Pressure-dependent elastic properties of sandstones, with applications to seismic reservoir characterisation and monitoring

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Abstract

Knowledge of the pressure dependence of rock properties is important for a diverse range of earth science problems, including seismic characterisation and monitoring of subsurface fluid flow processes, common in hydrocarbon, groundwater, and CO₂ sequestration reservoirs. This thesis focuses on developing a better understanding of the pressure-dependent elastic properties of unconsolidated and partially consolidated sandstones. The key contribution of this thesis is to improve the prediction and interpretation of pressure-dependent rock properties and their effects in seismic data.

A long-standing research problem is that theoretical models of velocity-pressure response often do not match laboratory measurements, and alternately, empirical regressions fit to lab data do not extrapolate accurately to wider pressure ranges since they have little or no underlying physical basis. In this thesis we develop a new model to describe the pressure sensitivity of the bulk and shear moduli for weakely cemented sedimentary rocks. The model incorporates effects of sedimentary compaction and the concept of critical porosity, including a relationship to account for porosity and density change with pressure. We demonstrate a method to estimate the critical porosity constraint at zero effective pressure using grain-size distribution data. The strong physical basis of this model, along with a unique two-stage model parameter fitting process, enables us to predict the elastic properties of unconsolidated sediments at a wide range of pressures, including low effective pressure when only data at higher pressures is available. The model is tested on laboratory measurements for various rock samples and fits well over a wide range of pressures. The new model should have implications for the improved prediction and interpretation of 3D and 4D seismic data, including for pressure prediction, quantitative AVO analysis, seismic reservoir characterisation, and time-lapse fluid-flow monitoring.
Additionally, we investigate further the reasons for observed discrepancies between theoretical predictions of unconsolidated sediment elastic properties and laboratory measurements. We show that grain contact heterogeneity and porosity variation (sorting and compaction) can explain the observed discrepancies, and develop a new modified grain contact theory model that incorporates these effects. We also show how porosity variations associated with sorting and compaction can explain observed Poisson’s ratios > 0.25, thereby overcoming weakness in previously published grain contact theory models. The calibration parameters of the new model agree with the observations of published granular dynamics simulations and provide an improved fit to laboratory data compared to existing models, including the ability to describe the correct variation in Poisson’s ratio with effective pressure, and values of Poisson’s ratio > 0.25.

Finally, we present a reservoir characterisation and monitoring case study, in which we analyse time-lapse seismic data acquired over a high-pressure water injector in the Carnarvon basin, offshore Australia. We show how rock physics diagnostics and seismic reservoir characterisation can be used to enhance the interpretation of 4D seismic data in terms of pressure and grain cementation effects. It is commonly assumed that certain rock properties remain constant during fluid production and injection, including porosity and grain contact cementation. However, in this thesis we show evidence from the Carnarvon basin case study that water injection at high pressure can damage or weaken grain contact cement in poorly consolidated sands, resulting in significantly larger time-lapse seismic anomalies than expected from initial feasibility work. These observations are important for assessing 4D seismic feasibility and interpretation in sedimentary rock in general, particularly in Macedon reservoir sands and other geologically analogous systems.

In summary, the work presented in this thesis allows for the improved analysis, prediction, and interpretation of pressure-dependent rock properties and their effects in seismic data, as a result of our development and analysis of new, more accurate relationships between stress-dependent elastic properties and seismic data.
Acknowledgements

I would like to thank all the friends, professors, and students that have made my time at UWA a unique and enjoyable experience.

I wish to express my sincere thanks to my coordinating supervisor W/Prof. David Lumley. This thesis profited greatly from his advice, and I am indebted to David for his continuous support throughout my time at UWA. David showed me the broad importance of understanding the pressure sensitivity of rock elastic properties, and his expertise in 4D seismic methods allowed me to quickly see the shortcomings in existing models. Together we were then able to develop the foundations of the work presented in this thesis. I greatly appreciate the opportunity David has given me to present the results of our research at a number of national and international conferences, along with his detailed and continual tips on how to improve my technical writing and presentation skills (culminating with winning Best Student Paper at the 2012 SEG Annual Meeting). I have learned a significant amount from David, including both technical and professional skills, that I will carry with me throughout my entire career.

David has setup an environment within the Centre for Petroleum Geoscience and CO₂ Sequestration (CPGCO2) that has meant my research has benefited greatly from regular discussions with other students in the research group. I wish to thank all former and present students within the group for their guidance, friendship and support.

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Life is great!
Publications and statement of candidate contribution

In accordance with the University of Western Australia’s regulations regarding Research Higher Degrees, this thesis is presented in part as a series of journal papers. The contribution of the candidature and co-author(s) for the papers comprising chapters 3, 4, 5, and 6 are hereby set forth.

**Paper 1.** The paper presented in Chapter 3 is first-authored by the candidate and co-authored by W/Prof. David Lumley, and is published as:


W/Prof. David Lumley introduced the candidate to the problem of being able to accurately predict the pressure sensitivity of rock elastic properties at low effective pressure. W/Prof. David Lumley suggested that a physical constraint should be placed at zero effective pressure when fitting available data. The candidate and W/Prof. David Lumley together proposed the use of the critical porosity constraint for uncemented sediments. The form of the SL model was developed by W/Prof. David Lumley and the candidate, including the idea to incorporate compaction within the model. The two-stage fitting procedure of the model was also developed together. This paper outlines the theoretical development of the model. All technical work, presentations at conferences, and writing was completed by the candidate.

**Contributions:** Candidate 60%, W/Prof. David Lumley 40%
**Paper 2.** The paper presented in Chapter 4 is first-authored by the candidate and co-authored by W/Prof. David Lumley and A/Prof. Jeffrey Shragge, and is published as:

Saul, M. J., D. Lumley, and J. Shragge, 2013, A practical method to predict the elastic properties of unconsolidated sands at low effective pressure: Geophysics, to be submitted.

The candidate and W/Prof. David Lumley developed the so-called “SL model” as in Chapter 3. This paper concerns the application of the SL model in a realistic setting to determine uncemented sediment elastic properties. The paper discusses in detail how to fit the SL model to available laboratory data using the two-stage fitting procedure. The paper also covers the candidates developed method to estimate the critical porosity constraint using grain-size distribution data. A/Prof. Jeffrey Shragge helped the candidate to code the developed fitting procedure in matlab. All technical work, presentations at conferences, and writing was completed by the candidate.

**Contributions:** Candidate 70%, W/Prof. David Lumley 20%, A/Prof. Jeffrey Shragge 10%

**Paper 3.** The paper presented in Chapter 5 is first-authored by the candidate and co-authored by W/Prof. David Lumley and A/Prof. Jeffrey Shragge, and is published as:

Saul, M. J., D. Lumley, and J. Shragge, 2013, Modeling the pressure sensitivity of uncemented sediments using a modified grain contact theory: Incorporating grain relaxation and porosity effects: Geophysics, 78, no.5, D327-D338.

W/Prof. David Lumley introduced the candidate to some of the issues associated with existing effective medium theory. The candidate developed the modified grain contact theory presented in this paper to overcome these issues, including investigating the effects of porosity variations. W/Prof. David Lumley and A/Prof. Jeffrey Shragge provided further valuable guidance and assistance during the work, to ensure it remained inline with the scope of the candidates thesis. All technical work, presentations at conferences, and writing was completed by the candidate.

**Contributions:** Candidate 80%, W/Prof. David Lumley 10%, A/Prof. Jeffrey Shragge 10%
Paper 4. The paper presented in Chapter 6 is first-authored by the candidate and co-authored by W/Prof. David Lumley, and is published as:


W/Prof. David Lumley organised for the Enfield dataset to be made available to the candidate from Woodside Energy Ltd. W/Prof. David Lumley gave the candidate an overview of the 4D seismic data and Enfield project, including discussing the unexplained anomalies associated with large pressure changes in the field. The candidate and W/Prof. David Lumley then together developed a suitable plan to interpret the 4D seismic data around one of the large water injectors in the field. The candidate proposed the use of rock physics diagnostics to infer the relative pressure sensitivity of sands within the field. The candidate proposed the idea that contact-cement must be mechanically weakening due to water injection, thus explaining the large observed 4D anomalies. The candidate also proposed the use of the velocity-pressure-cement model in order to forward model the effects of changing pressure and cement volume. All technical work, presentations at conferences, and writing was completed by the candidate.

Contributions: Candidate 70%, W/Prof. David Lumley 30%

I certify that, except where specific reference is made in the text to the work of others, the contents of this thesis are original and have not been submitted to any other university.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Publications and statement of candidate contribution</td>
<td>vii</td>
</tr>
<tr>
<td><strong>1 Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Research objective</td>
<td>1</td>
</tr>
<tr>
<td>1.2 How do seismic properties vary with pressure?</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Background and motivation</td>
<td>2</td>
</tr>
<tr>
<td>1.3.1 Seismic wave propagation</td>
<td>4</td>
</tr>
<tr>
<td>1.3.2 4D seismic fundamentals</td>
<td>6</td>
</tr>
<tr>
<td>1.4 Approach - chapter descriptions</td>
<td>9</td>
</tr>
<tr>
<td><strong>2 Rock physics models</strong></td>
<td>19</td>
</tr>
<tr>
<td>2.1 Foreword</td>
<td>19</td>
</tr>
<tr>
<td>2.2 Introduction</td>
<td>19</td>
</tr>
<tr>
<td>2.3 Porosity and mineralogy</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Cementation and sorting</td>
<td>23</td>
</tr>
<tr>
<td>2.5 Rock compressibility and fluid substitution</td>
<td>25</td>
</tr>
<tr>
<td>2.6 Pressure</td>
<td>27</td>
</tr>
<tr>
<td>2.7 Conclusions</td>
<td>30</td>
</tr>
<tr>
<td><strong>3 A new velocity-pressure-compaction model for uncemented sediments</strong></td>
<td>37</td>
</tr>
<tr>
<td>3.1 Foreword</td>
<td>37</td>
</tr>
<tr>
<td>3.2 Abstract</td>
<td>37</td>
</tr>
<tr>
<td>3.3 Introduction</td>
<td>38</td>
</tr>
<tr>
<td>3.4 Theory and model development</td>
<td>39</td>
</tr>
<tr>
<td>3.4.1 Porosity-depth trends in uncemented sands</td>
<td>39</td>
</tr>
<tr>
<td>3.4.2 Depth-pressure relationship</td>
<td>40</td>
</tr>
<tr>
<td>3.4.3 Moduli-porosity trends in uncemented sands</td>
<td>41</td>
</tr>
<tr>
<td>3.4.4 A new seismic velocity-pressure model</td>
<td>44</td>
</tr>
<tr>
<td>3.5 Application to laboratory data</td>
<td>45</td>
</tr>
<tr>
<td>3.6 Discussion and conclusions</td>
<td>50</td>
</tr>
<tr>
<td>3.7 Acknowledgments</td>
<td>53</td>
</tr>
<tr>
<td><strong>A Variable transform $\phi(z)$ to $\phi(P_{\text{eff}})$</strong></td>
<td>59</td>
</tr>
<tr>
<td><strong>B Derivation of proposed model</strong></td>
<td>61</td>
</tr>
</tbody>
</table>
## Contents

### 4 Predicting the elastic properties of unconsolidated sandstones at low effective pressure: A practical method

4.1 Foreword ........................................... 63
4.2 Abstract ........................................... 63
4.3 Introduction ....................................... 64
4.4 Background theory ................................ 67
   4.4.1 The SL model ................................ 67
4.5 Application to data ................................ 69
   4.5.1 Estimating sediment critical porosity ..... 69
   4.5.2 Fitting the SL model ......................... 70
   4.5.3 Prediction at low effective pressure ..... 72
4.6 Discussion ......................................... 76
4.7 Conclusions ....................................... 79
4.8 Acknowledgements ................................. 80

### 5 Modelling the pressure sensitivity of uncemented sediments using a modified grain contact theory

5.1 Foreword ........................................... 87
5.2 Abstract ........................................... 87
5.3 Introduction ....................................... 88
5.4 Theoretical background for EMT models .... 92
   5.4.1 Bulk modulus ................................ 93
   5.4.2 Shear modulus ................................ 94
   5.4.3 Poisson’s ratio ............................... 95
   5.4.4 Practical issues .............................. 95
5.5 Porosity variation in uncemented sediments 97
   5.5.1 Sorting trends ................................ 97
   5.5.2 Compaction-induced porosity loss ........ 99
5.6 Proposed modified GCT model ......... 103
   5.6.1 Incorporating grain relaxation and porosity effects 103
5.7 Proposed method for calibrating model parameters 106
   5.7.1 Calibration parameter relations for well-sorted sediments 108
5.8 Discussion ......................................... 113
5.9 Conclusions ....................................... 115
5.10 Acknowledgements .............................. 115

### 6 The effects of pressure and cementation on 4D seismic data - a NW Australia example

6.1 Foreword ........................................... 121
6.2 Abstract ........................................... 121
6.3 Introduction ....................................... 122
6.4 Field overview .................................... 123
   6.4.1 Geology ...................................... 124
6.5 Initial 4D seismic feasibility ............. 126
6.6 4D seismic data analysis ............. 127
   6.6.1 4D amplitudes ................................ 133
   6.6.2 4D time shifts ............................... 133
   6.6.3 New rock physics model .................... 138
6.7 Rock physics diagnostics of rock texture and diagenesis 141
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7.1</td>
<td>A new modelling approach for pressure sensitivity</td>
<td>149</td>
</tr>
<tr>
<td>6.8</td>
<td>4D interpretation and forward modelling</td>
<td>152</td>
</tr>
<tr>
<td>6.8.1</td>
<td>Amplitudes</td>
<td>153</td>
</tr>
<tr>
<td>6.8.2</td>
<td>Time shifts</td>
<td>157</td>
</tr>
<tr>
<td>6.9</td>
<td>Discussion</td>
<td>159</td>
</tr>
<tr>
<td>6.10</td>
<td>Conclusions</td>
<td>162</td>
</tr>
<tr>
<td>6.11</td>
<td>Acknowledgements</td>
<td>163</td>
</tr>
<tr>
<td>7</td>
<td>Conclusions</td>
<td>169</td>
</tr>
<tr>
<td>7.1</td>
<td>Summary</td>
<td>169</td>
</tr>
<tr>
<td>7.2</td>
<td>Contribution and future research</td>
<td>172</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Global energy demand by fuel type (International Energy Agency, 2012). . 3

1.2 4D seismic sensitivity to initial dry-rock compressibility and fluid compressibility contrast. Weak 4D signals are produced when the dry-rock compressibility is low and the fluid compressibility contrast is small. Conversely, strong 4D signals are produced when the dry-rock compressibility is high (e.g. unconsolidated sands) and when the fluid compressibility contrast is large (e.g. CO₂ injected into brine). Reproduced from Lumley (2010). . . . 8

