the stiffness of the soil and the pile in the form of \( \xi_s \).

The unknown function \( u(x) \) is obtained after substituting the boundary conditions into the general solution of Eq. 4. The boundary conditions are given by

\[ u(0) = 0 \quad \text{and} \quad \sigma(L) = 0. \]  

This yields the following solution

\[ u(x) = \frac{\alpha \Delta T \sinh(\psi x)}{\psi \cosh(\psi L)}. \]  

where \( L \) is the length of the pile and

\[ 0 \leq x \leq L. \]  

Eq. 7 implies that

\[ u(L) = \frac{\alpha \Delta T}{\psi} \tanh(\psi L). \]  

Combining Eq. 2 and Eq. 7 gives

\[ \varepsilon(x) = \alpha \Delta T \frac{\cosh(\psi x)}{\cosh(\psi L)}. \]  

Axial strain has global extremum at \( x = 0 \), which is given by

\[ \varepsilon_{\text{ext}} = \frac{\alpha \Delta T}{\cosh(\psi L)}. \]  

An axial strain at the pile head is given by

\[ \varepsilon(x) = \alpha \Delta T. \]  

Axial stress in the pile is obtained by combining Eq. 3 and Eq. 10. It is given by

\[ \sigma(x) = E \alpha \Delta T \left[ \frac{\cosh(\psi x)}{\cosh(\psi L)} - 1 \right]. \]  

Axial stress has global extremum at \( x = 0 \), which is given by

\[ \sigma_{\text{ext}} = E \alpha \Delta T \left[ \frac{1}{\cosh(\psi L)} - 1 \right]. \]
Thus, Eq. 14 gives the minimum and maximum axial stresses in the case of heating and cooling respectively. Specifically, in the case of cooling Eq. 14 gives the maximum tensile axial stress that develops at the pile tip. Thus, the normalized maximum tensile axial stress depends on the Young’s modulus of pile, coefficient of subgrade shear reaction, perimeter, cross sectional area, and length of the pile. The implied normalization is with respect to $\alpha\Delta T$.

It is noted that the interface shear stress is directed downwards or upwards in the case of heating or cooling respectively. The resultant skin friction force ($Q_s$) is given by

$$Q_s = N_b = AE\alpha\Delta T\left[\frac{1}{\cosh(\varphi L)} - 1\right].$$  \hspace{1cm} (15)

And Eq. (15) implies that skin friction force is negative in the case of heating while it is positive in the case of cooling.

**Application**

The above solution is illustrated by using four different soils that are present in the soil profile below the Swiss Federal Institute of Technology in Laussane, Switzerland as described in Laloui et al. [2]. It is noted that the actual soil profile at the site comprises four different layers named A1, A2, B and C. Nevertheless, the above analytical solutions are presented for the hypothetical soil profile whereby a single soil layer is present above the bedrock.

The HEP has diameter of 1m and it is 26 m long. Its modulus of elasticity is equal to 29.2 GPa. The stiffness of soil subsituting shear springs was provided in Knellwolf et al. [3]. It is equal to 16.7 Mpa/m, 10.8 Mpa/m, 18.2 MPa/m and 121.4 Mpa/m respectively for soils A1, A2, B and C.

Laloui et al. [2] carried out test T1 prior to the construction of the building by applying a single heating/cooling cycle. Fig. 2 depicts the predicted axial stress in the pile at the temperature difference between the pile and soil equal to 13.4 °C. The stiffer the soil the larger the magnitude of the axial stress generated in the pile. Since experimental data were obtained for the actual layered profile the results presented here are not diretly comparable to them.
If thermal load were to start with cooling rather than heating then tensile stress would be generated in the pile. The maximum tensile stress would occur at the pile tip. The normalized maximum value of the tensile stress at the pile tip can be obtained from Eq. 14 as

$$\frac{\sigma_{ext}}{\Delta T} = \alpha E \left[ \frac{1}{\cosh(\psi L)} - 1 \right]$$

(16)

Fig. 3 shows the magnitude value of normalized minimum and maximum axial stress in case of heating and cooling respectively against the value of the parameter $\psi$, whereby four points represent the four different soils (A1, A2, B and C). Thus, the stiffer the soil the larger the maximum tensile stress. The coefficient of thermal expansion of the HEP is equal to $1 \times 10^{-5}$ °C$^{-1}$. 

