Fluid-pipeline-soil interaction at the seabed

by

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Signed,
Abstract

Pipeline, cable and riser systems are used in the offshore industry to transfer material, power and data between facilities, both across the seafloor and from the seafloor to the ocean surface. Important design challenges for these systems involve prediction of pipeline, cable and riser movements on or near the seabed and assessment of the consequences of these movements. The fundamental physics inherent in these problems fall upon a spectrum of fluid-structure-soil interactions. In this thesis, two particular pipeline, cable and riser design issues are considered: (i) the on-bottom stability and safe operation of subsea pipelines and cables, and (ii) predicting seabed trenching within the touchdown zone of risers connected to floating vessels.

The first three main chapters in this thesis focus on soil-structure interaction with regard to the soil resistance available to prevent pipeline movement – with ‘pipeline’ being used hereafter to represent all cylindrical objects including cables and risers. First, in Chapter 2 finite element analyses are used to establish drained combined loading envelopes for the general case of a wished-in-place, surface-laid pipeline. This study parametrically explores the effect of sand density on the breakout resistance. The results show that under typical loading conditions, limit analysis using a modified set of soil strength parameters provides a close approximation to both non-associated flow finite element results and also experimental evidence. Chapter 3 explores the resistance of buried pipelines to upheaval movement and the effects of external fins fitted to the pipeline perimeter. Radial fins are experimentally shown to increase the uplifting resistance by up to 25%. An analytical method for predicting the changes is derived from limit equilibrium, which predicts the resistance to within 13% percent of the measurements. Particle image velocimetry is also used to examine the failure mechanisms in loose sand; and notably, these are found to comprise a localised mechanism that differs subtly from the analytical and accompanying numerical results. Chapter 4 numerically quantifies the changes to pipeline breakout resistance that occur through natural evolution of the seabed due to sediment transport processes, based on observations of an operating pipeline. Collectively, these chapters provide a better basis for predicting the interaction forces between pipelines and sandy seabeds, and give insights on how stability is enhanced, either via natural sedimentation or via engineer appendages.

The following two chapters shift focus to fluid-structure interaction, focusing on the fluid mechanics around an oscillating two-dimensional cylinder representative of a riser near the touchdown zone. Specific focus is placed on understanding how the local flow field is affected by the seabed, which is represented as a rigid wall. The first study (Chapter 5) uses computational fluid dynamics to directly solve the Navier-Stokes equations in two-dimensions at a Reynolds number of 150; while the latter (Chapter 6) uses particle
image velocimetry to visualise the flow field at higher Reynolds numbers, ranging from 1000 to 5000. Collectively, these studies indicate that for relatively small oscillation amplitudes the flow field remains symmetric about the oscillation axis and can generally be described with potential flow theory. In contrast, above a certain oscillation amplitude the flow field becomes asymmetric and the interaction of vortices with the wall plays a key role in the near-wall flow behaviour. The results also show that the inline hydrodynamic forces on the cylinder increase when near the wall and that this increase can be predicted using a modification factor based on potential flow theory.

In the final study (Chapter 7), the fluid-structure-soil interaction triumvirate is completed with an experimental study of sediment transport beneath an oscillating cylinder; thus extending the work in Chapters 6 and 7 to consider trenching beneath a riser. For this work, the cylinder is oscillated above a sandy seabed without touching, which allows the independent effects of fluid flow-induced sediment transport to be isolated from soil-structure interaction effects. For small oscillation amplitudes and small seabed-cylinder gaps, the threshold of sediment motion is well predicted using continuity arguments and an oscillatory boundary layer assumption. Sediment motion for larger oscillation amplitudes is observed to be driven primarily by shed vortices interacting with the seabed, and the transition between these behaviours is consistent with the observations in Chapters 6 and 7. The trench depth and rate of trench development are found to generally increase as a function of oscillation amplitude and maximum cylinder velocity. The combined observations in the final three studies give new insight into the driving mechanisms of trench formation beneath risers. New analyses that have been benchmarked against the experimental work, predict the spatial extent and formation rate of seabed trenches that are created solely due to riser motion without direct interaction with the seabed.
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Thesis Format and Authorship

In accordance with the University of Western Australia’s regulations regarding Research Higher Degrees, this thesis is presented as a series of academic papers, comprising work completed during the course of PhD candidature. The contribution of the candidate and co-authors for the papers comprising Chapters 2, 3, 4, 5, 6 and 7 are hereby set forth. Chapters 1 and 8 form introductory and concluding sections and were completed solely by the candidate. The contributions to Appendices A-C are also set forth.


The candidate conducted the numerical analyses, post-processed the results and conducted the majority of the interpretation. The paper was written by the candidate and reviewed and edited by the co-author. The paper was submitted for publication to the *Journal of Geotechnical and Geoenvironmental Engineering* and was under review as of the thesis submission date. Estimated contribution - 95%.


This paper comprises work completed in collaboration with industry co-sponsors, A. Haghighi and A. Maconochie, under the supervision of C.D. O’Loughlin and D.J. White. The candidate conducted the experiments (test plan devised by C.D. O’Loughlin and D.J. White in collaboration with A. Haghighi and A. Maconochie), completed post-processing of the data, conducted the numerical analyses and derived the analytical solution presented based on previous work by D.J. White. The paper was written by the candidate and reviewed and edited by the co-authors. The paper was published as of the thesis submission date. Estimated contribution - 90%.


This paper comprises work completed in collaboration with another PhD student, S.H.F. Leckie, both supervised by D.J. White and S. Draper. The candidate received field data processed and interpreted by the second author. The candidate conducted the numerical analyses and conducted the majority of the interpretation. The paper was primarily written by the candidate (estimated contribution - 80%) and the second author (estimated contribution - 15%) and reviewed and edited by the remaining co-authors.
The candidate presented the paper at the conference. The paper was published as of the thesis submission date and was also included in the collaborating PhD student’s thesis. The collaborating PhD student has given consent for this paper to also be included in the candidate’s thesis. Estimated contribution - 80%.

**Chapter 5:** Tom, J.G., Draper S., Milne I. and Zhao, M. (2018). Pumping and vortex shedding due to a cylinder oscillating normal to a plane wall. *To be submitted.*
The candidate conducted the numerical analyses (based on the code developed by the final author), post-processed the results and conducted the majority of the interpretation. The paper was written by the candidate and reviewed and edited by the co-authors. The paper has not been submitted for publication as of the thesis submission date. Estimated contribution - 85%.

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This paper comprises an early conference publication discussing work later incorporated into Chapter 6. The candidate conducted the experiments, post-processed the results and conducted the majority of the interpretation. The paper was written by the candidate and reviewed and edited by the co-author. The candidate presented the paper at the conference. The paper was published as of the thesis submission date. Estimated contribution - 95%.

This paper comprises a conference publication describing work in collaboration with an industry partner (M. O’Neill). The candidate conducted the experiments, post-processed the results, conducted the majority of the interpretation, conducted the probabilistic analyses described and wrote much of the literature review therein. The paper was co-written by the candidate and the second and third co-authors and reviewed and edited...
by the final co-author. The candidate presented the paper at the conference. The paper was published as of the thesis submission date. Estimated contribution - 80%.


This paper comprises a conference publication investigating a tangential aspect of the work described in Chapter 3. The candidate conducted the numerical analyses, post-processed the results and conducted the majority of the interpretation. The paper was written by the candidate and reviewed and edited by the co-author. The candidate presented the paper at the conference. The paper was published as of the thesis submission date. Estimated contribution - 95%.

Signed,

Joe G. Tom Jr., Candidate
August 2018

The stated contributions by the candidate for each of these publications has been agreed with the co-authors of each paper, and full permission has been granted by each co-author to include the relevant paper within this thesis.

Signed,

Associate Professor Scott Draper, Coordinating Supervisor
Chapter 1

Introduction

Production of energy in the offshore environment, through the extraction of hydrocarbon fuel sources or the use of renewable energy systems, requires pipelines, cables and risers to transfer material, power and data between locations, either laterally across the seafloor or vertically from the seafloor to floating facilities on the ocean surface. One of the primary design challenges for such connecting infrastructure is predicting and accounting for the consequences of pipeline movement, where ‘pipelines’ is used as a general term to include cables and risers, on or near the seabed. The interface with the seabed and the near-bottom environment represents a particularly challenging area of engineering design due to the complex fluid-pipeline-soil interactions involved. Although each individual two-way interaction has direct implications for pipeline systems, fully coupled interaction between all three components (i.e. fluid, pipeline and soil) is the most demanding for engineers, both in terms of the physics involved as well as the practical implications.

This thesis deals with a range of engineering problems that fall upon the spectrum of those involving fluid-pipeline-soil interactions at the seabed. The interactions within two areas of engineering design are given particular attention: (i) the on-bottom stability and safe operation of subsea pipelines and cables, and (ii) predicting seabed trenching within the touchdown zone of risers connected to floating vessels. The thesis itself comprises six main chapters that work towards advancing the state of knowledge in these areas and is formatted as a collection of technical papers with each chapter representing an individual paper. Therefore, each chapter is intended to be self-contained and begins with an abstract and an introduction with relevant literature summary, as customary.

This introductory chapter outlines the motivations for exploring these particular topics and a brief background for each that may be useful to put the subsequent chapters into context. Additionally, the specific objectives and a brief outline of each chapter is provided. This includes the placement of each chapter within the overall framework of fluid-pipeline-soil interactions and where each sits within the overall objectives of the thesis. Note that throughout the following chapter the abbreviation FPSI is used for ‘fluid-pipeline-soil interaction’.

1.1 Research motivation and background

In this section, introductory background reviews are given for each of the two design aspects of interest (i.e. on-bottom stability and safe operation of subsea pipelines and
Chapter 1 Introduction

trenching within the touchdown zone of pipeline risers) to provide context to the relatively self-contained paper-based chapters that follow. For each of these areas, the research aims addressed in the remainder of this thesis are also outlined.

1.1.1 Subsea pipeline systems

The evolution of the local seabed condition around a subsea pipeline or cable over the design life, and the consequences of these changes on the geotechnical response, affects the reliability of pipeline systems in a number of areas. In terms of the direct in-service design of pipelines, these areas include on-bottom stability (e.g. Draper et al. 2015), management of global buckling due to thermal expansion (e.g. Bransby et al. 2014), flow assurance (e.g. Zakarian et al. 2012) and cathodic protection of the pipeline structure (e.g. DNV 2005). Beyond operational issues, the pipeline burial condition also has implications for biodiversity in the offshore environment (e.g. McLean et al. 2017) and the potential for decommissioning offshore infrastructure by leaving it in situ.

Ensuring that on-bottom pipelines remain stable whilst withstanding environmental hydrodynamic loading requires that each of the individual components of FPSI be considered. The environmental loading itself occurs as a result of wave and/or current forcing induced by ocean dynamics and in this sense comprises fluid-pipeline interaction. To maintain stability, the pipeline is then required to resist motion when loaded, either via primary stabilisation (i.e. solely the soil resistance provided by the pipeline itself during operation) or secondary stabilisation (e.g. anchors or rock dumping). It is also well known that environmental fluid flows that are amplified local to seabed pipelines can lead to sediment transport and subsequent changes in the pipeline embedment over time, particularly in sandy materials (e.g. Sumer and Fredsoe 2002; Leckie et al. 2016). This further complicates the design of pipeline systems, which requires knowledge of the soil resistance throughout its design life, since the soil resistance is known to be strongly related to the pipeline embedment and variations in the soil properties (e.g. Verley and Sotberg 1994). As an additional complication, predicting the soil resistance in sandy materials that respond, under normal working conditions, as drained, frictional materials is challenging in that the response is non-associated (i.e. the direction of stress and plastic strain increment are not coincident) and not necessarily amenable to straightforward analysis using limit theorems (in contrast to undrained, clay materials). Traditionally, this has meant that estimates have been done empirically, which limits the flexibility of analyses in terms of geometry and soil properties.

Global buckling of pipelines is a phenomenon resulting from the expansion of pipelines induced by temperature or pressure effects. Conceptually, if a surface laid pipeline cannot freely expand, it may displace laterally once sufficient axial forces develop due to thermal or pressure-induced expansion of the pipeline. The global movement is notably distinct from local structural buckling, although it is local failure that represents the ultimate failure state if too much pipeline ‘feeds in’ to an uncontrolled buckle or if the pipeline is overstressed by buckling not occurring. Management of pipeline global buckling, like on-bottom stability, is directly impacted by the individual FPSI components, in

2
that knowledge of pipeline-soil interaction is required to ensure the structural integrity of buckling pipelines and this interaction again may be affected by fluid-pipeline-soil interaction in the form of pipeline scour or sedimentation. In practice, pipelines laid directly on the seabed are designed to buckle laterally in a controlled fashion, at specific locations and times during the design life; and buckle initiators (i.e. locations or in-line structures with specified levels of out-of-straightness) are often used to ensure buckles fire reliably. To ensure the reliability of buckling systems, the location and frequency of buckling must be known. The designer therefore requires knowledge of the soil resistance to movement, which like on-bottom stability, can be complicated by FPSI effects over time. An alternative approach is to bury the pipeline to prevent onerous displacement (now vertical as opposed to horizontal) and overstraining. The primary design issue for buried pipelines in this case is solely pipeline-soil interaction in terms of reliably predicting the soil resistance to upheaval movement.

For the other design issues mentioned (e.g. flow assurance, cathodic protection), FPSI effects other than pipeline-soil interaction are important but play more indirect roles in design. In particular, fluid-pipeline-soil interaction related to embedment changes due to sediment transport is important for ensuring operating efficiency and long-term serviceability of pipeline transport systems. For instance, changes in the pipeline burial condition affect the thermal insulation provided by the pipeline (e.g. Zakarian et al. 2012) and the electrochemical capacity of anodes (e.g. DNV 2005), which directly impact flow assurance and cathodic protection, respectively. However, since the FPSI aspects related to scour processes for stationary pipelines have been studied extensively in recent years (e.g. Zhang 2015; Leckie et al. 2016), these aspects are not considered further herein.

Research aims for subsea pipeline systems

As highlighted for both on-bottom stability and buckling management, reliable engineering design requires the available pipeline-soil resistance to be accurately known or predicted over the design life. The designer must therefore be able to estimate the soil resistance and understand how this resistance may change over the design life of the system. However, in certain circumstances, this may not be practicable and hence alternative engineered solutions, such as burying the pipeline by first trenching the seabed and then backfilling, might be preferable. In order to predict how the resistance may vary in practice, the research herein aims to explore the use of relatively simple analysis techniques for calculating the seabed resistance to pipeline movement (also known as the pipeline bearing capacity) and to apply these techniques to two design scenarios where the resistance changes due to changes in the seabed geometry from natural processes or due to changes in the pipeline geometry by the addition of engineered appendages. This is achieved by:

1. Numerically investigating the effect of non-associated flow on the combined vertical-horizontal bearing capacity of pipelines shallowly embedded in a frictional material, which provides insight into methods for predicting the bearing capacity across a range of embedment levels, soil friction angles and soil densities;
Chapter 1 Introduction

2. Numerically estimating the change in predominantly horizontal bearing capacity (or breakout resistance) for a pipeline laid on a sandy seabed subject to sediment transport effects, based on the changes in seabed geometry observed during field surveys of an operating pipeline;

3. Experimentally and numerically investigating the enhancement of uplift resistance attained by the addition of radial fins placed on the perimeter of a pipeline buried in loose sand.

1.1.2 Riser systems

For pipeline risers used to transfer materials from subsea extraction infrastructure to processing facilities on the ocean surface (or vice versa), the touchdown zone on the seabed where the risers first contact the seabed plays a key role in the structural fatigue of risers, particularly steel catenary risers (or SCRs). One of the main drivers for SCR design is structural fatigue of the system due to repeated loading over time. Such loading occurs primarily as a response to vessel and riser motions when subject to environmental loading. Structural fatigue is often most onerous at the positions where the SCR is connected to the vessel and where the SCR touches down on the seabed (e.g. Queau 2015). The structural response in the touchdown zone (TDZ), Figure 1.1, is primarily affected by (i) the geotechnical response to vertical and lateral riser motion (e.g. Thethi and Moros 2001; Randolph and Quiggin 2009) and (ii) trench formation beneath the riser and how this affects the geometry and bending moment profile of the riser (e.g. Bridge 2005; Randolph et al. 2013). The latter issue comprises the primary focus of the second portion of this thesis.

At the touchdown point, SCRs experience significant motions both during the installation and operation phases of their design lives. That these motions lead to sediment transport in a variety of materials has been suggested indirectly via observations of operational risers (e.g. Bridge and Howells 2007) and directly via video observations of pipelines during the lay process (e.g. Westgate et al. 2010). Bridge and Howells (2007)
presented field observations of trenches at the touchdown of SCRs in the Gulf of Mexico and offshore Brazil. These observations indicated trenches of up to 4.5 riser diameters in depth and widths of up to 10 riser diameters in primarily fine-grained sediments. Westgate et al. (2010) similarly observed trenches forming up to 1 diameter in depth for portions of pipelines where downtime lead to repeated oscillation at a particular location over a few hours. Bhattacharjee et al. (2014) reported trenches in deepwater clays in offshore West Africa around the mooring lines for semi-taut mooring systems that extended to a depth of up to 7 m below seabed.

The physics of trench formation are expected to comprise all three components of FPSI. Clearly, plastic deformation of the seabed may be a primary component of trenching. However, previous researchers exploring the riser-seabed response (i.e. the soil stiffness observed, its evolution with cycling and the effect on riser response) in model experiments for both fine-grained (e.g. Elliott et al. 2013; Yuan et al. 2016) and coarse-grained (e.g. Hodder and Byrne 2010) sediments reported trench depths less than 1 diameter. These studies were not intended to specifically explore trenching, which may explain some of the differences in trench depth between the experiments and the field observations; but the differences in trench magnitude nevertheless suggest that plastic deformation during riser-seabed contact may not completely explain the size of the trenches observed in the field. In addition to the observed trench sizes, plastic deformation alone does not explain the complete removal of sediment from trenches observed by Bridge and Howells (2007) - that is, open trenches. Fluid flows caused by riser motion are likely to contribute to this sediment removal, and they may also be strong enough to cause sediment motion by itself. Chiew et al. (2016), for instance, presented experiments specifically exploring trenching, including riser-seabed contact. However, they also observed trenches to form limited to less than about 1 diameter in depth (albeit in a sandy seabed). The Chiew et al. (2016) experiments do provide valuable insight into trench formation but are complicated by riser-seabed contact, which does not allow the proportionate influences of fluid-soil and pipeline-soil interactions to be ascertained independently. To fully explain the physics behind trench formation, the specific contribution of fluid-soil interactions (i.e. the how and whether the fluid motions caused by riser lead to sediment transport beneath the riser) must first be understood.

Although the second part of this thesis is chiefly concerned with detailing the fluid-pipeline and fluid-soil processes involved with trenching (as outlined in the following subsection), there is a secondary, but possibly important, effect that riser-seabed interaction may contribute, which is not explored in this thesis but is worth noting for context. This effect is the potential for softening of the sediment due to riser-seabed interaction, which may reduce the erosion resistance of the sediment and lead to increased trench extent and growth rates. Repeated loading has been shown to reduce the strength in both fine-grained (e.g. Andersen 2009; Hodder et al. 2013) and coarse-grained (e.g. Idriss and Boulanger 2008) sediments in a variety of contexts. As related to the resistance of the sediments to erosion, repeated, large-strain shearing of the sediment by a SCR during operation may impact sediment transport rates and trenching propensity via two potential mechanisms: (i) remoulding of the local seabed (leading to changes in effective
stress due to excess pore pressure generation) and (ii) entrainment of water into the seabed (leading to changes in sediment density/void ratio).

Remoulding occurs as a result of excess pore pressure generation when sediment is cyclically sheared as an undrained process. This leads to reductions in soil strength around an SCR as it moves into the seabed soil repeatedly over time. Hodder and Byrne (2010) showed that reductions in strength are significantly larger if free water is allowed to entrain into the sediment during cycling, such as may occur around an SCR near the bed surface. For instance, they showed that for kaolin clay strength reduction ratios (or sensitivities) of 2.4 were measured when cycling a T-bar at a minimum depth of 2 diameter into the seabed; however, when the cycling occurred at the same amplitude but only penetrating to a maximum depth of 0.5 diameters, the sensitivity was measured to be 7.5. In this case, water entrainment into the sediment presumably decreases the soil density, leading to reductions in strength, which does not occur on when cycling occurs contained within the seabed.

Shiri (2014) and others have demonstrated that the increased penetration due to the strength loss expected due to remoulding does not explain the trench depths observed in the field. However, given that remoulding and entrainment significantly reduce the undrained strength of soil, it is expected that these changes also lead to reductions in the erosion properties of the soil, leading to increased levels of transport and augments trench growth. A reduction in the threshold shear stress, for example, through changes in void ratio (e.g. Mitchener and Torfs 1996) or permeability (e.g. Mohr et al. 2016), would allow sediment to be more easily mobilised, suspended and convected away due to infrastructure motion. Therefore, it might be expected that both of these mechanisms (i.e. excess pore pressure generation due to remoulding and water entrainment at the seabed surface) may act together with the hydrodynamics produced by riser motions to lead to free standing trenches. This is believed to be the case because (a) remoulding and entrainment does not lead to removal of sediment on their own; and (b) field evidence (e.g. Bridge and Howells 2007) does not suggest obvious berm heave adjacent to trenches (relative to the amount of material not present in the trenches), as might be expected if trench formation were solely due to plastic deformation of the seabed.

**Research aims for riser systems**

To achieve more robust approaches to account for trenching, the physics of trench formation must first be understood. This thesis takes the approach of isolating certain components of this problem to enable better understanding of these individual features with regards to their contribution to trench formation. In particular, focus is given to exploring the fluid-pipeline interactions that occur near the seabed and how these interactions then affect sediment transport, in isolation to pipeline-soil interactions. Hence, the research herein aims to:

1. Investigate the effect of the seabed on the flow field around an oscillating cylinder. The focus here is on how the seabed (idealised as a rigid, plane wall) changes the flow regimes (e.g. vortex shedding regimes) and how the corresponding flow features
affect the near-wall velocity responsible for sediment transport. This is done with two approaches:

a) Numerical modelling at low Reynolds number to explore the effect of the wall in a parameter space that has been thoroughly described, both experimentally and numerically, for a free cylinder away from the wall;

b) Experimental testing at higher Reynolds numbers using flow visualisation techniques to ascertain the flow features for a more practically applicable parameter space.

2. Experimentally investigate sediment transport and trenching behaviour beneath a cylinder oscillating normal to a sandy seabed. This includes qualitative observations of the transport mechanisms, linking these to the flow field observations obtained, as well as quantitative measurements of motions required to cause transport and the subsequent size of trenches and rate of trench formation.

1.2 Thesis outline

The main thesis chapters are outlined within the context of FPSI on Figure 1.2.

Chapter 2 investigates the resistance to pipeline movement of a shallowly embedded pipeline for drained soil conditions using finite element analyses. This work explores the behaviour over a range of practical soil conditions and describes the changes in soil resistance with soil friction angle and density. The numerical approach presented in this chapter informs the analyses in the subsequent two chapters, which explore the soil response for two specific design scenarios.

Chapter 3 investigates the effect of external, radial fins on the uplift resistance of
buried pipelines. A series of model experiments were conducted with fins of various
lengths relative to the pipeline diameter, and numerical analyses are then compared with
the measured uplift resistance as well as observed soil failure mechanisms during loading.

Chapter 4 describes the changes to combined vertical and horizontal loading of seabed
pipelines that are subject to sediment transport-induced changes to their embedment
condition. In this work, observations of changes to the embedment condition of a pipeline
on the North West Shelf of Australia are analysed to assess how the breakout resistance
of the pipeline changes over time.

Chapter 5 and Chapter 6 comprise a fundamental exploration of the hydrodynamics
associated with a cylinder oscillating normal to a rigid, plane boundary. Chapter 5
presents a numerical study exploring the effect of a nearby wall on the flow patterns
and vortex shedding regimes for a cylinder oscillating normal to the wall, as compared
to a cylinder oscillating far from a wall. This study is conducted by directly solving
the two-dimensional Navier-Stokes equations at a relatively low, constant Reynolds
number of 150 and comparing the results both with and without a nearby wall with
previously published studies. Chapter 6 presents a series of flow visualisation experiments
to observe the flow patterns, vortex shedding features and near-wall flow quantities at
higher Reynolds number closer to the range of practical interest for offshore application.
Force measurements are also presented in this chapter to demonstrate the effect of the
wall on the hydrodynamic forcing of pipelines near the seabed. Both the numerical
and experimental studies indicate some similarities in the observed changes to the flow
patterns and provide insight into the overall behaviour used to interpret the subsequent
the sediment transport studies in Chapter 7.

Chapter 7 is an investigation into the mechanics of sediment transport and trench
development beneath a circular cylinder undergoing forced oscillation normal to an initially
plane bed of sand. A series of model experiments are described, and the mechanisms
of sediment transport are found to be consistent with the predominant pipeline-fluid
interactions observed in the preceding two chapters. The initiation of sediment motion
beneath the cylinder as well as the rate and depth of trenches that result from cylinder
motions were measured over a range of cylinder amplitudes and periods. It is shown that
initiation of motion is predicted reasonably well for low Keulegan-Carpenter numbers
using control volume arguments and an oscillatory boundary layer assumption. The rate
of trench development and the trench depths measured are shown to be a function of
both the amplitude and velocity of cylinder motion as well as of the minimum distance
to the bed.

Chapter 8 forms the concluding main chapter of this thesis, summarising the primary
contributions of this work and identifying areas for future research.

The thesis also comprises three appendices, which comprise various ancillary pieces
of work conducted in parallel to the primary work in this thesis. These include early
publications regarding the flow field observations as well as separate work focusing on
scour around subsea structures and the effect of drainage on the upheaval resistance of
buried pipelines.
References


Chapter 1 Introduction


Chapter 2

Drained bearing capacity of shallowly embedded pipelines

Abstract This study establishes the drained bearing capacity of pipelines embedded up to one diameter into the seabed subject to combined vertical-horizontal loading. Non-associated flow finite element analyses are used to calculate the peak breakout resistance in a non-associated flow, frictional Mohr-Coulomb seabed. Critical state friction angles and dilation angles ranging from 25° to 45° and 0° to 25°, respectively, are considered. Analytical expressions have been fitted to the results as a function of embedment depth and soil properties, and compare well with experimental measurements from previous studies. The horizontal bearing capacity at small vertical loads is also predicted well via upper bound limit analysis using the Davis reduced friction angle that accounts for the peak friction and dilation angles. The analytical relationships presented in this study provide simple predictive tools for estimating the bearing capacity of pipelines on free-drained sandy seabeds. These fill a void in knowledge for pipeline stability and buckling design by providing general relationships between drained strength properties and pipeline bearing capacity. The insight gained through the good comparison with limit analysis techniques also gives confidence in the use of simple numerical techniques to predict the bearing capacity of pipelines for more wide-ranging (i.e. non-flat) seabed topography.

This chapter has been submitted for publication as:
Chapter 2 Drained bearing capacity of shallowly embedded pipelines

2.1 Introduction

The bearing capacity of subsea pipelines is a primary input for many design areas, including on-bottom stability and global buckling management. This paper is concerned with the drained bearing capacity of a subsea pipeline that is subjected to combinations of vertical and horizontal loading.

If a pipeline has insufficient geotechnical bearing capacity (or ‘breakout resistance’) to resist externally-applied environmental or other operational loads then significant movements may occur, jeopardising the integrity of the pipeline. Accurate assessment of the available resistance can lead to significant cost savings in capital expenditure for offshore projects if pipeline stabilisation measures can be optimised. High temperature and pressure oil and gas pipelines also undergo operational expansions during start-up and shutdown cycles, which must be safely accommodated to prevent pipeline damage. Global buckling design is particularly complicated because the geotechnical resistance must be bracketed: a conservative design may rely on either an upper or lower estimate depending on the context.

Pipeline bearing capacity is further complicated by the fact that either drained or undrained (or intermediate, partially drained) conditions can prevail during breakout. Drainage conditions depend on the consolidation properties of the soil, the rate and duration of loading and the embedment condition of the pipeline. Drainage affects both the shear strength of the soil as well as the kinematics at failure. During undrained loading volume change does not occur, and associated flow conditions prevail at failure. The resulting volumetric and kinematic constraints allow exact bearing capacity solutions to be bounded using limit theorems (Martin and White 2012). Under drained conditions volume change may occur at failure, and the soil strength is controlled by friction. For drained failure the mobilised shear strength varies throughout the failure mechanism, and the resulting kinematics are complicated by the occurrence of volumetric strains due to non-associated flow.

The current understanding of drained pipeline bearing capacity is based primarily on experimental studies. Verley and Sotberg (1994) summarised three datasets from testing on silica sands and proposed a power law relationship to calculate the peak breakout resistance, which is a function of the applied vertical load and the pipeline embedment:

\[
\frac{H}{\gamma' D^2} = \left(5.0 - 0.15 \frac{\gamma' D^2}{V}\right) \left(\frac{w}{D}\right)^{1.25} + 6 \frac{V}{\gamma' D^2} \quad \text{for} \quad \frac{\gamma' D^2}{V} \leq 20
\]

\[
\frac{H}{\gamma' D^2} = 2.0 \left(\frac{w}{D}\right)^{1.25} + 0.6 \frac{V}{\gamma' D^2} \quad \text{for} \quad \frac{\gamma' D^2}{V} > 20
\]

(2.1)

where \(H\) and \(V\) are the vertical and horizontal loads (per unit length) at failure, \(\gamma'\) is the soil effective unit weight, \(D\) is the pipeline diameter, \(w/D\) is the normalised pipeline embedment measured from the pipeline invert (Figure 2.1). This method was based on tests conducted for embedments less than 35% of the pipeline diameter and no data was provided regarding the friction angle or other strength characteristics of the materials tested.
2.1 Introduction

Zhang (2001) and Zhang et al. (2002) describe centrifuge tests on pipelines embedded in calcareous sands. Based on these results, Zhang et al. (2002) presented a plasticity-based macro-element model for calculating the vertical-horizontal ($V - H$) failure envelope as well as the non-associated plastic potential surface. Zhang et al. (2002) defined the failure envelope shape as a generalisation of the envelope set out by Butterfield and Gottardi (1994):

$$H = \mu (V - V_{\text{min}})(1 - V/V_{\text{max}}) \tag{2.2}$$

where $\mu$ is a parameter controlling the gradient of the envelope at low $V$, $V_{\text{min}}$ is the vertical uplift capacity and $V_{\text{max}}$ is the purely vertical bearing capacity. This envelope implies that the maximum horizontal bearing capacity occurs at $V/V_{\text{max}} = 0.5$. Zhang et al. (2002) indicate that $V_{\text{max}}$ is a function of pipeline embedment and is determined either from vertical load-penetration curves or estimated from the conventional vertical bearing capacity overburden factor, $N_q$, as:

$$V_{\text{max}} \approx k_{vp}w = \gamma'N_qwD \tag{2.3}$$

where $k_{vp}$ is the gradient of the vertical bearing capacity increase with depth (units of kN/m/m). The friction parameter $\mu$ was suggested by Zhang et al. (2002) to be only a function of pipeline embedment:

$$\mu = 0.4 + 0.65w/D \tag{2.4}$$

based on calibration to their centrifuge data. Zhang et al. (2002) indicated that the model also provides reasonable fit to some of the silica sand results from the Verley and Sothberg (1994) database. However, the Zhang et al. (2002) model, like the Verley and Sothberg (1994) model, does not include any direct influence of soil friction angle or dilation angle (i.e. density) on the vertical bearing capacity or the horizontal breakout resistance at low vertical loads, other than that implied by Eq. 2.3.

Sandford (2012) conducted a set of experiments and non-associated flow finite element analyses of drained pipeline breakout in silica sand. Compared to Zhang et al. (2002), the overall response from the experiments and numerical analyses were of similar magnitude and produced similar envelope shapes but covered a limited range of soil properties and pipeline embedment levels. Beyond the work of Sandford (2012), the other published work to link drained pipeline bearing capacity to soil properties is by Gao et al. (2015), who presented a general slip-line solution for the ultimate drained vertical bearing capacity of pipelines. However, they did not consider the effect of non-associated flow on the
response.

The previous work exploring pipeline breakout in sand has not generalised the response to enable direct soil input to consider different friction and dilation angles or was focused on a limited range of soil properties and embedment levels. This paper expands upon the previous work by conducting non-associated flow finite element analyses (FEA) of the bearing capacity of shallowly embedded pipelines up to one diameter in embedment ($w$ on Figure 2.1). The analyses cover a wider range of friction and dilation angles (i.e. relative density) than previously explored. The friction and dilation angles are consistently linked by the strength-dilatancy relationship presented by Bolton (1986). The results provide insight into scenarios when non-association is most important and in what scenarios simple limit analysis techniques with the use of a reduced friction angle accounting for non-association may be sufficiently accurate.

2.1.1 Bearing capacity on drained soil with non-associated flow

The non-associated flow of sands at failure has a significant effect on the limiting capacity of geotechnical systems (e.g. Drescher and Detournay 1993; Frydman and Burd 1997). For associated flow, plasticity theorems enable the bearing capacity of boundary value problems to be bounded uniquely for a given set of boundary conditions and failure criteria. However, for non-associated flow, these bounds are no longer valid, other than that the upper bound of an equivalent associated flow problem (i.e. same friction angle) also forms an upper bound on the solution of the non-associated problem (Davis 1968). The literature on non-associated flow analyses suggests that non-association introduces two primary consequences: (i) that bifurcation/localisation of failure planes results in non-uniqueness and (ii) a general reduction in the bearing capacity of the system as compared to associated flow. Bifurcation implies a switch from a homogeneous solution to the governing equations to a non-homogenous (localised) one. Hence, a range of localised solutions to the governing equations are possible for non-associated flow problems (Krabbenhoft et al. 2012). In practice for numerical analyses, such non-uniqueness often manifests through sensitivity of the solution to mesh conditions and an irregular (unsteady) response in the limiting load with continuing displacement (e.g. Loukidis and Salgado 2009). By contrast, associated flow problems theoretically have a unique solution.

The second consequence of non-association is the general tendency for the load bearing capacity of the non-associated boundary value problem to be reduced as compared to an equivalent associated flow problem. This concept can be understood by analogy if one considers the sliding resistance of a rigid block with a purely frictional interface, or equivalently a direct shear test. In this case, the values of normal and shear stress acting on the horizontal interface do not necessarily lie on the plane of maximum obliquity to the Mohr’s circle of stress ($\phi_{IF}$ on Figure 2.2), or in other words the operative friction angle on the horizontal plane may be less than the tangent friction angle. However, from the boundary constraints, lateral extension strain in the horizontal direction is zero. If it assumed that the directions of principal stress and principal strain increment are coaxial
for soil undergoing plastic deformation (Roscoe 1970), Mohr’s circles of stress and strain increment can be drawn as on Figure 2.2. The actual stresses acting on the interface plane can be determined from the Mohr’s circles constructed on Figure 2.2 for a given set of Mohr-Coulomb soil properties and the dilation angle of the interface material. Noting that \( \sin(\phi_{MC}) = t/s \) and taking advantage of the sine rule to determine the interface friction angle, \( \phi_{IF} \), some rearrangement yields:

\[
\tan(\phi_{IF}) = \frac{\sin(\phi_{MC})\cos(\psi)}{1 - \sin(\phi_{MC})\sin(\psi)}
\] (2.5)

From Eq. 2.5, only when \( \psi = \phi_{MC} \) does \( \phi_{IF} = \tan(\phi_{MC}) \), so only under associated flow is the friction along a shear plane equal to the classical \( \tan(\phi_{MC}) \) result. For \( \psi < \phi_{MC} \), the friction ratio is lower - when \( \psi = 0^\circ \), \( \tan(\phi_{IF}) = \sin(\phi_{MC}) \) as first shown by Hill (1950). These relations simply mean that within a soil continuum there exists some element on which the combination of \( \tau/\sigma = \tan(\phi_{MC}) \) acts, but this stress ratio does not necessarily act on the shear plane itself.

Drescher and Detournay (1993) took advantage of this finding and proposed an approach to calculating the bearing capacity of a non-associated problem by using such modified material strength parameters within the framework of upper bound limit analysis. This enables a solution to be calculated that estimates the effect of non-association but cannot be a rigorous solution. The approach has been shown to provide reasonable estimates to various problems compared to finite element analyses (e.g. Michalowski and Shi 1995; Yin et al. 2001); however, Krabbenhoft et al. (2012) identified that, for certain problems, such as vertical uplift of buried anchors or pipelines, the use of modified parameters in an associated framework can overestimate the resistance. This is because the failure mechanism corresponding to associated flow can vary significantly from that of the non-associated case.
2.2 Methodology

2.2.1 Analysis software

The analyses described in this paper were performed using OptumG2, a commercially available finite element and finite element limit analysis software (OptumCE 2017). Associated flow analyses were conducted for both the upper and lower bound capacity using finite element limit analysis methods described by Lyamin and Sloan (2002). OptumG2 incorporates adaptive remeshing procedures, which enable automated optimisation of failure mechanisms in terms of the size, position and orientation of the mesh elements. For non-associated flow analysis, elastoplastic finite element analysis was used with Mohr-Coloumb soil elements, as described briefly below.

Krabbenhoft et al. (2012) proposed a method for numerical analysis of non-associated flow problems that involves recasting the non-associated problem into variational form that can be solved using numerical procedures developed for associated flow problems. This recasting improves some of the numerical convergence issues reported for non-associated flow (e.g. Loukidis and Salgado 2009) and allows both the local strength (friction angle) and kinematic (dilation angle) criteria for a non-associated Mohr-Coulomb material to be satisfied at failure. First, the Mohr-Coulomb failure criterion is converted to an algebraically equivalent form:

\[ F(\sigma) = \tau - \phi_{MC} \sigma - c \] (2.6)

\[ F^*(\sigma) = \tau - \psi \sigma - c^*(\sigma) \]
\[ c^*(\sigma) = c + (\phi_{MC} - \psi) \sigma \] (2.7)

where \( c \) is any ‘true’ cohesion and \( c^* \) is an apparent cohesion. Figure 2.3 illustrates the two failure criteria showing that the apparent cohesion \( (c^*) \) at a given instant is specified such that at the current normal stress level the same shear stress at failure results from both Eq. 2.6 and Eq. 2.7. Applying the assumption of associated flow to these two failure criteria, the normal direction to Eq. 2.7 corresponds to the dilation angle and thus non-associated plastic flow at failure is achieved. From Eq. 2.7b, the normal stress is required to calculate \( c^* \). Therefore, \( c^* \) must be explicitly calculated incrementally over a series of substeps for each calculation load increment. By using small substep increments, errors between \( F \) and \( F^* \) arising from differences in elastic and plastic stress states between the two can be minimised (Krabbenhoft et al. 2012). Explicit substep calculation of \( c^* \) allows its value to be known and \( F^* \) can then be used directly in implicit solution methods or solved in terms of variational principles. This approach does not alleviate the issue of bifurcation and localisation or non-uniqueness of solution. Therefore, use of such an approach remains approximate and should be compared with relevant experimental results.
2.2 Methodology

Figure 2.3 Frictional Mohr-Coulomb failure criteria unmodified $F$ (Eq. 2.6) and modified for substepping $F^*$ (Eq. 2.7).

Table 2.1 Adopted friction and dilation angle parameter sets

<table>
<thead>
<tr>
<th>Critical state friction angle $\phi_{cs}$ (°)</th>
<th>$\phi_{peak}$ − $\phi_{cs}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0 10 20</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0 12.5 25</td>
</tr>
<tr>
<td>35</td>
<td>0 12.5 25</td>
</tr>
<tr>
<td>$\phi_{peak}$</td>
<td>0 12.5 25</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0 12.5 25</td>
</tr>
</tbody>
</table>

2.2.2 Soil and pipeline parameter ranges

Analyses have been conducted for a range of pipeline embedment ($w/D = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0$) assuming a pipeline outer diameter of 1 m (although all results are presented non-dimensionally). In all cases, the pipeline was modelled as weightless and rigid; and pipe rotation is prevented during analysis. The pipeline was initially modelled as a polygon with a minimum side length of $0.1D$; however, the adaptive remeshing procedure locally refines the mesh in areas (including the pipeline perimeter) where more intense shearing occurs. This refinement achieved an approximately circular border at the pipe perimeter by the final remeshing step. The soil domain generally extended at least a distance of $3D$ on either side of the pipeline and $1.5D$ below the pipeline but was extended to minimise boundary effects when necessary.

The soil was modelled as a cohesionless Mohr-Coulomb soil, with a constant unit weight of 10 kN/m$^3$. A Young’s modulus of 1000 MPa and a Poisson’s ratio of 0.3 were assumed for all analyses, although the selected stiffness had no effect on the ultimate loads. The initial $K_0$ value for each analysis was based on the peak friction angle corresponding to Jaky’s equation, $K_0 = 1 − sin(\phi_{peak})$. The soil-pipeline interface condition was modelled as fully rough with the same soil properties as the surrounding material.

Peak friction angles ranging from $25°$ to $60°$ for both associated and non-associated flow analyses are considered. For the non-associated analyses, variations in dilation angle are linked to peak friction angle following Bolton (1986), leading to the nine cases shown in Table 2.1. This range of friction and dilation angles is expected to cover a practical range of relevant soil properties and spans relative density from approximately 20% to 100%.
Chapter 2 Drained bearing capacity of shallowly embedded pipelines

2.2.3 Analysis approach

For associated flow limit analysis, a final mesh of 15,000 elements was adopted, with 4 remeshing iterations during each analysis. The high number of elements was adopted for associated flow analyses to achieve a targeted error between upper and lower bound results of 2%. If this criterion was not achieved, further adaptation steps were conducted to reduce the error, although in some cases at high friction angle the minimum achievable error was 10%. Associated flow results are presented as the average of the upper and lower bounds.

For non-associated flow finite element analysis, a mesh convergence study was first conducted by calculating the purely vertical bearing capacity of a pipeline on soil with properties, $\phi_{peak} = 45^\circ$, $\psi = 25^\circ$, and varying the total number of elements in the model. In all cases, the model was remeshed for three iterations every three steps of the analysis. The pipeline embedment was varied from 0.1 to $1 \, D$ with total numbers of elements, after refinement, ranging from 1,000 to 6,000. The results of this study indicate that the difference in the calculated bearing capacity between cases with 3,000 and 6,000 elements is less than 5% (Figure 2.4), although notably the refinement curves are not monotonic due to the generally oscillatory load response. Therefore, 3,000 elements has been selected to provide a balance between computational cost and reasonable mesh convergence.

The bearing capacity envelopes under combined vertical-horizontal loading were determined by first calculating the uniaxial vertical downward and uplift bearing capacities. Further analyses are then conducted by applying a small initial constant vertical load to the pipeline (2 kN per unit length) and then applying 11 different combinations of horizontal and vertical load to failure, distributed between purely downward and purely upward. To provide additional detail of the envelope shape at low vertical load, further analyses were conducted by applying purely horizontal failure loads under constant vertical loads of 5 kN/m and 10 kN/m.

Figure 2.4 Sensitivity of vertical bearing capacity with total number of elements. $\phi_{peak} = 45^\circ$, $\psi = 25^\circ$. 
Each analysis is divided into 5 elastic load steps and 10 plastic load steps for each loading scenario. The presented limit loads are calculated as the average of the final 5 plastic load steps. Some analysis runs with large $V/V_{\text{max}}$ did not reach a steady state, where for the final 5 steps the ratio of the mean plus standard deviation to the mean was less than 5%, within the standard number of loading increments. In this case, additional plastic steps were added until a steady oscillatory response was achieved. For some cases, particularly for $\phi_{\text{peak}} \geq 55^\circ$, this criteria was not able to be achieved, and results with oscillation ratios larger than 5% of the mean have generally been excluded from the envelope interpretations described later.

### 2.2.4 Dimensionless groups

The results are presented as dimensionless loads:

$$\bar{V} = \frac{V}{\gamma D^2}; \quad \bar{H} = \frac{H}{\gamma D^2}$$

Practical ranges of $\bar{V}$ can be estimated for pipelines resting with only their self-weight acting on the seabed from:

$$\bar{V} = \frac{V}{\gamma D^2} = \frac{\pi}{4} (SG - 1)$$

where $SG$ is the specific gravity of the pipe – a term commonly used in pipeline engineering. A pipe that is neutrally buoyant in water has $SG = 1$ meaning it applies zero vertical load to the seabed. Typical values of $SG$ for gas pipelines and umbilical cables – which represent light and heavy extremes – are 1.2 and 3, which correspond to $\bar{V} = 0.2$ and 1.5 respectively. At the ends of a pipeline span, where the weight of the whole span is carried by a short length at the ‘abutments’, the vertical load may be increased by an order of magnitude. Similarly, when a pipe is laid on the seabed, the stress concentration at the touchdown point may increase $\bar{V}$ by a factor of 2-10, with higher values applying on stiff sandy soils.

