ANALYTICAL AND NUMERICAL SOLUTIONS FOR PILE FOUNDATIONS

by

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This research has investigated the performance of piles in non-homogeneous elastic-plastic media subject to vertical or torsional loading, the time-dependent response of a vertically loaded pile due to either creep or reconsolidation subsequent to pile driving, and the behaviour of vertically loaded pile groups. Closed form solutions have been established accordingly, and numerical programs, GASPILE and GASGROUP have been developed.

The closed form solutions were firstly developed for vertically loaded single piles. Secondly, in a similar manner, solutions for single piles subject to torsion were generated, in light of a newly established torsional load transfer model. The effect of non-linear soil stress-strain properties modelled using a hyperbolic stress-strain law, has been investigated through the program, GASPILE, for both vertical and torsional loading. Thereafter, the solutions for vertically loaded piles were extended to account for visco-elastic response, with a newly established visco-elastic model.

All the solutions have been developed to incorporate accurate modelling of the soil stiffness profile described by a power law of depth, and also with appropriate attention to the gradual development of slip between pile and soil.

Although the solutions are based on the load transfer approach, treating each soil layer independently from neighbouring layers, the accuracy has been extensively checked by more rigorous numerical approaches, against which load transfer factors have been extensively calibrated. Appropriate load transfer factors have been developed, allowing for the effect of the following parameters: pile slenderness ratio, ratio of the depth of underlying rigid layer to pile length, soil Poisson's ratio, and non-homogeneous soil profile.

One of the major concerns has been the variation of soil properties with time following pile installation. This variation has been simulated through a newly established visco-elastic radial consolidation theory.
The solutions for a single pile have then been eventually extended to evaluate settlement behaviour of large pile groups, in light of the principle of superposition.

All the solutions established have been substantiated by previous numerical and experimental results. Parametric analyses were undertaken extensively and a number conclusions were drawn.

In particular, non-linear analysis using a hyperbolic stress-strain model does not lead to appreciable differences from a simple elastic, perfectly plastic analysis. Therefore, the closed form solutions based on an elastic-plastic model can be applied directly to the non-linear case, without significant lose of accuracy.
DECLARATION

I certify that, except where specific reference is made in the text to the work of others, the content of this thesis are original and have not been submitted to any other university or institute. This thesis is the result of my own work and contains nothing which is the outcome of work done in collaboration.

Wei Dong Guo
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NOTATION

Roman

\( A \) = a coefficient for estimating shaft load transfer factor;
\( A(t) \) = time-dependent part of the shaft creep model;
\( A_2 \) = a parameter from rate process theory;
\( A_c \) = a parameter for the creep function of \( J(t) \);
\( A_g \) = constant for soil shear modulus distribution;
\( A_h \) = a coefficient for estimating \( \Lambda \), accounting for the effect of \( H/L \);
\( A_n \) = coefficients for predicting excess pore pressure;
\( A_{oh} \) = the value of \( A_h \) at a ratio of \( H/L = 4 \);
\( A_p \) = cross-sectional area of an equivalent solid cylinder pile;
\( A_t \) = a constant for shaft friction profile;
\( A_v \) = a constant for shaft limit stress distribution;
\( B \) = a coefficient for estimating shaft load transfer factor;
\( B_2 \) = a parameter from rate process theory;
\( B_c \) = a parameter for the creep function of \( J(t) \);
\( C_t(z) \) = a function for assessing torsional stiffness at a depth of \( z \);
\( C_{lo} \) = the limiting value of \( C_t(z) \) as \( z \) approaches zero;
\( c_v \) = coefficient of soil consolidation ;
\( C_v(z) \) = a function for assessing pile stiffness at a depth of \( z \), under vertical loading;
\( C_{vo} \) = limiting value of the function, \( C_v(z) \) as \( z \) approaches zero;
\( C_{v2} \) = limiting value of the function, \( C_{v2}(z) \) as \( z \) approaches zero;
\( C_\lambda \) = a coefficient for estimating \( \Lambda \), accounting for the effect of \( \lambda \);
\( d(r_0) \) = diameter (radius) of a pile;
\( E \) = Young's modulus of soil;
\( E_2 \) = Young's modulus of soil for spring 2 (Chapter 2);
\( E_p \) = Young's modulus of an equivalent solid cylinder pile;
\( E_{il} \) = initial Young's modulus of soil at pile base level;
\( E_L \) = Young's modulus of soil at pile base level;
\( f_{bii} \) = the displacement influence coefficient for the node at the pile base;
\( f_{bij} \) = the displacement influence coefficient at the pile base;
\( f_{sii}^k \) = the flexibility coefficient for pile shaft in layer \( k \) due to unit load the layer \( k \) in the same pile \( i \);
\( f_{sij} \) = the average settlement flexibility coefficient for shaft elements at pile \( i \) due to unit head load at pile \( j \);
\( f_{sij}^k \) = the displacement influence coefficient for pile shaft in layer \( k \) denoting the settlement of the shaft at pile \( i \) due to a unit load at pile \( j \), within the layer \( k \);
\( F(t) \) = the creep compliance derived from the generalised creep model;
\[ F^k \] = flexibility matrix of order \( n_g \times n_g \) for layer \( k \);
\( F_\phi \) = modification factor accounting for pile-soil relative slip;
\( G \) = scant shear modulus at radius, \( r \) (Chapters 3 and 8);
\( G \) = elastic shear modulus (Chapters 2 and 6);
\( G_{\text{ave}} \) = average shear modulus over the pile embedded depth;
\( G_b \) = shear modulus at just beneath pile base level;
\( G_c \) = soil shear modulus at a depth of \( z = L_c \);
\( G_i \) = initial soil shear modulus;
\( G_L \) = shaft soil shear modulus at just above the pile base level;
\( G_{ib} \) = initial shear modulus at just beneath pile base level;
\( G_{ib}(t) \) = time-dependent initial shear modulus at just beneath pile base level;
\( G_{ibj} \) = initial shear modulus at just beneath pile base level for spring \( j \) (\( j = 1, 2 \));
\( G_{IL} \) = initial shaft soil shear modulus at just above the pile base level;
\( G_{IL/2} \) = initial soil shear modulus at depth of \( L/2 \);
\( G_{ij} \) = the instantaneous and delayed initial shear modulus for elastic spring \( j \) (\( j = 1, 3 \));
\( G_{io} \) = initial soil shear modulus at mudline level;
\( G_{ij} \) = shear modulus at distance, \( r \) away from the pile axis for elastic spring \( j \);
\( G_{ro} \) = initial soil shear modulus at pile-soil interface;
\( G_p \) = shear modulus of an equivalent solid cylinder pile;
\( G_D \) = shear modulus for deviatoric stress-strain relationship;
\( G_v \) = shear modulus for volumetric stress-strain relationship;
\( G_T \) = initial soil shear modulus at strain \( \gamma \);
\( G_{\gamma j} \) = initial soil shear modulus at strain \( \gamma_j \) for spring \( j \) (\( j = 1, 3 \)) within the creep model;
\( G_{1\%} \) = shear modulus at a shear strain of 1\%;
\( H \) = the depth to the underlying rigid layer;
\( I \) = settlement influence factor for single piles subjected to vertical loading;
\( I_{o} \) = settlement influence factor for pile groups subjected to vertical loading;
\( I_{m}, I_{m-1} \) = Modified Bessel functions of the first kind of non-integer order, \( m \) and \( m-1 \) respectively;
\( I_{pp}, I_{ps} \) = new settlement influence factors for estimating base settlement;
\( I_\phi \) = torsional influence factor;
\( J \) = a creep parameter defined as: \( J = 1/G_{\gamma 1} + 1/G_{\gamma 2} \);
\( J(t) \) = a creep function defined as \( \zeta \phi /G_{\text{il}} \);
\( J_i \) = Bessel functions of the first kinds and of order \( i \) (\( i = 0, 1 \));
\( J_p \) = polar moment of inertia of a pile;
\( k \) = permeability of soil;
$k_j$ = a factor representing soil non-linearity of elastic spring $j$;

$k_s$ = a factor representing pile-soil relative stiffness;

$k_{L}$ = non-dimensional shaft stiffness factor;

$k_t$ = ratio of pile length, $L$, to the critical pile length, $L_c$;

$K_b$ = relative pile-soil stiffness ratio between Young's modulus of a pile and the initial soil Young's modulus at just above the base level, $E_p/E_{il}$;

$K_m$ = Modified Bessel functions of the second kind of non-integer order, $m$;

$K_{m-1}$ = Modified Bessel functions of the second kind of non-integer order, $m-1$;

$K_p$ = pile-head stiffness defined as $P_t/w_t$;

$K_T$ = relative pile-soil torsional stiffness ratio;

$l$ = pile segment length;

$L$ = embedded pile length;

$L_1$ = the depth of transition from elastic to plastic phase, the slip part length of a pile under vertical or torsional loading;

$L_2$ = length of the elastic part of a pile under a given load;

$L_c$ = the critical pile length of a pile under torsion;

$m$ = $1/(2+n)$;

$m_c$ = a creep parameter for the empirical creep model;

$m_2$ = ratio of shear moduli, $G_{y1}/G_{y2}$;

$m_3$ = ratio of shear moduli, $G_{y1}/G_{y3}$;

$N$ = SPT value;

$\overline{N}$ = the average value of the SPT values over a pile embedded depth;

$n$ = power of the shear modulus distribution, non-homogeneity factor;

$n_c$ = power of a creep model (Chapter 2);

$n_g$ = total number of piles in a group;

$n_{max}$ = maximum ratio of pile head load and the ultimate shaft load (Appendix C);

$n_p$ = ratio of pile head load and the ultimate shaft load (Appendix C);

$P_{10}$ = the pile-head load required to cause a head settlement of 10% of pile diameter;

$P_b$ = load of pile base;

$P_{bj}$ ($P_{bi}$) = base load at pile $j$ (i);

$P(z)$ = axial force of pile body at a depth of $z$;

$P_e$ = axial load at the depth of transition ($L_1$) from elastic to plastic phase;

$P_f$ ($P_{ult}$) = ultimate pile bearing load;

$P_{fb}$ = ultimate base load;

$P_s$ = ultimate shaft load of a pile;

$P_j$ = load on pile $j$, which is in a group of $n_g$;

$P_G$ = load exerted on a pile group;
\( P_s \) = shaft load of a pile;
\( P_s(z) \) = shaft load at a depth of \( z \);
\( P_{sl} \) = total shaft load of a pile;
\([P^k_i]\) = shaft load vector for layer \( k \);
\( P^k_{ij} (P^k_{si}) \) = shaft load at layer \( k \) at pile \( j \) (i);
\( P_l \) = load acting on pile head;
\( P_{ult} \) = the ultimate total pile capacity;
\( R \) = the radius beyond which the excess pore pressure is initially zero;
\( R_b \) = ratio of settlement between that for pile and soil caused by \( P_b \), base settlement ratio (Appendix C);
\( R_f \) = failure ratio of a hyperbolic model, curve-fitting constant;
\( R_{bf} \) = a hyperbolic curve-fitting constant for pile base load settlement curve;
\( R_{fj} \) = a hyperbolic curve-fitting constant, \( \tau_{fj}/\tau_{ultj} \), for the elastic element \( j \) within the creep models;
\( R_{fs} \) = ratio of limiting and ultimate shaft shear stress;
\( R_s \) = settlement ratio for pile groups;
\( r \) = distance from normal axis of pile body;
\( r_g \) = semi-width of the pile groups;
\( r_0 \) = pile radius;
\( r_m \) = radius of zone of shaft shear influence;
\( r_{mg} \) = radius of zone of shaft shear influence for pile groups;
\( r^* \) = the radius at which the excess pore pressure, by the time they reach there, are small and can be ignored;
\( s \) = argument of the Laplace transform;
\( s \) = pile centre-centre spacing;
\( s_{ij} \) = pile centre-centre spacing between pile \( i \) and pile \( j \);
\( S_{ij} \) = deviatoric stress;
\( su \) = undrained shear strength of soil;
\( t \) (\( t^* \)) = time elapsed;
\( t_i \) = normalising time constant;
\( t \) = power of the shaft friction distribution (Chapter 8);
\( t_k \) = a critical time at which the Voigt element 2 starts to work;
\( T_{50}, T_{90} \) = non-dimensional times for 50% and 90% degree of consolidation respectively;
\( T \) = relaxation time, \( \eta/G_{12} \);
\( T_2 \) = relaxation time, \( \eta_{12}/G_{r2} \);
\( T_3 \) = relaxation time, \( \eta_{12}/G_{r3} \);
$T_b$ = torque at the pile base;
$T(z)$ = torque in the pile body at a depth of $z$;
$T_e$ = torque at the depth of transition ($L_1$) from plastic to elastic phase;
$T_n(t)$ = the time for the reconsolidation theory;
$T_t$ = torque acting on a pile head;
$T_u$ = ultimate torque acting on a pile head;
$u(z)$ = axial pile deformation;
u = vertical displacement along depth (Chapter 5 only);
$u$ = pore water pressure (Chapter 6 only);
u = radial soil movement (Chapter 8 only);
uo = initial pore water pressure (Chapter 6 only);
uo(r) = initial excess pore water pressure at radius $r$;
v = circumferential movement (Chapter 8 only);
$V_i$ = cylinder function of $i$-th order;
w = local shaft deformation at a depth of $z$;
w_1 = settlement of a single pile under unit head load;
w_b = settlement of pile base;
w_{bi} = the overall settlement of the soil at the base of pile $i$ due to loading on itself and on neighbouring piles;
$(w_b)_2$ = base settlement of a pile in a group of two piles;
$(w_b)_j$ = base displacement of the $j$th pile;
w_c = the creep part of the local deformation;
w_e(w*) = limiting elastic shaft displacement calculated by using $\tau_{\text{max}}$;
w_G = settlement of a pile group;
w_i = settlement of any pile $i$ in a group;
w_p = displacement of a pile under head load, with rigid base resistance only;
w_{pp} = settlement of the base by the load transmitted at the pile base;
w_{ps} = settlement of the base due to the load transmitted along the pile shaft;
w_s = shaft displacement;
$(w_s)_2$ = shaft settlement of a pile in a group of two piles;
$(w_s)_j$ = shaft displacement of the $j$th pile;
w^k = the overall settlement of the soil at the pile shaft of pile $i$ within a soil layer, $k$ due to loading on itself and on neighbouring piles;
w_t = pile-head settlement;
w(r) = settlement at a distance of $r$ away from the pile axis;
w(z) = deformation of pile body at a depth of $z$ for a given time;
$[w^k_s] = $ shaft displacement vector for layer $k$;
y_R = a radius beyond which the excess pore pressure is initially zero;
$Y_i$ = Bessel functions of the second kinds and of order $i$ ($i = 0, 1$);
$z$ = depth.

**Greek**

$\alpha$ = average pile-soil adhesion factor in terms of total stress;
$\alpha_{bij}$ = base interaction factor between pile $i$ and pile $j$;
$\alpha_c$ = non-dimensional creep parameter for standard linear model;
$\alpha_{ij}$ = a parameter for the empirical creep model (Chapter 2);
$\alpha_{ij}^p$ ($\alpha_{ps}^i$) = interaction factors for assessing base settlement;
$\alpha_s$ = ratio of the total shaft and pile-head load;
$\alpha_{sij}$ = shaft interaction factor between pile $i$ and pile $j$;
$\alpha_{12}$ = pile-pile interaction factor;
$\alpha_y$ = a creep parameter obtained from rate process theory;
$\beta$ = average pile-soil adhesion factor in terms of effective stress (Chapter 6);
$\beta$ = non-dimensional shaft stiffness factor ($= \sqrt{\pi_3}$);
$\beta_0$ = non-dimensional shaft stiffness factor, $\beta(1 - \mu)$ (Appendix B);
$\beta_b$ = ratio of pile base and head load;
$\beta_c$ = a parameter for the empirical creep model;
$\beta^*$ = modified non-dimensional shaft stiffness factor, $1.15\beta$ (Chapter 2);
$\beta_y$ = a creep parameter obtained from rate process theory;
$\gamma$ = shear strain;
$\gamma_1$ = shear strain at time $t_1$ (Chapter 2);
$\gamma_j$ = shear strain for elastic spring $j$;
$\gamma_w$ = the unit weight of water;
$\dot{\gamma}$ = shear strain rate;
$\dot{\gamma}_j$ = shear strain rate for elastic spring $j$;
$\delta\sigma_r$ = mean total stress;
$\delta\sigma_r$ ($\delta\sigma_\theta$) = increments of the effective stress during consolidation in radial and circumferential directions;
$\Delta$ = stress distribution factor;
$\Delta t$ = time increment;
$\Delta u_a$ = an ambient component of excess pore pressure due to pile driving,
$\Delta u_s$ = a shearing component of excess pore pressure due to pile driving,
$\Delta w$ = displacement increment;
$\varepsilon_r$, $\varepsilon_\theta$, $\varepsilon_z$ = shear strain in the radial, circumferential and depth directions;
$\varepsilon_v$ = the volumetric strain;
\( \varepsilon_2 (\dot{\varepsilon}_2) \) = shear strain and its rate (Chapter 2);

\( \zeta \) = shaft load transfer factor;

\( \zeta_c \) = a non-dimensional creep function (Chapter 5 only);

\( \zeta_j \) = non-linear measure of the influence of load transfer for spring j (j = 1, 2) within the creep models;

\( \zeta_2 \) = shaft load transfer factor for two piles (Chapter 7 only);

\( \eta \) = homogeneity factor by Poulos (Chapter 3 only);

\( \eta \) = creep parameter for the visco-elastic model, shear viscosity for the dash;

\( \eta_1, \eta_2 \) = viscosity parameters for the model by Komamura and Huang (1974);

\( \eta_{\gamma_2} \) = shear viscosity for the dash at strain \( \gamma_2 \);

\( \eta_{\gamma_3} \) = shear viscosity for the dash at strain \( \gamma_3 \);

\( \eta_D \) = shear viscosity of the dash for deviatoric stress-strain relationship;

\( \eta_v \) = shear viscosity of the dash for volumetric stress-strain relationship;

\( \theta \) = power of the depth for limiting shaft stress profile;

\( \kappa \) = radial shear modulus non-homogeneity factor;

\( \lambda \) = relative stiffness ratio between pile Young's modulus and the initial soil shear modulus at just above the base level, \( E_p/G_{IL} \);

\( \lambda \) = relative stiffness ratio between pile shear modulus and the initial soil shear modulus at the depth of one pile radius, \( \lambda = G_p/(A_\pi r_o^n) \) (torsional case);

\( \lambda_n \) = the n-th root for the Bessel functions;

\( \lambda_r \) = \( P_{10}/P_{ult} \), load capacity reduction factor;

\( \mu \) = degree of pile-soil relative slip;

\( \nu_p \) = Poisson's ratio of a pile;

\( \nu_s \) = Poisson's ratio of soil;

\( \xi \) = shaft stress softening factor, when \( w > w_0 \);

\( \xi_b \) = pile base shear modulus non-homogeneous factor, \( G_{IL}/G_{ib} \);

\( \xi_r \) = outward radial movement;

\( \pi_1 \) = normalised pile displacement (Appendix C);

\( \pi_1^* \) = normalised local limiting displacement (Appendix C);

\( \pi_2 \) = normalised depth with pile length (Appendix C);

\( \pi_3 \) = normalised pile-soil relative stiffness factor (Appendix C);

\( \pi_4 \) = normalised pile-soil relative stiffness for plastic case (Appendix C);

\( \pi_{2p} \) = normalised depth with slip length (Appendix C);

\( \pi_t \) = non-dimensional relative torsional stiffness factor;

\( \rho_g \) = ratio of soil shear moduli at depths L/2 and L;

\( \sigma, \sigma_0 \) = total stress and its critical value (Chapter 2);

\( \sigma_2 \) = stress acted on the dashpot for the model by Murayama & Shibata (1961);

\( \sigma \) = effective stress;
\[ \sigma_{kk} = \text{volumetric stress}; \]
\[ \sigma_{vo} = \text{effective overburden pressure}; \]
\[ \tau(\dot{\tau}) = \text{shear stress (shear stress rate)}; \]
\[ \tau(\tau_{f0}) = \text{shear stress due to torsional loading}; \]
\[ \dot{\tau}_1 = \text{shear stress rate for spring 1 in the creep model}; \]
\[ \tau_{ave} = \text{average shear stress for equivalent homogeneous case}; \]
\[ \tau_c = \text{the fraction of shear stress causing flow}; \]
\[ \tau_f = \text{limiting local shaft stress}; \]
\[ \tau_{fj} = \text{(maximum) undrained (pile-soil) adhesion (j = 1, 3)}; \]
\[ \tau_j = \text{shear stress on elastic spring j (j = 1, 3)}; \]
\[ \tau_0 = \text{shear stress on pile soil interface}; \]
\[ \tau_{0(t)} = \text{shear stress on pile soil interface at the time of t}; \]
\[ \tau_{oj} = \text{shear stress on pile-soil interface at elastic spring j (j = 1, 2)}; \]
\[ \tau_p = \text{peak shear stress (Chapter 2)}; \]
\[ \tau_{ult} = \text{ultimate local shaft stress}; \]
\[ \tau_t = \text{ultimate local shaft stress for torsional case (Chapter 2)}; \]
\[ \tau_{ultj} = \text{ultimate (soil) shear stress for spring j (j = 1, 3) respectively}; \]
\[ \phi = \text{a fictitious stress system (Chapter 2)}; \]
\[ \phi = \text{local angle of twist of a pile}; \]
\[ \phi(z) = \text{angle of twist of pile at a depth of z}; \]
\[ \phi_b = \text{angle of twist of pile base}; \]
\[ \phi_e = \text{limiting elastic shaft rotation}; \]
\[ \phi_t = \text{pile head rotation or rotation at the transition level, z = L_1}; \]
\[ \chi = \text{a ratio of shaft and base stiffness factors for torsional loading}; \]
\[ \chi_v = \text{a ratio of shaft and base stiffness factors for vertical loading}; \]
\[ \chi_{v2} = \text{a ratio of shaft and base stiffness factors for a pile in a group of two piles}; \]
\[ \psi = \text{non-linear factor (\tau_0R_{f\theta}/\tau_f), stress level due to torsional loading}; \]
\[ \psi_j = \text{non-linear stress level for spring j (j = 1, 3) within the creep models}; \]
\[ \psi_o = \text{non-linear factor (\tau_0R_{f\theta}/\tau_f), stress level}; \]
\[ \psi_{oj} = \text{non-linear factor (\tau_{oj}R_{f\theta}/\tau_{max}), stress level on pile soil interface for spring j (j = 1, 2)}; \]
\[ \omega = \text{a pile base shape and depth factor}; \]
\[ \omega_b = \text{an empirical base modification factor}; \]
\[ \omega_h = \text{a coefficient for estimating \text{‘\omega’}, accounting for the effect of H/L}; \]
\[ \omega_{oh} = \text{the value of \omega_h at a ratio of H/L = 4}; \]
\[ \omega_v = \text{a coefficient for estimating \text{‘\omega’}, accounting for the effect of \nu_s}; \]
\[ \omega_{ov} = \text{the value of \omega_v at a ratio of \nu_s = 0.4}; \]
$\omega_2$ = base load transfer factor for two piles.

**Principal subscripts**

ave = average value
b = value for pile base;
e = at the transition depth from elastic to plastic zones;
f = failure;
max = maximum;
i = initial;
j = element number for the creep models;
p = pile;
s = soil;
t = pile head;
ult = ultimate value.
1. INTRODUCTION

1.1 BACKGROUND

Many numerical approaches and various closed form solutions have been proposed for analysis of single piles and, more particularly, for pile groups. However, for analysing a large pile group, it is rarely practicable, and in many cases impossible, to use rigorous numerical analysis alone, due to limitations in computing capacity, and time and cost constraints. Therefore hybrid load transfer numerical approaches have been proposed, which take advantage of the strength of numerical and analytical solutions to produce a complete numerical analysis. Such approaches are generally more efficient than other methods currently available. However, the approaches rely on the availability and accuracy of closed form solutions, which are of tremendous importance to practical pile group analysis.

Closed form solutions for a single pile subjected to vertical (or torsional) loading have been based either on point load solutions, e.g. Mindlin's solution (and Chan's solution), which is strictly only valid for homogeneous (and layered homogeneous), and elastic soil conditions, or on load transfer relationships relating the shear stress mobilised along the pile shaft to the local displacement. The load transfer approach appears to offer adequate accuracy and greater flexibility for considering visco-elasticity, non-linearity and heterogeneity of soil. The approach can be readily adapted to estimate pile group behaviour as well, and it requires much less computer storage compared with other approaches based on point load solutions. Therefore the development of closed form solutions should mainly be based on this approach.

Early empirical approaches for estimating load transfer curves have been extended and linked to more fundamental soil properties through the use of elastic or hyperbolic stress-strain models for the soil and the concentric cylinder approximation of shearing around the pile. However, the link is dominated by the load transfer factor, which in turn is significantly influenced by the following four factors: (a) non-homogeneous soil profile, (b) soil Poisson's ratio, (c) pile slenderness ratio, and (d) the relative ratio of embedment depth of the underlying rigid layer to the pile length. Therefore, it is essential to explore the effect of these four factors, so as to facilitate the application of load transfer analysis.
For a pile in a non-homogeneous soil, whether it is subjected to vertical or torsional loading, no exact closed form solutions are available except for a pile in an infinite homogeneous and/or Gibson soil.

In practical applications, piles may be subjected to time-varying loading, hence visco-elastic or creep response of the soil may be important. As shown by numerous experimental results, the deformation and strength of a soft soil is significantly time-dependent, due to the pronounced visco-elastic or creep properties. Similar response is demonstrated for piles in a clay, particularly at high load levels. The effect of load levels on the time-dependent response of piles needs to be clarified and quantified.

Driven piles normally generate excess pore pressures in the surrounding soil. Dissipation of the pore pressures following driving is predicted currently by available elastic theory. However, viscosity is pronounced for many soft clays, therefore its effect should be suitably accounted for. The gradual increase in pile capacity is dominated by the dissipation of excess pore pressure as has been widely explored both experimentally and theoretically. To predict the load-settlement response, the variation of pile-soil stiffness with the dissipation of pore pressure must also be quantified.

Currently available closed form solutions for assessing the settlement of pile groups are not unified in respect of either the pile-soil relative stiffness or the number of piles within a group (as shown in Chapter 2). The solutions are generally limited to piles in infinitely deep layer. The effect of a finite depth of compressible soil is not included. Non-homogeneity of the soil profile has been considered approximately, but needs to be handled more accurately, since a slight difference in estimating pile-soil-pile interaction factors may have considerable effect on the prediction of the overall response of large pile groups.

It is not yet fully clear how the torsional pile response is affected by the non-homogeneous soil profile and elastic-plastic soil response. Therefore some efforts are devoted to this direction.

1.2 OBJECTIVES

The aim of this research was to tackle the problems referenced above, specifically to establish:
(1) closed form solutions for a pile in non-homogenous elastic-plastic media under vertical loading, in terms of load transfer models;

(2) formulae for estimating load transfer factors, calibrated against more rigorous numerical analysis, particularly to explore the rationality of the load transfer approach;

(3) a non-linear visco-elastic load transfer model, which is a logical extension of the elastic model, allowing the elastic solutions established previously to be readily extended to account for visco-elastic effects;

(4) visco-elastic soil consolidation theory for the radial dissipation of pore water pressure following pile installation, so that the overall performance of a pile during the phase of reconsolidation may be quantified;

(5) unified exact solutions for estimating the settlement of (large) pile groups enabling the effects of the four factors discussed in Section 1.1 to be considered;

(6) closed form solutions for a pile subjected to torsional load in non-homogeneous elastic-plastic soil.

The particular form of soil non-homogeneity addressed in the thesis is that the soil shear modulus and limiting shaft shear stress vary as a power of depth. For vertically loaded piles, the new load transfer factors have been calibrated against more rigorous numerical analysis for a variety of soil and pile parameters, allowing the closed form solutions to be automatically extended to most cases of practical interest.

1.3 CLOSED FORM AND NUMERICAL SOLUTIONS

The closed form solutions are all expressed in the form of Bessel functions, for which, the numerical estimation in this thesis has been performed by Mathcad™ and newly designed spreadsheet programs operating in Windows EXCEL.

A non-linear load transfer analysis operating in Windows EXCEL has been developed, which enables the overall response of a single pile to be predicted for the instances of either vertical or torsional loading. The program has been utilised to verify the closed form solutions, and explore the influence of non-linearity of soil stress-strain.
Chapter 1

1.4 Introduction

To verify pile-head stiffness predicted by the closed form solutions outlined above, numerical analysis has been performed using the finite-difference program FLAC (Itasca, 1992). Load transfer factors have been back-figured extensively to consider the effect of the four factors discussed in Section 1.1, through comparisons between the FLAC analysis and the closed form solutions. The back-estimation has been undertaken through a program written in FORTRAN. In light of the back-figured load transfer factors, the rationality of the load transfer approach has therefore been extensively re-examined.

1.4 ORGANISATION OF THE DISSERTATION

A review of the literature pertaining to this research is presented in Chapter 2, which covers the performance of single piles subjected to vertical and torsional loading and pile groups subjected to vertical loading, with particular attention being paid to time-dependant, non-homogeneous soil properties.

Closed form solutions for vertically loaded piles in non-homogeneous elastic-plastic media have been established and compared extensively with previous numerical analyses as shown in Chapter 3. Non-linear stress-strain effect has been explored numerically.

Load transfer factors have been extensively calibrated using FLAC analysis, and have been provided in simple formulae in Chapter 4. The influence of different soil and pile parameters on the values of load transfer factors, and the sensitivity of pile-head stiffness to the load transfer factors have been explored. Finally, the rationality of the load transfer analysis has been clarified.

A non-linear visco-elastic load transfer approach has been proposed in Chapter 5. Both closed form and numerical solutions for single pile response are generated and compared with more rigorous numerical analysis. The effect of the time-scale of loading has been explored.

New closed form solutions governing visco-elastic soil consolidation around a driving pile have been produced in Chapter 6. In terms of several case studies, the effect of reconsolidation on the pile-soil interaction stiffness has been explored, allowing the time-dependant load-settlement response to be identified.
Chapter 1

1.5 Introduction

Pile group behaviour in non-homogenous media has been explored by a new unified approach, focusing particularly on the settlement of large pile groups. This is provided in Chapter 7.

Closed form solutions for torsional pile response in non-homogenous media have been established in a similar form to those for vertically loading piles, and are presented in Chapter 8. The effect of non-linear soil stress-strain response is explored as well.

The major conclusions and recommendations arising from this research are summarised in Chapter 9. Areas that may be studied further are highlighted

A number of relevant algebraic details have been provided in Appendix A to F. In particular, a program called GASPILE has been designed, which is shown in Appendix A, for estimating the load-settlement behaviour of a pile subjected to either vertical or torsional loading. The difference and similarity of the pile responses due to the two kinds of loading are explored. Non-dimensional closed form solutions for vertically loaded piles in strain-softening soil have been provided in Appendix C. Closed form solutions for radial consolidation in a radially non-homogeneous medium has been illustrated in Appendix E.
2. LITERATURE REVIEW

2.1 INTRODUCTION

Analysis of piles can be broadly classified into: (1) empirical methods, (2) numerical methods, (3) closed form solutions, and (4) a combination of these methods, (e.g., the hybrid method, which is a combination of (2) and (3)). Empirical methods and numerical approaches have been widely proposed, developed and refined. Nevertheless, relatively few closed form solutions have been proposed.

This thesis aims at the development of closed form solutions for piles, as mentioned in Chapter 1, that can capture the non-linear, non-homogeneous and visco-elastic properties of soil. In order to achieve such solutions, it is necessary to perform a review of the relevant literature, which has been organised according to the problems listed in the previous chapter. Particularly, key numerical and empirical methods will be summarised, as these will be used for comparison and verification of the current research.

2.2 VERTICALLY LOADED SINGLE PILES

A number of procedures have been proposed for predicting overall pile response, namely:

(1) Numerical analyses or simple closed form solutions based on either empirical load transfer curves or theoretical load transfer curves derived using a concentric cylinder approach.

(2) Numerical procedures based on hypothetical shaft and base load-settlement relationship respectively.

(3) Various rigorous numerical approaches, e.g., finite element analysis (FEM), boundary element method (BEM), and variational method (VM).

The research performed so far has been generally concerned with the pile-head stiffness, and the load and settlement distribution along the pile and the manner in which these quantities are affected by (a) non-homogeneity of either the soil shear modulus profile or the assumed shaft stress distribution, (b) the pile-soil relative stiffness, (c) relative thickness of the compressive soil layer compared with the pile length, (d) non-linear soil stress-strain response, and (e) slip development along the pile-soil interface.
Chapter 2  

2.2 Literature Review

The load transfer approach will be addressed first.

2.2.1 Load Transfer Approach

Load transfer analysis is an uncoupled approach that treats the shaft and base as independent elastic springs, Fig. 2-1(a). The behaviour of the elastic springs can be based on either empirical or theoretical relationships, referred to conventionally as t-z (shaft) and q-z (base) load transfer curves.

2.2.1.1 Empirical (1D) Load Transfer Approaches

The load transfer approach was originally based on direct measurement of local load-displacement response at different depths along the pile-soil interface (Fig. 2-1b) as reported by many researchers, e.g. Seed and Reese (1957), Coyle and Reese (1966), Coyle and Sulaiman (1967). Various functions have been proposed to fit the measured shaft and base load displacement data, namely:

(1) exponential functions by Kezdi (1957), Liu and Meyerhof (1987), Vaziri and Xie (1990), Georgiadis and Saflekou (1990);
(2) empirical functions by Reese et al. (1969), and Vijayvergiya (1977);
(3) elastic, perfectly plastic model by Satou (1965), and Fujita (1976);
(4) hyperbolic functions by Hirayama (1990);
(5) tri-linear function by Frank and Zhao (1982), Frank, et al. (1991), Zhao (1991), Tan and Johnston (1991), and Kodikara and Johnston (1994);
(6) Ramberg-Osgood function, as shown in Fig. 2-2, by many researchers, e.g. Abendroth and Greimann (1988), Armaleh and Desai (1987), O'Neill and Raines (1991).

Some of these transfer functions have been summarised in Tables 2-1 and 2-2 for axial pile analysis. The coefficients governing these functions are adjusted to simulate the measured data. However, as evidenced later, the local load transfer behaviour is mainly affected by the following four factors:

(a) soil Poisson's ratio;
(b) relative layer thickness ratio, that is the ratio of the depth of the underlying stiff stratum below the groundline, H to the pile length, L, H/L;
(c) shear modulus value and its variation with depth;
(d) pile geometry (e.g., pile slenderness ratio).
Therefore, in principle, those factors should be used as variables to fit the measured data rather than the irrelevant empirical curve fitting coefficients. In addition, all those empirical curves based on fitting measurement on the pile-soil interface reaction cannot reflect the soil reaction around the pile. Thereby, these curves obtained from a single pile test should not be utilised to predict behaviour of pile groups. Therefore, the analysis based on these groups of curves can be regarded as one-dimensional (1D) empirical approach.

By directly using a measured load transfer curve, a satisfactory evaluation of the pile behaviour might be obtained, compared with that measured (Coyle and Reese, 1966). (Note: that is probably why so many empirical functions have been proposed, as shown in Table 2-1.) However, the good comparison is the adoption of an correct value of the tangential shaft stiffness, $\tau/w$ for the specific cases. For subsequent reference, the shear modulus and/or limiting shaft shear stress might be back-figured from the measured load transfer curves, and should suitably account for the effect of the four factors.

2.2.1.2 Theoretical (2D) Load Transfer Models

(a) Shaft Model

The early empirical approaches shown in Table 2-1 have been extended and linked to more fundamental soil properties through a load transfer function. This function for the shaft may be derived from the stress-strain response of the soil using the concentric cylinder approach, which itself is based on a simple $1/r$ variation of shear stress around the pile (where $r$ is the radius), (e.g. Frank, 1974; Cooke, 1974; Randolph and Wroth, 1978). For a hyperbolic stress-strain model, the local stress and displacement relationship can be expressed as (Randolph, 1977; Kraft et al. 1981)

$$ w = \frac{\tau_0 r_0}{G} \zeta $$

(2-1)

where

$$ \zeta = \ln\left[\frac{(r_m/r_o - \psi_o)/(1 - \psi_o)}{(r_m/r_o - \psi_o)/(1 - \psi_o)}\right] $$

(2-2)

where $G$ is shear modulus at any depth; $\zeta$ is the shaft load transfer factor; $\tau_o$ is the local shaft shear stress; $r_o$ is the pile radius; $\psi_o = R_{fs} \tau_o / \tau_f$, which is the stress level on the pile-soil interface; $R_{fs} = \tau_f / \tau_{ult}$, a parameter which controls the degree of non-linearity; $\tau_{ult}$ is the ultimate local shaft stress; $r_m$ is the maximum radius of influence of
the pile beyond which the shear stress becomes negligible, and may be expressed in
terms of the pile length, L, as
\[ r_m = A\rho_g(1 - \nu_s)L + B \rho_0 \]  

(2-3)

where \( \rho_g \) is non-homogeneity factor, \( A = 2 \) to \( 2.5 \) (Randolph and Wroth, 1978, 1979a),
\( B = 0 \) to \( 5 \) (Randolph, 1994). The value of \( A \) may be adjusted to allow for the effect of
an underlying rigid layer, with the value decreasing as the depth to the rigid layer
decreases. Randolph (1994) has suggested increasing the value of \( B \) from \( 0 \) (applicable
for most piles) to \( 5 \) for piles where the length to diameter ratio is less than \( 10 \).

When the shear stress at the pile-soil interface exceeds the limiting shaft stress, \( \tau_f \), the
relationship between the shear stress and displacement has generally been determined
by the following ways: (1) direct shear simulation (Kraft et al. 1981); (2) an assumed
strain-softening curve (Randolph, 1986); (3) an assumed constant of \( \xi \tau_o \) (\( 0 < \xi \leq 1 \)).
For instance if \( \xi = 1 \), an ideal plastic load transfer is assumed upon reaching the plastic
stage, as demonstrated in Fig. 2-3; (4) an extension from the elastic empirical curves as
shown in Table 2-1.

Singh and Mitchell (1968) proposed an empirical creep model. For pile analysis, it has
been re-cast in the form (Ramalho Ortigao and Randolph, 1983)
\[ \Delta w = \beta_c w^* (\Delta t/t)^{mc} \exp(\alpha_c \tau_o/\tau_f) \]  

(2-4)

where \( \Delta w \) is displacement increment, \( \Delta t \) is time increment, \( w^* \) is the displacement to
mobilise peak skin friction (at the load transfer curve). Typical values for the constants
are: \( \alpha_c = 6 - 8 \), \( m_c = 0.75 - 1.2 \), \( \beta_c = 0 - 0.01 \) (Singh and Mitchell, 1968; Randolph,
1986). As illustrated in Fig. 2-4, the creep process has been regarded as a stress
relaxation process, and therefore the load transfer curve is shifted by a small amount
over each time increment. However, the creep is assumed to occur only above the yield
point (\( \tau_o \geq \xi \tau_p \), \( \tau_p \) = peak stress), as implemented in load transfer analysis of RATZ
(Randolph, 1986).
(b) **Base Interaction Model**

The base settlement can be estimated from the solution of a rigid punch resting on an elastic half-space

\[ w_b = \frac{P_b(1 - \nu_s)\omega}{4r_0G_b} \tag{2-5} \]

where \( G_b \) is the shear modulus just below the pile tip level; \( P_b \) is the mobilised base load; \( \omega \) is the pile base shape and depth factor, referred to as base load transfer factor, which is generally chosen as unity (Randolph and Wroth, 1978; Armaleh and Desai, 1987).

(c) **Comments on the Load Transfer Factors**

The shaft stiffness, \( T/w \) can be expressed explicitly by an equivalent value of \( G_i/r_0 \zeta \) as from Eq. (2-1). The shear modulus can also be back-figured by Eq. (2-1), once the factor, \( \zeta \) and the stiffness, \( T/w \) are known. The effect of the four factors, listed earlier in the section 2.2.1.1, on the assessment of shear modulus (or stiffness) can be explicitly accounted for by \( \zeta \). Therefore Eq. (2-1) is preferred to other empirical (1D) functions.

The key factors of \( \zeta \) and \( \omega \), referring to Eqs. (2-1) and (2-5), should be back-figured in terms of the stress and displacement obtained from more rigorous numerical analysis, (e.g., a continuum based Fast Lagrangian Analysis of Continua (FLAC) (Itasca, 1992)). As shown previously, Randolph and Wroth (1978) provides the simple way of estimating the shaft load transfer factor by Eq. (2-3), taking \( \omega = 1 \), which normally predicts pile-head stiffness sufficiently accurate in terms of their simplified formula, namely Eq. (2-7) as shown later, for a pile in a infinite layer. However it does not accurately reflect the distribution of pile load and settlement along the pile (Rajapakse, 1990) or the behaviour of an end-bearing pile subjected to downdrag (Lim et al. 1993). As explored later in Chapter 4, load transfer factors are considerably influenced by the listed factors of (a) to (d) (section 2.2.1.1), and even the closed form equation (accurate or approximate). Therefore, the suitability of the load transfer factors should be examined with respect to the corresponding closed form solution compared with continuum based numerical analysis under the desired conditions.
(d) **Shaft Limiting Stress and Stiffness**

The shaft model expressed by Eq. (2-1) represents a two dimensional (2D) simulation of pile-soil interaction, which considers the horizontal non-linear soil contribution by the integrated factor, $\zeta$. For the vertical dimension, two key facets of pile-soil interaction need to be accounted for, namely: the profiles of stiffness and limiting strength on the pile-soil interface. The former parameter controls the pile elastic response, while the latter offers evaluation of the limiting shaft displacement, hence the plastic pile-soil interaction.

Prediction of the limiting strength on the pile-soil interface is one of the most popular subjects. A number of empirical formulas have been proposed, as summarised previously by many researchers (e.g., Kraft et al. 1981; Poulos, 1989), which are briefly described here as:

1. a total stress method (\(\alpha\)-method), in which the shaft stress is correlated to the undrained shear strength, \(s_u\) through the empirical parameter \(\alpha\) (e.g. Woodward and Boitano, 1961; Tomlinson, 1957, 1970; Flaate, 1972; McClelland, 1974);
2. an effective stress method (\(\beta\)-method), where the shaft stress is correlated to the initial effective overburden stress, \(\sigma_{vo}'\) in terms of the empirical parameter \(\beta\) (Zeevaert, 1959; Eide et al. 1961; Chandler, 1968; Clark and Meyerhof, 1972, 1973; Burland, 1973; Mayerhof, 1976; Flaate and Selnes, 1977; Burland and Twine, 1988);
3. the Lambda method, as proposed by Vijayvergiya and Focht (1972), which is related to a combination of \(s_u\) and \(\sigma_{vo}'\); and
4. empirical formula considering the overconsolidation effect (Randolph and Murphy, 1985; Azzouz et al. 1990).

Generally these formulae are deduced from the equilibrium of a rigid pile, and are mainly concerned about the soil behaviour, (e.g. the strength, overconsolidation ratio and overburden vertical stress). To account for pile length effect, Kraft (1981) correlated the \(\lambda\) value (the Lambda method) to a non-dimensional pile-soil relative stiffness ratio proposed by Murff (1980), on the basis that decreasing capacity with increasing length was associated with a strain-softening load transfer curve and progressive failure. Poulos (1982) has argued that the length effect noted from pile load test may be largely attributed to the definition of failure at a pile-head displacement of 10% of the pile diameter. This is illustrated in Fig. 2-5. In the figure, \(\lambda_r\) is the load capacity reduction factor.
\[ \lambda_r = \frac{P_{10}}{P_{\text{ult}}} \]  

where \( P_{10} \) is the pile-head load required to cause a head settlement of 10% of pile diameter, \( P_{\text{ult}} \) is the ultimate total pile capacity. Randolph (1983) found that the length effect is largely attributed to the development of pile-soil relative slip, combining with the pile-soil relative stiffness (Fig. 2-6).

As a consequence, a realistic value of the limiting strength might be back-figured, based on known soil modulus and measured pile load-settlement response, through sophisticated numerical or closed form approaches, which should account for:

1. equilibrium of a pile-soil system;
2. pile-soil deformation compatibility;
3. realistic pile-soil load transfer behaviour.

Such numerical or analytical approaches have been established in Chapter 3.

Soil shear modulus can be estimated through field tests, for example, standard penetration test (SPT), Cone penetration test (CPT), self-boring pressuremeter tests, screw plate tests and seismic methods. Laboratory tests generally give lower values than from field tests. Many researchers have attributed this difference to sampling disturbance, although there is also a significant sample size effect, which can affect the stress condition within the sample and hence the measured stiffness (Yin et al. 1994). The effect of pile size (dimensions) on response of a loading test might be simulated through the \( \zeta \) in Eq. (2-1) in the load transfer approach.

In short, the main challenge in predicting the axial performance of piles lies in establishing the load transfer functions for the shaft and base, which are linked to fundamental properties of the soil and yet which allow for non-homogeneity, non-linearity and time dependence of the soil response; and the challenge in generating load transfer factors suitable for various conditions, which result in close agreement with results from continuum based numerical analysis, similar to the analysis by Randolph and Wroth (1978) for a pile in an infinite layer.

The load transfer approach based on the 2D model can lead to closed form solutions for a pile in a non-homogeneous media, and the solutions for estimating pile group behaviour. Similarly, the solutions for a single pile can also be implemented into
hybrid analysis, allowing for analysis of large pile groups. All these will be reviewed in later relevant sections.

2.2.2 Closed Form Solutions

Establishment of solutions for vertically loaded single piles in closed form has been based on Mindlin’s (1936) solution and load transfer approach.

2.2.2.1 Based on Mindlin’ Solution

Nishida (1957), Przystanski (1963) developed approximate elastic solutions for piles, based on Mindlin's solution for a vertical point load in a homogeneous, isotropic elastic half-space. D’Appolonia and Romulldi (1963) explored load transfer mechanism of end-bearing piles, using Mindlin’s solution. Mindlin’s solutions later formed the basis for numerical solutions for pile response, as discussed in detail in section 2.2.4.

2.2.2.2 Based on Empirical (1D) Model

Solutions based on the (1D) load transfer model first appeared in the middle of the 1960s. With a linear elastic, perfectly plastic shaft and base model, closed form solutions for single piles in homogeneous soil media were systematically derived, (e.g. Satou, 1965; Murff, 1975). The solutions required homogeneous soil constants, with uniform pile-soil shaft interaction stiffness and limiting shaft stress. For non-homogeneous case, equivalent values of stiffness and limiting stress had to be found. To obtain these values of stiffnesses and limiting stresses for non-homogeneous soil, Fujita (1976) generated empirical formulae as shown in Table 2-1, based on a database of about 30 pile loading tests and corresponding in situ SPT test results.

Some progress in the 1D based load transfer approach has been attempted during the past 20 years, in considering "non-linear" and stress-strain softening behaviour (e.g. Murff, 1980; Kodikara and Johnston, 1994). However, none of the approaches proposed so far can handle accurately the effect of a non-homogeneous soil profile.

Murff (1975) generated non-dimensional closed form solutions for a pile in a homogeneous elastic-plastic media. Later, he extended it to account for strain softening behaviour (Murff, 1980) by taking the shaft stress as \( \xi \tau_f \) (0< \( \xi \) ≤1), once the stress exceeds the peak shaft strength, \( \tau_f \).
Kodikara and Johnston (1994) extended the solutions by Murff (1980) to account for a tri-linear shaft load transfer model as shown in Fig. 2-7, where three different stages have to be considered along the pile, Fig. 2-8.

Motta (1994) reported a consideration of elastic-plastic behaviour for a pile in Gibson soil. A number of assumptions made are listed here: (1) Tip resistance is ignored; (2) Pile-soil interface stiffness, $\tau/w$ is taken as an equivalent constant, which is an average value for the upper length of 25 pile diameters; (3) a sufficiently large extent of elastic zone exists. As long as the above conditions are satisfied, the approximate solution (Motta, 1994) can be used, and the accuracy will be within 20% (Motta, 1994) for the prediction of the pile-head response. As a matter of fact, the solutions are essentially identical to those proposed by Satou (1965).

Castelli et al. (1993) proposed solution for a single pile in a homogeneous elastic media in a new form, which is essentially identical to those given by Satou (1965) and Murff (1975). They suggested to account for non-linear pile-soil interaction by decreasing the global shaft load transfer factor, which is equivalent to the $\sqrt{\pi^3}$ by Murff (1975), as pile-head load level increases. The load level is defined as the ratio of pile-head load to the sum of the ultimate shaft and base load. The pile-head load-settlement can be predicted numerically by this approach. However, the global factor is generally reduces with the development of pile-soil relative slip as shown in Appendix B.

2.2.2.3 Based on Theoretical (2D) Model

From the mid 1970s to the early 1980s, the load transfer mechanism was explored both theoretically (e.g. Randolph and Wroth, 1978; and Kraft et al. 1981) and experimentally (e.g. Cooke, 1974). This work led to the theoretical load transfer relation, Eq. (2-1), that empirically links the gradient of the load transfer curve to the elastic shear modulus of the soil. Randolph and Wroth (1978) also provided an approximate estimation of the pile-head stiffness, which is defined as

\[ \frac{P_t}{(G r_w)} \]

Note that except where specified, pile-head stiffness will be referred to as the value of $P_t/(G r_w)$ in this thesis.
\[
\left( \frac{P_t}{G_L \rho_0 w_t} \right)_1 = \frac{4}{1 - \nu_s} + \frac{2 \pi \rho_g L \tanh \beta}{L \rho_0 \beta} - \frac{\zeta}{\pi \lambda} \frac{1 - \nu_s}{1 - \nu_s} \tan \frac{\beta}{\rho_0} \left( \frac{\lambda}{\rho_0} \right) \tan \frac{\beta}{\rho_0} \left( \frac{\lambda}{\rho_0} \right)
\]

where \( \beta = \sqrt{2 / (\zeta \lambda) L / \rho_0} \), \( P_t, w_t \) are the pile-head load and settlement respectively, \( G_L \) is the shear modulus at depth \( L \), \( \lambda = E_p / G_L \). This approximate equation is essentially identical to that by Murff (1975), where \( \tau / w = G / (\rho_0 \zeta) \), \( \beta^2 = \pi_3 \). However, Eq. (2-7) is directly comparable with more rigorous continuum based numerical analysis.

The theoretical load transfer approach offers the greater flexibility and sufficient accuracy compared with more rigorous numerical approaches. Besides, if a suitable load transfer model can be established, solutions in closed form can be formulated even for visco-elastic, non-homogeneous case as shown in Chapters 3, 4 and 5.

2.2.3 Numerical Solutions Based on Discrete Element

2.2.3.1 Load Transfer Approach

\( (a) \) Based on Empirical (1D) Model

Seed and Reese (1955) presented an analytical method of predicting pile load-settlement curves, by using the measured relationship between pile resistance and the pile movement at various points along the pile as provided previously in Fig. 2-1. They divided the pile into small sections and considered the equilibrium of each section separately. Coyle and Reese (1966) developed Seed and Reese's method. The load-settlement curve for the pile head is synthesised by numerical integration of the different load transfer relations.

Kiousis and Elansary (1987) presented a simple method to calculate the load-settlement relation for an axially loaded pile, which resembles the method presented by Coyle and Reese (1966), but in contrast, the equilibrium of the pile during loading is considered globally. An example comparison shows that the pile-head stiffness predicted from global equilibrium is slightly higher than that by local equilibrium of each sections.

