Non-Hydrostatic Modelling of Stratified Flow in Lakes and Reservoirs

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Abstract

The study of hydrodynamics in lakes involves a broad spectrum of time and space scales that define how mass, momentum and energy are transported throughout lake ecosystems. As a result, hydrodynamics are inherently linked to the two major energy fluxes paths in lake ecosystems: the flux of mechanical energy through the water motion and the flux of chemical energy through the aquatic food web. The ability to simulate the interplay between processes occurring at this wide range of scales is a major challenge, given that some processes are slowly varying and occur at the basin scale and other processes are rapidly varying and occur at scales of a few meters. In particular, internal waves exist between two frequency limits of the spectrum: the low limit defined by the basin-scale waves and the high limit defined by the buoyancy-frequency.

At the high frequencies (and high wave numbers) of the spectrum the vertical acceleration becomes increasingly significant and the hydrostatic approximation commonly employed to simulate the low-frequency flow field is no longer valid. In order to simulate the lake hydrodynamics throughout this broad range of scales, two complications arise. First, there is a need for high model resolution to resolve the smaller scales of the motion; and second, a fully non-linear and sparse elliptic equation has to be solved to calculate the non-hydrostatic pressure. The main goal of this work was to devise a non-hydrostatic three-dimensional model to efficiently compute the motions of lakes at field scales encompassing the whole range of the internal-wave energy spectrum.

The model was validated in the simulation of the degeneration of internal waves in a laboratory tank. To save computational time, a grid-switching strategy taking advantage of the time scales on which non-hydrostatic processes manifest was developed. The non-hydrostatic model could reproduce the energy cascade from the basin scale waves to soliton-like waves, a process that hydrostatic models could not simulate. The grid switching strategy efficiently reduced computational time but introduced an instantaneous source of numerical diffusion in the simulations. Recommendations for field applications were made with the intent of using the grid switching technique efficiently.
The model was then applied to reproduce the high-frequency waves in a small and stratified lake. Different horizontal grid resolutions from 100x100 m to 1x1 m were used to simulate waves with frequencies just below the buoyancy frequency limit. The simulations results showed that the waves were part of unstable shear modes that developed in the weakly stratified surface layer at the commencement of wind events. The grid-switching strategy was used to provide a hot start for the simulation in the high-resolution grids. Because the shear instabilities developed on the background shear, the simulations with the hot start could reproduce the high-frequency waves, whilst the simulation with the cold start failed. While part of the energy was used to excite basin-scale low-frequency waves, the energy used to trigger the shear instabilities was directly used to mix the diurnal surface layer and lost to viscous dissipation.

Lastly, the model was used in conjunction with data from a field experiment to build an inventory of the influence of physical processes on the dissolved oxygen (DO) dynamics of a long and narrow stratified reservoir that presents hypoxia in its surface layer. Different grid resolutions were used to simulate the dynamics of wind-inflow interactions, which provided a mechanism for the generation of internal waves that propagated towards the upper end of the reservoir. The lake slopes provided collapsing breakers for the observed internal waves and, as such, most of the internal wave energy was dissipated with very little mixing efficiency at the lake slope. The wind re-aeration provided the main flux of DO to the hypoxic region and was one order of magnitude larger than other physical DO fluxes. The DO flux resulting from vertical mixing at the thermocline balanced the difference between incoming and outgoing advective DO fluxes. Net biological production rates were then estimated from the DO budget. It was hypothesised that the wind-inflow interactions affected the time and spatial scales of the observed DO and plankton patches in the reservoir.
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This thesis is dedicated to my grandmother, Vó Dagui, who did not live long enough to see me obtaining this degree.

"Se fosse só sentir saudade, mas tem sempre algo mais..."

(Renato Russo)
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Preface

This thesis results from the work contained in three distinct papers aimed for publication in peer-reviewed journals. Each of the papers is self-contained and presents a review of the relevant literature on which the research work was built on. This research work is wholly my own although carried out under supervision of J. Imberger. Chapter 1 provides an introduction to place each of the papers in the context of the overall work while Chapter 5 compiles the collective conclusions of the papers to direct future work.

Chapter 2 presents the research paper entitled “A grid-switching strategy for computing high-frequency, high wave number motions embedded in geophysical flows”, by D. A. Botelho, J. Imberger, C. Dallimore and B. R. Hodges. This paper is currently under revision based on comments of two anonymous referees and will be soon re-submitted to International Journal for Numerical Methods in Fluids. This work is my wholly own, J. Imberger provided scientific supervision, C. Dallimore provided instrumental help with the numerical code while B. R. Hodges provided thoughtful comments on the final manuscript.

Chapter 3 presents the research paper “Down-Scaling Model Resolution to Illuminate the Internal Wave Field in a Small Stratified Lake”, by D. A. Botelho and J. Imberger. This paper has been accepted for publication by ASCE Journal of Hydraulic Engineering.

Chapter 4 was accepted for publication by Limnology and Oceanography as “Dissolved oxygen response to wind-inflow interactions in a stratified reservoir”, by D. A. Botelho and J. Imberger. Some minor changes on the published manuscript were made to address the comments of one of the examiners of this thesis, Prof. Heidi Nepf. The works on Chapters 3 and 4 are my wholly own and carried out under supervision of J. Imberger.
Chapter 1

1. Introduction

Lakes and reservoirs are very important for mankind as they provide water that is used for potable supplies, crop irrigation, hydroelectricity production, effluent disposal, commercial fisheries and recreation. In addition to these well-known anthropogenic uses, lakes also function as the habitat for numerous species that are part of an intricate ecosystem. Needless to say, mankind and ecological perspectives on lake functionality are often in conflict. In principle, knowledge of the ecosystem dynamics provides the basis for sustainable management of lakes that considers all perspectives.

Sunlight is the primary energy source to the lake ecosystem because sunlight provides the energy for photosynthesis, which is then transferred to other organisms through the food chain (Wetzel 1983). Sunlight also provides the thermal energy that induces stratification and stability of the water column (Fischer et al. 1979) as well as the thermal energy that drives the atmospheric circulation and ultimately the local wind circulation, which very often is the main forcing of lake motion and controls the heat exchange between the lake and the atmosphere (TVA 1972). The water motions, in turn, control the rates at which mass, momentum and heat are transported in lake ecosystems (Fischer et al. 1979). As a result, the water motion exerts a large control on the lake ecosystem dynamics, because it dictates the spatial and time scales on which niches form to advantage living organisms at different trophic levels. Indeed, formation of such specific niches causes the ubiquitous patchiness observed in aquatic ecosystems (Martin 2003).

The ability to quantify and predict the water motion at a wide range of temporal and spatial scales is very useful if one intends to investigate the dynamics of these ecological patches. Moreover, this ability provides a means of understanding the energy (thermal, mechanical and biochemical) flux paths in aquatic ecosystems. In order to quantify and predict the water motion, solutions of simplified forms of the Navier-Stokes equations are generally used. Among these, solutions from numerical models offer some advantages over analytical models as the former have the flexibility to incorporate the combined effects of irregular lake geometry, density stratification and forcing (Gomez-Giraldo 2006). In particular, numerical solutions of the Navier-Stokes equations applied to lake
hydrodynamics generally use the hydrostatic approximation, neglecting the effects of the vertical accelerations. Such approximation is based on the premise that the horizontal scales of the motion are much larger than the vertical scale (Griffies et al. 2000) and, in fact, the hydrostatic approximation often suffices to describe the low-frequency basin-scale motions in lakes (Hodges et al. 2000; Rueda and Schladow 2003; Appt et al. 2004). However, for some important motions, especially high-frequency fine-scale motions, vertical acceleration becomes significant. Under these circumstances, to properly simulate the flow, a non-hydrostatic model that considers the effects of vertical accelerations is required.

There are two particular complications with a non-hydrostatic model. First, the solution of the non-hydrostatic pressure is a stiff elliptical problem. Fully explicit solutions of the non-hydrostatic pressure do not work because the constraint imposed by the incompressibility condition in the continuity equation forces an infinitely fast transmission of the pressure signal throughout the domain (Skamarock and Klemp 1993). The implicit solution of the non-hydrostatic pressure, on the other hand, requires the inversion of a sparse matrix, which has the size of total grid cells of the domain. For usual lake dimensions, the matrix inversion is not amenable to direct inversion techniques such that iterative techniques that are computationally expensive need to be used to solve the sparse system. Second, the scales on which the non-hydrostatic effects develop are small, requiring small grid sizes. The minimum grid aspect ratio (vertical grid size to horizontal grid size) to produce non-hydrostatic solutions that are effectively different from hydrostatic solutions is 1:100 (Wadzuk and Hodges 2003). For typical lake simulations, vertical grid sizes are generally the order of 0.5 to 1.0 m, which indicates that horizontal grid sizes should be less than 50 to 100 m, and preferably smaller (1 to 10 m). As a result, the number of grid points required for non-hydrostatic simulations in usual lake domains is large (about 100-1000 cells in each horizontal direction), further increasing the computational burden of the expensive non-hydrostatic pressure solution.

Due to the constraints posed above, applications of non-hydrostatic models to field scales have been limited. Moreover, most applications did not have sufficient resolution to produce an appreciable difference between hydrostatic and non-hydrostatic simulations (Mahadevan et al. 1996; Yamashiki et al. 2003) or, perhaps, simply did not sufficiently detail the regions where non-hydrostatic effects were influent (Casulli and Stelling 1998; Koçyigit and Falconer 2004; Fringer et al. 2006; Sato et al. 2006). However, simulations at
laboratory scales have shown where the main problems associated with the hydrostatic approximation exist, particularly for surface waves problems (Casulli and Stelling, 1998; Casulli 1999; Chen 2003; Koçyigit et al. 2002; Zijlema and Stelling 2005), and to a lesser extent, for internal wave problems (Fringer and Street 2003; Daily and Imberger 2003; Wadzuk and Hodges 2003), and fronts of gravity currents in lock-exchange problems (Casulli and Stelling 1998; Chen 2003). Hydrostatic models lack the dispersive effects imposed by the horizontal gradients of vertical acceleration because the vertical acceleration is neglected in hydrostatic models. As a result, there is no transfer of vertical momentum due to vertical accelerations to the horizontal momentum. The effect of this lack of momentum transfer is the excessive non-linear steepening of relatively large amplitude waves (those that are too steep to be smoothed out by viscous effects, Wadzuk and Hodges 2003) and the inability to make density interfaces roll (Özgökmen et al. 2003). This over steepening of waves in hydrostatic models induces numerical diffusion due to the large horizontal scalar gradients produced. As a consequence, hydrostatic models produce waves that are numerical artefacts and have no physical significance (Hodges et al. 2006).

The main objective of this thesis is to construct an efficient model that is able to simulate the water motion in lakes of different basin shapes and subject to a wide range of forcing. Focus is on the ability to simulate density-stratified flows and the associated mechanical energy transfer throughout the whole internal-wave frequency range. Chapter 2 introduces the non-hydrostatic model that is validated with the simulation of degeneration of internal basin-scale waves into internal surges and packets of solitary waves (Horn et al. 2001). A grid-switching strategy is proposed to alleviate the computational effort associated with the expensive non-hydrostatic solution. Comparisons with the results of the laboratory experiments and simulations of different grid resolutions are made together with some recommendations for field applications and improvements of the strategy.

In Chapter 3, the grid switching strategy is used to produce initial conditions for non-hydrostatic simulations of the flow in a small and stratified lake with the intent of reproducing observed internal waves with frequencies near the buoyancy-frequency limit that were forced during strong wind pulses. Required horizontal grid resolutions to simulate the observed waves and details of the wave’s generation mechanism are provided.
Chapter 4 presents the application of the non-hydrostatic model to a long and narrow stratified reservoir with the intent of showing how the inflows combine with the wind forcing to create favourable conditions for oxygen depletion in the surface layer of the upper reaches of the reservoir. Simulation results are combined with an extensive field data set to estimate the relative contribution of physical and biochemical processes to the observed dissolved oxygen depletion and to gain an understanding of how physical processes dictate the time and spatial scales on which biochemical processes can act in order to produce the observed ecological patches along the reservoir.

In Chapter 5, the main conclusions from Chapters 2 to 4 are drawn with recommendations for future work.
1.1. References


Chapter 2

2. A grid-switching strategy for computing high-frequency, high wave number motions embedded in geophysical flows

2.1. Abstract

Hydrostatic and non-hydrostatic models were used to simulate the generation of internal surges and associated soliton-like trailing waves from the non-linear steepening of low-frequency basin-scale waves. Results confirmed that the process cannot be modelled using the hydrostatic approximation, however the computational cost of the non-hydrostatic model was much larger than the hydrostatic. A grid switching strategy was developed to reduce the simulation run time of the non-hydrostatic model. In the strategy, a low-resolution grid using a hydrostatic computation of the flow field is dynamically switched to a high-resolution grid in the region of propagation of the leading internal surge, using a non-hydrostatic computation of the flow field. The strategy takes advantage of the small time scale required for non-hydrostatic effects to become important such that a high-resolution grid is invoked only when and where the non-hydrostatic effects become large. Run time reduction, conservation of the interpolation scheme involved in the grid switching and strategies for field scale studies were addressed. The grid switching strategy was shown to be able to predict the phase speed and the amplitude of the leading internal surge with greater agreement with laboratory experiments than the uniform-grid models however; the trailing soliton-like waves lost their signature due to smoothing effects of the interpolation scheme. All non-hydrostatic models predicted the essential features of the energy flux path between low and high-frequency waves showing that the grid switching strategy can be a viable alternative to simulate non-hydrostatic flows where computational power poses a limitation.

2.2. Introduction

Internal waves in enclosed basins, such as lakes, have received attention since the end of the 19th century (Mortimer 1974). Although internal waves are ubiquitous in most stratified lakes, they vary widely in origin, length and time scales, and in depth and
Grid-Switching Strategy

stratification on which they propagate. Here we confine ourselves to internal waves in a two-layer system that are not affected by the Earth’s rotation; waves that reside in small to medium sized water bodies. At the lower end of the frequency spectrum basin-scale waves (often-called seiches) form after the thermocline is tilted in response to the application of a surface wind stress. Upon relaxation of the stress, internal waves form and distribute the wind’s energy throughout the lake basin. Often the response involves the generation of higher frequency internal waves that are free from boundary constraints and are observed to propagate freely within the metalimnetic wave guide. These waves either dissipate within the interior of the lake or shoal and break on the perimeter of the basin (Hamblin 1977; Wiegand and Carmack 1986; Saggio and Imberger 1998; Boegman et al. 2003). The energy pathways giving rise to these high-frequency waves have recently received renewed attention (Imberger 1998; Wüest et al. 2000; Horn et al. 2001; Boegman et al. 2005a) as they provide the linkage between the energy introduced by the forcing (generally the wind in lakes) and the energy converted into mixing in both the lake interior and the benthic boundary layer (BBL) ultimately, controlling the nutrients redistribution and availability from the BBL to the surface-layer productive zones. Horn et al. (2001) showed that one of the energy flux pathways to the high-frequency internal wave field originates from non-linear steepening of basin-scale waves degenerating into an internal surge (also called rarefaction and hereafter also referred to as leading soliton) that evolves into a packet of soliton-like waves (hereafter also referred to as solitons or solitary waves). In Horn et al. (2001), a time-scale analysis for a two-layer system was used to identify five different regimes characterising the energy transfer from basin-scale waves to higher frequency waves in terms of the slope of the initial pycnocline tilt and the relative depths of the pycnocline to the total basin depth. When the basin-scale waves propagate over a sloping bottom this steepening is accelerated (Saggio and Imberger 1998; Boegman et al. 2003) and most of the energy transferred to the soliton-like waves is dissipated or converted to mixing when the latter waves break over the sloping boundaries of the lake (Michallet and Ivey 1999; Boegman et al. 2005b).

High-frequency internal waves may be excited by a variety of mechanisms, including non-linear basin-scale steepening (Saggio and Imberger 1998; Horn et al. 2001), shear instabilities (Hamblin 1977; Antenucci and Imberger 2001), shoaling of internal waves (Saggio and Imberger 1998; Boegman et al. 2005b), wave-wave interactions (Teoh et al. 1997; Tabaei et al. 2005) and mixing-induced internal wave pumping (Maxworthy et al.
Figure 2.1 reproduces Lake Biwa observations before and after a typhoon passed over the lake (Saggio and Imberger 1998). Increase in the isothermal displacement power spectra at both the low and high-frequency bands was observed with two clear peaks at high frequencies; the highest-frequency peak was due to shear instabilities and the other was shown to be due to basin-scale steepening (Horn et al. 2001). It is interesting to note that the intermediate parts of the spectra did not show a similar increase in their energy content (Saggio and Imberger 1998), suggesting a direct energy transfer from the basin-scale waves to the high-frequency waves (Boegman et al. 2003a). Computing the generation and propagation of high-frequency waves is challenging due to their small length scale, relative to the basin-scale waves. Simply refining the grid to allow for such small-scale features is still computationally unrealistic (Wadzuk and Hodges 2003) and high-frequency internal wave closure schemes have so far not been derived.

The transfer of energy from basin-scale internal waves to high-frequency waves can be reasonably modelled by a balance between non-linear steeping and non-hydrostatic dispersion which is the basis on the weakly nonlinear Korteweg de Vries equation (Benney 1966; Bogucki et al. 1997; Horn et al. 2002; Boegman et al. 2003a). Therefore, to simulate this degeneration process a numerical model requires a correct reproduction of the non-linearities and a non-hydrostatic pressure code in order to allow for the correct dispersion of high-frequency wave energy (Wadzuk and Hodges 2003). However, the small scale of high-frequency waves requires a small grid compared to the wavelength of the evolving high-frequency waves. In general, the wavelength of the high-frequency waves would depend on the depth and stratification of the fluid as well as the intensity of the forcing however, field observations indicate that these wavelengths can be smaller than 20 m in a strongly stratified lake (Gomez et al. 2006. Mixing in a stratified lake. Part 1: Wind-shear generated high-frequency waves as a precursor to mixing in Lake Kinneret, submitted to Limnology and Oceanography). The high-resolution grid in turn currently requires a computational effort beyond even the fastest workstations (Wadzuk and Hodges 2003) because of the sparse system that arises with the Poisson-type pressure equation (Mahadevan et al. 1996; Casulli and Stelling 1998; Shen and Evans 2004) in any non-hydrostatic algorithm. It is not surprising that the development of efficient algorithms to deal with 3-D hydrodynamic models for environmental free-surface flows is the focus of current research (Kocyigit and Falconer 2002; Wadzuk and Hodges 2003; Yuan and Wu 2004; Stelling and Zijlema 2005). However, while attention has been mostly directed
Grid-Switching Strategy

towards simulation of free-surface gravity waves (Casulli 1999; Casulli and Zanolli 2002; Chen 2003; 2005), numerical solutions of the three-dimensional non-hydrostatic Navier-Stokes equations for internal wave motions have not received detailed attention (exceptions being Daily and Imberger 2003; Wadzuk and Hodges 2003, Fringer and Street 2003; Shen and Evans 2004).

Table 2.1 introduces the different time scales of the basin-scale low-frequency motions and the small-scale high-frequency motions. The disparity between the large time taken for the steepening due the nonlinear effects and the relative short time required for the high-frequency waves to attain equilibrium with the non-hydrostatic dispersive effects (and become solitons) is used as the basis for the development of a grid-switching technique for the simulation of high-frequency waves. In order to simulate this basin-scale wave degeneration process, a three-dimensional model, for which the hydrostatic form was successfully used for computation of basin-scale waves in lakes (Hodges et al. 2000; Laval et al. 2003a; Laval et al. 2003b), can have the computational effort significantly reduced, provided a refined grid is introduced when a given degree of wave steepness is achieved in the basin-scale wave field. The simulation is then switched to a non-hydrostatic model with locally refined spatial grid when the flow characteristic requires the new dynamical description. This new numerical technique which dynamically switches both grid size and the incorporation of the hydrostatic pressure assumption in the momentum equations is used to simulate the non-linear internal-wave steepening in a laboratory flume (Figure 2.2) as detailed in (Horn et al. 2001). In the next section we will present the model details and the grid refinement algorithm. Results of model tests and comparisons of the models with the laboratory experiments in Horn et al. (2001) and the spectral analysis in Boegman et al. (2005a) are then presented. This is followed by discussion of the model results and how this methodology can be applied to flows in lakes.

2.3.Numerical Model Description

The model CWR-ELCOM (Centre for Water Research – Estuary, Lake and Coastal Ocean Model) was the basis for the numerical codes used in this paper. ELCOM solves the three-dimensional incompressible Boussinesq RANS equations (Reynolds Averaged Navier Stokes). In comparison to the model presented in (Hodges 2000) a non-hydrostatic formulation using a fractional step method as proposed in Casulli and Stelling (1998) and Casulli (1999) is employed. The numerical schemes adopted in ELCOM have been
extensively described in References (Casulli and Cheng 1992; Casulli and Cattani 1994; Casulli and Stelling 1998; Casulli 1999; Hodges 2000; Laval et al. 2003a, b; Wadzuk and Hodges 2003) and the reader should refer to the References cited throughout this section for further details. The equations are expressed below using tensor notation where the Einstein summation convention is adopted

\[
\frac{\partial u_z}{\partial t} + u_i \frac{\partial u_a}{\partial x_i} = -g \frac{\partial \eta}{\partial x_a} - \frac{g}{\rho_0 x_3} \int_0^\eta \frac{\partial \rho'}{\partial x_a} \, dx_3' - \frac{\partial P_{NH}}{\partial x_a} + \frac{\partial}{\partial x_i} \left( \nu_t \frac{\partial u_a}{\partial x_i} \right) \tag{2.1}
\]

\[
\frac{\partial u_3}{\partial t} + u_i \frac{\partial u_3}{\partial x_i} = -\frac{\partial P_{NH}}{\partial x_3} + \frac{\partial}{\partial x_i} \left( \nu_t \frac{\partial u_3}{\partial x_i} \right) \tag{2.2}
\]

\[
\frac{\partial u_j}{\partial x_j} = 0 \tag{2.3}
\]

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x_a} \int_0^\eta u_a \, dx_3 = 0 \tag{2.4}
\]

\[
\frac{\partial S}{\partial t} + u_i \frac{\partial S}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \kappa_t \frac{\partial S}{\partial x_j} \right) \tag{2.5}
\]

\[
P = P_H + \rho_0 P_{NH} \tag{2.6}
\]

\[
\frac{\partial P_H}{\partial x_3} = -\rho g \tag{2.7}
\]

\[
\rho = \rho(S,T) \tag{2.8}
\]

\( u_i \) is the velocity in the \( i \)th direction, \( t \) represents time, \( x_i \) is the coordinate in the \( i \)th direction, \( g \) is the acceleration due to gravity, \( \rho_o \) is the reference density, \( \rho' \) is the density fluctuation from the reference density at level \( x_3' \). \( P \) is the pressure, which is separated into a hydrostatic component \( (P_H) \) and non-hydrostatic component \( (\rho_0 P_{NH}) \) where the \( \rho_0 \) factor is used for convenience. \( \nu_t \) is the coefficient of turbulent eddy viscosity, \( \eta \) is the free-surface elevation and the sub index \( \alpha (\alpha=1, 2) \) indicates horizontal directions. \( z_b \) is the bottom elevation. \( S \) is salinity and \( T \) is temperature, which is constant in the present study. \( \kappa_t \) is the coefficient of turbulent eddy diffusivity. Equations (2.1) and (2.2) express the conservation of linear momentum and equation (2.3), the conservation of mass. Integrating equation (2.3) in the vertical from the bottom to the free surface, using kinematic boundary conditions, leads to equation (2.4). Equation (2.5) is the mass transport equation and
equation (2.8) is an equation of state to calculate the local density $\rho$. Equations (2.6) and (2.7) are used to obtain the deviation from the hydrostatic pressure field. The first and second terms in the right hand side of equation (2.1) represent the barotropic and the baroclinic acceleration terms, respectively, that originate from the horizontal hydrostatic pressure gradients contribution to the motion. Equations (2.1) to (2.7) are discretized and with equation (2.8) are solved by ELCOM. The numerical method is based on a finite-volume semi-implicit operator-splitting scheme on a staggered $z$-coordinate Arakawa-C grid stencil.

Following the methods described in (Casulli and Stelling 1998; Casulli 1999) we use the fractional step method to decompose the flow field in a hydrostatic and hydrodynamic counterpart. The hydrostatic flow field is used as the first approximation for the final hydrodynamic flow field when the non-hydrostatic pressure is considered. To obtain the hydrostatic flow field, the terms that include the hydrodynamic pressure contribution in the horizontal momentum equations are ignored and equation (2.7) approximates the vertical momentum equation. A brief description of the schemes used in the hydrostatic step is given below.