### 2.2.5 Validation of analysis methodology

Figure 2.5 compares elastoplastic analysis in OptumG2 for vertically loaded, rough strip footings with previous numerical results for both associated (Martin 2003; Lyamin et al. 2007) and non-associated soils (Loukidis et al. 2008). The associated flow results are all within 5% of the previously reported values, and the calculated non-associated collapse loads are about 10% lower than the Loukidis et al. (2008) results. These comparisons suggest that: (a) the mesh and loading discretisation for the elastoplastic finite element analyses are appropriate given that the associated flow results are within a small margin of known solutions; and (b) the non-associated flow calculation approach and discretisation provides similar but lower bearing capacities compared to the Loukidis et al. (2008) results over a range of friction and dilation angles, as expected from the relatively higher mesh density utilised herein.
Figure 2.5 Comparison of vertical bearing capacity factors for strip footing with previously published results.

Figure 2.6 Comparison of undrained $V - H$ envelopes with Martin and White (2012): $\gamma D/s_u = 1$. Solid circles – current analysis. Black lines – Martin and White (2012).

An additional validation is provided by pipeline bearing capacity analyses using the same analysis strategy combined with the undrained Tresca model. These results are compared with limit analysis results by Martin and White (2012) on Figure 2.6, for a fully rough pipeline interface with full tension allowed and a soil undrained strength of $\gamma D/s_u = 1$. The current results are generally within 5% of Martin and White (2012). Further confirmation of the appropriateness of the current approach for drained resistance can be found in Tom et al. (2017), where a similar approach is used with good success for back-calculating the uplift resistance of buried pipelines in relatively loose sand of known friction and dilation angles.

2.3 Results

2.3.1 Vertical bearing capacity

Normalised vertical bearing capacity results are shown on Figure 2.7 for both the associated flow and non-associated flow cases up to a normalised embedment of 1.0. Upper bound
estimates using a reduced friction angle (following Eq. 2.5) are also shown. Bearing capacity predictions from the recommendations of Zhang et al. (2002) as per Eq. 2.3 are shown for comparison, with \( N_q \) (Reissner 1924) calculated as:

\[
N_q = e^{\pi \tan(\phi)} \tan \left( \frac{45^\circ + \phi}{2} \right)^2
\]  

Eq. 2.10 estimates \( N_q \) values within 0.01% of exact values provided by Martin (2005).

The bearing capacity results generally increase slightly non-linearly with depth (i.e. the tangent stiffness reduces with depth). The results from limit analysis using Eq. 2.5 tend to underpredict the resistance compared to the non-associated flow results corresponding to the same combination of peak friction and dilation angles. This underprediction is particularly evident for high friction angles.

Using least-squares fitting, a power law relationship is fitted to the results with the corresponding fits also shown on Figure 2.7 following:

\[
V_{max} = A \left( \frac{w}{D} \right)^B
\]  

The fitted \( A \) coefficient for each analysis set, which represents \( V_{max} \) at \( w/D = 1 \), are plotted versus soil friction angle on Figure 2.8. The \( A \) coefficient increases with friction angle but the value at a given friction angle reduces with dilation angle. The coefficients on Figure 2.8 are grouped by the equivalent critical state friction angle. When grouped in this fashion, the results show consistent trends for each critical state friction angle. As a result, the following function has been fitted using least squares to the sets for each critical state friction angle (and to the associated flow as a separate fitting):

\[
A = C_1 \left( e^{\phi_{peak}} C_2 \right) C_3 \phi_{peak}
\]  

where \( C_1, C_2 \) and \( C_3 \) are additional fitting coefficients and angles are given in degrees.

Eq. 2.12 allows estimation of the \( A \) coefficient for various associated flow friction angles, as shown on Figure 2.8 using coefficients tabulated in Table 2.2, although the fit was weighted for friction angles less than 45° and the values for higher friction angles are underpredicted. For non-associated flow, the \( C \) parameters are found to be linear functions of \( \phi_{cs} \), where a trend can be fitted by:

\[
C_i = I_{c,i} + \phi_{cs} S_{c,i}
\]  

where \( C_i \) are the three \( C \) coefficients, \( I_{c,i} \) is the fitted intercept at \( \phi_{cs} = 0 \) for each \( C_i \) as a function of \( \phi_{cs} \) and \( S_{c,i} \) is the slope of the \( C_i \) trend with \( \phi_{cs} \). Fitted values of \( I_{c,i} \) and \( S_{c,i} \) for each \( C_i \) are tabulated in Table 2.2 and shown on Figure 2.8.

The \( B \) coefficient shows less variation than \( A \) with respect to dilation angle and is primarily a function of \( \phi_{peak} \). Hence, a simple linear relationship to approximate this variation with peak friction angle is shown on Figure 2.9 corresponding to:

\[
B = 1.3067 - 0.0123 \phi_{peak}
\]
For small $\phi_{\text{peak}}$ the coefficient is close to unity, which corresponds to the vertical capacity increasing linearly with depth. As $\phi_{\text{peak}}$ increases, $B$ reduces indicating that the tangential stiffness of vertical capacity reduces with depth.

The vertical bearing capacity results can be compared with experimental and numerical results presented by Sandford (2012), who presented a series of experiments investigating the vertical bearing capacity with embedment. Figure 2.10 shows the vertical bearing capacity measured in model experiments and the corresponding predictions based on Eq. 2.11 to 2.14 using the density information varying with depth as provided by Sandford (2012) and assuming the critical state friction angle to range from 34 to 38°. Although
2.3 Results

Figure 2.8 Variation in $A$ coefficient and $C$ coefficients with friction angle (note: relationships for $C$ coefficients are based on critical state friction angle; Eq. 2.12 function for $A$ coefficient is based on peak friction angle).

Table 2.2 Fitted coefficients for vertical capacity (Eq. 2.12)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>4.95</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.22</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$8.36 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.07</td>
</tr>
<tr>
<td>$I_{c_1}$</td>
<td>1.75</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.0163</td>
</tr>
<tr>
<td>$I_{c_2}$</td>
<td>0.6467</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$-5.97 \times 10^{-5}$</td>
</tr>
<tr>
<td>$I_{c_3}$</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

Table 2.2 presents the fitted coefficients for vertical capacity (Eq. 2.12). The 36° critical state angle appears to provide the best fit, this is slightly higher than the 34.3° value reported by Sandford (2012). Nevertheless, the vertical response predictions compare reasonably well with the measured experimental data, given the uncertainties in measuring sand density and operative friction angle.

2.3.2 Overall failure envelope shape

Bearing capacities corresponding to different combinations of vertical and horizontal load vectors are presented on Figure 2.11 and 2.12 for 84 material and geometry combinations for associated and non-associated flow, respectively. Results are normalised by the maximum vertical bearing capacity, $V_{max}$. For non-associated parameter combinations, portions of some envelopes are poorly defined for large values of $V$ due to irregular load-displacement response and difficulties in achieving numerical convergence. This was particularly problematic for $\phi_{peak} \geq 55^\circ$, and hence some load cases are excluded from these results. Other than variability due to these issues, both sets of results indicate that the ratio $H_{max}/V_{max}$ increases with embedment and generally converges with increasing $\phi_{peak}$ or $\psi$. This trend means that for large $\phi_{peak}$ the vertical bearing capacity increases
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Figure 2.9 Variation in $B$ coefficient with $\phi_{peak}$. Black Circles - non-associated flow. Blue squares - associated flow.

with embedment at a higher rate than the horizontal capacity.

Each envelope is also fitted (using a non-linear least squares approach) with a modified version of the envelope suggested by Zhang et al. (2002):

$$
\frac{H}{V_{max}} = \mu \left( \frac{V}{V_{max}} + \beta \right)^n \left( 1 - \frac{V}{V_{max}} \right)^m \tag{2.15}
$$

where $\beta$ represents the maximum vertical uplift (tension) capacity as a proportion of the maximum (downward) vertical capacity, $\mu$ is a constant proportional to $H_{max}/V_{max}$ for constant values of $m$ and $n$, which are exponents that control envelope shape at low and high vertical loads, respectively.

Figures 2.13a and 2.13b show the variation in parameters $n$ and $m$ grouped by $\phi_{peak}$ as a function of $w/D$, where $\mu$, $n$ and $m$ are all kept as independent variables in Eq. 2.15 (i.e. the fits corresponding to Figures 2.11 and 2.12) and $\beta$ is taken directly as $|V_{min}/V_{max}|$. Parameter $n$ increases slightly with $w/D$ but generally falls within a relatively small range from approximately 0.5 to 0.8. Parameter $m$ takes a larger range of values for the non-associated results with a slight increasing trend with $\phi_{peak}$.

A simplified method of describing the trends in fitting parameters has been adopted to provide a first order approximation of the non-associated envelopes from these analyses. To implement this approach, we take advantage of the relatively small variation in $n$ and the approximately linear relationship observed for $m$ with respect to $\phi_{peak}$ - $n$ is taken as a constant value corresponding to the mean of the non-associated results (i.e. 0.64) and $m$ assumed to be:

$$m = 0.38\phi_{peak} + 0.014 \tag{2.16}$$

With these assumptions for $n$ and $m$, Eq. 2.15 reduces to a two variable fitting problem for $\mu$ and $\beta$. Figure 2.14a shows the resulting fitted $\mu$ coefficients (with $\beta$ assumed directly from the results) grouped by $\phi_{peak}$ as a function of $w/D$. There is a general trend, with some variation, of increasing $\mu$ with $w/D$ and decreasing $\mu$ with $\phi_{peak}$, which
2.3 Results

Figure 2.10 Vertical penetration response, assuming relative density of 20%, compared with experimental results from Sandford (2012). Solid lines – predictions based on Eq. 2.11 to 2.14. Solid circles – values reproduced from Sandford (2012).

Figure 2.11 Combined loading failure envelopes for the associated flow sets. Circles – $w/D = 0.1$. Squares – $w/D = 0.2$. Upward triangles – $w/D = 0.4$. Dots – $w/D = 0.6$. Downward triangles – $w/D = 0.8$. Crosses – $w/D = 1.0$. Lines – fitted envelopes based on least squares to Eq. 2.15.

is qualitatively consistent with Figure 2.12. The resulting values of $\mu$ are fitted with a
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Figure 2.12 Combined loading failure envelopes for the non-associated flow sets. Circles – \( w/D = 0.1 \). Squares – \( w/D = 0.2 \). Upward triangles – \( w/D = 0.4 \). Dots – \( w/D = 0.6 \). Downward triangles – \( w/D = 0.8 \). Crosses – \( w/D = 1.0 \). Lines – fitted envelopes based on least squares to Eq. 2.15.

linear relationship via:

\[
\mu = 0.2w/D + \mu_0 \tag{2.17}
\]

where the slope 0.2 is assumed constant corresponding approximately to the slopes for \( 25^\circ \leq \phi_{peak} \leq 55^\circ \) and \( \mu_0 \) is the intercept at \( w/D = 0 \). Figure 2.14b shows \( \mu_0 \) as a function of \( \phi_{peak} \), which is also fit reasonably well by:

\[
\mu_0 = -0.00428\phi_{peak} + 0.42 \tag{2.18}
\]

Eq. 2.17 and 2.18 are similar to the relationship proposed by Zhang et al. (2002), except that \( \mu_0 \) is a linear function of \( \phi_{peak} \), whereas in Zhang et al. (2002) it was taken as constant for the range of soils considered.

The resulting coefficients following Eq. 2.16-2.18 (and \( n = 0.64 \)) allow envelopes to be inferred for different combinations of \( w/D \) and \( \phi_{peak} \). The appropriateness of this methodology can be seen by comparing the estimated values of \( \overline{H}/\overline{V} \) with those calculated directly from the numerical results. Figure 2.15a shows non-associated
2.3 Results

Figure 2.13 Fitted $n$ and $m$ coefficients for Eq. 2.15 for associated (top row) and non-associated (bottom row) results. Circles – $w/D = 0.1$. Squares – $w/D = 0.2$. Upward triangles – $w/D = 0.4$. Dots – $w/D = 0.6$. Downward triangles – $w/D = 0.8$. Crosses – $w/D = 1.0$.

$H_{\text{max}}/V_{\text{max}}$ calculated from Figure 2.12. Figure 2.15b compares $H_{\text{max}}/V_{\text{max}}$ using Eq. 2.16-2.18 with the values from Figure 2.15a. Good comparison is achieved using the relatively simple estimation relationship, which confirms that an approximation of the envelope shape and $H_{\text{max}}/V_{\text{max}}$ can be attained using this approach.

2.3.3 Low $V/V_{\text{max}}$ response

The previous section described the overall failure envelope response; however, the parameter space for practical applications is generally limited to a range of $V < 10$, as described in Section 2.2.4. Furthermore, achieving a reasonable fit of Eq. 2.15 to the overall envelope does not provide sufficient accuracy to fit the results at small $V/V_{\text{max}}$, which converge more consistently than at larger $V/V_{\text{max}}$. Therefore, in this section the horizontal capacity results at small $V/V_{\text{max}}$ are presented directly, without an overall envelope fitting framework.

At small $V/V_{\text{max}}$, the horizontal bearing capacity is often defined by the ratio of horizontal to vertical load at failure – $H/V$. Figure 2.16 shows $H/V$ for $V < 10$ for the considered parameter space. The non-associated results on Figure 2.16 indicate that $H/V$ increases with embedment, density (i.e. $\phi_{\text{peak}} - \phi_{\text{cs}} \approx \psi$) and $\phi_{\text{cs}}$ but reduces non-linearly as $V$ increases.

Figure 2.16 also shows equivalent upper bound limit analysis results assuming a reduced friction angle following Eq. 2.5. These results show good comparison with the non-associated FEA results over the range of $w/D$ and $\phi_{\text{peak}}$ considered. Also shown on
Chapter 2 Drained bearing capacity of shallower ly embedded pipelines

Figure 2.14 Fitted $\mu$ coefficients based on fitted values of $n$ and $m$ as per non-associated results on Figure 2.13. (a) Fits to numerical results. Circles – $\phi_{\text{peak}} = 25^\circ$. Squares – $\phi_{\text{peak}} = 35^\circ$. Upward triangles – $\phi_{\text{peak}} = 45^\circ$. Asterisks – $\phi_{\text{peak}} = 55^\circ$. Downward triangles – $\phi_{\text{peak}} = 60^\circ$. (b) Intercept to linear fits to (a), $\mu_0$.

Figure 2.16 are estimations due to a reinterpreted version of Eq. 2.4:

$$\frac{H}{V} = \tan(\phi_{\text{peak}}) + \frac{1 + \sin(\phi_{\text{peak}})}{1 - \sin(\phi_{\text{peak}})} \frac{w}{D}$$  \hspace{1cm} (2.19)

Eq. 2.19 comprises a superposition of frictional and passive resistance where the latter corresponds to a classical passive earth pressure multiplied by the pipeline embedment. This is similar to the relationship suggested by Zhang et al. (2002) for $\mu$, except that soil $\phi_{\text{peak}}$ is a direct input and passive resistance varies with $\phi_{\text{peak}}$ instead of being solely a linear function of embedment. Similarly, the inclusion of soil strength properties in Eq. 2.19 also differentiates it from that suggested by Verley and Sotberg (1994), Eq. 2.1. As embedment increases, Eq. 2.19 does a reasonable job of estimating $H/V$ at very small $V$, particularly for small $\phi_{\text{peak}}$ but underestimates the response increasingly as $w/D$ and $\phi_{\text{peak}}$ increase. Eq. 2.19 also clearly cannot account for the variation in $H/V$ with $V$.

The comparisons with simplified methods suggest that for relatively small values of $V$, limit analysis with a reduced friction angle provides better prediction of the calculated non-associated resistances and captures the variation with $V$. Good comparison is attained for small $V$ load cases because the failure mechanism at these load levels is similar for both the associated and non-associated flow cases, which allows the associated flow approach suggested by Drescher and Detournay (1993) to reasonably capture the kinematics at failure. Comparison between the failure mechanisms is shown on Figure 2.17 along with comparison of failure envelopes for $\phi_{\text{peak}} = 45^\circ$, $\psi = 12.5^\circ$ for $w/D = 0.2$ and 0.8. The calculated bearing capacities are most disparate when the failure mechanisms differ most significantly. This comparison also reveals that the non-associated envelope at negative $V$ is found to often be concave for $w/D > 0.5$, taking a heart shaped form with symmetry about the $V$ axis. For the case of $w/D = 0.8$ on Figure 2.16, the vertical (uplift) bearing capacity component for at least two load vectors ($160^\circ$ and $130^\circ$) is higher than that for purely vertical loading. This response is common across the range of $\phi_{\text{peak}}$ and $\psi < \phi_{\text{peak}}$ considered. This is not necessarily surprising as although associated flow
yield surfaces must conform to a convex shape (Drucker 1953), no such guarantee exists for non-associated flow.

Some insight into the origin of the relatively higher vertical capacities is gained by comparing the vertical component of the soil volume lifted during the initial few uplift load vectors. Figure 2.18 shows the variation in this component calculated by integrating the volume between the pipeline and contours representing the failure planes. For associated flow, the vertical component is maximum for pure uplift (i.e. 0° from vertical) and reduces as the loading angle increases. This reduction for associated flow occurs because the volume of soil lifted for pure uplift is similar to that for cases with small non-zero loading angles, due to the angles that the failure planes extend from the pipeline always being approximately $\phi_{peak}$. For non-associated flow, the vertical component of lifted soil increases over the first two increases in loading angle. From Figure 2.17 this increase corresponds to a larger increase in the volume of soil encompassed by the failure mechanisms relative to the pure uplift case. This mainly occurs because the failure planes extending from the pipeline are much smaller, similar to $\psi$, and hence relatively much less soil is lifted in pure uplift loading for non-associated flow. However, the differences between associated and non-associated flow reduce with increasing loading angle as the mechanisms converge to become more similar.

2.4 Conclusions

This paper describes a series of finite element and limit analysis results describing the effects of non-associated flow, and by inference soil density, on the bearing capacity of shallowly embedded pipelines. The analyses cover a range of soil parameters relevant
for practical application. Due to inherent non-uniqueness in analysis of non-associated materials, these results form only one particular solution to each considered scenario. However, the results compare favourably with other numerical results available in the literature as well as the limited experimental data that exists in the public domain with sufficient soils information to enable reasonable comparison. Therefore, some conclusions can be made from these results towards improving the current state of pipeline engineering practice.

The vertical bearing capacity was found to be strongly affected by non-association and using a reduced friction angle within a limit analysis framework does not appear to provide a satisfactory method to account for this. The increase with depth was found to consistently follow a power law relationship that is approximately linear at small \( \phi_{peak} \) and becomes non-linear (with a power reducing less than unity) with increasing \( \phi_{peak} \). A series of relationships to predict the variation in vertical bearing capacity for given combinations of \( \phi_{cs} \) and \( \psi \) have been provided, which provide good comparison with the experimental results of Sandford (2012).

The overall shape of the combined V-H loading envelopes was found to be similar
2.4 Conclusions

Figure 2.17 Example low $V$ results highlighting uplift component and displacement mechanisms. $\phi_{\text{peak}} = 45^\circ$, $\psi = 12.5^\circ$ for $w/D = 0.2$ and 0.8. Solid lines – non-associated FEA. Dashed lines – associated flow limit analysis with friction angle as per Eq. 2.5.

to that described previously by Zhang et al. (2002) but with the peak horizontal load occurring at a relatively smaller proportion of the maximum vertical bearing capacity. The calculated values of maximum horizontal load were found to generally increase with embedment as a proportion of the maximum vertical bearing capacity. As friction angle increases, the rate of increase in $H_{\text{max}}/V_{\text{max}}$ reduces because the vertical bearing capacity increases at a faster rate with friction angle than the horizontal bearing capacity. A modified version of the envelope suggested by Zhang et al. (2002) was shown to fit to the analysis results well, and a simplified methodology for first order predictions of the overall envelope shape have been provided.

The response at small values of vertical load have been interpreted in terms of the variation in the ratio $H/V$ with $V$. For loading scenarios with a predominantly horizontal load component, the effect of non-association is well predicted by using a reduced friction angle in limit analysis. This is a useful practical finding, given that increasing density results in much larger values of $H/V$ relative to critical state conditions for the same embedment level, because this indicates that relatively simple limit analysis calculations may be used to describe the variation in response for practical scenarios with different, site-specific seabed geometries.
Figure 2.18 Vertical component of the area of soil lifted for first three load vector angles. \( \phi_{\text{peak}} = 45^\circ, \psi = 12.5^\circ \) for \( w/D = 0.2 \) and 0.8. Solid lines – non-associated FEA. Dashed lines – associated flow limit analysis with friction angle as per Eq. 2.5.

Finally, it was also found that the shape of the non-associated flow envelopes for \( w/D > 0.5 \) can be concave due to the differences in the area of soil mobilised during loading at relatively small loading angles relative to vertical.
References

Chapter 2 Drained bearing capacity of shallowly embedded pipelines


Chapter 3

The effect of radial fins on the uplift resistance of buried pipelines

Abstract This paper investigates the potential for increasing the uplift resistance of buried pipelines through the addition of radial fins on the pipe circumference. Experiments conducted in loose sand showed that fins extending by 20% of the pipe diameter increase the vertical peak uplift resistance by up to 25%, depending on embedment depth and fin configuration. A limit equilibrium solution – based on known values of peak friction and dilation angles – predicts the uplift resistance within 13% of the measurements. The trends of peak uplift resistance with embedment and fin configuration were also replicated in numerical analyses conducted using a non-associated Mohr–Coulomb soil model. The numerically predicted peak uplift resistances were within 10 and 21% of the experimental values for rough and smooth interfaces, respectively. Soil failure mechanisms from the numerical analyses were broadly consistent with that assumed in the limit equilibrium solution. However, the experimentally observed mechanisms differed subtly, with a limited extent of lifted soil above the pipe and circulatory flow occurring from above to beneath the pipe. This mechanism was approached in the numerical analyses for a smooth interface by specifying a small negative dilation angle, which had minimal effect on the predicted peak uplift resistance.

This chapter has been published as:
3.1 Introduction

The soil resistance during upward movement of a buried pipeline is a key design consideration for much oil and gas infrastructure and many urban lifelines. Offshore pipelines are often buried for protection. Burial provides constraint against upheaval buckling for pipelines operating at high temperature and pressure. Onshore pipelines in earthquake-prone areas often rely on soil uplift resistance to avoid flotation in liquefaction events. It is therefore necessary to ensure adequate uplift resistance in design, while avoiding an excessive cover (burial) depth.

For sandy soils, previous work has established various prediction methods for uplift resistance, based on the soil unit weight, friction angle and sometimes also using a dilation angle. These methods are based empirically on model tests (Trautmann et al. 1985), or use plasticity limit analysis (Merifield and Sloan 2006) or limit equilibrium solutions (White et al. 2008a). In drained conditions, limit analysis can be unrealistic due to the assumption of associated flow. Drescher and Detournay (1993) suggested modifications to classical limit analysis to estimate limit loads, and Smith (2012) provided example solutions for buried plate anchors. For pipelines, numerical studies have also explored the effect of non-association on uplift resistance (e.g. Vanden Berghe et al. 2005). Prediction methods for peak uplift resistance in sand are generally based on a failure mechanism involving straight slip planes from the pipe to the soil surface, with different assumptions regarding the inclination of these planes and the normal stress on them.

Pipeline burial design typically involves optimisation of pipeline cover depth for given soil and pipeline properties. To avoid expensive deep burial, alternative methods of increasing the uplift resistance are sought.

This experimental and numerical study investigates the potential use of radial fins on a pipeline (where the fins would extend longitudinally along a section of the pipe) to increase the uplift resistance. The study compares bare pipelines with pipelines featuring fins of different sizes and orientations, and also provides detailed insight into the mobilised soil displacement mechanism during uplift. These results inform the efficiency of fins to add uplift resistance, while also providing insight into the validity of the slip plane model used to assess peak pipe uplift resistance.

3.2 Methodology

3.2.1 Experimental Arrangement

The experiments were conducted at the University of Western Australia (UWA) in test containers with internal dimensions of 226 mm by 338 mm by 299 mm. Two of the opposing vertical end walls of the container were manufactured from 25 mm thick Perspex panels. This allowed photography of the exposed soil plane using a five-megapixel resolution machine vision camera controlled by in-house software (Stanier and White 2013). The images were subsequently analysed using Particle Image Velocimetry software (Stanier et al. 2016) to determine the soil displacement patterns. The experimental
3.2 Methodology

(a) General arrangement of test container  (b) close up view of loading apparatus

Figure 3.1 Experimental equipment.

arrangement is shown on Figure 3.1a.

Uplift movement was controlled by an actuator mounted on the test container and controlled via Labview software (De Catania et al. 2010). A loading arm was attached rigidly to the actuator and an S-shaped load cell with a measurement range of 500 N was attached to the base of the loading arm to measure the vertical load using an in-house data acquisition system (Gaudin et al. 2009). A lifting cradle was attached below the load cell, and the actuator axes adjusted such that this cradle was positioned under a lifting rod connected to the buried pipeline, allowing the pipeline to be loaded vertically without introducing moment (Figure 3.1b).

3.2.2 Model pipelines

Five cylindrical model pipelines with an external diameter of 50 mm and a length of 225 mm were fabricated from aluminium for the experiments. The arrangement of radial fins used on four of the pipelines are shown on Figure 3.2 (with dimensions as described on Figure 3.3).

3.2.3 Test materials and model preparation

A commercially produced uniform ‘super fine’ silica sand with a median particle size ($D_{50}$) of $\approx 0.19$ mm was used, with properties given in Table 3.1. All samples were prepared by air pluviation of dry sand to a target relative density of $R_d = 25\% \pm 2\%$.

3.2.4 Experimental program

A total of 18 uplift tests were conducted. A series of 15 tests were conducted with the model pipe oriented with the ends away from the container walls to allow for reliable quantification of the uplift resistance without the influence of sidewall friction. Each pipe configuration was tested at three different pipeline cover ratios ($C/D = 2, 3, 4$),
Pipe 1: No fins  
Pipe 2: E/D=0.1 – centre-up  
Pipe 3: E/D=0.2 – centre-up  
Pipe 4: E/D=0.1 – centre-down  
Pipe 5: E/D=0.2 – centre-down

Figure 3.2 Tested pipeline configurations

as described schematically on Figure 3.3. Further tests were conducted with pipe ends oriented towards the Perspex walls for observation and measurement of soil movement during uplift at a cover ratio, $C/D = 3$.

Smooth end caps were used to cover the pipe ends during the non-PIV tests. The pipe ends were covered with flexible foam during PIV testing, to prevent the ingress of sand between the pipe and the window.

The pipes were displaced vertically at a constant velocity of 0.2 mm/s. Images were captured in PIV tests at a rate of 5 frames per second.

3.2.5 Numerical analysis approach

Parallel numerical simulations of a subset of the model tests were undertaken using version 2016.05.27 of the finite element software OptumG2 (OptumCE 2017). Elastoplastic analyses were undertaken modelling the soil with 6-node Gauss elements as cohesionless non-associated Mohr-Coulomb material with specified constant peak friction angle, dilation angle and soil unit weight. Strength parameters were selected to reproduce the stress and dilation conditions of the experiments. Table 3.2 summarises the relevant material parameters adopted, which include values of peak friction and dilation angles based on Bolton (1986), calculated from the relative density and constant volume friction angle. The pipeline was modelled as a weightless rigid material. The soil domain boundaries
### Table 3.1 Engineering properties of super fine silica sand (Lee 2009)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median particle size</td>
<td>$D_{50}(\text{mm})$</td>
<td>0.19</td>
<td>-</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>$G_s(-)$</td>
<td>2.65</td>
<td>-</td>
</tr>
<tr>
<td>Maximum void ratio</td>
<td>$e_{\text{max}}(-)$</td>
<td>0.747</td>
<td>Corresponds to a dry unit weight of 14.9 kN/m$^3$</td>
</tr>
<tr>
<td>Minimum void ratio</td>
<td>$e_{\text{min}}(-)$</td>
<td>0.449</td>
<td>Corresponds to a dry unit weight of 18.0 kN/m$^3$</td>
</tr>
<tr>
<td>Critical state friction angle</td>
<td>$\phi_{\text{cv}}(\circ)$</td>
<td>31</td>
<td>See White et al. (2008b)</td>
</tr>
</tbody>
</table>

**Figure 3.3** Definition of geometric notation

were 2.5 below and 3.5 diameters to the side of the pipe centre for all analyses. Sensitivity studies showed that this soil domain extent was sufficient to eliminate boundary effects.

### 3.3 Results

#### 3.3.1 Uplift response

The uplift response observed for the tests with the plain pipe and the larger fins, in each orientation, are shown on Fig. 3.4. The following features are observed:

1. The overall response involves an approximately quadratic reduction in uplift resistance with pipe embedment, which reflects the standard uplift model of shear on slip planes with effective stresses that are proportional to depth; as the depth reduces so does the length of, and mean stress on, the slip planes, leading to a quadratic profile.

2. During the initial 10% of a diameter of movement, a small brittle peak in resistance is also evident, suggesting mobilisation of a peak friction angle. Mobilisation distances to peak uplift resistance are in the range $\delta z_{\text{pipe}}/C = 0.5 - 1\%$ for all cover ratios. When expressed as a fraction of the pipe diameter, this corresponds to $\delta z_{\text{pipe}}/D$ increasing from $1 - 2\%$ at $C/D = 2$ to $4 - 8\%$ at $C/D = 4$. These values lie within the range reported in the literature (e.g. as summarised by Ivanovic and Oliphant 2014), and are higher than industry guidelines (DNV 2007) at higher cover ratios.
Table 3.2 Engineering properties of super fine silica sand (Lee 2009)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight</td>
<td>$\gamma'$</td>
<td>kN/m$^3$</td>
<td>15.7</td>
</tr>
<tr>
<td>Peak friction angle</td>
<td>$\phi_{peak}'$</td>
<td>$^\circ$</td>
<td>37.7</td>
</tr>
<tr>
<td>Peak dilation angle</td>
<td>$\psi_o$</td>
<td>$^\circ$</td>
<td>8.3</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_m$</td>
<td>MPa</td>
<td>20</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3.2 continued:

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.2</td>
</tr>
</tbody>
</table>

Figure 3.4 Load displacement response during pipe uplift: (a) overall response; (b) close up of initial response

3. Oscillations in resistance are evident from immediately after the peak is mobilised. This phenomenon has been observed in previous studies, and is linked to intermittent slumping of sand around the pipe periphery into the voids beneath (Trautmann et al. 1985, Dickin 1994, Cheuk et al. 2008, O’Loughlin and Barron 2012).

3.3.2 Comparison of observed and calculated peak uplift resistance

The values of peak uplift resistance ($P_{peak}$) from Fig. 3.4 are summarised in Table 3.3. The uplift factors ($N_\gamma$) in Table 3.3 are calculated as:

$$N_\gamma = \frac{P_{peak}}{\gamma'D(C + \frac{D}{2})}$$

(3.1)
### Table 3.3 Summary of key uplift test results

<table>
<thead>
<tr>
<th>Pipe type</th>
<th>Configuration description</th>
<th>Cover Ratio, C/D (-)</th>
<th>Peak resistance - experiment, ( P_{\text{peak}} (N) )</th>
<th>Uplift resistance factor, ( N_\gamma )</th>
<th>Experimental ( N_\gamma ) normalised by Pipe 1 (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No fins</td>
<td>2 49.2</td>
<td>2.15 2.12/2.07 1.98</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 80.1</td>
<td>2.60 2.65/2.71 2.45</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 124.4</td>
<td>3.12 3.17/3.33 2.88</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( E/D = 0.2 ) centre-up</td>
<td>2 55.6</td>
<td>2.55 2.81/3.09 2.57</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 94.2</td>
<td>3.10 3.35/3.76 3.01</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 152.5</td>
<td>3.89 3.71/4.31 3.43</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( E/D = 0.2 ) centre-down</td>
<td>2 45.9</td>
<td>2.08 2.01/2.13 1.87</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 77.5</td>
<td>2.53 2.50/2.43 2.34</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 125.6</td>
<td>3.19 2.94/3.11 2.78</td>
<td>1.02</td>
<td></td>
</tr>
</tbody>
</table>

(1) Numerical results for rough/smooth interface.

**Figure 3.5** Variation in peak uplift resistance relative to Pipe 1: (a) experimental results; (b) numerical analysis and limit equilibrium results for Pipe 3 and Pipe 5

Fig. 3.5a shows the change in \( N_\gamma \), relative to Pipe 1 caused by the addition of the fins. The centre-up fin arrangement leads to an average increase in \( N_\gamma \) by 13% and 21% for the small (\( E/D = 0.1 \)) and large (\( E/D = 0.2 \)) fins, respectively. In contrast, the centre-down fin arrangement actually leads to an average reduction in uplift resistance, although this is less evident for the larger fins.

These trends have been replicated in two ways: via a limit equilibrium solution for uplift resistance that has been modified to include the fins, and via numerical modelling.

The limit equilibrium solution involves straight shear planes inclined at the angle of dilation to the vertical, following White et al. (2008a). The normal stress on the shear planes is assumed to remain equal to the in situ value given by \( K_0 \) conditions. For the pipes with fins, the shear planes are assumed to originate from the tip of the fins (Fig. 3.6). The peak shear stress mobilised on developed shear planes is:

\[
\tau_{\text{peak}} = \gamma' z \tan(\phi'_{\text{peak}}) \left[ \frac{1 + K_0}{2} - \frac{1 - K_0}{2} \cos(2\psi) \right]
\] (3.2)
Integrating this over the length of assumed shear planes and including the weight of soil required to be lifted during uplift, the total peak uplift resistance per unit length, $P_{\text{peak}}$, is calculated as:

$$P_{\text{peak}} = H D' \gamma' - \left( \frac{D}{2} + E \right)^2 \sin(\theta) \cos(\theta) \gamma' - \left( \frac{\pi}{2} - \theta \right) \frac{D^2}{4} \gamma' + H^2 \tan(\psi) \gamma' + H^2 \gamma' \left[ \tan(\phi_{\text{peak}}) - \tan(\psi) \right]$$

$$\times \left[ 1 + \frac{K_0}{2} - \frac{1 - K_0}{2} \cos(2\psi) \right]$$

where $\theta$, $E$, $H'$ and $D'$ are variables defined by the pipeline geometry as defined on Fig. 3.3. This method is applicable for pipelines with lateral extent of fins ($D'$) greater than the original diameter ($D$).

Normalising this result to get the dimensionless uplift factor, $F_{\text{up}}$, and simplifying, Eq. 3.3 reduces to:

$$\frac{P_{\text{peak}}}{H D'} = N_\gamma = \frac{D'}{D} \frac{\left( \frac{D}{2} + E \right)^2 \sin(\theta) \cos(\theta)}{H D} - \frac{\left( \frac{\pi}{2} - \theta \right) D}{H} + F_{\text{up}} \frac{H^2}{H D}$$

where the uplift factor, $F_{\text{up}}$, is defined as:

$$F_{\text{up}} = \tan(\psi) + \left[ \tan(\phi_{\text{peak}}) - \tan(\psi) \right] \left[ \frac{1 + K_0}{2} - \frac{1 - K_0}{2} \cos(2\psi) \right]$$

This limit equilibrium solution gives good predictions of the change in uplift resistance due to the fins, as shown on Fig. 3.5b (for $Rd = 25\%$). It also provides a simple explanation for the relative performance of the centre-up and centre-down arrangements. The centre-up arrangement triggers longer and deeper shear planes that encompass a higher weight of soil, whereas the centre-down arrangement leads to shorter, shallow shear planes lifting less soil. The numerical results provide a further comparison and are also shown on Fig. 3.5b for the rough interface. These analyses used the same soil strength and dilation parameters as the limit equilibrium solution, and assumed elastic parameters (Table 3.2). The variation in resistance due to the fins is similar to the experimental and limit equilibrium results.

As well as accurately predicting the relative resistance with and without fins, the limit
3.3 Results

Figure 3.7 Peak measured normalised uplift factor: (a) Pipe 1; (b) Pipe 3; (c) Pipe 5

The limit equilibrium solution also gives reasonable absolute estimates of the uplift resistance, when using the parameters given in Table 3.1 and taking a relative density range from 20% to 30% (Fig. 3.7). The numerical results are also shown on Fig. 3.7 for both the smooth interface (open diamonds) and rough interface (open triangles) conditions.

In summary the predictions are generally consistent with the experimental values, including the relative resistance with and without fins in each orientation, apart from the smooth interface numerical results for Pipe 3, which overpredict the experimental results by about 11-21%. The numerical results tend to overpredict the experimental resistance, while the limit equilibrium calculations underpredict the results and generally provide a conservative estimate of the available resistance.
Contours of the soil vertical displacement field at peak resistance normalised by the pipe displacement (i.e. $\delta z_{\text{soil}}/\delta z_{\text{pipe}}$) for Pipes 1, 3 and 5 are shown on Figure 3.8, where the left hand sides of each figure shows OptumG2 numerical results and the right hand side shows PIV results. OptumG2 results on Figure 3.8 were calculated using the dilation angles listed in Table 3.2 and a smooth interface.

The experimentally determined mechanism involves a zone of uplifted soil above the pipe that extends to less than $2D$ above the pipe, with a reduction in soil displacement with height above the pipe and circulation of soil from above to beneath the pipe. The circulation mechanism is most evident for Pipe 1, whereas the fins appear to limit the flow of soil to beneath the pipe for Pipes 3 and 5. A similar mechanism was observed by Bransby and Ireland (2009) in experiments with a plain pipe in loose sand. Whilst the observed mechanism is not identical to that assumed in the inclined slip-line model
(Figure 3.6), it is more consistent with the concept of lifting a block of soil ahead of the pipe than the flow-around mechanism that is generally assumed for clay, and that is also expected for a much more deeply buried pipe in sand.

Overall, the numerical and experimental failure mechanisms are similar, although the volume of soil mobilised above the pipe tends to be overestimated numerically. Figure 3.9 shows that a second set of numerical results obtained using a dilation angle of $-2^\circ$ yields mechanisms that are more consistent with those observed experimentally. This change reduces the calculated resistance for the smooth interface by 14% (Pipe 1), 6% (Pipe 3) and 9% (Pipe 5) relative to the calculations obtained using the parameters in Table 3.2, and varies from the experimental results by -10% (Pipe 1), 15% (Pipe 3) and 13% (Pipe 5). Vanden Berghe et al. (2005) adopted a similar approach to elicit a localised flow-round mechanism in their numerical analyses of pipe uplift.

For the current numerical analyses, a combined vertical lifting and circulation mechanism was only initiated with a fully smooth pipe interface, whereas a rough interface resulted in a purely vertical slip wedge-type failure. Figure 3.7 suggests that the difference between these two interface conditions, and by extension the corresponding mechanisms, is within 14% for all cases.

### 3.4 Conclusions

A series of model tests were conducted in loose silica sand to investigate the effect of radial fins extending by up to 20% of the pipe diameter on the uplift resistance of buried pipelines. The tests showed a reasonable improvement in the purely vertical peak uplift resistance ranging from 8 to 25%, depending on embedment depth and fin length, when oriented in the centre-up arrangement.

The observed trends of changing fin configuration and embedment depth were captured both numerically and using a limit equilibrium solution. Predicted peak uplift resistances using both methods were broadly consistent, with the limit equilibrium solution providing a slightly conservative bias. In this context – and for the pipe cover ratios and soil densities considered here – the limit equilibrium solution is considered an appropriate method to predict the uplift resistance of shallowly embedded pipelines and to quantify how this resistance may be affected by geometry adjustments.
References

Chapter 4

Drained breakout resistance of a pipeline on a mobile seabed

Abstract This paper describes a numerical study investigating the effect of sediment transport and associated changes in the local seabed profile on the drained breakout resistance of subsea pipelines. Limit analyses were conducted assessing the breakout response of a pipeline placed on a cohesionless Mohr-Coulomb material considering different seabed profiles around the pipeline. These profiles were determined from surveys of a pipeline on an erodible seabed. The parametric study shows the relative importance of various parameters describing the seabed profile geometry, including the local pipe embedment and the adjacent slope of the seabed. Significant changes in drained resistance occur due to changes in local pipeline embedment resulting from scour induced pipeline lowering and/or sedimentation. The seabed slope local to the pipeline also has a strong impact. The assumption of a flat seabed can lead to predicted seabed resistance that differs significantly from the actual value, accounting for a more natural seabed profile.

This chapter has been published as:
4.1 Introduction

Seabed pipelines are used to transport materials or provide communication connections between locations for the offshore oil and gas industry, among others. These pipelines are often laid on mobile seabeds, such as sandy soils in relatively shallow waters ranging from the coastline out to the continental margin. In these waters the surficial materials may be actively mobile for some portion of the design life and placement of an obstruction to the natural flow, such as a pipeline, will affect the sediment transport regime. Sedimentation and pipeline scour may occur, resulting in changes to the local seabed profile along the length of the pipeline and over the design life of the pipeline. This phenomenon has been recognized for some time within the pipeline industry (e.g. Palmer 1996, Bruschi et al. 1997). The mechanisms of pipeline scour, for instance the onset of scour below pipelines, are reasonably well understood and a review is provided by Sumer and Fredsøe (2002).

Recently, Leckie et al. (2015) reviewed a series of in service surveys of a pipeline on the North West Shelf of Australia and described in detail the changes in pipeline embedment/spanning as a result of sediment mobility around the pipeline. That work illustrates that changes in pipeline embedment can occur rapidly (on the order of months) along the entire length of pipeline. Importantly, the results of that review confirm that significant changes occur in both local pipeline embedment (ranging from zero to almost one pipeline diameter) and the seabed profile around the pipeline, over the life of the pipeline. These variations are illustrated on Figure 4.1 and Figure 4.2, and form the major motivation for this current work. Throughout this work, ‘local’ embedment \( e_L \) refers to the elevation of soil in contact with the pipeline as compared to ‘far-field’ embedment \( e_F \), which would be defined as the ambient seabed elevation away from influence of the pipeline. ‘Mid-field’ embedment \( e_M \) refers to some intermediate seabed elevation as defined in the next section.

Although significant advances have been made recently in understanding pipeline scour and burial mechanics, how these phenomena affect the resistance to pipeline movement provided by the seabed has not yet been fully quantified. Proper understanding of the resulting changes to pipeline breakout resistance plays a key role in design of pipelines for on-bottom stability and thermal expansion induced buckling management.

For on bottom stability, DNV (2007) recommends the use of a slightly modified version of the force-displacement model presented by Verley and Sotberg (1994). Verley and Sotberg (1994) developed a relationship for breakout resistance based on a database of model pipeline breakout tests. These tests were conducted for pipelines covering a range of normalized far field embedments, \( e_F/D \), ranging from 0.01 to 0.35, where \( e_F \) (+ve down) is the pipeline embedment relative to the ambient seabed level and \( D \) is pipeline outer diameter (see Figure 4.3). However, a shortcoming of these tests is that they only report the far-field (not local) embedments and the range of embedment is limited in comparison to that expected for pipelines that experience scour and sedimentation. For instance, it is evident from the field data presented in Leckie et al. (2015) that \( e_L/D \) can approach unity for a real pipeline on a mobile seabed, while Verley and Sotberg (1994) considered only embedments less than 0.35 and did not consider the effect of speeded...
slope. Zhang et al. (2002) presented a kinematic hardening plasticity macroelement model of drained pipeline breakout resistance. However, their experimental investigations in carbonate sands did not consider seabed profiles resulting from sediment transport, such as large ranges of local embedment and seabed slope.

In summary, there has been limited investigation into seabed resistance for pipelines that have experienced sediment transport. One exception is the study by Bransby et al. (2014) who presented numerical analyses to demonstrate that the seabed profile can have a significant effect on the soil resistance. That investigation was conducted for undrained soil conditions and the present work considers drained seabed resistance. In this paper we also focus on seabed profiles informed directly from field observations.

Based on the foregoing, this paper has two primary aims: (1) to estimate the effect of changes in seabed profile (due to sediment mobility) on the pipeline breakout resistance and (2) to determine the relative influence of the parameters defining the local seabed profile geometry on the breakout resistance. These aims are investigated utilizing profiles from field data of an operating pipeline.

### 4.2 Seabed profiles around pipelines

The seabed profiles adopted for analysis in this paper are based on field data presented by Leckie et al. (2015), which span an 8 year period for a pipeline off the coast of Australia.
This paper focuses on the parts of the pipeline that remain in contact with the seabed. The sections of pipeline in free spans are not discussed herein.

For the current work, representative pipeline/seabed cross sections were established based on remotely operated vehicle (ROV) video and side scan sonar surveys. The seabed profile around the pipeline was essentially flat beyond a distance of about 3 diameters from the pipe centreline. Adjacent to the pipe, there was sometimes local embedment and a sloping seabed extending from the pipe to ~3 diameters away. The local embedment was, however, not always the same on both sides of the pipeline.