Fig. 2. Axial stress versus depth for homogeneous soils A1, A2, B and C and experimental data (OF-optical fiber, VWE-vibrating wire extensometer) for $\Delta T=13.4$° C
Conclusions

Analytical solutions have been derived for the end bearing HEP surrounded by a single homogeneous soil that is exposed to thermal load. Thermal load is accommodated by generation of shear stresses at the soil pile interface and axial stresses in the HEP. The maximum compressive and tensile stresses develop at the pile tip in the case of heating and cooling respectively.

The larger the shear stiffness of the soil the larger the magnitude of the axial stress in the pile. In particular, tensile axial stresses are generated in the case that the pile experiences cooling with respect to its initial temperature. Thus, it is vitally important to place a sufficient reinforcement into the pile to prevent any cracking.

References


VII  LARGE DEFORMATIONS, RESIDUAL STRESS AND SHAPE MEMORY
Microfabrication by Severe Plastic Deformation: Architecturing of Multi-Phase Materials by Deformation-Induced Patterning

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Keywords: Severe plastic deformation, ultrafine-grained materials, hybrid materials, microfabrication, patterning, strength

Abstract. Over the last decades, a range of forming processes, collectively known as severe plastic deformation (SPD) techniques, have come to the fore as a means of improving the mechanical properties of metallic materials by extreme grain refinement – down to submicron range. The uptake of these manufacturing technologies by industry has been somewhat sluggish, however. The main reason for that lies in the difficulties of upscaling SPD processes for industry-level production. We proposed to use downscaled versions of SPD processes and apply them for microfabrication whereby the benefits of grain refinement are two-fold: (i) improvement of the strength characteristics of the parts produced and (ii) increase in their resistance to failure. In this article, potential applications of SPD processing in microfabrication are presented. In particular, the possibilities of using SPD techniques for creating novel multi-phase materials with desirable inner architectures are discussed. Special emphasis is put on SPD-induced synthesis of new materials based on pattern formation during SPD at various length scales.

Introduction

Among the new developments in the contemporary materials science, a promising trend is clearly recognisable – the trend to designing hybrid materials whose inner architecture provides an additional handle on improving their mechanical performance [1]. Among the many ways in which such ‘architecturing’ can be achieved, a group of techniques initially developed for metal forming stands out as a means to simultaneously produce a large-scale inner architecture of a hybrid material and induce profound microstructural changes in its constituents. These techniques, collectively known as severe plastic deformation (SPD) processing [2-4], involve a giant shear deformation combined with a high hydrostatic pressure. The resulting outcome of processing is commonly extreme grain refinement – often down to submicron scale or, in some exceptional cases, even nano scale. This gives rise to a spectacular increase in the mechanical strength of most metals and alloys. The price to be paid, however, is poor ductility of SPD-processed materials. This drawback can be alleviated by choosing the appropriate architecture of an armor component, which provides additional ductility to the hybrid material. Thus, a favourable combination of strength and ductility can be achieved. We have been promoting this materials design philosophy in the recent years [5,6] and will outline some promising strategies in the subsequent sections. We see microfabrication as a particularly promising field of application of such SPD-induced materials synthesis concept [7]. Some thoughts about SPD-enabled microfabrication will be shared below.
Combating Brittleness by Macro Scale Architecturing

**Embedded planar layers.** A lesson engineers have learnt from Nature is that ductility and fracture toughness of a material can be achieved through integrating in its brittle matrix some soft material. A celebrated prototype for this kind of design, which has inspired a great deal of biomimetic work, is the structure of nacre [8,9]. Perhaps the simplest way to implement this idea for enhancement of the performance characteristics of a brittle material without sacrificing its strength is by embedding *planar soft layers* (Fig.1), as suggested in our earlier work [6]. A specific characteristic we considered is the sensitivity to overloads, which describes the tolerance of the materials to loads exceeding design specifications.

![Fig. 1. Engineered materials with embedded soft layers, after [6].](image)

Such designs are of particular importance in the context of SPD-processed ultrafine-grained materials, which are generally brittle otherwise.

