VORTEX-INDUCED VIBRATION OF TWO CYLINDERS WITH DIFFERENT DIAMETERS IN CLOSE PROXIMITY

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ABSTRACT

This thesis presents an investigation of vortex-induced vibrations (VIV) and vortex shedding of two circular cylinders with different diameters in a close proximity. The effect of the proximity of cylinders on their vortex regimes and induced vibrations is discussed. The cylinders are coupled to each other either mechanically or hydrodynamically. These two types of coupling are often found in subsea piggyback pipeline and umbilical systems. The possibility of collision of the cylinders when they are free to oscillate in proximity of each other is also investigated. Wide lock-in ranges and high vibration amplitudes outside their lock-in ranges are the direct results of their hydrodynamic coupling.

The Petro-Galerkin finite element method is implemented in two-dimensional space to solve the RANS equations. The implemented meshes are quadrilateral and are generated using ANSYS software. The developed CFD code is equipped with ALE method to facilitate the mesh movements near the solid surfaces. The SST $k-\omega$ turbulence model is used to deal with inherited turbulence features of the fluid flow at high Reynolds numbers. The results of the simulations are utilised to investigate the interactions between fluid flow and the two cylinders under different flow conditions.

The second Chapter deals with the VIV of a piggyback pipeline system. The diameter ratio of the cylinders in the piggyback pipeline system is fixed at 0.1. The gap between the cylinders and the orientation of the small cylinder are varied in a finite range of values based on industry applications. The effects of the different positions of the small cylinder on the VIV and hydrodynamic force coefficients of the bundle are also investigated. High vibration amplitudes of the bundle in both the in-line and cross flow directions are observed in staggered arrangements. Furthermore it is found that both orientation and the gap ratio of the cylinder have significant effects on the vibrations.

In Chapter 3, the two cylinders are allowed to oscillate independently in the crosswise direction. They are coupled to each other only hydrodynamically. Their VIVs and vortex shedding regimes are studied when they are placed initially in a side-by-side arrangement and the gap between them is equal to the small cylinder diameter. The possibility of collision is investigated by varying the natural frequencies of the two cylinders. Furthermore the effects of collision on VIV of the cylinders are investigated in detail. It is found that the collision of cylinders occurs when the natural frequency of the small cylinder is larger than the large cylinder.

The lock-on behaviours of the cylinders in the side–by-side arrangement are studied in Chapter 4. The comparison of the lock-on ranges of the cylinders with an isolated cylinder
and their mutual interactions on each other are discussed. The simultaneous lock-on of both cylinders and their consequent collisions are also modelled and analysed. Wide lock-in ranges and beating phenomena are observed in the vibration analyses of both cylinders. Physical interpretations of the link between the duration and direction of induced forces on the cylinders and vortex shedding are presented.

Conclusions of this study and recommendations for future research on this topic are presented in Chapter 5 of this thesis.
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CHAPTER 1

INTERODUCTION AND LITERATURE REVIEW

1.1 Introduction

Flow around bluff bodies is one of the interesting and classical subjects in fluid mechanics. The mutual interaction of the fluid and bluff body gives rise to vortex shedding from the bluff body which cause a fluctuating hydro/aerodynamic force on the structures. If the bluff body is free to oscillate and its natural frequency matches the frequency of the hydrodynamic force induced on the bluff body, then significant vibration of the bluff body will occur, which is normally termed as the vortex-induced vibrations (VIVs). VIVs occur in many engineering applications. Structures such as tension leg platforms, drilling risers, catenary and submarine pipelines, power transmission lines, chimneys and bridges experience vibrations induced by fluid flow passing across them. These induced vibrations shorten the durability of the structures. The qualitative and quantitative estimations of these vibrations are important for the evaluation of the fatigue life of the structures.

1.2 Vortex shedding from a bluff body

Some relevant parameters in the study of VIV of a bluff body, or specifically a cylinder, in steady flow are Reynolds number $\text{Re} = \frac{U_{\infty}D}{\nu}$, mass ratio $m^* = \frac{m}{m_d}$, the structural damping ratio of the cylinder, $\zeta = \frac{\delta}{2\pi}$ and reduced velocity $U_r = \frac{U_{\infty}}{f_nD}$, where $\nu$ is the kinematic viscosity of the fluid, $U_{\infty}$ is the free stream velocity, $D$ the diameter of the cylinder, $m$ the mass of cylinder, $m_d$ the mass of displaced fluid by the cylinder, $\delta$ the natural logarithmic decrement of the oscillating cylinder and $f_n$ the natural frequency of the system. Based on the reports of Roshko (1954), Lienhard (1966) and Williamson (1996) at very low Re (Re<5, high viscous and low inertia forces) flow passing a circular cylinder is perfectly symmetric and streamlined and the flow patterns resemble the inviscid potential flow. By increasing the Re (5<Re<40) a fix pair of symmetric vortices forms behind the cylinder. Further increase of Re (Re>40) contributes to the instability of these vortices and their alternative shedding from both sides of the cylinder. The vortex shedding process is described by Gerrard (1966). According to him the vortex on the topside of the cylinder, Fig. 1.1 (a), is fed by its connected shear layer till it attains enough strength to draw the downside shear layer toward itself. The counter-clockwise vortex from the downside cut off the supplied shear layer to the topside vortex, Fig. 1.1 (b). The disconnected clockwise vortex is then shed and moves downstream. In the next steps (Fig.1.1 (c) and (d)) the counter clockwise vortex in the downside is cut off by the clockwise vortex from the top side. As
these vortices are shed from different sides of the cylinder the surface pressure on the cylinder surface varies and as a consequence the cylinder experience varying hydrodynamic lift and drag forces. Lighthill (1979), Lighthill (1986) explained the relation of the these hydrodynamic forces to the strength and the varying distance of the shed vortices from the cylinder.

![Vortex shedding mechanism](image)

Fig. 1.1: Vortex shedding mechanism. (a) and (b) show the development and shedding of clockwise vortex (blue). (c) and (d) show the shedding of anti-clockwise vortex (red) from the bottom side.

The shedding vortices and the boundary layer prior to flow separation from the cylinder have different characteristics (regimes) for Re>40. The transition between these regimes does not occur at sharp Re and vary according to different authors. For Reynolds number in the range of 300<Re<1.5×10^5 the boundary layer remains laminar however the shed vortices are turbulent. This range of Reynolds number is called subcritical. Most of the hydrodynamic applications occur in this regime. Further information about the hydrodynamics force coefficients of stationary cylinders and factors affecting them can be found in the monographs presented by Sumer and Fredsøe (2006), Blevins (1977) and Zdravkovich (1997, 2003).

Strouhal in 1878 found that the vortex shedding frequency \(f_v\) is proportional to the free stream velocity divided by the diameter of the cylinder \(U_\infty/D\). The constant of proportionality is known as Strouhal number, \(St = f_v D/U_\infty\). The consensus value of St is about 0.2 for a circular cylinder in subcritical Reynolds number region, Blevins (1977). The lift force frequency is the same as the Strouhal frequency. However the drag force frequency
is twice the Strouhal frequency. The variation of the St number with different parameter has been discussed by Sumer and Fredsøe (2006) in detail.

1.3 Vortex-induced vibrations of a single cylinder

Flow-induced lift force on an elastically mounted cylinder is a function of vortex shedding frequency. This varying lift force contributes to the cross flow vibrations of the cylinder with more regular and stronger shed vortices compared to a stationary cylinder (Davies, 1976). The closeness of the vortex shedding frequency to the natural frequency of the cylinder results into large amplitude vibrations of the cylinder. These self-limited relatively high amplitude vortex induced vibrations are of practical interest mostly because of the fatigue hazard that they can cause to engineering structures. The vortex shedding frequency locks into the vibration frequency of the cylinder which contributes to vibration of the cylinder at its natural frequency. The lock-in range of these frequencies and the amplitude of vibration depend on the reduced velocity, damping and mass ratio of the cylinder. The shedding frequency departs from the natural frequency of the cylinder in higher reduced velocities and assumes its Strouhal values. Transverse oscillation of an elastically mounted rigid cylinder in free stream has been discussed by many researchers. Bearman (1984, 2011), Williamson and Govardhan (2004), Sarpkaya (2004) and Gabbai and Benaroya (2005) gave extensive reviews on the VIV of a circular cylinder.

Feng (1968) measured the cross-flow vibration of a flexibly mounted cylinder in the air flow ($m^*=248$ and $\zeta=0.00103$) for different reduced velocities. He observed the lock-in phenomena for $5 \leq U_r \leq 7$ in his experiments. An interesting feature of his experiment was the sudden transition (hysteresis) in the amplitude response of the cylinder for a critical value of the reduced velocity. The highest oscillation amplitude was about $0.57D$ during the increasing the reduced velocity (initial branch of the response curve). However, the amplitude for the same reduced velocity was about $0.38D$ when the reduced velocity was decreasing (lower branch of the response curve). Systematic analysis of Zdravkovich (1982) showed later that timing of vortex shedding changes at the discontinuity of the amplitude response curve. Williamson and Roshko (1988) found that the modes of vortex shedding during the initial and lower branches are different. They observed two shed vortices (2S) per each vibration cycle of the cylinder during the initial branch. However for the lower branch two pairs of shed vortices were observed per each cycle (2P). The sudden change of vortex shedding regime from 2S to 2P during increase of the reduced velocity was considered as the reason of hysteresis by Williamson and Roshko (1988). Brika and Laneville (1993) tests showed very similar results to the Feng’s tests. Both of these tests were carried out in the air and their damping and mass ratios were of the same order.
The experimental studies by Khalak and Williamson (1996, 1997, 1999) about 1-DOF VIV of an elastically mounted rigid cylinder showed higher amplitude of oscillations and wider lock-in range compared to the tests of Feng and Brika and Laneville, Fig. 1.2. The system that they used for their experiments had lower mass and damping ratios (e.g. $m^*=2.4$ and $2\pi\zeta=0.00862$) than previously mentioned ones. They categorized the amplitude response curve to three different parts, initial, upper and lower branches. The highest amplitude occurred in the upper branch. The reason for transition between lower and high branches is the change in the vortex regime (2S to 2P). The study by Govardhan and Williamson (2000) did not show any change in vortex regime during the transition from the upper to the lower branch.

![Fig. 1.2: Comparison of the amplitude responses of the Feng and Williamson group tests. $U^*$ is the same as the $U_r$.](image)

Numerical simulation of VIV of a rigid cylinder with 1-DOF is carried out by many researchers. Blackburn et al. (2001) compared the results of water tunnel experiments of a rigid cylinder with 1-DOF with numerical results of two-dimensional and three-dimensional VIV at $Re=500$ using the direct numerical simulation. Their two-dimensional investigation did not predict successfully the experimental results. They did not use any turbulence model for two-dimensional simulation. However their three-dimensional simulation predicted very well the presence of 2P regime in the lower branch. The numerical simulation of Pan et al. (2007) could not predict the upper branch found by the experiments of Williamson’s group. Statistical stability of the RANS simulation which erases the randomness disturbance in the
experiments was blamed for this conspiracy. The numerical results of Wanderley et al. (2008) captured both the high amplitudes and vortex regimes of Khalak and Williamson (1996) very well. The proper combination of the applied turbulence and the RANS models was considered as the key point in achieving compliant results with experiments.

Moe and Wu (1990) carried out experiments on an elastically mounted rigid cylinder oscillating in water. Oscillation could be forced or free in both in-line and cross flow directions (2-DOF) or just in cross flow direction (1-DOF). The ratio of the horizontal natural frequency to the vertical one was 2.178. For 2-DOF free oscillation they noticed the dominant presence of the third harmonic in addition to the low amplitude peaks at second and fifth harmonics in the lift force spectrum. The cylinder experienced more random lift force when it was constrained to oscillate only in cross flow direction however its lock-in range were more restricted compared to the 2-DOF free oscillations. According to Jeon and Gharib (2001) the main effect of the in-line oscillation of the cylinder on its crosswise oscillation is the shift change between lift force and oscillations. This change affects the energy transfer to the cylinder. Jauvtis and Williamson (2004) showed that letting a cylinder (with mass ratios less than 6) to oscillate in 2-DOF contributes to the emergence of one more branches, super upper branch in the amplitude response curve. The high amplitude of the cylinder in cross-flow direction, $1.5D$, in this branch found to be the result of new shedding vortex regime of 2T (three shed vortices per each half cycle). The other result of Jauvtis and Williamson (2004) was the derived relations between the shed vortices, cylinder motion and energy transfer to the cylinder (based on the Lighthill (1986) study). Experiments of Fujarra et al. (2001) about VIV of a cantilever beam also revealed that letting the beam to oscillate in 2-DOF will led to high amplitude of vibration even outside the lock-in region. However the experiment of Sanchis et al. (2008) showed that freedom to oscillate in 2-DOF does not affect the cross flow vibration amplitude of the systems with low mass ratios (for their experiments, $m^* = 1$) when the damping is high. The experimental studies by Someya et al. (2010) on the VIV of a cylinder in water with 2-DOF focused on the excitation frequencies of the cylinder in both in-line and cross flow directions. The considered reduced velocities were less than 4. The measured vibration frequencies in both directions did not follow the Strouhal law. Furthermore no meaningful relationships were observed between the in-line and cross flow vibration frequencies. Someya et al. (2010) observed this “anisotropy” in vibration frequencies in their other tests with different blockage ratio, damping coefficient and cylinder diameter. Their investigation showed that the main reason of their different results from other investigators is the three dimensionality of the flow due to free end gap which can affect the added mass in the in-line and cross flow directions. Singh and Mittal (2005) investigated the effects of Re and $U_r$ on free oscillation of a cylinder with variable
natural frequency for Re number less than 500. They found that by fixing the Re during the resonance, hysteresis effects can occur on both ends of the lock-in range. Furthermore, new vortex regime was found for varying Re (larger than 300) when the $U_r$ fixed at 4.92. This new vortex regime consists of a single and one pair vortices per one oscillation cycle. Modulation of the lift force time history was observed during this vortex regime.

1.4 Fluid flow around two parallel cylinders

The fluid flow and fluid-structure interaction study gets more complicated when two parallel cylinders are present in the fluid flow domain. More parameters should be considered for the analysis when the fluid flow around each cylinder is influenced by the presence of another cylinder. These types of interactions occur in many engineering problems and have been subjects of many studies. The arrangement of two parallel cylinders can be different. They can be positioned side-by-side (their centreline is perpendicular to the free stream flow), tandem arrangement (the centreline is parallel to the free stream flow) and staggered arrangement (for which the plane passing the centrelines of the cylinders is neither perpendicular nor parallel to the upstream fluid flow. According to Zdravkovich (1985, 1987) flow interference between two equal diameter cylinders occurs in three cases: when they are sufficiently close to each other (proximity interference), one of the cylinders is fully or partially submerged in the wake of the other (wake interference) and the overlapping of the two already mentioned (proximity + wake). The different type of flow interference between cylinders which induce fluctuating hydrodynamic forces on the cylinders can be jet-switch, gap-flow-switch and vortex induced vibration. The first two mentioned interferences can be bistable in some regions and cause high amplitude vibrations. The mechanism of inducing vibration by vortex induced vibration is different from the jet-switch and gap-flow-switch. The VIV is self-limited and stable. Other mechanisms inducing vibration (not by flow interference) are the wake-displacement and galloping which are beyond our discussion.

Sumner (2010) reviewed extensively the available literature on fluid flow around two infinite cylinders of equal diameters.

Experiments of Assi et al. (2006) on two tandem cylinders showed that for $m^*=1$ and 2 with $3.0 < S/D < 5.6$, where S is the pitch distance between two cylinders, the vibration mechanism is galloping and the amplitude of vibration increases continuously with increasing the reduced velocity. However for higher pitch distance, Brika and Laneville (1999) found VIV as the dominant vibration mechanism for higher mass ratios. Alam et al. (2005) discussed flow configuration and fluid forces on two staggered cylinders with different angular positions and gap ratios at Re=$5.5\times10^4$. They found that for small gap ratios the lift forces are mostly dependent on the flow between cylinders and the effect of
angular position is negligible. The intermittent formation and collapse of separation bubbles on cylinder surfaces were considered as the reason of the bi-stability of the flow. The highest reported fluctuating lift force on the downstream cylinder occurred at $\alpha = 25^\circ$ for $2.1 < T/D < 5$, where $T$ is the direct distance between two centres. The highest fluctuating drag force on the downstream cylinder occurred at $\alpha = 10^\circ$ when $2.4 < T/D < 3$. Kim et al. (2009) carried out experiments on 1-DOF induced vibration of two tandem cylinders for $0.1 < G/D < 3.2$. The oscillation of each cylinder when the other one was fixed and the simultaneous oscillations of both cylinders were investigated in this test. For gap ratios less than $0.2D$, no oscillation was observed without the application of initial perturbation. For gap ratios between $0.2D$ and $0.6D$, violent vibration was observed. For gap ratios between $0.6D$ and $2D$, both cylinders had fairly the same similar high amplitudes in the range of reduced velocities between 6 and 7. For gap ratios larger than $2.7D$, the downstream cylinder achieved higher amplitude than the upstream one. The experiments of Kim et al. (2009) showed how fixing or leaving one of the cylinders to oscillate affect the vibration of the other one dominantly. Prasanth and Mittal (2009) studied numerically the 2-DOF flow induced vibration of two tandem cylinders with equal diameter. The Reynolds number in their simulations was 100 and the centre-to-centre distance between the two cylinders was $5.5D$. They found that the presence of the downstream cylinder increases the oscillation amplitude of the upstream one compared to an isolated cylinder and this high amplitude shifts toward lower reduced velocities. The oscillation amplitude of the downstream cylinder was about twice the value of the upstream one and this high amplitude occurs at higher reduced velocities than corresponding to the upstream cylinder. Even beyond the lock-in range ($U_r > 15$) the downstream cylinder undergoes high amplitude vibrations. Flow interference between two cylinders with equal diameters at $Re=150$ with variable incident flow angle, $\alpha = 0^\circ$ (tandem arrangement) to $\alpha = 90^\circ$ (side-by-side arrangement), has been discussed by Bao et al. (2011). In their numerical investigation the upstream cylinder was fixed and the other one was set free to oscillate in cross flow direction. Their results showed that outside the wake region of the fixed cylinder the downstream one behaves like an isolated cylinder. However, in the wake region ($\alpha < 30^\circ$) the behaviour of the oscillating cylinder changed drastically because of its continuous interaction with the shed vortices of the upstream cylinder. The vibration amplitude reached higher values than an isolated cylinder and persisted beyond the lock-in region. The long period of the positive momentum transfer to the downstream cylinder (by the induced hydrodynamic lift force) was considered as the reason of high amplitude of oscillations by Bao et al. (2011).

The experimental results of King and Johns (1976) for cross flow vibrations of two mechanically couples cylinders in tandem arrangement revealed the presence of two peaks.
for the gap ratios less than 6. Vibrations ceased outside the lock-in range ($U_r > 10$) and the order of the peaks and their corresponding reduced velocities changed with the gap between cylinders. The effects of mechanical coupling on the vortex induced vibration of two cylinders with the same diameter in tandem arrangement with variable spacing were discussed by Brika and Laneville (1997). Vibrations of the cylinders could be in-phase or out-of-phase. The tests were carried out by increasing (decreasing) the reduced velocities while the coupled cylinders were vibrating steadily at their previous reduced velocity. Hysteresis and discontinuity were present in all amplitude responses but the in-phase vibration of cylinders at $G/D=25$. Hysteresis was not observed in this case and the dynamic responses for both increasing and decreasing reduced velocities were the same. Jester and Kallinderis (2003) studied numerically the fluid flow around two fixed cylinder pair in tandem, side-by-side and stagger arrangements. They implemented different strategies in their numerical simulations and their results were quantitatively comparable with experiments. In another study, Jester and Kallinderis (2004) modelled incompressible flow about transversely oscillating (both forced and free oscillations) cylinder pairs in tandem and side-by-side arrangements. Their numerical results predict the second peak in the dynamic response of rigidly connected tandem cylinders which was found previously in the experiments of King and Johns (1976) and Brika and Laneville (1999). The reason for high amplitude of vibration for the second peak was the comparative high rate of the net positive transfer of energy to the downstream cylinder by the shed vortices from the upstream one.

Numerical study of two tandem cylinders with different diameters (diameters ratio=0.5) was carried out by An et al. (2008) at a fixed reduced velocity of 8. The mass ratios and damping coefficients of both cylinders were the same as used in Khalak and Williamson (1996). The small cylinder was placed both in the upstream and downstream of the large one. The minimum and maximum considered gaps between cylinders were $0.5D$ and $3D$ respectively, where $D$ was the diameter of the large cylinder. Different phenomena such as lock-in of one of the cylinders or both of them and beating of the small cylinder were observed in this study for different gaps between cylinders. The large cylinder experienced the highest vibration amplitude ($1D$) and hydrodynamic force coefficients when the small cylinder was in upstream side and $G/D= 1.5$. The phase difference between cylinder oscillations was decreasing from 180° to 0° when the large cylinder was located in upstream and the gap between cylinders was increasing from $0.5D$ to $3D$.

Zdravkovich (1977) and Sumner et al. (1999) categorized flow around two side-by-side cylinders according to the gaps between them. The experimental study of Sumner et al. (1999) showed that two side-by-side cylinders in contact with each other behave similar to single bluff body with a single vortex street shed from their outer sides. Two dominant
shedding frequencies were observed in the power spectra analysis of fluid flow at downstream of the cylinders. The vortices were irregular in shape and disintegrate into small vortices. The PIV tests carried out in the water tunnel by Sumner et al. (1999) on side-by-side cylinders in contact with each other showed that the vortex formation length downstream of the cylinders is not fixed. For small gap ratios between cylinders, the single bluff body behaviour slightly changed. The presence of fluid flow between the cylinders contributed to lower drag force on cylinders and more extended vortex formation behind them. Three types of behaviour have been observed for this range of gaps. (I) A gap flow parallel to shear layers; (II) Asymmetrical near-wake region with a biased gap flow and (III) No gap flow. For intermediate gap ratios, biased flow pattern was found. The flow was deflected continuously toward one of the cylinders with a squeezed wake along it. The deflection in biased gap flow varies with the gap between cylinders. The power spectra presented by Sumner et al. (1999) shows the presence of two dominant shedding frequencies in the wake behind cylinders for average gap ratios. For large gap ratios the biased flow disappears and vortex shedding from cylinders behave more independently and similar to an isolated cylinder. The vortex shedding behaviour of the cylinders can be anti-phase or in-phase. According to Brun et al. (2004) the reason for the instability of the gap flow and its “flopping behaviour “could be the development of the Kelvin-Helmholtz instabilities which have nearly the same sizes as the shedding vortices.

Free vibration of two side-by-side elastic cylinders exposed to steady flow is discussed by Zhou et al. (2001). Cylinders had the same diameters and were fixed at both ends. Experiments were carried out for different gap ratios. Their measurements showed the co-occurrence of the second and third mode resonances which were more violent than the first mode. Another interesting result was the different nature of the lock-in phenomena from the rigid cylinder. The natural frequency of the cylinders varied rapidly near the resonance,” A dip followed by a rise”. This feature occurred also for a single elastic cylinder. On top of that, in contrast to rigid cylinder, the natural frequencies of the elastic cylinders were modified during resonance to adapt to St frequency.

1.5 Fluid flow around a circular cylinder in proximity of a plane

Submarine pipeline laid on the seabed are likely to experience scouring during their life time. The scouring or the unevenness of the seabed contributes to the emergence of free spans below the pipelines. These suspended pipelines may experience vortex-induced vibration. In general the distance between the pipeline and the plane surfaces affect the fluid flow behaviour and hydrodynamic forces on the pipeline effectively. The fluid flow characteristics and vibration behaviour of a cylinder close to another surface have been
discussed both numerically and experimentally by different authors. Zdravkovich (1985) found that for the flows with Re number in the subcritical range the lift force on a cylinder close to the plane wall depends strongly on the gap ratio \((G/D)\) where \(G\) is the shortest distance between the cylinder surface and the plane wall and \(D\) is the diameter of the cylinder. However the drag force is mostly affected by the ratio of the gap \((G)\) to the wall boundary layer thickness. Sarpkaya (1977) carried out some laboratory tests to find the values of the hydrodynamic force coefficients of a fixed cylinder near a plane wall. He observed the increase in both lift and drag force coefficients for \(0<G/D<0.5\). The force coefficients were a function of Reynolds number however reducing the gap at a fixed Reynolds number resulted in their increase. The values of the lift force coefficients of a cylinder in steady flow close to a plane wall was predicted by Fredsøe and Hansen (1987). They developed a modified version of the potential flow theory for a shear free steady flow close to a plane wall. Their results match the experimental values very well. Their experiments also showed that the shear in free stream reduces the lift force on the cylinder close to a plane wall compared to shear free flow. Sumer et al. (1994) showed experimentally that the cross flow oscillation of the cylinder near a plane doubles both the lifts and drag coefficients of vibrating cylinder compared to the corresponding fixed cylinder.

The hydrodynamic forces on the subsea pipelines resting on the seabed can be predicted by different models. Morison equation is the simplest one which is used widely in industry. The modified Fourier method and wake model are more developed methods proposed by Sorenson et al. (1986) and Lambrakos et al. (1987) respectively. These methods have been calibrated and adjusted using different experimental and Fourier decomposition methods. They can be applied for different types of flows (wave, wave and current and irregular wave) and their accuracy have been approved by other authors (Bryndum et al., 1992 and Verley and Reed, 1989).