Based on review of the pipeline survey data, representative seabed profiles have been analysed. These profiles are described using 3 parameters (or 6 if both sides of the pipeline are considered independently), defined as per Figure 4.3:

1. $e_L$: The local pipeline embedment, i.e. the elevation of soil adjacent to the pipeline relative to the pipe invert ($e_{L,P}$, $e_{L,S}$);

2. $S$: The seabed slope between the soil level adjacent to the pipeline and the mid-field embedment ($e_{M,P}$, $e_{M,S}$) at some distance ($W$) from the pipeline centreline;

3. $W$: Influence zone width, i.e. the distance from pipe centreline to the mid-field embedment point, beyond which the seabed is relatively flat: this is taken to be 3 pipeline diameters in this work but may be different for other pipelines, seabed conditions or metocean environments.

The definitions of these parameters are described schematically on Figure 4.3 with an example seabed profile from the surveys plotted in black behind. These parameters provide reasonable linear approximations of the real profiles. Further details on the development of these profiles are provided in Leckie et al. (2018).

To determine a realistic range of local embedment and seabed slope, two 200 m sections of pipeline have been analysed, denoted as Section 1 and Section 2 in the remainder of this paper. These two sections illustrate the two primary pipeline lowering mechanisms identified for this pipeline as described in Leckie et al. (2015): (i) lowering by sinking in Section 1 and, (ii) lowering by sagging in Section 2. The seabed soils along both sections of the pipeline are relatively uniform, comprising carbonate silty sand to carbonate sandy silt. For Section 1 the soil is relatively siltier ($d_{50} \sim 0.06$ mm) and for Section 2 relatively...
4.2 Seabed profiles around pipelines

For each of the pipeline sections, values of local pipeline embedment $e_L$ and seabed slope $S$ have been recorded at each meter along the pipeline (for the sections of pipeline not in free span). These values have been collected for two survey years, 2002 and 2006, which were 6 months and 4 years after pipe lay, respectively.

The embedment and slope data is plotted on Figure 4.4 and Figure 4.5 for both the port (offshore side) and starboard (onshore side) sides of the pipeline. Figure 4.4 shows the data for pipeline Section 1, and Figure 4.5 shows the data for pipeline Section 2. In 2002 both pipeline sections have similar seabed profiles, with some local sedimentation around the pipeline. However, Section 2 has relatively higher local and mid-field embedment (represented by the embedment at three diameters) than Section 1. Further detailed discussion of the seabed profiles is provided in Leckie et al. (2018).

On each figure, a linear least squares fit to the data is shown. $P_{10}$, $P_{50}$ and $P_{90}$ values of local embedment are plotted with corresponding values of seabed slope based on a linear fit to the data. Additionally, a point corresponding to the independent medians of

Figure 4.4 Pipeline embedment survey results - Section 1 (from Leckie et al. (2018))

Figure 4.5 Pipeline embedment survey results - Section 2 (from Leckie et al. (2018))
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Table 4.1 Specific local embedment values and associated best-fit slope values - port side

<table>
<thead>
<tr>
<th>Pipe sect.</th>
<th>Year</th>
<th>$P_{10}, \epsilon_{L,P}/D$ (m)</th>
<th>$S_P^{(1)}$ (°)</th>
<th>$P_{50}, \epsilon_{L,P}/D$ (m)</th>
<th>$S_P^{(1)}$ (°)</th>
<th>$P_{90}, \epsilon_{L,P}/D$ (m)</th>
<th>$S_P^{(1)}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2002</td>
<td>0.23</td>
<td>-4.50</td>
<td>0.48</td>
<td>-12.4</td>
<td>0.62</td>
<td>-17.0</td>
</tr>
<tr>
<td></td>
<td>2006</td>
<td>0.20</td>
<td>11.4</td>
<td>0.59</td>
<td>-1.10</td>
<td>0.73</td>
<td>-5.7</td>
</tr>
<tr>
<td>2</td>
<td>2002</td>
<td>0.28</td>
<td>2.60</td>
<td>0.60</td>
<td>-8.30</td>
<td>0.71</td>
<td>-12.0</td>
</tr>
<tr>
<td></td>
<td>2006</td>
<td>0.43</td>
<td>-0.70</td>
<td>0.66</td>
<td>-11.1</td>
<td>0.76</td>
<td>-16.0</td>
</tr>
</tbody>
</table>

(1) Best fit value (Figures 4.4, 4.5) for specified $\epsilon_{L,P}(m)$

Table 4.2 Specific local embedment values and associated best-fit slope values - port side

<table>
<thead>
<tr>
<th>Pipe sect.</th>
<th>Year</th>
<th>$P_{10}, \epsilon_{L,S}/D$ (m)</th>
<th>$S_S^{(1)}$ (°)</th>
<th>$P_{50}, \epsilon_{L,S}/D$ (m)</th>
<th>$S_S^{(1)}$ (°)</th>
<th>$P_{90}, \epsilon_{L,S}/D$ (m)</th>
<th>$S_S^{(1)}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2002</td>
<td>0.21</td>
<td>1.10</td>
<td>0.42</td>
<td>-5.40</td>
<td>0.54</td>
<td>-9.20</td>
</tr>
<tr>
<td></td>
<td>2006</td>
<td>0.24</td>
<td>16.3</td>
<td>0.73</td>
<td>3.80</td>
<td>0.87</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>2002</td>
<td>0.30</td>
<td>4.30</td>
<td>0.60</td>
<td>-5.40</td>
<td>0.71</td>
<td>-9.00</td>
</tr>
<tr>
<td></td>
<td>2006</td>
<td>0.49</td>
<td>1.25</td>
<td>0.63</td>
<td>-2.90</td>
<td>0.75</td>
<td>-6.40</td>
</tr>
</tbody>
</table>

(1) Best fit value (Figures 4.4, 4.5) for specified $\epsilon_{L,S}(m)$

the local embedment and seabed slope of each data set is plotted, which may be viewed as an approximate representation of the ‘average’ condition of the pipeline over each 200 m length. Based on this, Section 1 experienced a much larger increase in the local embedment from 2002 to 2006, as compared to Section 2.

From the data set presented on Figures 4.4 and 4.5, representative cross sectional profiles have been derived to represent the range of conditions for each section and year. For each section and year, the local embedment values below which the 10th ($P_{10}$), 50th ($P_{50}$), and 90th ($P_{90}$) percentiles of the data fall have been calculated. For each of these local embedment values the corresponding seabed slope has been calculated based on the least-squares fit to the data having the same local embedment. The derived geometric parameter values are tabulated in Table 4.1 (port side) and Table 4.2 (starboard side).

The resulting seabed cross sectional profiles corresponding to these parameters are illustrated on Figure 4.6 (pipeline Section 1) and Figure 4.7 (pipeline Section 2).

Inspection of the individual pipeline section profiles between 2002 and 2006 also suggests some differences in scour and pipeline lowering behaviour between the sections. Significant increases in local embedment, seabed slope and the volume of soil around the pipeline occur for pipeline Section 1, and the pipeline appears to have undergone significant general lowering between 2002 and 2006. On the other hand, relatively minimal changes in these profiles are apparent for Section 2, although the local embedment does marginally increase. The difference in behaviour between the two sections can be linked to differences in the mechanism of pipeline lowering.

Section 1 has more closely spaced scour initiation points, and this leads to more significant and spatially consistent sinking of the pipeline. Section 2, on the other hand, has more widely spaced initiation points, which results in the formation of longer spans, leading to intermittent pipeline sagging and subsequent burial. This mechanism is
believed to cause less consistent lowering with a reduced final embedment. This above interpretation is consistent with Leckie et al. (2015), who suggest these differences may result from soil property variations between the sections.

In addition to relative changes in local embedment between pipeline sections and over time, both pipeline sections generally have more soil volume and mid field embedment on the starboard side than the port side in 2006. Possible causes of this asymmetry are discussed by Leckie et al. (2018). The effect of this feature on the available soil resistance is shown in the results presented later in this paper.

4.3 Methodology

The analyses were performed using OptumG2, a commercially-available finite element and finite element limit analysis software (OptumCE 2017). Failure envelopes were calculated as the mean of the upper- and lower-bound results from finite element limit analysis assuming associated flow. OptumG2 incorporates adaptive remeshing (Lyamin et al. 2005; Lyamin et al. 2004), which automatically optimizes the size, position and orientation of mesh elements used in each analysis scenario.

The analyses used the parameters given in Table 4.3 for a drained Mohr-Coloumb soil. The pipe was modelled in the first (mesh adaptation) iteration of each analysis as a polygon with a minimum side length of 0.1 m. The adaptive remeshing procedure locally densifies the mesh size in areas where more intense energy dissipation occurred in previous mesh calculation steps. As a result, the areas along the pipeline perimeter where the shearing occurs become modelled as a polygon with progressively smaller side lengths inscribing a circle of 1 m diameter. In all cases, pipe rotation is prevented.

The soil domain was modelled a distance of 5 diameters on either side of the pipe.
### Table 4.3 Parameters adopted for OptumG2 analyses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipeline outer diameter</td>
<td>(D)</td>
<td>1 m</td>
</tr>
<tr>
<td>Soil friction angle</td>
<td>(\phi')</td>
<td>30°</td>
</tr>
<tr>
<td>Interface friction angle</td>
<td>(\delta)</td>
<td>30°</td>
</tr>
<tr>
<td>Soil effective unit weight</td>
<td>(\gamma')</td>
<td>10 kN/m(^3)</td>
</tr>
<tr>
<td>Interface tension</td>
<td>-</td>
<td>none</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>(E)</td>
<td>1000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>(\nu)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(a) Section 1 - 2006 \(P_{10\%\text{L}}\) (based on Fig. 4.6b)  
(b) Section 1 - 2006 \(P_{90\%\text{L}}\) (based on Fig. 4.7b)

**Figure 4.8** Typical adapted mesh and failure mechanism

centreline and 1.5 diameters below the pipeline invert, which was sufficiently remote to prevent interaction with the failure mechanism. 15,000 elements with 4 remeshing iterations on each loading step were adopted, which was sufficient in most cases to achieve a targeted relative discrepancy between the upper and lower bounds of 2%.

The typical analysis setup used in OptumG2 is shown for two example cases on Figure 4.8, which also show converged meshes resulting from the adaptive meshing method with failure mechanisms for two scenarios with a combined \(V : H\) loading vector as explained in the next section.

For all analyses a small initial vertical dead load was applied to the pipeline (2 kN per unit length) and the pipeline was then loaded at various ratios of horizontal and vertical load to failure (corresponding to small pipeline displacements). For each profile analysed, 28 load vector combinations were used to calculate the failure envelope.

The soil properties adopted for these analyses only consider spatially uniform/homogenous soil properties, which are also invariant with time and have the same properties as the in situ seabed. For example, ‘winnowing’ of the soil during erosion and deposition, i.e. the removal of fines from the soil matrix, that can be observed in model testing and field observations is not considered.

#### 4.4 Results

The analyses described are presented in this section as combined horizontal-vertical (\(H-V\)) load failure envelopes calculated for each representative seabed profile. Collapse load combinations are presented in terms of normalized vertical and horizontal loads, which are defined as:

\[
\begin{align*}
\bar{V} &= \frac{V}{\gamma'D^2} \quad (4.1) \\
\bar{H} &= \frac{H}{\gamma'D^2} \quad (4.2)
\end{align*}
\]
4.4 Results

Figure 4.9 Failure envelopes – Section 1. (a) Full envelope; (b) Low V

where $V$ is the vertical force per unit length of pipe, $H$ is the horizontal force per unit length of pipe, $\gamma'$ is the soil effective unit weight and $D$ is the outer diameter of the pipeline.

Calculated failure envelopes are presented on Figure 4.9, for pipeline Section 1, and Figure 4.10, for pipeline Section 2. Also plotted on each figure are ‘close up’ views of the envelopes for normalized vertical loads which are of particular interest in practice (i.e. normalized vertical loads less than 3). On Figure 4.8 horizontal loading to the right (starboard) is positive. On each ‘close up’ figure (with $V<3$) a hypothetical load path is drawn for a pipeline with an on-bottom weight corresponding to $V=1.5$ and a positive horizontal loading trajectory, $V:H$, of 1:1. This diagonal load path represents a condition with equal hydrodynamic lift and drag.

Results for Section 1 presented on Figure 4.9 show significant increases in overall resistance to pipeline movement in 2006, relative to 2002. In particular, for the $P_{50}$ and $P_{90}$ local embedment profiles the horizontal resistance is almost doubled for $V<3$. The local embedment increases much less for the $P_{10}$ case than the other cases; however, in 2006 the pipeline is situated within a scoured trench. Therefore, although the relative increase in horizontal breakout resistance between the surveys is less significant than the $P_{50}$ or $P_{90}$ embedment profiles, some increase is still expected for vertical loads of practical interest.

In contrast to Section 1, the failure envelopes for Section 2 show much lower increases in resistance over time (Figure 4.10), with no significant change between 2002 and 2006. As discussed previously, this section of pipeline saw a smaller overall increase in embedment because the mechanism of pipeline lowering was more representative of sagging than sinking.

In both sections, some asymmetry or rotation of the failure envelopes to the starboard (positive $H$) direction is evident, particularly for the 2006 envelopes. The rotation results from greater embedment on the starboard side of the pipeline.
4.4.1 Effect of changing seabed profile on breakout resistance

To further explore the changes in breakout resistance, this section assesses the horizontal breakout resistance for each seabed profile for an assumed (but realistic) loading path. For this study, the breakout resistance for each envelope is calculated for pipelines with an on-bottom weight of, $V$, of 1.5 undergoing a combined loading at a constant ratio of vertical uplift to horizontal load of -1:1. This path is illustrated on Figure 4.9 and Figure 4.10. For reference the initial normalized vertical load (1.5) might be appropriate, for instance, for a pipeline with about 50% of its length in free spans (assuming the weight of the spanning portions is shed to the portion in contact with the seabed), a pipeline $SG$ of 1.6 and an effective soil unit weight of 6.5 $kN/m^3$.

The corresponding breakout resistance is calculated as the point where the loading vector intersects the failure envelope. The horizontal breakout resistance ($H_{brk}$) is normalized by the corresponding vertical load at failure (i.e. $H_{brk}/V_{brk}$). For simplicity only loading in the positive horizontal (starboard) direction has been considered for this example. The resulting normalized breakout resistances are plotted versus local pipeline (starboard) embedment on Figure 4.11.

Figure 4.11 shows the Section 1 breakout resistance increases significantly for all local embedment percentiles considered. For the $P_{50}$ and $P_{90}$ embedment levels, the previously
4.4 Results

Table 4.4 Adopted parameters for seabed slope study

<table>
<thead>
<tr>
<th>Local embedment, $e_{L,P} = e_{L,S}(m)$</th>
<th>Seabed slope, $S_P = S_S(^o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>-20</td>
</tr>
<tr>
<td>0.75</td>
<td>-20</td>
</tr>
</tbody>
</table>

described increases in local and mid-field embedment lead to commensurate increases in normalized breakout resistance (by multiples of 2.6 and 3.1, respectively). Interestingly, the resistance for the $P_{10}$ local embedment profile increases by a multiple of 1.6, even though local embedment increases only marginally. Inspection of Figure 4.7b suggests this increase arises from the increased seabed slope in 2006 compared to 2002; i.e. the pipe sits within a scoured trench instead of a flat seabed with a similar local embedment.

For comparison, a simple method of estimating the horizontal force for a pipeline moving up a sloping seabed is:

$$H = \tan \phi' + \theta$$

(4.3)

where $\phi'$ is the friction angle of the soil and $\theta$ is the slope angle (defined in the same way as $S_P$ and $S_S$). Following Equation 4.3 and considering the slope angle for the $P_{10}$ profile for Section 1 in 2006, the calculated ratio between the sloping seabed and a flat seabed is 1.7, which is similar to the calculated estimate of 1.6. This result illustrates the effect that changes in the seabed profile can have on breakout resistance even if the local embedment of the pipeline is nominally unchanged.

For Section 2, after the 4 year period, only the embedment and breakout resistance of the $P_{10}$ profile increases substantially. No significant changes in either embedment or breakout resistance were calculated for the $P_{50}$ and $P_{90}$ profiles. However, the average resistance for the section does increase due to reduced range of breakout resistance (i.e. $P_{10}$ increasing).

4.4.2 Effect of local embedment and seabed slope

In the foregoing discussions both the local embedment and seabed slope affect the calculated resistance. However, a primary focus is often given to local embedment when assessing pipe-seabed resistance. To investigate the relative contributions of embedment and seabed slope, a systematic study is presented in this section.

The adopted profile parameters cover the range analyzed for the surveyed pipeline. These parameters are outlined in Table 4.4, and the resulting profiles are illustrated on Figure 4.12 for symmetric seabed profiles (i.e. port = starboard profile).

Assuming, as before, a pipeline with an on-bottom weight of 1.5 and a constant uplift load vector, $V : H = 1:1$, the normalized horizontal breakout resistance has been calculated for each of the profiles. These calculated breakout resistances, normalized by the horizontal resistance for a flat seabed slope with the same local embedment, are presented on Figure 4.13 as a function of seabed slope and on Figure 4.14 in terms of the normalized breakout resistance (or ‘friction factor’) as a function of local embedment with a flat seabed.

The results on Figure 4.13 show a virtually linear trend of increasing resistance ratio.
with increasing slope. This is consistent with the increased soil volume beside the pipeline (for a given local embedment) leading to increasing breakout resistance. For positive slope angles, the calculated ratios are consistently lower than the trend suggested by Equation 4.3. This relatively lower resistance is partially caused by the local pipeline embedment, which provides additional passive resistance not considered by Equation 4.3. As embedment reduces the ratio would converge towards the limiting solution provided by Equation 4.3. Since loading direction also affects resistance in combined $H - V$ loading, some deviations from Equation 4.3, which assumes purely horizontal loading, remain.

Figure 4.14 illustrates the individual effect of local embedment by comparing the normalized breakout resistance with a flat seabed for various local embedments. These analyses show that an increase in local embedment from 0.25 $D$ to 0.5 $D$ increases the normalized resistance by a factor of 2. Furthermore, increasing the local embedment from 0.25 $D$ to 0.75 $D$ increases the resistance by nearly a factor of 4.

In summary, these parametric studies reveal some important conclusions regarding the breakout resistance of pipelines subject to sediment transport. First, modelling a flat seabed corresponding to the measured local embedment may lead to 50% under- and overestimations of the resistance compared to more realistic sloping profiles, for pipelines with a range of local embedment. Secondly, the effect of seabed slope shows similar trends to that predicted by Equation 4.3 but this comparison is not exact due to other factors, such as local pipeline embedment and loading direction. Finally, the local embedment is shown to have an even stronger effect on the resistance than the seabed slope, increasing the resistance by up to factor of 4 in these analyses compared to factors of up to 1.5 for seabed slope. Nevertheless, both local embedment and seabed slope are shown to
4.5 Conclusions

A series of finite element limit analyses have been conducted investigating the effect on the drained breakout resistance of changes in seabed profile around pipelines. The seabed profiles analyzed represent simplifications based on the results of ROV surveys of an operating pipeline offshore Australia. The evolving geometries for two different 200 m long sections of the pipeline have been modelled considering changes occurring over a period of 4 operational years.

The results demonstrate the significant effect that these changes have on pipe breakout resistance. Several key conclusions can be made based on these analyses and observations:

1. Significant increases in estimated local breakout resistance (more than a multiple of two) occurred as pipeline scour and subsequent pipeline lowering occurred over the four years of observation.
2. The particular mechanism of pipeline lowering appears to strongly affect the level of resistance increase:

   a) Pipeline Section 1 comprised finer sediments and scour initiated at relatively even and closely spaced locations, resulting in more even ‘sinking’ of the pipeline. This led to larger changes in seabed profiles and larger increases in breakout resistance between surveys compared to Section 2.

   b) Pipeline Section 2 comprised coarser materials. Scour initiation locations in this section occurred more intermittently and at wider spacings. This behavior caused larger spans and pipeline ‘sagging’, leading to more uneven lowering. The overall magnitude of embedment and resistance increase between surveys was relatively less than Section 1.

3. Portions of pipeline lowered into a scoured trench, but without increased local embedment, show increased breakout resistance compared to pipelines embedded to the same local embedment with a flat seabed. Conversely, pipelines with significant local embedment but lower mid field embedment (e.g. deposition berms) have significantly lower resistance than a similar embedded pipeline with a flat seabed. This implies that:

   a) Assuming a flat seabed around an embedded pipeline may lead to significant error in resistance estimates compared to more realistic sloping seabed profiles.

   b) Standard methods for estimating breakout resistance assuming a flat seabed should be used with caution if sediment transport around the pipeline is likely to occur.

4. Using Equation 4.3 to estimate the effect of seabed slope (for instance for a pipe within a trench) captures the general trend of resistance with increasing seabed slope but can overestimate the effect by more than 50%. This overestimation is due to local pipeline embedment and the influence of loading direction.

5. Both the local embedment and seabed slope strongly affect the breakout resistance. Local embedment is shown to have a relatively larger impact on the estimated breakout resistance.

Certain simplifications and assumptions are made in this work. The seabed profiles are based directly on field observations, but other pipelines may not experience the same profiles. This paper also only focuses on changes in the seabed resistance owing to sediment transport, but the level of hydrodynamic loading will also be changed. Finally, the analyses do not consider the effect of non-association of plastic flow on the failure or plastic potential envelopes, although this does not affect the general conclusions of the work.
References


Chapter 5

Pumping and vortex shedding due to a cylinder oscillating normal to a plane wall

Abstract Two-dimensional direct numerical simulations were performed to explore the hydrodynamics due to a circular cylinder that oscillates normal to a plane, rigid wall for moderate Keulegan-Carpenter ($KC$) numbers at low Reynolds number ($Re$). Similar flow regimes to that observed for an isolated cylinder are identified, except that the temporal irregularity associated with Regime E is suppressed in the presence of a nearby wall. For $KC \lesssim 5.25$ the flow beneath the cylinder resembles a one-dimensional ‘pumping’ flow driven by pressure gradients in phase with cylinder motion. As $KC$ number increases, symmetry about the oscillation axis is broken and the near-wall velocity field becomes asymmetric for much of the cycle due to vortex shedding. Potential flow and theoretical control volume arguments, which imply pumping velocities to be a function of cylinder velocity and minimum distance to the wall, capture the near-wall velocity field at low $KC$ numbers ($\lesssim 5.25$) for parts of the cycle where vortex dynamics do not play a significant role. The inline hydrodynamic force is found to increase when the cylinder is near the wall, and the relative increases for the portions of the cycle when the cylinder is closest to the wall are estimated well by potential flow. The near-wall flow features observed contribute to understanding the mechanisms driving sediment transport beneath near-seabed riser pipelines and the amplification of hydrodynamic forces that affect the fatigue life of risers in the near seabed zone.
5.1 Introduction

Oscillation of a circular cylinder in otherwise still fluid creates flow patterns that vary with the amplitude and frequency of motions. For a cylinder that oscillates normal to a plane wall, the boundary constraints imposed by the wall may influence these patterns. These variations in flow patterns, and the near-wall flow behaviour they produce, are of interest for the behaviour of offshore infrastructure located at the seabed, such as pipelines and risers, where such motions may lead to sediment transport in the seabed beneath these structures. The changes in the flow field created by the wall may also impact the hydrodynamic forces acting upon the cylinder, which plays a role in the structural design of pipeline and riser systems. Thus, both the flow field and forces are important to characterise.

The flow field and hydrodynamics around a sinusoidally oscillating cylinder away from any nearby boundaries (i.e. an 'isolated' cylinder) has been explored extensively by other workers due to its general application to loading of offshore structures. Even without considering the effect of nearby boundaries, this problem displays a broad spectrum of varying physics particularly pertaining to the flow regimes observed and how these flow regimes vary with oscillation amplitude and frequency. Tatsuno and Bearman (1990), for instance, observed a family of flow regimes related to the formation of vortices that depend primarily on two parameters describing cylinder motion. The parameters are the Keulegan-Carpenter ($KC$) number, defined as:

\[ KC = \frac{2\pi A}{D} \]  

where \( A \) is the amplitude of motion and \( D \) is the cylinder diameter and the Reynolds number based on the cylinder diameter and maximum velocity:

\[ Re = \frac{U_m D}{\nu} \]  

where \( U_m \) defines the maximum cylinder velocity during oscillation and \( \nu \) is the fluid kinematic viscosity. A third group, commonly known as \( \beta = Re/KC \), is also often defined with a constant \( \beta \) corresponding to a constant oscillation period.

Tatsuno and Bearman (1990) identified eight flow regimes based on experimental observations of the flow patterns that occur primarily relating to vortex dynamics for \( \beta < 160 \) and \( KC < 15 \). The ranges observed for these regimes are illustrated for \( Re < 200 \) and \( KC < 10 \) on Figure 5.1. The regimes can be interpreted to mainly depend on: (i) the symmetry of vortices; (ii) the three-dimensionality of vortices; (iii) whether vortices shed during oscillation or detach at the end of half cycles; and (iv) the temporal repeatability of vortex patterns with continued oscillation. In general, as \( KC \) number increases, vortices that form were observed to become more asymmetric in strength and more likely to shed during oscillation; and as the \( \beta \) number increases, the vortices were observed to have more three-dimensional characteristics and be more temporally irregular over a number of cycles. Regimes A and $A^*$ represent two-dimensional flow fields that are symmetric about the cylinder oscillation axis; and B is also a symmetric regime but
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Figure 5.1 Flow regimes reproduced from Tatsuno and Bearman (1990) for an isolated cylinder. Circles represent parameter cases in the present study.

with a three-dimensional characteristic defined by the ‘streaked’ flow response described by Honji (1981). The remaining regimes are three-dimensional and asymmetric, with Regimes C, E and G (not shown on Figure 5.1) characterised as having the side on which the dominant vortex forms switching with continued oscillation. For Regime C, this switching occurs with a regular period; whilst for Regimes E and G, the switching is intermittent with an irregular period. Regimes D and F do not exhibit switching behaviour and are differentiated by whether a vortex sheds during each half cycle. In Regime D, the dominant vortex remains attached and forms an oblique vortex street upon detaching after cylinder reversal. For Regime F, the vortex sheds before the end of each half cycle and the vortex street is observed to be broadly parallel to the oscillation axis.

Williamson (1985) also experimentally observed that for $\beta < 220$ the flow patterns are primarily a function of $KC$ number. For $KC < 7$, the vortices formed on the trailing side of the cylinder but only detached upon cylinder reversal. The vortex pairs were observed to be symmetric for $KC < 4$ and asymmetric in strength for $4 < KC < 7$. For $KC > 7$, vortex shedding was observed during each half cycle with a transverse or oblique vortex street occurring for $KC < 15$ (equivalent to Regime G described by Tatsuno and Bearman 1990). For higher $KC$ numbers, increasing numbers of vortex pairs shed during each cycle (increasing by a pair per increase in $KC$ of approximately 8), leading to a family of flow regimes. Williamson (1985) also highlighted the interaction between different flow features observed and the hydrodynamic force characteristics that act on the cylinder. The observed link between vortex dynamics and force behaviour suggests the flow regimes are relatively constant over a large range of $Re$, since, for instance, based on observations by Sarpkaya (1976) and Justesen (1989) the transverse force frequency remains relatively consistent from $O(10^3)$ to $O(10^5)$.

The inline forces on oscillating cylinders have also been experimentally explored by several authors (e.g. Sarpkaya 1976; Obasaju et al. 1988) and have been found to be a strong function of both $KC$ and $Re$. Unlike the flow field regimes and transverse
(lift) force response, the inline forces are very sensitive to $Re$, at least over the range $Re < O(10^6)$. The inline forces are commonly described as an additive function (Morison et al. 1950) comprising a velocity dependent drag term and an acceleration dependent inertia term as:

$$F = \frac{1}{2} \rho DC_DU|U| + \frac{1}{4} \pi \rho D^2 C_M \dot{U}$$  \hspace{1cm} (5.3)

where $F$ is the inline force per unit length, $\rho$ is the fluid density, $D$ is the cylinder diameter, $U$ is the velocity of the cylinder and $C_D$ and $C_M$ are fitting coefficients representing hypothetical drag and inertial terms. The $C_M$ inertial term can also be represented as $C_M = C_m + 1$ where $C_m$ is the so called ‘added mass’ term, which represents the force required to accelerate the fluid in the path of the cylinder. The additional unity term is associated with the Froude-Krylov force, which results from pressure gradients driving the flow past a stationary cylinder. Alternatively, Maull and Milliner (1978), among others, have suggested the use of Blasius’ equation to calculate forces based on the complex potential $W$ in the complex $z = x + iy$ plane of the flow around the cylinder:

$$X - iY = \frac{1}{2} i\rho \oint \left( \frac{dW}{dz} \right)^2 dz - i\rho \frac{\partial}{\partial t} \oint W dz$$  \hspace{1cm} (5.4)

where $X$ and $Y$ are the inline and transverse components of the hydrodynamic force and $\rho$ is the fluid density. The latter approach in principle allows the forces to be theoretically predicted; however, in the vortex shedding regime, the location and strength of vortices must be predicted, which can be challenging. Nevertheless, such an approach works well for sufficiently low $KC$ numbers, where the flow field remains attached with negligible levels of vortex growth and the near-surface boundary layer is laminar (e.g. Bearman et al. 1985).

Much of the previous numerical work for isolated cylinders has focused on the behaviour at relatively low $Re$, where experimental and numerical approaches can both yield reasonable accuracy. For instance, Dütsch et al. (1998), Tong et al. (2015) and others solved the two-dimensional Navier-Stokes equations for $Re \lesssim 200$ and successfully reproduced the flow regimes observed by Tatsuno and Bearman (1990). Tong et al. (2015) noted that three-dimensional regimes in this $Re$ range, except for Regime B, were able to reproduced well using two-dimensional analyses. This finding is supported at higher $Re$ by An et al. (2015) who showed length-averaged forces and qualitative flow field observations from three-dimensional analyses were consistent with two-dimensional results. Similarly, Justesen (1991) conducted two-dimensional analyses for $KC < 26$ and $196 < \beta < 1035$ without a turbulence closure model and achieved reasonable qualitative agreement with the flow visualisation results of Williamson (1985) and good agreement with force measurements, at least for $KC < 3$. However, as the flow becomes more turbulent, the need for turbulence closure models increases. Saghafian et al. (2003) modelled oscillatory flow at $\beta = 1035$ comparing four different turbulence models and attained reasonably good comparison with experimental data in terms of hydrodynamic loads with the non-linear eddy-viscosity model proposed by Craft et al. (1996), although they note that previous work by Celik and Shaffer (1995) using linear $k – \omega$ models
did not provide good match with force measurements for steady flow at relatively high Reynolds number. Saghaian et al. (2003) also presented select flow fields but did not compare these results directly with available flow visualisation experiments.

Experimental results by Sumer et al. (1991) showed that a nearby wall affects the flow regimes and force characteristics of an oscillating cylinder when the cylinder oscillates parallel to the wall. They showed that the transverse vortex street, which was noted by Williamson (1985) to form for $KC > 7$ (i.e. Regime G), moves parallel to the wall when the cylinder oscillates within about $1.7D$ of the wall. Flow visualisation and force measurements also indicated that vortex shedding is suppressed for oscillations occurring at distances less than about $0.1D$ when $KC < 20$. These changes to the flow features occur due to confinement of the flow on one side of the cylinder and illustrate that the proximity to the wall is an important parameter in addition to $Re$ and $KC$ numbers.

The effect of the wall on inline forces has been explored primarily through theoretical means in the limit of either an inviscid, ideal fluid or viscous, Stokes flow at very small $Re$. Yamamoto et al. (1974), Wilde et al. (1995) and others have utilised potential flow theory, which indicates that oscillation normal to a rigid wall increases the theoretical inline force coefficient due to a suction force induced by the potential field. Others, such as Jeffrey and Onishi (1981), have utilised Stokes flow to find similar conclusions. Clarke et al. (2005) and Clarke et al. (2015), for instance, investigated the problem of microcantilevers vibrating near rigid or flexible planar surfaces. Such theoretical approaches provide insight into the expected increase in inline forces but do not allow the effects of vortex dynamics, and how this is changed by the presence of the wall, to be ascertained. However, Sumer et al. (1991) did experimentally show that the inline force increases in magnitude for oscillation parallel to a wall, but it is important to acknowledge that the problem comprises a different geometry.

The present study aims to attain initial insight into how a wall alters the flow regimes and forces at low $Re$ for vertical oscillations by using two-dimensional computational fluid dynamics and comparing with the vast literature available for isolated cylinders. Future work will then look at these effects at higher $Re$ and explore their application to predicting sediment transport behaviour beneath oscillating pipelines and risers. This study therefore focuses on $KC \leq 9$ and $Re = 150$, based on the maximum cylinder velocity, and $h_{min}/D = 0.125$ and 0.5. This parameter space, which has been thoroughly
Chapter 5 Pumping and vortex shedding - numerical analysis

studied in the past for the isolated cylinder, enables insight to be gained into the primary features affecting the physics of near-wall oscillation and how the wall changes the flow regimes for an oscillating cylinder. In this study, two-dimensional direct numerical simulations are used and cylinder motion is assumed to be purely sinusoidal and normal to the wall, where the vertical position of the cylinder centre is defined by:

\[ y(t) = h_{\text{min}} + \frac{D}{2} + A - A \cos \left( \frac{2\pi t}{T} \right) \]

where \( y(t) \) is the position in time, \( T \) is the oscillation period and \( h_{\text{min}} \) is the minimum distance between the cylinder and the wall as defined on Figure 5.2.

5.2 Numerical modelling

The flow field has been modelled by solving the two-dimensional Navier-Stokes equations using a modified Petrov-Galerkin finite element method, as described in detail by Zhao et al. (2009). Computations were conducted for an isolated cylinder and for a cylinder with a plane wall located at a minimum distance of either 0.125\( D \) or 0.5\( D \) from the cylinder. The size of the numerical domain used for the isolated cylinder analyses measured 55\( D \) in both dimensions. For cases with a nearby wall, the mesh extended 30\( D \) in the direction parallel to the wall and 15\( D \) in the direction normal to the wall. Example meshes are shown on Figure 5.3 for both the isolated cylinder and nearby wall cases. Cylinder oscillations were accommodated by locally stretching the mesh in the direction of motion. For the wall case, the mesh was stretched by interpolating nodal positions between two meshes corresponding to the minimum and maximum gap heights. The stretching was computed at each time step as a function of cylinder position based on:

\[ \nabla \cdot (\gamma \nabla S_i) = 0 \]

where \( S_i \) represents nodal displacements in each coordinate direction and \( \gamma \) is a parameter controlling element deformation. The latter was set to the inverse of the element area following Zhao and Cheng (2008). The fluid velocity gradients and turbulence gradients were set to zero in the normal direction on the outflow boundaries. On the cylinder surface and on the wall, a smooth non-slip boundary condition was adopted.

The isolated cylinder simulations were validated by comparing to direct numerical simulation results by Dütsch et al. (1998) and Tong et al. (2015) for \((KC, Re) = (5, 100)\) and \((12, 200)\), respectively. Table 5.1 shows results for \( C_d \) and \( C_m \) fitted by the least squares method, ensemble-averaged over 5 cycles, for three different mesh densities. It can be seen that for forces acting on the isolated cylinder, the medium and dense mesh both provide good comparison with previously published values, and are within 3% for both parameter sets. Based on this, the isolated cylinder results throughout the remainder of this paper are based on the F2 mesh.

For the wall cases, mesh sensitivity was conducted for two meshes with \( h_{\text{min}}/D = 0.125 \) corresponding to approximately the same mesh densities as F2 and F3 for the isolated
5.2 Numerical modelling

(a) Isolated cylinder, F2
(b) Wall case with $h_{\text{min}}/D = 0.125$, B2

Figure 5.3 Example computational meshes.

Table 5.1 Details of computational meshes for the isolated cylinder. Parameter $N_{\text{total}}$ and $N_{\text{circ}}$ denote the total number of elements in the domain and the number of elements along the cylinder circumference.

| Mesh | Domain size   | $N_{\text{total}}$ | $N_{\text{circ}}$ | $U \Delta t/D$ | $KC = 5$ | $KC = 12$

$Re = 100$ | $Re = 200$

|     |       |         |          |              | $C_D$ | $C_M$ | $C_D$ | $C_M$

| F1  | $55D \times 55D$ | 25348   | 96       | 0.001    | 2.05  | 2.24  | 1.84  | 1.86 |
| F2  | $55D \times 55D$ | 38596   | 144      | 0.001    | 2.06  | 2.47  | 1.82  | 1.86 |
| F3  | $55D \times 55D$ | 54388   | 216      | 0.001    | 2.06  | 2.47  | 1.83  | 1.86 |
| D1  | $-\times 55D$   | 98304   | 384      | 0.0014   | 2.09  | 2.45  | -     | -    |
| T1  | $100D \times 100D$ | 186264 | 180      | 0.001    | 2.11  | 2.42  | 1.80  | 1.88 |

cylinder. Computational details are provided in Table 5.2. Sensitivity analyses were conducted for two cases: $KC = 5, 9$ and $Re = 150$. Since it will later be shown that the inline forces with a wall are asymmetric about zero, the root-mean-square ($RMS$) of the inline force coefficient has been used instead of drag and inertia coefficients. For both $KC$ numbers, the $RMS$ force coefficients are within 3% of each other for the two meshes, indicating limited mesh sensitivity. Hence, the relatively coarser mesh, which is approximately consistent in density with the isolated cylinder case, was selected.

5.2.1 Parameter space

The parameter space considered in this study spans $KC = 2 - 9$ for $Re = 150$ and is illustrated on Figure 5.1 along with the flow regime map identified for an isolated cylinder by Tatsuno and Bearman (1990). The $Re$ of 150 has been chosen because it coincides with a large family of flow regimes for the isolated cylinder (namely Regimes A*, A, C, D, E and F) that were shown by Tong et al. (2015) to be successfully modelled using two-dimensional simulations in agreement with experiments. This $Re$ is also lower than that expected to contain Regime B, which Tong et al. (2015) suggested two-dimensional simulations cannot distinguish from A*. Analyses were conducted at all $KC$ numbers for
Table 5.2 Details of computational meshes for \( h_{\text{min}}/D = 0.125 \). Parameter \( N_{\text{total}}, N_{\text{circ}} \) and \( N_{\text{gap}} \) denote the total number of elements in the domain, the number of elements along the cylinder circumference the number of vertical layers modelled in the gap.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Domain size</th>
<th>( N_{\text{total}} )</th>
<th>( N_{\text{circ}} )</th>
<th>( N_{\text{gap}} )</th>
<th>( U\Delta t/D )</th>
<th>( KC = 5 ) ( Re = 150 )</th>
<th>( KC = 9 ) ( Re = 150 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( C_{I,\text{RMS}} )</td>
<td>( C_{I,\text{RMS}} )</td>
</tr>
<tr>
<td>B1</td>
<td>30D \times 15D</td>
<td>70000</td>
<td>144</td>
<td>80</td>
<td>0.001</td>
<td>2.454</td>
<td>2.471</td>
</tr>
<tr>
<td>B2</td>
<td>30D \times 15D</td>
<td>130180</td>
<td>216</td>
<td>112</td>
<td>0.001</td>
<td>2.471</td>
<td>1.695</td>
</tr>
</tbody>
</table>

each \( h_{\text{min}}/D \) considered: 0.125, 0.5 and \( \infty \).

5.3 Vortex dynamics and flow regimes

This section presents the results in terms of the overall flow features and comparison with the flow regimes described for the isolated cylinder. Figure 5.4 shows streaklines and instantaneous vorticity fields for analyses at four \( KC \) numbers, which for the isolated cylinder correspond to Regimes A, D, E and F, respectively. Results are also shown for the two \( h_{\text{min}}/D \).

5.3.1 Isolated cylinder

The flow fields visualised by streaklines for the isolated cylinder cases (\( h_{\text{min}}/D = \infty \) are generally consistent with previously published results (Tatsuno and Bearman 1990; Tong et al. 2015) and reproduce the expected regimes, particularly for Regimes A, D and F, which are temporally regular.

For the isolated cylinder, Regime A (\( KC = 4.5 \)) corresponds to the presence of symmetric, attached trailing vortices. These vortices detach at the end of each half-cycle leading to a symmetric vortex street parallel to the oscillation axis. This flow field corresponds to the ‘basic state’ noted by Elston et al. (2006) and maintains three spatial and spatio-temporal symmetries: symmetry about the oscillation axis (\( K_x \)); spatio-temporal symmetry with mirror reflection of the flow field about the axis perpendicular to oscillation when \( t_2 = t_1 + 0.5T \) (\( H_1 \)); and spatio-temporal symmetry with diagonal reflection of the flow field about the axis perpendicular to oscillation when \( t_2 = t_1 + 0.5T \) (\( H_2 \)).

In both Regime D (\( KC = 6.0 \)) and F (\( KC = 9.0 \)), \( K_x \) symmetry is broken but \( H_1 \) (for Regime D) and \( H_2 \) (for Regime F) symmetries are maintained for the isolated cylinder. The difference between these regimes consists in whether the dominant vortex sheds during each half cycle. For Regime D, the dominant vortex does not shed during the cycle and hence convects diagonally across the cylinder axis into the trailing wake during the subsequent half cycle. At higher \( KC \) numbers in Regime F, the dominant vortex has significant self-inertia from shedding, pairs with the adjacent trailing vorticity and is convected in a direction broadly parallel to the oscillation axis. As a consequence of the shed vortex coupling, the dominant vortex forms on the opposing side during the next half cycle.
5.3 Vortex dynamics and flow regimes

Regimes C (not shown) and E ($KC = 7.5$) for the isolated cylinder represent transitions between Regimes A and D (i.e. $K_x$ and $H_1$ symmetries) and D and F (i.e. $H_1$ and $H_2$ symmetries), respectively, where all three symmetries are broken. For these regimes, the side of the cylinder on which dominant vortex occurs switches over a number of cycles. For Regime C, the switching is periodic; but for Regime E the switching occurs aperiodically, which is consistent with the observations of Tatsuno and Bearman (1990), and the resulting flow field appears chaotic. For the isolated cylinder, periodic switching representing Regime C was identified only for $KC = 5.5$, which is not shown on Figure 5.4.
Figure 5.5 Schematic of flow field dynamics, \( KC = 4.5 \); \( h_{\text{min}}/D = 0.125 \). LHS: Colours represent instantaneous vorticity; solid lines – vortices; dashed lines – pumping or circulation flows. RHS: Colours represent instantaneous horizontal velocity.

5.3.2 Cylinder in proximity to a plane wall

The introduction of a wall into the problem (at \( y/D = 0 \) on Figure 5.4) breaks \( H1 \) and \( H2 \) symmetries (if present for the isolated cylinder) since flow is prevented for \( y < 0 \). However, the streaklines near the wall for \( KC = 4.5 \) maintain \( K_x \) symmetry reflected in the equivalent isolated cylinder regime. The vortex dynamics are illustrated schematically for \( h_{\text{min}}/D = 0.125 \) on Figure 5.5, 5.6, 5.5 and 5.9 for each \( KC \) number shown on Figure 5.4, along with instantaneous vorticity and horizontal velocity fields.

\( KC = 4.5 \)

For low \( KC \) numbers, a similar flow regime to Regime A is observed in the presence of the wall, with the formation of symmetric vortices that detach at the end of each half-cycle. However, unlike the isolated cylinder, the dominant flow feature with a nearby wall is a
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Figure 5.6 Schematic of flow field dynamics, $KC = 6.0$; $h_{\text{min}}/D = 0.125$. LHS: Colours represent instantaneous vorticity; solid lines – vortices; dashed lines – pumping or circulation flows. RHS: Colours represent instantaneous horizontal velocity.

Predominantly horizontal, symmetric flow that occurs in the gap between the cylinder and the wall. This flow resembles a pumping of fluid into and out of the gap beneath the cylinder, and hence is referred to as ‘pumping’ flow throughout the remainder of this paper. The second feature of note for low $KC$ are circulation cells of equal size that form on either side of the cylinder centreline. These cells are an amalgamation of detached vortices which form into circular patterns because they cannot convect vertically away from the cylinder.