More sophisticated material architectures of the simple or complex brick-and-mortar type were studied by et al. [10] and Djumas et al. [11]. Although they were produced by 3D-printing using hard and soft polymers, rather than SPD techniques, the outstanding levels of fracture toughness achieved with these designs demonstrate the general potential of bio-inspired architecturing of materials.

**Spiral-shaped armour.** It has been recognized [12,5] that by embedding spiral reinforcements in a material, or by inducing some of its elements to transform to a helical geometry through plastic deformation, one can attain higher ductility. In combination with enhanced strength due to grain refinement associated with SPD processing, a desired property profile of a material can be achieved. The SPD techniques suitable for producing the targeted spiral geometries are high-pressure torsion [5], high-pressure torsion extrusion (HPTE) [13] and twist extrusion [14].

The formation of a helical architecture by twist extrusion is possible owing to a vortex flow of the deforming material this process induces [15]. The flux characteristics and especially the degree of vorticity depend on the geometry of the matrix with a spiral channel [14]. This makes it possible to vary the helical architecture of the product by changing the cross-section shape of the matrix channel and its
slope line angle. This exemplified by Fig. 2, which shows the variation of the shape of straight fibres upon multi-pass twist extrusion obtained by numerical simulation [14]. Experimental results demonstrating the formation of hybrid materials with helical inner architecture were presented in [16].

A ‘handle’ on controlling the parameters of the inner spirals formed by HPTE is provided by the ratio of the axial translation velocity, $V$, and the twist velocity, $W$ [13]. In this way, various inner structures can be formed by using the same tool geometry. In this deformation process, both the coaxially arranged fibres and those inclined to the axis acquire a spiral shape. An important feature of the HPTE process is that a spiral structure can be formed in a single pass. Figs 2c,d shows the transformation of initially straight embedded fibres for the $V/W$ ratio of six.

Fig. 2. Spiral-shaped armour architecture produced from straight fibres by multi-pass twist extrusion (a,b) and high-pressure torsion extrusion (c,d): (a,c) initial configuration; (b,d) configuration after processing.

**Patterning by SPD.** An interesting aspect of severe plastic deformation is its ability to produce geometrically interesting patterns that result from plastic instabilities during processing [17]. Intelligent utilisation of patterning can give rise to novel material architectures [18]. Pattern formation can be a purely mechanics effect, such as the loss of stability of plastic flow, or stem from SPD-induced mechano-chemical phenomena associated with heterogenous reactions in the deforming medium. Examples illustrating the former type of patterning in high-pressure torsion of multilayered specimens are given in Fig. 3.

Fig.3 Examples of patterns obtained by HPT of multilayered specimens: (a) folds in Ni-Al; (b) a vortex in Cu-Al.
In our opinion, self-organisation within a structure resulting from heterogeneous reactions in a deforming medium has a much greater potential with regard to the formation of new patterns, but at present it is just a hypothetical one. To date, spatio-temporal structures that emerge in fluids due to the effect of heat and mass transfer on the kinetics of heterogeneous reactions [19, 20]. An adequate description of this kind of patterning, which is often observed in Nature, is offered by the reaction-diffusion-convection modelling approach [21].

Synergetic phenomena in solids are observed more rarely than in fluids and have therefore been less studied. The reason is that self-organisation implies that the characteristic reaction time ($\tau_1$) and the characteristic time of heat and mass transfer ($\tau_2$) are of the same order of magnitude, so that

$$\tau_1 \approx \tau_2$$

(1)

For solid state reactions, the condition $\tau_2 >> \tau_1$ typically holds, which precludes self-organisation from happening. The situation becomes entirely different, however, if a solid undergoes large plastic deformations. Under such conditions, deformation-induced intensive mass transfer occurs, and internal stresses and crystal lattice defects emerge. These effects reduce the magnitude of $\tau_2$ decisively. As a result, the relation (1) can be fulfilled, which gives rise to the occurrence of self-organised structures and patterning. Pertinent examples of such processes are abundant in metamorphic formations of the Earth crust [22].