Based on one-dimensional idealisation of a pile, Armaleh and Desai (1987) performed a one-dimensional finite element analysis for axially loaded piles. Non-linear Winkler springs were adopted to represent the response of the soil along the shaft and at the pile
A generalised Ramberg-Osgood model, as shown in Fig. 2-2, was used to simulate the shaft and base non-linear response. Good comparison with measured load-settlement curves resulted for piles in sand. In a similar way, Abendroth and Greimann (1988) carried out FEM studies which includes material and geometric non-linearity, utilises two-dimensional beam elements for the pile, and uncoupled non-linear Winkler soil springs for shaft and tip response. Based on curve-fitting measured strain data, they also developed the soil resistance and the displacement relationship in the form of Ramberg-Osgood expressions. The Ramberg-Osgood model does offer a flexible fitting for shaft and base load transfer behaviour (Armaleh and Desai, 1987; Abendroth and Greimann, 1988; O'Neill and Raines, 1991), but the t-z function generally varies with the four factors shown in the section 2.2.1.1, even within the same site. Therefore, it is difficult to choose suitable coefficients for the model for future design.

(b) Based on Theoretical (2D) Model

Randolph (1986) developed a load transfer based program, RATZ in which the theoretical load transfer models of Eqs. (2-1) and (2-6) are adopted. The predicted load-settlement relationships normally compare well with more rigorous continuum based analysis. The advantage of this analysis is that

1. The parameters, e.g., soil shear modulus, can be directly obtained;
2. Based on measured load-settlement relationships, shear modulus of soil can be back-figured through the program.

However, the program is confined to Fortran environment, therefore a spreadsheet program operating in EXCEL has been developed in this research.

2.2.3.2 Direct Hyperbolic Load Transfer Approach

Fleming (1992) proposed a numerical procedure for estimating pile load-settlement behaviour based on separate hyperbolic laws (Chin, 1970; Chin and Vail, 1973) for the shaft and base responses, the responses were then combined making due allowances for elastic shortening of the pile. A program called CEMSET was developed to facilitate the analysis. Although the predictions are satisfactory, compared with the measured response of many piles, several concerns need to be explored

1. A hyperbolic soil stress-strain relationship (Duncun and Chang, 1970) does not lead to a hyperbolic load transfer curve (Kraft et al. 1981; Randolph, 1994);
hence, particularly for a rigid pile, it cannot result in an integrated shaft load-displacement that may be modelled as a hyperbolic curve.

(2) The consequence of using hyperbolic models for shaft and base respectively leads to a result that violates the hyperbolic load-settlement relationship (Chin, 1970; Chin and Vail, 1973; Poskitt et al. 1993).

(3) The parameters used in the model are not directly related to soil properties, therefore the parameters have to be back-figured through the program only;

(4) Due to (3), CEMSET analysis is difficult to be directly checked with a more rigorous analysis.

(5) The method cannot be used to predict load distribution down a pile.

In essence, the principle of this method is identical to the “Empirical (1D) Load Transfer Approach”, but uses one element for the whole pile shaft.

2.2.4 Rigorous Numerical Analysis based on Continuum Media

As it is well known, a number of numerical procedures have been developed, and applied in the analysis of axial pile response.

2.2.4.1 Boundary Element Approach Based on Mindlin's Solution

(a) Butterfield and Banerjee (1971)

The essence of the boundary element approach is to find a fictitious stress system $\phi$ which, when applied to the boundaries of the figure inscribed in the half space, will produce displacements of the boundaries which are identical to the specified boundary conditions of a real pile system of the same geometry and also satisfy identically the stress boundary conditions on the free surface of the half space. The stress $\phi$ are fictitious in that they are to be applied to the boundaries of the fictitious half space figure and are therefore not necessarily the actual stresses acting on the real pile surfaces. However, once the $\phi$ values have been determined it is a simple matter to calculate the actual stresses and displacements they produce anywhere in the half space, including those on the real pile boundaries. The total vertical and radial displacements at a point due to a pile loaded vertically are expressed through integral equations as functions of $\phi$ and coefficients derived from Mindlin's solution (Butterfield and Banerjee, 1971). Radial displacement compatibility is ignored, since it generally produces negligible effects on the total load required for a given settlement. The integral equations are then estimated numerically, in a way that the pile shaft is divided
into n equal segments and the base into m rings. With this approach, Butterfield and Banerjee, (1971) provided the relationships between pile-head stiffness and the pile slenderness ratios for single piles and different pile groups.

(b) Banerjee and Davies (1977)

Banerjee and Davies (1977) reported non-dimensional load displacement behaviour of axially loaded pile embedded in Gibson soil by utilising a boundary integral method (BI). They showed the substantial effect of soil profiles on pile-head stiffness, load distribution down the pile and pile-soil-pile interaction factors, and hence pile group behaviour. The approach, however based on Mindlin's (1936) solutions are not strictly valid for a non-homogeneous, elastic half space.

(c) Poulos (1979)

Poulos (1979) adopted a boundary element approach (BEM) to analyse a single pile in non-homogeneous soil. As shown in Fig. 2-9, the method involves division of the pile into a number of elements, each acted upon by an unknown interaction stress. The vertical displacements of the pile at each location are expressed in terms of the unknown interaction stresses and the pile properties while the soil displacements are expressed in terms of the interaction stresses and the soil properties. If no slip occurs at the pile-soil interface, the expressions for pile-soil displacement can be equated and the resulting equations solved for the interaction stresses, the displacement along the pile can then be evaluated. The displacement influence factor may be evaluated by integration of the Mindlin equation for vertical displacement due to a vertical subsurface point load acting within a semi-infinite mass.

For the non-homogeneous condition, an equivalent value of shear modulus has been adopted, which is an average of the soil modulus at elements i and j. The soil non-homogeneous property below the pile tip has been considered approximately by an extension of the Steinbrenner approximation (1934). This analysis is generally consistent with that by BI analysis (Banerjee and Davies, 1977), except for short piles. In fact, for short piles, the BI analysis is reported to overestimate the pile-head stiffness by 20% (Rajapakse, 1990). Shear stress distribution along a pile is considerably affected by the soil profile, as shown in Fig. 2-10. However, for a stiff pile, the distribution of the shear stress down the pile is similar to the shear modulus profile, implying uniform shear strain with the depth. The settlement influence factor is
generated for given pile-soil relative stiffness of various slenderness ratio in a soil layer of $v_s = 0.3$, $H/L = 2$ ($H$ is depth of rigid layer).

(d) **Poulos (1989)**

Poulos (1989) reported an analysis of a pile load-settlement behaviour in a homogeneous soil based on the boundary element (BEM) analysis described above. Three different interface models have been adopted, namely: an elasto-plastic continuum based interface model, a hyperbolic continuum based interface model, and a load transfer model respectively. The analyses showed that except for the case of extremely high pile Young's modulus (e.g. $E_p = 30,000$ GPa), load transfer analysis provides an excellent prediction of pile load-settlement compared with the continuum based approaches and also the FEM analysis by Jardine et al. (1986).

2.2.4.2 **Boundary Element Approach Based on Chan's Solution**

Chin et al. (1990) reported a simplified elastic continuum boundary element method, in which the soil flexibility coefficients were evaluated using the analytical solutions for a layered elastic half space (Chan et al. 1974). The use of such solutions is theoretically more correct than the approximate procedures using Mindlin's homogeneous solutions. Radial displacement compatibility at the pile-soil interface was not included as it does not influence significantly the pile response (Mattes, 1969). Two kinds of idealisations of the pile-soil forces were adopted; a circular "patch" load over the cross-sectional area at the pile nodes and that of a "ring" load over the outer circumferential area of the pile elements. The pile-head stiffness against the pile slenderness ratio was provided for both homogeneous and Gibson soil by both "patch and ring" approaches. Finite layer effect was also explored and expressed as settlement influence factor against pile slenderness ratio.

2.2.4.3 **Finite Element Method**

Randolph and Wroth (1978) performed a comprehensive numerical exploration of load transfer behaviour of a single pile. In particular, two kinds of numerical analyses for rigid piles are: (1) Integral equation analysis for a rigid pile of various slenderness ratios in a soil of two Poisson's ratios: $v_s = 0$, 0.5; and (2) Finite element analysis for a rigid pile in a soil of Poisson's ratio: $v_s = 0.4$, and a profile of either homogeneous or Gibson types. Pile-head stiffness and its radial distribution away from the pile axis
have been presented. Shear stress distribution for the two types of soil profiles has been explored. The effect of Poisson's ratio and non-homogeneity of soil profile on a pile response has been investigated. For instance, for a rigid pile embedded in a Gibson soil, at the midpoint of the pile, the shear stress is of the order of half that for the homogeneous case. Therefore, they inferred that the horizontal influenced radius, $r_m$, is about half its value for the homogeneous case. This analysis led to the simple equation of Eq. (2-3) for the load transfer factor. Later, through a finite element analysis, they (Randolph and Wroth, 1979a) improved the equation to account for the effect of non-homogeneity and end-bearing on pile-head stiffness. However, the influenced radius could be more accurately calibrated by the comparison between numerical analysis and closed form solution.

Three dimensional FEM analysis has been performed by Trochanis et al. (1991), which included interface elements for representing slippage and pile-soil separation, based on an elasto-plastic (a generalised Drucker-Prager) model. The analysis showed that pile-soil slippage is practically the only source of non-linear behaviour under purely axial loading.

### 2.2.4.4 Variational Element Method

Rajapakse (1990) proposed a variational formulation (VM) coupled with a boundary-integral representation of the linearly increasing shear modulus with depth (non-homogeneous), and incompressible soil medium. He found that the approximate solution, Eq. (2-7) provides an estimation of pile-head stiffness sufficiently accurate for slenderness ratio exceeding 20. However, appreciable differences are noted in the prediction of pile base load, and hence, the load distribution down the pile.

In a word, the results from the BEM, VM and FEM analyses are generally consistent with each other except those reported by Banerjee and Davies (1977), which gives higher stiffness than others reported. The non-linear pile-soil interaction can be considered by pile-soil relative slip alone for purely axial loading.

### 2.2.5 Consideration of Non-homogeneity

Numerical analysis for elastic case (Banerjee and Davies, 1977; Poulos, 1979) showed that the non-homogeneity of shear modulus has significant influence on the pile-head stiffness, and load distribution down the pile. For a slender pile, relative pile-soil slip
could be developed (Randolph, 1983). Therefore, the non-homogeneity of the shaft limiting stress gains importance. Analysis of a pile should generally embrace the non-homogeneity of both the shear modulus and the limiting shaft shear stress.

Consideration of non-homogeneity in closed form solutions is currently confined to elastic stage and based on either shear modulus non-homogeneous factor or shaft stress distribution factor.

2.2.5.1 Based on Shear Modulus

Owing to the fact that given identical conditions, shaft stiffness defined as \( \frac{P_s}{(G_1r_0w_t)} \) \( (P_s = \text{shaft load}) \) of a rigid pile in a non-homogeneous soil will be reduced by a factor of \( \rho_g \) compared with that in homogeneous soil. Randolph and Wroth (1978) suggested to use the factor to predict approximately the head stiffness of a compressible pile, as shown in Eq. (2-7). This treatment gives sufficiently accurate prediction of the stiffness as evidenced by the FEM analysis. Probably due to the fact that as long as \( \lambda \) is high, as illustrated in Fig. 2-10 for \( \lambda = 2600 \), the similarity between the profiles of shear stress and modulus exists, similar to that for a rigid pile. However, for lower \( \lambda \), the similarity no longer exists, as demonstrated by the results from Rajapakse (1990). Therefore, the accuracy of the treatment for predicting the pile-head stiffness decreases as pile-soil relative stiffness reduces. Obviously, the treatment is not feasible for evaluating load and settlement distribution down the pile.

2.2.5.2 Based on Stress Distribution

(a) Bearing Capacity Estimation

Current design of single piles, particularly for offshore structure, is generally based on API Recommended Practice 2A, which, however, underpredicts the capacities of short piles and overpredicts the capacities of long piles (Olson, 1990). Many factors can attribute to the misleading prediction (Iskander and Olson, 1992). Mainly speaking, the API recommended practice,

(1) adopts the simplified approach of setting upper limits on side shear and end bearing, which however, should vary with depth (Vesic, 1967; Kulhaway, 1984; Briaud et al. 1987; Toolan et al. 1990; Kraft, 1991; Randolph et al. 1994), and take as non-homogeneous media;

(2) takes no account of pile-soil relative slip;
(3) concerns nothing about the load settlement behaviour.

For piles in sand, there is no plunging failure load, load and displacement just continue increasing; while for piles in clay, as it happens for most conventional onshore development, with the main purpose to satisfy a serviceability of deformation, the settlement prediction becomes more important than that of bearing capacities (Khan et al. 1992).

(b) Settlement Prediction

Based on an assumed load distribution down a pile, Vesic (1965, 1970, 1977) suggested a very simple way of predicting pile-head displacement. He considered pile settlement as three components: (1) the axial deformation of the pile shaft, \( w_s \), induced by axial load along a pile; (2) settlement of the base, \( w_{ps} \), by the load transmitted along the pile shaft; and (3) settlement of the pile base, \( w_{pp} \), due to the load transmitted at the base. Therefore, pile-head settlement, \( w_t \), equals

\[
w_t = w_s + w_{ps} + w_{pp}
\]  

(2-8)

Shaft displacement, \( w_s \) is expressed by the elastic shortening of a pile under a load of \( P_t \). Vesic (1965)

\[
w_s = \left( \alpha_s \Delta + \beta_b \right) \frac{P_t L}{E_p A_p}
\]

(2-9)

where \( P_t = P_{sl} + P_b \), \( P_t, P_b \) are the total head and base load respectively; \( \alpha_s = P_{sl}/P_t \), \( P_{sl} \) is the total shaft load; and \( \beta_b = P_b/P_t \). The non-uniform distribution of shaft load (stress) is represented by a constant called stress distribution factor \( \Delta \), which has been rewritten in a general form by Chen and Song (1991)

\[
\Delta = \frac{1}{L} \int_0^L \left(1 - \frac{P_s(z)}{P_{sl}}\right) dz
\]

(2-10)

where \( P_s(z) \) is the shaft load at depth \( z \). For friction distribution of triangle, inverse-triangle and rectangle, \( \Delta \) is found to be 0.67, 0.33, and 0.5 respectively (Vesic, 1977). For a two layered soil profile, the \( \Delta \) has been generated by Leonards and Lovell (1979) for the two cases as shown in Figs. 2-11 and 2-12 for the two different shaft friction patterns and relative thickness. The solution has been extended to a three-layered soil
system as well (Schmertmann, 1987). This stress distribution factor is, in fact, a representation of the non-homogeneity.

More practically, given a ratio of pile-head load $P_t$ and ultimate load $P_{ult}$, the distribution figure of skin friction along a pile can be approximately represented by a number of small sections, which are trapezoidal, rectangular or triangular. The distance of the geometrical centre of each figure from the ground surface can be estimated. The stress distribution factor defined by Eq. (2-10) is estimated to be the ratio of the distance of the geometrical centre of the whole figure to the corresponding pile length. Following this procedure, based on measurement from a database of 84 instrumented piles, Chen and Song (1991) recently deduced the pile shaft distribution factor at load levels of $P_t/P_{ult} = 0.5$ and 1 for concrete driven piles, steel piles, and bored piles respectively.

The base displacement, which consists of the last two components in Eq. (2-8), may be given by the following empirical equation

$$w_b = \omega_b P_b L/A_p E_p$$

(2-11)

where $\omega_b$ is an empirical base modification factor (Chen and Song, 1991). More rigorously, the base displacement may be estimated by the following equations

$$w_{pp} = P_{pp} d I_{pp}/A_p E_s$$

(2-12)

$$w_{ps} = \tau_{ave} d I_{ps}/E_s$$

(2-13)

where $\tau_{ave}$ is the average shaft friction; $I_{pp}$, $I_{ps}$ are the new settlement influence factors as given by Polo and Clemente (1988). Pile-head and base displacements have been obtained by FEM analysis for the following four different pile shaft stress distributions (Polo and Clemente, 1988), which are triangular decreasing with depth, triangular increasing with depth, parabolic and uniform with depth. FEM analysis has been undertaken by the following procedures:

1. idealising the pile as a hollow cylinder, and treating the soil as a homogeneous, linear elastic, isotropic half-space;
2. applying the different stress distribution along the inner surface of the pile.

The displacement obtained is then split into the three components. Thereby, with Eqs. (2-9), (2-12) and (2-13), the factors are back-figured respectively. As would be
expected, the factors achieved are shown to be different from those derived from Mindlin's solutions, due to the idealisation of the pile and the stress condition.

The major concern for the FEM analysis is the fact that the stress distributions assumed are not always compatible with the shear modulus profile used. The effect of this incompatibility may need to be clarified. Although many empirical stress distribution profiles have been reported (Vesic, 1967; Toolan et al. 1990; Kraft, 1991; Randolph et al. 1994), these are generally suggested for predicting pile capacity, rather than for settlement. As shown later in Chapter 3, settlement prediction is much more sensitive to the stress profile than bearing capacity prediction. For instance, settlement is a parabolic function of the stress at full pile-soil slip case, (see later, Eq. (3-14)). Therefore, using error analysis, it may be shown that the accuracy for estimating settlement may be more readily attained if using the shear modulus profile rather than using a stress profile. In fact, only for a rigid pile, can this approach based on stress distribution ensure compatibility between shear modulus and shear stress. Hence the prediction is theoretically reliable.

Following the above arguments, the key for using the approach is to choose a suitable stress distribution factor. The following effects may need to be considered beforehand.

(1) The distribution factor varies with load levels. Particularly for a slender pile and/or a higher ratio of $P_t/P_{ult}$, pile-soil relative slip might be developed. Therefore, the distribution factor is a coupled reflection of the stress distribution non-homogeneity of the elastic and the plastic parts.

(2) The distribution factor changes with the pile Young's modulus or the total pile deformation (Van Impe, 1988). Once the pile-soil relative stiffness or head displacement changes, the shear stress distribution is bound to be different.

However, using shear modulus based analysis (Chapter 3), the above concerns may be avoided.

As stated previously, the shear modulus non-homogeneity causes both stress and strain distribution alteration. Whether the shear modulus non-homogeneity factor (or the reduction factor), or the stress distribution factor mentioned previously can not be applied to the slip part. Thus a suitable formulation should be developed to account for the shear modulus non-homogeneity prior to slip and the shaft stress non-homogeneity posterior to the slip.
2.2.5.3 *Pile-Soil Relative Stiffness Factor*

The pile-soil relative stiffness factor is normally defined as a ratio of pile Young modulus and soil Young or shear modulus at the pile base level (Banerjee and Davies, 1977; Poulos, 1979; Randolph and Wroth, 1978). Pile-head stiffness, in terms of this definition as shown in Eq. (2-7), can be considerably altered for different degree of non-homogeneity (Banerjee and Davies, 1977; Randolph and Wroth, 1979), due to a corresponding difference in the average soil shear modulus over the whole pile embedded depth.

Poisson's ratio, \( \nu_s \), in fact, represents compressibility of a soil (e.g. a value of \( \nu_s = 0.5 \) implies that the soil is incompressible). Its variation could lead to significant change in pile-head stiffness (as explored in Chapter 4). To avoid this effect, as argued by Randolph and Wroth (1978), pile-soil relative stiffness might be suitably defined by the shear modulus rather than Young's modulus.

If the influence of shear modulus distribution alone on a pile behaviour is to be explored, pile-soil relative stiffness might be more reasonably defined as the ratio of pile Young modulus to the average soil shear modulus over the pile length. Accordingly, the shear modulus of "GL" in Eq. (2-7) should be replaced by the average shear modulus as well.

2.3 *TIME-DEPENDENT EFFECT*

2.3.1 *Soil Strength*

The disturbance of clays as a result of pile driving was first detailed by Casagrande (1932). As a result of this remoulding and subsequent reconsolidation, settlements and negative skin friction would develop on the pile. Investigating a large pile group driven through soft clay, Cummings et al. (1950) found that the effect of remoulding was limited and the reduction of strength was rapidly eliminated as a result of reconsolidation. They also found that strength observed 1 month after driving was about equal to the strength of the intact clay, and after 11 months it was considerably greater. Generally the undrained shear strength of soft clays is reduced immediately following driving, followed by subsequent increase in the strength with time after the end of driving, resulting in strengths equal to or greater than the initial values (Orrje and Broms, 1967; Flaate, 1972; Fellenius and Samson, 1976; Bozozuk et al. 1978).
Such a remoulding and strength variation are accompanied with the radial soil movement at constant volume (Francescon, 1983).

There are no solutions for predicting variation of soil strength due to reconsolidation. However, the theory of reconsolidation for pore pressure dissipation may be directly used with sufficient accuracy to simulate the variation as detailed in Chapter 6.

2.3.2 Excess Pore Pressure

The pore pressure caused by driving a pile in clay was first observed by Bjerrum et al. (1958). Generally the maximum excess pore pressures immediately after driving were equal to or exceeded the total overburden pressure in overconsolidated clays (Orrje and Broms, 1967; Koizumi and Ito, 1967; Clark and Meyerhof, 1972; Fellenius and Samson, 1976). The maximum pore pressures occur only in a limited volume of soil in the immediate vicinity of the pile wall. The magnitude of the driving pore pressures decreases rapidly with the distance from the pile wall, and becomes negligible at a distance in the order of 10-20 pile diameters (Bjerrum and Johannessen, 1960; Lo and Stermac, 1965).

The maximum excess pore pressure may be obtained through (1) a triaxial test-based empirical equation; (2) a cylinder expansion theory; and (3) strain path method.

The increase in pore pressure may be divided into an ambient component, $\Delta u_a$, and a shearing component, $\Delta u_s$, which are caused respectively by an increase in the ambient total stress and the shearing of the soil to large strains around the pile. The maximum ambient component, $\Delta u_a$, may be taken as the difference between vertical and horizontal effective stresses (Lo and Stermac, 1965). The maximum shearing component, $\Delta u_s$, may be taken as a product of vertical effective stress and a normalised maximum pore pressure, with the normalised value being estimated by consolidated-undrained triaxial tests. This approach gives very good comparison with field measurements for normally consolidated clay (Lo and Stermac, 1965). As for overconsolidated clays, the maximum shearing component, $\Delta u_s$, may be taken as a product of preconsolidation pressure and the normalised maximum pore pressure (Roy et al. 1981).

The maximum pore pressure can be reasonably predicted by a cylinder expansion (Randolph and Wroth, 1979b; Roy et al. 1981), which is based on the expansion of a
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cylinder cavity from zero radius to a radius of \( r_0 \) (\( r_0 \), the radius of the pile) in an ideal elastic, perfectly plastic material, characterised by a shear modulus, \( G \) and an undrained shear strength, \( s_u \). The theory was originally employed in the analysis of pressuremeter tests (Gibson and Anderson, 1963), but the suitability to simulate the installation of a pile has been extensively explored by Randolph and Wroth (1979b), Carter et al. (1979), in particular for overconsolidated soil by Randolph et al. (1979).

The maximum excess pore pressure may also be obtained by strain path method (Baligh, 1985a), which is based on a kinematically admissible soil deformation around a simple pile. The method can well account for the effects of the strain history on the principal stress directions of inelastic soils, but cannot ensure equilibrium everywhere in the soil (Baligh, 1986a; Teh and Houlsby, 1991). Therefore, the results from this method are generally considered to be approximate.

Analysis using strain path method by Baligh (1986b) shows that around pile shafts, cylindrical cavity expansion solutions can provide reasonable estimates of the soil conditions in the far field, where inelastic soil behaviour is negligible. Near the shaft, cylindrical expansion solutions may be used as well, except that they tend to overpredict the excess pore pressure.

2.3.3 Reconsolidation Process

The rate of pore pressure dissipation in a clay around a pile after driving is a radial reconsolidation process. Similar to any consolidation process (Randolph and Wroth, 1979b; Randolph et al. 1979), the radial reconsolidation will generally be affected by the following factors:

1. non-linear soil stress-strain relationship;
2. soil viscosity property;
3. soil (shear modulus) non-homogeneity.

Davis and Raymond (1965) developed a non-linear theory of consolidation for an ideal normally consolidated soil by assuming a linear relationship between void ratio and \( \log \sigma' \) (\( \sigma' \) = effective pressure) rather than a linear relationship between void ratio and \( \sigma' \) as adopted by Terzaghi (1943). They demonstrated that at high ratios of final to initial effective pressure, the pore pressure in a normally consolidated soil can be expected to be considerably higher at any particular time than that predicted by the Terzaghi theory.
Later, Vaid (1985) extended the non-linear theory of consolidation to the case of constant rate of loading. The difference between the results from linear and non-linear theories has been explored with regard to the rate of loading, sample thickness, and the value of effective stress from where the consolidation is initiated.

Merchant (1939) (referenced via Christie, 1964) first proposed the theory of visco-elastic consolidation in his thesis by using a standard linear model. Later, Gibson and Lo (1961) presented similar solutions for visco-elastic consolidation, using an identical soil model (Visco-elastic analysis based on other models has been reviewed in section "Visco-elastic behaviour"). As is well known, these kinds of theories were developed to simulate secondary compression.

Schiffman and Gibson (1964) explored the effect of non-homogeneous soil properties on consolidation behaviour. The analytical and numerical analyses showed that the difference in time-settlement relationship due to consolidation is quite appreciable between a nonhomogeneous soil and homogeneous soil.

For radial consolidation, so far there are no publications dealing with the effect of the (1) to (3) factors. To account for the effect of the three factors, a new non-linear visco-elastic model has been proposed by using hyperbolic stress-strain law coupled with the Mechant's model. The new model is then used to established new closed form solutions for radial consolidation. The radial soil (shear modulus) non-homogeneity must affect radial consolidation of soil following pile driving, To assess this effect, closed form solutions have been established by assuming the shear modulus is a power of the radial distance away form pile axis, which have been detailed in Appendix E.

In addition to the above-mentioned three factors, radial consolidation is significantly affected by the following two factors:

(4) soil shear stiffness expressed by a rigidly index as $\sqrt{G/s_u}$;
(5) overconsolidation ratio.

Teh and Houlsby (1991) showed that the initial excess pore pressure (hence the consolidation process) is significantly dependent on soil shear stiffness ($\sqrt{G/s_u}$). In order to provide a consistent pore pressure dissipation curve for different values of the soil shear stiffness, they introduced a new time factor.
Randolph et al. (1979) reported that the maximum pore pressure (normalised by $s_u$) decreases slightly as the overconsolidation ratio (OCR) increases. The time process of consolidation was reported to be affected slightly by the value of OCR, depending on the shear modulus.

Two basic approaches are commonly used for analysing consolidation problems. The first was developed from diffusion theory by e.g. Terzaghi (1943) and Rendulic (reported by Murray, 1978). The second was developed from elastic theory by e.g. Biot (1941), and more recently by Randolph and Wroth (1979b) for dissipation of pore pressure generated due to pile driving.

The diffusion theory is generally less rigorous than the elastic theory. However, the diffusion theory is mathematically much simpler to apply, and can be readily extended to account for complex conditions. e.g. soil visco-elasticity, soil shear modulus non-homogeneity. In fact, the diffusion theory is different from the elastic theory in that (1) the mean total stress is assumed constant in the diffusion theory; (2) the coefficients of consolidation derived for the two theories are generally different (Murray, 1978). However, for radial consolidation, the rate of change of mean total stress happens to be zero for elastic soil response (Chapter 6); thus, the only difference between the two theory is the coefficients of consolidation. Therefore, using a coefficient from elastic theory to replace the coefficient in the solution of the diffusion theory, the solution from the diffusion theory is readily converted into a rigorous solution.

Soderberg (1962) first proposed a numerical solution of the reconsolidation process. Later, Torstensson (1975) developed a solution based on a combination of the theory of cavity expansion and of Terzaghi's consolidation theory. Randolph and Wroth (1979b) proposed a closed form solution for reconsolidation, with the initial excess pore pressure around a pile described by a law of logarithmic variation away from the pile axis, which itself is obtained from the theory of cylinder expansion.

Once the dissipation process of pore pressure is known, the variation of pile capacity with the process can be readily deduced. The experiments by Fellenius (1972) showed that a significant negative skin friction develops on the pile during reconsolidation. The rate of development of this negative friction in the clay appears closely related to the rate of pore pressure dissipation in the clay in the vicinity of the pile. The analyses by Soderberg (1962), Randolph and Wroth (1979b) showed that the measured rate of development of pile capacity in clay appears to be consistent with the rate of pore
pressure dissipation in the clay close to the pile. In fact, all the variation of relevant soil properties due to reconsolidation may be assessed by using a radial consolidation theory as further detailed in Chapter 6.

2.3.4 Visco-elastic Behaviour

Numerous publications show that visco-elastic or time dependent creep property significantly affects the soil settlement and strength behaviour (Buisuman, 1936; Lee, 1955, 1956; Lee et al. 1959; Qian, 1985). However, most of the research so far has been confined to the visco-elastic consolidation based on hypothetical rheological models.

Murayama and Shibata (1961) modified the standard linear model by implementing a critical stress in parallel with the Kelvin element (also called Voigt element), and hence proposed a rheological model as shown Fig. 2-13. Based on the rate process theory, they obtained the shear strain rate, \( \dot{\varepsilon}_2 \) (for the elastic spring 2) and the viscosity parameter \( \eta_2 \) (for the dashpot) respectively as shown below

\[
\dot{\varepsilon}_2 = A_2 (\sigma - \sigma_0) \sinh \left( \frac{B_2 \sigma_2}{\sigma - \sigma_0} \right)
\]  

\[
\eta_2 = \frac{1}{A_2} \sinh \left( \frac{B_2 \sigma_2}{\sigma - \sigma_0} \right)
\]  

where \( \sigma \) is the total stress; \( \sigma_0 \) is the critical stress; \( \sigma_2 \) is the stress acted on the dashpot; \( A_2, B_2 \) are the creep parameters. Therefore, the total shear strain can be obtained as

\[
\varepsilon_2 = \frac{\sigma - \sigma_0}{E_2} - \frac{2(\sigma - \sigma_0)}{B_2 E_2} \tanh^{-1} \left[ \exp \left( -A_2 B_2 E_2 t \right) \tanh \left( \frac{B_2}{2} \right) \right]
\]  

where \( E_2 \) is the Young’s modulus of the element 2. The model is then implemented into conventional consolidation theory to yield formulae for predicting pore pressure and settlement. The predicted pore pressure and settlement compare well with the experimental results. It seems reasonable that when \( \sigma_0 = 0 \), the model reduces to the standard linear model. However, once the total stress equals the critical value of \( \sigma_0 \), there is a singularity in Eqs. (2-14) and (2-15).
In contrast to the model by Murayama and Shibata (1961), Christensen and Wu (1964) proposed a rheological model shown in Fig. 2-14, by implementing an elastic spring in series with the dashpot within Kelvin element. They also utilised rate process theory to obtain shear strain rate for spring 1, which is related to the response of the dashpot by

\[ \dot{\gamma}_1 = \beta_\gamma \sinh \alpha_\gamma \tau_c \]  

(2-17)

where the symbols are shown in the figure, \( \tau, \tau_c \) are the shear stress on the system and the fraction causing flow respectively. \( \alpha_\gamma, \beta_\gamma \) are the creep parameters. Equations for evaluating the stress, \( \tau \), strain, \( \gamma \), and the stress, \( \tau_c \), have been generated, in terms of the model. The solutions were then extended to three-dimensional stress systems. The predictions of normalised strain and stress compare well with those measured from triaxial tests (Christensen and Wu, 1964; Wu et al. 1966). However, the suitability for analysing the pile-soil interaction may need to be identified further.

Similar to the empirical creep function by Mitchell and Solymar (1984), Murff and Schapery (1986) assumed that the shear strain, \( \gamma \) can be simulated by

\[ \gamma = \left( \frac{t}{t_1} \right)^{n_c} \gamma_1 \]  

(2-18)

where \( \gamma_1 \) is the strain at \( t = t_1 \), and is a function of the loading intensity, \( t \) is the time under load, \( t_1 \) is the normalising time constant, and \( n_c \) is a constant. Based on Eq. (2-18), Murff and Schapery (1986) extended Murff's (1975) non-dimensional closed form solution approximately to the time-dependent case. However, the solution is only approximately valid for the instance of slow loading.

Soydemir and Schmid (1967, 1970) obtained some visco-elastic solutions by replacing the elastic constants with the corresponding visco-elastic parameters in available elastic solutions. The visco-elastic parameters were derived by utilising a single Kelvin model and Maxwell model to simulate the volumetric and deviatoric stress-strain components respectively as shown in Fig. 2-15 (a) and (b). In the figure, \( \sigma_{kk} \) is the volumetric stress; \( \eta_v, G_v \) are the model parameters; \( S_{ij} \) is the deviatoric stress; and \( \eta_D, G_D \) are the model parameters as well. The solutions are limited to the ideal time-dependent models, but as long as elastic solutions are explicitly expressed, the corresponding time-dependent expressions can be readily formulated.
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Komamura and Huang (1974) proposed a new rheological model for a sliding soil. As illustrated in Fig. 2-16, the model consists of the Bingham and Voigt models in series. The model expresses the visco-plasto-elastic behaviour of a material and gives a total strain by

\[ \varepsilon = \frac{\sigma - \sigma_0}{\eta_1} - \frac{\sigma}{E} \left( 1 - e^{-\frac{E}{\eta_2} t} \right) \]  

(2-19)

where \( E \) is Young’s modulus; \( \sigma \) is total stress; \( \eta_1, \eta_2 \) are the viscosity parameters. The model compares well with the experiment performed, and gives good prediction of the time-dependent behaviour of a landslide. However, the critical stress, \( \sigma_0 \), adopted in the model varies with the water content. Therefore, it may only be evaluated with a lot of tests. Besides, the instant elasticity as indicated by spring 1 in Fig. 2-13 is not included in this model.

With extensive experiments regarding soil secondary compression by odometer tests, Lo (1961) showed that, for most soils, a standard linear visco-elastic model is sufficient to represent the secondary time-deformation behaviour. The advantage of the model is that it can be readily extended to account for non-linear soil behaviour. The parameters used can be readily measured by conventional test, as shown in Chapter 5.

2.3.5 Time-dependent Load Settlement Behaviour

Generally the time-scale loading of a test can be represented by (1) step loading, (2) ramp type loading, or (3) a combination of step and ramp type loading. The corresponding response shows as time-dependent visco-elastic interaction or creep processes. For instance, pile stiffness and capacity varies with the time-scale loading (e.g. Wiseman and Zeitlen, 1971; Bergdahl and Hult, 1981; Ramalho Ortigao and Randolph, 1983; Edil and Mochtar, 1988; and Liu, 1990). The degree of time-dependent response is mainly dependent on load (stress) levels. At low stress levels, visco-elastic response dominates the time process as shown by the model tests (Edil and Mochtar, 1988), and many field tests (e.g. Eide et al. 1961; Konrad and Roy, 1987; Bergdahl and Hult, 1981). At high load levels, or for long slender piles where the load transfer is concentrated near the pile head, the viscosity can lead to significant creep movement of the pile-head at constant load (Eide et al. 1961), and even a gradual reduction in shaft capacity, which may be due to the shaft stress reaching the long term soil strength as argued in Chapter 5 and shown by experiments on soil (Geuze and Tan,
1953; Murayama and Shibata, 1961; Leonard, 1973). Ramalho-Ortigao and Randolph (1983) reported an apparent difference of some 30% in the tension capacity of a pile loaded at a constant displacement rate leading to failure in about 40 seconds, compared with a similar pile subjected to a maintained load test over a period of 40 days.

A few numerical analyses are available for the pile creep analysis (Booker and Poulos, 1976; Yuan, 1994). A simple empirical approach has been recently proposed (England, 1992).

Booker and Poulos (1976) implemented the standard linear visco-elastic model into Mindlin's solution for boundary element analysis of the creep behaviour of a single pile. Non-dimensional charts of the settlement influence factor have been produced for step loading case.

England (1992) extended the hyperbolic approach of pile analysis described by Fleming (1992) to allow the effects of time to be incorporated into axial pile analysis, with separate hyperbolic laws being used to describe the time-dependency of the (average) shaft and base response. This phenomenological approach is limited by the difficulty of linking the parameters required for the model to fundamental and measurable properties of the soil.

Time-dependent behaviour can arise from either reconsolidation due to disturbance from installation of a pile or consolidation induced by loading. However, the current research level is not permitting the distinguishing of the effect of the reconsolidation from that of the consolidation.

A realistic prediction of creep behaviour, above all, should be a logical extension from an elastic solution but also linked to fundamental soil properties.

2.4 VERTICALLY LOADED GROUP PILES

A large body of information is available for analysing pile groups. Generally the performance of pile groups can be predicted by the following procedures

(1) empirical methods (Terzaghi, 1943; Skempton, 1953; Meyerhof, 1959; Vesic, 1967; Kaniraj, 1993);
(2) load transfer approaches, based on either simple closed form solutions (Randolph and Wroth, 1978, 1979; Lee, 1993a) or discrete layer approach (Chow, 1986b; Lee, 1991);

(3) elastic continuum based methods, e.g. boundary element analysis (Poulos, 1968; Butterfield and Banerjee, 1971; Chin et al. 1990), infinite layer approach (Guo et al. 1987; Cheung et al. 1988); FEM analysis (Ottaviani, 1975; Valliappan et al. 1974; Pressley and Poulos, 1986);

(4) hybrid load transfer approach (O'Neill et al. 1977; Chow, 1986a; Lee, 1993b; Clancy and Randolph, 1993), which takes advantage of both numerical and closed form approaches, renders the possibility of analysing large group piles.

2.4.1 Empirical Approaches

A number of non-dimensional parameters have been introduced to describe pile group behaviour. One of the parameters is the settlement ratio, $R_s$, which was defined as the ratio of the average group settlement to the settlement of a single pile carrying the same average load (Poulos, 1968). For this particular parameter, empirical formulae were proposed by several researchers (e.g. Skempton, 1953; Meyerhof, 1959).

The empirical formulae were generally established from the comparison of full-scale or model test results between the settlement of a single pile group and that of a single pile in sands (Skempton, 1953; Meyerhof, 1959), but only the group geometry was taken into account. In addition, test results (Kaniraj, 1993) show that the settlement ratios generally decrease as the pile spacing decreases, but the empirical formulae (Skempton, 1953; Meyerhof, 1959) indicated an opposite trend. Therefore, the empirical formulae may be used only for the cases where the overall conditions are similar to those on which these formulae are based.

Kaniraj (1993) modified the definition of settlement ratio by Poulos (1968), and defined a new settlement ratio as a ratio of the settlement of a pile group to that of a single pile when the average stress on their respective load transmitting area is identical. The load transmitting area is the area at the pile base level, estimated through the dispersion angle ($\approx 7^\circ$ as reported by Berezantzev et al. 1961) as illustrated in Fig. 2-17. This new settlement ratio was presented in the form of semi-empirical equations, and was compared with the measured values. The equations give better estimations of the settlement ratios than the previous empirical formulae (e.g. Skempton, 1953;
Meyerhof, 1959), although generally the estimations are higher than the measured values.

In fact, as has been partly explored by different numerical analyses published, settlement ratio is dependent on the following factors: pile spacing, the number of piles in a group, pile-soil relative stiffness, the depth of the underlying rigid layer, and the profile of shear modulus both vertically and horizontally. Therefore, the empirical formulae may need to be improved to account for these factors, as shown in Chapter 7.

2.4.2 Interaction Factor and Superposition Principle

The influence of the displacement field of a neighbouring identical pile was represented by an interaction factor between pairs of incompressible piles (Poulos, 1968). The interaction factor reflects the increase in settlement of a pile due to the displacement field of a similarly loaded neighbouring pile, which can be expressed as

\[
\alpha_{ij} = \frac{\text{Pile - head stiffness of a single pile}}{\text{Pile - head stiffness of a pile in a group of two}} - 1
\]  

(2-20)

where \( \alpha_{ij} \) is the interaction factor between pile i and pile j. The interaction factor originally defined for two identical piles is then extended to unequally loaded piles. The shaft displacement increase due to a displacement field of a similarly loaded neighbouring pile may be represented by (Lee, 1993a)

\[
\alpha_{sij} = \frac{\text{Shaft displacement for a pile in a group of two}}{\text{Shaft displacement for a single pile}} - 1
\]  

(2-21)

where \( \alpha_{sij} \) is the shaft interaction factor between pile i and pile j. Similarly, a base interaction factor may be defined. In fact, the interaction factor can be defined in other forms depending on the manner used for estimating displacements. For instance, consistent with the settlement prediction for a single pile using Eq. (2-8), Polo and Clemente (1988) introduced two pile-pile interaction factors, \( \alpha_{ps}^{ij} \) and \( \alpha_{pp}^{ij} \) for base settlement estimation: the interaction factor, \( \alpha_{ps}^{ij} \), reflects an increase in the settlement at the base of pile j due to the load transmitted along the shaft of pile i; the interaction factor, \( \alpha_{pp}^{ij} \), reflects an increase in the settlement at the base of pile j due to the load transmitted at the base of pile i.
With a known displacement field or pile-soil-pile interaction factors, the behaviour of a pile in a group can be readily evaluated, using the principle of superposition. The results using the principle of superposition are generally the same as those by analysing the entire pile group. Even for general pile group analysis, the principle of superposition is approximately valid (Clancy, 1993). The validity of the superposition approach both to the estimation of the pile settlement and to the determination of the load carried by each pile was confirmed by Cooke et al. (1980) through a number of field tests. In the following sections, it may be noticed that the principle of superposition is utilised for all the load transfer based analyses.

2.4.3 Displacement Field Around a Single (Group) Pile

2.4.3.1 A Single Pile

From a vertically loaded single pile analysis, it was shown that (Cooke, 1974; Frank, 1974; Randolph and Wroth, 1978), the settlement around the pile shaft is given by

\[ w(r) = \frac{\tau_o \sigma_o}{G} \ln(r_m/r) \]  

(2-22)

The settlement at the pile base level away from the pile axis is approximated by

\[ w(r) = w_b \frac{2 \sigma_o}{\pi r} \]  

(2-23)

2.4.3.2 Two Piles

Using the superposition principle, the displacement field for a pile in a group of two may be obtained by superimposing the local displacement field (Randolph and Wroth, 1979c); therefore the settlement around each of the pile shafts, \((w_s)_2\), is given by:

\[ (w_s)_2 = \frac{\tau_o \sigma_o}{G} \left( \ln\left(\frac{r_{mg}}{r_o}\right) + \ln\left(\frac{r_{mg}}{s}\right) \right) \]  

(2-24)

where \(s\) is pile centre-centre spacing; \(r_{mg} = r_m + r_g\); \(r_g\) is the half the maximum distance between any two piles in the group. The base settlement of each of the pile base, \((w_b)_2\), is
Lee (1993a) modified Eq. (2-7) and gave the following equation for the average shaft displacement, \((w_s)_2\), around each pile in a group of two

\[
(w_s)_2 = \frac{P_b (1 - \nu_s)}{4r_o G} \left(1 + \frac{2 r_o}{\pi s}\right)
\]  

With Eq. (2-26), the shaft interaction factor by Eq. (2-21) may be expressed as

\[
\alpha_{sij} = \frac{1}{\ln(r_{mg}/r_o)} \ln\left(\frac{r_{mg}/s_{ij}}{r_o}\right)
\]  

where \(s_{ij}\) is pile centre-centre spacing between pile \(i\) and \(j\), \(s_{ij} = r_o\) for \(i = j\). And the base interaction factor may be expressed as

\[
\alpha_{bij} = \frac{2r_o}{\pi s_{ij}}
\]  

where \(\alpha_{bij}\) is base interaction factor for two piles corresponding to spacing between pile \(i\) and pile \(j\); \(s_{ij} = 2r_o/\pi\) for \(i = j\).

### 2.4.3.3 Muti-Piles

Generally, for a group of \(n_g\) piles, shaft displacement of the \(j\)th pile, \((w_s)_j\), may be obtained by Eq. (2-29) (Randolph and Wroth, 1979c)

\[
(w_s)_j = \frac{1}{G} \sum_{i=1}^{n_g} \left((\tau_o)_i (r_o)_i \ln(r_{mg}/s_{ij})\right)
\]  

Similarly, the base displacement, \((w_b)_j\), for \(j\)th pile may be given by

\[
(w_b)_j = \sum_{i=1}^{n_g} (w_b)_{ij} = \frac{(1 - \nu_s)}{4r_o G_L} \frac{2}{\pi} \sum_{i=1}^{n_g} \frac{(P_b)_i}{s_{ij}}
\]

The shaft and base displacements for each pile are estimated separately.
2.4.4 Simple Closed Form Approaches

Randolph and Wroth (1979c) provided a simple approach for predicting settlement of pile groups of \( n_p \) piles. The \( n_p \) values of pile shaft displacement, \( w_s \), was related to the \( n_p \) values of \( \tau_0 \) by Eqs. (2-29) to give a matrix equation of

\[
[w_s] = [F_s][\tau_0] \tag{2-31}
\]

Similarly, from Eq. (2-30), a base matrix may be formulated as

\[
[w_b] = [F_b][P_b] \tag{2-32}
\]

For rigid piles, \( w_s = w_b \), and for a rigid pile cap, \( (w_s)_i = (w_b)_i \). Therefore, it is straightforward to obtain shaft stress and base load for a given pile cap displacement by inverting the shaft and base matrices. The method generally furnishes good predictions compared with more rigorous numerical analyses (e.g. Butterfield and Banerjee, 1970). However, for compressible pile groups, the shaft displacement, \( w_s \), and base displacement, \( w_b \), for each pile is different, therefore an additional equation is needed for each pile to correlate these two displacements. This approach treating the shaft and base interaction effects separately generally requires an iteration for analysing compressible pile groups. However, should the shaft and base displacements be dealt with together, the analysis will be greatly simplified as shown in Chapter 7.

Lee (1993a) assumed the denominator of Eq. (2-7) as unity, but tried to compensate the assumption by replacing the \( \beta \) with \( \beta^* \) (\( = 1.15\beta \)); therefore the stiffness of a single pile is a simple sum of total base and shaft stiffness as

\[
\frac{P}{G_Lr_o w_t} = \frac{4}{1 - \nu_s} + \frac{2\pi\rho_s L \tanh\beta^*}{\zeta r_o \beta^*} \tag{2-33}
\]

The shaft stiffness is then implemented into Eq. (2-22) to yield an average displacement field around the pile, as given by Eq. (2-26). Therefore, the total stiffness for each pile in a group of two piles, \( \left( \frac{P}{G_Lr_o w_t} \right)_2 \), can be obtained as

\[
\left( \frac{P}{G_Lr_o w_t} \right)_2 = \frac{4}{1 - \nu_s} + \frac{1}{1 + 2r_o/\pi s} \frac{2\pi\rho_s L \tanh\beta}{r_o} \frac{1}{\ln \left( \frac{r_{mg}}{r_o} \right) + \ln \left( \frac{r_{mg}}{s} \right)} \tag{2-34}
\]
With the stiffness by Eq. (2-34) and that for a single pile by Eq. (2-33), the interaction factor defined by Eq. (2-20) was expressed explicitly and compared with more rigorous numerical analyses (Lee, 1993a). In turn, with this interaction factor, pile group behaviour was predicted, using the superposition principle. The predictions were then compared with more rigorous numerical analyses and field test results.

Using the average displacement field of Eq. (2-26), the approach only partly accounts for the effect of pile-soil relative stiffness, although the effect of the stiffness might be limited as shown later in Chapter 7. Another major concern is that even though Eq. (2-7) has been modified as Eq. (2-33), the resulting interaction factor still does not always compare satisfactorily with more rigorous numerical analysis as the pile centre to centre space changes.

In fact, load transfer factors embracing the neighbouring pile effect may be implemented directly into Eq. (2-7), to achieve pile-head stiffness of a pile in a group of two. In this manner, pile-pile interaction factor by Eq. (2-20) may be obtained as illustrated in Chapter 7.

### 2.4.5 Numerical Approaches

#### 2.4.5.1 Boundary Element (Integral) Approach

Using the BEM (BI) approaches described in Section 2.2.4:

1. Butterfield and Banerjee (1971) explored extensively pile-head stiffness for different pile groups of rigid cap at various pile slenderness ratios, and pile-soil relative stiffness;
2. Poulos (1968) introduced the pile-soil-pile interaction factor as mentioned earlier. With the interaction factor, values of the settlement ratio, $R_s$, and load distribution within a group have been obtained. The influence of pile spacing, pile length, type of group, depth of layer and Poisson’s ratio of the layer on the settlement behaviour of pile groups was examined;
3. Chin et al. (1990) reported pile-soil-pile interaction factors, in terms of Chan’s solution (Chan et al. 1974) for various pile spacing, relative stiffness and slenderness ratios.
2.4.5.2 Infinite Layer Approach

Guo et al. (1987) and Cheung et al. (1988) proposed an infinite layer approach. The stress analysis for a single pile embedded in layered soil was performed through a cylindrical co-ordinate system. Each soil layer was represented by an infinite layer element and the pile by a solid bar. The displacements of the soil layer were given as a product of a polynomial and a double series. The strain-displacement and stress-strain relations were established from the displacement fields and therefore the total stiffness matrix could be readily formed.

The interaction between two piles, which are called pile 1 and 2 respectively as shown in Fig. 2-18, is simulated through the following procedure:

1. Replacing pile 2 with a soil column of the same properties as the surrounding soil. The settlement of pile 1 as well as the soil due to the action of unit load on the pile is then computed by the single pile model. Ignoring the change in the displacement field due to the existence of pile 2, the force acting along pile 2 can be readily calculated by multiplying the displacement vector and the stiffness matrix of pile 2. The differences between the forces on the pile and those computed from the infinite layer model are regarded as residual forces, which are applied in the opposite direction along pile 2 to maintain the equilibrium of the whole system.

2. If the forces are applied to pile 2, pile 1 is replaced by a soil column. Similarly, the soil movement and residual forces induced in pile 1 are computed.

3. The whole procedure (1) to (2) is repeated by applying the residual forces of each step on pile 1 and pile 2 accordingly until the changes in the displacement of both piles due to the loading are negligible. By this analysis, the resulting interaction factors for two identical piles embedded in homogenous soil were found to be generally consistent with those by Poulos (1968).

2.4.5.3 Non-linear Elastic Analysis

Trochanis et al. (1991) studied the response of a single pile and pairs of piles by undertaking a 3-dimensional FE analysis using an elastoplastic model. The results demonstrated that as a result of the non-linear behaviour of the soil, the pile-soil interface interaction, especially under axial loading, is reduced greatly compared to that for an elastic soil bonded to piles. The commonly used methods for evaluating pile-soil-pile interaction, which are based on the assumption of purely elastic behaviour, can substantially overestimate the degree of interaction in realistic situations. In load
transfer analysis, this non-linear effect may be modelled by using elastic interaction factors and adding non-linear components afterwards (Randolph, 1994; Caputo and Viggiani (1984) referenced via Mandolini and Viggiani (1996)).

2.4.5.4 Discrete Element Analysis - Layer Model

"Layer model" for analysing pile groups is illustrated in Fig. 2-19(a) (Chow, 1986b). The piles are modelled using discrete elements with an axial mode of deformation, and the soil is treated as independent horizontal layers; therefore the interaction between piles takes place within each soil layer only. Assuming that the shear stress remains constant within each pile segment, 1, the deformation at a radius of r from the axis of a vertically loaded single pile is approximated by Eq. (2-22) ($\tau_o = \frac{P_s}{\pi d l}$, $P_s$ is the pile shaft load). Eq. (2-22) is applicable to each of the NL layers along the pile shaft.

For any pile i in a group of $n_g$ piles, the overall settlement of the soil at the pile shaft of a particular pile within a soil layer, $k$, due to loading on itself and on neighbouring piles, $w_{si}^k$, is given by

$$w_{si}^k = \sum_{j=1}^{n_g} f_{sij}^k P_{sj}^k$$  \hspace{1cm} (2-35)

where $P_{sj}^k$ is the shaft load at layer k at pile j (i); $f_{sij}^k$ is the displacement influence coefficient for pile shaft in layer k denoting the settlement of the shaft at pile i due to a unit load at pile j, within the layer k. $f_{sij}^k$ may be obtained from Eq. (2-22) (Chow, 1986b). Eq. (2-35) may be written for each of the $n_g$ piles in the group giving the following matrix equation

$$\{w_s^k\} = [F_s^k]\{P_s^k\}$$  \hspace{1cm} (2-36)

where $\{w_s^k\}$ is shaft displacement vector for layer k; $[F_s^k]$ is flexibility matrix of order $n_g \times n_g$ for layer k; and $\{P_s^k\}$ is shaft load vector for layer k. This procedure is repeated for each soil layer along pile shaft i.e. for $k = 1, 2, ..., NL$.