$KC = 6.0$

As $KC$ number increases to 6.0, asymmetric vortices form during oscillation, with the dominant vortex during each half cycle forming on the same side of the cylinder at both $t/T = 0$ and $t/T = 0.5$. This vortex behaviour is similar to that observed for Regime D
for the isolated cylinder. Pumping flow is still evident for portions of the cycle but the asymmetry of the trailing vortices also affect the flow field in various ways. First, as the dominant vortex core (A on Figure 5.6b) moves around the cylinder towards the wall, relatively strong recirculation is evident along the inner edge of the vortex. As this flow approaches the wall, it is diverted both ways along the wall. This interaction combined with the coupling of vortices A and C contribute to the formation of near-wall circulation cells that are asymmetric in size. The side on which the larger circulation cell forms and the size of the asymmetry appears to depend on $h_{\text{min}}/D$ (see Figure 5.4). For $h_{\text{min}}/D = 0.5$, the dominant vortex detaches, couples with opposing sign vorticity and begins to move across the cylinder before interacting with the wall (i.e. the initial stages of the oblique vortex street for the isolated cylinder). As a result, the detached vortices tend to move away from the their originating side and the cumulative effect causes larger circulation to occur on the side opposite the dominant vortex. The increased circulation on the opposite side for $h_{\text{min}}/D = 0.5$ also appears to coincide with the direction of the vortex street away from the wall moving more parallel to axis of oscillation. In contrast, when the cylinder is very close to the wall ($h_{\text{min}}/D = 0.125$), the paired dominant vortex reaches the wall earlier during the subsequent half cycle and hence on the same side of the cylinder as it forms during the previous half cycle. The earlier wall interaction, combined with increased pumping at small $h_{\text{min}}/D$, causes circular motion of detached vortices to occur in a more symmetric fashion.

The second feature evident is non-zero horizontal flows in the gap when the cylinder is halted at $t/T = 0, 1$. This flow results from an overall circulation in which fluid moves (on Figure 5.6) from below the cylinder, as it moves towards the wall, around the side of the cylinder opposite the dominant vortex before eventually feeding into the recirculation flow between the trailing vortices and back towards the side of the dominant vortex. The overall circulation patterns provides the impetus for flow near the wall at the end of the cycle that is not directly associated with pumping.

$KC = 7.5$

Compared to the other regimes, the flow field behaviour for $KC$ numbers associated with Regime E for the isolated cylinder differ most significantly in the presence of the wall. In contrast to the isolated cylinder for $KC = 7.5$, switching of the dominant vortex side appears to be significantly suppressed when oscillating in proximity to the wall, and the resulting flow features appear similar to those described for $KC = 6.0$. This behaviour is illustrated on Figure 5.8 in terms of transverse forces for $KC = 7.5$. The chaotic behaviour for isolated cylinder Regime E can be clearly seen from the force time history. The isolated cylinder transverse force frequency spectrum shows a broad, unsmooth spectrum, although with two relatively distinct modulation peaks about two times the oscillation frequency. For $h_{\text{min}}/D = 0.5$, the frequency spectrum is much smoother: one primary peak at two times the oscillation frequency (indicating the dominant vortex side stays on one side) with two secondary modulation peaks on either side of that peak that emerge from secondary period variation of the dominant vortex position (when viewed at the same phase over subsequent cycles). For $h_{\text{min}}/D = 0.125$ and $KC = 7.5$, ...
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Figure 5.7 Schematic of flow field dynamics, $KC = 7.5$; $h_{min}/D = 0.125$. LHS: Colours represent instantaneous vorticity; solid lines – vortices; dashed lines – pumping or circulation flows. RHS: Colours represent instantaneous horizontal velocity.

The transverse force record converges towards a single, primary peak at two times the oscillation frequency.

$KC = 9.0$

As $KC$ number increases further, a regime similar to Regime F is observed in which the side of the dominant vortex at the end of each half cycle forms on the opposite side to the previous half cycle. In this regime, the near-wall behaviour is broadly similar to that described as similar to Regime D ($KC = 6.0$), but the side of the larger circulation region appears to be relatively less affected by the minimum distance to the wall. This insensitivity is probably due to the tendency for the vortex street to form parallel to the oscillation axis, instead of moving across the cylinder upon reversal. In this regime, the dominant vortex at the end of the cycle is more defined than at lower $KC$ and has
already shed. This results in more local interaction of the vortex pairs with the wall when the cylinder is away (E-B on Figure 5.9).

Although there are clear differences in the vortex dynamics between different $KC$ numbers, there remains a spectrum of behaviour both within and between these regimes. For instance, although $KC=7.5$ and 9.0 ($h_{min}/D = 0.125$) are both similar to Regime F due to Vortex B being dominant at $t/T = 0$, 1 and Vortices O-A having already shed on Figures 5.7 and 5.9, the dominant direction of dominant flow around the cylinder depends on the stage of Vortex B development. For $KC = 7.5$, the dominant flow direction around the cylinder is towards Vortex C; whilst for $KC = 9.0$, it is towards Vortex B. In both cases the direction of the overall circulation pattern from beneath to above the cylinder remains anti-clockwise; but since the dominant flow from the vortices around the cylinder is opposite this direction, $KC = 9.0$ does not exhibit the large horizontal gap velocities at $t/T = 0$ evident for $KC = 7.5$.

5.3.3 Transverse force frequency characteristics

Transverse force spectra are shown on Figure 5.10 for each $h_{min}/D$ and $KC > 5$. For the isolated cylinder from Regime C and higher, $K_x$ symmetry is broken and the flow field is asymmetric. This asymmetry results in a non-zero transverse force, which can be
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Figure 5.9 Schematic of flow field dynamics, $KC = 9.0$; $h_{\text{min}}/D = 0.125$. LHS: Colours represent instantaneous vorticity; solid lines – vortices; dashed lines – pumping or circulation flows. RHS: Colours represent instantaneous horizontal velocity.

inferred from peaks in the transverse force spectra. It follows that asymmetry is evident for $KC$ numbers higher than 5.25 for the isolated cylinder and higher than 5.0 for the two wall cases. For $KC$ numbers below these threshold, symmetric flow occurs (i.e. flow regimes similar to A and $A^*$).

These spectra also provide insight into how the flow regimes vary with minimum distance to the wall. For the isolated cylinder, defined peaks can be seen for $5.75 \leq KC \leq 6.0$ and $8.5 \leq KC \leq 9$ (i.e. Regimes D and F), although the dominant frequency transitions from 2 (and 4) to 3 (and 1) times the oscillation frequency as a consequence of changes in the transverse loading direction during the cycle. For $KC$ numbers outside these ranges (but $\geq 5.5$), the frequency spectra are more widely distributed, representing modulation in the force time histories indicative of Regimes C and E. Regime E for the isolated cylinder is noted from $6.5 \leq KC \leq 8.0$, and Regime C for $KC = 5.5$ only. The Regime C response
Figure 5.10 Transverse force spectrum for $Re = 150$ with increasing $KC$. 
5.3 Vortex dynamics and flow regimes

The transverse forces and observed flow field response, Figure 5.12 shows the variation in flow regimes, as a function of $KC$ number.
and $h_{\text{min}}/D$. On this figure, the regimes for $h_{\text{min}}/D = 0.5$ and $0.125$ are noted as the equivalent isolated cylinder regime that they most closely resemble, although they are not technically identical due to the presence of the wall. For instance, from $6.0 \leq KC \leq 8.0$, $h_{\text{min}}/D = 0.5$ is noted as Regime D because the flow field is consistently asymmetric with a constant dominant vortex side even though the location of the vortex varies between cycles (if viewed at the same phase). The isolated cylinder regime boundaries differ somewhat from those previously reported ranges primarily in that the range for Regime D is found to be smaller than suggested previously, with $6.5 \leq KC \leq 8.5$ all being found to be temporally irregular with regards to both the transverse forces and flow field. The threshold between Regimes A∗-like and A-like has not been specifically defined.

### 5.4 Inline hydrodynamic forces

Figure 5.13 shows normalised inline forces ensemble-averaged over 20 cycles for several $KC$ numbers along with potential flow predictions for each wall distance, where the forces are normalised as per:

$$C_I = \frac{F_I}{0.5DV_{\text{in}}^2\rho} \quad (5.7)$$

where $F_I$ is the inline force per unit length. The numerical results are also compared with predictions via potential flow theory following Carpenter (1958). In potential flow, a cylinder oscillating near a wall may be represented as an infinite series of image doublets, given by:

$$w = \sum_{k=0}^{\infty} (w_k + w'_k) = V \sum_{k=0}^{\infty} \beta_k \left( \frac{1}{z - f_k} + \frac{-1}{z - (f - f_k)} \right) \quad (5.8)$$

$$f_k = \frac{b^2}{f - f_{k-1}}; \quad f_0 = 0 \quad (5.9)$$
5.4 Inline hydrodynamic forces

Figure 5.12 Flow regime map for $Re = 150$ for various $h_{min}/D$. Solid circles – analyses in current work. Note that the regimes identified for $h_{min}/D < \infty$ correspond to the isolated cylinder regime that is most similar, but these are not technically equivalent due to the wall.

\[ \beta_k = b^2 \prod_{i=1}^{k} f_i^2; \quad \beta_0 = b^2 \]  \hspace{1cm} (5.10)

where $w = \phi + i\psi$ is the velocity potential, $b$ is the cylinder radius, $f$ is twice the distance between the cylinder centre and the wall (i.e. $2 \left[ h_{min} + b + A - A \cos \left( \frac{2\pi t}{T} \right) \right]$), $z = x' + iy'$ defines the spatial coordinates and $k$ is the image doublet number. The velocities are calculated as:

\[ u = -\frac{\partial \text{Im}(w)}{\partial x'} = -\frac{\partial \psi}{\partial x'} = -U \sum_{k=0}^{\infty} \beta_k \text{Im} \left( \frac{i}{(z - f_k)^2} - i(z - (f - f_k))^2 \right) \]  \hspace{1cm} (5.11)

\[ v = \frac{\partial \text{Re}(w)}{\partial x'} = -\frac{\partial \phi}{\partial x'} = U \sum_{k=0}^{\infty} \beta_k \text{Re} \left( \frac{i}{(z - f_k)^2} - i(z - (f - f_k))^2 \right) \]  \hspace{1cm} (5.12)

where $Re$ and $Im$ are the real and imaginary components of the potential. The force on the cylinder can be calculated by integrating around its circumference following Eq. 5.4.

Both the numerical and potential flow results on Figure 5.13 indicate that the wall increases the inline force when the cylinder is near the wall. This occurs because the beginning of the cycle corresponds to the maximum cylinder acceleration and therefore the maximum time derivative of the potential. In the presence of the wall, this causes a pressure gradient with a minimum or maximum beneath the cylinder centreline to form. The pressure gradient causes inward pumping flow as the cylinder moves away, but the negative relative pressure beneath the cylinder also causes an additional negatively-directed suction force on the cylinder.

From Figure 5.13, the inline force component in phase with the cylinder acceleration near the beginning of each cycle also appears to increase with proximity to the wall regardless of the $KC$ number. In general apart from this initial increase, the force time histories for a given $KC$ number appear to be similar for both $h_{min}/D$. This similarity
suggests that the primary effect of the wall on inline forces is the suction force associated with pumping. Other aspects of the vortex dynamics influenced by the wall appear to have relatively little effect on the force response. The exception to this is for $KC = 7.5$, where the time history for each $h_{\text{min}}/D$ shows a distinct form. This occurs because distinct flow regimes result for the isolated cylinder and each near wall case (see Figures 5.4 and 5.8): the isolated cylinder flow field has irregular switching of the dominant vortex side, and the two wall cases are each at different stages of vortex shedding.

Since the observed increases for both the numerical and potential flow results are similar, a simple method of capturing the force increases is to factor the numerical isolated cylinder result by the ratio of the potential flow force near the wall to the isolated cylinder case. The symbols on Figure 5.13 show estimates using this method, which appears to provide a reasonable correction for most cases, including $KC = 9$. This reasonable agreement is not surprising, since the wall influence is primarily evident in the suction effect when the vortex dynamics are not significantly affected by the wall. Accordingly, the estimates are comparably poorer for $KC = 7.5$ where the flow fields are distinct.

Although the force at the beginning of the cycle increases as $h_{\text{min}}/D$ decreases for all
5.5 Near-wall velocities

From the observations in Section 5.3, the predominant flow mechanism at low $KC$ numbers for oscillation near the wall is a symmetric pumping flow and at larger $KC$ numbers is an asymmetric flow driven by vortex shedding. These flow features interact differently with the wall and therefore have differing effects on near-wall fluid response. In this section, the near-wall velocity response is explored with regard to its quantitative variation with $KC$ number, how changes in the vortex dynamics affect the response and whether the behaviours can be reasonably predicted using theoretical means. This provides insight into how these mechanisms may differently drive sediment transport beneath oscillating pipelines.

At small $KC$ in particular, pumping flow is the dominant mechanism and should be well described by potential flow theory. Potential flow therefore provides a useful analogue to describe the flow field. Comparison between the numerical results and potential flow is shown for $KC = 2$ on Figure 5.15, where the left and right hand sides are the numerical and potential flow fields, respectively. There is good qualitative comparison between the results, especially in the gap.

The pumping flow on Figure 5.15 is approximately one-dimensional (i.e. horizontal) for sufficiently small $h_{min}/D$. This suggests a further theoretical simplification to describe the flow by assuming quasi-steady, purely horizontal flow and taking a control volume in
Figure 5.15 Example flow fields for $KC = 2.0$, $b_{min}/D = 0.125$. Left hand side of each figure is the numerical result. Right hand side of each figure is potential flow theory. Contours represent horizontal velocity: Red contours – positive velocity; blue contours – negative velocity. Contour increments ±0.1, 0.3, 0.5, 0.7, 0.9.

Figure 5.16 Control volume between the cylinder and a rigid wall. The gap, as described on Figure 5.16. The depth-averaged velocity in the gap corresponds to the flow passing through the right hand boundary of the control volume, due to symmetry, and continuity is satisfied if:

$$\frac{\partial \Omega(x,t)}{\partial t} = \overline{u}(x,t) h(x,t) = x \frac{2\pi A}{T} \sin \left( \frac{2\pi t}{T} \right)$$

(5.13)

where $\Omega(x,t)$ represents the volume (per unit length) of the control volume, $\overline{u}(x,t)$ is the depth-averaged horizontal velocity moving across the boundary of the control volume and $h(x,t)$ is the height at the right-hand boundary of the control volume, defined by

$$h(x,t) = h_{min} + A - A \cos \left( \frac{2\pi t}{T} \right) + \frac{D}{2} - \frac{D}{2} \left( 1 - \left( \frac{2x}{D} \right)^2 \right)^{0.5} \text{ for } x \leq \frac{D}{2}$$

(5.14)

Integrating the gap height from 0 to $x$ and taking the derivative with respect to time
5.5 Near-wall velocities

\[ \frac{\partial \Omega(x, t)}{\partial t} = xV_m \sin \left( \frac{2\pi t}{T} \right) \]  

(5.15)

Inserting Eqs. 5.14 and 5.15 into Eq. 5.13 and rearranging gives:

\[ \frac{\Omega(x, t)}{V_m} = \frac{x}{\pi} \sin \left( \frac{2\pi t}{T} \right) \left[ \frac{h_{\text{min}}}{A} + \left( 1 - \cos \left( \frac{2\pi t}{T} \right) \right) + \frac{\pi}{KC} \left[ 1 - \left( 1 - \left( \frac{2x}{D} \right)^2 \right)^{0.5} \right] \right] \]  

(5.16)

for \(-0.5D < x < 0.5 < D\). This result is consistent with the depth-averaged horizontal velocity in the gap from potential flow (Eq. 5.8-5.12). Eqs. 5.13-5.16 are only applicable for low \(KC\) and \(h_{\text{min}}/D\), where the flow remains symmetric and can be approximated as one-dimensional (i.e. primarily horizontal).

Figure 5.17 compares numerical and theoretical estimates of the horizontal velocity at a specific location near the wall for \(KC = 2\). For \(h_{\text{min}}/D = 0.125\) at this \(KC\), there is relatively good agreement between all three results for much of the cycle, particularly for \(t/T \lesssim 0.1\) and \(t/T \gtrsim 0.9\). Agreement between the numerical and theoretical results is worse during other portions of the cycle when the gap flow is more two-dimensional. The control volume result significantly overpredicts the velocity for \(h_{\text{min}}/D = 0.5\), although the potential flow still provides a reasonable estimate. The control volume divergence with increasing \(h_{\text{min}}/D\) is consistent with the assumption of one-dimensional horizontal flow, which becomes less appropriate as the gap increases. One notable difference between the results is the vertical offset evident from the numerical results at around \(t/T = 0.5\). The offset corresponds to outward-directed horizontal flows that arise from the circulation cells that form from the amalgamation of detached vortices. These flows and others.
Figure 5.18 Variation in horizontal velocity over a cycle at $x/D = 0.4$, $y/D = 0.07$, for different $KC$ numbers. Black symbols – numerical result. Black solid line – potential flow theory. Black dashed line – control volume result. Inset labels on subfigures refer to corresponding flow schematics on Figure 5.5 to 5.9.

related to vortex dynamics are not captured by the potential flow or continuity arguments alone since they are viscous effects.

As the $KC$ number increases, vortices increasingly influence the overall flow field and the near wall velocities. The velocity time history over a cycle at $x/D = 0.4$; $y/D = 0.07$ (as before) is shown on Figure 5.18 for $2 \leq KC \leq 9$. On the figure, labels are placed at relevant phases where the velocity characteristics can be readily referenced from the flow features described on Figures 5.5-5.9. Although the theoretical estimates increasingly differ from the numerical results as $KC$ increases, for most cases there are portions of the cycle where the theoretical velocities remain similar to the numerical results. The velocities are most similar just as the cylinder moves away from the wall, which suggests that when $h_{\text{min}}/D$ is small the potential-like pressure gradients induced as the gap width changes in time dominate over viscous effects. Across all cases, potential flow theory generally provides a better estimate of the horizontal velocity than the control volume result, with the control volume giving a higher velocity; this inaccuracy is expected since
5.5 Near-wall velocities

Figure 5.19 Maximum horizontal velocity magnitude at $x/D = 0.4$, $y/D = 0.07$ with $KC$. Solid circles are numerical results. Black symbols – numerical result. Black solid line – potential flow theory. Black dashed line – control volume result.

the control volume is a depth-averaged velocity. The overestimate of the control volume result is particularly evident for larger $h_{min}/D$, since the flow field in that case is generally more two-dimensional. The agreement in the velocity is less favourable for $KC = 6.0$ and 7.5 with $h_{min}/D = 0.5$. This is attributed to the overall circulation that develops in the flow field (Figure 5.7a), which increases the local horizontal velocity above that due solely to continuity. However, the comparison with theory improves for $KC = 9.0$, even though vortex shedding still occurs. This suggests that whether potential flow remains applicable for portions of the cycle depends on the development of vortices and their effect on the overall flow field.

Two other notable features are evident on Figure 5.18. The first is the non-zero horizontal velocity at $t/T = 0$, when the cylinder is halted, which is particularly evident for $KC = 6.0$ and 7.5. Reference to Figure 5.7 suggests that these flows develop due to the overall circulation moving fluid from beneath the cylinder to above it as it moves towards the wall (Figure 5.7a). The subsequent strong flows between trailing vortices on the opposite side of the cylinder (Figure 5.7b) result in fluid flow towards the bed, which is then diverted along the bed into the gap. This residual horizontal velocity is less prominent for $KC = 9.0$ because of the differences in vortex development. The second notable feature is the apparent velocity bias around $t/T = 0.5$ that is particularly evident for $KC > 2$. This bias occurs due to the circulation cells that form near the wall, which can be seen even for $KC = 2$ at $t/T = 0.5$ on Figure 5.15b. For $KC \geq 6.0$ the offset is more variable over $0.25 < t/T < 0.75$ (as compared to being relatively constant for $KC = 4.5$) due to the asymmetric size and temporal movement of the circulation cells.

From Figure 5.18, the maximum near-wall horizontal velocity over a cycle clearly varies with $KC$ number and $h_{min}/D$. Comparison of the maximum horizontal velocity at the same position as Figure 5.18 is shown on Figure 5.19 along with potential flow and control
volume predictions. As a general trend, the maximum near-wall velocity, as a proportion of the maximum cylinder velocity, reduces with KC number. These trends are consistent with the theoretical predictions: potential flow theory generally provides a low estimate of the peak velocity for all KC and both \( h_{\text{min}}/D \). The control volume result as expected provides a much better estimate of the velocity for \( h_{\text{min}}/D = 0.125 \) than \( h_{\text{min}}/D = 0.5 \).

For both \( h_{\text{min}}/D \), the trend in maximum velocity appears to change above \( KC \sim 5.25 \), with relatively larger peak velocities calculated compared to the theoretical trends, at least for \( 5.25 < KC \lesssim 8.5 \). This KC number corresponds approximately to the point where significant asymmetry is evident on Figure 5.11. As a whole, these findings suggest that the initiation of flow asymmetry and the interaction of shed vortices cause the relatively higher near-wall velocities. On the other hand, peak velocities for \( KC \geq 8.5 \) are not significantly higher than predicted by potential theory, and this trend also holds on the opposite side of the cylinder. This suggest that the precise position and development of the vortices, and their resulting effects on the overall circulation, play a significant role in whether the near wall velocity is significantly amplified.

### 5.6 Concluding remarks

#### 5.6.1 Overall findings

Numerical simulations of an oscillating circular cylinder have been presented for \( KC \leq 9 \) and \( Re = 150 \) exploring the effect of a nearby rigid wall on (a) the flow regimes and vortex shedding dynamics, (b) the hydrodynamic forces acting on the cylinder and (c) the near-wall velocity response.

The wall provides an additional boundary constraint to the overall flow field and serves to confine vortex streets that otherwise extend away from the cylinder. The confinement of vortex convection causes the vortex streets to wrap into localised patterns that appear as circulation cells near the wall. For both \( h_{\text{min}}/D = 0.125 \) and 0.5: for \( KC < 5.25 \), the cells are symmetric about the axis of cylinder oscillation; and for \( KC > 5.25 \), the cells are asymmetric in size due to vortex asymmetry. The position of these cells appears to influence the angle of the oblique vortex street away from the wall for \( KC \) numbers where the flow field is similar to isolated cylinder Regime D. This is somewhat analogous to the parallelisation of the transverse street reported by Sumer et al. (1991) for cylinders oscillating parallel to a wall.

The wall appears to regulate and suppress switching of the dominant vortex side for \( KC \) numbers where the isolated cylinder Regime E is expected. The increased temporal consistency of the dominant vortex side appears to reduce with increasing \( h_{\text{min}}/D \) towards the limiting chaotic response of Regime E for the isolated cylinder. There may be increased switching in the dominant vortex side for higher \( Re \), where increased turbulence may increase instabilities in the flow field, or in experiments, since the vortices for Regime E have been experimentally observed in previous work to have a three-dimensional characteristic.

The predominant flow mechanism characterising both the overall and near-wall flow
5.6 Concluding remarks

behaviour is found to depend on the $KC$ number:

1. $KC < 5.25$:
   a) Symmetric near-wall pumping flows in the gap occur in unison with cylinder motion.

2. $KC > 5.25$:
   a) Pumping flow still occurs for most $KC$ numbers but only for portions of the cycle and depends on $h_{min}/D$;
   b) Interaction of trailing vortices with the wall causes larger near-wall horizontal velocities, relative to the equivalent pumping flow predicted from theory;
   c) Asymmetric horizontal flow is evident even when the cylinder is halted at the end of the cycle. The strength of the asymmetric flows depends on the $KC$ number and consequently the position and development of trailing vortices.

The $KC$ numbers where different behavioural regimes occur and when asymmetry initiates may be expected to differ with $Re$. By contrast, potential flow and control volume predictions may more accurately represent pumping flow with increasing $Re$, since the flow field (at least for symmetric flow) would tend to become more potential-like as viscous effects reduce with increasing $Re$. Other aspects of the current findings may vary more significantly at higher $Re$ numbers. In particular, the findings regarding flow similar to Regime F (i.e. vortex shedding during half cycles) are less applicable to higher $Re$, where Regime G is typically apparent for $KC > 7$ for the isolated cylinder. Regime G is significantly more three-dimensional than Regime F and comprises a transverse vortex street, as opposed to the parallel vortex street shown for Regime F. This means that the flow field is likely to involve more switching of the dominant vortex side and be more chaotic than Regime F.

5.6.2 Implications for practical application

Both pumping flow and vortex interaction are expected to play important roles in cylinder-wall interaction processes, such as seabed trenching beneath riser pipelines. The current findings in principle suggest a number of characteristics about trench development and ultimate geometry. First, at low $KC$, the flow field is expected to be symmetric and able to be reasonably modelled by theoretical means (i.e. potential flow and control volume arguments). Hence, the corresponding trench geometry would be expected to be symmetric and for theory to provide a reasonable means of predicting when trenching occurs. Second, at higher $KC$, the flow field was found to be driven by vortex interactions and asymmetric near-wall flows. These imply that vortex interaction, which causes relatively larger near-wall horizontal velocities, may drive sediment transport and that the trench geometry may, at stages, be asymmetric as a result.

The main finding on hydrodynamic forces herein is that the force time history is reasonably well represented by a superposition of the force time history for the isolated cylinder and a correction accounting for the suction force associated with pumping flow,
based on potential flow theory. This suggests that the inline force time history may be estimated in practice by applying a factor to the isolated cylinder force time history (for instance calculated using the Morison equation and empirical relationships) to account for the relative increase near the seabed indicated by potential flow theory.
References


Chapter 6

Observations of pumping and vortex shedding due to a cylinder oscillating normal to a plane wall

Abstract This study describes a series of experiments to investigate the fluid dynamics associated with oscillation of a circular cylinder normal to a plane wall. Flow visualisation experiments and force measurements were conducted with the ratio ($\beta$) of Reynolds number ($Re$) to Keulegan-Carpenter number ($KC$) maintained at a value of 500, with $KC$ varying from 2 to 12, representative of near-seabed motions of pipeline risers during operation. The minimum distance between the cylinder and wall is maintained at 12.5% of the diameter for flow visualisation and 12.5 to 50% of the diameter for force measurement. The flow visualisation experiments reveal three primary flow regimes that depend on $KC$ number. For $KC \leq 5$, the flow field is approximately symmetric about the cylinder centreline and the velocity field between the cylinder and the wall resembles a pumping flow in phase with cylinder motion, which is well predicted by theoretical arguments for most of the cycle. For $5 < KC < 8$, the flow field becomes increasingly asymmetric and there is frequent switching of the dominant vortex side between cycles. For $KC \geq 8$, the flow field is more consistently asymmetric due to vortex shedding, which remains transverse to the oscillation axis. The near-wall velocity beneath the cylinder is increasingly asymmetric and larger in magnitude, relative to theoretical predictions, with $KC$ number. Force measurements indicate that the inline force increases when the cylinder is near the wall due to a suction force caused by pumping flow. Inline force increases are found to reduce as the minimum distance to the wall increases and can be approximated as a superposition of the increase due to potential flow theory with the measured isolated cylinder force. The findings in this study provide insight into the fluid mechanics that may drive trenching beneath pipeline risers and contribute to riser fatigue.
6.1 Introduction

It is well known that the oscillation of a circular cylinder in otherwise still fluid induces flow patterns that vary depending on the amplitude and frequency of motions (e.g. Tatsuno and Bearman 1990). With the addition of a nearby wall, it is expected that these flow patterns can be further modified, depending on the proximity of the wall. Understanding this modified fluid-structure interaction problem is important for a number of practical applications, including modelling the near-seabed behaviour of pipeline risers, which are used to connect offshore floating infrastructure to the seabed, the installation of subsea equipment and the behaviour of free-spanning pipelines undergoing dynamic loading. For example, in the case of risers, Bridge (2005) examined a number of oil and gas developments worldwide and found that significant trenches can develop in the seabed where the risers touch down. Since risers often experience significant operational motions responding to connected vessel movements, the trenching phenomenon is believed to be caused by a combination of fluid flow generated by the riser motion and physical interaction of the riser with the soil. Understanding the near-seabed velocities are a key step in explaining the development of these trenches.

The fluid mechanics of a cylinder moving normal to a rigid wall in otherwise still fluid has been explored previously under the assumption of either inviscid (potential) flow or very low Reynolds number (Stokes) flow. Carpenter (1958), for instance, calculated the potential flow field for a single cylinder moving normal to a wall as an infinite series of image pair doublets. Yamamoto et al. (1974) and Wilde et al. (1995), amongst others, have subsequently used this solution to calculate the forces on pipelines oscillating near the seabed. Jeffrey and Onishi (1981) and Clarke et al. (2005) derived similar solutions for the flow field and forces, assuming Stokes flow. Each of these existing solutions illustrate that the wall can have a significant effect on the flow field and the forces. However, since these solutions are restricted to inviscid or Stokes flow, they provide limited insight into the intermediate Reynolds number flow regimes expected for many subsea applications.

A number of authors, including Williamson (1985) and Tatsuno and Bearman (1990), have experimentally explored the flow field around a sinusoidally oscillating cylinder far from any boundaries (i.e. an ‘isolated’ cylinder) at a range of Reynolds numbers for which boundary layer separation was observed. In these studies, the authors varied the Keulegan-Carpenter ($KC$) number (defined as $KC = 2\pi A/D$, where $A$ is the cylinder motion amplitude and $D$ is the cylinder diameter) and the $\beta$ parameter (defined as the ratio $\beta = Re/KC$, where $Re$ is the Reynolds number defined in terms of the maximum cylinder velocity). Across their experiments, Tatsuno and Bearman (1990) observed a family of different flow regimes, with the existence of each regime being dependent on both $KC$ number and $\beta$ when $\beta \lesssim 160$. In contrast, for larger $\beta$ numbers, the different regimes were observed to depend predominantly on the $KC$ number only. Williamson (1985) explored the same problem of an isolated oscillating cylinder across the parameter space $KC < 60$ and $\beta < 730$. Williamson (1985) found that for $KC < 4$ a pair of approximately symmetric vortices form on the trailing side of the cylinder, which do not shed during motion but detach upon cylinder reversal. Between $4 < KC < 7$, the
vortices become increasingly asymmetric in strength and still do not shed during a half cycle. From $7 < KC < 13$, the oscillation amplitude is sufficient for vortex shedding to occur; and a transverse vortex street forms with two vortices per cycle convecting away approximately normal to the oscillation axis. For $13 \geq KC \geq 15$, the vortex street moves oblique to the oscillation axis because the second vortex forming at the end of each half cycle is sufficiently strong to detach upon reversal. At larger $KC$ numbers, additional vortices shed during each half cycle, leading to a family of additional flow regimes. Over the parameter space $100 < \beta \lesssim 10,000$, Sarpkaya (1976) and Justesen (1989), measuring lift force frequency per cycle, showed that $KC$ number continues as the dominant parameter and the threshold $KC$ numbers between regimes remains relatively constant at least until $Re$ approaches $O(10^5)$. Since the lift force frequency is shown by Williamson (1985) to be related to vortex shedding frequency, the general description of vortex dynamics should also be primarily driven by $KC$ over this range.

The presence of a boundary close to an oscillating cylinder limits the flow normal to the boundary and therefore changes the vortex dynamics around the cylinder. Sumer et al. (1991) experimentally studied a cylinder oscillating parallel to a rigid wall in an otherwise still fluid. They noted that the symmetric, attached vortices that form for $KC < 4$ for an isolated cylinder become increasingly asymmetric as the distance between the cylinder and the wall reduces. For $7 < KC < 15$, they found that the transverse street moves parallel to the wall as the cylinder approaches the wall; and for $KC > 10$, vortex shedding was found to be suppressed when the cylinder is in close proximity to the wall ($< 0.1D$). These findings do not directly transfer to motion normal to the wall, since problem geometry and relevant boundary conditions are different but confirm that the vortex dynamics are modified in the presence of the wall.

Although the previous discussion has focused on theoretical models and experiments, numerical models are increasingly used to investigate flow regimes in detail for a range of scenarios. A number of researchers (e.g. Justesen 1991; Dütsch et al. 1998; Tong et al. 2015) have used computational fluid dynamics techniques to simulate the oscillatory flow field around an isolated cylinder and have shown that the flow field patterns and variations with $KC$ and $\beta$ described by Tatsuno and Bearman (1990) can be well reproduced for small $\beta$. Tom et al. (2018a) simulated oscillation normal to a rigid wall using two-dimensional direct numerical simulations for $Re = 150$. They found that the presence of the wall (located at a minimum distance from the cylinder of less than 0.5 times the cylinder diameter) prevents vortices from convecting vertically away from the cylinder in the direction of the wall. Instead, the vortex street wraps around upon itself, leading to local circulation cells near the wall, which are symmetric at low $KC$ and asymmetric above $KC \sim 5.5$. Flow field asymmetry and interaction of shed vortices with the wall was also found to increase the maximum near-wall velocities above that predicted by potential flow for $KC > 5.5$ and to lead to non-zero horizontal flows near the wall even when the cylinder is halted. For larger $\beta$, numerical analysis is complicated by the need for a suitable turbulence closure model (or onerously large computational requirements). Saghafian et al. (2003), for example, achieved reasonable success using non-linear eddy-viscosity models, but the use of empirical turbulence models does require accurate quantitative
experimental data to calibrate for a particular application.

This paper builds upon the previous experimental and numerical work investigating flow around oscillating cylinders oscillating normal to a rigid wall. The experiments described are limited to $KC < 12$, which is representative of typical motions experienced by submarine risers and pipelines during lay (e.g. Tom et al. 2018b), and to a single $\beta = 500$, on the basis that the vortex dynamics regimes are relatively insensitive to $\beta$ over a large range. Tests are conducted with a single minimum distance between the cylinder and the wall, $h_{min}/D = 0.125$ as defined on Figure 6.1. This distance is representative of parallel testing exploring sediment transport beneath an oscillating cylinder described by Tom et al. (2018b) and was shown by Tom et al. (2018b) to be sufficiently small to elicit sediment motion in sand within the $KC$ number range considered herein. The work is primarily focused on two aspects: (a) whether, for the considered parameter space, the presence of the wall notably changes the vortex shedding regimes (as was found by Sumer et al. 1991); and (b) quantifying how the identified vortex regimes influence the near-wall velocities relevant for sediment transport and whether simplified theoretical considerations can be used to predict these velocities.

6.2 Experimental arrangement

6.2.1 Motion control

Velocity field and force measurement experiments were conducted in a section of tank with a length of 15 m and cross-sectional dimensions of 0.4 m in width by 0.5 m in height. Cylinders were attached vertically via a 3-axis load cell (when used for force measurement tests) to a belt-driven linear actuator, which is attached to the top of the tank. Horizontal oscillations were produced by actuation along the length of the tank during testing. The cylinder motions considered were harmonic such that the vertical position in time is:

$$y(t) = h_{min} + A - A \cos\left(\frac{2\pi t}{T}\right)$$

where $y(t)$ is the distance from wall to the cylinder invert, $A$ is the oscillation amplitude, $T$ is the oscillation period, $h_{min}$ is the minimum distance from the cylinder invert to the wall and $t$ is time. These parameters are described schematically on Figure 6.1, where $D$ is also noted as the cylinder diameter.

Example test motions measured using a spring-loaded string potentiometer sampled at rate of 100 Hz are shown on Figure 6.2. These results have been filtered using a low pass 6th-order Butterworth filter with a cutoff frequency of 10 Hz and are presented as ensemble-averaged results over at least 20 cycles, compared with ideal sinusoidal motion. The recorded motion is not perfectly sinusoidal but improves with increasing motion amplitude ($KC$ number). The effect of these motion variations on the measured flow field and forces are explored in later sections.

A 20 mm thick Perspex plate was positioned across the tank and clamped in place during testing to form a plane, rigid wall. A gap of 5 mm was left between the end of the
6.2 Experimental arrangement

Figure 6.1 Problem definition.

Figure 6.2 Measured experimental motions for \( KC \) 2 to 12. Dashed line – ideal motion. Solid lines – ensemble-averaged measured motions increasing in \( KC \) with colour intensity.

cylinder and the bottom of the flume. The water depth in each test was set at 0.385 m. The test setup for the flow visualisation tests is schematically shown on Figure 6.3.

6.2.2 Velocity field measurements

For velocity field measurements, clear acrylic cylinders of 13, 25 and 40 mm in diameter were used. A 5-Watt continuous wave Argo-ion laser was used for illumination, producing an approximately 1-2 mm thick light sheet. Synthetic polycrystalline particles with median particle diameter of approximate 1 to 5 \( \mu m \) were suspended in the water for tracking. Images were captured using a high speed Photron camera (FASTCAM SA3), with a typical resolution of 768 px by 512 px at a frame rate of 500 frames/s and an exposure time of 1/1000 s.

Particle image velocimetry (PIV) analyses were conducted using the freely available software GeoPIV-RG (Stanier et al. 2016), which incorporates first-order subset deformation shape functions and inverse compositional Gauss-Newton sub-pixel interpolation to examine cross-correlation of image pairs. For tests where specific focus on the flow
between the cylinder and the boundary was assessed, adjacent image pairs (i.e. 1/500 s time difference) were analysed with 32 px by 32 px interrogation patches and 50% overlap. This corresponds to a patch size of about 3.3 mm with the adopted field of field, which is sufficient to describe the overall flow behaviour and velocity characteristics but is not sufficient to, for instance, extract detailed information regarding boundary layers or turbulence. For tests examining the larger flow field features (which generally used a smaller cylinder oscillated at higher frequency to maintain $\beta$ similarity), 48 px by 48 px resolution was generally used.

Some results in the following are presented as ensemble-averaged values or as long exposure images. Ensemble-averaging (also known as phase-averaging) has been calculated by averaging the quantity of interest (e.g. velocity) at a given time in the cycle over the total number of cycles where data is available. In some cases, this averaging was not done over all the available cycles but for a specified set of cycles, which is noted where relevant. For two-dimensional vector fields, the ensemble-average of the field corresponds to the ensemble-average at each spatial position. Long exposure images presented were artificially generated from the high speed images taken for PIV analysis. These images were created by adding the image intensities of 10-20 images, with individual exposure times of 1/1000 s, covering relatively short portions of the oscillation period. The number of images and the time between selected images was varied for different experiments to achieve optimal visual clarity to illustrate the flow features.

### 6.2.3 Force measurements

Hydrodynamic forces were measured using a 3-axis piezo-electric load cell attached to the top of the cylinder. An in house software (De Catania et al. 2010) was used for data measurement and forces were recorded at a rate of 200 Hz. The two load axes
of interest (i.e. parallel to and perpendicular to the direction of motion) were first statically calibrated by applying known loads at different distances from the load cell to attain a distance-calibration relationship for each axis. The resulting calibration relationships were found to be linear with distance from the load cell. These calibration relationships were confirmed by comparison of measured inertial mass of the system during dynamic oscillations of the cylinder in air. During post-processing, it was assumed that hydrodynamic forces act at the mid-depth of the immersed cylinder. If the location of the force resultant were assumed to act at either the full length of the cylinder or at the water surface, the force calibration factors would vary by $\pm 10\%$ from the currently assumed value. Recorded force time histories were filtered in post-processing using a 6th-order Butterworth filter. A low-pass filter was first used to eliminate high-frequency noise, with a cutoff frequency of 4 times the oscillation frequency. A high-pass filter was subsequently used, with a cutoff frequency of half the oscillation frequency, to correct for long-term drift in the load measurements. Force measurements were not explicitly corrected for blockage or end effects.

6.3 Overall flow field

Observations of the overall flow field for $2 \leq KC \leq 12$ suggest three flow regimes for oscillation normal to a wall based primarily on flow field symmetry: (1) approximately symmetric flow maintaining symmetry about the oscillation axis, (2) asymmetric flow with temporally intermittent switching of the dominant vortex side and (3) asymmetric flow with vortex shedding and more temporal consistency in the side of the dominant vortex. Each of these regimes is described in more detail in the following sections.

6.3.1 Approximately symmetric flow - $KC \leq 5$

The vortex dynamics and overall flow patterns typically observed for $KC \leq 5$ are summarised schematically on Figure 6.4. In this regime, the trailing vortices remain attached during each half-cycle and only detach upon cylinder reversal. This behaviour is similar to Regime A described by Tatsuno and Bearman (1990) and by Williamson (1985) for $KC < 4$. The flow field remains generally symmetric about the cylinder axis of oscillation, which means that $K_x$ symmetry, as described by Elston et al. (2006), is maintained. The other symmetries described by Elston et al. (2006), namely $H_1$ and $H_2$, are not maintained since the wall forms a boundary only at one end of the oscillation. Thus, the flow field at $t/T = 0$ is not spatially reflective of $t/T = 0.5$.

When oscillating normal to a plane wall, a few primary flow features emerge due to the wall. First, when the cylinder moves in the vicinity of the wall, a generally symmetric (about the oscillation axis) ‘pumping’ flow is observed in the gap between the cylinder and wall. The gap velocity is primarily horizontal along the wall and responds in phase with the cylinder motion. Second, this pumping flow combines with overall recirculation into trailing vortices to cause large scale circulation patterns to develop. The third prominent flow feature of note arises due to vortex pairs that detach towards the end
Chapter 6 Pumping and vortex shedding - experimental observations

Figure 6.4 Vortex dynamics for approximately symmetric flow ($KC \lesssim 5$). Solid lines represent vortices. Dashed lines represent pumping or overall circulation features.

of each cycle (e.g. Vortices O, P, C and D on Figure 6.4). Since these vortices cannot convect vertically away from the cylinder, they remain near the wall and, combined with recirculation into newly attached vortices (e.g. Vortices A and B at $t/T \sim 0.5$), result in two counter-rotating circulation cells forming on either side of the cylinder (e.g. Figure 6.4, $t/T = 0.5$). These cells induce outward directed flow from the centreline near the wall. Examples of each of these features from long exposure instances can be observed on Figure 6.5, with the vortices and overall dynamics labelled as on Figure 6.4.

The origin of these flow features is further illustrated on Figure 6.6 by comparing ensemble-averaged flow fields with and without a wall located at $y/D = 0$ for $KC = 4$. For the isolated cylinder, there is significant vertical motion away from the cylinder as it moves downwards. By contrast, the wall prevents this movement and instead this motion is diverted horizontally along the wall leading to pumping motions. Although the flow imaging shown on Figure 6.5 indicates that the vortices are not perfectly symmetric on a given cycle at this $\beta$ number, the vortex features are broadly symmetric when ensemble-averaged over a number of cycles (Figure 6.6d).

6.3.2 Intermittently asymmetric flow – $5 < KC < 8$

As $KC$ number increases, the trailing vortices become increasingly asymmetric during each half cycle, but the motion amplitude is not sufficient for vortex shedding to occur. The dominant vortex still detaches upon cylinder reversal and moves past the cylinder and obliquely across the trailing side of the cylinder. For an isolated cylinder, this motion is similar to that described by Tatsuno and Bearman (1990) as Regime D/Regime E and by Williamson (1985) to occur within the range $4 < KC < 7$. $K_x$ symmetry is broken
6.3 Overall flow field

Figure 6.5 Example long exposure images for $KC = 4$. Solid lines represent vortices with cores denoted by dots. Dashed lines represent pumping or overall circulation features.

(a) Isolated cylinder, $t/T = 0.75$

(b) $h_{min}/D = 0.125$, $t/T = 0.75$

(c) Isolated cylinder, $t/T = 0.5$

(d) $h_{min}/D = 0.125$, $t/T = 0.5$

Figure 6.6 Ensemble-averaged flow fields. $KC = 4$. Colours represent horizontal velocity, $u/V_m$.

for the isolated cylinder due to asymmetric vortex formation but $H_1$ mirror symmetry remains (except in Regime E, which is temporally irregular between cycles). However, in the presence of a wall, all spatial and spatio-temporal symmetries are broken.

The observed flow features and vortex dynamics near the wall are illustrated schematically on Figure 6.7. As described, a dominant vortex forms towards the end of each half cycle (e.g. Vortices E and H on Figure 6.7). Over this $KC$ number range, the dominant vortex often appears to de-agglomerate into two companion vortices of the same vorticity sign. For example on Figure 6.7, Vortex E splits into E and F, where F is the vortex closer to the cylinder and E, further away, is labelled the same as the original single vortex. Observations suggest that the second of these vortices (i.e. the one further away from the cylinder) tends convect past the first vortex after cylinder reversal due to the overall circulation that sets up as a result of the general vortex dynamics (see Vortices B and E on Figure 6.7).

In the overall flow field, a series of circulatory flows form around the dominant vortex: fluid feeds into the inner recirculation zone of the dominant vortex coming from the
opposite side of the cylinder, and a cross-flow in the opposite direction corresponds to
the outer edge of the vortex. For this $KC$ number range where vortex shedding does not
occur, the circulation appears to mainly involve local conservation of fluid whereby fluid
in front of the cylinder flows to the trailing side, as opposed to fluid flowing in from the
far field. Due to this flow pattern, relatively symmetric pumping flow is still observed
below the cylinder for portions of the cycle, particularly as the cylinder approaches the
wall.