**Microfabrication with SPD-processed materials.** As mentioned in the Introduction, microfabrication is seen as a promising field of application of SPD technology. Specifically, we suggest using SPD-processed materials as precursors for manufacturing parts and miniaturised devices with the size in the range from about 10 $\mu$m to about 10 mm. Examples are found in parts of mini and micro electrical engines for robots and drones and combustion engines for model aircraft, gas turbine engines for power generators for personalized use, elements of mini motor reductors, elastic parts of MEMS, medical implants, etc. The requirements on such articles in terms of strength and precision are very demanding. In this regard, SPD-processed materials with their ultrafine-grained microstructures offer an exciting prospective in terms of the quality of high-precision articles. A further advantage is the technological viability of SPD processes as applied to small-size products and a low required output as compared to traditional metallurgical processes, which is conducive for microfabrication.

**Summary**

The considerations presented above prompt a conclusion that SPD processing is just starting to show its true potential. Not only does it produce ultrafine microstructure providing known materials with new properties, but it can also create novel multi-phase materials with desirable inner architectures. This opens up fascinating avenues in advanced manufacturing, especially in microfabrication.

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**References**


Keywords: additive manufacturing, deflection, selective laser melting, modelling

Abstract. For quality control of additively manufactured products, it is important to develop a capability to numerically model the manufacturing process for assessment of residual stresses and distortions. This paper reports on preliminary modelling results of an AlSi10Mg cantilever beam produced by selective laser melting. A macroscale model was used to simulate the global heat transfer and structural responses by using a super layer modelling technique. The numerical results presented in this study correlated well with the experimental data. It has been noted that annealing through remelting and reheating plays an important role in the development of residual stresses and distortions. Further work is in progress to incorporate baseplate preheating and improve modelling of annealing.

Introduction

Additive manufacturing (AM) is rapidly gaining attention within the defence industry, due to its agility and universality in producing structural components on demand. AM can be used to produce parts in the field where they are urgently needed, or produce a few replacement parts for support of legacy military platforms. In these circumstances, traditional means of production are either impractical or too expensive. Reference [1] gives a comprehensive review of additive manufacturing technologies including selective laser melting.

One of the main challenges of using additive manufacturing to build metallic parts is to achieve the desired mechanical properties, the dimensional precision and the required surface finish. To control the quality of AMed products it is, therefore, important to be able to numerically model the manufacturing process for assessment of residual stresses and distortions. Other challenges include porosity prediction.

There are two approaches in literature for modelling residual stresses and distortion, depending on the length scale. Mesoscale modelling simulates the details on the length scale of the melt pool, and generally has higher fidelity but is computationally more expensive [2-5]. Macroscale modelling combines multiple powder bed layers into super-layers, and applies a uniform heat flux. The method has lower fidelity but is computationally cheaper [6-8]. Using a macroscale approach, [9] simulated the deflection of a 3 mm beam, and compared the results to the test data reported in [10]. In this paper, we report on some preliminary results of modelling cantilever deflection for all five cantilever beam thicknesses.

Background of Experimental Data

To gain a better appreciation of the technical challenges and develop a capability for modelling residual stresses and deflection in AM, we chose to study the deflection of cantilever beams produced by SLM. An experimental study of this problem has been reported in [10], which focused on studying the effect of preheating the baseplate on residual stress and deflection, and numerical modelling has been reported in [9]. Our objectives are to replicate and expand the results reported in [9] and, through the process, identify key areas for further improvement.
Figure 1 a. Geometry of the cantilever beam; b. The deflection after the beam was removed from the substrate; c. The measured deflection from the centre of symmetry. The images were from [10], and the data in c. were digitised from [10].

Figure 1 shows the geometry of the cantilever beam of AlSi10Mg. Reference [10] reported the experimental results for the cantilever beam with five thicknesses: 0.5, 1, 2, 3 and 5 mm. The beams were built using the SLM process, and the deflection was measured after the support structure was removed from the substrate, as indicated in Figure 1b. While the objective of [10] was to investigate the effectiveness of using preheating to reduce distortion, we chose the non-preheated results for our model validation. The corresponding set of deflection data for the intact cantilever beams is shown in Figure 1c.

The relevant process parameters are laser power $P=195$ W, powder layer thickness $h=30 \mu$m, the laser scanning speed $v=800$ mm/s, which gives a linear heat input of $H=243.75$ J/m. These values fall into the processing window for continuous AlSi10Mg scan vectors as outlined in [11].

