Empirical examination of investment under uncertainty

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Thesis Declaration

I, Kazuki Tomioka, certify that:

(i) This thesis has been substantially accomplished during enrolment in the degree.

(ii) This thesis does not contain material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution.

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July 7, 2018

Kazuki Tomioka
Date
Authorship Declaration: Co-authored Publications

This thesis contains work that has been prepared for potential, future publication.

- **Title:**
  Empirical evidence on the dynamics of investment under uncertainty in the US

- **Details of the work:**
  This paper studies the US investment dynamics under uncertainty, using a vector autoregression (VAR) model with drifting parameters, constructed based on an investment Euler condition. We show that short-run uncertainty and investment are negatively related, while long-run uncertainty and trend investment growth are positively related. These findings shed light on the theoretical debate about the ‘sign’ of the relation between investment and uncertainty. Furthermore, we find that the effect of uncertainty shocks on the economy have become more permanent since the Great Recession, which reflects the slow recovery in investment growth since then. We argue that the effects of uncertainty shocks are amplified when the economy is at the zero lower bound.

- **Location in thesis:**
  Chapter 3

- **References:**
  In this thesis, references are organized per chapter. Thus, full reference list for this material can be found under the section References for Chapter 3.

- **Student contribution to work:**
  The analytical derivation of the investment Euler condition, the construction and execution of statistical regression model and the interpretation of results.
Student signatures and dates:

Kazuki Tomioka Date

July 7, 2018

Co-author signatures and dates:

Leandro M. Magnusson Date

July 7, 2018

I, Rod Tyers, certify that the student statements regarding their contribution to the work listed above are correct.

Rod Tyers Date

July 7, 2018
Empirical examination of investment under uncertainty

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Abstract

This thesis presents two essays investigating the empirical relationship between investment and uncertainty. The common denominator in the two essays is that I investigate the dynamic relationship between investment and uncertainty using state-of-the-art mathematical and statistical techniques.

Introductory chapter is offered at the start of the thesis to motivate the research on investment under uncertainty.

The second chapter of the thesis is concerned with the impact of uncertainty on the economy. It first reviews the recent developments in the literature and then presents two empirical analyses. First, I investigate claims in the recent literature about the countercyclicality of uncertainty proxies to the business cycle and hence to investment. Second, I use the vector autoregression (VAR) model to analyze the dynamic impact of uncertainty shocks on investment. Remarkably, both empirical analyses point to the same following conclusions that: i) uncertainty proxies in general are negatively related to investment; and ii) some notable changes to their relationship occurred during and since the financial crisis in 2008.

The third chapter, coauthored with Leandro M. Magnusson, uses a VAR model with drifting parameters to study the time variations in US investment dynamics under uncertainty. We show that when the VAR is constructed in accordance with the investment Euler condition, our modeling strategy enables the reconciliation of the theoretical debate
about the ‘sign’ of the relation between investment and uncertainty. Furthermore, we find that the effects of uncertainty shocks on the economy becomes more permanent since the Great Recession, reflecting on the slow recovery in investment growth since then. We argue that the effects of uncertainty shocks are amplified when the economy is at the zero lower bound.

Concluding remarks are made in the final chapter, with discussions of potential future research on uncertainty.

Word count: approx. 30,000 words
“Even if the constants which economists wish to determine were less
numerous, and the method of experiment more accessible, we should still
be faced with the fact that the constants themselves are different at
different times. The gravitation constant is the same always. But the
economic constants—these elasticities of demand and supply—depending,
as they do, upon human consciousness, are liable to vary. The
constitution of the atom, as it were, and not merely its position, changes
under the influence of environment.”

Arthur Cecil Pigou, 1877 – 1959
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Coauthored by: Leandro M. Magnusson and Kazuki Tomioka

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Chapter 1

General introduction

1.1 Motivation and research question

The very first economic identity taught to undergraduate students in Macroeconomics is the National Income Identity of the form

$$Y = C + I + G,$$

where $Y$ is the aggregate output or the Gross Domestic Product (GDP) of the economy; $C$ is the aggregate consumption; $I$ denotes the aggregate investment; and $G$ is the Government expenditure. According to the data provided by the US Bureau of Economic Analysis (BEA),\(^1\) the consumption share of output in the US was 69.1%, and that for investment was 16.6% in 2017.

It is a stylized fact that business cycle fluctuations are distributed unevenly over the components of the output. While the investment share of GDP is substantially lower than that of consumption, its contribution to the business cycle fluctuations is higher (see, for e.g., Table 5.2 of Romer (2012), p.191). This is also evident from Figure 1.1, which depicts the time varying weighted log deviation of consumption and investment away from the Hodrick and Prescott (1997) (HP) trend (a.k.a., cyclical component).\(^2\) The solid black line depicts the weighted cyclical component of consumption and the dotted

---

\(^1\)Data sources are reported in detail in Appendix 1.A.

\(^2\)Details about how the depicted series are calculated can also be found in Appendix 1.A.
blue line depicts the weighted cyclical component of investment. The standard deviation of the depicted series (in percentages) is 0.77 for consumption and 1.13 for investment. This implies that understanding the dynamics of investment is central to understanding aggregate fluctuations.

[Figure 1.1 here]

Indeed, the volatility of investment has been studied in the macroeconomic literature as early as Pigou (1927). Keynes (1936) used the term ‘Animal Spirits’ to characterize the movements in macroeconomic aggregates over the Great Depression in the 1930s. It is often said that history repeats itself. Fast-forwarding some 70 years, the Great Recession, triggered by the financial crisis in 2008, is understood to be enlarged in magnitude by some combination of the presence of ‘risk’ and/or ‘uncertainty’ (Stock and Watson, 2012).

Within the academic literature, Knight (1921) first distinguished the related concepts of ‘risk’ and ‘uncertainty’. From a theoretical point of view, ‘risk’ has traditionally been thought of as an important element linking the financial markets with the economy in the modern literature (see, for e.g., Kiyotaki and Moore (1997), Bernanke et al. (1999), and Smets and Wouters (2007)). The ‘risk’ mechanism is captured by the riskiness in borrowing from a third party to finance investment, termed the ‘external finance premium’ or simply the ‘risk premium’.

Figure 1.2 depicts a common empirical proxy for risk premium, the spread between the Moody’s Baa and Aaa rated corporate bond yields. As evident from Figure 1.2, the risk premium spikes up during economic downturns and gradually lowers during upturns. This mechanism reflects the high default probability of lower grade corporations during recessions. It is worth mentioning now that this kind of variable is referred to as ‘counter-cyclical’, since it increases during business cycle downturns.

[Figure 1.2 here]

Since the financial crisis in 2008, policymakers and, the financial press have frequently referred to future ‘uncertainty’ as deterrent to investment. For example, in 2009 Olivier Blanchard, then the Economic Counsellor and Director at the International Monetary Fund (IMF) wrote in an article in The Economist (Blanchard, 2009):
Uncertainty is largely behind the dramatic collapse in demand. Given the uncertainty, why build a new plant, or introduce a new product? Better to pause until the smoke clears.

Lawrence Summers, former Director of the National Economic Council for President Obama, expressed similar views to Blanchard in a speech (Summers, 2009):

... unresolved uncertainty can be a major inhibitor of investment. If energy prices will trend higher, you invest one way; if energy prices will be lower, you invest a different way. But if you don’t know what prices will do, often you do not invest at all.

In line with the above anecdotal evidence, Bloom (2009) advocated ‘uncertainty’ as an alternative driver of the economy, by claiming that uncertainty reduces firm investment and hiring. Bloom’s claim has since, spawned a literature that applies new methods and enriched datasets to quantify uncertainty proxies. For example, Jurado et al. (2015) extracts a common volatility component of the $h$-steps-ahead forecast errors of a large number of macroeconomic and financial time series. There are more traditional measures for uncertainty, such as realized stock volatility, defined as the standard deviation of the realized daily stock returns over a month or quarter. Given this definition, these type of financial uncertainty proxies are referred to as second moment proxies.

Figure 1.3 depicts a common empirical uncertainty proxy, computed as the standard deviation of the daily returns of S&P500 index averaged over a quarter. At first glance, like the Baa-Aaa spread, uncertainty proxies appear to be countercyclical, increasing during economic downturns. However, there are periods in which the proxy spikes upward despite the fact that the economy is not in a recession. The most notable of such spike occurred in 1987, and is said to have been caused by the Black Monday stock market crash (Bloom, 2009).

[Figure 1.3 here]

The idea that future uncertainty reduces firm investment has gained empirical support (see, for e.g., Basu and Bundick (2017), Caggiano et al. (2017), and Muntaz and Theodoridis (2018)). However, as illustrated in Figure 1.3, there are periods where uncertainty spikes up sharply during economic upturn.
Consistent with the above observation, there exists theoretical debate regarding the direction of the response (positive/negative) of investment to uncertainty. The strand of literature that advocates for the negative response of investment to uncertainty credits the real options mechanism, where firms exercise the wait-and-see option (Bernanke, 1983). The positive response, called the Oi-Hartman-Abel effect (Oi, 1961; Hartman, 1972; Abel, 1983) is said to materialize in the long-run (Bloom, 2014).

In light of the observations that there are empirical discrepancies and a lack of theoretical consensus regarding the effects of uncertainty on investment, this thesis poses the question:

Does future uncertainty really deter investment?

To answer this question, I apply econometric methods that are popular in the traditional business cycle literature, but are yet to be applied to the literature investigating the effects of uncertainty. This allows us to: i) deepen our understanding about the properties of uncertainty proxies; and ii) examine if the aforementioned negative relationship found in the earlier empirical literature is robust to other econometric methods and specifications. The main objective of this thesis is to investigate, and, specifically to provide novel empirical evidence on the relationship between investment and uncertainty.

To achieve the objective, two econometric methods are considered: i) a variant of Bry and Boschan (1971)'s business cycle dating algorithm, termed BBQ by Harding and Pagan (2002, 2006); and ii) variants of the vector autoregression model developed by Cogley and Sargent (2005) and Primiceri (2005).

The thesis contributes to the recent literature that investigates the effects of uncertainty on the US economy by:

1. characterizing uncertainty proxies into “uncertainty cycle” (high and low states) using business cycle dating procedure to infer their relationship with investment.

2. analyzing time variations in the relationship between investment and uncertainty, particularly around the Great Recession.
1.2 Thesis outline

The thesis provides empirical evidence on the dynamics of investment under uncertainty for the US economy at the aggregated level from multiple perspectives. Chapter 2 reviews and extends upon the empirical results reported in the current literature with a focus on the descriptive, bivariate link between investment and uncertainty. Specifically, two econometric methods (nonparametric and parametric) are used to analyze if uncertainty does indeed affect investment negatively. First, I use the algorithm of Bry and Boschan (1971), augmented by Harding and Pagan (2002, 2006), to characterize uncertainty proxies and investment as two-state switching Markov processes. This method adds statistical rigor to the analysis of uncertainty proxies, and allows for comparability with the uncertainty definition of Bloom (2009). Through this framework, I provide an alternative perspective on the so-called ‘Bloom dating’ of uncertainty peaks. Second, I use the vector autoregression model to investigate if the results are consistent with those derived from Harding and Pagan’s method. Remarkably, the results from both the econometric methods indicate that the relationship between investment and uncertainty changed around the time of financial crisis in 2008.

Chapter 3 entails an explicit modeling of the time variations of the relationship between investment and uncertainty using the time-varying vector autoregression (time-varying VAR). This modeling strategy is motivated by the previous chapter which revealed changes in their relationship around the time of financial crisis. The time-varying VAR is flexible enough to encompass structural breaks (if any) in this relationship around the time of financial crisis and the extraordinary events accompanying it, such as the zero lower bound (ZLB) of the nominal interest rate.

While some important extensions to the empirics of the relationship between investment and uncertainty are offered in Chapter 2, the focus was on the pure empirical relationship. Chapter 3 goes further by discussing the theoretics of investment dynamics. In particular I derive the investment Euler condition from the variant of Christiano et al. (2005)’s New Keynesian model with capital accumulation described in Groth and Khan (2010). Thus, I estimate the time-varying VAR based on the derived investment Euler condition to add theoretical consistency.

The results obtained from the time-varying VAR are presented in the following two
ways: i) reduced form, descriptive statistics provide the time varying bivariate relation between investment and uncertainty; and ii) structural analyses provide the mechanism by which uncertainty influences investment.

Regarding i), following Cogley and Sargent (2001) and Cogley et al. (2010), among others in the trend inflation literature, I decompose the variables into long-run trend and short-run cyclical components using the time-varying VAR to show that the long-run uncertainty and trend investment growth are positively related while their respective short-run components are negatively related.3 This descriptive evidence sheds light on the theoretical predictions of Oi (1961), Hartman (1972), and Abel (1983) which postulate a positive relationship between investment and uncertainty.

Regarding ii), I argue that the structural mechanism by which uncertainty shocks propagate to investment has changed since the Great Recession, when the nominal interest rate was constrained by the ZLB. This finding corroborates that of Caggiano et al. (2017), who report that the contractionary effects of uncertainty shocks are greater when the economy is at the ZLB.

Overall, our empirical findings lend support to the simple mechanism that, when the future return on capital investment is uncertain and the economy is at the ZLB, private agents simply do not invest in capital or in bonds, and instead prefer to hold cash. Indeed, such increased cash holdings by US multinational corporations since the financial crisis has been reported in the finance literature (see, for e.g., Pinkowitz et al. (2012)) and has been termed the ‘high cash holdings puzzle’.

The general conclusion of the thesis is presented in Chapter 4.

3Note that I define uncertainty as the sum of short-run and long-run uncertainty, following Barrero et al. (2017).
Figures

Figure 1.1: Weighted detrended consumption and investment

Note: Series reported in percentages. Gray shaded regions in background depict recession periods dated by the National Bureau of Economic Research (NBER).
Source: US Bureau of Economic Analysis, retrieved from the Federal Reserve Bank of St. Louis website.
Figure 1.2: Moody’s Baa and Aaa Spread

Note: Original bond yields reported in not seasonally adjusted monthly percent, but are averaged to quarterly for visualization purposes. Gray shaded regions in background depict recession periods dated by the National Bureau of Economic Research (NBER).
Source: Moody’s Daily Corporate Bond Yield Averages, retrieved from the Federal Reserve Bank of St. Louis website.
Figure 1.3: Uncertainty proxied via the volatility in S&P500 returns

Note: Gray shaded regions in background depict recession periods dated by the National Bureau of Economic Research (NBER).
Source: The Chicago Board of Options Exchange.
Appendices for Chapter 1

Appendix 1.A  Data

1.A.1  Data sources

Data used in the chapter is collected from a variety of publicly available sources. This data appendix lists the sources of the data. All data used in this chapter is retrieved from the Federal Reserve Bank of St. Louis Economic Research website, (a.k.a., FRED), https://research.stlouisfed.org/fred2/ and thus the codes used by FRED are reported.

- **Real Personal Consumption Expenditures:**
  Consumption is measured by the Real Personal Consumption Expenditures, sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [PCECC96], https://fred.stlouisfed.org/series/PCECC96. This series is reported at quarterly frequency, in billions of chained 2009 dollars, seasonally adjusted annual rate.

- **Shares of Gross Domestic Product: Personal Consumption Expenditures:**
  Share of consumption is measured by the Shares of Gross Domestic Product: Personal Consumption Expenditures, sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [DPCERE1Q156NBEA], https://fred.stlouisfed.org/series/DPCERE1Q156NBEA. This series is reported at quarterly frequency, in percent, not seasonally adjusted.

- **Real Gross Private Domestic Investment:**
  Investment is measured by the Real Gross Private Domestic Investment, sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code
This series is reported at quarterly frequency, in billions of chained 2009 dollars, seasonally adjusted annual rate.

- **Shares of Gross Domestic Product: Gross Private Domestic Investment:**
  Share of investment is measured by the Shares of Gross Domestic Product: Gross Private Domestic Investment, sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [A006RE1Q156NBEA], https://fred.stlouisfed.org/series/A006RE1Q156NBEA. This series is reported at quarterly frequency, in percent, not seasonally adjusted.

- **Moody’s Seasoned Baa Corporate Bond Yield:**
  Baa corporate bond yield is measured by the Moody's Seasoned Baa Corporate Bond Yield, sourced from the Moody’s and retrieved from FRED with code [BAA], https://fred.stlouisfed.org/series/BAA. This series is reported at monthly frequency (averaged to quarterly), in percent, not seasonally adjusted.

- **Moody’s Seasoned Aaa Corporate Bond Yield:**
  Aaa corporate bond yield is measured by the Moody’s Seasoned Aaa Corporate Bond Yield, sourced from the Moody’s and retrieved from FRED with code [AAA], https://fred.stlouisfed.org/series/AAA. This series is reported at monthly frequency (averaged to quarterly), in percent, not seasonally adjusted.

- **Realized Volatility:**
  In Chapter 1, uncertainty is proxied by realized volatility, and the data for this is retrieved from the Chicago Board Options Exchange (CBOE). We compute the standard deviation of the daily returns of Standard & Poor’s 500 Stock Index (S&P500) averaged over a quarter.

### 1.A.2 Data transformation

Figures reported in the chapter are subject to standard treatment. This section provides the details of the way in which the series was created for visualization purposes, figure by figure.
Figure 1.1:

The Hodrick-Prescott (HP) filter is used to detrend both the consumption and investment series. Drawing on Enders (2015), the HP filter assumes that time-series data consists of a cyclical component and a trend component. Thus, for a generic variable, $X_t$, its cyclical component can be expressed as

$$X^c_t = X_t - X^t_t,$$

where $X^t_t$ is the trend component; and $X^c_t$ is the cyclical component. The HP filter minimizes the variance of the above while penalizing excessive changes in the trend component. This can be expressed in terms of the following minimization problem

$$\min_{\{X_t\}_{t=1}^T} \sum_{i=1}^T \left\{ (X_t - X^t_t)^2 + \lambda \left[ (X^t_{t+1} - X^t_t) - (X^t_t - X^t_{t-1}) \right]^2 \right\}$$

where $\lambda$ is the smoothing parameter.

In Figure 1.1, the HP filter is applied to the natural logarithm of consumption and investment to identify their respective trend and cyclical component, with the conventional smoothing parameter value of $\lambda = 1,600$ for quarterly data. The cyclical components are then weighted by their respective time varying shares

$$x^c_t = \frac{X^c_t}{W_{X,t}},$$

where $x^c_t \in \{c^c_t, i^c_t\}$ is the cyclical component of consumption and investment weighted by $W_{X,t} \in \{W_{C,t}, W_{I,t}\}$, their respective shares in GDP.

Figure 1.2:

The risk premium is proxied via the spread between the Baa and the Aaa corporate bond yield

$$R^p_t = Baa_t - Aaa_t,$$

where $R^p_t$ denotes the spread; $Baa$ and $Aaa$ respectively denote their yields. Note that the series are averaged from monthly to quarterly for visualization purposes and to match the consumption and investment series.
Figure 1.3:

Uncertainty is proxied via what is usually called *realized volatility*, defined as the standard deviation of daily stock (S&P500) returns averaged over a month or quarter. Figure 1.3 shows realized volatility averaged over a quarter for visualization purposes and to match the consumption and investment series.
References for Chapter 1


Chapter 2

What do we know about investment dynamics under uncertainty and to what end?

2.1 Introduction

In the wake of the financial crisis in 2008, policymakers and the financial press have frequently referred to future uncertainty as a deterrent to aggregate demand. Since empirical efforts have been made to quantify proxies for uncertainty to investigate the effects of uncertainty on the economy. In this chapter, I revisit the empirical relation between investment and uncertainty, questioning recent empirical evidence that reports a negative relation between them for various recently proposed empirical proxies for uncertainty (see, for e.g., Bloom (2009), Jurado et al. (2015), Basu and Bundick (2017), and Gilchrist et al. (2014)). This research question stems from the theoretical debate regarding the direction of the response (positive/negative) of investment to uncertainty. In a literature survey, Bloom (2014) documents the existence of theories that predict a positive response, called the Oi-Hartman-Abel effect (Oi, 1961; Hartman, 1972; Abel, 1983).

In this chapter, I employ two different econometric methods to tackle the research question, with the objective of providing an alternative view of their empirical relationship using four recently developed proxies for uncertainty. The chapter unfolds by first
employing the so called BBQ algorithm, which is the augmentation of Bry and Boschan (1971)’s nonparametric algorithm to dating cycles as in Harding and Pagan (2002, 2006). I employ this algorithm to ‘dissect’ and characterize uncertainty proxies and investment into a two-state switching Markov process. This framework enables us: i) to provide an alternative perspective on the so called ‘Bloom dating’ of uncertainty peaks, and ii) examine the ‘cyclical association’ between investment and uncertainty, to use Harding and Pagan’s term.

The second econometric method I employ is vector autoregression (VAR), which dates back to Sims (1980). I employ the VAR model to identify and distinguish the reduced form relationship from external shocks striking the relationship between investment and uncertainty. Recent literature has supported the claims of Bloom (2009) that ‘uncertainty shocks’ are contractionary shocks which aggravate economic condition. Yet, a limited number of studies have focused on assessing the impact of uncertainty shocks on investment. Moreover, to the extent of my knowledge, no paper has examined whether, and to what degree, uncertainty shocks can explain the fluctuations in investment. Thus, I use the structural dimension of the VAR model to analyze the impact and contribution of uncertainty shocks on investment in a bivariate framework to directly infer the dynamics of investment under uncertainty.