For $t/T = 0.0$ to $0.1$ as the dominant vortex (with accompanying recirculation flow
on its inner side) moves past the cylinder, vertical flow below $y/D = 0$ is prevented by
the wall. Hence, the recirculation flow subsequently impinges into and is diverted along
the wall. The occurrence of these impingements to the side of the cylinder along with
the presence of asymmetric vortices leads to locally asymmetric flow near the wall (i.e.
non-zero horizontal velocities across the cylinder centreline). The strength and frequency
of the asymmetric flows increases with $KC$ number; but over this $KC$ number range, these
instances are not generally sustained over many cycles. This inter-cycle inconsistency
occurs because the side on which the dominant vortex forms switches frequently in this
$KC$ range, as shown by two examples on Figure 6.8.

Intermittent switching of flow field asymmetry causes smoothing of distinct flow modes
when ensemble-averaged because the flow field is temporally irregular but geometrically
symmetric (about the oscillation axis for instance). Figure 6.9 compares ensemble-
averaged results for 96 cycles with those corresponding to ensemble-averaged results
where the cycles were mode-selected such that the dominant vortex is either on the left
or right sides of the cylinder. Mode-selection was conducted by selecting cycles where the
centreline horizontal velocities measured at $y/D = 0.1$ (for $t/T = 0.1$) was $|u/V_m| > 0.05$.
Using this criteria, approximately 15% of the cycles were left side dominant and 47% were
6.3 Overall flow field

right side dominant (the remaining 38% did not satisfy the specified velocity criterion at \( t/T = 0.1 \)). Whilst these proportions suggest a preference over the total number of cycles for right side dominant vortices (which is borne out in the centre column results on Figure 6.9), they do illustrate the variable nature of the flow field in this \( KC \) range.

The mode-selected results also help illustrate the flow features described on Figure 6.7. In both left side and right side dominant modes, potential-like pumping flow is evident in both directions beneath the cylinder for \( t/T \sim 0.75 \). However, there is also a preference at this phase for slightly stronger pumping flow in the direction opposite the dominant vortex. At \( t/T = 0.5 \), the spatial region where relatively large magnitude flows occur is confined to a small horizontal extent beyond the cylinder (approximately 1\( D \)). This indicates that the vortex motions are generally localised to a relatively small area around the cylinder axis, consistent with a lack of transverse vortex street in this \( KC \) number range.

6.3.3 Asymmetric flow with vortex shedding – \( 8 \leq KC < 12 \)

From \( 8 \leq KC < 12 \), cylinder motion amplitudes are sufficient for a vortex to fully shed during each half cycle. The primary flow features evident with a nearby wall for this \( KC \) number range are schematically described on Figure 6.10. In contrast to previous regimes, a single dominant vortex (e.g. Vortices A on Figure 6.10) forms each half cycle that has sufficient momentum to convect across the front of cylinder upon reversal, instead of across the back. The dominant vortex then detaches and convects away in a direction broadly transverse to oscillation axis (as opposed to obliquely away). Due to the increased strength of vortices and their transverse direction of movement, the overall circulation in this regime comprise significantly more fluid moving from the far field on one side of the cylinder across the oscillation axis to the other opposing side, as compared to previous regimes. This regime is therefore similar to the isolated cylinder Regime G described by Tatsuno and Bearman (1990) and that described by Williamson (1985) for \( 8 \leq KC < 13 \) as comprising a transverse street.

Inspection of the instantaneous flow fields (e.g. Figure 6.11) again reveals that the dominant vortex tends to de-aggregate into separate vortices (e.g. Vortices A to A+C and B to B+C). However, in this case, the de-aggregation tends to be more evident after the vortex has shed following cylinder reversal. For instance on Figure 6.11, Vortex B has not de-aggregated to a significant extent by \( t/T = 0.25 \). This vortex does typically split (e.g.
into Vortices Q and R on Figure 6.11, $t/T = 0.5$) subsequently during the half cycle. The split vortices tend to convect away in different directions. The presence of Vortex R on Figure 6.10 in particular appears to contribute to the tendency for asymmetric near wall flows during the last quarter of the cycle. Asymmetric flow across the cylinder centreline from $0.8 \leq t/T \leq 1.0$ is typically observed over this $KC$ number range, including non-zero horizontal velocities when the cylinder is momentarily halted at $t/T = 0, 1$.

Asymmetric near-wall flows are also typically followed by significant (negative) vertical velocities due to the vortex recirculation impinging on the wall, as described for the intermittent regime. In this regime, the side of the cylinder on which these impingements occur, the direction of asymmetric gap flow and the direction of vortex shedding all appear to be significantly more consistent between cycles than for smaller $KC$ numbers. This consistency is due to the increased background flow across the oscillation axis that results from vortex shedding. The direction of shedding typically remains constant over at least $\mathcal{O}(10)$ cycles. However, switching is still observed to occur occasionally, which is consistent with the observations of Williamson (1985) for the isolated cylinder.

### 6.3.4 Summary of flow regimes

The observed flow regimes and their approximate $KC$ number ranges are qualitatively similar to those identified by Williamson (1985) at a similar $\beta$ number. Therefore, the
6.3 Overall flow field

Figure 6.10 Vortex dynamics for asymmetric flow with vortex shedding ($8 \leq KC < 12$). Solid lines represent vortices with cores denoted by dots. Dashed lines represent pumping or overall circulation features.

Figure 6.11 Example long exposure images for $KC = 10$. Solid lines represent vortices. Dashed lines represent pumping or overall circulation features.

current observations suggest that oscillation normal to a wall does not significantly alter the overall vortex shedding behaviour. Oscillation normal to a wall also does not appear to limit or suppress vortex shedding, at least for $h_{min}/D = 0.125$. Vortex shedding is maintained because cross-axis flows are not limited by a wall being present at one end of the oscillation.

The three identified regimes can also be interpreted as somewhat equivalent to the isolated cylinder flow regimes identified at lower Reynolds number by Tatsuno and Bearman (1990): (1) the symmetric regime similar to Regime A; (2) the intermittently asymmetric regime similar to Regime E (which is intermittently asymmetric for the isolated cylinder); and (3) the predominantly asymmetric regime similar to Regime G. Although these comparisons bear some resemblance, there are some inconsistencies with the comparisons. Tatsuno and Bearman (1990) describe Regime A as comprising symmetric vortex detachment, but only observed for $\beta < 50$. This $\beta$ range is notably (and suggested by the authors to be) in contrast to Williamson (1985). Secondly, the classification chart given by Tatsuno and Bearman (1990) suggests that at least at $\beta = 150$ and $KC = 4$, Regime E was observed. Since Regime E was defined as asymmetric with an
intermittent oblique vortex street, this is also in contrast to the observations of Williamson (1985). The current observations indicate that the flow regime for both the isolated cylinder and wall cases the attached vortices are symmetric in an ensemble-averaged sense, which is consistent with Williamson (1985). However, even at low \( KC \) the vortex pairs were typically not perfectly symmetric on a given cycle. Regime G (i.e. a transverse vortex street) remains descriptive of the vortex shedding dynamics (other than the effect of the wall) observed at higher \( KC \) numbers.

To quantitatively differentiate between regimes that maintain oscillation axis symmetry or not with a nearby wall, flow asymmetry has been assessed by comparing the horizontal velocity beneath the centreline of the cylinder when it is close to the wall (e.g. \( t/T = 0 \)). Figure 6.12a shows the root-mean-square (RMS) value of the centreline velocity at \( t/T = 0 \) for a range of \( KC \) numbers. For purely symmetric flow, the centreline horizontal velocity is zero. The RMS velocity thus represents the typical deviation from symmetry averaged over the total number of cycles. At low \( KC \), there is relatively little asymmetry. For \( KC > 5 \) there is an evident increase in asymmetry. This suggests that the threshold between Regime A-like and Regime E-like flow occurs at a \( KC \) number of approximately 5 to 6.

The qualitative differentiation between Regime E-like and G-like mainly lies in whether vortex shedding occurs. To quantify this transition, the direction of horizontal flow on the centreline at \( y/D = 0.07 \) and \( t/T = 0.0 \) is tabulated for each cycle. Then, the horizontal velocity direction is compared between subsequent cycles. Figure 6.12b shows the percentage of cycles where the horizontal velocity changes sign over subsequent cycles. For \( KC = 6 \), there is frequent switching between cycles. However, for \( KC \geq 8 \), the switching is less frequent. Although there is a lack of definition between 6 and 8, a clear change occurs for \( KC \geq 8 \) in terms of both the switching and the magnitude of asymmetric flows. This \( KC \) number threshold of 8 for the initiation of full vortex shedding is the similar to that qualitatively observed by Williamson (1985).

The foregoing discussion highlights that temporal variation in flow direction commonly occurs over a certain range of \( KC \) numbers, generally \( 5 < KC < 8 \). The relatively frequent switching for \( 5 < KC < 8 \) is in contrast to Tom et al. (2018a) where for \( h_{min}/D = 0.125 \) the side of the dominant vortex was not found to switch between cycles.
(for $Re = 150$). This difference likely arises because, although the wall regulates the flow to some extent, increased turbulence and three-dimensional effects are more influential at $\beta = 500$. These effects were not modelled by Tom et al. (2018a). However, the asymmetry becoming more temporally regular for $KC \geq 8$ is consistent with the findings of Tom et al. (2018a) whereby Regime F-like flow also demonstrated regularity near the wall.

6.4 Near-wall flow

The foregoing flow field observations suggest that two primary flow mechanisms drive the flow response when oscillating normal to a wall: symmetric pumping flow and the influence of vortex dynamics. These features are expected to also characterise the near-wall velocity response. The near-wall response is expected to in turn directly affect the occurrence and geometry of trenching beneath cylinders that oscillate above the seabed offshore. Therefore, useful physical insight into trenching phenomena may be attained by quantitatively exploring how the velocity response varies due to these observed flow features and with $KC$ number.

6.4.1 Potential flow theory and control volume simplifications

Potential flow theory

In the inviscid limit, where flow separation is negligible, Carpenter (1958) represented the potential flow of a cylinder oscillating near a wall as an infinite series of image doublets. The velocity potential can then be defined as:

$$w = \sum_{k=0}^{\infty} (w_k + w'_k) = V \sum_{k=0}^{\infty} \beta_k \left( \frac{1}{z - f_k} + \frac{-1}{z - (f - f_k)} \right)$$  \hspace{1cm} (6.2)

$$f_k = \frac{b^2}{f - f_{k-1}}; \quad f_0 = 0$$  \hspace{1cm} (6.3)

$$\beta_k = \frac{b^2 \prod_{i=1}^{k} f_i^2}{b^{2k}}; \quad \beta_0 = b^2$$  \hspace{1cm} (6.4)

where $w = \phi + i\psi$ is the velocity potential, $f$ is twice the distance between the cylinder centre (i.e. $2 \left[h_{\text{min}} + b + A - A \cos \left(\frac{2\pi t}{T}\right)\right]$), $b$ is the cylinder radius, $z = x + iy$ is the spatial coordinate and $k$ is the image doublet number. The velocity field is then calculated as:

$$u = -\frac{\partial \text{Im}(w)}{\partial x'} = -\frac{\partial \psi}{\partial x'} = -U \sum_{k=0}^{\infty} \beta_k \text{Im} \left( \frac{i}{(z - f_k)^2} - i(z - (f - f_k))^2 \right)$$  \hspace{1cm} (6.5)

$$v = \frac{\partial \text{Re}(w)}{\partial x'} = -\frac{\partial \phi}{\partial x'} = U \sum_{k=0}^{\infty} \beta_k \text{Re} \left( \frac{i}{(z - f_k)^2} - i(z - (f - f_k))^2 \right)$$  \hspace{1cm} (6.6)

where $Re$ and $Im$ are the real and imaginary components.
Figure 6.13 Control volume between a cylinder and a plane wall.

One-dimensional control volume

An even simpler model can be developed by utilising control volume arguments. This further simplification is useful because it clearly demonstrates how various geometrical problem quantities affect pumping flow. Such an approach may also enable, within its limits of applicability, straightforward application to estimate the velocity response for predicting trenching beneath a cylinder.

To derive this model, a control volume is taken (assuming symmetry) comprising the volume between the cylinder and the wall from the centreline outwards, as described on Figure 6.13. The outer control volume height at a given position and time is defined as:

\[ h(x, t) = h_{\text{min}} + A - A \cos \left( \frac{2\pi t}{T} \right) + D - D \left( 1 - \left( \frac{2x}{D} \right)^2 \right)^{0.5} \text{ for } x \leq D \] 

(6.7)

Integrating Eq. 6.7 from 0 to \( x \) and then taking the derivative with respect to time to attain the volume rate of change with time:

\[ \frac{\partial \Omega(x, t)}{\partial t} = xV_m \sin \left( \frac{2\pi t}{T} \right) \] 

(6.8)

where \( \Omega(x, t) \) is the control volume (per unit length) and \( V_m \) is the maximum cylinder velocity. To satisfy continuity it follows that:

\[ \frac{\partial \Omega(x, t)}{\partial t} = xV_m \sin \left( \frac{2\pi t}{T} \right) = h(x, t)\bar{u}(x, t) \] 

(6.9)

Substituting in Eq. 6.7 and rearranging:

\[ \frac{\bar{u}(x, t)}{V_m} = \frac{\frac{x}{A} \sin \left( \frac{2\pi t}{T} \right)}{h_{\text{min}} \frac{h_{\text{min}}}{A} + \left( 1 - \cos \left( \frac{2\pi t}{T} \right) \right) + \frac{\pi}{Kc} \left( 1 - \left( \frac{2x}{D} \right)^2 \right)^{0.5}} \] 

(6.10)

for \(-0.5D < x < 0.5D\) where \( \bar{u} \) is the horizontal velocity averaged over the height \( h(x, t) \)
bounded by the cylinder and the boundary:

$$\n(x, t) = \frac{1}{h(x, t)} \int_0^{h(x, t)} u(x, y, t) dy$$

(6.11)

This result is consistent with the depth-averaged horizontal velocity in the gap from potential flow (Eq. 6.2 to 6.6). The control volume velocity derivation is applicable in the limits of small $KC$ and $h_{min}/D$, where the flow remains symmetric and can be approximated as one-dimensional (i.e. primarily horizontal).

From this derivation, the horizontal pumping velocity (as a proportion of the maximum cylinder velocity) is expected to decrease as $KC$ increases, for a constant gap, and to decrease as $h_{min}/A$ increases, all else constant. This formulation also indicates a local maximum occurs during each half cycle, which implies a corresponding maximum velocity in space and time during each half cycle.

### 6.4.2 Near-wall horizontal velocity results

In this section, the near-wall velocity is considered in detail to ascertain: (a) the prevalence of pumping flow, (b) the influence of vortices on the near-wall flow. In both cases, in order to apply these findings to seabed trenching, the results are interrogated as to whether and over what $KC$ number range simple models can explain the key physics.

Figure 6.14 summarises horizontal velocity time series, as measured by PIV analysis over at least 70 cycles, for two points near the wall representative of the behaviour in the gap beneath the cylinder: $x/D = 0$, $y/D = 0.07$ and $x/D = 0.4$, $y/D = 0.07$. The results are presented as ensemble-averages for all available cycles with bounds showing ±1 standard deviation, as well as results corresponding to cycles mode-selected for left side dominant vortices. For $x/D = 0.4$, the potential flow and control volume theoretical predictions are also shown (i) with perfectly sinusoidal motion and (ii) with the motion inferred from the experiments by tracking the cylinder position from the images. The latter prediction uses the ensemble-averaged motion inferred from the cylinder position in the images for each set of tests.

**Pumping flow**

The response at the lateral position for all $KC$ numbers generally follows a pattern of negative (inward-directed) velocity after the cylinder begins to move away from the wall and positive (outward-directed) velocity as the cylinder approaches the wall at the end of the cycle. These general trends are indicative of pumping flow. For $KC \leq 6$, the theoretical predictions capture this response reasonably well, although the control volume result is typically higher and tends to underestimate the velocity. For $KC \geq 8$, the measured velocity magnitudes differ from the theoretical predictions for portions of the cycle when the cylinder is near the wall. However, the measured and theoretical velocity time series still follow a similar general pattern. Even though the measured horizontal velocity is significantly larger than theory for $KC \geq 8$, the potential flow prediction incorporating the inferred cylinder motion tracks the variation in the irregular response
Figure 6.14 Horizontal velocity time histories with increasing KC number. Left column - $x/D = 0.0$. Right column - $x/D = 0.4$. Solid blue lines – ensemble-averaged result. Dashed blue lines – mean ±1 standard deviation. Solid green lines – $KC = 8$, mode-selected for left side dominant vortex. Solid black lines – sinusoidal motion, potential flow. Solid red lines – ensemble-averaged cylinder motion, potential flow. Dashed black lines – sinusoidal motion, control volume.

surprisingly well (but not the magnitude) when the cylinder is near the wall (with the notable exception of $KC = 2$ for which it was difficult to track the cylinder motion accurately). This suggests that pumping remains a dominant driver of the near-wall response throughout this $KC$ number range, at least for portions of the cycle, and that the irregular motion does not significantly affect the pumping physics observed.

Figure 6.15 shows measurements of the horizontal velocity magnitude at $x/D = 0.4$;
6.4 Near-wall flow

Figure 6.15 Maximum horizontal velocity magnitude with $KC$ number. $x/D = 0.4$. Solid lines – potential flow. Dashed lines – control volume simplification. (a) Solid black circles – experimental results mode-selected for left side dominant vortex. (b) Black line – measured ensemble-averaged values. Red line – filtered ensemble-averaged values. Vertical tick mark spacing $- u/V_m = 0.5$ (c) Grey – $y/D = 0.07$. Black – $y/D = 0.14$. Blue – $y/D = 0.25$.

$y/D = 0.07$ for different $KC$ numbers, where the velocity time histories were smoothed by applying a 6th-order low-pass Butterworth filter with a cutoff frequency of 6 times the oscillation frequency (Figure 6.15b for $KC = 2, 6, 10$). Figure 6.15a indicates that from $2 \leq KC \leq 5$ there is a trend of decreasing velocities relative to $V_m$ with increasing $KC$ number, consistent with theory. However for $KC \geq 6$, the measured horizontal velocities are typically high relative to theory, due to the increasing influence of vortices on the near-wall flow. Similar trends are evident regardless of the height above the wall where measurements are taken, as shown on Figure 6.15c. However, both the experimental and
potential flow results shift towards the control volume prediction as measurement height increases.

Both the time series and maximum velocity results indicate that the control volume approach typically overpredicts the measured velocity. This is due to differences between depth-averaged (control volume) and height-varying (potential flow) velocity estimates. However, the control volume estimate remains useful in observing the effect of non-dimensional parameters on the peak horizontal velocity. For instance, Eq. 6.10 suggests that the velocity is inversely proportional to $h_{\text{min}}$ and reduces with $KC$ number. These observations are illustrated through predictions on Figure 6.16a for different $h_{\text{min}}/D$.

Figure 6.16b shows that the relative differences between the two theoretical prediction methods increases with $h_{\text{min}}/D$ and that $KC$ number has a relatively smaller effect. As $h_{\text{min}}/D$ increases, the predictions increasingly diverge due to the flow becoming more two-dimensional. At large $h_{\text{min}}/D$, the horizontal velocity near the wall approaches zero; while the depth-averaged control volume velocity always includes some horizontal component near the cylinder.

**Influence of vortices**

Although pumping flow plays a role throughout this $KC$ number range, vortex interactions also influence the near-wall velocities even at small $KC$ numbers. For instance at $KC \leq 6$, the velocity at the lateral position on Figure 6.14 indicates a bias directed away from the centreline when the cylinder is away from the wall ($t/T \sim 0.4$ to 0.6). This bias is the quantitative expression of the circulation cells described in Section 6.3 and also appear with the opposite sign on the other side of the cylinder. These features are not captured by theory since they arise from viscous effects due to vortex detachment. Relatively small centreline bias at small $KC$ number is consistent with the circulation cells being approximately symmetric.
As $KC$ number increases, vortex dynamics increasingly dominate the response; and
the velocity bias is strongly evident at the centreline deriving from the effect of larger
shed vortices (e.g. Vortices Q and R on Figure 6.10). However, the ensemble-averaged
results (for all cycles) for $KC = 6$ and 8 suggest an approximately zero centreline velocity
throughout the cycle. This appears to be in contrast to Figure 6.12a. On the contrary,
inspection of the mode-selected results (Figure 6.9) reveals that the flow field is typically
asymmetric on individual cycles. The flow field only becomes symmetric when averaged
over many cycles due to fluctuation in the dominant vortex side.

The fluctuating asymmetric behaviours for $KC = 6$ and 8 are distinct from each other.
The variation in velocity for $KC = 6$ occurs due to frequent inter-cycle switching in
dominant vortex side (Figure 6.12a). For $KC = 8$, the flow field is typically asymmetric
to the same direction over the course a continuous test (i.e. $\sim 14$ cycles per test set)
but the asymmetry direction sometimes varied when the test was halted to download
PIV images. The latter behaviour suggests that the flow field for $KC = 8$ tends to be
consistently asymmetric but that the side on which vortex formation occurs depends
on the initial test conditions. Hence, Figure 6.12b shows switching between consecutive
cycles to be relatively rare for $KC = 8$.

By contrast, $KC = 10$ and 12 both indicate vortex formation on a consistent side
throughout all tests. For $KC = 10$ and 12, the (all cycles) ensemble-averaged results on
Figure 6.14 generally show a consistent directional bias coincident with the mode-selected
results. For $KC \geq 8$, the results also show a notable non-zero horizontal velocity at
t/T = 0, 1 that arises from the general vortex and circulation dynamics (Figure 6.10).

The near wall response for $KC \geq 8$ also indicates that large centreline velocities are
common from $0.2 < t/T < 0.6$. These instances correspond to flow forced across the
centreline following impingement of vortices/recirculation flow due to shed vortices. This
response, which is conceptually consistent with vortex dynamics described on Figure 6.10,
is quantitatively evident through probability distributions of the position and phase of
maximum (negative) vertical velocity instances for $KC = 10$ and 12, shown on Figure
6.17. The distributions indicate that the maximum vertical velocity towards the wall
typically occurred in these tests on the left side of the cylinder and for $t/T < 0.4$. Hence,
relatively high centreline velocities directed to the right of the cylinder (at least for
$0.4 < t/T < 0.6$) are coincident with and preceded by strong vertical velocities occurring
on the left of the cylinder.

### 6.5 Inline forces

Example measured inline hydrodynamic force time series are shown on Figure 6.18
each ensemble-averaged over at least 20 cycles for a range of $h_{min}/D$. Inline forces are
normalised as per:

$$C_I = \frac{F_I}{0.5 D V_m^2 \rho}$$  \hspace{1cm} (6.12)

where $F_I$ is the measured inline force per unit length. Also plotted on Figure 6.18 are
predictions based on potential flow theory where the hydrodynamic forces are calculated
Chapter 6 Pumping and vortex shedding - experimental observations

Figure 6.17 Probability distributions of the horizontal location and phase of the maximum (negative) vertical velocity instances at $y/D = 0.07$.

by integrating around the circumference following Blasius theorem:

$$X - iY = \frac{1}{2} i \rho \oint \left( \frac{dW}{dz} \right)^2 dz - i \rho \frac{\partial}{\partial t} \oint W d\tau$$  \hspace{1cm} (6.13)

where $X$ and $Y$ are the inline and transverse components of the hydrodynamic force and the potential is calculated via Eq. 6.2 to 6.4.

For the isolated cylinder at relatively low $KC$ numbers, the peak inline force (in the first half of the cycle) generally occurs towards the beginning of the cycle (i.e. $t/T \sim 0.1$). This indicates an inertia-driven force more in phase with the cylinder acceleration. As the $KC$ number increases, the phase of maximum inline force shifts to more closely align with the phase of maximum cylinder velocity, due to increased effect of vortices on the flow field.

The influence of the wall on the time series is primarily evident towards the beginning of the cycle over this $KC$ number range. For all $h_{\text{min}}/D$ tested, the wall generally increases the inline force for $t/T < 0.2$ as compared to the isolated cylinder measurements. Since force increases due to the wall are primarily confined when the cylinder is close to the wall, the increases are suggested to be due to the flow constraint that leads to pumping flow as opposed to vortex effects. Comparison with the phases where potential flow also indicates force increases due to the wall further confirm this observation. Both the measurements and potential flow predictions indicate that the wall influence reduces with increasing $h_{\text{min}}/D$, broadly approaching the limit of the isolated cylinder. Together, these observations, along with flow comparisons in Section 6.4, suggest that a reasonable analogy for the near-wall force increase may be drawn with potential flow even at larger $KC$ numbers.
6.5 Inline forces

Figure 6.18 Example inline force coefficients for various $h_{\text{min}}/D$. Black - $h_{\text{min}}/D = \infty$. Green - $h_{\text{min}}/D = 0.5$. Red - $h_{\text{min}}/D = 0.25$. Blue - $h_{\text{min}}/D = 0.125$. Solid lines - numerical results. Dashed lines - potential flow. Open circles calculated by factoring the numerical isolated cylinder result by the ratio of potential flow force at various $h_{\text{min}}/D$ to the potential flow force for the isolated cylinder.

A simple method for predicting the inline force increase due to the wall at a given $h_{\text{min}}/D$ is to factor the measured isolated cylinder time series by the relative increase predicted by potential flow:

$$C_{I,\text{corr}} = C_{I,\infty} \frac{C_{I,P-h_{\text{min}}/D}}{C_{I,P-\infty}} \quad (6.14)$$

where $C_{I,\text{corr}}$ is the corrected force time series for a given $h_{\text{min}}/D$, $C_{I,\infty}$ is the measured isolated cylinder force time series, $C_{I,P-h_{\text{min}}/D}$ is the potential flow prediction at a given $h_{\text{min}}/D$ and $C_{I,P-\infty}$ is the potential flow prediction for the isolated cylinder.

Figure 6.19a compares the maximum measured inline force during the first half of the cycle with that inferred from Eq. 6.14 for three different $h_{\text{min}}/D$. For $KC > 6$ the experiments only indicate small increases in the maximum inline force, regardless of $h_{\text{min}}/D$. As such, except for $KC = 4.7$, Eq. 6.14 generally overpredicts the increase in maximum inline force over the first half of the cycle due to the wall. The poor comparison occurs because as $KC$ increases, the time when the maximum inline force occurs increases (Figure 6.19b) due to the increased influence of the separation-driven force component in phase with the cylinder velocity. However, from Figure 6.18 for all $KC$ numbers, the
Figure 6.19 Comparison of inline force measurements and predictions as a function of $h_{\text{min}}/D$ and $KC$ number. Solid circles - measurements. Lines - predictions following Eq. 6.14. Black - $h_{\text{min}}/D = \infty$. Green - $h_{\text{min}}/D = 0.5$. Red - $h_{\text{min}}/D = 0.25$. Blue - $h_{\text{min}}/D = 0.125$. Open symbols - isolated cylinder results from Yuan (2013) (◦); Sarpkaya (1986) (□); Obasaju et al. (1988) (△).

effect of the wall is primarily evident for small values of $t/T$ when the cylinder is near the wall.

On the other hand, Figure 6.19c shows that at $t/T = 0$ there is generally an increase in the inline force over this range of $KC$, although for $KC > 8$ the magnitude is generally small in all cases. The magnitude of inline force increase is predicted relatively well by Eq. 6.14 for $t/T = 0$ because the dominant pumping mechanism driving the suction force at this phase is well described by potential flow. This means that potential flow can be used to predict the inline force increases, due to a nearby wall, for portions of the cycle where the component in phase with cylinder acceleration dominates, assuming that the isolated cylinder force response is known.

Using Eq. 6.14 provides a method to account for the generally asymmetric natural of the wall force time series. Without such a correction, utilising harmonic formula, such as
the Morison equation, is not appropriate to describe the force response. However, the asymmetry can be accounted for to some extent by estimating the inline force time history using various empirical measurements of drag and inertia coefficients described by the Morison equation (e.g. Sarpkaya 1986; Obasaju et al. 1988) and then modifying this for various $h_{\text{min}}/D$ according to Eq. 6.14. The results suggest that such an approach is most applicable for relatively small $KC$ numbers where the maximum force is closer to being in phase with cylinder acceleration. However, at larger $KC$ numbers, this correction does not provide a good indicator of the maximum inline force over the course of a cycle.

6.6 Conclusions

Flow visualisation experiments investigating the flow field around a cylinder oscillating normal to a plane wall have been described in this paper. The main observations regarding the flow field are summarised as follows:

1. Vortex flow regimes for oscillation normal to a wall are qualitatively similar to those identified by Williamson (1985) for isolated cylinders in that:
   a) For $KC \lesssim 5$, the flow field remains generally symmetric about the cylinder centreline, with trailing vortices of approximately equal strength remaining attached during oscillation.
   b) For $5 < KC < 8$, trailing vortices become asymmetric in strength but remain attached during motion, detaching only at the end of each half cycle. The side of the dominant vortex and overall asymmetric circulation flow switches irregularly between cycles.
   c) For $KC \gtrsim 8$, asymmetric vortex shedding occurs during oscillation and shed vortices tend to convect away in a direction broadly transverse to the oscillation axis. However, there appears to be some tendency for vortices to convect away along the wall. More consistent asymmetry in the flow field (between cycles) leads to sustained asymmetric horizontal velocities in the gap between the cylinder and the wall even when the cylinder is momentarily halted at the end of the cycle.

2. Confinement of flow between the cylinder and the wall, when the cylinder oscillates near the wall, causes pumping flow beneath the cylinder that is primarily horizontal along the wall at small $h_{\text{min}}/D$, approximately symmetric about the cylinder centreline and in phase with cylinder motion.
   a) Pumping flow response is apparent for portions of the cycle when the cylinder is close to the wall for all $KC \leq 12$ for $h_{\text{min}}/D = 0.125$.
   b) Pumping flow magnitude near the wall is described reasonably well by potential flow for $KC \leq 6$. For $KC > 6$ the variation in pumping flow is captured by potential flow but the magnitude of pumping flow is not captured due to asymmetric vortex dynamics.
c) Trends in the pumping flow magnitude are also captured by simpler one-dimensional control volume arguments, but this approach typically overpredicts the velocity magnitude and increasingly diverges from potential flow as $h_{min}/D$ increases.

3. For $KC > 5$, asymmetric vortex dynamics increasingly affect the near-wall flow field. This is characterised by asymmetric near-wall flow across the cylinder axis and instances of high velocity flows directed at the wall corresponding to vortex impingement.
   a) The direction of asymmetric flows and vortex impingement switches often and irregularly for $5 < KC < 8$, in contrast to the findings of Tom et al. (2018a) for $Re = 150$.
   b) For $KC \geq 8$ the direction of near-wall flow and side of vortex impingement is more regular over longer numbers of cycles ($O(10)$).

Force measurements were also described with a cylinder oscillating with various minimum distances between the cylinder and the wall. The main observations regarding the effect of the wall on the inline forces are:

1. The inline force generally increases when oscillating near to a wall.
2. The force increase is in phase with the cylinder acceleration due to suction caused by the flow confinement introduced by the wall. The increases are limited to a relatively small range of time towards the beginning of the cycle.
3. The increase in inline force appears to generally increase with reducing $h_{min}/D$.
4. The effect of inline force increases on the peak inline force over the cycle are most prominent for $KC < 7$, where the peak force occurs more in phase with the cylinder acceleration.
5. Potential flow predicts the increases in inline force (relative to the isolated cylinder case) reasonably well for $t/T \sim 0.0$ over the range of $KC$ numbers tested but generally does not predict the maximum inline force well over the range of $KC$ numbers considered.
References


Chapter 7

Sediment transport and trench development beneath a cylinder oscillating normal to a sandy seabed

Abstract The purpose of this study is to explore the conditions in which trenches form beneath oscillating cylinders – such as pipelines, cables or idealised chains - close to the seabed. Experiments are conducted by oscillating a circular cylinder in a direction normal to an initially flat sandy bed. Across a relatively wide parameter space, the transport patterns and trench geometries reveal three transport regimes that are linked to vortex dynamics and depend primarily on the ratio of oscillation amplitude to cylinder diameter ($KC$ number). For $KC \lesssim 4$ sediment motion results in bedload transport that is symmetric about the cylinder centreline. This leads to the formation of two parallel trenches with a prominent ridge forming directly beneath the cylinder. For $4 \lesssim KC \lesssim 9$ sediment motion occurs via localised transport events, which are associated with the motion of vortices shed from the cylinder. These transport events are irregular but occur on both sides of the cylinder and lead to the formation of a symmetric trench geometry. For $9 \lesssim KC \lesssim 12$ the sediment motion is characterised by localised transport events and asymmetric bedload transport driven by overall vortex dynamics. In terms of trench size, the maximum (equilibrium) depth is found to increase with $KC$ and a mobility number ($\psi$) defined in terms of the maximum cylinder velocity. The initial rate of trench development also increases with $KC$ number and $\psi$, with an additional dependency on the cylinder $\beta$ number. The cylinder motions required to initiate trenching are predicted well using continuity arguments and an oscillatory boundary layer assumption, provided the $KC$ number and minimum gap between the cylinder and the bed are relatively small. The findings in this study provide insight into the mechanisms and prediction of trench formation. In particular, this study reveals that significant trenches can form in sandy seabeds solely due to fluid flow induced by pipeline/cable/chain motion without direct seabed contact, which has implications for structural fatigue.

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7.1 Introduction

Objects that oscillate above the seabed cause local fluid motions that have the potential to cause sediment transport even without the influence of externally-applied flows. This transport can result in the formation of a trench beneath the object, which can impact the design of engineered near-seabed infrastructure in the offshore industry. For example, full utilisation of the fatigue life of steel catenary risers (SCRs) in deep-water offshore developments is limited by the inability to predict the seabed geometry due to oscillation of the riser at the touchdown zone Bridge and Howells (2007). This has implications on how the stresses in the riser vary close to the touchdown point over the lifetime of the system. Similarly, the significant trenches believed to result from soil-fluid-structure interaction around mooring line chains can significantly reduce the capacity of anchoring systems Bhattacharjee et al. (2014). For both SCRs and mooring lines, vertical oscillation in absence of background flows occurs if motion of the connected surface facilities is triggered by a metocean climate that involves minimal near-bed flow – for instance surface waves in deep water environments. Similarly, isolated near-bottom motion occurs during installation of pipelines and cables from surface pipe-lay vessels Westgate et al. (2010b).

Despite the practical importance of trench formation, limited previous work has studied the mechanics of scour owing solely to object motion. Chiew et al. (2016) performed an experimental study describing trench geometries that form in a sandy bed beneath a spring-mounted cylinder undergoing forced vibrations normal to the bed and allowed to contact the bed. The study focused on two oscillation frequencies and a targeted oscillation amplitude of 1.4 diameters. For these two cases the observed flow field was reported to be symmetric and the trench that formed was symmetric about the centreline of the cylinder. Interestingly, the maximum width of the trench was found to increase with increasing oscillation frequency, whilst the maximum depth decreased with increasing oscillation frequency. Chiew et al. (2016) provided important insight into trenching mechanics, motivating the wider parametric range of the current experiments.

In contrast, there exists a substantial body of work regarding scour beneath pipelines due to externally applied hydrodynamic forcing, such as currents and/or waves, as opposed to object motion (e.g. Sumer and Fredsøe 2002). The basic features of wave-induced pipeline scour are (a) that the equilibrium scour depth is a function of the Keulegan-Carpenter ($KC$) number, which is proportional to the orbital amplitude that a fluid particle moves during a cycle relative to the pipeline diameter; and (b) the time scale of scour formation is inversely proportional to the dimensionless skin friction shear stress applied to the seabed due to the wave forcing. Previous studies have also focused on scour beneath a vibrating pipeline or riser in the presence of currents or waves, which has relevance to vortex-induced vibrations Chiew et al. (2014), Gao et al. (2006), and Sumer et al. (1988). These and similar studies found that the scour depth generally increases with pipeline oscillation (relative to the stationary case) near the bed.

Based on classical fluid mechanics, it is well-known that if the oscillation amplitude of an object is not small compared with its diameter, vortex shedding may significantly influence the local flow field. Williamson (1985) and Tatsuno and Bearman (1990) documented the
vortex shedding regimes around a cylinder oscillating far from any boundary. Williamson (1985) described the changes in flow regime that are observed as $KC$ number increases (where the $KC$ number is now proportional to the amplitude of cylinder motion divided by cylinder diameter), for $\beta = Re/KC < 230$, where $Re$ is the Reynolds number defined in terms of the maximum cylinder velocity. At $KC < 4$ a pair of vortices with approximately equal strength form on the trailing side of the cylinder and do not shed, but detach at the end of each half cycle. As $KC$ increases to $\sim 7$, the strength and size of the vortex pair become increasingly asymmetric, but do not shed due to insufficient amplitude of motion. Above $KC \sim 7$, one vortex sheds per half cycle leading to the formation of a vortex street transverse to the axis of oscillation. At higher $KC$ numbers, additional vortices are able to shed per half cycle, leading to a family of additional flow regimes Sumer and Fredsøe (2006).

With the introduction of a rigid wall, a third parameter (in addition to the $KC$ and $\beta$ numbers) influences the local flow field: the minimum ‘gap’ distance between the cylinder and the wall. Sumer et al. (1991), for example, has shown that the transverse vortex street transitions to a parallel vortex street when a cylinder is oscillated parallel and sufficiently close to a wall. Sumer et al. (1991) also showed that vortex shedding is suppressed at small gap distances (less than $0.1D$). For the case of a cylinder oscillating normal to a rigid wall, Tom et al. (2018b), using particle image velocimetry experiments, showed that at low $KC$ ($\lesssim 5$ but also up to at least 12 for some portions of the cycle) and $\beta = 500$ the presence of a wall leads to relatively large near-boundary velocities (or pumping) that can be predicted well by potential theory and continuity arguments. For $KC \gtrsim 7$ vortices shed in the transverse direction but the confinement introduced by the wall causes horizontal flows near the wall to be strongly asymmetric. The influence of vortices was also suggested to increase the near-wall velocity, relative to that for symmetric pumping, for $KC \gtrsim 5$. Tom et al. (2018b) also showed that for a minimum gap height of 12.5% of the cylinder diameter, the wall does not appear to suppress transverse motion of shed vortices from a normally oscillating cylinder. The numerical results presented by Tom et al. (2018b) over this $KC$ range focused on a constant $Re = 150$. The results showed similar vortex-wall interaction features to the experimental results for larger $Re$ and showed that comparison between continuity arguments and near-wall velocity is good provided the minimum gap ratio and $KC$ number are sufficiently small.

Building on this previous body of research, the principle aim of this paper is to experimentally investigate trench formation beneath an oscillating cylinder over a relatively wide parameter space that is relevant to submarine pipelines, cables and mooring chains during installation and operation. As an idealisation of a typical riser system, a circular cylinder is oscillated sinusoidally in time normal to the bed (see Figure 7.1). A focus is placed on relating general observations of the trenching processes back to measurements of the velocity field obtained by Tom et al. (2018b) to elucidate the driving mechanisms of sediment transport. Three aspects of the problem are investigated quantitatively: (i) the range of cylinder motions that cause sediment transport, to provide a prediction on when trenching may occur; (ii) the rate of trench formation and (iii) the extent and depth of the trench as it progresses.
To investigate the mechanics of trench formation due to fluid motions alone, the experiments reported in this paper are for cylinder motions that do not contact the bed. The geotechnical aspects of trench formation due to pipe-seabed contact are thus avoided. Similarly, the present experiments only use sandy, coarse-grained sediment. This restriction allows for interpretation of trench formation from the well-established standpoint of sand transport. It is anticipated that future work considering finer grain sediments may utilise the insights gained from the present experiments.

The remainder of this paper is structured as follows. Section 7.2 describes the experimental setup utilised and Section 7.3 outlines the relevant parameter space that has been investigated. Section 7.4 then presents an overview of the sediment transport mechanisms and the trench geometries that were observed across the experimental parameter space. The effect of cylinder motions on the initiation, rate and depth of trench formation are then quantitatively investigated in Section 7.5. Discussions and conclusions are presented in Section 7.6 and 7.7, respectively.

7.2 Experimental methodology

The experiments were conducted in a 660 mm long section of a 395 mm wide tank at the University of Western Australia, which has a total length of 15 m. The water depth was 250 mm above the bed surface and the sand bed was placed to a total depth of 200 mm. The cylinder was oscillated using an electrical actuator whereby the prescribed vertical displacement in time was:

$$y(t) = h_{\text{min}} + \frac{D}{2} + A - A \cos \left( \frac{2\pi t}{T} \right)$$  \hspace{1cm} (7.1)

where $A$ is the oscillation amplitude, $T$ is the period and $h_{\text{min}}$ ($> 0$) is the minimum gap distance. Since the motion is defined by a cosine function, the lowest point in the cycle occurs at $t/T = 0, 1, 2, \ldots$ and the cylinder is at its furthest position at $t/T = 0.5, 1.5, \ldots$

To verify the cylinder motion, a linearly variable differential transformer was used to measure the cylinder displacement.
7.3 Experimental parameter space

Table 7.1 Properties of sediment used in experiments.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median particle size, $d_{50}$</td>
<td>(mm)</td>
<td>0.180</td>
</tr>
<tr>
<td>Coefficient of uniformity, $C_u$</td>
<td>(-)</td>
<td>2.000</td>
</tr>
<tr>
<td>Particle specific gravity, $s$</td>
<td>(-)</td>
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</tr>
<tr>
<td>Critical Shields parameter, $\theta_{cr}$</td>
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<td>0.051</td>
</tr>
<tr>
<td>Critical shear stress for erosion, $\tau_{cr}$</td>
<td>(Pa)</td>
<td>0.159</td>
</tr>
</tbody>
</table>

Three PVC cylinders were used (with diameters of 21 mm, 51 mm and 89 mm), which extended 385 mm across the width of tank. Additional tests were also conducted with 650 mm long cylinders (extending along the tank section) to investigate end effects. Prior to each experiment, the sediment was levelled by dragging a thin, rectangular piece of PVC material over the test bed. The leveller extended across the width of the tank and was suspended below two railings placed atop the sediment container. The initial bed surface, denoted $\xi(x,z,0)$, was then scanned using a SICK Ranger 3-D laser scanner. During testing cylinder motion was stopped at intervals and the bed was rescanned to recover $\xi(x,z,t)$. Scans were taken with the laser placed across the width of the tank and actuated along its length.

The sediment used in the experiments was commercially available silica sand, which has been employed in a number of previous scour studies (e.g. Leckie et al. 2016). Table 7.1 presents the properties of the sediment drawing on erosion parameters determined by Mohr et al. (2016) (who refer to this sediment as SS2).

7.3 Experimental parameter space

Dimensional considerations indicate that trench development beneath the cylinder is dependent on a number of variables related to the cylinder motion, fluid properties and sediment properties. The maximum trench depth $S_0 = \xi_0 - \min(\xi)$, for example, may be written functionally as:

$$S_0 = f(D, A, T, h_{min}, \nu, d_{50}, g(s-1), w_s)$$

(7.2)

where $\nu$ is the fluid kinematic viscosity and $d_{50}$ is the sediment median particle size. $g(s-1)$ is an independent quantity defined from $s = \rho_s/\rho$ and gravitational acceleration, $g$, where $\rho$ and $\rho_s$ are the fluid and sediment (particle) densities, respectively. Finally, although particle fall velocity ($w_s$) may be derived from the other parameters if the particle shape is known, it has also been included in Eq. 7.2 as an independent parameter for generality.

Noting that there are three primary dimensions, Eq. 7.2 can be rewritten as:

$$S_0/D = f\left(\frac{2\pi A}{D}, \beta = \frac{D^2}{T\nu}, h_{min}/D, \psi = \frac{(2\pi A/T)^2}{2\pi A/T}, \frac{(2\pi A/T)^2}{g(s-1)d_{50}}, \frac{2\pi A/T}{w_s}\right)$$

(7.3)

The first dimensionless group is $KC$ number, the second is the ratio of $Re$ to $KC$ number (commonly referred to as $\beta$), the third is the minimum gap ratio, the fourth describes
a sediment mobility number $\psi$ written in terms of the maximum cylinder oscillation velocity ($V_{\text{max}} = 2\pi A/T$) and the final group is the ratio of maximum cylinder velocity to fall velocity (somewhat analogous to the Dean Number; Hughes 1993). Across the first four independent groups, the present experimental program covers a realistic range for a single sand sediment. Although the final group is not explored systematically in this work, it is included for future reference since mode of erosion is known to affect trench geometry and rate of development (e.g. Mohr et al. 2016).

At full scale, the relevant motion of risers, pipelines or cables depends on aspects such as water depth, system configuration, system geometry and structural properties, operating metocean conditions, the response of an attached vessel and the geotechnical response of the seabed (e.g. for pipe installation or laying, see Westgate et al. 2010b). Limited published data in the literature exists regarding actual motions at the seabed, but inferences may be made to form a representative parameter range for testing. For example:

1. Full scale riser tests conducted during the STRIDE JIP (Bridge 2005) investigated vertical motions of a 0.168 m diameter riser corresponding (at 10 m above seabed) to a $[KC, T]$ of [1.9, 6 s] and [15, 25 s] for operating and extreme motions of the attached floating system, respectively. These motions were adopted based on riser analyses conducted as part of the JIP.