Fredsøe et al. (1987) indicated that vortex shedding frequency of a cylinder with 1-DOF in cross flow direction in proximity of a plane surface is very close to that of fixed cylinders for reduced velocities less than 3 and gap ratios larger than 0.3\(D\). The vibration frequency of the cylinder was higher than the St frequency for the reduced velocities between 3 and 8 for \(0<G/D<1\). For small gap ratios oscillations did not initiated without initial excitements of the cylinders. According to Fredsøe et al. (1987), the vortex shedding is not the only reason for the induced vibration and also the high lift force on the cylinder close to a plane wall. The fluctuations of the pressure on the bottom side of the cylinder are the main contributor to the cylinder oscillations and its high lift force coefficients for small gap ratios. The reason for these is the varying flow rate underneath the oscillation cylinder. The effects of the gap ratio on the vortex shedding were also investigated by Nishino et al. (2007) in a series of
experiments. The Effects of boundary layer thickness and span wise flow were minimized in this study by using the end plates and letting the ground below the cylinder to move with the same speed as the free stream flow. For the gap ratios larger than 0.5\(D\) vortex shedding was observed without any intermittency and the drag coefficient was constant at about 1.3. For lower gap ratios, 0.35<\(G/D\)<0.5, the vortex shedding became intermittent and the drag coefficient plummeted. The vortex shedding ceased completely for \(G/D<0.35\) and a dead fluid zone bounded with two parallel shear layer was created downstream of the cylinder. The drag coefficient in this range was insensitive to the gap ratio and remained constant slightly below one. Nishino et al. (2007) concluded that the reduction of the drag force on a cylinder near the plane wall in a shear free flow is the result of the vortex shedding suppression.

The experimental study of Yang et al. (2009) the VIV of a pipeline close to an erodible bed revealed that the onset of pipeline vibration with lower gap ratios demands higher reduced velocities. Their test pipeline did not experience any cross-flow vibration till the scour depth underneath reached a certain value. Numerical study of Kazeminezhad et al. (2010) about the effect of bed proximity on the vortex shedding frequency and forces on pipeline showed that both the RMS lift coefficient and St number (based on free stream velocity) are strongly affected by the gap ratio. Zhao and Cheng (2011) modelled numerically the 2-DOF VIV of a cylinder close to a rigid plane wall. The collision and bouncing back of the cylinder was also simulated in this study. They found that vortex shedding exist even for a very small gap ratios. Different vortex shedding modes from the gap were found based on the reduced velocity of the cylinder.

1.6 Fluid flow around two circular cylinders with different diameters

Using multiple flow lines in offshore industries gets more popular due to both technical and economic reasons. Zhao et al. (2007) studied numerically the turbulent fluid flow around two fixed cylinders with different diameters in steady current. The effects of the gap ratio (\(G/D=0.05, 0.1, 0.2\) and 0.4) and the angular position of the small cylinder (\(\alpha = 0^\circ\)to 180\(^\circ\)) on the vortex shedding and hydrodynamics forces were investigated in this experiment. The effects of the gap between cylinders were negligible on vortex shedding process when the cylinders were arranged in near tandem arrangement or the small cylinder was placed in the upper upstream side of the large cylinder. The highest drag and lift coefficients of the large cylinder occurred when \(\alpha = 90^\circ\) (side-by-side arrangement) and \(\alpha = 120^\circ\) respectively. This maximum lift was in downward direction.
The piggyback configuration is the case where the diameters of the flow lines are not equal and are strapped to each other in such a way that they oscillate as a one unit. Drag and lift coefficients of a piggyback near the plane wall was investigated experimentally by Kalghatgi and Sayer (1997). The diameter ratio of the two cylinders and the gap between them were 0.25 and $G=d=0.025\text{m}$, respectively, where $d$ was the diameter of the small cylinder. Their experiments covered a range of gaps between the large cylinder and the wall. Both subcritical and critical flows regimes were investigated in this investigation. They found that the effects of the small cylinder diameter and the gap between two cylinders on the drag force were more important than the effect of the gap between the large cylinder and wall. The direction of the lift force was dependent on the flow regime. For subcritical flows its direction was from small cylinder toward the large one and the wall and for the critical regime the lift force was acting in the reverse direction. However for the space ratio ($G/D$) equal to 0.15, the lift force was pointing permanently toward the plane wall. The direction of the lift force for an isolated cylinder close to the plane wall ($G/D<0.4$) was always toward seabed in subcritical flow regimes. However its magnitude was not a unique value for some gap ratios.

Zhao and Cheng (2008) discussed the effects of the gap ratio in piggyback (two cylinders with different diameter coupled mechanically side by side) configuration on the scour below the piggyback pipeline. They observed that for a critical value of the gap the scour gap below the piggyback reach its highest value and shift toward downstream. Rahmanian et al. (2012) studied the 2-DOF VIV of two mechanically coupled cylinders with different gap ratio and angular position of the small cylinder. The diameter ratio and reduced velocity of the cylinders were kept constant at 0.1 and $8 \times 10^3$ respectively. Thorough investigation of bundle oscillation, hydrodynamic force fluctuations and instantaneous vorticity field carried out in this study. The highest cross flow and in–line vibration amplitudes occurred when $G/D=0.1$ and $\alpha=67.5^\circ$, where $\alpha$ is the angular position of the small cylinder measured from the upstream side. The fluid flow was unsteady and the main reason for that was its intermittency between the cylinders. Large variations in oscillation amplitude in both in-line and cross flow direction were observed in this study.

Numerical study of Mittal and Raghuvanshi (2001) confirmed the experimental results of Strykowski and Sreenivasan (1990). Their results showed how the placement of a control rod in specific staggered position ($P/D=2$ and $T/D=0.8$, where $P$ is the horizontal and $T$ the vertical distance between centres respectively) downstream of main cylinder can suppress the vortex shedding of both cylinders in laminar flow regime ($Re=80$). The control rod stabilized the vortex streets locally. However the drag forces on both cylinders were increased. Dalton et al. (2001) also did a very similar numerical study to previous mentioned
one. The Reynolds numbers of 1000 and 3000 were considered in this study. The diameter ratio of the cylinder was 10 and the angle of attack of the upstream flow was changing to simulate different arrangement of the cylinders. The gap between two cylinders in horizontal direction varied for different Reynolds numbers (the horizontal gap between cylinder centres varied between 1.2\(D\) and 1.6\(D\), where \(D\) is the diameter of the large cylinder). Their numerical study confirmed the suppression of vortex induced vibration provided that the control cylinder is positioned appropriately.

Numerical simulation of 1- DOF oscillation of two cylinders in close proximity in steady flow has been discussed by Rahmanian et al (2012). In this study the possibility of collision of two cylinders with different diameters \((d/D=10)\) have been discussed. They found the possibility of collision of two cylinders with the same mass ratio for different natural frequencies. Collision could occur when the natural frequency of the larger cylinder was less than the small ones. Lee et al. (2012) studied the mutual interaction and the wake structure behind two side-by-side cylinders with different diameters \((D/d=2)\). The gap ratio between the two cylinders was 0.75\(D\). The excitation frequency and its one-third sub-harmonic were the two dominant lock-on frequencies which were recognized downstream of the excited cylinders. Excitations were in the form of small amplitude rotation applied to the large cylinder.

1.7 Research objective

Presence of a secondary cylinder of different diameter in close proximity of the main one makes the VIV analysis of the cylinders more complicated than that of an isolated cylinder or two parallel cylinders with equal diameters. This research addresses some interesting and practical issues that have not been investigated properly yet. The first objective of the present study is to investigate the effect of different arrangement of two mechanically coupled cylinders on VIV of the bundle. The second objective is to classify the VIVs and vortex shedding regimes of two proximity cylinders with different diameters. The possibility of collision is also investigated. A systematic approach is considered in this dissertation. The VIV of two mechanically coupled cylinders with a fixed diameter ratio is investigated initially for different arrangement of the cylinders. Then the possibility of collision of the two cylinders arranged side-by-side and its effect on the VIVs of both cylinders are discussed. Finally the lock-in phenomena of two elastically mounted cylinders in side-by-side arrangement is investigated comprehensively.
1.8 Thesis outline

This thesis consists of five chapters. In Chapter 1 the detailed physical definitions of the parameters in the field of VIV are defined and some of the relevant previous VIV studies are reviewed. Chapter 2 discusses the VIV of two mechanically coupled cylinders with a fixed diameter ratio. The vortex shedding regimes and VIV of the bundle are discussed in detail for different angular position and gap ratios of the cylinders.

Chapter 3 deals with the VIVs of two side-by-side cylinders. The possibility of collision is investigated by considering different natural frequencies for each cylinder. The physical interpretation of the results and comprehensive study of the vortex shedding regimes are also carried out. Chapter 4 discusses different lock-on cases of two cylinders in side-by-side arrangement. For the sake of comparison the diameter ratio of the cylinders is kept constant as in previous chapters. However the Reynolds number is varied by changing the upstream velocity. Chapter 5 summarises the results found in the previous chapters and future researches and challenges are suggested.

1.9 References


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CHAPTER 2

TWO DEGREE-OF-FREEDOM VORTEX INDUCED VIBRATION OF TWO MECHANICALLY COUPLED CYLINDERS OF DIFFERENT DIAMETERS IN STEADY CURRENT

ABSTRACT

Two-degree-of-freedom vortex-induced vibration of two mechanically coupled cylinders with a diameter ratio of 0.1 in steady current is investigated numerically. The study is aimed to investigate the effects of gap ratio \( G/D \) between the two cylinders and the angular position \( \theta \) of the small cylinder relative to the large one on the vibration amplitude and frequency. The force coefficients of the cylinder bundle are also investigated at typical gap ratios and angular positions of the small cylinder. It is found that the vibration frequency components, the amplitude of the vibration and the force coefficients of the two-cylinder bundle are very sensitive to the gap ratio and the angular position of the small cylinder. The maximum cross-flow vibration amplitude occurs when the cylinders are arranged in the staggered configuration \( \theta=67.5^\circ \) at the gap ratio of 0.1. The minimum cross-flow and in-line vibration amplitudes occur at \( \theta=112.5^\circ \) when the gap ratio is 0.3. The maximum in-line vibration amplitudes occur at the staggered positions of \( \theta=67.5^\circ \) as \( G/D=0.1 \) and \( \theta=135^\circ \) as \( G/D=0.2 \). Furthermore, it is found that the instantaneous position of the bundle relative to the last generated vortices has prominent effect on the force coefficients.

2.1 Introduction

Vortex-induced vibration (VIV) of bluff structures in fluid flow is of practical interest because it causes fatigue failure of these structures. Cylindrical structures, such as sub-sea pipelines, risers and cables etc, are common in offshore engineering applications. Therefore many studies about VIV have been focused on vibration response of a cylinder in currents. It has been understood that VIV of a cylinder is affected by a number of parameters including: (1) mass ratio defined by \( m^* = m/m_d \) with \( m \) and \( m_d \) are the masses of the cylinder and the displaced fluid by the cylinder, respectively; (2) structural damping ratio defined by \( \zeta = \delta / 2\pi \) with \( \delta \) being the logarithmic decrement; (3) Reynolds number defined by \( \text{Re} = U_\infty D/\nu \) with \( U_\infty \) being the free stream velocity, \( D \) the diameter of the cylinder and \( \nu \) the kinematic viscosity of fluid and (4) reduced velocity defined as \( U^* = U_\infty / f_n D \) with \( f_n \)
being the structural natural frequency of the cylinder in both horizontal and vertical directions.

Most studies on VIV have been focused on one-degree-of-freedom (1-DOF) vibration of a circular cylinder in the transverse direction of the flow (or cross-flow direction). Feng (1968), Brika and Laneville (1993), Anagnostopoulos (1994), Kozakiewics et al. (1997), Khalak and Williamson (1999) are only a few of them to mention. Sarpkaya (2004) and Williamson and Govardhan (2004) presented extensive reviews about transverse oscillation of a circular cylinder in steady flows. It was reported that the mass damping ratio \( \frac{m^* \zeta}{m} \) has significant influence on the VIV modes, Khalak and Williamson (1999). However, the more practical case of VIV of a cylinder in two-degree-of-freedom (2-DOF) has not been addressed as thoroughly as the 1-DOF VIV. Jauvtis and Williamson (2004) showed that for the mass ratios greater than 6 the response amplitude in the in-line direction was negligibly smaller than that in the cross-flow direction. Their test cylinder had the same mass ratios and natural frequencies in the in-line and the cross-flow directions. However, they found a dramatic increase in the cross-flow vibration amplitude for smaller mass ratios (less than 6) in the 2-DOF VIV. They defined a “super upper regime” of the reduced velocity, where the amplitude in the cross-flow direction reaches about 1.5 times the cylinder diameter. The vortex street in the “super upper regime” consists of three pairs of vortices that are shed from the cylinder in a cycle of vibration. In addition, they found the third harmonic component in the vibration time histories of the cylinder. Sanchis et al. (2008) reported that the 2-DOF system with a low mass ratio \((m^* = 1.04)\) cannot produce the “super upper branch” if the system mass damping \(m^* + C_a \zeta\) is high, where \(C_a\) is the added mass coefficient of the cylinder.

Fujarra et al. (2001) concluded that the observed high transverse amplitude outside the lock-in region was due to the coupling between the in-line and transverse motions of the test cylinder. Moe and Wu (1990) found that the additional degree of freedom in the 2-DOF VIV contributes to the larger cross-flow vibration amplitudes and the wider lock-in region. They also found that the randomness of lift force becomes strong when the in-line motion is inhibited. Furthermore, their experiments showed the existence of the high-order harmonics in the force spectra for the case of 2-DOF free vibration. Jeon and Gharib (2001) studied the effects of the in-line movement of the cylinder on the phase difference between the lift force and the cylinder motion and the vortex shedding regime. They found that one pair of vortices are shed from the cylinder per cycle instead of 2P mode for 2-DOF motions. Sarpkaya (1995) investigated the effects of frequency ratio (in-line to cross-flow direction) on the cross-flow amplitude and in-line force coefficient for 2-DOF vibrations of a circular
cylinder. Among the three implemented frequency ratios \( \frac{f_{nx}}{f_{ny}} = 1, 2 \) and \( \infty \), where \( f_{nx} \) and \( f_{ny} \) are the natural frequencies in the in-line and cross-flow directions, respectively) the largest vibration amplitude (about 1.1\( D \)) happened at the frequency ratio of 1. He also found that the mean value of drag coefficient for 2-DOF vibration is significantly higher than that of 1-DOF vibration.

Higher harmonic hydrodynamic forces of 2-DOF VIV have been reported by a few researchers including Jauvtis and Williamson (2004), Jhingran and Vandiver (2007), Constantinides and Oakley Jr (2009), Marzouk and Nayfeh (2009) and Modarres-Sadeghi et al. (2010). The numerical simulations by Constantinides and Oakley Jr (2009) showed that the third harmonic, namely the \( 3f \)-component, of the hydrodynamic force is the results of the in-line motion of a vibrating cylinder. Modarres-Sadeghi et al. (2010) explained the contributions of the trajectory of the cylinder and its frequency ratios in both the in-line and the cross-flow directions to the \( 3f \)-component of lift frequency. It was found that the proximity of the generated vortices to the cylinder during its free motion led to high-frequency lift forces (higher than the dominant Strouhal frequency).

There are many engineering applications involving VIV of multiple circular cylinders. The interaction among these cylinders may change the fluid flow structure prominently. Zdravkovich (1977) classified the interference of two rigid cylinders into four categories according to their arrangement with respect to the stream direction. Wake interference occurs when one of the cylinders is partially or fully submerged in the wake of the other one. Proximity interference occurs when the two cylinders are close to each other. Overlap interference is a combination of both interference and wake regimes. The fourth type of interference is the “no interference” regime which excludes aforementioned interferences. Many researchers also studied steady flow around two rigid stationary cylinders with different diameters, e.g., Zhao et al. (2007), Lee et al. (2004), Tsutsui et al. (1997) and Dalton et al. (2001).

Most of the studies on VIV of two cylinders concern two of identical diameters, e.g., King and Johns (1976), Brika and Laneville (1997), Mittal and Kumar (2001), Alam (2005) and An et al. (2008). However there are many engineering applications where two bluff bodies of different size are placed close to each other in fluid flow, such as a piggyback pipeline that comprises two pipelines of different diameters. Flow around two circular cylinders of different diameters is relevant to these applications (refer to Zhao et al. (2007)). Due to the technical and economic considerations, the two pipelines are usually strapped together at regular intervals and laid together. The bundled pipelines can be treated as a single object if the interval between two strap points is short. The position and orientation of
the small-diameter pipeline can affect the forces and vibration of the whole assembly. However, according to the best knowledge of the authors there are not special rules about the optimized configuration of piggyback pipelines in terms of VIV. Zhou and Lalli (2009) found that the vibration amplitude of a piggyback pipeline depends significantly on their arrangement. It was found that the highest amplitude occurs when the cylinders are in side-by-side arrangement at small gap ratios.

Laneville and Brika (1999) discussed the in-phase and out-phase oscillations of two identical mechanically coupled cylinders in tandem and staggered arrangements experimentally. They found that in-phase vibration of the mechanically coupled tandem cylinders spaced 25 times of the cylinder diameter is un-hysteresis and the dynamic response of the system is the same for both increasing and decreasing air velocities. In this experiment the vibration of the system ceased if \( U^* > 7.2 \). Furthermore, they reported two peaks in the amplitude-versus-reduced velocity curve. King and Johns (1976) conducted experiments of VIV of two identical cylinders in tandem arrangement for a range of water depths, Reynolds numbers between \( 10^3 \) and \( 10^4 \) and gaps between the two circular cylinders ranging from \( 0.25D \) to \( 6D \). Their results showed the existence of two different vibration amplitude peaks in the cross flow direction. The values of these peaks and their corresponding reduced velocity were a function of the gap between the cylinders. The numerical study by Jester and Kallinderis (2004) successfully predicted the two peaks reported by Laneville and Brika (1999) and King and Johns (1976).

In this study, 2-DOF vortex-induced vibration of two mechanically coupled cylinders of different diameters is studied numerically. The main aim of the study is to understand the effect of the position of the small cylinder on the VIV and force coefficients of the cylinder bundle in the lock-in region. This goal is achieved through a series of numerical experiments at a typical reduced velocity in the lock-in region. The work covered in this study was prompted as a result of interaction with local oil and gas industry where piggyback pipe lines have been deployed.

### 2.2 Problem description

Fig. 2.1 shows a sketch of the two-cylinder system investigated in this study. A small cylinder of diameter \( d \) is coupled mechanically to a large cylinder of diameter \( D \). In all the numerical simulations, the diameter ratio of the two cylinders is kept constant at \( d/D = 0.1 \). In Fig. 2.1, the angle \( \alpha \) is used to represent the position along the cylinder surface. The mechanical coupling between the cylinders forces them to oscillate as one rigid. The stiffness and damping coefficients in the in-line direction are the same as their counterparts in the cross-flow direction, respectively. The gap between the two cylinders, \( G \), ranges from
0.1D to 0.4D and the angular positions of the small cylinder, $\theta$, ranges between 0° to 180° with a 22.5° increment. The cylinder pair is free to oscillate in both the in-line and the cross-flow directions.

![Fig. 2.1: Definition sketch](image)

The mass ratio $m^*$ and the damping ratio $\zeta$ of the bundle (in both the in-line and the cross-flow directions) are 2.4 and 0.000863 respectively. These two parameters are identical to those employed in the experiments by Khalak and Williamson (1996, 1997, 1999) and Jauvtis and Williamson (2004). Furthermore, the implemented numerical model has been validated for a single cylinder based on these factors (refer to Zhao and Cheng (2010) and Rahmanian et al. (2011)). The Reynolds number and reduced velocity based on the large cylinder diameter are kept constants at $310^8 \times$ and 8 respectively for all the simulations. The considered reduced velocity is approximately in the middle of lock-in region.

### 2.3 Governing equations

#### 2.3.1 Flow equations

The flow in the wake of a stationary cylinder is three-dimensional if the Reynolds number (Re) exceeds 200 according to Williamson (1988). It is expected that the flow around a vibrating cylinder for Re in the turbulent regime will also be three-dimensional. Although the three-dimensionality cannot be captured by two-dimensional (2D) models, the 2D models based on Reynolds-Averaged-Navier-Stokes (RANS) equations have been demonstrated to provide good prediction of VIV and also the vortex shedding modes (e.g. Anagnostopoulos (1994), Guilmineau and Queutey (2004), Wanderley et al. (2008)). Since a
2D simulation consumes much less computational time than a 3D model, systematic parametric study can be carried out at affordable computational time.

RANS equations are the time averaged Navier-Stokes equations based on the instantaneous velocity tensor and pressure magnitude. The instantaneous values of velocities in the $i$-th direction and the pressure magnitude are considered as $u_i = U_i + u'_i$ and $p = P + p'$ where the capital letters refer to the mean values of the velocity and pressure and prime scripted ones are their fluctuating counterparts. The two dimensional incompressible RANS equations and the continuity equation are the governing equations for the problem considered in this study. The Arbitrary Lagrangian Eulerian (ALE) scheme is applied to deal with the moving boundary problem. The ALE scheme allows the computational mesh around the cylinders moves with the cylinders. The ALE formulation of the RANS equations for incompressible flows are expressed as

$$
\frac{\partial U_i}{\partial x_i} = 0 \quad \text{Eq (2.1)}
$$

$$
\frac{\partial U_i}{\partial t} + (U_j - \bar{u}_j) \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\nu S_{ij} - u'_i u'_j \right) \quad \text{Eq (2.2)}
$$

where $t$ is time, $u'_i u'_j$ is the averaged value of the product of fluctuating velocities, $\rho$ is the fluid density, $\nu$ is the kinetic viscosity, $\bar{u}_j$ is the mesh velocity and $S_{ij}$ is the mean strain tensor.

The product $-\rho u'_i u'_j$ is interpreted as the Reynolds stress, $\tau_{ij}$, which is responsible for the momentum losses of the flow and can be approximated using a turbulence model. In this paper, the shear stress transport (SST) $k-\omega$ turbulence energy model proposed by Menter (1994) is applied. The SST $k-\omega$ model is based on the Boussinesq approximation:

$$
\overline{u'_i u'_j} = 2\nu_s S_{ij} - (2/3)k \delta_{ij}
$$

where $\nu_s$, $k$ and $\delta_{ij}$ are the eddy kinematic viscosity, turbulent kinetic energy and the Kronecker delta, respectively. The SST $k-\omega$ model approximates the eddy viscosity using the kinetic energy ($k$) and its specific dissipation rate ($\omega$) transport equations.

### 2.3.2 Equations of motion of the cylinder bundle

Since the cylinders are mechanically coupled to each other, they oscillate at the same frequency and amplitude. The non-dimensional equation of motion of the cylinder bundle is
\[
\ddot{X}_i + \frac{4\pi C_i}{U^*} \dot{X}_i + \left( \frac{2\pi}{U^*} \right)^2 X_i = \frac{2C_i}{\rho m^*}
\]
Eq (2.3)

where \(\ddot{X}_i, \dot{X}_i, X_i\) and \(C_i\) are acceleration, velocity, displacement and hydrodynamic force coefficient in the \(i\)-th direction, respectively. The diameter of the large cylinder is used for normalizations in this study. The natural frequency in the \(x\)-direction is same as that in the \(y\)-direction. Hydrodynamic force on the cylinders is calculated by integrating the pressure and shear stress over the cylinder surfaces. It is normally decomposed into force components in the in-line direction (the drag force, \(F_D\)) and the cross-flow direction (the lift force, \(F_L\)). The drag and lift coefficients are defined as

\[
C_D = \frac{F_D}{\frac{1}{2} \rho U^*_\infty^2 D}, \quad C_L = \frac{F_L}{\frac{1}{2} \rho U^*_\infty^2 D}
\]
Eq (2.4)

### 2.3.3 Numerical method

In this paper, a modified isoparametric formulation of the Petrov-Galerkin finite element method in the ALE formulation, Zhao et al. (2009) is used to solve the governing equations. The modified isoparametric formulation has been demonstrated to be robust for convection dominated flows, Zhao et al (2009). The domination of the convective terms over diffusion terms in Navier-Stokes equations is dependent on Reynolds number. The higher the Reynolds number, the stronger is the convection. The negative diffusion introduced by the traditional Galerkin Finite Element Method (FEM) leads to unsuccessful and spurious results for convection dominated flows, Heinrich and Pepper (1999). The reason is the additional contribution of the convection terms to the non-diagonal elements of the derived stiffness matrix, Heinrich and Pepper (1999). An efficient way to overcome the weakness of the Galerkin FEM method is adding additional balancing diffusivity to the diffusion terms of the momentum equations, which can remedy the inherited under-diffusivity of the Galerkin FEM. Streamlined-Upwind/Petrov-Galerkin method introduced by Brooks and Hughes (1982), where balancing diffusion acts only in the flow direction, has been one of the successful techniques to stabilize the FEM schemes for convection dominated flows. The balancing diffusion is included into the weighting functions in the derived weighted residual method. The weighting functions of the new Petrov-Galerkin weighted residual method are different from shape functions. The new weighting functions are applied to all the terms in the weak form of equations.
2.3.4 Boundary conditions

No-slip condition on the cylinder surfaces is maintained, i.e. the fluid velocity on the cylinder surface is equal to the velocity of the cylinder motion. The velocity and the displacement of the cylinders are obtained by solving the equation of motion after finding the hydrodynamic forces in each time step. The fourth-order Runge-Kutta algorithm is utilized to solve the equation of motion. The dimensionless turbulent kinetic energy \( k \) and its specific dissipation rate \( \omega \) at the inlet boundary are \( 10^{-3} \) and 1, respectively. Gradients of velocity, pressure and turbulent quantities in the stream-wise direction are set as zero at the downstream boundary of the computational domain. Zero gradients of horizontal velocity, turbulent quantities and pressure in the cross-flow direction are implemented on two lateral boundaries of the computational domain.

2.4 Results and Discussion:

2.4.1 Mesh dependence check and model validation

The numerical model used in this study has been validated against the experimental results of VIV of a circular cylinder in steady current at low mass ratio by Zhao and Cheng (2010, 2011). In the numerical studies by Zhao and Cheng (2010, 2011), 2-DOF VIV of a circular cylinder in steady flow was compared with the experimental results reported in Jauvtis and Williamson (2004) under identical flow conditions e.g. mass ratio, \( m^* = 2.6 \) and damping factor, \( \zeta = 0.000575 \). It was found that the numerical results agree well with the experimental data. Since the model used in this study is identical to the model used by Zhao and Cheng (2010, 2011), no additional validation was carried out. The mesh dependence study of this study is reported in Rahmanian et al. (2011) and will not be repeated here.