For any pile i in a group of $n_g$ piles, the overall settlement of the soil at the base of pile i due to loading on itself and on neighbouring piles, $w_{bi}$, is given by
Chapter 2

2.37 Literature Review

\[ w_{bi} = \sum_{j=1}^{n_g} f_{bij} P_{bj} \]  
\[ (2-37) \]

where \( P_{bj} (P_{bi}) \) is the base load at pile \( j \) (i); \( f_{bij} \) denotes the displacement influence coefficient at the pile base. For \( i = j \), \( f_{bij} \) may be obtained from Eq. (2-5) and for \( i \neq j \), from Eq. (2-23). Eq. (2-37) may be written for each of the \( n_g \) piles in the group giving the following matrix equation, similar to Eq. (2-32)

\[ \{w_b\} = [F_b]\{P_b\} \]  
\[ (2-38) \]

where \( \{w_b\} \) is base displacement vector; \( [F_b] \) is flexibility matrix of order \( n_g \times n_g \) for pile base; and \( \{P_b\} \) is base load vector.

The stiffness matrices of the soil layers and that of the soil at the pile base are assembled together with the pile discrete element matrices to yield the total stiffness matrix of the pile group system governing pile load and displacement relationship. In this manner, the soil stratification can be dealt with, but the continuity of the soil medium is ignored. Generally the approach tends to underestimate the interaction except for piles with large slenderness ratios (Chow, 1986b).

2.4.5.5 Hybrid Load Transfer Approach

In the ‘layer model’, if the pile-soil-pile interaction between different layers is taken into account, the model is referred to as a continuum model (Chow, 1986a), which is illustrated in Fig. 2-19(b). The approach based on this model is more popularly called “hybrid load transfer approach” (O’Neill et al. 1977; Clancy and Randolph, 1993; Lee, 1991), and is a simple efficient method for analysing pile groups.

In the hybrid load transfer approach, the displacements given by Eqs. (2-35) and (2-37) may be decomposed into two components respectively

\[ w^k_{si} = \sum_{j=1}^{n_g} f^k_{sij} P_{sj} = f^k_{sik} P_{si} + \sum_{j=1}^{n_g} f^k_{sij} P_{j} \]  
\[ (2-39) \]

\[ w_{bi} = \sum_{j=1}^{n_g} f_{bij} P_{buj} = f_{bik} P_{bi} + \sum_{j=1}^{n_g} f_{bij} P_{j} \]  
\[ (2-40) \]
where $f_{sij}$ is the flexibility coefficient for the pile shaft in layer $k$ due to unit load at the layer $k$ in the same pile $i$; $f_{sij}$ is the average settlement flexibility coefficient for shaft elements in pile $i$ due to unit head load at pile $j$. The factor, $f_{sii}^k$ may be estimated by (Chow, 1986a)

$$f_{sii}^k = \ln\left(\frac{r_m}{r_o}\right)/2\pi G_l$$

(2-41)

and similarly, the flexibility coefficients for the node at the pile base is given by

$$f_{bij} = \frac{1 - \nu_s}{4G_l r_o}$$

(2-42)

While $f_{sij}^k = 0$ for loadings at nodes $j$ which are associated with the same pile as node $i$, and for $j \neq i$; the coefficients, $f_{sij}$ ($f_{bij}$), may generally be estimated by utilising the analytical point load solutions for soil displacement at each element along pile $i$ due to the loading acting at each element along every pile $j$ ($i \neq j$). The point load solutions may be based on Mindlin's solution for a vertical point in a homogeneous, isotropic elastic half-space (Chow, 1986a). The "hybrid" approach based on Mindlin's solution maintains the continuity of the soil, but handles the soil non-homogeneity in an approximate way. Lee (1991) reported the application of the "hybrid" method (Chow, 1986a) in analysing piles in layered soil media.

The interaction between pile $i$ and $j$ may be also accounted for by the displacement field around a pile of Eq. (2-24). Lee (1993b) found that the two components, $f_{sij}$, $f_{bij}$, of displacement influence coefficients in Eqs. (2-39) and (2-40) may be predicted by

$$f_{sij} = w_i \alpha_{sij}$$

(2-43)

$$f_{bij} = w_i \alpha_{bij}$$

(2-44)

where $w_i$ is the settlement of a single pile under unit load in a group. More importantly, Lee (1993b) found that the coefficient, $f_{sij}$, may be assumed to be the same for all the shaft elements in pile $i$ due to unit head load at pile $j$. Therefore, the computer storage required and the computing time reduces substantially, in comparison with the evaluation by utilising the analytical point load solutions. Besides, using the new coefficient, $f_{sij}$, the analysis can furnish sufficiently accurate results.

The average shaft and base flexibility coefficients have been evaluated by either Mindlin's solution (Chow, 1986a), or the pile-soil-pile interaction factors (Lee, 1993b).
Both solutions yield satisfactory results compared with more rigorous numerical approaches, using the continuum model (Fig. 2-19(b)). Therefore, the core of the hybrid analysis is to find an accurate, simple solution for pile-soil-pile interaction, which is able to account for various soil properties, e.g. non-homogeneity, and the effect of finite layer depth.

2.4.6 Influence of Non-homogeneity

2.4.6.1 Vertical Non-homogeneity

Vertical soil non-homogeneity significantly affects pile group behaviour (Guo and Randolph, 1996a), although it has limited effect on single pile response (Motta, 1994; Chapter 3), provided the average shear modulus along the pile depth is identical. Fig. 2-20 presents a comparison of the interaction factors for homogeneous soil by Poulos and Davis (1980) and Gibson soil by Lee (1993b). The differences in the pile-pile interaction factor may be partly attributed to the variation of the average shear modulus, and partly to the fact that the influenced zone is about twice as large for a homogeneous soil compared with a Gibson soil (Randolph and Wroth, 1978).

2.4.6.2 Horizontal Non-homogeneity

Horizontal non-homogeneity considered so far has been limited to the shear modulus alteration caused by pile installation (Randolph and Wroth, 1978; Poulos, 1988). This alteration leads to a significant change in the load transfer factor, and therefore normally results in a lower value of the pile-pile interaction factor as noted experimentally by O'Neill et al. (1977), and obtained numerically by Poulos (1988), Lee and Poulos (1990). Horizontal non-homogeneity may be readily incorporated into the "\( \zeta \)" (refer to Eq. (2-1)), in a manner used by Randolph and Wroth (1978). Therefore, closed form solutions as developed in this thesis may be directly used to account for the non-homogeneity.

2.4.6.3 Shear Stress Non-homogeneity

Using the interaction factors defined earlier in section 2.4.2, equations for analysing settlement of pile groups have been produced (Polo and Clemente, 1988).
Current elastic analyses for pile groups generally adopt an implicit assumption that shaft load over point load ratio is a constant throughout loading of individual piles. The assumption, however, is not realistic at high load levels. Based on measurements of the shaft loads of a single pile as it is loaded, and the new settlement interaction factors, $I_{pp}$, and $I_{ps}$, by Polo and Clemente, (1988), Clemente (1990) proposed a method to capture the effect of the variation of the ratio of shaft load over the base load on predicting settlement of pile groups. The method is based on known shaft stress distribution. However, pile-shaft stress distribution is significantly affected by the soil shear modulus profile and the pile-soil relative stiffness (Rajapakse, 1990). Therefore, how to choose a suitable stress profile consistent with the soil shear modulus profile is a major concern, prior to using this approach.

2.5 TORSIONAL PILES

2.5.1 Load Transfer Analysis

Similar to the analysis for vertical loading, the load transfer analysis for a pile subjected to torsion can be based on either analytical or numerical approaches.

The analytical approach was first proposed by Randolph (1981). Load transfer models were established in a similar way to that for vertical loading. With the models, closed form solutions for piles subjected to torsion in both elastic-plastic homogeneous and Gibson soil have been generated. Pile-head stiffness is defined and presented in a closed form solution. However, the head-stiffness is dependent on pile slenderness ratios. Randolph's (1981) solutions were extended into two layered soil cases (Hache and Valsankar, 1988), but the numerical results for the two layered soil were expressed in two new non-dimensional factors, namely, pile-soil relative stiffness and pile-head torsional influence factors. The advantage is that the relationship between the two new factors is independent of the pile slenderness ratio. However, the relationship is sensitive to the soil non-homogeneity; therefore a more accurate modelling is essential.

A discrete element approach was proposed by Chow (1985). The pile is discretized into a series of elements connected at the nodes. The soil is treated as horizontal layers, each with a modulus of subgrade reaction. In other words, the approach is an uncoupled analysis. However, as would be expected, the approach provides excellent comparison with that from finite element analysis. This gives further confidence to develop the solutions based on a load transfer approach as shown in Chapter 8.
2.5.2 Continuum Based Numerical Approach

Poulos (1975) proposed a boundary element approach (BEM) for analysing a torsionally loaded pile, which is similar to that for analysing a pile under vertical loading. The pile is discretized into a number of elements. The soil rotation at the midpoint of each element is obtained from elastic theory (Mindlin’s solution for a horizontal subsurface point load) in terms of the unknown interaction stress. The corresponding pile rotation is then expressed in terms of these interaction stresses by considering the pile as a circular cylinder. While conditions at the pile-soil interface remain elastic, the expressions for soil and pile rotations can be equated and solved, together with the equilibrium equation, to obtain the interaction stresses and thus pile rotation. To allow for the possibility of pile-soil slip, limiting shaft and base skin friction are specified. When the shaft stress, \( \tau \) reaches or exceeds the limiting value \( \tau_t \), the rotation compatibility equation for that element is replaced by the condition of, \( \tau = \tau_t \) and the solution is recycled until \( \tau \leq \tau_t \) at all elements.

The rotation behaviour of a pile in both a uniform soil and a Gibson soil, in which shear modulus and pile-soil adhesion increase linearly with depth, has been analysed. Design charts for torsional influence factor were provided, but unfortunately, were presented in different pile-soil relative stiffnesses for elastic and plastic stages respectively. Also the relation between the influence factor and pile-soil stiffness is dependent on the slenderness ratio. Therefore for practical design, a trial and error approach is needed. This approach is strictly not valid for non-homogeneous soil.

2.6 SUMMARY

Following the literature review presented in this Chapter, a number of weaknesses in the existing approaches have been revealed, as briefly summarised below.

2.6.1 Single Piles

(1) None of the current empirical formulas for prediction of pile capacity and/or settlement can accurately account for soil non-homogeneity, and pile-soil relative slip.

(2) The theoretical load transfer approach based on concentric cylinder approach approximates the pile-soil interaction in a simple 2-dimensional form, and it is readily comparable with other rigorous numerical approaches. Therefore, it is preferred to all other curve fitting approaches. Only with the load transfer model
can accurate closed form solutions be formulated, which can be readily extended to other cases as well.

(3) Settlement prediction methods are preferred to be based on shear modulus non-homogeneity than the stress distribution factor. Since using a stress distribution factor, the stress distribution along a pile cannot be ensured to be compatible with the shear modulus profile. In addition, the stress distribution factor can be readily obtained by closed form solutions based on shear modulus.

(4) If the influence of the shear modulus distribution is to be considered alone, then the definition of pile-soil relative stiffness should be based on an average shear modulus along a pile, rather than the modulus at the tip level.

In view of the above points (1), (2) and (3), closed form solutions for vertically loaded single piles have been established as shown in Chapter 3. The effect of point (4) has also been explored.

(5) The load transfer factors are dependent on the four factors listed earlier in section 2.2.1.1.

Following point (5), a comprehensive investigation of the suitability and rationality of load transfer analysis has been undertaken in Chapter 4. Load transfer factors have been calibrated against more rigorous numerical (FLAC) analysis, and have been provided in statistical forms in respect of the four factors.

2.6.2 Time-Dependent Effect

(6) Time dependent behaviour of a pile is governed by visco-elastic soil response when at lower load levels and/or for a short pile, or by creep when at high load levels and/or for a long pile (hence by long term soil strength).

(7) The available analysis on step loading cases shows a significant time-dependent pile response; however, the effect of other time-scale loading on pile behaviour remains to be further explored.

Following points (6) and (7), a visco-elastic shaft model has been developed which can well account for the effect of stress levels, and yet is a logical extension of an elastic load transfer model. This study can be found in Chapter 5.

(8) The effect of visco-elastic soil properties on pile capacity and settlement following driving is not yet clear. How to predict the overall pile response due to the installation of the pile in a clay is left to be explored.
Following point (8), a rigorous radial consolidation theory has been established in Chapter 6. A number of predictions accounting for time effects have been provided.

2.6.3 Pile Groups

(9) Numerical analysis is generally expensive and confined to the analysis of small pile groups. Hybrid analysis has the potential to be applied for analysing large pile groups, but it relies on the feasibility and availability of the closed form solutions.

(10) Non-homogeneity of the soil shear modulus has considerable effect on the pile-soil-pile interaction, and therefore the overall pile group response.

(11) The methods of using stress distribution factors to estimate the settlement of pile groups is subject to further research, since the effect of the incompatibility between shear modulus and stress distribution is not yet known.

Following points (9), (10) and (11), a rigorous closed form solution for pile-soil-pile interaction factor has been proposed. Settlement of (large) pile groups has been investigated extensively using the interaction factors, as detailed in Chapter 7.

2.6.4 Torsional Piles

(11) Current design charts for the torsional response of piles is dependent on slenderness ratios. Therefore, an trial and error is needed for practical design.

(12) The non-homogeneity effect of the soil profile has been explored only for the cases of homogeneous and Gibson soil.

An ideal approach should reflect any degree of non-homogeneity, and elastic-plastic behaviour but be independent of slenderness ratios. Such solutions have been established in Chapter 8.
### TABLE 2-1 Summary of Available Shaft Load Transfer Curves

<table>
<thead>
<tr>
<th>Formulations</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| Kezdi (1957)  | \( \tau = \sigma \tan \varphi \left[ 1 - \exp \left( -k \frac{w}{w_e - w} \right) \right] \)  
Non-linear elastic \( \tau \)-\( w \) relation. | \( \tau \) is the shear stress required for producing a displacement \( w \) at normal stress \( \sigma \); \( \varphi \) is the angle of full shearing resistance; \( w_e \) is the shear displacement necessary for the development of full friction; \( k \) is the initial tangent of the \( \tau/\sigma \) versus \( w \) curve. The maximum ratio of \( \tau/\sigma \) is equal to \( \tan \varphi \). |
| Reese et al. (1969) | \( \tau = k \left[ 2 \left( \frac{w}{w_e} - \frac{w}{w_e} \right) \right] \)  
Non-linear elastic \( \tau \)-\( w \) relation. | \( \tau \) is the local stress, kPa; \( k = 2.74N \), a stress transfer factor, kPa; \( N \) is the number of blows of SPT test; \( w_e = 2d\varepsilon_f \) m; \( \varepsilon_f \) is the average failure strain (%), obtained from unconfined compression tests run on soil samples near the pile tip. |
| Fujita (1976)  | \( \tau/w = 4\overline{N} \) \( (w \leq w_e) \)  
\( \tau = \tau_f \) \( (w > w_e) \)  
Ideal elastic-plastic \( \tau \)-\( w \) curve | \( \tau/w \) is the gradient of the load transfer curve, kPa/cm; \( \overline{N} \) is the average \( N \) values; \( w_e \) is the shaft displacement at the transition depth from elastic to plastic stage, average, cm; \( \tau_f = 13\overline{N}^{0.5} \), maximum local stress, kPa. |
| Armaleh and Desai (1987) | \( \tau = \frac{(k_{os} - k_{fs})w}{M_s} + k_{fs}w \)  
where \( M_s = \left[ 1 + \left( \frac{(k_{os} - k_{fs})w}{P_{fs}} \right)^{m_s} \right]^{1/m_s} \) | \( k_{os}, k_{fs} \) are initial and final spring stiffnesses respectively; \( P_{fs} = K_h \sigma' \tan \varphi \), load at yield point, which equals to \( \tau_{max}(z) \); \( K_h \) is the coefficient of earth pressure; \( \sigma' \) is effective normal stress and \( m_s \) is the order of the curve, taken as unity; \( k_{fs}(z) = 0.005k_{os}(z) \), \( z \) is the depth below ground surface. |
| Hirayama (1990) | \( \tau/w = (a_s + b_s w)^{-1} \)  
Hyperbolic \( \tau \)-\( w \) curve | \( a_s = 0.0025 / \tau_f \); \( b_s = 1/\tau_f \), where \( \tau_f \) (kPa) is correlated to SPT and/or CPT value for bored pile. For sand, \( \tau_f = 5N(\leq 200 \text{ kPa}) \)  
for clay, \( \tau_f = 10N \) or \( s_u(\leq 150 \text{ kPa}) \) |
<p>| Randolph and Worth (1978), Kraft et al. (1981), and Chow (1986) | Theoretical load transfer approach, refer to this chapter |</p>
<table>
<thead>
<tr>
<th>Formulations</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| **Fujita (1976)**<br>
\[ P_b = A_b k_s w_b^n \] For linear case, \( n = 1 \), nonlinear case, \( n = 0.5 \).<br>
(1) Linear case, \( k_s = 40\overline{N}_b \); for nonlinear case, \( k_s = 80\overline{N}_b \);<br>
(2) Linear case, \( k_s = 100\overline{N}_b \); for nonlinear case, \( k_s = 10\overline{N}_b^{1.5} \). | \( A_b = \) base area; \( k_s = \) pile tip resistant factor, unit for linear case, kPa/cm, for nonlinear case, kPa/cm\(^{0.5}\); \( \overline{N}_b = \) average N values for 3 meter above the tip. When shaft stiffness factor is correlated with the average SPT value over the pile, formulas shown in (1) should be adopted; When shaft stiffness factor is related to pile length, formulas in (2) should be used. |
| **Armaleh and Desai (1987)**<br>
\[ P_b = \frac{(k_{ob} - k_{fb}) w_b}{1 + \left(\frac{k_{ob} - k_{fb}}{P_{fb}}\right)^{m_b} P_{fb}} \] \( k_{ob}, k_{fb} = \) initial and final spring stiffness respectively, \( k_{fb} = 0.005 k_{ob} \); \( P_b = \) pile tip resistance and \( m_b = 1 \), the order of the curve; \( \lambda_b = 2.6 \); For very dense sand when \( L/d \geq 20 \) or for very loose sand when \( L/d \geq 10 \), the yield point \( P_{fb} \) is estimated by, \( P_{fb} = q_f A_p \) otherwise, \( P_{fb} = N_q^* \sigma^*_v A_p \). \( N_q^* = \) bearing capacity for deep circular or square foundation. |
| **Wang (1987)**<br>
\[ k_b = 0.267 g_c \sqrt{0.3/d} \] | \( g_c = \) average tip friction from CPT between the depth 4d above pile tip level and 1d below the level. \( d = \) diameter, m. |
| **Hirayama (1990)**<br>
\[ P_b = w_b (a_b + b_b w_b)^{-1} \] Hyperbolic \( P_b-w_b \) curve | For sandy layer, \( a_b = 0.25 d_b/P_{ultb} \); \( P_{ultb} \) has been related to SPT and/or CPT value. |
| **Randolph and Worth (1978), Chow (1986)**<br>
\[ k_b = \frac{4G_b r_o (1 - P_b R_{fb}/P_{fb})^2}{1 - \nu_s} \] | Refer to this chapter. The case for \( R_{fb} = 0 \) was proposed by Randolph and Wroth (1978). |

Note that except where defined previously, all the symbols in Tables 2-1 and 2-2 are not included in the NOTATION.
3. VERTICALLY LOADED SINGLE PILES

3.1 INTRODUCTION

The load-settlement response of single piles and pile groups is significantly affected by non-homogeneity in stiffness and strength of the ground. Three aspects of the response may be identified: (a) the pile-head settlement at working load; (b) the distribution of load down the pile; and (c) the degree of the interaction between piles within a group. While, for piles of moderate slenderness ratio, the settlement under working load is primarily a function of the average stiffness of the ground over the depth of embedment of the pile, variations in stiffness and limiting shaft friction with depth have an increasing influence as the slenderness of the pile increase. For all piles, however, the relative homogeneity of the soil is critical in determining the load distribution along the pile and also interaction between piles.

Analytical methods for piles fall into two main categories: continuum-based, such as the boundary or finite element methods; or load transfer approaches. The latter category quantifies interaction between pile and soil through a series of independent springs distributed along the pile and at the base (Coyle and Reese, 1966). For pile groups and piled rafts, increasing use is being made of a so-called ‘hybrid’ approach, where load transfer springs are used to obtain the response of each single pile, while a continuum model is used to assess effects of interaction between different piles and with the pile cap or raft (Chow, 1986a, 1986b; Clancy and Randolph, 1993).

The load transfer approach is attractive in its flexibility, enabling non-linear and heterogeneous soil conditions to be incorporated easily. At the other extreme, closed form solutions may be obtained for homogeneous elastic-perfectly plastic load transfer spring stiffness (Murff, 1975; Kodikara and Johnston, 1994; Motta, 1994), which may also be related to the elastic properties of the soil in order to simulate continuum-based solutions (Randolph and Wroth, 1978).

The present chapter addresses the response of axially loaded piles in a generic non-homogeneous soil where stiffness and strength vary monotonically with depth. The main aims of the chapter are:

1) to calibrate the relationship between load transfer spring stiffness and elastic soil properties, extending the work of Randolph and Wroth (1978) to consider the effect of non-homogeneity on the relationship;
(2) to present new closed form solutions for the case of elastic-perfectly plastic soil response with stiffness and strength varying as a power law of depth.

(3) to develop a spreadsheet program called GASPILE, which is then adopted to explore the difference between non-linear elastic-plastic and the elastic-perfectly plastic analyses.

The study uses continuum analyses from previous published work, and from an extensive parametric study undertaken using the finite difference program, FLAC (Itasca, 1992), to verify the closed form solutions. The solutions are also used to back-analyse field data, allowing comparison of computed and measured load distributions.

3.2 LOAD TRANSFER MODELS

3.2.1 Expressions of Non-homogeneity

The soil profile concerned and the relevant non-dimensional parameters adopted in this chapter are briefly described below.

(1) The initial soil shear modulus ($G_i$) distribution down a pile is assumed as a power function of depth (Booker et al., 1985)

$$G_i = A_g z^n$$

where $A_g$ and $n$ are constants; $z$ is the depth below the ground surface. The average shear modulus down the pile can be estimated by

$$G_{ave} = A_g L^{n/(n+1)}$$

where $L$ is the pile embedded length. Below the pile tip level, the shear modulus is kept as a constant, $G_{ib}$. Therefore, the pile base shear modulus jump is expressed by the ratio, $\xi_b (= G_{ib}/G_{il})$, where $G_{il}$ is shear modulus just above the pile base level; $\xi_b$ is referred to as the end-bearing factor, which is assumed to be unity in this chapter except where specified. Fig. 3-1 shows examples of the shear modulus distribution.

(2) Generally, it is assumed that the ratio of the limiting shaft friction to shear modulus falls into a narrow range (Randolph and Wroth, 1978), particularly for a
given soil and pile combination. The limiting shaft friction may be expressed, in a similar manner to the shear modulus, as

\[ \tau_f = A_\nu z^\theta \]  

where \( A_\nu \) and \( \theta \) are constants. In this thesis attention will be restricted to the case of \( n \) being equal to \( \theta \). Therefore, the ratio of modulus to limiting shaft stress is invariant with the depth, and is equal to \( A_g/A_\nu \).

(3) Non-homogeneity factor is expressed by, (a) \( \rho_g = G_{ave}/G_{iL} = 1/(n + 1) \), which was referred to as the shaft non-homogeneity factor (Randolph and Wroth, 1978); (b) \( \eta = G_{io}/G_{iL} \) (Poulos, 1979), where \( G_{io} \) = shear modulus at the mudline level. This definition is suitable for a Gibson soil, in which the soil modulus increases linearly with depth; or (c) simply by the power \( n \).

(4) Pile-soil relative stiffness ratio may be expressed as (a) the ratio of pile Young's modulus, \( E_p \) and the base level soil Young's modulus, \( E_{iL} \) (Poulos, 1979), i.e.

\[ K_b = E_p/E_{iL} \]  

(3-4)

or (b) the ratio of \( E_p \), to the shear modulus at pile base level, \( G_{iL} \), (Randolph and Wroth, 1978), i.e.

\[ \lambda = E_p/G_{iL} \]  

(3-5)

As discussed in Chapter 2, the non-homogeneity factor \( n \), and the relative stiffness factor \( \lambda \) are adopted in the current research. For ease of comparison, the previously published results will be converted and expressed in terms of these non-dimensional parameters later.

### 3.2.2 Elastic Stiffness

Theoretical load transfer models were developed for a homogeneous or Gibson soil (Randolph and Wroth, 1978), where the stiffness of the load transfer relationship for soil along the pile shaft and at the pile base was expressed in terms of the elastic properties of the soil. These models are extended further to more general non-homogeneous soil profile as described below.
3.2.2.1 Shaft Load Transfer Model

The shaft displacement, \( w \) is related to the local shaft stress, \( \tau_o \), and initial shear modulus, \( G_i \) by (Randolph and Wroth, 1978)

\[
w = \frac{\tau_o r_o \zeta}{G_i}
\]  

(3-6)

where \( r_o \) is the radius of the pile and \( \zeta \) is a parameter given by

\[
\zeta = \ln \left( \frac{r_m}{r_o} \right)
\]  

(3-7)

where the parameter, \( r_m \), represents the maximum radius of influence of the pile beyond which the shear stress becomes negligible, and is discussed further below and in Chapter 4. Using a hyperbolic law to model the soil stress-strain relationship, the parameter, \( \zeta \), then can be expressed as (Randolph, 1977; Kraft et al. 1981)

\[
\zeta = \ln \left[ \frac{(r_m/r_o - \psi_o)/(1 - \psi_o)}{r_m/r_o} \right]
\]  

(3-8)

where the term, \( \psi_o \) (\( \psi_o = R_{fs}\tau_o / \tau_f \)), represents the non-linear stress level at the pile-soil interface (the limiting stress being assumed to be equal to the failure shaft stress); \( R_{fs} \) is a parameter controlling the degree of non-linearity.

The critical value of the maximum radius of influence of the pile beyond which the shear stress becomes negligible was expressed in terms of the pile length, \( L \), as (Randolph and Wroth, 1978)

\[
r_m = 2.5\rho_g (1 - \nu_s) L
\]  

(3-9)

where \( \nu_s \) is the Poisson's ratio. This estimation of \( r_m \) is generally valid for a pile embedded in an infinite layer. More generally, it can be expressed as

\[
r_m = A \frac{1 - \nu_s}{1 + n} L + B r_o
\]  

(3-10)

Values of \( A \) and \( B \) for different pile geometry, pile-soil stiffness, and various thickness of finite soil layer are explored in Chapter 4.
The purpose of this chapter is to establish closed form solutions, using the load transfer approach. The assessment of the load transfer method, and the suitability of Eq. (3-10) are explained in detail in Chapter 4, where it is shown that the load transfer factor, \( \zeta \) can be taken as approximately constant with depth.

As the pile-head load increases, the mobilised shaft shear stress will reach the limiting value (\( \tau_f \)). The local limiting displacement can be expressed as

\[
we = \zeta r_0 \frac{A_y}{A_g}
\]  

(3-11)

Thereafter, as the pile-soil relative displacement exceeds the limiting value, the shear stress is kept as \( \tau_f \) (i.e., an ideal elastic, perfectly plastic load transfer response is assumed). Due to the assumed similarity of the limiting shaft stress and the shear modulus distribution, the limiting shaft displacement is a constant down the pile.

The load transfer response may be taken as elastic, by assuming a constant value of \( \psi_0 \), (referred to as 'simple linear analysis (SL)'). Alternatively, non-linearity may be incorporated, expressing the parameter \( \psi_0 \) as a function of stress level and the constant \( R_{fs} \) in Eq. (3-8) (hence, \( \zeta \) is dependent on stress level). This is referred to as non-linear (NL) analysis. As an example, Fig. 3-2 shows the non-dimensional shear stress versus displacement relationship for NL (\( R_{fs} = 0.9 \)) and SL (\( \zeta = \text{constant with } \psi_0 = 0.5 \)) analyses, with \( \tau_f/G_i = 350 \), \( L/r_0 = 100 \), and \( \nu_s = 0.5 \). Note that full mobilisation of shaft friction occurs at a displacement of 1 - 2 % of the pile radius, which accords with experimental evidence (Whitaker and Cooke, 1966).

### 3.2.2.2 Base Pile -Soil Interaction Model

The base settlement can be estimated through the solution for a rigid punch acting on an elastic half-space, as suggested by Randolph and Wroth (1978)

\[
w_b = \frac{P_b(1 - \nu_s)\omega}{4r_0G_{ib}}
\]

(3-12)

where \( P_b \) is the mobilised base load; \( \omega \) is the pile base shape and depth factor which is generally chosen as unity (Randolph and Wroth, 1978; Armaleh and Desai, 1987). This parameter will be assessed in detail in Chapter 4. Using a hyperbolic model, the base load displacement relationship can be given by (Chow, 1986b).
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\[ w_b = \frac{P_b(1 - \nu_s)\omega}{4\tau_0 G_{ib}} \frac{1}{(1 - R_{fb} P_b/P_{fb})^2} \]  

(3-13)

where \( P_{fb} \) is the limiting base load; \( R_{fb} \) is a parameter controlling the degree of non-linearity.

3.3 OVERALL PILE SOIL INTERACTION

Generally, a pile is assumed to behave elastically, with constant diameter and Young's modulus. Therefore, the governing equation for pile-soil interaction can be written as (Randolph and Wroth, 1978)

\[ \frac{d^2u(z)}{dz^2} = \frac{\pi d\tau_0}{E_p A_p} \]

(3-14)

where \( E_p, A_p \) are the Young's modulus and its cross-sectional area of an equivalent solid cylinder pile, and \( u(z) \) is the axial pile deformation.

3.3.1 Elastic Solution

Within the elastic stage, the shaft stress in Eq. (3-14) can be expressed by the local displacement as prescribed by Eq. (3-6), in which the load transfer factor, \( \zeta \) is estimated by Eq. (3-8) with \( \psi_0 = 0 \). Therefore, the basic differential equation governing the axial deformation for a pile fully embedded in the non-homogeneous soil described by Eq. (3-1) is found to be

\[ \frac{d^2u(z)}{dz^2} = \frac{A_g}{E_p A_p} \frac{2\pi}{\zeta} z^n w(z) \]  

(3-15)

The axial pile displacement, \( u(z) \) should equal the pile-soil relative displacement, \( w(z) \) when ignoring any external soil subsidence. Normally the load transfer factor \( \zeta \) can be taken as a constant along a pile depth (Randolph and Wroth, 1978). Therefore, Eq. (3-15) can be solved in terms of Bessel functions of non-integer order

\[ w(z) = \left( \frac{z}{L} \right)^{1/2} (A_1 I_m(y) + B_1 K_m(y)) \]  

(3-16)
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\[ \frac{dw(z)}{dz} = - \frac{P(z)}{E_p A_p} = k_s \left( \frac{z}{L} \right)^{1/2} - \frac{n}{2} (A_1 I_{m-1}(y) - B_1 K_{m-1}(y)) \]  

(3-17)

where \( w(z) \), \( P(z) \) are the displacement and load at a depth of \( z \) \((0 < z \leq L)\); \( m = \frac{1}{n + 2} \). The variable \( y \) is

\[ y = 2m \frac{L}{r_0} \sqrt{\frac{2}{r_0}} \left( z \right)^{1/2} \]

(3-18)

and the stiffness factor \( k_s \)

\[ k_s = \frac{L}{r_0} \sqrt{\frac{2}{r_0}} \left( \frac{1}{L} \right)^{1/2} \]

(3-19)

The constants \( A_1 \) and \( B_1 \) can be found in terms of the stress and deformation compatibility conditions at the pile base

\[ w(L) = w_b \]

(3-20)

\[ \left[ \frac{\partial w}{\partial z} \right]_{z=L} = - \frac{P_b}{A_p E_p} = - \frac{4}{(1 - \nu_s)\omega \xi b \lambda} \frac{1}{r_0} w_b \]

(3-21)

where \( P_b \) has been expressed in terms of \( w_b \), through Eq. (3-12). Therefore, the coefficients \( A_1, B_1 \) can be expressed respectively as

\[ A_1 = w_b \left( K_{m-1} - \chi_v K_m \right) / \left( K_{m-1} I_m + \chi_v K_m I_{m-1} \right) \]

(3-22)

\[ B_1 = w_b \left( I_{m-1} + \chi_v I_m \right) / \left( K_{m-1} I_m + \chi_v K_m I_{m-1} \right) \]

(3-23)

where \( I_m, I_{m-1}, K_{m-1} \) and \( K_m \) are the values of the Bessel functions for \( z = L \). The ratio \( \chi_v \) is given by

\[ \chi_v = \frac{P_b}{w_b} \frac{L}{E_p A_p} \left( k_s L^{n/2+1} \right) = \frac{2\sqrt{2}}{\pi (1 - \nu_s) \omega \xi_b \lambda} \frac{\xi}{\lambda} \]

(3-24)

Substituting the expressions for \( A_1 \) and \( B_1 \) into Eqs. (3-16) and (3-17), the displacement and load at any depth of \( z \) can be expressed respectively as
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\[
\begin{align*}
w(z) &= w_b \left( \frac{z}{L} \right)^{1/2} \left( \frac{C_3(z) + \chi_v C_4(z)}{C_3(L)} \right) \\
P(z) &= k_s E_p A_p w_b z^{n/2} \left( \frac{z}{L} \right)^{1/2} \left( \frac{C_1(z) + \chi_v C_2(z)}{C_3(L)} \right)
\end{align*}
\]

(3-25) (3-26)

where

\[
\begin{align*}
C_1(z) &= -K_{m-1} l_{m-1}(y) + K_{m-1}(y) l_{m-1} \\
C_2(z) &= K_{m} l_{m-1}(y) + K_{m-1}(y) l_{m} \\
C_3(z) &= K_{m-1} l_{m}(y) + K_{m}(y) l_{m-1} \\
C_4(z) &= -K_{m} l_{m}(y) + K_{m}(y) l_{m}
\end{align*}
\]

(3-27)

At any a depth \( z \), the stiffness can be derived as

\[
\frac{P(z)}{G_{IL} w(z) r_0} = \sqrt{2 \pi} \sqrt{\frac{\lambda}{\zeta}} C_v(z)
\]

(3-28)

where

\[
C_v(z) = \frac{C_1(z) + \chi_v C_2(z)}{C_3(z) + \chi_v C_4(z)} \left( \frac{z}{L} \right)^{n/2}
\]

(3-29)

At the ground surface, where \( z = 0 \), it is necessary to take the limiting value of \( C_v(z) \) as \( z \) approaches zero. This will be referred to as \( C_{vo} \). From Eqs. (3-25) and (3-26), the base settlement can be written as a function of pile load and displacement as well. The accuracy of the above closed form solutions (CF) have been checked by Mathcad™. Further corroboration by continuum-based finite difference analysis is shown later.

3.3.2 Elastic-Plastic Solution

As the pile-head load increases, pile-soil relative slip is assumed to commence from the ground level and at any stage during loading may be taken to have developed to a depth called transition depth (\( L_1 \)), at which the shaft displacement, \( w \), corresponds to the local limiting displacement. The upper part of the pile, above the transition depth, is in a plastic state, while the lower part below this depth is in an elastic state. Within the plastic state, the shaft shear stress in Eq. (3-14) should be replaced by the limiting shaft
stress Eq. (3-3). Pile-head load and settlement are, therefore, expressed respectively as a sum of the elastic part represented by letters with subscript of "e", and the plastic part

\[ P_t = P_e + \frac{2\pi \rho_o A_v L_1^{\theta+1}}{\theta + 1} \]  \hspace{1cm} (3-30)

\[ w_t = w_e + \frac{L_1}{E_p A_p} \left( \frac{2\pi \rho_o A_v L_1^{\theta}}{1 + \theta} + P_e \right) \]  \hspace{1cm} (3-31)

where \( \mu = L_1/L \) is defined as the degree of slip (0 < \( \mu \) ≤ 1); \( L_1 + L_2 = L \); \( L_2 \) is the length of the lower elastic part, which equals \( L(1 - \mu) \). The pile load at the transition depth is written as \( P_e \). Since \( w(z) = w_e \) at the transition depth of \( L_1 \), \( P_e = P(L_1) \) can be readily estimated from Eq. (3-28), therefore, Eq. (3-30) can be rewritten as

\[ P_t = w_e k_s E_p A_p L^{n/2} C_v(\mu L) + \frac{2\pi \rho_o A_v (\mu L)^{1+\theta}}{1 + \theta} \]  \hspace{1cm} (3-32)

Similarly, as a result of Eq. (3-31), and substituting for \( P_e \), the pile-head settlement is expressed as

\[ w_t = w_e \left[ 1 + \mu k_s L^{n/2+1} C_v(\mu L) \right] + \frac{2\pi \rho_o A_v (\mu L)^{2+\theta}}{E_p A_p} \]  \hspace{1cm} (3-33)

These solutions provide three important results as shown below:

1. For a given degree of slip, the pile-head load and settlement can be estimated by Eqs. (3-32) and (3-33) respectively, therefore the full pile-head load-settlement relationship may be obtained;

2. For a given pile-head load, the corresponding degree of slip of the pile can be back-figured by Eq. (3-32);

3. The distribution profile of either load or displacement can be readily obtained, at any stage of the elastic-plastic development. Within the upper plastic part, at any depth of \( z \),

   (i) from Eq. (3-30), the load, \( P(z) \) can be predicted by

   \[ P(z) = P_e + 2\pi \rho_o A_v \frac{(\mu L)^{\theta+1} - z^{\theta+1}}{\theta + 1} \]  \hspace{1cm} (3-34)

   (ii) from Eq. (3-31), the displacement, \( w(z) \) can be obtained by
The current analysis is limited to the case of n = 0, but the physical implications of n (related to elastic stage) and θ (to plastic stage) are completely different, therefore, both parameters are presevered in the equations.

3.4 PILE RESPONSE WITH HYPERBOLIC SOIL MODEL

3.4.1 A Program for Non-linear Load Transfer Analysis

A program operating in Windows EXCEL called GASPILE has been developed to allow analysis of pile response in non-linear soil. The analytical procedure is similar to that proposed by Coyle and Reese (1966) for computing load-settlement curves of a single pile under axial load. The pile is discretised into elements, each of which is connected to a soil load transfer spring. The load transferred is divided into two parts: the shaft where Eq. (3-6) is adopted; and the base where Eq. (3-13) is employed. The input parameters include (1) limiting pile-soil friction distribution down the pile; (2) initial shear modulus distribution down the pile; (3) the end-bearing factor and soil Poisson's ratio; and (4) the dimensions and Young's modulus of the pile. Comparison (referred to in Appendix A) shows that the results from GASPILE are consistent with those from RATZ (Randolph, 1986).

To explore the effect of the non-linear soil model, GASPILE has been used to analyse a typical pile-soil system: The pile is assumed to have dimensions of L = 25 m, r₀ = 0.25 m and E_p = 2.9 GPa, in a soil of G_ave = 20 MPa, Poisson's ratio, ν_s = 0.4; the ratio of modulus and strength, G_i/τ_f = 350; The end-bearing factor has been taken as, ξ_b = 1 and the ultimate base load as, P fb = 1.2 MN. The pile is discretized into 20 segments, although in practice the results are very similar to those using 10 segments.

3.4.2 Shaft Stress-Strain Non-linearity Effect

GASPILE analyses have been performed respectively by using both the non-linear model (NL, R_f = constant) model and the simple linear model (SL, ψ = constant) as described previously and shown in Fig. 3-2. To explore the difference in the shaft component of the pile response between the analyses using NL and SL models, for both analyses the base soil models have been taken identically by using a value, R fb of 0.9.
Fig. 3-3 shows that the analyses from the NL \((R_{fs} = 0.9)\) and SL \((\psi_o = 0.5)\) models are generally consistent for (1) non-dimensional load and displacement distribution down the pile (two different base displacement, \(w_b = 1.5, 3.0\) are provided), and (2) the pile-head response (note that base behaviour is identical). This is probably because the shaft NL and SL models, as illustrated in Fig. 3-2, are generally consistent with each other for stress levels below about 0.6 to 0.7. The difference in load transfer due to the variation of the degree of non-homogeneity can be sufficiently reflected by the simplified model (SL). For most realistic cases, the effect of non-linearity is expected to be significant only at load levels close to failure (e.g. Fig. 3-3(c) for which \(R_{fs} = 0.9\)). Therefore, it is generally sufficient to use the simplified model (SL) resulting in a constant value of \(\zeta\).

### 3.4.3 Base Stress-Strain Non-linearity Effect

The influence of base stress level is obvious only when a significant settlement occurs (Poulos, 1989). If the base settlement, \(w_b\) is less than the local limiting displacement, \(w_e\), the base soil is generally expected to behave elastically except when the underlying soil is less stiff than the soil above the pile base level \((\xi_b > 1)\). However, in the case of \(\xi_b > 1\), the base contribution becomes less important. Therefore an elastic consideration of the base interaction before full shaft slip is generally adequate.

The effect of the non-linear stress-strain relationship on pile-head response is further illustrated in Fig. 3-4 for soil of different profiles, with the results presented separately for clarity. Each presentation provides the comparison between closed form solutions with two constant values of \(\psi_o = 0, 0.5\) and the NL \((R_{fs} = 0.9)\) analyses by GASPILE.

### 3.5 VERIFICATION OF THE THEORY

To verify the elastic solutions outlined previously, a continuum-based numerical analysis was performed with the finite-difference program FLAC, while the elastic-plastic solutions were substantiated by the available continuum-based solutions (Poulos, 1989; Jardine et al. 1986).

#### 3.5.1 FLAC Analysis

For the current FLAC analysis, a typical axisymmetric grid generated is shown in Fig. 3-5. The width of the grid was the maximum of \(2.5L\) and \(75r_0\). The effect of the ratio of the depth to the underlying rigid layer, \(H\) and the pile length, \(L\) has been explored. As
demonstrated in Fig. 3-6, a decrease in H/L generally leads to an increase in the pile-head stiffness, particularly for the case of higher relative pile-soil stiffness. However, the difference becomes negligible, when the value of H/L exceeds 4. Therefore in the following analysis H is kept at 4L (H/L = 4) to minimise the boundary effect. In addition, Poisson's ratio of the pile has been taken as, \( v_p = 0.2 \).

### 3.5.2 Pile-head Stiffness and Settlement Ratio

The closed form solution for the pile response, which is later referred to as CF, depends on the load transfer parameter, \( \zeta \), which in turn depends on the \( \psi_o \) and \( r_m \). The solutions presented below have been based on values of: \( A = 2, B = 0, v_s = 0.4, \xi_b = 1, \psi_o = 0, \) and \( \omega = 1 \). Note that these values will be adopted in all the following analyses, except where specified. Justification for the choice of A and B will be presented in Chapter 4.

The pile-head stiffness predicted by Eq. (3-28) has been plotted against the result from FLAC analyses in Fig. 3-7. The head stiffness obtained by the simple analysis (SA) (Randolph and Wroth, 1978) has been shown as well, albeit with \( A = 2.5 \) (other parameters being identical to those adopted in Eq. (3-28)). The results show that:

1. The CF approach is reasonably accurate but generally slightly underestimates the stiffness in comparison with the FLAC results. However the difference is less than 10%;
2. The SA analysis progressively overestimates the stiffness with either increase in non-homogeneity factor \( n \) (particularly, \( n = 1 \)), or decrease in pile-soil relative stiffness factor. However, the difference is less than 20%;
3. For a pile in homogenous soil (\( n = 0 \)), the CF and SA approaches are exactly the same as illustrated in the Appendix B. However, since different values of A have been adopted, the two approaches predict slightly different values of the head stiffness.

The ratio of pile head and base settlement estimated by Eq. (3-25) has been compared with those from the FLAC analyses in Fig. 3-8. For extremely compressible piles, the CF solutions diverge from the FLAC results, probably because the displacement prediction becomes progressively more sensitive to the neglecting of the interactions between each horizontal layer of soil.
3.5.3 Load Settlement

The load settlement relationship furnished by Eqs. (3-32) and (3-33) will be verified by continuum-based analyses in this section. Detailed results are compared for a particular set of pile and soil parameters, concentrating on the elastic-plastic response of the pile.

3.5.3.1 Homogeneous Case

A pile of 30 m in length, and 0.75 m in diameter, is located in a homogeneous soil layer 50 m deep. The initial tangent modulus of the soil (for very low strains) is 1056 MPa, Poisson's ratio is taken as 0.49, and a constant limiting shaft resistance of 0.22 MPa is adopted over the pile embedded depth. The Young's modulus of the pile is taken as 30 GPa. The numerical analyses by GASPILE and the closed form solutions for the load settlement curves are shown in Fig. 3-9, together with the results from a finite element analysis involving the use of a non-linear soil model (Jardine et al. 1986), and from two kinds of boundary element analyses utilising an elastic-plastic continuum-based interface model and a hyperbolic continuum-based interface model respectively (Poulos, 1989). The results demonstrate that the load transfer analysis is very consistent with other approaches. However, as noted by Poulos (1989), the response of very stiff piles (e.g., a value of Young's modulus of the pile being 30,000 GPa), obtained using an elastic, perfectly plastic soil response, can differ significantly from that obtained using a more gradual non-linear soil model.

Generally speaking, except for short piles, the pile-head response is only slightly influenced by base shear behaviour, as shown in Fig. 3-10 for two end-bearing factors of $\xi_b = 1$ and 2.5. The non-linear base behaviour, as illustrated by the difference between the non-linear GASPILE and linear (closed form) analyses, will become obvious only when local shaft displacement at the base level exceeds the limiting displacement, $w_e$.

3.5.3.2 Non-homogeneous Case

Previous analyses (e.g. Banerjee and Davies, 1977; Poulos, 1979; Rajapakse, 1990) have reported that pile-head stiffness substantially decreases as the soil shear modulus non-homogeneity factor (n) increases. This is partly because the modulus at the pile tip level was kept constant. Therefore as n increases, the average shear modulus over the pile length decreases (see Eq. (3-2)). To explore the effect of the distribution alone,
when the non-homogeneity factor \( \theta = n \) is changed, the average shaft shear modulus should be kept as a constant and also all the other parameters be identical. In such a way, the closed form prediction by \( \psi_0 = 0 \) (linear elastic-plastic case) has been shown in Fig. 3-11 (a). In this particular case, only about 30% difference due to variation in the \( n \) is noted within the elastic stage. Both the pile-head load and settlement, at which slip is initiated, decreases as \( n \) increases, as demonstrated in Fig. 3-11 (b) and (c). Once pile-soil relative slip develops (\( \mu > 0 \)), the pile-head load and displacement for \( \theta = 1 \) increases with the slip degree, \( \mu \) at a higher gradient than that for \( \theta = 0 \). Therefore at some degree of the slip, the load will be identical irrespective of the shaft stress non-homogeneity factor, \( \theta \). For a purely frictional pile, the degree is unity. However, generally due to the base contribution, the degree is less than unity, and the corresponding load occurs before the full development of the pile-soil slip.

### 3.6 SETTLEMENT INFLUENCE FACTOR

The above analysis is further substantiated for different slenderness ratio and relative pile-soil stiffness cases.

#### 3.6.1 Settlement Influence Factor

The settlement influence factor, \( I \) is defined as the inverse of a pile-head stiffness, therefore

\[
I = \frac{G_{il} w_1 r_0}{P_t} \quad (3-36)
\]

where \( I \) is the settlement influence factor. While within the elastic stage, the factor can be derived directly from Eq. (3-28)

\[
I = \frac{1}{\pi \sqrt{2C_{vo}}} \sqrt{\frac{\zeta}{\lambda}} \quad (3-37)
\]

It is also straightforward to obtain an elastic-plastic formula for the factor, in terms of Eqs. (3-32) and (3-33). The settlement influence factor is mainly affected by pile slenderness ratio, pile-soil relative stiffness factor, the degree of the non-homogeneity of the soil profile, and the degree of pile-soil relative slip.
3.6.2 Pile Slenderness Ratio Influence

Fig. 3-12 shows the settlement influence factor for a pile of different slenderness ratios in a Gibson soil, at a constant relative stiffness factor ($\lambda = 3000$), together with the BEM analysis based on Mindlin's solution (Poulos, 1989), BEM analysis of three dimensional solids (Banerjee and Davies, 1977), and the approximate closed form solution by Randolph and Wroth (1978). The effect of slenderness ratio on the settlement influence factor reflected by the closed form solution is generally consistent with those provided by the other approaches.

3.6.3 Pile-Soil Relative Stiffness Effect

Using identical values as mentioned previously, the settlement influence factor, $I$, was estimated by Eq. (3-37) for a list of given $\lambda$, which are shown in Fig. 3-13 for four different slenderness ratios, in comparison with the boundary element (BEM) analysis by Poulos (1979) and the current FLAC analysis. The BEM analysis is for the case of $H/L = 2$, while this CF solution is corresponding to the case of $H/L = 4$. As presented early in Fig. 3-4, increase in the value of $H/L$ can lead to decrease in pile-head stiffness and thus an increase in the settlement influence factor. For the case of $L/r_0 = 50$, $\lambda = 26,000$ ($K_b = 10,000$), $n = 0$, an increase in $H/L$ from 2 to infinity can lead to an increase in the settlement factor by up to 21% (Poulos, 1979). In view of the $H/L$ effect, the closed form solutions are generally quite consistent with the numerical analysis.

3.7 CASE STUDY

The non-homogeneous soil property and the pile-soil relative slip can be readily taken into account by the established solutions. An example analysis is demonstrated as follows. The test reported by Gurtowski and Wu (1984) describes the detailed measured response of a pile. The pile was 0.61 m wide octagonal prestressed concrete hollow pile with a plug at the base, and was driven to a depth of about 30 m. For the current analysis, the parameters used by Poulos (1989) have been adopted directly as shown below: Young's modulus of the pile was 35 GPa; soil Young's modulus is approximated by 4N MPa ($N =$ SPT value); $N$ increases almost linearly with depth from 0 at ground surface to 70 at a depth of 30 m. The limiting shaft stress is taken as 2N kPa, the base limiting stress is 0.4 MPa, and the soil Poisson's ratio is 0.3. The pile-head and base load-settlement predictions by GASPILE with $R_{fs} = 0.9$, $G_i/\tau_f = 769.2$ $R_{fb} = 0.9$ and by the closed form solutions with $\psi_o = 0.5$ are shown in Fig. 3-14 together with those
predicted by boundary element analysis (Poulos, 1989). Good comparisons have been demonstrated between the current predictions and the measured results, except at failure load levels. The divergence at high load levels between the current predictions and those of Poulos (1989) is because the assumed ultimate base stress of 0.4 MPa in the current GASPILE and closed form analyses is different from that used in the boundary element analysis.

### 3.7.1 Load Displacement Distribution Down a Pile

Load and displacement distribution below and above the transition depth may be estimated in terms of the elastic and elastic-plastic solutions. Under a given pile-head load, the depth of the transition is expressed by the degree of slip, and can be estimated by Eq. (3-32) (i.e., by using Mathcad), which is also affected by the non-homogeneity factor n. At a depth below the transition point, the local shaft displacement must be less than the limit displacement, \(w_e\). Therefore, the distributions may be estimated by Eqs. (3-25) and (3-26) respectively. Otherwise, with an estimated \(\mu\), the distribution of load and displacement within the upper plastic zone can be evaluated by Eqs. (3-34) and (3-35) respectively. If a pile-head load exceeds the load corresponding to the full shaft slip, then the difference should be attributed to the base load.