2. Westgate et al. (2010a) reported video footage of 0.32 m diameter pipeline motions during the lay process of up to $KC = 5.0$ in the near-bed region with an average wave period of 5 s. That pipeline had a recorded trench depth (where sediment was suggested to have been eroded) in some locations of 1 to 1.5 diameters.

3. Bridge and Howells (2007) report riser trenches in the field ranging from 2 to 4.5 diameter deep; this suggests the $KC$ number may reach up to 6 to 14 if it is assumed the riser returns to the original seabed level and contacts the bed in a cycle.

4. Geotechnical centrifuge testing undertaken by Elliott et al. (2013) considered a maximum $KC$ value of about 14 and a period $T$ of 20 s (but also 1 s, in an attempt to scale the motions for centrifuge modelling) defined at prototype scale for a point on the riser 5 m above the seabed. The riser had a prototype diameter of 0.508 m.

5. Hejazi (2018) provided the authors with example motions from dynamic riser analyses for a 0.228 m diameter riser in 1,000 m water depth for typical operational sea states corresponding to 0.4 m vertical amplitude motions of a semi-submersible host. The motions for this specific example in the touchdown zone resulted in a $KC$ number of about 6.9 and an oscillation period $T$ of 13 s.

From these references it appears that $KC < 15$ is broadly representative of pipeline and riser motions induced by a floating host near the touchdown zone. For pipelines being installed, the range is likely to be smaller (i.e. $KC < 5$) since pipelay occurs in relatively calm conditions. The range in quoted oscillation period suggests $Re \sim \mathcal{O}(10^4 - 10^5)$ or $\beta \sim \mathcal{O}(10^3 - 10^4)$. 

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7.4 Results: General Observations

For the present study, two series of tests were conducted: the first (main) series focused on observations and measurements of trench formation, whilst the second (supplementary) series focused on the threshold of sediment motion. The experiments that were performed in the first series are listed in Table 7.2. These experiments were chosen to cover a representative range in $KC$ number and $\beta$ number. The mobility number was varied whilst fixing the minimum gap ratio at a reference value of approximately 0.1 to ensure a range of near bed velocities.

For the second supplementary series, $KC$ was varied up to 12 and $\psi$ was varied so as to achieve sediment motion. Three different pipe diameters were adopted to vary $\beta$ between $\sim O(10^2 - 10^3)$, and the minimum gap height was varied between 5 to 20% of the cylinder diameter.

7.4 Results: General Observations

A range of different trench geometries are observed across the parameter space investigated in the main experiments. Figure 7.2 shows a selection of seabed cross-sections recorded during the experiments. Figure 7.3 shows the corresponding centreline bed level progression over time. General observations can be made as follows:

1. Sediment transport leading to trench formation is generally confined to the seabed local to the cylinder, with the width of the seabed affected by sediment transport generally increasing in size with increases in $KC$ number and $\psi$, up to $\pm 2D$.

2. Trench formation coincides with the formation of berms above the initial bed level, with similar cross-sectional area to the trench indicating that the sediment moves primarily in bed load or, if suspended, it is deposited locally.

3. Trenching occurs quickly in the early stages of cycling and continues in most cases at a reducing rate towards an equilibrium geometry. These findings suggest that transport patterns are linked to the scale of the fluid flow induced by cylinder oscillations and depend significantly on the motion of the cylinder.

In addition to these general observations, there are noticeable differences in both the trench profiles and how the bed evolves with time. These changes are primarily dependent on $KC$ number since this parameter primarily controls changes in the overall vortex shedding dynamics (see Section 7.1). Three primary regimes of trench formation have been identified based on the main experimental results. These are described in further detail in the following sections.

7.4.1 Symmetric central ridge ($KC < 4$)

For $KC < 4$ a central ridge of sediment forms beneath the centreline of the cylinder, together with two trenches on either side (see Figure 7.2a, b). At the commencement of each experiment, sediment motion occurs primarily via bedload transport when the cylinder is close to the bed (i.e. $0.75 < t/T < 1.2$). This sediment motion is directed away
Table 7.2

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<th>Viscous parameter</th>
<th>Mobility number</th>
<th>Critical mobility number</th>
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Note: (1) Approximate value estimated from same diameter and KC number on Figure 7.9.
7.4 Results: General Observations

Figure 7.2 Measured trench profiles after progressively increasing numbers of cycles. Cylinder represents lowest position in cycle.
Figure 7.3 Measured trench depth progression at the centre of the cylinder (i.e. $S_{CL} = \xi_0 - \xi(0,0,t)$). Lines grouped by colour for similar $KC$ numbers.

from the centreline when the cylinder approaches the bed and towards the centreline as the cylinder moves upward. The transport is symmetric about the cylinder centreline but is not time-symmetric over a cycle. This results in net transport towards the centre of the cylinder. Over multiple oscillations this net transport leads to formation of a ridge (see Figure 7.4a).

To illustrate these observations, Figure 7.4b-e provides sketches of the sediment transport observed at different instances during a cycle. Also shown on this figure (in grey lines) are the flow features identified via flow visualisation experiments for a cylinder oscillating with similar $KC$ number above a rigid wall (Tom et al. 2018b). The main flow features include the vortex dynamics and ‘pumping’ of water into and out of the gap beneath the cylinder. For this regime, these flow features are consistent with the observed sediment transport mechanisms. In particular, symmetric sediment motion either side of the cylinder is consistent with pumping flow beneath the cylinder, which is symmetric and peaks at a similar phase during the oscillation (Tom et al. 2018b). In addition, the time period over which sediment motion occurs during the second half of the cycle (i.e. cylinder moving towards the bed) was slightly longer than the first half cycle. This difference is attributed to the background circulation in the flow field directed outward near the bed (indicated by the circulation cells on Figure 7.4b-e), which prolongs transport as the cylinder moves towards the bed.

7.4.2 Symmetric trench with intermittently asymmetric transport

$(4 < KC < 9)$

For the experiments where $4 < KC < 9$, the eventual trench geometries on Figure 7.2 are symmetric about the cylinder centreline, with generally no prominent ridge above the initial bed level. However, although the final geometries are qualitatively similar throughout this regime, the dominant mechanisms of sediment transport observed during individual cycles varies.

For $4 < KC < 6$, bedload transport similar to that described in the previous subsection
7.4 Results: General Observations

(a) Example video still from $KC = 3.6$, $\psi = 17.6$ after 2000 cycles.

(b) Idealised flow and transport schematic

Figure 7.4 Regime 1 – Example video still and idealised representation of flow dynamics and trench formation. Solid grey lines represent vortices with cores denoted by dots. Dashed grey lines represent pumping or overall circulation features.
still contributes significantly to sediment movement over the time interval $t/T = 0.75$ to 1.2. However, sediment movement also occurs due to spontaneous localised transport events in which sediment is temporarily suspended within a localised area (e.g. Figure 7.5a). These localised events occur on either side of the cylinder, although not generally on both sides at the same position along the cylinder during a cycle. The events occur at a slightly different time interval in the cycle compared to the bedload transport ($0.1 < t/T < 0.5$, as the cylinder moves away from the bed). For some cycles, the regions of sediment associated with the localised events are sufficiently large to encompass sediment at the centreline of the cylinder, which appears to inhibit ridge formation. With increasing $KC$ number and $\psi$, the contribution to total sediment transport due to the localised events appears to increase relative to that due to bedload transport.

To summarise these observations for $4 < KC < 6$, Figure 7.5b-e presents sketches similar to Figure 7.4, which also include flow features simplified from Tom et al. (2018b) for a similar $KC$ number range. These flow features indicate the formation of asymmetric trailing vortices during each half cycle, which detach when the cylinder reverses direction. When the larger of these detached vortices sheds at the lowest point of the cylinder motion (i.e. Vortex A on Figure 7.5c and 7.5d), it advects back towards the bed and induces a jet-like flow between itself and the newly forming vortex on the same side of the cylinder (Vortex C on Figure 7.5d). At around $t/T \sim 0.1$, the jet-like flow impinges on the bed and is believed to cause the localised sediment suspension events.

In terms of net sediment transport, the asymmetry in the vortex shedding shown on Figure 7.5 was observed to switch frequently in Tom et al. (2018b), such that jet flow often occurs on either side of the cylinder on different cycles. This switching is consistent with the observation of localised transport events occurring on both sides of the cylinder but not at the same position. This frequent switching is believed to be the reason why the final trench geometry is approximately symmetric.

For larger $KC$ numbers in this regime (i.e. $6 < KC < 9$), localised transport events are also observed; but a key additional observation is the occurrence at $t/T \sim 0.1$ of uni-directional (i.e. asymmetric) bedload transport across the cylinder axis on some (but not every) cycle. This asymmetric transport removes sediment from the centre of the cylinder. The direction of bedload transport is usually away from regions with localised transport, and appears to switch direction intermittently over tens of cycles. This intermittency is consistent with results reported by Tom et al. (2018b) suggesting that for $5 \leq KC < 8$, $\beta = 500$ the side on which the dominant trailing vortex forms and the direction of near-wall horizontal flow at the cylinder centreline varies between individual cycles. Analogously, Tatsuno and Bearman (1990) described a flow regime (Regime E) wherein the direction of asymmetric vortex behaviour periodically switches sides. This was linked to three-dimensional flow structures moving irregularly along the cylinder in time. The general observations in this study for $6 < KC < 9$ are conceptually similar to those illustrated on Figure 7.4b-e, except that the bedload transport becomes more asymmetric beneath the cylinder.
7.4 Results: General Observations

(a) Example video still from $KC = 6.1, \psi = 33.5$ after 1 cycle.

(b) Idealised flow and transport schematic

Figure 7.5 Regime 2 - Example video still and idealised representation of flow dynamics and trench formation. Solid grey lines represent vortices with cores denoted by dots. Dashed grey lines represent pumping or overall circulation features.
7.4.3 Asymmetric trench \((9 < KC < 12)\)

For \(KC > 9\), the trench geometry is asymmetric throughout much of its development. To quantify this shift from symmetric to asymmetric geometry, Figure 7.6 shows the horizontal location of the deepest point in the trench on the left and right hand sides of the cylinder (averaged along the cylinder length). Results are shown for all tests for which > 1000 cycles were completed (some of which are not detailed in Table 7.2). For a symmetric trench, \(|x_{\text{max,RS}} + x_{\text{max,LS}}| = 0\), so the vertical axis in this figure would be zero. Since \(x_{\text{max,RS}} \geq 0\) and \(x_{\text{max,LS}} \leq 0\), large values of \(|x_{\text{max,RS}} + x_{\text{max,LS}}|\) indicates that the deepest position on one side of the cylinder is further from the centreline than the deepest position on the other side. For tests conducted at \(KC \lesssim 9\), the trench profiles measured are approximately symmetric after many cycles. In contrast, for \(KC \gtrsim 9\), \(|x_{\text{max,RS}} + x_{\text{max,LS}}|\) is often larger than a cylinder diameter, indicating large asymmetry in the trench profile.

The general mechanisms of sediment transport observed in this asymmetric regime are similar to that observed for \(6 < KC < 9\) and comprise asymmetric bedload transport and localised transport events. However, for \(KC > 9\) the sediment motion during the initial cycles (both the direction of bedload transport and the location of localised events) are much more uniform along the cylinder (see Figure 7.7a) and maintain the same sense of asymmetry for tens to hundreds of cycles without switching. This observation is consistent with Tom et al. (2018b), who showed that over this \(KC\) range consistent vortex shedding in a single primary direction was also maintained over \(O(10)\) to \(O(100)\) of cycles coincident with the development of a transverse vortex street.

To explain the observed trends in the sediment motion, Figure 7.7b-e again documents...
7.4 Results: General Observations

(a) Example video still from $KC = 12.3$, $\psi = 33.4$ after 1 cycle. (not listed in Table 7.2)

(b) Idealised flow and transport schematic

Figure 7.7 Regime 3 – Example video still and idealised representation of flow dynamics and trench formation. Solid grey lines represent vortices with cores denoted by dots. Dashed grey lines represent pumping or overall circulation features.

The flow field and sediment motion during a cycle. This flow field is similar to that shown on Figure 7.5, except that the dominant trailing vortex (labelled Vortex A on Figure 7.7c) now detaches prior to the cylinder reversing direction. This results in the formation of a transverse vortex street. However, the jet-like flow that forms between this detached vortex and the newly forming trailing vortex (Vortex B on Figure 7.7d, e) is still evident and causes localised transport. The asymmetric flow near the bed at $t/T \sim 0$ (Figure 7.7c) coincides with the overall circulation dynamics observed by Tom et al. (2018b) for $KC > 8$. This asymmetry is similar to but more consistent over multiple cycles than that described for Regime 2.
7.4.4 Uniformity of trench geometry

The preceding results focused on only 2D features of sediment transport and the flow field. For low to moderate $KC$ numbers (i.e. $KC < 9$) these features dominate, with uniform trenches forming along the cylinder length throughout the trenching process (see Figure 7.8). In contrast, for $KC > 9$ the trenches are more non-uniform along their length and appear to be affected by the end conditions, even for tests with $L/D = 30$. Figure 7.8d, for example, shows the trench width and depth varying along the cylinder axis. Increased transport also occurs near the ends of the cylinder, suggesting end-effects; and for large $KC$ the sediment motion is clearly three-dimensional. Quantitative results presented in this paper for $KC > 9$ should therefore be treated cautiously. However, the qualitative discussion of the trenching mechanics seen at the mid-plane of the flume is representative of this $KC$ number regime regardless.
7.5 Quantitative results: trench formation

7.5.1 Threshold of sediment motion

To predict trenching, the conditions under which sediment motion occurs must first be assessed. Experiments were undertaken to assess the cylinder motions for which sediment was observed to be mobilised. These experiments were conducted systematically for specific values of $h_{\text{min}}/D$ and $KC$ by oscillating the cylinder above an initially flat bed for approximately 100 cycles at a constant value of $\psi$. This process was then repeated for increasing $\psi$. During these oscillations, sediment motion was defined as occurring when one of two observations were made:

1. Method 1: grains were transported in bedload a distance exceeding one grain diameter during the cycle over more than 50% of the cylinder length; or

2. Method 2: sediment was mobilised (in bedload or suspension) due to localised transport events at least once per cycle over more than 10 cycles.

Mobility via the first observation was most prevalent at low $KC$ numbers. The second observation was more common at higher $KC$ numbers where localised transport events are more prevalent. For a given value of $KC$ number, the smallest value of $\psi$ that corresponds to occurrence of sediment motion (i.e. the critical mobility number) is denoted as $\psi_{cr}$ throughout the remainder of this paper.

Figure 7.9 summarises the threshold of motion observations in terms of $KC$ number and $\psi$ for different diameters and values of $h_{\text{min}}/D$. In all cases for $KC < 4$ and approximately constant $h_{\text{min}}/D$, the required $\psi$ for sediment motion increases with $KC$. This implies that for this range in $KC$ number, as amplitude of motion increases, the cylinder must move faster to achieve a near-bed fluid velocity sufficient to mobilise the sediment. The results also indicate that in the range of $\psi$ near the sediment mobility threshold the transport regime primarily involves symmetric bedload transport for $KC < 4$.

In contrast, for $KC \geq 4$ the required $\psi$ for sediment motion reduces slightly for $4 < KC < 6$ (at least for $D = 51$ mm) and then changes only slightly for $KC > 6$. This reduction suggests that as $KC$ increases there is an apparent amplification in the near-bed velocities, which enables sediment to be mobilised at relatively lower $\psi$. Initiation of sediment motion in this regime also coincides with the occurrence of localised transport events described previously.

Aside from the trends with $KC$ number, Figures 7.9c, d and e also indicate that for $KC < 4$ the cylinder motions required for mobility are dependent on the minimum distance between the cylinder and the seabed. When the minimum gap is larger, the rate of volume change in the gap beneath the cylinder decreases. Thus, faster cylinder motions are required to cause the same near bed fluid velocities at the same cycle phase. At higher $KC$ numbers (see Figure 7.9b), this dependency is less clear because the sediment motion is caused by localised events.
Chapter 7 Trench development beneath an oscillating cylinder

Figure 7.9 Threshold of motion. Squares - Method 1. Circles - Method 2. Solid symbols - observed sediment mobility. × - no motion. Black symbols - $h_{\text{min}}/D < 0.07$. Blue symbols - $0.07 < h_{\text{min}}/D < 0.14$. Red symbols - $h_{\text{min}}/D \geq 0.14$. Shaded areas - approximate ranges for regimes in Section 7.4. Blue shaded areas - predictions based on Eq. 7.7 to 7.13 for measured range in $h_{\text{min}}/D$. 

(a) $D = 21\text{mm}$  
(b) $D = 51\text{mm}$  
(c) $D = 89\text{mm}, h_{\text{min}}/D < 0.07$  
(d) $D = 89\text{mm}, 0.07 \leq h_{\text{min}}/D < 0.14$  
(e) $D = 89\text{mm}, h_{\text{min}}/D \geq 0.14$
7.5 Quantitative results: trench formation

Figure 7.10 Control volume between an oscillating cylinder and a rigid wall.

Prediction of threshold of motion at low KC number

Provided the KC number is small (i.e. $KC \lesssim 4$) and $h_{\text{min}}/D$ is small, Tom et al. (2018b) showed that continuity arguments can be used to estimate the flow velocity beneath the cylinder. This is possible because small $KC$ ensures the flow is symmetric and free from significant vortex shedding influence. Additionally, small $h_{\text{min}}/D$ ensures the flow is relatively uniform with height in the gap between the cylinder and wall, such that the depth-averaged velocity deduced from a control volume analysis is representative of that at any height within the gap. In this subsection, such estimates of flow velocity are used to predict the threshold of motion beneath the cylinder.

To obtain an expression for the velocity beneath the cylinder using continuity arguments, a control volume can be introduced (dashed region on Figure 7.10). The change in size of the control volume during an oscillation can be equated to the flow passing through its boundary. Due to symmetry only the right-hand boundary in Figure 10 allows flow to pass, and hence continuity is satisfied if

$$ \frac{\partial \Omega(x, t)}{\partial t} = \bar{u}(x, t) h(x, t) $$

(7.4)

where $\Omega(x, t)$ represents the volume (per unit length) of the control volume, $h(x, t)$ is the height at the right-hand boundary of the control volume and $\bar{u}(x, t)$ is the depth-averaged horizontal velocity moving across the boundary of the control volume.

Given that the cylinder is moving sinusoidally in time, it follows that

$$ \frac{\partial \Omega(x, t)}{\partial t} = x \frac{2\pi A}{T} \sin \left( \frac{2\pi t}{T} \right) $$

(7.5)

and

$$ h(x, t) = h_{\text{min}} + A - A \cos \left( \frac{2\pi t}{T} \right) - \frac{D}{2} \left( 1 - \left( \frac{2x}{D} \right)^{2} \right)^{0.5} \quad \text{for} \quad x \leq \frac{D}{2} $$

(7.6)
Substituting Eq. 7.5 and 7.6 into 7.4 gives, after some rearrangement,

\[
\frac{\pi(x,t)}{V_{\text{max}}} = \frac{\frac{\pi}{A} \sin \left( \frac{2\pi t}{T} \right)}{h_{\text{min}}/A + \left(1 - \cos \left( \frac{2\pi t}{T} \right) \right) + \frac{\pi}{K}} \left[ 1 - \left( \frac{2\pi t}{T} \right)^2 \right]^{0.5} \tag{7.7}
\]

where \( V_{\text{max}} \) is the maximum cylinder velocity. Although the form of Eq. 7.7 is complicated, it illustrates several key points. Firstly, when the minimum gap between the cylinder and the bed is small relative to the amplitude of motion (i.e. \( h_{\text{min}}/A << 1 \)), the pumping velocity increases. Secondly, for a constant gap height, the pumping velocity decreases relative to the maximum cylinder velocity as \( K \) increases. Thirdly, it can be readily shown that Eq. 7.7 has a local maximum during each half cycle, implying a maximum near bed velocity in space and time (\( u_{\text{max}} \)) during each up and down stroke of the cylinder.

Eq. 7.7 is used to predict the threshold of motion by assuming that (i) the peak seabed shear stress due to the pumping of fluid beneath the cylinder is related to \( u_{\text{max}} \) and (ii) that, although Eq. 7.7 implies that the near-bed velocity is not perfectly sinusoidal, the framework introduced by Soulsby (1997) may be used as a first estimate of the maximum shear stress. Adopting this approach, the maximum shear stress is then given by

\[
\tau_{\text{max}} = \frac{1}{2} \rho f_w u_{\text{max}}^2 \tag{7.8}
\]

where \( f_w \) is the (wave) friction factor and \( u_{\text{max}} \) is the maximum near-bed velocity amplitude. Following Soulsby (1997), the friction factor is calculated according to

\[
f_w = \max\left(f_{w,\text{rough}}, f_{w,\text{smooth}}\right) \tag{7.9}
\]

where

\[
f_{w,\text{rough}} = 0.237r^{-0.52} \tag{7.10}
\]

for rough turbulent flows, and:

\[
\begin{align*}
    f_{w,\text{smooth}} &= 2R_w^{-0.5} \quad \text{for } R_w \leq 5 \times 10^5 \\
    f_{w,\text{smooth}} &= 0.0521R_w^{-0.187} \quad \text{for } R_w > 5 \times 10^5
\end{align*}
\tag{7.11}
\]

for laminar and smooth turbulent flows, respectively. In these equations \( R_w \) represents the wave Reynolds number and is given by:

\[
R_w = \frac{u_{\text{max}} A_p}{\nu} \tag{7.12}
\]

where \( A_p \) is the half-cycle maximum particle excursion calculated by \( A_p = u_{\text{max}} T/2\pi \), and \( r \) is a relative roughness given by:

\[
r = \frac{A_p}{k_s} \tag{7.13}
\]

where \( k_s \) is the Nikuradse equivalent roughness of the bed, which is assumed to be equal to 2.5\( d_{50} \).
Finally, to infer mobility, the shear stress computed from Eq. 7.8 can be compared with the critical shear stress for sediment transport. This is given in Table 7.1 for the present experimental sediment.

Figure 7.9 compares the measurements with estimates based on Eq. 7.7 to 7.13, shown by the blue shaded area bounding the predictions for the measured range in $h_{\min}/D$. The comparison indicates that the theoretical prediction works well in the region where the cylinder motions are consistent with the basic model assumptions (i.e. small $KC$ and $h_{\min}/D$). When these assumptions are violated, sediment mobility is observed at different values of $\psi$ for a given $KC$ than predicted by the model. For instance, when $KC > 4$ sediment mobility is observed at values of $\psi$ smaller than predicted theoretically. This implies an apparent amplification of the bed shear stress due to complex vortex-bed interactions. In other words, vortices increase the effectiveness of cylinder motion at initiating sediment transport. Similarly, at small $KC$ with larger $h_{\min}/D$, the agreement is poorer. This difference occurs because as $h_{\min}/D$ increases, relatively larger horizontal velocities occur near the cylinder than near the wall, which a depth-averaged velocity does not account for. Over-prediction of the near-wall velocity by continuity arguments, increasing with $h_{\min}/D$, is consistent with the findings of Tom et al. (2018a) comparing to numerical simulations.

### 7.5.2 Extent of trench formation

Table 7.2 lists the tests conducted where trench profiles were measured over least 10,000 cylinder oscillations. For tests in Regime 1 ($KC < 4$) and Regime 3 ($KC > 9$) there was either no trench at the centreline of the cylinder, or the trench was non-uniform along the cylinder. However, for Regime 2 ($4 < KC < 9$) a uniform trench was generally observed along the cylinder. For this regime, the trench depth can be reasonably approximated by an exponential function of the form (see Figure 7.11 and 7.12):

$$S(t) = S_0 \left[ 1 - \exp \left( - \left( \frac{N}{N_t} \right)^{n_e} \right) \right]$$  \hspace{1cm} (7.14)

where $S_0$ is the equilibrium trench depth, $N$ is the number of cycles, and both $N_t$ and $n_e$ are empirical coefficients controlling the rate of trench depth development and the non-linearity of the trench development during the initial stages. The results were fitted using a non-linear least-squares method and model coefficients are reported in Table 7.2. Although Eq. 7.14 describes trench depth progression well within the measured range of cycles, there is uncertainty in extrapolating to an equilibrium trench depth for some of the tests, particularly those with relatively low numbers of cycles and high $KC$. For these tests, the equilibrium trench depths may be less reliable but the fitted coefficients are still useful for comparing the trends over the initial $O(10^4)$ cycles.

The results in Table 7.2 show that $n_e$ has little variation across the experiments, which implies a self-similar trench development in time. In contrast, there is larger variation in $S_0$ and $N_t$. To explore $S_0$, Figures 7.13 and 7.14 show the measured trench depths at $N = 500, 1,000, 10,000$ and equilibrium conditions as a function of $KC$ and $\psi$. In
agreement with the qualitative observations in Section 7.4, for each diameter of cylinder there is an evident increase in the trench depth with $KC$ and, to a lesser degree, $\psi$ for each number of cycles. However, there is significant variation for a constant $KC$ or $\psi$, and the trends (particularly for $\psi$) appear to vary between the cylinder diameters. Closer inspection of the data on Figures 7.13 and 7.14 also indicates that over the range of experimental conditions tested the $KC$ number influences the trench depth at larger cycle numbers and at equilibrium conditions (compare Figure 7.13d and Figure 7.13e with Figure 7.13a), whereas $\psi$ influences the behaviour at relatively small cycle numbers (compare Figure 7.14a with Figure 7.14d). This result suggests that $\psi$ is most closely linked to the transport rate per cycle at the start of the trenching process, which would be expected to control the initial trench depth. In contrast, the $KC$ number appears to be a better predictor of the equilibrium trench depth. This implies that the equilibrium depth depends primarily on the scale of near-bed fluid motions.

To capture the combined influence of $KC$ and $\psi$ on trench depth, Figure 7.15 presents
7.5 Quantitative results: trench formation

The trench depth as a function of both $KC$ and $\psi/\psi_{cr}$ for each cylinder diameter. The exponents $\alpha$ and $\beta$ are selected to best collapse the data and $\psi_{cr}$ chosen based on the threshold of motion experiments for each $KC$ number. Provided both $KC$ and $\psi/\psi_{cr}$ are used in combination the trench depth data collapses well regardless of cycle number. Thus for a given cylinder diameter (or, approximately, Reynolds number) and gap ratio, both $\psi$ and $KC$ affect the trenching process. Furthermore, as the cycle number increases $KC$ becomes more prominent in controlling the trench depth compared with $\psi$. Figure 7.16d similarly shows the fitted equilibrium trench depths as a function of $KC$ and $\psi/\psi_{cr}$ but with $\alpha$ and $\beta$ chosen to correspond to the best fit results for $N=10,000$ (Figure 7.15d). These trends generally agree well, although there is some scatter due to uncertainty in fitting the equilibrium depth from a limited number of cycles. There does appear to be a cylinder diameter or $Re$ dependency of the equilibrium results using this empirical fitting. However, the differences in equilibrium depth magnitude between diameters are relatively
Figure 7.14 Variation in maximum trench depth for different numbers of cycles with $\psi$. small; and notably, there appears to be no significant difference between diameters for depth as a function of $KC$ number alone (Figure 7.13).

The equilibrium results from Figure 7.13d have been reproduced on Figure 7.16 with the best fit relationship from Figure 7.13d corresponding to:

$$\frac{S_0}{D} = 0.1KC - 0.4 \quad \text{for} \quad 4 < KC < 9, \quad \text{and} \quad \psi/\psi_{cr} > 3.0 \quad (7.15)$$

This fit implies no trench directly beneath the cylinder for $KC \leq 4$. The relationship is also limited to $KC < 9$, due to the paucity of results at larger $KC$ and uncertainty related to changes in vortex shedding regimes as $KC$ increases beyond this range. The requirement for $\psi/\psi_{cr} > 3.0$ reflects the range of parameters considered in the current experiments; for smaller $\psi/\psi_{cr}$ (particularly for $\psi \approx \psi_{cr}$), Eq. 7.15 would overestimate the trench depth. The results and fitted relationship are also compared with the empirical curve derived by Sumer and Fredsøe (1990) for scour below fixed pipelines in waves on
7.5 Quantitative results: trench formation

Figure 7.15 Trench depth at various cycle number as a function of $KC$ and $\psi/\psi_{cr}$.

Figure 7.16. The empirical result is based on experiments in live bed conditions (i.e. the motions are sufficiently strong to induce transport of sediment located far from the pipeline) and is in terms of a $KC$ number defined based on the orbital excursion of a water particle near the bed rather than the amplitude of the cylinder motion. At low $KC$ number, the equilibrium scour depth under a fixed cylinder is generally larger than that observed for the present experiments, regardless of mobility number. By contrast, the observed trench depths exceed the fixed pipe results at higher $KC$ numbers ($KC > 6$). This trend illustrates the difference in eroding mechanisms between fixed and oscillating cylinders. For a fixed cylinder, fluid motion is primarily parallel to the bed and is amplified beneath the cylinder, leading to mobilisation directly below the cylinder via flow amplification. This is in contrast to the oscillating cylinder, where the frequency and intensity of vortex-bed interactions drive erosion and appear to control trench depth. The results therefore indicate that the frequency and intensity of vortex-bed interactions scale differently with $KC$ number compared to oscillatory flow under a fixed pipeline and
imply that at least for $6 \lesssim KC < 8$ the net transporting effect of vortices due to cylinder oscillation is larger than that due to amplification beneath a fixed cylinder.

### 7.5.3 Initial rate of trench formation

The initial rate of trench formation across all of the experiments can be compared to the derivative of Eq. 7.14, which is

$$\frac{dS}{dN} = S_0 \left[ n_e N^{n_e-1} \frac{N}{N_t} \exp \left( - \left( \frac{N}{N_t} \right)^{n_e} \right) \right]$$

At $N = 1$, since $N/N_t \ll 1$, this simplifies to

$$\frac{dS(t)}{dN} = S_0 \frac{n_e}{N_t^{n_e}}$$

which describes the initial rate of trench development with cycles. For the current problem, such an evaluation provides more insight into the trench formation rate than the values of $N_t$ and $n_e$ alone, which are more sensitive to the fitting method and varied more across the experiments. Since the influence of vortex impingements on bed shear stresses cannot be predicted accurately at present, methods to correct the initial rate, or $N_t$ and $n_e$, using an effective mobilised time approach (e.g. Larsen et al. 2017) have not been attempted but would be an area of interest in further work.

Figure 7.17 shows the variation in initial rate of trenching as a function of $KC$ number and $\psi/\psi_{cr}$. There is scatter with respect to both parameters but a general trend of increasing rate with both $KC$ number and $\psi/\psi_{cr}$. These trends indicate that the near
bed velocities and shear stresses due to vortex impingement events, which drive transport rates, increase with both $KC$ and $\psi/\psi_{cr}$. In general initial trench development rate increases at a faster rate with $\psi/\psi_{cr}$ for the smaller diameter cylinder, for which $\beta < 700$, suggesting a dependency of the trench development rate on $\beta$.

### 7.6 Discussion

In this paper, sediment transport and trench development beneath an oscillating cylinder has been explored experimentally and shown to be driven by two main mechanisms, which depend primarily on the $KC$ number: (i) pumping of fluid in the gap leading to bedload transport and (ii) impingement of jet-like flows between vortices that occur as the cylinder moves away from the bed causing localised transport events. Observations indicate that a one-dimensional control volume model with an oscillatory boundary layer assumption predicts $\psi$ required for sediment motion at low $KC$ and for small $h_{min}/D$ when pumping dominates. At larger $KC$ numbers, vortex-bed interactions cause sediment motion to occur at relatively smaller $\psi$ values than predicted by the one-dimensional model.

The trench depth and initial growth rate trends appear proportional to both $KC$ and $\psi$, with a stronger dependency of equilibrium trench depth on $KC$. For $KC < 9$, Figure 7.16 indicates that the relationship between $KC$ and $S_{0}/D$ given by Eq. 7.15 provides an estimate of the trench depth, encompassing the variation across $400 < \beta < 3800$, which covers the lower range of typical field conditions described in Section 7.3. Consequently, Eq. 7.15 may be used to predict the equilibrium trench depth in the field, provided $\psi/\psi_{cr} \gg 1$ (i.e. sediment is mobilised). For motions where $\psi/\psi_{cr} \approx 1$, Eq. 7.15 is likely to represent an upper bound estimate.

An important aspect of the experiments presented in this paper is that only the effect of cylinder motion on transport and trench development has been considered. This was
done to deliberately isolate the effect of fluid-structure interaction (i.e. the induced flow field) from the effects of structure-seabed interaction. The results show that trenches up to \(0.4D\) can form without structure-seabed interaction, provided that \(\psi\) is sufficiently large for sediment mobility to occur; but the resulting trench geometry depends on \(KC\) number. For \(KC < 4\), net transport over multiple cycles forms a prominent central ridge beneath the cylinder due to symmetry. In practice, such ridge formation may be hindered due to structure-seabed interactions if vertical motion amplitudes vary leading to the cylinder plastically deforming the ridge material. At larger \(KC\) numbers, sediment mobility is enhanced by vortex-seabed interaction; but for \(KC < 9\) transport due to vortices occurs on both sides of the cylinder causing an eventually symmetric trench extending below the initial bed level beneath the cylinder. The vortices enhance the peak seabed flow velocities, and therefore the growth rate and final depth of trenches, and continuity arguments alone are not able to explain the increases. Empirical results offer an initial estimate of the likelihood of trench formation and both the depth and rate of trench formation at higher \(KC\).

To fully model the behaviour of an oscillating near-seabed structure, there is a need to incorporate structure-seabed interaction either experimentally or numerically. However, with respect to numerical modelling, the complexities of the transport mechanisms observed in this work suggest that if computational fluid dynamics simulations are used to explore this then they must be able to correctly simulate the vortex dynamics induced by cylinder motion (for \(KC > 4\)). The fluctuating vortex dynamics on a cycle by cycle basis are important to resolve to capture the eventual symmetry of the trench profile (for Regime 2) and detailed time development of trenching. For higher \(KC\) (i.e. \(> 9\)), the trench observations showed significant three-dimensional effects, which suggests the need to capture three-dimensional flows in any numerical modelling and to ensure that an experimental facility is adequate to avoid (or quantify) end effects.

In the context of previous literature, the results suggest that although the current phenomena are ostensibly similar to wave-induced scour of fixed pipelines, there are key differences in the mechanisms of sediment transport for each scenario. For a fixed pipeline, the funnelling of externally applied flows through a narrow gap beneath the cylinder amplifies the near-bed velocity and applied bed shear above those felt elsewhere along the bed away from the cylinder. This has the capacity to transport sediment relatively far from the pipeline. In contrast, for a normally oscillating cylinder, the near-bed pumping velocity beneath the cylinder for \(h_{\text{min}}/D = 0.125\) is on the order of the maximum cylinder velocity for \(KC < 4\) (see Eq. 7.7 and Tom et al. 2018b). For this reason, the trench depths for an oscillating cylinder are less than for a fixed cylinder with an equivalent \(KC \lesssim 6\). However, when vortices enhance the near-bed fluid motions relative to pumping flow at larger \(KC\), trench development intensifies as a stronger function of \(KC\).

The relatively symmetric equilibrium trench geometries identified for Regime 2 are consistent with the findings of Chiew et al. (2016), who showed similarly shaped and sized trenches beneath an elastically mounted cylinder that was allowed to contact the bed and had an approximate \(KC\) number of 8 to 9. Chiew et al. (2016) note that vortex interactions drive the transport, which is in agreement with the findings in the present
work. They also suggest that the flow field, when ensemble-averaged, is symmetric about
the cylinder and that this symmetry leads to the symmetric trench shape. With respect
to the latter, the observations in the current body of work indicate that the evolution of
the trench during individual cycles is driven by instantaneously asymmetric transport
even though the trench shape eventuates to become symmetric. Hence, although when
averaged over many cycles, the flow field appears symmetric (Tom et al. 2018b), this is
only because the flow asymmetry varies frequently between cycles and along the cylinder
for $4 < KC < 9$. The current findings expand on the work by Chiew et al. (2016) by
specifically focusing on the bed dynamics that result without contacting the bed and
the physics that drive trenching over a larger parameter space than previously explored.
The understandings gained from these observations provide a phenomenological basis
to assess trench development for a number of applications. The $KC$ number range
considered covers practical applications ranging from pipelines undergoing lay installation
to operational motions of SCRs.

7.7 Conclusions

The main conclusions from this study are:

1. The sediment transport behaviour beneath oscillating cylinders in otherwise still
water is governed by the near-bed flow field, which varies as a function of $KC$, $\psi$,
$h_{min}/D$ and $\beta$. For the range of cylinder motions and the sediment considered,
trench development occurs local to the cylinder and progresses quickly during the
initial stages of cycling before becoming increasingly slow as the trench develops
towards an equilibrium.

2. The influence of vortex shedding is found to enhance the trench extent and growth
rate, relative to continuity-driven pumping flow.

3. For $KC < 12$ three regimes were identified in the transport behaviour and trench
formation:
   a) $KC \lesssim 4$ where the flow and transport is generally symmetric and driven by
      pumping of fluid in the gap between the cylinder and bed. A prominent central
      ridge with two parallel trenches forms in this regime.
      i. Control volume arguments and an oscillatory boundary layer assumption to
         infer the bed shear stress provide reasonable agreement with the measured
         $\psi$ required to mobilise sediment at a given $KC$ number. This agreement
         reduces as $h_{min}/D$ increases.
   b) $4 \lesssim KC \lesssim 9$ where the flow and transport is periodically asymmetric over
      tens of cycles and localised transport events attributed to jet-like flow between
detached vortices becomes increasingly important in driving transport. The
      trench profiles in this regime are transiently asymmetric but generally tend
      towards a symmetric equilibrium.
i. Simple control volume arguments are not able to predict sediment motion due to the influence of detached vortices, which increase in intensity with both $KC$ and $\psi$. Control volume arguments generally underpredict the potential for sediment motion for these $KC$ numbers.

ii. For $N < 10,000$ the trench depths and initial growth rates increase as functions of both $KC$ and $\psi/\psi_{cr}$. At small cycle numbers, $\psi/\psi_{cr}$ primarily controls; and $KC$ number increases in importance with cycling.

iii. Equilibrium trench depth is found to be a function (primarily) of $KC$ number and reasonably represented by Eq. 7.15 for $KC < 9$ and $\psi/\psi_{cr} > 3.0$.

c) $KC \gtrsim 9$ where the flow and transport are predominantly asymmetric. The direction of the asymmetry switches sides over the course of relatively long time periods (tens to hundreds of cycles), and the trench profiles are significantly asymmetric. The direction of trench asymmetry can still vary over many cycles.
References


Chapter 7 Trench development beneath an oscillating cylinder


Chapter 8

Conclusions

8.1 Summary of findings

This thesis has explored various fluid-pipeline-seabed interaction processes that affect the design of pipeline systems near, on or in the seabed. The work herein has particularly focused on how movement of pipelines (including risers and idealised chains) (a) is resisted by the seabed and how this resistance may change either due to evolution of the seabed or through engineered intervention; and (b) causes changes in the seabed topography due to motion-induced fluid flow and trenching.

This chapter briefly summarises the outcomes of each study comprising this thesis and provides recommendations for future work.

8.1.1 Pipeline-soil interaction in drained materials

Chapters 2 through 4 were chiefly concerned with exploring the bearing capacity (and in particular, the horizontal breakout resistance) of pipelines on sandy seabeds.

In Chapter 2 the combined (vertical-horizontal) bearing capacity of pipelines embedded into a drained material (e.g. sand) was explored numerically using finite element analysis. The analyses covered a range of pipeline embedments (less than one pipeline diameter) and soil properties covering a wide range of soil friction angles and relative densities (i.e. dilation angles) covering those that may be encountered in practice. The particular focus of these analyses was to explore the effect of non-associated flow on the bearing capacity of the pipeline and how this response changes with embedment and friction and dilation angles. Comparison of the non-associated flow results was made with the approach suggested by Drescher and Detournay (1993), which uses upper bound limit analysis with a reduced friction angle to account for non-associated flow.

The vertical bearing capacity was found to be approximately linear with depth for small friction angles, becoming non-linear (approximated as a power law function) as friction angle increases. Relationships were provided to allow prediction of the vertical bearing capacity for known values of $\phi_{cs}$, $\psi$ and assuming the Bolton (1986) relationship to calculate $\phi_{peak}$, which were able to predict experimental vertical bearing capacity data presented by Sandford (2012) within a range corresponding to $\phi_{cs} = \pm 2^\circ$. The approach by Drescher and Detournay (1993) however led to significant underprediction of the vertical bearing capacity calculated using non-associated finite elements. This is because the mechanisms for associated and non-associated flow are dissimilar for vertical bearing
capacity, and hence an associated mechanism does not capture the correct kinematics at failure. Therefore, the relationships provided based on the non-associated finite element analyses are expected to provide a more accurate prediction of the embedment that results from purely static lay processes, while the method by Drescher and Detournay (1993) may overpredict the embedment in practice leading to relatively higher horizontal breakout resistance predictions.

The maximum horizontal capacity for the overall combined loading envelopes (relative to the maximum vertical bearing capacity) was found to generally increase with embedment, but the level of capacity increase due to an increment of increased embedment reduces with increasing friction angle. The general increase with embedment is consistent with the findings of Zhang et al. (2002), but Zhang et al. (2002) did not identify a reducing increase as friction angle increases. The ratio of horizontal to vertical capacity ($\frac{H}{V}$) at low normalised vertical loads relevant for practical application is found to be well predicted using the approach suggested by Drescher and Detournay (1993) whereby an upper bound limit analysis is used with a reduced friction angle to account for non-associated flow and $\psi < \phi_{\text{peak}}$. The better comparison for primarily horizontal loading at low vertical loads occurs because the failure mechanisms between non-associated and associated flow are more similar than for primarily vertical loading conditions. This finding suggests that simple limit analysis approaches can be utilised to assess bearing capacity failure envelopes for non-associated materials at least for relatively small values of normalised vertical load ($V \lesssim 5$), which comprise most practical scenarios. This also suggests that the approach by Drescher and Detournay (1993) may be used with reasonable accuracy for predicting the breakout resistance of pipelines with varying seabed geometries, for instance due to sediment transport (Chapter 4).

In Chapter 3 the variation in uplift resistance of a pipeline buried in dry, loose sand due to the attachment of radial fins was explored experimentally and numerically. This study comprised a series of model scale experiments where the uplift resistance of pipelines with and without radial fins of varying lengths and orientations was compared. Three radial fins oriented $120^\circ$ from each other were considered with (a) one fin oriented in the positive vertical direction at the top of the pipeline or (b) one fin oriented in the negative vertical direction at the bottom of the pipeline. The orientation with one fin in the positive vertical direction was found to increase the uplift resistance by 8 to 25% over the no fin case. Increasing the fin length from 10% to 20% of the pipeline diameter approximately doubled the increase in uplift resistance relative to the no fin case. The orientation with one fin in the negative vertical direction was found to either slightly reduce or have negligible effect on the uplift resistance. An analytical expression based on limit equilibrium was derived to estimate the change in uplift resistance due to radial fins, which limits to the solution by White et al. (2008) in the case of no fins. The limit equilibrium result provides good estimation of the measured changes for both orientations and may be used to predict the resistance for other fin orientations and pipeline sizes. Taken together, these findings suggest that
fins may be used to increase the uplift resistance of buried pipelines, but that caution is needed to ensure that the most optimal orientation of fins is achieved. For instance, if a pipeline has any rotational distortion along its length, then any gains achieved by adding fins in the optimal orientation may be counteracted to some extent by slight reductions in the uplift resistance at other sections.

Particle image velocimetry was also conducted for a set of these experiments to observe the soil displacement mechanisms at failure. The measured capacities and observed failure mechanisms were then compared with finite element analyses. The observations of the soil displacement field at failure reveal that in loose sand, the failure mechanism for the no fin case comprised a zone of uplifted soil extending less than \( 2D \) above the pipeline. This is contrast to the commonly assumed failure mechanism for loose sand whereby the failure planes extend to the surface at a small angle from the vertical (as done in the limit equilibrium solution). Finite element analysis using a Mohr-Coulomb model provided reasonably good comparison to the magnitude of uplift resistances measured in the experiments. The failure mechanism for the measured soil properties were however not consistent with the experimental measurements. Better comparison of the failure mechanisms was attained by assuming a negative dilation angle \( (\psi = -2^\circ) \), which still resulted in resistance estimations within 15\% of the measurements.