Momentum advection is calculated using the Euler-Lagrange CFL-based-hybrid scheme (Hodges 2000) that is third-order accurate in space (Wadzuk and Hodges 2003). The baroclinic terms discretization uses a first-order spatially accurate forward Euler scheme (Casulli and Cattani 1994; Hodges 2000). The vertical turbulent mixing closure scheme based on the turbulent kinetic energy budget developed by Hodges et al. (2000) is used for the vertical eddy viscous terms. However, we use the partial mixing algorithm developed by Laval et al. (2003a, b) and further refined with the inclusion of mixing regimes dependent on the vertical gradient Richardson number (Simanjuntak et al. 2006, Benthic boundary layer and interfacial mixing in a shallow salt-wedge estuary. To be submitted). As shown in Laval et al. (2003a, b), the model also includes the effects of drag with the bottom to introduce turbulent kinetic energy into the mixing model and to parameterize the momentum loss induced by the bottom shear stresses. Although this closure scheme was designed for typical flows in lakes and estuaries, the turbulent kinetic energy budget considered in the closure scheme extracts turbulent energy not only from surface and bottom drags, but also from convective instabilities and from the mean shear flow, which is the main source of turbulent kinetic energy and mixing in the laboratory flows considered in this paper (Horn et al. 2001). Horizontal viscous terms account only for the molecular
contributions and use a simple explicit second-order spatially accurate centred scheme. Free-surface elevation is calculated in a similar manner to the method presented in Casulli and Cheng (1992). However, the terms involving the free surface elevation in equations 2.1 and 2.4 incorporate a backward implicit discretization. The penta-diagonal system of equations that arises for the free surface is solved using a pre-conditioned conjugate gradient method incorporating an incomplete Cholesky pre-conditioner (Casulli and Cheng 1992). Horizontal velocities in the hydrostatic step can be calculated once the free-surface elevation is known (see Casulli and Cheng 1992; Hodges 2000).

The final vertical velocities in the hydrostatic model (not in the hydrostatic step) are diagnostically calculated from the discrete approximation of the conservation of volume, equation (2.3), integrated vertically from the known bottom boundary condition. The bottom vertical velocity is enforced (generally set to zero for impermeable boundaries) and with the horizontal velocity field, vertical velocities are calculated from the bottom up to the top in each water column using

$$w_{i,j,k+1/2} = \frac{1}{\Delta x_{i,j,k} \Delta y_{i,j,k}} \left[ u_{i+1/2,j,k} \Delta z_{i,j+1/2,k} - u_{i-1/2,j,k} \Delta z_{i,j-1/2,k} \right] \Delta y_{i,j,k} +$$

$$\left( v_{i+1/2,j,k} \Delta z_{i,j+1/2,k} - v_{i-1/2,j,k} \Delta z_{i,j-1/2,k} \right) \Delta x_{i,j,k} \int w_{i,j,k-1/2}$$

where $u, v$ are the horizontal velocities and $w$ is the vertical velocity in the discrete system. $i, j$ and $k$ are the indices of the computational cells, $\Delta x, \Delta y$ are the horizontal grid sizes and $\Delta z$ is the vertical grid size. $\Delta x, \Delta y$ and $\Delta z$ are allowed to vary in their respective directions. The one-half terms summing the indices subscripts indicate values computed at the faces of the cells. Velocities and free surface calculated so far correspond to the hydrostatic model. The resulting velocity field is used for computation of the salinity and scalars transport in the hydrostatic model.

In the non-hydrostatic model, equation (2.2) has to be considered to complete the vertical momentum equation. The Euler-Lagrange hybrid scheme of Hodges (2000) was adopted for the advection terms in equation (2.2). While the vertical velocity in the hydrostatic model is computed using equation (2.9), an approximate vertical velocity is required in the fractional step method. This approximate velocity is computed neglecting the non-hydrostatic pressure term in equation (2.2) and for this reason we will consider the
calculation of this approximate velocity as part of the hydrostatic step, however it has to be clear that this velocity is not part of the hydrostatic model calculations. For the horizontal viscous terms in equation (2.2), the procedure is analogous to what is used for the horizontal momentum equations.

In the non-hydrostatic step described below, the non-hydrostatic pressure is determined at the same time we enforce the divergence-free velocity field required by the continuity equation (2.3). The calculation of $P_{NH}$ follows the same method presented in (Casulli and Stelling 1998; Casulli 1999) however, we allow for the inclusion of varying horizontal grid sizes. The solution of $P_{NH}$ form a discrete Poisson equation for the hydrodynamic pressure field divided by the reference density, which are solved iteratively with the Bi-CGSTAB method of Van der Vorst (1992). The method can be used for non-symmetrical systems that appear with the use of varying horizontal grid sizes (Chen 2003). Similar to what is done with the free-surface system, we also use an incomplete Cholesky pre-conditioner in the solution of the $P_{NH}$ system of equations. Tests (not shown here for brevity) indicate that the convergence criterion for the Poisson equation solver suggested in Zijlema and Stelling (2005) is not appropriate for internal waves because only a couple of iterations on average were required per time step. These few iterations did not guarantee convergence. The criterion proposed in Wadzuk and Hodges (2003) guaranteed convergence in the same tests. However, this second criterion required 89 iterations per time step on average. The criterion given in Wadzuk and Hodges (2003) indicates a solution that cannot be further improved as the maximum accuracy of the scheme used to calculate $P_{NH}$ is reached (see all details of the criteria and tests in Wadzuk and Hodges 2003 and Stelling and Zijlema 2005).

After the iteration process is completed and the $P_{NH}$ field is defined, the free-surface correction is applied and the final horizontal velocity fields are obtained as described in Casulli (1999).

The final vertical velocities in the non-hydrostatic model are obtained with equation (2.9) using the horizontal velocities computed in the non-hydrostatic step thus, assuring a divergence-free velocity field. This velocity and $P_{NH}$ fields with the corrected free-surface form the solution in the non-hydrostatic model. Salinity (and tracers) transport is computed with the same schemes for both hydrostatic and non-hydrostatic models. This transport is treated in three different stages. First, turbulent vertical mixing of scalars is accomplished together with the vertical mixing of momentum (Hodges 2000; Laval et al. 2003a, b). After the solution of the flow field, the advection of the scalars is computed using the
ULTIMATE-QUICKEST scheme (Leonard 1991; Falconer and Lin 1997). Similar to momentum, horizontal eddy transport of scalars is neglected and only molecular diffusion is included for which a simple explicit-centred, second-order-spatially-accurate scheme is used. Whenever salinity fields are calculated, density is updated by the UNESCO (1981) equation of state for equation (2.8).

Specifications of boundary conditions are needed to conclude the model’s description. Free-slip and no-slip boundary conditions are enforced at the lateral and bottom boundaries, respectively (Laval et al. 2003a, b). The velocities normal to all boundaries were set to zero and scalar flux was not allowed at the closed boundaries. The hydrodynamic pressure is set to zero at the free-surface cells while a zero-gradient condition for the hydrodynamic pressure (i.e. Neumann conditions) are used at the closed boundaries. The algorithm used in the grid-switching strategy is given below.

**Grid switching**

In the non-hydrostatic model the values of $P_{NH}$ for every point of the domain need to be solved simultaneously via the iterative solution of the Poisson-type equation and, as can be seen later, this is where most of the computational effort resides. Noting that in our study we were concerned with the non-hydrostatic pressure gradient in the steep front of the basin-scale internal wave where nonlinear steepening leads to an increase of magnitude of the dispersive terms, solution of $P_{NH}$ can be carried out more efficiently if we devise a grid with high resolution only in this region. One simple (and not computationally demanding) way this can be done for structured-grid models is by using a set of pre-defined variable-cell-size grids thus, avoiding grid re-construction in a given time step. It is therefore necessary to choose an appropriate set of grids, define how the variables are interpolated from one grid to another and some criterion to choose an appropriate grid-switching time.

The switching between grids was achieved by interpolating the values from an “original” grid to a “target” grid in five stages. First, the free-surface heights were interpolated from the “original” grid to the “target” grid so we can define which are the wet cells (i.e. the cells that are under water for which values of variables are effectively computed) in the target” grid. Second, an “intermediate” grid that had the same horizontal resolution as the “original” grid and the same vertical resolution as the “target” grid was created. Third, in each water column the values of horizontal velocities and tracers from the “original” grid to the “intermediate” grid were linearly interpolated in the vertical. Fourth, the variables from the “intermediate” grid to the “target” grid were interpolated in the
horizontal. Fifth, equation (2.9) was used to compute the vertical velocities in the “target” grid in order to maintain a velocity field that is divergence-free. Figure 2.3 presents a diagram illustrating the five-step procedure to transfer the variables from the original to the target grid. Time integration then resumes in the target grid. Horizontal interpolations (i.e. free-surface height from the “original” to the “target” grid and variables from the “intermediate” grid to the “target grid”) are calculated as:

\[
S_{m,k} = \frac{\sum_{n=1}^{N_k} \frac{S_{n,k}}{d_n^2}}{\sum_{n=1}^{N_k} \frac{1}{d_n^2}}
\]

(2.10)

$S_{m,k}$ is the particular field at the wet cell $m$ in the “target” grid, $S_{n,k}$ is the particular field in the “intermediate” grid at the wet cell $n$, $N_k$ is the number of wet cells in layer $k$ in the intermediate grid, $d_n$ is the horizontal distance from the location of the variable being interpolated from cell $n$ in the intermediate grid to the location of the variable in the cell $m$ in the target grid. The velocity field is not directly interpolated, rather we interpolate

\[
S = \text{sgn}(U_i)U_i^{2}
\]

(2.11)

where $U_i$ is horizontal velocity field with $i$ equals 1 for velocities in the transversal direction and $i$ equals 2 for velocities in the longitudinal direction. The velocity field in the target grid is then simply recovered by $U_i = \text{sgn}(S)\sqrt{|S|}$. This approach has better kinetic energy conservation properties than interpolating the velocity field directly. It is important to note that this interpolation method is computationally demanding however; other schemes (i.e. bi-linear interpolation) can be easily adapted to reduce the effort and enforce conservation, if required.

For simplicity, we constructed four pre-defined variable-cell-size grids with finer resolution in each of the quarters of the tank and two horizontal uniform grids, which had both the same resolution as the coarsest and finest resolution cell of the irregular grid domains (Table 2.2 and Figure 2.4). All grids used the same vertical grid resolution. For the purposes of investigating the effectiveness of grid/model switching we established a set of
four base solutions for each initial-condition case comprehending simulations using the hydrostatic and non-hydrostatic models in the coarse and fine-resolution, horizontally-uniform grids. These solutions were also used to choose the time to switch to the appropriate variable-resolution grid according to the position of the internal surge front. Possible criteria for automatic grid switching are presented later in the Discussion section. In the grid-switching strategy, simulations were started in the coarse-resolution uniform grid. The first grid switching, which also involved the change between the hydrostatic solution and the non-hydrostatic solution, was determined by visually observing the time when the density field in the two solutions (i.e. hydrostatic and non-hydrostatic) started to deviate. In other words, when the wave front steepened sufficiently to produce visible dispersive effects, the variable-resolution grid with high resolution in the location of the wave front was activated and used for the computation until the wave front arrived at the boundary of the adjacent quarter of the tank. Then, the appropriate variable-resolution grid was invoked. Results of the different algorithms described in this section are presented below.

2.4. Results

Results presented hereafter illustrate the model’s ability to reproduce the basin-scale internal wave degeneration caused by internal wave steepening and dispersion. In the tests presented, the lower-layer thickness to depth ratio \( h_l/H \) was held constant (equals 0.3) and the forcing was parameterized by the inverse Wedderburn number, i.e. the initial basin-scale wave amplitude to the lower-layer thickness ratio \( W = \eta_0/h_l \), see Horn et al. 2001) which was varied from 0.15 to 0.90 (Table 2.2). Table 2.3 shows the time scales used to define the basin-scale degeneration regimes given by Horn et al. (2001). The experimental conditions modelled herein are within degeneration regimes 1 and 2 (see also Horn et al. 2001). These regimes are solely defined by the comparison between the dampening time scale, \( T_d \) (Table 2.3) and the steepening time scale, \( T_s \) (Tables 2.1 and 2.3). In the following, we present a comparison between hydrostatic and non-hydrostatic model’s performances before finally illustrate the advantages, limitations and problems of the grid switching strategy in comparison to the uniform-grid non-hydrostatic models.

The grid switching algorithm presented in section 2.3, using the grids shown in Figure 2.4 and Table 2.2, was used in all simulations where the basin-scale wave degenerated into solitons. Although the dynamics in the tank is essentially two-dimensional, the number of
cells required to maintain a horizontal grid aspect ratio close to one was used in the transversal direction. A time step $\Delta t = 0.05$ s was used in all simulations. This time step provided sufficient time resolution for comparisons with the laboratory experiments and maintained the Courant-Friederich-Levy condition (CFL) based on the internal wave speed and based on the advection always smaller than 0.1 and 0.08, respectively, for any simulation. We did not have the initial density or salinity distribution in the tank; however we used the information given in Horn et al. (2001) to create similar conditions of the laboratory experiments. While the initial temperature distribution was considered constant and equals to 20 °C the salinity profile was composed of three layers. The top layer had a salinity of 9.00 ppt and the lower layer had a salinity of 35.3 ppt. The density difference between layers with this temperature and salinity distribution was 19.96 Kg/m$^3$. An interfacial layer of 0.5 cm half-width with a linear variation of salinity was assumed in the interface. The initial position of the interface was simply a linear tilt of the pycnoclyne in order to produce the values of $W^l$ considered.

**Hydrostatic vs. non-hydrostatic model - internal wave evolution**

Figure 2.5 shows the interfacial displacements at the centre of the tank for a case that is characterized by Regime 1 of Horn et al. (2001). The initial density interface tilt was small ($W^l = 0.15$) and, as can be seen in Table 2.3, neither shear instabilities ($T_{kh} < T_s/4$) nor supercritical conditions occurred ($T_b < T_s/4$); the viscous damping time scale was shorter than the non-linear steepening time scale ($T_d < T_s$) and no high-frequency waves were observed. Because the gauge B (WGB) was approximately at the seiche nodal position zero displacement would be expected for an exactly linear wave (Boegman et al. 2005a), however the wave steepens and a very small amplitude progressive internal surge forms (Boegman et al. 2005a) creating small interfacial displacements ($\eta/\eta_o < 0.16$, $\eta_o = 1.3$ cm). The differences between model and laboratory results are in general smaller than the experimental accuracy of the interface height measurements ($\pm 2$mm, see Horn et al. 2001), however the magnitude of the interfacial displacements presented an increasing trend which differed from the lab observations. The reasons for this discrepancy are discussed later in the Discussion section. Although the results are in closer agreement with the lab for the fine resolution non-hydrostatic model, the coarse resolution hydrostatic model requires only 8% of the run-time required by the former and provides an internal surge with similar period as given by the other models. However, the amplitude of the internal surge for the coarse
resolution hydrostatic model was up to 54% larger than the non-hydrostatic fine resolution model.

For larger initial basin-scale wave amplitudes, non-linear effects become more important and part of the energy from the initial basin-scale wave was transferred, as described in Horn et al. (2001), to the internal surge and then to the solitons (Boegman et al. 2005a). The case presented in Figure 2.6 illustrates the interfacial displacements at wave gauge A (WGA) resulting from a moderate initial seiche amplitude ($W^2 = 0.45$), which falls in Regime 2 of Horn et al. (2001) (i.e. $T_s < T_d$, Table 2.3), where the basin-scale waves steepen until dispersive effects transfer energy to the solitons. In Figure 2.6 it can be seen that at ~0.9$T_s$ a train of solitary waves appear in the records of WGA. In agreement with the findings in Wadzuk and Hodges (2003), the non-hydrostatic model was able to reproduce these oscillations with similar amplitude ($\eta = 0.25 \eta_0 \sim 0.77 \eta_0$ range, $\eta_0$=3.9 cm), wavelength (~20.5$\eta_0$) and frequency ($f \sim 15.26 f_i$, where $f_i = 9.17 \times 10^{-2}$ Hz is the initial basin-scale wave frequency) as observed in the laboratory (Figure 2.6) until about 3$T_s$. The leading soliton corresponded to the internal surge and was well reproduced by the hydrostatic model runs. However, because there are no dispersive terms to balance the wave steepening, the internal surge signature evolved into a shape resembling a bore, but with the steepening tendency being counteracted by numerical diffusion. As shown in Wadzuk and Hodges (2003), with finer resolution there is less numerical diffusion and a steeper front forms. High-frequency waves also formed in the hydrostatic model but they had much smaller wavelengths (~6.4$\eta_0$-10.2$\eta_0$), smaller amplitudes (~0.25$\eta_0$) and higher frequencies ($f > 21.8f_i$). Confirming the results shown in Wadzuk and Hodges (2003), the resolution of the hydrostatic model greatly affected the properties of these waves (Figures 2.6a and 2.6b). By comparison, the resolution in the non-hydrostatic models (at least until 3$T_s$) only effected the phase and the number of solitons generated by the models and, at the fine resolution, the non-hydrostatic model correctly simulated the evolution of the solitons (Figures 2.6a and 2.6b); the crest of the leading soliton in the fine uniform grid simulation was in phase with the laboratory experiments until 3.5$T_s$, while the results of the coarse uniform grid model showed a lag of ~1.5s (10.6x10^{-3}T_s, for $T_s = 142$s). The non-hydrostatic model slightly over-predicted the amplitudes of the solitons, producing progressively larger amplitudes after each reflection from the end-walls, more noticeably after 3$T_s$. Past 3.5$T_s$, the amplitude of the leading soliton in the coarse grid model was approximately 40% larger than the
amplitude of the leading soliton observed in the experiments, while the amplitudes in the fine grid model was about 52% larger. Following results in Wadzuk and Hodges (2003) we assume that larger numerical diffusion in the low-resolution grid created a lower amplitude growth of the solitary waves in comparison to the high-resolution grid. Wadzuk and Hodges (2003) also put forward that the large modelled amplitudes in comparison to lab experiments is caused by the absence of drag at the sidewalls of the tank. The fact that the amplitude had an increase rate larger after reflection from the end walls (i.e. when the leading solitons are close to each other in Figure 2.6 as at $2.2T_i < \text{Time} < 2.6T_i$) indicates that the absence of drag at the end-walls may have an effect as large or possibly larger than the absence of drag at the sidewalls in the model.

In Figure 2.7 we present the resulting interfacial displacements for case $W^I = 0.90$ with the largest non-linearity and which fell between Regime 2 and 4 ($T_{kh} = T_i < T/4$, Table 2.3) of Horn et al. (2001) (note that although it was classified into Regime 2 in Horn et al. 2001 the observations showed a rapid evolution and decay of turbulence - Horn et al. 2001 - which indicated Regime 4 characteristics as suggested by the time scales in Table 2.3). As described in Horn et al. (2001), the seiche piled up against the vertical wall opposite to the side where the initial interface was elevated, steepening sufficiently to give birth to a packet of solitons that started crossing WGA at $\sim 0.6T_s$ (Figure 2.7a). The uniform, coarse-grid non-hydrostatic simulation did not capture the initial evolution of these solitons, however at $\sim 0.9T_s$ the fine resolution non-hydrostatic model showed excellent agreement for the first and second leading solitons crossing WGB with well predicted amplitudes ($< 0.15\eta_o$ and $0.09\eta_o$ difference, respectively, for $\eta_o = 7.83$ cm) and timing (difference $< 4.2 \times 10^{-3} T_s$ and $11.3 \times 10^{-3} T_s$, respectively, for $T_s = 71$ s) compared to that observed in the laboratory (Figure 2.7b). By contrast, with the coarse resolution model only the leading soliton was predicted to cross WGB and with a lag $> 32.3 \times 10^{-3} T_s$ (Figure 2.7b). As shown in Table 2.4, increasing the resolution increased the numbers of solitary waves and the timing of their formation and, in general, the numerical results converged to the results observed in the laboratory; between 7 and 8 horizontal grid cells are required to resolve the horizontal scales of the leading soliton and obtain the correct number of trailing soliton-like waves resulting from internal wave steepening. The number of grid cells per soliton-like wave is an important result for determining the grid refinement required in applications of the grid switching strategy.
**Hydrostatic vs. non-hydrostatic model - energy cascade**

The energy cascade through the internal wave field is illustrated with the normalised wavelet power spectra (see Torrence and Compo 1998 for methodology) of interfacial displacements (Figure 2.8). We present the wavelet maps in a non-dimensional time vs. non-dimensional frequency space instead of a time vs. period domain. Figure 2.8 shows the wavelet analysis for WGB records of the case $W^r/l=0.90$. The wavelet spectrum at higher frequencies ($4.4\times10^3/f_i$, for $f_i = 9.2\times10^{-3}\text{Hz}$) had a quite discernible flat pattern in the non-hydrostatic model and in the laboratory experiments however; the hydrostatic model presented a Christmas-tree-like repetitive pattern. The high-frequency oscillations in the hydrostatic model occurred just behind the excessively steepened internal surge and did not possess the dispersive characteristics of the solitons. Thus, the spectrum was filled up in a broad range of frequencies every time the basin-scale seiche crossed the wave gauge. In the case of the laboratory data and the non-hydrostatic model, the dispersive nature of the solitons created an increasing number of waves filling the whole domain space (e.g. Figures 2.9 and 2.10) thus, because there was a wave passing the wave gauge all times, a flatter pattern (as opposed to the Christmas-tree-like pattern in the hydrostatic model) of spectra with time could be observed at the $8.7\times10^0/f_i-2.2\times10^1/f_i$ frequency range. Contrastingly, the hydrostatic model produced physically insignificant waves of frequencies of up to $1\times10^2/f_i$. As a consequence, the resulting global wavelet spectrum of the hydrostatic model covered a larger range of frequencies with a cut-off at approximately $7.6\times10^1/f_i$ (Figure 2.8f) while the laboratory data and the non-hydrostatic model had a cut-off at about $2.2\times10^1/f_i$ (Figures 2.8b and 2.8d). The appearance of these “numerical” waves in the hydrostatic model was also pointed out in Wadzuk and Hodges (2003) and Hodges et al. (2006). Although this is obviously not the case in our results, Hodges et al. (2006) shows that an accidental collusion of nonlinear steepening and numerical dissipation/diffusion may give the false impression that hydrostatic models are able to reproduce the internal wave degeneration process. Therefore, computationally efficient strategies for the calculation of the non-hydrostatic equations are required.

**Grid switching strategy – run-time performance**

The main aim of the grid switching strategy was to reduce simulation run-time whilst still producing results that were in agreement with the lab experiments in comparison to the
Grid-Switching Strategy

fine-resolution grid non-hydrostatic model and with better agreement in comparison to the coarse resolution model. For case $W^r = 0.15$ the grid switching strategy was not employed because the wave dispersion effects were small (Figure 2.5), steepening time scale was large compared to dampening time scale (Table 2.3) and the train of solitary waves did not form during the course of the simulation. Table 2.5 presents the total run-time of the different scenarios studied in this paper. In all cases grid switching produced run times ranging from one third to one half of the time required for the fine-resolution uniform-grid simulations. The larger the non-linearity and initial basin-scale wave amplitude the longer the run times obtained with the grid switching strategy, the earlier the wave steepening occurred and therefore, the earlier the non-hydrostatic solution was required. As a result, more grid switches were needed (Table 2.6) to finish the time integration. Once the non-hydrostatic non-uniform-grid calculations started, between 5 and 6 grid switches per basin-scale wave period were needed. Table 2.6 presents the ratio between simulated time and run time (which hereafter will be called simply time ratio) for different initial values of $W^r$.

The time ratios shown for the non-uniform-grid non-hydrostatic simulations in Table 2.6 account for the run-time in a grid plus the time taken to interpolate the variables from the prior grid used in the simulation. The sum of all grid-switching times in a single case never exceeded 1% of the total run-time and was in the worst case ($W^r = 0.90$) 0.52% of the total run time of the fine-grid non-hydrostatic simulation. In the worst and best performances, the time ratio of the variable grid simulation (plus the grid-switching time) were respectively, 26.9% larger than the time taken using the fine grid model (Table 2.6) for a grid 29% smaller (Table 2.2) and 117.2% (Table 2.6) for a grid 32% smaller (Table 2.2). However, the main improvement of time ratio was due to the initial use of a hydrostatic model in the coarse uniform grid and, in the case $W^r = 0.30$, the time ratio was almost identical to the time ratio of the non-hydrostatic coarse-uniform-grid simulation. As can be seen in Tables 2.5 and 2.6, the run-time performance of the grid-switching strategy was satisfactory because, in the worst case, run-time in the non-uniform-grid non-hydrostatic simulation was reduced approximately at the same rate of the grid size reduction. In the next section we present the grid-switching performance relative to the uniform-grid non-hydrostatic models in reproducing the laboratory experiments.

Grid switching strategy – internal wave evolution

The interfacial displacements calculated using the grid switching strategy are presented for comparisons in Figures 2.6 and 2.7. Results at WGA (located at the downwelled side of
the initial basin-scale wave displacement) reveal that the phase lags of the internal surge in
the grid-switching model are smaller in comparison to the laboratory results than the other
models using uniform grids (Figure 2.6). The internal surge in the grid switching model
was still in phase with experimental observations at 3.5\(T_s\), although there was a phase lag of
\(-0.016T_s\) (for \(T_s = 142\) s) immediately before reflection from the end wall. For the
experiment with greatest nonlinearity (\(W' = 0.90\)) the same pattern was repeated at WGA
(Figure 2.7a), however the lag in phase occurred earlier, just after 1.0\(T_i\) with a phase lag of
\(-0.01T_s\) (for \(T_s = 71\) s) in comparison to the laboratory experiment. The phase lags, at the
same time, at the other wave gauges were smaller for the grid-switching model than for the
two uniform grid models (Figure 2.7). This phase lag reduction is due to the initial
formation of the solitons occurring earlier with the grid switching strategy (see the initial
development of the solitons at \(-0.6T_s\) at WGA, Figure 2.7a). Because the model was
hydrostatic before the first grid switching at \(-0.3T_s\) wave steepening was faster as there
were no dispersive terms to slow the steepening.