Interestingly, the results from these two different approaches to modeling investment and uncertainty point to similar conclusions. In fact, the results from both econometric methods confirm that investment and uncertainty are negatively related. However, it is to be noted that the financial crisis in 2008 has contributed to strengthening the negative relationship between them. The inference that the financial crisis in 2008 has strengthened their negative relationship is also reported elsewhere in the literature. For example, Caldara et al. (2016) using VAR report that the significance of the impact of uncertainty shocks on the economy has intensified due to the financial crisis.

The remainder of the chapter is structured as follows. Section 2 describes the uncertainty proxies used and presents the cycle characteristics of uncertainty in relation to investment. Section 3 provides details of the VAR model and the related estimates.

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1Bloom (2009) detrends his uncertainty proxy using the Hodrick and Prescott (1997) filter and reports the dates that exceed the detrended mean by 2 standard deviations. Although reasonably plausible events are assigned to those dates, it is unclear if such detrending is appropriate, given some concerns about the HP filter (see, for e.g., Cogley and Nason (1995) and Hamilton (2017)).
Conclusions are offered in Section 4.

2.2 Dissecting investment and uncertainty cycles

The inference that investment is counter-cyclical to uncertainty has tended to be based on simple cross-correlations or visual inspection against the National Bureau of Economic Research (NBER) recession dates. This section offers a formalization, using Harding and Pagan (2002, 2006)’s augmentation of Bry and Boschan (1971)’s algorithm, which allows the revelation of the cycle properties of uncertainty proxies, and their association with the investment cycle.

I use four recently developed uncertainty proxies collected from a variety of sources over the period 1986:1 to 2015:3 as outlined in Table 2.1. All proxies are reported in monthly frequency. For notational convenience we refer to $U_{j,t}$, for $j \in \{\text{VOL, VXO, GSZ, JF3}\}$, as denoting a generic proxy for uncertainty, specific proxies being the elements $j$. For investment, I use Private Non-Residential Investment, which I deflate by the Gross Domestic Product: Implicit Price Deflator, sourced from the U.S. Bureau of Economic Analysis. This series is reported quarterly, rather than monthly. As I will show, however, the proposed method offers a simple solution to such data frequency issues. A detailed description of the data sources is provided in Appendix 2.A.

2.2.1 Do uncertainty proxies cycle?

To motivate the analysis, we first construct an uncertainty index by computing the first principal component score\(^2\) of the uncertainty proxies described in Table 2.1. Figure 2.1 depicts the estimates of the demeaned and standardized first principal component score, with US recessions shaded vertically in dark gray. Interestingly, the principal component exercise reveals that around 78.5% of the total variability in uncertainty proxies is explained by the first principal component.

\(^2\)Note that I first demean and standardize all proxies reported in Table 2.1 before conducting a principal component analysis.
From this figure we note that the constructed uncertainty proxy is indeed cyclical. Moreover, it appears that uncertainty is counter-cyclical to the business cycle, though there are notable periods of discrepancy, suggesting some degree of time variation. I shall formalize this visual interpretation in the coming subsections using statistical techniques.

2.2.2 Algorithm for dating cycles

The Harding-Pagan’s augmentation of Bry and Boschan (1971)’s nonparametric time series dating algorithm allows the sorting of time series data into two states, via switches between 0 and 1. It requires no explicit assumptions about the underlying data generating process (DGP).\(^3\) The algorithm, which is typically referred to as BBQ, is appealing in this application for two reasons.\(^4\) First, the two-state switching characterization of uncertainty is in line with Bloom (2009)’s theoretical formulation of uncertainty, which switches between ‘low’ and ‘high’. Second, sorting the series into two states mitigates the data frequency issue. The investment series is easily converted to monthly once it is filtered through the BBQ algorithm.\(^5\)

The core of BBQ algorithm lies in the detection of local turning points over a pre-specified window. For a generic sequence of observations, \(\{x_t\}_{t=1}^T\), preliminary turning points (peaks and troughs) are established at \(t\) if

Peak: \(x_t = \max\{x_{t-K}, \cdots, x_{t-1}, x_t, x_{t+1}, \cdots, x_{t+K}\} \quad \forall \ k = 1, \cdots, K\)

Trough: \(x_t = \min\{x_{t-K}, \cdots, x_{t-1}, x_t, x_{t+1}, \cdots, x_{t+K}\} \quad \forall \ k = 1, \cdots, K\),

where \(K\) denotes the length of pre-specified window parameter. In other words, a turning point is established at \(t\) if \(x_t\) is higher/lower than it is for \(K\) preceding and succeeding observations.

After the detection of preliminary turning points, a set of censoring rules ensure that

---

\(^3\)Note that this procedure has been applied to time series data that vary in their DGP, including to commodity prices. See Cashin et al. (2002) for example.


\(^5\)This improves on standard interpolation techniques, such as those of polynomial spline or Chow-Lin.
the original sequence is characterized as a recurrent event in which a phase, defined as an alternating sequence from trough/peak to peak/trough. Recurrence from phase to phase then defines the complete cycle. The censoring rules include pre-specified restrictions on the minimum duration of phases and cycles and a threshold restriction that overrules and bypasses minimum phase restriction where series show abrupt changes. Uncertainty proxies and investment series are filtered through on a log scale.\textsuperscript{6}

For investment, I apply restrictions consistent with the standard definitions used in the business cycle literature. The pre-specified window parameter is set to two quarters with minimum phase and cycle duration restrictions of two and five quarters respectively. The threshold restriction is set at 0.15. Establishing cycle definitions for uncertainty proxies poses more difficulty. To this end, I specify two different cycle definitions for uncertainty proxies. First, as a baseline, I use cycle definitions based on US stock prices, to which BBQ is applied by Pagan and Sossounov (2003). Drawing upon the Dow Theory, they set the window parameter to 8 months, the minimum phase and cycle restrictions at 4 and 16 months, and the threshold restriction at 0.2. This selection is appropriate for my uncertainty proxies since they are second moment uncertainty proxies derived from the financial market data. Second, I use standard business cycle definitions at the monthly frequency. This is reasonable given the aforementioned claims in the literature that uncertainty is counter-cyclical to aggregate measures. These censoring rules are summarized in Table 2.2.

\textsuperscript{6}Note that uncertainty proxies were demeaned and standardized before obtaining the first principal component score. Thus, all proxies are modified such that they are all positive. Appendix 2.A details the data transformation.

2.2.3 Salient features of uncertainty cycles

I report a set of baseline results and relegate other cases to Appendix 2.C, since these offer similar interpretation. Figure 2.2 illustrates my baseline application of the BBQ algorithm to uncertainty proxy FPC. The periods of increasing uncertainty are denoted by gray shaded regions and the periods of decreasing uncertainty remain unshaded. The bar chart beneath the main plot illustrates the corresponding investment cycles, where black shaded regions represent the investment contraction periods and the unshaded regions denote the
expansion periods.

Visual inspection of Figure 2.2 reveals three intuitive features of the uncertainty proxy. First, the notion of Bloom (2009) that uncertainty spikes during extreme events is supported. My approach identifies the Black Monday stock market crash in 1987 and the 2008 financial crisis as peaks. Second, relative to investment cycles, uncertainty undergoes more frequent switches in phases. There are also periods when uncertainty enters an increasing phase while investment continues to expand. Third, for the periods that investment is identified to contract, uncertainty enters the increasing phase some time earlier, except for the 2008 financial crisis period.

[Figure 2.2 here]

The cyclical properties of uncertainty proxies on the one hand and investment on the other are reported in Table 2.3. Note that durations for uncertainty proxies are denoted in months, while that for investment is denoted in quarters. The expansion phases for uncertainty proxies are more evenly spread than investment, which spends about 1:3 ratio in favor of expansion. Investment expansion durations are more than twice as numerous as contractions, while uncertainty proxies have more equal numbers of expansions and contractions. Whilst it may seem counterintuitive that the economy is in an uncertain state half the time, I observe subsequently that it is natural for uncertain states to be more durable than investment contractions.

Across uncertainty proxies, average durations in phases differ substantially, by up to a year. For instance, FPC has the longest average duration in the increasing uncertainty phase with 23 months, while GSZ has the shortest with 12 months. Likewise, in the decreasing uncertainty phase, FPC has the longest duration with 34 months and VOL has the shortest with 20 months. Dissecting cycles in a triangular fashion implies that amplitudes measure the scale of each phase, from one turning point to another. All uncertainty proxies have amplitudes that are similar to themselves in both phases. In particular, FPC has the largest amplitudes of all the uncertainty proxies in both phases.

The excess measure in Table 2.3 gives the degree of departure of the actual path from the hypotenuse of the triangle approximation, where the duration of a phase and amplitude

\footnote{Readers are referred to Appendix 2.B for detailed explanation of this procedure.}
are treated as the base and height of a triangle respectively. This captures the pattern of change over the phase, such that a constant rate of change in an uncertainty proxy between trough and peak implies zero excess. Since the actual time paths of the series often diverge from the triangle approximation this index offers a convenient measure. Across proxies, the excess is negative during the increasing uncertainty phase and positive during the decreasing phase. That is to say, the rate of acceleration (deceleration) continuously increases (decreases) before reaching the next peak (trough). This pattern is the opposite of that observed for investment.

[Table 2.3 here]

2.2.4 Coincidence in investment and uncertainty cycles

To provide a new dimension to the cyclical analysis of investment and uncertainty, I compute the pairwise concordance index, as proposed by Harding and Pagan (2006) between uncertainty proxies and investment. This index is a convenient way to measure the pairwise comovement, since the cointegration technique in the sense of Engle and Granger (1987) is not applicable given the stationarity of uncertainty proxies.

The degree of pairwise concordance between generic sequences of observations \{i_t\}_{t=1}^T and \{j_t\}_{t=1}^T is measured by

\[ C_{i,j} = \frac{1}{T} \left\{ \sum_{t=1}^{T} S_{it} S_{jt} + \sum_{t=1}^{T} (1 - S_{it})(1 - S_{jt}) \right\}, \]

(2.1)

where \(i \in \{\text{INV}\}; \ j \in \{\text{VOL}, \cdots, \text{FPC}\}\); and \(S_{it}\) and \(S_{jt}\) are binary state variables\(^8\) that take the value of 0 in a contractionary phase and 1 in an expansionary phase. Thus, \(C_{i,j} = 1\) implies perfect synchronization, and occurs if and only if \(\{i_t\}_{t=1}^T\) and \(\{j_t\}_{t=1}^T\) are in the same phase at any given point in time. In other words, perfect synchronization occurs if \(S_{it}\) and \(S_{jt}\) are identical, \(S_{it} = S_{jt} \forall t\). Likewise, \(C_{i,j} = 0\) when \(S_{it} = (1 - S_{jt})\), and implies non-synchronous movement. The degrees of synchronization based on these concordance indexes are presented in the first row of Table 2.4. For most proxies, the concordance between investment ranges 0.40 to 0.50. Note that concordance of 0.40 implies that they are in an opposite phase for approximately 60% of the time.

\(^8\)It is worth noting that these state variables are high order stationary ergodic Markov Chains, which exhibit serial correlation even when none are present in the original sequences.
An alternative way to capture\textsuperscript{9} the degree and significance of pairwise comovement between $S_{it}$ and $S_{jt}$, is what is called the ‘cyclical correlation’, $\rho_S$, which can be estimated based on the following regression

$$\frac{S_{it}}{\hat{\sigma}_S \hat{\sigma}_j} = \alpha + \rho_S \left[ \frac{S_{jt}}{\hat{\sigma}_S \hat{\sigma}_j} \right] + u_t,$$

where $\hat{\sigma}_S$ and $\hat{\sigma}_j$ respectively denote the standard deviations of $\{S_{it}\}_{t=1}^T$ and $\{S_{jt}\}_{t=1}^T$. I test the null of no correlation, $\rho_S = 0$, using Newey and West (1987) heteroscedastic and autocorrelation consistent (HAC) standard errors with Bartlett weights. Lags are set in accordance with their nonparametric bandwidth selection procedure. The resulting cyclical correlation statistics for each uncertainty proxy is reported in the second row of Table 2.4.

It can be inferred from these results that there are disparities across the uncertainty proxies regarding their relationship with investment. Some are significantly negatively correlated, while the others are positively correlated but the relationship is not statistically significant. Although cyclical correlation coefficients are only significant in the negative direction for any proxy, there remains a mystery with regards to their relationship with investment. We further analyze their relationship in the subsequent section.

### 2.3 Dynamics of investment under uncertainty

The results from the previous section offer the preliminary interpretation that investment and uncertainty may be negatively related. There is a growing body of empirical literature suggesting that their relationship varies with time. Indeed, as depicted in Figure 2.2, there are periods in which uncertainty proxies enter an increasing phase but investment does not contract. Moreover, for the periods in which investment does contract, uncertainty enters into an increasing phase before investment starts to contract, except for the financial crisis period where investment contracts first. In fact, the observation that some differences and changes in the relationship between investment and uncertainty has been reported in the

\textsuperscript{9}The concordance index is known to be biased by the magnitude or the expected value of binary state variables.
literature. For example, Caldara et al. (2016) report that the impact of uncertainty shocks on the economy have been intensified by the financial turmoil in 2008. Conversely, Muntaz and Theodoridis (2018) report the opposite and claim that effects of uncertainty shocks on the economy have weakened in magnitude over time.

Given these recent literature, I conduct two 'experiments' using VAR. Specifically, I estimate bivariate (investment and uncertainty) VAR models over two different sample periods. The first sample period spans 1986:I to 2005:IV, thereby excluding the investment contraction periods associated with the 2008 financial crisis. The second sample spans 1986:I to 2015:I, which is the full sample used in the previous sections of the chapter.

2.3.1 The VAR framework

A VAR of order $P$ in a structural form can be expressed as

$$
Ay_t = \phi + \sum_{p=1}^{P} \Phi_p y_{t-p} + \varepsilon_t,
$$

where $y_t$ is the $n \times 1$ vector of endogenous variables; $\phi$ denotes the $n \times 1$ vector of intercept; $A$ and $\Phi_p$, $\forall p = 1, 2, \cdots, P$ are $n \times n$ matrices of coefficients; $\varepsilon_t$ is $n \times 1$ vector of unstandardized structural shock that follow $\varepsilon_t \sim N(0, \Sigma \Sigma')$.

Pre-multiplying both sides of equation (2.3) by $A^{-1}$ yields a reduced form VAR of order $P$

$$
A^{-1}Ay_t = A^{-1}\phi + \sum_{p=1}^{P} A^{-1}\Phi_p y_{t-p} + A^{-1}\varepsilon_t.
$$

By defining $c \equiv A^{-1}\phi$; $B_p \equiv A^{-1}\Phi_p$, $\forall p = 1, 2, \cdots, P$; and $u_t \equiv A^{-1}\varepsilon_t$, the reduced form VAR can be expressed as

$$
y_t = c + \sum_{p=1}^{P} B_p y_{t-p} + u_t, \quad u_t \sim N(0, \Omega).
$$

Note that $\Omega$ is a positive semi-definite variance-covariance matrix of the reduced form

---

10Note that peak of the investment is identified to be 2006:I using the BBQ and I therefore cut the rest of the sample out to minimize the effect of the financial crisis.
Residual

\[ \Omega \equiv \begin{bmatrix}
\sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\
\sigma_{21} & \sigma_2^2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \sigma_{n-1n} \\
\sigma_{n1} & \cdots & \sigma_{nn-1} & \sigma_n^2
\end{bmatrix}. \]

Performing a standard triangular factorization on the variance-covariance matrix of the form

\[
E(\varepsilon_t \varepsilon_t') = E(Au_t u_t' A') = A E(u_t u_t') A' = A \Omega A',
\]

\[ = \Sigma \Sigma', \]

implies that

\[
E(u_t u_t') = \Omega = A^{-1} \Sigma \Sigma'(A^{-1})',
\]

(2.5)

where \(A^{-1}\) is a lower triangular matrix that captures the contemporaneous relations among the variables and \(\Sigma\) is a diagonal matrix of standard deviation

\[
A^{-1} \equiv \begin{bmatrix}
1 & 0 & \cdots & 0 \\
\tilde{a}_{21} & 1 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \tilde{a}_{n-1n} \\
\tilde{a}_{n1} & \cdots & \tilde{a}_{nn-1} & 1
\end{bmatrix}, \quad \Sigma \equiv \begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \sigma_n
\end{bmatrix}.
\]

Without loss of generality, I form a \(n(nP + 1) \times 1\) vector \(\beta\) by stacking the elements in the rows of \(c\) and \(B_p\) in equation (2.4) and denote \(X_t \equiv I_n \otimes (1, y'_{t-1}, y'_{t-2}, \ldots, y'_{t-P})\), where \(\otimes\) denote the Kronecker product. Then equation (2.4) can be expressed as a seemingly unrelated regression model

\[
y_t = X_t \beta + A^{-1} \Sigma \varepsilon_t,
\]

(2.6)

where \(\Sigma \varepsilon_t = \varepsilon_t \sim \mathcal{N}(0, \Sigma \Sigma')\), the unstandardized structural shock; and \(\varepsilon_t \sim \mathcal{N}(0, I_n)\) is the standardized structural shock.\(^{11}\)

\(^{11}\)Since \(\varepsilon_t \sim \mathcal{N}(0, I_n)\), the following holds

\[
E(\varepsilon_t \varepsilon'_t) = E(\Sigma \varepsilon_t \varepsilon'_t \Sigma') = \Sigma E(\varepsilon_t \varepsilon'_t) \Sigma' = \Sigma I_n \Sigma' = \Sigma \Sigma'.
\]

Therefore

\[
\Omega = A^{-1} \Sigma \Sigma'(A^{-1})'.
\]
2.3.2 Model specification

I analyze the dynamics of investment under uncertainty using the same set of data employed in the previous section. Nevertheless, I transform the data such that it is more suitable for VAR analyses. First, I take the demeaned and standardized uncertainty proxy, FPC as my measure of uncertainty and convert it from monthly to quarterly frequency by taking the average of within quarter observations. Second, investment is transformed from log level to log changes, so as to achieve stationarity. Specifically, investment enters the VAR as \( \tilde{I}_t = 400 \times \log(I_t/I_{t-1}) \), which is the quarterly log change at the annualized rate.

Following the literature on uncertainty (e.g., Bloom (2009), Basu and Bundick (2017), and Jurado et al. (2015)), I rely on the Cholesky factorization\(^\text{12}\) of the reduced form variance-covariance matrix to identify the structural shocks. The endogenous variables enter the VAR as follows

\[
y_t = \begin{bmatrix} U_t, \tilde{I}_t \end{bmatrix}',
\]

where \( U_t \) is the uncertainty proxy; and \( \tilde{I}_t \) is the quarterly log change of investment at the annualized rate. The ordering reflects Basu and Bundick (2017), where they order uncertainty proxy before real variable like investment. They argue that this ordering is consistent with their theoretical New Keynesian model. Note that the bivariate framework adopted here is somewhat restrictive as it abstracts from the general macroeconomic VARs that include the rate of inflation, interest rate and so on. Nevertheless, I still view this exercise important so as to embellish the empirical relationship between investment and uncertainty.

With this ordering, the VAR model is estimated using the ordinary least squares (OLS) method with one lag, set in accordance with the Bayesian information criterion. Note that I conduct a robustness check using a different uncertainty proxy, the result of which can be found in Appendix 2.D. I confirm that the qualitative interpretations of the result are robust to the proxy used.

\(^{12}\)Note that factorization presented in equation (2.5) is the most general case of factoring the variance-covariance matrix. See Kilian (2011) for further discussion.
2.3.3 Analyses via innovation accounting

Figures 2.3 and 2.4 illustrate the impact of uncertainty proxy and investment growth shocks\textsuperscript{13} on the uncertainty proxy and investment, respectively, where the shock size is one standard deviation for both uncertainty and investment. For both figures, the top row depicts the impulse responses computed using pre-financial crisis sample, while the bottom row depicts the impulse responses computed using the entire sample. For both the top and the bottom rows, the left panel depicts the response of uncertainty while the right panel depicts the response of investment growth. The black solid line depicts the median and the dotted line around the median are 1,000 bootstrapped 68% bands.

The impulse response functions depicted in Figures 2.3 and 2.4 lend support to the widespread view in the empirical literature that uncertainty shock is a contractionary shock that aggravates economic activity. More interestingly, there are clear differences between the responses of each variables to each other’s shocks for pre-crisis sample and full sample.\textsuperscript{14} For the pre-crisis sample period, neither shocks in uncertainty nor investment cause significantly negative responses to each other. However, when I consider the entire sample, which includes the 2008 financial crisis and the subsequent period, the significance of shock to each other becomes apparent.

To delve more deeply into the contribution of uncertainty to investment, I decompose the forecast error variances. Figure 2.5 depicts the contribution of uncertainty and investment to the forecast error variance of investment, while Figure 2.6 depicts the contribution of uncertainty and investment to the forecast error variance of uncertainty. As with the impulse response plots, the top row depicts the decomposition of the forecast error variance based on the pre-crisis sample, while the bottom row depicts that of the entire sample.

\textsuperscript{13}Theoretically, the investment growth shock I refer to could be interpreted as investment-specific technology shock (Greenwood et al., 1997, 2000). However, the identified shock may not be an accurate depiction of such shock since the specified VAR is bivariate.