2.1 Physical interpretation of the Hashin-Shtrikman bounds for the bulk modulus of a two-phase material. Upper bound when \( K_1 > K_2 \) and lower bound when \( K_1 < K_2 \). Reproduced from Mavko et al. (1998). . . . . . . . . . . . 22

2.2 Saturated compressional velocity measurements versus porosity for a number of samples, compared with the Reuss and Voigt bounds. Blue circles show consolidated sandstone samples from Han et al. (1986), red circles show uncemented sediment and glass bead samples from Zimmer (2003), and green circles show suspension data from Hamilton (1956). . . . . . . . . . . . . . . 23

2.3 Schematic diagram of three theoretical models for high-porosity sands. The thickening of circles represents the addition of contact cement from the initial sand pack. Reproduced from Avseth et al. (2005). . . . . . . . . . . . 25

2.4 (a) Pressure path for a typical sample, "Sa 35% small" (Zimmer, 2003). (b) Porosity versus effective pressure for each pressure step. (c) Dry \( V_P \) versus effective pressure. (d) Dry \( V_S \) versus effective pressure. We see the porosity and velocity exhibit hysteresis due to the loading-unloading pressure steps. 29

3.1 (a) Porosity versus effective pressure. The data are measured by Zimmer (2003) on the dry, unconsolidated Pomponio Beach sand. The data are selected from the loading cycles of the measurements. The solid black line in (a) shows the proposed fit to the data from equation 3.7. The black dashed line shows the fit of the form of equation 3.2. The black line in (b) shows the corresponding prediction of the density-pressure relationship from equation 3.9. . . . . . . . . . . . . . . 42

3.2 Normalised saturated bulk modulus, \( K_{sat}/K_m \), versus porosity, coloured by effective pressure, for unconsolidated samples from Zimmer (2003). The saturated bulk modulus has been calculated from the dry ultrasonic velocity measurements and Gassmann fluid substitution. Blue curves show equation 3.11 for various values of normalised pore-space compressibility, \( K_\phi/K_m \). For a constant effective pressure, the bulk modulus varies as the inverse of a linear function of porosity. The red line is the Reuss bound. . . . . . . . 43
3.3 (a) Fit of proposed ‘SL’ model to dry bulk and shear moduli for the Pomponio Beach sand sample from [Zimmer, 2003]. Data are for the loading cycles of the experiment. (b) Corresponding prediction of dry compressional and shear velocities calculated from the moduli fits in (a) and equation 3.17. 46

3.4 Fit of proposed ‘SL’ model to dry bulk and shear moduli for the Galveston Beach sand sample of [Zimmer, 2003]. Data are for the loading cycles of the experiment. Predictions from the Hertz-Mindlin and Walton Smooth models are shown for direct comparison with the proposed model. 47

3.5 Dry compressional and shear velocities for the Galveston Beach sand sample of [Zimmer, 2003]. Data are for the loading cycles of the experiment. Predictions for the SL, Hertz-Mindlin, and Walton Smooth models are calculated from the moduli fits in Figure 3.4 and equation 3.17. The fit of the empirical single exponential model of [Yan and Han, 2009] shows that the model fails to fit the data at low effective pressures. 48

3.6 Dry bulk to shear modulus ratio for the Galveston Beach sand sample of [Zimmer, 2003]. Data are for the loading cycles of the experiment. The ratio varies significantly at low effective pressures, which cannot be predicted with the theoretical Hertz-Mindlin and Walton Smooth models. 49

3.7 Fit of proposed SL model to Ottawa Beach sand data from [Domenico, 1977]. (a) Porosity versus effective pressure with the fit of equation 3.7. (b) Dry compressional and shear velocity versus effective pressure with predictions calculated from proposed model. 49

3.8 Diagram of loading and unloading trends for uncemented sedimentary rocks. (a) Effective pressure versus saturated compressional velocity. (b) Effective pressure versus porosity. 51

4.1 Dry (triangles) and saturated (circles) $V_p/V_S$ ratio as a function of effective pressure, coloured by porosity, for uncemented sediment samples [Zimmer, 2003]. Saturated $V_p/V_S$ ratio has been calculated using the dry measurements and [Gassmann, 1951] fluid substitution. 66

4.2 Normalised saturated bulk modulus, $K_{sat}/K_m$, versus porosity, coloured by effective pressure, for unconsolidated samples from Zimmer (2003). The saturated bulk modulus has been calculated from dry ultrasonic velocity measurements and [Gassmann, 1951] fluid substitution. The blue curve is the Reuss bound (equation 4.3). 68

4.3 (a) Critical porosity versus Trask sorting coefficient for 48 unconsolidated samples from Beard and Weyl (1973). The generalised relationship of Scherer (1987) (equation 4.8) is shown in green. (b) Actual versus predicted critical porosity for unconsolidated samples from Zimmer (2003). 71

4.4 Two-stage fitting process of the SL model for the Galveston Beach sand sample from [Zimmer, 2003]. (a) Linear fits to low and high effective pressure ranges in log bulk modulus versus effective pressure space. (b) The linear fits to the low and high pressure ranges in (a) are extended over the full desired pressure range in bulk modulus versus effective pressure space. (c) The functions in (b) are added in a least-squares sense using equation 4.10 to determine the final SL fit of bulk modulus versus effective pressure. 73
List of Figures

4.5 (a) Saturated compressional velocity versus effective pressure, calculated with SL fit to dry moduli and Gassmann fluid substitution. The velocity approaches the correct suspension velocity given by Wood’s equation (coloured triangles on vertical axis) for the calculated critical porosity of each sample (0.428 for Galveston Beach and 0.339 for glass beads). (b) Saturated $V_p/V_S$ ratio versus effective pressure. Saturated $V_p/V_S$ ratio approaches infinity at zero effective pressure as shear velocity approaches zero.

4.6 Fitting SL model to predict moduli and porosity at low effective pressures (0 – 4 MPa) when only data at high effective pressures (4 – 20 MPa) is available. Data used in the fit is shown as blue triangles, with actual data in the low pressure range shown in cyan. (a) to (c) involve the same fitting steps as outlined in Figure 4.4. (d) Porosity versus effective pressure with fit of equation 4.4. The critical porosity at zero effective pressure is estimated using grain size distribution data and equation 4.7.

4.7 (a) Comparison between SL model (solid line) and the single exponential model of Yan and Han (2009) (dashed line) in predicting water saturated compressional velocity at low effective pressure using only data in the range of 4 – 20 MPa (dark blue triangles). (b) Water saturated $V_p/V_S$ ratio versus effective pressure.

4.8 (a) Dry compressional velocity versus effective pressure for unconsolidated samples (Zimmer, 2003). (b) Porosity versus effective pressure for the same samples in (a).

4.9 Dry bulk modulus versus effective pressure. Result of fitting SL model to predict data at low effective pressure when data is only available in the range 4 – 20 MPa. The dashed line shows the result of extending the high pressure exponential fit to 2 MPa within the two-stage fitting process, to exploit the observation that the ‘rollover’ in elastic properties with effective pressure usually occurs at approximately 2 MPa (see Figure 4.8).

5.1 (a) Dry Poisson’s ratio as a function of effective pressure, coloured by porosity, for un cemented sediment samples (Zimmer, 2003). Data are from the loading cycle of the experiments. (b) Fraction of nonslip contacts as a function of effective pressure, coloured by porosity, for the same samples in (a). Negative fractional values are unphysical.

5.2 Friable-sand model applied to bulk and shear moduli of un cemented samples from Zimmer (2003), calibrated to the highest porosity sample. Data are from the loading cycle of the experiments. Friable-sand model with the well-sorted end member calculated using the HM model and the coordination number relation of Murphy (1982) for (a) dry bulk and (b) shear moduli. The friable-sand model with the well-sorted end member is calculated using the BA model for (d) dry bulk and (d) shear moduli.

5.3 Porosity versus effective pressure, coloured by Poisson’s ratio, for un cemented samples from Zimmer (2003). Data are from the loading cycle of the experiments. The highest porosity sample (Galveston Beach sand) has been fit with equation 5.20 (black line).
List of Figures

5.4 Effect of packing changes on bulk and shear moduli. Laboratory measurements at different effective pressures (colour) for different loading cycles are shown as triangles (Zimmer, 2003). The solid curves show the modified friable-sand model after fitting the high-porosity-data point using the BA model as in Figure 5.2. Dashed lines show the BA model, in which porosity is changed in the model directly. ................................................. 102

5.5 Dry bulk modulus versus effective pressure, coloured by porosity, for the Pomponio Beach sand sample (Zimmer, 2003). Data are from the loading cycle of the experiment. The solid red curve shows the prediction by the HM model. Accounting for changes in coordination number with pressure within the HM model gives the red dashed curve. Blue curves show the result of accounting for compaction-induced porosity loss with the MHSLB as in the friable-sand model. In all cases, quartz mineral elastic properties have been used as input to the HM and friable-sand models. ....................... 103

5.6 Investigation of the weight factor on the dry Poisson’s ratio for the proposed modified GCT model (solid curve). Comparison to the BA model (dashed curve) shows that the new model can predict Poisson’s ratio values > 0.25. ......................... 105

5.7 Calibration parameters for the proposed modified GCT model, for three uncedmented samples (Zimmer, 2003). Circles are Galveston Beach sand, squares are Santa Cruz aggregate, and triangles are glass beads (GB 35% Tiny 2 from Zimmer (2003)). Data are from the loading cycle of the experiments. (a) Inverted weight factor w, fit with equation 5.23. (b) Inverted calibration parameter c, fit with equation 5.24. (c) Porosity-pressure data fit with equation 5.20. (d) Calibration parameter c versus $P_{\text{eff}}$. ......................... 108

5.8 Inverted weight factor w versus effective pressure, for the Galveston Beach sand sample (Zimmer, 2003). The relationship for the grain relaxation correction factor determined from the GD simulation study of Sain (2011) is shown as the dashed curve. ......................................................... 109

5.9 Fit of proposed modified GCT model to (a) bulk modulus, (b) shear modulus, and (c) bulk to shear modulus ratio, for uncemented samples from Figure 5.7. Circles are Galveston Beach sand, squares are Santa Cruz aggregate, and triangles are glass beads. Data are from the loading cycle of the experiments, and quartz mineral elastic properties have been used as input to the model. ......................................................... 110

5.10 Comparison between the proposed modified GCT model and Dutta et al. (2010) for the Pomponio Beach sand sample (Zimmer, 2003). Data are for the loading cycle of the experiment, and quartz mineral elastic properties have been used as input to the model. Comparisons for the fits to (a) $K_{\text{dry}}$, (b) $G_{\text{dry}}$, and (c) $K_{\text{dry}}/G_{\text{dry}}$ are shown, along with corresponding predictions of (d) $V_p$, (e) $V_S$, and (f) $V_p/V_S$. Two predictions of shear modulus are required for the Dutta et al. (2010) model, one for calculating $V_p$ (Dutta Vp) and one for calculating $V_S$ (Dutta Vs). ......................................................... 111

5.11 Comparison between the proposed modified GCT model and Dutta et al. (2010) for predicting elastic properties of a glass bead sample (Zimmer, 2003). Data are from the loading cycle of the experiment. (a) Predictions in moduli space. (b) Predictions in velocity space. (c) Log-log plot of dry velocity versus effective pressure. ......................................................... 112

6.1 Exmouth Sub-basin and Enfield location map. From (Hamp et al., 2008). ......................................................... 124
6.2 Enfield reservoir map and well locations. The area of interest for this study, focused around Injector_1, is in the red square. Adapted from [Hamp et al. 2008].

6.3 Initial rock physics pressure model. Normalised $V_P$ and $V_S$ as a function of normalised effective pressure. Measurements made on a dry core sample from an Enfield appraisal well are shown as points. Normalised $V_P$, $V_S = 1.0$, and normalised effective pressure = 1.0 represent initial baseline conditions. Curves show $V_P$ and $V_S$ fit using the Macbeth empirical regression model [MacBeth 2004] for dry and brine-saturated conditions. Brine-saturated conditions are calculated using modified Gassmann fluid substitution, with fluid-pressure effects calculated using equations of Batzle and Wang (1992). Normalised brine velocities are shown relative to dry rock velocities at in-situ effective pressure. Adapted from Wulff et al. (2008).

6.4 We calculate acoustic impedance change (%) in the water leg due to a change in reservoir pore pressure with the initial rock physics pressure model in Figure 6.3.

6.5 We forward model the reservoir amplitude response as a function of pressure change using measured logs in Well_2 (Gamma ray shown in pink) and the rock physics model shown in Figure 6.3. Each seismic trace represents a pore pressure increase of 0.4 MPa above the initial pressure (labelled). In the top panel, wiggles with area fill show the modelled baseline seismic response, while unfilled wiggles show the modelled monitor response (near-stack trace 8 – 19° where the strongest 4D AVO effects are expected due to pressure). The middle panel shows the top reservoir maximum trough amplitude for the modelled baseline and monitor responses. The bottom panel shows the difference traces (monitor-baseline) at the same scale as the top panel.

6.6 4D AVO intercept and gradient difference plot, based on our modelling from appraisal wells. Conditions at 0, 0 are for initial reservoir pressure ($P_{eff}$ = 20 MPa, $P_P$ = 20MPa) and oil saturation (80%). Pressure and saturation effects plot in different quadrants meaning they may be separable with 4D AVO analysis [Lumley et al. 2003].

6.7 Modelled 4D AVO Gathers using log data from Well_3. (a) Modelled AVO gather for Baseline conditions (20 MPa pore pressure), and an increase in pore pressure of 10 MPa above initial reservoir pressure. The 4D difference shows that pressure effects should be observed greatest at near offsets (angles). (b) Modelled AVO gather for Baseline condition (80% oil saturation), and a change in saturation to water. The 4D difference shows that saturation effects should be observed greatest at far offsets (angles). We also note that in each case, Class III AVO behaviour is observed on the baseline and monitor gathers.