For this typical example, the degrees of slip at \(P_t = 1.8\) MN are 0.058, 0.136, 0.202, 0.258, and 0.305 for \(n = 0, 0.25, 0.5, 0.75,\) and 1.0 respectively. For \(P_t = 3.45\) MN, \(\mu = 0.698, 0.723, 0.743, 0.758, \) and 0.771 correspondingly. For \(P_t = 4.52\) MN, the base should take a load of 1.07 MN. The closed form predictions of load and displacement distributions down the pile are generally consistent with those from non-linear GASPILE analysis. For the three different soil profiles of \(n = 0, 0.5\) and 1.0, the settlement (only at two load levels) and load transfer are shown in Fig. 3-15 together with the those from GASPILE. A summary of the closed form solutions and that from GASPILE (\(n = 1\) case only) are shown in Fig. 3-16 a, b, c in conjunction with those by Poulos (1989) and the measured data. The results show that the linear correlations of the soil strength and shear modulus with the values of SPT (\(n = 1\)) yield reasonable predictions of the pile response in comparison with the measured.

### 3.8 CONCLUSIONS

The analysis outlined in this chapter has attempted to provide a more rigorous approach to the analysis of a pile in a non-homogenous soil medium. The accuracy of the
solutions based on the load transfer approach is very good compared with those from continuum and non-linear load transfer analyses. The following conclusions can be drawn:

(1) A non-linear elastic-plastic analysis makes only a slight difference from that of the simplified linear elastic-plastic analysis. Therefore the newly established closed form solutions based on the simplified elastic-plastic soil response are sufficiently accurate even for estimation of the non-linear response.

(2) The previously reported significant influence of \( n \) on pile-head stiffness or settlement influence factor is partly caused by the alteration of average shear modulus over the pile length, and partly by the distribution. For a constant average shear modulus, the influence of the \( n \) on the pile-head stiffness become relatively minor.

(3) The effect of the pile-soil relative slip on estimating load-settlement behaviour including the load and displacement distributions down the pile can be readily simulated by the newly established theory.

From the second conclusion, it may be inferred that even for a complicated shear modulus profile, the non-homogeneity factor, \( n \) may be adjusted to fit the general trend of the modulus with depth. The solution presented here may thus still be applied with reasonable accuracy.
4. LOAD TRANSFER IN FINITE LAYER MEDIA

4.1 INTRODUCTION

Load transfer analysis is an uncoupled analysis, which treats the pile-soil interaction along the shaft and at the base as independent springs (Coyle and Reese, 1966). The stiffness of the elastic springs, expressed as the gradient of the local load transfer curves, may be correlated to the soil shear modulus by load transfer factors (Randolph and Wroth, 1978). Given suitable load transfer factors, the analysis provides a close prediction to a continuum based numerical analysis as reported by Randolph and Wroth (1978) for piles in an infinite elastic half-space. The question is whether the load transfer factors are significantly affected by a number of features: (a) non-homogeneous soil profile, (b) soil Poisson's ratio, (c) pile slenderness ratio, and (d) the relative ratio of the depth of any underlying rigid layer to the pile length. How these features affect the final pile prediction has not yet fully explored. In addition, the assumption of proportionality of load transfer gradient to the soil shear modulus, uniformly down the pile, has not been rigorously explored.

Ideally, load transfer analysis should give identical results to that of a continuum based numerical analysis. Therefore, continuum based analysis should be used to calibrate the load transfer factors.

This purpose of this chapter is devoted to (1) investigating the adequacy of the load transfer approach; (2) calibrating the load transfer factors by undertaking Fast Lagrangian Analysis of Continua (FLAC) (Itasca, 1992) considering the above mentioned (a) to (d) conditions. The results are then expressed in statistical formulae.

The back-figured factors are thereafter adopted in the new closed form (CF) solutions to estimate pile-head stiffness, and the ratio of pile base and head load over a wide range of slenderness ratios, finite layer ratios, soil Poisson's ratios, non-homogeneity and relative pile-soil stiffness factor, with the purpose to re-examine the suitability and the accuracy of the load transfer approach through comparison with previous publications and the current FLAC analysis.
4.2 RATIONALITY OF LOAD TRANSFER APPROACH

4.2.1 Calibration Procedures

The core of the closed form (CF) solutions is the two critical load transfer factors, $\zeta$ and $\omega$, which are essentially dependant on each other. As stated later, the shaft load transfer factor can reasonably be assumed as a constant with depth. It is thus reasonable to back-estimate the two factors with the CF solutions through comparison with a more rigorous numerical (FLAC) analysis. The base behaviour will be calibrated uniquely against Eq. (3-12), while the shaft load transfer factor can be calibrated against a number of non-dimensional ratios, e.g.

(a) pile-head stiffness, defined as $P^\prime/(G_LW_{t_0})^1$;
(b) the ratio of base and head loads;
(c) the ratio of pile head and base settlement.

If the load transfer approach is accurate compared with the FLAC analysis, then identical values of $\zeta$ and $\omega$ should be obtained irrespective of the ratios used for the calibration.

For the calibration against pile-head stiffness (referred to as matching pile-head stiffness), the procedure can be detailed as:

(i) Using a desired set of soil and pile parameters, FLAC analysis is performed.
(ii) In terms of the FLAC analysis, pile-head stiffness, and the ratio of base to head loads can be obtained. Therefore, $\omega$ may be obtained by Eq. (3-12).
(iii) For the same problem, using a guessed initial value of $\zeta$, a pile-head stiffness can be estimated by Eq. (3-28), with the ‘$\omega$’ obtained above.
(iv) The initial value of $\zeta$ may be adjusted iteratively, so that the estimated pile-head stiffness can match (within a desired accuracy) with that obtained from FLAC analysis. Therefore, $\zeta$ is obtained.

This process has been fulfilled through a purpose written program in FORTRAN 77. To examine the accuracy of the load transfer approach, calibration against the load ratio between pile base and head (referred to as load ratio) has been performed as well. This approach is similar to that of matching pile-head stiffness. However, in steps (iii) and (iv), load ratio rather than stiffness has been estimated by Eq. (3-26), and matched against that from FLAC analysis.

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1 Except where specified, all the symbols are identical to those defined previously in the thesis.
4.2.2 FLAC Analysis

A continuum based finite difference (FLAC) analysis has been used to explore the validity of the load transfer approach, and to assess optimum values of load transfer factors, $\xi$ and $\omega$. In the simulation of the problem, two kinds of boundary conditions were used:

(a) normal boundary: For the radial boundary, the radial displacements of all the nodes are restrained, while for the base boundary, only vertical displacements are constrained;
(b) fixed boundary: The vertical and radial displacements of the nodes along the boundary are restrained.

Prior to any analysis being performed, a number of factors affecting the load transfer factors and pile head-stiffness have been explored for a pile embedded in a soil with shear modulus described by Eq. (3-1). Except where specified, this pre-exploration was based on the following conditions:

- The mesh adopted was a $21 \times 50$ grid, which is identical to that used in Chapter 3.
- Normal boundary was adopted.
- The width of the grid was chosen as the maximum of 2.5L and 75$\rho_0$.

This pre-exploration gave the following points:

1. The effect of the Poisson's ratio of the pile on this analysis may be ignored as illustrated in Table 4-1.

2. The width of the $21 \times 50$ grid may affect the pile-head stiffness. Table 4-2 shows that pile-head stiffness obtained from using a width of mesh of 2.5 L is barely different from that using 75$\rho_0$. However, as shown in Fig. 4-1 (in the figure, the current equation is referred to Eq. (4-5) as shown later), the width of the mesh should be increased to 2.5L ($L/\rho_0 > 40$) to avoid the effect on back-estimated $\xi$.

3. A width of 2.5L may be used to avoid the boundary effect. To show the effect of the width of the mesh combining with boundary condition, a radial width of 75$\rho_0$ for the mesh has been adopted for a set of analysis based on fixed boundary. As shown in Fig. 4-2, the fixed boundary comes into effect as pile slenderness ratio ($L/\rho_0$) exceeds 40, and results in lower values of $\xi$ than those from the normal boundary with 2.5L. However, as long as the width of the mesh is sufficient, the boundary effect can be ignored.
Chapter 4 4.4 Load Transfer Factors

(4) Boundary conditions become important when the ratio of H/L exceeds about 4. Fig. 4-3 shows a comparison of the effect of the boundary on pile-head stiffness and the ratio of head and base settlements (The widths of the meshes are identical, with a value of 2.5L.) When the ratio of H/L exceeds 4, fixed boundary offers constant values of pile-head stiffness, and ratio of head and base settlements. Therefore, the fixed boundary seems to be a reasonable assumption for the prediction of the pile-soil response rather than the normal boundary, which has been further substantiated by the following points:

(a) As illustrated in Fig. 4-3(a), as the ratio of H/L increases, pile-head stiffness obtained from using a normal boundary keeps decreasing (for lower values of n), which does not seem to be real, in contrast with that obtained from using fixed boundary.

(b) Fig. 4-3 (b) shows that the ratios of pile head and base settlements from using a normal boundary diverge progressively (when H/L > 4), which does not seem realistic.

(5) As the ratio of H/L increases, the effect of the density of the mesh becomes obvious. Fig. 4-4 shows the results obtained by using two different kinds of grids. A grid of 21 x 100 will give accurate value of shaft and base load transfer factors. Using a grid of 21 x 50, reasonable values of shaft factors may be obtained for H/L < 4, but incorrect trends for the base load transfer factor are evident for larger values of H/L.

(6) FLAC analysis gives a slightly higher pile-head stiffness. Table 4-3 shows comparisons among boundary element analysis (BEM, Randolph and Wroth, 1978), variational method (VM, Rajapakse, 1990) and the FLAC analysis for single piles in homogeneous soil (n = 0, H/L = 4). The FLAC and BEM analyses were based on a Poisson’s ratio of 0.4, while in the VM analysis, the soil Poisson’s ratio was selected as 0.5. Since a higher Poisson’s ratio generally leads to higher stiffness (as shown later), the stiffness from FLAC analysis is slightly higher than the predictions from the other approaches. Table 4-4 shows a further comparison between FEM analysis (Randolph and Wroth, 1978), and the FLAC analysis for both homogeneous (n = 0) and Gibson soil (n = 1). Note that the results shown in Table 4-4 were based on a constant shear modulus below the pile base level.

In terms of the above exploration, in the present FLAC analysis

(i) generally a grid of 21 x 50 was used, following point (5);
(ii) the Poisson's ratio of the pile was taken as $v_p = 0.2$, following point (1);
(iii) the width of the grid was chosen as the maximum of $2.5L$ and $75r_0$, following points (2) and (3);
(iv) when the ratio of $H/L$ is less than 4, the normal boundary was used, otherwise the fixed boundary was adopted, following point (4);
(v) base load transfer factor was estimated in terms of a grid of $21 \times 100$, following point (5).

The analyses have explored the effect of the four factors discussed earlier.

### 4.2.3 Variation of Shaft Load Transfer Factor With Depth

FLAC analysis is utilised to find the base and shaft load transfer factors. The base factor, $\omega$ is directly back-figured by Eq. (3-12), in which the base load, $P_b$ was estimated through linear extrapolation from the stresses of the last two segments from the base of the pile, and the base displacement, $W_b$ was taken as the node displacement of the pile base. The shaft load transfer factor has been evaluated in terms of the shaft load transfer model and the closed form solutions for single pile (Chapter 3) as described respectively below.

(1) Variation of $\zeta$ with depth: With the shaft shear stress and displacement along a pile obtained by FLAC analysis, the shaft load transfer factor has been back-figured by using Eq. (3-6), as represented by 'FLAC' in Fig. 4-5(a).

(2) Average value of $\zeta$ over the pile embedded depth: Taking $\zeta$ as a constant with depth, the value of the factor, $\zeta$ has been back-figured in light of the calibration procedures described previously in section 4.2.1, and has been illustrated in Fig. 4-5(a) as matching 'load ratio' and pile head 'stiffness'.

Fig. 4-5(a) shows that the variation of $\zeta$ with depth can be approximately taken as a constant. With average values of $\zeta$, the predicted loads and displacements along the pile, using Eqs. (3-25) and (3-26) respectively, are very close to those from the FLAC analysis, as illustrated in Fig. 4-5 (d) and (e) respectively. Therefore, the shaft factor, $\zeta$, can generally be assumed as a constant with depth.

### 4.3 Expressions for Load Transfer Factors

As just mentioned in section 4.2, the base factor, $\omega$ may be back-figured directly from Eq. (3-12). The shaft load transfer factor may be taken as a constant with depth, and so may be backfigured directly by either matching 'load ratio' or pile-head 'stiffness'.
With these approaches, load transfer factors have been obtained in terms of the FLAC analysis, and have been given in the form of simple expressions.

### 4.3.1 Base Load Transfer Factor

Load distribution prediction is sensitive to the base load transfer factor. Thus, a more accurate value of the factor has been provided here as

\[
\omega = \frac{\omega_h \omega_v}{\omega_{oh} \omega_{ov}}
\]  

(4-1)

where \(\omega_h, \omega_v\) are the parameters that reflect the effect of H/L and soil Poisson’s ratio; \(\omega_{ov}\) is \(\omega_v\) at \(v_s = 0.4\), \(\omega_{oh}\) is \(\omega_h\) at H/L = 4.

The inverse of the factor ‘\(\omega\)’ reflects the base stiffness \((P_b(1-v_s)/(4G_b\rho_0\omega_b))\). Therefore, all the figures will be illustrated in the form of ‘1/\(\omega\)’ to be consistent with that for pile-head stiffness. The following conclusions have been observed:

1. The ratio of ‘1/\(\omega\)’ generally increases slightly with the pile slenderness ratio, when the ratio of \(L/r_0\) is higher than 20, as shown in Fig. 4-6. As the non-homogeneity factor, \(n\) increases from 0 to 1, the factor ‘1/\(\omega\)’ increases by about 0.15. Therefore, it can be approximated by

\[
1/\omega_o = 0.67 - 0.0029 L/r_0 + 0.15n \quad (L/r_0 < 20)
\]

\[
1/\omega_o = 0.6 + 0.0006 L/r_0 + 0.15n \quad (L/r_0 \geq 20)
\]

(4-2)

2. As Poisson’s ratio increases, ‘1/\(\omega\)’ decreases slightly. However, once \(v_s\) exceeds 0.4, it increases as shown in Fig. 4-7. Thereby

\[
1/\omega_v = 1/\omega_o + 0.3(0.4 - v_s) \quad (v_s \leq 0.4)
\]

\[
1/\omega_v = 1/\omega_o + 1.2(v_s - 0.4) \quad (v_s > 0.4)
\]

(4-3)

3. The ‘1/\(\omega\)’ calibrated is sensitive to the grid used for the case of different values of H/L, as described previously. Following careful exploration, it maybe concluded that ‘1/\(\omega\)’ can be predicted by the following equation
4.7 Load Transfer Factors

\[ 1/\omega_h = (0.1483n + 0.6081) \left( 1 - e^{\frac{H}{L}} \right)^{0.1008e^{-0.2406}} \]

Eq. (4-4) compares well with those from FLAC analysis, as shown in Fig. 4-4.

As demonstrated in the Figs. 4-8, increase in the pile-soil relative stiffness can lead to a slight increase in the value of ‘1/\omega’, particularly at high pile-soil relative slenderness ratios. However, it is generally sufficiently accurate to ignore the effect of pile-soil relative stiffness.

4.3.2 Shaft Load Transfer Factor

Back-figured shaft load transfer factors are slightly different, as noted before, depending on the back-estimation procedures of either matching the pile head-stiffness or load ratio. In this section, expressions for estimating the values of \( \zeta \) will be obtained through curve fitting those values back-figured from the process of matching ‘pile-head stiffness’, although the corresponding values back-figured from matching load ratio will be attached for comparison as well.

The shaft load transfer factor is mainly affected by the combination of pile slenderness ratio \( L/r_0 \), the soil non-homogeneity factor \( n \), and the soil Poisson’s ratio \( \nu_s \). The shaft load transfer factor, \( \zeta \), can be approximated by the following expression (Chapter 3)

\[ \zeta = \ln \left( A \left( \frac{1 - \nu_s L}{1 + n r_0} \right) + B \right) \]

The parameters \( A \) and \( B \) have been estimated through fitting Eq. (4-5) to the values of \( \zeta \) obtained by the approach of matching pile-head stiffness. Fig. 4-9 shows the variation of \( \zeta \) with pile slenderness ratio and soil non-homogeneous profile described by Eq. (3-1), at \( \nu_s = 0.4, H/L = 4 \) and \( L/r_0 = 40 \). The curve fitting to this variation results in \( B = 1.0 \) and

\[ A = \frac{1}{1 + n} \left( \frac{2}{1 - 0.3n} \right) \]

The prediction from Eq. (4-5), with ‘\( A \)’ from Eq. (4-6) and \( B = 1 \), has been shown in the figure and termed as ‘Current equation’. Eq. (4-6) is limited to Poisson’s ratio, \( \nu_s = 0.4 \). Generally the factor, \( \zeta \) varies with Poisson’s ratio in a way as shown in Fig. 4-10, which may be simulated by a modification of Eq. (4-6), so that ‘\( A \)’ may be rewritten as
Chapter 4

4.8 Load Transfer Factors

\[ A = \frac{1}{1+n} \left( \frac{0.4 - \nu_s}{n + 0.4} + \frac{2}{1 - 0.3n} \right) + C_\lambda (\nu_s - 0.4) \]  

(4-7)

where \( C_\lambda = 0, 0.5 \) and 1.0 for \( \lambda = 300, 1000 \) and 10000. Eq. (4-5) with ‘A’ from Eq. (4-7) offers a reasonably good fit as illustrated in the figure.

The shaft load transfer factor decreases as the finite layer ratio decreases. This effect can be accounted for by simply decreasing the parameter, A. To accommodate this adjustment, the parameter, A, may be rewritten in the following format

\[ A = A_h \left( \frac{1}{1+n} \left( \frac{0.4 - \nu_s}{n + 0.4} + \frac{2}{1 - 0.3n} \right) + C_\lambda (\nu_s - 0.4) \right) \]

(4-8)

where \( A_h \) is \( A \) at a ratio of \( H/L = 4 \), \( A_h \) is given by the following equation

\[ A_h = 0.1236 e^{2.23 p_s} \left( 1 - e^{\frac{H}{L}} \right) + 1.0 e^{0.107n} \]

(4-9)

where \( p_s = 1/(1+n) \). Estimation of Eq. (4-9) is simpler than it looks. It has no physical implication but compares well with that back-figured from FLAC analysis, as illustrated in Fig. 4-11.

The shaft factor, \( \zeta \) is only slightly affected by the pile-soil relative stiffness factor, \( \lambda \) as shown in Fig. 4-12 (and expressed by the factor of \( C_\lambda \) in Eq. (4-8)), and therefore may be approximately taken as independent of \( \lambda \).

4.3.3 Accuracy of Load Transfer Approach

The back estimation of ‘A’ has been based on matching either load ratio or head-stiffness. There are some difference in the values of the back-figured ‘A’ from the two approaches, particularly for the following listed cases: (1) homogeneous soil profile, and (2) cases of higher slenderness ratio but lower stiffness, e.g. \( \lambda = 300 \) (Figs. 4-9 and 4-12). In these cases, as shown previously, the accuracy of load transfer approach might be less than that for other cases. However, generally, the values of ‘A’ from the two methods are consistent with each other.
4.3.3.1 Using ‘A=2.5’ for a Pile in an Infinite Layer

When the ratio of H/L is less than 4, boundary (fixed or normal) conditions have negligible effect on the analysis of pile response. However, once the ratio H/L exceeds 4, boundary conditions progressively affect the final pile-head stiffness (Fig. 4-3(a)), and affect significantly the values of ‘A’, as shown in Fig. 4-13 for case I (shear modulus by Eq. (3-1) across the entire depth, H) at \( \lambda = 1000 \). It has been argued before that at a higher ratio of H/L (> 4), analysis using fixed boundary is more realistic, which gives a value of ‘A = 2.1 (n = 0)’ for infinite layer case (H/L = \( \infty \)). Therefore, for the case of H/L = \( \infty \) and n = 0, a discrepancy arises between the value of ‘A = 2.1’ by the current calibration and the previous suggestion of ‘A = 2.5’ (Randolph and Wroth, 1978). Many researchers have reported that a value of ‘A = 2.5 (n = 0)’ gave excellent comparison with most of the available numerical approaches for single piles (shown late in this chapter) and pile groups (shown in Chapter 7). It may be due to the fact that FLAC analysis gives a higher pile-head stiffness for a pile in an infinite layer than most of other approaches, and therefore gives a lower value of ‘A = 2.1 (n = 0)’.

Using (incorrect) normal boundary, as H/L increases from 2 to 12, a significant increase in the back-figured ‘A’ of about 90 and 10% individually for n = 0 and 1 is noted. However, corresponding to which, the head stiffness changes only about 11 and 0.5% respectively for n = 0 and 1 (Table 4-5). Therefore, using an ultimate ‘A’ value of 2.5 (corresponding to the incorrect normal boundary), when H/L > 2, should generally underestimate the stiffness by less than 11% (n = 0) compared with that obtained by FLAC analysis using (correct) fixed boundary. In view of this fact and that FLAC analysis generally overestimates the head stiffness for a pile in an infinite layer, a value of ‘A = 2.5’ may still be taken.

Generally even a 30% difference in choice of ‘A’ value, leads to a less than 10% difference in the prediction of head stiffness from Eq. (3-28). However, the accuracy of ‘A’ becomes important when estimating pile-pile interaction factors, as noted by Guo and Randolph (1996a) and further explored in Chapter 7.

4.3.3.2 Effect of Base Load Transfer Factor

The base load transfer factor can generally be taken as unity for predicting pile-head stiffness. As shown above, 1/\( \omega \) varies generally between 0.6 and 1.0, with an average of about 0.8. The base contribution to the pile-head stiffness is generally less than 10%. Therefore, taking \( \omega \) as a unity will result in less than 6% difference in the predicted pile-
head stiffness. Fig. 4-14 shows an example for two extreme cases of higher \((L/r_0 = 80)\) and lower \((L/r_0 = 10)\) values of \(\omega\), and the prediction by the current equation, Eq. (4-5).

4.4 VALIDATION OF LOAD TRANSFER APPROACH

The current solutions of pile head-stiffness and the load ratio are in the form of modified Bessel functions as illustrated in Chapter 3. The numerical evaluation of the solutions have been performed through a spreadsheet program, which operates through a macro sheet in Microsoft EXCEL, with the shaft load transfer factor given by Eq. (4-7) and the base load transfer factor generally given by Eq. (4-1). Except for comparison with the FLAC analyses, a value of unity for base load transfer factor has been used. All the following CF solutions result from this program.

4.4.1 Comparison with Existing Solutions

Table 4-6 shows that the predicted pile-head stiffness by Eq. (3-28) and the ratio of pile base and head load by Eq. (3-26) are shown to be consistent with those from other approaches. However, soil Poisson’s ratio was selected as 0.4 in the current FLAC and CF analyses, rather than 0.5 as used in the other approaches. In view of the effect of the Poisson’s ratio, the FLAC analysis offers a slightly higher head stiffness. Particularly, at higher pile-soil relative stiffness \((e.g., \lambda > 10000)\), the FLAC and current CF approaches yield appreciably (about 10%) higher stiffness than those from other approaches as shown in Table 4-7.

4.4.1.1 Slenderness Ratio Effect

Fig. 4-15(a) shows the variation of pile-head stiffness with slenderness ratio obtained for a pile in a homogeneous, infinite half space by Butterfield and Banerjee (1971), Chin et al. (1990). To simulate the infinite half space condition, a value of 2.5 for the ‘A’ is assumed in the closed form prediction of Eq. (3-28), which offers a very good comparison of pile-head stiffness with those from the more rigorous numerical approaches shown in the figure. Fig. 4-15(b) provides a further comparison of the head stiffness for a pile in a Gibson soil \((n = 1)\) obtained by Chow (1989), Banerjee and Davies (1977) and the present closed form solutions. Increase in slenderness ratio as shown in Fig. 4-15 can lead to an increase in pile-head stiffness, but this tendency is limited to a certain value, beyond which any an increase in the slenderness ratio will lead to an negligible difference in the pile-head stiffness.
4.4.1.2  **Soil Poisson's Ratio Effect**

Poisson's ratio reflects the compressibility of a soil; the more incompressible (higher Poisson's ratio) the soil is, the higher is the pile-head stiffness, as shown in Fig. 4-16. The difference in the stiffness due to variation of Poisson’s ratio between 0 and 0.5 can be as high as 25%.

4.4.1.3  **Finite Layer Effect**

The finite layer ratio, $H/L$ can influence the pile response, if it is less than a limiting value (e.g., $H/L = 4$ when $L/r_0 = 40$). The limiting value of $H/L$ is affected by the critical pile slenderness ratio, beyond which any increase in the pile slenderness ratio results in negligible increase in pile-head stiffness. Fig. 4-17 shows that as the ratio of $H/L$ increases from 1.25 to 4, the head stiffness incurs about 15% reduction, but only a slight decrease in base load is noted (not shown). At this particular slenderness ratio ($L/r_0 = 40$), the percentage in reduction of stiffness with $H/L$ is consistent with those reported by Poulos (1974) and Valliappan et al. (1974) (not shown). The effect of the ratio of $H/L$ can be well represented for different slenderness ratios by the current load transfer factors, as illustrated in Fig. 4-18, together with the predictions by Butterfield and Douglas (1981).

The overall comparison demonstrates that Eq. (4-1) for $\omega$ and Eq. (4-5) for $\zeta$ are sufficiently accurate for load transfer analysis.

4.5  **EFFECT OF SOIL PROFILE BELOW PILE BASE**

The above analysis is generally based on the shear modulus described by Eq. (3-1) across the entire depth, $H$, and is referred to as case I. If the modulus is taken as a constant below the tip level, it will be referred to as case II.

For piles of different slenderness ratios embedded in a soil profile of $H/L = 4$, Poisson's ratio = 0.4, FLAC analysis for case I and II shows that

1. For short piles ($L/r_0 \leq 20$), the pile-head stiffnesses diverge progressively for the two cases, as $n$ increases (Fig. 4-19(a));

2. The ratio of head and base settlement differs more obviously in case II than case I for different non-homogeneity factors (Fig. 4-19(b)), but the difference is minor.
(3) The difference in the values of the load transfer factor, ‘A’ (as the gradient of the plot in Fig. 4-20) between case II and I becomes progressively more pronounced as the value of ‘n’ increases. To accommodate this effect, ‘B’ may still be taken as unity, and Eq. (4-6) may be replaced with,

\[ A = 2.1 - \frac{n}{3.53n - 0.03} \]  

Eq. (4-10) is based on H/L = 4 and \( \lambda = 1000 \), which offers a close prediction of the \( \zeta \) variation as shown in Figs. 4-21 (a) to (d) represented by ‘Equation for case II’.

(4) For case II (\( \lambda = 3000 \)), ‘A’ decreases as H/L reduces (Fig. 4-22). Particularly, when H/L < 2, ‘A’ decreases very sharply. At higher values of H/L, ‘A’ approaches a constant. The variation of ‘A’ for the case II may be simulated by the following equation (to replace Eq. (4-9) for case I),

\[ A_h = 0.9064e^{0.4172\lambda + 0.6429e^{0.2205n}} \]  

This ‘A’ based on \( \lambda = 3000 \) is generally about 10% (for n = 0) lower than that obtained by Eq. (4-10), as illustrated in Fig. 4-23. The effect of pile-soil relative stiffness (\( \lambda = 1000 \) for case I, \( \lambda = 3000 \) for case II) on \( \zeta \) is relative small as illustrated in Fig. 4-22 for the case of n = 0. Therefore, the effect of the soil profile below pile base level is assessed to be obvious as clearly demonstrated by the figure for the case of n = 1.

In summary, generally the ‘A’ for case II may be estimated by

\[ A = \frac{A_h}{A_{oh}} \left( 1 + \frac{0.4 - \nu_s}{n + 0.4} \right) + \left( 2.1 - \frac{n}{3.53n - 0.03} \right) + C_\lambda (\nu_s - 0.4) \]  

where \( A_{oh} \) is \( A_h \) at a ratio of H/L = 4, \( A_h \) is given by Eq. (4-11).

(5) The base load transfer factor (1/\( \omega \)) for case II generally lies in the range 0.55 to 1.0 (not shown), increasing with L/r_0, H/L (for H/L > 2) and pile-soil stiffness ratio, \( \lambda \), and decreasing as \( \nu_s \) increases. The values of ‘1/\( \omega \)’ are about 20% lower than those previously reported (Guo and Randolph, 1996a), because a new mesh of 21\( \times \)100 grid has been adopted in the present analysis, rather than a grid of 21\( \times \)50 as used previously.
4.7 CONCLUSIONS

In this Chapter, an extensive numerical analysis has been undertaken using the FLAC program. With these numerical results, the suitability and rationality of load transfer analysis has been explored extensively.

Preliminary numerical check showed that a grid of $21 \times 100$ was necessary to obtain accurate estimation of the base load transfer parameter, $\omega$. Also, while the radial boundary condition made no difference for $H/L < 4$, fixed boundary was essential for $H/L > 4$. With the fixed boundary, it was found that $H/L = 4$ may be considered as effectively an infinite deep soil layer.

The numerical analysis shows that the effect of choosing soil Poisson's ratio can be equally as important as the ratio of $H/L$ and should be taken into consideration. The finite layer ratio of $H/L$ can only lead to about 15% increase in head stiffness when $H/L$ decreases from 4 to 1.25, but the increase in soil Poisson's ratio from 0 to 0.499 can result in about a 25% increase in pile-head stiffness.

The calibration using load transfer model shows that, generally, shaft load transfer factor can be taken as constant with depth. With average values of the shaft load transfer factor, the load transfer approach yielded close predictions of overall pile response compared with those obtained by FLAC analysis.

The calibration using the closed form solutions demonstrates that shaft load transfer factor (1) increases with increase in pile slenderness ratio; (2) decreases with increase in Poisson's ratio; (3) increases slightly with increase in the ratio of $H/L$ ($H/L \leq 4$), but (4) is nearly independent of the pile-soil relative stiffness.

The difference in the values of shaft load transfer factors, calibrated against pile-head stiffness and ratio of base and head load, implies that the load transfer approach is less accurate in the cases of (1) homogeneous soil profile; and (2) higher pile slenderness ratio but lower pile-soil relative stiffness. However, an appreciable (e.g. 30%) difference in selection of the value of 'A' generally leads to a slight (e.g. about 10%) difference in the predicted pile-head stiffness of a single pile. Therefore, generally load transfer analysis is sufficiently accurate for practical analysis.

The backfigured load transfer factors have been expressed in the form of simple formulas and also implemented in a spreadsheet program. In comparison with the current FLAC analysis and relevant rigorous numerical approaches, the simple formulas
can well account for the effects of various relative thickness ratio of H/L ($\leq 4$), Poisson's ratio and pile slenderness ratio. In the case of an infinite layer, it seems that a value of ‘$A = 2.5$’ gives good comparison with most of the available numerical predictions.

The shear modulus distribution below the pile tip level can significantly alter the value of the shaft load transfer factor. To account for this effect, (1) for the case of shear modulus varying as a power law of depth across the entire depth, $H$, Eq. (4-8) may be used, otherwise (2) for the case of a constant value below the tip level, Eq. (4-12) may be used.
TABLE 4-1  Comparison of the effect of Poisson's ratio of the pile  
\[v_p = 0 \text{ and } 0.2, \; v_s = 0.49, \; L/r_0 = 40, \; \lambda = 1000\]

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{P_t}{G_L r_0 w_t} )</td>
<td>59.08</td>
<td>51.93</td>
<td>46.64</td>
<td>42.62</td>
<td>39.56</td>
</tr>
<tr>
<td>(\frac{w_t}{w_b} )</td>
<td>1.64</td>
<td>1.60</td>
<td>1.58</td>
<td>1.55</td>
<td>1.53</td>
</tr>
<tr>
<td>(\frac{P_b}{P_t} )</td>
<td>7.08</td>
<td>8.75</td>
<td>10.5</td>
<td>12.07</td>
<td>13.68</td>
</tr>
<tr>
<td>Note: numerator and denominator for (v_p = 0.2) and 0 respectively</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4-2  Comparison of radial boundary effect  
\[v_p = 0.2, \; v_s = 0.4, \; L/r_0 = 40, \; \lambda = 1000\]

<table>
<thead>
<tr>
<th>Items</th>
<th>n</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{P_t}{G_L r_0 w_t} )</td>
<td>75r_0</td>
<td>53.7</td>
<td>47.85</td>
<td>43.37</td>
<td>39.93</td>
<td>37.24</td>
</tr>
<tr>
<td>(\frac{w_t}{w_b} )</td>
<td>75r_0</td>
<td>1.55</td>
<td>1.54</td>
<td>1.52</td>
<td>1.50</td>
<td>1.49</td>
</tr>
<tr>
<td>(\frac{P_b}{P_t} )</td>
<td>75r_0</td>
<td>6.4</td>
<td>7.86</td>
<td>9.33</td>
<td>10.78</td>
<td>12.2</td>
</tr>
<tr>
<td>Note: numerator and denominator for (v_p = 0.2) and 0 respectively</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 4-3 Comparison of FLAC analysis with other approaches (n = 0)

<table>
<thead>
<tr>
<th>( \frac{P_t}{G_L w_t r_0} )</th>
<th>FLAC</th>
<th>BEM</th>
<th>VM</th>
<th>FLAC</th>
<th>BEM</th>
<th>VM</th>
<th>FLAC</th>
<th>BEM</th>
<th>VM</th>
<th>FLAC</th>
<th>BEM</th>
<th>VM</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>69.70</td>
<td>65.70</td>
<td>72.2</td>
<td>64.38</td>
<td>61.3</td>
<td>65.1</td>
<td>53.60</td>
<td>52.00</td>
<td>54.9</td>
<td>36.51</td>
<td>36.80</td>
<td>38.7</td>
</tr>
<tr>
<td>**</td>
<td>1.05</td>
<td>1.12</td>
<td>1.19</td>
<td>1.18</td>
<td>1.15</td>
<td>1.19</td>
<td>2.92</td>
<td>2.66</td>
<td>3.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>***</td>
<td>109.0</td>
<td>102.2</td>
<td>1000</td>
<td>85.0</td>
<td>85.2</td>
<td>85.2</td>
<td>61.6</td>
<td>61.6</td>
<td>61.6</td>
<td>36.2</td>
<td>38.0</td>
<td></td>
</tr>
<tr>
<td>( \frac{w_t}{w_b} )</td>
<td>1.18</td>
<td>1.54</td>
<td></td>
<td>2.96</td>
<td></td>
<td></td>
<td>7.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{P_t}{G_L w_t r_0} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{w_t}{w_b} )</td>
<td>1.16</td>
<td>1.54</td>
<td></td>
<td>2.68</td>
<td></td>
<td></td>
<td>6.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda = \frac{E_p}{G_L} )</td>
<td>10000</td>
<td>3000</td>
<td>1000</td>
<td>300</td>
<td>1000</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * \( L/r_0 = 40, \ ** L/r_0 = 80, \ *** \) rigid pile.

VM analysis was based on \( v_s = 0.5 \), while BEM and FLAC analyses were based on \( v_s = 0.4 \). Also for FLAC analysis, \( H/L = 4 \).

### Table 4-4 Comparison between FEM and FLAC analyses (n = 0, 1)

<table>
<thead>
<tr>
<th>( \frac{P_t}{G_L w_t r_0} )</th>
<th>FEM</th>
<th>FLAC</th>
<th>FEM</th>
<th>FLAC</th>
<th>FEM</th>
<th>FLAC</th>
<th>FEM</th>
<th>FLAC</th>
<th>FEM</th>
<th>FLAC</th>
<th>FEM</th>
<th>FLAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>43.95</td>
<td>56.84</td>
<td>63.89</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>**</td>
<td>29.89</td>
<td>37.21</td>
<td>39.53</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{P_t}{G_L w_t r_0} )</td>
<td></td>
<td>25.0</td>
<td>34.8</td>
<td>35.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L/r_0 )</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>( n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Both FEM and FLAC analyses were based on \( v_s = 0.4, \lambda = 1000 \). However, \( H/L = 2 \) for FEM and \( H/L = 2.5 \) for FLAC analyses.
### TABLE 4-5 Influence of ‘A’ on pile-head stiffness (L/r₀ = 40, vₜ = 0.4)

<table>
<thead>
<tr>
<th>Cases</th>
<th>Normal boundary</th>
<th>Fixed boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indexes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>A</td>
<td>P₀/(GᵢLₜ₀w₀r₀)</td>
</tr>
<tr>
<td>0</td>
<td>1.69</td>
<td>0</td>
</tr>
<tr>
<td>Increase (%)</td>
<td>93.0</td>
<td>10.6</td>
</tr>
</tbody>
</table>

### TABLE 4-6 Comparison with the previous analyses (L/r₀ = 40, n = 0)

<table>
<thead>
<tr>
<th></th>
<th>P₀/(GᵢLₜ₀w₀r₀)</th>
<th>P₀/Pₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>36.46</td>
<td>38.7</td>
</tr>
<tr>
<td>1000</td>
<td>52.72</td>
<td>54.9</td>
</tr>
<tr>
<td>3000</td>
<td>62.32</td>
<td>65.1</td>
</tr>
<tr>
<td>10000</td>
<td>68.43</td>
<td>72.2*</td>
</tr>
</tbody>
</table>

Note: * rigid pile.

Present CF and FLAC analyses were based on H/L = 4, vₜ = 0.4.
TABLE 4-7  Comparison with the previous analyses ($L/r_o = 40$)

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$P_t/(G_{il}w_{it}r_o)$ ($n = 0.25$)</th>
<th>$P_t/(G_{il}w_{it}r_o)$ ($n = 1.0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>31.19</td>
<td>31.3</td>
</tr>
<tr>
<td>1000</td>
<td>46.41</td>
<td>45.8</td>
</tr>
<tr>
<td>3000</td>
<td>55.46</td>
<td>54.4</td>
</tr>
<tr>
<td>10000</td>
<td>61.17</td>
<td>58.6</td>
</tr>
</tbody>
</table>

Note: Present CF and FLAC analyses were based on $H/L = 4$, $v_s = 0.4$. 
5. NON-LINEAR VISCO-ELASTIC LOAD TRANSFER MODELS FOR PILES

5.1 INTRODUCTION

Numerical solutions for axial pile response, based on elasticity, have been extended to allow for non-homogeneity of the soil (e.g. Banerjee and Davies, 1977; Poulos, 1979), relative slip between pile and soil (e.g. Poulos and Davis, 1980), and visco-elastic response of soil (Booker and Poulos, 1976). However, a load transfer approach appears to offer adequate accuracy and distinctly much greater flexibility to yield unified compact closed form solutions to take into account all of these factors (Randolph and Wroth, 1978; Guo and Randolph, 1996c).

Time can have an important effect on the response of piles in clay. For driven piles, dissipation of the excess pore pressures generated during driving leads to an increase in shaft friction and in the stiffness of the surrounding soil (e.g. Bergdahl and Hult, 1981; Trenter and Burt, 1981). In addition, creep or viscoelastic response of the soil leads to variations in stiffness and capacity depending on the time-scale of loading. At high load levels, or for long slender piles where the load transfer is concentrated near the pile head, creep can lead to significant pile head movement at constant load, and even a gradual reduction in shaft capacity. Ramalho-Ortigao and Randolph (1983) report an apparent difference of some 30% in the tension capacity of a pile loaded at a constant displacement rate leading to failure in about 40 seconds, compared with a similar pile subjected to a maintained load test over a period of 40 days.

England (1992) has extended the hyperbolic approach of pile analysis described by Fleming (1992) to allow the effects of time to be incorporated into axial pile analysis, with separate hyperbolic laws being used to describe the time-dependency of the (average) shaft and base response. This phenomenological approach is limited by the difficulty of linking the parameters required for the model to fundamental and measurable properties of the soil.

Creep displacement can be induced by any of the following factors: (1) a prolonged step loading; (2) a vibration or (3) a change of temperature. For conventional pile loading tests (e.g, Maintained loading test and Constant rate load test), the time-scale of loading can be simulated sufficiently accurately by the two kinds of commonly encountered loading: 1-step loading, and ramp (linear increase followed by sustained) loading. In the present chapter, visco-elastic shaft and base load transfer models have been proposed for the two types loading respectively. With the models, the previous closed form
Chapter 5 5.2 Visco-elastic Load Transfer Models

solutions for a pile in an elastic-plastic non-homogeneous media (Chapter 3) have been extended to account for visco-elastic response. A previously designed program called GASPILE has been extended to allow the time-dependent pile response to be computed. The solutions have been compared extensively with the numerical analysis by Booker and Poulos (1976) for the case of 1-step loading. The overall pile response for the two commonly encountered loading types has been explored. Finally, two example analyses are compared with measured pile responses to illustrate the validity of the proposed theory to practical applications.

5.2 SHAFT BASE PILE-SOIL INTERACTION

The main challenge in predicting the axial performance of piles lies in establishing load transfer functions for the shaft and base, which are linked to fundamental properties of the soil and yet which allow for non-linearity and time dependence of the soil response. Load transfer functions for the shaft may be derived from the stress-strain response of the soil using the concentric cylinder approach, which itself is based on a simple 1/r variation of shear stress around the pile (where r is the distance from pile axis) (e.g. Frank, 1974; Cooke, 1974; Randolph and Wroth, 1978). The treatment below extends those functions to allow for visco-elastic response of the soil.

5.2.1 Non-linear Visco-elastic Stress-Strain Model

A pile in clay under a sustained load usually undergoes additional settlement, the amount of which varies from soil to soil and which is thought to be due to changes in the stress-strain behaviour with aging (Mitchell and Solymar, 1984). Such creep behaviour, which occurs in the soil surrounding the pile as well as on the pile-soil interface itself, has been well recognised (Edil and Mochtar, 1988). A model consisting of Voigt and Bingham elements in series can account well for the creep behaviour of several soils (Komamura and Huang, 1974). However determination of the slider threshold value for the Bingham model is difficult, and an alternative is to adopt a hyperbolic stress-strain model as shown by experiment (Zen, 1991; Feda, 1992). Such a treatment can lead to a modified intrinsic time dependent non-linear creep model (Fig. 5-1(a)), which can be expressed as

\[ \gamma = \gamma_1 + \gamma_2 \]  
\[ \tau_j = \gamma_j G_0 k_j \]  
\[ \tau_3 = \eta_3 \dot{y}_3 \]
where $\gamma_j$ is the shear strain for the elastic spring 1, 2 and dashpot 3 ($j = 1, 2$ and 3) respectively; $\gamma$ is the total shear strain; $G_{ij}$ is the instantaneous and delayed initial elastic shear modulus ($j = 1, 2$) respectively; $\dot{\gamma}_3$ is the shear strain rate for the dashpot ($\gamma_3 = \gamma_2$); $\eta_{i3}$ is the shear viscosity at a strain rate of $\dot{\gamma}_3$; $\tau_j$ is the shear stress acted on spring 1, 2 and dashpot 3 ($j = 1, 2$ and 3) respectively; $k_j$ is the coefficient for considering non-linearity of elastic springs 1 and 2 ($j = 1, 2$) respectively.

In terms of rate process theory, the shear strain rate, $\dot{\gamma}$, can be expressed in different forms related to absolute temperature and/or deviatoric shear stress (Murayama and Shibata, 1961; Mitchell, 1964; Christensen and Wu, 1964; Mitchell, et al. 1968). However, none of the expressions available can account for the non-linearity of the soil creep. A non-linear hyperbolic model of soil shear stress-strain relationship can offer a good comparison with the measured stress-strain relationship at different time (Feda, 1992); therefore, the model is employed as expressed by Eq. (5-5), where the coefficient $k_j$ is expressed as

$$k_j = 1 - \psi_j$$

where $\psi_j = \frac{R_0 \tau_j}{\tau_{0j}}$ ($j = 1, 2$), and $R_0$ is originally defined as $\frac{\tau_{0j}}{\tau_{ult}}$ ($\tau_{ult}$ is the ultimate and failure local shaft stress for spring $1$ and $2$ respectively) for the hyperbolic model only (Duncan and Chang, 1970).

From Eqs. (5-1) to (5-4), it follows that

$$\tau_1 + \tau_2 + \tau_3 = \gamma + \frac{\eta_3}{G_2} \dot{\gamma}$$

where $J = 1/G_1 + 1/G_2$; $G_{ij} (= G_{ijk})$, is the instantaneous and delayed elastic shear modulus at a strain of $\gamma_j$ ($j = 1, 2$) respectively; $\dot{\tau}_1, \dot{\gamma}$ are the shear stress rate and shear strain rate respectively. Integration of Eq. (5-6) with respect to time, considering the initial conditions: $t = 0, \gamma = 0$, leads to

$$\gamma = \frac{\tau_1}{G_1} \left( 1 + \frac{G_{y_1} G_{y_2}}{\eta_{y_3}} \int_0^t \frac{\tau_1(t')}{\tau_1} \exp \left( -\frac{G_{y_2}}{\eta_{y_3}} (t - t') \right) dt' \right)$$
where $\tau_1$, $\tau_1(t^*)$ are the soil shear stress at time $t$ and $t^*$ respectively. The total shear strain in Eq. (5-7), obtained from the non-linear soil model by Eqs. (5-1) to (5-4), reflects two types of responses to stress: instantaneous elasticity ($G_{i1}$) and delayed elasticity ($G_{i2}$). At the onset of loading, only the elastic part of shear strain exists, but as time passes, some creep displacement (delayed elasticity) on and/or around the pile-soil interface is anticipated.

Generally speaking, secondary compression of all remoulded and undisturbed clays obtained by oedometer tests can be sufficiently accurately predicted by the model of Eq. (5-7) for the elastic case ($\psi_1 = \psi_2 = 0$), except for a soil of loose structure that is susceptible to breakdown, where a Voigt element has to be added in series with Mechant's model (Lo, 1961). As for pile foundations, since remoulding of the soil around the piles is inevitable due to construction, the model proposed herein and expressed by Eqs. (5-1) to (5-4) may be adequate to simulate the creep behaviour as shown later.

In pile analysis, $\tau_0$ is taken as $\tau_{ult}$, and the $R_0$ is used as a parameter to control the degree of non-linearity for spring 1 and 2 ($j = 1, 2$) respectively. Strictly speaking, the limiting value of shear stress at the pile-soil interface, $\tau_{ult}$, may be larger than the failure shear stress, $\tau_n$, in the hyperbolic model of the soil response. Appropriate values of $\tau_n$ ($\tau_{ult}$) may be correlated with the shear strength of the soil, or with the effective overburden stress (e.g., API RP2A; Tomlinson, 1970; Randolph and Murphy, 1985), or estimated through the correlation to the CPT, SPT (e.g. Hirayama, 1990) and vane shear test (Tomlinson, 1970; McClelland, 1974; Meyerhof, 1976).

Normally, as time passes, the stress initially taken by the dashpot redistributes to the elastic spring 2 (Fig. 5-1 a), until finally all the stress is transferred, and the time dependent creep ceases. During the transferring process, if the shear stress on spring 2 exceeds the failure stress, the spring will yield and a larger fraction of the stress has to be endured by the dashpot, which could lead to a non-terminating creep and eventually trigger a failure. Therefore, the stress $\tau_{ult}$ ($\tau_{ult2}$) must be the long term value, which is lower than $\tau_n$ ($\tau_{ult1}$), as reported by many researchers, e.g. Geuze and Tan (1953), Murayama and Shibata (1961), Leonard (1973). Reduction in soil strength is linearly related to the logarithmic time elapsed (Casagrande and Wison, 1951), which has also been formulated by Leonard (1973). Based on a number of creep tests at an approximate constant rate of loading, Murayama and Shibata (1961) report that the ratio of $\tau_{ult2}/\tau_{ult1}$ is about 0.71, while the values themselves $\tau_{ult2}$, $\tau_{ult1}$ increase logarithmically as the water content decreases.
5.2.2 Shaft Displacement Estimation

5.2.2.1 Visco-elastic Shaft Estimation Formula

Local shaft displacement can be predicted through a concentric cylinder approach, which itself is based on elastic theory (Randolph and Wroth, 1978; Kraft et al. 1981). The correspondence principle (Lee, 1955; Lee et al. 1959) states that the analysis of stress and displacement field in a linear visco-elastic medium can be treated in terms of the analogous linear elastic problem having the same geometry and boundary conditions. However, for the case of non-linear elastic soil, the principle is invalidated. Therefore, a shaft model reflecting non-linear visco-elastic response might have to be directly obtained from the generalised visco-elastic stress strain relationship of Eq. (5-7), with suitable shear modulus. Model pile tests show that load transfer along a model pile shaft leads to a nearly negligible volume change (or consolidation) in the surrounding soil (Edil and Mochtar, 1988). Approximately, the vertical displacement, \( u \) along depth, \( z \) ordinate may be ignored. Therefore it follows that

\[
\gamma = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \approx \frac{\partial w}{\partial r} \tag{5-8}
\]

where \( w \) is the local displacement of shaft element at time \( t \). Based on the concentric cylinder approach, the shaft displacement is obtained by integration from \( r_0 \) to the maximum radius of influence, \( r_m \)

\[
w = \int_{r_0}^{r_m} \frac{\partial w}{\partial r} \, dr \tag{5-9}
\]

With the shear stress, \( \tau_1 \) at a distance of \( r \) away from the pile axis given by, \( \tau_1 = \frac{\tau_0 r_0}{r} \), and substituting Eq. (5-7) into the above equation

\[
w = \int_{r_0}^{r} \frac{\tau_0 r_0}{r} \frac{1}{G_{r_1}} \, dr + \int_{r_0}^{r} \frac{\tau_0 (t^*) r_0}{r} \frac{1}{G_{r_2}} \exp\left(-\frac{G_{r_2}}{\eta_{r_3}} (t - t^*)\right) \, dt^* \, dr \tag{5-10}
\]

where \( \tau_0, \tau_0(t^*) \) are the shear stress on the pile soil interface at time \( t \) and \( t^* \) respectively. \( G_{r_j} \) is the shear modulus at distance, \( r \) away from the pile axis for elastic spring 1 and 2 (\( j = 1, 2 \)). Although the shear modulus and the viscosity parameter are functions of the stress level, the relaxation time, \( \frac{G_{r_2}}{\eta_{r_3}} \) may be taken as a constant as shown in the
Chapter 5  5.6  Visco-elastic Load Transfer Models

experiment by Lo (1961). Hence it is replaced with $1/T$, $(1/T = G_{12}/n, \eta = \text{the value of } \eta_3$ at strain $\gamma_3 = 0\%$).

Due to the inverse linear reduction of shear stress away from a pile (Frank, 1974; Cooke, 1974; Randolph and Wroth, 1978), with Eq. (5-5), a variation of shear modulus with distance, $r$ can be resulted and expressed as

$$G_{nj} = G_{nj} \left(1 - \frac{r_0}{r} \psi_{nj}\right) \quad (5-11)$$

where $r_0$ is the pile radius; $\psi_{nj} = R_{nj} / \tau_{nj}$, which is the non-linear stress level on the pile-soil interface; $\tau_{nj}$ is the shear stress on pile-soil interface ($j = 1, 2$).