**Modelling Methodology**

To simplify the modelling process, the material was modelled as a homogenous solid. The discrete modelling of the powder, to account for powder melting, consolidation or material phase transformation, was not considered, nor was the localised stress concentrations caused by porosity defects. The enhanced heat transfer due to Marangoni, or thermo-capillary, convection within the melt pool was not modelled using CFD.

The model geometry used in this study is shown in Figure 2a. Figure 2b shows the base-plate, the support structure and the beam. The beam itself is composed of five sub-components, with thicknesses of 0.5, 0.5, 1.0, 1.0 and 2.0 mm, from bottom to top, as shown in the inset of Figure 2b. This configuration was used to simplify the geometry model. In this configuration the first component from the bottom represents a beam with a thickness of 0.5 mm; the first two components represent a beam with a thickness of 1; and so on. The analyses of the five beams can be conducted using this single geometry model.

In simulation, the five components were each divided into $n$ super-layers, with each super-layer representing a number of powder layers in the SLM process. The number of super-layers was decided by considering a trade-off between the computational burden and accuracy in results, as discussed in the
results section. With this model set-up, the deflections for the five different beams were obtained in a single simulation run, using the ABAQUS restart.

When a layer is activated, an equivalent body heat flux is applied to melt it. This heat flux may also remelt the adjacent layers below the current layer. This localised heating and reheating is the root cause for residual stress and global distortion in the built parts. The super-layer activation uses the element birth-death method for material deposition, using the “*Model change” card [12].

As the heat input is uniform across the layer, a quarter of the cantilever and baseplate were simulated, with symmetric geometrical boundary conditions applied, as indicated in Figure 2a. The baseplate geometry and boundary conditions were not included in [10], so a 10 mm thick baseplate with pinned boundary condition on the bottom of the baseplate was used. The mounting plate was assumed to have a fixed temperature thus an isothermal boundary condition, 25 °C in this study, was applied on the bottom surface of the baseplate. The radiation and convective heat transfer from top surface and the conductive heat transfer and mechanical support from the surrounding powder bed were not modelled.

Figure 2 a. The geometry and applied symmetry of the cantilever used for simulation; ABAQUS model with the baseplate and geometric partitions for layer activation and EDM cut shown.

The heat transfer and static simulations were sequentially coupled. The advantage of using sequential coupling over fully coupled were that a single heat transfer analysis could be fed into multiple static simulations. Another advantage is that it allowed early step termination of heat transfer analysis if the cooling rate fell below a threshold. Not doing so would lead to convergence issues in the static analysis as the temperature gradient became small.

The first step deactivates all the elements that represent the cantilever, with subsequent steps activating the elements when heating occurs. When an element is activated a body heat flux is applied. Once the heating step ends, the equivalent body heat flux is no longer applied and the part is cooled until the assembly falls below a cooling rate, defined as <5K/s. After the part was fully built, the support structures between 1-2 mm were deactivated, as shown in Figure 2b by the light shaded layer, to simulate the physical cutting.

A thermoelastoplastic material model with plastic annealing was implemented using the *Elastic, *Plastic and *Anneal temperature cards. An isotropic hardening plasticity model was used to model the strain hardening caused by the thermal strains. To account for the loss of plasticity history due to material melting [13], the equivalent plastic strain was set to zero once the temperature of the integration point exceeds the annealing temperature, set to 600 °C.

Thermophysical data. The following thermophysical properties, thermal conductivity, specific heat under constant pressure, density, latent heat of fusion, of AlSi10Mg (LM9) were unavailable in recently published literature, so the properties for AlSi7Mg (LM25) were used instead [14]. The AlSi10Mg solidus and liquidus temperatures were taken to be 560 °C and 600 °C respectively [10]. To account for
the Marangoni convection effects on heat transfer, an artificial thermal conductivity ratio value of 5 was used in this study.