6.8 Full stack RMS amplitude maps at top reservoir. (a) Baseline survey. (b) M1 survey. (c) 4D difference (M1-Baseline). Area of interest around Injector_1 in red box.
6.9  RMS amplitude maps at top reservoir showing AVO response around Injector_1. (a) Baseline near stack. (b) Baseline mid stack. (c) Baseline far stack. (d) M1 near stack. (e) M1 mid stack. (f) M1 far stack. (g) Near stack difference (M1-Baseline). (h) Mid stack difference (M1-Baseline). (i) Far stack difference (M1-Baseline). The strong Class 3 AVO response of the Baseline and M1 surveys is consistent with modelling at the wells (e.g., Figure 6.7). The red 4D AVO anomalies are consistent with increases in pore pressure between Baseline and M1 (bright at near offsets, dimming to far offsets - e.g., Figure 6.7b). Also, the blue 4D AVO anomalies are consistent with increases in water saturation (dim at nears, brightening to far offsets - e.g., Figure 6.7b). .................................................. 134

6.10  Baseline reservoir time-thickness map (ms) showing the thick channel sand (> 20ms) to the north of Injector_1. 4D amplitude and time shift data is analysed within the black polygon (above max tuning thickness of ~ 12 ms). Approximate OWC shown in pink. .......................... 135

6.11  4D RMS amplitude maps at top reservoir. (a) Near-stack difference map (M1-Baseline). (b) Mid-stack difference map (M1-Baseline). (c) Near-stack difference map (M1-Baseline) with areas affected by tuning (< 12ms time thickness between top and base reservoir) masked out (grey). (d) Mid-stack difference map (M1-Baseline) with areas affected by tuning masked out (grey). We also note that the reservoir thickness in Well_2 and Well_3 is above tuning thickness. ........................................ 136

6.12  Histogram of extracted maximum trough amplitude within the thick channel sand polygon (e.g., Figure 6.10) for (a) near and (b) mid stacks. Baseline amplitudes are shown in blue and M1 amplitudes in red. Amplitude increases between near and mid stacks, consistent with the Class III AVO observed in Figure 6.9 and modelled in Figure 6.7. The near-stack 4D difference (M1-Baseline) is greater than the mid-stack 4D difference, also consistent with modelled 4D AVO behaviour (Figure 6.7). At M1, a 60% increase in mean amplitude is observed compared to Baseline for the near stack, while a 50% increase is observed for the mid stack. ............................ 137

6.13  (a) Baseline near-stack seismic section through area of interest and Injector_1. (b) M1 near-stack seismic section. (c) 4D seismic difference section (M1-Baseline) highlighting strong amplitude anomaly around the water injector and the effects of associated time shifts. Red and cyan arrows point to events representing the effects of time shifts in the red and cyan-dashed events, respectively, in (a) and (b). The thick channel sand analysed in this study can be seen just down-dip from the injector. ........................................ 139

6.14  4D time shifts around Injector_1. (a) Time shift measured as the 4D increase in time thickness between top (yellow dashed interpretation in Figure 6.13) and base reservoir (red dashed interpretation in Figure 6.13) in ms. In the polygon we extract an average time shift of 6 ms. (b) Time shift measured as the 4D increase in time thickness between top reservoir and a deeper negative event below base reservoir (cyan interpretation in Figure 6.13) in ms. In the polygon we extract an average time shift of 4 ms, implying that the source of the 4D time shifts is maximum in the reservoir, and decreases or "heals" in the seismic images with depth below reservoir. The time shifts show a background noise level of ± 2 ms. ........................................ 140
6.15 Analysis of ‘fracture/crumbled’ velocity-pressure model of Wulff et al. (2008): Normalised $V_P$ and $V_S$ as a function of normalised effective pressure. Measurements made on a dry core sample from an Enfield appraisal well are shown as solid circles. Normalised $V_P$, $V_S = 1.0$, and normalised effective pressure = 1.0 represent baseline conditions. Curves show $V_P$ and $V_S$ predictions using the MacBeth (2004) empirical model, where the model fitting parameters have been adjusted to achieve a model-to-seismic match around the water injectors. We also plot data (triangles) from measurements on unconsolidated sands from Zimmer (2003). We normalise data from Zimmer (2003) relative to Enfield dry rock velocities at baseline effective pressure. Due to irreversible changes to the reservoir rock, the velocity-pressure response for a reduction in pore pressure from M1 should follow a flatter trend than the Macbeth model, as indicated by the black arrow. Also, the velocity of a cemented rock cannot decrease below the velocity of an uncedented sample at a given pressure (assuming same porosity and composition).

6.16 Log data for Well_2 and Well_3 appraisal wells. Vertical axis shows two-way travel time. Well_2 is in the aquifer, while Well_3 is in the oil leg. We calculate Well_3 elastic logs for brine saturation with Gassmann fluid substitution. Different zones of interest are highlighted, including the Upper and Lower Macedon sands.

6.17 Schematic diagram of three theoretical models for high-porosity sands. The thickening of circles represents the addition of contact cement from the initial sand pack. Adapted from Avseth et al. (2005).

6.18 Saturated (a) compressional and (b) shear log velocity versus volume of shale for the Upper and Lower Macedon sands ($V_{shale} < 0.5$) from Well_2 and Well_3 appraisal wells. Colours correspond to the different zones of interest in Figure 6.16.

6.19 Saturated (a) compressional and (b) shear log velocity values versus porosity for the Upper (blue) and Lower (green) Macedon reservoir sands. Also shown is the friable-sand model (Dvorkin and Nur, 1996) which we calibrate to uncedented sediment samples from Zimmer (2003) using the grain contact theory model of Saul et al. (2013), along with the contact-cement (Dvorkin and Nur, 1996) and constant-cement (Avseth et al., 2000) models. The red triangle shows the location of the core sample used in the 4D seismic feasibility study.

6.20 Schematic diagram of dry compressional velocity versus effective pressure (Saul and Lumley, 2013). The maximum pressure sensitivity of a rock is given by the loading curve when the rock is uncedented (solid red curve). If the rock has some contact cement then velocity will be higher and less sensitive to changes in effective pressure (solid blue curve). In this graph, unloading refers to a decrease in confining pressure, whereas loading refers to a decrease in effective pressure due to an increase in pore pressure.
List of Figures

6.21 Saturated compressional log velocity versus porosity, coloured by pore-space incompressibility, for the Macedon sands (triangles). The sands diagnosed to have higher contact-cement volume have the highest pore-space incompressibility, and should therefore be less sensitive to changes in pressure. Unconsolidated samples from Zimmer (2003) show much lower pore-space incompressibilities (circles). The highest pressure measurements from Zimmer (2003) track along the friable-sand model curve, and correspond to the in-situ reservoir pressure at Enfield. 148

6.22 Water saturated (a) compressional and (b) shear velocity versus effective pressure for sandstones of varying contact-cement volume, calculated using a modified version of the velocity-pressure-cement model of Avseth and Skjei (2011). The uncemented end-member has been calibrated to data from Zimmer (2003) (circles). The cemented end-member has been calibrated such that the average contact-cement volumes of the Upper and Lower Macedon reservoir sands (cyan circle) agree with the volumes determined from rock physics diagnostics (Figure 6.19). 152

6.23 Forward modelled percent change in maximum trough amplitude at top reservoir (for an 11.7 MPa pressure change) versus contact-cement volume. Red and black circles show modelled near and mid-stack amplitudes, respectively. Observed percent change in near and mid-stack amplitude (polygon within thick channel sand - Figure 6.11) between Baseline and M1 surveys are shown as red and black-dashed lines, respectively. Average initial contact-cement volume is interpreted to be 2.5% from rock-physics diagnostics (Figure 6.19). 154

6.24 Forward modelled near-stack seismic data from Well_2, for varying amounts of contact-cement weakening using the model in Figure 6.22. In each case the increase in pore pressure is kept constant (increase of 11.7 MPa from initial reservoir pressure, as measured in Injector_1 at the time of M1), and the volume of contact cement is varied. The scale for each synthetic trace is the same as in Panel A. 155

6.25 Forward modelled mid-stack seismic data from Well_2, for varying amounts of contact-cement weakening using the velocity-pressure-cement model in Figure 6.22. 156

6.26 Forward modelled time shift (4D increase in time thickness between top and base reservoir) in ms versus contact-cement volume for an increase in reservoir pressure of 11.7 MPa. Red circles show modelled time shift on the near-stack using Well_2 (reservoir thickness ~ 20 – 25 m). Blue square and blue circle show modelled time shifts using equation 6.12 for h = 40 m and h = 50 m, respectively. Observed average 4D time shift on near stack (within thick channel sand polygon - Figure 6.14) between Baseline and M1 surveys is shown as red-dashed line. Average initial contact-cement volume is interpreted to be 2.5% from rock-physics diagnostics (Figure 6.19). 158

6.27 Porosity versus effective pressure for an uncemented sand sample from Zimmer (2003). Data are from the loading cycle of the measurements. The data has been fit with the SL porosity-pressure model (Saul and Lumley, 2013). Based on this model, a realistic porosity variation associated with the pore-pressure increase around Injector_1 is 1 – 2% (porosity units). This is strictly for an uncemented sediment, so the porosity increase at Enfield could be less; therefore, 1 – 2% is an upper-bound on the porosity change. 161
6.28 4D AVO modelling. (a) $\Delta G$ versus $\Delta R_0$ for decreasing contact-cement volume. (b) $\Delta G$ versus $\Delta R_0$ for increasing reservoir pressure (decreasing effective pressure). Average overburden shale properties taken from Well_2 and Well_3. 162
1 | Introduction

“Our ability to predict the sensitivity to pressure from first principles is poor.”

– Per Avseth

1.1 Research objective

The main objective of this thesis is to develop a better understanding of the pressure dependent elastic properties of unconsolidated to partially consolidated sandstones, and to develop new relationships between such elastic properties and effective pressure that can be used to improve seismic reservoir characterisation and monitoring. This general objective contains several more specific goals: (1) to develop a new model that relates elastic properties to effective pressure that can be used to predict such elastic properties outside measured data ranges, particularly at low effective pressures; (2) to investigate causes for the discrepancy between theoretical predictions of elastic properties and laboratory data, and to improve such theoretical models to incorporate these effects; and (3) to apply these newly developed pressure-dependent models to a real data case study from the North West Shelf, offshore Australia, to improve the understanding of 3D and 4D seismic data responses associated with high pressure water injection. The overall research challenges and motivations for each of these objectives are described below.
1.2 How do seismic properties vary with pressure?

Knowledge of the pressure dependence of elastic rock properties is useful for the analysis of rocks in sedimentary basins, including for the prediction of pore pressure and geomechanical effects, and for time-lapse monitoring of subsurface fluid flow as occurs in hydrocarbon, groundwater, geothermal, and CO₂ sequestration reservoirs. Quantifying how elastic properties vary as a function of effective pressure has been the focus of research studies spanning several decades (e.g., Mindlin, 1949; Domenico, 1977; Han et al., 1986; Walton, 1987; Zimmer, 2003; Saul and Lumley, 2012b, 2013; Saul et al., 2013). These studies include the development of theoretical and empirical models for the pressure sensitivity of sediment elastic properties. The most pressing problem is that theoretical models often do not adequately predict elastic properties measured in the laboratory (e.g., Makse et al., 1999; Zimmer, 2003; Bachrach and Avseth, 2008; Sain, 2011; Saul et al., 2013), and alternately, empirical regressions fit to measured laboratory data have little or no physical basis and thus often fail to predict the elastic properties beyond the limited range of available data (e.g., Eberhart-Phillips et al., 1989; Lumley, 2001; Meadows et al., 2002; Avseth et al., 2005; Saul and Lumley, 2012).

To address the above problem we need to: (1) understand which properties of an unconsolidated sedimentary rock control the pressure sensitivity of elastic parameters; (2) place physical constraints on empirical models fit to measured laboratory data in order to ensure we can accurately predict elastic properties beyond measured data ranges; and (3) investigate reasons for the discrepancy between theoretical models and measured laboratory data. Finally, to have confidence in our models we must test them against real data, including 3D and 4D seismic measured over a sedimentary basin in which pressure is significantly changing.

1.3 Background and motivation

The global demand for energy will continue to rise for the foreseeable future, driven mainly by the growth of emerging nations and population increase (International Energy Agency, 2012). Fossil fuels will remain a major part (80%) of the global energy mix (Figure 1.1).
1.3. **Background and motivation**

Figure 1.1: Global energy demand by fuel type (International Energy Agency, 2012).

with the improved recovery efficiency of existing fields becoming essential to meet the increased demand over the coming decades (Lumley, 2010). Secondary or tertiary recovery processes, such as water or gas flooding, are often necessary to improve recovery; however, owing to reservoir complexity and heterogeneity, these processes can still bypass significant volumes of hydrocarbons. Time-lapse (4D) seismic reservoir monitoring will play a key role in monitoring and optimising the recovery process of these fields, since the technology makes it possible to map areas of injected water and bypassed hydrocarbons in 3D.

Along with the increased global demand for hydrocarbon-based energy, there is an ever increasing demand to reduce global CO₂ emissions. The introduction of carbon taxes in many developed countries further increases the motivation for large energy companies to sequester CO₂, a common by-product of oil and gas/LNG production, back into subsurface reservoirs, a process known as Geosequestration. The International Energy Agency (IEA) sees Geosequestration as a key option to mitigate CO₂ emissions in the energy sector (IEA World Energy Outlook, 2012). Measurement, monitoring and verification (MMV) of carbon storage programs is vital to ensure the sequestered CO₂ is injected into the correct location, remains safely stored in the pore space over time, and does not interfere with other subsurface resources (e.g., Eiken et al, 2000; IPCC, 2005; Quanlin et al, 2008; Lumley, 2010b). Time-lapse (4D) seismic will play a vital role in the MMV process, since the extent of the CO₂ plume can be imaged in 3D over the life of the project (Lumley, 2010b).
1.3. Background and motivation

In order to qualify/quantify the observed 4D seismic anomalies associated with hydrocarbon production and/or CO₂ sequestration it is essential to understand how these production/injection related processes affect the propagation of seismic waves. For example, the injection (withdrawal) of fluids into (from) porous sedimentary rock results in changes in pore pressure within the reservoir. These pressure changes have an effect on the seismic rock properties of the reservoir and therefore on the recorded 4D seismic data. Production/injection related variations in other reservoir properties, such as saturation and geomechanical changes, also affect seismic rock properties. These will be discussed in parts of this thesis; however, the main focus is on how changes in pressure affect seismic rock properties. It is important to note that although we have only discussed the specific examples of hydrocarbon production and CO₂ sequestration above, the theory and models developed in this thesis apply equally to other earth science problems, including pore pressure prediction analysis using seismic data (e.g., Dvorkin et al., 1999; Prasad, 2002; Hofmann et al., 2005), 3D seismic reservoir characterisation studies (e.g., Avseth et al., 2005), and for time-lapse seismic monitoring of groundwater and geothermal projects (Lumley, personal communication). An overview of seismic wave propagation and the fundamentals of 4D seismic are given in the sections below.