The reduction of amplitudes of the solitons generated by the grid-switching model in
correlation to the other two models was particularly noticeable for the peaks \(2T_i\) and \(T_i\) for
the cases \(W' = 0.45\) (Figure 2.6) and \(W' = 0.90\) (Figure 2.7), respectively. As discussed
below, the grid-switching technique has the tendency to act as a smoothing filter, therefore
dispersing the wave field and decreasing the wave amplitudes.

*Grid-switching strategy – interpolation effects*

Figure 2.9 shows the interpolation effects on the modelled salinity field for the case
\(W' = 0.60\) between 0.4\(L\) and 0.85\(L\) (\(L = 6\) m) in the longitudinal section of the tank. The
amplitude of the internal surge in the grid switch presented in Figure 2.9 was reduced by
\(-0.15\eta_o\) (for \(\eta_o = 5.2\) cm). The reduction in amplitude of the waves was a direct effect of the
chosen interpolation scheme and low-resolution grid size in the location of the trailing
solitary waves in the grid-switching strategy. The interpolation scheme uses the whole
horizontal domain in each layer of the “original grid” to define the dependent variables
values in the “target grid” (see equation 2.10). Smoothing was more noticeable for the
trailing waves because the larger horizontal grid spacing enhances the smoothing effect.
Note that the solitons trailing the internal surge have smaller amplitudes. Indeed, tests
showed (not included in this paper for brevity) that smoothing occurred more due to the
increase in grid spacing away from the leading internal surge than due to the interpolation
scheme.
In some instances, spurious oscillations occurred in the finer portion of the non-uniform grids after the interpolation was performed. These oscillations could be seen as very low amplitude and high-wavenumber waves that were quickly damped out as the model continued to run (Figure 2.10). Figure 2.10a shows the internal wave field just before the interpolation of the ninth grid switch in the simulation of case $W^l = 0.60$. After about 20 time steps (1s, i.e. $\sim 0.01T_s$) spurious oscillations (wavelength of $2\sim 3\Delta x$, i.e. $2.3\sim 3.4\eta_0$ and amplitudes of $1\sim 2\Delta z$, i.e. $0.04\sim 0.07\eta_0$) appeared (Figure 2.10b) and then after 80 time steps (4s, i.e. $\sim 0.04T_s$) oscillations were completely damped out as shown in Figure 2.10c. Although these oscillations were quite small in comparison to the wave’s amplitudes, they indicated that the interpolation scheme used is non-conservative.

**Grid-switching strategy – non-conservative characteristics**

We quantified conservation of tracer volume and energy components for the times when the grid switches were invoked. Tracer volume was quantified by calculating the evolution of the total salt volume ($TS$) in the domain.

$$TS = \frac{1}{1000} \sum_{i=1}^{N} S_i Vol_i$$  \hspace{1cm} (2.12)

where $i$ represents the grid cells, $S$ is the salinity and $Vol$ is the cell volume. $N$ is the total number of grid cells. The total potential energy ($TPE$) of the system was calculated by

$$TPE = \sum_{i=1}^{N} \rho_i gz Vol_i$$  \hspace{1cm} (2.13)

Only part of $TPE$, the so-called available potential energy ($APE$) can be used to energise the internal wave motion. $APE$ was calculated from the difference between $TPE$ and the background potential energy ($BPE$), which in the absence of mixing and external sources of energy (the case of the grid-switching under analysis) should remain unchangeable (see e.g. Laval et al. 2003c). Any changes in $BPE$ were expected to result from errors associated with the switching procedure. $BPE$ was computed by first, resorting the density and volumes of each cell in decreasing order of density. Cumulative volumes are then calculated starting from the volume of the densest cell. For each cell height in the domain, we calculate the cumulative volume thus, creating a correspondence between
cumulative volume and height in the domain. The sorted density field was then interpolated to the density field $\rho^*$ corresponding to the cumulative volumes associated with each height in the domain. $BPE$ was finally computed from

$$BPE = \sum_{i=1}^{N} \rho_i^* g z_i Vol_i$$

(2.14)

$BPE$ was calculated for all initial conditions and grid switches ($n=146$ samples). We estimated the error in calculating $BPE$ by calculating the difference in mass between the sorted density field used to compute $BPE$ and the actual density field. This mass difference corresponded to a very steady value of $(-6.9\pm0.5) \times 10^{-5}M$ ($M$ is the total water mass in the tank). Note that this error is due to the $BPE$ calculations only and are not related to the grid switching. Kinetic energy (KE) was calculated as

$$KE = \frac{1}{2} \sum_{i=1}^{N} \rho_i Vol_i \left( u_i^2 + v_i^2 + w_i^2 \right)$$

(2.15)

Figure 2.11 presents the evolution of $TS$ for all $W$ cases. ELCOM conserves the salt volume in between every grid switch thus; all changes in total salinity were an increase due to the grid switching. The larger the $W$ values, the less conservative the interpolation scheme became. Changes introduced by each grid-switch were as high as 0.05%, 0.1%, 0.2% and 0.4% of $TS_0$ (the subscript $o$ indicates the initial conditions) for $W$ equals 0.30, 0.45, 0.60 and 0.90, respectively, while the changes accumulated throughout the simulation were, respectively, 0.63%, 1.4%, 1.9% and 2.9% of $TS_0$. These changes occurred in the elevations of the interfacial displacements where the salinity variations were large. This indicates that, for many switches, error redistribution may be required to balance salt volume or methods with better tracer conservation properties should be used. Although the salt volume is relatively small compared to the water volume in the tank, the salt included might have modified the potential energy of the system. Results of the energy balance are given below.

For all $W$ cases Figure 2.12 presents the variation of $KE$, $APE$ and the sum of $KE$ and $APE$. Following Wadzuk and Hodges (2003) and Hodges et al. (2006) this sum will be called dynamic energy ($DE$). $KE$, $APE$ and $DE$ were normalised by $DE_0$, which is equal to
APE. Note that for each point in time (normalised by $T_i$) in Figure 2.12 there are two values of $KE$, $APE$ and $DE$, one corresponding to the original grid and the other corresponding to the target grid, however, because of the scale of the plots, they are almost indistinguishable. Also note, that for different cases, the times for switching were different because the solitary waves formed at different times. In all cases, the changes in normalised energy were larger for the smaller $W^{-1}$ because $DE_o$ was small compared to the other cases, so any errors in the computation of $BPE$ had an increased effect in the results of $APE$ and $DE$. We estimated the errors to be, at maximum (considering all the mass difference in the same height as the maximum initial basin-scale displacement height), around 59%, 38%, 27% and 15% of $DE_o$ for $W^{-1} = 0.30$ to $0.90$, respectively. These error estimates are likely too large as most part of the error is distributed throughout the vertical region of displacement and not concentrated on the maximum basin-scale wave height. Moreover, $BPE$ is under-estimated as the mass difference was always negative thus, implying that $APE$ and $DE$ were overestimated. Nevertheless, such errors were present in both calculations for original and target grids, amounting to very similar errors in the calculation of $BPE$ for the grids involved in the same switching. In the two smaller values of $W^{-1}$, an increase in $DE$ with time could be observed (Figure 2.12), indicating that the $BPE$-derived estimates were not reliable because $DE$ should decrease during the course of simulation (see Wadzuk and Hodges 2003 for the grid-size dependence of the energetic evolution). For cases $W^{-1} = 0.60$ and $0.90$, $DE$ decrease due to grid-switching was of the order of 0.5-1% of $DE_o$ and always smaller than 2.2% of $DE_o$, which is about the same decrease as the minimum decrease observed in between two grid switches. $KE$ calculations were completely independent of $BPE$ therefore; the changes in $KE$ were only due to grid switching. Generally, $KE$ increased due to the smoothing nature of the interpolation scheme producing small velocities in the quiescent regions. These small velocities produced the vertical velocities that induced the development of the spurious oscillations discussed earlier. Nonetheless, $KE$ changes were, at maximum, 0.3% of $DE_o$. The energy losses were thus, mostly caused by $APE$ reduction. We estimate that $\sim 0.11DE_o$ and $\sim 0.13DE_o$ (in a total of $\sim 0.70DE_o$ and $\sim 0.80DE_o$ energy losses over the entire simulations, Figure 2.12) were due to grid switching for cases $W^{-1} = 0.60$ and $0.90$, respectively. We can conclude that although the effect of a single switch is relatively small, the cumulative effect of many grid switches can amount to considerable (numerical) $APE$ energy losses over the course of
the simulation. Note that this applies for the interpolation form in the switching employed in our study.

**Grid switching strategy – energy cascade**

Figure 2.13 shows the wavelet power spectrum of the grid-switching model with the laboratory and non-hydrostatic uniform-grid models. The spectra were computed at WGA for the case $W^1 = 0.45$. Because WGA was located off the nodal position, the signal of the horizontal mode 1 basin-scale seiche ($f_1$) could only be seen in Figure 2.13 but not in Figure 2.8. At the nodal position WGB, the signal had a larger power spectra at the even horizontal modes frequencies (e.g. $2f_1$, $4f_1$, ... Figure 2.8) as revealed by the study of Boegman et al. (2005a). The wavelet power spectra of the grid switching and non-hydrostatic simulations (Figure 2.13e and 13g) showed energy levels and frequency peaks closer to the laboratory results. In the fine resolution non-hydrostatic model, the higher frequency waves ($\sim 1.1 \times 10^2 f_1$) had too much energy peaking at levels larger than the internal surge ($\sim 3.3 f_1$) (Figures 2.8b and 2.13b). These peaks did not appear in the laboratory experiments and they were likely due to the excessive amplitude of the solitons after they reflected at the end walls and interacted with each other, as can be seen in the time evolution presented in Figures 2.6 and 2.7. For the first peaking of energy at high frequencies ($0.8 T_s < \text{Time} < 1.7 T_s$) in Figure 2.13, the grid-switching and the coarse-resolution simulations, the length of time over which the energy is held at the highest frequency is significantly underestimated in comparison to the lab experiments and the fine resolution simulation. In the coarse-resolution grid simulation, this underestimation is because the number of solitons was smaller than the fine-resolution grid while in the grid-switching simulation the trailing waves were severely damped with many grid switches. It follows that the subsequent peaks in the grid-switching simulation were considerably underestimated even in comparison to the coarse-grid resolution model. As discussed earlier, the grid-switching introduced a numerical mechanism to dissipate energy and, in our results, the net result is the reduction of the internal wave amplitude of the internal surge to amplitudes similar to the laboratory experiments and excessive damping of the train of solitary waves. Also observed in the grid-switching model were brief (localized in time) bursts of energy peaking at very high frequencies ($> 10^2 f_1$), which were associated with most of grid switches (Figure 2.13c). These bursts correspond to the high-frequency numerical waves originated with the grid switching and probably amount to numerical errors in the course of the simulation. Although we acknowledge the problems, we have not pursued them any further.
2.5. Discussion

The results shown above illustrate the application of a fully non-hydrostatic model to reproduce the non-linear evolution of basin-scale waves in a laboratory tank. The numerical solution of the non-hydrostatic pressure poses two major challenges for numerical modellers of flows in lakes and enclosed basins. First, it requires the solution of a sparse system that is not amenable to direct inversion of the matrices involved in the set of equations. Second, the solution requires high-resolution grids to resolve the non-hydrostatic effects. The grid-switching technique presented above aimed to address both issues, assuming that non-hydrostatic effects are not a global feature of the flow field but rather a localised feature. Under these circumstances, the number of equations in the set of equations was reduced and a high-resolution grid was effectively employed only when and where it was required. The performance of the hydrostatic and non-hydrostatic models, the grid-switching technique and directions for future developments are discussed below.

Internal solitary waves of permanent form were previously studied using a model similar to ours (Daily and Imberger 2003), however these authors neglected the advective terms in the vertical momentum equations. Daily and Imberger (2003) showed that a non-hydrostatic model could reproduce solitary waves, but they did not attempt to model the basin-scale seiche degeneration process. Our model results were contrasted to laboratory experiments carried out by Horn et al. (2001), showing reasonable agreement, i.e. amplitudes, phases, time of emergence and number of solitons. While our model is based on the solution of the full RANS equations, the interfacial displacements can also be modelled by the solution of the KdV equation (Horn et al. 2002). Both Horn et al.'s (2002) and our models showed less agreement with the laboratory experiments with increased non-linearity of the system, predominantly after the solitons reflected at the end-walls and interacted with the other solitons in the packet. These non-linear interactions between the solitary waves were not modelled accurately although the main features of the interactions were reproduced qualitatively (not shown here for brevity). In lakes, up to 70% of the solitons energy can be dissipated at the lake boundaries (Michallet and Ivey 1999; Boegman et al. 2005a). In such lakes where the slope introduces considerable energy loss, non-linear effects of wave-wave interactions would be of smaller importance, however, in
lakes with nearly vertical boundaries (dam walls in reservoirs; for example), wave reflections without energy loss may become difficult to reproduce with great precision.

Our comparisons between hydrostatic and non-hydrostatic uniform-grid simulations confirm the results presented in Wadzuk and Hodges (2003). Although our numerical scheme is very similar to the numerical scheme presented in Wadzuk and Hodges (2003), the internal wave evolution test described in Wadzuk and Hodges (2003, p.22) was used to assess the performance of the Poisson solver (note that for the test we used the inviscid assumption and used the same form of the free-surface correction used in Wadzuk and Hodges 2003) in the models. The number of iterations performed by the Bi-CGSTAB method to achieve convergence is 89 iterations on average, which is much less than 180 obtained in the SOR employed in Wadzuk and Hodges (2003). However, the amount of work per iteration is larger in the Bi-CGSTAB. In comparison to SOR, Bi-CGSTAB requires $12N$ ($N$ is the number of grid cells) flops for vector updates plus four more inner products (Van der Vorst 1992). For these operations, the extra computation time in the Bi-CGSTAB would depend on machine architecture and compiler optimizations and a direct test is required to determine which method is indeed more efficient. The advantage of Bi-CGSTAB over SOR is that convergence is guaranteed under any circumstances including the case of non-uniform grids that lead to non-symmetric systems (Van der Vorst 1992; Chen 2003). In fact, to our knowledge, this is the first time that results of simulations of this type of non-hydrostatic models (i.e developed upon Casulli and Stelling’s 1998 approach) for internal wave modelling using non-uniform grids have been presented.

Given that run time is an issue in the computation of non-hydrostatic flows, we developed a simple grid switching strategy in order to improve run-time performance. In general, the time chosen for the switch between hydrostatic and non-hydrostatic solutions varied between $0.2T_s$ and $0.5T_s$. Although $T_s$ is the time for the emergence of the solitons (Horn et al. 2001) the non-hydrostatic effects start playing a role in the motion before the actual emergence of the solitons and in some cases (e.g. Figure 2.7), solitons emerged earlier than $T_s$, which is quite consistent with the error bars in the observation of Horn et al. (2001) (see their Figure 2.7) and the temporal horizon for steepening advanced in Hodges et al. 2006). When computational resources restrain a desirable model resolution one can tweak the non-hydrostatic solution in time scales near $T_s$ in order to set a more precise time for soliton emergence. Although substitute one form of error for another cannot be considered good modelling practice, this tweaking can allow for more vigorous basin-scale
wave steepening and compensate for lack of resolution to resolve the non-hydrostatic pressure.

Although the grid switching strategy did not show evolution of unstable modes with integration in time, two main initial caveats were identified: 1- non-hydrostatic effects and soliton generation occur in distinct parts of the domain (such as during solitons reflection and interaction) and not only around the leading soliton (Wadzuk and Hodges 2003) and 2- the fine grid model had to be run previously in order to identify the position of the leading soliton and thus the time for grid switching. As shown in Wadzuk and Hodges (2003), the occurrence of non-hydrostatic regions in the internal wave motion has a spatial and temporal character, respectively. Wadzuk and Hodges (2003) have developed a screening parameter that identifies such non-hydrostatic regions in the flow field. This screening parameter can be used to choose a numerical grid from a set of predefined grids, essentially the same as what is done in this paper but with an automatic recognition of the target grid. This approach is slightly different from nesting grid techniques used in ocean modelling (see e.g. Kowalick and Murty 1993 p.201-205; Skogen et al. 1998; Kantha and Clayson 2000 p. 206-211; Heggelund and Bernsten 2002) because we are employing several different grids and marching forward in time with the new grid, while nesting techniques such as the two-way nesting (see e.g. Heggelund and Bernsten 2002) in general, telescope in a single region of interest and march forward in time with sub-time steps of a coarse grid that completely encompasses the nested fine grid. Because the non-hydrostatic field evolves in space, grids that can automatically change with time are desirable as long as they do not undertake large computational effort. The adaptive mesh refinement (AMR) (Berger and Oliger 1984; Skamarock and Klemp 1993; Blayo and Debreu 1999) coupled with the screening parameter of Wadzuk and Hodges (2003) may be a good combination for future developments. The screening parameter can be used to select regions where telescoping is needed while the AMR technique is relatively simple and robust (Blayo and Debreu 1999) for structured grid finite-volume codes such as ELCOM. For three-dimensional non-hydrostatic (however compressible) atmospheric flows, Skamarock and Klemp (1993) used the AMR technique with relative success (they required user interaction to define where resolution should be placed) however we are not aware of its utilization in non-hydrostatic
(incompressible) ocean and lake modelling. Nonetheless, we do not discard the possibility of using two-way nesting as modelling alternatives. Together with parallelization of the code, we believe these are the next steps in the development of ELCOM for high-resolution investigation of non-hydrostatic flows in large-scale enclosed basins.

The run-time improvement was dependent on the number of grid switches in a given simulation, thus the application of the simulation will define how efficient the grid switching strategy algorithm can be. In the case presented here, the reduction in run time was not very significant (i.e. only about 60% reduction in comparison to the fine-grid simulation and less than one order of magnitude as reported by the use of AMR in Skamarock and Klemp 1993). In real lake simulations the low frequency basin-scale motions can be accurately simulated with low resolution models with grids of the order $10^{2-3}$ m. Therefore, very significant computational time saving can be achieved using a grid switching model with grids of order $10^1$ and $10^2$ m in comparison to a fine grid model of order $10^{0-1}$ m being used throughout the whole simulation. We believe that in such grid arrangements run-time reduction can be about or more than one order of magnitude, given that with many grid switches and only a 30% reduction in grid cells gave us about 40% run-time reduction. The advantage of the grid switching over the coarse-resolution grid simulation was the prediction of the phase evolution of the internal surge. The grid-switching model also modelled the amplitude of the waves better than the uniform-grid models in comparison to the laboratory experiments, however we do not consider this as an advantage because these large wave amplitudes are mainly associated with dissipation mechanisms not accounted for in the model, which include the drag at the side, top and end-walls. The grid-switching introduced numerical mechanisms for dissipation of energy (see the effect in Figure 2.12) which for the internal surge were probably comparable to the drag with the walls in the laboratory when few grid switches were invoked. However, with increasing number of grid switches, there was excessive damping of the trailing internal waves, which produced a poor performance even in comparison to the coarse-resolution simulations. Nonetheless, for most present computational capabilities, it is unlikely that non-hydrostatic codes will be used for long time simulations due to computational

* Note that Skamarock and Klemp (1993, p. 798) do not recommend the use of AMR for elliptic solvers which appears in the case of the incompressible RANS, an issue that will probably require further adaptation of the approach.
Grid-Switching Strategy

constraints (Wadzuk and Hodges 2003), thus the technique is attractive for studying single localised events where few grid switches would be invoked. Further, the interpolation scheme used in the current grid switching can be easily modified such that filter techniques (see e.g. Laval et al. 2003c) and other interpolation techniques (e.g., simple bi-linear interpolation in the horizontal) can be easily adapted in the grid-switching in order to alleviate the problems with tracer and energy conservation.

2.6. Conclusions

We have presented a non-hydrostatic version of the Estuary, Lake and Coastal Ocean Model (ELCOM) with a new capability of switching grids to resolve the regions where non-hydrostatic effects are considerably large (although the time for switching and the grid to be used needed to be known a priori). The non-hydrostatic model was able to replicate the dynamics of the internal wave degeneration process due to non-linear steepening as observed in laboratory experiments, in agreement with the findings in Wadzuk and Hodges (2003). The grid-switching strategy saves computational time without much degradation of the internal wave field solution if few grid switches are employed, being quite attractive for the simulation of single localized events in lakes where non-hydrostatic effects of the motion are large. Future fusion of the present interpolation scheme with automatic recognition of the location of non-hydrostatic effects as proposed in Wadzuk and Hodges (2003) and, possibly, with automatic grid refinement may lead to a viable alternative to compute non-hydrostatic flows where computer resources represent a limitation.

2.7. Acknowledgements

We would like to thank the constructive comments made by Kevin G. Lamb and Keisuke Nakayama. We also thank David Horn for making the laboratory data available to us. Daniel Botelho greatly acknowledges the PhD scholarship granted by CAPES (Ministry for Education and Culture, Brazil). Ben Hodges was partially funded by Office of Naval Research Young Investigator Award N00014-01-1-0574 during this project. We also thank C. Torrence and G. Compo for providing the wavelet analysis software. This manuscript forms Centre for Water Research reference ED 1657-DB.
2.8. References


Wadzuk B. M., and Hodges, B. R. (2003). Hydrostatic and non-hydrostatic internal wave models. CRWR online report 04-09, University of Texas at Austin.