In Chapter 4 the changes in the horizontal bearing capacity of a pipeline placed on a mobile seabed was explored through finite element limit analysis, leveraging the findings in Chapter 2 that associated flow provides reasonable estimates of the breakout resistance for loading cases with low applied vertical load. The study assessed the change in resistance inferred from changes in the seabed geometry, due to scour and sediment transport, observed over a period of several years for an operating pipeline on the North West Shelf of Australia. The analyses reveal that more than a twofold increase in the estimated breakout resistance may occur due to changes in embedment from natural pipeline scour and lowering processes over the four years of observation. However, the mechanism of pipeline lowering affected the amount of breakout resistance increase estimated to occur. For the pipeline section with finer sediments, scour initiated at relatively even and closely spaced locations along the pipeline, which appeared to cause more longitudinally consistent ‘sinking’ of the pipeline with larger changes in seabed profiles and larger increases in breakout resistance between surveys. The pipeline section with coarser materials experienced scour initiation at more intermittent and widely spaced intervals. This appeared to lead to larger spans and pipeline ‘sagging’ with longitudinally more uneven lowering. The overall magnitude of embedment and resistance increase between surveys was relatively less than for the finer grained section.

Importantly, both the local embedment of the pipeline and the geometry of the seabed was found to affect the breakout resistance of the pipeline. For a pipeline lowered into a scoured trench, but without increased local embedment from subsequent sediment deposition, the breakout resistance increased relative to a pipeline wished-in-place to the same local embedment but with an overall flat seabed. Conversely, a pipeline with
increased local embedment but lower mid-field embedment (e.g. deposition berms) has significantly lower resistance than a pipeline embedded to the same local embedment but with a flat seabed otherwise. This means that assuming a flat seabed in design may introduce a significant error compared to more realistic sloping seabed geometries, and that standard methods of estimating the breakout resistance, which assume a flat seabed, should be used with caution if the sediment is potential mobile over the design life of the pipeline.

8.1.2 Hydrodynamics of cylinders oscillating normal to a plane wall

In Chapter 5 the fluid mechanics associated with a circular cylinder oscillating normal to a plane wall were studied using two-dimensional direct numerical simulations. The analyses were conducted at $Re = 150$ and covered a range of $KC \leq 10$ and $h_{min} = 0.125$ and 0.5. Particular focus was placed on how the wall changes the flow regimes compared with a cylinder oscillating far from a wall. The observations indicate that the presence of the wall forces the vortex street (in the direction of the wall) to wrap into localised circulation patterns near the wall. The symmetry of these circulation cells and the stage of vortex development for different $KC$ numbers defines the flow regimes observed for oscillation near a wall. For $KC < 5.25$ these circulation cells are symmetric about the cylinder centreline. This is consistent with symmetric flow field patterns observed for an isolated cylinder at similar $KC$ numbers (i.e. Regime A and A*). For $KC > 5.25$ the circulation cells are asymmetric, due to asymmetries in vortex formation and shedding at higher $KC$ numbers. The overall vortex dynamics were found to depend on $KC$ number and $h_{min}/D$ but showed behaviour similar to the isolated cylinder Regimes D and F. The temporally irregular vortex shedding Regime E for the isolated cylinder was observed to be suppressed in the presence of the wall. The side on which vortices form tended to occur more regularly on a single side, not switching over cycles.

The dominant flow mechanisms were observed to vary depending on the $KC$ number and hence flow regime. For $KC < 5.25$, symmetric near-wall pumping flow occurs in the gap in unison with cylinder motion. For $KC > 5.25$, pumping flow still occurs for most $KC$ numbers but only for portions of the cycle and varying with $h_{min}/D$. Interaction of shed vortices with the wall increase the near-wall horizontal velocities relative to pumping flow. Asymmetric near-wall flow occurs even when the cylinder is halted at the end of the cycle, with the strength of asymmetry depending on $KC$ number and vortex development. Pumping flow was found to be explained well using potential flow theory. Control volume arguments also capture the trends in near-wall velocity for pumping flow but this comparison becomes poorer as $KC$ and $h_{min}/D$ increase. Hydrodynamic forces were found to be reasonably well represented by a superposition of the force time history for the isolated cylinder with a correction accounting for suction forces induced by pumping flow in the presence of a wall based on potential flow theory.

In Chapter 6 the fluid mechanics explored in 5 were extended to higher $Re$ through flow
8.1 Summary of findings

In general, the flow regimes identified were qualitatively similar to those shown by Williamson (1985) for the isolated cylinder, except that flow away from the cylinder is confined by the wall, and broadly similar to those identified at smaller $Re$ in Chapter 5. For the higher $\beta$ tests described in this chapter, the side on which the dominant vortex forms was found to switch frequently for $KC \lesssim 8$. By contrast, for $KC \gtrsim 8$ switching of the dominant vortex side was found to be consistent over at least $O(10^{1-2})$ cycles.

In this chapter, observations also focused on the physics driving the near-wall velocity behaviour over a parameter space more representative of practical conditions. This provides insight into the mechanisms causing sediment transport when oscillation occurs over a sediment bed (Chapter 7). Pumping flow similar to that identified at lower $Re$ in Chapter 6 was also observed for portions of the cycle for all $KC \leq 12$ and is again described reasonably well by potential flow theory and control volume arguments. The near-wall velocity field is observed to be significantly affected by vortex interactions, including the occurrence of relatively high velocity flows directed toward the wall due to vortex shedding events.

8.1.3 Sediment transport and trenching beneath an oscillating cylinder

In Chapter 7 sediment transport and trenching that occurs as a result of cylinder oscillation above a sandy seabed was explored experimentally. Experiments were conducted to determine the motions required to cause sediment to move and the rate and extent of trenches that form as a result. Sediment transport behaviour was found to vary with $KC$ numbers, $\psi$ and $h_{min}/D$. For $KC \lesssim 4$, sediment motion was primarily observed in bedload, occurring symmetrically about the cylinder and in unison with cylinder motion. The threshold of sediment motion in this parameter range was found to be well predicted (at least for $h_{min}/D < 0.14$) using control volume arguments and a oscillatory boundary layer assumption. At small $KC$ numbers, a prominent central ridge tended to form beneath the cylinder accompanied by two parallel trenches to either side. However, it is likely that such ridges may be less evident in practical scenarios where the cylinder motion is not purely sinusoidal. In particular, varying amplitude movement may lead to pipeline-seabed interaction that may limit ridge formation over time.

For $KC \gtrsim 4$, the influence of vortex interactions was found to be a primary driver in sediment motion. Due to the influence of shed vortices, sediment mobility occurs at relatively smaller cylinder velocities than predicted by the combined control volume and oscillatory boundary layer approach, and hence this approach does not appear to applicable for larger $KC$ numbers. Trenches beneath the initial bed level tended to form for cylinder motions sufficient to cause sediment movement. Even though sediment motion due to vortex-driven flows is inherently an asymmetric process (Chapter 6), for $KC < 9$ the final trench geometries were symmetric about the centreline. This occurs because the side on which the vortices form switches very frequently, which when averaged
over many cycles leads to approximately equal amounts of net transport on either side of
the cylinder.

The rate of trench formation was found to primarily be a function of $\psi/\psi_{cr}$, while the
equilibrium trench depth was a stronger function of $KC$ number. However, the trends
between different cylinder diameter (i.e. $Re$ number ranges) were found to be different,
although the magnitudes of trench depth were similar. Whether these differences with
$Re$ are a true $Re$ dependency or a reflection of the differing rates of trench formation
between the two cylinder diameters requires further investigation.

8.2 Recommendations for future research

The work undertaken in this research project has revealed a number of areas where
additional research is warranted to further develop the findings of this thesis.

1. The combined vertical-horizontal pipeline bearing capacity analyses described in
Chapter 2 were validated against a set of experimental data presented by Sandford
(2012). However, further experiments exploring in detail the effect of density and
friction angle on the pipeline response would be useful to further validate the results.
In particular, the variation in breakout resistance at low vertical loads, over a wider
range of densities and sediment types, would provide insight into the applicability
of the relationships proposed and be a valuable contribution providing fundamental
insight beyond other databases of pipeline breakout response (e.g. Verley and
Sotberg 1994).

2. The experiments described in Chapter 3 provide insight into the potential for
increasing pipeline uplift resistance by the addition of engineering appendages. An
alternative idea along similar lines is to use vertically-oriented plate(s) beneath
seabed-lain pipelines to (a) increase the resistance to lateral pipeline movement
stability and (b) reduce potential for/gain certainty on the initiation of pipeline
scour leading to changes in the embedment due to seabed mobility.

3. Visualisation of the soil failure mechanisms for buried pipelines in Chapter 3
indicates that the mechanism for a pipeline buried in loose sand at $H/D = 3$ does
not comprise either a deep-flow mechanism or a wedge-type mechanism extending to
the soil surface. Instead, the mechanism appeared to be an intermediate mechanism
comprising a relatively localised zone of soil movement limited to approximately $2D$
above the pipeline. However, although the mechanism was quite different from the
analytical limit equilibrium method proposed and the numerical results, the ultimate
capacities were similar. This discrepancy, which has been noted occasionally in the
literature, may be due to counter-balancing errors in commonly used approaches
that happen to achieve approximately the same level of work dissipation (and hence
are somewhat successful in practice). Further investigation into this discrepancy
may provide insight into the appropriateness of commonly used analyses throughout
geotechnical engineering.
4. The analyses described in Chapter 4 have contributed to a recent body of work reported elsewhere aimed at accounting for and potentially taking advantage of the sediment transport-induced changes in pipeline embedment in the on-bottom stability design of subsea pipelines. However, limited work has been done on the effect of sediment transport-induced embedment changes on the global buckling design of pipelines. The latter problem is arguably more challenging because the changes to pipeline breakout resistance must be reliably considered over both the initial and later stages of operation in order for pipeline systems to function reliably. In the case of global buckling, increased embedment may not necessarily be advantageous for the pipeline designer.

5. The CFD modelling described in Chapter 5 focused on the fundamental response at a relatively low $Re$ number that is below typical values relevant for many engineering applications. Although these results still provide valuable insight into the fundamental physics, it would be useful to further this work at more practical $Re$ numbers. However, this would require the use of an appropriately calibrated turbulence closure model and three-dimensional analysis to model the behaviour accurately.

6. The analyses of Chapter 5 could also be furthered (at the same and/or higher $Re$) by considering the geometry of a scoured trench, based on the experiments described in Chapter 7 or from field observations. Analyses could be conducted statically using a predefined trench profile, or utilising more advanced modelling techniques to simulate the evolution of the trench geometry based on the imposed bed shear stresses in a time-stepping manner. However, the use of appropriate turbulence closure models and three-dimensional analyses will make further studies challenging.

7. The flow visualisation study described in Chapter 6 could be expanded by studying the three-dimensional flow field between an oscillating cylinder and flat boundary. In the current work, two-dimensional flow asymmetries were apparent; however, similar asymmetries have been noted (e.g. Tatsuno and Bearman 1990) to correspond to the movement of three-dimensional vortex cells along the cylinder axis. It would be insightful to determine the influence of similar three-dimensional effects on the frequency of vortex mode-switching described in Chapter 6 for $KC \lesssim 8$.

8. Further flow visualisation studies could be conducted for other shaped objects, such as chains or rectangular foundation-like structures, and also for infrastructure oscillating within a pre-formed trench to experimentally assess the applied bed shear stress induced by motions for trenches that are approximately at equilibrium.

9. A significant area of further research lies in exploring trenching in fine-grained, consolidated materials. Trenching in fine-grained materials is complicated because the effects of pipeline-seabed interaction, potentially leading to pore pressure generation, liquefaction and loss of shear stress, are likely to play dominant roles
in the ability of sediment to be mobilised. A first step into this work would be to synthesise the work done on water entrainment in the near-surface seabed to assess the level of strength loss likely to occur during pipeline-seabed interaction and then explore how this affects the erosion properties of different sediments.

10. Lastly, numerical analyses and experiments should be conducted to explore the level of transferability between results for circular cylinders (i.e. pipelines) and differently-shaped objects, such as chains and rectangular structures. This should be done both in terms of the hydrodynamics of the structures - how appropriate would it be to assume other objects have an equivalent circular diameter for estimating near-bed velocities as well as hydrodynamic forces - and trenching phenomena - and are the results in terms of sediment motion thresholds and trenching rates and extents applicable to other geometries.
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Appendix A

Flow between a plane wall and an oscillating circular cylinder in still water at low KC and Reynolds number

Abstract The flow field that results between a plane wall and a normally oscillating cylinder is explored through a series of particle image velocimetry (PIV) experiments. Sinusoidal cylinder motion is considered for Keulegan Carpenter (KC) numbers between 1 – 10 and Reynolds numbers (Re) less than 5000 (holding $\beta = Re/KC$ constant). A constant minimum gap ratio between the cylinder and wall equal to 0.125 is adopted for all experiments. For sufficiently small $KC$ and $Re$, the measured flow velocities beneath the cylinder show good comparison with both analytical predictions based on continuity arguments and on potential flow theory. At larger $KC$ number asymmetry results, which is not captured in the analytical predictions. Over the full parameter space the results are used to explore the relationship between the motion of the cylinder and the flow velocity near the wall. It is believed that this relationship is important for quantifying the sediment transport beneath offshore infrastructure such as riser pipelines and mooring line chains, which oscillate normal to the seabed.

This chapter has been published as:
A.1 Introduction

Oscillation of circular cylinders normal to a plane wall has engineering applications relating to: the behaviour of pipeline risers near the touchdown zone Bridge and Howells (2007); the vibration of seabed pipelines; mooring line behaviour; and, in the medical industry, microcantilevers Clarke et al. (2005). The fluid flow characteristics around oscillating cylinders without the presence of a wall have been extensively studied in recent decades (Williamson 1985, Sumer and Fredsøe 2006, Lam et al. 2010). However, little work has investigated the effect of a nearby boundary on the hydrodynamics of a normally oscillating cylinder, other than work focussing on vortex induced vibrations in the presence of external hydrodynamic flows.

The presence of a wall is expected to change the flow patterns around oscillating objects in at least two ways:

1. Lateral ‘pumping’ of fluid will occur beneath the object as it approaches the wall and moves away from the wall, and

2. The presence of the wall may affect vortex shedding caused by oscillation.

These effects are explored in this paper for a cylinder oscillating normal to a wall with position (see also Figure A.1)

\[ y(t) = H_{\text{min}} + A - A\cos(2\pi t/T) \]  

(A.1)

where \( H_{\text{min}} \) is the minimum distance to the wall, \( A \) is the amplitude of the motion and \( T \) is the period of the motion. The cylinder is closest to the wall when \( t/T = 0, 1 \).
A.1 Introduction

A.1.1 Application to Steel Catenary Risers

A motivation for the work in this paper is to study the flow velocity near the wall so as to better understand sediment transport and ‘trenching’ that is often observed at the touchdown zone of Steel Catenary Risers (SCRs) on mobile seabeds Bridge and Howells (2007). Predicting the occurrence of sediment transport and the rate (and ultimate depth) of these trenches requires predictions of the velocities induced in the near seabed environment by oscillating objects. The motions of an SCR at any particular location along its length varies depending on vessel motions, location, metocean conditions and the proximity to the touchdown zone. However, the amplitude of motion close to the touchdown zone, which is the focus of this work, is normally on the order of 1-2 diameters. Similar motions are also typical of pipelines during installation in normal operating conditions Westgate et al. (2012). Therefore, motions with Keulegan Carpenter ($KC$) number in the range $1 < KC < 12$ are relevant to oscillating risers or pipelines near the seabed, with

$$KC = \frac{2\pi A}{D}$$  \hspace{1cm} (A.2)

where $D$ is the object diameter (see Figure A.1).

This paper explores the flow induced by a circular cylinder oscillating normal to a plane boundary such that $KC < 10$. This investigation has been undertaken by performing experiments in which a cylinder is oscillated sinusoidally according to A.1 in otherwise still water. The resulting flow field is captured using particle image velocimetry (PIV). In each experiment the Reynolds numbers ($Re$) was held at less than 5000, corresponding to a constant $\beta = Re/KC$ of 500. The Reynolds is given by

$$Re = \frac{U_m D}{\nu}$$  \hspace{1cm} (A.3)

where $U_m$ represents the peak velocity ($= 2\pi A/T$) and $\nu$ the fluid viscosity.

The primary aims of the experiments were to: (a) investigate near bed flow field in the range $1 < KC < 10$; and (b) to demonstrate the applicability of various theoretical solutions to predict flow field beneath the cylinder at different $KC$. To explore the flow beneath the cylinder, both the lateral velocity (parallel to the wall) and the Eulerian particle excursion in the same direction (experienced over the course of a cycle at a given point beneath the cylinder) are analysed, where the latter is made non-dimensional so that

$$KC^+ = \frac{d^+}{D}$$  \hspace{1cm} (A.4)

where $d^+$ is the maximum Eulerian displacement during a cycle. This latter metric, which quantifies the integrated velocity over a half cycle, may also be relevant to the net displacement of sediment from beneath an SCR and, in turn, the dimensions of a trench which might form on a mobile seabed.
Appendix A  Flow between a plane wall and an oscillating circular cylinder

A.2 Theoretical solutions for the velocity near the wall

At small $KC$ number potential flow may be used to predict the flow velocity beneath the riser, since flow separation is expected to be limited. Carpenter (1958) presented a potential flow solution for two cylinders moving in an infinite ideal fluid, represented as an infinite series of image doublets, i.e. two cylinders with the same diameter moving in line with each other is analogous to a single cylinder moving normal to a smooth wall. The complete solution is not reproduced here for brevity; however results computed based on the Carpenter (1958) solution are presented later in this paper. For these solutions we found that three image doublets was sufficient to achieve reasonable convergence (where the minimum gap distance was 12.5% of the pipeline diameter). More doublets are required for small gaps.

An even simpler approximation to estimate the lateral velocity beneath the cylinder can be found from continuity arguments. Taking a Control Volume (CV) beneath the cylinder as a function of distance from the centreline and time (dashed line Figure A.1), the incremental change in volume is equal to the mean flow rate leaving the control volume. Utilising symmetry and assuming sinusoidal motion, it follows that

$$
\bar{u}(x, t) = xU_m \sin \left(\frac{2\pi t}{T}\right) \left( H_{\text{min}} + A - Acos\left(\frac{2\pi t}{T}\right) + \frac{D}{2} - \left(\frac{D^2 - x^2}{2}\right)^{1/2}\right)^{-1}
$$

(A.5)

where $\bar{u}(x, t)$ is the horizontal velocity averaged over the height $h(x, t)$ bounded between the wall and the cylinder, i.e.

$$
\bar{u}(x, t) = \frac{1}{h(x, t)} \int_0^{h(x, t)} u(x, y, t) dy
$$

(A.6)

Equation A.5 is applicable provided $|x| < D/2$. As expected the solution from Eq. A.5 agrees with the mean horizontal velocity beneath the cylinder derived from potential flow. However, at any point beneath the cylinder the two solutions may differ because the velocity obtained from continuity represents a mean velocity.

The theoretical solutions due to continuity and potential flow may approximate the flow when vortex shedding is minimal. For a free-field cylinder, Williamson (1985) described ranges of $KC$ where various vortex shedding regimes occur. Below $KC$ 4, the vortices form symmetrically on the trailing side of the cylinder but do not shed during half-cycles. It is therefore expected that the theoretical solutions should provide a reasonable approximation in this range of motion, before vortex shedding becomes prominent.

A.3 Experimental setup

Experiments were conducted in a section of a recirculating wave flume at the University of Western Australia. The flume has a width of 0.4m and height of 0.5m. Acrylic cylinders with a diameter of either 25 or 40mm were oscillated using a belt-driven linear actuator. The cylinder was attached vertically to the actuator and oscillated horizontally along the flume. A 20mm thick Perspex wall was positioned across the flume and clamped in place.
A.4 Experimental results

during wall tests. A 5-Watt continuous wave Argo-ion laser was used for illumination, producing an approximately 1-2 mm thick light sheet. Synthetic polycrystalline particles with median particle diameter of approximately 1 to 5 $\mu m$ were used. Images were captured using a high speed Photron (FASTCAM SA3), with typical resolution of 768 pixels by 512 pixels at a frame rate of 500 frames/s and an exposure time of 1/1000s. Images were recorded to cover a minimum of 10 cycles for each test combination and the results are presented as ensemble-averaged at various phases. PIV analyses were conducted using the freely available software GeoPIV-RG (Stanier et al. (2016)), which incorporates first-order subset deformation shape functions and inverse compositional Gauss-Newton sub-pixel interpolation to examine cross-correlation of image pairs. For these analyses, subsequent image pairs (i.e. 1/500s time difference) were analysed with 32 $\times$ 32 interrogation patches with 50% overlap. This corresponds to a patch size of about 3.5 mm with the adopted field of field, which is sufficient to describe the overall flow behaviour and velocity characteristics but is not sufficient to, for instance, extract detailed information regarding boundary layers or turbulence.

A.4 Experimental results

A.4.1 Near wall flow

The experimental results are first assessed focusing on the behaviour near the wall to investigate the lateral ‘pumping’ velocities beneath the cylinder. Figure A.2 shows velocity vectors for $KC_4$ (A.2a, A.2b) and $KC_{10}$ (A.2c, A.2d) at two different phases during the cycle. Results for $KC_4$ show that the flow responds to the direction of the cylinder motion symmetrically about the centre of the cylinder. For $KC_{10}$, the flow direction is clearly asymmetric, especially at $t/T=0.1$. Figure A.2c also shows evidence of the formation of a trailing vortex as the cylinder moves away from the wall.

Figure A.3 shows the phase-averaged lateral velocity obtained from PIV at a point located at $x/D = 0.25$ and $y/D = 0.075$ (as identified on Figure A.2) for $KC_4$ and $10$ as a function of cylinder velocity (i.e. at different points in the cycle). The predicted velocities based on potential flow and continuity are also presented. All sets of results generally show a similar asymmetric lemniscate shape. The PIV results for $KC_4$ suggest that the velocity is increased compared to the potential flow predictions when the cylinder is furthest away from the wall. This is believed to be caused by the formation of circulation cells, which will be explored in detail in the next section. Otherwise, the results for $KC_4$ generally show the measured flow velocity agrees reasonably well with the potential flow predictions and the solution based on continuity, particularly as the cylinder approaches the bed.

For $KC_{10}$ a similar figure eight shape is evident but the flow is significantly stronger in the positive direction than predicted by potential flow or continuity. This behaviour is caused by asymmetry in the overall flow field where the vortex formed just before the end of the cycle continues in the previous direction of motion and wraps around the temporarily halted cylinder. For a free cylinder, the vortices are able to convect...
Appendix A Flow between a plane wall and an oscillating circular cylinder

Figure A.2 Example fluid velocity vectors: Vector magnitudes in units of mm/s all scaled by 1/500. Location of velocity comparison noted in black – $x/D = 0.25$; $y/D = 0.075$

Figure A.3 Measured fluid velocity at $x/D = 0.25$, $y/D = 0.075$

away from the cylinder Williamson (1985); however, the presence of the wall prevents this and instead concentrates flow in the gap beneath the cylinder, causing a ‘slingshot’ amplification effect.

The results in Figure A.3 may also be assessed in terms of maximum Eulerian particle excursion over the course of a cycle ($d^+$), which better quantifies the integrated magnitude and direction of the flow. A $10^{th}$ order polynomial was fit to the PIV velocities (measured at the previously specified point) in time - horizontal velocity space. The integration of this polynomial over the time when the velocity is positive outward corresponds to the maximum Eulerian excursion over one (ensemble-averaged) cycle.

The results of the Eulerian excursions are shown on Figure A.4 for $KC$ 4 and 10, as well as other values of $KC$ less than 10, normalised by the cylinder diameter. At low $KC$ the excursion $KC^+$ is relatively small and gradually increases with increasing $KC$ up to 6. Above $KC$ 6 there is an obvious increase in the excursion and divergence from theoretical predictions with increasing $KC$, which is consistent with the previous observations based on Figure A.3b.
A.4 Experimental results

Figure A.4 Eulerian fluid particle displacement at $x/D = 0.25, y/D = 0.075$ normalised by cylinder diameter

Figure A.5 Comparison of horizontal velocities with potential flow: LHS – Experiment; RHS – Potential Flow

A.4.2 Overall flow

Trends in the overall flow behaviour can be observed on Figure A.5, which present velocity vector results: the LHS of each subfigure shows PIV results and the RHS shows the corresponding potential flow result.

At low $KC$ the pumping action beneath the cylinder was evident from Figure A.2a and A.2b. Figure A.3a also demonstrates this to be consistent with potential flow theory, particularly for motion towards the wall. However, this does not explain the outward bias of the flow when the cylinder is away from the wall. Figure A.5a shows ensemble-averaged vectors for $KC = 4$ at $t/T = 0.55$. The presence of an attached trailing vortex is evident but importantly the remnants of the previous half-cycle trailing vortices are seen near the wall. These remnant vortex pairs appear to contribute to circulation cells with the same vorticity direction as the previous trailing vortices, increasing the outward velocity near the wall. The wall prevents these vortex pairs from moving away, which would normally occur for a free-field cylinder. This motion is obviously not captured by potential flow, leading to the negative velocity bias in Figure A.3a.

Velocity vectors for $KC = 10$ are shown on Figure A.5b and A.5c for $t/T = 0.9$ and
0.05, respectively. In both of these figures, a shed vortex is seen above the cylinder, which progresses in the negative direction between A.5b and A.5c. The approximately transverse vortex street appears to remain in the presence of the wall. The street may not be interfered with significantly because the vast majority of vortex formation physical occurs between the extremes of the oscillation.

From Figure A.5b, the wake behind the cylinder is evident from the nascent second vortex forming in the half cycle. Beneath the cylinder the flow appears relatively similar to the potential flow at this stage. However, after the end of the cycle and upon resumption of motion, the preceding wake appears to accelerate around the cylinder and the velocity becomes localised beneath the cylinder, as shown on Figure A.5c. This creates the magnified velocity and asymmetry shown on Figures A.3 and A.4.

A.5 Conclusions

In this paper the flow features around and beneath a cylinder oscillating perpendicular to a wall have been investigated through PIV analyses. Experiments were conducted at low \( KC \) (< 10) and \( Re \) (< 5000).

Results indicate that for \( KC < 4 \) symmetric ‘pumping’ occurs as the cylinder approaches and moves away from the wall. The magnitude and time variation of lateral velocities associated with pumping are reasonably predicted using potential flow theory and continuity arguments. Potential flow diverges from measured velocities near the wall when the cylinder is far from the wall due to counter-rotating circulation cells fed by released vortices following reversal at the end of cycles.

For \( KC > 4 \) vortex shedding starts to dominate the velocity near the wall over the majority of the oscillation cycle. The lateral velocities near the wall become directionally asymmetric and their magnitude significantly amplified compared to predictions based on potential flow or continuity (for a symmetric flow) due to the impact of the trailing wake formed on the previous half cycle as the cylinder approaches the wall.

These findings suggest that potential flow and continuity arguments may be appropriate for predicting fluid motions and sediment transport beneath oscillating objects for low \( KC \) motions but not motions of higher amplitude. Example calculated Eulerian particle excursions are provided, which provide insight into the flows at higher \( KC \).
References


Appendix B

Risk-based assessment of scour around subsea infrastructure

Abstract Scour poses a significant risk to infrastructure placed on mobile seabeds. Seabed mobility is common on the North West Shelf of Australia, in parts of the North Sea, and also occurs in the deepwater Gulf of Mexico, due to loop currents. Scour can undermine structures and, for shallow-skirted mudmat foundations, there can be significant consequences including excessive settlements, tilt and loss of bearing and sliding capacity. However, scour mitigation via engineered protection is costly, and to be avoided if possible.

This paper describes a new quantitative risk-based approach for assessing the susceptibility of subsea infrastructure to scour processes. This probabilistic scour assessment accommodates measurable uncertainties in metocean and seabed conditions, using new characterization techniques. The approach allows operators and owners to better assess the optimum strategy to address scour risk, selecting from mitigation during installation or in-service monitoring, prediction and remediation.

The paper describes (i) best practice approaches for assessing scour susceptibility and propagation rates with and without engineered protection, (ii) new methods for determining the applicable seabed and metocean inputs, (iii) a probabilistic framework for encompassing uncertainties, and (iv) how this approach can be applied in project applications.

Our probabilistic method of assessing and presenting scour risk produces a distribution of estimates of scour depth and time rate. By capturing and quantifying the full range of uncertainties, this method facilitates decision-making by showing the range of potential outcomes and allowing the associated costs and consequences to be evaluated. This approach is superior to deterministic ‘worst case’ calculations, which are often used to assess scour susceptibility.

In summary, this paper provides operators and owners with an improved methodology to unlock Capex and Opex savings through more accurate and informed scour assessments.

This chapter has been published as:
Appendix B Risk-based assessment of scour around subsea infrastructure

B.1 Introduction

Seabed scour around subsea structures occurs when near-bed fluid flow around the structure is sufficiently strong to remove sediment (e.g. Figure B.1). Flow blockage due to the structure causes local amplification of fluid velocities around the structure. This leads to preferential removal of sediment adjacent to the structure if the bed shear stress applied by the fluid is greater than the soil resistance to sediment motion. The spatial distribution and temporal development of scour therefore depends on the flow conditions near the seabed, the geometry of the structure, and the soil erosion properties.

The consequences of scour occurring around subsea infrastructure depend on the magnitude and rate of the scour as well as the criticality of the structure, the type of foundation used, and the primary design drivers for the foundation. For instance scour may have a relatively small impact on the capacity of deep piled foundations, and can be straightforwardly considered in design (Li et al. 2009). In contrast, structures supported by shallow foundations, which may or may not be skirted, have more significant consequences if scour occurs. For instance, the size of subsea infrastructure is often governed by the requirement for sufficient horizontal capacity. Figure B.2 shows failure mechanisms – with the white lines indicating slip planes – beneath a shallow foundation under vertical loading and under predominantly horizontal loading. The latter case is more common for subsea foundations that are loaded from connected equipment and hydrodynamic loading. Figure B.2 shows that the shallow zone of soil adjacent to the foundation makes a significant contribution to the sliding capacity, both via passive resistance against the foundation side, and also as a contribution to the mechanism extending from the base of the foundation. As a consequence, the performance of a shallow foundation could be significantly affected if it is undermined due to scour. Even without failure occurring, vertical settlements may occur if the scour extent is significant, or differential settlements may result if different levels of scour occur around the perimeter of the structure. Both of these types of settlements may be onerous for subsea infrastructure, which often has stringent settlement criteria due to connections to other seabed equipment (e.g. flowlines, jumpers, or spools).

In broad terms, sites with sandy sediments are generally susceptible to scour under most design scenarios, assuming the metocean conditions are not completely quiescent. Scour may occur relatively rapidly and be deep and wide, particularly for sites where the sediment is mobile in the free-field (away from the influence of the structure). However, the spatial distribution and time development of scour around the structure will depend on the particular flow conditions and structural geometry, since the distribution of amplification around the structure varies with these characteristics. This means that scour design will usually focus on the extent and type of scour protection required to prevent scour, with additional attention given to predicting scour that occurs in the intervening time between structure installation and placement of protection.

Sites with silty or muddy sediments generally experience slower scour development compared to sandy sites. This is because the erosion resistance of finer sediments is improved, due to a mechanism analogous to undrained failure in soil mechanics (Mohr
B.1 Introduction

Figure B.1 Scour around a subsea structure foundation (reproduced from Whitehouse et al. 2011)

Figure B.2 Comparison of foundation failure mechanisms – purely vertical load vs $H/V = 0.5$

et al. 2016; Winterwerp and Van Kesteren 2004; Whitehouse et al. 2000), and other chemical and biological effects may influence erosion properties in addition to the particle size and packing density. Rates of fine-grained material erosion depend on the bulk sediment properties of the material.

As a result of the slower development of scour and reduced overall extent of scour over the design life, intelligent ‘observational’ design approaches can be applied to more efficiently consider scour in foundation design on silty or muddy sediments. Options for protection optimization include: (1) exclusion of scour protection if scour is shown to not be a significant risk; (2) spatial optimization of scour protection; and (3) use of Inspection, Monitoring and Remediation (IMR) procedures to reduce protection requirements at installation, by adding protection later where required. Such optimization approaches (described in more detail later in this paper) provide useful tools to complement project scour management plans, such as those described by Harris and Whitehouse (2012).

Such scour protection optimization can only be confidently undertaken if the associated risks are quantified and communicated to project stakeholders. However, this is complicated by significant uncertainties in the input parameters required to quantify scour and
in the prediction models themselves (Briaud et al. 2013). Whether scour will occur, and its extent over time, is often not a straightforward design consideration. Uncertainties exist in the relevant metocean and sediment properties, as well as residual uncertainty associated with prediction methods.

Quantifying these uncertainties to allow better more-informed decision-making forms the motivation of this paper, which presents a practical design approach to account for uncertainties in a probabilistic risk-based scour assessment framework. The optimizations and probabilistic approaches considered could conceivably be applied to sandy sites but this paper focuses on silty and muddy sites where the methodology has been found to offer most benefit. This work draws on concepts recently utilized on multiple projects at silty and muddy sites worldwide.

First, a general model for estimating both the extent and rate of scour for a given set of input parameters, appropriate to a silty or muddy site, is described. This basic model is then compared with a set of new model tests conducted at the University of Western Australia measuring scour development for a model rectangular subsea structure placed on fine-grained materials. These results illustrate the models that now exist to quantify the erosion properties of fine-grained soils and the translation of these properties into predictions of the rate and extent of scour.

Next, the uncertainties in both the inputs to these calculations and in the prediction models themselves are discussed. A probabilistic approach to account for these uncertainties is then introduced and an example implementation is presented. Finally, the design implications and project benefits of adopting a probabilistic approach are discussed.

**B.2 Scour of subsea structures on fine-grained soils**

Typical design practice to assess scour comprises two basic calculation components: (1) equilibrium scour depth and extent; and (2) the initial rate of scour, or characteristic time scale of the scour process. These components may then be combined to determine the scour development in time, and to determine if and when scour protection is required. The quantity and quality of input data to characterize the metocean and sediment properties affects the reliability of these assessments, which can also be backed up by local experience that allows some calibration and verification of scour assessments. The cost and availability of different scour mitigation solutions also plays a role in the design decision-making process.

**B.2.1 Predicting scour in a given flow condition**

The equilibrium scour depth is the depth to which scour would develop in a given flow condition, if the condition was maintained for an infinitely long period of time. This depth is generally taken as a function only of structural geometry and flow condition, i.e. the current and/or wave velocities and directions. Focusing on current only conditions (which are usually most relevant for subsea structures placed in deeper water on silty or muddy seabed), Briaud et al. (1999), for example, presented data suggesting that the
equilibrium scour depth around surface-piercing cylindrical piles in either sand or clay is
the same for a given diameter and flow velocity. Briaud et al. (1999) also suggested a
formula based on model scale tests relating the equilibrium depth to the flow Reynolds
number:
\[ S_{eq} = 0.18 \times Re^{0.635} \] (B.1)
where \( Re \) is the Reynolds number (defined as \( Re = VD/\nu \) where \( D \) is the pile diameter,
\( V \) is the average upstream flow velocity, and \( \nu \) is the kinematic viscosity of water) and
\( S_{eq} \) is the equilibrium depth (in units of mm)).

Although this equation is dimensionally inconsistent, it provides a reasonable fit
to lab testing data in the literature for both sand and clay in currents (Sumer and
Fredsøe 2002). Other potential relationships exist for predicting equilibrium scour depth,
including correlations with Froude number (see, for example, Oh 2009 and Briaud 2014)
Nevertheless, the adopted relationship in Eq. B.1 will be compared with experimental
results and to a database of field test results (bridge pier scour) later in this paper. In
future work predictive formulas different to Eq. B.1 may be used to perform a probabilistic
scour assessment using the same methodology as that outlined in this paper.

Since subsea infrastructure is often significantly shorter, relative to the water depth,
than the surface-piercing columns considered by Briaud et al. (1999) (among others, e.g.
Oh 2009; Sumer and Fredsøe 2002), the predicted equilibrium scour depth due to Eq. B.1
must be reduced to account for the lower flow blockage provided by shorter structures
(e.g. Zhao et al. 2010b, Zhao et al. 2012). Based on model test results, Sumer and Fredsøe
(2002) suggested a reduction of the form:
\[ S'_{eq}/S_{eq} = 1 - \exp(-\beta \frac{H}{D}) \] (B.2)
where \( S'_{eq} \) is the corrected equilibrium scour depth, \( S_{eq} \) is the depth for an infinitely tall
structure, \( H \) is the structure height, \( D \) is the structure diameter normal to the flow and
\( \beta \) is an empirical correction coefficient. The \( \beta \) parameter can be estimated, with some
residual uncertainty, based on the available data. Suggested values of \( \beta \) range from 0.55
(Sumer and Fredsøe 2002) to 2.1 (Zhao et al. 2010b). Recent work at UWA, shortly to be
published, suggests an intermediate range of 0.75 to 1 (Yao 2016).

The second component of a scour assessment is the time-dependent development of
scour and is related to the rate of sediment erosion. Generally speaking, fine-grained soils
(i.e. those with a predominant fraction of grains smaller than 75\( \mu \)m) erode mainly in
suspended transport, as opposed to bedload. This has a strong effect on the rate of scour
development because mobilized sediment is only transported away from the system, not
into it.

Briaud et al. (1999) suggested that a hyperbolic formulation for the rate of erosion fits
well to experimental data, such as:
\[ R_i = f(\tau_{max}) \] (B.3)
where \( S(t) \) is the scour depth at a given time, \( t \) is the elapsed time, and \( R_i \) is the initial
Appendix B Risk-based assessment of scour around subsea infrastructure

Figure B.3 (a) Example of erosion rate data obtained from the laboratory. (b) Example specimen in O-tube and resulting eroded sample from Mohr et al. (2016). Note that flow is directed to the right in the two photographs.

rate of scour. Similar formulations have been shown to also fit well with scour data beneath pipelines on fine-grained soils (Mohr et al. 2016).

In Eq. B.3 the parameter controlling the rate of scour is \( R_i \). Assuming that the initial scour rate in a given flow condition is equivalent to the erosion rate of the sediment at the location of maximum amplified local shear stress close to the structure, this parameter may be estimated empirically according to

\[
R_i = f(\tau_{\text{max}}) \tag{B.4}
\]

where \( \tau_{\text{max}} \) is the maximum shear stress close to the structure and ‘f’ denotes a functional relationship. One approach to obtain this functional relationship is to perform laboratory erosion tests to measure the erosion rate of a sediment at different values of shear stress. Figure B.3, for example, provides data derived from laboratory erosion testing on a single sample of sediment using the mini O-tube flume at UWA (see Mohr et al. 2013 for more details). Also shown on Figure B.3 is an empirical equation of the form

\[
f(\tau) = M(\tau - \tau_{\text{cr}})^n \tag{B.5}
\]

in which \( \tau_{\text{cr}} \) is the critical shear stress, and \( M \) and \( n \) are empirical fitting parameters. It can be seen that the data in Figure B.3 can be fitted well by the empirical Eq. B.5 with \( \tau_{\text{cr}} = 1.5 \text{Pa} \), \( M = 0.84 \) and \( n = 2.5 \). The right hand side of Figure B.3 shows an example test setup of the mini O-tube and an example eroded specimen.

If erosion testing data like that shown in Figure B.3 is not available, the empirical formula in Eq. B.5 may sometimes be used together with unbiased estimates of \( M \), \( n \) and \( \tau_{\text{cr}} \) if published results exist for similar sediments. For example, Roberts et al. (1998) presents fitted coefficients for quartz sediments and Mohr et al. (2016) presents fitted coefficients for sediment sourced from the North West Shelf of Australia. However, in adopting this approach, if published results must be extrapolated the range of uncertainty in the fitted parameters is likely to be considerably larger than if sample specific erosion testing was performed.
The maximum shear stress close to the structure, $\tau_{max}$, is commonly interpreted in terms of a shear stress amplification relative to the free-field shear stress (Whitehouse 1998). An advantage of this interpretation is that the amplification factor (in currents at least) is often almost invariant to the actual velocity magnitude. This means that the bed shear stress in the free-field (away from the influence of the structure) may be calculated for a chosen design flow condition, and then the amplification factor may be applied to this value to estimate the maximum shear stress close to the structure.

Because of its invariance to velocity magnitude, the amplification factor depends primarily on the structure geometry and the direction of the flow relative to the structure. The highest amplification will generally occur next to the structure and reduces away from the structure.

Following Whitehouse (1998), a general amplification factor of 4 to 6 appears to be appropriate for rectangular structures. Li et al. (2009) presented a basic set of results describing how the amplification factor reduces for relatively short structures. These results only considered cylindrical structures and showed a lower limiting reduction of about 60% for very short structures.

Soulsby (1997) reviewed methods for calculating the free-field shear stress depending on the flow and seabed conditions. For current-only conditions, the free-field shear stress can be calculated assuming a logarithmic profile as:

$$U(z) = \frac{u_* \kappa \ln \left( \frac{z}{z_0} \right)}{z_0}$$

$(z)$ is the height above seabed, $z_0$ is the roughness length of the seabed, $\kappa$ is von Karman’s constant ($\sim 0.4$), $U(z)$ is the flow velocity, and $u_*$ is the friction velocity. Hence, provided the velocity is known at some reference height above the seabed, and the bed roughness may be estimated (following Soulsby 1997, for example) then the friction velocity may be evaluated and the free-field shear stress may be determined as $\tau = \rho u_*^2$.

The basic deterministic methodology outlined above may be used to calculate scour for a given set of assumed input conditions. To investigate how well this methodology works in practice, the predictions are compared with results from experiments undertaken to replicate scour close to an offshore subsea structure in the ‘Experimental Program’ section later in this paper. These experiments are novel in that they consider scour around subsea structures (i.e. short columns) in fine sediment, and therefore they build on work presented previously for bridge piers (see, for example, Oh 2009 and Briaud 2014).

### B.2.2 Predicting cumulative scour over multiple flow conditions

To estimate the cumulative scour depth across the project life of a structure a methodology is needed to incorporate multiple flow conditions. One methodology that may be used is the procedure outlined in Briaud et al. (2001) which was introduced to estimate scour around bridge piers across multiple flood events. This procedure is equivalent to using Eq. B.3 for the first metocean event capable of causing scour, and then computing the
Appendix B Risk-based assessment of scour around subsea infrastructure

scour in a second event according to:

\[ S(t) = \frac{t_s}{\left( \frac{1}{R_{i,2}} + \frac{t_s}{S'_{eq,2}} \right)} \]  

(B.7)

where \( R_{i,2} \) and \( S'_{eq,2} \) are, respectively, the initial erosion rate and equilibrium scour depth associated with the second metocean event (computed irrespective of the amount of scour that has already occurred). The time \( t_s \) is an effective time, taken to be:

\[ t_s = t + t' = t + \frac{S_{R_{i,2}}}{1 - \frac{S}{S'_{eq,2}}} \]  

(B.8)

where \( S \) is the cumulative scour which has resulted up to the end of the first metocean event.

Repeated using of Eqs. B.7 and B.8 for subsequent metocean events allow a complete life cycle of metocean events to be accounted for in the scour estimate, although not the potential for backfill. For example, to account for the third metocean event, ‘2’ would become ‘3’ in Eq. B.7 and Eq. B.8, and \( S \) would represent the cumulative scour depth after the second metocean event.

It is worth noting that the non-linear form of Eqs. B.7 and B.8 implies that the total cumulative scour depth due to a series of different metocean conditions is dependent on the order of occurrence of the metocean events. For instance, calculating scour due to the smallest current first (for the appropriate duration) and then adding successively the additional scour due to larger and larger currents (i.e. adding in order of ascending current velocity) typically leads to a higher final estimate of scour depth than if the currents were considered to occur in a random order or in the opposite order (i.e. descending current velocity). For this reason, care should be taken to construct appropriate realizations of metocean events over the life time of the structure.

B.3 Experimental program

B.3.1 Experimental setup

Model experiments were conducted in a recirculating flume (known as the mini O-tube) at the University of Western Australia (An et al. 2013; Mohr et al. 2016). The primary test section is 200 mm wide, 190 mm high and 1800 mm long (see Figure B.4). Rectangular PVC blocks were used as model structures. Tests were conducted with blocks of structural dimensions 40 mm wide, 80 mm long and either (a) 40 mm tall or (b) 180 mm tall. The blocks were placed against the wall of the O-tube working section (see Figure B.4). Assuming symmetry, the corresponding total width of the structures normal to the flow was therefore 80 mm. Only steady currents were considered in this experimental program. Sediment placed in the flume extended to at least 200 mm past the edge of the structure.