The domain considered for all calculations is a rectangle with the length and the width of 70\( D \) and 40\( D \) respectively. The centre of the large cylinder is positioned initially 30\( D \) downstream of the inlet boundary and is on the centre line in the transverse direction. Linear 4-node quadrilateral elements are used to discretise the computational domain. The total number of elements varies between 30,000 and 34,000. The total element numbers along the surfaces of the large and the small cylinders are 280 and 110 respectively. The minimum size of the elements occurs adjacent to the small cylinder surface. It is about 0.0004\( D \) in the radial direction. The maximum aspect ratio of the elements is 28 for the elements in the radial direction adjacent to the surface of the large cylinder between two cylinders centres.
2.4.2 Effects of $\theta$ and $G$ on vibration amplitude

The effects of the angular position of the small cylinder ($\theta$) and the gap ratio ($G/D$) on vibration amplitude of the bundle are investigated by conducting a series of numerical tests. Fig. 2.2 shows the variations of the deflections and vibration amplitudes in the in-line and the cross-flow directions with the angular position of the small cylinder for different gap ratios. The in-line ($X_{\text{max}}$) and the cross-flow ($Y_{\text{max}}$) deflections are defined as the maximum displacements in the in-line and the cross-flow directions respectively. The in-line vibration amplitude ($X_{\text{max}}-X_{\text{min}})/D$ and the cross-flow ($Y_{\text{max}}-Y_{\text{min}})/2D$ are defined as the difference between the maximum and minimum displacements in the in-line and cross-flow directions respectively. For an isolated cylinder with the mass ratio, the structural damping and the stiffness the same as their counterparts in the cylinder bundle, the $X_{\text{max}}/D$ and $(X_{\text{max}}-X_{\text{min}})/D$ are 0.63$D$ and 0.15$D$ respectively. The maximum in-line deflections of the bundle (about 1.2$D$ and 1.0$D$) occur at $\theta=67.5^\circ$ and $\theta=90^\circ$ respectively both at $G/D=0.1$, while the minimum in-line deflections occur at $\theta=90^\circ$ and 112.5$^\circ$ both at $G/D=0.3$. The maximum in-line deflection in the tandem arrangements ($\theta=0^\circ$ and 180$^\circ$) does not appear to be affected significantly by the variation of the gap ratio, although it tends to increase slightly with the increasing gap ratio. This is in contrast to other arrangements where the maximum deflection varies noticeably with the gap ratio. The effect of the position angle on the deflections appears to become less prominent as the gap ratio increases. The variation of the in-line vibration amplitude with the position angle appears to follow similar trends to those of the in-line deflection. When the small cylinder is either in front of or behind the larger one, the vibration amplitude in the $x$-direction is very small. However the in-line vibration amplitude for other cylinder arrangements varies noticeably with the gap ratio. The maximum amplitude of the in-line vibration occurs at $\theta=67.5^\circ$ and $\theta=135^\circ$ when $G/D=0.1$ and 0.2 respectively, whereas the minima occur at $\theta=90^\circ$ and $\theta=112.5^\circ$ both as $G/D=0.3$. The effects of the position angle on the in-line vibration amplitude appear to be most significant at $G/D=0.2$. Variations of both maximum in-line deflection and vibration amplitude with gap ratio are large between $\theta=67.5^\circ$ and $\theta=90^\circ$.

The cross-flow vibration amplitude for an isolated cylinder with the same mass ratio, stiffness and structural damping is 0.67$D$. The trends of the cross-flow deflection of the cylinder bundle (Fig. 2.2 (c)) are similar to those for the in-line deflection shown in Fig. 2.2 (a). The maximum cross-flow deflection (also about 1.2$D$ and 1.0$D$) occurs at $\theta=67.5^\circ$ and 90$^\circ$ both as $G/D=0.1$. The cross-flow deflection for the tandem arrangements ($\theta=0^\circ$ and 180$^\circ$) is almost independent of the gap ratio, although its value for $\theta=180^\circ$ is consistently larger than its counterpart for $\theta=0^\circ$ at all gap ratios investigated. Both the cross-flow deflections for
\( \theta = 67.5^\circ \) and 90\(^\circ\) arrangement decrease rapidly as the \( G/D \) increases from 0.1 to 0.2. The cross-flow deflections for \( \theta = 45^\circ \) and 135\(^\circ\) arrangements follow a similar trend where the maximum cross-flow deflections reach their peak values of about \( 0.9D \) at \( G/D = 0.2 \) and decrease monotonically as \( G/D \) either increases or decreases from \( G/D = 0.2 \). The minimum cross-flow deflections for different \( G/D \) occur at \( \theta = 112.5^\circ \).

The variation of the cross-flow vibration amplitude with the position angle and gap ratio appears to follow similar trends to those of the cross-flow deflection. The cross-flow vibration amplitudes for the tandem arrangements (\( \theta = 0^\circ \) and 180\(^\circ\)) are almost independent of the gap ratio, although the maximum cross-flow vibration amplitude for \( \theta = 180^\circ \) is consistently larger than its counterpart for \( \theta = 0^\circ \). The maximum cross-flow vibration amplitude occurs at \( \theta = 67.5^\circ \) and \( G/D = 0.1 \) and it decreases rapidly as \( G/D \) increases from 0.1 to 0.2. Another large cross-flow amplitude that occurs at \( \theta = 157.5^\circ \) does not change significantly with the increasing gap ratio. The variation of the maximum cross-flow vibration amplitude with \( G/D \) for \( \theta = 67.5^\circ \) and that for 157.5\(^\circ\) follow a similar trend. The minimum cross-flow amplitude occurs at \( \theta = 112.5^\circ \) and \( G/D = 0.3 \).

![Figure 2.2: Variations of deflections and vibration amplitude with the position angle \( \theta \). (a) In-line deflection, (b) in-line vibration amplitude, (c) cross-flow deflection, (d) cross-flow vibration amplitude](image-url)
2.4.3 Flow characteristics of the cylinder bundle

2.4.3.1 Isolated cylinder

Before discussing the results of the cylinder bundle, the results of an isolated cylinder are discussed in this section. They are obtained at the same Re and $U^*$ as for the cylinder bundle and can serve as the base case to investigate the effects of small cylinder on flow characteristics.

Fig. 2.3: Vorticity contours of a single cylinder with 2-DOF
Fig. 2.3 shows the vorticity contours behind an isolated cylinder in two successive oscillation cycles. One can observe that three vortices are shed in each cycle, two clockwise in frames \(b\) and \(e\) (\(j\) and \(n\) in the second cycle) and one counter clockwise in frame \(h\) (\(q\) in the second cycle). One of the two clockwise vortices is shed after the cylinder starts to move downward and another one is shed when the cylinder is at the lowest position. The counter-clockwise vortex is detached from the cylinder during upward motion of the cylinder. The corresponding time histories of force coefficients and cylinder oscillation trajectories are shown in Fig. 2.4. The maximum vibration amplitudes in the \(x\)- and the \(y\)-directions observed in Fig. 2.4 (a) are 0.085\(D\) and 0.67\(D\), respectively. The second harmonics observed in the lift coefficient time history (Fig. 2.4 (a)) is attributed to the three vortices that are shed in each vibration cycle. The peak or the trough values of the drag coefficients observed in Fig. 2.4 (a) actually occur (1) when the vertical oscillation either changes its direction or passes the balanced position (\(y/D=0\)) and (2) at the moment when the in-line motion changes its direction (point \(b, k, o\) and \(f\) for peaks, and \(d, i\) and \(m\) for troughs). The peak value of the

![Graphs showing force coefficients and vibration trajectories](image-url)
lift coefficient occurs at the end of the downward motion of the cylinder and just before it changes its direction ($m$ and $d$), except for the small peak due to the second harmonics. These two locations are also located nearly at the most downstream positions of the cylinder counter-clockwise motion trajectory ($d$ and $m$ in Fig. 2.4 (c) and (d)). The trough values of the lift coefficient happens shortly after the cylinder commencing the downward motion and just before it reaches to its most upstream position ($a$ and $j$).

![Graph of lift coefficients](image)

Fig. 2.5: FFT analysis of an isolated cylinder ($f_{ys}$ is the highest peak frequency of the cross-flow displacement for an isolated cylinder).

The results of the Faster Fourier Transform (FFT) of the time histories of the in-line and the cross-flow displacements and hydrodynamic force coefficients of an isolated cylinder are shown in Fig. 2.5. The frequency in Fig. 2.5 is normalized by the highest peak-frequency in the cross-flow direction $f_{ys}$ for a single cylinder. The value of $f_{ys}$ is found to be 1.1 times the structural natural frequency of the cylinder. The main aim of the FFT analysis is to examine the harmonics that happen consistently in the in-line vibration, cross-flow vibration and force coefficient spectra. Even though the main non-dimensional frequency of the cross flow vibration ($f/f_n$) happens at 1.1, the lift and drag force coefficient oscillations are combinations of harmonics. The presence of the normalized frequency, $f/f_{ys}=0.5$ is prominent in the spectrum of in-line vibrations. The reason for the presence of this harmonic component is the shedding of one more clockwise vortex in each vibration cycle. Shedding of the clockwise vortices induces lift force in negative direction (Jauvtis and Williamson (2004)). Shedding of two vortices of opposite directions leads to one cycle of vibration. The shedding of the second clockwise vortex in each cycle contributes to the extra frequency component of forces of a period twice the oscillation period.

### 2.4.3.2 $θ=0^o$

The effects of positioning the small cylinder upstream the big cylinder are discussed in this section. Vortex shedding structures and vibration characteristics around the pipeline bundle with $G/D=0.4$ are discussed here due to some distinct flow features that cannot be
easily observed in other gap ratios. Fig. 2.6 shows the successive instantaneous vorticity contours of the cylinder bundle at $G/D=0.4$ in one typical vibration cycle. The vortex shedding from both cylinders can be clearly identified in Fig. 2.6 Interaction among the vortices that are shed from the small and the large cylinders is observed to be affected by the motion of the cylinder bundle. Since the amplitude in the $x$-direction is much smaller (less than 10%) than that in the $y$-direction (as shown in Fig. 2.8), the cylinder bundle mainly moves vertically. Although it is altered by the existence of the large cylinder, the vortex street behind the small cylinder always exists. When the cylinders move from top to bottom, vortices from the small cylinder are shed towards the top side of the large cylinder and vice versa.

Fig. 2.6: Vorticity contours within a vibration cycle at $\theta=0^\circ$ and $G/D=0.4$

Fig. 2.7 shows the results of FFT analysis of both in-line and cross-flow displacements. The results of the in-line displacement spectra of the cylinder bundle show either two or three dominant normalized frequencies depending on the gap ratio. Two dominant frequencies are observed for small gap ratios ($G/D=0.1$ and 0.2) while three dominant frequencies are observed for large gap ratios ($G/D=0.3$ and 0.4). The third frequency component observed at the large gap ratios has a normalized frequency smaller than the other two dominant normalized frequencies. The amplitude corresponding to this low frequency component increases as the gap ratio is increased from 0.3 to 0.4. This low frequency component has the highest peak value in the displacement spectra of $G/D=0.4$. The unique flow feature of this gap ratio ($G/D=0.4$), i.e. the bias of the vortex street behind the small cylinder toward the top and bottom of the large cylinder periodically, are likely the
reason for the high amplitude at this low beating frequency. Varying the gap ratio does not appear to affect the spectra of cross-flow displacement significantly, although small noticeable peaks around the dominant normalized frequency are observed in the spectra of $G/D=0.4$.

![Fig. 2.7: FFT analysis of the displacement of two-cylinder bundle at $\theta=0^\circ$ for different gap ratios. (a) in-line displacement, (b) cross-flow displacement.](image)

![Fig. 2.8: Time histories of the vibration displacement and the $XY$-trajectories of the vibration at $\theta=0^\circ$. (a) time history of the in-line displacement, (b) time history of cross-flow displacement, (c) $XY$-trajectories.](image)

Fig. 2.8 (a) and (b) shows the time histories of the displacement of the cylinder bundle. For the small gap ratio of $G/D=0.1$, the in-line variation remains regular and periodic. However the variation of displacement for $G/D=0.4$ is irregular. The maximum displacement of the cylinder bundle in the cross-flow direction for $\theta=0^\circ$ occurs at $G/D=0.4$ (refer to Fig. 2.2 (d)). It is speculated that the irregularity of the vibration displacement at $G/D=0.4$ is
due to the influence of the vortex shedding from the small cylinder (as shown in Fig. 2.6). Fig. 2.8 (c) shows the trajectories of the vibration at different gap ratios. The vibration amplitude of the cylinder bundle for all gap ratios is lower than that of the isolated cylinder (compared with Fig. 2.4 (d)). Similar to the single cylinder case, the trajectories are asymmetric, although the configuration is symmetric with respect to the $X$-axis. At $G/D=0.1$ the vibration trajectory is similar to that of an isolated cylinder because the very small gap between the two cylinders suppresses the vortex shedding from the small cylinder.

### 2.4.3.3 $\theta=22.5^\circ$

Fig. 2.9 shows the results of FFT analysis of both the in-line and the cross-flow displacements. Comparison between Fig. 2.7 (a) and Fig. 2.9 (a) shows that the increase in the angular position of the small cylinder from $\theta = 0^\circ$ to $22.5^\circ$ leads to the occurrence of the low frequency amplitude at low gap ratio of $G/D=0.2$. While there are two dominant frequencies in the spectrum of $G/D=0.1$, the spectra for higher gap ratios include three dominant frequencies each. Suppression of the lowest frequency component in the spectrum of $G/D=0.1$ and $G/D=0.2$ at $\theta = 0^\circ$ (Fig. 2.7 (a)) shows the dependence of this low frequency on both the gap between cylinders and the angular position of the small cylinder. The vortex shedding regime and amplitude response of $\theta = 22.5^\circ$ are very similar to those for $\theta = 0^\circ$. Fig. 2.9 (b) shows the presence of single dominant frequency in the spectra of the cross-flow displacement. Fig. 2.10 shows the streamlines superimposed on the vorticity contours near the cylinder bundle at four different instants. The trajectory of the cylinder bundle and its instantaneous position are shown inside the large cylinder in each picture. One can observe that the oscillation of the cylinder bundle leads to the streamlines stop passing between the cylinders in Fig. 2.10 (a) and (d). The flow between the cylinders is intermittent during the oscillation and it has to pass over the bundle during its oscillation between positions $d$ and $a$. This intermittency occurs only for $G/D=0.1$ and does not occur for wider gap ratios.

### 2.4.3.4 $\theta=45^\circ$

Fig. 2.11 shows the vorticity contours at eight instants in a vibration cycle for $G/D=0.2$. At the gap ratio of $G/D=0.2$ two distinct vortex streets (one from each cylinder) exist which interact with each other. It is found that the vortex shedding from the small cylinder can be suppressed or strengthened, depending on the moving direction of the cylinder bundle. When the bundle moves downward (Fig. 2.11 (a) to (d)), vortex shedding occurs behind the small cylinder. However there is not vortex shedding from the small cylinder during upward
and leftward movement of the bundle (Fig. 2.11 (e) to (h)). The suppression of the vortex shedding from the small cylinder is due to the relative velocity of the fluid to the cylinder bundle. When the cylinder bundle is moving downwards, the vortex shedding occurs behind the small cylinder because the fluid velocity relative the cylinder bundle is in the top-right direction and there is ample space in the wake of the small cylinder for vortex shedding.

Fig. 2.9: FFT analysis of the displacement of two-cylinder bundle at θ=22.5° for different gap ratios. (a) in-line displacement, (b) cross-flow displacement.

Fig. 2.10: Instantaneous vorticity contours and streamlines of the cylinder bundle at four different instants at θ=22.5° and G/D=0.1.
However, when the cylinder bundle is moving upwards, the fluid attacks the cylinder bundle in the bottom-right direction, leading to the small cylinder directly located at the upstream of the large cylinder. The vortex shedding from the small cylinder is suppressed because no enough space is available in the gap between the two cylinders for vortex shedding.

Fig. 2.12 shows the spectra of the displacement in both the in-line and the cross-flow directions. Different from other gap ratios, multi-frequency components are found in the spectra of both $X$ and $Y$ displacements as $G/D=0.2$. The lowest normalized peak frequency of the in-line displacement is the dominant one at this gap ratio. This low normalized frequency is unique and does not occur at other gap ratios, is likely due to the unique vortex shedding
pattern at this gap ratio. The proximity of the cylinders as $G/D=0.1$ does not allow any vortex shedding from the small cylinder even during the downward motion of the bundle. However as $G/D=0.3$ and 0.4 two separate vortex shedding always exist throughout a whole vibration cycle, although the strength of the vortices that are shed from the small cylinder varies with the direction of the movement of the cylinder bundle. For cross-flow direction (Fig. 2.12 (b)) the dominant frequency remains the same for all gap ratios. However, one can observe two other frequency components at $G/D=0.2$.

Fig. 2.13 (a) and (b) shows the time histories of the in-line displacements at $G/D=0.2$ and 0.4 and the cross-flow displacement at $G/D=0.2$, respectively. The highest componential amplitude of the FFT analysis at $G/D=0.2$ is $0.45D$, which is smaller than that at $G/D=0.1$ (about $0.55D$) and that at $G/D=0.3$ (about $0.5D$). A clear beating phenomenon of the cylinder bundle is observed in Fig. 2.13 (b). The beating frequency is about one third of the peak oscillation frequency of the cylinder bundle. The trajectories of the vibration for different gap ratios are shown in Fig. 2.13 (c). The trajectories in Fig. 2.13 (c) are quite repeatable, although each of them contains more than one up-and-down movement. Different from other gap ratios, the vibration trajectory for $G/D=0.2$ comprises three up-and-down movements of the cylinder bundle.

**Fig. 2.13:** Time histories of the vibration displacement and the $XY$-trajectories at $\theta=45^\circ$. (a) time history of the in-line displacement, (b) time history of the cross-flow displacement, (c) $XY$-trajectories.
2.4.3.5 \( \theta=67.5^\circ \)

It was shown in Fig. 2.2 that the highest deflection and amplitude for both the cross-flow and the in-line oscillations occur at \( \theta=67.5^\circ \) and \( G/D=0.1 \). The results of the FFT analysis of the in-line and cross-flow displacements for this particular case are shown in Fig. 2.14. The presence of the highest amplitude components at \( G/D=0.1 \) is clear in both Fig. 2.14 (a) and (b). For the in-line direction (Fig. 2.14 (a)) the highest amplitude component occurs at normalized frequency less than one at \( G/D=0.1 \). The dominant normalized frequencies at \( G/D=0.2 \) is also less than one. However the highest peak amplitude at \( G/D=0.2 \) is only about one-third of that at \( G/D=0.1 \). The highest peak amplitude is close to 1 at gap ratios greater than 0.2.

![Fig. 2.14: FFT analysis of the vibration displacement of the two-cylinder bundle at \( \theta=67.5^\circ \) for different gap ratios. (a) in-line displacement, (b) cross-flow displacement.](image)

Fig. 2.15 shows the streamlines superimposed on the pressure contours near the cylinders surface for different gap ratios when the bundle is at its highest vertical position. No streamlines pass through the gap between the cylinders for \( G/D=0.1 \) and 0.2. It seems that the bundle acts nearly as a single body at these two gap ratios when it is at the highest position in each cycle. The streamlines on the top side of the small cylinder direct toward the large cylinder in these two cases. However for higher gap ratios of \( G/D=0.3 \) and 0.4, the fluid flows through the gap without biasing toward large cylinder. Fig. 2.16 shows the vibration trajectory of the cylinder bundle for \( G/D=0.1 \). It is interesting to see that the cylinder bundle vibrates in a V-shaped track. The vibration is very repeatable and the direction of the movement of the bundle is from \( a \) to \( h \).
Fig. 2.15: Instantaneous pressure contours and streamlines for different gap ratios at $\theta=67.5^\circ$.

Fig. 2.16: $XY$-trajectory of the cylinder boundle at $\theta=67.5^\circ$ and $G/D=0.1$. 
2.4.3.6  \( \theta = 90^\circ \)

It is observed in Fig. 2.2 that there are significant reductions of in-line and cross-flow deflections and vibration amplitudes at this angular position as the gap ratio increases from \( G/D = 0.1 \) to \( G/D = 0.2 \). To understand the reasons for such significant reductions of in-line and cross-flow deflections and vibration amplitudes at this angular position, flow characteristics around the cylinder bundle with \( G/D = 0.1 \) and \( G/D = 0.2 \) in two successive repeatable vibration cycles are investigated in Fig. 2.17 and Fig. 2.18 respectively. The time instants corresponding to the flow structures shown in Fig. 2.17 and Fig. 2.18 can be found in Fig. 2.19 and Fig. 2.20.

Fig. 2.17: Vorticity contours within two vibration cycles at \( \theta = 90^\circ \) and \( G/D = 0.1 \).
Fig. 2.18: Vorticity contours within one vibration cycle at $\theta=90^\circ$ and $G/D=0.2$. 
Fig. 2.19: Time histories and trajectories of the two-cylinder bundle at $\theta=90^\circ$ and $G/D=0.1$. (a) force coefficient and displacement time histories, (b) drag coefficient variation with in-line displacement, (c) lift coefficient variation with cross-flow displacement, (d) $XY$-trajectory.

It was found that vortex shedding around a cylinder near a plane wall is suppressed for gap ratios smaller than 0.2-0.3, Bearman and Zdravkovich (1978) and Taniguchi and Miyakoshi (1990). One can see that the vortex shedding exists around the small cylinder even for gap ratio of $G/D=0.1$ of the cylinder bundle. This is because the ratio of gap to the diameter of the small cylinder is one in this case which is greater than the critical gap ratio (0.2 to 0.3) for vortex shedding. For the case of $G/D=0.1$ shown in Fig. 2.17, as the cylinder bundle moves up (e.g. from Fig. 2.17 (c) to (g)) vortex shedding from the small cylinder is clearly visible. The vortices that are shed from the small cylinder are forced into the back side of the large cylinder due to the upward movement of the cylinder bundle. The interactions of the vortices from the small cylinder with the large cylinder are seen through high frequency oscillations in the time history of force coefficients $C_D$ and $C_L$ as shown in Fig. 2.19 (b) and (c). During the same period, the horizontal displacement of the cylinder increases from about 0.66 to about 1.2 as the vertical displacement changes from about -0.7 to about 1.2. The large amplitudes in both directions during this upward moving period are due to the strong interaction between the vortex street of the small cylinder and the large cylinder. The upward movement of the cylinders shortens the distance between the vortex...
street of the small cylinder and the surface of the large cylinder and as the result the large
cylinder is attracted into the vortex street as it moves up. The effect of the small cylinder on
the trajectory of the large cylinder’s motion is clearly shown by comparing the trajectory of
the upward motion of a single cylinder shown in Fig. 2.4 (d) (from point d to i) and the
trajectory of upward motion shown in Fig. 2.19 (d) (from point c to g) for the side-by-side
arrangement at $G/D=0.1$. When the cylinders move downwards (e.g. from g to k in
Fig. 2.17), the large cylinder moves away from the vortex street from the small cylinder that
was formed during the upward motion. During this period, it doesn’t appear to be vortex
shedding from the small cylinder. Therefore there is almost no interaction between the
vortex street behind the small cylinder and the large cylinder, leading to the smooth time
history curves of force coefficients $C_D$ and $C_L$ shown in Fig. 2.19 (b) and (c) and the smooth
vibration curves shown in Fig. 2.19 (a).

![Figure 2.20](image)

Fig. 2.20: Time histories and trajectories of the two-cylinder bundle at $\theta=90^\circ$ and $G/D=0.2$. (a) force
coefficient and displacement time histories, (b) drag coefficient variation with the in-line
displacement, (c) lift coefficient variation with the cross-flow displacement, (d) XY-trajectory.

The flow characteristics and cylinder response for $G/D=0.2$ (Fig. 2.18 and Fig. 2.20) are
more or the less similar to those observed for the case with $G/D=0.1$ shown in Fig. 2.17 and
Fig. 2.19. The major difference between the two gap ratios are: (1) the vortex street formed
behind the small cylinder at $G/D=0.2$ does not get as close to the large cylinder surface as in
the case of \( G/D = 0.1 \), (2) vortex shedding from the small cylinder appear to exist during both upward and downward motions of the cylinder bundle at \( G/D = 0.2 \) and (3) the amplitudes of both horizontal and vertical displacements are significantly smaller than their counterpart observed in case of \( G/D = 0.1 \), especially the amplitude of horizontal displacement. The presence of the small cylinder filters out the \( 2f \)-frequency component in the lift coefficient of an isolated cylinder (see Fig. 2.4 (a) and Fig. 2.20 (a)). The crest values of the lift coefficient for both \( G/D = 0.1 \) and 0.2 occur just before the bundle reaches the lowest point of the \( XY \)-trajectories in the vertical directions (instants \( k, c \) and \( s \) for \( G/D = 0.1 \) and \( c \) for \( G/D = 0.2 \)). However the trough value of lift coefficients occurs just after the cylinder bundle starts moves downwards (instants \( p, h \) and \( s \) for \( G/D = 0.1 \) and \( h \) for \( G/D = 0.2 \)). The magnitude of these values for \( G/D = 0.1 \) varies in each cycle. The \( XY \)-trajectory of \( G/D = 0.2 \) resembles an 8-shape, as shown in Fig. 2.20 (d) with the in-line amplitude about 5% of the cross-flow amplitude.