**Mechanical data.** The as-built mechanical properties, such as elastic modulus and the Poisson ratio, yield strength, ultimate tensile strength and elongation before fracture [15-18], all vary with the build direction and magnitude of linear heat input. A consistent experimental finding of AlSi10Mg tensile tests [15-18] is that the Z-build direction results in a lower elongation before fracture compared to XY-build direction. Due to the build conditions of the cantilever, the supports are expected to be Z direction oriented, due to column like bending, while the beam itself would be XY oriented, due to longitudinal bending. As the region of interest in modelling is the beam, the XY mechanical properties were used throughout this study.

The Young’s modulus, Poisson ratio and stress-strain data were taken from [17], which had similar build conditions to the experimental paper, 67° rotated scanning pattern and H=247.5 J/m. The flow stress behaviour of AlSi7Mg (A356), at elevated temperatures shows that the ultimate tensile strength has a non-linear dependency on temperature [19]. A simplification of thermomechanical behaviour is to assume a linear dependency between flow stress and temperature. The experimental data for temperature dependent AlSi10Mg mechanical properties was unavailable, thus the yield stress and Young’s modulus were set to a tenth of its values at the solidus temperature.

**Modelling Results and Discussion**

The simulation was performed using a computer with Intel Xeon (X5690@3.47GHz) with random access memory of 96 GB. Preliminary simulations were performed to gauge parameter sensitivity. It was noted that the deflection has dependence on the mesh size, but the results were stable for mesh sizes between 0.45 and 0.60 mm. Subsequent simulations used a global mesh size of 0.45 mm. The magnitude of heat flux has shown significant influence on the results. Six different heat flux magnitudes were studied: 0.5, 1.0, 1.5, 2.0, 2.2 and $2.4 \times 10^{13}$ W/m$^3$. An arbitrary fixed heating time of 0.2 ms was used. It was found that the 0.5 mm and 1 mm beam thicknesses were sensitive to the magnitude of heat fluxes used and that values 1 and $1.5 \times 10^{13}$ W/ m$^3$ provided the best fit to experimental results.

The sensitivity in deflection due to n-layers is shown in Figure 3, using a heat flux magnitude of $1.5 \times 10^{13}$ W/m$^3$. When only one layer was used, the model results correlated poorly with the experimental results, as shown in Figure 3a. When the number of layers is three or above, the correlation improved significantly, as shown in Figure 3b, which shows the results of using four layers.

![Figure 3](image_url)

**Conclusion and Future Work**

A numerical study was conducted to model the deflection of a cantilever beam built using the selective laser melting process, using a macroscale modelling approach. Using the parameters determined through sensitivity studies, the simulation results obtained from ABAQUS analysis correlate well with the experimental results reported in open literature, for all the beam thicknesses considered.
The modelling may be further improved by developing high fidelity input data, e.g. the stress-strain data under different temperatures, and a mechanical constitutive model that can account for the stress relaxation due to material state changes and phase transitions. Future mesoscale modelling can inform the upscaling of thermal loads and temperature dependent material properties, rather than the trial and error procedure presented.

References


Shape Memory Effect of Polymer Glass and Its Application in Molding Process: A Numerical Simulation Study

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Keywords: Polymer glass, shape memory effect, finite element simulation, molding, residual stress.

Abstract. Most of the thermally-deformed polymer glasses can recover to their original shapes after reheating, but the underlying mechanisms and potential applications have not been fully explored. With the aid of the finite element method, this paper studies the shape memory mechanism of polymer glass PMMA and proposes a new molding technique: shape memory molding. The simulation results showed that after the cooling down from a thermal compression at a high temperature, the polymer sample can maintain its deformed shape and withhold significant residual stresses due to its large viscosity and stiffness. In the reheating process, however, the residual stresses could be released at around 90 °C and make the sample recover to its original shape. Based on this, a numerical platform was established to explore the feasibility of shape memory molding. The shape recovery capacity of the polymer was first evaluated at different pre-treatment temperatures and strains to demonstrate the molding process via shape memory. It was found that the pre-treatment strain could significantly affect the shape recovery capacity of the polymer. A higher pre-treatment strain and contact pressure would result in better shape recovery.

Introduction

Shape memory polymers (SMPs) are polymeric materials that are capable of recovering from their deformed shapes under an external stimulus, such as temperature and electric and magnetic fields [1]. Compared with the traditional shape memory alloys/ceramics, SMPs can retain their shape memory capacity at a very large deformation up to 800% [1, 2]. Moreover, such capacity can be tailored by changing the compositions of polymers.