\textsuperscript{14}Note tangentially that the degree of uncertainty is reduced to a shock in investment growth. This finding, structurally interpreted implies that exogenous changes to investment growth, like the aforementioned investment-specific technological shock (Greenwood et al., 1997, 2000) influences the degree of uncertainty in the economy. This result is inline with the recent work of Ludvigson et al. (2018) that report the potential that variation in uncertainty may be an endogenous result of traditional shocks that drives the economy (e.g., total factor productivity shock).
I find convincing evidence of an increasing variance contribution of uncertainty to investment as the forecast horizon increases. The similarity in the shape of the contribution of each shock to forecast error variance is to be noted between the pre-crisis sample and the entire sample. The maximal contribution is reached after five to six quarters for both samples. Nevertheless, the contribution of an uncertainty shock to investment forecast error variance appears to double in magnitude when I consider the full sample, as compared with the pre-crisis sample. This finding corroborates with the earlier conclusion drawn from the impulse response analyses: that the significance of an uncertainty shock on investment increases when the financial crisis and the subsequent period is included in the sample.

Overall, this variance decomposition analysis reinforces the earlier conclusions from the impulse response analyses. It is evident that the financial crisis enhanced the negative relationship between investment and uncertainty.

2.4 Conclusion

This chapter offers some extensions to the approaches adopted in the previous literature to better understand, and to provide alternative perspectives on, the relationship between investment and uncertainty. Specifically, I employ two econometric methods to test if the empirical findings reported in the literature is a robust feature of the data.

By focusing on four recently developed financial uncertainty proxies, I first identify their cycle characteristics so as to observe their association with investment cycles. To achieve this, I use the method of Harding and Pagan (2002, 2006). This method enables the inference of the degree of synchronization of their cyclical relationship, which are indirectly apparent from earlier empirical findings reported in the literature. Second, I complement the results from the cyclical analyses by employing a VAR model to reveal the importance of ‘uncertainty shocks’ in explaining investment fluctuations.

The results obtained from the two separate analyses corroborate one another and complement preceding studies on investment under uncertainty. The results offer strong confirmation that: i) investment and uncertainty cycles are negatively synchronized; ii)
measured uncertainty had negative impacts on investment throughout the entire sample period; and iii) the impact of uncertainty shocks on investment during the 2008 financial crisis strengthened their negative relationship.
Figures

**Figure 2.1**: Uncertainty proxies

*Note*: Data spans from 1986:1 to 2015:3 monthly frequency. FPC denotes the first principal component of the four uncertainty proxies and VOL is the implied stock volatility. Both measures are demeaned and standardized.
Figure 2.2: Uncertainty and investment cycles

Note: The black solid line on the main panel depicts the original, demeaned and standardized uncertainty proxy, FPC. The gray shaded region in the background is when uncertainty is in an increasing phase of the cycle, dated by BBQ. The black region in the bar graph below shows the contraction phase of investment dated by BBQ.
Source: Author’s compilation.
Figure 2.3: Responses of uncertainty proxy and investment growth to an uncertainty shock

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands. Shock size is one standard deviation.
Source: Author’s compilation.
Figure 2.4: Responses of uncertainty proxy and investment growth to an investment shock

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands. Shock size is one standard deviation. 
Source: Author’s compilation.
Figure 2.5: Forecast error variance contribution: uncertainty

**Pre-2008 financial crisis sample**

![Graph showing variance contribution](image)

**Entire sample**

![Graph showing variance contribution](image)

*Note:* The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands.

*Source:* Author’s compilation.
Figure 2.6: Forecast error variance contribution: investment growth

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands.
Source: Author’s compilation.
### Table 2.1: Mnemonics and summary of uncertainty proxies

<table>
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<tbody>
<tr>
<td>VOL</td>
<td>Realized volatility, a proxy for uncertainty computed as a standard deviation of daily S&amp;P500 returns averaged over a month.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>VXO</td>
<td>Implied S&amp;P 100 volatility, used instead of the VIX for the same reasons as in other literature, to increase the time span.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>GSZ</td>
<td>Risk-adjusted proxy for uncertainty that captures the common shocks in the idiosyncratic volatility of equity returns.</td>
<td>Gilchrist et al. (2014) and Caldara et al. (2016)</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>FPC</td>
<td>The first principal component scores of the above 4 uncertainty proxies.</td>
<td>Author’s computation</td>
<td>1986:1 – 2015:3</td>
</tr>
</tbody>
</table>
Table 2.2: Specifications of censoring rules in the BBQ algorithm

<table>
<thead>
<tr>
<th>Monthly</th>
<th>Imposed restrictions on censoring rules</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Window</td>
<td>Phase</td>
<td>Cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specification 1</td>
<td>8</td>
<td>4</td>
<td>16</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>Specification 2</td>
<td>6</td>
<td>6</td>
<td>15</td>
<td></td>
<td>0.20</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Quarterly</th>
<th>Window</th>
<th>Phase</th>
<th>Cycle</th>
<th></th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td></td>
<td>0.15</td>
</tr>
</tbody>
</table>

*Note: Numbers in the censoring rules are denominated in their respective frequencies, monthly and quarterly.*
**Table 2.3: Cyclical statistics on uncertainty proxies**

<table>
<thead>
<tr>
<th>Proxies</th>
<th>Decreasing uncertainty / Contraction</th>
<th>Increasing uncertainty / Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portion</td>
<td>Duration</td>
</tr>
<tr>
<td>VOL</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.00</td>
<td>−0.39</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>(−0.68)</td>
</tr>
<tr>
<td>VXO</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21.42</td>
<td>−0.30</td>
</tr>
<tr>
<td></td>
<td>0.54</td>
<td>(−0.62)</td>
</tr>
<tr>
<td>GSZ</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.50</td>
<td>−0.37</td>
</tr>
<tr>
<td></td>
<td>0.43</td>
<td>(−0.52)</td>
</tr>
<tr>
<td>JF3</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28.86</td>
<td>−0.27</td>
</tr>
<tr>
<td></td>
<td>0.61</td>
<td>(−0.63)</td>
</tr>
<tr>
<td>FPC</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>34.17</td>
<td>−0.46</td>
</tr>
<tr>
<td></td>
<td>0.57</td>
<td>(−0.37)</td>
</tr>
<tr>
<td>INV</td>
<td>0.30</td>
<td></td>
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<tr>
<td></td>
<td>11.67</td>
<td>−0.15</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>(−0.81)</td>
</tr>
</tbody>
</table>

*Note: Cyclical statistics reported here are for complete cycles only. Data on uncertainty proxies spans from 1986:1 to 2015:3 at monthly frequency, while investment spans the same period in quarterly frequency. Durations are measured in months for uncertainty proxies and quarters for investment, amplitudes and excess are expressed in percentage changes. Coefficient of variations reported in parentheses.*
Table 2.4: Concordance and cyclical correlation between uncertainty proxies and investment

<table>
<thead>
<tr>
<th>Statistics</th>
<th>FPC</th>
<th>VOL</th>
<th>VXO</th>
<th>GSZ</th>
<th>JF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordance</td>
<td>0.40</td>
<td>0.39</td>
<td>0.49</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>Cyc. correl</td>
<td>-0.13***</td>
<td>-0.16***</td>
<td>-0.08*</td>
<td>0.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>

*Note: Data spans from 1986:1 to 2015:3 at monthly frequency. Refer to equations (2.1) and (2.2).
Appendices for Chapter 2

Appendix 2.A  Data

2.A.1  Data sources

Data used in this chapter is collected from a variety of publicly available sources. Some of the data used in this chapter is retrieved from the Federal Reserve Bank of St. Louis Economic Research website, (a.k.a., FRED), https://research.stlouisfed.org/fred2/ and thus the codes used by FRED are reported.

Uncertainty proxies

\textbf{VOL}: The data for this proxy is retrieved from the Chicago Board Options Exchange (CBOE), and I compute the standard deviation of the daily returns of S&P 500 index averaged over a month.

\textbf{VXO}: The data for this proxy is sourced from CBOE and retrieved from FRED, https://fred.stlouisfed.org/series/VXOCLS. I use this proxy instead of the volatility index, VIX, since the VXO, [VXOCLS] spans a longer time horizon. The correlation between the VIX and the VXO is high and use of the VXO instead of the VIX in the uncertainty literature has become standard.

\textbf{GSZ}: This proxy is introduced in Gilchrist et al. (2014) and retrieved from the Online Appendix of Caldara et al. (2016).

\textbf{JF3}: This is financial uncertainty measure proposed in Jurado et al. (2015), retrieved from Sydney C. Ludvigson’s personal website, https://www.sydeyludvigson.
Investment

In this chapter, investment is proxied via the seasonally adjusted Private Nonresidential Fixed Investment (in billions of US dollars), deflated to 2009 dollars by the Gross Domestic Product: Implicit Price Deflator (GDP deflator).

Private Nonresidential Fixed Investment:
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [PNFI], https://fred.stlouisfed.org/series/PNFI. This series is reported at quarterly frequency in billions of dollars, seasonally adjusted annual rate.

Gross Domestic Product: Implicit Price Deflator:
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [GDPDEF], https://fred.stlouisfed.org/series/GDPDEF. This series is reported at quarterly frequency in index, 2009=100, seasonally adjusted.

2.A.2 Data transformation

For cycle analyses, uncertainty proxies are filtered through the BBQ algorithm on a log scale in monthly frequency. However, since uncertainty proxies have been demeaned and standardized to retrieve the first principal component score, I transform the series in the following way

\[ \tilde{U}_{j,t} = U_{j,t} + 2 \times \min \{U_{j,t}\}, \quad \forall j \]

where min \{U_{j,t}\} denotes the minimum value of the demeaned and standardized uncertainty proxies. This happens to be about 2.05 and occurs for the uncertainty proxy GSZ.

Appendix 2.B  BBQ algorithm

Harding and Pagan (2002, 2006)'s augmentation of Bry and Boschan (1971)'s algorithm,\textsuperscript{15} termed BBQ, is implemented in this chapter to infer the relationship between investment

\textsuperscript{15}As recommended by Adrian Pagan, we use the MATLAB version of the code written by James Engle, available at http://www.ncer.edu.au/resources/data-and-code.php.
and uncertainty proxies. This section of the appendix describes the details of the statistics reported in the main text.

The idea behind the BBQ algorithm is to ‘dissect’ a time series of a variable into two states through a ‘triangle approximation’. Hence, the cycle statistics can be thought of as statistics that inform us about the degree of proximity and/or diversion from a triangle. Figure 2.A.1 replicates the figure used in Harding and Pagan (2002) to illustrate a stylized downturn (recession) phase, where point A is the peak and C is the trough. The height of the triangle is the amplitude; the base is the duration and the hypotenuse is the triangle approximation of the time path of a series.

Adopting the same notations as in Harding and Pagan, I designate the duration of the $i^{th}$ phase as $D_i$, and the corresponding amplitude as $A_i$, the triangle approximation of the cumulative movement of the series is

$$C^T_i = 0.5(D_i \times A_i).$$

The actual path of a series often diverge from the triangle approximation and I measure the degree of diversion via

$$E_i = \frac{1}{D_i}(C^T_i - C_i + 0.5 \times A_i),$$

where the term $0.5 \times A_i$ is added so as to eliminate bias that arises in using a sum of rectangles to approximate a triangle. The excess, as referred to in the main text, is measured by the mean of the $E_i$’s and is computed separately for upturn and downturn phases.

The path of a series depicted in Figure 2.A.1 is characterized by a rapid initial change that decelerates towards the next turning point. In this specific example, the nature of the series is characterized by a positive excess. One should note that the path of the series over a phase does not always lie inside or outside the triangle and the actual path may switch within a phase. In these instances the excess index will simply reflect whether the area inside or outside the triangle is larger.
Appendix 2.C Additional results from cycle analyses

This section of the appendix reports additional results derived from different specifications of the censoring rules in the BBQ algorithm. Readers are referred to Table 2.2 for the specifics of the censoring rules.

Additional results derived from specification 2 is reported. Table 2.A.1 reports the respective cycle statistics, while the measures of concordance and cyclical correlation between investment and uncertainty proxies are reported in Table 2.A.2.

[Table 2.A.1 and 2.A.2 here]

It is evident from above tables that the broad contour of the analyses presented in the chapter holds, although some peculiarities are present. The magnitude of concordance and cyclical correlation for uncertainty proxies FPC and VXO strengthens in the negative direction, while for others, it weakens.

Appendix 2.D Additional results from VAR analyses

As a robustness check to the VAR results presented in the chapter, I replicate the impulse responses and forecast error variance decomposition plots using VXO as a proxy for uncertainty. Impulse responses are depicted in Figures 2.3 and 2.4 and the decomposition of forecast error variances are depicted in Figures 2.5 and 2.6.

[Figure 2.3 and 2.4 here]

[Figure 2.5 and 2.6 here]

It is clearly evident from above figures that the consistency of the analyses presented in the chapter holds. That is, including the financial crises years into the sample period strengthens the negative relation between investment and uncertainty.
Appendix figures

Figure 2.A.1: Stylized downturn phase

Note: Replication of Figure 1 of Harding and Pagan (2002).
Figure 2.A.2: Responses of uncertainty proxy (VXO) and investment growth to an uncertainty shock

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands. Shock size is one standard deviation.
Source: Author's compilation.
Figure 2.A.3: Responses of uncertainty proxy (VXO) and investment growth to an investment shock

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands. Shock size is one standard deviation.
Source: Author’s compilation.
Figure 2.A.4: Forecast error variance contribution: uncertainty (VXO)

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands.
Source: Author’s compilation.
Figure 2.A.5: Forecast error variance contribution: investment growth (VXO)

Pre-2008 financial crisis sample

Entire sample

Note: The black solid line depicts the median and the dotted lines around the median are 68% bootstrapped bands.

Source: Author’s compilation.

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## Appendix tables

### Table 2.A.1: Cyclical statistics on uncertainty proxies

<table>
<thead>
<tr>
<th>Proxies</th>
<th>Portion</th>
<th>Duration</th>
<th>Amplitude</th>
<th>Excess</th>
<th>Portion</th>
<th>Duration</th>
<th>Amplitude</th>
<th>Excess</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>0.58</td>
<td>40.80</td>
<td>−0.32</td>
<td>14.42</td>
<td>0.42</td>
<td>19.00</td>
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<td>(0.83)</td>
<td>(−0.73)</td>
<td>(1.46)</td>
<td>(0.60)</td>
<td>(0.63)</td>
<td>(−0.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VXO</td>
<td>0.43</td>
<td>33.20</td>
<td>−0.34</td>
<td>37.46</td>
<td>0.57</td>
<td>26.80</td>
<td>0.32</td>
<td>−47.34</td>
</tr>
<tr>
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<td>(0.75)</td>
<td>(−0.67)</td>
<td>(0.27)</td>
<td>(0.63)</td>
<td>(0.78)</td>
<td>(−0.51)</td>
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<td></td>
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<td>(1.00)</td>
<td>(0.49)</td>
<td>(0.49)</td>
<td>(−1.52)</td>
<td></td>
<td></td>
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<td>JF3</td>
<td>0.61</td>
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<td></td>
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<td>(1.05)</td>
<td>(0.73)</td>
<td>(0.84)</td>
<td>(−0.37)</td>
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<td>FPC</td>
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<td>(−0.53)</td>
<td>(1.51)</td>
<td>(0.37)</td>
<td>(0.51)</td>
<td>(−0.17)</td>
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<td></td>
</tr>
</tbody>
</table>

*Note:* Cyclical statistics reported here are for complete cycles only. Data on uncertainty proxies spans from 1986:1 to 2015:3 at monthly frequency and thus durations are measured in months. Amplitudes and excess are expressed in percentage changes. Coefficient of variations reported in parentheses.
<table>
<thead>
<tr>
<th>Statistics</th>
<th>FPC</th>
<th>VOL</th>
<th>VXO</th>
<th>GSZ</th>
<th>JF3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordance</td>
<td>0.35</td>
<td>0.41</td>
<td>0.34</td>
<td>0.59</td>
<td>0.52</td>
</tr>
<tr>
<td>Cyc. correl</td>
<td>-0.20***</td>
<td>-0.03</td>
<td>-0.25***</td>
<td>0.18***</td>
<td>0.11**</td>
</tr>
</tbody>
</table>

*Note:* Data spans from 1986:1 to 2015:3 at monthly frequency. Refer to equations (2.1) and (2.2).
References for Chapter 2


Chapter 3

Empirical evidence on the dynamics of investment under uncertainty in the US

Coauthored by: Leandro M. Magnusson and Kazuki Tomioka

3.1 Introduction

The volatility of investment has been studied in the macroeconomic literature since the dawn of business cycle theory (Pigou, 1927). The slow recovery in investment (Gutiérrez and Philippon, 2017) and policymakers’ frequent use of the term ‘uncertainty’ in the years following the 2008 financial crisis have reinvigorated interest in the relationship between investment and uncertainty. Bloom (2009) popularized uncertainty as an alternative driver of the economy, by asserting that uncertainty reduces firms’ investment and hiring. Subsequent empirical research, much of it conducted via vector autoregression (VAR) has reaffirmed Bloom’s original claim. For example, Gilchrist et al. (2014) find negative response of investment to uncertainty shocks. Similar negative responses in other measures of real activity following uncertainty shocks are reported in Jurado et al. (2015) and Katayama and K.H. Kim (2018) among others.
Overall, there is a nascent consensus among researchers that uncertainty about the future is likely to cause adverse effects on the economy. However, as noted by Bloom (2014), there exists a theoretical debate regarding the direction (positive/negative) of the response of investment to uncertainty. The positive response of investment to uncertainty is said to materialize in the long-run. The literature has taken two different approaches in search of empirical evidence regarding the response and association of investment and uncertainty.

One of such approach has been taken by Barrero et al. (2017), who decompose uncertainty measured by stock volatility into a short and a long-run component to investigate the effect of each component on investment. They find both the short and the long-run uncertainty to be negatively related with investment. To the extent of our knowledge, no other research has attempted to investigate the long-run relationship between investment and uncertainty.

A number of other studies have attempted to decompose the relationship between investment and uncertainty over the time horizon. Using risk adjusted volatility in equity returns as a proxy for uncertainty, Caldara et al. (2016) compare the impulse responses of various real variables to uncertainty shocks derived from VARs. They show that negative effects of uncertainty shocks on real variables intensifies when the sample period spans over the 2008 financial crisis, relative to when the sample period end prior to the onset of the financial crisis. Conversely, Mumtaz and Theodoridis (2018), using an extended, factor augmented VAR, claims that the negative impact of uncertainty shocks on real activity have gradually declined. Thus, whether the relationship between investment and uncertainty differs in the long-run from short-run, and whether it has changed over time are still open questions in the literature.

This paper attempts to fill these two gaps, by constructing a time-varying VAR model as in Cogley and Sargent (2005) and Primiceri (2005), to capture the relationship between investment and uncertainty over time. This model is flexible enough to encompass possible disruptions or breaks (if any) of this relationship. To determine what variables enter the VAR, we rely the investment Euler condition derived from a variant of Christiano et al. (2005)’s New Keynesian model with capital accumulation. There are several advantages with this theoretical framework.

First, as exemplified in Groth and Khan (2010), we can eliminate the presence of the
marginal $Q$ in the investment Euler condition. Hayashi (1982) showed that the marginal $Q$ and the average $Q$ are equivalent under certain regularity conditions. However, subsequent empirical studies have revealed such approximations to be unsatisfactory due to measurement issues (Erickson and Whited, 2000). Second, the specification of adjustment cost function enables the introduction of inertia in investment in a micro-founded way, which is empirically important (Eberly et al., 2012). To the extent that our empirical model is a VAR, such inertia in investment can be captured by its lag structure.

Our presentation of the results is twofold: i) descriptive, reduced form results that focus only on the bivariate relationship between investment and uncertainty, and ii) structural analyses that investigate mechanisms whereby uncertainty influences investment. First, regarding i), we show in a bivariate framework that the correlation between investment and uncertainty varies substantially over time. Moreover, following Cogley and Sargent (2001) and Cogley et al. (2010) in the trend inflation literature, we decompose the variables into long-run trend and short-run cyclical components using the time-varying VAR. The results suggest that the long-run uncertainty and trend investment growth are positively related while their respective short-run components are negatively related. The evidence that long-run uncertainty and trend investment growth are positively related sheds light on the theoretical predictions of Oi (1961), Hartman (1972), and Abel (1983) that postulate positive relation between investment and uncertainty.

Regarding ii), we first argue that the structural mechanism by which uncertainty shocks propagate to investment has systematically changed since the Great Recession, when the nominal interest rate was constrained by the zero lower bound (ZLB). This finding corroborates that of Caggiano et al. (2017) who report that the contractionary effects of uncertainty shocks are greater when the economy is at the ZLB. Second, we estimate a statistic that measures the persistence in uncertainty developed by Cogley et al. (2010). The result is suggestive of lower persistence in uncertainty during recessions, or, in others words the predictability of uncertainty is low during recessions. Third, we attempt to ‘replay history’ via a counterfactual simulation, in which we mute uncertainty shocks and fix the parameters in the investment equation to their 1995 level. The simulation results differ sharply from those of history. Absent uncertainty shocks and conditional on investment adjusting as it did in 1995, average investment growth from 2008 to 2015 would

\footnote{Note that we define uncertainty as the sum of short-run and long-run uncertainty following Barrero et al. (2017).}
have been higher by as much as 97%. Most of the gains in investment growth appear
to stem from shutting off uncertainty shocks. Fourth, model comparison of time-varying
VAR and its nested constant counterparts via the root mean squared forecast error and
the log predictive density scores reveal that allowing for time variations in the parameters
are important in explaining the dynamics of investment and uncertainty.