The results of the FFT analysis of the in-line and the cross-flow displacements are shown in Fig. 2.21. It can be seen that different gap ratios result in different vibration frequencies both in the in-line and the cross-flow directions. The highest peak values for different gap ratios differ from each other slightly. These highest peak values in the in-line and the cross-flow directions for \( G/D = 0.1 \) are about 0.04\( D \) and 0.5\( D \), respectively. Although the shape of the spectra at the lowest frequency of the in-line vibration for \( G/D = 0.1 \) is not well defined, there is indeed a dominant low frequency component with an average amplitude about 0.55, which is about half of the dominant frequency in the cross-flow vibration. The wavelet analysis of the in-line vibration for \( G/D = 0.1 \) (Fig. 2.21 (c)) shows the variation of amplitude of the in-line displacement in the time-frequency domain. The amplitude corresponding to the lowest dominant frequency is the largest during the dimensionless time of about 120 to 180 and smallest during dimensionless time from 420 to 450.

The time series of the in-line and the cross-flow displacements for \( G/D = 0.1 \) are shown in Fig. 2.22 (a) and (b). Even though no repeatability is identified in the in-line vibration, the oscillation becomes steady as the gap ratio increases to \( G/D = 0.4 \). It can be seen in Fig. 2.22 (b) that the cross-flow oscillation is not symmetric and is biased toward positive vertical direction. The reason for this is the mean positive lift force on the cylinder bundle. This can be seen by referring to Fig. 2.19 (a). The \( XY \)-trajectories for \( \theta = 90^\circ \) are shown in Fig. 2.22 (c). The unsteadiness of the vibration at \( G/D = 0.1 \) is clearly shown in this figure, while regular vibrations are observed at other three gap ratios.
Fig. 2.21: (a) Variation of dominant frequencies with gap ratio for the in-line displacement at $\theta=90^\circ$ (b) variation of dominant frequencies with gap ratio for the cross-flow displacement at $\theta=90^\circ$ (c) Wavelet analysis of the in-line vibrations at $\theta=90^\circ$ as $G/D=0.1$.

Fig. 2.22: Time histories of the displacement and $XY$-trajectories of the cylinder bundle at $\theta=90^\circ$. (a) time history of the in-line displacement, (b) time history of the cross-flow displacement, (c) $XY$-trajectory.
2.4.3.7 \( \theta = 112.5^\circ \)

The smallest in-line and cross-flow vibration amplitudes occur at this angular position. The minimum vibration amplitudes in both directions are less than that of an isolated cylinder. Fig. 2.23 shows the FFT analysis of both the in-line and the cross-flow displacements. It can be seen that amplitude of the bundle at \( G/D = 0.3 \) is the smallest compared with the amplitudes in other cases. Another interesting feature is that the dominant vibration frequencies of the bundle as \( G/D = 0.3 \) for both the in-line and the cross-flow vibrations are larger than their counterparts at other gap ratios. Pressure contours and velocity fields of the bundle for \( G/D = 0.1 \) and 0.3 are shown in Fig. 2.24. The three instants in Fig. 2.24 correspond to the highest, the middle and the lowest positions of the bundle during its downward motion as shown in the trajectories in Fig. 2.25. While the streamlines between the gap are intermittent for \( G/D = 0.1 \), they are continuous for \( G/D = 0.3 \). The position of the core of the main vortex at \( G/D = 0.1 \) is also different from that at \( G/D = 0.3 \). For \( G/D = 0.1 \) it is far downstream the bundle, while for \( G/D = 0.3 \) it is very close to the bundle. The trajectories shown in Fig. 2.25 show that changing the gap from 0.1\( D \) to 0.3\( D \) reduces both the in-line and the cross-flow amplitudes significantly. The trajectories for both cases are closed loops that do not cross themselves during oscillation.

![Fig. 2.23: FFT analysis of the displacement of two-cylinder bundle at \( \theta = 112.5^\circ \) for different gap ratios. (a) in-line displacement, (b) cross-flow displacement.](image)

2.4.3.8 \( \theta = 135^\circ \)

Fig. 2.26 shows the instantaneous vorticity contours for \( G/D = 0.2 \), where the in-line amplitude is the highest among other gap ratios (as shown in Fig. 2.2). Vortex shedding from the small cylinder in Fig. 2.26 only occurs when the cylinder bundle moves upwards. When cylinder bundle moves downwards, the vortex shedding from the small cylinder is suppressed because the small cylinder is immersed in the wake of the large cylinder.
Fig. 2.24: Streamlines and pressure contours at $\theta = 112.5^\circ$ for $G/D = 0.1$ and $0.3$.

Fig. 2.25: $XY$-trajectories of the cylinder bundle at $\theta = 112.5^\circ$ for $G/D = 0.1$ and $0.3$. $a$, $b$, and $c$ are for $G/D = 0.1$ and $d$, $e$, and $f$ for the $G/D = 0.3$.

Fig. 2.27 shows the time histories of the vibration displacement, the hydrodynamic force coefficients and the vibration trajectories of the cylinder bundle. The maximum in-line amplitude of the motion happens during its upstream excursion from (instant i to o in Fig. 2.27 (d)). The trajectory shows that the movement of the bundle towards the most upstream location and return from this position is almost horizontal. During this in-line motion, the lift coefficient drop to its minimum value sharply and then increase sharply again. A minimum value of the drag coefficient occurs during this in-line motion ($o$, $p$ and $q$). One can also observe sharp changes in the drag coefficient and relative large in-line
displacement during the vertical upward motion of the bundle from $d$ to $i$. The cylinder bundle moves towards the vortex street behind the small cylinder during this period. The sharp drag change and the large in-line displacement during this upward movement are dependent on the instantaneous position of the bundle relative to the vortex street behind the small cylinder.

Fig. 2.26: Vorticity contours at $\theta=135^\circ$ and $G/D=0.2$.

Fig. 2.28 shows the results of the FFT and wavelet analysis of the vibration displacement. The spectra of both in-line and cross-flow displacements at $G/D=0.2$ contain many small peaks. Wavelet analysis of both the in-line and the cross-flow displacements for $G/D=0.2$ also confirm the unsteadiness of the vibration with wide frequency bands near the dominant frequencies. The time histories of the in-line displacement for $G/D=0.1$ and 0.2 and cross-displacement at $G/D=0.1$ are shown in Fig. 2.29 (a) and (b), respectively. The regular vibrations in the in-line direction can be seen at $G/D=0.1$. Both the in-line and the cross-flow displacements at $G/D=0.2$ shows strong beating behaviour with beating frequencies varying with time. Fig. 2.29 (c) shows the trajectories of the cylinder motion. Trajectories for three higher gap ratios are delineated by different style lines for better illustrations. For the case $G/D=0.1$ and 0.4 the closed loops show the repeatability of the trajectories. The semi-circular oscillations of the bundle as $G/D=0.1$ is noticeable.
Fig. 2.27: Time histories and trajectories of the two-cylinder bundle at $\theta=135^\circ$ and $G/D=0.2$. (a) force coefficient and displacement, (b) drag coefficient variation with the in-line displacement, (c) lift coefficient variation with the cross-flow displacement, (d) $XY$-trajectory.

2.4.3.9 $\theta=157.5^\circ$

The most dominant feature at $\theta=157.5^\circ$ is the high amplitude of vibration in the cross-flow direction. The FFT analysis of the VIV in both the in-line and the cross-flow directions are shown in Fig. 2.30. This is the only angular position where the normalized vibration frequencies of the bundle (except $G/D=0.4$) in the cross-flow direction are very close to (or slightly below) 1. Therefore the amplitude in the cross-flow direction is the highest among other position angles. The normalized vibration frequencies of the bundle for all other angles are greater than one. One of the interesting features at this angular position is the high amplitude of the in-line vibration for $G/D=0.1$. It was shown in Fig. 2.2 that the highest in-line amplitude at this configuration occurs at $G/D=0.1$ and the second highest in-line amplitude takes place for $G/D=0.4$. The FFT spectra in Fig. 2.30 (a) do not show a well-established frequency behaviour for $G/D=0.4$ which means the vibration is irregular. In contrast, the frequency response for the case $G/D=0.1$ demonstrates the steadiness of the in-line displacement time history.
Fig. 2.28: FFT analysis and wavelet analysis of the displacement of two-cylinder bundle at $\theta=135^\circ$ for different gap ratios. (a) FFT of in-line displacement. (b) FFT of the cross-flow displacement, (c) Wavelet of the in-line displacement at $\theta=135^\circ$ and $G/D=0.2$, (d) Wavelet of the cross-flow displacement at $\theta=135^\circ$ and $G/D=0.2$.

2.4.3.10 $\theta=180^\circ$

Fig. 2.31 shows the instantaneous vorticity contours superimposed on the streamlines at two different instants a and b in one cycle of vibration at $G/D=0.1$. The trajectory and instantaneous positions of cylinder are shown inside the large cylinder. The instant a is right after the cylinder bundle starts moving downwards and b is right after the bundle start moving upwards. These two captured instances are the ones with complete flow circulations behind the large cylinder. Flow circulation in different direction can be seen. One can observe that even for this gap ratio the flow exist between cylinders. That could be the reason why the flow characteristics at all gap ratios at this angular position are fairly similar. There is not intermittent flow between the gaps for all gap ratios investigated at this angle. There is no vortex shedding from the small cylinder because it is totally immersed in the wake of the large cylinder. The changes of flow direction experienced by the small cylinder appear to have prevented the vortices developing from the small cylinder.
Fig. 2.29: Time histories of vibration displacements and the $XY$-trajectories of the bundle at $\theta=135^\circ$. (a) time history of the in-line displacement, (b) time history of the cross-flow displacement, (c) $XY$-trajectory.

Fig. 2.30: FFT analysis of the displacement of two-cylinder bundle at $\theta=157.5^\circ$ for different gap ratios. (a) FFT of in-line displacement. (b) FFT of the cross-flow displacement.
Fig. 2.31: Instantaneous vorticity contours and streamlines within one vibration cycle at $\theta=180^\circ$ and $G/D=0.1$.

Fig. 2.32 shows the results of FFT analysis of the VIV displacements. Placing the small cylinder in the downstream of the large cylinder at small gap ratios $G/D=0.1$ and 0.2 suppresses the low frequency components that exists in the in-line spectrum of a single cylinder (compare Fig. 2.32 (a) and Fig. 2.5). For higher gap ratios of $G/D=0.3$ and 0.4, the irregularities in the vortex shedding period of the bundle prohibit the presence of a distinct low normalized frequency for these cases (beating frequency), see Fig. 2.32 (a). The presence of the dominant frequency of 2.15 for all gap ratios is clear in Fig. 2.32 (a). The amplitude and the dominant frequency for cross-flow vibrations remains fairly the same for all gap ratios in Fig. 2.32 (b).

As shown in Fig. 2.2, the highest in-line and cross-flow amplitude of oscillations at $\theta=180^\circ$ occur respectively as $G/D=0.4$ and 0.1. The in-line and cross-flow time histories of pipe bundle motion for these gap ratios are shown in Fig. 2.33 (a) and (b). One can see the unsteadiness of the in-line displacements at $G/D=0.4$. However the in-line vibration at $G/D=0.1$ and cross-flow vibrations at both gap ratios are regular. The XY-trajectories of the cylinder bundle motion are shown in Fig. 2.33 (c). For a clear illustration of the trajectories, different style lines are used in Fig. 2.33 (c). The trajectories as $G/D=0.1$ and 0.2 assemble
the closed loops which show the repeatability of the oscillations in both directions. However, the open loops trajectories at other two gap ratios indicate the increased level of interactions between the two cylinders.

Fig. 2.33: Time histories of the vibration displacement and the $XY$-trajectories at $\theta=180^\circ$. (a) time history of the in-line displacement, (b) time history of the cross-flow displacement, (c) $XY$-trajectories.

### 2.5 Force Coefficients

Fig. 2.34 shows the force coefficients of the bundle, which are normalized by the diameter of the large cylinder. The mean drag and the root mean square lift coefficients of an isolated cylinder are 1.32 and 0.28 respectively. It can be seen that the large values of the mean drag coefficients of the bundle, $C_D$, occur when the cylinders are in the side by side arrangement and the gap ratio is $G/D=0.1$ and 0.2. The proximity of the cylinders contributes to a high blockage of the flow and consequently high pressure on the upstream side of the cylinders. Checking the r.m.s lift coefficient of the bundle (Fig. 2.34 (b)), $C_{\text{rms}}$, it is clear that the values of the r.m.s lift coefficient for the side by side arrangements at $G/D=0.1$ and 0.2 are about two times larger than those at gap ratios of 0.3 and 0.4. One can also observe that the change in the gap ratio from 0.2 to 0.3 results in a sudden decrease in lift coefficient on the bundle at $\theta=90^\circ$. Furthermore, both the $C_D$ and $C_{\text{rms}}$ at $G/D=0.4$ are larger than their counterparts at $G/D=0.3$. 
Fig. 2.34: Force coefficients of the bundle. (a) mean drag coefficient, (b) r.m.s lift coefficient.

Fig. 2.35 shows the contribution of each cylinder to the total lift and drag coefficients for different cases. Fig. 2.35 (a) and (b) show the lift coefficients of each cylinder separately at \( \theta = 90^\circ \) as \( G/D = 0.1 \) and 0.3, respectively. Comparing them, it can be seen that the proximity of the small cylinder to the large cylinder suppress the vortex shedding from the small cylinder. The high frequency component of the lift coefficient at \( G/D = 0.1 \) (Fig. 2.35 (a)) is much smaller and intermittent compared with that for \( G/D = 0.3 \) (Fig. 2.35 (b)).

Fig. 2.35: Force coefficients of each cylinder in the two-cylinder bundle. (a) lift coefficient at \( \theta = 90^\circ \) and \( G/D = 0.1 \), (b) lift coefficient at \( \theta = 90^\circ \) and \( G/D = 0.3 \), (c) lift coefficient at \( \theta = 135^\circ \) and \( G/D = 0.2 \), (d) drag coefficient at \( \theta = 135^\circ \) and \( G/D = 0.2 \).
Fig. 2.35 (c) and (d) show the drag and lift coefficients of each cylinder separately at \( \theta = 135^\circ \) and \( G/D = 0.2 \). Each cylinder affects the force coefficients of the other one. Referring to Fig. 2.26 and Fig. 2.27, it can be seen that strong interactions between the two cylinders exist when they are at positions \( f \) to \( i \), \( s \) and \( t \). No vortices are shed from the small cylinder during the quasi horizontal motion of the bundle. Furthermore the shed vortices from the top side of the large cylinder are diminished by the small cylinder.

### 2.6 Conclusions

Two-degree-of-freedom vortex induced vibration of two mechanically coupled cylinders is modelled numerically using the finite element method. The gap ratio between the two circular cylinders and angular position of the small cylinder vary between \( 0.1 < G/D < 0.4 \) and \( 0^\circ < \theta < 180^\circ \), respectively. The diameter ratio between the two cylinders is kept constant at \( d/D = 0.1 \). Following major conclusions are obtained.

1. The VIV results of an isolated cylinder show the presence of the second harmonic (2f) in the lift force coefficient. Attaching a small cylinder to a large cylinder affects the frequencies and amplitudes involved in the frequency spectra. The vibration amplitudes depend on both the angular position and the gap ratio.

2. The highest vibration amplitude in the cross-flow direction occurs at \( G/D = 0.1 \) and \( \theta = 67.5^\circ \) which is followed by the amplitudes at \( G/D = 0.1 \) and \( \theta = 157.5^\circ \), and at \( G/D = 0.1 \) and \( \theta = 90^\circ \) sequentially. The highest vibration amplitude in the in-line direction also occurs at \( G/D = 0.1 \) and \( \theta = 67.5^\circ \), followed by those at \( \theta = 135.5^\circ \) with \( G/D = 0.1, 0.2 \) and \( 0.3 \), and that at \( G/D = 0.1 \) and \( \theta = 90^\circ \). The minimum cross-flow vibration amplitude occurs at \( \theta = 112.5^\circ \) with \( G/D = 0.3 \).

3. The continuous interaction between the two coupled cylinders leads to very irregular vibration of the bundle at large gap ratios. Large variations of displacements in both the in-line and the cross-flow directions are observed. The reason for these high-amplitude variations is the flow intermittency between the cylinders.

4. The vortex shedding from the small cylinder appears to affect the responses of the cylinder bundle significantly for \( \theta = 90^\circ \) and \( 135^\circ \). Sharp changes in the drag coefficient and relative large in-line displacement are observed during the period that cylinder bundle moves towards the vortex street behind the small cylinder. The observed sharp drag change and the large in-line displacement are dependent on the relative instantaneous position of the bundle to the vortex street behind the small cylinder.
2.7 References


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CHAPTER 3

VOXET INDUCED VIBRATION AND VORTEX SHEDDING
CHARACTERISTICS OF TWO SIDE-BY-SIDE CIRCULAR
CYLINDERS OF DIFFERENT DIAMETERS IN CLOSE
PROXIMITY IN STEADY FLOW

ABSTRACT

This paper presents results of a numerical study of vortex-induced vibrations of two
side-by-side circular cylinders of different diameters in steady incompressible flow. The two
dimensional Reynolds-averaged Navier-Stokes equations with a SST $k-\omega$ turbulence model
closure are solved using the Petrov-Galerkin finite element method and the Arbitrary-
Lagrangian-Eulerian scheme. The diameter ratio of the two cylinders is fixed at 0.1 and the
mass ratio of both cylinders is a constant of 5.0. Both cylinders are constrained to oscillate in
the transverse direction only. The Reynolds number based on the large cylinder diameter and
free stream velocity is fixed at 5000. The effects of the reduced velocities of the cylinders on
the vibration amplitude and vortex shedding regimes are investigated. It is found that for the
range of parameters considered, collision of the cylinders is dependent on the difference of
the reduced velocities of the cylinders. Presence of the small cylinder in the proximity of the
large cylinder appears to have significant effects on the vortex shedding regime and
oscillation amplitude of the large cylinder.

3.1 Introduction

Vortex-induced vibration (VIV) of cylindrical structures is relevant in many engineering
applications such as marine risers and subsea pipelines. It has been understood that VIV of a
circular cylinder is affected by a number of parameters including: (1) mass ratio,
$m^* = m/m_d$, with $m$ and $m_d$ being the masses of the cylinder and the fluid displaced by the
cylinder, respectively; (2) structural damping ratio (based on the free decay test), $\zeta = \delta / 2\pi$, with $\delta$ being the logarithmic decrement; (3) Reynolds number, $Re=U_\infty D/\nu$, with
$U_\infty$ being the free stream velocity, $D$ the diameter of the cylinder and $\nu$ the kinematic
viscosity of fluid and (4) reduced velocity, $U_r = U_\infty / f_n D$, with $f_n$ being the structural
natural frequency of the cylinder.

A large number of studies have been carried out on flow-induced vibration of an
elastically mounted cylinder in a cross-flow. The classical experimental study of one-degree-
of freedom vibration of a circular cylinder in the cross-flow direction with the mass-
damping-ratio of $m^*\zeta = 0.25$ by Feng (1968) showed that the vortex shedding frequency of
the cylinder synchronizes into the natural frequency of the system over a range of reduced
velocity of $5 \leq U_r \leq 7$. The experiments of Brika and Laneville (1993) with $m^*\zeta$ of the
same order as that in Feng’s experiment showed similar results to Feng’s. The amplitude
response of the cylinder in these studies was categorized into two branches, namely the
initial and the lower branches. Two single vortices (2S) are shed from the cylinder per cycle
of oscillation in the initial branch. However, during the lower branch two pairs of vortices
(2P) are formed in each vibration cycle. The highest amplitude observed during these
experiments was slightly greater than 0.5D. In addition to the initial and the lower branches,
Khalak and Williamson (1996) found the third branch, namely the upper branch in the
amplitude response of a flexibly-mounted circular cylinder with low mass-damping ratios
($m^*\zeta = 0.013$). The highest amplitude observed in the upper branch was about 1.1D. Later,
Jauvtis and Williamson (2004) found a very high vibration amplitude of about 1.5D in the
super upper branch in an experiment where the cylinder is allowed to vibrate in both
horizontal and vertical directions at low mass ratios (less than 6) and low damping ratios.
Three pairs of vortices were shed from the cylinder in each cycle of the vibration in the
super upper branch.

Sarpkaya (1995) investigated the effects of natural frequency ratio (in-line to cross-flow
direction) on the vibration amplitude and force coefficient of the 2-DOF vibration of a
circular cylinder experimentally. Among the implemented frequency ratios ($f_{nx}/f_{ny} = 1$, 2
and $\infty$, where $f_{nx}$ and $f_{ny}$ are the natural frequencies in the in-line and the cross-flow
directions, respectively) the largest vibration amplitude of about 1.1D in the cross flow
direction was observed at the frequency ratio of 1.0. Another important finding was that the
drag coefficient was larger in 2-DOF vibrations than in 1-DOF vibrations. Singh and Mittal
(2005) observed hysteresis effects in their numerical study of 2-DOF vibrations of a circular
cylinder at a low Reynolds number of 100. They found that the vortex shedding regime was
dependent on the Reynolds number at the reduced velocity of $U_r=4.92$.

A number of experimental and numerical investigations about the vortex shedding and
VIV of two-cylinder systems have been carried out by many researchers, e.g. King and
Johns (1976), Brika and Laneville (1997), Mittal and Kumar (2001), Assi et al. (2006) and
Alam et al. (2005), Mahbub Alam and Kim (2009). All of them focused on two identical
cylinders placed far from each other to prevent the collision.
It was found in Rahmanian et al. (2012) that the flow characteristics around a cylinder coupled mechanically to a large one at small gap ratios is very similar to the flow characteristics around a cylinder near a plane boundary. The vortex shedding and VIV response of a cylinder near the fixed plane boundary have been investigated by many researchers. Bearman and Zdravkovich (1978), Tsahalis and Jones (1981), Fredsøe (1985) and Zhao and Cheng (2011) are few to mention. Suppression of the vortex shedding for the small gap distances was reported by most of them. Various interference regimes have been defined for flow around a pair of cylinders. Zdravkovich (1977) categorized the interference of two rigid cylinders into four groups based on the centre-to-centre distance and the direction of the approaching flow. Proximity interference occurs if the two cylinders are close to each other and one of them is not submerged in the wake of the other. Wake interference occurs when the downstream cylinder is partially or fully submerged in the wake of the upstream one. Overlap interference is a combination of the proximity and the wake interference regimes. The fourth type of interference is the “no-interference” regime with negligibly weak interference between the two cylinders.

Steady flow around two rigid stationary cylinders of different diameters has also been studied by many researchers, e.g. Tsutsui et al. (1997), Dalton et al. (2001), Lee et al. (2004) and Zhao et al. (2007). It is found that only one vortex street forms behind the cylinders if the two cylinders are in close proximity. It was shown (Dalton et al. (2001)) that both drag and lift forces on a cylinder were reduced by placing a small-diameter cylinder in the wake of the large cylinder. It was observed that the small cylinder suppressed the vortex shedding from the large cylinder. Numerical study of Mittal and Raghuvanshi (2001) showed that the placement of a small cylinder at a specific position of $(P/D, T/D) = (2.0, 0.8)$ downstream of the large cylinder, where P and T are the horizontal and vertical distances between two cylinder centres, suppressed the vortex shedding of both cylinders in laminar flow regime ($Re=80$). Their study confirmed the experimental results of Strykowski and Sreenivasan (1990). Williamson (1985) realized that reducing the gap between the two side-by-side cylinders to a distance of 3.5 diameters contributes to a significant magnification of the maximum lift and drag forces of both cylinders over their isolated values. Large fluctuations and impulses were observed in the recorded lift forces of both cylinders. Lakshmana Gowda and Deshkulkami (1988) investigated the effects of the presence of a rigid cylinder on the cross-flow vibration of an elastically mounted cylinder for the fixed reduced velocity of 7. Effect of the diameter of the rigid cylinder on vibration amplitude of the elastically mounted cylinder was investigated at different gaps between the two cylinders. For the side by side arrangement the amplitude of vibration of the elastically mounted cylinder showed a distinct response when the fixed rigid cylinder diameter was $0.5D$ (where $D$ was the diameter of the
elastically mounted cylinder) and the gap between cylinders was 0.25\(D\). The vibration amplitude experienced a sudden decrease when the gap was increased to the next step value (0.5\(D\)). Placement of a rigid cylinder of different diameters (\(d/D=1.0, 1.5\) and 2.0) close to the elastically mounted cylinder did not initiate any oscillations. The range of gap ratios over which the vibration amplitude is affected was different for different diameter ratios. The lock-on behaviour of two side by side cylinders with a diameter ratio of 2 and a gap ratio of 0.75\(D\) \((D\) is the diameter of the large cylinder) was discussed by Lee et al. (2012). Based on comparison of the large cylinder excitation frequency and the measured wake frequencies of both cylinders and the phase difference (between vortex formation on cylinders and their shedding on different side of each cylinder) across the large cylinder excitation frequencies, they concluded that the large cylinder oscillation and vortex shedding lock on at two frequencies, namely the natural shedding frequency and the frequency of its third sub-harmonics. The flopping gap flow observed by previous researchers (Brun et al. (2004)) for two equal diameter cylinders above the critical Reynolds number (1700 to 1900) was not observed in this study.

Flow past two side-by-side elastically mounted cylinders of different diameters is investigated numerically at different reduced frequencies in the present study. Although this problem is expected to be three-dimensional at the Reynolds number considered in this study, the use of a two-dimensional numerical model is actually a compromise between computational efficiency and numerical accuracy. On one hand, the computational cost of a three dimensional model is prohibitive for the large number of test runs conducted in this study. On the other hand, two dimensional numerical models have enjoyed reasonable success in modeling VIV problems. Two-dimensional models have been applied to simulate various VIV problems over a rich wide range of Reynolds numbers. For examples, Anagnostopoulos (1994) investigated VIV of an elastically mounted cylinder at Reynolds number= 130 with a laminar wake at its behind; Ding et al. (2013) simulated VIV of two elastically mounted cylinders in tandem arrangement for 30000<Re<105000; Zhang and Dalton (1996) simulated VIV of an elastically mounted cylinder for Reynolds number=13,000. The two-dimensional Reynolds-Averaged Navier-Stokes (RANS) equations have been found to be able to predict VIV and the vortex shedding modes at relatively large Reynolds numbers in the order of \(10^4\), e.g. Cox et al. (1998), Guilmineau and Queutey (2004) and Mittal and Kumar (2004). The numerical results of one-degree-of-freedom VIV by Wanderley et al. (2008) and those of the two-degree-of-freedom VIV by Zhao and Cheng (2011) agreed very well with the experimental results of Khalak and Williamson (1996) and Jauvtis and Williamson (2004), respectively. The numerical results
of VIV of a circular cylinder close to a plane boundary by Zhao and Cheng (2011) also agreed with the experimental data by Yang et al. (2009).