With the shear modulus variation rule of Eq. (5-11), Eq. (5-10) can be simplified as

$$w = \frac{\tau_0 r_0}{G_{i1}} \left(\zeta_1 + \zeta_2 \frac{G_{i2}}{G_{i1}} A(t)\right) \quad (5-12)$$

with the time dependent part $A(t)$ being related to stress level by

$$A(t) = \frac{1}{T} \int_0^t \frac{\tau_0 (t')}{\tau_0} \exp\left(-\frac{(t-t')}{T}\right)dt' \quad (5-13)$$

The radial shear influence can be determined by

$$\zeta_j = \ln \left(\frac{r_m/r_0 - \psi_{nj}}{1 - \psi_{nj}}\right) \quad (5-14)$$

where $\zeta_j$ is a measure of the shear influence for stress level, $\psi_{nj}$ is the non-linear stress level for elastic springs 1 and 2 ($j = 1, 2$) respectively; $r_m$ = the maximum radius of influence of the pile beyond which the shear stress becomes negligible, and may be expressed in terms of the pile length, $L$, as (Randolph and Wroth, 1978; Chapter 3)

$$r_m = A \frac{1 - \nu_s}{1 + n} L + B r_0 \quad (5-15)$$

where generally $A = 2$, $B = 0$, as shown in Chapters 3 and 4; $\nu_s$ is Poisson's ratio of the soil; $L$ is the embedded pile length; $G_{i1}$, $G_{i2}$ are the initial shear moduli of the soil just above the level of the pile tip, and that beneath the pile tip, with the ratio given by $\zeta_b =$
The estimation of \( A(t) \) depends on the shear stress-time relationship. For most practical loading tests, the shear stress is likely caused by a ramp type loading, which is a combination of constant rate of loading (addition of load, even though it might be limited to a short duration of \( t_c \)) Fig. 5-1(c)) and sustained loading (corresponding to a creep process). Within the elastic stage, the shear stress should follow a similar pattern of time-dependency to the loading. Therefore at any time, \( t^* \) in between 0 and \( t_c \), it follows that

\[
\tau_o(t^*)/\tau_o(t) = t^*/t \quad (5-16)
\]

Afterwards, when \( t^* > t_c \), the stress ratio stays at unity. Therefore, if the total loading time, \( t \) exceeds \( t_c \), Eq. (5-13) may be integrated, allowing \( A(t) \) to be written as

\[
A(t) = \frac{t^*}{t} \exp\left( -\frac{t-t_c}{T} \right) - \frac{1}{T} \exp\left( -\frac{t-t_c}{T} \right) - \exp\left( -\frac{t}{T} \right) + 1 - \exp\left( -\frac{t-t_c}{T} \right) \quad (5-17)
\]

Otherwise, if \( t \leq t_c \), it follows that

\[
A(t) = 1 - \frac{T}{t} \left( 1 - \exp\left( -\frac{t}{T} \right) \right) \quad (5-18)
\]

where \( t_c \) is the time at which a constant load commences. If \( t_c = 0 \), it is reduced to 1-step loading (Fig. 5-1(b)), which can be simply described by

\[
A(t) = 1 - \exp(-t/T) \quad (5-19)
\]

The non-dimensional local displacement and stress level for non-linear visco-elastic case (NLVE) is shown in Fig. 5-2, for a pile of \( L/r_0 = 50 \) in a clay of \( \tau_f/G_{ij} = 0.04 \) (\( j = 1, 2 \)), \( V_s = 0.5 \), \( n = 1 \), \( G_{ii}/G_{i2} = 1 \), and pile-soil interaction factors of \( \zeta_2/\zeta_1 = 1 \), and \( \xi_b = 1 \). The creep is supposed to be initiated at a stress level of \( (\tau_o/\tau_n =) 0.5 \) for 1-step loading, or initiated at the beginning and held at a prolonged load level of 0.8 from time \( t_c \) (Fig. 5-1) for ramp loading. The results from linear elastic (LE) and non-linear elastic (NLE) load transfer model have been illustrated in the Fig. 5-2 as well. For ramp type loading, the relative ratio of the duration of constant rate of loading, \( t_c \) and total loading time, \( t \) can have significant effect on the stress-displacement response, particularly as \( t_c \).
earlier (Fig. 5-3) by giving a constant of $t_c/T$ but varying $t/T$.

5.2.2.3 Discussion on Local Shaft Stress-Displacement Relationship

From Eq. (5-12), a shaft displacement can be expressed by

$$w = \frac{\tau_c r_c}{G_{11}} \zeta_c \zeta_e$$  \hspace{1cm} (5-20)

where

$$\zeta_e = 1 + \frac{\zeta_c}{\zeta_1} \frac{G_{11}}{G_{12}} A(t)$$ \hspace{1cm} (5-21)

Eq. (5-20) is called non-linear visco-elastic load transfer ($t$-$z$) model. Estimation of the shear measure of influence is divided into two entities which can be evaluated separately in a rational and systematic manner. The displacement calculation embracing non-linear visco-elastic behaviour still retains the simplicity and pragmatism of the previous formulas suggested by Randolph and Wroth (1978), and Kraft et al. (1981).

As discussed by Randolph and Wroth (1978), typical values of the parameter $\zeta_1$ are about 4 for $\psi = 0$. Fig. 5-4 shows how the parameter varies with the shear stress level, $\psi$. It may be seen that, at failure, the secant stiffness of the load transfer curve is approximately half the initial tangent value for values of $R_f$ in the region of 0.9, and in fact the whole shape of the curve may be approximated closely by a parabola (Randolph, 1994). The pile behaviour within the time taken for consolidation of 90% degree under working load might normally be treated by elastic analysis ($\zeta_e = 1$), which has been explored earlier (Chapter 3).

Except where specified in this chapter, the ratio of $\zeta_2/\zeta_1$ is assumed to be unity, which is based on the correspondence principle for linear visco-elastic media, with identical shaft failure stress for both springs 1 and 2. Accordingly only secondary deformation of clay is concerned. Generally, as evidenced by experiments (Geuze and Tan, 1953; Murayama and Shibata, 1961), $\tau_{ult2}$ is lower than $\tau_{ult1}$. Therefore, the stress level on spring 2 must be higher than that on spring 1 at the same degree of shaft displacement mobilization. When the pile-soil interface stress level reaches the limiting shear stress $\tau_{ult2}$ (but lower than $\tau_{ult1}$), the parameter $\zeta_2$ estimated by Eq. (5-14) can be significantly larger than $\zeta_1$.  

At this stage, the pile would not yield, but significant creep displacement can be induced, particularly for long piles.

The variation of the creep modification factor, $\zeta_c$ with non-dimensional time $t/T$ for various modulus ratios, $G_{i1}/G_{i2}$ is shown in Fig. 5-5 (a) for step loading. The effect of the $t_c/t$ of the ramp loading (giving $G_{i1}/G_{i2} = 1$) on the value of $\zeta_c$ is illustrated in Fig. 5-5(b). When $t > t_c$, the increase in $\zeta_c$ with time is accelerated compared with that of $t < t_c$ case.

Particularly for the step loading case, in terms of Eq. (5-19), a creep function $J(t)$ in a general form can be inferred

$$J(t) = A_c + B_c e^{-t/T}$$  \hspace{1cm} (5-22)

where $A_c = 1/G_{i1} + \zeta_{2}/G_{i2}\zeta_{1}$; $B_c = -\zeta_{2}/G_{i2}\zeta_{1}$. The function is equivalent to that adopted by Booker and Poulos (1976), and will be used later for comparison. In addition, comparing Eq. (5-21) with (5-22), it follows that,

$$J(t) = \zeta_c/G_{i1}$$  \hspace{1cm} (5-23)

This relationship enables Eq. (5-20) to be written as a function of $J(t)$ as well:

$$w = \tau_o r_o \zeta_{2} J(t)$$  \hspace{1cm} (5-24)

From Eqs. (5-20) and (5-21), the creep part of displacement for step loading case can be expressed as

$$w_c = \frac{\tau_o r_o \zeta_{2} A(t)}{G_{i2}} = \frac{\psi_{i2} \tau_{f2} r_o \zeta_{2}}{R_{f2} G_{i2}} \left(1 - \exp\left(\frac{-t}{T}\right)\right)$$  \hspace{1cm} (5-25)

where $w_c$ is the local creep displacement at time $t$. In terms of Eq. (5-25), the rate of creep displacement of a frictional pile is proportional to the diameter of the pile and the stress ratio, which has also been well founded theoretically and/or empirically in previous publications (Edil and Mochtar, 1988; Mitchell, 1964). It seems plausible that pile slenderness ratio, the shaft non-homogeneity factor and Poisson's ratio (expressed by $\zeta_{2}$) could have some influence on creep behaviour. The time-displacement relationship given by Eq. (5-25) is different from the statistical formula by Edil and
Mochtar (1988). However the next section will demonstrate that it does well fit to the experimental data.

5.2.2.4 Verification of the Shaft Load Transfer Model

The shaft displacement can be easily determined from Eq. (5-20) which includes the non-linear elastic part obtained by using $\zeta_e = 1$ and the creep part, e.g. by Eq. (5-25) for step loading. Since the theoretical verification for non-linear case has been made previously (Randolph and Wroth, 1978; Kraft et al. 1981; Guo and Randolph, 1996c), only experimental verifications of Eq. (5-25) are given below. To allow such a comparison, the following parameters need to be known: (a) the initial elastic and delayed shear moduli; (b) the ultimate (failure) shaft shear stress for the springs 1 and 2; (c) the relaxation time; and (d) the geometry and elastic property of the pile.

Evaluation of ultimate (failure) shaft shear stress, $\tau_{\Omega}$ and $\tau_{\Omega}$ has been described previously.

To assess $G_{\Omega}$, the most reasonable way is by fitting the measured local shear stress-displacement relationship with Eq. (5-20). As a first approximation, the following principle might be used as proposed by Kuwabara (1991): The equivalent modulus to evaluate a pile settlement of 1% of the pile radius can be taken as three times the shear modulus at a shear strain of 1%. When pile settlement is larger than 1% of the pile radius, a smaller value should be taken. For normally consolidated clays, the shear modulus at a shear strain of 1%, ($G_{1\%}$) and 0% ($G_{\Omega}$) can be obtained respectively as

$$G_{1\%} = (80-90)\sigma_u \quad (5-26)$$

and

$$G_{\Omega} = (400-900)\sigma_u \quad (5-27)$$

whether using non-linear elastic or elastic form ($\psi = 0$) of Eq. (5-20) generally results in a slight discrepancy of the overall pile response over a loading level between 0 and 0.75 (Chapter 3). Therefore, initial shear modulus, $G_{\Omega}$ can generally be chosen as 1 to 3 times the corresponding shear modulus estimated by field measurement or empirical formulas (e.g. by Fujita, 1976).

For estimating the development of the local displacement with time, the rate factor, $1/T$, should be ascertained for a range of relevant loading level. Three examples from laboratory tests (Edil and Mochtar, 1988) are cited here. Settlement time relationships
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5.11 Visco-elastic Load Transfer Models

from the tests are presented for the head of the piles. The local shaft displacement time relationship at the top level may be assumed to be identical to these relationships, since the model piles are relatively short and rigid. Comparison between the predicted and the measured behaviour has been shown in Fig. 5-6 where the "calculated" represents the prediction by Eq. (5-25), for which the corresponding adopted parameters and information are given in Table 5-1. The initial part of the comparison is not good irrespective of fitting with Eq. (5-25) or Edils and Mochtar's statistical formula, which implies a possible existence of non-linear elastic displacement in the creep tests and reflects the hydrodynamic period of consolidation process (Lo, 1961). The relaxation time of 1/T back-figured is quite consistent with those from other publications, e.g. values of 1/T, (1.71 to 3.29)×10^-5 s^-1(second^-1, simplified as s^-1) have been adopted by Qian et al. (1992) in estimation of vacuum preloading.

It is also convenient to back-estimate the creep parameters for a given load from measured settlement versus time relationship of a load test, which is similar to that proposed by Lee (1956). In terms of Eq. (D-1) (referred to Appendix D), an example of the fitting between the data reported by Ramalho Ortigao and Randolph (1983) and that calculated by Eq. (5-25) is plotted in Fig. 5-7 for two load levels, and a very similar range of values of 1/T are deduced, in the range of 0.36 to 0.664×10^-5s^-1).

Based on a comprehensive study of the secondary compression of Sodium Bentonite clay, remoulded London clay, Grangemouth clay and both undisturbed and remoulded Fornebu clay, Lilla Edet clay, Lo (1961) shows that generally the rate factor, 1/T lies in between 0.2 and 0.4 (×10^-5s^-1), and is a constant for a definite clay. The compressibility index ratio, G_{11}/G_{12}, lies in between 0.05 and 0.2, and is only slightly influenced by the soil water content. Variation of the above two ratios (factors) with stress level is negligible, except that for clay of loose structure such as Lilla Edet clay, the value of G_{11}/G_{12} increases, e.g. a value of 1.4 to 1.6 is recorded, when the consolidated pressure exceeds slightly the soil preconsolidated pressure. However, the individual values of G_{11}, G_{12} and η vary with load (stress) level. Parametric analysis on settlement time relationship under given load levels shows that an average value of 1/T over a zone of working load should be assessed and employed in Eq. (5-20); such a simplification does not affect very much pile-soil response over the range of pile working load.
5.2.3 Base Pile-Soil Interaction Model

The base settlement can be estimated through a rigid punch as shown in Chapter 3. Supposing that a hyperbolic model for base load settlement is adopted, it follows (Chow, 1986b)

\[ w_b = \frac{P_b (1 - v_s) \omega}{4r_0 G_{ib}(t)} \left(1 - \frac{P_b}{P_{lb}}\right)^2 \]  

(5-28)

where \( P_b \) is the mobilised base load; \( \omega \) is the pile base shape and depth factor, which is generally chosen as 1.0 (Randolph and Wroth, 1978; Armaleh and Desai, 1987), but more accurately can be estimated by the empirical equations shown in Chapters 3 and 4; \( r_0 \) is the pile radius; \( P_{lb} \) is the limiting base load; \( R_{nb} \) is a parameter which controls the degree of non-linearity. The time dependent shear modulus can be estimated by

\[ G_{ib}(t) = \frac{G_{ib1}}{1 + \frac{G_{ib1}}{G_{ib2}} A(t)} \]  

(5-29)

where \( G_{ib1}, G_{ib2} \) are the shear modulus just beneath the pile tip level for spring 1 and 2 respectively. The ratio of \( G_{ib1} \) and \( G_{ib2} \) can be taken the same as that of \( G_{i1}/G_{i2} \).

5.3 VALIDATION OF THE THEORY

5.3.1 Closed Form Solutions

Closed form solutions for a pile in an elastic-plastic non-homogeneous soil have been generated in Chapter 3. Under the circumstance of a constant of \( C_c \), these solutions can be readily extended to account for visco-elastic response of soil by simply replacing: (a) the non-linear elastic load transfer, \( \zeta_t \) with the new load transfer factor, \( \zeta_c \zeta_t \); (b) the base shear modulus, \( G_{ib} \) with the time dependent \( G_{ib}(t) \). Therefore, load ratios of pile base and head can be predicted by

\[ \frac{P_b}{P_t} = \frac{4r_0 G_{ib}(t)}{(1 - v_s) \omega} \frac{1}{k \cdot E_p A_p z_t} \left(\frac{z_t}{L}\right)^{n/2} \left(\frac{C_1(z_t) + \chi_v C_2(z_t)}{C_3(L)}\right) \]  

(5-30)

where \( E_p, A_p \) are Young's modulus, and its cross-sectional area of an equivalent solid cylinder pile.
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\[
\begin{align*}
C_1(z) &= -K_{m-1}I_{m-1}(y) + K_{m-1}(y)I_{m-1} \\
C_2(z) &= K_mI_{m-1}(y) + K_{m-1}(y)I_m \\
C_3(z) &= K_{m-1}I_m(y) + K_m(y)I_{m-1} \\
C_4(z) &= -K_mI_m(y) + K_m(y)I_m
\end{align*}
\]  

(5-31)

with the modified Bessel functions \(I_m(y), I_{m-1}(y), K_{m-1}(y),\) and \(K_m(y)\) being written as \(I_m, I_{m-1}, K_{m-1},\) and \(K_m\) at \(z = L\). The ratio \(\chi_v\) is given by

\[
\chi_v = \frac{2\sqrt{2}}{\pi(1-\nu_s)\omega_s b} \sqrt{\frac{\xi_1 \xi_2}{\lambda}}
\]

(5-32)

The variable \(y\) is given by

\[
y = 2m\frac{L}{r_o} \sqrt{\frac{2}{\lambda \xi_2 \xi_1}} \left( \frac{z}{L} \right)^{1/2m}
\]

(5-33)

and the stiffness factor, \(k_s\) is provided by

\[
k_s = \frac{L}{r_o} \sqrt{\frac{2}{\lambda \xi_2 \xi_1}} \left( \frac{1}{L} \right)^{1/2m}
\]

(5-34)

The settlement influence factor, \(I\), can be estimated by

\[
I = \frac{G_{Ht}w_r t_o}{P_t} = \frac{1}{\pi C_v(z_t)} \sqrt{\frac{\xi_1 \xi_2}{2\lambda}}
\]

(5-35)

where \(G_{Ht}\) is the shear modulus at the level just above the pile tip. The coefficient of \(C_v(z_t)\) is given by

\[
C_v(z_t) = \frac{C_1(z_t) + \chi_v C_2(z_t)}{C_1(z_t) + \chi_v C_4(z_t)} \left( \frac{z_t}{L} \right)^{n/2}
\]

(5-36)

As the pile head load increases, the mobilised shaft shear stress will reach the limiting shaft stress, \(\tau_f\)

\[
\tau_f = A_v z^g
\]

(5-37)
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where $A_v$ is a constant for limit shear stress distribution, $\theta$ is a constant determining the shaft limiting stress distribution, normally taken as equal to the constant $n$. Therefore, the local limiting displacement, $w_e$ can be obtained from Eq. (5-20) as

$$w_e = \frac{A_v}{A_g} \frac{d}{2} \zeta_c \zeta_l$$  \hspace{1cm} (5-38)

Pile-head load, $P_t$ and settlement, $w_t$ can be expressed as the slip degree, $\mu = L_1/L$ ($L_1$ = slip length) by

$$P_t = w_e k_s E_p A_p L^{n/2} C_v (\mu L) + \pi d A_v \left( \frac{\mu L}{1 + \theta} \right)$$ \hspace{1cm} (5-39)

$$w_t = w_e \left[ 1 + \mu k_s L^{n+1} C_v (\mu L) \right] + \frac{\pi d A_v}{E_p A_p} \frac{\mu L^{2+\theta}}{2 + \theta}$$ \hspace{1cm} (5-40)

For the pile at high stress levels and/or of a higher slenderness ratio, $\zeta_c$ is no longer a constant. Therefore, Eqs. (5-39) and (5-40) are no longer valid. In this case, it is desirable to use a numerical analysis, e.g. the GASPILE program, to account for the variation of $\zeta_c$.

5.3.2 Validation

Booker and Poulos (1976) have incorporated a linear visco-elastic model into Mindlin's solution for analysing creep behaviour of a vertically loaded pile. They show the variation of the settlement influence factor and the ratio of base and head loads affected by the following three variables: (a) pile-soil relative stiffness; (b) the ratio of long-term and short-term soil response $J(\infty)/J(\theta)$, and (c) non-dimensional time, $t/T$. Except that the effect of the viscosity on the Poisson's ratio has been ignored, the numerical analysis is rigorous and hence has been adopted to validate the current closed form prediction. Fig. 5-8 shows a comparison of the settlement influence factor for the case of two different relative stiffnesses at a ratio of $J(\infty)/J(\theta) = 2$. Fig. 5-9 illustrates that the ratio of base and head load predicted by Eq. (5-30), both considering and ignoring the effect of base creep. Base creep significantly affects the load ratio, but it has negligible effect on the settlement influence factor as shown in Fig. 5-10. For a higher ratio of $J(\infty)/J(\theta)$, for instance, a value of 10 (corresponding to a ratio of $G_{ii}/G_{i2} = 9$), the difference between the response predicted by Eq. (5-35) and the numerical solution (Booker & Poulos, 1976) becomes apparent (Fig. 5-10). Fortunately, the ratio of $G_{ii}/G_{i2}$ is normally...
lower (Lo, 1961) and generally less than 5 as backfigured from a few different field tests.

Fig. 5-11 shows that the pile-head load and settlement predicted by Eqs. (5-39) and (5-40) respectively is normally consistent with the numerical prediction by GASPILE.

### 5.4 COMPARISON BETWEEN THE TWO KINDS OF LOADING

The time dependent behaviour of a pile subjected to 1-step and ramp loading has been examined. Comparison of the settlement influence factor using the closed form solution, Eq. (5-35) for the two types of loading has been presented in Fig. 5-12 (a), (b). It demonstrates that a larger settlement occurs for the case of step loading as would be expected. The relative time ratio of $t_c/t$ has significant effect on the pile settlement. By controlling the time $t_c$ (hence the loading rate), significant secondary pile settlement can be prevented. Similarly, a slightly higher percentage of base load over the head load for the step loading case in comparison with that for the ramp loading as predicted by Eq. (5-30) is demonstrated, which has been illustrated in Fig. 5-13 (a), (b).

### 5.5 APPLICATION

In general, two kinds of time dependent loading tests on piles are frequently reported:

1. A series of loading tests are performed at different time intervals following installation of a pile. For each step of the loading tests, a sufficient time is given.

2. Only one loading test is performed and will be undertook only when the destructed soil around the pile has been fully reconsolidated. However, when the test is undertaken, the time for each step of loading is allowed as required.

The first kind of test reflects the recovery of the soil strength (modulus) with reconsolidation, its simulation will be discussed in Chapter 6. Whereas the second kind of test reflects purely the pile response due to loading. The response can be simulated by either the closed form solutions of Eqs. (5-39), (5-40) or the numerical GASPILE program. Normally, if the test time for each step loading is less than that required for a 90% degree of consolidation $t_{90}$ for the soil, the pile response has been assumed to behave elastically. While the effect of an extra long time has been attributed to the visco-elastic response. Unfortunately, the current criteria for stopping each step of a loading test is based on the settlement rate (e.g. Maintained Loading Test) rather than
5.5.1 Case 1: Tests reported by Konrad and Roy (1987)

Konrad and Roy (1987) reported the results of an instrumented pile loaded to failure at intervals after driving. The closed-ended steel pipe pile of outside radius 0.219 m, 8.0 mm thick wall was jacked vertically to a depth of 7.6 m below the ground level. The Young's modulus was $2.07 \times 10^5$ MPa and the cross sectional area was 53.03 cm$^2$. Therefore the equivalent pile modulus can be inferred as 29,663 MPa. The test was performed at a site consisting of 0.4 m of topsoil, 1.2 m of weathered clay crust, 8.2 m of soft silty clay of marine origin, 4.0 m of very soft clayey silt and a deep layer of dense sand extending from a depth of 13.7 m to more than 25 m. The profile of the soil undrained shear strength, $S_u$, increased nearly linearly from 18 kPa at a depth of 1.8 m to 28 kPa at 9 m. The pile was loaded to failure in 10 to 15 increments of 6.67 kN. Each load was maintained for a period of 15 min. The soil shear modulus is taken as $270 S_u$. With the data tabulated in Table 5-2, the elastic prediction of load-settlement relationship by GASPILE and the closed form solutions are shown in Fig. 5-14, together with the immediate elastic response measured at different days. At a load level higher than about 70%, a non-linear relationship between the initial load and settlement prevails with increasing curvature as failure approaches. This non-linearity principally reflects the effect of the base non-linearity, since by simply using a non-linear base model ($R_{fb} = 0.95$ in Eq. (5-28)), an excellent prediction using GASPILE is achieved. Time dependent creep predictions for the test at 33 days after completion of the driving have been obtained by the visco-elastic analysis, with $G_1/G_2 = 2$. As shown in Fig. 5-15, the analytical results are generally very good compared with those measured at a number of time intervals, 0, 15 and 90 minutes. However, at higher load levels, the factor, $\zeta_2$, is not a constant as adopted in the prediction, or else the effect of the base non-linearity becomes important; thus, the closed form solution cannot furnish a good prediction.

5.5.2 Case II: Visco-elastic Property Predominated Compressive Loading

Two driven wooden piles were tested in a site about 20 km west of Stockholm (Bergdhl and Hult, 1981), in which the subsoil consisted mainly of postglacial organic clay. The undrained shear strength was 9 kPa at a depth of 4 to 5 m and increased almost linearly to 25 kPa at 14 m. Both piles (termed as B$_1$ and B$_2$) were 100 mm square sections and 15 m lengths. The two piles gave consistent results, therefore only pile B$_1$ will be
analysed herein. The Young's modulus of the piles is taken as $10^4$ MPa. Other relevant information for the analysis has been tabulated in Table 5-3 for numerical GASPILE analysis. An equivalent shear modulus distribution of $G_{ave} = 755.6$ kPa, $n = 0.75$ is obtained to perform the closed form predictions. The load settlements predicted by the non-linear elastic analyses both by numerical GASPILE program and the closed form solutions are compared with the measured data in Fig. 5-16(a). The creep behaviour was monitored by maintained load tests, with the load increased in steps of $1/16$th of the estimated bearing capacity of the pile every 15 minutes. This creep displacement is obtained theoretically as the difference between the non-linear visco-elastic (NLVE) and the non-linear elastic (NLE) analysis. It has been shown in Fig. 5-16 (c) in comparison with the measured creep displacement. The corresponding load distribution down the pile is illustrated in Fig. 5-16 (b). In this instance, the secondary deformation due to the viscosity of the soil can be sufficiently accurately predicted by a visco-elastic analysis over a loading level of 75% of the ultimate bearing capacity as determined by constant rate of penetration (CRP) test. Afterwards, considerable creep occurs as shown in the tests.

5.6 CONCLUSIONS

The proposed shaft and base pile soil interaction models can account well for non-linear visco-elastic soil property at any stress levels. Based on these analytical models, the overall pile response under 1-step and the ramp type loading can be readily estimated through either the closed form solutions or the GASPILE program. Nevertheless, the closed solutions are only valid for normal working loads, e.g. less than 70% of ultimate load level (hence for estimation of the secondary consolidation), because a constant load transfer factor, $\zeta_c$ is adopted. At a higher stress level, $\zeta_c$ is no longer a constant. Therefore, numerical analysis (e.g. by GASPILE analysis) of the creep behaviour is necessary. The ratio of initial and delayed elastic shear moduli, and the relaxation time factor can be ascertained from measurements of time settlement relationships of a pile under a load or from soil creep tests. Both the variation of the shear modulus, and failure shear stress with depth, might be simply obtained from current empirical formulas or more accurately by field tests. A suitable control of the ramp type loading can avoid excessive secondary settlement. Step loading should be avoided wherever possible.
TABLE 5-1  Curve Fitting Parameters for Fig. 5-6

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$G_{ij}/\tau_{i2}$</th>
<th>$G_{ij}/\tau_{i1}$ (10^{-5} \text{ Sec.}^{-1})$</th>
<th>$\xi_{i1}$</th>
<th>$\xi_c$ for 0/15/90 Minutes</th>
<th>$\omega$</th>
<th>$\xi_{sb}$</th>
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<tr>
<td>38</td>
<td>175</td>
<td>0.5</td>
<td>115.6</td>
<td>10.1</td>
<td>0.91</td>
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<td>32</td>
<td>175</td>
<td>0.55</td>
<td>90.4</td>
<td>17.0</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>500</td>
<td>2.67</td>
<td>77.5</td>
<td>26.7</td>
<td>0.68</td>
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</tr>
</tbody>
</table>

TABLE 5-2  Parameters for Creep Analysis of Case I

<table>
<thead>
<tr>
<th>$G_{ii}/s_u$</th>
<th>$G_{ii}/\tau_{i1}$</th>
<th>$\nu_s$</th>
<th>$\zeta_{i1}$</th>
<th>$\zeta_c$</th>
<th>$\omega$</th>
<th>$\xi_{sb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>260</td>
<td>270</td>
<td>0.4</td>
<td>4.60</td>
<td>1/1.13/1.666</td>
<td>1</td>
<td>1.0</td>
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</tbody>
</table>

TABLE 5-3  Parameters for Creep Analysis of Case II (Pile B1)

<table>
<thead>
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<th>$G_{ii}/\tau_{i1}$</th>
<th>$\nu_s$</th>
<th>$\zeta_{i1}$</th>
<th>$G_{ii}/G_{ij}$</th>
<th>$\zeta_c$</th>
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<tr>
<td>47.5</td>
<td>80</td>
<td>0.4</td>
<td>6.27</td>
<td>0.025</td>
<td>1.025</td>
<td>1</td>
<td>1.0</td>
</tr>
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</table>
6. PERFORMANCE OF A DRIVEN PILE IN VISCO-ELASTIC MEDIA

6.1 INTRODUCTION

Installation of a driven pile in a clay generally leads to a remoulding of the soil, some loss in the strength and an increase in pore water pressure in the vicinity of the pile. Increase in strength with time, subsequent to driving, results in the final soil strength being equal to, or greater than the initial value (Orrje and Broms, 1967; Flaate, 1972; Fellenius and Samson, 1976; Bozozuk et al. 1978), accompanied by a gradual decrease in water content in the clay adjacent to the pile, and increase in the bearing capacity of the pile (Seed and Reese, 1955).

The maximum pore pressure occurs immediately following driving, and may approximately equal, or exceed the total overburden pressure in overconsolidated soil (Koizumi and Ito, 1967; Flaate, 1972). The magnitude of the pore pressures induced due to driving decreases rapidly with distance from the pile wall, and becoming negligible at a distance of 5 to 10 pile diameters. This distribution of excess pore pressure around a driven pile may be simulated with sufficient accuracy using the cylindrical cavity expansion analogy (Randolph and Wroth, 1979b) or the strain path method (Baligh, 1985, 1986a, 1986b). The former theory, though, is a one-dimensional analysis, has generally provided sufficient accuracy, compared with the latter analysis. A particular advantage of the approach is that it can be readily extended to the case of visco-elastic soil response.

Using a radial consolidation theory, Soderberg (1962), Randolph and Wroth (1979b) show that the measured rate of development of pile capacity in soft clay appears to be consistent with the rate of pore pressure dissipation. Therefore, with the assumption of an impervious pile, the problem of predicting the variation of capacity becomes one of predicting the hydrostatic excess pressures at the pile shaft as a function of time.

Dissipation of the excess pore pressures generated during driving leads to an increase not only in shaft friction but also in the stiffness of the surrounding soil (e.g. Eide et al. 1961; Flaate, 1972; Flaate and Selnes, 1976; Bergdahl and Hult, 1981; Trenter and Burt, 1981). Accurate prediction of pile behaviour requires determination of the profile of pile-soil interaction stiffness and limiting shaft friction, which are generally treated as invariants with time. However, the soil strength is normally significantly altered by pile driving, which means that the overall pile-soil interaction should be treated as a time-
dependent problem. Many researchers have emphasised the importance of predicting the load-settlement behaviour (Olson, 1992; Fleming, 1992; Randolph, 1994), particularly where piles act as settlement reducers. However, most research conducted to date has concentrated on the time-dependent bearing capacity, rather than how the overall response is affected by soil reconsolidation following pile driving.

Two basic approaches are commonly used for analysing consolidation problems. The first was developed from diffusion theory by e.g. Terzaghi (1943) and Rendulic (reported by Murray, 1978). The second was developed from elastic theory by e.g. Biot (1941), and more recently by Randolph and Wroth (1979b) for dissipation of pore pressure generated due to pile driving.

The diffusion theory is generally less rigorous than the elastic theory. However, the diffusion theory is mathematically much simpler to apply, and can be readily extended to account for complex conditions. e.g. soil visco-elasticity, soil shear modulus non-homogeneity. In fact, the diffusion theory is different from the elastic theory in that (1) the mean total stress is assumed constant; (2) the coefficients of consolidation derived for the two theories are generally different (Murray, 1978). If the mean total stress change happens to be zero, the only difference between the two theory is in the coefficients of consolidation. Therefore, a coefficient from elastic theory may be used to replace the coefficient in the solution of the diffusion theory, then the solution is converted into a rigorous solution.

In this chapter:

(1) A generalised non-linear visco-elastic stress-strain model is first generated.

(2) A governing equation from the diffusion theory is established for radial reconsolidation of a visco-elastic medium. By comparing with an available rigorous elastic solution (Randolph, 1977), a rigorous solution for a visco-elastic medium is obtained by using a suitable coefficient of consolidation. Alternatively, rigorous visco-elastic solutions have been obtained by using the correspondence principle (Mase, 1970), in light of the available elastic solutions.

(3) Equations for radial consolidation for a given logarithmic variation of initial pore pressure are provided.

(4) Three case studies are described to illustrate the time variation of pore pressure, pile capacity, average pile shaft cohesion, and average shear modulus.
6.2 NON-LINEAR VISCO-ELASTIC STRESS-STRAIN MODEL

A non-linear visco-elastic model (simply called Mechant's model) has already been described in Chapter 5 as illustrated in Fig. 6-1(a). In principle, the model is directly adopted herein, except that a Voigt element is added in series with Mechant's model. This addition leads to a generalised visco-elastic model, as detailed in Fig. 6-1(b). For a prolonged constant loading, the stress-strain relationship for the generalised model can be expressed by

\[ \gamma = \tau F(t) \] 

(6-1)

where \( \tau, \gamma \) are the total shear stress and strain respectively for the model. The creep compliance, \( F(t) \) is given by (Lo, 1961)

\[ F(t) = \frac{1}{G_{\gamma 1}} \left( 1 + m_2 \left( 1 - \exp\left( -\frac{t}{T_2} \right) \right) + m_3 \left( 1 - \exp\left( -\left( t - t_k \right)/T_3 \right) \right) \right) \] 

(6-2)

where \( m_2 = G_{\gamma 1}/G_{\gamma 2} ; m_3 = G_{\gamma 1}/G_{\gamma 3} ; 1/T_2 = G_{\gamma 2}/\eta_{\gamma 2} ; 1/T_3 = G_{\gamma 3}/\eta_{\gamma 3} ; \eta_{\gamma 2}, \eta_{\gamma 3} \) are the shear viscosity at visco elements 2 and 3 respectively; \( G_{\gamma j} \) is the shear modulus for each of the elastic springs; \( t_k \) is a critical time used to determine when the Voigt element is in action. The value of \( t_k \) can be assessed by experiment (Lo, 1961). If the elapsed time, \( t \) is less than \( t_k \), the Voigt element 2 is not in effect. Therefore, \( m_3 \) is zero, and the generalised model reduces to the Mechant’s standard linear model as shown in Fig. 6-1(a).

To account for the soil non-linear response, the shear modulus for each of the elastic springs, \( G_{\gamma j} \) is derived through using a hyperbolic stress-strain model, and may be expressed by

\[ G_{\gamma j} = G_{\gamma j} k_j \] 

(6-3)

where \( k_j = 1 - \psi_j, \ \psi_j = R_{\gamma j} \tau_j / \tau_{\gamma j}, k_j \) is the coefficient for considering the non-linear effect on the shear moduli of the elastic springs 1, 2 and 3 \( (j = 1, 2, \text{and} \ 3) \) respectively; \( R_{\gamma j} \) is a parameter, which controls the degree of non-linearity for springs 1 to 3 respectively; \( \tau_j \) is the shear stress on element \( j \); \( \tau_{\gamma j} \) is the local failure shaft stress for springs 1 to 3; \( G_{\gamma j} \) and \( G_{\gamma j} \) are the initial and the average secant shear moduli up to a strain level of zero and \( \gamma_j \) respectively for each of the springs 1 to 3.
A number of conclusions about using Mechant's model have been obtained in Chapter 5. These conclusions as described below are generally valid for the current generalised model as well; as such they are directly adopted in this Chapter:

1. The limiting shaft stress, \( \sigma_{\text{ult}} \), and the failure stress, \( \sigma_f \), are assumed identical.

2. An appropriate value of \( \sigma_{\text{ult}} \) can be correlated with the shear strength of the soil, or with the effective overburden stress (e.g. API RP2A; Tomlinson, 1970; Randolph and Murphy, 1985). The failure stresses of \( \sigma_{\text{f2}} \) and \( \sigma_{\text{f3}} \) may be assessed through experiments, and the stresses are generally correlated with the \( \sigma_{\text{f1}} \), e.g. \( \sigma_{\text{f2}} \) was reported to be approximately equal to \( 0.7 \sigma_{\text{f1}} \) (Geuze and Tan, 1953; Murayama and Shibata, 1961).

3. With the model, two types of responses to stress are reflected: instantaneous elasticity \( G_{\text{y1}} \) and delayed elasticity \( G_{\text{y2}} \) and/or \( G_{\text{y3}} \). At any given time, e.g. at the onset of loading, the stress-strain response may be modelled as a non-linear hyperbolic curve. Under a specified stress, the displacement develops as a creep process.

4. According to the experiment by Lo (1961), generally secondary deformation of all remoulded and undisturbed clays can be modelled sufficiently accurately by the linear standard model, with \( m_3 = 0 \). For a soil of loose structure, the generalised model may be used, with the values of \( t_k \) and \( m_3 \) determined by experiment.

6.3 GOVERNING DIFFUSION EQUATION FOR RECONSOLIDATION

The effect of driving a pile into clay can be simulated by expansion of a long cylindrical cavity under undrained conditions in an ideal visco-elastic, perfectly plastic material, characterised by the shear moduli, \( G_{\text{yj}} \) (\( j = 1, 2 \) and 3) and the undrained shear strength, \( s_u \). Experiment shows that the expansion is a plane strain problem for the middle part of the pile (Clark and Meyerhof, 1972). The soil properties and the stress state immediately following pile driving have been simplified and illustrated in Fig. 6-2.

6.3.1 Volumetric Stress-strain Relation of Soil Skeleton

In this section, volumetric effective stress-strain relationship is first given for an elastic medium and then the relationship is converted into that for a visco-elastic medium. The plane strain version of Hooke's law is
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\[ \varepsilon_r = \frac{1}{2G} \left[ (1 - v_s) \delta \sigma'_r - v_s \delta \sigma'_\theta \right] \]

\[ \varepsilon_\theta = \frac{1}{2G} \left[ -v_s \delta \sigma'_r + (1 - v_s) \delta \sigma'_\theta \right] \]

\[ \varepsilon_z = 0 \]

(6-4)

where \( G \) is the elastic soil shear modulus; \( \delta \sigma'_r \), \( \delta \sigma'_\theta \), \( \delta \sigma'_z \) are the increments of the effective stresses during consolidation in the radial, circumferential and depth directions, with \( \delta \sigma'_z = v_s (\delta \sigma'_r + \delta \sigma'_\theta) \). Combining Eq. (6-4) and the effective stress principle, the volumetric effective stress-strain relationship for plain strain cases may be written as

\[ \varepsilon_v = \frac{1 - 2v_s}{G} \left( \delta \theta - (u - u_0) \right) \]

(6-5)

where \( v_s \) is the Poisson's ratio of the soil; \( \delta \theta \) is the total mean stress change, \( \delta \theta = 0.5 (\delta \sigma'_r + \delta \sigma'_\theta) \); \( u \) is the excess pore pressure; \( u_0 \) is the initial value following driving (Randolph and Wroth, 1979b).

Eq. (6-5) is valid for an elastic medium. Similar volumetric expression for visco-elastic media may be directly transformed from Eq. (6-5), using the correspondence principle (Mase, 1970), by the following procedures:

1. Applying the Laplace transform to Eqs. (6-1) and (6-2) respectively, allowing the shear modulus \( \mathcal{G} \), to be related to the compliance, \( \mathcal{F}(t) \) by

\[ \mathcal{F}(t) = \frac{1}{s \mathcal{G}} \]

(6-6)

2. Applying the Laplace transform to Eq. (6-5), and using \( \mathcal{F}(t) \) to replace the transformed modulus, \( \mathcal{G} \), to give

\[ \overline{\varepsilon_v} = \left( 1 - 2v_s \right) s \mathcal{F}(t) \left( \delta \theta - (u - u_0) \right) \]

(6-7)

where \( s \) is the argument of the Laplace transform.

3. Applying the inverse Laplace transform to Eq. (6-7), to obtain the final expression of the volumetric strain for visco-elastic media as
\[ \varepsilon_v = \frac{1 - 2v_s}{G_{\gamma_1}} \left( \delta \theta - (u - u_0) + G_{\gamma_1} \int_0^t (\delta \theta - (u - u_0)) \frac{dF(t - \tau)}{d(t - \tau)} \, d\tau \right) \]  \hspace{1cm} (6-8)

where \( \varepsilon_v \) is the volumetric strain.

The Poisson's ratio is regarded as a constant, and the effect of this assumption is generally ignored even for numerical analysis (Booker and Poulos, 1976). In fact, considering the viscous effect on Poisson's ratio would lead to a formidable inverse Laplace transform.

The total mean stress change, \( \delta \theta \) is generally not zero (a constant) with time during consolidation (Mandel, 1957; Cryer, 1963; Murray, 1978). However, taking it as a constant (zero) as assumed by Terzaghi (1943) and Rendulic (1936), will significantly simplify the solution of the problem, and the solution generally compares very well with the corresponding rigorous solution (Davis and Poulos, 1968; Christian and Boehmer, 1970; Murray, 1978). In fact, as noted by Murray (1978), many of the currently popular theories are based on this assumption, for instance, the sand drain problem solved by Barron (1948).

To simply the current problem, it is assumed that \( \delta \theta = 0 \). In terms of Eq. (6-8), the changing rate of volumetric strain may then be expressed by

\[ \frac{\partial \varepsilon_v}{\partial t} = -(1 - 2v_s) \frac{1}{G_{\gamma_1}} \left( \frac{\partial u}{\partial t} + G_{\gamma_1} \int_0^t \frac{\partial u}{\partial t} \frac{dF(t - \tau)}{d(t - \tau)} \, d\tau \right) \]  \hspace{1cm} (6-9)

It worth noting that Eq. (6-9) is derived from the stress-strain relationship.

### 6.3.2 Flow of Pore Water and Continuity of Volume Strain Rate

The volumetric strain rate may be obtained by considering the flow of pore water and continuity of volume. The pore water velocity may be related to the pressure gradient by Darcy's law. For continuity, the rate of volumetric strain must be related to the flow of pore water into and out of any region by (Randolph and Wroth, 1979b)

\[ \frac{\partial \varepsilon_v}{\partial t} = -\frac{k}{\gamma_w \rho \tau} \frac{\partial}{\partial \tau} \left( \frac{\partial u}{\partial \tau} \right) \]  \hspace{1cm} (6-10)

where \( k \) is the permeability of the soil; and \( \gamma_w \) is the unit weight of water.
Eqs. (6-9) and (6-10) may be combined to yield

\[
\frac{kG_{y_1}}{\gamma_w(1-2\nu_s)} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t} + G_{y_1} \int_0^t \frac{\partial u}{\partial t} \frac{dF(t-\tau)}{d\tau} d\tau
\]  
(6-11)

This is the governing equation for radial consolidation. In fact, it is a diffusion equation, and does not necessarily satisfy radial equilibrium. If the soil is treated as an elastic medium, then \( \frac{dF(t-\tau)}{d(t-\tau)} = 0 \); hence Eq. (6-11) reduces to that for the elastic case.

\[
c_v \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t}
\]  
(6-12)

where

\[
c_v = \frac{kG_{y_1}}{\gamma_w(1-2\nu_s)}
\]  
(6-13)

In the following parts of this chapter, the subscript "\( y_1 \)" in \( G_{y_1} \) will be dropped, unless required for emphasis. As illustrated later, solutions of Eq. (6-12) are identical to those given by Randolph (1977) for the case of constant total vertical stress.

### 6.4 BOUNDARY CONDITIONS

The boundary conditions for radial consolidation of an elastic medium around a rigid, impermeable pile have been detailed previously by Randolph and Wroth (1979b). These conditions are generally valid for the visco-elastic case as well, and hence are re-stated here:

\[
u|_{t=0} = u_o(r) \ (t = 0, \ r \geq r_0)
\]  
(6-14a)

\[
\frac{\partial u}{\partial r}|_{r=r_0} = 0 \ (t \geq 0)
\]  
(6-14b)

\[
u|r=r_0 = 0 \ (t \geq 0)
\]  
(6-14c)

\[u = 0 \ \text{as} \ t \to \infty \ (r \geq r_0)
\]  
(6-14d)
where \( r^* \) is some radius beyond which the excess pore pressures are zero. Initially, \( u = 0 \) for \( r \geq R \) (\( R \) is the width of plastic zone). However, during consolidation, outward flow of pore water will give rise to excess pore pressures in the region \( r > R \), and generally it is necessary to take \( r^* \) as 5 to 10 times \( R \).

### 6.5 GENERAL SOLUTION

Solution for an visco-elastic problem can be achieved by either (1) a direct solution; (2) using the correspondence principle, in terms of the available elastic solutions.

#### 6.5.1 Direct Solution of the Diffusion Equation

The general solution to Eq. (6-11) may be obtained by separating the variables for time dependant and independant parts, i.e.

\[
u = wT(t)
\]

(6-15)

With a separation constant of \( \lambda_n^2 \), it follows

\[
\frac{\partial^2 w}{\partial t^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \lambda_n^2 w = 0
\]

(6-16)

\[
\frac{dT(t)}{dt} + G_{\gamma_1} \int_0^t \frac{dT(t)}{dt} \frac{dF(t - \tau)}{d(t - \tau)} d\tau + \alpha_n^2 T(t) = 0
\]

(6-17)

where

\[
\alpha_n^2 = c_v \lambda_n^2
\]

(6-18)

The parameter, \( \lambda_n \), is one of the infinite roots satisfying Eq. (6-16), which may be expressed in terms of Bessel functions as

\[
w_n(r) = A_n J_0 (\lambda_n r) + B_n Y_0 (\lambda_n r)
\]

(6-19)

where \( A_n \) is dependent on the boundary conditions. The functions \( J_0, Y_0, J_1, Y_1 \) are Bessel functions of zero order and first order, with \( J_1 \) being Bessel functions of the first kind, and the \( Y_1 \) being Bessel functions of the second kind.

Cylinder functions, \( V_i(\lambda_n r_0) \) of \( i \)-th order (McLachlan, 1955) may be expressed as
Based on the boundary condition of Eq. (6-14b), \( B_n = -A_n \frac{J_1(\lambda_n r_0)}{Y_1(\lambda_n r_0)} \). Thus, from Eq. (6-19),

\[
w_n(r) = A_n V_0(\lambda_n r) \tag{6-21}
\]

\[
\frac{dw_n(r)}{dr} \bigg|_{r=r_0} = A_n \frac{V_1(\lambda_n r)}{Y_1(\lambda_n r_0)} = 0 \tag{6-22}
\]

Also, with Eq. (6-14c), \( u = 0 \) for \( r \geq r^* \), it follows

\[
V_0(\lambda_n r^*) = J_0(\lambda_n r^*) - \frac{J_1(\lambda_n r_0)}{Y_1(\lambda_n r_0)} Y_0(\lambda_n r^*) = 0 \tag{6-23}
\]

Eqs. (6-22) and (6-23) render the cylinder functions to be defined. There is an infinite number of roots of \( \lambda_n \) satisfying these equations, since the Bessel functions are periodic.

The time-dependent solutions are dominated by the creep model. For the generalised creep model, Eq. (6-17) can be solved as (Appendix E, Abramowitz and Stegun, 1964)

\[
T_n(t) = E_n \exp(a_n t) + F_n \exp(b_n t) + G_n \exp(c_n t) \tag{6-24}
\]

where

\[
E_n = \frac{a_n^2 + H_n a_n + I_n}{(a_n - b_n)(a_n - c_n)}, \quad F_n = \frac{b_n^2 + H_n b_n + I_n}{(b_n - a_n)(b_n - c_n)}, \quad G_n = \frac{c_n^2 + H_n c_n + I_n}{(c_n - a_n)(c_n - b_n)} \tag{6-25}
\]

and

\[
H_n = \left( \frac{m_2/T_2 + m_3/T_3 + \alpha_k m_3/T_2 + 1/T_2 + 1/T_3}{m_3 \alpha_k + 1} \right)
\]

\[
I_n = \left( \frac{m_2 + m_3 + 1}{m_3 \alpha_k + 1} \right) T_2 T_3
\]

\[
a_n = -p_n/3 + \Delta_1(n) + \Delta_2(n)
\]

\[
b_n = -p_n/3 - (\Delta_1(n) + \Delta_2(n))/2 + (\Delta_1(n) - \Delta_2(n))\sqrt{-3}/2
\]

\[
c_n = -p_n/3 - (\Delta_1(n) + \Delta_2(n))/2 - (\Delta_1(n) - \Delta_2(n))\sqrt{-3}/2
\]
\[ \Delta_1(n) = \sqrt{\frac{b_1(n)}{2} + \frac{b_1^2(n)}{4} + \frac{a_1^3(n)}{27}} \quad \Delta_2(n) = -\sqrt{\frac{b_1(n)}{2} + \frac{b_1^2(n)}{4} + \frac{a_1^3(n)}{27}} \]

\[ a_1(n) = \frac{(3q_n - p_n^2)}{3} \quad b_1(n) = \frac{(2p_n^3 - 9p_nq_n + 27r_n)}{27} \]

\[ p_n = (m_2/T_2 + m_3/T_3 + m_3/T_2 \alpha_k + \alpha_n^2 + 1/T_2 + 1/T_3)/(m_3 \alpha_k + 1) \]

\[ q_n = \frac{(m_2 + m_3 + 1)/T_2T_3 + \alpha_n^2(1/T_2 + 1/T_3)}{(m_3 \alpha_k + 1)} \]

\[ r_n = \frac{\alpha_n^2/(m_3 \alpha_k + 1)T_2T_3}{, \alpha_k = 1 - \exp(t_k/T_3)} \]

The delayed time, \( t_k \), is expressed by the coefficient, \( \alpha_k \). For the standard linear visco-elastic model, since \( m_3 = 0 \), \( 1/T_3 = 0 \), it follows that (Appendix E)

\[ T_n(t) = \frac{(\omega_1(n) - \alpha_c)\exp(-\omega_1(n)t) - (\omega_2(n) - \alpha_c)\exp(-\omega_2(n)t)}{\omega_1(n) - \omega_2(n)} \]  (6-26)

where

\[ \omega_1(n) = \frac{\alpha_c + \alpha_n^2}{2} + \frac{1}{2}\sqrt{(\alpha_c + \alpha_n^2)^2 - 4\alpha_n^2/T_2} \]  (6-27)

\[ \omega_2(n) = \frac{\alpha_c + \alpha_n^2}{2} - \frac{1}{2}\sqrt{(\alpha_c + \alpha_n^2)^2 - 4\alpha_n^2/T_2} \]  (6-28)

\[ \alpha_c = \frac{(1 + m_2)}{T_2} \]  (6-29)

For the elastic case, in terms of Eqs. (6-26), (6-27) and (6-28), \( \alpha_c = 0 \), \( \omega_1(n) = \alpha_n^2 \), \( \omega_2(n) = 0 \), it follows

\[ T_n(t) = e^{-\alpha_n^2 t} \]  (6-30)

The full expression for pore pressure, \( u \), will be a summation of all the possible solutions

\[ u = \sum_{n=1}^{\infty} A_n V_e(\lambda_n r)T_n(t) \]  (6-31)

Normally the first 50 roots of the Bessel functions are found to give sufficient accuracy.

With Eqs. (6-14a) and (6-31), it follows
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$$A_n = \int_{r_0}^{r} u_o(r) V_o(r \lambda_n) r dr / \int_{r_0}^{r} V_o^2(r \lambda_n) r dr$$  \hspace{1cm} (6-32)

6.5.2 Rigorous Solutions for the Radial Reconsolidation

For the elastic case, the above established solutions reduce to the rigorous solutions from the elastic theory by Randolph (1977) for the case of constant total vertical stress. The difference in the solutions from the current diffusion theory and the elastic theory (by Randolph and Wroth, 1979b) is just the coefficient of consolidation, since $\delta \theta = 0$. Therefore, the above solutions may be readily transformed into the case of plane strain deformation by simply replacing the $c_v$ of Eq. (6-13) with

$$c_v = \frac{k}{\gamma_w} \frac{2(1-v_s)G_y}{1-2v_s}$$  \hspace{1cm} (6-33)

Considering non-linear soil stress-strain response, a lower value of the shear modulus will generally result, as shown by Eq. (6-3). Therefore, the consolidation time increases by a factor of $1/(1-\psi_j)$ as demonstrated by Eq. (6-30). The stress level, $\psi_j$ here is an average value for the domain concerned. For convenience, it may be taken as 0.5 as argued previously in Chapter 3.