1.3.1 Seismic wave propagation

To leading order, seismic waves propagate in the subsurface as a function of the saturated rock’s bulk modulus \(K\) (incompressibility), shear modulus \(G\), and bulk density \(\rho_b\). This is evident in the following form of the elastic wave equation for linear isotropic materials (Aki and Richards, 1980):

\[
\rho_b \ddot{\mathbf{u}} = \mathbf{f} + (K + 4G/3) \nabla(\nabla \cdot \mathbf{u}) - G \nabla \times (\nabla \times \mathbf{u}),
\] (1.1)

where \(\mathbf{u}\) is the wave particle displacement vector and \(\mathbf{f}\) is the source function vector. Solutions of equation 1.1 result in three seismic waves; a longitudinal compressional wave and two orthogonally polarised transverse shear waves, that propagate at compressional
1.3. Background and motivation

$V_P$ and shear $V_S$ velocities, respectively:

$$V_P = \sqrt{\frac{K + \frac{4}{3}G}{\rho_b}} \quad \text{and} \quad V_S = \sqrt{\frac{G}{\rho_b}}. \quad (1.2)$$

Along with the travel times and velocities of the propagating seismic waves, we are also interested in the wave amplitudes. Reflection amplitudes recorded in seismic data are a result of material property contrasts in the subsurface scattering/reflecting downward propagating waves back to the surface. The amplitude of a recorded wave is related to the reflection coefficient at the scattering interface. A common form of the P-wave reflection coefficient as a function of incidence angle is given by the linearised form of Zoeppritz equation (Aki and Richards, 1980):

$$R_{PP}(\theta) \approx \frac{1}{2} \left( 1 - 4p^2V^2_{Sa} \right) \left( \frac{\Delta \rho_b}{\rho_{ba}} \right) + \frac{1}{2 \cos^2(\theta)} \frac{\Delta V_P}{V_{Pa}} - 4p^2V^2_{Sa} \frac{\Delta V_S}{V_{Sa}}, \quad (1.3)$$

where:

$$p = \frac{\sin \theta_1}{V_{P1}},$$

$$\Delta \rho_b = \rho_{b2} - \rho_{b1},$$

$$\Delta V_P = V_{P2} - V_{P1},$$

$$\Delta V_S = V_{S2} - V_{S1},$$

$$\rho_a = (\rho_2 + \rho_1)/2,$$

$$V_{Pa} = (V_{P2} + V_{P1})/2,$$

$$V_{Sa} = (V_{S2} + V_{S1})/2,$$

$$\theta = (\theta_1 + \theta_2)/2 \approx \theta_1.$$  

In equation 1.3, which is linearised with respect to small material property contrasts, and small incident and transmission angles, $p$ is the ray parameter, $\theta_1$ is the angle of incidence, $\theta_2$ is the transmission angle, and $V_{P1}$ and $V_{P2}$ are the P-wave velocities above and below a given interface, respectively. Similarly, $V_{S1}$ and $V_{S2}$ are the S-wave velocities, while $\rho_{b1}$ and $\rho_{b2}$ are the bulk densities. Equation 1.3 is often called the AVO (or AVA) equation because it describes the Amplitude-Versus-Offset (or Angle) of a reflected P-wave (Ostrander, 1984; Castagna, 1993).

It is clear from equations 1.1 to 1.3 that seismic travel time, velocity, and amplitude all depend on the elastic properties of the medium in which the waves are propagating/scattering. We can also imagine that these elastic properties will be a function of the
physical rock properties of the medium, including porosity, mineralogy, fluid saturation, and pressure. The science that describes the link between seismic elastic properties and physical rock properties is termed rock physics (e.g., Mavko et al. 1998) (see Chapter 2 - Rock physics models). Combining relevant rock physics relationships with equations 1.1 to 1.3 allows us to simulate and analyse propagating wavefields in terms of the rock properties of the medium of interest.

1.3.2 4D seismic fundamentals

Time-lapse (4D) seismic monitoring involves repeatedly imaging the subsurface over calendar time to detect and quantify dynamic changes associated with fluid flow (e.g., Lumley 2001). The concept was first introduced by Nur (1982), where it was predicted from rock physics lab measurements that steam injection for enhanced oil recovery (EOR) should be detectable with repeat surface seismic surveys. During the following decade, several key worldwide field tests showed significant anomalies associated with enhanced recovery are detectable using 4D seismic (e.g., Greaves and Fulp 1987, Lumley 1995). Monitoring of oil-water systems has been the focus of 4D studies since the mid-late 1990s (e.g., Sonneland et al. 1997, Johnston et al. 1998, Lumley et al. 1999, Gan et al. 2004, Smith et al. 2008, Johnston and Laugier 2012). Recent projects have focused on using 4D seismic techniques to monitor CO₂ sequestration, for example at the Sleipner field where CO₂ has been injected since 1996 (e.g., Eiken et al. 2000, Arts et al. 2007, Lumley et al. 2008). Lumley et al. (1997, 2000) discuss factors affecting a 4D seismic response and develop techniques to assess the risk of 4D projects for a variety of production scenarios. Research continues to focus on the improved acquisition, processing, and interpretation of 4D seismic data (e.g., Lumley et al. 2003, Widmaier et al. 2004, Hatchell and Bourne 2005, Cantillo et al. 2010, Vanorio et al. 2010, Duffaut et al. 2011, Tatanova and Hatchell 2012), including quantifying changes in pressure and saturation associated with production and/or injection (e.g., Landro 1999, Tura and Lumley 1999, Landro 2001, Lumley 2001, Lumley et al. 2003b, Cole et al. 2002).

As discussed above, seismic wave propagation in the subsurface is a function of the rock’s bulk modulus $K$, shear modulus $G$, and density $\rho_b$. Before production ($t_1$), the
1.3. Background and motivation

seismic wavefield $u_1$ is a function of the pre-production saturated rock properties $m_1 = (K, G, \rho_b)_1$. During production/injection, if the subsurface saturated rock properties were to change, either due to the withdrawal or injection of fluids, the seismic wavefield recorded at a second calendar time $t_2$ will be a function of the new saturated rock properties $m_2 = (K, G, \rho_b)_2$ (Lumley, 2001, 2011). Thus, recording the seismic wavefield at different calendar times during production can provide us with information on how the subsurface model properties vary due to production:

$$\Delta u_{k+1,k} = (u_{k+1} - u_k) = [u(x, t_{k+1}) - u(x, t_k)]. \quad (1.4)$$

The change in subsurface model properties over the interval $\Delta t = (t_{k+1} - t_k)$ can be obtained from equation [1.4] by creating time-lapse images of reflectivity $\Delta R = (R_{k+1} - R_k)$, time-lapse images of the change in two-way traveltime $\Delta T = (T_{k+1} - T_k)$, or by time-lapse inversion directly to changes in velocity $(\Delta V_P, \Delta V_S)$ and saturated rock properties $\Delta m = (\Delta K, \Delta G, \Delta \rho_b)$.

The more compressible the initial dry-rock frame is, and the larger the contrast between the initial and saturating fluid, the greater the change in the 4D seismic saturated rock properties (Figure 1.2) (Lumley, 2010). The injection (withdrawal) of fluids into (from) porous sedimentary rock can also alter the rock compressibility over production time. This can be due to changes in pore pressure, porosity, geomechanical deformation, and/or geochemical reactions. In this thesis, the effect of changes in saturation, pore pressure, porosity, and some aspects of geomechanical deformation are discussed. Geochemical reactions are outside the scope of this thesis, but should be considered in certain circumstances, such as the injection of CO$_2$ into calcium carbonate bearing rocks (Lumley, 2010; Vanorio et al., 2011; Vialle and Vanorio, 2011).

The ability to detect a 4D signal depends not only on the magnitude of the signal, but also on the noise level of the data. Typically, the largest source of noise in 4D seismic data is "non-repeatable" noise, which results from not being able to perfectly replicate the 3D seismic acquisition experiment and environmental conditions. A common measure of the non-repeatable noise between two seismic surveys is given by the normalised root-mean-
1.3. Background and motivation

**Figure 1.2:** 4D seismic sensitivity to initial dry-rock compressibility and fluid compressibility contrast. Weak 4D signals are produced when the dry-rock compressibility is low and the fluid compressibility contrast is small. Conversely, strong 4D signals are produced when the dry-rock compressibility is high (e.g. unconsolidated sands) and when the fluid compressibility contrast is large (e.g. CO₂ injected into brine). Reproduced from Lumley (2010).

An NRMS value of 0 indicates perfect repeatability, while a value of 2 indicates the data sets are identical but polarity-reversed. If the two data sets consist of random, uncorrelated noise then NRMS=1.4. An NRMS value of < 0.2 is considered excellent for 4D seismic monitoring purposes (Lumley, 2010). Improving repeatability is of critical importance in order to detect and image small 4D signals.

This section has given the background and motivation for the research contained within this thesis, including an overview of the fundamentals of seismic wave propagation and 4D seismic. Chapter 2 (Rock physics models) will go into specific details of how different reservoir rock properties (static and dynamic) affect seismic elastic properties.
1.4 Approach - chapter descriptions

This thesis consists of seven chapters, including this introduction. The chapters are linked together and represent the analysis of the pressure sensitivity of unconsolidated to partially consolidated sandstone elastic properties, including the development and application of new models.

Rock physics is the science that relates geophysical observations (e.g., laboratory measurements and seismic data) to rock properties. Chapter 2 gives an overview of the key rock physics models (static and dynamic) used throughout this thesis, including how such models relate to 4D seismic data modelling and interpretation. Since this thesis is presented as a series of papers, more detailed information on specific rock physics models is also presented in the introduction and theory sections of each chapter.

Chapter 3 introduces a new velocity-pressure-compaction model (the so-called “SL model”) to describe the pressure sensitivity of uncemented sediment elastic properties. The model is built-up from observed compaction theory and the concept of critical porosity. The strong physical basis of the model enables accurate determination of model fitting parameters outside measured data ranges. The model also includes a relationship to account for porosity and density changes with pressure. The chapter is concerned with the theoretical development of the new model and includes examples of the excellent fit to measured laboratory data, including comparisons to existing theoretical and empirical models. The developed model allows for improved predictions and interpretations of rock properties and seismic data because it more accurately relates changes in velocity and porosity due to pressure variation. The work was presented at the AGU Fall Meeting in San Francisco (Saul and Lumley 2011), the 22nd ASEG Conference in Brisbane (Saul and Lumley 2012), and at the 82nd Annual International SEG Conference in Las Vegas (winner Best Student Paper) (Saul and Lumley 2012b). The full text from this chapter has been published in *Geophysical Journal International* (Saul and Lumley 2013).

Uncertainty in elastic properties at low effective pressure often occurs due to our poor ability to predict pressure sensitivity from first principles, and because making laboratory measurements at low effective pressure (i.e., high pore pressure) is generally not practical.
Chapter 4 builds on the discussion in Chapter 3, and includes details on how to calibrate the SL model to available laboratory data. The chapter covers how the SL model can be applied in a realistic setting to determine sediment elastic properties outside measured data ranges, particularly at low effective pressures where existing models typically fail. The chapter also details a method to estimate critical porosity from grain-size distribution data, and therefore calculate sediment elastic properties at zero effective pressure. The approach has implications for the improved prediction and interpretation of 3D and 4D seismic data, including for overpressure prediction and seismic reservoir characterisation with AVO-based methods. Parts of the chapter were presented at the AGU Fall Meeting in San Francisco (Saul and Lumley, 2011), the 22nd ASEG Conference in Brisbane (Saul and Lumley, 2012), and at the 82nd Annual International SEG Conference in Las Vegas (winner Best Student Paper) (Saul and Lumley, 2012b). The full text from this chapter is to be submitted to Geophysics.

Commonly used rock physics models for unconsolidated sediments (e.g., the Hertz-Mindlin and friable-sand models) tend to over predict shear modulus (Makse et al., 1999; Zimmer, 2003; Dutta et al., 2010). To correct for this, the shear modulus is often multiplied by an ad-hoc correction factor: e.g., the fraction of frictionless contacts (Bachrach and Avseth, 2008); or by using separate coordination number-porosity relations to estimate bulk and shear modulus for the same rock (Dutta et al., 2010). Our goal (Chapter 5) is to improve rock physics models of unconsolidated sediments by using an appropriate weighting factor to account for grain heterogeneity effects, and for the weighting factor to be consistent with granular dynamic (GD) simulations that solve a constitutive set of physics relations. This results in a more physical scaling of shear modulus than existing published approaches. The chapter also investigates in detail the effect of variations in porosity on unconsolidated sediment elastic properties and discusses how such variation helps to explain observed discrepancies between theoretical models and laboratory measurements. The new modified grain contact theory model should help to improve the prediction and interpretation of elastic properties as a function of depth/effective pressure. The work was presented at the 75th EAGE Conference and Exhibition in London (Saul et al., 2013b) and has been published in Geophysics (Saul et al., 2013).
Chapters 3, 4, and 5 deal with pressure-dependent elastic properties in unconsolidated sediments, including the development of new models to describe such properties. We propose that these models should allow for improved prediction and interpretation of 3D/4D seismic data in terms of changes in effective pressure. The implications could span a wide-range of earth-science problems, including for prediction of pore pressure and geomechanical effects, and for 3D/4D seismic monitoring of hydrocarbon, groundwater, geothermal, and CO₂ sequestration reservoirs. In Chapter 6 we choose a specific case study example, with the objective to improve the interpretation of 4D seismic data acquired over a high pressure water injector in the Carnarvon Basin, offshore Western Australia. The study also includes the rock physics diagnostics (Avseth et al., 2000) of Macedon reservoir sands, to investigate the impact that contact cement has on pressure sensitivity within the field. The observations are important for assessing 4D seismic feasibility and interpretation in Macedon reservoir sands, as well as in other geologically analogous systems. The work from this chapter is in preparation to be submitted as a technical paper to a relevant journal.

Chapter 7 provides general discussions and conclusions, including contributions from the work presented in this thesis and recommendations for further research.
1.4. Approach - chapter descriptions
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2 | Rock physics models

“Essentially, all models are wrong, but some are useful”

– George E.P. Box

2.1 Foreword

This section gives a brief overview of rock physics models used throughout this thesis, including how they relate to the modelling and interpretation of 3D and 4D seismic data. Since this thesis is presented as a series of papers, further information on rock physics models related to pressure sensitivity can be found in the introduction and theory sections of each chapter.