The major engineering applications of two side-by-side cylinders concerned in this study are in offshore oil and gas engineering where two pipelines are often bundled and laid together to save installation costs. The large diameter pipeline is used to transport oil or gas products while the small diameter pipeline is often used to transport control fluids for subsea equipment. The diameter ratio of the two pipelines is often in the range of 0.1 to 0.2. A small gap of approximately 10% of the diameter of the large pipeline is required structurally to attach the small pipeline to the large diameter pipeline. Since the primary aim of this study is to investigate the responses of the two cylinders at close proximity, the cylinders are restricted to the cross-flow vibration only. Although the interaction between the two cylinders is expected to be influenced by the kinematic and structure dynamic parameters of the system such as cylinder diameter ratio, mass, natural frequency, initial gap ratio as well as flow Reynolds number, only the effect of cylinders’ natural frequencies on the interaction is examined in this study. The effects of other parameters on the interaction will be addressed in subsequent studies as a part of an overall research effort on the topic.

3.2 Problem description

Fig. 3.1 shows the sketch of flow past two elastically mounted side-by-side cylinders close to each other. The centre of the large cylinder is placed at the origin of the Cartesian coordinate system initially. Both the small and large cylinders (of diameter \( d \) and \( D \)) are constrained to oscillate in the cross-flow direction only. The diameter ratio of the two cylinders is 0.1. The initial gap between the two cylinders, \( G \), is fixed at 0.1\( D \). The flow direction is in the horizontal direction (i.e. positive \( x \)-direction). The mass ratios of the large and the small cylinders are \( m^1 = m^2 = 5 \). The damping ratios of both cylinders are fixed at 0.000863. The Reynolds number based on the large cylinder diameter is a constant of 5000 for all simulations. Both the reduced velocities of the large and the small cylinders are defined based on the large cylinder diameter as \( U_{r1} = U_\infty / (f_{n1} D) \) and \( U_{r2} = U_\infty / (f_{n2} D) \), where \( f_{n1} \) and \( f_{n2} \) are the structural natural frequencies of the large and the small cylinders, respectively. The reduced natural frequencies \( f^*_{n1} \) and \( f^*_{n2} \) are defined as \( f^*_{n1} = 1 / U_{r1} = f_{n1} D / U_\infty \) and \( f^*_{n2} = 1 / U_{r2} = f_{n2} D / U_\infty \), respectively.
3.3 Governing equations and numerical methods

3.3.1 Governing equations

The governing equations for the problem of interest are the two-dimensional Reynolds-averaged Navier-Stokes (RANS) equations with the Shear Stress Transport (SST) $k$-$\omega$ turbulence model due to Menter (1994). The Arbitrary Lagrangian Eulerian scheme is applied to deal with the moving boundary problem. The two-dimensional RANS equations in the ALE scheme are expressed as

$$\frac{\partial U_i}{\partial t} = 0 \quad \text{Eq (3.1)}$$

$$\frac{\partial U_i}{\partial t} + (U_j - \hat{u}_j) \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\nu S_{ij} - \hat{u}_i \hat{u}_j \right) \quad \text{Eq (3.2)}$$

where $x_i (i=1,2)$ are the in-line and cross-flow directions of the flow respectively in the Cartesian coordinate system, $\rho$ is the fluid density, $\nu$ is the kinematic viscosity, $\hat{u}_j$ is the mesh velocity and $t$ is the time. $S_{ij}$ is the mean strain tensor defined as $S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} / 2$. The averaged value of the product of fluctuating velocities $\overline{u_i'}u_j'$ is calculated using the Boussinesq approximation, i.e. $\overline{u_i'u_j'} = 2\nu_t S_{ij} - \left( 2/3 \right) k \delta_{ij}$, where $\nu_t$, $k$ and $\delta_{ij}$ are the eddy kinematic viscosity, turbulent kinetic energy and the Kronecker delta, respectively. The SST $k$-$\omega$ turbulence model by Menter (1994) is used to close the RANS equations. The governing equations are discretised using the modified streamline Petrov-Galerkin method developed by Zhao et al. (2009). A rectangular computational domain is
employed in this study. The domain in the flow direction is 70\(D\) and the cylinder is placed 30\(D\) downstream from the inlet boundary. The width of the domain in the cross-flow direction is 40\(D\) with the large cylinder at its midpoint. No-slip boundary condition is implemented on the cylinder surfaces. The specific dissipation of turbulent kinetic energy \((\omega)\) and turbulent kinetic energy \((k)\) are 1 s\(^{-1}\) and 0.001\(\rho U_{in}^2\), respectively at the inlet boundary. The gradient of turbulent quantities and velocity at the outlet boundary are set to be zero. Zero gradients of horizontal velocity, pressure and turbulent quantities are considered at the two side boundaries.

3.3.2 Equations of motion of cylinders

The motion of each cylinder is governed by the following equation:

\[\ddot{Y}_i + 4\pi f_m^2 \zeta \dot{Y}_i + 4\pi^2 f_m^2 Y_i = F_{Li}/m_i\]  
Eq (3.3)

where \(\ddot{Y}_i\), \(\dot{Y}_i\), \(Y_i\) are the acceleration, velocity and displacement of the \(i\)-th cylinder in the \(y\)-direction respectively, \(m_i\) and \(f_m\) are the mass and natural frequency of \(i\)-th cylinder, \(F_{Li}\) is the total lift force on the \(i\)-th cylinder. The fourth-order Runge-Kutta algorithm is used to solve Eq (3.3).

3.3.3 Mesh movement technique and boundary conditions

The mesh movement technique considered for this study is based on a modified Laplace equation:

\[\nabla \cdot (\gamma \nabla V_j) = 0\]  
Eq (3.4)

where \(V_j\) \((j=1, 2)\) is the displacement of a nodal point. The displacement of the nodal points on each cylinder surface is same as the displacement of the corresponding cylinder. The other boundaries of the computational domain are fixed and their displacements are set to zero. The coefficient \(\gamma\) controls the distribution of the finite element nodes. Numerical experiments showed that \(\gamma = 1/A\) with \(A\) being the area of the element, leads to satisfactory results, Zhao and Cheng (2008).

One of the aims of this study is to investigate the collision between the two cylinders. When the two cylinders are very close to each other \((G/D<0.1)\), the mesh quality in the gap between the two cylinders cannot be guaranteed by using Eq (3.4) and sometimes the elements overlap with each other, making simulation impossible. In this study, the
displacements of the mesh outside a circle of \( x^2 + y^2 = \alpha^2 \) (\( \alpha \) is selected according to the gap distance between cylinders, in this study it is equal to 3D) are determined based on Eq (3.4). The coordinates of the nodal points inside this circle are found using an interpolation scheme based on two predefined meshes. A total of 32000 structured elements are used to discretise the fluid flow domain. The node number along the circumferences of the large and the small cylinders are 280 and 100 respectively. The aspect ratios of the elements adjacent to the cylinder surfaces (when \( G/D = 0.1 \)) vary between 3 and 10 for large cylinder and between 2 and 13 for the small one. The non-dimensional distance from the wall 
\[ y^+ = \Delta_1 u_f / \nu, \]
where \( \Delta_1 \) is the thickness of the first mesh layer adjacent to the cylinder surface and \( u_f \) is the frictional velocity) varies between 0.5 and 3.0 for both cylinders. Fig. 3.2 shows the two meshes for two extreme situations. The two cylinders are very far from each other in mesh (a) and very close to each other in mesh (b). The structure and the element number of mesh (a) are exactly same as their counterparts in mesh (b). The only difference between mesh (a) and mesh (b) is in the coordinates of nodal points. The coordinates of a node in a mesh with a gap \( G \) ranging between the two predefined meshes are determined based on a geometric interpolation scheme.

Fig. 3.2: Computational meshes at two extreme positions. (a) largest gap (b) smallest gap.

To avoid extreme mesh distortion (i.e. mesh size in circumferential direction is excessively greater than that in the radius direction), the gap ratio of \( G/D = 0.002 \) is considered to be the lowest limit of the gap ratio that is practically possible, Zhao and Cheng (2010, 2011). When the gap between the two cylinders reaches to \( G/D = 0.002 \), the two cylinders are considered to collide with each other in the numerical model. After a collision, the cylinders are bounced back with their new velocities calculated according to the conservation law of the momentum. Zero damping during collision is assumed in this study (i.e. no loss of energy).
3.4Results and discussion

3.4.1 Mesh dependence check and model validation

The validation of the model for simulating VIV of a single cylinder has been carried out before by Zhao and Cheng (2010, 2011). Zhao and Cheng (2011) showed that numerical results of 2-DOF VIV of a circular cylinder were in good agreements with the test results by Jauvitis and Williamson (2004) and the numerical results of 2-DOF VIV of a circular cylinder close to a plane boundary by Yang et al. (2009). The mesh dependency study for the reduced velocity used in this study was reported previously in Rahmanian et al. (2011) and will not be repeated here. Fig. 3.3 compares the calculated instantaneous vorticity contours for an isolated cylinder with the experimental results of Govardhan and Williamson (2000). The vortex shedding is in "2P" mode in both Fig. 3.3 (a) and (b). The present numerical model appears to reproduce what was observed in the experiment well.

3.4.2 VIV response of the two cylinders

In this study, the Reynolds number is kept at a constant of 5000 and the reduced natural frequency is varied by varying the natural frequencies of the two cylinders. The possibility of the collision of the two cylinders for different reduced natural frequency of each cylinder is investigated by conducting a series of numerical tests. The variation of the vibration amplitudes and vortex shedding patterns are also studied. The reduced natural frequency of the large cylinder is in the lock-in region to facilitate the collision of the two cylinders via the high amplitude of vibration of the large cylinder. Presence of the small cylinder (which
oscillates independently in cross flow direction) in the proximity of the large one with different reduced frequencies facilitates the investigation of its effect on the collision of the cylinders.

The collision of the cylinders is defined by the near-zero gap ($G/D$ reaching 0.002 in the numerical simulation) between the two cylinders. Preliminary simulation results showed that the collision of the two cylinders does not occur when the reduced natural frequency of the small cylinder is smaller or marginally larger than the reduced natural frequency of the large cylinder. For examples, for those cases where $f_{n1}^* = 0.2$, collision was not observed for $f_{n2}^* \leq 0.24$, and for the cases where $f_{n1}^* = 0.4$, collision did not occur for $f_{n2}^* \geq 0.36$. Fig. 3.4 shows the variation of the gap between the two cylinders with the normalized time (Time=$tU_\infty /D$) in four different cases where the reduced natural frequency of the large cylinder is greater than that of the small cylinder. It can be seen that the gap between the two cylinders was always greater than zero for the cases shown in Fig. 3.4. The high frequency components appeared in the gap variation in Fig. 3.4 (c) and (d) are caused by vortex shedding from the small cylinder. It seems that higher reduced natural frequencies of both cylinders result in a reduction of the gap between the cylinders and more interactions of the shed vortices.

Fig. 3.4: Variation of gap with normalized time (Time=$tU_\infty /D$) in four cases where the two cylinders do not collide with each other (the reduced frequency of the larger cylinder is higher than the reduced frequency of the small cylinder). The high frequency components in (c) and (d) explain the effects of the vortex shedding of the small cylinder, Rahmanian et al (2011).
Three groups of numerical tests have been carried out to investigate potential collisions between the two cylinders. The ranges of the reduced natural frequency are shown in Table 1, where \( f_{n1}^* \) and \( f_{n2}^* \) are the reduced natural frequencies for the large and small cylinders respectively. In the first group of numerical tests, the reduced natural frequency of the small cylinder \( f_{n2}^* \) is kept constant at 0.2 and the reduced natural frequency of the large cylinder \( f_{n1}^* \) varies from 0.08 to 0.28. The variation of vibration amplitudes, phase angle difference between the cylinders’ oscillations, the maximum and minimum gaps between the two cylinders with \( f_{n1}^* \) are shown in Fig. 3.5. The normalized amplitude of a cylinder is defined as \( A_y/D=(Y_{\text{max}}-Y_{\text{min}})/2D \), where \( Y_{\text{max}} \) and \( Y_{\text{min}} \) are the maximum and minimum cross-flow displacements of the cylinder respectively. No collision was observed in the Group 1 tests. The minimum gap of 0.05\(D\) between the two cylinders in the Group 1 tests occurs at \( f_{n1}^* = 0.26 \). The amplitude of the large cylinder increases gradually from a value slightly greater than zero at \( f_{n1}^* = 0.08 \) to a value very close to 0.6\(D\). The amplitude of the large cylinder reaches its maximum in the lock-in region of \( f_{n1}^* = 0.16 \) to 0.26. The variation of the amplitude of the small cylinder with \( f_{n1}^* \) is similar to that of the large cylinder. The maximum amplitude of the small cylinder is about 0.3\(D\). The difference between the minimum and maximum gaps between the two cylinders \( G_{\text{max}}-G_{\text{min}} \) is less than 0.1\(D\) at \( f_{n1}^* = 0.08 \). The magnitude of \( G_{\text{min}} \) at \( f_{n1}^* = 0.08 \) is about 0.37\(D\), which suggests the neutral positions of the cylinders are moved away from each other. The largest gap between cylinders approaches 0.8\(D\) at \( f_{n1}^* = 0.28 \). The phase differences between cylinders’ displacements in the range of 0.14 \( \leq f_{n1}^* \leq 0.26 \) explain why cylinders do not collide with each other though their vibration amplitudes are high. The small cylinder moves upward before the large cylinder collides with it and also move downwards after the large cylinder starts its downward stroke.

Table 3.1: Numerical tests groups and reduced natural frequencies.

<table>
<thead>
<tr>
<th>Group</th>
<th>( f_{n1}^* )</th>
<th>( f_{n2}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08 - 0.28</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.2 – 0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.2 – 0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Fig. 3.5: Vibration amplitudes, maximum and minimum gaps between two cylinders and the phase difference between cylinders oscillations (right vertical axis) for $f_{n2}^*=0.2$.

The reduced natural frequency of the large cylinder is kept constant at $f_{n1}^* = 0.2$ and $f_{n2}^*$ is varied from 0.2 to 0.4 with an increment of 0.04 in the Group 2 tests. The reduced natural frequency of $f_{n1}^* = 0.2$ was selected because of its closeness to the Strouhal frequency of the large cylinder (in the range of resonance). In the Group 3 tests the reduced natural frequency of the small cylinder $f_{n2}^*$ is kept a constant at 0.4 and $f_{n1}^*$ is increased from 0.2 to 0.4 with an increment of 0.04. In both Group 2 and Group 3 tests, the natural frequency of the small cylinder is kept to be higher than that of the larger cylinder in order to investigate the collision between the cylinders.

Fig. 3.6 shows the normalized time histories of vibration amplitude of both cylinders and the gap between the cylinders. Occurrence of the collision between the two cylinders is examined by checking the gap ratio $G/D$ between the cylinders. It was observed that collision does not occur if the difference in the reduced natural frequencies is small, e.g. $0 \leq (f_{n2}^* - f_{n1}^*) \leq 0.04$. The collision appears to affect the trajectories of the small cylinder significantly (and consequently the gap) while the large cylinder trajectory remains almost sinusoidal. The presence of the small cylinder appears to increase the vibration amplitude of the large cylinder slightly if the reduced frequency of latter is in the lock-in region ($f_{n1}^* = 0.2$) and $(f_{n2}^* - f_{n1}^*) \geq 0.08$. 
Fig. 3.6: Time histories of cross flow displacement of the large cylinder ($Y_{\text{large}}/D$), small cylinder ($Y_{\text{small}}/D$) and the gap between the two cylinders ($\text{Gap}/D$) for different reduced frequencies.
Fig. 3.7: Normalized amplitudes of the large and small cylinders for (a) $f_{n1}^* = 0.2$, $f_{n2}^* = 0.2$ to 0.4 and (b) $f_{n2}^* = 0.4$, $f_{n1}^* = 0.2$ to 0.4, respectively, and the maximum and minimum values of the gap ratio for (c) $f_{n1}^* = 0.2$, $f_{n2}^* = 0.2$ to 0.4 and (d) $f_{n2}^* = 0.4$, $f_{n1}^* = 0.2$ to 0.4, in the order.

Fig. 3.7 shows the vibration amplitudes of the two cylinders, the maximum and minimum gaps between the cylinders. In order to quantify the vibration amplitude increase caused by the presence of the small cylinder, VIV of an isolated cylinder of identical diameter to the large cylinder was simulated under the exact the same conditions as the two-cylinder system at two reduced natural frequencies of $f_{n1}^* = 0.2$ and 0.4, which correspond to $U_r = 5$ and 2.5 respectively. The simulated vibration amplitudes for $U_r = 5$ and 2.5 are 0.57 and 0.046 respectively. It can be seen from Fig. 3.7 (a) that the maximum normalized amplitude $A_y/D$ of the large cylinder for a constant $f_{n1}^* = 0.2$ is about 0.65, which occurs at $f_{n2}^* = 0.28$, 0.32 and 0.4. This is about 15% greater than that of an isolated large cylinder under the identical flow conditions. However for $f_{n2}^* = 0.24$ the amplitude is less than that of an isolated cylinder by 16%.

The large vibration amplitudes of both cylinders occurring at $f_{n1}^* = 0.2$ and $f_{n2}^* = 0.4$ shown in Fig. 3.7 (a) and (b) were likely due to the collision of the cylinders. The reason for high amplitude oscillation of the small cylinder is the forced upward displacement of the small cylinder by the large cylinder during the upward motion of the large cylinder. The large vibration amplitude of the large cylinder is caused by the strong semi-periodic vortex...
shedding from the colliding cylinders and the high net positive transfer of energy from fluid to the large cylinder. A further explanation of the high vibration amplitudes of both cylinders will be given later on.

The results shown in Fig. 3.7 (c) and (d) confirm that the collision of the two cylinders occurs if the difference of reduced natural frequencies between the cylinders exceeds or equals to 0.08. The zero gap between the cylinders is observed in the third column of Fig. 3.6 when collision occurs. It can be seen in Fig. 3.7 (a) and (b) that vibration amplitudes of the small cylinder follow similar trends to the vibration amplitudes of the large cylinder. When \( f'_{n2} \) is fixed at 0.4, the increase of \( f'_{n1} \) leads to a continuous decrease of the normalized gap between the cylinders. It can be seen in Fig. 3.7 (d) that the increase in the reduced frequency of the large cylinder results in the increase of the minimum gap and the decrease of the maximum gap between the cylinders.

It is interesting to see that the presence of the small cylinder intrigues the vibration of the large cylinder for reduced velocity \( f'_{n1} \) outside of the lock-in regime. For example, at \( f'_{n1} = 0.4 \) (see Fig. 3.7 (b)) the vibration amplitude of the large cylinder is 0.28D, in comparison with 0.05D of an isolated cylinder under same conditions. The amplitudes of the large cylinder for both \( f'_{n1} = 0.32 \) and 0.36 are very close to 0.33D in Fig. 3.7 (b), which is also much greater than their counterparts in the single cylinder case. The reason for the wider lock-in range of the large cylinder is likely induced by the close proximity of the cylinders during the upward motions of the cylinders. Under certain conditions the two cylinders moves upwards together after collision (refer to Fig. 3.6). The natural frequency of the two-cylinder system increases as the result of the increase of the stiffness of the system. In addition, the vortex shedding and VIV characteristics around the two cylinders of close proximity during the upward motion resemble those around a bluff body of a larger diameter. The increased natural frequency and size of the cylinder system during the upward motions result in the observed wide lock-in range of large cylinder.

The amplitude spectra of the large cylinder based on the Fast Fourier Transformation (FFT) analysis are shown in Fig. 3.8. The normalized vibration frequency \( f^*_L \) is defined as \( f^*_L = f_L D/U_n \), where \( f_L \) is the dimensional vibration frequency of the large cylinder. The presence of one dominant vibration frequency is clear in both Fig. 3.8 (a) and (b). The spectra in Fig. 3.8 show that collision of the two cylinders makes the spectrum more broadband \( (f^*_n = 0.2 \text{ to } 0.32 \text{ curves in Fig. 3.8 (a) and } f^*_{n2} = 0.28 \text{ to } 0.4 \text{ in Fig. 3.8 (b))} \). In each spectrum in Fig. 3.8 (a), the dominant vibration frequency is always smaller than the
reduced natural frequencies of the large cylinder. The reason for this is the presence of the small cylinder with a higher reduced natural frequency in the proximity of the large cylinder.

Fig. 3.8: Amplitude spectra of the large cylinder. \( f_L^* = f_L D / U_\infty \) is the normalized vibration frequency of the large cylinder (a) \( f_{n2}^* = 0.4 \); (b) \( f_{n1}^* = 0.2 \).

The reduced velocity of the small cylinder based on the small cylinder diameter varies between 25 and 50 in the simulations. This range of reduced velocity is far outside the lock-in region. Therefore, the vibration of the small cylinder should be negligibly weak if it were isolated from the large cylinder. When the difference in the reduced natural frequency between the two cylinders is less than 0.04, the displacement of the small cylinder is mainly towards the positive \( y \)-direction. One can observe this by inspecting the periodic oscillation of the small cylinder for the cases of \( f_{n1}^* = f_{n2}^* = 0.2 \) and \( f_{n1}^* = f_{n2}^* = 0.4 \) (Fig. 3.6). In these cases the small cylinder oscillates mainly about its neutral position \((y/D=0.65)\).

The vibration amplitude of the small cylinder reached a value higher than 5 times of its diameter when \( f_{n1}^* = 0.2 \) and \( f_{n2}^* = 0.28, 0.32 \) (Fig. 3.7 (a)). This value is extremely large, given the maximum cross-flow VIV amplitude observed for an isolated cylinder is expected to approximately two times of the cylinder diameter (King (1974, 1977)). To explain the reason of such high vibration amplitudes, the normalized time histories of the displacement \((Y)\), velocity \((\dot{Y})\), acceleration \((\ddot{Y})\) of the small cylinder and the calculated lift force on it \((F_L/m, m\) here is the mass of the small cylinder) are shown in Fig. 3.9 for two different situations. Fig. 3.9 (a) shows the normalized times history of the small cylinder during its downward motion after the collision for the case \( f_{n1}^* = 0.2, f_{n2}^* = 0.4 \). Fig. 3.9 (b) belongs to the case where the small cylinder oscillates mainly about its neutral position, \( f_{n1}^* = f_{n2}^* = 0.2 \). The same scaling has been used in both figures for the convenience of
It can be seen that collision contributes to higher velocities of the small cylinder as it passes the neutral (zero) position in the negative vertical direction. The scaled velocity \((50\ddot{Y}/U_c)\) at \(Y=0\) in Fig. 3.9 (a) is about -38, however its corresponding value in Fig. 3.9 (b) is about -15. The forced movement of the small cylinder by the large one during their upward motion increases its elastic potential energy. The release of this energy during the downward motion of the large cylinder contributes to a larger displacement and velocity of the small cylinder in downward direction. Suppression of vortex shedding of the small cylinder is another interesting feature. One can notice that the small amplitude wiggles that occur on the velocity curve in Fig. 3.9 (b) do not exist during downward motion of the small cylinder in Fig. 3.9 (a). This is because vortex shedding from the small cylinder is suppressed after collision during its downward motion with the large cylinder. While the large cylinder is moving downward, the vortex shedding behind the small cylinder occurs and its effects on the velocity of the small cylinder is clearly seen in Fig. 3.9 (b).

Fig. 3.9: Comparison of the small cylinder non-dimensional displacement \((100\ddot{Y}/D)\), velocity \((50\ddot{Y}/U_c)\), acceleration \((25D/U_c^2)/\dddot{Y}\) and force, \((25D/U_c^2)/F_L/m\) for (a) \(f^*_{n1} = 0.2, f^*_{n2} = 0.4\) (b) \(f^*_{n1} = f^*_{n2} = 0.2\).

It was observed that the dominant vibration frequency of the small cylinder is approximately the same as the vibration frequency of the large cylinder (see Fig. 3.6). This was even the case when collision does not occur and the difference in natural frequencies is large. For example, the smaller cylinder vibrates at the natural frequency of the larger cylinder in Fig. 3.6 with \(f^*_{n1} = 0.2\) and \(f^*_{n2} = 0.2\) and 0.24. When collision happens, such behaviour is expected. The question is when collision does not occur, why the small cylinder does not vibrate at its own natural frequency? To answer this question, the normalized time history of the lift force coefficient of cylinders, their velocity and the gap between them for
the case of \( f_{a1}^* = f_{a2}^* = 0.2 \) are plotted in Fig. 3.10. In this case the cylinders do not collide with each other. It can clearly be seen that the lift force on the small cylinder has a variable frequency and amplitude. Both the frequency and amplitude range of the lift force coefficient vary with the distance between the cylinders. The frequency and amplitude of the lift on the small cylinder are higher at small gap ratios than those at large gap ratios.

Fig. 3.10: Variations of the lift forces of the cylinders, their velocities and the gap between them with normalized time (Time). Both the amplitude range and the variation frequency of the lift force coefficient of the small cylinder vary with the gap between cylinders.