<table>
<thead>
<tr>
<th>Process</th>
<th>Time Scale (s)</th>
<th>Length Scale (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basin-scale Seiche*</td>
<td>$T_s = \frac{2L}{\sqrt{g' \frac{h_1 h_2}{h_1 + h_2}}}$</td>
<td>$L_s = 2L$</td>
</tr>
<tr>
<td></td>
<td>1 hour - 1 day</td>
<td>10 - 100 Km</td>
</tr>
<tr>
<td>Internal Wave Steepening</td>
<td>$T_s = \frac{L}{\frac{3n_s}{2} \sqrt{g' \left( \frac{h_2 - h_1}{h_1 + h_2} \right)^2}}$</td>
<td>$L_s = L$</td>
</tr>
<tr>
<td></td>
<td>1 hour - 1 day</td>
<td>5 - 50 Km</td>
</tr>
<tr>
<td>Internal Wave Shoaling</td>
<td>$T_{sh} = \frac{1}{S \sqrt{g' \frac{h_1}{h_1 + h_2}}}$</td>
<td>$L_{sh} = h_1/S$</td>
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<td>10 min - 6 hours</td>
<td>1-10 Km</td>
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<td>Shear Instabilities**</td>
<td>$T_{bh} = \frac{L}{n_0 \sqrt{g' \delta_p}}$</td>
<td>$L_{bh} = \delta_p$</td>
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<td></td>
<td>10 min - 10 hours</td>
<td>1 - 10 m</td>
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<tr>
<td>Wave-Wave Interactions***</td>
<td>$T_{ww} = \frac{(\lambda_1 + \lambda_2)}{2 \sqrt{g' \frac{h_1 h_2}{h_1 + h_2}}}$</td>
<td>$L_{ww} = \lambda_1 + \lambda_2$</td>
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<td></td>
<td>1 min - 1 hour</td>
<td>100 m - 10Km</td>
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</tbody>
</table>

Table 2.1 - Time and length scales of basin-scale internal waves and small-scale process in usual geophysical flows. Typical field scales for the process are also shown. Two layer stratification assumed. $g'$ is the reduced gravity, $h_1$ and $h_2$ are the lower layer and upper layer thickness, respectively, $n_0$ is the initial basin-scale interfacial displacement, $\delta_p$ is the interface thickness, $Q = 0.5 \{1 + (1 + 4(vT_{kh}/S_p^2)^2)^{1/2}\}$ is a viscous damping factor, $v$ is the kinematic viscosity, $S$ is the bottom slope, $L$ is the basin length and $\lambda_1$ and $\lambda_2$ are the wavelength of the two interacting waves. $T_{kh}$ and $T_s$ are derived in Horn et al. (2001). ‘Horizontal mode 1 internal seiche assumed. **Shear instabilities caused by the shear induced at the nodal position of the seiche. ***Wave-wave interaction properties are considered in the vertical plan of their interaction and for weak interactions. Linear internal wave speeds $c_i = \sqrt{g' \frac{h_2 h_1}{h_1 + h_2}}$ considered for the approximations of $T_s$, $T_{sh}$ and $T_{ww}$. 
<table>
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<th>Grid</th>
<th>Resolution</th>
<th>Number of cells in the horizontal</th>
<th>Horizontal grid size (m)</th>
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<td></td>
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<td></td>
<td>(1st quarter)</td>
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Table 2.2 – Dimensions and characteristics of the grids used in the tests. All have the same vertical configuration of 113 with cell sizes varying from 2mm (from bottom to the maximum elevation of the density interface) to 10 mm at the top of the tank. The quarter of the tank where model resolution is increased is specified inside the parenthesis.
<table>
<thead>
<tr>
<th>$W^{-1}$</th>
<th><strong>Time Scales (s)</strong></th>
<th>Regimes</th>
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</thead>
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<tr>
<td></td>
<td>0.25$T_i$</td>
<td>$T_s$</td>
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<tr>
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<tr>
<td>0.90</td>
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</tr>
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</table>

Table 2.3 - Time scales and degeneration regimes for the experimental conditions (see Horn et al. 2001 for details). $T_s, T_{kh}, T_b$ are given in Table 2.1, $T_d = T_d/\gamma_d$ is the dampening time scale, $\gamma_d = \frac{1}{2} \left( \frac{\pi \delta_b A_b}{V} + \frac{\nu H T_i}{\delta_b h_1 h_2} \right)$ is the internal wave modulus of decay, $\delta_b = (\nu T/\pi)^{1/2}$ is the laminar boundary layer thickness, $A_b$ is the area of the solid boundaries of the tank, $V$ is the volume of the tank and $T_b$ is the bore formation time scale. Regime 1 - $T_s > T_d; T_{kh} > T/4; T_b > T/4$. Regime 2 - $T_s < T_d; T_{kh} > T/4; T_b > T/4$. Regime 4 - $T_s < T_d; T_{kh} < T/4; T_b > T/4$. The table shows the time scales and regimes for different $W^{-1}$ values, indicating the conditions under which each regime applies.
## Length Scales Grid Points per Solitary Wave

<table>
<thead>
<tr>
<th>$W^j$</th>
<th>Vertical</th>
<th>Horizontal</th>
<th>Vertical</th>
<th>Grid Points per Solitary Wave</th>
<th>Horizontal (model resolution)</th>
<th>$N_M/N_L$ (model resolution)</th>
<th>$N_L$</th>
</tr>
</thead>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>50</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>0.30</td>
<td>0.22 $\eta_o$</td>
<td>19.50 $\eta_o$</td>
<td>3</td>
<td>6</td>
<td>13</td>
<td>-</td>
<td>1.3</td>
</tr>
<tr>
<td>0.45</td>
<td>0.41 $\eta_o$</td>
<td>12.01 $\eta_o$</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>-</td>
<td>0.8</td>
</tr>
<tr>
<td>0.60</td>
<td>0.61 $\eta_o$</td>
<td>6.32 $\eta_o$</td>
<td>16</td>
<td>3</td>
<td>6</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>0.90</td>
<td>0.46 $\eta_o$</td>
<td>3.96 $\eta_o$</td>
<td>19</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 2.4 - Characteristics of the simulated solitary waves: Vertical and horizontal length scales of the leading solitary waves, number of grid points per solitary wave in each model and ratio between solitary waves formed in the model ($N_M$) and in the lab ($N_L$).
<table>
<thead>
<tr>
<th>Inverse Wedderburn Number ($W^J$)</th>
<th>Run Time (hours)</th>
<th>Coarse Grid</th>
<th>Fine Grid</th>
<th>Variable Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>N-H</td>
<td>H</td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td>1.08</td>
<td>4.93</td>
<td>4.28</td>
</tr>
<tr>
<td>0.30</td>
<td></td>
<td>0.90</td>
<td>3.65</td>
<td>3.03</td>
</tr>
<tr>
<td>0.45</td>
<td></td>
<td>1.09</td>
<td>4.68</td>
<td>3.83</td>
</tr>
<tr>
<td>0.60</td>
<td></td>
<td>1.08</td>
<td>4.28</td>
<td>4.08</td>
</tr>
<tr>
<td>0.90</td>
<td></td>
<td>0.90</td>
<td>4.09</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Table 2.5 - Run time model performances for different scenarios. H stands for hydrostatic and N-H for non-hydrostatic. In the last column, the number of grid switches needed is shown inside the parenthesis.
<table>
<thead>
<tr>
<th>$W^I$</th>
<th>Coarse Grid</th>
<th>Fine Grid</th>
<th>Variable Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>N-H</td>
<td>H</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1028</td>
<td>0.0225</td>
<td>0.0260</td>
</tr>
<tr>
<td>0.30</td>
<td>0.1235</td>
<td>0.0304</td>
<td>0.0366</td>
</tr>
<tr>
<td>0.45</td>
<td>0.1021</td>
<td>0.0238</td>
<td>0.0290</td>
</tr>
<tr>
<td>0.60</td>
<td>0.1031</td>
<td>0.0260</td>
<td>0.0272</td>
</tr>
<tr>
<td>0.90</td>
<td>0.1230</td>
<td>0.0272</td>
<td>0.0317</td>
</tr>
</tbody>
</table>

Table 2.6 - Ratio between simulated time and run time for different scenarios. H stands for hydrostatic, N-H for non-hydrostatic, Max H for maximum ratio achieved in the hydrostatic part of the grid-switching run, Min for the minimum ratio achieved by the non-hydrostatic part of the grid-switching run plus the time taken for interpolation of values from the previous used grid, Max for the maximum ratio achieved by the non-hydrostatic part of the grid-switching run plus the time taken for interpolation of values from the previous used grid and Total for the ratio achieved by the entire run.
Figure 2.1 – Power spectra of an isotherm displacement in Lake Biwa, Japan for a 10-day period before (dotted line) and a 10-day period after (solid line) a typhoon passed over the lake. Reproduced from Saggio and Imberger (1998). Note the increase in the spectral density at the frequency band of both the basin-scale waves and the non-linear waves and the spectral gap left for the free-gravity waves frequency band.
Figure 2.2 – Schematic diagram of the experimental set up. a) The ultrasonic wave gauges were located at the positions marked A, B and C. b) and c) The tank and the density structure immediately before and after an experiment commences. After Horn et al. (2001)
Figure 2.3 – Sketch of the five-step interpolation method used in the grid-switching strategy. a) Original grid, b) intermediate grid and c) target grid. Note that the intermediate grid has the same vertical resolution of the target grid but maintains the same horizontal resolution of the original grid.
Figure 2.4 - Side view of the grids used in the simulations. a) Coarse resolution grid. b) Fine resolution grid. c) Variable resolution grid with increased resolution in the first quarter of the tank. d) Variable resolution grid with increased resolution in the second quarter of the tank. The other two grids (not shown) have a similar arrangement to grids c) and d) but with resolutions in the fourth and third quarters, respectively.
Figure 2.5 – Normalized interfacial displacements registered at wave gauge B located near the centre of the longitudinal section of the tank for the case $W^{-1}=0.15$. Displacements are normalized by the maximum basin-scale wave initial displacement $\eta_{0}$. The time in the lower axis is normalized by $T_{s}$ while in the upper axis time is normalized by $T_{r}$. The solid vertical lines indicate each half period of the basin-scale wave and the dot-dashed vertical line indicates the dampening time scale. Dotted vertical lines to appear in the next plots indicate the steepening time scale.
Figure 2.6 – Normalized interfacial displacements registered at wave gauge A located approximately at the third quarter of the longitudinal section of the tank. Colour code, normalization of time and normalization of displacements are as in Figure 2.5. The plots are for the case $W^*=0.45$. a) Using grid of fine horizontal resolution. b) Grid of coarse horizontal resolution. c) Variable resolution using grid switching. The fine grid non-hydrostatic is also plot in c) for comparison. The text “GS” in plot c) indicates the times when grid switching was invoked.
Figure 2.7 – Normalized interfacial displacements registered for the case $W^{-1}=0.90$ using grid of fine and coarse resolutions. a) At wave gauge A. b) At wave gauge B. c) At wave gauge C. Colour code, normalization of displacement and normalization of time are as in Figure 2.5. Results of the grid switching model and the laboratory data are staggered in the plot with a $0.77 \eta/\eta_0$ shift for a better visualization. The text “GS” in the plots indicate the times when the grid switching was invoked.
Figure 2.8 – Normalized wavelet power spectrum (panels a, c and e) and normalized global power spectrum (panels b, d and f) of interfacial displacements at wave gauge B and case $W^{-1}=0.90$. a) and b) Laboratory data. c) and d) Non-hydrostatic fine resolution model. e) and f) Hydrostatic fine resolution model. The time was normalized by $T_s$ (upper axis in wavelet plots) and by $T_t$ (lower axis in wavelet plots). The frequency was normalized by the initial basin-scale frequency $f_0$. In the wavelet plots, contour lines indicate the 95% significance levels, while the line linking the top left corner to the right top corner indicates the area where edge effects due to padded zeroes in the Fourier transform have an influence on the results (see Torrence and Compo 1998). In the power spectrum plots the dotted line indicates the 95% level of significance.
Figure 2.9 – Smoothing effect of the interpolation scheme. The wave is propagating from the right hand side of the tank to the left. Top panel shows the wave field before interpolation and bottom panel shows the wave field after the interpolation. Vertical lines show the horizontal grid resolution. Effects of smoothing are more noticeable for the trailing waves as resolution is decreased. Vertical scale is larger than horizontal scale to highlight the smoothing effect. The vertical coordinate axis is non-dimensionalized by the tank height (H = 0.29 m), the horizontal coordinate axis is non-dimensionalized by the tank length (L = 6 m) and the colour scale in ppt.
Figure 2.10 – Spurious oscillations in the fluctuating density field introduced by the interpolation scheme. Case shown is $W^1 = 0.60$ and the waves are propagating to the right. Top panel: fluctuating density field after the interpolation at time = 191 s. Centre panel: fluctuating density field after 1 s at time = 192 s showing the spurious oscillations that appeared between $y/L = 0.83$ and $y/L = 0.95$. Bottom panel: fluctuating density field after 5 s at 196 s showing that the oscillations practically damped out. Axis scales are equal and use the same space non-dimensionalization as Figure 2.9. Colour scale is in Kg m$^{-3}$. Vertical lines show the horizontal grid spacing.
Figure 2.11 – Evolution of the total salt volume in the grid-switching simulations. The total salt volume, TS is non-dimensionalised by the initial total volume of salt in the tank. Time is non-dimensionalised by the basin-scale wave period $T_i = 109$ s.
Figure 2.12 – Evolution of dynamic energy components in the grid-switching simulations. 
a) Kinetic energy $KE$. b) Available potential energy $APE$. c) Dynamic energy $DE$. Energy is non-dimensionalised by the initial $DE_0$. Time is non-dimensionalised by the basin-scale wave period $T_i = 109$ s.
Figure 2.13 – Same as Figure 2.8 but for wave gauge A and case \( W^{-1}=0.45 \). a) and b) Laboratory data. c) and (d) Non-hydrostatic fine-resolution grid simulation. e) and f) Non-hydrostatic coarse-resolution grid simulation. g) and h) Grid-switching simulation. Vertical dot-dashed lines in g) correspond to the times when switching was invoked.
3. Down-Scaling Model Resolution to Illuminate the Internal Wave Field in a Small Stratified Lake

3.1. Abstract

This paper presents the application of hydrostatic and non-hydrostatic three-dimensional hydrodynamic models to a stratified lake. Focus was given to the multi-scale response of the internal wave field to strong wind gusts exceeding 20 m s\(^{-1}\). Simulations were performed using different horizontal grid resolutions with uniform grid sizes varying from 100x100 m to 10x10 m. Results of the hydrostatic models were used to investigate the large-scale features of the internal wave motion. With the intent of investigating the high-frequency waves observed results of these simulations were used as initial conditions for non-hydrostatic simulations using smaller grids. Wavelength of the high frequency waves decreased with grid resolution however; none of the uniform grids were sufficiently fine to capture the waves of the highest frequency. Simulations performed using a non-uniform grid produced internal waves of similar frequency of the waves observed in the field. The simulations showed that these waves were shear unstable modes and that their vertical and horizontal length scales were in close agreement with results from linear stability analysis.

3.2. Introduction

Three-dimensional hydrostatic models have been commonly used to describe basin-scale internal wave evolution in density-stratified lakes (e.g. Hutter et al. 1998, Hodges et al. 2000, Laval et al. 2003, Rueda and Schladow 2003, Appt et al. 2004). However, internal wave phenomena can be highly nonlinear, especially at smaller spatial and temporal scales, and the hydrostatic assumption may not be valid (Hodges et al. 2006). Extensive field measurements have shown that detecting the generation mechanism and propagation of these non-linear waves can be quite difficult (Thorpe et al. 1996). Thus, models that are able to simulate such non-linear waves can improve our understanding of internal wave dynamics and provide further insight into the flux path of energy and mass in lakes. On the other hand, it is a big challenge to calculate the full internal wave spectrum given the large disparity between the wave’s length scales, from 1000-10000 m for the basin-scale waves.
down to ~100-1000 m for soliton-like waves and to about 10-100 m for shear instabilities. The role played by each of the internal waves in the energy flux path (Imberger 1998) from the work done by the wind to turbulent dissipation is only now being revealed (Imberger 1998, Wüst et al 2000, Boegman et al. 2005, Venayagamoorthy and Fringer 2005, Gomez-Giraldo et al. 2006). Furthermore, the implications of the high-frequency (small-length scale) waves to the ecological patchiness of lakes are still poorly understood. For practical purposes, ability in modelling the full wave spectrum will provide an invaluable tool for management and research in lakes and enclosed basins.

Extensive progress has been made to derive three-dimensional numerical models that are able to solve the Navier-Stokes equations for free-surface flows without the hydrostatic approximation. As a method to solve for non-hydrostatic pressure, the fractional step method (Kim and Moin 1985; Casulli and Stelling 1995) has received considerable attention over the last decade. The method first resolves a flow field using the hydrostatic approximation and uses this approximate hydrostatic field as an initial guess for the final flow field that is obtained with the solution of the Poisson-type equation that arises from the formulation of the hydrodynamic pressure restricted by the divergence-free velocity field condition. The method introduces a splitting error because advection and hydrodynamic pressure gradients are computed separately, leading to first-order time accuracy and damping of the free-surface waves (Zijlema and Stelling 2005). This problem can be circumvented with further corrections of the free-surface level and velocity field (Casulli 1999, Chen 2003) or application of pressure-correction methods (Li and Fleming 2001, Zijlema and Stelling 2005). The fractional step method is a natural extension of previous 3-D hydrostatic models (Casulli and Cheng 1992, Hodges 2000) and has been adapted to work with Cartesian vertical coordinates (Casulli 1999) and with sigma coordinates (Mahadevan et al. 1996). Most schemes are semi-implicit when resolving the free-surface elevation in the hydrostatic step. Unlike explicit schemes, implicit and semi-implicit schemes are unconditionally stable if the implicitness parameter in the discretisation of the Navier-Stokes equations varies between 0.5 and 1.0 (Casulli and Catani 1994). As a result, they do not require the small time steps imposed by the Courant-Friederich-Levy (CFL) stability condition based on the propagation speed of the rapidly evolving free-surface waves (Casulli and Catani 1994).

While model improvements have targeted more accurate and efficient simulation of free-surface waves, i.e. by reducing the number of vertical grid cells without loss of
Illuminating the internal-wave field

accuracy (Zijlema and Stelling 2005), the use of non-hydrostatic models to simulate the internal waves in density stratified water bodies remains limited. Moreover, fine vertical grids are required to delineate the internal modal structure. Daily and Imberger (2003) used a simplified non-hydrostatic model (considering only local vertical acceleration) to reproduce the propagation of soliton-like waves in a linearly stratified fluid. Extension of the model to the full vertical momentum equations was used to simulate long internal wave degeneration in a laboratory tank (Wadzuk and Hodges 2004, Botelho et al. Chapter 2). Other three-dimensional non-hydrostatic models that incorporate a rigid lid and did not necessarily use the fractional step method have been used to study internal flows at similar laboratory scales, i.e. few meters in the horizontal direction and tens of centimetres in the vertical scale (Fringer and Street 2005, Venayagamoorthy and Fringer 2005).

Despite the applicability of the model to these small scales, Wadzuk and Hodges (2004) could not successfully extend their non-hydrostatic model to field scales due to uncontrollable instabilities in the calculations. Applications of three-dimensional non-hydrostatic models to field scales have been limited (note that here we are not considering the large-scale oceanic applications calculated in massive parallel systems). Notable exceptions are the work of Mahadevan et al. (1996) in the Gulf of Mexico, Casulli and Stelling (1998), which is actually only quasi-hydrostatic, and Casulli and Zanolli (2002) in Venice Lagoon, Yamashiki et al. (2003) in Lake Biwa, Koçyigit and Falconer (2004) in Esthwaite Water and, most recently, Sato et al. (2006) in Isahaia Bay. Among these, Sato et al. (2006), Yamashiki et al. (2003) and Mahadevan et al. (1996) are the only works that applied a three-dimensional non-hydrostatic model to a stratified water body; however, none of these models were used to investigate internal wave motion and the latter two did not have sufficient horizontal grid resolution for depicting non-hydrostatic effects (Wadzuk and Hodges 2004).

For medium to large lakes high resolution is a severe limitation as typical computational resources (Wadzuk and Hodges 2004) and technical expertise for setting up parallel computing systems are not readily available. Recently, Botelho et al. (Chapter 2) have developed a grid switching strategy aiming to decrease computational run-time of non-hydrostatic models. In this strategy, a hydrostatic model is used for simulation spin-up and for reproducing large-scale low frequency motions, i.e. basin-scale internal waves. When the motion becomes non-hydrostatic the flow field is interpolated to a higher resolution grid where non-hydrostatic effects are large and the simulation is continued with
this new grid using a non-hydrostatic model. The model was validated by simulating the
degeneration of basin-scale waves into high-frequency waves in a laboratory flume
(Botelho et al. - Chapter 2).

The objectives of the research presented in this paper are to show that (i) it is possible,
given some limitations to be addressed in the text, to simulate the full spectrum of internal
waves at field scales; (ii) a non-hydrostatic simulation is only required when non-
hydrostatic effects become significant and; (iii) our non-hydrostatic model can reproduce
(and elucidate) the generation mechanism and spatial scales of high-frequency waves
observed in the field. A simple spatial downscaling with grid-switching approach (see text)
was used to illuminate the characteristics of the internal wave flow field under analysis.
The paper is organized as follows. First, we briefly describe the modelling approach, the
main characteristics of Lake Pusiano, and the grid switching (down-scaling) strategy. The
model results are then compared to field measurements and used to analyse the high-
frequency internal wave dynamics in the lake. Next we present a discussion of the model
results, with respect to previous studies, directions for future developments and
applicability to ecological processes.

3.3. Hydrodynamic Model and Lake Pusiano Characteristics

ELCOM

The Estuary, Lake and Coastal Ocean Model (ELCOM) developed at CWR (Centre for
Water Research) was used in the simulations presented in this paper. ELCOM solves the
Boussinesq approximated Reynolds-averaged Navier-Stokes equations using a semi-
implicit formulation on a finite-volume framework that incorporates structured hexahedral
cells in an Arakawa-C grid stencil (Hodges 2000). Numerical techniques coded in the
hydrostatic version of ELCOM have been extensively documented in Casulli and Cheng
(1992), Casulli and Catani (1994), Hodges (2000), Hodges et al. (2000), Laval and
Imberger (2003) and Laval et al. (2003). An operator-splitting treatment of the momentum
equations is used to calculate each of the terms. Advection of momentum is treated with an
Euler-Lagrange CFL-based hybrid scheme (Hodges 2000) that is third-order spatially
accurate (Wadzuk and Hodges 2004). Horizontal viscous terms are treated with a second-
order spatially-accurate centred scheme. Vertical mixing of momentum and scalars is
treated simultaneously with the vertical mixing layer model described in Hodges et al.
(2000), Laval and Imberger (2003) and Laval et al. (2003) but with the inclusion of
Illuminating the internal-wave field

gradient-Richardson-number based regimes to compute the mixing time scales. Description of the closure scheme is beyond the scope of this paper such that the details of the scheme will be presented elsewhere (Simanjuntak MA et al. 2006 Benthic and interfacial mixing in a shallow salt-wedge estuary. Submitted to J. Hydr. Eng. ASCE). The free-surface elevation in the hydrostatic step and in the hydrostatic model is calculated following Casulli and Cheng (1992) using a forward-spatial, fully-implicit discretisation of the free-surface terms. A forward-in-space explicit-in-time discretisation is used for the hydrostatic baroclinic pressure (Casulli and Stelling 1998; Hodges 2000). Coriolis terms involving vertical velocities in the horizontal momentum equations as well as all Coriolis terms in the vertical momentum equations are neglected. Advection of scalars is accomplished with the ULTIMATE-QUICKEST scheme (Leonard 1991). The UNESCO (1980) equation of state is used to compute density every time temperature is calculated. The non-hydrostatic baroclinic term is calculated with the fractional step method (Kim and Moin 1985, Casulli and Stelling 1995) with posterior free-surface correction (Casulli 1999) and permits the use of non-uniform grids in all directions (Botelho et al. – Chapter 2). Calculation of the non-hydrostatic pressure uses a Bi-CGSTAB solver (Van der Vorst 1992) with an incomplete Cholesky pre-conditioner. When required (see below), grid switching was performed with the interpolation procedure described in Botelho et al. (Chapter 2). Boundary conditions for velocities and scalars were the same as in other similar studies (e.g. Laval and Imberger 2003; Rueda and Schladow 2003), i.e., no-slip at the bottom and free-slip at the side boundaries. The land boundaries are considered impermeable and no-flux boundary conditions are set for momentum and scalars. Wind and bottom shear stresses are computed with quadratic drag laws based on the shear velocity (Laval and Imberger 2003). Surface heat transfer considers the atmospheric boundary layer stability conditions subject to wind stress, evaporation/condensation, conduction and short and long wave radiation (CWR - ELCOM web manual 2006, available at http://www2.cwr.uwa.edu.au/~ttfadmin/cwrsoft/doc/elcom_science/index.html). In the present study, no adjustment of parameters was performed in ELCOM in a calibration-free exercise. Lake characteristics and the different grids used to illuminate the internal wave field in Lake Pusiano are described below.

Lake Pusiano

Lake Pusiano is a small lake located in the northern Italy (48.80°N, 9.27°E). An intense field experiment was carried out from 22 (day 203) to 30 (day 211) July 2003 with the aim
of understanding the interaction between physical mechanisms and the dominance of the cyanobacteria *P. rubescens* in the Lake. Seasonal distribution of algae species, competitive advantages of *P. rubescens* over other algae and trophic evolution of the lake conditions from a previous experiment are thoroughly discussed in Legnani et al. (2005) and references therein. In the present paper, we discuss only the numerical modelling efforts to reproduce the internal wave field generated in response to an episodic wind event, as first discussed by Boegman et al (2005). The main topographical characteristics of the lake are given in Table 3.1. The deepest point of the lake basin is eccentric to the north and the bathymetry has steeper northern slopes and more gradual southern slopes (Figure 3.1a). A high-resolution (0.1 Hz sample rate) LDS (Lake Diagnostic System) was deployed approximately at the deepest point of the basin (Figure 3.1a) to collect water temperature, wind speed and direction, relative humidity, air temperature and net radiation. Temperature data were collected with thermistors (accuracy of ±0.01 °C) spaced 0.75 m apart starting from 0.30 m depth in the water column.

Occasionally, strong pulses of northerly winds (Figure 3.2a) with gusts exceeding 20 m s⁻¹ excited internal waves in the thermally stratified lake (Figure 3.2c). We will concentrate on the internal wave response forced by a wind event of short duration (Figure 3.2) that occurred in the first few hours of day 205 (July 24). Similar wind events occurred at this time of the year (i.e. at the end of day 208). Wind events of longer duration that lead to upwelling also occurred (i.e. at night of day 209, July 28) but are not analysed in this paper. The wind event analysed occurred at night (thus, no short-wave radiation heat fluxes) and were accompanied by sudden temperature decreases (Figure 3.2) and subsequent humidity increases (not shown), suggesting a large exchange of latent heat from the lake surface to the atmosphere, turbulent stirring of the surface waters, build-up of a surface shear layer and the evolution of a strong convective motion in the surface layer (Imberger 1985). Figure 3.2c shows the temperature field for the 8 days of observations. The wavelet analysis of the 18°C isotherm displacements (Figures 3.2d and 3.2e) depicted high-frequency low-amplitude waves (Figure 3.2d and in detail in 3.2e) concurrent with the increase of wind velocities (Figure 3.2a). Further details of the wave field are given in the Results and Discussion sections. In order to simulate the small scales we resorted to different grid configurations to represent the lake (Figure 3.1).
Lake Pusiano Model

Simulations were carried out in grids of different horizontal resolutions and a fixed vertical resolution (Figure 3.1). In the vertical, the grid was non-uniform with the highest resolution of 0.25 m where the temperature gradient was largest, between 3.3 m and 8.8 m depths from the mean lake surface level (Figure 3.1c). Towards the surface, the grid size was incremented by 0.05 m until it reached 0.65 m and was similarly increased towards the bottom to 0.95 m below which six 1 m and two 1.1 m grid-size cells filled the remaining domain (Figure 3.1c). An improved representation of the internal wave motion could be achieved without excessively increasing simulation run-time using this vertical grid configuration in comparison to a 1 m uniform vertical grid size throughout.