2.2 Introduction

The science of rock physics attempts to relate geophysical observations to the underlying physical properties of rocks (Mavko et al., 1998). Rock physics plays a key role in reservoir characterisation (Avseth et al., 2005; Sayers, 2013), and is especially important for quantification and interpretation of 4D seismic signatures during reservoir depletion and injection (Nur, 1982; Lumley, 1995, 2001; Hatchell et al., 2003; Calvert, 2009; Avseth and Skjei, 2011). Rock physics models can be empirical relations based on laboratory measurements (e.g., Han et al., 1986; Eberhart-Phillips et al., 1989), or theoretical models based on physics and idealized sediment microstructure (e.g., Mindlin, 1949; Walton, 1987).
2.3. **Porosity and mineralogy**

Several authors have studied the seismic properties of rocks and developed important relationships between elastic and reservoir properties, such as porosity and clay content (e.g., Han et al. 1986; Marion 1990; Yin 1992), diagenesis (e.g., Jizba 1991; Dvorkin and Nur 1996), lithology (e.g., Castagna et al. 1985; Blangy 1992; Greenberg and Castagna 1992), pore fluids (e.g., Wang and Nur 1990; Batzle and Wang 1992), and pressure (e.g., Mindlin 1949; Han et al. 1986; Walton 1987; Yan and Han 2009; Saul and Lumley 2013; Saul et al. 2013). We can see from equations 1.1 to 1.4 that such relations allow us to simulate and interpret propagating wavefields in terms of the rock properties of the medium of interest.

In terms of reservoir characterisation and monitoring, we are interested in rock physics models for both static (remain constant over calendar time) and dynamic (vary over calendar time) reservoir properties. Static properties generally include mineralogy, porosity, and cement volume, whereas dynamic properties typically include saturation and pressure. In very unconsolidated materials, or in reservoirs with significant pressure changes, geomechanical changes can occur resulting in time-lapse variations in porosity and rock consolidation (e.g., Geertsma 1973; Hatchell et al. 2003).

### 2.3 Porosity and mineralogy

Rock bulk modulus $K$, shear modulus $G$, and bulk density $\rho_b$ can vary significantly with the mineral content and porosity of a rock, where porosity is defined as the volume of the void space normalised by the total volume of the rock (Mavko et al. 1998). If we only know the porosity and elastic moduli of the mineral constituents of a rock, the best we can predict are the upper and lower bounds of the seismic elastic properties (Mavko et al. 1998; Avseth et al. 2005).

Two useful bounds are the Voigt (1928) upper and Reuss (1929) lower bounds. The Voigt bound can be expressed as an arithmetic average:

$$M_V = \sum_{i=1}^{N} f_i M_i,$$

where $f_i$ is the volume fraction of the $i$th rock component and $M_i$ is the elastic modulus.
The variable $M_V$ can represent any modulus (e.g., bulk or shear). The Voigt bound represents the stiffest way a mixture of constituents can be put together. Conversely, the softest way a mixture of constituents can be put together is given by the Reuss bound via the harmonic average:

$$\frac{1}{M_R} = \sum_{i=1}^{N} \frac{f_i}{M_i}. \quad (2.2)$$

Slightly narrower bounds for an isotropic elastic mixture of two constituents are given by the Hashin-Shtrikman bounds (Hashin and Shtrikman, 1963):

$$K^{HS\pm} = K_1 + f_2 \left(\frac{K_2 - K_1}{K_1 + 4G_1/3}\right)^{-1},$$

$$G^{HS\pm} = G_1 + f_2 \left(\frac{G_2 - G_1}{G_1 + 2G_1/(5G_1(K_1 + 4G_1/3))}\right)^{-1}, \quad (2.3)$$

where $K_1, K_2$ are the bulk moduli, $G_1, G_2$ are the shear moduli, and $f_1, f_2$ are the volume fractions of the individual phases, respectively. Generally, equation (2.3) gives the upper bound when the stiffest material is subscripted $1$, and the lower bound when the softest material is subscripted $1$. The separation between the upper and lower bounds depends on how "elastically different" the two constituents are. The physical interpretation of the Hashin-Shtrikman bounds is shown schematically in Figure 2.1. The rock is made of spheres of material 2, surrounded by a shell of material 1, with each sphere and its shell having volume fractions $f_1$ and $f_2$, respectively. The upper bound is realised when the stiffer material forms the shell, while the lower bound is realised when it is the core (Mavko et al., 1998).

For a given volume fraction of two constituents the effective modulus will fall somewhere between the upper and lower bounds, depending on the geometric details of their arrangement. A number of theoretical effective medium models make assumptions of the geometric details of the mineral constituents, and therefore attempt to make more accurate predictions of elastic moduli. More consolidated rocks are often approximated as an elastic block of mineral perturbed by inclusions (e.g., Norris, 1985; Zimmerman, 1991b; Berryman, 1992, 1995; Mavko et al., 1998). Unconsolidated rocks are often described as separate elastic grains in contact (e.g., Mindlin, 1949; Walton, 1987). In practice, it is very
2.3. Porosity and mineralogy

**Figure 2.1:** Physical interpretation of the Hashin-Shtrikman bounds for the bulk modulus of a two-phase material. Upper bound when $K_1 > K_2$ and lower bound when $K_1 < K_2$. Reproduced from Mavko et al. (1998).

difficult to determine the geometric details of a given rock and thus theoretical models often fail to accurately describe the elastic properties of rocks (Makse et al., 1999; Avseth et al., 2005; Mavko et al., 1998; Zimmer, 2003; Sain, 2011; Saul et al., 2013).

Figure 2.2 shows compressional velocity versus porosity for a number of water-saturated samples, along with Reuss and Voigt bounds computed for a quartz and water mixture using equations 2.1 and 2.2 respectively. Prior to deposition, a sediment consists of quartz particles suspended in water. Since the Reuss bound describes the elastically softest way to mix two phases, we expect the compressional velocity of such suspensions to plot on this lower bound (Avseth et al., 2005). When the sediment is deposited at the seafloor such that the sediment grains are in a fluid suspension, the elastic properties are still expected to plot close to the Reuss bound since the sediment is still very unconsolidated (Hamilton, 1956). The porosity at deposition is controlled by the grain-size distribution (sorting) of the sediment, with more well-sorted sediments having higher depositional porosities (Hamilton, 1956; Beard and Weyl, 1973; Zimmer, 2003). This depositional porosity is often termed the critical porosity $\phi_c$ (Nur et al., 1995). Critical porosity is defined as the porosity at which a rock’s mechanical and elastic behaviour is separated into two distinct domains: for porosities less than $\phi_c$ the grains within the rock are load bearing, while for porosities greater than $\phi_c$, the grains are in a non-load-bearing suspension (Mukerji et al., 1995). As a sediment is buried, mechanical compaction, diagenesis (e.g., cement), and increases
2.4. Cementation and sorting

In pressure (stress) act to reduce porosity and increase $K$, $G$, and $\rho_b$, and thus velocity (Figure 2.2).

2.4 Cementation and sorting

As we see in Figure 2.2, the slope of the velocity-porosity trend greatly depends on the geologic processes that are controlling porosity loss. Porosity loss due to compaction and diagenesis (e.g., cementation) results in a much steeper velocity-porosity trend than porosity loss due to variations in sorting (Dvorkin and Nur, 1996; Avseth et al., 2000, 2005). Cementation has more of a stiffening effect than sorting because grain contacts are essentially "glued" together (Avseth et al., 2005).

Dvorkin and Nur (1996) introduced two theoretical models to describe the diagenetic and sorting trends for high-porosity sands. The friable-sand model describes porosity loss due to deteriorating sorting, while the contact-cement model describes porosity loss due to the deposition of cement on the surface of grains. Avseth et al. (2000) introduced the constant-cement model to describe sands with constant cement volume, but varying sorting.
2.4. Cementation and sorting

(it is assumed that sands that have a variation in porosity due to sorting have the same amount of contact cement). The model is a combination of the contact-cement model and the friable-sand model (see schematic in Figure 2.3). Firstly, it is assumed that porosity reduces from that of the initial sand pack (solid circle in Figure 2.3) to the initial-cement porosity $\phi_b$ due to the deposition of contact cement (open circle in Figure 2.3). The bulk $K_b$ and shear $G_b$ moduli at this initial-cement porosity are calculated with the contact-cement model [see equations in Dvorkin and Nur [1996]]. The elastic moduli at a lower porosity (due to deteriorating sorting) is then calculated using a modified Hashin-Shtrikman lower bound (MHSLB) as in the friable-sand model:

\[
K_{\text{dry}} = \left( \frac{\phi/\phi_b}{K_b + 4G_b/3} + \frac{1 - \phi/\phi_b}{K_m + 4G_b/3} \right)^{-1} - 4G_b/3
\]
\[
G_{\text{dry}} = \left( \frac{\phi/\phi_b}{G_b + z} + \frac{1 - \phi/\phi_b}{G_m + z} \right)^{-1} - z, \quad \text{where} \quad z = \frac{G_b}{6} \frac{9K_b + 8G_b}{K_b + 2G_b}
\]

and $K_m$ and $G_m$ are the bulk and shear moduli of the mineral phase, respectively. These models can be superimposed on moduli-porosity cross-plots to diagnose sands into three groups: (1) friable (unconsolidated) sands; (2) sands with porosity variation associated with contact cement; and (3) sands with constant cement volume but varying porosity due to deteriorating sorting (or clay content).

The biggest shortcoming of the contact-cement model is that it does not include pressure sensitivity. This is because the model assumes all grain contacts have the same amount of cement, and that grains immediately lose pressure sensitivity once cementation begins. However, a number of studies have shown that cemented reservoirs can still have significant pressure sensitivity, as contact cement is not usually deposited evenly on all grain contacts (e.g., Avseth et al. [2009], Avseth and Skjei [2011]). Given this shortcoming, the model is still very useful for diagnosing reservoirs at a near-constant depth (pressure), because the relative ambiguity between contact-cement volume and pressure for samples at different depths is removed. The absolute cement volume calculated with the model may have some uncertainty; however, the relative variation in cement volume between sands provides valuable information during reservoir characterisation and interpretation. Accounting for variations in sandstone elastic properties due to depositional and diage-
2.5 Rock compressibility and fluid substitution

Fluid substitution is the rock physics problem of predicting how rock elastic properties, and therefore seismic velocities, change as a function of pore fluid type. As shown by Walsh (1965), Zimmerman (1991), and Mavko et al. (1998) the compressibility of a dry rock can be expressed as

\[ \frac{1}{K_{\text{dry}}} = \frac{1}{K_m} + \frac{\phi}{K_\phi}, \]

\[ (2.5) \]
2.5. **Rock compressibility and fluid substitution**

where $K_{\text{dry}}$ is the dry rock bulk modulus, $K_m$ is the mineral bulk modulus, $\phi$ is the porosity, and $K_\phi$ is the pore-space stiffness. Rocks that are unconsolidated or are at low effective pressure generally have small values of $K_\phi$ and are therefore highly compressible (Avseth *et al.* 2005). Conversely, rocks that are well cemented or at high effective pressure tend to have large values of $K_\phi$ and are therefore relatively incompressible.

The compressibility of a saturated rock can be expressed as

$$\frac{1}{K_{\text{sat}}} = \frac{1}{K_m} + \frac{\phi}{K_\phi + K_f},$$

which can be written approximately as

$$\frac{1}{K_{\text{sat}}} \approx \frac{1}{K_m} + \frac{\phi}{K_\phi + K_f},$$

where $K_f$ is the pore-fluid bulk modulus (Mavko *et al.* 1998). From equations 2.5 and 2.7, it is clear that changing the pore fluid has the effect of altering the effective pore-space stiffness. It is also clear that a rock with a large pore-space stiffness will have a small sensitivity to fluids, while a rock with a small pore-space stiffness will be more sensitive to fluids. This confirms our earlier statement, that the more compressible the dry-rock frame is (i.e., low value of $K_\phi$) and the greater the contrast between the initial and saturating fluid, the stronger the resulting 4D seismic signal (Figure 1.2).

Gassmann’s (1951) relations can be obtained by combining equations 2.5 and 2.7 together:

$$\frac{K_{\text{sat}}}{K_m - K_{\text{sat}}} = \frac{K_{\text{dry}}}{K_m - K_{\text{dry}}} + \frac{K_f}{\phi(K_m - K_f)}.$$

Equation 2.8 relates the saturated bulk modulus to the dry bulk modulus and the bulk modulus of the fluid within the pore space. It is assumed that no change in shear modulus is associated with changes in pore-fluid fill since most (non-viscous) fluids cannot support shear stress:

$$G_{\text{sat}} = G_{\text{dry}}.$$
2.6. Pressure

Changing the pore fluid will also result in a change in the rocks bulk density $\rho_b$:

$$\rho_b = (1 - \phi)\rho_m + \phi\rho_f,$$

(2.10)

where $\rho_m$ is the mineral density, and $\rho_f$ is the fluid density.

The compressional and shear velocities of the saturated rock can then be calculated with equation 1.2. Within equations 2.5 to 2.10 the fluid bulk modulus and densities can be calculated as a function of temperature and pressure using empirical regression equations published by Batzle and Wang (1992). In the case that more than one mineral type is present, the effective mineral moduli can be calculated using the Voigt-Reuss-Hill average (Mavko et al., 1998).

It is important to note that Gassmann’s relations were derived for an incremental compression; therefore, they are only valid at frequencies low enough such that wave-induced pore pressures have time to equilibrate during a seismic period/wavelength (Mavko et al., 1998). It is generally assumed that Gassmann’s relations are appropriate for 3D surface seismic frequencies (1-100 Hz), and in most cases log frequencies (1-100kHz) (Avseth et al., 2005). For the purposes of this thesis we will assume that when more than one pore fluid is present they are mixed uniformly with an effective fluid bulk modulus given by the Reuss (1929) average. Variations in spatial saturation distribution (e.g., “patchiness”, Mavko et al., 1998) can affect rock elastic properties, however this is outside the scope of this thesis. For further discussion on the assumptions and limitations of Gassmann’s relations, the reader is directed to Avseth et al. (2005).

2.6 Pressure

The main objective of this thesis is to investigate how changes in effective pressure affect the elastic properties of unconsolidated and partially consolidated sandstones. There are a number of ways that changes in pressure can influence rock elastic properties, including: (1) reversible, elastic effects on the dry-rock frame; (2) porosity loss due to compaction, packing changes, and diagenesis; (3) prevention of diagenesis from overpressure; and (4) changes in fluid properties due to changes in pore pressure (Avseth et al., 2005). The
2.6. Pressure

research contained in this thesis focuses on points (1) and (2).

Effective pressure $P_{\text{eff}}$ is defined as the difference between overburden (confining) pressure $P_{\text{ov}}$ and pore pressure $P_{\text{p}}$, such that

$$P_{\text{eff}} = P_{\text{ov}} - nP_{\text{p}},$$  \hspace{1cm} (2.11)

where $n$ is the effective stress coefficient, typically assumed to be 1.0 for many unconsolidated rocks (e.g., Hofmann et al., 2005).