6.5.3 Solution By Correspondence Principle

The above visco-elastic solutions may be readily obtained, in terms of the elastic solutions, by the correspondence principle. From Eq. (6-31)

$$\bar{u} = \sum_{n=1}^{\infty} A_n \bar{T}_n \bar{V}_o (\lambda_n r)$$  \hspace{1cm} (6-34)

For the visco-elastic analysis given by the standard linear model, the time-dependant part should be replaced with (Appendix E),

$$\bar{T}_n = \frac{s + \alpha_c}{(s + \omega_1(n))(s + \omega_2(n))}$$  \hspace{1cm} (6-35)

For the elastic analysis, the time-dependant part should be replaced with

$$\bar{T}_n = 1/(s + \alpha_n^2)$$  \hspace{1cm} (6-36)
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The inverse Laplace transform of Eqs. (6-35) and (6-36) (referred to Appendix E) leads to Eqs. (6-26) and (6-30) respectively. That is to say, the visco-elastic solutions can be formulated by simply replacing the time-dependent part of the elastic solutions with that for the visco-elastic case.

6.6 CONSOLIDATION FOR LOGARITHMIC VARIATION OF $u_0$

The initial stress state for radial consolidation of a visco-elastic medium around a rigid, impermeable pile is similar to that of an elastic medium (Randolph and Wroth, 1979b), as described below:

1. For a cavity expanded from zero radius to a radius of $r_0$ (pile radius), the stress change, $\delta \theta$ within the plastic zone ($r_0 \leq r \leq R$ as shown in Fig. 6-2) is given by

$$\delta \theta = s_u \left( \ln(G/s_u) - 2 \ln(r/r_0) \right) \quad (6-37)$$

2. The width of the plastic zone is given by

$$R = r_0 \left( \frac{G}{s_u} \right)^{1/2} \quad (6-38)$$

3. Under undrained conditions, if the mean effective stress remains constant, the initial excess pore pressure distribution away from pile wall varies according to

$$u_0(r) = 2s_u \ln(R/r) \quad r_0 \leq r \leq R$$

$$u_0 = 0 \quad R < r < r^*$$

where $R$ is the radius, beyond which the excess pore pressure is initially zero.

In light of the initial pore pressure distribution of Eq. (6-39), the coefficients can be simplified as

$$A_n = \frac{4s_u}{\lambda^2} \frac{V_o(\lambda_n r_0) - V_o(\lambda_n R)}{r^2 V_o(\lambda_n r^*) - r_0^2 V_o(\lambda_n r_0)} \quad (6-40)$$

With these values of $A_n$, the pore pressure can be readily estimated with Eq. (6-31). Evaluation of these functions has been carried out in a spreadsheet.
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6.13 Visco-elastic Consolidation

With the correspondence principle, the elastic solution of the outward radial movement, $\xi_r$, by Randolph and Wroth (1979b) can be extended to the visco-elastic case as

$$\xi_r = \frac{1}{2G^*} \left[ \sum_{n=1}^{\infty} \frac{A_n T_n(t) V_1(\lambda_n r)}{\lambda_n} + s_u \left( r \ln\left(\frac{R^*}{R}\right) - \frac{r_0^2}{r} \ln\left(\frac{r_0}{R^*}\right) \right) \right]$$  \hspace{1cm} (r_0 \leq r \leq R) \hspace{1cm} (6-41)

$$\xi_r = \frac{1}{2G^*} \left[ \sum_{n=1}^{\infty} \frac{A_n T_n(t) V_1(\lambda_n r)}{\lambda_n} + s_u \left( R^2 \ln\left(\frac{R}{R^*}\right) - \frac{r_0^2}{R^*} \ln\left(\frac{r_0}{R^*}\right) \right) \right]$$  \hspace{1cm} (R \leq r \leq r^*) \hspace{1cm} (6-42)

where $G^* = \frac{G_{y1}}{1 - 2\nu_s}$, $R^* = R \sqrt{e}$. The $T_n(t)$ is given by Eqs. (6-24), (6-26) and (6-30) respectively, dependent on which model is adopted.

The rate of consolidation may be expressed by the following non-dimensional variable (Soderberg, 1962),

$$T = \frac{c_v t}{r_0^2}$$ \hspace{1cm} (6-43)

The visco-elastic effect may be represented by the factor,

$$T_c = \frac{1 + m^2}{(T_2 c_v)}$$ \hspace{1cm} (6-44)

A parametric study has been undertaken for the solutions based on standard linear model. Fig. 6-3 shows the consolidation expressed as $u(r_0)/s_u$ ($u(r_0)$ is the pore pressure on pile-soil interface) for a soil with $G_{y1}/s_u = 50$, but at different ratios of primary and secondary shear moduli, $G_{y1}/G_{y2}$ and various values of the viscosity factor, $T_c$. Provided that other input parameters are identical, variation of the relaxation factor, $1/T_2$ can only shift the dissipation curve of pore pressure at the middle stage of the process, but not the initial or the final stages. Generally speaking, the viscosity effect becomes obvious only at a later stage.

Fig. 6-4 shows a set of plots of the non-dimensional times for 50% ($T_{50}$) and 90% ($T_{90}$) degree of consolidation to occur at different values of $u_0(r_0)/s_u$ and $G_{y1}/G_{y2}$ ($u_0(r_0)$ is the initial pore pressure on pile-soil interface immediately following pile installation). Considering the secondary consolidation by the ratio of $G_{y1}/G_{y2}$, higher values of $T_{50}$ and $T_{90}$ are obtained compared with those from elastic analysis ($G_{y1}/G_{y2} = 0$). Accordingly longer consolidation times and higher displacements occur compared with the elastic case.
The rules shown in Figs. 6-3 and 6-4 are applicable for both cases of constant total stress and plane strain deformation. The corresponding coefficient of consolidation, $c_v$, may be used for each case.

6.7 VISCO-ELASTIC BEHAVIOUR

6.7.1 Parameters for the Creep Model

The magnitude of the relaxation time has been provided in Table 6-1, based on the relevant publications (Edil and Mochtar, 1988; Qian et al. 1992; Ramalho Ortigao and Randolph, 1983).

In particular, as reviewed in the previous chapter, the experiment by Lo (1961) showed that

1. The rate factor, $G_{r2}/\eta_{r2}$, is generally a constant for a given clay. For the clays tested, it lies between 0.2 and 0.4 ($\times 10^5$ s$^{-1}$).
2. The compressibility index ratio, $G_{r1}/G_{r2}$, is only influenced by the soil water content, and generally lies between 0.05 and 0.2, except for a soil of loose structure.
3. The individual values of $G_{r1}$, $G_{r2}$ and $\eta_{r2}$, however, vary with load (stress) level.

6.7.2 Prediction of the Ratio of Modulus and Limiting Shaft Stress

Experimental results (Clark and Meyerhof, 1972) show that:

1. during a loading test, the change in pore water pressure along the shaft of the pile is insignificant;
2. the magnitude of the total and effective radial stress surrounding the pile is primarily related to the stress changes brought about when the pile is driven and during subsequent consolidation. Changes with time due to loading are insignificant relative to the initial values.

Therefore, the ratio of $G_{r1}/\tau_{fl}$ may generally be assumed to be a constant during a loading test, so that it can be estimated from the measured load-settlement curve by fitting theoretical solutions (e.g. by GASPILE). Soil stress-strain non-linearity has only limited effect on such a back-analysis. Because the overall response of a pile by elastic analysis are barely different from that by a non-linear elastic as shown in Chapter 3.
The recovery of the soil strength and modulus during reconsolidation was investigated previously (e.g. Trenter and Burt, 1981) by a series of loading tests performed at different time intervals following installation of a pile. Through fitting the measured load-settlement curve with the theoretical solutions (GASPILE analysis), the soil strength and modulus are back-figured. From the series of loading test results, a series of the strength and the corresponding shear modulus are obtained corresponding to the test time interval; and then these values of strength and modulus are normalised by the initial values respectively.

**Test Reported by Trenter and Burt (1981)**

Four driven open ended pile load tests were performed in Indonesia, mainly by maintained load procedure (Trenter and Burt, 1981). The basic pile properties are shown in Table 6-2; Young's modulus of the pile is assumed as 29,430 MPa (the effect of this assumption is explored later). The undrained shear strength of the subsoil at the site varies basically according to \( s_u = 1.5z \) (\( s_u \), kPa; \( z \), depth, m). The initial shear modulus is taken as a multiple of the undrained shear strength, \( s_u \), the ratio \( G/s_u \) being back-analysed from the test data. There is no information about the values of the creep parameters. However, based on previous publications, shear modulus ratio, \( G_{y1}/G_{y2} \), may be reasonably taken as 0.15, and rate factor, \( G_{y2}/\eta_{y2} \), taken as, \( 0.5 \times 10^{-5} \) (s\(^{-1}\)). Using these assumed values, the variation of normalised soil strength and shear modulus with time during consolidation is not affected, as justified later.

The accuracy of the load transfer factor, \( \zeta_1 \), given by Eq. (3-7) has been testified (against the available rigorous numerical solutions shown in Fig. 4-15 in Chapter 4) up to a slenderness ratio, \( L/r_0 \), of 180. As for a higher slenderness ratio, the slenderness ratio used in the Eq. (3-7) may be replaced with a critical pile slenderness ratio, which is defined as \( 3\sqrt{\lambda} \) (Fleming et al. 1992), but this definition is recursive. For the current example, the interest is to find the normalised variations of shear strength and modulus with time. Therefore, the accuracy of the load transfer factor becomes relatively unimportant. For convenience, the load transfer factor is simply estimated with \( A = 2.5 \) (for infinite layer case), and the pile slenderness ratios. The input parameters have been detailed in Table 6-2. Using GASPILE analysis, the relevant average values are back-figured and shown in Tables 6-3 to 6-5. From the measured load-settlement curves, pile-head displacements have reached 4% and 6.3% of the diameter for pile 4 and 3.
respectively, when the piles reaches their ultimate capacities. Therefore, according to Eq. (5-26) in Chapter 5, the value of $G_{yi}/\tau_{fl}$ for pile 3 (Table 6-4) should be lower than that for pile 4 (Table 6-3).

In light of the measured data, the back-analysis of the overall pile response by GASPILE program has been illustrated in Fig. 6-5 (pile 2), Fig. 6-6 (pile 4) and Fig. 6-7 (pile 3) individually. A list of the abbreviations used in the figures has been detailed in Table 6-6. For pile 4 at 1.7 and 10.5 days, the following analyses have been undertaken: non-linear elastic (NLE), non-linear visco-elastic (NLVE), linear elastic (LE), and linear visco-elastic (LVE). However the difference amongst these analyses are so small, as shown in Fig. 6-5, that only non-linear visco-elastic and linear elastic analyses are shown in the other cases.

The shaft resistance was analysed in terms of total and effective stress using the following expressions

$$\tau_{fl} = \alpha s_a$$ \hspace{1cm} (6-42)

$$\tau_{fl} = \beta \sigma'_{vo}$$ \hspace{1cm} (6-43)

where $\sigma'_{vo}$ is the effective overburden pressure; $\tau_{fl}$ is the limiting shaft stress, $\alpha$ is the average pile soil adhesion factor in terms of total stress; $\beta$ is the average pile soil adhesion factor in terms of effective stress. The corresponding parameters ($\alpha$, $\beta$) have been estimated by Trenter and Burt (1981) as tabulated in Table 6-7. Using each of the estimated data at 1.7 days to normalise the rest, a consistency between the increase in the non-dimensional strength and shear modulus with consolidation of soil is demonstrated as shown in Table 6-8. More generally, strength increases logarithmically with time (Bergdahl and Hult, 1981; Sen and Zhen, 1984), but obviously the increase should be limited.

Analysis shows that to fit a measured load-settlement curve by GASPILE analysis through selecting (1) different Young's modulus of a pile, (2) different ratio of creep moduli, $G_{yi}/G_{y2}$, and even (3) the load transfer factor, $\zeta$, only the initial shear modulus needs to be changed. Young's modulus of the pile, creep moduli and the load transfer factor have been taken constants for each of pile at different stages of consolidation, in the back-analysis of initial shear modulus from the load-settlement curves; hence, the obtained relationships of non-dimensional values versus time, as shown in Table 6-8, are not affected by the selected values.
This example demonstrates that (1) the pile-soil interaction stiffness increases simultaneously as soil strength regains; (2) secondary compression of clay only accounts for a small fraction of the settlement of the pile.

6.8 CASE STUDY

Theoretical prediction of the pore pressure dissipation is provided using the radial consolidation theory, and compared with the following non-dimensional parameters:

1. the difference of the measured (if available) pore pressure, \( u_0 - u \) normalised by the initial value, \( u_0 \);  
2. the back-figured shear modulus normalised by the value at \( t_{90} \);  
3. the back-figured limiting shear strength normalised by the value at \( t_{90} \).  
4. the measured time-dependent pile bearing capacity normalised by the value at \( t_{90} \).

The shear modulus and limiting strength with time have been back-analysed in a similar manner as described in section 6.7.2.1, using the measured load-settlement curves at different times following pile driving. Also the values at \( t_{90} \) were obtained through interpolation. The theoretical predictions are based on assumption of plane strain deformation for each case study and are expressed in the form of \( (u_0 - u)/u_0 \).

6.8.1 Tests reported by Seed and Reese (1955)

To assess the change in pile bearing capacity with reconsolidation of soil following pile installation, Seed and Reese (1955) performed instrumented pile loading tests at intervals after driving. The pile, of radius 0.0762 m, was installed through a sleeve, penetrating the silty clay from a depth of 2.75 to 7 m. The Young's modulus is \( 2.07 \times 10^5 \) MPa and the cross sectional area is 9.032 cm\(^2\). Therefore, the equivalent pile modulus can be inferred as 10,250 MPa.

Through fitting the measured load-settlement response by the GASPILE program analysis (Fig. 6-8), values of \( G_{yi}, \tau_{fl} \) were back-figured from each load-settlement curves. These values are tabulated in Table 6-9. In terms of Poisson's ratio, \( \nu_s = 0.49 \), permeability \( k = 2 \times 10^{-6} \) m/s, \( c_v = 0.0529 \) m\(^2\)/day, the 90% degree of reconsolidation is estimated to occur at \( t_{90} = 8.76 \) days \( (T_{90} = 74.43, G_{yi}/\tau_{fl} = 350) \). From Table 6-9, at the time of \( t_{90} \), the shear modulus is about 3.55 MPa, which is less than 90% of the maximum value of 4.5 MPa at 33 days. The shaft friction is estimated as about 11.6 kPa.
compared with the final pile-soil friction of 12.6 kPa, which is a fraction of the initial soil strength of 18 kPa, due to soil sensitivity.

The back-figured shear modulus and the limiting strength have been normalised by the values at $t_{90}$ and plotted in Fig. 6-9(a) together with the normalised measurements and predicted dissipation of pore water pressure.

Assuming $G_{yi}/G_{y2} = 1$, the visco-elastic analysis leads to $t_{90} = 16.35$ days ($T_{90} = 148.85$, $G_{yi}/\tau_{fl} = 350$). Also from Table 6-9, at this $t_{90}$, the corresponding shear modulus is about 4.06 MPa, which is about 90% of the final value, 4.5 MPa at 33 days. The shaft friction is about 12.54 kPa. The normalised data by these visco-elastic estimations are shown in Fig. 6-9(b) together with the measurement and predictions.

Elastic analysis can give reasonable predictions of shear strength or pile capacity variation, but not the overall pile behaviour, particularly the deformation, as further explored in the next case study.

### 6.8.2 Tests reported by Konrad and Roy (1987)

Konrad and Roy (1987) reported the results of an instrumented pile, loaded to failure at intervals after driving. The pile, of outside radius 0.219 m, and wall thickness 8.0 mm, was jacked closed-ended to a depth of 7.6 m below ground level. The Young’s modulus is $2.07 \times 10^5$ MPa and the cross sectional area is about 53.03 cm$^2$. Therefore, the equivalent pile modulus is inferred as 29,663 MPa.

The increase in shaft capacity was reported by Konrad and Roy (1987) and has been normalised by that at 2 years after installation. The normalised values are shown to be generally consistent with the normalised dissipation of pore pressure measured at three depths of 3.05, 4.6, and 6.1 m as illustrated in Fig. 6-10(a) and (b).

In terms of conventional elastic analysis, from the initial load-settlement response measured at different time intervals after pile installation, an initial value of $G_{yi}/s_u = 270$ has been back-analysed (Chapter 5). From the final load-settlement relationships at different time intervals, the values of $G_{yi}/s_u$ are found to be almost a constant of 210-230, with a value of $G_{yi}/G_{y2} = 2$. The shear modulus and the shear strength (being assumed to increase linearly with depth) have been back-figured through fitting the
measured response with the visco-elastic GASPILE analysis, and are tabulated in Table 6-10.

The visco-elastic analysis gives a satisfactory agreement with the measured response at low load levels of about 70% ultimate load (Fig. 6-11). The visco-elastic analysis gives a better prediction than the elastic analysis (Fig. 6-12) in comparison with the measured response, particularly at higher load levels (being greater than 70%). Due to the marked non-linear soil response of the base (Chapter 5), the visco-elastic analysis is still significant different from the measured response at high load levels. The prediction may be improved, if the non-linear base response and the variation of $\zeta_1$ with load level is accounted for. However, the current analysis is sufficiently accurate for assessing the interested values: the variation of shear strength and modulus with reconsolidation.

The value of the coefficient of consolidation has been estimated as $c_v = 0.0423 \text{ m}^2/\text{day}$ ($\nu_s = 0.45$, Konrad and Roy, 1987). Using elastic analysis, the time factor for 90% degree of consolidation, $T_{90}$, is about 65, with $G_{y1}/S_u = 230$ from Fig. 6-4; hence, $t_{90} \approx 18$ days. From Table 6-10, at the time of $t_{90}$, the shear strength at the pile base level is estimated to be 22.4 kPa, which is in good agreement with the value of 23.0 kPa obtained as 90% of limiting stress, $\tau_{fl}$ ($\tau_{fl} = 25.64$ kPa), while the corresponding modulus is 4.79 MPa, which is slightly lower than 5.08 MPa obtained as 90% of the maximum shear modulus (5.64 MPa). Using visco-elastic analysis, with $G_{y1}/G_{y2} = 2$, $T_{90} = 205.1$, $t_{90}$ is estimated to be about 57 days, the corresponding values are $\tau_{fl} = 23.99$ kPa and $G_{y1} = 5.16$ MPa by interpolation from the data given in Table 6-10.

With the above estimated values at $t_{90}$, the normalised variations are plotted in Fig. 6-11(a) and (b) respectively for elastic and visco-elastic analyses, together with the theoretical curves of dissipation of pore water pressure. The predicted values at the initial stage are lower than the measured data, probably due to the radial soil non-homogeneity (Appendix E). Radial non-homogeneity can also retard the regain in the average shear modulus (at some distance away from the pile axis). Therefore, a comparatively higher value of $t_{90}$ for the modulus regain is expected than the current prediction. Using modulus at the $t_{90}$ (interpolated by Table 6-10) to normalise the rest values can only lead to a lower trend than the current prediction as shown in the figure.

Soil strength may increase due to reconsolidation, but it may also decrease due to creep (Creep causes soil strength reduce, until finally the strength approaches to a long term strength, which is about 70% of the soil strength). The effect of reconsolidation and
creep on the soil strength may offset in this particular case. However, since the creep leads to an increase in settlement, only with a visco-elastic analysis, (e.g. with a value of $G_{r1}/G_{r2} = 2$ for the current example), can an excellent prediction be made as shown in Fig. 6-11 for final settlement and Fig. 5-15 (Chapter 5) for a time-dependent process.

### 6.8.3 Comments on the Current Predictions

The back-figured values of $G_{r1}/G_{r2}$ for the two case studies are higher than those reported by Lo (1961). The reason for this may be that the former are based on field tests, while the latter are based on confined compression (oedometer) tests. The current radial consolidation theory is based on a homogeneous medium. However, as just argued, radial non-homogeneity can alter the shape of the time-dependent curve at initial stage and increase the time for regain of shear modulus.

### 6.9 CONCLUSIONS

The research outlined here has attempted to offer a prediction of the overall response of a pile following driving, rather than just the pile capacity. A number of important conclusions can be drawn:

1. Visco-elastic solutions can be obtained by (a) solving diffusion theory and then using an accurate coefficient of consolidation; or (b) the available elastic solutions using the correspondence principle.

2. The viscosity of a soil can significantly increase the consolidation time, hence increase the pile-head settlement. However, it has negligible effect on soil strength or pile capacity.

3. Almost all the case studies show that the variation of the normalised pile-soil interaction stiffness (or soil shear modulus) due to reconsolidation is consistent with that of the pore pressure dissipation on the pile-soil interface and that of increase in soil strength. Therefore, the time-dependent properties following pile installation can be sufficiently accurately predicted by the radial consolidation theory. With the predicted time-dependent parameters, it is also straightforward to obtain load-settlement response at any times following driving by either GASPILE analysis or the previous closed form solutions (Chapters 3, 4 and 5).
### Table 6-1 Summary of the Relaxation Factor for Creep Analysis

<table>
<thead>
<tr>
<th>Authors</th>
<th>$G_y/\eta_2$ ($\times10^{-5}$ s$^{-1}$)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lo (1961)</td>
<td>0.2 to 0.4</td>
<td>Oedometer test</td>
</tr>
<tr>
<td>Edil &amp; Mochtar (1988)</td>
<td>0.5 to 2.67</td>
<td>Creep test on model piles</td>
</tr>
<tr>
<td>Qian et al. (1992)</td>
<td>1.71 to 3.29</td>
<td>Vacuum preloading</td>
</tr>
<tr>
<td>Ramalho Ortigao &amp; Randolph (1983)</td>
<td>0.36 to 0.664</td>
<td>Field pile test</td>
</tr>
</tbody>
</table>

### Table 6-2 Parameters for the Analysis of the Tests by Trenter and Burt (1981)

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>Diameter (mm)</th>
<th>Wall thickness (mm)</th>
<th>Penetration (m)</th>
<th>$\zeta_1$ ($\psi = 0$)</th>
<th>$\zeta_1$ ($\psi = 0.5$)</th>
<th>$\omega/\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>400</td>
<td>12</td>
<td>24/30.3</td>
<td>4.5/4.73</td>
<td>5.2 /5.4</td>
<td>1.0/2.</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>12</td>
<td>53.5/54.5</td>
<td>5.31</td>
<td>6.0</td>
<td>1.0/2.</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>12</td>
<td>43.3</td>
<td>4.4</td>
<td>5.08</td>
<td>1.0/2.</td>
</tr>
</tbody>
</table>

### Table 6-3 Parameters for Analysis of Pile 4 Tested by Trenter and Burt (1981)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>$G_{\gamma 1}$ (MPa)</th>
<th>$\tau_{fl}$ (kPa)</th>
<th>$G_{\gamma 1}/\tau_{fl}$</th>
<th>$G_{\gamma 2}/\eta_2t$</th>
<th>$G_{\gamma 1}/G_{\gamma 2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>6.11</td>
<td>20.18</td>
<td>303</td>
<td>12.96</td>
<td>0.15</td>
</tr>
<tr>
<td>10.5</td>
<td>7.64</td>
<td>25.28</td>
<td>302</td>
<td>12.96</td>
<td>0.15</td>
</tr>
<tr>
<td>20.5</td>
<td>8.43</td>
<td>27.12</td>
<td>311</td>
<td>12.96</td>
<td>0.15</td>
</tr>
<tr>
<td>32.5</td>
<td>8.43</td>
<td>27.5</td>
<td>306</td>
<td>12.96</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### Table 6-4 Parameters for Analysis of Pile 3 Tested by Trenter and Burt (1981)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>$G_{\gamma 1}$ (MPa)</th>
<th>$\tau_{fl}$ (kPa)</th>
<th>$G_{\gamma 1}/\tau_{fl}$</th>
<th>$G_{\gamma 1}/G_{\gamma 2}$</th>
<th>$G_{\gamma 2}/\eta_2t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>3.62</td>
<td>20.717</td>
<td>175</td>
<td>0.15</td>
<td>12.96</td>
</tr>
<tr>
<td>3.0</td>
<td>3.69</td>
<td>21.13</td>
<td>175</td>
<td>0.15</td>
<td>12.96</td>
</tr>
<tr>
<td>4.2</td>
<td>3.9</td>
<td>22.308</td>
<td>175</td>
<td>0.15</td>
<td>12.96</td>
</tr>
</tbody>
</table>
### TABLE 6-5  Parameters for Analysis of Pile 2 Tested by Trenter and Burt (1981)

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>$G_Y1$ (MPa)</th>
<th>$\tau_f$ (kPa)</th>
<th>$G_Y1/\tau_f$</th>
<th>$G_Y1/G_Y2$</th>
<th>$G_Y2/\eta_Y2$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.0</td>
<td>8.46</td>
<td>20.225</td>
<td>418</td>
<td>0.15</td>
<td>$0.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>30.3</td>
<td>7.75</td>
<td>23.865</td>
<td>308</td>
<td>0.15</td>
<td>$0.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

### TABLE 6-6  Explanation of the Abbreviations Used in the Figs.(6-5) to (6-7)

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLVE</td>
<td>Non-linear visco-elastic analysis, by choosing $\psi = 0.5$ in Eq. (3-8), given values of compressibility factor, $G_Y1/G_Y2$ and rate factor of $G_Y2/\eta_Y2$. The prefix is referred to the time for creep (e.g. $2.5\text{hr}$ means a 2.5 hours has been adopted in the estimation).</td>
</tr>
<tr>
<td>LVE</td>
<td>Linear visco-elastic analysis. Every parameter is exactly the same as used in NLVE except choosing $\psi = 0$ in Eq. (3-8).</td>
</tr>
<tr>
<td>NLE</td>
<td>Non-linear elastic analysis. Every parameter is exactly the same as used in NLVE except choosing $G_Y1/G_Y2 = 0$.</td>
</tr>
<tr>
<td>LE</td>
<td>Linear elastic analysis. Every parameter is exactly the same as used in NLVE except choosing $G_Y1/G_Y2 = 0$ and $\psi = 0$ in Eq. (3-8).</td>
</tr>
<tr>
<td>Pb</td>
<td>Calculated base load-settlement relationship</td>
</tr>
<tr>
<td>Mea</td>
<td>Measured pile-head load-settlement relationship</td>
</tr>
</tbody>
</table>

### TABLE 6-7  Parameters for Empirical Formulas (from Trenter and Burt 1981)

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>4</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (days)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.7</td>
<td>10.5</td>
<td>20.5</td>
<td>32.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.63</td>
<td>0.81</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>0.51</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.16</td>
<td>0.20</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>0.13</td>
<td>0.13</td>
<td>0.14</td>
</tr>
</tbody>
</table>
### TABLE 6-8  Comparison of the Parameters for Bearing Capacity Predictions

<table>
<thead>
<tr>
<th>Pile No.</th>
<th>4</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (days)</td>
<td>1.7</td>
<td>10.5</td>
<td>20.5</td>
</tr>
<tr>
<td>$\alpha/\alpha_0^*$</td>
<td>1.0</td>
<td>1.286</td>
<td>1.381</td>
</tr>
<tr>
<td>$\beta/\beta_0^*$</td>
<td>1.0</td>
<td>1.25</td>
<td>1.375</td>
</tr>
<tr>
<td>$G_i/G_{io}^*$</td>
<td>1.0</td>
<td>1.25</td>
<td>1.38</td>
</tr>
</tbody>
</table>

Note: *$\alpha_0$, $\beta_0$, $G_{io}$ are the values of $\alpha$, $\beta$, $G_i$ at 1.7 days*

### Table 6-9  Back-figured Parameters from the Measured by Seed and Reese (1955)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>.125</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>14</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{fl}$ (kPa)</td>
<td>2.26</td>
<td>5.71</td>
<td>8.4</td>
<td>11.3</td>
<td>12.52</td>
<td>12.68</td>
</tr>
<tr>
<td>$G_{y1}$ (MPa)</td>
<td>.6</td>
<td>1.6</td>
<td>2.1</td>
<td>3.4</td>
<td>4</td>
<td>4.5</td>
</tr>
</tbody>
</table>

### Table 6-10  Back-figured Parameters from the Measured by Konrad and Roy (1987)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>4</th>
<th>8</th>
<th>20</th>
<th>33</th>
<th>730</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{fl}$ (kPa)</td>
<td>5.58*/12.93**</td>
<td>6.56/19.49</td>
<td>7.75/23.0</td>
<td>8.06/23.93</td>
<td>8.63/25.61</td>
</tr>
<tr>
<td>$G_{y1}$ (MPa)</td>
<td>1.07/3.19</td>
<td>1.44/4.29</td>
<td>1.65/4.9</td>
<td>1.73/5.15</td>
<td>1.9/5.64</td>
</tr>
</tbody>
</table>

Note: *numerators for ground level, **denominators for the pile base level.*
7. SETTLEMENT OF PILE GROUPS IN NON-HOMOGENEOUS SOIL

7.1 INTRODUCTION

Various numerical approaches have been proposed for analysing the settlement of pile groups. Generally, the approaches are based on either (a) a direct and complete analysis of the whole pile group, or (b) the superposition principle through using interaction factors.

Direct analysis is generally achieved through boundary element approach, for example, Butterfield and Banerjee, (1971), and Butterfield and Douglas, (1981). The analysis is relatively accurate and rigorous, but requires long computation time and large computer storage space. Therefore, so far, only relative small groups, e.g., 8 × 8, have been analysed. The approach is therefore limited for practical analysis of large piled groups.

Using interaction factors (e.g. Poulos and Davies, 1980), analysis based on the superposition principle is generally more efficient and straightforward. However, at present, a numerical technique such as the boundary element approach (BEM) is usually adopted for direct analysis of two equally loaded piles, so as to obtain the interaction factors. Randolph and Wroth (1979c) suggested a simple way to estimate the interaction factors. However, in their approach, the shaft and base components were considered separately; thus, an iterative procedure is needed for compressible pile groups. Based on simple solutions by Randolph and Wroth (1978), Lee (1993a) gave an approximate equation for direct evaluation of the interaction factors for both rigid and compressible pile groups.

Mandolini and Viggiani (1996) proposed a numerical approach for estimating the settlement of large piled groups. They also used BEM analysis to obtain the value of the pile-pile interaction factors, from which the superposition principle is then utilised to estimate the settlement of each pile in a group, assuming either a rigid or fully flexible pile cap.

For a pile group in a finite layer, the pile-pile interaction should reduce significantly due to the reduction of the shaft load transfer factors. However, none of the closed solutions available can account for the reduction.
This chapter presents

1. an extension of the exact closed form solutions for the response of single piles (Chapter 3) to piles within a group, with the soil stiffness increasing with some power of depth (Booker et al. 1985);

2. closed form expression for interaction factors for two identical piles;

3. a numerical program, GASGROUP, for analysing large piled groups, using the superposition principle, with interaction factors being given by the closed form expression;

4. a number of case studies.

The expression of interaction factors based on load transfer approach is verified extensively by the results from more rigorous numerical analyses provided by Poulos and Davis (1980), Cheung et al. (1988), Chin et al. (1990) and Lee (1993a). Pile group stiffness obtained by the GASGROUP program is compared with that from the more rigorous numerical approach by Butterfield and Banerjee (1971), Banerjee and Davies (1977), and Poulos (1989) for groups in an infinite layer; and by Butterfield and Douglas (1981) for pile groups embedded in different finite layers.

### 7.2 ANALYSIS OF A SINGLE PILE IN A GROUP

Closed form solutions for a pile in a non-homogeneous soil have been generated in Chapter 3 for the case where the elastic shear modulus of the soil varies with depth according to

\[ G = A_g z^n \]  \hspace{1cm} (7-1)

where n is the power of the depth variation (referred to as the shaft non-homogeneity factor) and \( A_g \) determines the magnitude of the shear modulus. The shear modulus below the base of the pile is assumed constant at \( G_b = G_l/\xi_b \) (with a value of \( \xi_b = 1 \) in this chapter).

In order to allow for the presence of neighbouring piles, following Randolph and Wroth (1979c), the resulting load transfer factors for a pair of piles are
Chapter 7 7.3 Settlement of Pile Groups

\[ \zeta_2 = \ln \left( \frac{r_m + nr_g}{r_0} \right) + \ln \left( \frac{r_m + r_g}{s} \right) \quad (7-2) \]

and

\[ \omega_2 = \omega \left( 1 + \frac{2 r_o}{s \pi} \right) \quad (7-3) \]

where \( r_g \) is the semi-width of the pile group (0.5s in the case of two piles) and \( s \) is the pile spacing; \( r_m \) is estimated by Eq. (3-8). The load transfer factors for estimating \( r_m \) and \( \omega \) (base factor) are evaluated with the expressions proposed in Chapter 4, while for a pile in an infinite layer, a simple value of \( A = 2.5 \) and \( \omega = 1 \) is used.

The solutions for a single pile can be readily extended to a pile in a group, through replacement of the load transfer factors, \( \zeta \), \( \omega \) for a single pile with the factors, \( \zeta_2 \), \( \omega_2 \) for a pile in a group. Therefore, the ratio of load, \( P \), and settlement, \( w \), at any depth, \( z \), may be expressed as (refer to Chapter 3)

\[ \left( \frac{P(z)}{G_L w(z) r_0} \right)_2 = \sqrt{2 \pi} \frac{\lambda}{\zeta_2} C_{v2}(z) \quad (7-4) \]

where the subscript ‘2’ refers to a pile in a group. The function, \( C_{v2}(z) \), is given by

\[ C_{v2}(z) = \frac{C_1(z) + \chi_{v2} C_2(z)}{C_3(z) + \chi_{v2} C_4(z)} \left( \frac{z}{L} \right)^{n/2} \quad (7-5) \]

The individual functions, \( C_j \), are given in terms of modified Bessel functions of fractional order:

\[ C_1(z) = -K_{m-1}I_{m-1}(y_2) + K_{m-1}(y_2)I_{m-1} \]
\[ C_2(z) = K_mI_{m-1}(y_2) + K_{m-1}(y_2)I_m \]
\[ C_3(z) = K_{m-1}I_m(y_2) + K_m(y_2)I_{m-1} \]
\[ C_4(z) = -K_mI_m(y_2) + K_m(y_2)I_m \]

\[ y_2 = 2m \frac{L}{r_o} \sqrt{\frac{2}{\lambda \zeta_2} \left( \frac{z}{L} \right)^{1/2m}} \quad (7-7) \]

The ratio \( \chi_{v2} \) is given by
where $\nu_s$ is Poisson's ratio. Note that the surface value of $C_{v2}$ must be taken as a limit, as $z$ approaches zero.

### 7.3 INTERACTION FACTOR

Influence of the displacement field of a neighbouring identical pile may be represented by interaction factors as described by Poulos (1968). The factor may be expressed as

$$\alpha_{12} = \frac{(G_{10}r_0^2 w_t / P_t)_{2}}{G_{1}r_0^2 w_t / P_t} - 1$$

(7-9)

where $\alpha_{12} = \text{the conventional interaction factor, which can be expressed explicitly from Eqs. (7-4) and (7-5)},$

$$\alpha_{12} = \frac{C_v}{C_{v2}} \sqrt{\zeta_2/\zeta} - 1$$

(7-10)

where $C_{v2}$ and $C_v$ are the limiting values of the function, $C_{v2}(z)$ in Eq. (7-5) as $z$ approaches zero, with values of $\zeta_2$, $\omega_2$ and $\zeta$, $\omega$ respectively.

For a pile in an infinite layer, the interaction factors predicted by Eq. (7-10) are shown in Fig. 7-1 and 2 for homogeneous ($n = 0$) and Gibson ($n = 1$) soil respectively at a number of slenderness ratios, together with previously published results. Generally the agreement is very good, except for very slender piles with $L/r_0 > 100$ (not shown). However, such cases are of limited interest, since pile-soil slip will generally occur in the upper region of slender piles, even at working loads. The corresponding elastic region of the pile would probably still fall within the range of a 'short' pile.

### 7.4 PILE GROUP ANALYSIS

#### 7.4.1 GASGROUP Program

The settlement of any pile in a group can be predicted using the superposition principle together with appropriate interaction factors. For a symmetrical group, the settlement $w_i$ of any pile $i$ in the group can be written as,
where $w_i$ is the settlement of a single pile under unit head load; $\alpha_{ij}$ is the interaction factor between pile $i$ and pile $j$ (for $i = j$, $\alpha_{ij} = 1$) estimated by Eq. (7-10), and $n_g$ is the total number of piles in the group. The total load applied to the pile group is the sum of the individual pile loads, $P_j$.

For a perfectly flexible pile cap, each pile load will be identical and so the settlement can be readily predicted with Eq. (7-11). For a rigid pile cap, with a prescribed uniform settlement of all the piles in a group, the loads may be deduced by inverting Eq. (7-11). This procedure for solving Eq. (7-11) has been designed in a program called GASGROUP.

In the present analysis, estimation of settlement of a single pile under unit head load, and the interaction factors, are based on closed form solutions. Therefore, the calculation is relatively quick and straightforward, e.g., for a 700 piled group, the calculation only takes about 5 minutes. All the present solutions referred to later are from predictions using the GASGROUP program, assuming a rigid cap.

### 7.4.2 Verification of the GASGROUP Program

A number of non-dimensional quantities so far have been introduced to describe the response of pile groups, these are

1. pile-head stiffness, which was defined as (a) $P_t/(G_L w_t)$ (Randolph and Wroth, 1979c), (b) $P_t/(G_L d w_t)$ (Butterfield and Banerjee, 1971), and (c) more recently as $K_p/(\sqrt{n_g} G_L)$, where $K_p = P_t/w_t$ (Randolph, 1994);

2. settlement ratio, $R_s$, which was defined as the ratio of the average group settlement to the settlement of a single pile carrying the same average load;

3. the settlement influence factor, $I_G$, which was defined as (Poulos, 1989)

$$I_G = w_G d E_L / P_G$$

where $P_G$ is the load exerted on the pile group; $E_L$ is soil Young's modulus at the pile tip level; $w_G$ is the settlement of the pile group.
These non-dimensional factors are used in the following comparisons.

### 7.4.2.1 Small Pile Groups in an Infinite Layer

For pile groups embedded in a homogeneous soil profile, the present solution was compared with that obtained using the boundary integral approach (BI) by Butterfield and Banerjee (1971) and is presented in

1. Fig. 7-3 for the pile-head stiffness of three symmetrical pile groups at different pile-soil relative stiffness; and
2. Fig. 7-4 for the sharing of load among the piles in a 3×3 symmetrical pile groups.

For pile groups embedded in a Gibson soil, the present solution is compared with that obtained using the boundary element approach of Lee (1993a), as illustrated in Fig. 7-5, which gives the sharing of load within a 3×3 pile group.

Values of settlement ratio, Rs, were estimated and compared with those obtained by Butterfield and Banerjee (1971), as shown in

1. Fig. 7-6 for different spacing ratios, s/r₀ for three symmetrical pile groups.
2. Fig. 7-7 for a symmetrical four pile group, accounting for the effect of the pile-soil stiffness ratio, λ.

Available values of settlement influence factor, IG, were used to substantiate the present solution for pile groups in a Gibson soil, as shown in Fig. 7-8.

All the above comparisons show that for pile groups in an infinite layer, the closed form approach as used in the GASGROUP program is capable of predicting a very similar response of different pile groups to those obtained previously by various numerical approaches.

### 7.4.2.2 Small Pile Groups in a Finite Layer

The most comprehensive rigorous analysis of pile groups in a finite layer is probably that provided by Butterfield and Douglas (1981). Using the PGROUP program (Banerjee and Driscoll, 1978 referenced via Randolph, 1994), Butterfield and Douglas (1981) obtained flexibility factors (the inverse of stiffness factors) for pile groups in homogeneous, finite (H/L = 1.5 and 3.0) and infinite layers, with the pile cap being treated as rigid and at ground level, but with no contact between the pile cap and the soil.
The normalised stiffness, \( K_p/\left(\sqrt{n} s G_L\right) \), obtained from the present solution is first compared with those obtained by Butterfield and Douglas (1981) for 2-pile groups at different centre-centre spacing embedded in an infinite layer, as shown in Fig. 7-9. The normalised stiffness obtained by the present solution is then compared with those obtained by Butterfield and Douglas (1981) for groups in homogenous soil layers of various values of \( H/L (= 1.5, 3.0 \text{ and infinite layer}) \) at pile centre-centre spacing of \( s/d = 2.5 \text{ and } 5 \). The later comparison for symmetrical pile groups is presented individually in

- a) Fig. 7-10 for 2x2 pile groups;
- b) Fig. 7-11 for 3x3 pile groups;
- c) Fig. 7-12 for 4x4 pile groups;
- d) Fig. 7-13 for 8x8 pile groups;
- e) Fig. 7-14 for 4x2 pile groups;
- f) Fig. 7-15 for 8x2 pile groups.

The comparison shows that

1. Generally the present solution is consistent with that of PGROUP analysis;
2. For large group of piles, e.g. 8x8, and at lower values of \( H/L \), e.g. 1.5, differences between the present solution and the PGROUP analysis become obvious. The PGROUP analysis for these cases was found to be unreliable (Butterfield and Douglas, 1981). The results from PGROUP analysis are independent of pile slenderness ratio, which do not seem to be realistic, while comparatively, the current prediction gives a reasonable trend.

7.4.2.3 Large Pile Groups in an Infinite Layer

For large pile groups, previous solutions are available only for groups embedded in an infinite layer. Fig. 7-16 shows a comparison of the solutions obtained by the following computer codes:

1. The analysis by Fleming et al (1992), based on the PIGLET program (Randolph, 1987);
2. The interaction factor approach derived from analysis using DEFPIG program (Poulos and Davis, 1980);
3. The most rigorous numerical results by Butterfield and Douglas (1981), based on the full BEM analysis incorporated in the PGROUP program.

The average of the first two approaches appears to offer reasonably accurate solutions (Randolph, 1994). The present solution is quite consistent with this average trend for close pile spacing \( (s/d = 2.5) \) as illustrated in Fig. 7-16(a), and approaches a limiting normalised stiffness of 4.5 corresponding to that of a shallow foundation. However, for a large pile centre-centre space \( (s/d = 5) \), the normalised stiffness by the present
GASGROUP analysis as shown in Fig. 7-16 (b) tends to decrease and becomes lower than that for a shallow foundation. Probably as noted by Cooke (1986), at a large pile centre-centre space (e.g. greater than 4d), the pile group performs in a different way from that of a densely spaced pile group.

7.5 APPLICATIONS

Settlements of a number of actual pile groups have been analysed using the present GASGROUP program. These cases are

1. full scale pile tests by Cooke (1974);
2. a tank supported by 55 piles, embedded in silt and very silty clay (Thorburn et al. 1983);
3. a 19-storey building supported by a group of 132 piles, embedded in sandy layer (Koerner and Partos, 1974);
4. a block of 40 cylindrical silos supported by a large group of 697 piles, embedded in a layer of interbedded sands and stiff clays (Goosens and Van Impe, 1991);
5. a 5-storey building supported by a group of 20 piles, embedded in a layer of stiff clays underlain by a medium to dense sand (Yamashita et al. 1993).

Input parameters for each analysis include (i) soil shear modulus distribution down the pile, Poisson’s ratio, and the ratio of H/L; (ii) the dimensions and Young’s modulus of the pile; (iii) the number of piles in the foundation and pile centre-centre space. There is no practical difficulty in estimating the exact centre-centre spacing for each pair of piles. However, for convenience, equivalent average pile spacing has been assessed and used for large pile groups. In the prediction of Rs, the irregular plans of large groups were converted to equivalent rectangular plans.

7.5.1 Full Scale Tests (Cooke, 1974)

Cooke (1974) reported the results of full scale tests on vertically loaded single piles, and a row of three piles spaced at s = 6r0, embedded in London clay at Hendon. The tubular steel piles, of external radius 84 mm and wall thickness 6.4 mm, were jacked to a depth of 4.5 m. The equivalent Young’s modulus of the piles is \( E_p = 30.8 \) GPa.

The load distribution in the piles as well as the vertical displacements at different levels below the ground surface were measured. From the test results of the central pile of the row of three piles, which was loaded before the installation of the two flanking piles,
the pile-head stiffness $P_t/w_t$ is 127,800 kN/m, and also from Cooke (1979), the shear modulus may be simulated by Eq. (7-1) with $n = 0.85$, $A_g = 12.48$ MPa m$^{-0.85}$, which leads to a pile-soil relative stiffness factor, $\lambda = 687.1$. Other relevant parameters have been estimated as presented in Fig. 7-17.

With these parameters, the predicted pile-pile interaction factors agree well with the measured values reported by Cooke (1979, 1980) (Fig. 7-17). This may be attributed to the more accurate selection of the A value of 1.66 for $H/L = 2$, as backfigured by FLAC analysis shown in Chapter 4. This 'A' value gives an excellent estimation of the maximum radius of influence of the pile-shaft shear, and the corresponding theoretical predictions of displacement are consistent with those measured, as shown in Fig. 7-18 for the single pile, and Figs. 7-19 (a) and (b) for pile groups of equal pile load and rigid pile cap respectively. The prediction of pile-head load-displacement relations are illustrated in Figs. 7-20 (a) and (b) for equal pile load and rigid pile cap respectively.

### 7.5.2 Molasses Tank (Thorburn et al, 1983)

The Molasses tank described by Thorburn et al (1983) was 12.5 m in diameter, and was supported by 55 precast concrete piles, each 0.25 m square, and 27 m long (effective length), laid out on a triangular grid at a spacing of 2.0 m. The strength profile of the subsoil may be written as

$$s_{u} \text{ (kPa)} = 6 + 1.8z \text{ (m)} \quad (7-13)$$

and the shear modulus was estimated as

$$G \text{ (MPa)} = 1.5 + 0.45z \text{ (m)} \quad (7-14)$$

From the single pile test, the measured initial elastic stiffness of $P_t/w_t$ was 88 MN/m. Young’s modulus of the pile was measured as 26 GPa. Therefore, with an assumed value of $n = 1$, the backfigured shear modulus was $G \text{ (MPa)} = 0.54z \text{ (m)}$.

By the GASGROUP analysis, taking the group as a rectangular array of $7 \times 8$, the estimated settlement ratio, $R_s$, was 5.43. Alternatively, taking the group as rectangular, $5 \times 11$, the estimated settlement ratio, $R_s$, was 6.08. At the average load per pile of 440 kN, the predicted elastic displacement of the single pile was 5 mm. Therefore, the predicted settlement of the pile group was in the range of 27.2 to 30.4 mm. This compares well with the measured settlements in the range of 29 - 30 mm.
7.5.3 19-storey R. C. Building (Koerner and Partos, 1974)

The 19-storey building described by Koerner and Partos, (1974) was founded on 132 permanently cased driven piles, covering an approximately rectangular area, about 24 m by 34 m. The piles were cased 0.41 m diameter and 7.6 m long, with an expanded base of 0.76 m. Dividing the total area by 132 piles results in a mean area of 6.18 m² per pile, and a pile ‘spacing’ of 2.48 m.

The SPT varies approximately with depth by a power of n = 0.5. From the single pile loading test results, a secant stiffness of \( P_t/w_t = 350 \text{ kN/mm} \) was obtained. With the ratio of H/L = 2.2, the shear modulus variation with depth may be approximated by

\[
G (\text{MPa}) = 16.43 z^{0.5}
\]  

(7-15)

Young’s modulus of the pile was measured as 30 GPa. With these parameters, the GASGROUP analysis gave a value of the settlement ratio, \( R_s \), of 19.85. The single pile settlement was computed to be 3.3 mm for the average load of 1.05 MN. Thus the average group settlement was computed to be 65.5 mm. The measured values ranged between a maximum of 80 mm near the centre, to 37 mm near the corners of the building, with an average of about 64 mm. The predicted settlement is therefore close to the average measured value.

7.5.4 Ghent Grain Terminal (Goosens and Van Impe, 1991)

A block of 40 cylindrical reinforced concrete grain silo cells was erected in Ghent, covering a rectangular area 34 m by 84 m, within a new terminal for storage and transit (Goosens and Van Impe, 1991). Each of the cells is 52 m high and 8 m in diameter. The silos were built on a 1.2 m thick slab, which in turn rested on a total of 697 driven cast in situ reinforced concrete piles. The piles are of 13.4 m in length, 0.52 m in shaft diameter, and incorporating an expanded base, which was estimated to be 0.8 m in diameter. The average working load for each pile was about 1.3 MN.

The average shear modulus near the centre of the site may be regarded as uniform with depth, with a value of 28.6 MPa (Poulos, 1993). Young’s modulus of the pile was assumed as 30 GPa. The average area per pile was estimated to be 4.1 m², giving a ‘pile spacing’ of 2.02 m. Using the GASGROUP analysis, the settlement ratio, \( R_s \), was estimated to be 59.15. At the average working load of 1.3 MN, the single pile displacement was estimated to be 3.15 mm. Therefore, the predicted settlement of the
pile group was 186.3 mm. At completion of the building, the measured settlement was 185.0 mm. The predicted settlement is quite consistent with the measured value.

7.5.6 5-Storey Building (Yamashita et al. 1993)

A piled raft foundation has been adopted in Japan for a five-storey building with plan area measuring 24 m by 23 m. A total of 20 piles were utilised to reduce the potential settlement (Yamashita et al. 1993). The piles were 16 m in length and 0.7 and 0.8 m in diameter, with pile centre to centre spacing of 6.3 to 8.6 times the pile diameter. The total working load was 47.5 MN.

The shear modulus profile adopted by Yamashita et al (1993) may reasonably be approximated by

\[
G \text{ (MPa)} = 9.5z^{0.8} \quad (7-16)
\]

Young’s modulus of the pile was assumed as 9.8 GPa. Using the GASGROUP analysis, the settlement ratio, \( R_s \), was estimated to be 2.7. At the average working load of 2.4 MN, the single pile showed about 5.0 mm displacement. Therefore, the predicted settlement of the pile group was 13.5 mm. At completion of the building, the measured settlements were in the range of 10 to 20 mm, with an average of about 14 mm. The predicted settlement is quite consistent with the measured value.

7.5.7 General Comments From the Case Study

Generally speaking, using an assumed pile Young’s modulus, the corresponding initial soil modulus may be backfigured, in terms of single pile test results. The parameters from the single pile analysis may then be used directly to predict the settlement of the pile group. In the case of using enlarged pile base (section 7.5.3), a secant stiffness from single pile test results may be used. Where an inclined underlain rigid layer exists, since the ratio of \( H/L \) varies across the pile group, different values of \( H/L \) may be used to assess the possible displacement range of the foundation.

7.6 CONCLUSIONS

This chapter was aimed at establishing a simple efficient approach for predicting settlement of large pile groups. A closed from expression for estimating pile-pile interaction factors was established, which was then used to predict behaviour of large
pile groups embedded in non-homogeneous, finite layer media. The current solutions have been compared extensively with the previous numerical analyses. A number of actual pile groups have been analysed. The main conclusions from this research are:

(1) The new closed form expression for interaction factors, using the modified load transfer factors, gives very good agreement with those obtained by more rigorous numerical analyses.

(2) The current approach for estimating pile group stiffness yields very good agreement with those obtained by rigorous numerical analysis for a range of different layer thickness ratio, H/L.

(3) The current program, GASGROUP, gives reasonable prediction in comparison with both rigorous numerical analyses and measured data. The program is very quick, efficient and can be readily run on a personal computer. Therefore, it may be used for practical engineering design.