In order to understand the gross features of the internal wave motion in the lake, initial simulations were performed using the hydrostatic version of ELCOM in a 100x100 m horizontal uniform grid (Figure 3.1b) and in a 20x20 m horizontal uniform grid (Figure 3.1d). The simulations using the 100x100 m horizontal grid were performed in a Pentium 4 2.4 Ghz processor with 1 GB of memory available. All other simulations were performed in a PowerPC G5 2 GHz running in only one processor with 4 GB of memory available.

Non-hydrostatic simulation for the whole period between day 203 and day 211 was impractical using the 20x20 m grid; the expected run-time for this 8-day simulation was around 44 days (compared to approximately 5 days for the hydrostatic simulation, Table 3.2). Non-hydrostatic simulations were not performed with the 100x100 m grid because this resolution was too coarse to resolve the scales on which non-hydrostatic effects become important (Wadzuk and Hodges 2004). Therefore, for the entire period (days 203 to 211), only hydrostatic simulations in the 100x100 m and in the 20x20 m grids were produced. Water temperature data and weather data from the LDS were used to supply initial water temperature conditions, wind stress and atmospheric forcing to the model; the forcing was applied uniformly over the lake surface.

Non-hydrostatic simulations were performed with the 20x20 m uniform (Figure 3.1 d), 10x10 m uniform (Figure 3.1e) and 1x1 m non-uniform (Figure 3.1f) grids and for a time span compatible with the response to a particular wind event (see below). For clarity, only the parts of the grids given by the detailing squares in Figures 3.1a, 3.1b and 3.1d were plotted in Figures 3.1d, 3.1e and 3.1f. The non-uniform grid had a high density of cells in the region where T1 was located (Figure 3.1a). Inside this 11x11 m region (Figures 3.1d and 3.1f), the horizontal grid size was 1x1 m and outside the region the grid expanded...
towards the lateral boundaries of the domain with an expansion coefficient of 1.10 (Figure 3.1f). The choice of resolution in this case was solely driven by the possibility of making comparisons with an appropriate data set, i.e., one with very high-resolution sampling depicting the characteristics of the highest frequency waves observed. The time steps were chosen to provide sampling resolution compatible to the thermistor chain and keep the internal wave Courant-Friederich-Lévy number (IW CFL) below the number required for stability, that is $< 0.71$ (Hodges 2000). Initial conditions of these non-hydrostatic simulations were obtained with a grid-switching algorithm (Botelho et al. – Chapter 2) that interpolated the velocity, free-surface elevation and scalars computed with the hydrostatic simulation on the 20x20 m grid to the other grids (not employed for the 20x20 m grid). Results of the numerical simulations on the 20x20 m, 10x10 m and 1x1 m (non-uniform) grids presented in this paper refer “only” to non-hydrostatic simulations. Table 3.2 presents all descriptive details of the simulations for which results are presented below.

3.4. Results

Dominant frequencies

Field measurements of the internal waves simulated in this paper are given by the isotherm displacements shown in Figure 3.2. The characteristic frequency of the waves was obtained by the wavelet power spectrum calculated using a Morlet mother wavelet using the same procedure described by Torrence and Compo (1998). There were clear peaks (within 90% confidence intervals) at $8 \sim 9 \times 10^{-3}$, $2 \times 10^{-4}$, $1 \times 10^{-4}$, and $5 \times 10^{-5}$ Hz (Figures 3.2d and 3.2e). The peak at $8 \sim 9 \times 10^{-3}$ Hz was the same order of $N$ ($1.5 \times 10^{-2}$ Hz), the maximum buoyancy frequency of the metalimnion, and appeared as the wind events commenced (Figure 3.2a). The peak at $1 \times 10^{-4}$Hz corresponded to the signal of the first vertical and horizontal mode (V1H1) basin-scale wave, consistent with the frequency of a V1H1 basin-scale wave in a continuous stratification model of an idealized two dimensional basin (see e.g. Münich et al 1992) of 1700 m length by 24 m depth with the same background temperature profile as measured in Pusiano. The peak at $2 \times 10^{-4}$Hz was approximately double the frequency of the third peak and corresponded to an internal surge as demonstrated by Boegman et al. (2005). The other lower frequency peak at $5 \times 10^{-5}$Hz was likely resulting from higher vertical mode waves and occurred out of our window of analysis (Figure 3.2e). We hypothesize that the highest-frequency peak was due to waves generated by wind-induced shear instabilities. Waves of similar frequency characteristics
concurrent with wind events are ubiquitous in lakes forced by the wind (Wiegand and Carmack 1986, Stevens 1999, Boegman et al. 2003). Analyses of observations in other lakes (Boegman et al. 2003, Gomez-Giraldo et al. 2006) suggest that these waves have wavelengths as small as 20 m; high-resolution models are required to depict the motion at such scales.

Model resolution

The temporal evolution of the isotherm displacements at the LDS location during the first wind event is shown for the field data and the different resolution model outputs with the wind forcing in Figure 3.3. Before plotting, data was interpolated from the model outputs to the same position of the thermistors in the field. All models outputs have the same time resolution as the field data. From Figures 3.3c to 3.3e, it may be seen that the higher the model resolution the finer the apparent metalimnion structure in the simulations. Notably, the 100x100 m resolution simulation had a much smoother wave field and could not detail the subtleties of the constriction and expansion of the metalimnion in the same way as the 10x10 m and the 20x20 m resolution simulations. Although the low to medium-frequency wave motion (i.e. $1 \sim 2 \times 10^4$ Hz) was clearly reproduced in all different resolution simulations (despite the limited time for simulation spin-up), the high-frequency signal (seen in enclosure in Figure 3.3b) that produced the dominant signal at $8 \sim 9 \times 10^{-3}$ Hz could not be observed in any of the grid configurations. These results implied that the spatial scales of these waves were smaller than the resolution these grids could resolve.

Figure 3.4 shows the three-dimensional spatial details of the internal wave field given by the 21.5°C isotherm displacements at time 5:13 on the 24th of July (time 205.2178). All grids represented the tilt of the upper portion of the metalimnion downwelling at the southern shore of the lake. This initial tilt generates the horizontal pressure gradients that trigger the (low-frequency) basin-scale waves. The higher the horizontal resolution, the more detailed the displacements of isotherms, the more numerous are the waves that appeared and the higher their wavenumbers (and likely, the higher their frequency) (Figures 3.4 and 3.5). These waves were present throughout the lake and had different structure according to their location (Figures 3.4 and 3.5). The waves had higher amplitude (Figure 3.4) and smaller horizontal scales in the shallows of the south/south-western shores (Figures 3.4b, 3.4c, 3.5a and 3.5c), with crests and troughs probably modulated by the shear induced by the bathymetry. At the north part of the lake, the waves had crests and troughs
roughly oriented in the crosswind direction (NE-SW direction) (Figures 3.4b, 3.4c, 3.5a and 3.5c), approximately at 150° from the north direction (Figure 3.5).

The temperature contour in the vertical plane that passes through points A-A' (i.e. in the direction of the wave propagation, Figures 3.5b and 3.5d) is plotted in Figures 3.5a and 3.5c. For comparison we chose a wave approximately 200 m from the LDS station that could be depicted by the 20x20 m resolution simulation (Figure 3.5). Note, however, that many smaller waves seen in the 10x10 m resolution simulation were absent in the lower resolution models. The horizontal scales were larger in the 20x20 m grid resolution simulation (wavelength $\lambda \approx 150$ m, Figure 3.5a) compared to the 10x10 m grid ($\lambda \approx 100$ m, Figure 3.5c) and the vertical scales, respectively, were smaller, amplitude $A \approx 0.50$ m compared to $A \approx 0.70$ m. With increased grid resolution waves with higher wave numbers were produced, however the high-frequency wave signals similar to field conditions were absent in Figure 3.3, indicating that even the 10x10 m resolution grid was not dense enough to fully resolve high-frequency waves. In our shear-instability waves hypothesis, the wavelength of the waves has to be approximately the same size of the fastest growing unstable modes (Sun et al. 1998). As shown later, linear stability analysis (e.g. Hazel 1972, Sun et al. 1998, Boegman et al. 2003, Gomez-Giraldo et al. 2006) revealed that the horizontal wavelength of the fastest growing vertical modes at the position of the LDS station were of the order of $\sim 10$ m. Clearly, the wavelengths reproduced by the model had much larger horizontal scales than predicted by linear stability analysis.

To assess the skill of the non-hydrostatic model to resolve the waves near the higher frequency limit a non-uniform grid with high resolution in the surrounding of the LDS (Figure 3.1 and Table 3.1) was used. Two simulations were made in the non-uniform grid using different initial conditions: first, a cold-start with no initial velocity field and second, a hot-start where the initial conditions were obtained from the interpolation of the velocity field simulated using a 20x20 m grid. Both initial conditions used a temperature field simulated on the 20x20 m grid that was interpolated to the non-uniform grid (see Botelho et al. – Chapter 2 - for the interpolation method of scalars and velocity field). These two configurations were required to test our hypothesis that the high-frequency waves were triggered by shear instabilities.

Figure 3.6 presents the isotherm displacements modelled using the non-uniform grid and observed in the field. The observed waves were not reproduced in the cold-start simulation (Figure 3.6c), as there was no background shear to drive instabilities. In
contrast, the hot-start simulation produced waves that were very similar to the field observations, having periods of approximately 100 s (Figure 3.6b) that were slightly (considering the wide range of frequencies of the internal wave spectrum) smaller than the observed periods of approximately 120 s. A delay of approximately 900 s can be seen between the waves observed in the field and the waves produced in the simulations. However, the waves persisted for a similar duration (~40 min) before subsiding. The delay is likely a result of slight differences between simulated and observed buoyancy frequency and vertical shear distributions in the location of the generation of these waves (see below). Especially, the vertical shear distribution is a very sensitive input for the linear stability analysis (Gomez-Giraldo, pers. comm.). The simulated interfacial displacements, given by the 21°C isotherms, had larger amplitudes (~1.1 m for the maximum excursions) compared to the field data (~0.6 m).

The horizontal scales of the simulated waves are presented in Figure 3.7a to 3.7c, using the 21°C isotherm surface produced by the hot-start model at time 205.2178. The small displacements that make the high-frequency internal wave field are not distinguishable in Figure 3.7a. As shown in Figure 3.6, the development of the high-frequency waves required the initial background velocity and temperature fields, which, in turn, were obtained with the interpolation of the flow field computed using the hydrostatic simulation on the 20x20 m uniform grids. Figure 3.7a shows that the initial background basin-scale internal wave field in the simulation using 1x1 m non-uniform grid was preserved after the interpolation of the results obtained with the hydrostatic model on the 20x20 m grid (Figure 3.4b). Figures 3.7b and 3.7c show the plan view of the 21°C isotherm surface illustrating the development of the crests and troughs of the waves aligned roughly in the W-E direction. The approximate direction of the propagation of the waves in the vertical plane through section B-B' was approximately 170° from the north. In Figure 3.7d, both horizontal ($\lambda \approx 12$ m) and vertical scales ($A \approx 1$ m) in the vertical plane B-B' can be observed. Given Nyquist spatial-scales considerations, it is clear that the 10x10 m and larger horizontal grid sizes were unable to resolve the spatial scales of these high-frequency waves, even though the time step used in this larger grid simulations (10 s) was six times smaller than the time step required to cut-off the Nyquist high-frequency signal (i.e. 60 s for 120 s period waves).

**Stability Analysis**

In order to verify the spatial scale of the waves produced by the model, we performed a linear stability analysis of the stratified shear flow at the LDS station. The stability analysis
consisted of the solution of the Taylor-Goldstein equation for a given set of real wavenumbers $K = (\kappa + i)^{1/2}$ (where $\kappa$ and $\lambda$ are the W-E and N-S wavenumber components, respectively) of infinitesimal wavelike disturbances imposed on a basic Boussinesq and hydrostatic stratified shear flow (see Antenucci et al. 2001, Boegman et al. 2003, Gomez-Giraldo et al. 2006). The existence of an imaginary part $c_i$ of the complex phase speed of the disturbances $c = c_r + ic_i$ (where $c_r$ is the real part of the phase speed) for a given $K$ indicates that the solution is unstable and the instability (in an inviscid fluid) have a growth rate $\omega_i = Kc_i > 0$ (see e.g. Sun et al. 1998, Kundu and Cohen 2002). The frequency of the unstable mode in a frame of reference moving with the mean flow is $\omega_k = Kc_r$. The fastest growing instability modes, i.e. largest $\omega_k$, are expected to dominate and being observed as the flow evolves (Sun et al. 1998). Solutions of this eigenvalue problem were investigated for wavelengths $\lambda = 2\pi/K$ varying from 2 m to 60 m in increments of 2 m (i.e. $K$ varying from $1.047 \times 10^{-1}$ rad m$^{-1}$ to $3.142$ rad m$^{-1}$) in vertical planes on which the horizontal velocities were projected. The planes varied in 11.25° increments from the south to north directions. We did not measure the flow velocities in the field; and therefore constrained our analysis to density and velocity profiles given by the non-uniform grid simulation. The velocity and density profiles were linearly interpolated to 10cm intervals from the surface to the bottom of the water column and averaged between times 205.1985 and 205.2015 (260 s) just prior the high-frequency oscillations appearing in our signal (Figure 3.6).

Figure 3.8 shows the background conditions for an angle of 168.75° from north and the results of the stability analysis. It can be seen that the fastest growing modes were at 2.9 m depth where the vertical shear was the greatest (0.15 s$^{-1}$, Figure 3.8b), the buoyancy frequency (squared) was relatively small ($7.7 \times 10^{-4}$ rad$^2$ s$^{-2}$) compared to the seasonal thermocline ($1.21 \times 10^{-2}$ rad$^2$ s$^{-2}$, Figure 3.8b) and the gradient $Ri$ (ratio between the buoyancy frequency squared to the vertical shear squared) was the smallest ($-0.025$, Figure 3.8c). A depth of 2.9 m was therefore the critical depth; where the velocity was equal to the fastest growing mode celerity (real part of the eigenvalue, $c_r$) and the instabilities developed (Figure 3.9). The mode structure and the phase shift for the fastest growing mode at the critical depth is shown in Figure 3.8d. The wavelengths of the modes associated with the largest growth rates ($-0.021$ s$^{-1}$) were equal to 6 and 8 m and their direction of propagation were south (180°) and south easterly (168.75°), respectively (Figures 3.8e and 3.8f). The latter direction coincided with the direction of propagation of the simulated waves.
whilst their wavelength was \( \sim 30\% \) smaller than the simulated (Figure 3.7d). The frequency of the fastest growing mode was 0.08 Hz (period of \( \sim 150 \) s), slightly (again, considering the wide range of frequencies of the internal wave spectrum) smaller than the frequency of the simulated high-frequency waves (Figure 3.6).

**Billowing**

As shown in Figure 3.8, the very low values of \( Ri < 0.05 \) and the ratio between the shear layer thickness to the density interface thickness (<2.4) indicates that Kelvin-Helmholtz (K-H) modes would have much larger growth rates than Holmboe modes (see e.g. Haigh and Lawrence 1999, Hogg and Ivey 2003), therefore K-H billows were expected to be the type of instabilities associated with the observed high-frequency waves. The K-H instabilities, in turn, would break down into secondary instabilities, turbulence and mixing at the critical depth (Thorpe 1973). Figure 3.9 shows the evolution of two billows (indicated by isotherms surfaces of the 26.3 and 26.42°C isotherms) simulated near the LDS and the motion of the 21°C isotherm below. The four frames in Figure 3.9 are 60 s apart but the simulation time step was only 0.1 s (Table 3.1); therefore it is apparent that time resolution was more than sufficient to capture the temporal evolution of the billow. The 21°C isotherm oscillated coherently with the revolving isotherms above, reflecting the vertical modal structure of the instability depicted in Figure 3.8. While the period and horizontal scales of the billows were similar to the period of the vertical interfacial displacements shown in Figure 3.6b and the horizontal scales shown in Figure 3.7c and 3.7d, the vertical scales of the billows grew from 1.9 to 3.5 m (Figure 3.9d), over twice the size of the shear layer thickness shown in Figure 3.8. Figures 3.9c and 3.9d show that the distance between the closely spaced isotherms increased due to mixing produced by the collapsing of the overturning billows. The 26.42°C surface above the billows (Figures 3.9a to 3.9c) deepened with time, which suggests that some convective motion originated from heat exchange at the free surface, and was then engulfed by the K-H billow. The combination of the billowing and convective motion depicted by the model agreed well with the conceptual model of an eroding diurnal thermocline (Imberger 1985).

**3.5. Discussion**

High-resolution three-dimensional non-hydrostatic models are now opening up avenues to investigate physical process at field scales, especially in oceanic applications where parallel computational resources are available. Most efforts have been directed to develop
mixing parameterizations for entrainment of overflow descents in large-scale hydrostatic models (e.g. Özgökmen et al. 2004, Legg et al. 2006 and references therein). While most non-hydrostatic models used to investigate high-frequency internal waves (other than basin-scale waves) were computed at laboratory scales (e.g. Wadzuk and Hodges 2004, Fringer and Street 2005, Botelho et al. – Chapter 2), to our knowledge, the present study forms the first attempt to simulate internal waves with frequencies approaching the buoyancy frequency limit with a three-dimensional model at lake scales (> 1Km). Our model results showed unambiguously that the waves near the high-frequency limit in Lake Pusiano were due to shear instabilities developed in the remnants of a diurnal thermocline. The results reinforce the interpretation of the internal wave spectrum given by Boegman et al. 2005. In Lake Pusiano, the energy imparted by the wind excites the basin scale waves, which degenerate into waves at the medium-frequency range (~2x10⁻⁴ Hz) as they shoal at the southern boundary of the lake (Boegman et al. 2005). The medium-frequency waves, in turn, produce mixing or dissipate most of the energy imparted into the basin-scale waves as they break in the shore (Boegman et al. 2005). The wind forcing also directly energises the waves near the high frequency limit. In this case, we have shown that the background shear in favour of the wind direction and a very low gradient Richardson number was a necessary requirement for the development of the waves in the simulations. Two-dimensional numerical experiments of the Pacific equatorial undercurrent showed the same generation mechanism required for the high-frequency waves (Skyllingstad and Dembo, 1994). The results provided further support to the findings of previous field (Antenucci et al. 2001, Gomez and Imberger 2006) and numerical (Skyllingstad and Dembo, 1994) studies that argued that the observed high-frequency waves were part of unstable shear modes that developed in regions of small $Ri$ due to the additional shear imparted by the wind forcing on the background basin-scale internal wave flow. Moreover, the billowing associated with the convective motion induced by surface heat exchange in the simulations was consistent with the conceptual model of erosion of the diurnal thermocline (Imberger 1985). A model using the hydrostatic approximation cannot make density interfaces roll (Özgökmen et al. 2003) and therefore, cannot simulate the highly non-linear instabilities leading to the wave breaking observed in our results. However, the hydrostatic models can, with reasonable confidence, reproduce the background motion of basin-scale internal waves on which the high-frequency waves develop (Hodges et al. 2000, Rueda et al. 2003).
Sufficiently high resolution to describe these waves numerically throughout the whole domain is still computationally prohibitive in a single processor with memory limitation. For example, we could not produce simulations in a 5x5 m grid due to insufficiency of memory in our system such that swap-space memory allocation seriously compromised simulation run time. Our objective, which was successfully reached, was to reproduce the high-frequency waves (~10^{-2} Hz) and show the generation mechanism of the waves. Given the relatively large run time of the simulation using the 1x1 m non-uniform grid (~2 days for an one-hour simulation) we did not attempt to test if simulations with horizontal grid spacing smaller than one meter would produce high-frequency waves with similar wavenumber and frequencies as produced by the simulation using the non-uniform grid. Considering a linear increase in run-time with number of grid cells and time steps reducing both the horizontal grid sizes and the time step by one-half would require an 8 times larger computation time. Given this is an optimistic approach (see RE:RU in Table 3.2), computation time in these circumstances would probably be at least 10 times larger. Therefore, parallelization of the ELCOM code is a necessary step for full-domain-resolving non-hydrostatic simulations. This is particularly necessary for larger lakes than Lake Pusiano. Such simulations would allow more precise lake-integrated estimates of the energy flux path (Imberger 1998) than is currently possible with field studies and low-resolution models. While our computational resources are still prohibitive for whole-lake high-resolution non-hydrostatic simulations, grid-switching (e.g. Botelho et al. – Chapter 2), adaptive-mesh-refinement (e.g. Blayo and Debreu 1999) or grid-nesting techniques (e.g. Heggelund and Bernstein 2002), all of which use non-uniform grids, have to be used for a detailed analysis of small-scale flow features. Note, however, that there are other possibilities emerging with non-hydrostatic non-structured grid based models (see e.g. Ocean Modelling 2004). A nesting technique approach has been very recently applied to study the effects of a density current generator in Isahaya Bay, Japan (Sato et al. 2006); however, the small-scale features of the model were not shown. In the study of Sato et al. (2006) the solution of the Poisson-type pressure equation was resolved only in a small local part of the domain however, the elliptic equation has to be solved for the whole domain simultaneously (Skamarock and Klemp 1998); nonetheless, rigorous test cases have to be developed to understand the physical and numerical repercussions of such an approach. It has to be acknowledged that large grid size variations in this study (∆x_{min}/∆x_{max} ~ 170, Table 3.1) are undesirable for long time simulations where the small-scale features are not
localised (e.g. Figure 3.5 showed high wavelength waves throughout the domain). It is very likely that the rate of deepening of the diurnal thermocline (indicated by the 25.5°C isotherm in Figure 3.7) was decreased away from the high-resolution portion of the grid due to sub-grid-scale (SGS) process. These processes are not resolved in the low-resolution part of the grid. It follows that SGS closure schemes must conform to the spatial variability of the grid in both horizontal and vertical directions.

Nevertheless, we are reaching the ability of reproducing most of motions from the very fine (1~10 m) up to the basin scales. This is now possible as we are solving the full non-hydrostatic equations but also significantly increasing the resolution of the simulations. Previous studies provide substantial evidence of the implications of both basin and fine structure motion in the ecological patchiness in lakes and the ocean (see e.g. Franks 1995, MacIntyre and Jellison 2001). Quoting MacIntyre and Jellison (2001), "... Major progress in understanding physical controls on lacustrine ecosystem dynamics will occur when we can relate the onset, spatial variability and intensity of turbulence to wind forcing and the resulting internal wave field." We have shown that, despite some limitations, the current modelling capabilities are arriving at the point to address these processes concomitantly in a lake-wide fashion. Certainly, when coupled with water quality/ecological models, these well-resolved models will become a powerful tool for a more thorough understanding of the processes leading to the ecological patchiness of lakes.

3.6. Conclusion

This paper shows that now it is possible to produce high-resolution simulations of an entire lake using a three-dimensional non-hydrostatic model computed in a readily available desktop machine. For the relatively small lake investigated in this paper, most of the spectrum of the internal wave field could be resolved with a uniform grid applied to the whole domain. However, some form of grid adaptation was required to illuminate the internal wave field at the finest scales, where the internal wave frequencies are close to the buoyancy frequency limit. Although model resolution is reduced away from the region of interest, the technique of illuminating the flow field provides a computationally simple but effective way of simulating the entire internal wave spectrum, accounting for the low-frequency background field imposed by the basin-scale motions up to highest internal wave frequencies permitted in a stratified lake.
3.7. Acknowledgments

The writers thank Andres Gomez-Giraldo for valuable help and discussion in the linear stability analysis presented in this paper. The Lake Pusiano data were collected by the Field Operations Group at CWR and the Italian Water Research Institute. We especially thank Diego Copetti for his valuable help. The authors also acknowledge C. Torrence and G. Compo for the wavelet software provided. The first author was supported by CAPES (Brazilian Ministry of Education) with a PhD scholarship (process BEX 1383-00/0) and by the Centre for Water Research with an Ad-hoc Scholarship. We also thank Chris Dallimore, Peter Yeates and Greg Lawrence for constructive comments on the final version of this manuscript. We also acknowledge that the quality of the paper was greatly improved by the comments of three anonymous reviewers.

3.8. References


Illuminating the internal-wave field


## Table 3.1 - Lake Pusiano topographic characteristics in relation to mean level altitude (adapted from Legnani et al. 2005).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Mean Level Altitude</td>
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</tr>
<tr>
<td>Perimeter</td>
<td>12.6 Km</td>
</tr>
<tr>
<td>Area</td>
<td>5.26 Km(^2)</td>
</tr>
<tr>
<td>Volume</td>
<td>69.2x10(^6) m(^3)</td>
</tr>
<tr>
<td>Maximum Depth</td>
<td>24.3 m</td>
</tr>
<tr>
<td>Mean Depth</td>
<td>13.2 m</td>
</tr>
<tr>
<td>Maximum Length</td>
<td>3.8 Km</td>
</tr>
<tr>
<td>Maximum Width</td>
<td>2.6 Km</td>
</tr>
</tbody>
</table>
**Table 3.2 - Details of each simulation.** H stands for hydrostatic and NH for non-hydrostatic. $\Delta t$ is the time step. Initial conditions refer to where initial conditions are obtained from. $IW\ CFL$ stands for internal wave Courant-Friederich-Lévy number. $RE:RU$ stands for the ratio between real time and run time of the simulation. *This simulation was carried out in a Pentium 4 2.4 GHz processor with 1GB of memory available.