Most polymer glasses possess shape memory capacity [3, 4]. This is due to their unique glass transition during which the deformation in the rubbery stage (above glass transition temperature $T_g$) can be frozen in the cooling process and can be released after reheating. Many studies have been done to design advanced polymer glass composites with extraordinary shape recovery capacity or low glass transition temperature suitable for biomedical applications [1, 2, 5, 6]. However, less effort has been made to reveal the shape memory effects of the commonly used polymer glasses and explore their applications.

Thermo-mechanical forming/hot embossing is a traditional method to fabricate polymer glass products, such as micro-lens arrays, functional surface patterns, micro-fluid channels/devices [4, 7-9]. A typical forming process requires a precision molding machine, which is costly and time-consuming. Considering that polymer glasses have the capacity to change its shape during reheating, it is possible to develop a new molding technique based on the shape memory effect.

This paper aims to reveal the underlying mechanisms of the shape memory effect of a commonly used polymer glass PMMA with the aid of the finite element method. The feasibility of shape memory molding will also be investigated.
Methods

Numerical simulations are conducted with Abaqus – a finite element code commercially available. The numerical analysis of the shape memory effect includes two steps: (1) introducing a pre-deformation at a high temperature above the $T_g$ of PMMA and freezing it at room temperature, i.e., “programming”; and (2) reheating the sample to a high temperature to trigger the shape memory process. Fig. 1a presents the finite element model used in programming step. It consists of upper and lower Tungsten Carbide plates with a thickness of 1 mm and radius of 10 mm, and a PMMA specimen with a thickness of 5 mm and radius of 3 mm. Considering the symmetry of the model, only a quarter of the model is established with symmetrical boundary conditions. The model is divided by the thermal-mechanical coupled element. Tungsten carbide is treated as a linear elastic material, with Young’s modulus 570 GPa, Poisson’s ratio 0.22, CTE $4.9 \times 10^{-6}/K$ and thermal conductivity $38 \text{ W/(m·s)}$. PMMA is simulated by a Prony-series type viscoelastic model. In brief, the deformation of PMMA can be divided into volumetric and deviatoric parts, i.e.,

$$\varepsilon_i = \varepsilon_i + \text{tr}(\varepsilon)\delta_{ij}/3, \quad \sigma_i = \sigma_i + \text{tr}(\sigma)\delta_{ij}/3,$$

where $\varepsilon_i$ and $\sigma_i$ are the deviatoric strains and stresses, respectively, $\text{tr}(\varepsilon)$ and $\text{tr}(\sigma)$ are the traces of the strain tensor and stress tensor, respectively, and $\delta_{ij}$ is the Kronecker delta. Considering the strong resistance to volumetric changes, the bulk viscosity of PMMA could be assumed to be infinite. Thus the relationship between the volumetric stress and strain can be written as

$$\frac{1}{3} \text{tr}(\varepsilon) - \alpha \Delta T = \frac{1}{9K} \text{tr}(\sigma)$$

where $K$ is the bulk modulus and $\alpha$ is the coefficient of thermal expansion (CTE). The relationship between deviatoric stress and strain can be described by a Prony series i.e.

$$S_i = G(t)\varepsilon_i$$

where

$$G(t) = G_0 - \sum_{i=1}^{\infty} G_i [1 - \exp(-t/\tau_i)]$$

The change of relation time with temperature was described by WLF equation

$$\log a_r = -C_1(T - T_g)/(C_2 + (T - T_g))$$

where $C_1$ and $C_2$ are constants. The corresponding parameters are listed in Table 1.