The remainder of the paper is structured as follows. In Section 2 we review a theo-
retical model of investment dynamics, which serves as a motivation for the econometric
model specification presented in the Section 3. Section 4 describes the data set, which is
followed by the empirical results reported in Section 5. Concluding remarks end this pa-
per. Supplemental materials regarding the data set, the Bayesian estimation method of the
time-varying VAR, derivation of the investment equation and some additional empirical
results are available in the Appendix.

3.2 Investment dynamics

In this section we review the key theoretical determinants of investment dynamics, which
will serve as a guide in the construction of our empirical model. To this end, we rely on the
New Keynesian model with capital accumulation, proposed by Christiano et al. (2005),
that explicitly models the dynamics of investment. For expositional purposes, we consider
a simplified variant of their model as in Groth and Khan (2010).

In addition to rational expectations, the model assumes that the representative house-
hold optimizes an intertemporal utility function subject to: i) a sequence of budget con-
straint that accounts for investment expenditure into physical capital (that earn a rate of
return from capital ownership) and ii) the capital accumulation equation of the form

$$K_{t+1} = (1 - \delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t,$$

where $K_t$ is the capital stock; $\delta$ is the depreciation rate; $I_t$ is investment; and $S(I_t/I_{t-1})$
is the convex investment adjustment cost function, with $S(1) = S'(1) = 0$ and $S''(1) \equiv
\kappa > 0$. Thus, households maximize their utility not only by weighing consumption and
the disutility of supplying labor, but also, indirectly, from investments that earn interest
from capital ownership, where this interest differs from the risk free rate received from
investing in bonds.
We solve this household’s problem analytically by setting up the Lagrangian and deriving the first order condition with respect to investment in terms of the marginal $Q$, defined as the ratio of the Lagrange multipliers (shadow prices). Log-linearizing the first order condition around the non-stochastic steady state yields the investment Euler condition of the form

$$\tilde{i}_t = \frac{1}{1 + \beta} \tilde{i}_{t-1} + \frac{\beta}{1 + \beta} \mathbb{E}_t \tilde{i}_{t+1} + \frac{1}{\kappa(1 + \beta)} \tilde{q}_t,$$

(3.1)

where $\tilde{i}_t$ is the log-deviation of investment from its steady state; $\beta$ is the discount factor; $\kappa$ is the inverse of the elasticity of investment with respect to 1% change in marginal $Q$; and $\tilde{q}_t$ is the log-deviation of the marginal $Q$ from the steady state. Similar treatment of the first order condition with respect to capital and bond jointly yields

$$\tilde{q}_t = - (\tilde{r}_t - \mathbb{E}_t \tilde{r}_{t+1}) + \frac{1 - \delta}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{q}_{t+1} + \frac{r_k^*}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{r}_{t+1},$$

(3.2)

where $\tilde{r}_t - \mathbb{E}_t \tilde{r}_{t+1}$ is the log-deviation of ex-ante real interest rate; $\tilde{r}_{t+1}^k$ is the log-deviation of the rate of return from capital; and $r_k^* = 1/\beta - 1 + \delta$. Combining (3.1) and (3.2) delivers the following representation of the Euler condition

$$\tilde{i}_t = \frac{1}{\kappa} \left[ \phi_k \mathbb{E}_t \tilde{r}_{t+1}^k - (\tilde{r}_t - \mathbb{E}_t \tilde{r}_{t+1}) \right]$$

$$+ \frac{1}{1 + \beta + \phi_q} \tilde{i}_{t-1} + \frac{\beta + \phi_q (1 + \beta)}{1 + \beta + \phi_q} \mathbb{E}_t \tilde{i}_{t+1} - \frac{\beta \phi_q}{1 + \beta + \phi_q} \mathbb{E}_t \tilde{i}_{t+2},$$

(3.3)

where $\phi_q = (1 - \delta)/(1 - \delta + r_k^*)$; and $\phi_k = r_k^*/(1 - \delta + r_k^*)$. The above equation (3.3) represents current investment as a function of the future expectation of the rental rate of capital, the real interest rate, and the lagged and future expectation of investment.

The benefit of equation (3.3) stems primarily from the removal of the marginal $Q$ out of the investment Euler condition, as presented in (3.1). Though Hayashi (1982) showed that under certain regularity conditions, such as perfect competition and constant returns to scale technology, that marginal $Q$ and average $Q$ are equivalent. This conflicts with the poor empirical performance of average $Q$ as a predictor of marginal $Q$, which is a widely observed finding in the literature (see, for e.g., the survey by Chirinko (1993)). Furthermore, equation (3.3) is advantageous over the traditional investment models because it

\footnote{Note that equation (3.3) is as presented in Groth and Khan (2010). We re-derived these log-linearized Euler conditions and found the resultant condition to be equivalent with theirs. The derivations can be found in the Appendix.}
introduces inertia in the investment equation in a micro-founded way, which is empirically important, see Eberly et al. (2012). To the extent that our empirical model is a VAR, such inertia in investment is captured by the VAR’s lag structure.

3.3 Econometric framework

Our primary aim is to empirically, investigate whether there were sudden disruptions and/or gradual changes in the relationship between investment and uncertainty. In particular, we are interested in comparing the short term and long term relationships between investment and uncertainty, and, moreover, the structural inter-relationships between the variables we include in the VAR. To achieve this aim, we use a VAR with time varying parameters. This section describes this VAR along with the estimation method, the variables included endogenously in the VAR, and the identification strategy of the structural shocks.

3.3.1 The time-varying VAR

A reduced form time-varying VAR of order \( p \) is expressed as

\[
y_t = c_t + \sum_{p=1}^{P} B_{p,t} y_{t-p} + u_t, \quad u_t \sim N(0, \Omega_t),
\]

(3.4)

where \( y_t \) is the \( n \times 1 \) vector of endogenous variables; \( c_t \) is a \( n \times 1 \) column vector of intercepts; and \( B_{p,t} \) is a \( n \times n \) matrix containing the \( p^{th} \) lag autoregressive coefficients.

Since our approach to modeling investment dynamics is a VAR, we abstract from the theoretical, investment Euler condition reported in equation (3.3) in two ways. First, by focusing on the backward looking dynamics. Second, by introducing uncertainty into the model, which we proxy via stock volatility measures (discussed later). In other words, we allow uncertainty and uncertainty shocks to influence investment and other endogenous variables in the VAR.

This multivariate time-varying VAR has several advantages over the single equation modeling traditionally used in estimating investment models. First, our hypothesis implies that modeling the time dimension of the relationship between investment and uncertainty is essential. Second, the model enables us to identify and distinguish internal system and
external shocks, which can further be decomposed with a set of identifying assumptions. Such decomposition allows us to observe less direct relations between the variables, not achievable in univariate regressions.

3.3.2 Model specification and estimation

With reference to equation (3.4), we form a vector \( \beta_t \) by stacking the elements of \( c_t \) and \( B_{1,t} \ldots B_{P,t} \) equation by equation such that
\[
\beta_t \equiv \text{vec}(c_t, B_{1,t}, \ldots, B_{P,t})
\]
and let \( X_t \equiv I_n \otimes (1, y_{t-1}', \ldots, y_{t-P}' ) \), where \( \otimes \) denote the Kronecker product. Then equation (3.4) can be recast as a measurement equation of the state space model

\[
y_t = X_t \beta_t + A_{t}^{-1} \Sigma_t \epsilon_t, \quad t = P + 1, \ldots, T,
\]

where \( A_t^{-1} \) is a time varying matrix capturing the contemporaneous relations; \( \Sigma_t \) is a time varying matrix comprising standard deviations of the structural shocks; and \( \epsilon_t \sim N(0, I_n) \) is a vector of standardized structural shocks. Notice from equation (3.4) that the variance-covariance matrix of reduced form disturbance term, \( \Omega_t \), is subscripted with \( t \), implying that \( \Omega_t \) is also time varying and are factored as \( \Omega_t = A_t^{-1} \Sigma_t \Sigma'_t (A_t^{-1})' \), where \( A_t^{-1} \) and \( \Sigma_t \), as above, are respectively defined as

\[
A_t^{-1} = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
\tilde{a}_{21,t} & 1 & \ldots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
\tilde{a}_{n1,t} & \cdots & \tilde{a}_{nm-1,t} & 1
\end{bmatrix}, \quad \Sigma_t = \begin{bmatrix}
\sigma_{1,t} & 0 & \ldots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \sigma_{n,t}
\end{bmatrix}.
\]

For future references, we represent equation (3.5) in a companion form

\[
z_t = v_t + M_t z_{t-1} + P_t \epsilon_t,
\]

where \( z_t \) is a vector containing endogenous variables up to the lag order of \( P - 1 \); \( v_t \) comprises the intercept term; \( M_t \) is the companion matrix; and \( P_t \) is a Cholesky matrix that factors the companion form of the reduced form variance-covariance matrix, \( V_t = P_t P'_t \).

To capture possible non-stationary dynamics, we let the time varying parameters to
evolve as random and geometric random walk processes

\[ \beta_{t+1} = \beta_t + \nu_t, \]  
\[ \alpha_{t+1} = \alpha_t + \zeta_t, \]  
\[ \log \sigma^2_{t+1} = \log \sigma^2_t + \eta_t, \]

where \( \beta_t \) comprises stacked reduced form coefficients; \( \alpha_t \) is a vector of non-zero and non-unitary elements of \( A_t \) which are stacked by rows, \( \alpha_t = (a_{21,t}, \cdots, a_{nn-1,t})' \); and \( \sigma^2_t \) represents the vector of the leading diagonal elements of the variance matrix \( \Sigma_t \Sigma'_t \) such that \( \sigma^2_t = (\sigma^2_{1,t}, \cdots, \sigma^2_{n,t})' \), with corresponding dimensions \( n(nP+1) \times 1 \), \( n(n-1)/2 \times 1 \) and \( n \times 1 \) respectively. Next, we make a distributional assumption on \( \epsilon_t, \nu_t, \zeta_t \) and \( \eta_t \), which we now vectorize as \( \psi = (\epsilon_t, \nu_t, \zeta_t, \eta_t)' \). Following Primiceri (2005), we assume \( \psi \) to be jointly Gaussian with mutually uncorrelated white noise, zero mean and variances defined by \( I_n \) and the hyper-parameters \( Q, S \) and \( W \), such that \( \psi \sim N[0, \text{diag}(I_n, Q, S, W)] \), where \( I_n \) is a \( n \)-dimensional identity matrix; and \( Q, S \) and \( W \) are all diagonal positive semi-definite matrices.\(^3\)

For the initial states of parameters, we place priors that are considered uninformative for this class of model. Specifically, \( \beta_{P+1} \sim N(0, 10I) \), \( \alpha_{P+1} \sim N(0, 10I) \) and \( \log \sigma^2_{P+1} \sim N(0, 10I) \). The corresponding hyper-parameters’ priors are respectively \( Q_i \sim G^{-1}(20, 0.001) \), \( S_i \sim G^{-1}(6, 0.01) \), \( W_i \sim G^{-1}(6, 0.01) \), where \( G^{-1} \) denotes the inverse Gamma distribution; \( Q_i \), \( S_i \) and \( W_i \) denote respective diagonal elements in \( Q, S \) and \( W \). Essentially, the hyper-parameters in the time-varying VAR governs the rate at which the parameters vary over time. We regard these priors as being conservative from the point of view that there are time variations in the relationship between investment and uncertainty.\(^4\)

The time-varying VAR is then estimated by augmenting the Markov chain Monte

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\(^3\)The diagonal assumption of \( S \) implies that the non-zero and non-unitary elements of the matrix \( A_t \) are assumed to evolve and hence be distributed independently across equations (rows), but correlated within equations (rows). We further assume \( Q \) and \( W \) to be diagonal matrices to economize on the estimation of parameters, given reports that suggest that these cross-equation restrictions are of minimal importance (Primiceri, 2005; Nakajima, 2011).

\(^4\)Note that priors set over the hyper-parameters for the time-varying VAR coefficients are more tighter/informative than the ones set for variance and covariance states, due to the comment given by Sims (2001) on Cogley and Sargent (2001)’s work. Note moreover, that these priors set here are comparable to the ones set by Nakajima et al. (2011) and Nakajima and West (2013) that consider respectively, the Japanese and the US macroeconomic variables, where their sample includes the period where the nominal interest rate was constrained by the ZLB.
Carlo (MCMC) routine developed by Nakajima (2011). We employ Nakajima (2011)’s routine for generating the posterior distributions. In particular, we employ the simulation smoother of de Jong and Shephard (1995) to sample the parameters $b^T = \{\beta_t\}_{t=P+1}^T$ and $a^T = \{\alpha_t\}_{t=P+1}^T$. Likewise, we use the block sampler of Shephard and Pitts (1997) (algorithm corrected by Watanabe and Omori (2004)) to sample the stochastic volatility, $h^T = \{\log \sigma_t\}_{t=P+1}^T$. Hyper-parameters of these parameters are then generated from the inverse Gamma distribution, conditioned on their respective parameters. Our augmentation of this MCMC routine follows Cogley and Sargent (2005). Specifically, we solve for the eigenvalues of the characteristic equation, $\lambda_t$, and reject draws that produce $\max(|\lambda_t|) > 1$, such that $\lambda_t$ are all inside the unit circle. This step ensures that our posterior comprises draws that produce stable VAR coefficients at every point in time. Applying this MCMC routine, we collect 10,000 posterior samples, and discard the first 2,000 draws to ensure the convergence of the chain. Interested readers can refer to the Appendix about the details of the MCMC routine and the evaluation of the posterior samples.

3.3.3 Identification strategy

We employ a recursive identification scheme to identify uncertainty shocks. That is, we rely on the timing assumption with which the structural shocks affect endogenous variables within the VAR. This timing assumption allows us to re-interpret the Cholesky factorization of the reduced form residual’s variance-covariance matrix as a mapping between the reduced form and the structural model. This is consistent with previous VAR literature such as Bloom (2009), Jurado et al. (2015), and Basu and Bundick (2017) among others, that identify uncertainty shocks based on timing assumptions. Furthermore, this is convenient since the model presented in equation (3.5) and the corresponding Cholesky decomposition can be directly used for structural analyses.

As we rely on the recursive identification scheme to orthogonalize the VAR innovations, the order by which the endogenous variables enter the VAR matters. The conventional wisdom is to enter the most endogenous variable last, such that the more endogenous variables affect the less endogenous variables with a lag. To this end, we rely on the theory that we have discussed in section 2 and on the previous VAR studies that incorporate uncertainty proxy to analyze the macroeconomic dynamics. In a nutshell, our baseline
ordering of the endogenous variables is

\[ y_t = \left[ s_t, r_t^k, i_t, m_t \right]', \]

where \( s_t \) is the demeaned and standardized uncertainty proxy; \( r_t^k \) is the annualized rate of return from capital; \( i_t \) is the log difference of investment at annualized rate; and \( m_t \) is the ex-post real interest rate. We will describe the details of the data used in the VAR in the next section.

The ordering utilizes the timing assumption and is meant to reflect the backward and forward looking nature of the Euler condition in equation (3.3), even though the forward looking nature is not directly modeled. Here, uncertainty is treated as the most exogenous process and is ordered first. Given that current investment is dependent on expectations of the future real rental rate of capital, it is ordered second, before investment. The real interest rate, a composite of nominal yield on short term bonds and future expectation of inflation is ordered last, so as to best capture the behavior and the reaction of the Federal Reserve. Thus, the mechanism here is such that, private agents making investment decisions are able to immediately change their decision given changes to forward looking uncertainty and the real rental rate of capital, and the real interest rate is allowed to respond accordingly. This ordering mimics the macro VAR presented in Basu and Bundick (2017).

Keeping uncertainty as the first variable in the VAR, we confirm that our results are robust to different orderings of the remaining variables. The results are also robust to prior choices and to the use of alternative proxies for uncertainty and investment. The robustness checks are reported in the Appendix.

3.4 Data

We use time series data, collected from several sources, for the US over the period 1986:I to 2015:I. Here, we outline the dataset used for producing the results in the paper and relegate further details of the transformations and the sources of the data used for robustness to the Appendix.

\footnote{Basu and Bundick argue that the ordering of uncertainty measured by stock volatility is consistent with their theoretical New Keynesian model.}
Our measure of investment follows that of Groth and Khan (2010) and Gilchrist et al. (2014) and is the seasonally adjusted Private Nonresidential Fixed Investment (in Billions of US dollars), deflated to 2009 dollars by the Gross Domestic Product: Implicit Price Deflator (GDP deflator). The ex-post real interest rate is used as a proxy for the ex-ante real interest rate and is the difference between the secondary market rate on three months treasury bills and the annualized rate of log change in the GDP deflator. Since investment data and GDP deflator is available only at quarterly frequency, the data on nominal interest rate is averaged to quarterly.

Following Christiano and Davis (2006) and Groth and Khan (2010), we proxy the real rental rate of capital as

$$r^k_{t+1} = 400 \times \log \left( \frac{(\text{real S&P composite price}_{t+1} + \text{real dividends}_t)}{\text{real S&P composite price}_t} \right).$$

The relevant data was sourced from Robert Shiller’s personal website. These series are reported in monthly and are thus averaged to quarterly to match the frequency of other variables. The rental rate of capital is scaled by the parameter $\phi_k$ with $\beta = 0.95$ and $\delta = 0.05$, which are well within the typical calibrated values, see equation (3.3). This scaling makes the real rate of return on capital comparable to the real interest rate, a useful feature when analyzing impulse response functions.

We consider uncertainty proxies that are second moment in nature, derived from the financial market. Four existing proxies fit this criteria, and are summarized in Table 3.1. They are: i) VOL: realized stock volatility, ii) VXO: implied stock volatility, iii) GSZ: the proxy estimated by Gilchrist et al. (2014), and iv) JF3: Ludvigson et al. (2018)’s financial proxy. As we are dealing with investment, we argue, from the point of view of the investors that these are the relevant ones and moreover, they are consistent with traditional investment literature (see, e.g., Leahy and Whited (1996) and Bloom et al. (2007)).

We reduce the dimension of these four proxies to one via the principal component method. This treatment enables the extraction of the most important element describing

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the four processes. We compute the first principal component from the monthly series, and then averaging within the quarter to match the investment series. Interestingly, we find that the first principal component explains 78.5% of the total variations.

Figure 3.1 compares implied volatility, denoted VXO and the estimated first principal component, FPC, where both of proxies are demeaned and standardized. The US recessions are shaded in dark gray. From this figure we note the similarity between VXO and FPC, with regards to the shape and apparent spikes in 1987, corresponding to the Black Monday stock market crash and the financial crisis in 2008.

3.5 Empirical results

Using the ordering and priors presented in Section 3, we estimate a time-varying VAR with two lags. The lag length is set in accord with Cogley and Sargent (2005) and Primiceri (2005) and the confirmed instability of the time-varying VAR coefficients at higher order of lag length, inferred from the rejection step in the MCMC. The results from this specification are presented in this section. Results with different specification of the model, such as ordering, priors and lag lengths are similar and can be found in the Appendix.

3.5.1 Descriptive analyses of investment and uncertainty

We start our analyses by reporting the posterior estimates of the correlation between investment and uncertainty for both the short (instantaneous) and the long-run. The left panel of Figure 3.2 depicts the short-run estimate and the right panel represents the long-run estimate.

The short-run correlation is computed from the variance-covariance matrix, $\Omega_t$, defined in equation (3.4). We obtain the long-run correlation estimates from the unconditional variance-covariance matrix, which is estimated by taking the limit of conditional variance

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7 Another possibility would be first averaging the series and then calculating the principal component. In this case, the first principal component explains 82% of the total variations. In any case, the resulting principal component series are very similar.
over the forecast horizon

\[ \mathbf{\Gamma}_t(0) = \sum_{h=0}^{\infty} \mathbf{\Phi}_{h,t} \Omega \mathbf{\Phi}'_{h,t}, \]  

(3.9)

where \( \mathbf{\Phi}_{h,t} \) is the \( h \)th matrix lag of the corresponding vector moving average (VMA(\( \infty \))) form of equation (3.4). Such rewriting is assured by the satisfaction of the stability condition, achieved by the rejection of the draws that produce explosive roots.

Consistent with the previous empirical literature that reports negative correlation between investment and uncertainty (e.g., Jurado et al. (2015)), we observe that the short-run correlation between investment and uncertainty is negative for the entire period. Furthermore, although their correlation is weak in magnitude during the ‘normal’ times, it intensifies in the negative direction during recessions. Contrary to the short-run correlation, their correlation in the long-run appear to be not as clear cut in interpretation. At the start of the sample, the long-run correlation is strictly negative but it gradually weakens up until about the turn of the century; at that point, the relation is weak and ambiguous. It then dips back into the negative region and the magnitude gradually intensifies, reaching the ‘trough’ at the time of the financial crisis in 2008. Immediately after the crisis, the correlation weakens again and settles around at the value of \(-0.2\) for the remainder of the sample.

For the short-run correlation estimates, it is fair to conclude that it is driven at the business cycle frequency, and that the relationship between investment and uncertainty strengthens during recessions. In contrast, the long-run correlation appears to capture other unobserved forces, although the 2008 financial crisis likely caused the long-run correlation to intensify, as it did the short-run correlation.