In cemented rocks, increasing effective pressure acts to stiffen the dry rock frame by closing compliant crack-like porosity. In this case, the stiffening is generally reversible in the sense that the rock would weaken by the same amount if the effective pressure were subsequently decreased to the original effective pressure. In unconsolidated (uncemented) rocks, increasing effective pressure acts to stiffen the dry rock frame by stiffening the contact between individual grains, and also by decreasing porosity associated with grain rotation, reorientation, and compaction. In the case of unconsolidated rocks there is often significant hysteresis depending on the pressure path (i.e., increase or decrease in effective pressure, Figure 2.4). It is important to note that in the case of unconsolidated rocks the degree of hysteresis also depends on how the effective pressure is varied (i.e., varying pore pressure versus overburden pressure in equation 2.11) (Siggins and Dewhurst, 2003; Saul and Lumley, 2013).

For both consolidated (cemented) and unconsolidated (uncemented) rocks, the variation in elastic properties with effective pressure is typically highly non-linear (e.g., Figure 2.4). In consolidated rocks this is due to compliant pores/cracks being open at low effective pressure, with further increases in effective pressure resulting in the pores/cracks subsequently closing (Zimmerman, 1991; Shapiro, 2003; Sayers, 2010). At a certain effective pressure, once all the compliant pores/cracks are closed, the rock becomes insensitive to further changes in pressure. In unconsolidated rocks this non-linearity is due to grains being loose (i.e., free to move) at low effective pressure, with further increases in effective pressure resulting in the grains become more tightly packed with stiffer grain contacts. At a certain effective pressure, the grains are tightly packed and the grain contacts reach their maximum stiffness.
2.6. Pressure

It is also helpful to think about the pressure sensitivity of rocks in terms of pore-space compressibility in Equation 2.5. We can imagine unconsolidated rocks at low effective pressure have a high pore-space compressibility, which will decrease as the effective pressure increases, resulting in a pressure-sensitive dry bulk modulus. This, along with the reduction in porosity associated with compaction, grain rotation, and re-orientation will result in a rock that is highly sensitive to changes in effective pressure. In contrast, a stiff rock (e.g., cemented) will have a very low pore-space compressibility, which will only slightly decrease with increases in effective pressure, resulting in dry bulk modulus that is only weakly sensitive to pressure. Unfortunately, it is difficult to measure the pore-space compressibility of a given rock directly (Mavko et al., 1998), and for high porosity unconsolidated rocks that are an aggregate of grains it is difficult to even conceptualise.

Owing to the granular nature of unconsolidated sands, the modelling of uncemented
sediment properties often invokes an idealised spherical pack approximation, which greatly facilitates the development of theoretical models (e.g., Mindlin 1949, Walton 1987) and computational granular dynamics simulations (e.g., Makse et al. 1999, Sain 2011). Such models attempt to relate the evolution of grain-scale microstructure during deformation to the bulk properties of the granular pack. Unfortunately, the predictions of most currently used theoretical models often do not adequately predict measured elastic properties (e.g., Makse et al. 1999, Zimmer 2003, Bachrach and Avseth 2008, Sain 2011, Saul et al. 2013). For this reason, the current state of the art requires that empirical regressions be fit to ultrasonic velocity measurements for core samples over a limited range of pressure values (e.g., Eberhart-Phillips et al. 1989, Yan and Han 2009). A problem that arises when making ultrasonic velocity measurements is the difficulty in making measurements at very low effective (high pore) pressures (e.g., in the range $0 - 4$ MPa). This is due to issues with transducer-sand coupling and the preparation of samples (Bachrach et al. 1998, Duffaut and Landro 2007, Avseth and Skjel 2011). In addition to this problem, empirical regressions typically lack any underlying physical basis, and thus often fail to accurately predict the pressure dependence of the elastic properties beyond the limited range of measured pressure values. For an overview of existing theoretical and empirical models for the pressure sensitivity of sediment elastic properties the reader is directed to Chapters 3, 4, 5, and 6 of this thesis.

2.7 Conclusions

Rock physics relationships attempt to relate geophysical observations to the underlying physical properties of rocks. Such relationships play a key role in reservoir characterisation and monitoring, because they make it possible to quantify and interpret 3D/4D seismic signatures in terms of the reservoir properties of interest (e.g., saturation and pressure variations). In this chapter we introduced the key rock physics relationships that are used throughout this thesis, including briefly some limitations of existing models for pressure-dependent elastic properties. The following four chapters discuss in detail the limitations of existing empirical and theoretical relationships for pressure-dependent elastic properties, and introduce new models to overcome some of the aforementioned problems.
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A new velocity-pressure-compaction model for uncemented sediments

3.1 Foreword

This chapter presents a new model for the pressure sensitivity of uncemented sediments. It outlines the theoretical development of the model and the complete text has been published as a technical paper in *Geophysical Journal International* - ([Saul and Lumley](#) 2013).

3.2 Abstract

Knowledge of the pressure dependence of rock properties is useful for a wide range of earth science problems, especially related to pore pressure changes caused by fluid injection or withdrawal, as often occurs in groundwater, hydrocarbon, and CO₂ sequestration reservoirs. A long-standing problem is that theoretical models of velocity-pressure response often do not match laboratory measurements, and alternately, empirical regressions fit to such data do not extrapolate accurately to wider pressure ranges since they have little or no physical basis. Accurate determination of the dry rock frame properties at low effective pressure is a key aspect of the problem, particularly when ultrasonic laboratory measurements are not available in this pressure range. We present a new model to describe the pressure sensitivity of the bulk and shear moduli for uncemented sedimentary rocks. Our model incorporates effects of sedimentary compaction and critical porosity, including a relationship to account for porosity and density change with pressure. The model is tested on laboratory measurements for various rock samples and fits well over a wide range of pressures. The new velocity-pressure model should be useful for improved prediction and
3.3. Introduction

The injection (withdrawal) of fluids into (from) porous sedimentary rock results in changes in pore pressure and stress, and therefore alters the elastic rock properties which are typically pressure sensitive. Knowledge of the pressure dependence of elastic rock properties is useful for analysis of rocks in sedimentary basins, including prediction of pore pressure and geomechanical effects (e.g., Dutta, 2002), and time-lapse monitoring of hydrocarbon, groundwater, geothermal, and CO₂ sequestration reservoirs (e.g., Lumley, 2001).

Although many studies of the velocity-pressure response in unconsolidated sands have been conducted (e.g., Domenico, 1977; Zimmer, 2003), and a number of theoretical models developed, for example, the Hertz-Mindlin and Walton models (Mindlin, 1949; Walton, 1975, 1987), the discrepancy between measured and predicted values of pressure dependent elastic properties remains an active area of research. Because theoretical models often do not adequately predict measured elastic properties, empirical regressions are fit to ultrasonic velocity measurements for core samples over a limited range of pressure values (e.g., Eberhart-Phillips et al., 1989; Yan and Han, 2009). However, empirical regressions typically lack any underlying physical basis, and thus often fail to accurately predict the pressure dependence of the elastic properties beyond the limited range of measured pressure values.

A number of factors contribute to the pressure sensitivity of a given sedimentary rock, including mineral composition, texture, grain size, porosity, degree of compaction, and state of cementation (Mayr and Burkhardt, 2006; Pervukhina et al., 2010). We develop a new velocity-pressure model that appears to accommodate realistic variations in these factors, excluding grain cementation, and show that we can accurately fit ultrasonic velocity data over a wide range of effective pressures. We also demonstrate that our model can be used to accurately predict the elastic rock properties of core samples beyond the range of measured pressure values. This paper presents the background theory and physical development of the new velocity-pressure model, and shows examples of the accurate fit to various laboratory measurements for uncemented sediments.
3.4 Theory and model development

3.4.1 Porosity-depth trends in uncemented sands

Geomechanical compaction is the dominant porosity-reducing mechanism for sedimentary rock during burial from the earth’s surface to depths of about 3 km (i.e., the approximate maximum depth limit of mechanical compaction). Geochemical alteration and diagenesis (grain cementation, re-mineralization etc.) can also be important, especially for burial depths greater than 3 km, but here we assume that the rocks have not undergone significant chemical change or grain cementation. Under typical lithostatic and hydrostatic pressure conditions it has been shown that compaction-induced porosity decrease approximately follows an exponential trend with depth:

$$\phi(z) = \phi_c e^{-cz}$$

(3.1)

where $\phi$ is the porosity at depth $z$, $\phi_c$ is defined as the critical porosity at $z = 0$ [Nur et al., 1995], and $c$ is a compaction rate constant [Athy, 1930].

However, at shallow burial depths, grain rotation and reorientation during compaction can also account for significant porosity loss [Berner, 1980]. This added component of porosity decrease due to compaction implies that equation (3.1) is not fully correct at shallow depths. To account for porosity reduction mechanisms at both shallow and deeper depths, Dutta et al. [2009] proposed a double exponential equation of the form:

$$\phi(z) = ae^{-bz} + ce^{-dz},$$

(3.2)

where $a$, $b$, $c$, and $d$ are fitting parameters. Since we are interested in porosity variation due to compaction over the depth range from the surface (i.e., critical porosity) to the maximum limit of mechanical compaction, we propose a slightly more general double exponential porosity-depth relationship:

$$\phi(z) = \phi_\infty + ae^{-bz} + ce^{-dz},$$

(3.3)
3.4. Theory and model development

where $\phi_\infty$ is the irreducible non-zero porosity at the limit of mechanical compaction, and $a$, $b$, $c$, and $d$ are fitting parameters, different to those in equation 3.2. In this model we assume no porosity loss due to grain crushing, chemical diagenesis or cementation. Note that the term $(\phi_\infty + a + c)$ is equal to the critical porosity $\phi_c$ at $z = 0$.

3.4.2 Depth-pressure relationship

The physical properties of a sedimentary rock (e.g., porosity and elastic moduli) are known to vary with effective pressure, which is defined as the difference between lithostatic overburden pressure $P_{ov}$ and pore fluid pressure $P_p$:

$$P_{eff} = P_{ov} - nP_p,$$  \hspace{1cm} (3.4)

where $n$ is the effective stress coefficient, which in general is porosity dependent but we herein assume to be 1.0, as is appropriate for the case of high porosity unconsolidated sands (e.g., Siggins and Dewhurst 2003, Hofmann et al. 2005). The relationship for effective pressure given by equation 3.4 can then be rewritten as

$$P_{eff} = g \int_0^z [\rho_b(z) - \rho_f(z)] \, dz,$$  \hspace{1cm} (3.5)

where $g$ is the gravitational acceleration (assumed to be approximately constant over the depth range $z$), $\rho_f$ is the fluid density, and $\rho_b$ is the bulk density of the saturated rock given by:

$$\rho_b = \phi \rho_f + (1 - \phi) \rho_m,$$  \hspace{1cm} (3.6)

where $\rho_m$ is the density of the mineral matrix (i.e., effective grain density).

Over the depth range of observed mechanical compaction in the subsurface, and under hydrostatic pressure conditions, it is reasonable to assume to first order that effective pressure increases linearly with depth (e.g., Dutta 2002, Chopra and Huffman 1965, Dutta et al. 2009). Given the linear relation between depth and effective pressure, we can make a linear variable transformation from $z$ to $P_{eff}$ in the porosity-depth relationship of equation
3.3 to obtain the following porosity-pressure relationship:

$$\phi(P_{\text{eff}}) = \phi_\infty + \alpha e^{-\xi P_{\text{eff}}} + \gamma e^{-\psi P_{\text{eff}}},$$  \hspace{1cm} (3.7)

where $\alpha$, $\xi$, $\gamma$, and $\psi$ are new fitting parameters. Details for the variable transformation are given in Appendix A. At $P_{\text{eff}} = 0$ the porosity is at the critical porosity value, given by:

$$\phi_c = \phi_\infty + \alpha + \gamma.$$  \hspace{1cm} (3.8)

By choosing optimal parameter values, equation 3.7 provides an excellent fit to porosity-pressure data for uncremented sediments, as shown by the Galveston Beach example from Zimmer (2003) (Figure 3.1a). The porosity-pressure data shown in Figure 3.1a are from loading cycle measurements, directly analogous to porosity loss due to geomechanical compaction (e.g., Zimmer, 2003; Dutta et al., 2009, 2010). The excellent fit to the data suggests the form of equation 3.7 is consistent with the physical mechanisms that account for porosity loss in uncremented sediments with increasing loading pressure. A fit of the form of equation 3.2 does not include a finite porosity value at the limit of mechanical compaction and thus is not consistent with the physical mechanisms that take place at high effective pressures (black dashed curve in Figure 3.1a).

Combining equation 3.6 with the porosity-pressure relationship of equation 3.7 allows us to derive the following density-pressure relationship:

$$\rho_b = \rho_m + (\rho_f - \rho_m)(\phi_\infty + \alpha e^{-\xi P_{\text{eff}}} + \gamma e^{-\psi P_{\text{eff}}}).$$  \hspace{1cm} (3.9)

The form of the density-pressure relationship in equation 3.9 is also in excellent agreement with density-pressure data for uncremented sediments (Figure 3.1b).

3.4.3 Moduli-porosity trends in uncremented sands

The bulk modulus $K$ and shear modulus $G$ tend to vary inversely with porosity such that for uncremented sediments, high-porosity rocks are softer and weaker than low-porosity rocks. As shown by Walsh (1965), Zimmerman (1991), and Mavko et al. (1998) the rock
3.4. Theory and model development

Figure 3.1: (a) Porosity versus effective pressure. (b) Density versus effective pressure. The data are measured by Zimmer (2003) on the dry, unconsolidated Pomponio Beach sand. The data are selected from the loading cycles of the measurements. The solid black line in (a) shows the proposed fit to the data from equation 3.7. The black dashed line shows the fit of the form of equation 3.2. The black line in (b) shows the corresponding prediction of the density-pressure relationship from equation 3.9.

The bulk modulus can be written as

\[
K = \left[ \frac{1}{K_m} + \frac{\phi}{K_\phi} \right]^{-1}
\]

(3.10)

\[
= \frac{1}{c_1 + c_2 \phi}
\]

(3.11)

where \(K\) can be either the dry or saturated bulk modulus, \(1/K_\phi\) is the pore-space compressibility under either dry or saturated conditions respectively, \(K_m\) is the bulk modulus of the mineral grains, \(c_1 = 1/K_m\), and \(c_2 = 1/K_\phi\). As equation 3.11 shows, \(K\) varies as the inverse of a linear function of porosity, as is also evident by the form of the Reuss lower bound (Reuss, 1929) and the lower Hashin-Shtrikman bound (Hashin and Shtrikman, 1963). For uncemented sands at constant effective pressure, the inverse linear dependence of bulk modulus on porosity is typically due to variations in pore-space compressibility associated with grain sorting and packing (Figure 3.2) (Avseth et al., 2000).