<table>
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<th>$E$ (GPa)</th>
<th>$\alpha$ (1/K)</th>
<th>$C$ (J/K)</th>
<th>$\nu$</th>
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<th>$G_2$ (GPa)</th>
<th>$\tau_1$ (s)</th>
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<td>17.44</td>
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In the programming process, the PMMA specimen was compressed by the upper plate with a constant velocity (1 mm/min) at temperature $T_p$ (pre-treatment temperature above $T_g$). When strain $\varepsilon_p$ (pre-treatment strain) was reached, the upper plate stopped moving and the whole model was cooled down to room temperature. To check whether the deformation produced at high temperature could be frozen at
room temperature, the force applied on the specimen was released by moving the upper plate up. In the reheating process, the specimen was reheated to $T_p$. As the applied force from the upper plate had been removed, the specimen should be able to expand freely. During the whole process, the thickness of the specimen and the internal residual stresses were examined. In the analysis, different values of $\varepsilon_p$ (43%, 26%, and 18%) and $T_p$ (130 °C, 140 °C, and 150 °C) were used to study their effects. Fig. 1b shows the typical profiles of the temperature and upper plate velocity during the whole process with $T_p = 140$ °C and $\varepsilon_p = 44\%$.

To evaluate the shape recovery abilities of the programmed PMMA specimens, a “restart” technique was used in the reheating process. In brief, all the information of the programmed specimen including stress, strain, and temperature were assigned to a new specimen model, which was placed between the established upper and lower flat dies, as shown in Fig. 1c. The positions of the dies were adjusted carefully to avoid any initial contact force. During the reheating, the expansion of the specimen was constrained by the dies and then their shape recovery capacity could be evaluated by monitoring the contact pressure. To explore the feasibility of shape memory molding, a concave lower die was also applied as shown in Fig. 1d.

![Fig. 1. (a) Finite element model for the programming step, (b) typical profiles of the temperature and upper plate velocity used in the analysis of shape memory effect, (c) finite element model for evaluating the shape recovery capacity of programmed PMMA, and (d) finite element model for conducting the shape memory moulding of PMMA.](image)

Results

Fig. 2 presents the changes of the shape and internal von-Mises stress distribution of the PMMA specimen in the programming and subsequent reheating steps, with $T_p = 140$ °C and $\varepsilon_p = 44\%$. As shown in Figs. 2a-2b, a significant internal stress was produced and withheld in the specimen during programming. After removing the upper die pressure, the shape and stress of the specimen do not change with time at room temperature, as shown in Figs. 2c-2d. When the specimen was heated to around $T_g$, ...
however, the stress started to release (Fig. 2e) and the shape recovered back to the original (Fig. 2f). Fig. 2g shows the thickness change of the specimen in the whole process, which clearly indicates that the specimen shape quickly recovered when the temperature reached to $T_g$.

![Image of stress distribution and temperature changes](image)

Fig. 2. The changes of shape and internal von-Mises stress distribution of the PMMA specimen with time, (a) 0s, 25 °C; (b) 134.4s, 94 °C; (c) 507s, 25 °C; (d) 1000s, 25 °C; (e) 1134s, 102 °C; (f) 1500s, 140 °C. (g) The corresponding thickness and internal temperature changes.

Fig. 3 shows the contact pressure variation between the specimen and the upper die, which provides information about the shape recovery capacity of the PMMAs at different $T_p$ and $\varepsilon_p$. It is clear that the pre-treatment strain $\varepsilon_p$ can significantly affect the shape recovery capacity of polymer. A higher pre-treatment strain and contact pressure leads to better shape recovery. The influence of the pre-treatment temperature $T_p$ is trivial. The major shape recovery occurs near the glass transition temperature during reheating.
Fig. 3. The changes of contact pressure (a) at different heat treatment temperature (pre-strain 34%) and (b) at different pre-strain (140 °C).

Fig. 4 shows the whole process of shape memory molding by using the concave lower die. It is clear that when the temperature reached $T_g$, part of the specimen above the concave feature of the die started to deform and fill into the cavity. After cooling, the formed feature was retained, indicating a successful molding process.

Fig. 4. A typical shape memory molding process of PMMA. (a) 0s, 25 °C; (b) 120s 94 °C; (c) 137s, 104 °C; (d) 158s, 116 °C; (e) 203s, 140 °C; (f) after cooling, 25 °C.

Conclusions

This paper has studied numerically the shape memory effects of polymer glass PMMA with the aid of the finite element method. It was found that the shape memory capacity of PMMA is attributed to the internal residual stress produced in the programming process. Upon reheating to beyond the $T_g$ of the material, the residual stress can be released, and the sample shape recovers. The pre-strain in the programming process can significantly affect the shape recovery capacity while the effect of programming temperature is negligible.
References

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