To better understand the relationship between investment and uncertainty, we decompose the respective variables into short and long-run components

\[ i_t = i_t^{LR} + i_t^{SR}, \]
\[ s_t = s_t^{LR} + s_t^{SR}, \]

where the superscripts \( SR \) and \( LR \) denote, respectively, the short-run and the long-run. For investment, the long-run component \( i_t^{LR} \) captures the trend growth in investment
and $i^{SR}_t$ captures the short-run cyclical deviation away from the trend investment growth. Similarly, we decompose uncertainty into the short and the long-run component, which follows the treatment in Barrero et al. (2017).

Recall the $z_t$ vector (comprising endogenous variables) from the companion form representation of the VAR. The vector of long-run component is defined as

$$
\mu_t = \lim_{h \to \infty} E_t (z_{t+h}),
$$

i.e., the value which the variable is expected to converge to in the long-run. Following the trend inflation literature (see, e.g., Cogley and Sargent (2005) and Cogley et al. (2010)), we approximate this via the unconditional mean of the variable $\mu_t \approx ( I_{np} - M_t )^{-1} \nu_t$. The short-run component can then be approximated by taking the difference between the observed and the long-run component.

The posterior medians are used to construct the short and the long-run components of investment and uncertainty, which are plotted against one another in Figure 3.3. As a guideline, we include the line of best fit together with the 90% credible interval.

[Figure 3.3 here]

It is evident from the left column of Figure 3.3 that the short-run components of investment and uncertainty are negatively related while the long-run components are positively related. The negative short-run relationship between investment and uncertainty is hardly a surprise given previous uncertainty literature. However, to the extent of our knowledge, this is the first empirical study to reveal a positive relationship between trend investment growth and long-run uncertainty, consistent with the theoretical prediction of Oi (1961), Hartman (1972), and Abel (1983), that postulates that uncertainty is beneficial to investment growth once the short-run capital and labor adjustment constraints faced by firms are not binding (Bloom, 2014).

Using the log ratio of quarterly capital expenditures per unit of perpetual inventories capital as a proxy for investment and, the log difference between 30 day and six months
implied volatility as a proxy for uncertainty, Barrero et al. (2017) find a negative relationship between long-run uncertainty and investment. We note a difference between our findings and Barrero et al. (2017)’s. We find a positive relationship between the long-run component of uncertainty and the ‘trend growth’ in investment, whereas Barrero et al. (2017) infer a negative relationship on the basis of a regression of observed investment on long-run uncertainty. Similar to their finding, we find that the correlation between identified long-run component of uncertainty and the observed investment growth is indeed negative, and at $-0.13$, weak in magnitude.

The slope of the short-run negative relation between investment and uncertainty is about $-9.3$, implying that one unit change in the cyclical component of uncertainty brings about $-9\%$ reduction in cyclical investment growth. We can infer that there are three points that tilt the line of best fit in the negative direction, which correspond to the period during the 2008 financial crisis when investment growth and uncertainty were at their historical low and high respectively. When these three points are removed, the value of the slope declines to $-6.0$.\footnote{We have also ran a Bayesian robust regression with 1 degree of freedom parameter on the $t$-distribution, keeping the three points in the sample. The posterior mean value of the slope changed to $-7.5$, with widening of the 90\% credible interval as the value of the cyclical component of uncertainty increase.} Conversely, the positive long-run relationship between investment growth and uncertainty appears to be unaffected by the financial crisis. We confirm that the slope is barely affected, reading 2.6 both with and without the financial crisis period. Thus, it appears that the financial crisis in 2008 is more of a short-run event than the one affecting the long-run relation between investment and uncertainty. We shall delve further into this issue by invoking the structural dimension of the VAR.

3.5.2 Evolution of investment dynamics under uncertainty

The impulse response function is used to characterize the hypothetical reaction of each variable to a shock in uncertainty. We apply this shock in every time period. To allow comparability between the computed impulse responses, we apply in each period the time series average of stochastic volatility for uncertainty, $\sigma_s = 1/(T - P) \sum_{t=P+1}^{T} \exp\left(\log \sigma_{s,t}^2 / 2\right)$. Thus, the impulse responses reported here depicts the responses of variables when an average sized shock strikes the VAR system.

Each columns in Figure 3.4 illustrates the responses of the impacts of uncertainty
shocks on investment, real rental rate of capital, and the real interest rate respectively, over zero (instantaneous), two and three year impulse horizon separated in rows.

[Figure 3.4 here]

We confirm, consistent with previous studies that uncertainty shocks are contractionary shocks that aggravate economic conditions. Investment is negatively affected instantaneously. The adverse effects of uncertainty shocks are evident also for the real rental rate of capital, which responds negatively with little time variations. The instantaneous response of the real ex-post interest rate is mostly ambiguous. At the two year horizon, we note that the financial crisis appears to have created a systematic difference in the response of investment following uncertainty shocks, indicating that, following an uncertainty shock, investment growth is lower in post financial crisis years compared to the Great Moderation period. Similar, but more distinct pattern is observed at the three year impulse response horizon. Some evidence of the overshooting effect of uncertainty on investment is observed during the Great Moderation period, but that appears to be missing post financial crisis.

The relative sluggish reversion of investment growth back to its equilibrium (or lack of overshooting) in the years since the financial crisis might be explained by the responses of the real interest rate to uncertainty shocks, as is depicted in the right column of Figure 3.4. After two years, the response of the real interest rate—a measure the opportunity cost of investment—is strictly negative across the sample, excluding the post-crisis period. Similar behavior for the real interest rate is observed at the three year horizon. In contrast to the real interest rate response, we note that the response of real rental rate of capital reverts back to zero quickly. In fact, the reversion back to zero appear to be faster in post-crisis years than pre-crisis years.\textsuperscript{11}

To reinforce these findings, we plot the joint posterior distribution of the IRFs accumulated over three years for 1995 and 2014. The year 2014 represents a post-crisis year, one sufficiently removed from the financial crisis and its associated recession. The year 1995 is the representative year for the Great Moderation period. We also note that these two years are when the uncertainty proxy, FPC, depicted in Figure 3.1, is below the historical

\textsuperscript{11}The real rate of capital is converted on to a similar scale to that of the real interest rate by the scaling parameter $\phi_k$, see equation (3.3).
mean, with both years having approximately the same in value. We note that the choice of the year 1995 is arbitrary and we can confirm that the changes in the results are negligible if a different year in the Great Moderation sample is selected. Each panel in Figure 3.5 illustrates the joint distribution of the IRFs. The black line depicts the 45° degree line, and the departure from this line is an evidence of systematic differences in the impulse responses.

[Figure 3.5 here]

There are noticeable differences in the response of investment growth and the real interest rate between 1995 and 2014. Most of the posterior mass of investment growth lies below the 45° degree line, whereas for real interest rate it is above the 45° degree line. This implies that response of investment to uncertainty shock is more negative in 2014 than in 1995, with the opposite being true for the real interest rate.

Complementary to Figure 3.5, Table 3.2 reports the probabilities of a response to an uncertainty shock being greater in 1995 than in 2014, for impulse horizons $h = 0 - 3$, $h = 4 - 7$ and $h = 8 - 11$.

[Table 3.2 here]

The probability that the response of investment growth following uncertainty shocks is higher in 1995 than in 2014 ($P(i_{1995} > i_{2014})$) holds essentially for the entire impulse horizon. When the shocks are accumulated over three years, this probability reaches 75%. Likewise, the same probability estimate for the real interest rate, $P(m_{1995} > m_{2014})$ is stable over the impulse horizon and exhibits significant differences between 1995 and 2014. These results imply that investment growth was lower in 2014 than in 1995 and, conversely, that the response of real rate following uncertainty shocks remained comparably higher in 2014 than in 1995.

Based on the results derived thus far, it is evident that the dynamics of investment following uncertainty shocks has changed over time, indicating a nonlinearity in the relationship between the determinants of investment at the ZLB. The impulse response functions suggest that these changes stem partly from the insufficient decline in the real interest rate (or, from insufficiently high inflation) in response to uncertainty shock.
From the above table, we also observe that the same probability estimate for uncertainty when accumulated over three years is $P(s_{1995} > s_{2014}) = 0.70$, indicating that the effect of uncertainty shocks on uncertainty dissipated more quickly in 2014 than in 1995. We explore this finding in the next subsection by investigating if the degree of ‘persistence’ of the uncertainty shock has changed over time.

**3.5.3 Has persistence of uncertainty proxy changed?**

Following Cogley et al. (2010), we define the measure of persistence as one minus the ratio of conditional (forecast error) variance over unconditional variance

$$R_t^2(H) = 1 - \frac{k_s \left[ \sum_{h=0}^{H-1} \Phi_{h,t} \Omega_t \Phi_{h,t}' \right] k_s'}{k_s \left[ \sum_{h=0}^{\infty} \Phi_{h,t} \Omega_t \Phi_{h,t}' \right] k_s'},$$

where $k_s$ is a selection (row) vector for uncertainty proxy; $\Phi_{h,t}$ and $\Omega_t$ are as defined before. By construction, $R_t^2(H)$ measures the persistence in the gap component, i.e., the difference between the observed and the long-run component. We view this as a useful metric, given that the short-run cyclical component is responsible for retarding investment. One caveat is that the measure reported here captures whether uncertainty has been more or less persistent, and not the reason for its persistence.

Figure 3.6 contains the 3-dimensional plot of the posterior median of $R_t^2(H)$. The remaining panels depict the estimates at 1, 4 and 8 quarter forecast horizons together with their respective 68% credible bands.

From the above figure we note, first, the substantial time variations in the persistence of uncertainty proxy at the business cycle frequency. Persistence of uncertainty is high during the ‘normal’ times, while it is low during recessions. In other words, the predictability of uncertainty is high during the normal times but low during recessions. This result reinforces the earlier finding that the nonlinear response in the growth of investment since the financial crisis stems from the ZLB. That is, the growth of investment did not recover as it had after recessionary periods, which were themselves characterized by the behavior of uncertainty returning to more predictable level in the post-recessionary periods.
3.5.4 Counterfactual simulation

Given previous results, we conduct a counterfactual simulation to assess the role of uncertainty over the financial crisis and the subsequent period. To this end, we consider counterfactual scenarios where we: i) mute the identified uncertainty shocks, and ii) draw the coefficients and covariance states of the investment equation in our time-varying VAR over 2008:I to 2015:I from the posterior of 1995, and iii) a scenario where we combine i) and ii). Thus, in the counterfactual simulation, we shut down uncertainty shocks by setting it to zero and/or let investment adjust as it did in 1995, when the US economy experienced stable growth.

Each panel in Figure 3.7 depicts both the actual and counterfactual path of the variable, under scenario iii) simulated from the time-varying VAR. At some points in time, the variables under the counterfactual scenario demonstrate substantial departure away from the recorded/historical paths of the variables. In other words, the counterfactual scenario where we shut down uncertainty shocks and allow investment to adjust as it did in 1995 would have changed the course of history.

Observe Figure 3.7, there are three periods where the rate of growth of investment in the counterfactual simulation was significantly higher (less negative) than the actual path. The first period occurs in the fourth quarter of 2008, where the uncertainty proxy peaks during the onset of the financial crisis. It is evident that, in the counterfactual scenario, the rise in the uncertainty proxy and the fall in the real rental rate of capital is not as extreme. The second period in which the growth of investment in the counterfactual scenario is significantly higher than the actual path occurs in the second quarter of 2010. This period corresponds to the ‘euro-zone crisis’ period as identified by Baker et al. (2016). The third period occurs in the third quarter of 2011 where the US government bonds were downgraded by Standard & Poor’s (S&P). The financial market reaction has been negative.

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12 1995 posterior means the posterior average (Monte Carlo average) over the four quarters in 1995. This choice is arbitrary, meant to be a representative year during the Great Moderation.

13 Note that there are no material difference in the results even if the volatility for uncertainty shock is kept constant. In other words, no material effect on the results are observed when the variance of the unstandardized structural shock, $\Sigma_i \Sigma_i'$ for uncertainty is fixed at the value observed in 1995 over 2008:I to 2015:I.
and the S&P500 lost 14% over the quarter, which is the largest loss second to the financial crisis. These onset of events since the financial crisis appear to have had notable effect on uncertainty (proxied via the stock volatility), real rental rate of capital and investment. The real interest rate appears to deviate away from the actual path during the financial crisis but the deviation in the third quarter of 2011 is minimal.

To tease out the contributions of scenarios (i) and (ii), Table 3.3 shows the actual average growth rates of variables from 2008:I to 2015:IV as well as the deviation away from the actual growth rates under the counterfactual scenario. It is evident that most of the gains in the higher growth of investment stems from the muting of the uncertainty shocks, rather than from fixing the investment adjustment process to function as it had in 1995. Nevertheless, there are some positive gains in investment growth under the counterfactual simulation (ii), suggesting that the investment adjustment process has indeed changed.

3.5.5 Model comparison

In order to examine the importance of time varying parameters in the VAR, we now consider a forecasting exercise motivated by the results of the previous sections. We achieve this by comparing the root mean squared forecast error (RMSFE) and log predictive density scores (LPDS) computed based on the pseudo-out-of-sample forecast estimates from the time-varying VAR and its nested constant counterparts. To this end, we follow the density forecasting framework for time-varying VAR models developed by Cogley et al. (2005) and forecast one to four quarters over the 2000:I to 2015:I sample period. This period splits the periods before and after the financial crisis into approximately equivalent lengths.

Adopting a similar notation to Cogley et al. (2005), let $y^T = [y_1', \ldots, y_T']'$ denote the vector of observables up to time $T$, and let $y^{T+1,T+h} = [y_{T+1}', \ldots, y_{T+h}']'$ denote the vector of the potential future paths of the variables. The time-varying VAR consists of two sets of parameters, one that drift over time, which we define it as $\theta^T = [b^T, a^T, h^T]'$, with corresponding forecasts $\theta^{T+1,T+h}$, and another set for constant parameters, $\omega = [Q, S, W]'$, comprising hyper-parameters. It is to be noted that the composition of $\theta^T$ and $\omega$ changes for the nested models of time-varying VAR.
Our forecast estimates are constructed from the posterior predictive density, which is the joint probability density over future paths of the data, conditioned on priors and the history of observables. This density is expressed as

\[
p(y^{T+1,T+h}|y^T) = \int \int p(y^{T+1,T+h}, \theta^{T+h}, \omega|y^T) \, d\theta^{T+h} \, d\omega,
\]

where \(p(y^{T+1,T+h}, \theta^{T+h}, \omega|y^T)\) is the joint posterior density, factored as

\[
p(y^{T+1,T+h}, \theta^{T+h}, \omega|y^T) = p(\theta^T, \omega|y^T) p(\theta^{T+1,T+h} | \theta_T, \omega, y^T) p(y^{T+1,T+h} | \theta^{T+h}, \omega, y^T),
\]

following Cogley et al. (2005). The first term on the RHS is the posterior density, for which we obtain 10,000 samples using the MCMC routine described in Section 3. The second term represents the restricted density of the future course of time varying parameters. Samples from this density are obtained by iterating on the transition equations in (3.6) to (3.8) for every sample obtained from \(p(\theta^T, \omega|y^T)\) with rejection of explosive draws. The last term is the predictive density of the VAR with known parameters, given draws from \(p(\theta^T, \omega|y^T)\) and \(p(\theta^{T+1,T+h} | \theta_T, \omega, y^T)\). Therefore, equation (3.5) can be used to sample from \(p(y^{T+1,T+h}|y^T)\). The marginal density \(p(y^{T+1,T+h}|y^T)\) is obtained by ignoring the nuisance parameters.

The metric we use to evaluate the accuracy of the point forecasts is RMSFE, defined

\[
\text{RMSFE}[y^{T+1,T+h}] = \left[ \frac{1}{T - h - t_0 + 1} \sum_{t=t_0}^{T-h} \left( y^{T+1,T+h} - E(y^{T+1,T+h}|y^T) \right)^2 \right]^{\frac{1}{2}},
\]

where \(y^{T+1,T+h}\) is the observed data; \(E(y^{T+1,T+h}|y^T)\) is the posterior mean of the realizations of the forecasts; \(h\) denotes the forecast horizons, which we forecast from 1 to 4 forecast horizons. We start the forecasting exercise from \(t_0=2000:1\). The density forecasts are evaluated at the observed data using the log predictive density scores

\[
\text{LPDS}[y^{T+1,T+h}] = \frac{1}{T - h - t_0 + 1} \sum_{t=t_0}^{T-h} \log \{ p(y^{T+1,T+h}|y^T) \}.
\]

Table 3.4 presents the resulting RMSFE and LPDS as a ratio and log ratio over the

---

14Note that the model considered in Cogley et al. (2005) does not allow time variations in the covariance states. Density forecasting using this framework for the more general time-varying VAR of Primiceri (2005) can be found in Cross and Poon (2016) and D’Agostino et al. (2013) among others.
constant parameter VAR respectively. The time varying models are distinguished by what parameters are allowed to vary through time, with these parameters identified in the table’s leftmost column.

[Table 3.4 here]

It is evident from Table 3.4 that the time varying models provide some improvement in the accuracy of point forecasts in the short forecast horizon, although this improvement appears minimal. However, significant improvement in favor of the time varying models are evident when Bayesian uncertainty around the point forecasts is taken into account. The LPDS in favor of the time-varying VAR against the constant VAR amounts to 73.57 at the one quarter forecast horizon, which is practically substantial.

For a set of time varying models and forecast period we consider, it is evident that the time-varying VAR outperforms rest of the nested models. Furthermore, we can infer that the VAR model with stochastic volatility proposed by Uhlig (1997) is outperformed by the standard, time invariant VAR for longer term forecasts. This result might be of some surprise, given the model comparison evidence from Chan and Eisenstat (2018) regarding the US macroeconomic variables. Chan and Eisenstat (2018) consider essentially the entire post World World II period and find that the gains from allowing for drifts in the variances are greater than for allowing changes in the VAR coefficients. This is likely true when one tries to model the transition of the US economy from the very volatile periods of the 70s and the early 80s to the Great Moderation period. However, when one tries to compare shorter time periods that exclude the volatile periods of the 70s, as is the case in our study, the results might change, particularly when the relative size of the period characterized by the ZLB increases. Indeed, our results suggest that it is more important to allow for coefficient drift than for drift in variance.

3.6 Conclusion

This paper studies the relationship between investment and uncertainty using the time-varying VAR. The use of time-varying VAR is motivated by preceding literature on the changes in the relationship between investment and uncertainty around the Great Recession.
The main findings of the paper are summarized as follows. First, the descriptive, reduced form relationship between investment and uncertainty varies over time, with the short-run components negatively related, and the long-run components positively related. Second, non-linearity in the response of investment to uncertainty shock is observed in the aftermath of the Great Recession. This coincides with the period in which the nominal interest rate was constrained by the zero lower bound, and the lack of negative response in real interest rate from uncertainty shock is evident. Third, evidence of time variations in the persistence in uncertainty is observed at the business cycle frequency. Fourth, counterfactual simulation reveals that shutting down uncertainty shocks would have raised the growth of investment. Finally, the model comparison exercise demonstrates the importance of time varying parameters. Taken together, we conclude that the effects of uncertainty shocks are amplified when the economy is at the zero lower bound.
Figures

Figure 3.1: Uncertainty proxy

Note: Data span from 1986:I to 2015:I, averaged from monthly to quarterly frequency. FPC denotes the first principal component of the four uncertainty proxies and VXO is the implied stock volatility. Both measures are demeaned and standardized.
Figure 3.2: Short and long-run correlation

Note: Short run correlation is computed from the instantaneous variance-covariance matrix; long run correlation is computed from the unconditional variance-covariance matrix. The black solid line and dotted lines respectively illustrate the median and 68% credible intervals. Gray shaded region in the background depicts NBER recessions.
Figure 3.3: Short and long-run relationship between investment and uncertainty

Note: The top panels depict the short and long-run components of investment and uncertainty. The bottom panels depict their respective relationships, where solid blue lines and blue transparent shaded regions illustrate the mean and 90% credible intervals.
Figure 3.4: Responses of $i_t$: investment, $r_t^k$: real rental rate of capital and $m_t$: real interest rate to uncertainty shocks

Note: Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering $y_t = [s_t, r_t^k, i_t, m_t]'$ adopted. The first column shows the response of $i_t$: investment, second column shows the response of $r_t^k$: real rental rate of capital, and the third column shows the response of $m_t$: real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
Figure 3.5: Joint distribution of accumulated IRF in 1995 and 2014

Note: The joint distribution of accumulated impulse responses over 3 years in 1995 and 2014. Black line depicts the 45° line.
Figure 3.6: Persistence of $s_t$ as measured by $R^2_t(H)$

Note: 3D plot depicts the posterior median of $R^2_t(H)$ over 12 quarters. Rest of the panels depicts the posterior median and 68% credible intervals at 1, 4, and 8 quarter horizon. Gray shaded region in the background depicts NBER recessions.
Figure 3.7: Counterfactual paths of variables

Note: Each panel compares the actual and counterfactual path of the variables. Black solid lines depict the actual path of the variables, and the dotted line is the posterior median. Blue shaded regions represent the posterior 68% credible interval, while the Gray shaded region in the background depicts NBER recessions. The panel under $r^c_t$ depicts the real rental rate of capital scaled by $\phi_k$. 