The high frequency, large amplitude lift force on the small cylinder at small gap ratios also result in large vibration velocities of the small cylinder. This suggests that vortex shedding is not the only cause for the vibration of the small cylinder. It appears that the vibration of the large cylinder dictates the vibration of the small cylinder. The same phenomenon was also observed previously Fredsøe et al. (1987). The instantaneous variation of the pressure between a cylinder and a plane wall was attributed to the cause of the variable lift force and higher (in positive range) amplitudes on the small cylinder (Fredsøe et al. (1987)). The effects of the small cylinder can also be observed on the large cylinder lift force during the proximity of the small cylinder.

Figure 3.11 shows the amplitude spectra of the small cylinder. In order to compare the vibration frequency of the small cylinder with that of the large cylinder, the non-dimensional vibration frequency of the small cylinder \( f_f^* = f_s D / U_o \) (where \( f_s \) is the dimensional vibration frequency of the small cylinder) is normalized based the large cylinder diameter. The leading frequencies in Fig. 3.11 are the same as their counterparts of the large cylinder shown in Fig. 3.8. The presence of the secondary harmonics is clear in Fig. 3.11.
Fig. 3.11: FFT analyses of the displacement time histories of the small cylinder. $f^*_S = f_SD/U_w$ is the normalized vibration frequency of the small cylinder (a) $f^*_n = 0.4$ (b) $f^*_n = 0.2$

The amplitude of the secondary harmonics appears to increase with the increase of the difference in the reduced velocities between the two cylinders. As discussed above, the cylinders do not collide with each other for $0.4 \leq (f^*_n - f^*_S) \leq 0.04$. The large difference of the reduced frequencies, for example $f^*_n \geq 0.08$, leads to repetitive collisions and bouncing backs of the cylinders. The collisions make the vibration of the smaller cylinder very different from the sinusoidal vibration, resulting in the second harmonic in the spectra.

Fig. 3.12 shows the FFT analyses result of the time histories of the gap between the two cylinders. The normalized variation frequency of the gap between cylinders in this figure is $f^*_G = f_GD/U_w$, where $f_G$ is the dimensional variation frequency of the measured gap between cylinders. One can notice the similarities between these spectra and those of the small cylinder, indicating that the gap between the two cylinders is induced mainly by the movement of the small cylinder.

The collision has significant influence on the movement of the small cylinder. This can be seen both in the velocity time histories of the small cylinder and gap between the two cylinders (see second and third columns of Fig. 3.6). Fig. 3.13 shows an example of the normalized time histories of non-dimensional displacement and velocity of both cylinders during collisions. The first two (or three) bouncing of the small cylinder can be identified by the zigzag shaped curve of the velocity curve of the small cylinder (before the circle in the figure, these also could be seen in the third column of Fig. 3.6). The presence of the large cylinder below the small cylinder does not allow it to move downward freely. Therefore, its downward motion occurs with continuous collision to the large cylinder as shown in the
zoomed-in view in Fig. 3.13. It can also be seen in Fig. 3.13 that the highest point of the large cylinder and the lowest point of the other follow the same path during the downward motion. In other words the small cylinder is literally stuck to the large cylinder during its downward motion.

![Fig. 3.12: FFT analyses of the gap time histories.](image)

Fig. 3.12: FFT analyses of the gap time histories. $f_G^* = f_G D / U_\infty$ is the normalized frequency of the gap variation between the cylinders (a) $f_{n2}^* = 0.4$ (b) $f_{n1}^* = 0.2$.

![Fig. 3.13: Velocity and displacement time histories of two cylinders during collision.](image)

Fig. 3.13: Velocity and displacement time histories of two cylinders during collision.

### 3.4.3 Vortex shedding:

The Helmholtz decomposition states that any vector field can be decomposed into an irrotational field (scalar potential) and a divergence free vector field (vector potential), Morino (1986). The change rate of the fluid flow momentum due to induced velocities by
these fields is equal to the hydrodynamic force applied on the structure. Lighthill (1986) divided the total hydrodynamic force on the structure into two parts, potential force which change linearly with ambient velocity and is caused by the scalar potential and the nonlinear part which depends on the vortex shedding and the convection of shed vortices. The nonlinear part (vortex force) is generated by the vector potential. Later, Jauvtis and Williamson (2004) showed that the energy transfer to an oscillating cylinder (with periodic motion) is mainly associated with vortex force. The total force applied on the structure is equal to the summation of the time rate of change of the momentum of the structure and the hydrodynamic force applied by the fluid on the structure. Due to “classical ambiguity in how to define the momentum of the flow”, the definition of impulse have been used for finding the force applied by structure on the fluid (which is the opposite of the force applied on the structure by fluid flow). The hydrodynamic force on the cylinder is equal to

\[-d(0.5\rho \int \mathbf{x} \times \omega \, dV) / dt\]

which is the negative of the time rate of change of the impulse. In the above mentioned equation, \(\mathbf{x}\) is the position vector, \(\omega\) is the additional vorticity (refer to Lighthill (1986)) and \(dV\) is the volume occupied by the vorticity element. According to the above explanation the vortex regime downstream of a structure can be used to investigate both the force applied on and the energy transferred to the structure. Furthermore vorticity contours explain how velocity and pressure distributions around the structure change.

The effects of reduced natural frequencies and the collision of cylinders on the structure of vortex shedding are discussed in this section. Three representative cases have been considered. In the first case, \(f_{n1}^* = f_{n2}^* = 0.2\), the large cylinder is in lock-in region. As it was found in the previous section the cylinders do not collide with each other in this case. In the second case, \(f_{n1}^* = 0.2\), \(f_{n2}^* = 0.4\), the large cylinder is in the lock-in region and the difference in reduced frequencies of the cylinders is the highest and the collision occurs. In the third case, \(f_{n1}^* = f_{n2}^* = 0.4\), both cylinders are out of lock-in regime. The vortex shedding of a single vibrating cylinder with \(f_n^* = 0.4\) (see Fig. 3.14) shows that the vortex shedding mode is 2S. Fig. 3.15 displays the instantaneous vorticity contours during one vibration cycle for the case \(f_{n1}^* = f_{n2}^* = 0.4\). The vortices that are shed from the small cylinder are engulfed by the clockwise vortices from the top of the large cylinder in most of the diagrams in Fig. 3.15. Due to the influence of the vortex street from the small cylinder, the negative vortices from the top of the large cylinder are in very irregular shape. The presence of the small cylinder at \(f_{n2}^* = 0.4\) contributes to a vortex pair comprising a weak clockwise vortex (yellow) and a counter clockwise vortex (light blue) below the wake centreline as shown in Fig. 3.15. The high amplitude of the large cylinder and the
interference of the anti-clockwise vortices from the small cylinder with clockwise rolled-up shear layer on the top side of the large cylinder makes the vortex shedding regimes in the wake of the two side-by-side cylinders different from that of an isolated cylinder \( f_{n1}^* = f_{n2}^* = 0.4 \). One can see in Fig. 3.15 (b) and (c) that the small cylinder is drawn toward the large one during the downward motion of the large cylinder. The minimum gap between the cylinders in this case is 0.046\( D \) (Fig. 3.7 (d)) and occurs at Fig. 3.15 (i). It can be seen that even at this minimum gap, the vortices are still shed from the small cylinder. This is because the duration of such a small gap is much smaller than the vortex shedding period from the small cylinder.

The number of vortices that are shed from the cylinders per cycle in the case with the maximum difference between the two reduced natural frequencies \( f_{n1}^* = 0.2, f_{n2}^* = 0.4 \) is different from that of the third case \( f_{n1}^* = f_{n2}^* = 0.2 \). The well-known 2P mode (two pair of vortices per cycle) found for an isolated cylinder does not occur in this case. However one repeatable flow cycle that comprises four cycles of the vibration of the large cylinder is observed as shown in Fig. 3.16. Fig. 3.17 shows the normalized time histories of the vibration of the cylinders. Instants correspond to the diagrams shown in Fig. 3.16 are marked in Fig. 3.17. It can be seen that each of the four successive vibration cycle of the large cylinder has different amplitude. Fig. 3.16 focuses on the instants when main vortices are disconnected completely from both cylinders. The numbers of clockwise and counterclockwise vortices shed in one shedding period (including four vibration cycles of the large cylinder) are 7 and 5 respectively, not including the shed vortices from the small cylinder. During the first oscillation of the large cylinder (Fig. 3.16 (a) to (d) and just before starting

Fig. 3.14: Instant vorticity contours for one cycle of vibration of a single cylinder. \( m^* = 5, f_{n}^* = 0.4 \) and Re=5000
Fig. 3.15: Instant vorticity contours for $f_1^* = f_2^* = 0.4$.

(e), check Fig. 3.17 (a)) three clockwise and one counter-clockwise vortices are shed from the cylinders which are stuck to each other. The $1_{cw}$ (cw: clock wise) and $1_{ccw}$ (ccw: counter clock wise) are shed during the upward motion of both cylinders in Fig. 3.16 (b). Two more clockwise vortices ($2_{cw}$ and $3_{cw}$) are separated from the topside of the small cylinder in Fig. 3.16 (e). The $2_{cw}$ and $3_{cw}$ are combined and move downstream together. The number of shed vortices in the second period (Fig. 3.16 (e) to (h)) is equal to two, i.e. one counter-clockwise ($2_{ccw}$) in Fig. 3.16 (e) and one clockwise ($4_{cw}$) in Fig. 3.16 (f) during the downward motion of the cylinders. There is one more separated clockwise vortex from the top side of the small cylinder in Fig. 3.16 (f), shown by $R_1$, which shrinks and combines to the attached clockwise vortex on the top side of the large cylinder in Fig. 3.16 (g). The third cycle (Fig. 3.16 (i) to (l)) includes two disconnected clockwise vortices ($5_{cw}$ and $6_{cw}$) in Fig. 3.16 (j) and one anti-clockwise shed vortex ($3_{ccw}$) in Fig. 3.16 (i). The vortices $5_{cw}$ and $5_{ccw}$ are combined with each other and are shed as one vortex ($5_{cw}$). The numbers of clockwise and anti-clockwise shed vortices in the last cycle are one and two,
respectively. The two anti-clockwise vortices (4ccw and 5ccw) are disconnected in Fig. 3.16 (l) during the upward motion of the large cylinder before it collides to small cylinder. The last clockwise vortex (7cw) is shed during the downward motion of the cylinders in Fig. 3.16 (n). While two clockwise vortices are separated instantly from the topside of the cylinders in Fig. 3.16 (m), one of them, shown in Fig. 3.16 (m) by R2, combines to the clockwise vortex attached to the cylinders. This reconnected vortex is shed later in Fig. 3.16 (q). Fig. 3.16 (q) has nearly the same structure as Fig. 3.16 (a). Comparison of the Fig. 3.16 (c), (f), (j) and (n) shows how the vortex shedding structures are different during the downward motion of the cylinders in each cycle. These differences contribute to different amplitude of vibration of the four mentioned cycles which repeat well then. It is noticed from both Fig. 3.17 and Fig. 3.6 (\(f'_{n1} = 0.2, f'_{n2} = 0.4\)) that the vibration time histories of both cylinders repeat well every four vibration cycles. The two cylinders are stuck to each other in each cycle when the large cylinder is in the top side of its mean position. When the large cylinder is at the bottom side of its mean position the small cylinder separate from the large one and clear vortex street can be seen in the wake of it. These vortices are merged into the clockwise vortices that are shed from the top of the cylinders system. Once the small cylinder is stuck to the large one, the vortex street behind it vanishes. The bouncing of the small cylinder and its proximity to the large cylinder during downward motion (most of it) has a prominent effect on the orientation and the strength of the vortices shed from the large cylinder.

The normalized gap between the cylinders, the lift coefficient force, the displacement and the normalized velocity of the large cylinder during four time interval mentioned by \(\alpha, \beta, \eta\) and \(\lambda\) in Fig. 3.17 are shown in Fig. 3.18. The arrows are to show the collision instants of the two cylinders. It can be observed that collision contributes to the increase of the lift force on the large cylinder. Among the four mentioned normalized intervals the highest amplitude occurs following the time interval \(\eta\) in Fig. 3.18 (c). Both lift force coefficient and the velocity of the large cylinder are positive during the time interval “\(\eta\)” which means the net positive transfer of energy from fluid to the large cylinder at this time interval. The velocity of the large cylinder is positive during other time intervals however the lift force coefficient is negative.

Fig. 3.19 shows the instantaneous vorticity contours and streamlines in one cycle of vibration for the case \(f'_{n1} = f'_{n2} = 0.2\). The number of vortices that are shed from the large cylinder is two in each cycle of vibration, one clockwise and one counter-clockwise. The high amplitude of displacement results in elongated clockwise shed vortices. After being shed from the large cylinder, the negative vortex from the top of the large cylinder is cut into
Fig. 3.16: Successive instantaneous vorticity contours within four vibration cycles, $f_{n1}^* = 0.2$, $f_{n2}^* = 0.4$

Fig. 3.17: Vibration time histories when $f_{n1}^* = 0.2$, $f_{n2}^* = 0.4$ (a) large cylinder (b) small cylinder

two parts in Fig. 3.19 (e) and (f), one of them joins the top row of the vortices and another one joins the bottom row. Because of the division of the negative vortices, 2P+S street is
observed behind the cylinder system. The positive vortex from the bottom of the large cylinder remains integrated after it is shed from the cylinder. However, the negative vortices in the bottom row dissipate very quickly and only one pair of vortices is seen after they dissipate. The vortex shedding behind the small cylinder always exists and the small vortices from the small cylinder are merged into the negative vortex from the large cylinder once the two cylinders become very close to each other (Fig. 3.19 (g) and (h)). The wide and distorted shear layer on the top side of the two-cylinder system forces the shedding of the counter-clockwise vortex at the bottom side of the large cylinder to occur farther from the large cylinder compared to the shed vortex from an isolated cylinder.

![Diagram](image_url)

**Fig. 3.18: Comparison of the lift force coefficient of the large cylinder, its cross flow normalized velocity, the gap between cylinders and the normalized amplitude of the large cylinder at four different time interval shown by \( \alpha, \beta, \eta \) and \( \lambda \) in Fig. 3.17. Both the lift force coefficient and velocity during the time interval \( \eta \) are positive which means the net positive transfer of energy from fluid to the large cylinder.**
3.5 **Conclusions**

Cross flow vibrations of two side-by-side cylinders are modelled numerically. The diameter of the small cylinder was fixed at $d=0.1D$. The Reynolds based on the large cylinder is fixed at 5000. The reduced natural frequencies of both cylinders (based on the large cylinder diameter) vary from 0.2 to 0.4 with an increment of 0.04, corresponding to reduced velocities varying between 5 and 2.5. The effects of the small cylinder on the vibration amplitude and vortex shedding mode of the large cylinder were investigated. The results can be summarized as follows.

1- The small cylinder has prominent effects on the vortex shedding regime of the large cylinder. Presence of the small cylinder at the side of a large cylinder not only increases the amplitude of the large cylinder but also widen the lock-in regime.
2- The collision of the two cylinders does not occur when the frequency difference is small \( 0 \leq (f_{n2}^* - f_{n1}^*) \leq 0.04 \). For greater reduced frequency differences \( (f_{n2}^* - f_{n1}^*) \geq 0.08 \) the cylinders collide with each other periodically. If the collision happens, the small cylinder will be stuck to the large cylinder every time when the large cylinder is moving downwards above its neutral position.

3- For the range of reduced natural frequency that was studied, the small cylinder locks hydrodynamically on to the large cylinder. Its dominant vibration frequency is the same as that of the large cylinder.

4- Vortex shedding from the small cylinder occurs throughout a whole vibration cycle if collision does not occur. If the collision occurs, during the time period when the small is stuck to the large cylinder, the vortex shedding from the small cylinder is suppressed because it is immersed in the shear layer of the large cylinder.

It should be noted the problem investigated in the present study is expected to be three-dimensional. Although the present study employs a two-dimensional model, it is able to reveal the major features of the cylinders’ interaction. Previous studies using two-dimensional models have been demonstrated that two-dimensional models are able to predict VIV and the vortex shedding modes at relatively large Reynolds numbers in the order of \( 10^4 \) (e.g. Anagnostopoulos (1994), Cox et al. (1998), Guilmineau and Queutey (2004), Mittal and Kumar (2004), Wanderley et al. (2008)). To fully understand the effect of three dimensionality of the flow on the responses of the cylinders, a detailed study using a three-dimensional model is recommended.

### 3.6 References


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CHAPTER 4

LOCK-IN STUDY OF TWO SIDE-BY-SIDE CYLINDERS OF DIFFERENT DIAMETERS IN CLOSE PROXIMITY IN STEADY FLOW

ABSTRACT

Lock-in of the vortex-induced vibration of two side-by-side circular cylinders of different diameters (diameter ratio $d/D = 0.1$) was investigated numerically. The cylinders are located in close proximity and free to oscillate in the cross-flow direction. The initial gap between the two cylinders is set the same as the small cylinder diameter ($d$). The mass ratios of both cylinders ($m^*$) are fixed to be 5 and the damping ratios are small enough to be considered negligible. Simulations are firstly carried out for two cases where the large-to-small-cylinder natural frequency ratio is 1. Case 1 is focused on the lock-in of the large cylinder and Case 2 is focused on the lock-in of the small cylinder, which is far narrower than that of the large cylinder. Then simulations are carried out at a natural frequency ratio (small-to-large-cylinder) of 0.1, where both cylinders are expected to lock on to their own natural frequencies (referred to be Case 3). The interference between the two cylinders under these conditions is investigated in detail. The widening of the lock-in range of the reduced velocities for the large cylinder in Case 1, the beating behaviour of the small cylinder in its lock-in range in Case 2 and the dual lock-in behaviour of the small cylinder during the simultaneous lock-in of both cylinders in Case 3 are some of the key findings of this study.

4.1 Introduction

Vortex-induced vibrations (VIVs) occur in many aerodynamics and hydrodynamics applications. Structures such as tension leg platforms, drilling risers, catenary and submarine pipelines, transmission lines, chimneys and bridges experience vibrations induced by fluid flow passing across them. Excessive vibrations weaken the durability and shorten the lives of the structures. The qualitative and quantitative estimations of these vibrations are important for the evaluation of structural fatigue failure.

VIV of a cylinder in steady flow is governed by a number of important parameters including Reynolds number $\text{Re} = \frac{U_\infty D}{\nu}$, mass ratio $m^* = \frac{m}{m_d}$, structural damping factor of the cylinder, $\zeta = \frac{\delta}{2\pi}$ and reduced velocity, $U_r = \frac{U_\infty}{f_n D}$ where $U_\infty$ is the free stream velocity, $D$ the diameter of the cylinder, $\nu$ the kinematic viscosity of the fluid,
The mass of cylinder, $m_d$ the mass of displaced fluid by the cylinder, $\delta$ the natural logarithmic decrement of the oscillating cylinder and $f_n$ the natural frequency of the system. For a stationary circular cylinder the non-dimensional vortex shedding frequency ($f$) is defined as the Strouhal number (St) by $St=fD/U_\infty$, with $f$ being the vortex shedding frequency. The consensus value of the Strouhal number is about 0.2 for a circular cylinder in the subcritical Reynolds number regime (Williamson, 1996). The vortex shedding frequency of an elastically mounted circular cylinder follows the Strouhal law outside the lock-in (resonance) regime. However when the reduced velocity is within the lock-in regime, the vortex shedding frequency locks onto the natural frequency instead of following the Strouhal law. It has been found that the lock-in occurs at a wide range of the reduced velocity and results in very high vibration amplitude, Feng (1968).

Studies of the lock-in phenomena at high mass ratios (Feng, 1968; Brika and Laneville, 1993) and low mass ratios (Khalak and Williamson, 1996, 1997, 1999) revealed the dependence of the dynamic response on the mass-damping ratio ($m^*\zeta$). The mass-damping ratio in the test of Khalak and Williamson (1996) was one order of magnitude less than that in the test by Feng (1968). Low mass-damping ratios result in wide lock-in regimes and high maximum oscillation amplitudes of the cylinder. Jauvtis and Williamson (2004) showed that allowing the cylinder to oscillate in 2-DOF also contributes to high amplitude of vibration provided that the mass ratio is less than 6. (Zhao and Cheng, 2010, 2011) compared numerical results with the experimental data by Jauvtis and Williamson (2004) and found that the numerical results agreed well with the experimental measurements. Transverse oscillations of an elastically mounted rigid cylinder in steady flow have been discussed by many researchers. Bearman (1984, 2011), Williamson and Govardhan (2004), Sarpkaya (2004) and Gabbai and Benaroya (2005) gave extensive reviews on the VIV of a circular cylinder.

Flow interference between two parallel cylinders occurs in many engineering problems and has been the subject of many studies. The arrangement of two parallel cylinders can be side-by-side, tandem, or staggered relative to the incoming fluid flow. Sumner (2010) reviewed extensively the available literature on fluid flow around two cylinders of equal diameters. According to (Zdravkovich, 1977, 1985), the interference between two cylinders occurs at three different patterns depending on the arrangement of the cylinders. Proximity interference occurs when the two cylinders are close to each other and placed side by side. Wake interference takes place if one of the cylinders is fully or partially submerged in the wake of another and the flow around it is significantly affected by the wake of the upstream cylinder. The third flow pattern is the overlapping of the proximity interference and the
wake interference. The flow interferences can be bi-stable in some regions and cause high amplitude vibrations. The mechanisms of resultant oscillating force coefficient by these interferences, jet-switch and gap-flow-switch were also discussed by (Zdravkovich, 1977, 1985).

Zdravkovich (1988) and Sumner et al. (1999) studied flow around two side-by-side cylinders at different gap ratios. It was found two side-by-side cylinders in contact with each other behave as a single bluff body and a single vortex street was found in the wake, except two vortex shedding frequencies were found due to the disintegration of large vortex structures of irregular shapes into small vortices. The PIV tests conducted by Sumner et al. (1999) in a water tunnel showed the variation of the vortex formation length. At small gap ratios \( G/D < 0.2 \) between two side-by-side cylinders, the single bluff body behavior changed slightly. A small gap between the two cylinders leads to a lower reduction of drag forces on cylinders and more extended vortex formations behind the cylinders. King and Johns (1976), Brika and Laneville (1999), Mittal and Kumar (2001), Assi et al. (2006), (Alam et al., 2005, Mahbub Alam and Kim, 2009) are a few among many studies discussing the VIV of two identical circular cylinders in tandem and staggered arrangement. In an experimental study of VIV of two tandem cylinders, Assi et al. (2006) observed galloping and continuous increase of the vibration amplitude with the reduced velocity at \( m^* = 1 \) and 2 with \( 3.0 < S/D < 5.6 \), where S is the pitch distance between the two cylinders. Brika and Laneville (1999) found that the dominant vibration mechanism of both cylinders at large pitch distances and high mass ratios is VIV. An experimental and numerical study by Dalton et al. (2001) on two cylinders with different diameters showed that both drag and lift forces on the large cylinder were reduced by purposely placing the small diameter cylinder in its wake. The small cylinder was found to weaken the vortex shedding from the large cylinder. Lee et al. (2012) studied the interaction and the wake structure behind two side-by-side cylinders with a diameter ratio of \( D/d = 2 \) and a gap ratio of 0.75D. Their experiments revealed the presence of two dominant lock-in frequencies in the wake region, namely the excitation frequency and its one third sub-harmonics.

While the aforementioned studies on two cylinder wakes are mainly concerned on the force characteristics and flow regime classifications, the study on VIV of the two side-by-side cylinders with different diameters is limited. This paper presents a study on the interference of two cylinders undergoing VIV. The cross flow vibrations of two elastically mounted side-by-side circular cylinders of different diameters in a steady current are the focus of this study. Individual and mutual lock-ins and potential collisions of the two cylinders are investigated.
Implementation of three-dimensional models to investigate the flow interaction between cylinders in the present study is expected to be more appropriate choice of the simulation. However the computational cost of three-dimensional study of the present research is restrictively high to be conducted for the considered cases. The use of a two-dimensional numerical model in this study is actually a compromise between computational efficiency and numerical accuracy and efficiency. The efficiency of two-dimensional models enables systematic study of a wide range of parameters at affordable computational costs. They have been applied successfully to simulate various VIV problems over a wide range of Reynolds numbers e.g. Cox et al. (1998), Guilmineau and Queutey (2004), Mittal and Kumar(2004), Wanderley et al. (2008) and Ding et al. (2013).

4.2 Problem description

The schematic diagram of the two side-by-side circular cylinders free to oscillate in the cross flow direction is shown in Fig. 4.1. The ratio of the small cylinder diameter to the large one \((d/D)\) is fixed at 0.1. The initial gap between the cylinders is equal to the small cylinder diameter \((G=d)\). The incoming flow is steady and is in the horizontal direction as shown in the Fig. 4.1. The mass ratios of both cylinders are identical with \(m_1^* = m_2^* = 5\). The damping ratios of both cylinders are fixed at 0.000863. The large cylinder diameter \((D)\) is used to define the Reynolds number. The reduced velocities of the large cylinder and small cylinder are \(U_r = U_\infty / f_{nl} D\) and \(U_{rs} = U_\infty / f_{ns} D\) where \(f_{nl}\) and \(f_{ns}\) are the structural natural frequencies of the large and the small cylinders, respectively.

![Schematic diagram of the flow past two side-by-side cylinders.](image)

**Fig. 4.1:** Schematic diagram of the flow past two side-by-side cylinders.

Lock-in of the two cylinders is studied in three different cases. In Case 1, the large-to-small cylinder natural frequency ratio is kept to be 1.0 and the reduced velocity of the large cylinder is varied from 1 to 15 by increasing the fluid flow velocity with the corresponding
Reynolds number in the range of \(10^3 \leq \text{Re} \leq 15 \times 10^3\). In Case 2, the natural frequency ratio is also fixed at 1.0 and the small cylinder’s reduced velocity varies between 1 and 15. In Case 3, lock-in of both cylinders is studied by setting the natural frequency ratio at \(f_{nl}/f_{ns}=0.1\). The reduced velocities based on both cylinders are the same and they vary between 1 and 15 with an interval of 0.5.