<table>
<thead>
<tr>
<th>Run</th>
<th>Calculation Type</th>
<th>Grid Size (m)</th>
<th>$\Delta t$ (s)</th>
<th>Initial Conditions</th>
<th>Start (day)</th>
<th>Finish (day)</th>
<th>$IW\ CFL$</th>
<th>$RE:RU$</th>
</tr>
</thead>
<tbody>
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<td>H</td>
<td>100 100</td>
<td>10</td>
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<td>211.000</td>
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<td>25.50*</td>
</tr>
<tr>
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<td>H</td>
<td>20 20</td>
<td>10</td>
<td>Station T1</td>
<td>203.000</td>
<td>211.000</td>
<td>0.29</td>
<td>1.6187</td>
</tr>
<tr>
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<td>Run 2</td>
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<td>205.398</td>
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<tr>
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<td>205.235</td>
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<td>0.0241</td>
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</table>
Figure 3.1 – a) Lake Pusiano bathymetry. Isobaths are the same as the values indicated in the scale bar. The black square indicates the limits in plots d) and e) and the dot indicates the position of the sampling station T1. b) 100x100 m horizontal grid. The white square indicates the limits in plots d) and e). c) Vertical grid configuration in a N-S cross-section of the 100x100 m grid at 1550 m from the west-most point of the grid. d) 20x20 m horizontal grid in the region detailed in the square in b). Black thick square indicates the region detailed in f). e) Same as d) but for 10x10 m horizontal grid. f) Non-uniform horizontal grid with high resolution around station T1 as indicated in d). Grid size in f) expands towards the boundaries with an expansion coefficients ~1.10.
Figure 3.2 – a) Wind vectors. b) Atmospheric Temperature. c) Water temperature measured at stations T1. d) Wavelet power spectrum of the 18°C isotherms displacements at stations T1. e) Detail of the wavelet power spectrum presented in panel d) showing the time window of analysis. Direction pointing upwards in a) is north and downwards is south. The dashed lines in a), c) and d) indicate the time window for which wind events are analyzed in this paper. Data in a) and b) are averaged over 10 minute intervals. Isotherms contours in c) are the same as indicated in the colour bars while the dots indicate the position of the thermistors. Contour lines in d) and e) indicate 90% of confidence in the spectrum, while the dot-dashed lines in d) indicate edge effects in the spectra (see Torrence and Compo 1998). Details of the wind forcing and water temperature in the window of analysis is shown later in Figure 3.3.
Figure 3.3 – Wind forcing, field temperature data and different-grid-resolution model temperature outputs for the time period detailed in the earlier time windows in Figure 3.2. a) Wind forcing averaged every two minutes, as in Figure 3.2. b) Field temperature data. The dashed lines indicate the time window detailed in Figure 3.6. c) 100x100 m resolution hydrostatic model simulation outputs. d) 20x20 m resolution non-hydrostatic model simulation outputs. e) 10x10 m resolution non-hydrostatic model simulation outputs. Isotherm contours are indicated in the scale at the side of the plots. Dots indicate the vertical position of the thermistors in the field on which the model data was interpolated to.
Figure 3.4 – Three-dimensional view of the 21.5°C isotherm surface at time 205.2178. a) 100x100 m horizontal grid resolution. b) 20x20 m horizontal grid resolution. c) 10x10 m horizontal grid resolution.
Figure 3.5 – Spatial scales of the internal waves in the lake at time 205.2178. a) Shadow image of the 21°C-isotherm-surface plan view in the simulation on a 20x20 m horizontal grid. b) Contour of isotherms in the vertical plane passing through A-A’ in the simulation on a 20x20 m horizontal grid. c) Shadow image of the 21°C-isotherm-surface plan view in the simulation on a 10x10 m horizontal grid. d) Contour of isotherms in the vertical plane passing through A-A’ in the simulation on a 10x10 m horizontal grid. The position of the station T1 is demarked by the dots in the plan view and by the dot-dashed line in vertical views. The horizontal scales given in the text are indicated in the plan view from the LDS station towards the NW shore (white line) and towards the SE shore (black line). In the vertical views the wider line highlights the 21°C isotherm contour. Both horizontal (130 m and 100 m in c) and d), respectively) and vertical scales (0.50 m and 0.70 m in c) and d), respectively) are shown by the horizontal and vertical line segments near the vertical displacements of the 21°C isotherm.
Figure 3.6 – Isotherm contours at LDS station. a) Field data. b) Simulation on the non-uniform grid with a hot-start. c) Simulation on the non-uniform grid with a cold-start. The labels on the colour scale at the side indicate contours in the plot. Dots indicate the vertical position of the thermistors on the LDS for which model results were interpolated to. The 21°C isotherm contour is highlighted with a thicker line. The horizontal bars indicate the length of time in seconds.
Figure 3.7 – Temperature field produced by the non-uniform grid hot-started simulation at time 205.2178. a) Three-dimensional view of the 21.5°C isotherm surface. b) Shadow image of the 21°C-isotherm-surface plan view. c) Zoomed view of b) in the surrounding of the LDS. d) Contour of isotherms in the vertical plane passing through B-B’. The meaning of the symbols in plots b), c) and d) is similar to Figure 3.5. Horizontal scales in plots c) and d) are 12 m and vertical scale in plot d) is 1 m.
Figure 3.8 – Background conditions and stability analysis results. a) Density (full line) and velocity (dotted line) background condition profiles for the plane 168.75° from the N direction. b) Buoyancy frequency squared (full line) and shear (dotted line) background condition profiles for the plane 168.75° from the N direction. c) Inverse of Ri (full line) and growth rates (dots) calculated by the linear stability analysis. d) Eigenfunction (full line) and phase (dotted line) calculated by the linear stability analysis for the wave number and growth rates that are indicated in the lower part of the plot. e) Growth rates of the fastest growing modes in each plane. The circle in e) corresponds to an 8 m wavelength.
Figure 3.9 – Billow evolution for different times. Velocity vectors are plotted in a cross section through the LDS station (position 1033 m N-S, 2076 m W-E). 21, 26.30 and 26.42 °C isotherms surfaces are plotted. Times are indicated in each of the frames. Vector magnitudes are indicated in the vertical (0.125 m s\(^{-1}\)) and horizontal (0.25 m s\(^{-1}\)) scales at the right hand side of the plots. Vectors are plotted every two points in the vertical for clarity. Transparent surfaces were used to reveal the velocity structure inside the billow.
4. Dissolved-oxygen response to wind-inflow interactions in a stratified reservoir

4.1. Abstract

Results of a field campaign and numerical simulations are used to show how physical mechanisms impose length and time scales that determine the dominant biogeochemical process. As an example, the dynamics of the Snake River inflows into Brownlee Reservoir is investigated to explain the onset and maintenance of an oxygen-depleted region (the oxygen block) in the surface layer of the upstream part of the reservoir. The oxygen block was located in a region of the reservoir in which the surface layer was warmer as a result of smaller wind stresses and reduced evaporation rates. Numerical simulations reproduced the hydrodynamic field observations resulting from inflow, outflow, wind stress, and atmospheric heat fluxes. When the wind stress opposed the inflow, the surface layer was arrested, forming a zone of convergence, stagnating the fluid and allowing the biological oxygen demand in the water to deplete the dissolved oxygen in the surface water; direct measurements showed that vertical mixing was small and contributed only marginally to the oxygen depletion. Net DO production in the water column was consistent with the observed variation with the buoyant inflow pattern, that is, a sink during overflows and overcast days and a source during interflows and intense sunlight. These observations provided further evidence that the water in this region was biologically isolated as confirmed by a scaling analysis. Modern numerical hydrodynamic simulations have reached a level of accuracy where they may be used to identify and quantify ecological niches.

4.2. Introduction

The physical system determines the rate at which nutrients, organic matter, and microbial communities are transported and mixed in an ecosystem and therefore, it is the
physical system that imposes, or at least greatly influences, the scales that determine which ecological processes are dominant. Reynolds et al. (2002) realized that this influence is strong enough to allow functional classification of phytoplankton based largely on the characteristics of the physical environment. Moreover, spatial and temporal variability of ecological processes in lakes and the ocean have been associated with a wide range of physical processes: longitudinal shear dispersion (Okubo 1980; Martin 2003), vertical shear dispersion (Martin 2003; Cowles 2004), internal wave oscillations (Franks 1995), wind-induced upwelling (Robarts et al. 1998), internal wave breaking (MacIntyre et al. 1999), and buoyancy driven inflows (Serruya 1974; Šimek et al. 2001) to name a few. In man-made lakes, the water quality and ecology is often regulated by the characteristics of the inflowing rivers carrying nutrients and organic matter into a lake (Comerma et al. 2001; Friedl and Wüst 2002; Masin et al. 2003).

The transition zone between the high organic-nutrient rich inflowing river waters and the deep-water lake environment is known to depend intimately on the flow conditions of the inflowing river (Kennedy et al. 1982; Friedl and Wüst 2002; Masin et al. 2003). The longitudinal variability imposed by the flow dynamics can be so large that different regions in the same lake may present distinct ecological niches at the same time (Kennedy et al. 1982; Lind et al. 1993; Šimek et al. 2001). While many factors can play a role in this longitudinal variability, the key factor is the local residence time of the surface water; a low-density surface overflow will be subject to direct influence of the wind stress which, in turn, will control the residence time, the longitudinal dispersion and mixing of the inflow as it enters a lake (Fischer et al. 1979; Alavian et al. 1992; Menéndez and Laciana 2006). An interflow, on the other hand, may redistribute pollutants, nutrients, and organic matter to deeper strata of the lake and will not be largely influenced by the wind stress (Gu et al. 1996). However, the wind may still provide the mechanism for generating basin scale waves that may redistribute nutrients to the surface layer at upwelling locations (Fischer and Smith 1983; Carmack et al. 1986). Moreover, these internal waves may steepen and break at the lake slope producing intense vertical mixing between the surface layer and the hypolimnion (MacIntyre et al. 1999, Boegman et al. 2005).

Brownlee Reservoir (Figure 4.1) experiences increased pressures from contamination in the upstream catchment where large loads of nutrients and organic matter enter the water streams (Myers et al. 1997). Several studies have shown the occurrence of strong longitudinal variability in the dissolved oxygen (DO) concentrations in the surface layer of
the upper reaches of the reservoir during summer, when the water becomes thermally stratified (Ebel and Koski 1968; Harrison et al. 1999; Nürnberg et al. 2001). Hypolimnetic anoxia normally develops in upstream reaches of reservoirs, however the surface layer, in general, has supersaturated DO levels as particles settles with the reduced inflow inertia and the increased light penetration fuels photosynthesis (Cole and Hannan 1990; Friedl and Wüest 2002). On occasions, however, the surface layer oxygen levels in Brownlee Reservoir drop sufficiently to cause fish kills (Ebel and Koski 1968; Harrison et al. 1999; Sullivan et al. 2003); Ebel and Koski (1968) were the first to report the problem calling it an oxygen block. Using the data from Ebel and Koski (1968), Cole and Hannan (1990) suggested that the oxygen block could be due to the formation of a convergent zone at the plunge point, similarly to high productive zones in oceanic fronts. However, they did not quantify this hypothesis. Harrison et al. (1999) noted that the oxygen block was located downstream of a high chlorophyll a (Chla) concentration zone which was fuelled by excessive nutrient inputs via the Snake River. Harrison et al. (1999) postulated several causes of the oxygen block: (1) decay of organic matter in the Snake River inflows, (2) upward mixing fluxes of deep anoxic water to the surface layer of the reservoir, (3) scouring and resuspension of (mainly inorganic) oxygen-demanding materials near the head of the reservoir and (4) off-gassing from anoxic sediments. While Harrison et al. (1999) discarded hypotheses 3 and 4, Nürnberg et al. (2001) concluded that “entrainment” of hypolimnetic water in the lake interior caused the oxygen depletion in the block region. However, the data of Nürnberg et al. (2001) did not provide any evidence of such mixing events.

In this paper, we present the results of an intensive field campaign and numerical simulations detailing the dynamics of the Snake River inflows into Brownlee Reservoir during the summer of 2002. The main objective of the present study was to present an example of how modern 3D numerical simulations maybe used to clarify the physical mechanisms forming ecological niches by imposing length and time scales that determine which biogeochemical process can dominate and, in this case, leading to what has been called the oxygen block. Use of such a model opens up a whole new machinery to the limnologists that, only a few years ago, did not exist; numerical models have made remarkable advances where it is now possible to resolve the transport and mixing patterns in a large lake down to a few meters. This makes it possible to identify, in a water body, water masses that are essentially isolated to a range of biological processes and thus,
provide a niche for these processes.

The paper is structured as follows: in the next section we give a description of the study site, then we describe the field program followed by an outline of the numerical model used. Results from both the experiment and the modelling are then presented, followed by a discussion of how physical processes influence ecological processes and contribute to the depletion of DO in the surface layer of the reservoir. Concluding remarks are presented at the end.

4.3. Study Site

Brownlee Reservoir is located in the southwest of the state of Idaho, USA, in a hilly semi-arid region along the Snake River watercourse where the total annual average precipitation is around 310 mm (NOAA. 2003. Normal Monthly Precipitation (inches). NOAA web-site – http://www.ncdc.noaa.gov oa/climate/online/ccd/nrmlprcp.html, 02 Dec.). The reservoir is used mainly for hydro-power generation and flood control. However, other uses include salmon spawning, public recreation and, to a lesser extent, domestic water supply. The reservoir is the most upstream in a cascade of three reservoirs referred to as the Hells Canyon Complex. It is a typical canyon-valley reservoir (Figure 4.1), narrow (< 1 km width), long (~80 km) and with increasingly depth from the upper reaches (~5 m) to the dam wall (~80 m). A summary of its morphometric characteristics (Nürnberg et al. 2001) is given in Table 4.1. The reservoir stratifies in spring and a sharp density gradient develops forming a stable metalimnion throughout the summer. Stratification is dependent on both the inflow, the level of the offtake and the water level regime under which the reservoir is operated for flood control (Ebel and Koski 1968). During low (high) flow years, when the level of the reservoir is maintained relatively high (low), seasonal stratification is stronger (weaker), and develops earlier (later), the oxygen block is less (more) pronounced and is also located further upstream (downstream) (Ebel and Koski 1968; Harrison et al. 1999).

4.4. Field Program and Numerical Modelling

Both a field program and numerical modelling were used to understand how the inflow dynamics forms niches for the oxygen block to form and recover. The field program included data from a routine monthly sampling programme and from a dedicated field investigation conducted from days 198 to 206 in 2002 (17 to 25 July, Figure 4.1 and Table
4.2. At that time, the reservoir was thermally stratified, the air temperatures were quite high, flow rates were low and the oxygen block was present (see below). Descriptions of the regularly monitored data, the field experiment schedule and the numerical model used in this investigation are given below.

**Inflow/outflow discharge and water quality parameters**

More than 97% of the inflow to Brownlee Reservoir comes from the Snake River. Only two other tributaries, the Burnt and Powder Rivers (Figure 4.1), flow regularly into the reservoir in summer, but their contribution is minor. Snake River flow rates from the USGS gauge no. 1326900 were sampled every 15 minutes. Flow rates of the Burnt and Powder Rivers, as well as the outflows at the dam wall, were provided by the Idaho Power Company (IPC). Sampling on the Burnt River was 1 hour, while the outflow discharges and Powder River flows were sampled at intervals of 15 minutes. IPC also provided inflow temperature data and water levels at the dam wall at 15-minute intervals and DO and temperature data obtained from a monthly regular monitoring program. Biological oxygen demand (BOD), Chla and nutrients were sampled twice a month in a station 35 km upstream of station BR01 (see below). These data were sampled separately from our measurements and were courtesy of Dr. Jack Harrison (University of Idaho).

**In-lake continuous sampling**

Three LDS's (Lake Diagnostic Systems; Gomez-Giraldo et al. 2006) were installed in the lake at locations shown in Figure 4.1. The middle LDS was equipped with a thermistor chain with precision thermistors every meter and a full meteorological sensor set 2 m above the water surface to measure the atmospheric heat fluxes. The other two LDS had identical thermistor chains, but were only equipped with wind speed and direction. Data was collected every 60 s and transferred to a shore computer via radio and cellular telemetry four times a day.

**Processes field experiment**

The main objective of the field experiment was to delineate the physical mechanisms contributing to the deficit of the oxygen in the lake surface layer forming the oxygen block, described above. Three mechanisms were investigated in detail: First, vertical mixing of anoxic hypolimnetic water into the surface layer (Harrison et al. 1999; Nürnberg et al. 2001), second, upwelling of anoxic bottom waters (Harrison et al. 1999; Nürnberg et al. 2001), third, advection in and out of a control volume surrounding the oxygen block and
oxygen transfers through the free surface (Harrison et al. 1999). Figure 4.1 and Table 4.2 summarize the equipment used, the time, and locations of sampling in the field experiment (note that data from the extended transect on day 206 and the microstructure profiles at BR06 and BR34 are not shown in this paper).

Vertical mixing was estimated from temperature microstructure data that was sampled using SCAMP (manufactured by Precision Measurements Engineering), the self-contained autonomous microstructure profiler (Carter and Imberger 1986). SCAMP has a high sampling rate providing temperature data every ~1 mm in the vertical. The microstructure data were used to derive $\varepsilon$, the rate of dissipation of turbulent kinetic energy (Luketina and Imberger 2001) and this was then used to obtain an estimate of the vertical eddy-mixing coefficient from well-known formulae (see e.g., Osborn 1980; Wüst et al. 2000; Saggio and Imberger 2001).

Upwelling and advection induced by the inflows was documented with fine scale transects that were constructed from a series of profiles sampled with the Fine-scale Probe (hereafter F-probe, Imberger and Head 1994, Anohin et al. 2006) from stations BR01 (most upstream) to BR30 (Figure 4.1) and with 3D modelling. On average, stations were 1.3 km apart and each transect covered a length of thalweg of about 40 km (Figure 4.1). Vertical resolution of the profiles was from 1 to 5 cm, depending on the probe descent velocity. In order to reveal the evolution during sub-daily time scales, the five transects sampled on days 200 and 201 (19 and 20 Jul) were collected at intervals of approximately six hours each. The F-probe data shown in this paper include temperature, salinity (derived from conductivity) and DO. The F-probe data were also used to provide the initial conditions for the modelling and to validate the numerical simulation results. Chla concentrations were estimated from fluorescence measurements excited at different light spectra with a BBE-Moldaenke Fluoroprobe® attached to the F-probe. While fluorescence from yellow substances was used to correct the Chla signal, the equipment was used with default factory conversion parameters to estimate Chla concentration from fluorescence measurements. The equipment was not calibrated for the reservoir conditions therefore the Chla results presented in this paper are only indicative of relative abundance of biomass and were not used for any quantitative biological analysis.

Hydrodynamic modelling

Three-dimensional numerical simulations were used to obtain the flow field resulting from the combination of different forcing: inflows, outflows, wind stresses, and surface
DO Response to Wind-Inflow Interactions

heat fluxes. The CWR-Estuary, Lake and Coastal Ocean Model (ELCOM) was used in the simulations presented in this paper. ELCOM solves the Reynolds Averaged Navier-Stokes equations with the Boussinesq approximation for density differences using a semi-implicit formulation on a finite-volume framework that incorporates structured hexahedral cells in an Arakawa-C grid stencil (Hodges 2000). The main structure of the numerical code is fully described in Hodges (2000) and the reader is referred to Casulli and Cheng (1992), Casulli and Catani (1994), and Hodges (2000) for details of the numerical algorithms. Vertical mixing of momentum and scalars was treated similarly to that described by Hodges et al. (2000) with the addition of a regime dependent vertical mixing layer model described in Simanjuntak et al. 2006 (Benthic and interfacial mixing in a shallow salt-wedge estuary, submitted to J. Hydr. Eng. ASCE). Wind stress and bottom drag were treated as described in Simanjuntak et al. (2006). Surface heat transfers included the atmospheric boundary layer stability correction (CWR - ELCOM web manual 2006, http://www.cwr.uwa.edu.au/services/models/legacy/model/elcom/elcom_science/ELCOM_Science/ELCOM_Science.html). The method was adapted from Imberger and Patterson (1990) and Rayner (1981). In the present study, no adjustment of parameters was performed in ELCOM. Both hydrostatic and non-hydrostatic high-resolution simulations are presented. The non-hydrostatic code was based on the fractional step method of Casulli and Stelling (1998) with corrections for the free-surface elevation (Casulli 1999). The non-hydrostatic pressure solver uses the Bi-CGSTAB method (Chapter 2).

Bathymetry approximation and grid resolution

Three different grid configurations were deployed to simulate the flows in Brownlee Reservoir (Figure 4.2). First, the bathymetry of the reservoir was straightened following Hodges and Imberger (2001). This grid was uniform with size of 100, 800, and 1m in the transversal, longitudinal, and vertical directions, respectively. Hereafter, we will call this grid the 800x100 m grid. The Powder River arm was not incorporated in this bathymetry. Second, the bathymetry was not straightened and the grid had 100-m resolutions in both horizontal directions and 1-m resolution in the vertical direction. Hereafter, we will call this grid the 100x100 m grid. The other configuration included a 20-m grid size in the convergence zone of the overflows (see Results section below) however, maintaining a 100x100 m resolution in the other parts of the domain. Hereafter, we will call this grid the 100x20 m grid. The large rectangle in Figure 4.2a shows the region where the high resolution was located. This 100x20 m grid was used with both hydrostatic and fully non-
hydrostatic versions of the model. For comparison of the grid sizes the small rectangle in Figure 4.2a is enlarged and shown in Figure 4.2c with the lines demarking the 100x100 m grid. The green rectangle shows the comparative size of the 800x100 m grid while the blue rectangle is further enlarged in Figure 4.2d to show the relative size of the 20x20 m grid in the 100x20 m grid. Time steps were equal to one minute for the 800x100 m grid and the 100x100 m grid. For the 100x20 m grid the time step was set to twelve seconds. These time steps were designed to provide output compatible with the LDS data and to produce stable simulations according to the internal wave CFL condition.

**Initial conditions**

The first F-probe transect was used for the initial model temperature and salinity fields. Each profile, and the data from the LDS, was first linearly interpolated onto the vertical layers of the domain. Then, values for each grid point in the domain were obtained by horizontal interpolation of the assigned points using an inverse of distance-squared weighting method. A cold start was used for the flow initial conditions and the starting time was the average time taken to complete the first transect sampled in the field. The non-hydrostatic simulation was performed between days 199 and day 201 and started from outputs of the hydrostatic simulation with the 100x20 m grid.

**Inflow and outflow boundary conditions**

The inflows of Burnt and Powder rivers, as well as the outflows at the dam wall, were specified from the flow rates measured by the IPC gauges. Simulations performed with the Snake River inflow measured by the USGS gauge presented a discrepancy between the measured and calculated reservoir levels. Therefore, the Snake River inflows were corrected on a daily basis to properly match the water balance in the reservoir taking into consideration the measured inflows (Burnt, Powder and Snake Rivers), outflows and volume (water-level) variations. The discrepancies were believed to be mostly due to small inaccuracies of the flow rate measurements, evaporation between the gauge and the boundary of the model domain and, mainly, flow diversions for irrigation between the gauge and the reservoir (USGS 2006 http://waterdata.usgs.gov/id/nwis/uv?13269000, 06 May). The Snake River inflow temperature measured at the IPC gauge was used at all flow boundaries. The salinity in the Snake River varied between 0.205 and 0.217, a value low enough not to influence the density of the inflow, yet valuable as a tracer. We set the average values of the salinity obtained at station BR01 to use at the Snake River inflow boundary condition. There were no data for the other inflows and these were set constant
and equal to 0.2 for the whole simulation.

**Wind and atmospheric forcing**

The atmospheric forcing (air temperature, humidity, net wave radiation, and short wave radiation collected at T1) at the free surface of the different domains was uniform. For the simulations with the 800x100 m grid, the wind was assumed uniform in the transversal direction; wind speeds and directions varied only in the longitudinal direction. In the other grids wind speeds and directions varied only in the N-S direction and were uniform in the E-W direction. The variation of the wind field was obtained from linear interpolation between the three LDS stations and assumed constant between T1 (T3) and the downstream (upstream) boundary of the domains.

### 4.5. Results

**The oxygen block**

Figure 4.3 shows the DO concentrations measured with the F-probe along transect 1 on days 198, 200, and 205 between stations BR01 and BR30. Some isotherms are also shown for reference. The oxygen block corresponded to the region of low oxygen in the neighbourhood of station T3. The region was defined as the water in which the oxygen concentration was between 1.5 and 3 mg L$^{-1}$ lines as indicated in Figure 4.3. This demarcation was arbitrary but clearly showed that the block presented a large variability in length (13–21 km) and depth (5–15 m). The downstream end of the block did not move much, however the upstream end moved with time, compressing the block to progressively smaller lengths of about 20 km on day 198 to 5 km on 205 (Figure 4.3). In the vertical, the 1.5 mg L$^{-1}$ isooxygen tracked the 24°C isotherm, which suggests that the processes leading to DO depletion in the surface layer were disconnected from the processes leading to anoxia in the hypolimnion. Below, we aimed to (i) detail how physical processes contributed to the DO balance in the block and (ii) how physical processes defined the time and length scales on which biogeochemical processes acted to influence the balance.