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### Table 3.1: Mnemonics and summary of uncertainty proxies

<table>
<thead>
<tr>
<th>Proxy</th>
<th>Description</th>
<th>Source</th>
<th>Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>Realized volatility, a proxy for uncertainty computed as a standard deviation of daily S&amp;P500 returns averaged over a month.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>VXO</td>
<td>Implied S&amp;P 100 volatility, used instead of the VIX for the same reasons as in other literature, to increase the time span.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>GSZ</td>
<td>Risk-adjusted proxy for uncertainty that captures the common shocks in the idiosyncratic volatility of equity returns.</td>
<td>Gilchrist et al. (2014) and Caldara et al. (2016)</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>FPC</td>
<td>The first principal component scores of the above 4 uncertainty proxies.</td>
<td>Author’s computation</td>
<td>1986:1 – 2015:3</td>
</tr>
</tbody>
</table>
Table 3.2: Difference in the posterior probability of IRFs

<table>
<thead>
<tr>
<th>Probability</th>
<th>Impulse horizon (h)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 0 - 3$</td>
<td>$h = 4 - 7$</td>
<td>$h = 8 - 11$</td>
<td>Accum</td>
</tr>
<tr>
<td>$s_1$: uncertainty proxy</td>
<td>$P(s_{1995} &gt; s_{2014})$</td>
<td>0.74</td>
<td>0.70</td>
<td>0.54</td>
</tr>
<tr>
<td>$r_1^k$: real capital rental rate</td>
<td>$P(r_{1995} &gt; r_{2014})$</td>
<td>0.46</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>$i$: investment</td>
<td>$P(i_{1995} &gt; i_{2014})$</td>
<td>0.71</td>
<td>0.64</td>
<td>0.77</td>
</tr>
<tr>
<td>$m_1$: real interest rate</td>
<td>$P(m_{1995} &gt; m_{2014})$</td>
<td>0.17</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>

*Note: Column under ‘Accum’ shows the probability for IRFs accumulated over 3 years.*
Table 3.3: The movements of variables under counterfactual simulations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Actual</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_t$: uncertainty proxy</td>
<td>0.30</td>
<td>-0.76</td>
<td>0.00</td>
<td>-0.76</td>
</tr>
<tr>
<td>$r_k^t$: real capital rental rate</td>
<td>1.10</td>
<td>0.52</td>
<td>0.00</td>
<td>0.52</td>
</tr>
<tr>
<td>$i_t$: investment</td>
<td>0.80</td>
<td>0.96</td>
<td>0.19</td>
<td>0.97</td>
</tr>
<tr>
<td>$m_t$: real interest rate</td>
<td>-1.23</td>
<td>-0.04</td>
<td>0.00</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Note: Column under ‘Actual’ shows the average of the observed growth rate over 2008:I to 2015:IV, while under ‘Counterfactual’, shows the deviation away from the actual growth rate under the counterfactual simulations (i), (ii) and (iii), using the posterior median. Specifically, it is computed by \[
\frac{(\text{counterfactual} - \text{actual})}{\text{actual}}.
\]
<table>
<thead>
<tr>
<th>Time varying parameter</th>
<th>RMSFE</th>
<th></th>
<th></th>
<th></th>
<th>LPDS</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td></td>
<td>0.93</td>
<td>0.95</td>
<td>0.94</td>
<td>0.93</td>
<td>49.82</td>
<td>33.20</td>
<td>30.31</td>
</tr>
<tr>
<td>$\log \sigma_t^2$</td>
<td></td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
<td>0.58</td>
<td>-2.80</td>
<td>-4.23</td>
</tr>
<tr>
<td>$\beta_t$, $\alpha_t$,</td>
<td></td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>62.58</td>
<td>38.22</td>
<td>33.06</td>
</tr>
<tr>
<td>$\log \sigma_t^2$</td>
<td></td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>73.57</td>
<td>36.34</td>
<td>28.52</td>
</tr>
</tbody>
</table>

*Note:* The model in first row mirrors the one developed by Cogley and Sargent (2001), while the model in the second row is by Uhlig (1997). Models in third and fourth rows respectively are developed by Cogley and Sargent (2005) and Primiceri (2005).
Appendices for Chapter 3

Appendix 3.A  Data

The chapter draws from a variety of publicly available data sources: the investment data comes from the US Bureau of Economic Analysis, and the proxies for uncertainty come from the Chicago Board Options Exchange, as well as the papers Gilchrist et al. (2014) and Jurado et al. (2015). Some of the data were retrieved from the Federal Reserve Bank of St. Louis Economic Research website, (a.k.a., FRED), http://research.stlouisfed.org/fred2/. For clarity on the mnemonics used for uncertainty proxies, Table 3.1 of the main text is reprinted as Table 3.A.1.

3.A.1  Data sources

Uncertainty proxies

  **VOL**: The data for this proxy is retrieved from the Chicago Board Options Exchange (CBOE), and I compute the standard deviation of the daily returns of S&P 500 index averaged over a month.

  **VXO**: The data for this proxy is sourced from CBOE and retrieved from FRED, https://fred.stlouisfed.org/series/VXOCLS. I use this proxy instead of the volatility index, VIX, since the VXO, [VXOCLS] spans a longer time horizon. The correlation between the VIX and the VXO is high and use of the VXO instead of the VIX in the uncertainty literature has become standard.
**GSZ:** This proxy is introduced in Gilchrist et al. (2014) and retrieved from the Online Appendix of Caldara et al. (2016).

**JF3:** This is financial uncertainty measure proposed in Jurado et al. (2015), retrieved from Sydney C. Ludvigson’s personal website, https://www.sydneyludvigson.com/data-and-appendixes/.

**Investment**

In the manuscript, investment is proxied via the seasonally adjusted Private Nonresidential Fixed Investment (in billions of US dollars), deflated to 2009 dollars by the Gross Domestic Product: Implicit Price Deflator (GDP deflator). Following Basu and Bundick (2017), robustness checks are conducted with using the seasonally adjusted sum of Private Fixed Investment and Personal Consumption Expenditures: Durable Goods (in billions of US dollars), deflated by the GDP deflator.

**Private Nonresidential Fixed Investment:**
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [PNFI], https://fred.stlouisfed.org/series/PNFI. This series is reported at quarterly frequency in billions of dollars, seasonally adjusted annual rate.

**Fixed Private Investment:**
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [FPI], https://fred.stlouisfed.org/series/FPI. This series is reported at quarterly frequency in billions of dollars, seasonally adjusted annual rate.

**Personal Consumption Expenditures: Durable Goods:**
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with code [PCDG], https://fred.stlouisfed.org/series/PCDG. This series is reported at quarterly frequency in billions of dollars, seasonally adjusted annual rate.

**Inflation and interest rates**

**Gross Domestic Product: Implicit Price Deflator:**
Sourced from the U.S. Bureau of Economic Analysis and retrieved from FRED with
code [GDPDEF], https://fred.stlouisfed.org/series/GDPDEF. This series is reported at quarterly frequency in Index, 2009=100, seasonally adjusted.

3-Month Treasury Bill: Secondary Market Rate:
Sourced from the Board of Governors of the Federal Reserve System (US) and retrieved from FRED with code [TB3MS], https://fred.stlouisfed.org/series/TB3MS. This series is reported in percent, not seasonally adjusted, averaged to quarterly.

2-Year Treasury Constant Maturity Rate:
Sourced from the Board of Governors of the Federal Reserve System (US) and retrieved from FRED with code [DGS2], https://fred.stlouisfed.org/series/DGS2. This series is reported in percent, not seasonally adjusted, averaged to quarterly.

Rental rate of capital

Real S&P Composite Price:

Real Dividends:

3.A.2 Data transformation

Data retrieved from the above sources are subject to standard transformations. In this subsection, we first outline the way in which the data were transformed for the analyses in the main manuscript, then likewise for the robustness.

Manuscript

Uncertainty: The first principal component of the demeaned and standardized monthly uncertainty proxies (four proxies as above) are taken. Then, the resulting series is again, demeaned and standardized before it is averaged to quarterly.
Rental rate of capital: Sum up a period ahead Real S&P Composite Price series and current period Real Dividends series. Then, divide the resulting series by the current period Real S&P Composite Price series and take logs. Multiply it by 400 to annualize. The series is then scaled by \( \phi_k = r^k_* / (1 - \delta + r^k_* + \delta) \), where \( r^k_* = 1/\beta - 1 + \delta \).

With \( \beta = 0.95 \) and \( \delta = 0.05 \), \( \phi_k = 0.0975 \).

Investment: Deflate the private non-residential fixed investment it into 2009 Dollars using the Gross Domestic Product: Implicit Price Deflator. Take the log ratio of the resulting series and multiply by 400 to convert it into annualized rate.

Real interest rate: Ex-ante real interest rate is proxied via the ex-post real interest rate, by taking the difference of current nominal, secondary market rate on the 3-months treasury bills and the period ahead realized inflation, using the GDP deflator. Note that inflation is defined as the log change in the GDP deflator multiplied by 400.

Robustness

Uncertainty: For robustness, I use the demeaned and standardized VXO following Basu and Bundick (2017).

Investment: Sum up Fixed Private Investment and Personal Consumption Expenditures: Durable Goods and then deflate it into 2009 Dollars using the Gross Domestic Product: Implicit Price Deflator. Take the log ratio of the resulting series and multiply by 400 to convert it into annualized rate.

Real interest rate: Ex-ante real interest rate is proxied via the ex-post real interest rate, by taking the difference of current nominal, secondary market rate on the 2 year treasury bills and the period ahead realized inflation, using the GDP deflator. Note that inflation is defined as the log change in the GDP deflator multiplied by 400.
Appendix 3.B Derivation of the Euler condition

We consider the following prototypical New Keynesian model adopted from Groth and Khan (2010), which is a simplified variant of the medium-scale model developed by Christiano et al. (2005).

A representative household’s lifetime utility, separable in consumption, $C_t$ and the hours worked, $L_t$ is expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),$$

(3.A.1)

where $\beta$ is the discount factor. Here, output is indexed following Dixit and Stiglitz (1977)

$$Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1,$$

where $\theta$ is the elasticity of substitution between the differentiated goods over $i \in [0, 1]$. The corresponding demand for each good is

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} Y_t,$$

where $P_t$ is the associated price index

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

and note the existence of technology that covert homogeneous goods one-for-one with consumption and investment goods.

The household is constrained by a period budget

$$C_t + I_t + \frac{B_{t+1}}{R_t P_t} = \frac{B_t}{P_t} + \frac{W_t L_t}{P_t} + \Pi_t + R^k_t K_t,$$

where $I_t$ is investment; $B_t$ is the amount of nominal risk-free bonds that pay a nominal gross interest rate of $R_t$; $W_t$ is a nominal wage; $\Pi_t$ is the lump-sum profit of the firm; and $R^k_t$ is the real rental rate of capital, $K_t$. Investment, $I_t$ induces capital evolution, which accumulate according to

$$K_{t+1} = (1 - \delta) K_t + \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$
where $\delta$ is the depreciation rate; and $S(I_t/I_{t-1})$ is the convex investment adjustment cost function.

In this class of models, the capital stock, $K_t$, is assumed to be owned by the household which lends it to firms at the rental rate, $R^k_t$. With this in mind, the Lagrangian associated with the household’s problem is

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ U(C_t, L_t) + \lambda_{1,t} \left( \frac{B_t}{P_t} + \frac{W_t L_t}{P_t} + \Pi_t + R^k_t K_t - C_t - I_t - \frac{B_{t+1}}{R_t P_t} \right) + \lambda_{2,t} \left\{ (1-\delta)K_t + \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t - K_{t+1} \right\} \right].$$

The resulting first order conditions with respect to $I_t$ and $K_{t+1}$ are

$$\frac{\partial L}{\partial I_t} = -\lambda_{1,t} + \lambda_{2,t} \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta \mathbb{E}_t \left\{ \lambda_{2,t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} = 0,$$

$$\frac{\partial L}{\partial K_{t+1}} = -\lambda_{2,t} + \beta \mathbb{E}_t \left[ \lambda_{1,t+1} R^k_{t+1} + \lambda_{2,t+1}(1-\delta) \right] = 0.$$  

Rewriting the above in terms of the marginal $Q$, defined as the ratio of Lagrange multipliers, $Q_t = \lambda_{2,t}/\lambda_{1,t}$, we have

$$1 = Q_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta \mathbb{E}_t \left\{ Q_{t+1} \frac{\lambda_{1,t+1}}{\lambda_{1,t}} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\},$$

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \left[ R^k_{t+1} + Q_{t+1}(1-\delta) \right] \right\}.$$  \hspace{1cm} (3.A.2)

Equation (3.A.2) implies that in the absence of adjustment costs, i.e. $S(\bullet) = 0$, the marginal $Q$ is one, and the model collapses to a standard model. In other words, at the steady state, this functional form imply $S(1) = S'(1) = 0$ and $S''(1) \equiv \kappa > 0$, with $\kappa$ denoting the adjustment cost parameter.

To log-linearize (3.A.2) about the steady state, consider first the partial derivatives of
arguments inside the expectation operator

\[ Q_{t+1} : \frac{\lambda_{1,t+1} + 1}{\lambda_{1,t}} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2, \]

\[ \lambda_{1,t+1} : \frac{Q_{t+1}}{\lambda_{1,t}} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2, \]

\[ \lambda_{1,t} : -Q_{t+1} \frac{\lambda_{1,t+1} + 1}{\lambda_{1,t}} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2, \]

\[ I_{t+1} : Q_{t+1} \frac{\lambda_{1,t+1} + 1}{\lambda_{1,t}} \left[ 2 \left( \frac{I_{t+1}}{I_t^2} \right) S' \left( \frac{I_{t+1}}{I_t} \right) + \left( \frac{I_{t+1}}{I_t} \right)^2 S'' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{1}{I_t} \right) \right], \]

\[ I_t : Q_{t+1} \frac{\lambda_{1,t+1} + 1}{\lambda_{1,t}} \left[ -2 \left( \frac{I_{t+1}}{I_t^2} \right) S' \left( \frac{I_{t+1}}{I_t} \right) - \left( \frac{I_{t+1}}{I_t} \right)^2 S'' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{1}{I_t} \right) \right], \]

and that outside the expectation operator

\[ Q_t : 1 - S \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right), \]

\[ I_t : Q_t \left[ S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{1}{I_t} \right) - S'' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{1}{I_{t-1}} \right) \right], \]

\[ I_{t-1} : Q_t \left[ -S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) + S'' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{1}{I_{t-1}} \right) \right]. \]

Observe that partial derivatives of \( Q_{t+1}, \lambda_{1,t}, \lambda_{1,t+1} \) are zero at the steady state, due to \( S(1) = S'(1) = 0 \). Now consider the first order Taylor expansion about the steady state, starting again with arguments inside the expectation operator

\[ I_{t+1} : \kappa \left( \frac{I_{t+1} - \bar{I}}{\bar{I}} \right) \bar{Q}, \quad (3.4.4) \]

\[ I_t : -\kappa \left( \frac{I_t - \bar{I}}{\bar{I}} \right) \bar{Q}, \quad (3.4.5) \]

where \( \bar{Q} \) and \( \bar{I} \) are steady state values of their respective variables. The arguments outside the expectation operator are

\[ Q_t : Q_t - \bar{Q}, \quad (3.4.6) \]

\[ I_t : -\kappa \left( \frac{I_t - \bar{I}}{\bar{I}} \right) \bar{Q}, \quad (3.4.7) \]

\[ I_{t-1} : \kappa \left( \frac{I_{t-1} - \bar{I}}{\bar{I}} \right) \bar{Q}. \quad (3.4.8) \]
Define
\[ \tilde{x}_t \equiv \frac{X_t - \bar{X}}{\bar{X}}, \]
and apply it to equations (3.A.4) to (3.A.8). Plugging the resultants back into (3.A.2)
yields the log-linearized investment Euler condition
\[ \begin{align*}
1 &= \tilde{Q} + \left( Q_t - \tilde{Q} \right) - \kappa \tilde{Q}_{1,t} + \kappa \tilde{Q}_{1,t-1} + \beta \mathbb{E}_t \left( \tilde{i}_{t+1} - \tilde{i}_t \right) \kappa \tilde{Q}, \\
\left( Q_t - \tilde{Q} \right) &= \kappa \tilde{Q} \left[ \left( \tilde{i}_t - \tilde{i}_{t-1} \right) - \beta \mathbb{E}_t \left( \tilde{i}_{t+1} - \tilde{i}_t \right) \right], \\
\frac{1}{\kappa} \tilde{q}_t &= \tilde{i}_t (1 + \beta) - \tilde{i}_{t-1} - \beta \mathbb{E}_t \tilde{i}_{t+1}, \\
\tilde{i}_t &= \frac{1}{1 + \beta} \tilde{i}_{t-1} + \frac{\beta}{1 + \beta} \mathbb{E}_t \tilde{i}_{t+1} + \frac{1}{\kappa (1 + \beta)} \tilde{q}_t, \\
\end{align*} \]
using the fact that marginal \( Q, \bar{Q} = 1 \) at the steady state. The inertia in investment, as
reflected by the lagged investment term is due to the specification of the adjustment cost
function.

In regard to the log-linearization of equation (3.A.3), move the discount factor \( \beta \) inside
the expectational operator to derive the first order partial derivatives
\[ \begin{align*}
\frac{\partial Q_t}{\partial \lambda_{1,t+1}} &= \frac{\beta}{\lambda_{1,t}} \left[ R^k_{t+1} + (1 - \delta) Q_{t+1} \right], \\
\frac{\partial Q_t}{\partial \lambda_{1,t}} &= -\frac{\lambda_{1,t+1}}{\lambda^2_{1,t}} \beta \left[ R^k_{t+1} + (1 - \delta) Q_{t+1} \right], \\
\frac{\partial Q_t}{\partial R^k_{1,t+1}} &= \beta \frac{\lambda_{1,t+1}}{\lambda_{1,t}}, \\
\frac{\partial Q_t}{\partial Q_{t+1}} &= \beta (1 - \delta) \frac{\lambda_{1,t+1}}{\lambda_{1,t}}. \\
\end{align*} \]
First order Taylor expansion about the non-stochastic steady state gives
\[ \begin{align*}
Q_t &\approx \beta \left[ R^k + \bar{Q} (1 - \delta) \right] + \frac{\beta}{\lambda_1} \left( \lambda_{1,t+1} - \bar{\lambda}_1 \right) \\
&\quad - \frac{\beta}{\lambda_1} \left[ R^k + \bar{Q} (1 - \delta) \right] \left( \lambda_{1,t} - \bar{\lambda}_1 \right) + \beta \left( R^k_{t+1} - \bar{R}^k \right) + (1 - \delta) \beta \left( Q_{t+1} - \bar{Q} \right), \\
\end{align*} \]
which can be tidied up to
\[ \begin{align*}
Q_t &\approx 1 + \bar{\lambda}_{1,t+1} - \bar{\lambda}_{1,t} + \beta \left( R^k_{t+1} - \bar{R}^k \right) + (1 - \delta) \beta \left( Q_{t+1} - \bar{Q} \right), \\
\end{align*} \]
by using $\tilde{x}_t \equiv (X_t - X)/X$ and noticing that at the steady state, $\tilde{Q} = 1$, which yields $\beta \left[ R^k + \tilde{Q}(1 - \delta) \right] = 1$ from (3.A.3). Substituting the above back inside the expectational operator gives

$$Q_t - \tilde{Q} = \mathbb{E}_t \left[ \tilde{\lambda}_{1,t+1} - \tilde{\lambda}_{1,t} + \beta \left( R^k_{t+1} - R^k_t \right) + (1 - \delta) \beta \left( Q_{t+1} - \tilde{Q} \right) \right] ,$$

which when multiplied by $1/\tilde{Q}$ and $R^k/\tilde{R}^k$ yields

$$\tilde{q}_t = \mathbb{E}_t \tilde{\lambda}_{1,t+1} - \tilde{\lambda}_{1,t} + \beta R^k \mathbb{E}_t \tilde{r}^k_{t+1} + (1 - \delta) \beta \mathbb{E}_t \tilde{q}_{t+1} .$$

(3.A.10)

The same procedures for bond is required to remove the shadow prices in the law of motion for $\tilde{q}_t$. The FOC w.r.t $B_{t+1}$ is

$$\frac{\partial L}{\partial B_{t+1}} = - \left( \frac{\lambda_{1,t}}{R_t P_t} \right) + \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) = 0 .$$

(3.A.11)

The partial derivatives of the above first order condition are

$$R_t : \quad \beta \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right) \left( \frac{P_t}{P_{t+1}} \right) ,$$

$$\lambda_{1,t+1} : \quad \beta R_t \left( \frac{1}{\lambda_{1,t}} \right) \left( \frac{P_t}{P_{t+1}} \right) ,$$

$$\lambda_{1,t} : \quad - \beta R_t \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right) \left( \frac{P_t}{P_{t+1}} \right) ,$$

$$P_t : \quad \beta R_t \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right) \left( \frac{1}{P_{t+1}} \right) ,$$

$$P_{t+1} : \quad - \beta R_t \left( \frac{\lambda_{1,t+1}}{\lambda_{1,t}} \right) \left( \frac{P_t}{P_{t+1}^2} \right) .$$