The concept of critical porosity was first presented by Nur et al. (1995). Critical porosity is defined as the porosity at which a rock’s mechanical and elastic behaviour is
3.4. Theory and model development

Figure 3.2: Normalised saturated bulk modulus, $K_{\text{sat}}/K_m$, versus porosity, coloured by effective pressure, for unconsolidated samples from [Zimmer (2003)]. The saturated bulk modulus has been calculated from the dry ultrasonic velocity measurements and Gassmann fluid substitution. Blue curves show equation 3.11 for various values of normalised pore-space compressibility, $K_\phi/K_m$. For a constant effective pressure, the bulk modulus varies as the inverse of a linear function of porosity. The red line is the Reuss bound.

separated into two distinct domains: for porosities less than $\phi_c$, the grains within the rock are load bearing, while for porosities greater than $\phi_c$, the grains are in a non-load-bearing suspension (Mukerji et al., 1995). We assume that the depositional porosity of an unconsolidated sediment at zero effective pressure is equal to the critical porosity. The bulk modulus $K_c$ of a sediment at critical porosity can be accurately calculated as the Reuss harmonic average of the fluid and mineral constituents (Marion et al., 1988):

$$K_c = \left( \frac{\phi_c}{K_f} + \frac{1 - \phi_c}{K_m} \right)^{-1}, \quad (3.12)$$

where $K_f$ is the bulk modulus of the pore fluid. For dry conditions, $K_f$ can be specified in various ways as per the discussion in Mavko et al. (1998, p. 274). At critical porosity, the shear modulus $G$ of the sediment is approximately zero since the fluid is load bearing and fluids have little or no shear strength. Marion et al. (1988) showed that in sediment suspensions the compressional velocity $V_P$ varies negligibly for porosity values greater than
3.4. Theory and model development

the critical porosity and is accurately approximated by Woods (1955) equation:

\[ V_P = \sqrt{\frac{K_c}{\rho_b}}. \]  
(3.13)

Laboratory measurements from Zimmer (2003) confirm that as the effective pressure approaches zero, the saturated bulk modulus approaches the Reuss bound (red curve in Figure 3.2). We use dry velocity measurements to avoid the effects of velocity-frequency dispersion (Mavko et al., 1998), and calculate the saturated bulk modulus using Gassmann fluid substitution (Gassmann, 1951). The fact that the bulk modulus approaches the Reuss bound at zero effective pressure confirms that porosity approaches the critical porosity value. Studies by others (e.g., Hamilton, 1971; Prasad, 2002; Vanorio et al., 2003) also confirm this result. We therefore suggest that velocity-pressure models for uncemented sediments should include a critical porosity constraint at zero effective pressure, as given by equation 3.13.

3.4.4 A new seismic velocity-pressure model

We are interested in developing an improved relationship for pressure-dependent elastic moduli. Equation 3.7 constitutes a new relationship for porosity variation with pressure, and we justify that \( K \) varies as the inverse of a linear function of porosity (equation 3.11). By combining the two relationships, we therefore propose a model of the following form to describe the pressure dependence of bulk modulus:

\[ K \propto \frac{1}{c_1 + c_2 \phi(P_{\text{eff}})} \]  
(3.14)

\[ = \frac{1}{\kappa + \sigma e^{-\beta P_{\text{eff}}} + \tau e^{-\delta P_{\text{eff}}}}, \]  
(3.15)

where \( \kappa, \sigma, \beta, \tau, \) and \( \delta \) are constants, in general different to those in equation 3.7. Applying a Taylor series expansion to equation 3.15 leads to our new model for the pressure sensitivity of bulk modulus:

\[ K(P_{\text{eff}}) = K_\infty - A e^{-B P_{\text{eff}}} - C e^{-D P_{\text{eff}}}, \]  
(3.16)
where $K_\infty$, $A$, $B$, $C$, and $D$ are in general fitting parameters, but have relationships back to depth-dependent porosity and pressure via the coefficients in equations 3.3, 3.7 and 3.15. The detailed expansion of equation 3.15 and derivation of equation 3.16 are shown in Appendix B. Note that the term $(K_\infty - A - C)$ is equal to $K_c$, the zero effective-pressure constraint on $K$ at critical porosity (equation 3.12). $K_\infty$ denotes the asymptotic value of the bulk modulus at high effective pressure. We use a relation analogous to equation 3.16 for the shear modulus $G$. We can then predict pressure-dependent velocities using the standard elastic velocity relations for isotropic media:

$$V_P = \sqrt{\frac{K + \frac{4}{3}G}{\rho_b}} \quad \text{and} \quad V_S = \sqrt{\frac{G}{\rho_b}}, \quad (3.17)$$

where $V_P$ and $V_S$ are the compressional and shear velocities, respectively.

### 3.5 Application to laboratory data

We fit our new model to the dry bulk and shear moduli of the Pomponio Beach sand sample from Zimmer (2003) (Figure 3.3a). The measured laboratory data are taken from loading cycles under dry conditions, and no smoothing is applied to the data prior to fitting. We also predict the corresponding pressure-dependent dry compressional and shear velocities using the fits to the dry moduli and equation 3.17 (Figure 3.3b). The model accurately describes the pressure sensitivity of the dry rock moduli and velocities, even to very low effective pressures. We compare the fit of our model with predictions from two commonly used theoretical models: the Hertz-Mindlin model (Mindlin, 1949) and the Walton Smooth model (Walton, 1987) (Figure 3.4). These theoretical models have the general form:

$$\lambda = \lambda_{\text{eff}} \frac{[2 - v - 2f_t(1 - v)]}{(2 - v)}, \quad (3.18)$$

$$G = G_{\text{eff}} \frac{[2 - v + 3f_t(1 - v)]}{(2 - v)}, \quad (3.19)$$
3.5. Application to laboratory data

![Graph](image)

**Figure 3.3:** (a) Fit of proposed ‘SL’ model to dry bulk and shear moduli for the Pomponio Beach sand sample from Zimmer (2003). Data are for the loading cycles of the experiment. (b) Corresponding prediction of dry compressional and shear velocities calculated from the moduli fits in (a) and equation 3.17.

where \( \lambda \) and \( G \) are the dry-rock Lamé parameters, \( v \) is the grain Poisson ratio, \( f_t \) is the fraction of grain contacts that have infinite friction, and

\[
\lambda_{\text{eff}} = \frac{C(1 - \phi_c)}{5\pi} \frac{\lambda_0}{(1 - v)} \left[ \frac{3\pi}{2} \frac{(1 - v)}{(1 - \phi_c)C} \frac{P_{\text{eff}}}{\lambda_0} \right]^{1/3},
\]

(3.20)

\[
G_{\text{eff}} = \frac{C(1 - \phi_c)}{5\pi} \frac{G_0}{(1 - v)} \left[ \frac{3\pi}{2} \frac{(1 - v)}{(1 - \phi_c)C} \frac{P_{\text{eff}}}{G_0} \right]^{1/3},
\]

(3.21)

where \( C \) is the coordination number (number of contacts per grain), and \( \lambda_0 \) and \( G_0 \) are the grain Lamé parameters. When \( f_t = 1 \), equations 3.18 and 3.19 are equivalent to the Hertz-Mindlin model (grains frictionally locked, zero grain-to-grain shear slip with pressure change), and when \( f_t = 0 \), equations 3.18 and 3.19 are equivalent to the Walton Smooth model (frictionless grain contacts, maximum grain-to-grain shear slip with pressure change). Here, we only show comparisons to the Hertz-Mindlin and Walton Smooth.
3.5. Application to laboratory data

**Figure 3.4:** Fit of proposed ‘SL’ model to dry bulk and shear moduli for the Galveston Beach sand sample of Zimmer (2003). Data are for the loading cycles of the experiment. Predictions from the Hertz-Mindlin and Walton Smooth models are shown for direct comparison with the proposed model.

models. Other authors (e.g., Bachrach and Avseth 2008; Dutta et al., 2010) have extended these models to include $f_t$ as a fitting parameter, as well as empirical $C(\phi)$ and $C(P_{\text{eff}})$ relationships to account for changes in coordination number with porosity and pressure. Although these extended models fit the data better than the original Hertz-Mindlin and Walton models, they require complex fitting methodologies and often yield non-physical results, such as the requirement for two separate $C(\phi)$ and $G$ estimates in order to fit both the compressional and shear velocity data for the same rock (Dutta et al., 2010). It should also be noted that non-physical values of $\phi_c$ are often required to fit the Hertz-Mindlin and Walton contact models, as well as the unlikely knowledge of the porosity at each pressure step in order to determine $C$ from the coordination-porosity relation of Murphy (1982).

Also of note is that the Hertz-Mindlin and Walton models result in a constant ratio of bulk to shear modulus versus pressure, which is typically not observed in laboratory measurements (Figure 3.6). A constant bulk to shear modulus ratio means that one can obtain a reasonable fit to one of the moduli as a function of pressure, but not the other, resulting in inaccurate velocity-pressure predictions. We calculate the predictions of compressional and shear velocity (equation 3.17) for the Galveston Beach sample data using the theories.
3.5. Application to laboratory data

**Figure 3.5:** Dry compressional and shear velocities for the Galveston Beach sand sample of [Zimmer, 2003](#). Data are for the loading cycles of the experiment. Predictions for the SL, Hertz-Mindlin, and Walton Smooth models are calculated from the moduli fits in Figure 3.4 and equation 3.17. The fit of the empirical single exponential model of [Yan and Han, 2009](#) shows that the model fails to fit the data at low effective pressures.

Of Hertz-Mindlin and Walton, and compare the results to our model (Figure 3.5). We also compare the fit of the velocity-pressure model of [Yan and Han, 2009](#):

\[
V(P_{\text{eff}}) = V_{\text{max}} \left( 1 - c_P e^{-\frac{P_{\text{eff}}}{b_P}} \right),
\]

where \(V_{\text{max}}, c_P,\) and \(b_P\) are fitting parameters. The [Yan and Han, 2009](#) model fails to fit the data at low effective pressures (Figure 3.5). Alternatively our model, with the zero effective pressure constraint (equation 3.13), predicts an excellent fit to the velocity data over the full pressure range. We have tested our model with equal success on a wide variety of uncemented rock samples, as shown for example by the fit to dry laboratory measurements on a classic Ottawa sand core sample from [Domenico, 1977](#) (Figure 3.7).
3.5. Application to laboratory data

**Figure 3.6:** Dry bulk to shear modulus ratio for the Galveston Beach sand sample of [Zimmer (2003)]. Data are for the loading cycles of the experiment. The ratio varies significantly at low effective pressures, which cannot be predicted with the theoretical Hertz-Mindlin and Walton Smooth models.

**Figure 3.7:** Fit of proposed SL model to Ottawa Beach sand data from [Domenico (1977)]. (a) Porosity versus effective pressure with the fit of equation 3.7. (b) Dry compressional and shear velocity versus effective pressure with predictions calculated from proposed model.
3.6 Discussion and conclusions

We develop a new velocity-pressure model that incorporates effects of sedimentary compaction and critical porosity. Our model accurately describes the behaviour of dry bulk and shear moduli, porosity, and density over a wide range of effective pressures, especially at low effective pressures where current models often fail. We have chosen to formulate our model in terms of the elastic moduli, rather than $P$- and $S$-wave velocities, since this formulation ensures that the resulting velocity-porosity-pressure relationships are physically realisable.

Beard and Weyl (1973) showed that sorting is the primary controlling factor for the depositional porosity of a sediment. Since we assume depositional porosity is equivalent to critical porosity, we can use grain size distribution data to obtain an accurate estimate of the zero effective pressure constraint (equation 3.12). Since our new model includes this zero effective pressure constraint, which is difficult to obtain directly with ultrasonic velocity measurements, our model can be used to accurately predict the velocity-pressure response beyond the available pressure measurement ranges.

The velocity-pressure model we have presented applies to depositional compaction or loading; however, the model may also be useful for geomechanical dilation or unloading under certain circumstances. Loading implies that the confining pressure is increased while the pore pressure remains approximately constant. An example of loading and compaction can be demonstrated by pressing down firmly on loose wet beach sand in a bucket. Geomechanical loading corresponds to geological compaction of sediments during burial at hydrostatic pressure conditions (e.g., Zimmer 2003, Dutta et al. 2009, 2010).

We could subsequently unload the sediment in two different ways which have asymmetric loading and unloading responses. If we were to unload the sediment by removing the applied force and thus decreasing the confining pressure, the sediment would not return to its original porosity and elastic moduli values, due to the irreversible changes associated with compaction (Figure 3.8). In the beach sand example, the sand would not return to its original height and shape after we stop pressing downward. Asymmetric loading and unloading in this manner is equivalent to burying a sediment to a certain compaction
3.6. Discussion and conclusions

Figure 3.8: Diagram of loading and unloading trends for uncemented sedimentary rocks. (a) Effective pressure versus saturated compressional velocity. (b) Effective pressure versus porosity. At point A velocity is given by equation 3.13 with porosity equal to the depositional (critical) porosity. Increasing effective pressure by sediment loading causes compaction, increases velocity and decreases porosity along the loading curve until point B. If we subsequently unload the sediment by decreasing the effective pressure via a reduction of the confining (overburden) pressure, the velocity and porosity will move along a new unloading curve to C. After such unloading, non-reversible compaction will result in a higher velocity and a lower porosity than the original depositional values, and thus exhibits asymmetric loading and unloading curves (hysteresis). From point B, if we instead decrease the effective pressure to unload the sediment by increasing the pore pressure, we contend that the resulting pore-space dilation in the uncemented sediment will cause the velocity and porosity to reverse back along the compaction loading trend, until at zero effective pressure the velocity and porosity will be approximately equal to the original depositional values, thus displaying symmetric loading and unloading curves.

Conversely, we could instead unload the sediment by increasing the pore pressure rather than decreasing the confining pressure. In the beach sand example, this could be achieved by injecting water into the sand - the compaction that occurred during loading will be reversed, and the sand will return approximately to its original conditions (symmetric loading...
and unloading). Unloading in this manner may similarly be achieved by injecting fluids at depth into unconsolidated sedimentary rock, thereby increasing the pore pressure and dilating the pore space (reversing the compaction that occurred during burial). This effect has been observed in unconsolidated reservoir rocks where large water injection wells significantly increase the pore pressure, sometimes to the point where the rock reaches zero effective pressure (e.g., Landro 2001; Smith et al. 2010). We contend that unloading by pore pressure increase in unconsolidated sediments can be described by reversing along the compaction loading curve (i.e., symmetric loading and unloading). Grain cementation makes the unloading process more complex and non-reversible (Dvorkin and Nur 1996; Siggins and Dewhurst 2003; Hofmann et al. 2005). Therefore, our model for velocity-pressure sensitivity is applicable not only for unconsolidated sediments undergoing compaction, but also for unconsolidated sedimentary rocks at depth undergoing unloading by pore pressure increase, such as that caused by fluid injection.