(4) Some guidelines for estimating settlement of pile groups have been provided, using GASGROUP program for a variety of different subsoil profiles.
8. TORSIONAL PILES IN NON-HOMOGENEOUS MEDIA

8.1 INTRODUCTION

Numerical and analytical solutions have been published for piles subjected to torsion, where the piles are embedded in elastic soil with either homogeneous modulus, or modulus proportional to depth (Poulos, 1975; Randolph, 1981). A more general, and often appropriate, class of soil is one where the depth variation of modulus may be described by a simple power law (see later, Eq. (8-1)), as investigated for shallow foundations by Booker et al. (1985). The nature of the power law, which encompasses the homogeneous and proportionally varying cases as well, can have a significant effect on the calculated pile head stiffness, particularly as the torsional load transfer is generally concentrated in the upper part of the pile (Poulos, 1975).

This chapter describes new analytical solutions for the torsional response of piles in non-homogenous soil deposits where the stiffness profile is modelled as a power law of depth. The solutions are expressed in terms of Bessel functions of non-integer order, and have been evaluated using Mathcad and also using a spreadsheet approach with the Bessel functions approximated by polynomial functions. Expressions for the critical pile length, beyond which the pile length no longer affects the pile head stiffness, are presented. The solutions have also been extended into the non-linear range, using a hyperbolic stress-strain response for the soil. At one extreme of the hyperbolic model, the stress-strain response becomes elastic, perfectly plastic, and for that case analytical solutions are presented giving the pile head response right up to complete torsional failure of the pile. Simple non-dimensional charts have been presented to facilitate hand calculation of the pile response.

In all the above solutions, a load transfer approach has been used, where each horizontal layer of soil is considered as independent of neighbouring layers. The resulting solutions have been checked against the more rigorous, continuum, solutions of Poulos (1975) for the extreme cases of uniform modulus and modulus proportional to depth.

8.2 TORQUE-ROTATION TRANSFER BEHAVIOUR

Torque transfer models have been presented by Randolph (1981) for elastic conditions, and incorporated into closed form solutions for the pile head response, assuming either uniform soil modulus, or modulus varying proportionally with depth. Here, those
solutions are extended for more general non-homogeneity of the soil, and also for non-linear soil response using a hyperbolic stress-strain law.

8.2.1 Non-homogeneous Soil Profile

The soil modulus profile is taken as a power law variation of depth, given by

\[ G_i = A_g z^n \]  

(8-1)

where \( G_i \) is the initial (tangent) shear modulus at depth \( z \); \( A_g \) is a modulus constant; and \( n \) is the depth exponent, referred to here as the non-homogeneity factor. Typically, the factor will lie in the range 0 (uniform soil) to 1 (stiffness proportional to depth).

The limiting shaft friction, \( \tau_f \), can also be expressed as a power law variation with depth, as

\[ \tau_f = A_t z^t \]  

(8-2)

where \( A_t \) is a constant that determines the magnitude of shaft friction, and \( t \) is the corresponding non-homogeneity factor. In this chapter, attention will be restricted to situations where the shear modulus and shaft friction profiles are similar (with \( n = t \)). The ratio of modulus to shaft friction is then constant through the profile, and equal to \( A_g/A_t \).

8.2.2 Non-linear Stress-Strain Response

Non-linear response of the soil may be modelled using a hyperbolic stress-strain law, where the secant shear modulus, \( G \), is given by

\[ G = G_i \left(1 - R_f \frac{\tau}{\tau_f}\right) \]  

(8-3)

where \( R_f \) is the hyperbolic parameter that controls the ratio of the secant modulus at failure, to the initial tangent modulus, \( G_i \). Note that it is assumed here that the limiting shear stress in the soil is the same as the limiting pile-soil shaft friction. While this is a simplification, the hyperbolic approach has sufficient flexibility to provide realistic non-linear response.

The 'concentric cylinder approach' (Frank, 1974; Randolph and Wroth, 1978; Randolph, 1981) may be used to estimate the radial variation of shear stress around a pile subjected
Chapter 8

8.3 Torsional Piles

to torsion. Assuming that the stress gradients longitudinally (parallel to the pile) are small by comparison with radial stress gradients, it may be shown that the shear stress (formally, \( \tau_{r\theta} \), but the double subscript is omitted here) at any radius, \( r \), is given by

\[
\tau = \tau_0 \frac{r^2}{r^2}
\]

where \( r_0 \) is the pile radius and \( \tau_0 \) is the shear stress mobilised at the pile. Combining this equation with Eq. (8-3) gives the radial variation of secant shear modulus as

\[
G = G_i \left( 1 - \frac{r_0^2}{r^2} \right)
\]

where \( \psi = R_f \tau_0 / \tau_f \), which defines the relative mobilisation of shaft friction at the pile surface. Note that \( R_f = 0 \) corresponds to a linear elastic case, while the upper limit on \( R_f \) is unity.

The above relationships are similar to those derived for axial loading of a pile (Kraft et al. 1981). However, the effect of non-linearity is much more localised close to the pile for the torsional case, as shown by Fig. 8-1 where the normalised shear modulus, \( G/G_i \), is plotted as a function of radius, \( r/r_0 \), for the two types of loading.

8.2.3 Shaft Torque-Rotation Response

The shear strain, \( \gamma_{r\theta} \), around a pile subjected to torsion may be written as (Randolph, 1981),

\[
\gamma_{r\theta} = \frac{\tau_{r\theta}}{G} = \frac{1}{r} \frac{\partial u}{\partial \theta} + r \frac{\partial}{\partial r} \left( \frac{v}{r} \right)
\]

where \( u \) is the radial soil movement, \( v \) is the circumferential movement, and \( \theta \) is the angular polar co-ordinate. From symmetry, \( \partial u / \partial \theta \) is zero, and so this equation may be combined with (8-4) to give

\[
\frac{\partial}{\partial r} \left( \frac{v}{r} \right) = \frac{\tau_0 r^2}{Gr^3}
\]

Substituting (8-5) and integrating this with respect to \( r \) from \( r_0 \) to \( \infty \) yields the angle of twist at the pile as
\[ \phi = \left( \frac{v}{r_0} \right) = \frac{\tau_0}{2G_1} \left( -\frac{\ln(1 - \psi)}{\psi} \right) \]  \hspace{1cm} (8-8)

This equation may be transformed into

\[ \phi = \frac{\tau_f}{G_1} \left( -\frac{1}{2R_f} \ln(1 - \psi) \right) \]  \hspace{1cm} (8-9)

which shows that the angle of twist depends logarithmically on the relative shear stress level.

Again, the form of the torque-twist relationship is similar to that for axial loading. However, as shown in Fig. 8-2, the degree of non-linearity (for a given \( R_f \) value) is somewhat more for the torsional case (\( w \) in the figure is the shaft displacement for vertical loading, while \( d \) is the pile diameter and \( \zeta \) is a load transfer parameter).

### 8.3 OVERALL PILE RESPONSE

The governing equations for the overall pile response have been documented by Randolph (1981), and may be written:

\[ \frac{d^2 \phi}{dz^2} = \frac{2\pi r_0^2}{(GJ)_p} \tau_0 \]  \hspace{1cm} (8-10)

where \((GJ)_p\) is the torsional rigidity of the pile. It is also helpful to introduce an equivalent shear modulus for a solid pile, \( G_p \), where

\[ G_p = \frac{2(GJ)_p}{\pi r_0^4} \]  \hspace{1cm} (8-11)

#### 8.3.1 Critical Pile Length and Pile-Soil Stiffness Ratio

For slender piles, transfer of torque is concentrated in the upper part of the pile, at least at low load levels, and the torque-twist stiffness of the pile is not affected by the overall pile length. It is useful to introduce the concept of a critical pile length, and hence a pile-soil stiffness ratio defined in terms of the pile length relative to that critical pile length. Randolph (1981) defined the critical pile length as that length beyond which the pile head torque-twist stiffness became independent of the overall pile length. The critical pile length was defined as
where $G_c$ is the shear modulus of the soil at a depth $z = L_c$. For general soil conditions, this definition is recursive. However, for the power law modulus variation considered here, the critical length may be written as

$$L_c \approx r_0 \left( \frac{G_p}{A_g r_0^n} \right)^m$$

(8-13)

where $m = 1/(2+n)$.

A pile-soil stiffness ratio may then be conveniently defined in terms of the ratio of the actual pile length, $L$, to the critical pile length, $L_c$. Thus

$$k_t = \frac{L}{L_c} = \frac{L}{r_0 \left( \frac{A_g r_0^n}{G_p} \right)^m} = L \left( \frac{\pi A_g d^2}{8(GJ)_p} \right)^m$$

(8-14)

However, it will be shown later that it is more convenient to adopt a stiffness ratio, $\pi_t$, which is larger than $k_t$ by a factor of $8^m$; thus

$$\pi_t = 8^m k_t = \frac{L}{r_0 \left( \frac{8A_g r_0^n}{G_p} \right)^m} = \left( \frac{\pi d^2 A_g L^{n+2}}{(GJ)_p} \right)^m$$

(8-15)

### 8.3.2 Elastic Solution

For fully elastic conditions, the angle of twist at any depth is directly related to the local shear stress mobilised at the pile shaft, by

$$\phi = \frac{\tau_o}{2G_i} = \frac{\tau_o}{2A_g z^n}$$

(8-16)

Substituting into Eq. (8-10) gives the governing differential equation as

$$\frac{d^2 \phi}{dz^2} = \frac{4\pi \tau_o^2 A_g z^n}{(GJ)_p^2} \phi = \left( \frac{\pi_t}{L} \right)^{1/m} z^n \phi$$

(8-17)

The solution of Eq. (8-17) is in the form of modified Bessel functions, $I_m$ and $K_m$, of fractional order, $m$ and $m-1$, where the pile twist, $\phi$, and twist gradient, $d\phi/dz$, are
\[ \phi(z) = \sqrt{\frac{z}{L}} [A_{Im}(y) + BK_m(y)] \]  
\[ \frac{d\phi}{dz} = \frac{1}{L} \left( \frac{\pi t z}{L} \right)^{1/2m} \left( \frac{z}{L} \right)^{-0.5} (A_{Im-1}(y) - BK_{m-1}(y)) \]

where the argument, \( y \), is given by
\[ y = 2m \left( \frac{\pi t z}{L} \right)^{1/2m} \]

The relative magnitude of the constants, \( A \) and \( B \), is found from conditions at the base of the pile, where (Randolph, 1981)
\[ \phi_b = \frac{3}{16 G_b r_0^3} = \frac{3 T_b}{2 G_b d^3} \]  
\[ D = K_{m-1} I_m + K_m I_{m-1} \]

The absolute values of the constants may be obtained from the boundary condition at the head of the pile, where
\[ \left( \frac{d\phi}{dz} \right)_{z=L} = -\frac{T_t}{(GJ)_p} = \frac{16 G_b r_0^3}{3 (GJ)_p} \phi_b \]  
\[ \phi(z) = \sqrt{\frac{z}{L}} [A_{Im}(y) + BK_m(y)] \]  
\[ \frac{d\phi}{dz} = \frac{1}{L} \left( \frac{\pi t z}{L} \right)^{1/2m} \left( \frac{z}{L} \right)^{-0.5} (A_{Im-1}(y) - BK_{m-1}(y)) \]

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\[ \frac{d\phi}{dz} = \frac{1}{L} \left( \frac{\pi t z}{L} \right)^{1/2m} \left( \frac{z}{L} \right)^{-0.5} (A_{Im-1}(y) - BK_{m-1}(y)) \]

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\[ \phi(z) = \sqrt{\frac{z}{L}} [A_{Im}(y) + BK_m(y)] \]  
\[ \frac{d\phi}{dz} = \frac{1}{L} \left( \frac{\pi t z}{L} \right)^{1/2m} \left( \frac{z}{L} \right)^{-0.5} (A_{Im-1}(y) - BK_{m-1}(y)) \]

where the argument, \( y \), is given by
\[ y = 2m \left( \frac{\pi t z}{L} \right)^{1/2m} \]

The relative magnitude of the constants, \( A \) and \( B \), is found from conditions at the base of the pile, where (Randolph, 1981)
Substituting the expressions for A and B into Eqs. (8-18) and (8-19), the ratio of torque and rotation at any depth z may be expressed as

\[ \frac{T(z)}{\phi(z)} = \pi t^{1/2m} C_t(z) \frac{(GJ)_p}{L} \]

where

\[ C_t(z) = \frac{C_1(z) + \chi C_2(z) \left( \frac{z}{L} \right)^{n/2}}{C_3(z) + \chi C_4(z)} \]

and

\[ C_1(z) = -K_{m-1}I_{m-1}(y) + K_{m-1}(y)I_{m-1} \]
\[ C_2(z) = K_{m}I_{m-1}(y) + K_{m-1}(y)I_{m} \]
\[ C_3(z) = K_{m-1}I_{m}(y) + K_{m}(y)I_{m-1} \]
\[ C_4(z) = -K_{m}I_{m}(y) + K_{m}(y)I_{m} \]

The torque-twist stiffness at the top of the pile may be evaluated by allowing z to approach zero.

### 8.3.3 Elastic-Plastic Solution

The elastic solution may be extended to the situation where partial slip occurs between pile and soil. As the angle of pile twist increases, the mobilised shear stress at the pile shaft will reach the limiting value given by Eq. (8-2), and slip will occur between pile and soil. For the case where the exponents \( n \) and \( t \) are identical, slip will always start at the pile head, and gradually progress down the length of the pile.

The limiting elastic shaft rotation, before slip occurs, may be written as

\[ \phi_e = \frac{A_t}{2A_g} \]

At any stage during partial slip of the pile, the total embedded length, \( L \), may be divided into a length, \( L_1 \), where slip has occurred, and a lower elastic region, \( L_2 \). The proportion of the pile that has slipped may then be expressed as \( \mu = L_1/L \).
From Eqs. (8-28) and (8-31), the torque, $T_e$, at the top of the elastic section of the pile may be written as

$$T_e = 0.5\pi_t^{1/2}mC_t(\mu L)\frac{A_t}{A_g}\frac{(GJ)_p}{L} \quad (8-32)$$

The response of the pile in the slipped zone may be obtained directly from the known shear stress acting on the pile shaft, as given by Eq. (8-2). Taking the modulus and shaft friction exponents, $n$ and $t$ respectively, as equal, the torque at the pile head, $T_t$, may be written as

$$T_t = T_e + 0.5\pi d^2 \frac{A_tL^{n+1}}{n+1} \quad (8-33)$$

Substitution of the shaft friction profile into Eq. (8-10) and integrating leads to an expression for the pile head twist of

$$\phi_t = \phi_e + \frac{L_1}{(GJ)_p} \left( T_e + 0.5\pi d^2 \frac{A_tL^{n+1}}{n+2} \right) \quad (8-34)$$

Substituting for $T_e$ and $\phi_e$ in these two expressions leads to final relationships of

$$T_t = 0.5\left[ \pi_t^{1/2}mC_t(\mu L) + \pi_t^{1/m} \mu_n^{n+1} \right] A_t \frac{(GJ)_p}{L} \quad (8-35)$$

$$\phi_t = 0.5\left[ 1 + \mu_t^{1/2}mC_t(\mu L) + \pi_t^{1/m} \mu_n^{n+2} \right] A_t \frac{A_t}{A_g} \quad (8-36)$$

### 8.4 VALIDATION OF THEORY

Numerical results from the solutions in the previous section may be presented in terms of the angle of twist at the pile head, $\phi_t$, expressed as

$$\phi_t = \frac{I_\phi}{F_\phi \frac{(GJ)_p}{L}} T_t \quad (8-37)$$

where $I_\phi$ is an elastic influence factor, and $F_\phi$ represents the relative reduction in pile-head stiffness due to partial slip between pile and soil. From Eq. (8-28), the elastic influence factor may be expressed as

$$I_\phi = \left( \pi_t^{1/2}mC_{t0} \right)^{-1} \quad (8-38)$$
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where $C_{to}$ represents the limit of $C_t(z)$ as $z$ tends to zero.

8.4.1 Relationship with Previous Published Elastic Solutions

Before presenting any numerical results, it is helpful to document the relationship between the present solutions, and previous solutions published for specific soil profiles, particularly those by Poulos (1975), Randolph (1981) and Hache and Valsangkar (1988). Essentially, the form of pile-soil flexibility ($\pi_t$) and influence factor ($I_\phi$) adopted in the present chapter are identical with those proposed by Hache and Valsangkar (1988). The relationships with pile-soil stiffness ratios and influence factors published by the other authors are shown in Table 8-1.

Table 8-1 Comparison of previous published approaches

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Pile-soil flexibility</td>
<td>[ \pi_t = \left( \frac{\pi d^2 A_g L^{n+2}}{(GJ)_p} \right)^m ]</td>
<td>[ \frac{1}{K_T} = \frac{A_g d^{n+4}}{(GJ)_p} ]</td>
<td>[ \lambda = \frac{2}{\pi A_g r_0^{4+n}} ]</td>
</tr>
<tr>
<td>Influence factor</td>
<td>[ I_\phi = \frac{\phi_t (GJ)_p}{T_t L} ]</td>
<td>[ T_t = \frac{\pi t A_g d^{n+3}}{I_\phi} ]</td>
<td>[ \frac{T_t}{\phi_t A_g r_0^{n+3}} ]</td>
</tr>
</tbody>
</table>

Numerical results obtained from the present closed form solution are compared with results from Poulos (1975) in Fig. 8-3, for values of $K_T$ (n = 0) between 1 and $10^5$ ($K_T'$, an equivalent stiffness for the case of n = 1, between $10^{-3}$ and $10^7$), and pile slenderness ratios in the range $1 < L/d < 100$. It may be seen that the two solutions agree over a wide range of parameters.
A key feature of the pile-soil flexibility ratio, \( \pi_t \), and influence factor, \( I_\phi \), is that design curves are essentially independent of the slenderness ratio, \( L/d \). This is illustrated in Fig. 8-4, (for elastic conditions) for extreme values of \( L/d = 5 \) and \( L/d = 150 \), for a range of soil profiles (\( n = 0, 0.25, 0.5, 0.75 \) and 1).

A number of characteristics of Fig. 8-4 are worthy of comment:

1. A value of \( \pi_t = 1 \) provides a break-point between two sections of the plot.

2. For values of \( \pi_t < 1 \), the gradient is very close to -1 for all values of the modulus exponent, \( n \).

3. For values of \( \pi_t > 1 \), the gradients approximate to \(-1/(n+2)\).

The pile-soil flexibility of unity represents a transition point between essentially rigid piles, and piles which are fully flexible (where negligible torsion is transmitted to the pile base). For rigid piles, Randolph (1981) has shown that the pile-head stiffness may be expressed as

\[
\frac{T_t}{A_g L n r_0 \phi_t} = \frac{16}{3} \frac{4 \pi}{n+1} \frac{L}{r_0}
\]

The first term on the right-hand side represents the contribution from the pile base, and typically contributes less than 10% of the total stiffness. Ignoring this contribution, the expression may be manipulated to give

\[
I_\phi = \frac{\phi_t (GJ)_{p,n+1}}{T_t L} = \frac{n+1}{\pi_t n + 2}
\]

At the other extreme, the pile-head stiffness of flexible piles may be estimated from the approximate approach of Randolph (1981):

\[
\frac{T_t}{G_c r_0 \phi_t} \approx \frac{\sqrt{2} \pi}{n+1} \left( \frac{G_p}{G_c} \right)^{0.5}
\]

where \( G_p \) and \( G_c \) have been discussed earlier (see Eqs. (8-11) to (8-13)). This expression may be transformed to give

\[
I_\phi = \frac{\phi_t (GJ)_{p,n+1}}{T_t L} = \frac{n+1}{(2\sqrt{2})^{n/(n+2)} \pi_t}
\]
This expression matches the curves shown in Fig. 8-4 very closely.

### 8.4.2 Elastic-Perfectly Plastic Response

The torque-twist relationship where partial slip occurs along the pile shaft may be obtained from the expressions for $T_t$ and $\phi_t$ in Eqs. (8-35) and (8-36). The yield correction factor to the elastic flexibility coefficient, $I_\phi$, may be written as

$$F_\phi = \frac{1}{C_{to}} \frac{C_t(\mu L) + \pi_t^{1/2} \frac{\mu^{n+1}}{n+1}}{1 + \mu \pi_t^{1/2} C_t(\mu L) + \pi_t^{1/m} \frac{\mu^{n+2}}{n+2}}$$

where $\mu L$ is the depth to which slip has occurred. This may be related to the proportion of the ultimate capacity, $T_u = 0.5 \pi d^2 A_t L^{n+1}/(n+1) + T_b$, where $T_b$ is the base torque taken as $(\pi d^3/12)A_t L^n$, by

$$\frac{T_t}{T_u} = \frac{(1+n)\pi_t^{-1/2} C_t(\mu L) + \mu^{1+n}}{1 + \frac{1+n}{6} \frac{d}{L}}$$

Fig. 8-5 shows the variation of $F_\phi$ with $T_t/T_u$, for different values of the flexibility ratio, $\pi_t$. The above solution is compared with values published by Poulos (1975), with the two sets of results showing excellent agreement. A fuller set of design curves for the correction factor, $F_\phi$, is given in Fig. 8-6. It may be seen that the correction factor is essentially independent of the slenderness ratio, $L/d$, of the pile.

### 8.5 PILE RESPONSE WITH HYPERBOLIC SOIL MODEL

The previous section presented solutions for partial slip along the pile, where the soil response was modelled as elastic-perfectly plastic. Here, the effect of a hyperbolic stress-strain response of the soil is explored.

#### 8.5.1 Rigid Piles

For rigid piles, the angle of twist, $\phi$, will be uniform down the pile, and so the torque-twist response at the pile head may be obtained directly by integrating the local torque transfer curve given by Eq. (8-9). The overall torsional stiffness may be written in the form adopted by Randolph (1981) as
8.5.2 Flexible Piles

For flexible piles, it is necessary to adopt a numerical approach in order to implement the non-linear torque transfer curve. A spreadsheet program, GASPILE, originally developed for axial loading (Chapter 3; Guo and Randolph, 1996c), has been extended to torsional loading. With it, non-linear analyses have been performed for hyperbolic soil response, as given by Eq. (8-9) taking $R_f = 0.95$. At low load levels, the computed influence factor, $I_\phi$, is essentially identical to the closed-form results, as indicated in Fig. 8-4(b). For the hyperbolic model, the overall torque-twist relationship for the pile head is indistinguishable from that obtained using an elastic-perfectly plastic model with the same initial shear modulus. This result, which has been noted for axial loading by Poulos (1989) is illustrated below in the case study.

8.6 CASE STUDY

An example analysis is given here, for torsional load tests reported by Stoll (1972). The two piles were steel pipes of external diameter 0.273 m, and wall thickness 6.3 mm, back-filled with concrete. Stoll (1972) reports the torsional rigidity $(GJ)_p$ of the two piles as 12.8 MNm$^2$.

Pile A-3 was driven to a penetration of 17.4 m through soil where the SPT value $(N)$ varied approximately linearly with depth according to,

$$N \approx 1.38z$$  \hspace{1cm} (8-46)

where $z$ is the depth in m. The other pile, pile V-4, was driven to 20.7 m at a location where the SPT value in the upper 2.4 m was very low, and below 2.4 m it varied linearly with depth according to

$$N \approx 2.62(z - 2.4)$$  \hspace{1cm} (8-47)

The SPT profiles suggest distributions of shear modulus and shaft friction which vary linearly with depth, giving $n = 1$. For pile V-4, an artificial ground surface at $z = 2.4$ m has been assumed, and the calculated pile head flexibility has been increased accordingly.

The ultimate torques measured in each case were 29.3 kNm and 52.1 kNm for piles A-3 and V-4 respectively. These lead to values of $A_t$ of 1.66 kPa/m and 2.66 kPa/m.
The ratios of shaft friction to N value are therefore 1.2 kPa and 1.1 kPa respectively, which are rather lower than the ratio for axial loading proposed by Meyerhof (1976), of 2 kPa.

The initial torsional stiffness of pile A-3 is \( \frac{T}{r_0 \phi} = 20 \text{MN/rad} \), and Pile V-4, \( 24.3 \text{MN/rad} \) at the depth of 2.4 m. Therefore based on Eq. (8-42), with \( n = 1 \), values of \( A_g \) were back-figured as 1.5 MPa/m (A-3) and 2.69 MPa/m (V-4) respectively, which give approximately Young's modulus of \( E = 3 \text{N MPa} \) (\( \nu_s = 0.4 \), pile A-3) and 2.88N MPa (pile V-4). Previous publications show that Young's modulus could be approximately estimated as 4N MPa (Poulos, 1989), 7N (Shibata et al.1989) or 2.8N (Randolph, 1981).

**Table 8-2 Hand Calculation of Torque-twist Relationship for Pile A-3**

<table>
<thead>
<tr>
<th>( \frac{T}{T_u} )</th>
<th>( F_\phi )</th>
<th>( \frac{T}{r_0 \phi} ) (MN/rad)</th>
<th>( \frac{T}{r_0} ) (kN)</th>
<th>( \phi ) (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.73</td>
<td>14.6</td>
<td>54</td>
<td>0.0037</td>
</tr>
<tr>
<td>0.5</td>
<td>0.53</td>
<td>10.6</td>
<td>109</td>
<td>0.0102</td>
</tr>
<tr>
<td>0.75</td>
<td>0.45</td>
<td>9.0</td>
<td>163</td>
<td>0.0181</td>
</tr>
<tr>
<td>1.0</td>
<td>0.38</td>
<td>7.7</td>
<td>217</td>
<td>0.0288</td>
</tr>
</tbody>
</table>

Hand calculation of the complete \( \frac{T}{r_0} \) versus \( \phi \) relationship may be achieved for the given values of \( A_g = 1.5 \text{ MPa/m} \) (giving \( \pi_t = 5.248 \)) and the yield correction factor from Fig. 8-6. The results are shown in Table 8-2, and also plotted in Fig. 8-8(a) for comparison with the computed solutions.

To assess the influence of the non-linear model, both linear elastic-plastic (LEP, \( R_f = 0 \) in Eq. (8-9)) and non-linear elastic-plastic (NLEP, \( R_f = 0.95 \)) analyses have been performed, using the GASPILE program. Fig. 8-8 shows respectively the pile head load and the angle of twist relationship for pile A-3 and V-4 predicted by GASPILE, the present closed form (CF) solution and the results obtained by Chow (1985), together with those measured by Stoll, (1972). The closed form prediction for V-4 pile has been based on an equivalent pile of length 18.3 m, with allowance for the twist originating from the upper 2.4 m. Only small differences may be seen between linear and non-
linear elastic-plastic analyses. However, none of the computed solutions provides a very good match with the measured data. The computed stiffness at loads of 50% of the ultimate is too low, and even increasing the soil modulus by a factor of 10 makes little difference, owing to the occurrence of slip at low load levels. A possible explanation lies in the choice of torsional rigidity of the piles, which may have been affected by load level, due to cracking of the concrete.

Fig. 8-9 shows profiles of shear stress, load \((T/r_0)\) and displacement \((\phi r_0)\) down the pile, predicted from non-linear and linear elastic-plastic numerical GASPILE analysis, and also by the closed form equations (see Appendix F). All profiles are very similar.

8.7 CONCLUSIONS

The analysis outlined in this chapter has attempted to provide a comprehensive approach for the analysis of piles subjected to torsion. Solutions have been presented for piles embedded in a non-homogeneous medium, where the shear modulus and limiting shaft friction are taken as power law functions of depth. Consideration has also been given to the form of torque-twist relationship arising from a hyperbolic stress-strain response of the soil.

The rapid decay of shear stress with distance from the pile entails that non-linear effects in the soil are limited to the immediate vicinity of the pile. The resulting torque-twist curve may be closely approximated by elastic-perfectly plastic response, even where the stress-strain response of the soil is markedly non-linear.

A general closed-form solution has been developed for elastic-perfectly plastic soil response. The solution is written in terms of fractional Bessel functions, and has been evaluated using Mathcad. An alternative numerical solution, which can handle more general non-homogeneous and non-linear stress-strain response of the soil, has been implemented in a spreadsheet program, GASPILE. Both solutions have been shown to agree with each other, and with solutions published previously for the limiting cases of soil with either uniform stiffness or stiffness proportional to depth.

Design charts have been presented in non-dimensional form for the pile-head influence coefficient, \(I_\phi\), and a modifying factor, \(F_\phi\), to allow for partial slip down the pile. Both factors are a function primarily of the pile-soil flexibility ratio, \(\pi_0\), and are essentially independent of the slenderness ratio, \(L/d\). A flexibility ratio of unity marks a transition point between very stiff piles, where the twist is uniform down the pile, and fully flexible piles where the torque at the pile base is negligible.
Chapter 8

8.15 Torsional Piles

The solutions have been applied to a case study, where two piles were loaded torsionally to failure. The soil parameters (shear modulus and shaft friction) back-analysed from the initial stiffness and ultimate torque capacity were found to be consistent with common practice. However, the overall agreement between calculated and measured response curves was relatively poor, indicating that some aspect of the pile-soil system was incorrectly modelled, possibly due to progressive cracking of the concrete interior of the pile.
Chapter 9 9.1 Conclusions

9. CONCLUSIONS

Closed form solutions have been established for a pile subject to either vertical or torsional loading in non-homogeneous media, where the shear modulus and limiting shaft friction are taken as power law functions of depth. The rationality and suitability of load transfer approaches have been extensively checked corresponding to various boundary conditions. Simple statistical formulas for estimating load transfer factors were given. Closed form solutions have been generated respectively to account for the effect of soil elastic-plastic, visco-elastic properties, and the reconsolidation due to pile installation. A load transfer analysis program, GASPILE, has been newly developed, to explore the effect of non-linearity and visco-elastic soil properties on a pile response for the instance of either vertically or torsional loading. A closed form expression for pile-pile interaction has been generated. Therefore, the solutions for analysing a single pile are then extended to that for pile groups. A numerical program called GASGROUP has been designed to facilitate estimation of settlement of large piled groups. Extensive comparisons with the previously publications have been made for every theories established. Relevant design charts have been produced. Case studies for each theories have been undertaken to illustrate the strength of the current research. Detailed conclusions arising from the research have been presented in each of the previous chapters, and the main findings are briefly summarised below.

9.1 VERTICALLY LOADED SINGLE PILES

Analytical solutions have been established for a pile in a non-homogenous elastic-plastic media. The accuracy of the solutions, which are based on the load transfer approach, is very good compared with those from more rigorous continuum based numerical analyses and the numerical load transfer analysis, GASPILE. The following conclusions were drawn

- A non-linear elastic-plastic analysis (NL) shows only slight differences from a simplified linear elastic-plastic analysis (SL); therefore the closed form solutions established based on the simplified elastic-plastic model can be directly utilised for the non-linear case.

- The significant influence of non-homogeneity of the soil profile on pile-head stiffness or settlement influence factor maybe attributed partly to the non-
homogeneity and partly to the variation of the average soil modulus over the pile length. In other words, the influence of non-homogeneity originated partly from the definition of relative pile-soil stiffness in terms of the soil modulus at the pile tip level.

- For a given average shear modulus over the pile length, but different distribution (n different), the final elastic pile-head stiffness (or the settlement influence factor) is not significantly different (e.g. less than 20%).

- For a long pile, pile-soil relative slip should be considered when estimating load-settlement behaviour and load distribution down the pile. A case study has shown that the pile response can be modelled well by the closed form solutions, right up to full shaft slip.

- Though only vertical non-homogeneity is considered in the current load transfer model, radial non-homogeneity due to disturbance from pile installation can be taken into account as well by a modification of the shaft load transfer factor.

- The power of the depth (refer to Eq. (3-1)) may be adjusted to fit more complicated shear modulus profile, allowing the analysis proposed here still to be used.

9.2 VERTICALLY LOADED SINGLE PILES IN A FINITE LAYER

Load transfer factors have been back-figured through FLAC analysis subject to a variety of boundary conditions. The suitability and rationality of load transfer analysis has been explored extensively. The following conclusions have been demonstrated

- A preliminary numerical check showed that a grid of $21 \times 100$ was necessary to obtain accurate estimation of the base load transfer parameter, $\omega$. Also, setting a radial boundary at a distance of $2.5L$, the radial boundary condition made no difference for $H/L < 4$, while fixed boundary was essential for $H/L > 4$. With the fixed boundary, it was found that $H/L = 4$ may be considered effectively as an infinitely deep soil layer.

- The numerical analysis shows that the effect of choosing soil Poisson’s ratio can be equally as important as the ratio of $H/L$ and should be taken into consideration. The finite layer ratio of $H/L$ can only lead to about 15% increase in head stiffness
when H/L decreases from 4 to 1.25, but an increase in soil Poisson's ratio from 0 to 0.499 can result in about a 25% increase in pile-head stiffness, when using a constant value of relative stiffness ratio defined as the ratio of pile Young's modulus to soil shear modulus.

- Calibration using the load transfer model shows that, generally, the shaft load transfer factor can be taken as constant with depth. With average values of the shaft load transfer factor, the load transfer approach yielded close predictions of overall pile response compared with those obtained by FLAC analysis.

- Calibration using the closed form solutions demonstrates that shaft load transfer factor (1) increases with increase in pile slenderness ratio; (2) decreases with increase in Poisson's ratio; (3) increases slightly with increase in the ratio of H/L (H/L < 4), but (4) is nearly independent of the pile-soil relative stiffness.

- The difference in the values of shaft load transfer factors, calibrated against pile-head stiffness and ratio of base and head load, implies that the load transfer approach is less accurate in the cases of (1) homogeneous soil profile; and (2) higher pile slenderness ratio or lower pile-soil relative stiffness. However, an appreciable (e.g. 30%) difference in selection of the value of 'A' (referred to Eq. (4-5)) generally leads to a slight (e.g. about 10%) difference in the predicted pile-head stiffness of a single pile. Therefore, generally load transfer analysis is sufficiently accurate for practical analysis.

- The backfigured load transfer factors have been expressed in the form of simple formulas and also implemented in a spreadsheet program. In comparison with the current FLAC analysis and relevant rigorous numerical approaches, the simple formulas can well account for the effects of various relative thickness ratio of H/L (≤ 4), Poisson's ratio and pile slenderness ratio. In the case of an infinite layer, it seems that a value of 'A = 2.5' gives good comparison with most of the available numerical predictions.

- The shear modulus distribution below the pile tip level can significantly alter the value of the shaft load transfer factor. To account for this effect, (1) for the case of shear modulus varying as a power law of depth across the entire depth, H, Eq. (4-8) may be used, otherwise (2) for the case of a constant value below the tip level, Eq. (4-12) may be used.
9.3 VISCO-ELASTIC RESPONSE OF SINGLE PILES

A new load transfer model has been established and substantiated by available rigorous numerical analysis. The numerical program, GASPILE, has been extended to account for the non-linear visco-elastic response. The major conclusions from this study were:

- The new non-linear visco-elastic shaft load transfer (t-z) model compares well with published field and laboratory test data.

- Generally, the numerical GASPILE program and the closed form solutions may be utilised to undertake creep analysis. However, at high stress levels (e.g. higher than 70% of ultimate pile capacity), shaft load transfer factor is no longer a constant, hence, the closed form solutions are no longer valid.

- At high load levels, pile response is mainly affected by soil long-term strength, while at lower load levels, the response is affected by the soil delayed (secondary) elastic shear property.

- Parametric studies on the two extreme types of time-scale loading, namely: step and ramp (linear increase followed by sustained) loading, show that the former incurs significantly higher displacement than the latter does, should other conditions be identical.

- The case studies show that excellent comparison with measured response can be made with the proposed theory.

9.4 PERFORMANCE OF DRIVEN PILES

A visco-elastic radial consolidation has been established and compared with previous theory for elastic case. A number of case studies have been undertaken. The study showed that:

- Visco-elastic consolidation theory can be obtained by (a) solving the diffusion theory and then using an accurate coefficient of consolidation; or (b) adapting the available elastic solutions using the correspondence principle;

- Viscosity of soil can significantly increase the reconsolidation time, hence increase the final pile settlement. However, it has negligible effect on soil strength or pile
capacity, which may be attributed to the offset between the reduction in the strength to the long terms value due to creep, and the increase in strength due to reconsolidation.

- The case studies show that normalised pile-soil interaction stiffness (shear modulus) variation, due to reconsolidation, is consistent with dissipation of pore pressure and increase in soil strength on the pile-soil interface. The time-dependent parameters for analysing a driven pile following installation can be sufficiently accurately predicted by radial consolidation theory. Therefore with the parameters predicted, it is straightforward to predict the load-settlement response for piles tested at different times following driving, by either GASPILE analysis or the previous closed form solutions (Chapters 3 and 5).

9.5 VERTICALLY LOADED PILE GROUPS

The new closed form solutions for single piles have been extended to the analysis of pile groups. Closed form expressions for pile-pile interaction factor have been established, yielding a unified approach for analysing pile group behaviour by means of the superposition principle. A number of analyses have led the following conclusions:

- The closed form expression for interaction factors gives very good comparison with those obtained by more rigorous numerical analyses, using the modified load transfer factors.

- The current approach yields very good comparison of the pile group stiffness with those obtained by rigorous numerical analysis, over a range of layer thickness ratios.

- The current program, GASGROUP, gives reasonable prediction in comparison with either more rigorous numerical analyses or measured data. The program is very quick, efficient and can be readily run in a personal computer. Therefore, it may be used for practical engineering design.

- Guidelines for estimating settlement of (large) pile groups have been provided, using GASGROUP program for a variety of different subsoil profiles.
9.6 TORSIONAL PILES

A comprehensive approach for analysing piles subjected to torsion has been attempted. Solutions have been presented for piles embedded in a non-homogeneous medium. The study showed that:

- The rapid decay of shear stress with distance from the pile entails that non-linear effects in the soil are limited to the immediate vicinity of the pile. The resulting torque-twist curve may be closely approximated by elastic-perfectly plastic response, even where the stress-strain response of the soil is markedly non-linear.

- A general closed-form solution has been developed for elastic-perfectly plastic soil response. The solution is written in terms of fractional Bessel functions, and has been evaluated using Mathcad™. An alternative numerical solution, which can handle more general non-homogeneous and non-linear stress-strain response of the soil, has been implemented in a spreadsheet program, GASPILE. Both solutions have been shown to agree with each other, and with solutions published previously for the limiting cases of soil with either uniform stiffness or stiffness proportional to depth.

- Design charts have been presented in non-dimensional form for the pile-head influence coefficient, and a modifying factor, to allow for partial slip down the pile. Both factors are a function primarily of the pile-soil flexibility ratio, πt, and are essentially independent of the slenderness ratio, L/d. A flexibility ratio of unity marks a transition point between very stiff piles, where the twist is uniform down the pile, and fully flexible piles where the torque at the pile base is negligible.

- The solutions have been applied to a case study, where two piles were loaded torsionally to failure. The soil parameters (shear modulus and shaft friction) back-analysed from the initial stiffness and ultimate torque capacity were found to be consistent with common practice. However, the overall agreement between calculated and measured response curves was relatively poor, indicating that some aspect of the pile-soil system was incorrectly modelled, possibly due to progressive cracking of the concrete interior of the pile.
9.7 RECOMMENDATIONS FOR FURTHER RESEARCH

The current research enables pile behaviour to be predicted under a range of working conditions as shown above. A direct extension of this research may be directed toward the following subjects:

- The solutions for vertical loading may be implemented directly into “hybrid analysis” for analysing pile-raft foundation in a non-homogeneous soil.

- For a pile in a layered or radially non-homogeneous soil, the load transfer factors may be back-figured, although using a value of load transfer factor predicted by the formulae shown in Chapter 3 may give a satisfactory result.

- Elastic-plastic response of a pile group might be investigated, adapting the current closed form solutions for a single pile under vertical loading.

- By prescribing a stress distribution along a pile, numerical analysis, similar to that by Polo and Clemente (1988), may be performed for a pile in a medium of a desired shear modulus profile, so that new values of load transfer factors may be back-figured. Therefore, the affect of incompatibility between the assumed stress distribution and the shear modulus profile might be examined.

9.8 CONCLUDING REMARKS

The current research has led to various closed form solutions. The suitability of the theoretical load transfer approach has been clarified. The overall pile response has been explored extensively focusing on the effect of non-homogeneous soil profiles, the development of pile-soil relative slip, visco-elastic pile-soil interaction, and soil reconsolidation subsequent to pile driving. The response of pile groups in a non-homogeneous medium has been modelled accurately by the new closed form solutions. Solutions for torsional pile-soil interaction have been achieved using an elastic-plastic soil model in combination with the non-homogeneous soil profile. The effect of non-linear soil response on the pile behaviour has been investigated in regards to both vertical and torsional loading by the newly developed load transfer programs.
APPENDIX A GASPILE: A Spreadsheet Program

A.1 INTRODUCTION

Load transfer approach refers to simulating the pile-soil interaction by a series of independent linear (or non-linear) elastic-plastic spring down the pile shaft and at the pile base, with the spring stiffness being evaluated by either direct experimental measurement (Coyle and Reese, 1966), or the theoretical load transfer model (Randolph and Worth, 1978; Kraft et al. 1981). Particularly, the theoretical load transfer approach can offer sufficient accuracy compared with more rigorous numerical analyses. Therefore it has been widely utilised to predict pile load settlement behaviour (e.g. Randolph and Wroth, 1978; Kraft et al. 1981; Lee, 1991). The advantage lies in its simplicity and valid to a variety of loading conditions, e.g. vertical, torsional loading, and group pile case as well. For linear, elastic-plastic case, analytical solutions have been established in the current thesis. But for the non-linear case, the prediction has to recourse to numerical approach. A few numerical programs have been developed previously (Coyle and Reese, 1966; Randolph and Wroth, 1978; Kraft et al. 1981; Kiousis and Elansary, 1987), but generally they are based on a sophisticated Language, (e.g. Fortran), and therefore restricted to the special environment. The post-analysis is generally time-consuming compared with a routine spreadsheet analysis. In addition, none of the analyses can account for the torsional behaviour.

This appendix aims at (1) exploring the difference and similarity between the torsional and vertical loading; (2) developing a spreadsheet program, which can be readily utilised to predict pile behaviour under either torsional or vertical loading. The newly designed program is apparently more efficient and accessible to design engineers.

A.2 LOAD TRANSFER MODELS

As may be seen from Chapters 3 and 8, there is a striking similarity of the governing equations between vertical and torsional loaded piles. Therefore, firstly, a general procedure and principle of load transfer analysis for either vertical or torsional loading is briefly presented, then the difference between the two kinds of loadings is explored.

A.2.1 The Similarity

The pile is discretised into a number of sections. For each sections, the pile-soil interaction is represented by a shaft (or base) load transfer model. The shaft model
Appendix A

A.2 A Spreadsheet Load Transfer Analysis

describing the local stress-displacement relationship are normally assumed to consist of two components: prior to failure and posterior to failure components.

(1) Prior to failure, the displacement versus shear stress relationship may be assessed by the cylinder concentric approach (Randolph and Wroth, 1978; Chapter 3), thus

\[ y = \frac{\tau_o}{\tau_f} = \frac{w}{\tau_o \zeta(y) \zeta_1} \frac{G_i}{\tau_f} \]  

(A-1)

where \( \tau_o \) is local shaft shear stress; \( r_o \) is the pile radius; \( y \) is the stress level on the pile-soil interface, \( y = \tau_o / \tau_f \).

(2) Once the shaft stress level reaches failure, the limiting stress is assumed to be equal to the failure stress, \( \tau_f \). To consider the stress softening behaviour (Appendix C), the stress level may be replaced with \( \xi \tau_f \) (Fig. 2-7, \( \xi \) is the stress softening factor, normally being less than 1).

For the case of vertical loading, it follows (Chapter 3)

\[ \zeta(y) = \ln\left(\frac{r_m/r_o - yR_{fs}}{1 - yR_{fs}}\right) \]  

(A-2)

where \( R_{fs} \) is a parameter controlling the degree of non-linearity; \( r_m \) is the maximum radius of influence of the pile beyond which the shear stress becomes negligible.

For the case of torsional loading, it follows that (Chapter 8)

\[ \zeta(y) = -\ln(1 - yR_{fs})/(2R_{fs}) \]  

(A-3)

Considering the visco-elastic behaviour by the shaft model shown in Fig. A-1, the modification factor, \( \zeta_1 \), is derived as (Chapter 5)

\[ \zeta_1 = 1 + \frac{G_{12}}{G_{11}} \left(1 + e^{-\frac{G_{12}/\eta}{\tau_f}}\right) \]  

(A-4)

where \( G_{12}/\eta \) is the relaxation time factor; \( G_{11}/G_{12} \) is the relative ratio of shear modulus.

For the case of purely a non-linear elastic medium, \( \zeta_1 = 1 \).

---

1 Note that all the symbols used in Appendixes are identical to those defined earlier, except where specified.
As shown in Fig. A-2, the increment of displacement, $\Delta w$ arising from the midpoint of the segment $n$ to that of $n+1$ consists of two parts

$$\Delta w = \Delta w_{AB} + \Delta w_{BC}$$ (A-5)

These parts may be estimated by (Coyle and Reese 1966)

$$\Delta w = w_{qn} + \frac{\tau_0 \pi d_n L_n}{8 [EA]_n} L_n$$ (A-6)

with

$$w_{qn} = \frac{Q_n L_n}{2[EA]_n} + \frac{Q_{n+1} + 3Q_n}{8} \frac{L_{n+1}}{[EA]_{n+1}}$$ (A-7)

where $Q_n$ is the axial force, subscript "n" for segment "n", $[EA]_n$ is the pile rigidity for segment $n$; $L_n$, $L_{n+1}$ are the segment lengths for segment $n$ and $n+1$ respectively; $d_n$, $d_{n+1}$ are the segment diameters for segment $n$ and $n+1$ respectively. The total shaft displacement of the segment is given by

$$w = w_{on} + \Delta w$$ (A-8)

where $w_{on}$ is the initial displacement at the segment of $\overline{AC}$.

Within the elastic stage, displacement by Eq. (A-1) should equal that by Eq. (A-8). Therefore, it follows

$$F(y) = w_{on} + w_{qn} + \frac{\gamma \tau \ell L_o}{8E_p A_p} \pi d_n L_n - \frac{\gamma \tau \zeta(y) \zeta_1 \tau_f}{G_i} = 0$$ (A-9)

The load transfer factor, $\zeta(y)$, is dependent on shear stress level, and is given by the following local stress-displacement relationship for a non-linear visco-elastic medium

$$f(y) = \zeta_1 \zeta(y) - \frac{1}{\gamma \tau_o} \frac{wG_i}{\tau_f} = 0$$ (A-10)

where $w$ is the relative pile-soil movement at the midpoint of the segment, $\overline{AC}$.

### A.2.2 The Difference

The difference between torsional and vertical loading has been summarised in Table A-i, which is an extension of the analysis presented in the Chapters 3 and 8.
Appendix A

A.4 A Spreadsheet Load Transfer Analysis

Table A-i Comparison of the Theories for Torsional and Vertical Loading Piles

<table>
<thead>
<tr>
<th>Items</th>
<th>Model /Formula</th>
<th>Torsional loading</th>
<th>Vertical loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td>$\tau_o = \frac{wG}{r_o \zeta(y)}$</td>
<td>Eq. (A-3)</td>
<td>Eq. (A-2)</td>
</tr>
<tr>
<td>Base</td>
<td>$P_b = \omega_m \frac{w_b}{r_o} G_{ab} r_0^2$</td>
<td>$\omega_m = 16/3$</td>
<td>$\omega_m = 4/(1 - \nu_z)\omega$</td>
</tr>
<tr>
<td>(1) Displacement compatibility</td>
<td>$\Delta w_{AB} = \frac{Q L_{AB}}{[EA]}$</td>
<td>$[EA] = \frac{(GJ)_p}{r_o^2}$</td>
<td>$[EA] = E_p A_p$</td>
</tr>
<tr>
<td>(2) Load equilibrium</td>
<td>$Q_n = P_b + P_s$</td>
<td>$Q_n = T_n/r_o$</td>
<td>$Q_n$</td>
</tr>
<tr>
<td>Variable 1</td>
<td>$w$</td>
<td>Circumferential displacement</td>
<td>Vertical displacement</td>
</tr>
<tr>
<td>Variable 2</td>
<td>$Q$</td>
<td>Torque shear force</td>
<td>Vertical axial load</td>
</tr>
</tbody>
</table>

* Note: $Q$, $L_{AB}$ are the average axial force and length between the point A and B respectively.

A.3 STRUCTURE OF THE PROGRAM

Following the above-mentioned principles, a numerical program called GASPILE has been designed. The program consists basically of two files: (1) the input and output spreadsheets, and (2) the corresponding macrosheets. To commence an analysis, you should

1. be in state of EXCEL;
2. open the spreadsheet: GASPARS1.xls and the macrosheet: GASP-2.xlm for vertical loading cases; or open the spreadsheet: Tor-A.xls, and the macrosheets: Tor-B.xlm, and Tor-C.xlm for torsional case.
3. input the necessary parameters including pile length $L$ (m), diameter $d$ (m), soil shear modulus $G_{it}$ (MPa), pile Young’s modulus $E_p$, ratio of the initial shear modulus and ultimate shear stress $G_{it}/\tau_u$, and the corresponding creep parameters ($G_{it}/G_{i2}$, $G_{it}/\eta t$);
Appendix A

A.5  A Spreadsheet Load Transfer Analysis

(4) input the value for base property, R_b, which was defined originally for vertically loaded pile by Murff (1975) and rewritten as (Appendix C)

\[ R_b = \frac{P_b L}{[EA] w_b} \]  \hspace{1cm} (A-11)

where the base load P_b and the stiffness [EA] has been provided in Table A-i.

A.4 VERIFICATION OF THE PROGRAM

Comparisons between GASPILE analyses and the previous analyses for vertical loading and torsional loading are presented in Chapters 3, 5 and 8. However, as an illustration, an example analysis by GASPILE is provided here, in comparison with the results from RATZ analysis (Randolph, 1986).

The analysed pile is assumed of E_p = 20 GPa; L = 40 m, r_0 = 1.0 m, and embedded in a soil of shear modulus, G varying in such a way that the non-homogeneous factor equals 2/3. Given an average shear modulus, G_{ave} = kL, (k = the gradient of the linearly increasing shear modulus with depth), then the shear modulus at the ground level is, G_{io} = 1/2kL, and that at the pile tip level is, G_{IL} = 3/2kL. The value of pile-soil relative stiffness therefore equals \( \lambda = \frac{E_p}{G_{ave}} = 500/k \). For a number of \( \lambda \), the pile-head load-settlement relationships have been predicted by GASPILE program, and are shown in Fig. A-3, together with the results by RATZ program. Obviously, as would have expected, the two programs gives reasonable consistent predictions.

A.5 SUMMARY AND CONCLUSIONS

Pile response could be readily predicted by simply changing the input data in the spreadsheet of the GASPILE program. The results are automatically generated and presented in form of both data and charts, which encompass:

(1) Pile-head load and settlement relationship.
(2) Load and displacement distribution down the pile.
(3) Pile shaft load transfer curves (up to 5 depths).

The program GASPILE may be modified to account for (1) negative friction caused by external subsidence; (2) the effect of the reconsolidation based on the theory presented in Chapter 6; (3) group pile interaction based on the theory illustrated in Chapter 7.
APPENDIX B

VERTICAL PILES IN HOMOGENEOUS SOIL

This appendix shows that the solutions for a pile in a homogeneous soil published previously can be achieved readily from the new theory established the Chapter 3.