Thus, the first order Taylor expansion of equation (3.A.11) about the steady state gives

$$1 \approx \beta R + \beta \left( R_t - \tilde{R} \right) + \beta \left( \frac{R}{\lambda_{1,t}} \right) \left( \lambda_{1,t+1} - \tilde{\lambda}_1 \right)$$

$$- \beta \left( \frac{R}{\lambda_{1,t}} \right) \left( \lambda_{1,t} - \tilde{\lambda}_1 \right) + \left( \frac{R}{P} \right) \left( P_t - \tilde{P} \right) - \left( \frac{R}{P} \right) \left( P_{t+1} - \tilde{P} \right) ,$$

which can be simplified by using $\tilde{x}_t \equiv (X_t - X)/X$ and the steady state condition that
\( \beta \bar{R} = 1 \)

\[ 0 \approx \beta \left( R_t - \bar{R} \right) + \tilde{\lambda}_{1,t+1} - \tilde{\lambda}_{1,t} - \tilde{\pi}_{t+1}, \]

where \( \tilde{\pi}_{t+1} = \tilde{p}_{t+1} - \tilde{p}_t \), the inflation gap. Substituting the above condition into the expectational operator gives

\[ 0 = \mathbb{E}_t \left[ \beta \left( R_t - \bar{R} \right) + \tilde{\lambda}_{1,t+1} - \tilde{\lambda}_{1,t} - \tilde{\pi}_{t+1} \right], \]

\[ \tilde{\lambda}_{1,t} = \beta \left( R_t - \bar{R} \right) + \mathbb{E}_t \tilde{\lambda}_{1,t+1} - \mathbb{E}_t \tilde{\pi}_{t+1}, \]

which when both sides are multiplied by \( \bar{R}/R \) gives

\[ \tilde{\lambda}_{1,t} = \tilde{r}_t - \mathbb{E}_t \tilde{\pi}_{t+1} + \mathbb{E}_t \tilde{\lambda}_{1,t+1}, \quad (3.A.12) \]

where \( \tilde{r}_t \) is the log-deviation of nominal interest rate from the steady state. Substituting (3.A.12) into (3.A.10) yields

\[ \tilde{q}_t = - (\tilde{r}_t - \mathbb{E}_t \tilde{\pi}_{t+1}) + \beta \bar{R}^k \mathbb{E}_t \tilde{r}_{t+1}^k + (1 - \delta) \beta \mathbb{E}_t \tilde{q}_{t+1}, \]

which by defining \( \bar{R}^k \equiv r_k^* = 1/\beta - 1 + \delta \) for consistency with the literature delivers

\[ \tilde{q}_t = - (\tilde{r}_t - \mathbb{E}_t \tilde{\pi}_{t+1}) + \frac{1 - \delta}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{q}_{t+1} + \frac{r_k^*}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{r}_{t+1}^k, \quad (3.A.13) \]

The following treatment is adapted from Groth and Khan (2010). To achieve the Euler condition reported in the main text, first rewrite equation (3.A.9) in terms of \( \tilde{q}_t \)

\[ \tilde{q}_t = \kappa (1 + \beta) i_t - \kappa i_{t-1} - \beta \kappa \mathbb{E}_t i_{t+1}, \quad (3.A.14) \]

which when iterated a period forward gives

\[ \tilde{q}_{t+1} = \kappa (1 + \beta) \mathbb{E}_t i_{t+1} - \kappa i_t - \beta \kappa \mathbb{E}_t i_{t+2}. \]

Substitute this resulting condition into equation (3.A.13)

\[ \tilde{q}_t = - (\tilde{r}_t - \mathbb{E}_t \tilde{\pi}_{t+1}) + \frac{1 - \delta}{1 - \delta + r_k^*} \mathbb{E}_t \left( \kappa (1 + \beta) \tilde{i}_{t+1} - \kappa \tilde{i}_t - \beta \kappa \tilde{i}_{t+2} \right) + \frac{r_k^*}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{r}_{t+1}^k, \]

\[ = - (\tilde{r}_t - \mathbb{E}_t \tilde{\pi}_{t+1}) + \frac{1 - \delta}{1 - \delta + r_k^*} \kappa (1 + \beta) \mathbb{E}_t \tilde{i}_{t+1} - \kappa (1 - \delta) \frac{1 - \delta + r_k^*}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{i}_{t+2} + \frac{r_k^*}{1 - \delta + r_k^*} \mathbb{E}_t \tilde{r}_{t+1}^k. \]
and equate it with equation (3.A.14), which when like terms are collected gives

\[
\kappa(1 + \beta + \delta) + \kappa(1 - \delta) \frac{1}{1 - \beta + r_k^*} \tilde{\iota}_t = -(\tilde{\rho}_t - E_t \tilde{\pi}_{t+1}) + \kappa \tilde{\iota}_{t-1} + \frac{r_k^*}{1 - \delta + r_k^*} E_t \tilde{\rho}_{t+1}^{k} + \left[ \frac{\kappa(1 - \delta)(1 - \beta)}{1 - \delta + r_k^*} + \beta \kappa \right] E_t \tilde{\iota}_{t+1} - \beta \kappa(1 - \delta) E_t \tilde{\iota}_{t+2}.
\]

Tidying up the coefficients delivers the investment Euler condition in the manuscript

\[
\kappa(1 + \beta + \phi_q) \tilde{\iota}_t = -(\tilde{\rho}_t - E_t \tilde{\pi}_{t+1}) + \kappa \tilde{\iota}_{t-1} + \phi_k E_t \tilde{\rho}_{t+1}^k + \kappa \phi_q (1 + \beta) + \beta \kappa(1 - \delta) \tilde{\iota}_{t+1} - \beta \kappa \phi_q \tilde{\iota}_{t+2},
\]

where \( \phi_q = (1 - \delta)/(1 - \delta + r_k^*) \); and \( \phi_k = r_k^*/(1 - \delta + r_k^*) \).

**Appendix 3.C Markov chain Monte Carlo method for sampling the VAR parameters**

In order to analyze the evolving dynamics of investment under uncertainty, the chapter employs a time-varying VAR where all model coefficients and covariances drift with time. The model has become standard in the macroeconometrics literature and a variety of Bayesian Markov chain Monte Carlo (MCMC) estimation methods have been proposed. Earlier applications of the model to US and Japanese macroeconomic data are in Cogley and Sargent (2005), Primiceri (2005), Nakajima (2011), and Nakajima et al. (2011) and they offer detailed posterior computation methods. Hence, in what follows, we only outline the MCMC procedures used in the chapter and refer the readers to the references provided for a more formal treatment of the individual sampling algorithms.

This chapter augments the MCMC routine developed by Nakajima (2011)\(^{15}\) to draw samples from the joint posterior density function, \( p(b^T, a^T, h^T, Q, S, W|y^T) \). This is achieved by the following MCMC routine that cycles through:

\(^{15}\)The original MATLAB program written by Jouchi Nakajima is available on his personal website, https://sites.google.com/site/jnakajimaweb/.
1. Initialize $b^T, a^T, h^T, Q, S, W$

2. Sample $b^T$ from $p(b^T|a^T, h^T, Q, y^T)$

3. Sample $Q$ from $p(Q|b^T)$

4. Sample $a^T$ from $p(a^T|b^T, h^T, S, y^T)$

5. Sample $S$ from $p(S|a^T)$

6. Sample $h^T$ from $p(h^T|b^T, a^T, W, y^T)$

7. Sample $W$ from $p(W|h^T)$

8. Compute $\text{det}(M_t - \lambda_t I_n P) = 0$ for $\lambda_t$:
   - if $\max(|\lambda_t|) > 1$, reject the draw
   - else $\max(|\lambda_t|) \leq 1$ store for posterior analyses

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Note that $p(\bullet|\bullet)$ denote conditional density functions, and recall that $b^T = \{\beta_t\}_{t=P+1}^T$, $a^T = \{\alpha_t\}_{t=P+1}^T$, $h^T = \{\log \sigma_t^2\}_{t=P+1}^T$, and $y^T = \{y_t\}_{t=P+1}^T$. The joint sampling of $b^T$ conditioned on rest of the parameters is much more efficient than to sample $\beta_t$ one at a time conditioned on $a^T, h^T, Q, y^T$ but also on $\beta_t$ for $\tau \neq t$. This is achieved by using the simulation smoother developed by de Jong and Shephard (1995).

3.C.1 Sampling coefficient ($b^T$)

The time varying VAR coefficient, $b^T$, is sampled from the conditional posterior density, $p(b^T|a^T, h^T, Q, y^T)$, with a prior on the initial state $\beta_{P+1} \sim \mathcal{N}(0, 10I)$, where $I$ is an identity matrix of the size $\text{dim}(\beta_t)$. The joint sampling of $b^T$ conditioned on rest of the parameters is much more efficient than to sample $\beta_t$ one at a time conditioned on $a^T, h^T, Q, y^T$ but also on $\beta_t$ for $\tau \neq t$. This is achieved by using the simulation smoother developed by de Jong and Shephard (1995).
3.C.2 Sampling simultaneous relation ($a^T$)

Consistent with the sampling of $b^T$, the time varying simultaneous relation, $a^T$, is also sampled from the conditional posterior distribution, $p(a^T|b^T, h^T, S, y^T)$ using the simulation smoother of de Jong and Shephard (1995) with $\alpha_{P+1} \sim \mathcal{N}(0,10I)$, where $I$ is an identity matrix of the size $\text{dim}(\alpha_t)$.

3.C.3 Sampling volatility ($h^T$)

Stochastic volatility, $h^T$, is sampled using the block sampler of Shephard and Pitts (1997), corrected by Watanabe and Omori (2004), which samples $h^T$ directly from the exact conditional posterior density, $p(h^T|b^T, a^T, W, y^T)$. We place a prior $\log \sigma^2_{P+1} \sim \mathcal{N}(0,10I)$ as the initial state, where $I$ is an identity matrix of the size $\text{dim}(\sigma^2_t)$. Parenthetically, note that the algorithm employed here departs from, perhaps the more popular mixture sampler of S. Kim et al. (1998), used by Primiceri (2005). The mixture sampler samples log volatility from the density function that approximates the posterior conditional distribution, while the appeal of the block sampler is its sampling from the exact conditional posterior.

3.C.4 Sampling hyper-parameters

The hyper-parameters on the time varying VAR can be drawn from their respective distribution conditional on their respective parameters. Our model takes the form where the hyper-parameters $Q, S$ and $W$ are diagonal. These assumptions are typically placed for convenience to simplify the estimation method and to economize on the estimation of parameters. The removal of cross-equation restrictions via diagonalization are reported to have minimal effect on the posterior estimates of the parameters (Primiceri, 2005; Nakajima, 2011).

The conditional posterior distribution of the diagonal elements $\forall \iota_i = Q_i, S_i, W_i$ given $\delta^T = b^T, a^T, h^T$ is

$$p(\iota_i|\delta^T) \sim \mathcal{G}^{-1}\left(\hat{v}_{\delta}^T, \hat{V}_{\iota,i}\right),$$

where $\mathcal{G}^{-1}$ denotes the inverse-Gamma distribution; $\hat{v}_{\delta}^T = v_{\delta}^T + T - P - 1$; and $\hat{V}_{\iota,i} =$
\[ V_{i,0} + \sum_{t=P+1}^{T-1} (\omega_i_{t+1} - \omega_i_t)^2. \] Notice that \( v_0 \) and \( V_{i,0} \) are the arguments for the initial states of the parameters, which in this chapter is set as \( \omega_i \sim G^{-1}(20, 0.002) \) for \( \omega_i = Q_i \) and \( \omega_i \sim G^{-1}(4, 0.02) \) for the rest.

Note that the diagonal assumption on \( W \) and the individual \( \log \sigma_{t,t}^2 \) together define traditional univariate stochastic volatility model (Jacquier et al., 1994) except we place a geometric random walk assumption rather than the stationary AR(1) process to allow the capture of non-stationary behaviors. This aids to economize on the estimation burden that hinders high dimensional model like the time varying VAR.

**Appendix 3.D  Evaluation of the posterior samples**

Figure 3.A.1 plots the sample traces of MCMC draws, their respective posterior distributions and the autocorrelation functions for selected parameters in the time varying VAR model. Note that \( \hat{\omega}_{1,50} \) denotes the first element of the parameters at \( t = 50 \), and \( \sqrt{Q_1}, \sqrt{S_1}, \sqrt{W_1} \) all denote the square roots of the \((1,1)\) elements in the respective matrices. We observe that the autocorrelation functions decay quickly and that their sample traces are stable, providing evidence for mixing of the chains.

[Figure 3.A.1 here]

Table 3.A.2 reports the posterior means, standard deviations, 95% credible intervals, \( p \)-values of convergence diagnostic tests and inefficiency factors. The convergence of the MCMC samples are assessed using Geweke (1992)’s finite draw convergence diagnostics test. The test statistic is

\[ CD = \frac{(\bar{x}_a - \bar{x}_b)}{\sqrt{\sigma_a^2/k_a + \sigma_b^2/k_b}}, \]

where \( k_a \) and \( k_b \) are respectively the first 10 percent and last 50 percent of the total draws, \( k = 10,000 \). The variable \( x_j \) is defined as \( x_j = 1/k_j \sum_{i=m_j+k_{j-1}}^{m_j+k_j-1} x^{(i)} \), where \( x^{(i)} \) is the \( i \)th draw; and \( \sqrt{\sigma_j^2/k_j} \) is the numerical standard error of \( x_j \) for \( j = a, b \). We set \( m_a = 1 \) and \( m_b = k_a + 1 \) where the \( \sigma_j^2 \) is obtained via a Parzen window with bandwidth \( B_m = 500 \). The \( p \)-values are computed using the fact that \( \lim_{k \to \infty} CD \xrightarrow{d} N(0,1) \).

The inefficiency factor (see Chib (2001) for more details) is defined as the inverse of

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Geweke (1992)’s measure of relative numerical efficiency

\[ IF = 1 + 2 \sum_{p=1}^{B_{max}} \rho_p, \]

where \( \rho_p \) is the sample autocorrelation at lag \( p \). The inefficiency factor measures the degree to which the chain mixes.

At the five percent level of significance, we do not reject the null and confirm the convergence of the chains to their respective posteriors virtually for all reported parameters. Furthermore, inefficiency factors for selected parameters are relatively low, which suggests our sampling methods are efficient.

To further validate and reiterate our efficiency in drawing our posterior samples, we plot histogram of hyper-parameters’ inefficiency factors in Figure 3.A.2. It is clear that hyper-parameters are efficiently drawn with only two hyper-parameters’ inefficiency factor exceeding the value of 20.

Appendix 3.E Posterior computation of various statistics

In this section, we describe in detail the statistics computed in the main text. Most of the statistics are time varying counterpart of what is described in detail in standard time series econometrics textbooks such as Hamilton (1994) and Lütkepohl (2005). Some are from academic sources are cited accordingly.

3.E.1 Computing unconditional means

Consider equation (3.4) of the main text reprinted here

\[ y_t = c_t + \sum_{p=1}^{P} B_{p,t} y_{t-p} + u_t, \quad u_t \sim N(0, \Omega_t), \]
where \( \mathbf{y}_t \) is the \( n \times 1 \) vector of endogenous variables; \( \mathbf{c}_t \) is a \( n \times 1 \) column vector of intercepts; and \( \mathbf{B}_{p,t} \) is a \( n \times n \) matrix containing the \( p^{\text{th}} \) lag autoregressive coefficients. This can be rewritten in companion form

\[
\mathbf{z}_t = \mathbf{v}_t + M_t \mathbf{z}_{t-1} + \mathbf{e}_t,
\]

(3.A.16)

which, with \( P = 2 \) is

\[
\begin{bmatrix}
\mathbf{y}_t \\
\mathbf{y}_{t-1}
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{c}_t \\
\mathbf{0}_{n,t}
\end{bmatrix} + 
\begin{bmatrix}
\mathbf{B}_{1,t} & \mathbf{B}_{2,t} \\
\mathbf{I}_{n,t} & \mathbf{0}_{n,t}
\end{bmatrix} 
\begin{bmatrix}
\mathbf{y}_{t-1} \\
\mathbf{y}_{t-2}
\end{bmatrix} + 
\begin{bmatrix}
\mathbf{u}_t \\
\mathbf{0}_{n,t}
\end{bmatrix}.
\]

Beveridge and Nelson (1981) defines the stochastic trend in \( \mathbf{z}_t \) as the value to which the series is expected to converge in the long run

\[
\mu_t = \lim_{h \to \infty} E_t(\mathbf{z}_{t+h}),
\]

which we approximate via the unconditional mean

\[
\mu_t \approx (I_n P - M_t)^{-1} \mathbf{v}_t,
\]

following the trend inflation literature (see, for e.g., Cogley and Sargent (2005) and Cogley et al. (2010)). Thus, elements of \( \mu_t \) are often interpreted as the ‘core’ or ‘trend’ of a variable.

3.E.2 Computing unconditional variances

The conditional mean, represented in companion form is

\[
\mathbf{v}_t = (I_n P - M_t) \mu_t,
\]

which when substituted it into (3.A.16) gives

\[
\mathbf{z}_t - \mu_t = M_t (\mathbf{z}_{t-1} - \mu_t) + \mathbf{e}_t,
\]

(3.A.17)

where, for notational convenience, denote \( \tilde{\mathbf{z}}_t = \mathbf{z}_t - \mu_t \), implying that the process \( \tilde{\mathbf{z}}_t \) has zero mean.

For a time invariant model, the unconditional variance-covariance matrix can be com-
puted by

\[ \tilde{\Omega}_{nP \times nP} = E(\tilde{\zeta}_t \tilde{\zeta}_t'), \]

\[
\begin{bmatrix}
\Gamma(0) & \Gamma(1) & \cdots & \Gamma(P - 1) \\
\Gamma'(1) & \Gamma(0) & \cdots & \Gamma(P - 2) \\
\vdots & \vdots & \ddots & \vdots \\
\Gamma'(P - 1) & \Gamma'(P - 2) & \cdots & \Gamma(0)
\end{bmatrix},
\]

where each individual element \(\Gamma(h)\) denotes the autocovariance matrix that has the relationship

\(\Gamma'(h) = \Gamma(-h)\).

To compute the unconditional variance-covariance matrix that time varies, \(\tilde{\Omega}_t\), we can proceed by substituting \(\tilde{\zeta}_t = M_t(\tilde{\zeta}_{t-1} + e_t)\)

\[ \tilde{\Omega}_t = E_t[(M_t \tilde{\zeta}_{t-1} + e_t)(M_t \tilde{\zeta}_{t-1} + e_t)'], \]

\[ = M_t E_t(\tilde{\zeta}_{t-1} \tilde{\zeta}'_{t-1}) M'_t + E_t(e_t e'_t), \]

\[ = M_t \tilde{\Omega}_{t-1} M'_t + V_t, \]

where \(V_t\) is the companion form of the reduced form variance-covariance matrix. Applying the vec operator and the Kronecker product rule\(^\text{16}\) to both sides yields

\[ \text{vec}(\tilde{\Omega}_t) = (M_t \otimes M_t)\text{vec}(\tilde{\Omega}_{t-1}) + \text{vec}(V_t), \]

hence

\[ \text{vec}(\tilde{\Omega}_t) = (I_{nP^2} - M_t \otimes M_t)^{-1}\text{vec}(V_t), \quad (3.A.18) \]

with the approximation \(\text{vec}(\tilde{\Omega}_t) \approx \text{vec}(\tilde{\Omega}_{t-1})\). Thus, the unconditional variance-covariance matrix can be approximated by solving equation (3.A.18) for every time period. When \(\text{vec}(\tilde{\Omega}_t)\) is converted back into a matrix form, the first \(n \times n\) elements are \(\Gamma_t(0) = K\tilde{\Omega}_t K'\), where \(K = [I_n : 0_{n \times (P-1)}]\) such that \(K'K = I_{nP'}\); and \(\Gamma_t(0)\) is the time varying unconditional covariance matrix. The diagonal elements characterize unconditional variance of variables, or put differently, the second moment counterpart of the Beveridge and Nelson (1981) trend.

\(^\text{16}\) Let \(A, B,\) and \(C\) be matrices whose dimensions are such that the product \(ABC\) exists. Then, the Kronecker product rule \(\text{vec}(ABC) = (C' \otimes A)\text{vec}(B)\) holds.
An alternative method to approximate the unconditional covariance matrix is to follow Cogley and Sargent (2001) and use $h = 120$ in equation (3.9) of the manuscript

$$\Gamma_t(0) = \sum_{h=0}^{120} \Phi_{h,t} \Omega_t \Phi_{h,t}'.$$

For our dataset we find that there are no material difference in the results when either either of the approximations are used.