4.3 Governing equations and numerical methods

The Reynolds-averaged Navier-Stokes (RANS) equations and the Shear Stress Transport SST \(k-\omega\) turbulence model developed by Menter (1994) are the governing equations to simulate the fluid flow in the present study. An Arbitrary Lagrangian Eulerian (ALE) scheme is applied to deal with the moving boundaries in this study. The two-dimensional RANS equations in the ALE scheme are expressed as

\[
\frac{\partial U_i}{\partial t} = 0
\]

\[\tag{4.1}
\]

\[
\frac{\partial U_i}{\partial t} + (U_j - \hat{u}_j) \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (2\nu S_{ij} - u'_i u'_j)
\]

\[\tag{4.2}
\]

where \(x_i (i=1,2)\) is the Cartesian coordinates in the in-line and cross-flow directions of the flow respectively, \(\rho\) is the fluid density, \(\nu\) is the fluid kinematic viscosity, \(\hat{u}_j\) is the mesh velocity, \(U_i\) is the mean flow velocity in the \(x_i\)-direction, \(P\) is the pressure and \(t\) is the time. \(S_{ij}\) is the mean strain tensor defined as \(S_{ij} = \frac{(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}{2}\). The averaged value of the product of fluctuating velocities \(\overline{u'_i u'_j}\) is calculated using the Boussinesq approximation, i.e. \(\overline{u'_i u'_j} = 2\nu S_{ij} -(2/3)k\delta_{ij}\), where \(\nu, k\) and \(\delta_{ij}\) are the eddy kinematic viscosity, turbulent kinetic energy and the Kronecker delta, respectively. The SST \(k-\omega\) turbulence model by Menter (1994) is used to close the RANS equations. The governing equations are discretised using the modified streamline Petrov-Galerkin method developed by Zhao et al. (2009). A rectangular computational domain is employed in this study. The domain in the flow direction is \(70D\) and the cylinder is placed \(30D\) downstream from the inlet boundary. The width of the domain in the cross-flow direction is \(40D\) with the large cylinder at its midpoint. No-slip boundary condition is implemented on the cylinder surfaces. The specific dissipation of the turbulent kinetic energy (\(\omega\)) and the turbulent kinetic energy (\(k\)) are \(1s^{-1}\) and \(0.001 \rho U_z^2\), respectively at the inlet boundary. The gradient of turbulent quantities and velocity at the outlet boundary are set to be zero. Zero gradients of horizontal velocity, pressure and turbulent quantities are considered at the two side boundaries.
The motion of each cylinder is governed by the following equation:

$$\ddot{Y}_k + 4\pi^2 \frac{f_{nk}^2}{m_k} Y_k + 4\pi^2 f_{nk}^2 Y_k = F_{lk} / m_k$$  \hspace{1cm} \text{Eq (4.3)}

where $\ddot{Y}_k$, $\dot{Y}_k$, $Y_k$ are the acceleration, velocity and displacement of the $k$-th cylinder in the $y$-direction respectively, $m_k$ and $f_{nk}$ are the mass and natural frequency of $k$-th cylinder, $F_{lk}$ is the total lift force on the $k$-th cylinder. The fourth-order Runge-Kutta algorithm is used to solve Eq (4.3).

The mesh movement technique considered in this study is based on a modified Laplace equation:

$$\nabla \cdot (\gamma \nabla V_i) = 0$$  \hspace{1cm} \text{Eq (4.4)}

where $V_i (i=1, 2)$ is the displacement of a nodal point in the $x_i$-direction. The displacement of the nodal points on the surface of each cylinder is the same as the displacement of the cylinder. On other boundaries the displacements of the nodal points are zero. The coefficient $\gamma$ controls the distribution of the displacements of the finite element nodes. Numerical experiments showed that $\gamma = 1/A$ with $A$ being the area of the element leads to satisfactory results (Zhao and Cheng, 2008).

The mesh quality in the gap between the two cylinders cannot be guaranteed by using Eq (4.4) when the gap between the two cylinders is either extremely small or too large. Excessive distortions of the finite element mesh can lead to breakdowns of simulations. This problem is overcome by a specifically-designed re-meshing strategy in this study. Equation (4.4) is only used for calculating the displacements of the mesh outside a circle of $(x/D)^2 + (y/D)^2 = \alpha^2$ ($\alpha = 3D$ is selected in this study according to the gap distance between cylinders). The coordinates of the nodal points inside this circle are calculated using an interpolation scheme based on two predefined meshes to deal with extreme conditions where the gap between two cylinders is either extremely small or two large. Fig. 4.2 shows the two predefined meshes, together with the initial mesh. The three meshes are of the same topographical structure but of different densities within the gap. A total of 32,000 structured elements are used to discretise the fluid flow domain. The node number along the circumferences of the large and the small cylinders are 280 and 100 respectively. The aspect ratios of the elements adjacent to the cylinder surfaces vary between 3 and 10 for the large cylinder and between 2 and 13 for the small one. The two cylinders are very close to each other in mesh (b) and far away from each other in mesh (c). The coordinates of the mesh
with a gap ($G$) are determined through linear geometric interpolations based on the two predefined meshes.

**Fig. 4.2:** The implemented structured mesh for simulations. (a) initial mesh, $G/D=0.1$ (b) cylinder are in contact with each other (c) cylinders are far away from each other. In all these meshes the node numbers and element numbers are the same.

To avoid extreme mesh distortions (i.e. mesh size in circumferential direction is excessively greater than that in the radius direction), the gap ratio of $G/D=0.002$ is considered to be the minimum gap ratio that is practically possible (Zhao and Cheng, 2010, 2011). When the gap between the two cylinders reaches $G/D=0.002$, the two cylinders are considered to collide with each other in the numerical model. After a collision, the cylinders are bounced back with their new velocities calculated according to the conservation law of the momentum. Zero damping is assumed for the collision of the two cylinders in this study (i.e. no loss of energy).

The present numerical model has been validated in previous studies by (Zhao and Cheng, 2010, 2011; Rahmanian et al., 2012). It was shown that the numerical results of
two-degree freedom (2-DOF) VIV of a circular cylinder obtained using the present numerical model agree well with experimental results reported by Jauvtis and Williamson (2004), Yang et al. (2009) and Govardhan and Williamson (2000). A mesh dependence study for the range of reduced velocity used in this study was also reported previously by Rahmanian et al. (2011). Therefore no further validation on the numerical model and mesh dependence is carried out in this study in light of the previous studies.

4.4 Results and discussions

4.4.1 Case 1: Large cylinder lock-in

Responses of the two cylinders of different natural frequencies are investigated through a series of numerical tests. For the convenience of discussions, some key parameters are defined. The natural frequency ratio is defined as $R_{fn} = f_{nl} / f_m$. Three different cases are considered in this study. In Case 1, simulations are conducted at a natural frequency ratio of $R_{fn} = 1$ and the reduced velocities $U_r$ ranging from 1 to 15 with an increment of 0.5. The lock-in of the larger cylinder is the focus of the Case 1 study. Case 2 is focused on the lock-in regime of the small cylinder. In order to identify the lock-in regime for the small cylinder, the reduced velocities of the large cylinder is varied between 0.1 and 1.5 with an increment of 0.05. The corresponding $U_r$ ranges from 1 to 15, which covers the lock-in regime of the small cylinder. In Case 3, the natural frequency ratio is $R_{fn} = 0.1$ so that the reduced velocities for both cylinders are the same. Both cylinders may lock-in due to the same reduced velocities in Case 3. In the following discussion, the responses of the cylinders in each of the three cases are discussed separately. In each case, the fluid velocity is increased gradually from 1 to 15 with the corresponding Reynolds number in the range of $10^3 \leq \text{Re} \leq 15 \times 10^3$. As the velocity increases slowly to each desired reduced velocity level, it is kept constant to allow the VIV reaches equilibrium. Then the response of the cylinders is recorded before the velocity is increased to a higher level.

4.4.1.1 Vibration amplitude

Variations of response amplitudes with the reduced velocity in Case 1 are shown in Fig. 4.3. It is observed that the variation of response amplitude of the small cylinder follows that of the large cylinder over the range of reduced velocities investigated. Both cylinders experience high vibration amplitudes in the reduced velocity range of $2.5 \leq U_r \leq 10$. The highest vibration amplitudes of the large and small cylinders occur at $U_r = 7.5$ and 4.5 with
the values of 0.66D and 0.38D respectively. The corresponding lock-in regime and the maximum vibration amplitude of an isolated large cylinder (based on the present numerical method) are $3.5 \leq U_r \leq 9.5$ and 0.63D, respectively. The existence of the small cylinder does not appear to have affected the lock-in regime and the response amplitude of the large cylinder significantly. The existence of the large cylinder, however, does affect the response amplitude of the small cylinder over the lock-in regime of the large cylinder. If there were not the large cylinder, the lock-in regime of the small cylinder would have been much narrower (0.35 $\leq U_r \leq$ 0.95 based on the large cylinder diameter) than the large cylinder lock-in regime ($3.5 \leq U_r \leq 9.5$) and the response amplitudes would have been smaller than those observed in Fig. 4.3 over the lock-in regime of the large cylinder. When the vortex shedding from the large cylinder locks into its natural frequency, the small cylinder passively oscillates with the large cylinder at very high amplitudes that are a few times of its own diameter. The resultant gaps between the two cylinders at different reduced velocities are shown in Fig. 4.4. It can be seen that before the lock-in starts $U_r \leq 2$ the gap remains

Fig. 4.3: Variation of maximum transverse displacement of both cylinders with reduced velocity (based on the large cylinder). The large cylinder experiences the lock-in.

Fig. 4.4: Variation of the maximum and the minimum induced gap between the two cylinders in Case (1) with the reduced velocity of the large cylinder.
almost constant and is close to its initial value \( G = 0.1D \) because the amplitudes of both cylinders are negligibly small. In the lock-in regime of the large cylinder the maximum gap between the two cylinders increases with the reduced velocity until \( U_r = 9 \), although the response amplitude of both cylinders does not vary much with the reduced velocity in the lock-in regime. It is also observed that the vibration of the small cylinder lags that of the large cylinder. This is likely because the small cylinder only follows the large cylinder passively. When the large cylinder moves towards the small cylinder, the slow reaction of the small cylinder makes the gap small. Inversely, the lag of the small cylinder’s reaction results in large gaps as the large cylinder is moving downwards. The phase differences between the responses of the two cylinders are shown in Fig 4.5. The phase difference varies from 10° to about 70° as the reduced velocity increases from 3.0 to 9.5. Since the vibration frequencies of the two cylinders are same, the maximum gap between the two cylinders increases with the phase difference. In the lock-in regime, the minimum value of the gap occurs at \( U_r = 3 \), where the phase-difference is the smallest and the maximum at \( U_r = 9 \), where the phase-difference is the largest. Because the minimum gap over the range of reduced velocities investigated never falls below the prescribed minimum gap \( 0.002D \), the collision between the two cylinders does not occur under the conditions investigated in Case 1. Outside the lock-in region \( U_r > 10 \) the cylinders vibration amplitudes reduce to a value close to zero. The gap between the two cylinders increases with the increasing reduced velocity outside the lock-in regime because of the increase in the repulsive force between the cylinders. The small cylinder has shifted significantly upwards relative to its neutral position at the highest calculated reduced velocity. The increased velocity pushes the small cylinder to higher positions due to the effect of the proximity of the large cylinder. The maximum gap is about \( 1D \), which occurs at the maximum calculated reduced velocities \( U_r = 14.5 \) and 15) in this study.

![Fig 4.5: Phase difference between the oscillation of the small cylinder and large cylinder at different reduced velocities.](image)
4.4.1.2 Vibration history

Fig. 4.6 shows the time histories of the displacement of both cylinders at $U_r = 4.5, 7.5, 8, 10.5$ and $12$. The large cylinder vibrates mainly about its neutral position with a small shift downwards. However the mean position of the small cylinder is moved upwards significantly. At $U_r = 4.5$ the amplitude of the small cylinder is about $0.2D$ and its maximum displacement is very close to that of the large cylinder. With the increase in the reduced velocity, the mean position of the small cylinder increases and its amplitude decreases due to the weakened influence from the large cylinder. It can be seen that at $U_r = 10.5$ the mean displacement of the small cylinder is above $0.6D$. The vibration time histories of both cylinders at $U_r = 10.5$ are significantly different from those at other reduced velocities. High frequency component that appears in the oscillations of the small cylinder at $U_r = 10.5$ shows the effect of the vortex shedding from the small cylinder on the vibration. The high frequency component does not appear at lower reduced velocities in the lock-in regime of the large cylinder. The effects of vortex shedding from the small cylinder on the response of the small cylinder can be observed clearly from its displacement time histories at $U_r > 10.5$. Fig. 4.6 (e) and (f) show that the response of the small cylinder at reduced velocities higher than 10.5 is the superposition of the vibration induced by the vortex shedding from the large cylinder and the vibration induced by vortex shedding from small cylinder.

4.4.1.3 Frequency analysis

The response frequencies of both cylinders are examined by the Fast Fourier Transform (FFT) analysis. The power spectrum contours of both cylinders are shown in Fig. 4.7. The normalized frequencies of both cylinders in Fig. 4.7 are consistently defined as the ratio of the cylinder vibration frequency ($f_v$) to the natural frequency of the large cylinder ($f_{nr}$) to facilitate direct comparisons between the response frequencies of the large and the small cylinders. It is observed from Fig. 4.7 that the natural frequency of the large cylinder dominates in the lock-in regime and the frequency follows the Strouhal law outside the lock-in regime. The similarity between Fig. 4.7 (a) and (b) shows that the small cylinder passively follows the vibration of the large one. The lock-in regime is located in the range of $2.5 \leq U_r \leq 10$ where the normalized frequencies are close to 1. The peak frequencies outside the lock-in regime fall into the straight line of a slope of about 0.21, which is the Strouhal number in the subcritical flow regime. It is obvious that multiple peak frequencies
are present in FFT spectrum of the small cylinder. This is because both the vibration of the small cylinder and the incoming ambient flow conditions for the small cylinder are affected by the vibration and vortex shedding from the large cylinder. The effect of vortex shedding from the small cylinder on its vibration is very weak both in and outside the lock-in regime due to the strong influence from the larger cylinder. This is further supported by the observation that the vibration frequency of the small cylinder follows the Strouhal law of the large cylinder outside the lock-in regime.

4.4.1.4 Vortex shedding

Fig. 4.8 shows the pressure distribution around the large cylinder and the vorticity contours around both cylinders at different instants within one vibration cycle at $U_r = 5$. Two large vortices are shed during each vibration cycle of the large cylinder, the clockwise vortex at instant (b) and the anticlockwise one at instant (f). The presence of the small cylinder results in an elongated clockwise vortex which contaminates a wide area downstream of the cylinders. The shed vortices from the small cylinder weaken the anticlockwise vortex and delay the vortex shedding from the top of the large cylinder. Fig. 4.9 shows the lift coefficient on both cylinders, the gap between them and the displacement of the large cylinder in the same vibration cycle (both of the lift coefficients are normalized using large cylinder diameter). Each instant in Fig. 4.8 is marked in Fig. 4.9. In order to understand how the proximity of the cylinders affects the pressure distribution on the small cylinder, pressure distributions around the small cylinder at the same instants as those shown in Fig. 4.8 are shown in Fig. 4.10. An outward vector stands for negative pressure and an inward vector denotes positive pressure. The stagnation point on the large cylinder is defined as the location with highest positive pressure that splits clockwise and counter clockwise vortices on the upstream side of the cylinder. During upward motion of the large cylinder, the stagnation point moves in clockwise direction towards the top of the large cylinder. The pressure vectors on the large cylinder in Fig. 4.8 (d) to (h) show clearly higher suction pressure on the top side than on the bottom side of the large cylinder. When the cylinder moves downward, the stagnation point moves towards the bottom side and the high suction pressure zone is located at the bottom side of the large cylinder (Fig. 4.8 (a), (b) and (i) to (l)). The shifting of the stagnation point is believed to be induced by the relative direction of the incoming flow velocity to the cylinder changes as the large cylinder undergoes the vertical vibration. The proximity of the small cylinder to the large cylinder appears to affect the local pressure distributions on the large cylinder as shown in Fig. 4.8 (g) and Fig. 4.8 (h). These fluctuations also can be observed at instants (g), (h) and (i) in Fig. 4.9. The lowest magnitude of the negative pressure on the top side of the large cylinder occurs during its
downward motion when it is at a higher position than its neutral position (when the gap between cylinders is increasing) as shown in Fig. 4.8 (a), (i), (j) and (k).

Fig. 4.6: Displacement time histories of both cylinders at different reduced velocity of the large cylinder in Case (1).
Fig. 4.7: FFT analyses of the displacement time histories in Case (1). The results are shown as the amplitude contours for different normalized frequencies at different reduced velocities of the large cylinder. (a) results for the large cylinder (b) results for the small cylinder.

The corresponding lift coefficient curve (Fig. 4.9) shows that the lift force coefficient of the large cylinder at these instants are relatively small compared to other instants. The stagnation point of the small cylinder appears to be always located at its lower upstream side in Fig. 4.10. The direction of the flow relative to the small cylinder biases towards upwards as the result of the influence of the large cylinder, resulting in a mean positive lift force on the small cylinder. It can be observed that the upward motion of the large cylinder results in higher pressure variation around the small cylinder (Fig. 4.10 (f), (g) and (h)).

Further inspection of Fig. 4.9 reveals that both the periodicity and the magnitude of lift force coefficient of the small cylinder depend strongly on the gap between cylinders. This explains why the small cylinder follows the large cylinder oscillations in Case 1. The maximum lift coefficient of the small cylinder occurs during upward motion of the large cylinder when the gap between the cylinders reaches small values at instants (f) and (g) in Fig. 4.9. The small cylinder lift coefficients experiences fluctuating upward lift force during the time that the gap between cylinders is small. The variation of the frequency of lift force coefficient of the small cylinder also can be observed in Fig. 4.9. A high frequency component is observed when the gap between the cylinders is small. Similar phenomena have been observed before by different researchers. Bearman and Wadcock (1973) reported high amplitude pressure fluctuations in the narrow gap between two equal diameter cylinders. Fredsøe et al. (1987) reported variation in the vibration frequency of an isolated cylinder near a plane wall for variable gap ratios for an elastically mounted cylinder close to the plane wall.

### 4.4.2 Case 2: Small cylinder lock-in
The simulations carried out in Case 2 aim to investigate the lock-in of the small cylinder. The natural frequency ratio is also fixed at 1 in Case 2. The lock-in regime for the small cylinder is found to be in the range of $2.5 < U_r < 6.5$, corresponding to the reduced velocity of the large cylinder in the range of $0.25 < U_r < 0.65$.

Fig. 4.8: Instantaneous vorticity contours of both cylinders and pressure distribution around the large cylinder for one cycle of vibration of the large cylinder for $U_r = 5$. The large cylinder is in lock-in range, Case (1). The instants mentioned inside the cylinders are labelled in Fig. 4.9.
Fig. 4.9: Induced lift force time histories of cylinders, the large cylinder vibration and the induced gap between cylinders. The instants shown in Fig. 4.8 are marked in this figure for both cylinders. Both lift coefficients are normalized by large cylinder diameter.

Fig. 4.10: Pressure distribution around the small cylinder at corresponding instants shown in Fig. 4.8 and Fig. 4.9
Since the simulations carried out in Case 1 were not able to identify the lock-in regime of the small cylinder, the simulations in Case 2 are conducted at a very small range of the reduced velocity of the large cylinder of $0.1 \leq U_r \leq 1.5$ with an interval of 0.05, corresponding to $1 \leq U_{rs} \leq 15$. This reduced velocity range is expected to cover adequately the lock-in regime of the small cylinder.

4.4.2.1 Amplitude response

The response amplitudes of both cylinders in Case 2 are shown in Fig. 4.11. The VIV of the small cylinder in its lock-in regime has little influence on the VIV of the large cylinder. The lock-in of the small cylinder occurs in the reduced velocity regime of the large cylinder between $U_r = 0.25$ and 0.65. The highest amplitude of the small cylinder is about $0.7d$ ($0.07D$) which occurs at $U_r = 0.4$, i.e., $U_{rs} = 4$. Outside the lock-in region ($U_r > 0.65$) of the small cylinder, the amplitude of the small cylinder decreases to a very small value of $0.1d$. In the considered range of reduced velocity the oscillation amplitude of the large cylinder increases gradually with the increasing reduced velocity as $U_r > 0.5$. The large cylinder oscillation experiences a local maximum at $U_r = 0.4$, which is clearly due to the influence of the lock-in of the small cylinder. The VIV amplitudes of the cylinders are in similar magnitudes at the highest simulated reduced velocity of $U_r = 1.5$. Collision of the cylinders did not occur in the investigated range of reduced velocity in Case 2.

4.4.2.2 Vibration history

Displacement time histories at five different $U_r$ of 0.25, 0.4, 0.55, 0.6 and 0.8 are shown in Fig. 4.12. Beating responses of the small cylinder observed in Fig. 4.12 (c) and (d) are distinctly different from the response of a single cylinder. The local maximum response amplitude of the large cylinder at $U_r = 0.4$ in Fig. 4.11 can be explained by observing Fig. 4.12 (b). Unlike those in Fig. 4.12 (c) and (d), the vibration frequency of the large cylinder in Fig. 4.12 (b) does not follow the Strouhal law and it is the same as that of the small cylinder. The large-amplitude oscillation of the small cylinder at this reduced velocity can be the reason for these high frequency vibrations of the large cylinder. According to Fredsøe et al. (1987) high frequency oscillation of the small cylinder contributes to instantaneous flow rate change between the two cylinders. These sudden changes in fluid flow between the two cylinders cause pressure fluctuations which can induce small amplitude oscillations whose frequency is very close to the natural frequency of the large cylinder (frequency ratio is 1.0 in Case 2). The proximity of these two frequencies (of
fluctuating pressure induced by small cylinder oscillation and the natural frequency of the large cylinder) is responsible for the modulation of the large cylinder at $U_r = 0.4$.

Amplitude modulation can also be seen in the vibration time histories of the small cylinder. The most obvious beating can be observed in the vibration history of the small cylinder at the reduced velocities of 0.55 and 0.6. These reduced velocities are in the lock-in range of the small cylinder and the beating demonstrates that different range of frequencies contributes to the vibration of the small cylinder. Beating phenomenon was observed before by Tsahalis and Jones (1981), Singh and Mittal (2005), Yang et al. (2009) and Bao et al. (2011) in the VIV analyses of an isolated cylinder, two cylinders of equal diameter in proximity, a vibrating cylinder close to a plane wall and an elastically mounted cylinder near a stationary cylinder. The large cylinder vibration amplitude and its vortex shedding frequency (which follows the Strouhal law) are too low to participate in the beating of the small cylinder. However the FFT analyses of the small cylinder oscillation confirm the presence of two different frequencies which are the frequency based on the Strouhal law and the natural frequency of the small cylinder. The large cylinder vibration amplitude and its vortex shedding frequency (which follows the Strouhal law) are too low to participate in the beating of the small cylinder. Beating phenomenon was observed before by Tsahalis and Jones (1981), Singh and Mittal (2005), Yang et al. (2009) and Bao et al. (2011) in the VIV analyses of an isolated cylinder, two cylinders of equal diameter in proximity, a vibrating cylinder close to a plane wall and an elastically mounted cylinder near a stationary cylinder. The large cylinder vibration amplitude and its vortex shedding frequency (which follows the Strouhal law) are too low to participate in the beating of the small cylinder. However the FFT analyses of the small cylinder oscillation confirm the presence of two different frequencies which are the frequency based on the Strouhal law and the natural frequency of the small cylinder. The alternative switching between these frequencies should be the reason of beating behaviour of the vibration history of the small cylinder. By increasing the reduced velocity to a value higher than 0.6 the oscillation amplitude plummets to a value about $0.1d$ as shown in Fig. 4.8 (e). The amplitude of its cross flow vibration remains fairly stable around $0.1d$. The oscillation frequencies of both cylinders outside the lock-in range are very close to their Strouhal law frequencies.

### 4.4.2.3 Vibration frequency

Fig. 4.13 shows the FFT amplitude spectra of both cylinders. It can be seen that the vibration frequency of the large cylinder follows the Strouhal law except at $U_r = 0.4$, while the small cylinder vibrates at the natural frequency of the cylinders (Normalized frequency=1). For reduced velocities outside the lock-in regime the peak frequencies of both cylinders are governed by the Strouhal law. For the small cylinder the lock-in region is characterized by the high amplitude contours in the regime of $0.25 < U_r < 0.65$. Checking the reduced velocities 0.55 and 0.6 it can be seen that the two frequencies contribute to the vibration and they are close to each other. This is the main reason for the beating responses as discussed in the previous section. When the reduced velocity of the small cylinder is beyond the lock-in region the vibration frequency of the small cylinder follows the Strouhal law of both cylinders. The normalized frequencies governed by the Strouhal law of the small cylinder occur at higher frequencies and are outside of the range of Fig. 4.13. Presence of these high frequencies is clearly observed in Fig. 4.12 (e).
Fig. 4.11: Variation of maximum transverse displacement of both cylinders with reduced velocity (based on the large cylinder). The small cylinder experiences the lock-in, Case (2).