B.1 ELASTIC SOLUTION

For a pile in an ideal non-linear homogeneous soil subject to vertical loading, since \( n = 0 \), the coefficients in Eq. (3-27) can be simplified as following

\[
\begin{align*}
C_1(z) &= C_4(z) = \frac{1}{k_s L} \sqrt{\frac{L}{z}} \sinh k_s (L - z) \\
C_2(z) &= C_3(z) = \frac{1}{k_s L} \sqrt{\frac{L}{z}} \cosh k_s (L - z)
\end{align*}
\]

Shaft displacement, \( w(z) \), and axial load, \( P(z) \), of the pile body at depth of \( z \) are expressed as

\[
\begin{align*}
w(z) &= w_b (\cosh k_s (L - z) + \chi_v \sinh k_s (L - z)) \\
P(z) &= k_s E_p A_p w_b (\chi_v \cosh k_s (L - z) + \sinh k_s (L - z))
\end{align*}
\]

where \( E_p A_p \) is the cross-sectional rigidity of an equivalent solid cylinder pile. Supposing load acted on the pile head is \( P_t \), thereby a clear understanding of the relationship among the force on a pile base and head, the base settlement, \( w_b \) and the shaft (base) settlement ratio can be established as by Eq. (B-3)

\[
P_b \cosh \beta + k_s E_p A_p w_b \sinh \beta = P_t
\]

Note that on the head of a pile (\( z = 0 \)), from Eq.(3-23), pile settlement \( w_t \)

\[
w_t = w_b (\cosh \beta + \chi_v \sinh \beta)
\]

where \( \beta = k_s L \). With Eq.(3-28), the non-dimensional relationship between the head load \( P_t \) (hence deformation, \( w_p \)) and settlement \( w_t \) is derived as
where \( \frac{w_p}{w_t} = \frac{P}{(\tan \beta + \chi_v) / (\chi_v \tan \beta + 1)} \) (B-6)

where \( w_p = \frac{P_t L}{E_p A_p} \). Eq. (B-6) can be expanded to that given by Randolph and Wroth, (1978), in which \( \beta \) is equal to the "\( \mu L \)" shown in their paper and \( \chi_v \) should be replaced with Eq. (3-24).

### B.2 ELASTIC-PLASTIC SOLUTION

Within the elastic-plastic stage, in terms of Eqs. (B-6), and (3-11), the pile load at transition depth is easily derived

\[
P_e = \frac{\pi d t_f L}{\beta} \left( \frac{\tan \beta + \chi_v}{\chi_v \tan \beta + 1} \right) \quad (B-7)
\]

where \( \beta = k_s L_2 = \beta (1 - \mu) \); for plastic zone \( 0 < z < L_1 \). Considering Eqs. (3-30) and (B-7), it follows that

\[
P_t = \pi d t_f L \left( \mu + \frac{1}{\beta} \frac{\tan \beta + \chi_v}{\chi_v \tan \beta + 1} \right) \quad (B-8)
\]

In terms of Eqs. (3-33) and (3-11), the pile-head settlement can also be written as

\[
w_t = w_e \left( 1 + \frac{\beta^2 \mu^2}{2} + \beta \mu \frac{\tan \beta + \chi_v}{\chi_v \tan \beta + 1} \right) \quad (B-9)
\]
C.1 INTRODUCTION

In this appendix, new closed form, non-dimensional elastic-plastic solutions for a pile in a non-homogeneous, stress-strain softening soil have been established.

C.2 LOAD TRANSFER ANALYSIS

C.2.1 The Soil Concerned

The distribution of the initial soil shear modulus, $G_i$ down a pile is assumed as a power function of depth (except it needs to be stressed that the subscript "i" will be dropped)

$$G_i = A_g z^n$$  \hspace{1cm} (C-1)

where $z$ is the depth below the ground surface; $A_g$ and $n$ are constants. The limiting shear stress with depth can be assumed as (Chapter 3)

$$\tau_f = A_\nu z^\theta$$  \hspace{1cm} (C-2)

where $A_\nu$ and $\theta$ are constants for limiting shear stress distribution. The $\theta$ is supposed to be equal to $n$, and called non-homogeneity factor.

C.2.2 Load Transfer Models

The shaft displacement may be approximated by the following expression (Randolph and Wroth, 1978)

$$w = \frac{\tau_o r_o \zeta}{G_i}$$  \hspace{1cm} (C-3)

where $w$ is the local shaft displacement; $\zeta$ is the shaft load transfer factor as detailed in Chapter 4. $\tau_o$ is the local shaft shear stress and $r_o$ is the pile radius. When the shaft stress exceeds $\tau_f$, the shear stress is kept as $\xi \tau_f$. $\xi$ ($0 < \xi \leq 1$) is the stress softening factor as defined by Murff (1975). Generally, $n = \theta$, $G_i/\tau_f$ is a constant (Chapter 3). Thus, the limiting shaft displacement, $w_e$, determined by replacing $\tau_o$ with $\tau_f$ in Eq. (C-3), is linearly proportional to the pile shaft radius and the shaft load transfer factor, $\zeta$. Since the factor $\zeta$ can be regarded as a constant over pile length (Chapter 4), accordingly, $w_e$ is a constant over the length.
Appendix C  

C.2 Non-dimensional Solutions

At the base of the pile, the elastic load-deformation relationship can be given by (Randolph and Wroth, 1978)

\[ w_b = \frac{P_b(1 - \nu_s)\omega}{4\tau_0 G_{ib}} \] (C-4)

where \( w_b \) is the base displacement; \( \nu_s \) is the soil Poisson's ratio; \( G_{ib} \) is the shear modulus just below the pile base level; \( P_b \) is the mobilised base load; \( \omega \) is the base load transfer factor, as detailed in Chapter 4.

C.3 NEW CLOSED FORM SOLUTIONS

The basic differential equation governing the axial deformation is derived as following for the elastic case (Murff, 1975)

\[ \frac{d^2 w}{dz^2} = \frac{\pi d}{E_p A_p w_e} \tau_f \] (C-5)

where \( E_p \) and \( A_p \) are the elastic modulus and a cross-sectional area of an equivalent solid pile respectively; \( d \) is the diameter of the pile. When any external subsidence is ignored, the axial pile displacement should equal the pile-soil relative displacement, \( w \) predicted by Eq. (C-3).

C.3.1 Elastic Solution

Introducing non-dimensional parameters, Eq. (C-5) can be transformed into

\[ \frac{d^2 \pi_1}{d\pi_2^2} = \pi_3^6 \pi_2^2 \pi_1 \] (C-6)

where \( \pi_1 = w/d \), \( \pi_2 = z/L \) \((0 < z \leq L)\), \( \pi_3 = \pi d A_v L^{2+\theta}/(E_p A_p w_e) \), and \( L \) is the pile length. Therefore, Eq. (C-6) can be solved as modified Bessel functions, \( I \) and \( K \) of the non-integer order \( m \) and \( m-1 \).

\[ \pi_1 = \pi_2^{1/2} \left( A_1 I_m(y) + B_1 K_m(y) \right) \] (C-7)

\[ \frac{d\pi_1}{d\pi_2} = \sqrt{\pi_3} \pi_2^{(1+\theta)/2} \left( A_1 I_{m-1}(y) - B_1 K_{m-1}(y) \right) \] (C-8)

where \( m = 1/(n + 2) \); \( y = 2m \sqrt{\pi_3} \pi_2^{1/2m} \). \( A_1 \) and \( B_1 \) are constants determined by boundary conditions. From Eq. (C-8), the pile-head load, \( P_t \) can be expressed as
Appendix C

C.3 Non-dimensional Solutions

\[ P_t = \frac{E_p A_p}{L} \left[ \left( \pi_2 |_{z=z_t} \right)^{n+1/2} \left( A_1 I_{m-1}(y_1) - B_1 K_{m-1}(y_1) \right) \right] \quad (C-9) \]

where \( y_t = y |_{z=z_t} \). The load at the pile base, \( P_b \) can be obtained from Eq. (C-7)

\[ P_b = R_b w_b = R_b d \pi_1 |_{z=L} = R_b w_b \left( \pi_2 |_{z=L} \right)^{1/2} (A_1 I_m + B_1 K_m) \quad (C-10) \]

where \( I_m \) and \( K_m \) are the values of the modified Bessel functions for \( z = L \). \( R_b \) is the base stiffness, which has been defined earlier by (Murff, 1975) as

\[ R_b = \frac{P_b}{w_b \frac{L}{E_p A_p}} \quad (C-11) \]

In terms of Eq. (C-4), it can be rewritten in the non-dimensional form as

\[ R_b = \frac{8}{(1 - \nu_s) \pi \omega} \frac{1}{\lambda \xi_b} \frac{L}{2 \tau_o} \quad (C-12) \]

where \( \lambda = E_p / G_\infty \), as the ratio of \( E_p \), and the shear modulus at pile base level, \( G_\infty \); \( \xi_b \) is taken as 1 for the current analysis. The coefficients \( A_1, B_1 \) are obtained as

\[ A_1 = -\frac{P_t L}{E_p A_p \sqrt{\pi_3}} \left( \frac{R_b K_m - \sqrt{\pi_3 K_{m-1}}}{R_b I_m + \sqrt{\pi_3 I_{m-1}}} \right) C_1 \quad (C-13) \]

\[ B_1 = \frac{P_t L}{E_p A_p \sqrt{\pi_3}} \frac{1}{C_1} \quad (C-14) \]

with \( C_1 \) being given by

\[ C_1 = \left( \pi_2 |_{z=z_t} \right)^{n+1/2} \left( \frac{R_b K_m - \sqrt{\pi_3 K_{m-1}}}{R_b I_m + \sqrt{\pi_3 I_{m-1}}} \left( y_1 + K_{m-1}(y_1) \right) \right)^{-1} \quad (C-15) \]

Substituting \( A_1 \) and \( B_1 \) into Eq. (C-7), the non-dimensional displacement, \( \pi_1 \) at a depth, \( \pi_2 \) can be derived as a function of the pile-head load

\[ \pi_1 = \frac{P_t L}{E_p A_p \sqrt{\pi_3}} \left( \frac{\pi_2 |_{z=z_t}^{1+n/2}}{\pi_2 |_{z=z_t}} \right) \frac{1}{C_v(z)} \quad (C-16) \]

where


\[ C_v(z) = \frac{C_1(z) + C_2(z) R_b / \sqrt{\pi_3}}{C_3(z) + C_4(z) R_b / \sqrt{\pi_3}} = n^{n/2} \]  

(C-17)

and

\[ C_1(z) = -K_{m-1}I_{m-1}(y) + K_mI_m(y) + K_{m-1}(y)I_{m-1} \]

\[ C_2(z) = K_{m-1}I_{m-1}(y) + K_mI_m(y) + K_{m-1}(y)I_{m-1} \]

\[ C_3(z) = K_{m-1}I_m(y) + K_m(y)I_{m-1} \]

\[ C_4(z) = -K_{m-1}I_m(y) + K_m(y)I_m \]  

(C-18)

The pile-head stiffness can be formulated as, from Eq. (C-16)

\[ \frac{P_1}{G_L w_t r_o} = \sqrt{\pi_3} \frac{r}{r_o} \frac{C_v(z_t)}{L} \]  

(C-19)

Within the elastic stage, the shaft displacement at the pile-head level, \( w_t \) equals the head settlement. For the pile head, the depth should be replaced with an infinitesimal small value \( z_t \). The sharing of the load between the pile base and head can be obtained as

\[ P_b = \frac{1}{P_1} \left[ \frac{C_1(L) + C_2(L) R_b / \sqrt{\pi_3}}{C_1(z_t) + C_2(z_t) R_b / \sqrt{\pi_3}} \right] \]  

(C-20)

The results from these non-dimensional solutions are identical to those from the dimensional solutions as shown in Chapter 3.

C.3.2 Plastic Solution

As the load increases, plastic yield is assumed to be initiated at the soil surface and propagates down the pile. Thus in the general case, a transitional depth, \( L_t \) exists along the pile at which the soil displacement, \( w \) equals \( w_e \), above which the soil resistance is plastic, below which it is elastic. For the upper plastic zone, the governing differential Eq. (C-6) reduces to

\[ \frac{d^2 \pi_1}{d\pi_{2p}^2} = \pi_4 \pi_{2p}^{\theta} \]  

(C-21)

where \( \pi_1 = w/d \), \( \pi_{2p} = z/L_t \), \( L_t \) is the length of the upper plastic zone; and \( \pi_4 = \pi A_p L_t^{2+\theta} / (E_p A_p) \). Integration of Eq. (C-21) leads to
Appendix C

C.5 Non-dimensional Solutions

\[ \pi_1 = \frac{\pi_4}{(1+\theta)(2+\theta)} \pi_{2p}^{2+\theta} + C_1 \pi_{2p} + C_2 \]  \hspace{1cm} (C-22)

The \( \pi_4 \) is positive where the pile is in compression and negative where it is in tension. In terms of the boundary conditions: (1) at the pile head, \( P(\pi_{2p})|_{z=L_1} = P_t \), and (2) at the transition depth, \( \pi_1|_{z=L_1} = \pi_1^* \). Therefore it follows that

\[ \pi_1 = \frac{\pi_4}{(1+\theta)(2+\theta)} \left( \pi_{2p}^{2+\theta} - 1 \right) + \frac{P_t L_1}{E_p A_p d} \left( \pi_{2p} - 1 \right) + \pi_1^* \]  \hspace{1cm} (C-23)

where \( \pi_1^* = w_e/d \).

C.3.3 Combined Solutions

It is convenient to express the load on the pile head \( P_t \), as a fraction of the ultimate adhesion or friction load, \( P_{fS} \), i.e. \( P_t = n_p \pi d A_v L^{1+\theta}/(1+\theta) \) (\( n_p \) is a ratio describing the mobilisation of pile shaft capacity). For the stress softening model described by Murff (1975, 1980), that is, once \( w > w_e \), the limiting shaft stress is replaced with the product of \( \xi \) and \( \tau_f \). The \( \pi_4 \) should be replaced with the product of the softening factor, \( \xi \) and \( \pi_4 \). Therefore, the pile-head deformation determined from Eq. (C-23) may be rewritten in the following form

\[ \frac{w}{w_e} = 1 - \frac{\xi \pi_4}{(1+\theta)(2+\theta)} \left( \frac{L_1}{L} \right)^{2+\theta} + \frac{n_p}{1+\theta} \pi_3 \left( \frac{L_1}{L} \right) \]  \hspace{1cm} (C-24)

At the depth of elastic-plastic interface (\( z = L_1 \)), the pile load, \( P_e \) can be estimated as

\[ P_e = P_t - \frac{\pi d A_v \xi}{1+\theta} L_1^{1+\theta} = \frac{\pi d A_v L_1^{1+\theta}}{1+\theta} \left( n_p - \left( \frac{L_1}{L} \right)^{1+\theta} \xi \right) \]  \hspace{1cm} (C-25)

and the displacement can be estimated by Eq. (C-16)

\[ \frac{w_e}{d} = \frac{P_e L}{E_p A_p d \sqrt{\pi_3} C_v(L_1)} \]  \hspace{1cm} (C-26)

Therefore, the ratio \( n_p \) for the capacity is (\( n = \theta \))

\[ n_p = \left( \frac{L_1}{L} \right)^{1+\theta} \xi + \frac{1+\theta}{\sqrt{\pi_3}} C_v(L_1) \]  \hspace{1cm} (C-27)
where $\mu = L_1/L$ is called the degree of slip. The non-dimensional load-settlement curves ($P_f$, ultimate pile frictional load) has been plotted in Fig. C-1. The load ratio is significantly influenced by the stiffness ratio, $\pi_3$ and the slip degree (Fig. C-2, $\xi = 1$). At some values of $\mu$ and $\xi$, $n_p$ reaches its maximum, $n_{\text{max}}$, which can be determined through the derivative of $n_p$ with respect to $\mu$

$$\frac{dn_p}{d\mu} = (1 + \theta) \left( \mu^{\theta} \xi v + L \frac{d}{dL_1} \left( \frac{C_v(L_1)}{\sqrt{\pi_3}} \right) \right)$$

(C-28)

Fig. C-3 shows that influence of the softening factor, $\xi$ on the maximum ratio of $P_f/P_b$ ($n_{\text{max}}$) for different relative stiffness $\pi_3$.

The CF load transfer analysis is determined by the ultimate shaft displacement, $w_e$ and base factor $R_b$, which in turn depend on the shaft ($\zeta$) and base ($\omega$) load transfer factors respectively.
APPENDIX D DETERMINATION OF CREEP PARAMETERS

This appendix shows how to back-estimate creep parameters from a maintained pile loading test by matching the time dependent settlement with that predicted by the theoretical load transfer model. From Eqs. (5-12) and (5-25), the creep settlement rate may be expressed as

$$\frac{dw_c}{dt} = \left[ \frac{1}{T} \exp\left(-\frac{t}{T}\right) \right] \left[ \frac{\tau_s \rho_2}{G_{12}} \zeta_2 \right]$$

(D-1)

For a given sustained load at the pile top, the variation of the creep settlement rate $\log(dw_c/dt)$ can be plotted against time. The response can be fitted by Eq. (D-1), and usually results in a straight line. Thus creep parameters are back estimated. An example is illustrated below:

A pile called pile I was tested in clay up to failure in an increment sustained tensile loading pattern (Ramalho Ortigao and Randolph, 1983). It was a closed ended steel pipe pile of 203 mm diameter and 6.4 mm wall thickness driven 9.5 m into a stiff overconsolidated clay. Young's modulus for the pile body was $2.1 \times 10^5$ MPa. Soil shear modulus was about 12 MPa from back estimation with the load settlement curve, and the failure shaft friction was about 41.5 kPa for the pile. The creep parameters for this pile I has been back-figured as illustrated below.

For estimation of the non-linear elastic load transfer measure, $\zeta$, an average pile stress level is used. From the loading tests, the ultimate load of pile I is 280 kN; thus the corresponding stress (load) level for the pile under loads of 200 and 240 kN would be 0.714 and 0.857 respectively. With the stress levels, the pile geometry, a soil Poisson's ratio of 0.3, the non-linear elastic measure, as predicted by Eq. (5-14), is 6.35 at load level 1 of 200 kN (referred to as $\xi_2 \zeta_1$) and 7.04 at level 2 of 240 kN (referred to as $\xi_2 \zeta_2$) respectively.

Based on the measurement of pile I by Ramalho Ortigao and Randolph (1983), a plot of the log creep settlement rate and time relationship shown in Fig. 5-7 demonstrates that for the pile under two different loading levels of 200 and 240 kN, the corresponding relaxation times, $1/T_1$ and $1/T_2$ are equal to $6.64 \times 10^{-6}$ and $3.6 \times 10^{-6}$ s$^{-1}$ respectively. The intersections in the creep settlement rate ordinate for the two loading levels are 0.00018 and 0.00035 mm/min.
In terms of these parameters and Eq. (D-1), at loading level 1,

\[
\left( \xi_2 \right)_1 \left( \frac{\tau_0 r_0}{G_{i1}} \right)_1 \left( \frac{G_{i1}}{G_{i2}} \right)_1 \frac{1}{T_1} = 0.00018 \text{ (mm/min)} \quad (D-2)
\]

with \( \psi_1 = 0.714 \), \( (\xi_2)_1 = 6.35 \), \( r_0 = 101.5 \text{ mm} \), \( 1/T_1 = 6.64 \times 10^{-6} \text{s}^{-1} \) and \( \tau = 41.5 \text{ kPa} \) therefore \( G_{i1}/G_{i2} = 0.2839 \), \( (G_{i2})_1 = 42.27 \text{ MPa} \).

At loading level 2,

\[
\left( \xi_2 \right)_2 \left( \frac{\tau_0 r_0}{G_{i1}} \right)_2 \left( \frac{G_{i1}}{G_{i2}} \right)_2 \frac{1}{T_2} = 0.00035 \text{ (mm/min)} \quad (D-3)
\]

with \( \psi_2 = 0.857 \), \( (\xi_2)_2 = 7.04 \), \( r_0 = 101.5 \text{ mm} \), \( 1/T_2 = 3.6 \times 10^{-6} \text{s}^{-1} \) and \( \tau = 41.5 \text{ kPa} \) therefore \( G_{i1}/G_{i2} = 0.7653 \), \( (G_{i2})_2 = 15.68 \text{ MPa} \). The initial shear modulus, \( G_{i1} \) generally increases with the process of consolidation of the soil, but can be regarded as a constant, once the primary consolidation is complete. The creep parameters, \( G_{i2} \) and \( \eta \) normally vary with the loading (hence stress) level (Fig. 5-7). For this particular example, a value of 2.69 is obtained for the ratio of delayed shear moduli between load level 1, \( (G_{i2})_1 \) and level 2, \( (G_{i2})_2 \). However, the ratio of \( G_{i1}/G_{i2} \) and \( G_{i2}/\eta \) are nearly constants within normal working load level, e.g. less than 70% of failure load level, of a pile of normal length. At higher load levels or for a long pile, the ultimate shaft stress for spring 2 is normally about 70% of that of spring 1, therefore, a higher value of \( \xi_2 \) than that of \( \xi_1 \) is generally resulted even if the pile does not yield, which may accompany by a higher value of \( G_{i1}/G_{i2} \). The ratio, \( G_{i2}/\eta \) influences the duration of creep time rather than the final pile head response. Therefore it can roughly be taken as a constant over the zone of general working load.
APPENDIX E  RADIAL CONSOLIDATION

This appendix gives

(1) the solution of the time-dependent equation, Eq. (6-17) for visco-elastic case;
(2) elastic solution for radial non-homogeneous case; and
(3) elastic solution for radial non-homogeneous case, with logarithmic variation of
initial pore pressure distribution.

E.1 SOLUTION FOR THE TIME-DEPENDENT EQ. (6-17)

The following time-dependent governing equation is solved in this section.

\[
\frac{dT_n(t)}{dt} + G_{\gamma 1} \int_0^t \left( \frac{dF(t-\tau)}{d(\tau-\tau)} \right) d\tau + \alpha_n^2 T_n(t) = 0
\]  
(E-1)

In terms of Laplace transform, it follows

\[
[sT_n - T_n(0)] + G_{\gamma 1} [(sT_n - T_n(0))(sF-F(0))] + \alpha_n^2 T_n = 0
\]  
(E-2)

Due to \(T_n(0) = 1\), Eq. (E-2) can be written as

\[
\bar{T}_n = \frac{G_{\gamma 1}(sF-F(0)) + 1}{G_{\gamma 1}(s^2 F-sF(0)) + \alpha_n^2 + s}
\]  
(E-3)

The flexibility factor, \(F(t)\), is given by Eq. (6-2), hence, the Laplace transform of \(F(t)\) is

\[
\bar{F} = \frac{1}{G_{\gamma 1}} \left[ \frac{1}{s} + m_2 \left( \frac{1}{s} - \frac{1}{s+1/T_2} \right) + m_3 \left( \frac{1}{s} - \frac{\exp(t_k/T_3)}{s+1/T_3} \right) \right]
\]  
(E-4)

Therefore

\[
G_{\gamma 1}(s^2 \bar{F} - F(0)) = \frac{m_2/T_2}{s+1/T_2} + \frac{s\alpha_k m_3 + m_3/T_3}{s+1/T_3}
\]  
(E-5)

This equation enables Eq. (E-3) to be written as

\[
\bar{T}_n = \frac{m_2/(T_2 s+1) + (s\alpha_k m_3 T_3 + m_3)/(sT_3 + 1) + 1}{s[m_2/(T_2 s+1) + (s\alpha_k m_3 T_3 + m_3)/(sT_3 + 1)] + \alpha_n^2 + s}
\]  
(E-6)

To facilitate the inverse Laplace transform, Eq. (E-6) is rewritten as
Appendix E

E.2 Radial Consolidation

\[
\overline{T}_n = \frac{s^2 + H_n s + I_n}{(s - a_n)(s - b_n)(s - c_n)} \tag{E-7}
\]

where all the parameters have been defined in the Chapter 6. Eq. (E-7) may be divided into three parts according the denominator, for each parts, an inverse Laplace transform can be readily obtained (Abramowitz and Stegun, 1964). Finally, Eq. (E-7) can be transformed into Eq. (6-24).

If \( m_3 = 0, \frac{1}{T_3} = 0, \alpha_k = 0 \), then

\[
\frac{s + (1 + m_2) / T_3}{s^2 + (\alpha_n^2 + (m_2 + 1) / T_2) s + \alpha_n^2 / T_2} = \frac{s + (1 + m_2) / T_3}{s^2 + (m_2 + 1) / T_2} \tag{E-8}
\]

The inverse of Eq. (E-8) is Eq. (6-26). Otherwise, if \( m_2 = m_3 = 0, \frac{1}{T_2} = 0, \alpha_k = 0 \), then Eq. (E-6) reduces to the following format

\[
\overline{T}_n = \frac{1}{s + \alpha_n^2} \tag{E-9}
\]

The inverse of Eq. (E-9) is Eq. (6-30).

E.2 SOLUTION FOR RADIAL NON-HOMOGENOUS CASE

For elastic case, from Eq. (6-2), \( \frac{dF}{dt} = 0 \). Therefore, if taking \( G_{\gamma 1} = G_{\gamma o} r^\kappa \), \( G_{\gamma o} = \) shear modulus at pile-soil interface. In the following parts, the subscript “ro” may be dropped, unless it need stressing; \( \kappa \) is a constant, \( 0 \leq \kappa \leq 1 \), Eqs. (6-16) and (6-17) may be rewritten as

\[
\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \lambda_n^2 r^{-\kappa} w = 0 \tag{E-10}
\]

\[
\frac{dT(t)}{dt} + \alpha_n^2 T(t) = 0 \tag{E-11}
\]

where

\[
\alpha_n^2 = \lambda_n^2 c_v \tag{E-12}
\]

\[
c_v = \frac{k}{r_w} \frac{G_m}{1 - 2\nu_s} \tag{E-13}
\]

The parameter, \( \lambda_n \), is one of the infinite roots satisfying Eq. (E-10), which may be expressed in terms of the Bessel functions as
where $A_n$ is dependent on the boundary conditions. The functions $J_o$, $Y_o$, $J_1$, $Y_1$ are Bessel functions of zero order and first order, with $J_1$ being Bessel functions of the first kind, and the $Y_1$ being Bessel functions of the second kind. The variable, $y$, is given by

$$y = \frac{2}{2-\kappa} r^{(2-\kappa)/2}$$

and the corresponding boundary value at $r = r_o$ is

$$y|_{r=r_o} = y_o = \frac{2}{2-\kappa} r_o^{(2-\kappa)/2}$$

Cylinder functions, $V_i(\lambda_n y_o)$ of i-th order (McLachlan, 1955) may be expressed as

$$V_i(\lambda_n y) = J_i(\lambda_n y) - \frac{J_1(\lambda_n y_o)}{Y_1(\lambda_n y_o)} Y_i(\lambda_n y)$$

Based on the boundary condition of Eq. (6-14b), $B_n = -A_n J_1(\lambda_n y_o)/Y_1(\lambda_n y_o)$. Thus, from Eq. (E-14),

$$w_n(y) = A_n V_o(\lambda_n y)$$

$$\frac{dw_n(y)}{dr}\bigg|_{y=y_o} = A_n V_1(\lambda_n y)|_{y=y_o} = 0$$

Also, with Eq. (6-14c), $u = 0$ for $y \geq y^* (r \geq r^*)$, it follows

$$V_o(\lambda_n y^*) = J_o(\lambda_n y^*) - \frac{J_1(\lambda_n y_o)}{Y_1(\lambda_n y_o)} Y_o(\lambda_n y^*) = 0$$

Eqs. (E-19) and (E-20) render the cylinder functions to be defined.

The solution of Eq. (E-11) is

$$T_n(t) = e^{-\alpha_n^2 t}$$

The full expression for pore pressure, $u$ will be a summation of all the possible solutions

$$u = \sum_{n=1}^{\infty} A_n V_o(\lambda_n y) T_n(t)$$
Appendix E

E.4 Radial Consolidation

Normally the first 50 roots of the Bessel functions are found to give sufficient accuracy. With Eqs. (6-14a) and (E-22), it follows

\[
A_n = \int_{y_0}^{y^*} u_0(y) V_0(y \lambda_n) y dy \int_{y_0}^{y^*} V_0^2(y \lambda_n) y dy
\]  

(E-23)

E.3 CONSOLIDATION FOR LOGARITHMIC VARIATION OF \( u_o \)

The initial stress state for radial consolidation of an elastic non-homogeneous medium around a rigid, impermeable pile is similar to that of an elastic medium, which are described below:

(1) The width of the plastic zone given by Eq. (6-38) may be rewritten as

\[
y_L = y_R = \frac{2}{2 - \kappa} \left( \frac{r_o \sqrt{G/s_o}}{(2 - \kappa)^{1/2}} \right)
\]  

(E-24)

(3) The initial excess pore pressure distribution by Eq. (6-39) may be rewritten as

\[
u_o(r) = \frac{4}{2 - \kappa} s_u \ln \left( \frac{y_R}{y_o} \right) \quad y_o \leq y \leq y_R
\]

\[
u_o = 0 \quad y_R < y < y^*
\]  

(E-25)

where \( y_R \) is the radius, beyond which the excess pore pressure is initially zero.

In light of the initial pore pressure distribution of Eq. (E-25), the coefficients, \( A_n \), can be simplified as

\[
A_n = \frac{2}{2 - \kappa} \frac{4s_u}{\lambda_n^2} \frac{V_o(\lambda_n y_o) - V_o(\lambda_n y_R)}{y^2 V_1(\lambda_n y^*) - y_o^2 V_0(\lambda_n y_o)}
\]  

(E-26)

With these values of \( A_n \), the pore pressure can be readily estimated using Eq. (E-22). Fig. E-1 shows an example of the effect of radial non-homogeneity on the dissipation of excess pore pressure. It demonstrates that radial non-homogeneity has significant effect on the value of the ratio, \( u(r_0)/s_o \), during the initial stage of the consolidation, but has negligible effect at the late stage of the consolidation process.
APPENDIX F  TORQUE AND TWIST PROFILE

This appendix shows the closed form solutions for the assessment of the torque and twist angle profile along a pile, the consistency of the stiffness prediction of the solutions shown in Chapter 8 with the previous solution for homogeneous cases.

Profiles of torque and twist angle down the pile, for elastic conditions have been formulated as

\[ \phi(z) = \left( \frac{z}{L} \right)^{1/2} \phi_b \left[ \frac{C_3(z) + \chi C_4(z)}{C_3(L)} \right] \]  \hspace{1cm} (F-1)

\[ \frac{T(z)}{(GJ)_p} = \pi_{t}^{1/2m} \left( \frac{z}{L} \right)^{(1+n)/2} \phi_b \left[ \frac{C_1(z) + \chi C_2(z)}{C_3(L)} \right] \]  \hspace{1cm} (F-2)

Within the plastic zone, it follows that

\[ T(z) = T_e + (GJ)_p \phi_e \pi_{t}^{1/2m} \frac{L_1^{1+n} - z^{1+n}}{1 + n} \]  \hspace{1cm} (F-3)

\[ \phi(z) = \phi_e + \frac{L_1 - z}{(GJ)_p} \phi_e \pi_{t}^{1/2m} \frac{(1 + n)L_1^{2+n} - (2 + n)zL_1^{1+n} + z^{2+n}}{(1 + n)(2 + n)} \]  \hspace{1cm} (F-4)

If the non-homogeneity factor \( n = 0 \), the coefficients can be simplified as follows

\[ C_1(z) = C_4(z) = \frac{1}{\pi_t} \sqrt{\frac{L}{z}} \sinh \pi_t \left( 1 - \frac{z}{L} \right) \]  \hspace{1cm} (F-5)

\[ C_2(z) = C_3(z) = \frac{1}{\pi_t} \sqrt{\frac{L}{z}} \cosh \pi_t \left( 1 - \frac{z}{L} \right) \]

Therefore, shaft rotation, \( \phi(z) \), and axial torque, \( T(z) \), of the pile body at depth of \( z \) are expressed respectively as

\[ \phi(z) = \phi_b \left[ \cosh \pi_t \left( 1 - \frac{z}{L} \right) + \chi \sinh \pi_t \left( 1 - \frac{z}{L} \right) \right] \]  \hspace{1cm} (F-6)

\[ T(z) = \frac{(GJ)_p \pi_{t}}{L} \phi_b \left[ \sinh \pi_t \left( 1 - \frac{z}{L} \right) + \chi \cosh \pi_t \left( 1 - \frac{z}{L} \right) \right] \]  \hspace{1cm} (F-7)
From the definition shown in Table 8-1 column 3 (Chapter 8), with Eq. (8-28), the pile-head stiffness can be rewritten as

\[
\frac{T_t}{G_t r_o^3 \phi_t} = \frac{(GJ)_p \pi_t \chi + \tanh(\pi_t)}{A_g r_o^3 L (1 + \chi \tanh(\pi_t))} \tag{F-8}
\]

Using the relationship between \(\pi_t\) and \(\lambda\) relationship shown in Table 8-1 and Eq. (8-27), the form of Eq. (F-8) can be transformed into an identical one to that for the linear case obtained by Randolph, (1981).
REFERENCES


References


R.10 References


Randolph, M. F. (1986). *RATZ, Load Transfer Analysis of Axially Loaded Piles*. Dept. of Civil Engrg. The University of Western Australia.


(a) Idealization of the pile-soil

(b) Shaft load transfer curve

Fig. 2-1 Load transfer approach (Seed and Reese, 1957)

Fig. 2-2 Schematic of t(\tau_0)-z(w) curves (Armaleh and Desai, 1986)
Fig. 2-3 Theoretical load transfer approach

Fig. 2-4 Modelling effect of soil creep by RATZ (Randolph, 1991)
Steel pipe pile
\( r_0 = 2 \text{ m} \)
50 mm wall thickness

Fig. 2-5 Variation of the load capacity reduction factor with pile length

For pile of
\( r_0 = 0.5 \) to \( 4.0 \) m

Dimensionless axial stiffness,
\[ K_{ax} = \frac{E_p A_p}{A_g L^3} \]

Fig. 2-6 Load capacity reduction factor as a function of the dimensionless axial stiffness (Poulos, 1982)
Fig. 2-7 Extended idealised \( \tau-z \) curve for piles in rock (Kodikara and Johnston, 1994)

Fig. 2-8 Conditions for the elastic-elastoplastic-plastic case (Kodikara and Johnston, 1994)
Interactive shear stresses acting on a soil

Interaction shear stresses acting on a pile

Distribution of soil with depth

Fig. 2-9 Analysis of a single pile (Poulos, 1979)
Fig. 2-10 Influence of non-homogeneity on interaction shear stress distribution along a pile (Poulos, 1979)
Shaft Friction Pattern

Ground surface

Ratio of \( \frac{L_1}{L} \)

Pile tip

Ratio of \( \frac{\tau_1}{\tau_2} \)

Fig. 2-11 Coefficient for various distributions of unit shaft friction (Leonards and Lovell, 1979)
Fig. 2-12  Coefficient for various distributions of unit shaft friction (Leonards and Lovell, 1979)
Fig. 2-13 Visco-elastic models adopted by Murayama and Shibata (1961)

Fig. 2-14 Rheological model adopted by Christensen and Wu (1964)

(a) Volumetric Kelvin model  (b) Deviatoric Maxwell model

Fig. 2-15 Visco-elastic models adopted by Soydemir and Schmid (1970)
Fig. 2-16 Rheological model for soil behaviour
(Komamura and Huang, 1974)

Fig. 2-17 Load transmitting area for (a) single pile, (b) pile group (Kaniraji, 1993)
Fig. 2-18 Model for two piles in layered soil (Guo et al. 1987)

Fig. 2-19 Discrete element models of pile groups (Chow, 1986a)
Fig. 2-20  Effect of soil non-homogeneity profile on interaction factor
Shear modulus, MPa

EP = 30 GPa
\( \varphi = 0.2 \)
\( r_0 = 0.25 \text{ m} \)
\( \lambda = 1000 \)
\( G_{IL} = 30 \text{ MPa} \)

H/L = 4

Underlying rigid layer simulated by constraining the vertical displacement

Fig. 3-1 Example pile-soil system for FLAC analysis

<table>
<thead>
<tr>
<th>Normalized shaft stress level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>0.8</td>
</tr>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.0</td>
</tr>
</tbody>
</table>

Fig. 3-2 Comparison of load transfer behaviour estimated by non-linear and simple elastic-plastic approaches
Fig. 3-3 Comparison of pile load and displacement behaviour between the non-linear (NL) and linear (SL) analyses (L/r₀ = 100)
Fig. 3-4 Comparison of pile-head load settlement relationship among the non-linear and simple linear ($\psi = 0.5$) GASPILE analyses and the CF solution ($L/r_o = 100$)
Fig. 3-5 An example mesh for FLAC analysis ($L/r_o = 20$)
Fig. 3-6 Influence of the H/L on the pile-head stiffness
(L/r_o = 40, v_s = 0.4)
Fig. 3-7 Comparison of pile-head stiffness by FLAC, SA (A = 2.5) and CF (A = 2) analyses
Fig. 3-8 Comparison between the ratio of head settlement and base settlement by FLAC analysis and the CF solution
Fig. 3-9 Comparison between various analyses of single pile load-settlement behaviour

Fig. 3-10 Effect of base end-bearing factor on P-S response
Fig. 3-11 Effect of slip development on pile-head response (n = θ)
Fig. 3-12 Comparison of the settlement influence factor by various approaches

Soil modulus proportional to the depth \( n = 1 \)
\[ \lambda = 3000 \quad v_s = 0.5 \]
Fig. 3-13 Comparison of settlement influence factor from different approaches
Fig. 3-14 Comparison among different predictions for load settlement (measured data from Gurtowski & Wu, 1984)
Fig. 3-15 Comparison between the CF and the non-linear GASPILE analyses (case study)

Fig. 3-16 Comparison among different predictions of the load distribution
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Fig. 4-2 Boundary effect on shaft load transfer factor

(H/L = 4.0, v_s = 0.4, matching load ratio)
Fig. 4-3  Comparison of the effect of the two boundaries
\( (L/r_0 = 40, \nu_s = 0.4, \lambda = 1000) \)
Fig. 4-4  Comparison of the effect of the two boundaries

\((L/r_0 = 40, \nu_s = 0.4 \ \lambda = 1000)\)
Fig. 4-5 Effect of different back-estimation procedures for $\zeta$ on the pile response ($L/r_0 = 40$, $v_s = 0.4$, $H/L = 4$)
Fig. 4-6 Base load transfer factor vs slenderness ratio relationship

(H/L = 4.0, v_s = 0.4, \( \lambda = 1000 \))
Fig. 4-7 Base load transfer factor vs Poisson's ratio relationship
\((L/r_o = 40, H/L = 4)\)

Fig. 4-8 Base load transfer factor vs pile-soil relative stiffness relationship
\((L/r_o = 40, H/L = 4)\)
Fig. 4-9 Load transfer factor vs slenderness ratio
(H/L = 4, νs = 0.4)
Fig. 4-10  Shear influence zone vs Poisson's ratio relationship  
(H/L = 4, L/r₀ = 40, matching head stiffness)
Fig. 4-11  Load transfer factor vs H/L ratio
(L/r_o = 40, v_s = 0.4)
Fig. 4-12 Load transfer factor vs relative stiffness
\((v_s = 0.4, \ H/L = 4)\)
Fig. 4-13 Effect of the two boundary conditions on shaft load transfer behaviour (Case I)

\( L/r_0 = 40, \nu_s = 0.4, \lambda = 1000 \)
Fig. 4-14 Variation of the load transfer factor due to using unity and the realistic value for base factor ω
Fig. 4-15  Comparison between pile-head stiffness vs slenderness ratio relationship
Legend

CF
FLAC
\( n = 0, 0.25, 0.5, 0.75, 1.0 \)

Fig. 4-16 Pile-head stiffness vs Poisson's ratio relationship
\( (L/r_o = 40, H/L = 4) \)
Fig. 4-17 Pile-head stiffness vs the ratio of H/L relationship
\( (L/r_o = 40, \nu_s = 0.4) \)
Fig. 4-18 Comparison between current closed form analysis (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981)
Fig. 4-19 Effect of shear modulus distributions below the pile base level on the pile response
Fig. 4-20 Effect of shear modulus distribution below pile base level on the relationship of shear influence zone vs slenderness ratio
(a) Effect of slenderness ratios

(b) Effect of pile-soil relative stiffness

(c) Effect of Poisson's ratio of the soil
(d) Effect of Pile-soil relative stiffness

Fig. 4-21 Comparison of the current equation with those back-figured for soil shear modulus being constant below the pile base.
Fig. 4-22  Effect of the soil profiles below the pile base level 
(using fixed boundary, $L/r_0 = 40$, $v_s = 0.4$)
Fig. 4-23 Effect of pile-soil relative stiffness on the shaft load transfer factor, $A$
($H/L = 4, v_s = 0.4, L/r_o = 40$)
Fig. 5-1 Creep model and two kinds of loading adopted in this analysis
Fig. 5-2 Local stress displacement relationship for 1-step and ramp loading

Fig. 5-3 Stress local displacement relationship for ramp loading
$L/r_o = 50$
$t/T = \infty$
$R_f = 0.99$

$n$ as shown below:

- —— 0
- —— 0.33
- —— 1

Fig. 5-4 Non-linear measure variation with stress level

Modification factor $\xi_c$ for $G_{11}/G_{12} = 1$

Creep time factor, $t/T$

(a) Step loading only

Modification factor $\xi_c$ for $G_{11}/G_{12} = 3$

Creep time factor, $t/T$

(b) Ramp loading

Fig. 5-5 Modification factor of load transfer measure for non-linear viscoelastic case
Fig. 5-6 Comparison of predicted shaft creep displacement with test results (data cited from Edil & Mochtar, 1988)

Fig. 5-7 Evaluation of creep parameters from time settlement relationship (data cited from Ramalho-Ortigao and Randolph, 1983)
Fig. 5-8 Comparison of the settlement influence factor predicted by the numerical and closed form approaches

\[ J(t) = A_c + B_c e^{-t/T} \]

\( \lambda = 260 \)

\( \lambda = 2600 \)

\( L / r_o = 100 \)

\( J(\infty) / J(0) = 2 \)

Booker & Poulos (1976)

Present closed form
Fig. 5-9 Comparison of the ratio of pile head and base load

\[ J(t) = A_c + B_c e^{-t/T} \]

\[ L / r_o = 100 \]

\[ J(\infty) / J(0) = 2 \]

- Booker & Poulos (1976)
- Present CF (ignoring base creep)
- Present closed form solution


\[ J(t) = A_c + B_c e^{-t/T} \]

- Booker & Poulos (1976)
- Present CF (ignoring base creep)
- Present closed form

Fig. 5-10 Comparison of the settlement influence factor
Fig. 5-11 Comparison between closed form and GASPILE analyses for different values of creep parameter: $G_{i2}/G_{i1}$

- $G_{i1}/G_{i2} = 0$
- $G_{i1}/G_{i2} = 1$
- $G_{i1}/G_{i2} = 2$

- $L/L_0 = 50$
- $\lambda = 1500$, $Ag/Av = 350$
- $1/T = 0.5 \times 10^{-5}$ sec.
- $t_c = 2$ hrs. $t = 10$ hrs.
Fig. 5-12 The effect of loading time $t_c/T$ on settlement influence
Fig. 5-13 The effect of the loading time \( t_c \) on ratio of \( P_b/P_t \)

(a) Comparison among three different loading cases

(b) Influence of relative \( t_c/T \)
Fig. 5-14 Comparison between the measured (Konrad and Roy, 1987) and predicted load and initial settlement relationship
Fig. 5-15  Visco-elastic predictions of load settlement for the tests (33 days) by Konrad and Roy (1987)
(a) Comparison between the calculated and the measured for pile B1

(b) Load distribution down the pile

(c) Comparison of creep between the predicted and the measured

Fig. 5-16 Analysis of pile creep response (measured data from Bergdahl and Hult, 1981)
(a) Standard linear model (Mechant's model)  
(b) Generalised creep model

Fig. 6-1 Visco-elastic creep model for radial consolidation analysis

Soil is assumed to deform
(a) elastically or
(b) visco-elastically governed
by: (i) standard linear model
(ii) the generalised creep model

Fig. 6-2 Diagram of radial consolidation around a driving pile
Fig. 6-3 Influence of creep parameters on the excess pore pressure
\(G/s_u = 50\)
Fig. 6-4 Variation of times for 50 and 90% degree of consolidation with the ratio $u_0(r_0)/s_u$.
Fig. 6-5 Comparisons between the calculated and the measured by Trenter and Burt (1981) for pile 2.
Fig. 6-6 Comparisons between the calculated and the measured by Trenter and Burt (1981) for pile 4
Fig. 6-7 Comparisons between the calculated and the measured by Trenter and Burt (1981) for pile 3
Fig. 6-8  Comparison between the calculated and measured (Seed & Reese, 1955) load-settlement curves at different time intervals after driving
Fig. 6-9 The normalised measured time-dependent properties (Seed & Reese, 1955) compared with the theoretical decay of excess pore pressure

(a) Elastic analysis:
- $t_{90} = 8.76$ days
- $(\tau_n)_{90} = 11.6$ kPa
- $(G_{\gamma})_{90} = 3.55$ MPa

(b) Visco-elastic analysis:
- $t_{90} = 16.35$ days
- $(\tau_n)_{90} = 12.54$ kPa
- $(G_{\gamma})_{90} = 4.06$ MPa
(a) Elastic analysis: 
\( t_{90} = 18 \text{ days} \)
At the base level 
\((\tau_{\text{fl}})_{90} = 22.4 \text{ kPa} \)
\((G_{\text{fl}})_{90} = 4.79 \text{ MPa} \)

Elastic prediction
- Pore pressure at 3.05 m
- at 4.6 m
- at 6.1 m
- Pile-soil adhesion
- Shear modulus

(b) Visco-elastic analysis: 
\( t_{90} = 57 \text{ days} \)
At the base level 
\((\tau_{\text{fl}})_{90} = 23.99 \text{ kPa} \)
\((G_{\text{fl}})_{90} = 5.16 \text{ MPa} \)

Visco-Elastic
- Pore pressure at 3.05 m
- at 4.6 m
- at 6.1 m
- Soil Strength
- Shear modulus

Fig. 6-10 The normalised measured time-dependent properties (Konrad and Roy, 1987) compared with the theoretical decay of excess pore pressure
Fig. 6-11 Comparison of load settlement relationship predicted by visco-elastic GASPILE analysis with the measured (Konrad and Roy, 1987)
Fig. 6-12 Comparison of elastic and visco-elastic load settlement relationship predicted by GASPILE
Fig. 7-1 Effect of pile spacing and pile-soil stiffness ratios on interaction factors in homogenous soil
Fig. 7-2  Effect of pile spacing and pile-soil relative stiffness on interaction factor in Gibson soil
Fig. 7-3 Comparison of pile-head stiffness for three different pile groups in homogenous soil

\[ \frac{P_f}{G(L_0, r_0, w_0)} \]

- BI (Butterfield & Banerjee, 1971)
- Closed form

- 4 pile group
- 3 pile group
- 2 pile group

- \( \lambda = 6000, \frac{s}{r_0} = 5, \nu_s = 0.5 \)

- \( \lambda = \infty, \frac{s}{r_0} = 5, \nu_s = 0.5 \)
Fig. 7-4 Comparison of pile-head stiffness in homogeneous soil (3×3 pile group)
Fig. 7-5 Comparison of pile-head stiffness in Gibson soil
(3x3 pile group)
Fig. 7-6 Comparison of settlement ratios for pile groups

Fig. 7-7 Influence of pile compressibility on settlement ratios
Modified CF approach (Lee, 1993a)
Randolph approach (Poulos, 1989)
Poulos (1989)
Present CF

\[ w_G = \frac{P_G I_G}{dE_{sl}} \]

\( L/r_0 = 80, \ s/r_0 = 6 \)
\( v_s = 0.5, \ \lambda = 3000 \)

Fig. 7-8 Comparison between solutions for group settlement in Gibson soil
Fig. 7-9 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (2×1 pile group)
Fig. 7-10 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (2x2 pile group)
Fig. 7-11 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (3x3 pile group)
Fig. 7-12 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (4x4 pile group)
Fig. 7-13 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (8×8 pile group)
Fig. 7-14 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (4×2 pile group)
Fig. 7-15 Comparison between present solution (dashed line) and the numerical result (solid line) by Butterfield and Douglas (1981) (8×2 pile group)
(a) \[ \lambda = 1000, \; \nu_s = 0.5 \]
\[ n = 0, \; s/d = 2.5 \]

Limiting stiffness

Square root of number of piles in group

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Fig. 7-16 Comparison of different pile groups analysis procedures
Fig. 7-17 Comparison of the measured interaction factors (Cooke et al. 1979) with the closed form predictions.

Fig. 7-18 Comparison of the measured vertical displacement (Cooke et al. 1974) around a loaded pile with that from the closed form prediction.
(a) Equal pile load
43.7 kN per pile
\(A=1.66, \frac{H}{L}=2, n=0.85\)
\(A_g=12.48\text{MPa/m}^{0.85}\)
\(v_s=0.5\)

(b) Rigid cap
Total load = 94.1 kN
\(A=1.66, \frac{H}{L}=2, n=0.85\)
\(A_g=12.48\text{MPa/m}^{0.85}\)
\(v_s=0.5\)

Fig. 7-19 Comparison with field test results by Cooke (1974)
Fig. 7-20 Measured (Cooke et al. 1979, 1980) and predicted load-settlement behaviour of pile groups
Fig. 8-1 Variation of shear modulus away from torsional or axial loading pile axis

Fig. 8-2 Local load transfer behaviour for torsional and vertical loading cases
Fig. 8-3 Comparison of elastic influence factor between present closed form solution and the numerical approach
(a) Influence of pile slenderness ratio

(b) Comparison between the three different approaches

Fig. 8-4 Elastic influence factor vs the relative stiffness relationship
Fig. 8-5 Comparison of yield correction factor between present closed form solution and the numerical approach.
Fig. 8-6 Yield correction factor vs $T_T/T_u$ relationship

- $\pi_t$ as shown
  - $L/d = 5$
  - $L/d = 150$
Fig. 8-7 Effect of shaft soil non-linearity on torque and twist relationship

$\pi_t = 5.0 \quad L/d = 25$

For base $R_f = 0.95$
Shaft $R_f$ as shown in legend
Fig. 8-8 Comparison of load and angle of twist predicted by different methods and the measured

Fig. 8-9 Comparison of load and circumferential displacement profile down the pile (A-3) predicted by CF and GASPILE methods
Fig. A-1 Shaft pile-soil interaction model for visco-elastic soil response

Fig. A-2 Displacement prediction for the segment \( \overline{AC} \)

\[
Q_{n-1}
\]

\[
A
\]

\[
\Delta w_{AB}
\]

\[
B
\]

\[
\Delta w_{BC}
\]

\[
C
\]

\[
d_n
\]

\[
Q_n
\]

\[
\Delta w_{AB} = \frac{Q_{AB}L_{AB}}{[EA]_n}
\]

\[
Q_{AB} = \frac{(Q_A + Q_B)}{2}
\]

\[
\Delta w_{BC} = \frac{Q_{BC}L_{BC}}{[EA]_{n+1}}
\]

\[
Q_{BC} = \frac{(Q_B + P_b)}{2}
\]

\( A, C = \text{midpoint of segments of } n \text{ and } n+1 \)
Fig. A-3 Comparison between GASPILE and RATZ analyses for an ideal frictional pile in Gibson Soil (L/r₀ = 160, E_p = 20 GPa)
Fig. C-1  Non-dimensional pile head load-displacement relationship
\[ (\pi_3 = 2.5, R_b = 0, n \text{ different}) \]
Fig. C-2 Non-dimensional load ratio $P_t/P_f$ versus $\pi_3$ relationship ($\xi = 1, R_b = 0$)

Legend

CF

$n$ 0 0.25 0.5 0.75 1.0

Fig. C-3 $n_{\text{max}}$ versus $\pi_3$ relationship ($\xi$ as shown and $R_b = 0$)

Legend

CF

$n$ 0 0.5 1.0
A normal radius
\( r_0 = 10 \) cm, and
\( G/s_u = 250 \)

Fig. E-1 Influence of radial non-homogeneity on dissipation of excess pore pressure (elastic analysis)