**Variations of surface heat exchange**

Figure 4.4a presents the 1 h-averaged wind shear stresses from day 198 to day 206 at stations T2 and T3, while Figure 4.4b presents the total heat flux at the water surface. The wind stresses ($\tau$) at station T3 showed little periodicity, whilst at station T2, $\tau$ was quite periodic and stronger, generally in excess of 0.15 N m$^{-2}$ for several hours, increasing from ~23:00 to ~11:00 h of the next day and then rapidly subsiding (Figure 4.4a). Noteworthy is
that between day 199 and 201 a cold front crossed the lake, the wind stresses were relatively uniform along the lake (Figure 4.4a), the air temperatures decreased and surface layer temperatures became more homogeneous (Figure 4.4c). The weather was overcast during this period, as indicated by the reduced surface heat fluxes (Figure 4.4b). Similar events occurred with a periodicity of approximately 10 days (not shown). As the weather warmed up, the larger wind stresses at T2 caused large latent heat fluxes and therefore, large evaporation rates (see e.g., TVA 1972) and re-aeration during the night periods (see Equation 4.9 in the DO balance below). Combined with the weaker wind stress near station T3 this difference in surface heat fluxes led to the observed longitudinal temperature gradients that, in turn, changed the inflow pattern to interflows (days 201 to 205). As the air temperatures rose the inflow heated more than the lake surface layer and inflows became overflows (after day 206, Figure 4.4c). This subtle change of inflow patterns, in response to both inflows and atmospheric forcing was the main driver for the associated observed oxygen concentration variations. Modelling was used to understand these flow complexities.

**Model validation**

A comparison between the water temperature records measured at stations T2 and T3 and the results of the different simulations are presented in conjunction with the power spectra of the 23°C-isotherms vertical displacements in Figure 4.5. The increased model accuracy with grid resolution is illustrated in the power spectra comparisons in Figure 4.5. The spectra of the isotherms followed the spectra obtained with the field measurements very closely from low frequencies to a frequency where the simulated spectra departed considerably from the field spectra. These are indicated in Figure 4.5 and we will denominate this frequency \( f_s \) as the frequency of model departure. \( f_s \) indicated the smallest time scales resolved by each of the simulations. The time scales of model departure \( t_s = 1/f_s \) at station T3 were, respectively, >3.6 days, ~7 hours, and 1.0 hour for the hydrostatic simulations on the 800x100 m grid, on the 100x100 m grid and, on the 100x20 m grid. For the non-hydrostatic simulation on the 100x20 m grid \( t_s \) was about 1.7 hours. For station T2 the respective \( t_s \) were 19.8 hours, 15.4 hours, and 8.7 hours for the hydrostatic simulations and 2.5 hours for the non-hydrostatic simulation. Although \( t_s \) of the hydrostatic simulation on the 100x20 m grid was smaller than on the non-hydrostatic simulation, the motion at these smaller time scales were not accurately described by a hydrostatic model (Hodges et al. 2006, Chapter 2). However, the computational cost associated with the increase of
model resolution and the use of a non-hydrostatic solver can be illustrated by the ratio between real time and run time. These time ratios for the simulations carried on a Power-PC G-5 using a single processor with 4 GB of memory were, respectively, 144.66, 14.37, 0.89, and 0.19 for the simulations using the 800x100 m, 100x100 m, 100x20 m (hydrostatic), and 100x20 m (non-hydrostatic) grids.

Figures 4.6 and 4.7 present the difference between the simulated and field temperature using the 100x20 m and 100x100 m grids and the field transects collected during an overflow on days 200 (transect 1) and during an interflow on day 205, respectively. Although in most regions the error was relatively small, in the range between 0.5 °C and 1.5 °C, the largest errors (measured as the difference between model results and field observations) were as large as 2.5 °C and occurred locally, downstream of the inflow lift-off point and downstream of T2 at the thermocline level (Figures 4.6c and 4.7c). These large errors are reflections of phase differences between the model and the field data in the internal seiching of the metalimnion in regions where there are large horizontal and vertical temperature gradients. The longitudinal variability in the surface layer temperature was modelled well, with errors smaller than 0.5 °C throughout the simulation period. Figures 4.6c and 4.7c also show that there was no significant error amplification from the early (Figure 4.6) to the final (Figure 4.7) stages of the simulation.

Figure 4.8 presents the velocity and temperature fields obtained from the non-hydrostatic simulations in the high-resolution area of the 100x20 m grid at the same depth for which \( f^* \) was obtained. Velocities at the location of T3 were approximately 0.05 m s\(^{-1}\), which implied a length scale of model departure of around \( l_s \approx 325 \) m, indicating that the model possessed about 15 to 20 (3 to 4) points per resolved patch in the high (low)-resolution area of the grid, in accordance with Nyquist frequency considerations (Arbic et al. 2004). These wavy features, highlighted as LSM in Figure 4.8b, have scales from 300-500 m and may be assumed to be real. Using the same velocity scale the length scale \( l_s \), for the 800 x 100 m simulation was >16 km and about 1.2 km for the 100x100 m hydrostatic simulations. As a result, the simulation on the 800x100 m grid did not provide sufficient accuracy to investigate the DO dynamics in the block. In contrast, the simulations on the 100x100 m provided spatial resolution similar to the F-probe measurements and the simulations on the 100x20 m grid provided spatial resolution sufficient to capture transversal variations in the flow. Therefore, the simulations on these two non-straightened grids were used to analyze the flow and interpret the DO dynamics in the oxygen block.
Hereafter, results from the simulation on the 100x20 m grid correspond solely to the non-hydrostatic simulation.

**Overflows and convergence**

Because of the hilly terrain, the wind was funnelled along the canyon of the reservoir and directed either northerly or southerly (Figures 4.8, 4.9 and 4.10). Particularly, the large wind stresses at station, T2 shown in Figure 4.4a, corresponded to northerly winds (Figures 4.8a, 4.9a, and 4.10a). In contrast, the Snake River flowed in the opposite direction, from the South. As shown in Figure 4.6, inflow and in-lake surface-layer temperatures were similar such that overflows were the main buoyant inflow pattern. Figure 4.8b, 4.9b, and 4.10b show the results of the simulation in the high-resolution region of the 100x20 m grid, where the evolution of the temperature and velocity fields at different levels in the surface layer were depicted at every three hours between days 199.000 and 200.500. The flow in the diurnal surface layer, as given by the top-most level (level 83), is shown in Figure 4.10b. The model successfully reproduced the reduction of the longitudinal temperature gradients (Figure 4.10b) due to the surface cooling (see also Figure 4.4). At the bottom part of Figure 4.10b (>38 km Southing), the velocity field was strongly correlated with the wind field at T3 while on top, the velocity field seemed more correlated with the wind at T2, suggesting that the velocity fields quickly adjusted and became oriented in the same direction of the imposed wind field. At level 77, ~6m below the surface, the currents were much smaller in magnitude (~8x on average, Figure 4.9b) in comparison to those in the diurnal surface layer. Between times 199.375 and 200.000, the wind stress became northerly at both wind stations in such a manner that no clear flow pattern could be distinguished; the flow essentially stagnated in this convergent zone (Figure 4.9b). After time 200.500, the wind subsided at both stations, allowing an inflow flow-driven pattern to be re-established (Figure 4.9b). In the interim period between 200.000 and 200.500 the motion is similar to Figure 4.8b but with a ~12 hour phase delay. As the flow streams, moving in opposite directions, compressed the convergence zone, the returning flow downwelled, inducing an apparent deepening of the surface layer (Figure 4.5). Because the deepening rate was larger at the downwind end, horizontal baroclinic pressure gradients formed in the metalimnion. Upon wind relaxation (or change of wind direction) long progressive waves propagated towards the upper end of the reservoir (Figures 4.6 and 4.8b). While the motion in the diurnal surface layer was primarily wind driven (Figure 4.10b), the motion in the seasonal surface layer was inflow driven however, under a large
influence of the superimposed internal-wave field (Figures 4.8b and 4.9b).

Internal waves

Some studies pointed out that wave breaking at the slope can be an important mechanism for redistribution of nutrients and mixing between hypolimnion and the surface layer (MacIntyre et al. 1999, Boegman et al. 2005). Wave breaking may result from two mechanisms: first, breaking of the long progressive waves mentioned above and second, breaking of waves that result from dispersive processes associated with the non-linear steepening of long progressive waves (Horn et al. 2001, Boegman et al. 2005). The internal Irrribaren $I_{\infty}$ number provides insight about the type of breaking that may be expected for internal waves shoaling at the sloping boundary (Boegman et al. 2005).

$$I_{\infty} = \frac{S}{\sqrt{a_{\infty}/\lambda_{\infty}}}$$

(4.1)

where $S$ is the slope of the reservoir. For the data considered here, largest amplitude of the observed waves was $a_{\infty} \sim 7$ m (Figure 4.5a, 4.5d and 4.5g), its wavelength was $\lambda_{\infty} \sim 14$ km (estimated from Figure 4.6a and times between profiles) and the bottom slope was $S \sim 1.3 \times 10^{-3}$, therefore $I_{\infty} = 0.060$. This value corresponds to spilling breakers, which provide nearly complete dissipation of the waves at the slope. Comparing the values obtained here with Figure 12b and 12d of Boegman et al. (2005), the reflection coefficient of these waves was smaller than 20% and the mixing efficiency was less than 5%.

Formation of dispersive waves can occur if the steepening time scale $T_{st}$ for the waves observed is smaller than the time taken for the waves to arrive at the lake slope. Assuming a two-layer system $T_{st}$ was approximately (Horn et al. 2001, Hodges et al. 2006, Chapter 2)

$$T_{st} = 0.7 \frac{\lambda_{\infty}}{2\alpha}$$

(4.2)

where

$$\alpha = 3 \frac{c_0}{2} \frac{h_2 - h_1}{h_2 h_1}$$

(4.3)

and $c_0$ is the linear internal wave speed

$$c_0 = \sqrt{\frac{g \rho_2 - \rho_1}{\rho_0} \frac{h_2 h_1}{h_2 + h_1}}$$

(4.4)

where $h_1$ and $h_2$ are the thicknesses of the top and bottom layer, respectively, $\rho_1$ and $\rho_2$ are the densities of the top and bottom layers, respectively and $\rho_0$ is a reference density. Using $h_1 = 12$ m, $h_2 = 20$ m, $\rho_2 - \rho_1 = 2$ kg m$^{-3}$, $\rho_0 = 1000$ kg m$^{-3}$, $c_0 = 0.38$ m s$^{-1}$, $\alpha = 0.019$ s$^{-1}$,
and $T_s \sim 3$ days. For the given $c_0$, the waves would reach the boundary much earlier than $T_{st}$ and no dispersive waves would form. This is confirmed by the numerical simulation results.

Thus, mixing associated with wave breaking, did not contribute to the DO depletion in the surface layer. In practical terms, the energy of the wind, which is used to arrest the inflow and "pump" these internal waves, does not produce waves with sufficiently large slopes to either break or degenerate into steeper waves that, in turn, would produce efficient mixing upon breaking at the lake slope. As temperatures increased, the inflow pattern changed to interflows ceasing the internal wave "pumping" mechanism that was produced by the interaction between the wind shear layer and the overflows.

Vertical mixing

Measurements of the dissipation of turbulent kinetic energy $\varepsilon$, at stations BR14 and BR25, are presented in Figure 4.11. On Figure 4.11a and 4.11c are data corresponding to the temperature microstructure profiles sampled during the night of day 203 and early morning of day 204 and on the right hand side of Figure 4.11b and 4.11d are data corresponding to the profiles sampled during the night of day 205 and early morning of day 206 (see Table 4.2). The dissipation rates varied appreciably over the depth, ranging from about $10^{-7}$ m$^2$ s$^{-3}$ at the diurnal surface layer to $<10^{-10}$ m$^2$ s$^{-3}$ in the metalimnion. Estimates of the dissipation values may be combined with the buoyancy frequency $N$ to obtain an upper bound for the vertical eddy diffusion coefficient $K_p$. This may be done in different ways (Yeates, P. et al., unpubl.), but a reasonable upper bound is obtained by the Osborn formulae

$$K_p \leq 0.2 \frac{\varepsilon}{N^2}$$  \hspace{1cm} (4.5)

From the temperature profiles and dissipation rates shown in Figure 4.11, the logarithmic average vertical eddy diffusion coefficient ($\overline{K_p}$) was computed from all segments within 2.5 m of the 24°C isotherm

$$\log \overline{K_p} = \log(\overline{K_p}) \pm \text{std}(\log(\overline{K_p}))$$  \hspace{1cm} (4.6)

where the overbar indicates the mean value of values sampled within 2.5 m of the 24°C isotherm and std indicates standard deviation. From a total of 143 samples, $\overline{K_p}$ was estimated to be $3.9 \times 10^{-6}$ m$^2$ s$^{-1}$, with logarithmic standard deviations ranging from $6.4 \times 10^{-7}$ to $2.3 \times 10^{-5}$ m$^2$ s$^{-1}$. This range of $K_p$ values was used when carrying out, below, a DO balance for the oxygen block region. The model simulations (not shown) produced
variations similar to those shown in Figure 4.11.

Chl-a and DO patchiness in response to the wind-inflow dynamics

Figure 4.12 presents transects of salinity superimposed on Chl-a as measured in the field on days 198, 200 (transect 3), and 205. The salinity field was chosen as a tracer, because it clearly illustrated the transition from overflows (days 198 and 200) to interflows (day 205) and the respective transects were chosen because they could be associated with the DO and temperature fields shown in Figure 4.3. All panels in Figure 4.12 present relatively large Chla concentrations upstream and downstream of the DO block region. In the block, there were two discernible patches of higher concentrations. These Chla patches were important because they coincided with patches of relatively large DO concentrations in the block (Figure 4.3), an indication that photosynthetic activity effectively contributed to the DO budget in the block. The first patch occurred over 5 km downstream of T3 whilst the second patch was much stronger and occurred just downstream of the plunge point. As discussed below, the patches were significantly more concentrated during the interflow period after day 201 (Figure 4.12).

The vertical length scale of the patches was ~4-5 m (Figure 4.12), i.e., the thickness of the diurnal surface layer. The horizontal length scales of the patches were approximately 5 km (i.e., observed across 4 sampling stations). The horizontal length scale of the first patch was approximately the length between the two bends in the convergent zone, which is shown between 35 and 39 km Southing (see Figure 4.8b for location of sampling stations in the convergent zone). This was a location less susceptible to wind-shear turbulence and where the water was still (Figure 4.10). The horizontal length scale of the second patch correlated with the varying location of the plunge point (Figure 4.12), which is probably a reflection of the mixing of nutrients at the plunge point (Armengol et al. 1999).

Dissolved oxygen balance

A Eulerian control volume was defined by the free surface on top, by the 24 °C isotherm (thermocline) below, by a cross-section located halfway between stations BR10 and BR11 at the upstream end of the block, and by a cross-section located half-way between stations BR23 and BR24 (see Figure 4.1) at the downstream end of the block. However, the control volume boundary was allowed to move with the isotherm at the bottom so that we could neglect the vertical advection in and out of the block. The DO balance in this control volume was computed as
\[ \frac{dOV}{dt} = (F_{up} - F_{dn}) + (F_{air} - F_{mix} + S) \] (4.7)

The net rate of change of oxygen in the block (term 1) resulted from advective fluxes in (term 2) and out (term 3) of the block, a gain of oxygen through the air-water interface (term 4), loss of oxygen due to diffusion through the thermocline (term 5), and net community DO production in the block (term 6).

We estimated the rate of change by calculating the volume integral of the DO measured with the F-probe and then obtaining the difference between successive profiles divided by the time taken between the first and the last profile of successive transects.

\[ F_{up} \text{ and } F_{dn} \text{ were computed using the time averaged flow rates above the 24°C isotherm multiplied by the time-and-depth-averaged DO concentration above the 24°C isotherm at stations BR11 and BR23, respectively. The flow rates were obtained from the simulation with the 100x100 m grid.} \]

\[ F_{air} \text{ was estimated from a standard gas-transfer model through the air-water interface (e.g., Wanninkhof 1992)} \]

\[ F_{air} = k_{av}(O_{air} - O_{wat})A_{fs} \] (4.8)

where \( k_{av} \) is the gas transfer velocity in m s\(^{-1}\), \( O_{air} \) is the DO concentration in the air phase at the air-water interface, \( O_{wat} \) is the concentration in the water phase and \( A_{fs} \) is the free surface area. \( O_{air} \) was estimated as in Riley and Skirrow (1974) where zero salinity was assumed. In order to provide boundaries to our \( F_{air} \) estimates, we used two models to calculate the gas-transfer velocity. The first model, proposed by Wanninkhof (1992), was obtained from measurements in the ocean and in the Red Sea for average wind speeds of 4.7-7.4 m s\(^{-1}\).

\[ k_{av}^w = 2.8 \times 10^{-5} \frac{U_{10}^2}{S_c^{1/2}} \] (4.9a)

where \( U_{10} \) is the wind velocity corrected to 10 m above the free-surface (measured in m s\(^{-1}\)) and \( S_c \) is the Schmidt number of DO in water. The second model, proposed by Crusius and Wanninkhof (2003), was obtained from measurements in a small lake for average wind speeds of 0.95-5.43 m s\(^{-1}\). For average wind speeds smaller than 3.7 m s\(^{-1}\),

\[ k_{av}^c = 2.0 \times 10^{-4} \frac{0.72U_{10}}{S_c^{1/3}} \] (4.9b)

where \( p=1/2 \) for wind speeds larger than 3.0 m s\(^{-1}\) and \( p=2/3 \), otherwise. For average wind speeds larger or equal to 3.7 m s\(^{-1}\),
\[ k_w^c = 2.0 \times 10^{-4} \left( \frac{4.33 u_0 - 13.3}{S_c^{1/2}} \right) \] (4.9c)

The second model (Equations 4.9b, c) is probably more suited to the conditions in Brownlee Reservoir (average wind speeds of 1.5-3.9 m s\(^{-1}\)). For both gas-transfer models, we used the wind speeds set for model boundary conditions and corrected them to the 10 m height. The flux values obtained from the profiles measured at each station were integrated over the surface area and averaged between two successive transects.

An eddy Fickian diffusion model was used to estimate the flux through the thermocline

\[ F_{mix} = -\overline{K_p} \frac{\partial O}{\partial z} A_{th} \] (4.10)

where \( \overline{K_p} \) is the eddy diffusivity computed from Equation 4.6 and \( A_{th} \) is the horizontal area at the depth of the 24°C isotherm. The vertical gradient of DO was obtained with the F-probe measurements. The flux values obtained from the profiles measured at each station were then integrated over \( A_{th} \) and averaged between two successive transects.

The net DO production term was then found from

\[ S = F_{mix} - F_{air} + F_{dn} - F_{up} + \frac{dOV}{dr} \] (4.11)

Figure 4.13 presents the results of the DO balance. \( F_{air} \) represented the largest flux of DO to the oxygen block. In Figure 4.13, the central value of \( F_{air} \) is the mean value of \( F_{air} \) obtained from the two gas-transfer models, while the error bars indicate the values computed with the WA gas-transfer model (larger values) and the values computed with the CW model (smaller values). \( F_{air} \) computed with the CW model was 49%-69% of the fluxes computed with the WA model however, for both gas-transfer models, \( F_{air} \) was still significantly larger than the other fluxes (67%-436% larger than \( F_{up} \), the largest of the other fluxes).

We were unable to calculate a range of values for \( F_{up} \) and \( F_{dn} \) because we did not measure the flow rates in the field (note that the model accuracy does not indicate model error in relation to field conditions, but simply the truncation error in the numerical scheme). However, for \( F_{up} \) to be about the same as \( F_{air} \), either the flow rate or the DO concentration would have to double, at least. Given the good agreement between the model results and our temperature measurements, the errors in the modelled flow rates were certainly not in a factor of two, which is the equivalent to a Snake River flow rate larger than 400 m\(^3\) s\(^{-1}\). The accuracy of the DO Sensor is about 2% of the saturation levels (<0.2
mg L\(^{-1}\)) therefore; DO measurement errors did not amount to a factor of two, either. Other errors such as the cross-sectional area calculation, the use of a single profile to represent the average DO concentrations of the whole section, etc. were also difficult to evaluate, but were also not in a factor of two. For the rate of change of DO \((d(OV)/dt)\), the same error evaluation problems arose i.e., use a single DO profile to represent the concentration of the whole volume, volume estimation error, etc. We speculate that the errors for \(d(OV)/dt\), \(F_{up}\) and \(F_{dn}\) were smaller than 50%.

In general, \(F_{mix}\) and \(F_{dn}\) balanced \(F_{up}\) (94-186\% of \(F_{up}\), considering the upper range value of \(\overline{K_p}\)), and \(F_{mix}\) (computed with the average \(\overline{K_p}\)) represented only between 15\% and 24\% of \(F_{dn}\) (Figure 4.13b). Note that the error bars for \(F_{mix}\) (computed using the standard deviations of \(K_p\), Equation 4.6) are not symmetrical, given the nature of the lognormal distribution for microstructure data (Baker and Gibson 1987). Note also that Equation 4.5 assumes an absolute maximum mixing efficiency (Ivey and Imberger 1991) thus; \(K_p\) values adopted in this analysis represent a very safe upper bound. Mixing in the lake interior, therefore, did not contribute significantly to the depletion of oxygen in the block.

\(S\) was most times negative in order to offset the large re-aeration influx, indicating that the net (biological) oxygen demand exceeded the net (photosynthetic) DO production. However, the analyses involving transects collected between days 200 and 201 (i.e., estimated in a time scale of ~9 hours) presented a large variability in the rate of change of DO that was consistent with the observed \(S\) variability (Figure 4.13a). The largest calculated consumption rates (negative \(S\)) and observed DO decline were observed on day 200 (Figure 4.13a) during an overcast period (Figure 4.4) of strong wind forcing arresting the surface overflows (Figure 4.10). In contrast, the block experienced positive rates of change of DO that required a net DO production (positive \(S\)) to satisfy the balance on day 201 (Figure 4.13a). In comparison to a similar period of day 200, this increase on day 201 resulted from the combination of a strong wind induced re-aeration (Figure 4.13a) and a 60\% increase in the average sunlight radiation (Figure 4.4). Light intensity therefore likely limited photosynthesis and controlled the role of the phytoplankton as either a net consumer or producer of DO. Although \(S\) was negative in order to satisfy the balance in the period on which interflows were observed (Figure 4.13a), this result was biased because of the large time over which the balance was considered, that is ~48 h in comparison to 9 h for the
balances during the overflow period. The fact that the rate of change of DO was positive and $F_{air}$ decreased during the interflow period (Figure 4.13a) indicated that the DO block became less depleted in response to interflows. The main reason was because the flow stagnation associated with the plunge point and the sheltered convergent zone provided niches for algae growth (Figure 4.12, see also Discussion) with better light conditions upon inflow plunging. During the whole period, the oxygen levels were recovering in the block, as indicated by the predominantly positive DO rates of change with time (Figure 4.13a). Results in Figure 4.13 presents the DO variations in time scales varying from 9 h to 7 d in the DO block and this allows us to understand how such oxygen block formed.

**DO-block inter-seasonal evolution**

Figure 4.14 illustrates the severe decline of DO in the surface layer as summer temperatures intensified. On day 156 (05 Jun), interflows were the main inflow pattern and the DO block was in its early stages of development (Figure 4.14a). Such characteristics could by identified by the warm signature of the convergent zone (also the conductivity signal, not shown) and the lower DO concentrations in the neighbourhood of station T2, respectively. From day 156 to day 182 (01 Jul), the DO concentrations in the core of the block reduced 36%, the lake surface layer and inflow temperatures increased about 5°C, the inflow flow rates decreased 43% and the position of the block moved 8 km to the location of the convergent zone (Figure 4.14). This increase in water temperatures was consistent with a gradual increase of air temperatures (not shown) leading to a continuous overflow inflow pattern (Figure 4.4) and this decrease in inflow flow rates led to the arrestment of the inflow in the convergent zone (Figure 4.10) where the wind stress was weaker and re-aeration rates were lower. In this interim period (days 164 to 192, 13 Jun to 11 Jul), measurements taken in the Snake River 35 km upstream of BR01 registered a two-fold increase (to ~7 mg L$^{-1}$) in the observed BOD$_5$ (five-day BOD), a 2~3-fold increase (to ~70 μg L$^{-1}$) in Chla concentrations, a drastic reduction in the ratio of NO$_3$ to TKN (total Kjehldal nitrogen; from 0.73 to 0.26) and a slight reduction in the ratio of DRP (dissolved reactive phosphorus) to TP (total phosphorus; from 0.27 to 0.07); about 80~95% of nutrients became incorporated into the algae biomass and the organic substrate (Figure 4.15). The following sequence of events could be used to explain how the DO block formed. As the water temperature (and associated sunlight radiation) increased, the BOD in the water column also increased in response to the algae bloom in the Snake River (Figure 4.15). Given that ammonia levels in the inflow, during this period, were always below
detection limit (Figure 4.15, see also Nürnberg et al. 2001), the oxygen demand in the water column could be attributed mostly to the respiration of the algae and associated bacteria growing on the algae substrate. At the same time, inflow rates drastically further reduced, decreasing the inflow inertia as the inflow approached the reservoir. As a result the phytoplankton biomass in the inflow sank in the proximities of the plunge/lift-off point (see Figure 4.12 and Discussion). The northerly winds, on the other hand, arrested the continuous overflows against the warm (and stagnant) convergent zone where the respiration of the bacterial community thrived on the dissolved (colloidal) organic matter culminating in the observed DO block (Myers et al. 1997).