A total of 5 scour tests were conducted and are described in Table B.1. The tests used two different sediments, as well as two different structures. In each test, a specified steady
B.3 Experimental program

Table B.1 Model experiments

<table>
<thead>
<tr>
<th>Test</th>
<th>Sediment</th>
<th>Width, W (mm)</th>
<th>Length, L (mm)</th>
<th>Height, H (mm)</th>
<th>Upstream (V_{0.01mASB}) (m/s)</th>
<th>Free-field shear stress (\tau) (Pa)</th>
<th>Equilibrium scour depth, (S'_e) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SS</td>
<td>80</td>
<td>80</td>
<td>180</td>
<td>0.18</td>
<td>0.12</td>
<td>54</td>
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<tr>
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<td>SS</td>
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<td>180</td>
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<td>3</td>
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<td>40</td>
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<td>0.12</td>
<td>24</td>
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<tr>
<td>4</td>
<td>CS</td>
<td>80</td>
<td>80</td>
<td>40</td>
<td>0.35</td>
<td>0.34</td>
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<tr>
<td>5</td>
<td>CS</td>
<td>80</td>
<td>80</td>
<td>40</td>
<td>0.43</td>
<td>0.50</td>
<td>21</td>
</tr>
</tbody>
</table>

Table B.2 Test sediment properties

<table>
<thead>
<tr>
<th>Sediment</th>
<th>Median grain size, (d_{50}) (mm)</th>
<th>Fines content, (f) (%)</th>
<th>Specific gravity, (G_s) (-)</th>
<th>Porosity, (\epsilon) (-)</th>
<th>(M) ((m/Pa^n.s))</th>
<th>(n) (-)</th>
<th>Critical shear stress, (\tau_{cr}) (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>0.19</td>
<td>0</td>
<td>2.67</td>
<td>0.41</td>
<td>NA</td>
<td>NA</td>
<td>0.159</td>
</tr>
<tr>
<td>CS</td>
<td>0.025</td>
<td>83.5</td>
<td>2.73</td>
<td>0.42</td>
<td>5.35e-6</td>
<td>2.49</td>
<td>0.120</td>
</tr>
</tbody>
</table>

current was imposed and the evolution of scour around the structure was monitored until negligible further scour was occurring.

Sediment properties for the two sediments used are presented in Table B.2. This data is based on test results reported by Mohr et al. (2016). The erosion properties (i.e. \(\tau_{cr}\), \(M\) and \(n\)) were derived by performing erosion tests in the mini O-tube using a bed of sediment (with no structure). The rate of erosion was recorded while the flow velocity was increased in discrete steps, following the methodology described by Mohr et al. (2016), leading to results of the form shown in Figure B.3.

The structure erosion test program involved the following variations in test parameters. Tests 1 and 3 were conducted on the silica sand to investigate the effect of structure height on equilibrium scour depth. Test 2 was a repeat of Test 1, but at a higher velocity (in which the free field was mobile) so as to investigate the effect of velocity (or Reynolds number) on the equilibrium scour depth. Test 4 and Test 5 then investigated scour development for a fine-grained material, with a particular focus on determining if the erosion rate for these fine grained soils is predicted well using Eq. B.3.

B.3.2 Experimental results

Figure B.5 presents the scour depth corresponding to the greatest depth below ambient seabed next to the structure, at various points in time. From these figures it is evident that scour develops quickly at first, before slowing towards an equilibrium value. This is qualitatively consistent with design formulas presented above. In the remainder of this section a focus is placed on how well these design formulas quantitatively predict both the equilibrium scour depth and the scour time development in the experiments.

Starting first with the equilibrium scour depth, Table B.1 presents the equilibrium scour depths identified in each experiment. In these results it can be seen that, for a
Appendix B Risk-based assessment of scour around subsea infrastructure

(a) O-Tube Facility test setup with a model structure placed against the wall of the O-tube working section.

(b) Photograph from the facility after some scour accumulation at the leading corner of the structure (flow direction indicated by arrow)

Figure B.4 Experimental test setup. (a) O-Tube facility; (b) photo of the experimental setup

given sediment, the equilibrium scour depth increases with velocity and structure height. To compare these measurements with Eq. B.1, Figure B.6 plots the measured results against Reynolds number (computed using the upstream velocity measured 0.1 m above the flume bed in the experiments). Figure B.6 also presents the same equilibrium scour values following correction for the finite height of the structure using Eq. B.2 and a value of $\beta = 0.75$. It can be seen that following this correction the experimental results agree well with Eq. B.1 and are within the scatter of a larger data set of results presented in Briaud et al. (1999). The experimental results therefore appear to be consistent with Eq. B.1 and Eq. B.2.

Secondly, to investigate the rate of scour in fine grained sediment, Figure B.7 presents the measured scour depth in Test 4 and Test 5 together with predictions based on Eq. B.3.
To form these predictions the erosion rate parameters $\tau_{cr}$, $M$ and $n$ in Table B.2 have been used for the CS sediment together with an amplification factor of 5 (in agreement with that suggested by Whitehouse (1998) for rectangular caissons) to compute $R_i$. It can be seen that with this amplification factor Eq. B.4 provides a very good prediction of the scour development in time. This prediction is much better than that which may be obtained using the empirical formula of Sumer and Fredsøe (2002), pg. 210, which was developed for circular columns in sand and does not account directly for the increased erosion resistance often associated with silty or muddy sediments. Consequently, the agreement shown between Eq. B.4 and the experimental measurements in Figure B.7 reinforces the importance of erosion property measurements in estimating scour rates in these finer sediments.

**B.4 Uncertainties in scour predictions**

The foregoing experiments demonstrate that design equations may be used to predict the scour process reasonably well for model subsea structures if all input parameters are known. In practice, however, uncertainties complicate these predictions. The uncertainties in scour prediction are twofold. First, the input properties – sediment erosion parameters and metocean conditions – are uncertain and have natural variability (in space and time). Second, the prediction models themselves contain uncertainties, which should be considered in prediction calculations. This section outlines these uncertainties and how they can be tackled rationally.

The primary uncertainties in the input parameters are the erosion properties of the seabed sediments, the bed characteristics (e.g. macro-scale roughness), and the metocean conditions applicable over the design life of the structure. The effect of these uncertainties on the development of scour over time is shown schematically on Figure B.8, and is discussed in this section. Figure B.8a shows how, for a given equilibrium scour depth, variation in the initial erosion rate can affect the scour depth during the early stage of the scour process, soon after installation. The erosion rate variation illustrated in Figure B.8a is typical, based on project scenarios in which a good quantity of erosion testing has
Figure B.6 Equilibrium scour depth as a function of Reynolds number. Square symbols represent raw data in Table B.1. Large circle symbols represent corrected scour depth using Eq. B.2. Small circle markers represent original data compiled by Briaud et al. (1999) for different seabed material. Solid black line is Eq. B.1

been performed. The range therefore illustrates the inevitable natural variability often found at sites.

Figure B.8b illustrates the range in long term scour depth (for a constant initial erosion rate) based on the variation in scour depth measurements from field bridge scour databases described later, noting that this parameter is principally controlled by the shape of the structure and the flow conditions, and not by the sediment properties. This uncertainty is discussed in terms of model uncertainty in the next section.

B.4.1 Input property uncertainty

Soil erosion properties. The erosion properties of the seabed soils include the critical (or threshold) shear stress, at which erosion of the sediment begins, and the erosion rate properties at shear stresses equal to or greater than critical. Both of these parameters will naturally vary spatially due to inherent soil variability – this variability can only be captured through sufficient soil exploration and testing. Figure B.9 shows the variation in these properties evident from a set of samples tested in the laboratory. Also shown are probabilistic distributions used to fit the resulting parameters used to parameterize the erosion rate curve (i.e. \( \tau_{cr} \), \( M \) and \( n \) from Eq. B.5). Figure B.9a also shows the resulting \( P_{10} \) and \( P_{90} \) output results for the erosion threshold (onset of erosion) and subsequent erosion rate adopted in the example calculations described later. These samples were taken from a field offshore Australia in the same surficial geological unit. Figure B.9b shows the corresponding probabilistic distributions adopted for the various soil parameters that define the erosion rate curve.

Metocean inputs. The key properties which define the metocean conditions are the magnitude, direction and duration of the near bed velocities. Combined with the
B.4 Uncertainties in scour predictions

Figure B.7 Scour development in time for fine sediments. Markers indicate experimental data, solid line is prediction due to Eq. B.4 and dashed line is prediction due to Sumer and Fredsøe (2002), pg. 210, using $\delta/D = 1$

Figure B.8 Sources of scour depth uncertainty: initial erosion rate (left) and equilibrium scour depth (right)

seabed roughness, these parameters both determine the free field shear stress and the amplification of shear stress near the structure; hence they are inputs to calculating the maximum shear stress $\tau_{\text{max}}$ and duration of scour.

Figure B.10 shows an example set of metocean inputs. These might typically be derived from metocean recordings over the relevant period or be an annual dataset. The example shown illustrates that both the direction and amplitude of current can vary significantly. This in turn would be expected to affect the location of scour and the amplification. Figure B.10b shows that the velocity varies significantly over measurement times and both high frequency, low velocity currents and low frequency, high velocity currents must be considered. Therefore, it is important to consider the full spectrum of potential metocean events for a given project setting and set of sediment properties. The scour behavior may not be controlled simply by the largest events, such as storms.

Figure B.10 shows the simplest possible representation of temporal variations in current speed and direction. In design analyses, the metocean input is considered in a more sophisticated way, for example by modelling annual sequences of monthly current
conditions, and by incorporating the joint occurrence likelihood of speed and direction. The ordering of metocean events will also affect the calculated scour depth because scour development for a given event will depend on the prior history of scour development (e.g. Eq. B.7 and B.8). Various iterations of realistic orderings may be required to sufficiently capture the behavior for risk-based assessments.

Seabed roughness enters into the calculation of the free-field shear stress (c.f. Eq. B.6) and depends on the seabed properties and the presence of any bedforms on the seabed. Soulsby (1997) presents a number of characteristic roughness values for a variety of seabed types. These characteristic values are based on a number of measurements summarized by Soulsby (1983), which show significant variation even for very similar seabed types. Figure B.11 shows a log-normal distribution fitted to the data presented by Soulsby (1983) for ‘mud’ and ‘silty’ seabeds. This was derived by statistically combining the reported mean and standard deviation values for the reported measurements above muddy and silty seabeds to evaluate equivalent values of mean and standard deviation for these seabeds. This can then be used to represent the hypothetical variation in seabed characteristics. If available, project-specific roughness values should be considered for each project setting.

### B.4.2 Prediction model uncertainty

The primary model uncertainties relate to the shear stress amplification factor and the equilibrium scour depth. Scour prediction methods are often based on experimental or numerical results. These methods have uncertainties based on their capability to capture all relevant physics. As more results become available in the literature (e.g. experimental results), these model uncertainties can be refined further.

**Shear stress amplification factor.** The distribution of shear stress amplification around a structure controls not only the overall scour rate but also the spatial distribution of where scour occurs. This is dependent on the shape of the subsea structure and also the type and direction of fluid flow relative to the structure. The amplification can be determined through computational fluid dynamics (CFD) analyses (e.g. Zhao
B.4 Uncertainties in scour predictions

Figure B.10 Example metocean parameters (simplest uncorrelated two-parameter representation)

Figure B.11 Example distribution of roughness length

et al. (2010a)) or model testing (e.g. Tavouktoglou et al. 2015). As noted earlier, Whitehouse 1998 summarized a number of previous studies into relevant amplification factors, suggesting values ranging from 4 to 6 for rectangular structures. Flow towards structure corners is suggested to result in higher values but this depends strongly on the structural geometry. CFD analyses can be used to assess more detailed relationships between flow direction and amplification for particular geometries. Additionally, the amplification will also tend to reduce with distance away from the structure (e.g. Zhao et al. 2010a), which can also be determined in detail using CFD and considered in design as shown in an example calculation later in this paper.

Equilibrium scour depth. The primary model uncertainty in equilibrium scour depth predictions is illustrated by comparing predictions to available field results. At this time, the only extensive prototype scale scour databases available are primarily based on bridge pier scour observations. For this paper, the database presented by Froehlich (1988) has been utilized to demonstrate the model uncertainty in Eq. B.1. This database comprises 83 case studies of scour around bridge piers in both fine and coarse-grained materials. The required correction between Eq. B.1 predictions and the field observations
are described by:

\[ S_{\text{obs}} = \theta z S_{\text{pred}} \]  \hspace{1cm} (B.9)

where \( S_{\text{obs}} \) is the observed equilibrium scour depth, \( S_{\text{pred}} \) is the predicted scour depth according to Eq. B.1 and \( \theta z \) is a correction factor to match the predicted values to the observed values. Figure 12a shows the resulting cumulative distribution of correction factors for piers with a cross-sectional diameter greater than 2 m. This suggests Eq. (1) provides an approximately \( P50 \) prediction of the database. It should be noted that a similar approach to define the model uncertainty in equilibrium scour depth could be adopted for a different predictive formula to that given in Eq. B.1.

In addition to uncertainty in the equilibrium scour depth for surface-piercing columns, there is an additional uncertainty in the depth reduction for shorter structures. Using the combined experimental database of Zhao et al. (2010b, 2012), the potential range of empirical parameter \( \beta \) is shown on Figure B.13a. Parameter values of 1.5 and 3.1 represent those between which 80% of the rectangular (Zhao et al. 2012) data fall. This range has then been used to derive the example distribution of the parameter shown on Figure B.13b.

### B.5 Probabilistic design

#### B.5.1 General approach

A probabilistic risk-based approach provides a rational basis for treatment of uncertainty in scour analysis. Probabilistic techniques are used as standard in determination of metocean design criteria and geotechnical engineering is increasingly embracing stochastic treatment of uncertainty in order to quantify levels of conservatism and risk in design. It is therefore logical to combine these emerging techniques in the analysis of scour, which is a process that lies (literally) at the boundary between metocean and geotechnics.

This section outlines an approach we have used on various projects to capture and
combine the various uncertainties in scour for design purposes and yield quantitative predictions of scour development to inform project decision-making.

The probabilistic techniques considered here are relatively simple Monte Carlo analyses. This technique uses random sampling to select input and model parameters, which are based on probability distributions developed from the available input data and the research and analysis that underlies each of the calculation steps. By making a sufficiently large number of deterministic calculations based on successive selections of random parameters from the corresponding probability distributions, the final results can be interpreted to provide a probabilistic assessment of scour development.

The reliability of Monte Carlo methods depends on the probability distributions assumed for the input parameters and the model parameters. In the case of input parameters for scour assessments, natural variability in soil properties and metocean forcing are quantities that can be measured on a site-specific basis. If this is done, the resulting distributions will be narrower than if they were based on purely regional data, and the final uncertainty will be lower.

Model uncertainty parameters on the other hand are inherent to the prediction models and attempt to account for variations in results in situations when the input parameters are known but the underlying calculation model may not fully represent all of the relevant physical processes. In principle, model uncertainty is negligible if all problem physics are understood and exact solutions are attainable. In practice this is almost never the case and most engineering problems, including scour predictions, contain model uncertainty. This uncertainty should not be overlooked and can easily be incorporated into Monte Carlo approaches.
Appendix B  Risk-based assessment of scour around subsea infrastructure

Table B.3  Example subsea structure dimensions

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>m</td>
<td>20</td>
</tr>
<tr>
<td>Width</td>
<td>m</td>
<td>10</td>
</tr>
<tr>
<td>Height</td>
<td>m</td>
<td>4</td>
</tr>
</tbody>
</table>

B.5.2 Example application

To illustrate this approach, illustrative calculations for an example subsea structure are presented. For this example, the adopted structural dimensions are typical for a subsea structure, such as a manifold or flowline end termination, and are summarized in Table B.3. These dimensions are simplified compared to actual structures – for instance the porosity of the structures with respect to the flow is not considered in this example but does impact the actual scour results and should be considered in design.

Based on the uncertainties described in the previous sections, example probabilistic distributions have been adopted within a Monte Carlo framework to calculate the probability of exceedance for scour depth around the structure. The probabilistic input variables considered in this example include: soil erosion properties \( \tau_{cr}, M \) and \( n \), seabed roughness length. The model uncertainty related to equilibrium scour depth (both the prediction for surface piercing columns and the finite height correction) is also captured through a probability distribution fitted for the correction factor defined in Eq. B.9 and a probability distribution for \( \beta \) based on Figure B.13. In addition, the metocean input includes an annual variation in flow direction and velocity. In practice, other properties can be considered when important, but this example is limited to these probabilistic variables for simplicity.

The adopted metocean inputs are based on conditions relevant to offshore Western Australia. An example set of current velocity measurements and cumulative time of measurement over the course of approximately 1 year was adopted. This information was available for 16 directions (i.e. N, NNE, NE, etc.). The cumulative times for opposing direction (e.g. N and S) were combined to simplify the results and the resulting frequency distribution of these measurements was applied over all time periods considered in the analyses (such as shown on Figure B.10). The amplification factor used in the example calculations was linked to the direction of applied current relative to the structure, with a maximum amplification factor of 6 and a minimum of 4. For this example, we have assumed scour is cumulative for flows that are parallel but in opposing directions. The structure was assumed to be oriented such that the long side of the structure is in the north-south direction. The adopted distributions of sediment erosion properties are typical for natural sediments that are present on Australia’s North West Shelf.

B.5.3 Example results

The results from the example Monte Carlo simulation are shown on Figure B.14. Figure B.14a shows the probability of exceeding a given scour depth for different periods of scour evolution. Figure B.14b shows the evolution of scour depth predicted for different exceedance probabilities. These results can be implemented into risk-based decision-
B.5 Probabilistic design

making to determine the probability of exceedance for a given design scour depth (beyond which the scour may be unacceptably high, for instance).

If scour is measured at a certain time after installation, then the probability of exceedance curves can be used to forecast the further development of scour and aid in determining whether or not proactive scour protection would be required. The Monte Carlo analysis can also be revisited – in a Bayesian manner – using the observation to refine the input parameters and generate updated forecasts of scour evolution.

Further design optimization can also be attained by combining such a probabilistic approach with more detailed amplification factor distributions from CFD analyses. One way that scour protection prevents scour is by blocking amplification from being sufficient to remove sediment. Therefore, amplification distributions inform the extent of scour protection required.

A practical implementation of this knowledge is to iterate the probabilistic analysis for successive refinements of scour protection geometry, using modified amplification factors based on the CFD results. This allows the scour protection to be tailored to provide only the required level of protection, reducing unnecessary seabed coverage. For instance, if provision of scour protection for areas close to the structure where CFD shows amplification greater than 3.5 were assumed, the amplification factor used on previous calculations could be updated, based on this information. Figure B.15 shows updated probabilistic results generated showing the reduced scour development, which provide updated likelihoods for excessive scour including some scour protection at installation.

The probabilistic results are also useful for design approaches in which an inspection regime with the potential for later installation of scour protection is used as a substitute for installation protection with the initial placement of the structure. This approach may prove more cost effective by reducing the overall costs if the protection proves unnecessary or delaying costs by transferring them from CapEx to OpEx, although installing scour protection around operating infrastructure can be difficult.

The probabilistic calculations can be used to assess the feasibility of this approach, which enables potential scour hazards to be observed and addressed after installation. Any
scour that occurs can be identified during inspection of structures following installation. The timeframe for these inspections – specifically, how soon after installation the first should be, and how frequently thereafter – can be selected based on results similar to those shown on Figure B.14. For a given inspection schedule, the results allow the likelihood that unacceptable scour could occur unobserved to be determined.

Furthermore, the probabilistic calculations can be extended to estimate the expected cost of the adopted scour mitigation strategy. Figure B.16 illustrates an extended analysis showing example predictions of scour evolution for Option A (upfront installation as CapEx) and Option B (inspection and remediation as OpEx), based on the example $P_{95}$ results from Figures B.14 and B.15. The particular probability cases that lead to scour equal to the design limit at the end of the design life are shown. For Option B, the full family of scour trends for all likelihoods (not shown in Figure B.16, but given in Figure B.15a) can be considered together to estimate (i) the probability that remediation is needed, (ii) the distribution of times at which that would occur, and (iii) the extents of scour protection that would be needed at those times in order to keep the scour within the design limit over the design life. A cost model overlain on these calculations then indicates the potential value in deferring the installation of scour protection as compared to installing it at the same time as the structure. Figure B.16 does not illustrate the alternative lower probability results (e.g. $P_{5}$) whereby protection could be avoided at installation and at any time during the design life.

B.6 Conclusions and recommendations

Seabed scour can significantly affect the performance of shallow skirted subsea infrastructure in locations with potentially mobile sediments. Provision of scour protection to mitigate these risks can often be cost intensive for projects. However, a robust physical understanding of the phenomenon can provide the opportunity for project cost savings by allowing for risk-based optimization of scour protection (or the potential for elimination
B.6 Conclusions and recommendations

This paper summarizes some of the physics of scour and methods for scour prediction. It demonstrates how the scour behavior of fine-grained materials is different compared to sands, with scour evolving more slowly. This can be accurately predicted using simple calculation methods combined with sediment-specific tests to characterize the erosion properties. When applied in a probabilistic approach, these tools can improve the basis of design decisions and reduce costs.

A set of scour experiments was conducted with model rectangular subsea structures in both sand and fine-grained silt materials. The results show that the prediction methods presented in this paper accurately predict the scour development in the case where all inputs are known. Importantly, the use of Eq. B.3 with laboratory erosion tests (to assess the sediment erosion properties) is shown to provide very good agreement with the measured scour rate for the fine-grained sediment. Having established these findings, the calculation method can be confidently extended to a probabilistic approach that accounts for other uncertainties in the scour process. A Monte Carlo probabilistic approach was presented that incorporates various uncertainties including input uncertainties (e.g. sediment erosion properties) and model uncertainties (e.g. comparing Eq. B.1 to a bridge pier field database).

In summary, the following recommendations are made, to provide a rational basis for risk-based scour protection design, in regions where subsea structures may be susceptible to scour:

1. Gather site-specific sediment erosion data via small scale flume testing of samples, to provide geotechnical input to the scour assessment
2. Assess – ideally via site-specific data – the annual or seasonal temporal and directional variation in near seabed wave and current velocities, to provide the metocean input to the scour assessment (the peak value alone is inadequate)
3. Use a probabilistic method – such as the Monte Carlo approach described here – to determine the likely evolution of scour depth with time. This must include both input uncertainties and model uncertainties.
4. If the analysis is marginally acceptable, use CFD to provide structure-specific...
seabed shear stress amplification factors. This will reduce the model uncertainty, and narrow the output range.

5. If the analysis indicates that scour protection is required, use iterative probabilistic analysis to determine the required extent of protection. CFD assists by accurately defining the shear stress amplification field around the structure, minimizing unnecessary protection.

These recommendations allow scour risk to be quantified, improving the basis on which decisions regarding the use and extent of scour protection can be made. This potentially allows reduced capital expenditure in cases where scour risk is marginal and can be mitigated during operation if the requirement eventuates.
References


Appendix C

Effect of drainage on upheaval buckling susceptibility of buried pipelines

Abstract. This paper investigates the effect of soil drainage on the uplift resistance of buried pipelines, and their susceptibility to upheaval buckling. The uplift resistance of buried pipelines is considered through analytical and numerical predictions for both drained and undrained conditions. Combinations of soil strength parameters for typical soils are estimated based on common correlations. For certain ranges of typical normally consolidated soil conditions, particularly those with high critical state friction angles, the drained uplift resistance may be lower than the undrained resistance. This observation is important because in typical practice only drained or undrained behaviour is considered depending on the general type of soil backfill used. In this case, the critical or minimum uplift resistance may be overlooked. Further, the changing undrained uplift mechanism between shallow and deep conditions is investigated. It is found that the common approach of considering the minimum of either a local (flow around) or global (vertical slip plane) failure can overestimate the uplift resistance in normally consolidated soils.

This chapter has been published as:
C.1 Introduction

High temperature and pressure oil and gas pipelines are often buried in the offshore environment to prevent upheaval buckling and to provide protection from the environment and fishing activity. The installation process may comprise mechanical trenching followed by pipe placement or jet-trenching of a pre-laid pipeline. The subsequent engineering properties of the backfill material will vary depending on the trenching method used as well as the environmental conditions and time after backfill placement. The backfill material by the commencement of operation may be a loose, normally consolidated soil; although the reconsolidation process can take significant amounts of time particularly for clayey materials. However, in addition to the changes in soil properties over time, the response of a buried pipeline will also vary depending on loading conditions and the resulting boundary conditions of the uplift problem.

The latest DNV Recommended Practice for design of buckling high temperature and pressure pipelines (DNV 2007) comments that both the drained and undrained uplift resistance of a buried pipeline should be checked and that the lower of these should be adopted for the lower bound resistance in design. In general, DNV (2007) notes that the drained resistance will provide an upper bound on the available resistance, particularly at depth when the deep flow-round mechanism dominates. However, it is useful to parametrically explore the variation in response with a coherent set of soil parameters to ascertain under what conditions these assumptions hold, and whether simply adopting the lowest calculated resistance is always appropriate.

From critical state soil mechanics, a soil element may, depending on its state relative to the critical state line, tend to behave in either a dilating or contracting state. For instance, a soil element loose of critical will generally have a drained strength greater than its undrained shear strength for a given stress path, all else equal. Similarly, for a dense of critical soil, the drained strength can be lower than the undrained shear strength owing to dilation and negative pore pressure generation. However, the collapse load of a loaded system does not necessarily follow this logic directly, as the overall response depends on the boundary value problem and the kinematics of the corresponding failure mechanism.

This paper explores the variation in the uplift response of buried pipelines for a range of typical soil conditions and burial depths ($H/D < 4$). In particular, the focus is placed on the differences in drained and undrained behaviour and the conditions under which typical assumed failure mechanisms hold.

C.2 Upheaval buckling

Early stage design for upheaval buckling of buried pipelines is often done following the analytical solution presented by Palmer et al. (1990). This method was developed by solving the beam bending equation for a range of pipeline properties and nondimensionalising the results to provide estimates of the uniform download required (i.e. pipeline weight plus burial cover resistance) to prevent buckling. The approach assumes that buckling
C.2 Upheaval buckling

The resulting design curve given by Palmer et al. (1990) is shown on Figure C.1. States that lie above the line are stable and states below the line are susceptible to upheaval buckling.
C.3 Uplift resistance of buried pipelines

Methods for calculating the uplift resistance of a buried pipe are usually based on limit analysis, limit equilibrium or finite element analyses. For undrained conditions, these methods generally provide similar calculated resistances, as shown for instance by Merifield et al. (2001). However, for drained conditions, substantial variations occur between the different methods, owing to the effect of flow rule of drained materials at failure. The various solution methods are contrasted in the following section.

The ability of limit analysis to accurately calculate a solution to a given problem is predicated on the assumption of associated flow of the material at failure (i.e. the plastic strain increments are normal to the yield surface in conjugate stress space). Since under undrained conditions associated flow is generally true, limit analysis approaches provide rigorous upper and lower bounds on the load at failure for a given set of soil properties. For this study, finite element limit analysis (FELA) was used within the commercially available software, OptumG2 (OptumCE 2017) to calculate the collapse loads, and the mean of the upper and lower bound results are presented throughout. For each run case, 5000 elements were used with 5 remeshing steps.

On the other hand, under drained conditions associated flow does not generally hold; and the use of limit analysis will significantly overestimate the uplift resistance of buried pipelines. Drescher and Detournay (1993) suggested the use of modified friction angles to account for non-associated flow. Although this approach can provide reasonable (though not rigorous) estimates of limit loads for many problems including lateral breakout of partially embedded pipelines (Tom and White 2018), Krabbenhoft et al. (2012) showed that this approach can overestimate the uplift resistance for buried anchors and pipelines. Therefore, other methods are required for accurate calculation of drained uplift resistance.

Limit equilibrium is another commonly utilised method for calculating the uplift resistance; however, these solutions do not have the mathematical rigour of the limit theorems. Instead, the flow rule at failure and the resulting failure mechanism can be modified to match experimental evidence, for instance. White et al. (2008) proposed a limit equilibrium approach for pipes buried in sand, which assumed an inclined slip mechanism (Figure C.2a) and assumed the stresses on the failure planes to be related to in situ $K_0$ conditions. Following this approach, the peak uplift force per unit length can be calculated by:

$$ P_{up,D} = \gamma' D H \left(1 - \frac{\pi D}{8H}\right) + \gamma' H^2 f_{up,D} \quad (C.3) $$

$$ f_{up,D} = \tan(\psi) + (\tan(\phi_{peak}) - \tan(\psi)) \left[1 + \frac{K_0}{2} - \frac{(1 - K_0) \cos(2\psi)}{2}\right] \quad (C.4) $$

This formula was found to provide good estimates for the uplift resistance for sand (i.e. drained failure) over a range of relative densities and embedment levels, compared with experiments.

Similarly, a limit equilibrium solution for undrained resistance can be derived in a straightforward manner by assuming straight slip surfaces from the edge of the pipeline to the surface, as shown on Figure C.2b. From this, it follows that the uplift resistance is
calculated as:

\[ P_{up,UD—shallow} = \gamma'HD\left(1 - \frac{\pi D}{8H}\right) + 2Hs_{u,ave} \]  \(\text{(C.5)}\)

where \(s_{u,ave}\) is the average undrained shear strength of the material above the pipe springline. This ‘shallow’ undrained failure mechanism is often compared to a ‘deep’ flow mechanism, where the failure is confined to the local area around the pipeline and does not extend to the soil surface:

\[ P_{up,UD—deep} = N_c s_{u,sp} D \]  \(\text{(C.6)}\)

where \(N_c\) is some undrained bearing capacity factor, usually taken as 9 to 12, depending on interface roughness and \(s_{u,sp}\) is the undrained shear strength at the pipeline springline. It is common practice to take as the operative undrained uplift resistance the lower of Eq. C.5 and C.6.

DNV (2007) recommends similar analytical approaches to calculating the uplift resistance. For the undrained resistance, the lesser of either shallow or deep failure (referred to as ‘global’ and ‘local’ in the RP) is assumed to apply. The global failure in DNV is the same as Eq. C.5; but the local mechanism corresponds to Eq. C.6 with the additional recommendation of a strength reduction factor (of between 0.55 and 0.8) on the resistance. Discussions in the DNV RP suggest that the local mechanism will generally dominate as embedment depth increases, but only after the adoption of the empirical strength reduction factor on the undrained shear strength. For drained resistance, DNV suggests using a similar approach to Eq. C.3 but with specified empirical values recommended instead of Eq. C.4 as per White et al. (2008).

Displacement-based finite element analysis can also be used to calculate the uplift resistance. In this study, the multiplier elastoplastic module in OptumG2 has been used calculate the drained, non-associated flow resistance as an additional comparison. A Mohr-Coulomb model using the same number of elements as the limit analysis results with 3 remeshing iterations for each time substep has been utilised. Details regarding this approach can be found from the program manual.

This paper aims to address two primary questions related to pipeline uplift design, given these typical design approaches:

1. How valid are analytical approaches such as Eq. C.3-C.5 as compared to more realistic FEA and FELA analyses?
2. For a given soil with a consistent set of drained and undrained soil parameters, how important is the effect of potential soil drainage on the pipeline design, and which drainage condition is critical?

C.4 Soil parameters

Appropriate soil parameters for determining buried pipe uplift resistance vary depending on soil type, deposition process, loading history and rate of loading, amongst others. From a high level perspective, the range in soil properties may be understood through a
Appendix C Effect of drainage on upheaval buckling

Table C.1 Adopted soil parameters

<table>
<thead>
<tr>
<th>Set</th>
<th>$\phi'_cs$</th>
<th>$\frac{s_u}{\sigma'_{v0}}$</th>
<th>$\gamma'$</th>
<th>$K_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>0.15</td>
<td>10</td>
<td>0.74</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>0.25</td>
<td>10</td>
<td>0.58</td>
</tr>
<tr>
<td>C</td>
<td>35</td>
<td>0.35</td>
<td>10</td>
<td>0.43</td>
</tr>
<tr>
<td>D</td>
<td>45</td>
<td>0.45</td>
<td>10</td>
<td>0.29</td>
</tr>
<tr>
<td>E</td>
<td>55</td>
<td>0.55</td>
<td>10</td>
<td>0.18</td>
</tr>
</tbody>
</table>

critical state framework, e.g. Schofield and Wroth (1968). Using the Modified Cam Clay model, a normally consolidated soil under conditions has an undrained shear strength defined as $s_u = q_f/2$. Since critical state conditions are defined by the critical state stress ratio:

$$M = \frac{q_{cs}}{p'_{cs}} = \frac{6 \sin(\phi'_cs)}{3 - \sin(\phi'_cs)}$$  \hspace{1cm} (C.7)

where $q_{cs} = q_f$ is the deviatoric stress at failure, $p'_{cs}$ is the mean effective stress at failure, which is defined as half the consolidation mean effective stress, $p'_{v0}$, and $\phi'_cs$ is the critical state friction angle. Under isotropic conditions, it follows from Eq. C.7 and the definitions of undrained shear strength and mean effective stress at failure (for Modified Cam Clay) that:

$$\frac{s_u}{\sigma'_{v0}} = M \approx \frac{\phi'_cs}{100} \text{ to } \frac{\phi'_cs}{95}$$  \hspace{1cm} (C.8)

where $\sigma'_{v0}$ is the initial (consolidation) vertical effective stress. Note that the range in denominator corresponds to the relative stiffness, $\kappa/\lambda$, adopted in Modified Cam Clay.

For simplicity, we adopt a value of $\phi'_cs/100$ for the remainder of this paper.

The lateral earth pressure coefficient at rest, $K_0$, which is required for drained analyses, has been estimated based on Jaky’s equation, as a function of critical state friction angle:

$$K_0 = 1 - \sin(\phi'_cs)$$  \hspace{1cm} (C.9)

This equation results in an increasingly small value of $K_0$ as friction angle increases. The appropriateness of this can be seen in the relative comparison of results for the limit equilibrium solution with the elastoplastic finite element results, which only uses $K_0$ for the initial stress development, but subsequently allows stresses to develop as appropriate.

For analyses conducted herein, the unit weight is assumed to be constant for all analyses. For the remainder of this paper, the soil is assumed to be normally consolidated and therefore for drained analyses, the peak friction angle is equal to the critical state friction angle and the dilation angle is zero. The parameters adopted for this study are shown in Table C.1. The friction angles (or undrained shear strength ratios) considered span the typical range from soft deep-water clays to carbonate silts, which can typically have critical state friction angles upwards of 45°.

### C.5 Implications for design - validity of uplift solutions

Common industry practice for calculating the undrained uplift resistance for pipelines buried in clay is to adopt the lesser of the calculated resistance assuming either shallow
C.5 Implications for design - validity of uplift solutions

Figure C.3 Undrained uplift resistance – comparison of numerical results and code guidance. Solid circles: shear strength gradient of $0.25\gamma'z$. Squares: uniform strength of $0.25\gamma'H$. Red symbols: shallow mechanism. Blue symbols: deep mechanism. Failure mechanisms represented by contours of vertical displacement at failure.

or deep undrained failure. DNV (2007) suggests a deep flow around (‘local’) mechanism, similar to Eq. C.6, with some empirical strength reduction factor and a shallow (or ‘global’) mechanism is similar to Eq. C.5. For the deep mechanism, the shear stress is assumed to equal the strength at the pipe springline; but for the global mechanism, the strength is the average strength between the soil surface and the pipe springline. The lower resistance resulting from the adoption of the strength reduction factor ($\eta$) is suggested to account for lower than expected resistances evident in experiments cited by DNV (2007).

In reality, there is a gradual transition of the mechanism from a ‘global’ to ‘local’ mode, rather than an abrupt change. To explore the transition from shallow (global) to deep (local) mechanism, a set of undrained limit analyses were conducted for a pipe at different embedment depths. In this case, the undrained shear strength ratio was assumed to be constant at 0.25 and pairs of analyses were conducted by adopting undrained shear strength profiles of: (a) strength gradient of $0.25\gamma'z$ and (b) uniform strength of $0.25\gamma'H$. Figure C.3 shows the limit analysis results for these parameters with the assumption of a rough interface on the top of the pipeline and a no tension interface on the bottom half (representing the possibility of free water being left after the pipe is laid, creating a water-filled zone at hydrostatic pressure). The limit analysis results with the strength gradient compare well with, but are slightly higher than, the shallow mechanism (Eq. C.5) up to an embedment depth of about 3. The slight overestimate is due to differences in interface condition on the bottom half of the pipe for the limit analysis, in which a purely smooth interface is not assumed which is different than that assumed by Eq. C.5 (for detailed discussion of these differences, see Houlsby and Puzrin 1999). However, for deeper embedments the global straight slip mechanism overestimates the capacity as
Appendix C  Effect of drainage on upheaval buckling

Compared to limit analysis, which is consistent with the findings of Martin and White (2012). For the strength gradient case, the deep flow around mechanism does not occur until an embedment of at least 10 times the pipe diameter. Instead, a more complex ‘global’ mechanism occurs, with slip planes that are not simply straight and vertical. The limit equilibrium predictions from the minimum of Equations C.5 and C.6 are inaccurate in this case, but the adoption of a strength reduction factor ($\eta$) happens to predict the resistance reasonably well. This is due to compensating errors – the incorrect failure mechanism (Eq. C.6) combined with an adjusted strength can give the correct resistance.

Figure C.3 also shows limit analysis results for the second set of parameters assuming a uniform strength corresponding to the strength at the pipe waist. These results are always higher than the strength gradient results. However, at embedment depths greater than about 5, the evident mechanism switches from a shallow-type mechanism to a predominantly deep flow mechanism. The transition for this assumed uniform strength profile scenario occurs at a shallower embedment depth than predicted by the DNV RP without a strength reduction and is approximately consistent with a strength reduction value of about 0.8, which is at the upper end of the recommended range in DNV (2007).

Figure C.4 compares the peak uplift response for the various analysis methods, for both drained and undrained conditions, as a normalised uplift factor, defined as:

$$N_\gamma = \frac{P_{up}}{\gamma'HD}$$ (C.10)

The undrained results show the global limit equilibrium results to generally underpredict the limit analysis results, as discussed for Figure C.3. This is, as mentioned previously, consistent with previous results in the literature and is simply due to the more simplified failure mechanism and interface conditions assumed in the limit equilibrium solution. Figure C.4 also shows comparison of the drained results for the various analysis techniques. Again, the global limit equilibrium method is generally lower compared to the finite element analysis results. This deviation is due to the assumption of the failure mechanism as well as the assumed initial stress distribution, which is likely to be more accurately modelled in the finite element results. The limit equilibrium solutions thus provide generally conservative estimates of the uplift resistance, assuming that lower resistance is conservative, which is the case for pipeline upheaval buckling analysis.

C.6 Implications for design - critical drainage condition

Limit equilibrium results for each soil parameter set are shown on Figure C.5 in terms of the ratio of undrained to drained resistance for different embedment ratios, $H/D$. These results suggest that regardless of embedment ratio less than 5 the ratio of undrained to drained resistance decreases with increasing critical state friction angle for normally consolidated (i.e. non-dilating) soils. Figure C.5 also shows select results from limit analysis and finite element analysis for combinations of soil parameters and embedment depths, and additionally results for the same friction angle but with the initial $K_0$ set to 0.5 and 0.7. The trends from these additional analyses are very similar to the limit
Osman and Randolph (2011) presented a closed-form analytical solution for consolidation around a laterally loaded pile, which is analogous to a buried pipeline undergoing sustained uplift loading. The authors show the form of the normalised excess pore pressure dissipation curve for an element of soil at the pile/pipe interface is relatively invariant over a range of soil properties and zones of influence. Although the dissipation at the pipe interface does not exactly capture the drainage behaviour of the relevant failure mechanism for this case, this does provide some insight into the relevant timeframe.

From Figure 7 of Osman and Randolph (2011), the normalised time to 90% excess pore pressure dissipation, $T_{90}$, is about 1.25 for a range of soil properties, where this
Appendix C  Effect of drainage on upheaval buckling

Increasing $I_{cs}$ from 15° to 55°

$K_0 = 0.5$  $K_0 = 0.7$

$K_0 = 1 - \sin(I_{cs})$

Figure C.5 Ratio of drained to undrained uplift resistance

Figure C.6 Contours of months required to achieve 90% excess pore pressure dissipation at pipeline interface

coefficient is defined as:

$$T_{90} = \frac{ct}{D^2}$$  \hspace{1cm} (C.11)

where $c$ is some coefficient of consolidation, $t$ is the time after loading and $D$ is the diameter of the pipe.

These results are represented on Figure C.6 in terms of coefficient of vertical consolidation versus pipeline diameter as contours of months of sustained load required to achieve 90% consolidation. These contours provide some indication of the relative timeframes over which drainage may occur, although the results are only approximate since the drainage on the actual mechanism is not capture precisely by this solution. The range of consolidation coefficient values shown approximately cover reported values in the literature for offshore clays and carbonate silts. The results suggest that for even relatively permeable soils and small diameter pipelines, a drained response may not occur until several weeks after switch on. In materials with high friction angle, where the drained resistance can be lower than the undrained resistance, this means delayed upheaval buckling should be considered as a possibility.


### Table C.2 Example pipeline properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $D$</td>
<td>m</td>
<td>0.32</td>
</tr>
<tr>
<td>Wall thickness, $t$</td>
<td>m</td>
<td>0.02</td>
</tr>
<tr>
<td>Coefficient of thermal expansion, $\alpha$</td>
<td>$1/C^o$</td>
<td>$1.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Young’s modulus, $E$</td>
<td>GPa</td>
<td>206</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>Vertical imperfection height, $\delta/D$</td>
<td>-</td>
<td>0.4 to 2.5</td>
</tr>
<tr>
<td>Imperfection wavelength, $L/\delta$</td>
<td>-</td>
<td>100</td>
</tr>
<tr>
<td>Pipeline submerged weight, $w'_0$</td>
<td>kN/m</td>
<td>2</td>
</tr>
<tr>
<td>Internal pressure, $p$</td>
<td>MPa</td>
<td>25</td>
</tr>
<tr>
<td>Operational temperature change, $\Delta T$</td>
<td>$C^o$</td>
<td>70</td>
</tr>
<tr>
<td>Pipeline embedment, $H/D$</td>
<td>-</td>
<td>3.5</td>
</tr>
<tr>
<td>Soil effective unit weight, $\gamma'$</td>
<td>kN/m$^3$</td>
<td>6</td>
</tr>
<tr>
<td>Soil critical state friction angle, $\phi'_{cs}$</td>
<td>$^o$</td>
<td>45</td>
</tr>
<tr>
<td>Undrained shear strength ratio, $S$</td>
<td>-</td>
<td>0.45</td>
</tr>
</tbody>
</table>

### C.7 Upheaval buckling

To illustrate the implications of these findings on the design of buried pipelines, a simplified design example is useful to show how the uplift behaviour influenced by drainage might affect decision-making for pipeline burial. Table C.2 lists relevant pipeline properties for this exercise, which represent feasible operating conditions in practice. The size of the vertical out-of-straightness feature, $\delta/D$, is taken to be a variable. This parameter can be determined from surveys of a pipeline after laying and before backfilling. The required backfill cover depth may be selected based on the size of out of straightness (OOS) features, defined by $\delta/D$. Figure C.7 shows the results for the adopted parameters for both the undrained and drained resistance as well as the corresponding download factor ratios for each of the two resistances. These results assume that the hypothetical imperfection wavelength, $L$, is linearly related to the vertical imperfection height, $\delta$, such that the ratio of $\delta/D$ is kept constant at 100.

For the scenario indicated on Figure C.7, upheaval buckling is predicted for vertical imperfection heights, $\delta/D$, between 0.91 and 1.3 at startup, which corresponds to undrained failure. However, in this case there is also a wider range of potential pipeline geometries (i.e. imperfection heights) from about 0.56 to 1.74 for which upheaval buckling could occur at some later stage, once drainage is able to occur over the failure mechanism. This simplified example highlights the need to consider the full range of upheaval buckling scenarios that could occur over the life of a pipeline system, which include variation in potential soil response, e.g. drainage.

### C.8 Conclusions

The uplift resistance of buried pipelines has been explored using rational sets of soil properties for normally consolidated soils, considering both drained and undrained conditions. For soils with relatively low friction angles, the undrained resistance is generally lower than the drained resistance; however as friction angle increases, the
drained resistance becomes lower than the undrained resistance. Both the undrained and drained resistance should be checked in situations where the uplift load may be sustained for some time, as illustrated by a design example using typical pipeline parameters.

Limit analysis results also find that the uplift resistance is not simply the minimum of the conventional shallow (vertical slip) and deep (flow around) mechanisms in undrained conditions. Instead, there is a gradual transition between the two types of mechanism, and the deep flow mechanism may not occur until a burial of more than 10 pipe diameters, for a linearly increasing soil strength profile. Current design codes recommend adopting the minimum of the vertical slip and flow around mechanisms, and apply an arbitrary strength reduction factor that compensates for the simplified choice of mechanism. However, a more reliable approach is to select uplift resistance factor that represent the true failure mechanism.
References