Fig. 4.12: Displacement time histories of both cylinders at different reduced velocities of the large cylinder in Case (2).
Fig. 4.13: FFT analyses of the displacement time histories in Case (2). The results are shown as the amplitude contours for different normalized frequencies at different reduced velocities of the large cylinder. (a) results for the large cylinder; (b) results for the small cylinder

4.4.2.4 Vortex shedding

Vortex shedding from both small and large cylinders in the lock-in regime \( U_{rs} = 0.55 \) is investigated. Fig. 4.14 shows the instantaneous vorticity contours and streamlines around the small cylinder at discrete instants within one beating period of the VIV of the small cylinder. The instants corresponding to the numbers shown on the large cylinder in Fig. 4.14 can be found from the vibration time history in Fig. 4.15. The instants in Fig. 4.14 cover a whole beating period. A global view of the vortex shedding from both cylinders is shown in Fig. 4.16. The high-frequency component of the vibration history of the large cylinder in Fig. 4.15 is due to the influence from the small cylinder. The first eight instants, 2, 4, 6, 7, 9, 11, 14 and 16 in Fig. 4.14 show the streamlines around cylinders and the formation and development of vortices due to the gradual increase of the small cylinder vibration amplitude. Emerging and shedding of a clockwise vortex downstream the large cylinder can be seen in these eight instants. Due to the influence of the negative vortex from the large cylinder, the incoming flow for the small cylinders bias upwards and small vortices behind small cylinder are convected downstream at the outskirt of the big vortex. Because the flow biases upwards, the vortices from the bottom of the small cylinder develop and shed freely. Instants 23, 26, 28 and 30 in Fig. 4.14 show the vorticity field and streamlines when the small cylinder reaches its highest range of displacement. It can be easily seen that the vortices with different orientation cut each other at shorter downstream distances compared to the previous 8 instances. The small vortices in contact with the large cylinder can easily be seen in these diagrams. Instants 36, 37, 38 and 40 are in the period when the amplitude of the small cylinder decreases. The counter clockwise vortex behind the large cylinder is separated from the large cylinder and the streamlines passing through the gap between
cylinders and those over the small cylinder deflect toward downstream bottom of the large cylinder. It can be seen that the effect of the large cylinder on the vortex shedding from the small cylinder is weak as the negative big vortex is growing in the wake of the large cylinder.

Fig. 4.14: Instantaneous vorticity contours and streamlines around the cylinders for $U_{rs}=0.55$ in Case (2). The corresponding vibration time histories of the labelled time instants are shown in Fig. 4.15.

4.4.3 Case 3: Both cylinders lock-in

The ratio of the natural frequencies of the cylinders in Case 3 is chosen to be $R_{fn} = 0.1$ to investigate the interactions of the two cylinders in the lock-in regime. The reduced velocities of the two cylinders are identical based on this natural frequency ratio. This
frequency ratio resembles practical situations where a small diameter stiff pipeline is in the proximity of a heavy large diameter pipeline.

Fig. 4.15: Induced vibration time histories of both cylinders. The instants shown in Fig. 4.14 are marked in this figure for both cylinders.

Fig. 4.16: Vortex shedding instances of the large cylinder during the same time showed in Fig. 14 and 15. Two vortices are shed per one vibration cycle of the large cylinder. The large cylinder is outside of its lock-in range.
4.4.3.1 Dynamic responses

The amplitude responses of both cylinders with \(1 \leq U_r \leq 15\) are shown in Fig. 4.17. The lock-in range of the reduced velocity is found to be much wider than those in Case 1 and Case 2 or that of an isolated circular cylinder. Collision of the cylinders occurs for \(2.5 \leq U_r \leq 14\) in this Case. This occurs when the large cylinder is moving upward and passes through its neutral position. Following the collision, both cylinders keep moving upward with each other. They finally separate from each other during their following downward motion. The cylinders collision and their simultaneous upward motion resemble a slightly larger diameter assembly with a higher natural frequency compared to both cylinders. The higher natural frequency of this assembly contributes to a wider lock-in range. In Case 3, high-amplitude vibrations of the cylinders occur in the reduced velocity range of \(2 < U_r < 14\). The response amplitude of the large cylinder increases with increasing \(U_r\) until \(U_r = 7\), where the response amplitude is about 0.6\(D\). Then, the response amplitude of the large cylinder decreases with increasing \(U_r\) until \(U_r = 14\). The maximum response amplitude of the large cylinder is smaller than that (0.66\(D\)) in Case 1. The vibration amplitude of the large cylinder decreases rapidly as \(U_r\) increases from 14 to 14.5. The vibration amplitude at reduced velocities larger than 14.5 is about 0.1\(D\). Small cylinder response curve has similar trend to the large one with the maximum amplitude of 0.26\(D\) which occurs at \(U_r = 7\).
4.4.3.2 Vibration history

Vibration time histories at $U_r = 4, 5, 6, 7, 8, 11.5, 14$ and $15$ are shown in Fig. 4.18. The mean position of the large cylinder shifts downwards and that of the small cylinder shifts upwards in Case 3. The reason for this can be the high lift force amplitudes on both cylinders. The time histories of the lift coefficient on both cylinders at $U_r = 14.5$ for Cases 3 ($R_{fl} = 0.1$) and Case 1 ($R_{fl} = 1.0$) are shown in Fig. 4.19 (a) and (b) respectively. The lift forces on both cylinders in Case 3 have higher amplitudes compared with their counterparts in Case 1. Fig. 4.19 (c) and (d) show the vortices around the cylinders when the large cylinder is at its high position. The entrainment of the shear layer from the top side of the small cylinder within the shear layer of large cylinder results in larger clockwise vortices in Case 3 (Fig. 4.19 (c)) compared to the Case 1 (Fig. 4.19 (d)). These large clockwise vortices result in the downward bias of the large cylinder oscillation. The dynamic response of the small cylinder comprises two frequency components in Case 3. The low frequency component is due to the influence from the large cylinder. The small cylinder starts clinging to the large one after the large cylinder moving upwards above its mean position. As the large cylinder moves downwards and passes its mean position, the small cylinder detaches from it. Then, the two cylinders keep separate from each other until the large cylinder moves upward from its lowest position and hits the small cylinder again. The small cylinder lock-in can be seen by its high frequency vibrations after it separates from the large cylinder. When the two cylinders are separated from each other, the small cylinder lock-in occurs, resulting in the high frequency component of its vibration. The averaged high-frequency oscillation amplitude of the small cylinder is about ten times of the low frequency oscillation amplitude in Fig. 4.18. After the large cylinder hits the small cylinder and the two cylinders are stuck to each other, how far the small cylinder can go upwards is governed by the large cylinder. After the two cylinders separate from each other, the small cylinder vibrates freely and its vibration lock-on to its natural frequency. The maximum vibration amplitude of the small cylinder during its free vibration (separated from the large cylinder) is $0.7d$. The amplitude of the large cylinder affects the high-frequency amplitude of the small cylinder. After the separation, the high potential energy of the small cylinder stored during its upward movement forced by the large cylinder is released and provide the initial speed for the lock-in vibration of the small cylinder. The detached vibration is defined to be the high-frequency vibration of the small cylinder after it separates from the large cylinder in this study. The varying amplitude of the large cylinder in Fig. 4.18 (a) results in alternative strong and weak...
Fig. 4.18: Displacement time histories of both cylinders at different reduced velocities in Case (3).
high-frequency detached vibrations of the small cylinder at $U_r = 4$. The steady high-frequency detached vibration of the small cylinder follows the high initial displacement induced by the large cylinder. The high initial forced displacement of the small cylinder results in an increase in the amplitude during detached high-frequency resonance and vice versa.

The numerical results show that for the reduced velocities higher than 5.5 and less than 12 no steady periodic results can be achieved as shown Fig. 4.18 (c), (d), (e) and (f). The modulations observed in the displacement time histories of both cylinders in Fig. 4.18 (e) and (f) are unsteady and do not repeat well. The unsteady beating of the large cylinder for $U_r = 8$ will be discussed later. The high amplitude vibration of the large cylinder persists until $U_r = 14$. The vibration of the large cylinder becomes steady and bias downwards as $U_r > 11.5$. Based on Fig. 4.10, the lock-in regime of the small cylinder was found in the range $2.5 < U_r < 6.5$. In Case 3, the lock-in regime of the small cylinder appears to be the
same as that of the large cylinder. At $U_r = 8$, after the two cylinders separate, the potential energy of the small cylinder is transferred to high initial velocity. Because the vibration is not in the lock-in regime, the high initial speed of the small cylinder does not last long and the amplitude of the free detached vibration of the small cylinder decreases gradually before the two cylinders come together again. At $U_r = 14.5$ and 15, the collision do not occur and the vibration frequency of each cylinder is governed by the vortex shedding frequency.

### 4.4.3.3 Vibration frequency

The results of FFT analyses of both cylinders are shown in Fig. 4.20. The dominant normalized frequency, $f_r / f_{ad}$, of the large cylinder in the range of $3 < U_r < 14.5$ is close to one. The secondary frequency as the reduced velocity exceeds 12 follows the Strouhal law. The FFT analyses of the small cylinder shows the presence of multiple frequency components. The forced motion of the small cylinder by the large one is presented in the FFT analyses by the normalized frequency close to 1. For the range of reduced velocities between 4 and 8, one can observe the normalized frequencies about 10. This high frequency component is due to the detached vibration of the small cylinder after it is detached from the large cylinder. For higher reduced velocities the FFT analysis shows a combination of harmonics of the small cylinder. These irregular combinations occur because the vibration is far different from sinusoidal.

![Fig. 4.20: FFT analyses of the displacement time histories in Case (3). The results are shown as the amplitude contours for different normalized frequencies at different reduced velocities. (a) results for the large cylinder; (b) results for the small cylinder.](image)

### 4.4.3.4 Vortex shedding

The displacement time histories of both cylinders for a period longer than one beating cycle at $U_r = 8$ are shown in Fig. 4.21. The letters in the figure represent the type of vortex
shedding regimes of each cycle which will be explained in Fig. 4.22. The time history of about three periods of beating shown in Fig. 4.18 (c) reveals some differences between beating periods. As mentioned before the recorded oscillation time histories of both cylinders are unsteady and vary with time for the range of the reduced velocities $5.5 < U_r < 12$. The vortex shedding is found to vary from cycle to cycle and the six observed vortex shedding regimes (labelled as a ~ f) of the large cylinder for the discussed period in Fig. 4.21 are shown in Fig. 4.22. Each row in Fig. 4.22 shows a different regime observed during cylinders oscillation. The first column of each row (regime) in Fig. 4.22 shows instantaneous vorticity contours and streamlines around the cylinders when they are at the peak positions. The cylinders are stuck to each other and act as a single body in the first column. The streamlines and the vorticity distribution in the first column suggest the presence of two large vortices with different orientations and a small vortex behind the small cylinder. The S-shape streamlines in the wake of the cylinders is the boundary between the two large vortices downstream of the cylinders. The direction of the S-shape streamlines behind the cylinders depends on the direction of the cylinder movements. During upward motion, the S-shape streamlines are generated by the deflection of the streamlines beneath the large cylinder. During the downward motion of the cylinders, the S-shape streamlines are however produced by the streamlines passing over the topside of the small cylinder. The clockwise vortex shedding from the top side of the small cylinder is affected by both the large and small anti-clockwise vortices in the first column. The instants that cylinders separate from each other during their downward motions are shown in the second column in Fig. 4.22. The effects of the large anti-clockwise vortex on the clockwise one can be clearly seen in this column. In general the clockwise vortex is divided into two smaller ones in this column. However, the timing of this splitting is different for different regimes. The small vortices in the first column are dissipated during the downward motion of the cylinders. The third column displays the instants when the cylinders are apart from each other and the large cylinder has commenced its upward motion. The second clockwise vortex reattachment (separation) to (from) the large cylinder can be observed for different regimes in this column. For regimes (a), (b) and (d) the second clockwise vortex is detached from the large cylinder. However for the regimes (b), (e) and (f) separation has not occurred yet. Different structure of the anti-clockwise vortices in the third column is also noticeable. The shedding of the first anti-clockwise vortex occurs in the regime (c). The vortex shedding from the small cylinder strongly affects the shape, development and the shedding time of clockwise vortex from the top side of the large cylinder. The effect can be observed clearly in regime (c). The deflection of the clockwise vortex by the shed vortices from the small cylinder result in the shedding of the anti-clockwise vortex. Column four in Fig. 4.22 shows the instants that the
cylinders are in contact with each other. The anti-clockwise vortex from the large cylinder is shed at this column (instants). The clockwise vortex in regime (e) is still attached to the large cylinder however it is going to separate in regime (f). In general the strength and number of shed vortices vary for different regimes. The number of shed vortices from the large cylinder for higher reduced velocities with unsteady vibration histories is three, two clockwise and one anti-clockwise. The strength and shedding time of these vortices vary in each cycle.

Fig. 4.21: Vibration time histories of both cylinders at $U_r = 8$. The letters in the figure represent the dominate vortex shedding regimes which will be shown in Figure 22.

Fig. 4.23 shows the vortex shedding patterns corresponding to the five instants marked on the vibration time history (Fig. 4.23 (f)) in about one cycle at $U_r = 12$. The vibration time history for this reduced velocity and higher reduced velocity is steady and their vortex shedding regimes repeat well. The two frequency components found in the FFT analysis (Fig. 4.20 (a)) and the large amplitude of vibration of the large cylinder for $12 \leq U_r \leq 14$ can be explained using this figure. The number of vortices that are shed from the large cylinder per vibration cycle at $12 \leq U_r \leq 14$ is four. Fig. 4.23 (a), (b), (c) and (d) show the instants that vortices are shed from the large cylinder. The instants corresponding to Fig. 4.23 (a) ~ (e) are marked on the time histories of the displacement and vibration speed of the large cylinder shown in Fig. 4.23 (f). It is found that the shedding period of the shed vortices (c) and (d) are varied by the upstream velocity according to Strouhal law. The downward motion of the shed vortices from the small cylinder and their interaction with the large cylinder near wake is the main reason for the cutting off of the shed vortices in (c) and (d). Both of these vortices are shed during the upward motion of large cylinder and their effect on both velocity and vibration time history can be clearly seen during upward motion of the large cylinder in each vibration cycle in Fig. 4.23 (f). The sudden fluctuation in the velocity of the large cylinder during upward motion is caused by these two vortices. The opposing transferred energy to the large cylinder by the shed vortex at instant (c) tries to
decelerate the large cylinder upward motion (Jauvtis and Williamson, 2004). The clockwise developed vortex downstream of the small cylinder was divided generally into two vortices in Fig. 4.22. However for $12 \leq U_r \leq 14$ one vortex with higher strength is shed periodically at the downstream of the small cylinder when both cylinders are moving downward simultaneously (Fig. 4.23 (a)).

Fig. 4.22: Different vortex regimes found during the vibration time history presented in Fig. 4.21. These regimes (a ~ f) are captured at four instants during the large cylinder oscillations.
Fig. 4.23: Vortex shedding patterns (a ~ e) corresponding to the five instants on the vibration time history (f) for $U_r = 12$. The clockwise vortex shed at instant (a) does not split into two separate vortices by the anticlockwise vortex at the bottom side (compare it with different regimes shown in Fig. 4.22).

Fig. 4.24 is proposed to explain how different shed vortices with different frequencies affect the vibration history of the large cylinder. The results of the FFT spectrum of the large cylinder vibration at $U_r = 12$ are implemented in Fig. 4.24 to reassemble the same vibration history of the large cylinder as shown in Fig. 4.23 (f). The two dominant features of the vibration time history of the large cylinder in Fig. 4.23 (f) are its downward bias and its smooth deflection during its upward motion. The vortices (c) and (d) shown in Fig. 4.23 (c) and (d) are shed during upward motion of the large cylinder where the displacement curve starts to deviate from its pure sinusoidal form. This suggests that the effects of the vortices (c) and (d) are mainly during upward motion of the large cylinder. The shedding of these two vortices occurs at the Strouhal frequency of the large cylinder during each shedding period.
of the vortices (a) and (b) with a period time close to the natural period of the large cylinder. Based on this assumption the two curves shown in Fig. 4.24 are proposed as the induced vibrations by two different set of vortices. The total vibration of the large cylinder can be considered as the summation of these two. The high amplitude oscillations are caused by vortices (a) and (b) and the small one is induced by vortices (c) and (d). It can be seen that the second set of vortices affects the oscillation mainly during upward motion of the large cylinder. The downward shift of the final oscillation (the summation of the two curves in Fig. 4.24) is caused by the small cylinder which contributes to the higher strength clockwise shed vortices and consequently higher net positive force during downward motion of the cylinders.

For reduced velocities higher than 14 two vortices are shed per vibration cycle and the shedding frequency is governed by Strouhal law. The shedding instants are shown in Fig. 4.25.

Fig. 4.24: The two presented graphs are proposed to show how the two sets of vortices ((a) and (b)) and ((c) and (d)) contribute to the total oscillation of the large cylinder.

Fig. 4.25: Vorticity shed instances for $U_r=14.5$. Two vortices are shed per period.
4.5 Conclusions

Vortex induced vibrations of two side-by-side elastically mounted circular cylinders in the cross-flow direction were investigated numerically. The diameter ratio of the cylinders was fixed at 0.1 and an initial gap was equal to 0.1 times the large cylinder diameter. VIV studies are focused on three different scenarios: The lock-in study of the large cylinder (Case 1), lock-in study of the small cylinder (Case 2) and simultaneous lock-in of both cylinders (Case 3). The main conclusions can be drawn as follows:

1- In Case 1, the small cylinder is forced to vibrate passively with the large cylinder. The lock-in regime and the vibration frequency in the lock-in regime of the small cylinder are the same as their counterparts of the large cylinder. The vibration amplitude of the small cylinder is smaller than that of the large cylinder but it is a number times of the small cylinder diameter, which is far greater than that of a single cylinder case.

2- In Case 2, the vibration of the large cylinder is very weak because it is outside of the lock-in regime and the small cylinder vibrates at a frequency much higher than that of the large cylinder. In Case 2, beating was a dominant feature during the small cylinder lock-in. Two dominant frequencies are observed in the FFT analysis of the vibration histories. These two frequencies are the vortex shedding frequency and the natural frequency of the small cylinder.

3- The beating phenomenon is observed in the large cylinder vibrations at $U_r = 0.4$ in Case 2. The corresponding vibration amplitude reaches to the maximum for both cylinders in Case 2. The two participating frequencies in the beating are the natural frequency of the large cylinder and the frequency of the oscillation of the small cylinder.

4- In Case 3 the vibration of the small cylinder comprises two parts: the forced vibration due to its proximity to the large cylinder and the vortex induced vibration of its own. The second part only exists when the small cylinder is detached from the large one. The vibration of the small cylinder after it is separated from the large cylinder is defined as the detached vibration. The forced vibration is observed in Cases 1 and 3. However, the detached vibration is observed only in Case 2 and Case 3. The lock-in regime of the reduced velocity in Case 3 is wider than those in Case 1 and Case 2. Two pair of vortices are shed per vibration cycle of the large cylinder for $12 \leq U_r \leq 14$ in Case 3.

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4.7 References


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CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Main Findings

This study investigated the interaction of steady fluid flow and two cylinders with different diameters ($D/d=10$). The cylinders were placed in proximity of each other. The completed research has resulted in improved understanding on three physical processes: (1) VIV of mechanically coupled cylinders and the behaviour of the bundle at different orientation and spacing of the small cylinder, (2) The possibility of collision of the two cylinders when they were positioned in a side-by-side arrangement and were allowed to oscillate independently in the cross flow direction, and (3) the lock-in behaviour of each cylinder in the presence of the other one. The vortex shedding regimes, induced vibrations and the consequences of proximity of the cylinders were studied in detail for the above mentioned cases.

5.1.1 Mechanically coupled cylinders

Chapter 2 discussed the 2-DOF VIV of two mechanically coupled cylinders in piggyback arrangement. The diameter ratio of the cylinders was fixed ($d/D=0.1$) and the gap and angular position of the small cylinders were varied. The Reynolds number was fixed at 8000 and angular position of the small cylinder was varied from $0^\circ$ to $180^\circ$ with $22.5^\circ$ increment. The gap between cylinders’ surfaces varied between $0.1D$ and $0.4D$ with $0.1D$ increment. The vortex shedding and VIV of the bundle were discussed in detail.

The in-line arrangements of the cylinders ($\theta=0^\circ$ and $180^\circ$) were found the least sensitive arrangements to the gap ratio. The vibration amplitudes did not vary significantly with the gap ratio at these arrangements. The fluid flow between cylinders was found to be an important factor affecting both the vortex shedding and vibration amplitudes in staggered and side-by-side arrangements of the cylinders. In general, the narrower the gap was in the side-by-side and staggered arrangements, the more significant the effect of angular position of the small cylinder on the bundle’s VIV was going to be.

The present results showed that the highest amplitudes of oscillation in the cross-flow direction occurred at ($\theta=67.5^\circ$ and $157.5^\circ$). The high oscillation amplitude of the bundle in the in-line direction for staggered configurations was also noticeable. It exceeded $0.6D$ at ($\theta=67.5^\circ$, $G/D=0.1$) and ($\theta=135^\circ$, $G/D=0.2$). The vibration amplitudes of the bundle in the
side-by-side arrangement change significantly with small variations in the gap ratio or angular position of the small cylinder.

5.1.2 Cylinders collision

The possibility of collision of the two side-by-side cylinders during their cross flow vibrations (1-DOF) was discussed in Chapter 3. The same mass ratios were considered for both cylinders. However their diameters were different ($d/D=0.1$). Collision of the cylinders occurred when the reduced natural frequency of the small cylinder was larger than that of the large cylinder. It was found that the hydrodynamic force on the small cylinder and its fluctuation frequency were affected substantially by the gap ratio. Both the frequency and amplitude of the oscillation were increased by decreasing the gap ratio. The simultaneous oscillation of both cylinders contributed to a sudden velocity fluctuation and the consequent variation of pressure force on the small cylinder. A thorough investigation of the shed vortices showed the presence of a semi-steady vortex shedding regime from the large cylinder when the difference in the reduced natural frequencies was the maximum at 0.1 ($f_{n1}^* = 0.2, f_{n2}^* = 0.4$). It was found that vortex shedding regime repeated well each four vibration cycles of the large cylinder. The maximum oscillation amplitude of the large cylinder occurred when both its lift force and velocity were in the upward direction and positive energy was transferred to the large cylinder. Finally it was found that collision of the cylinders contributed to the sudden increase of the lift force on the large cylinder.

5.1.3 Lock-in of elastically mounted cylinders

The resonances of two elastically mounted cylinders with different diameters in a side-by-side arrangement were examined in Chapter four. Three cases were discussed. In all three cases the small cylinder was positioned initially on top of the large one and the initial gap between cylinders was 0.1D. The first case discussed the lock-on of the large cylinder when the small one was outside of its own lock-on range. The proximity of the cylinders forced the small cylinder to oscillate with the large one. The cylinders did not collide with each other in this case. The lock-in range of the large cylinder was wider than that of an isolated cylinder. The interactions between them were obvious for narrow gaps with the small cylinder experiencing a large lift force. The phase difference between the oscillation time histories of the cylinders inhibited the collision during their upward motion.

The simultaneous monitoring of the pressure distribution and the vortex shedding of the large cylinder clearly showed the concurrent appearance of the strong vortex shedding from each side of the cylinder with the lower pressure region on the other side. This explained
why the vortex shedding from each side of the cylinder was coincident with an application of the force on the cylinder toward the opposite direction.

The second case discussed the resonance of the small cylinder when the underneath large cylinder was outside of its lock-on range. The large cylinder was free to oscillate and its vibrations were expected to be governed by its Strouhal frequency. However the high frequency oscillations and beating behaviour of the large cylinder (outside its Strouhal frequency range) were found for a range of reduced velocities. It was found that high frequency oscillations of the small cylinder in proximity of the large one induced fluctuating pressure force with frequency being very close to the natural frequency of the large cylinder and this closeness of frequencies resulted in the high amplitude vibration of the large cylinder (compared to nearby reduced velocities). Modulation in the oscillation time history of the small cylinder also was present. Strouhal frequency and natural frequency of the small cylinder were found in the FFT spectrum analysis. The closeness of these frequencies resulted in the modulation of the small cylinder oscillations.

The last case discussed the simultaneous resonance of both cylinders. The practical situation similar to this case is when a stiff pipeline is in proximity of a less stiff and larger diameter pipeline. Both cylinders experienced the same reduced velocity due to their different natural frequencies. The oscillation of the small cylinder in the lock-in range consisted of two parts. First part was the low frequency and high amplitude vibrations induced by the large cylinder. The large cylinder collided with the small one and pushed it upward. The small cylinder only detached from the large one after initiation of the downward motion of the latter one. The largest oscillation amplitude of the small cylinder was about 6 times of its diameter in this part. The second part was the detached vibrations of the small cylinder occurred during the time that cylinders were separated from each other. The highest amplitude of the small cylinder was about 0.7 of its diameter during detached oscillations. The oscillation behaviour of the small cylinder in the detached mode (the second part) was dependent on it forced displacements by the large cylinder in the first part. The detached vibrations of the small cylinder (initiating after its separation from the large cylinder) damped very fast outside the lock-in range. However in the lock-in range, the amplitude of the detached vibrations increased with time after the separation of the cylinders.

The lock-in range of the large cylinder in the last case was wider than that in the first case. The reason for this wider lock-in range was found to be due to the shedding of one pair of vortices (instead of one vortex) during the upward motion of the large cylinder outside the normal lock-in range of an isolated cylinder \((11.5 \leq U_r \leq 14)\). Furthermore the continuous collision of cylinders during upward motions resembled a bluff body with a
larger diameter which also contributed to the wider lock-in range. Vibration of both cylinders was found to be unsteady for $6 \leq U_r \leq 11$. For higher reduced velocities the vibrations of both cylinders were steady. Collisions of the cylinders were observed in the range of $2 < U_r < 14.5$.

5.2 Further research

5.2.1 2-DOF oscillations of two cylinders in close proximity in two and three dimensional spaces

The present research has provided new results about the physics of VIV of two proximity cylinders in 2D space. It would be useful to investigate the effects of the third dimension on the VIVs of two proximity pipelines. Furthermore, 2-DOF interactions of two cylinders with different diameters may also deserve more research efforts. Use of the overset grid method may be a good option to overcome difficulties associated with meshing. However keeping the mass conservation in different grids would be a challenge. The proximity of cylinders can exacerbate the situations and an innovative solution for this case is badly needed.

5.2.2 Experimental study

The developed code used the conservation of momentum theory to deal with collision of cylinders. A damping coefficient was assumed in the simulations to simulate the collision. However this was not validated experimentally. This may be an area of further study.