By contrast to the above scenario of DO depletion, Figure 4.16 shows how DO concentrations changed as the reservoir approached stratification-overturn period. From days 206 to 232, after a collapse of the Snake River algae biomass between days 192 and 199 (18 Jul), inflow temperatures (~8°C) and Chlα concentrations (~50%) reduced (Figure 4.15). Although BOD30 remained roughly constant, the rapid organic matter decay associated with the BOD5 was reduced by 26%. As BOD measurements are corrected to 25°C, the actual BOD reduction in the water was probably larger. Such a reduction in the inflow BOD enhanced the effects of wind re-aeration in the DO block and produced a net source of DO to deeper strata in the lake (Figure 4.16).

At the same time that temperatures decreased, the surface layer deepened, the inflow plunged and the DO block disappeared (Figure 4.16). Interflows became a continuous inflow pattern, favouring growth of the in-lake algae patches to such an extent that supersaturated DO concentrations existed in the convergent zone (Figure 4.16b, see Wetzel 1983 for DO solubility as a function of temperature). Moreover, both inflow and outflow flow rates increased and the wind shear layer no longer arrested the interflow and as a result, the inflow residence time became smaller (Figure 4.16). Further reduction of the residence-time occurred when the thermocline reached the level of the offtakes and the winter withdrawal operation initiated forming a through flow between the inflow and the outflow (Ebel and Koski 1968).

4.6. Discussion

The problem of oxygen depletion in the surface layer of the upper reaches of Brownlee Reservoir has puzzled many investigators for the last 40 years (Ebel and Kosky 1968; Myers et al. 1997; Nürnberg et al. 2001) and no definite conclusions about the causes of the
problem have been provided. The work presented in this paper is the first to rely on detailed measurements and simulations of physical processes. Under the conditions experienced during this study and from the available data, we concluded that oxygen depletion occurs predominantly due to oxygen demand in the inflow and the formation of a convergence zone in the reservoir where the water was essentially stagnant, and wind re-aeration rates were relatively small in comparison to other regions of the lake. Hence, in this convergent zone, the oxygen demand was large enough to overcome all other processes that would bring an increase of the oxygen concentrations. In addition, we showed that the DO demand was due to respiration of the planktonic community accompanied by a Chla decrease in the Snake River approaching the reservoir. Chla and DO decrease along the inflow was consistent with observations made during summer in other temperate canyon-valley reservoirs (Comerma et al. 2001; Masin et al. 2003). Despite this trend and the realization of the importance of bacterial metabolism, the origin of the organic matter serving as substrate for the bacterioplankton growth and respiration in these other reservoirs was not clearly specified, that is, if the organic matter was composed mainly by labile DOC (dissolved organic carbon) associated with the phytoplankton metabolism and senescence or by recalcitrant DOC associated with humic substances. Given DO production is directly dependent on the primary production rates, it is worth discussing the possible causes for Chla decrease along the inflow.

First, there is strong evidence that in summer, temperature no longer limits heterotrophic bacterial growth such that carbon and nutrient resources, notably inorganic phosphorus, become limiting and bacteria effectively out compete algae for nutrients (Currie 1990; Felip et al. 1996; Brett et al. 1999). Specifically, negative (positive) phytoplankton (bacteria) growth rates were observed in an eutrophic low-residence-time reservoir (Chrzanowski and Grover 2001). Under nutrient limitation, especially dissolved phosphorus (Figure 4.15), bacteria can uptake inorganic nutrients more efficiently than algae (Currie and Kalff 1984; Rothaupt and Güde 1992; Chrzanowski and Grover 2001). In principle, nutrients enter the microbial loop and do not become available to the algae. Our data does not permit us to verify this hypothesis, which, however, is further supported by the collapse of Chla concentrations observed between days 192 and 199 (Figure 4.15).

Second, our Fluoroprobe measurements indicated that diatoms dominated the Snake River algae population (Figure 4.17a). This dominance had been previously verified in Brownlee Reservoir (Harrison et al. 1999) and was consistent with an algae group that
requires water turbulence to remain in the water column (Reynolds et al. 2002). With the reduced inflow inertia experienced with the inflow lift-off and the northerly winds arresting the convergence zone, diatoms become progressively less adapted to the environment and sink to the sediment (Friedl and Wüest 2002). Significant sinking rates of river-borne diatoms have been observed in many other reservoirs (see e.g., Shermann et al. 1998, Garnier et al. 2000, Tsujimura and Okubo 2003). While the relative contribution of algae/bacteria competition for nutrients and diatom sinking rates still need to be quantified, both mechanisms may explain the reduction of Chla (and associated DO decline) as the Snake River inflow approached Brownlee reservoir.

However, the lowest DO concentrations occurred 7–12 km downstream of the lift-off point (Figure 4.3), which is a few kilometres downstream of the position where the diatoms effectively sank (Figures 4.12 and 4.17a). Straskabova and Komarkova (1979) observed that peaks of bacterial abundance in a stratified reservoir sometimes lagged phytoplankton maxima in a time scale of about 10 days. This is exactly the same time scale of the travel time to cover the 35 km from the station where the inflow measurements in Figure 4.13 were made to the approximate location of the DO block core at T3 (assuming the Snake River travels at 0.04 m s⁻¹, Figure 4.10). The decline in the DOC signal within the inflow downstream of the lift-off point (Figure 4.18) provided a further indication that respiration of heterotrophic bacteria was responsible for the DO block formation. While this scenario may explain the main reasons for DO block formation within the inter-seasonal time scale, the large variability of net DO consumption rates within the block itself deserves further attention.

The patches of increased DO concentrations in the block were strictly related to an increase of Chla concentrations. Contrasting to the diatom dominance in the river section, a combination of cyanophytes and cryptophytes groups dominated the patch of Chla downstream of the plunge point (Figure 4.17). Cryptomonas, Microcystis, and Anabaena species have all been previously counted in expressive numbers in the reservoir in similar periods and locations of our study (Harrison et al. 1999). This shift in dominance has been previously attributed to the commencement of seasonal stratification in several other reservoirs (Sherman et al. 1998, Garnier et al. 2000, Tsujimura and Okubo 2003), however, to our knowledge, this is the first time a shift in dominance was related to spatial variations in the flow field. Such a shift from river diatoms to lake cyanophytes downstream of the plunge point was consistent with the functional classification of the phytoplankton.
As suggested by the DOC signal in Figure 4.18, light intensity increased downstream of the plunge point. Such a change in light climate is of great importance to the local ecosystem. Light availability has recently been pointed out as the main controlling factor of the relative contribution of the phytoplankton and bacteria on total community respiration (Roberts and Howarth 2006). While bacterial respiration rates remain roughly constant under differing light conditions, increased light intensity can increase the respiration rates of the phytoplankton to over 90% of the total planktonic respiration (Roberts and Howarth 2006). Therefore, the apparent improvement of DO concentrations in the block could be quickly offset under reduced light intensity as occurred on days 199 and 200 (Figures 4.4 and 4.13). The change in inflow patterns may have further implications in the relationship between algae and bacteria, given inflow nutrients and substrate are only partially transported to the surface layer via mixing at the plunge point (Armengol et al. 1999). If we note that bacteria remains aggregated to the organic substrate and that the mineralization rate of the labile dissolved organic carbon is larger than 0.5 day\(^{-1}\) (Wetzel 2003), at first order, it takes about 2 days after the inflow started plunging for labile DOC to reduce to less than 35% of its initial levels. As a result, bacteria that had been previously transported with the overflows progressively became carbon limited. Therefore, a bacterial community that was previously out-competing the diatom algal community for nutrients in a carbon-rich river-like environment would have to change strategy to survive in a carbon-limited lake-like environment. This strategy change is consistent with the two contrasting mutualism/competition relationships between algae and bacteria proposed by Currie (1990). It is therefore not surprising that the inflow dynamics in canyon-valley reservoirs may not only affect the plankton niches and trophic levels along the transition zone of reservoirs (Kennedy et al. 1982; Lind et al. 1993; Šimek et al. 2001; Comerma et al. 2001) but it also affects the whole functioning of the ecosystem, i.e., the interaction between planktonic communities and associated fluxes of carbon, oxygen and nutrients. Moreover, our results suggest that such a shift in the ecosystem functioning could occur at several scales depending on the constancy of inflow pattern, that is, the patch scale during the 5-day inflow pattern variation and at the “inflow-zone” basin scale when interflows became a continuous inflow pattern.

It is yet to be proven if such a remarkable shift in the ecological relationships of planktonic communities can occur in relatively small time scales (i.e., 2 days) following
subtle changes in the physical and biogeochemical forcing of the system, in this case, to differing inflow patterns resulting from longitudinal variations of the wind stress. Field experimentation to understand the relative importance of physical (i.e., inflow pattern, light limitation) and biological (i.e., competition for nutrients, carbon limitation) processes is not trivial and requires a large logistic apparatus, human resources and cost considerations. We have shown that our numerical simulations reproduced the motion in the reservoir at scales (~500 m) that are small enough to capture variability at the patch level (~5 km). Under some constraints these resolvable scales can be reduced to the order of 10 m (Chapter 3). It is also yet to be shown that ecological models have the sophistication required to reproduce the ecosystem dynamics at such scales. Our final goal is to establish a coupled hydrodynamic and ecological model that can provide reliable answers to these questions at a reasonable cost. Such a model, if based on sound validation, would open new avenues to understand how physical processes affect the chemical energy flux in aquatic ecosystems.

4.7. Conclusions

In this paper we have shown how precision field work can be combined with modern 3D hydrodynamic modelling to identify and quantify water column environmental niches where certain biological processes may be isolated. In the example presented we show how the interaction between the wind stress and inflows in a canyon-valley stratified reservoir facilitated depletion of DO in the surface layer. While the surface overflows provided organic matter to the surface layer of the reservoir the wind stress acted preferentially in the opposite direction of the inflow flow direction. The direct effect of the wind stress was to create a convergent zone, where the inflowing water was brought to rest, allowing the oxygen demand in the water to continue to deplete the oxygen levels whilst favouring the decline of a photosynthesising diatom population. To first order, the oxygen budget in this stagnant water was an oxygen increase due to photosynthetic production and aeration through the surface layer and depletion due to decaying organic matter. At second order, vertical mixing across the thermocline was shown to balance the difference between horizontal advective fluxes coming across the end boundaries. The work has demonstrated that limnology is at the point where the physical and biogeochemical signal maybe be separated to advantage, giving order to the biogeochemical variations.
4.8. Acknowledgments

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4.9. References


Crusius, J., and R. Wanninkhof. 2003. Gas transfer velocities measured at low wind speeds
DO Response to Wind-Inflow Interactions


Centre for Water Research.


Rothaupt, K. O., and H. Güde. 1992. The influence of spatial and temporal concentration gradients on phosphate partitioning between different size fractions of plankton: further


### Table 4.1 - Morphometric characteristics of Brownlee Reservoir.

<table>
<thead>
<tr>
<th>Alt. (m)</th>
<th>$A_w$ (Km$^2$)</th>
<th>$A_s$ (Km$^2$)</th>
<th>$A_w/A_s$</th>
<th>$z_{max}$ (m)</th>
<th>$\bar{z}$</th>
<th>$\sqrt{\bar{z}}/A_s$</th>
<th>Vol. ($10^6$m$^3$)</th>
<th>Len. (km)</th>
<th>Width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>631</td>
<td>$189 \times 10^3$</td>
<td>47.5</td>
<td>3891</td>
<td>85</td>
<td>32.3</td>
<td>4.7</td>
<td>1533</td>
<td>80</td>
<td>300-900</td>
</tr>
</tbody>
</table>

Note: *Alt.* = Mean pool level altitude a.s.l.; $A_w$ = watershed area; $A_s$ = mean pool leve surface area; $z_{max}$ = maximum depth; $\bar{z}$ = mean depth at mean pool level; Vol. = volume at mean pool level; and Len. = length
<table>
<thead>
<tr>
<th>Day in 2002</th>
<th>Instruments</th>
<th>Stations</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>198</td>
<td>F-PROBE</td>
<td>BR01 to BR30</td>
<td>Time to profile: 12:20-17:32</td>
</tr>
</tbody>
</table>
| 200        | F-PROBE     | BR01 to BR30 | Time to profile: 07:09-11:16  
|            |             |            | Time to profile: 13:16-17:54  
|            |             |            | Time to profile: 19:23-23:18              |
| 201        | F-PROBE     | BR01 to BR30 | Time to profile: 07:28-12:42  
|            |             |            | Time to profile: 14:44-20:01              |
| 203        | F-PROBE     | BR01 to BR30 | Time to profile: 11:07-15:58                |
| 203        | SCAMP       | BR06, BR14  | 10 profiles between 19:01-20:59  
|            |             |            | 10 profiles between 21:29-23:31           |
| 204        | SCAMP       | BR25        | 10 profiles between 00:08-02:35             |
| 205        | F-PROBE     | BR01 to BR30 | Time to profile: 10:02-14:42               |
| 205        | SCAMP       | BR06, BR14, BR25 | 1 profiles at 23:47  
|            |             |            | 10 profiles between 15:28-18:56  
|            |             |            | 10 profiles between 20:52-22:59           |
| 206        | F-PROBE     | BR01, BR04, BR06, BR10  
|            |             | BR14, BR16, BR19, BR22, BR25, BR28, BR31 to BR36 | Time to profile: 11:31-15:26              |
| 206        | SCAMP       | BR06, BR34  | 9 profiles between 00:01-01:19  
|            |             |            | 4 profiles between 16:03-17:57             |

Table 4.2 - Field data collected during the intensive field campaign.
Figure 4.1 - Brownlee Reservoir map (44.5°N, 117.1° W) with sampling stations. Only LDS stations (T1, T2, and T3), SCAMP stations (BR06, BR14, BR25, and BR34), and initial (BR01) and final (BR30 and BR36) F-probe longitudinal transects stations are annotated. The other stations not annotated in the map (BR02 to BR05, BR07 to BR13, BR15 to BR24, BR26 to BR29, BR31 to BR33, and BR35) increase their station number (i.e., BRXX, where XX is the station number) from the upstream (Snake River inflow) to the downstream (dam wall) Snake River flow direction. SCAMP stations were also used as F-probe stations.
Figure 4.2 - (a) Bathymetry of Brownlee Reservoir with the 100x100 m grid. The large rectangle defines the region where the 20x20 m grid shown in panel (d) and Figures 4.8 to 4.10 was used. The small rectangle is shown in detail in panel (c). (b) Straightened bathymetry used in the 800x100 m grid. (c) Detail of the small rectangle in panel (a) to show the 100x100 m grid. The blue square is shown in detail in panel (d). The green rectangle was drawn to compare the grid sizes of the 800x100 m grid with the 100x100 m
grid. (d) Detail of the blue square in panel (c) to show the 20x20 m high-resolution used in the 100x20 m grid. Southing and longitudinal axes in panels (a and b) are kept at the same scale for comparison.
Figure 4.3 - Field data - fine scale dissolved oxygen concentrations and water temperature. (a) Data collected on day 198, 17 July 2002, (b) data collected on day 200, 19 July 2002 (transect 3) and (c) data collected on day 205, 24 July 2002. White dashed lines represent the location of thermistor chains and the circles at the top part of the plots indicate the position of each profile. The white solid lines show selected isotherms from 8 to 26°C in increments of 2°C. The black solid lines show the 1.5 and 3 mg L⁻¹ isoconcentrations. Colour scale represents DO concentrations in mg L⁻¹.
Figure 4.4 – (a) Wind shear stress at station T2 and T3. (b) Surface heat fluxes at T2 and T3. (c) Water temperatures measured at the inflow gauge, T2 and T3 and air temperature. Grey solid line indicates data at T3, black solid line data at T2, and the dot-dashed line data at the inflow. The dotted line indicates air temperature for which the scale is at the right hand side of the plot. Vertical dashed lines in (panel c) indicate the time from which interflows and overflows were observed.
Figure 4.5 - (a, b, c, g, h, i) Time evolution of modelled and measured temperature field at the LDS stations. (d, e, f, j, k, l) Modelled and measured power spectra of the vertical
displacements of the 23°C isotherm. (a, b, c, d, e, f) Comparison at station T3 with the simulations (a, d) in the 800x100 m grid, (b, e) in the 100x100 m grid, and (c, f) in the 100x20 m grid. (g, h, i, j, k, l) Same as (panels a to f) but for station T2, (g, j) in the 800x100 m grid, (h, k) in the 100x100 m grid, and (i, l) in the 100x20 m grid. Black solid lines indicate hydrostatic simulation results, green solid lines in (panels f and l) indicate non-hydrostatic simulation results, and red solid lines indicate field data. Isotherms shown range from 15 to 25°C in increments of 2°C for station T3 and from 13 to 25°C for station T2. Horizontal ticked lines in (panels c and i) indicate the troughs of internal waves mentioned in the text. The departing frequencies are indicated in the power spectra plots.
Figure 4.6 - (a) Field data - fine scale temperature on 19 July 2002, day 200, transect 1. White dashed lines represent the location of thermistor chains and circles indicate the position of each cast. Isotherm contour interval is 1°C from 7 to 25 °C (labelled) and 0.25 °C from 25.25 to 26 °C (not labelled). (b) Same as (panel a) but for simulation results using the 100x20 m grid. (c) Difference in temperature between simulation and field data. Colour scale in (panels a and b) represents temperature while in (panel c) it represents the difference in °C.
Figure 4.7 - Same as Figure 4.6 but for day 24 July 2002, day 205.
Figure 4.8 - Wind forcing and flow field in the surface layer obtained from the non-hydrostatic simulation in the high-resolution area of the 100x20 m grid. (a) 10-minute averaged wind velocities at station T2. (b) Temperature and velocity field at the level 70 (at the interface between metalimnion and surface layer) from time 199.000 to 200.500 at
every 3 hours (0.125 days). (c) Same as (panel a) but for station T3. In (panels a and c) the negative values indicate purely northerly winds and the positive values indicate purely southerly winds, as suggested by the N and S labels, and vertical lines indicate the times shown on (panel b). In (panel b) each frame is offset by 2 km, the time for each of the frames is displayed in the inferior part of the panel, the velocity scales are given in the right hand side of the panel and the temperature scale is given in °C. In (panel b) the yellow dot indicates the location of station T3 and the red dots indicate stations BR14 (south most) to BR20 (north most).
Figure 4.9 - Same as Figure 4.8 but for level 77. (a) Same as Figure 4.8a. (b) Same as Figure 4.8b but for level 77. (c) Same as Figure 4.8c.
Figure 4.10 - Same as Figure 4.8 but for level 83. (a) Same as Fig 4.8a. (b) Same as Figure 4.8b but for level 83 (surface). (c) Same as Figure 4.8c.
Figure 4.11 - Measured dissipation rates and temperature profiles. (a) At BR14 on day 203, 22 July 2003. (b) At BR14 on day 205, 24 July 2003. (c) At BR25 on day 204, 23 July 2003. (d) At BR25 on day 205, 24 July 2003. See Table 4.2 for time details. Bars are the log-means of the dissipation rates that intersect each of the 1-m depth bins. Lines are the temperature profiles.
Figure 4.12 - Field data - fine scale Chla concentrations and salinities. (a) Data collected on day 198, 17 July 2002, (b) data collected on day 200, 19 July 2002 (transect 3) and (c) data collected on day 205, 24 July 2002. White dashed lines represent the location of thermistor chains and the circles at the top part of the plots indicate the position of each profile. The white solid lines show selected isohalines from 0.15 to 0.20 in increments of 0.01 in the metalimnion and from 0.200 to 0.218 in increments of 0.002 in the surface layer. Extension of the DO block is shown for comparison with Figure 4.3. The colour scale indicates Chla concentrations in mg L$^{-1}$.
Figure 4.13 - DO fluxes in and out of the DO block. (a) $\frac{d(OV)}{dt}$, $F_{air}$, and $S$. (b) $F_{up}$, $F_{dn}$, and $F_{mix}$. Data was split in two panels to highlight the difference in magnitude between the fluxes. $F_{air}$ error bars indicate the results of different gas-transfer models (see text). $F_{mix}$ error bars indicate the results of fluxes calculated with the range of $K_p$ computed with Eq. 6 (see text). $S$ error bars indicate maximum and minimum values of the net DO production rates computed from the combination of different $F_{air}$ and $F_{mix}$ value ranges.
Figure 4.14 - IPC data - fine scale dissolved oxygen concentrations and water temperature. Same legend as Figure 4.3. (a) Data collected on day 156, 05 June 2002, (b) data collected on day 182, 01 July 2002 (transect 3). The black solid lines show the 4.5 and 6 mg L$^{-1}$ isoconcentrations in (panel a) and the 2.0 and 3.5 mg L$^{-1}$ isoconcentrations in (panel b). White dashed lines represent the location of thermistor chains and the circles at the top part of the plots indicate the position of each profile. Note the change in colour scale in comparison to Figure 4.3. Snake River inflow discharges are indicated in the plot. The colour scale represents DO concentrations in mg L$^{-1}$. 
Figure 4.15 - Nutrients in the Snake River 15 km upstream of station BR01. (a) BOD$_{30}$, BOD$_{5}$, and Chla, (b) ammonia, nitrate, and total Kjehldahl nitrogen (TKN), (c) dissolved reactive phosphorus (DRP), total phosphorus (TP), and inflow temperature.
Figure 4.16 - IPC data - fine scale dissolved oxygen concentrations and water temperature. Same legend as Figure 4.14. (a) Data collected on day 253, 10 September 2002, (b) data collected on day 280, 07 October 2002 (transect 3). The black solid lines in (panel b) show the level of the outflows. Snake River inflow and outflow discharges are indicated in the plot. The colour scale represents DO concentrations in mg L\(^{-1}\).
Figure 4.17 - Field data - fine scale algae-group normalized Chla concentrations and salinities on day 205, 24 July 2002. (a) diatoms, (b) cryptophytes and (c) cyanophytes. Chla concentration is normalized by the total Chla concentration (Figure 4.12c). White dashed lines represent the location of thermistor chains and the circles at the top part of the plots indicate the position of each profile. The white solid lines show selected isohalines from 0.15 to 0.20 in increments of 0.01 in the metalimnion and from 0.200 to 0.218 in increments of 0.002 in the surface layer. Extension of the DO block is shown for comparison with Figure 4.3.
Figure 4.18 - Field data - fine scale fluorescence of yellow substances on day 205, 24 July 2002. White dashed lines represent the location of thermistor chains and the circles at the top part of the plots indicate the position of each profile. The white solid lines show selected isohalines from 0.15 to 0.20 in increments of 0.01 in the metalimnion and from 0.200 to 0.218 in increments of 0.002 in the surface layer. Extension of the DO block is shown for comparison with Figure 4.3. Units of fluorescence are r.u.
Chapter 5

5. Conclusions and Future Work

This thesis contributes to the current ability in reproducing multi scale phenomena in density-stratified lakes by using an efficient strategy to account for non-hydrostatic effects in the water motion; therefore, opening up new ways of understanding how different physical processes interact and impact on energy fluxes paths and ecological patchiness observed in lake ecosystems.

The numerical model and grid-switching strategy implemented in this research was well suited to a wide range of scales, simulating internal waves with scales ranging from the basin-scale to the buoyancy frequency limits of the spectrum. Current conventional computer resources, however, still restrict our ability in reproducing the small-scale phenomena in a lake-wide fashion. Nonetheless, given increasing efficiency and improvement of computational technology, resources in the near future may allow the application of the numerical model to a high-resolution discretization of a whole-lake domain. In this case, the current numerical model seems to be sufficiently robust to simulate the flow field at all scales of the internal wave spectrum.

Despite these limitations, the numerical model was shown to reproduce two distinct flux paths in the internal wave energy cascade, the cascade from basin-scale waves to the internal surge and the packet of solitary waves, and the transfer of energy from the wind forcing directly to high-frequency waves near the buoyancy-frequency limit. Noting that the spatial scales and nature of forcing is strongly variable among different lakes, the model provides a new tool to investigate the combined effects of physical processes acting at different scales.

Results presented in Chapter 4, for example, show how the effect of the variable wind forcing along the lake acting upon the inflow forcing can induce the evolution of ecological patches and the formation of different ecological niches associated with these patches. Understanding of these peculiar mechanisms can lead to a more effective management of lakes taking into consideration both the economic and ecological perspectives in the functioning of the lake ecosystem.
Chapter 4

The work contained in this thesis still has large potential for improvement. Tests of different interpolation schemes for the grid-switching strategy are likely to considerably reduce the numerical error during each grid switch, therefore making the technique more appealing to long time integrations. Further, automatic grid switching based on the non-hydrostatic flow characteristics would skip the requirement of previous knowledge of the flow field in order to plan grid-switching simulations. I believe that such high-resolution models if coupled to ecological and water quality models will provide new ways of examining and understanding how the interaction between physical and biogeochemical processes occur at different spatial and time scales.