### 3.E.3 Computing impulse response functions

Consider equation (3.5) of the main text reprinted here

$$y_t = X_t \beta_t + A_t^{-1} \Sigma_t \epsilon_t, \quad t = P + 1, \ldots, T,$$

and note the decomposition of the reduced form residual term

$$u_t = A_t^{-1} \Sigma_t \epsilon_t.$$

To achieve this decomposition in the companion form of the residual, $e_t$, first define

$$P_t = \begin{bmatrix} A_t^{-1} \Sigma_t \\ O_n \end{bmatrix},$$

such that the standard Cholesky factorization $P_t P_t' = V_t$ holds. Therefore, equation (3.A.16) can now be rewritten

$$z_t = \nu_t + M_t z_{t-1} + P_t \epsilon_t.$$

Recall from Appendix 3.C, the rejection of unstable draws that produce an explosive root. The satisfaction of the stability condition allows the rewriting of the VAR into the vector moving-average, VMA($\infty$) notation (a.k.a., Koyck transformation)

$$y_t = K z_t = K \mu_t + K \sum_{h=0}^{\infty} M_h \epsilon_{t-h},$$

where, as before, $K = [I_n; 0_{n \times n(P-1)}]$ is a $(n \times nP)$ selection matrix with the property
that $K'K = I_nP$. Thus the above equation admits the following representation

$$y_t = K\mu_t + \sum_{h=0}^{\infty} KM_t^hK'Ke_{t-h},$$

$$= K\mu_t + \sum_{h=0}^{\infty} KM_t^hK_tP_t\epsilon_{t-h},$$

$$= K\mu_t + \sum_{h=0}^{\infty} \Xi_t(h)\epsilon_{t-h},$$

where $\Xi_t(h) = KM_t^hK'KP_t = KM_t^hK'A_t^{-1}\Sigma_t$ for $h \geq 0$ is a $(n \times n)$ impulse response matrix; $\epsilon_t \sim N(0, I_n)$ is the standardized structural shock vector; and note that $\Phi_{h,t} = KM_t^hK'$ denotes the $h$th matrix lag of the VMA($\infty$) representation. More neatly, the orthogonalized impulse response function can be represented as

$$\frac{\partial y_t(h)}{\partial \epsilon_t} = \Xi_t(h), \quad h = 0, 1, 2, \ldots$$

where

$$\Xi_t(h) = \begin{bmatrix}
\Xi_{11,t}(h) & \Xi_{12,t}(h) & \cdots & \Xi_{1n,t}(h) \\
\Xi_{21,t}(h) & \Xi_{22,t}(h) & \cdots & \Xi_{2n,t}(h) \\
\vdots & \vdots & \ddots & \vdots \\
\Xi_{n1,t}(h) & \Xi_{n2,t}(h) & \cdots & \Xi_{nn,t}(h)
\end{bmatrix},$$

such that $\Xi_{ij}(h)$ denotes the response of variable $i$ to a shock in variable $j$ after $h$ periods. Note that with Cholesky factorization of the reduced form covariance matrix, the upper triangular elements in $\Xi_t$ are zero for $h = 0$. Likewise, the simultaneous response of a variable $i$ to shock in variable $j$ is, characterized by the elements of $A_t^{-1}\Sigma_t$.

The impulse response functions are computed for every MCMC draw by applying the posterior mean of the standard deviation over the sample period, $\sigma_i = 1/(T - P) \sum_{t=P+1}^{T} \exp\left(\log \sigma_{i,t}^2/2\right)$, at each sample period. This treatment of the impulse response allows the comparability across time, which is required since $\Sigma_t$ is time varying. Finally, the impulse responses are computed with the assumption that VAR parameters remain constant at their current values, which corroborate with the notion of bounded rationality and learning, as stressed by Cogley et al. (2010).
3.E.4 Computing persistence

Following Cogley et al. (2010), persistence in the chapter is defined as one minus the ratio of conditional variance (forecast error variance) to unconditional variance

\[ R^2_{q,t}(H) = 1 - \frac{k'_q \left[ \sum_{h=0}^{H-1} \Phi_{h,t} \Omega \Phi_{h,t}' \right] k_q}{k'_q \left[ \sum_{h=0}^\infty \Phi_{h,t} \Omega \Phi_{h,t}' \right] k_q}, \]

where \( k_q \) is a selection vector. The persistence measure we refer to here measures the ‘gap’ persistence, that is, the persistence of the difference from the observed value to the unconditional mean.

3.E.5 Computing density forecasts

Let \( y^T = [y'_1, \ldots, y'_T]' \) denote the vector of observables up to time \( T \), and let \( y^{T+1,T+h} = [y'_{T+1}, \ldots, y'_{T+h}]' \) denote the vector of the potential future paths of the variables. The time-varying VAR consists of two sets of parameters, one that drifts over time, which we define it as \( \theta^T = [b', a', h]' \), with corresponding forecasts \( \theta^{T+1,T+h} \); and another that is constant, \( \omega = [Q, S, W]' \), comprised of hyper-parameters.

Our forecast estimates are constructed from the posterior predictive density, which is the joint probability density over future paths of the data, conditioned on priors and the history of observables. This density is expressed as

\[ p(y^{T+1,T+h} \mid y^T) = \int \int p(y^{T+1,T+h}, \theta^{T+h}, \omega \mid y^T) \, d\theta^{T+h} \, d\omega, \]

where \( p(y^{T+1,T+h}, \theta^{T+h}, \omega \mid y^T) \) is the joint posterior density, which is factored as

\[ p(y^{T+1,T+h}, \theta^{T+h}, \omega \mid y^T) = p(\theta^T, \omega \mid y^T)p(\theta^{T+1,T+h} \mid \theta^T, \omega, y^T)p(y^{T+1,T+h} \mid \theta^{T+h}, \omega, y^T), \]

(3.A.19)

following Cogley et al. (2005). Each individual factor on the RHS of (3.A.19) represents a distinct sources of uncertainty. The first term, \( p(\theta^T, \omega \mid y^T) \) is the joint posterior density function obtained via the MCMC recursions described in Appendix 3.C and resembles the in-sample parameter uncertainty. The second term, \( p(\theta^{T+1,T+h} \mid \theta^T, \omega, y^T) \) represents the uncertainty about the future course of time varying parameters away from its terminal
values in $\theta_T$.\textsuperscript{17} The third term, $p(y_{T+1:T+h}\mid \theta_{T+h}, \omega, y_T)$, represents the posterior density of the future observables conditioning on the future path of the parameters and the data.

Practically, density forecasting within the time varying VAR is conducted in a recursive format that mirrors the reality facing forecasters, requiring a re-estimation of the model and simulation of forecasts as new data are acquired. That is, given $y_T$, we estimate the model and simulate forecast for time $T + 1 : T + h$. With the acquisition of new (observed) data for time $T + 1$, we re-estimate the model with the inclusion of the updated data $y_{T+1}$ and simulate the resulting forecast for $T + 2 : T + h + 1$. The process is repeated until the data runs out.

Appendix 3.F Robustness checks

In this section, we report results from alternative specifications of the time-varying VARs. The baseline specification is the specification reported in the manuscript and the robustness checks are conducted as departures from the baseline. Thus, unless otherwise stated, the baseline specification will be assumed. For convenience, we first describe the baseline specification before we move on.

The baseline model specification:
The model is a standard time-varying VAR, and the model is estimated with two lags. The structural shocks are identified based on the causal ordering mechanism, implying that the order by which the variables enter the VAR matter. The baseline ordering of the endogenous variables are as follows

$$ y_t = \begin{bmatrix} s_t, r^k_t, i_t, m_t \end{bmatrix}', $$

where, $s_t$ is the demeaned and standardized uncertainty proxy; $r^k_t$ is the annualized rate of return from capital; $i_t$ is the log difference of investment at annualized rate; and $m_t$ is the ex-post real interest rate.

\textsuperscript{17}Notice the marginalization with respect to $\theta^{T-1}$ given the Markovian property of $\theta_t$. 

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The following priors are considered as a baseline

\[
\begin{align*}
\beta_{P+1} & \sim N(0, 10I), \quad Q_i \sim G^{-1}(20, 0.001), \\
\alpha_{P+1} & \sim N(0, 10I), \quad S_i \sim G^{-1}(6, 0.01), \\
\log \sigma^2_{P+1} & \sim N(0, 10I), \quad W_i \sim G^{-1}(6, 0.01),
\end{align*}
\]

where, \( N \) and \( G^{-1} \) respectively denote the Gaussian and inverse Gamma distributions. With this set up, the model is estimated using the MCMC method outlined in Appendix 3.C.

We conduct the following robustness checks: i) prior sensitivity, ii) different ordering of the endogenous variables in the time-varying VAR, iii) different lag length and iv) different proxy for short term nominal interest rate. Each of the following subsections presents a brief explanations of the specific changes made relative to the baseline, as well as the accompanying results.

### 3.F.1 Prior sensitivity

Here, we provide results with different priors set over the parameters and hyper-parameters. Specifically, the following priors are considered

\[
\begin{align*}
\beta_{P+1} & \sim N(0, 4I), \quad Q_i \sim G^{-1}(6, 0.02^2/2), \\
\alpha_{P+1} & \sim N(0, 4I), \quad S_i \sim G^{-1}(6, 0.02), \\
\log \sigma^2_{P+1} & \sim N(0, 4I), \quad W_i \sim G^{-1}(6, 0.02),
\end{align*}
\]

where, as in the manuscript, \( N \) and \( G^{-1} \) respectively denote the Gaussian and inverse Gamma distributions.

Recall that our baseline priors were set such that we allowed for relatively less movements in the VAR coefficients than in the variances. Here, we further exaggerate this by setting an even tighter prior on the hyper-parameter \( Q_i \) and, moreover, doubling the size of the scaling parameters in the inverse Gamma distribution for \( S_i \) and \( W_i \). Below, we replicate the impulse response functions and model comparison exercise under the above priors. Figure 3.A.3 depicts the impulse response function and Table 3.A.3 shows the model comparison exercise results.
3.F.2 Different ordering

Here, we provide results with a different ordering of the endogenous variables in the time-varying VAR, with the priors the same as in the manuscript. Specifically, the following ordering is considered

\[ y_t = [s_t, r_t^k, m_t, i_t]', \]

where, as in the manuscript, \( s_t \) is the demeaned and standardized uncertainty proxy; \( r_t^k \) is the annualized rate of return from capital; \( i_t \) is the log difference of investment at annualized rate; and \( m_t \) is the ex-post real interest rate.

This ordering allows the investors to observe the interest rate movements in both the rate of return from capital and short-term bonds. Below, we replicate the impulse response function under the above ordering.

3.F.3 Different lag length

Here, we provide results from the time-varying VAR with three lags. Following Cogley and Sargent (2005) and Primiceri (2005), in the manuscript, we set the lag length to two. The selection of three lags as opposed to other lag length are due to Nakajima and West (2013) that report the results with three lags for the US macroeconomic variables. We replicate the impulse response function below in Figure 3.A.3 with three lags.

3.F.4 Different proxies for endogenous variables

Here, we report results from the time varying VAR using different proxies for uncertainty, investment and the nominal interest rate. With reference to Table 3.1 and the FRED

[Table 3.A.3 here]
codes in Appendix 3.A, the following changes are made

\[
s_t : \text{FPC} \rightarrow \text{VXO},
\]

\[
i_t : \text{PNFI} \rightarrow \text{FPI} + \text{PCDG},
\]

\[
m_t : \text{TB3MS} - \pi \rightarrow \text{DGS1} - \pi.
\]

We replicate the impulse response functions for each changes made above in Figures 3.A.6, 3.A.7 and 3.A.8 respectively.

[Figure 3.A.6, 3.A.7 and 3.A.8 here]

It is clear from impulse response functions depicted that the results presented in the manuscript is not an artifact of the prior calibration, ordering or the lag length. Moreover, it is also evident that broad contour of the result interpretation presented in the manuscript holds even when the different proxies for respective endogenous variables are used.
Figure 3.A.1: Estimation results for the baseline model presented in the chapter

Note: trace of the sample (top row); and histogram of posterior draws (middle row); sample autocorrelation functions (bottom row) depicted for selected parameters.
Note: Plots the histogram of hyper-parameters' inefficiency factors in the time varying VAR model.
Figure 3.A.3: Responses of \( i_t \): investment, \( r^k_t \): real rental rate of capital and \( m_t \): real interest rate to uncertainty shocks (prior sensitivity)

Note: Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering \( y_t = [ s_t, r^k_t, i_t, m_t ]' \) adopted. Different priors set over the parameters and hyper-parameters. The first column shows the response of \( i_t \): investment, second column shows the response of \( r^k_t \): real rental rate of capital, and the third column shows the response of \( m_t \): real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
**Figure 3.A.4:** Responses of $i_t$: investment, $r^k_t$: real rental rate of capital and $m_t$: real interest rate to uncertainty shocks (different ordering used)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Instantaneous</strong></td>
<td><strong>2 years ahead</strong></td>
<td><strong>3 years ahead</strong></td>
</tr>
<tr>
<td>$i_t$</td>
<td>$r^k_t$</td>
<td>$m_t$</td>
</tr>
</tbody>
</table>

Note: Instantaneous and 2 and 3 years ahead impulse responses. Different identification ordering $y_t = [s_t, r^k_t, m_t, i_t]$ adopted. The first column shows the response of $i_t$: investment, second column shows the response of $r^k_t$: real rental rate of capital, and the third column shows the response of $m_t$: real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering $y_t = [s_t, r_t^k, i_t, m_t]$ adopted. Time-varying VAR estimated with three lags. The first column shows the response of $i_t$: investment, second column shows the response of $r_t^k$: real rental rate of capital, and the third column shows the response of $m_t$: real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
Figure 3.A.6: Responses of $i_t$: investment, $r_t^k$: real rental rate of capital and $m_t$: real interest rate to uncertainty shocks (different proxy for uncertainty)

Note: Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering $y_t = [s_t, r_t^k, i_t, m_t]$ adopted. Uncertainty proxied via the VXO instead of FPC. The first column shows the response of $i_t$: investment, second column shows the response of $r_t^k$: real rental rate of capital, and the third column shows the response of $m_t$: real interest rate. The gray shaded regions represent the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
Note: Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering \( y_t = [ s_t, r^k_t, i_t, m_t ] \) adopted. Different proxy for investment used. The first column shows the response of \( i_t \): investment, second column shows the response of \( r^k_t \): real rental rate of capital, and the third column shows the response of \( m_t \): real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
Figure 3.A.8: Responses of $i_t$: investment, $r^k_t$: real rental rate of capital and $m_t$: real interest rate to uncertainty shocks (different proxy for real interest rate used)

```
Note: Instantaneous and 2 and 3 years ahead impulse responses. The baseline identification ordering $y_t = [s_t, r^k_t, i_t, m_t]'$ adopted. The first column shows the response of $i_t$: investment, second column shows the response of $r^k_t$: real rental rate of capital, and the third column shows the response of $m_t$: real interest rate. The gray shaded regions represents the 68% posterior credible intervals around the posterior median depicted in solid, dashed, and dashed-dotted lines.
```

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## Appendix tables

### Table 3.A.1: Mnemonics and summary of uncertainty proxies

<table>
<thead>
<tr>
<th>Proxy</th>
<th>Description</th>
<th>Source</th>
<th>Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOL</td>
<td>Realized volatility, a proxy for uncertainty computed as a standard deviation of daily S&amp;P500 returns averaged over a month.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>VXO</td>
<td>Implied S&amp;P 100 volatility, used instead of the VIX for the same reasons as in other literature, to increase the time span.</td>
<td>Chicago Board Options Exchange, CBOE</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>GSZ</td>
<td>Risk adjusted proxy for uncertainty that captures the common shocks in the idiosyncratic volatility of equity returns.</td>
<td>Gilchrist et al. (2014) and Caldara et al. (2016)</td>
<td>1986:1 – 2015:3</td>
</tr>
<tr>
<td>FPC</td>
<td>The first principal component scores of the above 4 uncertainty proxies.</td>
<td>Author’s computation</td>
<td>1986:1 – 2015:3</td>
</tr>
</tbody>
</table>
Table 3.A.2: Estimation and convergence diagnostics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
<th>Geweke</th>
<th>Ineff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{1,11,50}$</td>
<td>0.5179</td>
<td>0.1205</td>
<td>0.2781</td>
<td>0.7537</td>
<td>0.967</td>
<td>1.93</td>
</tr>
<tr>
<td>$a_{21,50}$</td>
<td>5.3526</td>
<td>0.5065</td>
<td>4.3766</td>
<td>6.3656</td>
<td>0.080</td>
<td>2.63</td>
</tr>
<tr>
<td>$\log \sigma_{2,50}^2$</td>
<td>-3.1396</td>
<td>0.7139</td>
<td>-4.6571</td>
<td>-1.8028</td>
<td>0.941</td>
<td>5.15</td>
</tr>
<tr>
<td>$\sqrt{Q_1}$</td>
<td>0.0072</td>
<td>0.0008</td>
<td>0.0058</td>
<td>0.0091</td>
<td>0.783</td>
<td>4.35</td>
</tr>
<tr>
<td>$\sqrt{S_1}$</td>
<td>0.0433</td>
<td>0.0096</td>
<td>0.0296</td>
<td>0.0665</td>
<td>0.002</td>
<td>11.04</td>
</tr>
<tr>
<td>$\sqrt{W_1}$</td>
<td>0.5970</td>
<td>0.2614</td>
<td>0.2461</td>
<td>1.2796</td>
<td>0.276</td>
<td>43.15</td>
</tr>
</tbody>
</table>

Note: Estimated with 2 lags and 12,000 iterations where the first 2,000 draws are discarded. Geweke refers to the convergence diagnostic test offered in Geweke (1992) and the $p$-values are reported. $\sqrt{Q_1}$ denotes the square root of an element (1,1) of matrix $Q$. 

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Table 3.A.3: RMSFE and LPDS ratios of time varying VARs to constant VAR (prior sensitivity)

<table>
<thead>
<tr>
<th>Time varying parameter</th>
<th>RMSFE</th>
<th>LPDS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \beta_t )</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>( \log \sigma^2_t )</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( \beta_t ), ( \alpha_t ), ( \log \sigma^2_t )</td>
<td>1.01</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>1.02</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Note: The model in first row mirrors the one developed by Cogley and Sargent (2001), while the model in the second row is by Uhlig (1997). Models in third and fourth rows respectively are developed by Cogley and Sargent (2005) and Primiceri (2005).
References for Chapter 3


Chapter 4

General conclusion

4.1 Concluding remarks

The thesis provides a novel interpretation of the relationship between investment and uncertainty, by analyzing their cyclical and time varying relationship. At the broadest level, this thesis has confirmed the previous literature findings that the relationship between investment and uncertainty is indeed negative. In addition, this thesis has established that the dynamics of investment under uncertainty has changed around the time of the financial crisis in 2008. It is intriguing that this finding is robust to various econometric methods and specifications.

In Chapter 2, I show that uncertainty proxies are counter-cyclical to the investment cycle, although at times they appear to be acyclical. Surprisingly, their levels of integration, as measured by ‘concordance’ and ‘cyclical correlation’ indexes, are weaker than one anticipates given the recent literature’s attention to and claims regarding uncertainty as a new driver of the business cycle. Furthermore, it is clear from visual inspection of the uncertainty and investment cycles that the relationship between them changed around the time of the financial crisis in 2008. I tested this claim using the vector autoregression (VAR) model. I show that the negative effects of uncertainty shocks on investment intensifies when the sample period spans over the 2008 financial crisis, relative to when the sample period end prior to the onset of the financial crisis. Taken together, I interpret the results to be due to some unobserved structural changes in the channel by which the
effects of uncertainty are transmitted to investment.

Given the conclusion offered in Chapter 2, Chapter 3 further investigates the relationship between investment and uncertainty. Specifically, three new features to the analyses of investment and uncertainty have been added: i) structural inter-relations; ii) time variations; and iii) long-run relations. I have argued in Chapter 2 that purely investigating investment and uncertainty in a bivariate framework has limitations. There are other determinants which private agents consider when making investment decisions. The chapter rectifies this shortcoming by deriving the investment Euler condition from a prototypical New Keynesian model with capital accumulation (Christiano et al., 2005) so as to guide the construction of the time-varying VAR model (Primiceri, 2005). The time-varying VAR is then used to model and capture the potential time variations that are difficult to establish by visual inspection alone.

The results from the time-varying VAR reveal that some considerable structural changes between investment and uncertainty have occurred due to the zero lower bound (ZLB) constraint on the nominal interest rate. Impulse responses of investment to uncertainty shocks during the ZLB period are shown to be more negative relative to preceding period. This finding lend support to the idea that, when the future return on capital investment is uncertain and the economy is at the ZLB, private agents simply do not invest in capital or in bonds, and instead prefer to hold cash. To the extent that such increased cash holdings by US multinational corporations since the financial crisis have been reported in the finance literature (see, for e.g., Pinkowitz et al. (2012)), this mechanism is likely. Overall, the initial question posed in the thesis: “Does future uncertainty really deter investment?” is affirmed by this research.

4.2 Future research

Future works on uncertainty literature fall into two areas. First, the interaction of uncertainty and financial friction, the so called ‘Finance-Uncertainty Multiplier’ (Alfaro et al., 2017). It is argued that the effects of uncertainty on the economy are enlarged in magnitude when coupled with financial frictions. Caggiano et al. (2017) investigates their potential time-varying interactions using the time-varying VAR and shows that the reaction of financial conditions to heightened uncertainty has amplified the effects of uncertainty
shocks during the financial crisis in 2007/08.

Second, in relation to the first point, the question whether uncertainty is an endogenous result of agent’s actions or an exogenous process remains unresolved. In other words, is it due to some exogenous reasons that the state of the world is uncertain, which in turn causes financial turmoil and hence recessions? Or is it, to the contrary, due to the endogenous actions of agents? As was made apparent in Figures 2.4 and 2.A.3 of Chapter 2, level of uncertainty do indeed respond to investment growth shock, which can potentially be interpreted as the investment-specific technology shock (Greenwood et al., 1997, 2000). If uncertainty is an equilibrium phenomenon that stems from agent’s decisions, policy experiments that treat uncertainty as exogenous process are subject to the Lucas (1976) critique. Surprisingly, limited amount of work along these lines currently exists (see, for e.g., Ludvigson et al. (2018) and Saijo (2017)) and further work on this will be beneficial.
References for Chapter 4


“To know that we know what we know and to know that we do not know what we do not know, that is true knowledge.”

Nicolaus Copernicus, 1473 – 1543
Biography

Kazuki Tomioka was born in Himeji, Japan. He received his bachelor's degree with First Class Honours in Economics from the University of Western Australia (UWA) in 2016. During his studies at UWA, he was awarded Research Training Program Scholarship funded by the Government of the Commonwealth of Australia, among other Travel Grants to attend academic conferences nationally and internationally. He has also served as teaching and research assistant for the Department of Economics at the UWA Business School.