Our velocity-pressure-compaction model addresses a long-standing problem in that theoretical relationships such as the Hertz-Mindlin and Walton models often do not accurately fit laboratory core measurement data, and alternately empirical regression models that can be adapted to fit the data often do not accurately predict the correct velocity-pressure behaviour outside of the range of measured pressure values. In contrast, our model appears to accurately fit measured velocity-pressure data over a wide range of sample types and pressure ranges for unconsolidated sedimentary rocks.

Our velocity-pressure model may have beneficial implications for the improved prediction and interpretation of pressure-sensitive rock properties and their effects in seismic data. For example, pore pressure prediction analyses (e.g., Dvorkin et al. 1999; Prasad 2002) typically make estimates of pore pressure change using velocity-pressure relationships and measured changes in log and seismic velocity data to predict anomalously over-pressured zones which can be hazardous to drilling operations (e.g., Deepwater Horizon disaster in the Gulf of Mexico, 2010). As another example, time-lapse 4D seismic is a well established technique to monitor fluid saturation and pore pressure changes in reservoir rock (e.g., Lumley 2001). The feasibility and interpretation of detecting a 4D seismic signal above the non-repeatable noise threshold depends in part on an accurate estimate of the velocity-
3.7 Acknowledgments

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3.7. Acknowledgments
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A | Variable transform $\phi(z)$ to $\phi(P_{\text{eff}})$

We assume that $P_{\text{eff}}$ varies linearly with depth:

$$P_{\text{eff}}(z) = kz + k_0,$$  \hspace{1cm} (A.1)

where $k$ and $k_0$ are constants. Therefore,

$$z = \frac{P_{\text{eff}} - k_0}{k}.$$  \hspace{1cm} (A.2)

Substituting this into equation 3.3 gives:

$$\phi\left(\frac{P_{\text{eff}} - k_0}{k}\right) = \phi_\infty + ae^{-b\left(\frac{P_{\text{eff}} - k_0}{k}\right)} + ce^{-d\left(\frac{P_{\text{eff}} - k_0}{k}\right)}$$  \hspace{1cm} (A.3)

$$= \phi_\infty + ae^{-b\frac{P_{\text{eff}}}{k}} e^{b\frac{k_0}{k}} + ce^{-d\frac{P_{\text{eff}}}{k}} e^{d\frac{k_0}{k}}.$$  \hspace{1cm} (A.4)

Therefore,

$$\phi(P_{\text{eff}}) = \phi_\infty + \alpha e^{-\xi P_{\text{eff}}} + \gamma e^{-\psi P_{\text{eff}}},$$  \hspace{1cm} (A.5)

where $\alpha = ae^{b\frac{k_0}{k}}$, $\xi = \frac{b}{k}$, $\gamma = ce^{d\frac{k_0}{k}}$, and $\psi = \frac{d}{k}$.
B | Derivation of proposed model

As equation 3.11 shows, \( K \) varies as an inverse function of porosity:

\[
K \propto \frac{1}{c_1 + c_2 \phi(P_{\text{eff}})} \quad (B.1)
\]
\[
= \frac{1}{\kappa + \sigma e^{-\beta P_{\text{eff}}} + \tau e^{-\delta P_{\text{eff}}}} \quad (B.2)
\]
\[
= \frac{1}{\kappa \left(1 + \frac{\sigma}{\kappa} e^{-\beta P_{\text{eff}}} + \frac{\tau}{\kappa} e^{-\delta P_{\text{eff}}})\right). \quad (B.3)
\]

Defining \( y = \frac{\sigma}{\kappa} e^{-\beta P_{\text{eff}}} + \frac{\tau}{\kappa} e^{-\delta P_{\text{eff}}} \) gives:

\[
K = \frac{1}{\kappa (1 + y)}. \quad (B.4)
\]

The Taylor series expansion is:

\[
\frac{1}{1 + y} = \sum_{n=0}^{\infty} (-1)^n y^n \quad (B.5)
\]
\[
= 1 - y + y^2 - y^3 + y^4 + \sum_{n=5}^{\infty} (-1)^n y^n \quad (B.6)
\]
\[
= 1 - y + y^2 - ... \quad (B.7)
\]

Thus,

\[
K = \frac{1}{\kappa \left(1 - y + y^2 - ...ight)} \quad (B.8)
\]
\[
= \frac{1}{\kappa} \left[1 - \frac{1}{\kappa} \left(\sigma e^{-\beta P_{\text{eff}}} + \tau e^{-\delta P_{\text{eff}}}\right) + \frac{1}{\kappa^2} \left(\sigma^2 e^{-2\beta P_{\text{eff}}} + \tau^2 e^{-2\delta P_{\text{eff}}} + \sigma \tau e^{-(\beta+\delta) P_{\text{eff}}}\right) - ...ight]. \quad (B.9)
\]
Making a first order approximation that truncates at $n = 1$ gives

$$ K = \frac{1}{\kappa} - \frac{\sigma}{\kappa^2} e^{-\beta_P e_{\text{eff}}} - \frac{\tau}{\kappa^2} e^{-\delta_P e_{\text{eff}}}. \quad (B.11) $$

Defining $K_\infty = \frac{1}{\kappa}$, $A = \frac{\sigma}{\kappa^2}$, $B = \beta$, $C = \frac{\tau}{\kappa^2}$, and $D = \delta$ we arrive at our new model for the pressure sensitivity of bulk modulus:

$$ K(P_{\text{eff}}) = K_\infty - A e^{-B_P e_{\text{eff}}} - C e^{-D_P e_{\text{eff}}}, \quad (B.12) $$

where $K_\infty$, $A$, $B$, $C$, and $D$ are all constants. Also the constraint $K_c = K_\infty - A - C$ from equation 3.12 must be satisfied. We note that making the first order approximation implies that $K_\infty \gg A$ and $K_\infty \gg C$, while the constraint from equation 3.12 means that $K_\infty \approx A + C$. Making the first order approximation, even when not fully satisfying $K_\infty \gg A$ and $K_\infty \gg C$, is sufficiently valid to describe rocks that are moderately compressible with pressure (i.e., where $K_\infty - K_c$ is not overly large). We find that this first order approximation appears to be valid for a wide variety of rocks, since we are able to accurately fit the form of equation 3.16 to a range of uncemented sedimentary rock samples and properties as shown in the text.
Predicting the elastic properties of unconsolidated sandstones at low effective pressure: A practical method

4.1 Foreword

The previous chapter covered the theoretical development of the Saul Lumley (SL) model, including detailed discussion on the applicability of the model for geoscience applications. The purpose of this chapter is to build on this discussion, to cover in detail how the SL model can be applied in a realistic setting to determine unconsolidated sandstone elastic properties. We discuss in practical terms how to fit the SL model to available laboratory data, including how to calculate critical porosity from grain size distribution data and therefore predict the elastic properties of unconsolidated sediments at low effective pressure. The complete text of this chapter is to be submitted as a technical paper to *Geophysics*.

4.2 Abstract

Determining the elastic properties of unconsolidated sandstones at low effective pressure is required for pore-pressure prediction, and for modelling/interpreting 4D seismic responses due to fluid and pressure changes. However, a greater uncertainty in elastic properties at low effective pressure (i.e., in the range 0–5 MPa) often occurs due to the difficulty in predicting pressure sensitivity from first principles, and because making laboratory measurements at low effective pressures (i.e., high pore pressures) is generally impractical. Using a two-stage fitting process of a double exponential model we present a practical
4.3. Introduction

method for predicting the elastic properties of unconsolidated sediments at low effective pressure, when only data at higher pressures (i.e., > 4 MPa) is available. The model includes a critical porosity constraint at zero effective pressure, estimated from grain-size distribution data, and a relationship accounting for changes in porosity with effective pressure. The former constraint ensures that the sediment compressional velocity approaches the value of a suspension (at zero effective pressure), with shear velocity approaching zero. The results show accurate predictions of elastic properties, including $V_P/V_S$ ratio, can be made at low effective pressure, even in the absence of data in the low effective pressure range. We assert that the proposed model has implications for the improved modelling and interpretation of 3D and 4D seismic data, including for pore-pressure prediction and 4D seismic reservoir characterisation.

4.3 Introduction

High pore pressure conditions within unconsolidated sedimentary rock often occur due to shallow water flows (SWF) (Huffman and Castagna, 2001) and to injection associated with hydrocarbon production and CO$_2$ sequestration (e.g., Lumley, 2001; Duffaut and Landro, 2007, Smith et al., 2010). SWFs can be a major hazard for drilling operations (e.g., Ostermeier et al., 2002), while high pore pressure due to fluid injection in a producing reservoir has potential hazardous implications for future infill drilling targets (e.g., Duffaut and Landro, 2007, Thomas and Smith, 2010), as well as implications for monitoring production/injection related effects using time-lapse (4D) seismic (e.g., Lumley, 1995, Landro, 2001, Lumley, 2001, Duffaut and Landro, 2007).

Elastic properties such as velocity are known to vary as a function of effective pressure $P_{\text{eff}}$ (e.g., Domenico, 1977, Prasad, 2002, Zimmer, 2003), defined as the difference between overburden pressure $P_{\text{ov}}$ and pore pressure $P_p$:

$$P_{\text{eff}} = P_{\text{ov}} - nP_p,$$

(4.1)

where $n$ is the effective stress coefficient, typically assumed to be 1.0 for high porosity unconsolidated sands (e.g., Hofmann et al., 2005). As such, high pore pressure conditions
associated with SWFs and injection at high pressures can result in low effective pressure conditions within unconsolidated sands. Because velocity varies with effective pressure, seismic velocities are often used for the prediction of high pore-pressure conditions (e.g., Pennebaker, 1968; Bowers, 1995; Grauls et al., 1995; Moos and Zwart, 1998; Dutta, 2002), and for interpreting production/injection-related effects in 4D seismic data (e.g., Nur, 1982; Lumley, 1995; Tura and Lumley, 1999; Lumley, 2001; Landro, 2001; Hatchell et al., 2003; Calvert, 2009). In order to quantify such pressure-related effects, a knowledge of the relationship between the elastic properties and effective pressure is required.

Because the ability to predict pressure sensitivity from first principles is poor (e.g., Mindlin, 1949; Walton, 1987; Makse et al., 1999; Zimmer, 2003; Avseth and Skjei, 2011), the relationship between velocity and effective pressure is typically determined by making ultrasonic velocity measurements on core samples over a range of effective pressures (e.g., Domenico, 1977; Prasad, 2002; Zimmer, 2003). Empirical models are then fit to the measurements (e.g., Eberhart-Phillips et al., 1989; Han et al., 1986; MacBeth, 2004; Yan and Han, 2009), with the regression used to forward model changes in dry rock-frame elastic properties outside the measured range. Due to issues with transducer-sand coupling and the preparation of samples, it is difficult to make ultrasonic-velocity measurements at very low effective pressure (e.g., 0 – 4 MPa) (Bachrach et al., 1998; Duffaut and Landro, 2007). In addition to this problem, empirical regressions fit to the available data typically lack any underlying physical basis and thus often fail to predict the velocity-pressure response at low effective pressure (Saul and Lumley, 2013). This could potentially lead to inaccurate pore pressure predictions from 3D and/or 4D seismic data, with potentially hazardous implications.

Conventional overpressure prediction studies use compressional velocities ($V_P$) to estimate pore pressure, because overpressure often results in an anomalous decrease in $V_P$; however, seismic velocity is not only a function of effective pressure, it also depends on pore fluids (e.g., Gassmann, 1951; Wang and Nur, 1990; Batzle and Wang, 1992), porosity and clay content (e.g., Han et al., 1986; Marion, 1990; Yin, 1992), diagenesis (e.g., Jizba, 1991; Dvorkin and Nur, 1996), and lithology (e.g., Castagna et al., 1985; Blangy, 1992; Greenberg and Castagna, 1992). Not accounting for these effects can lead to further inac-
4.3. Introduction

Figure 4.1: Dry (triangles) and saturated (circles) $V_P/V_S$ ratio as a function of effective pressure, coloured by porosity, for uncemented sediment samples (Zimmer, 2003). Saturated $V_P/V_S$ ratio has been calculated using the dry measurements and Gassmann (1951) fluid substitution.

Accuracies in estimating pressure from compressional velocities. Recent work on AVO-based overpressure prediction utilises the $V_P/V_S$ ratio to resolve some of the ambiguity associated with using $V_P$ alone (e.g., Pigott and Tadepalli, 1996; Dvorkin et al., 1999; Bachrach et al., 2000; Dutta, 2002; Prasad, 2002; Duffaut and Landro, 2004; Kao et al., 2010). Ultrasonic velocity measurements show slightly increasing $V_P/V_S$ ratios as effective pressure decreases in dry or gas saturated rocks, and significant increases in $V_P/V_S$ ratios for water saturated rocks with decreasing effective pressure (Figure 4.1). Figure 4.1 also shows the effect that porosity has on the $V_P/V_S$ ratio, with lower porosity samples generally having higher values of $V_P/V_S$ ratios.

The objective of this study is to develop a practical method to predict the elastic properties (compressional and shear) of unconsolidated sediments at low effective pressure (e.g., in the range 0 – 4 MPa) when only data at higher effective pressures (e.g., in the range 4-20 MPa) is available. We extend on previous work by Saul and Lumley (2013), and propose a unique two-stage fitting procedure of their model, which incorporates the effects of sorting and compaction induced porosity loss, to estimate the elastic properties of unconsolidated sediments at low effective pressure. We begin with an overview of the
“SL model”, including details on the proposed zero effective pressure constraint. Next, we apply the model to real data examples to show how to estimate the zero effective pressure constraint using grain size distribution data, and also how to fit available velocity-pressure data using the two-stage fitting procedure. We show how to use the estimated zero effective pressure constraint and the two-stage fitting procedure to predict the elastic properties of unconsolidated sandstones at low effective pressures, when only data above 4 MPa is available. Finally, we discuss implications of the new approach, including for interpreting 4D seismic AVO data, and pore pressure data.

4.4 Background theory

4.4.1 The SL model

Saul and Lumley (2013) present a generalised double exponential model (“SL model”) to describe the pressure-dependent elastic properties of uncemented sediments. The model incorporates effects of sedimentary compaction and critical porosity, and also includes a relationship to account for porosity and density change with pressure. Saul and Lumley (2013) propose the following relationship to describe the pressure-dependent dry bulk modulus:

\[ K(P_{\text{eff}}) = K_\infty - Ae^{-BP_{\text{eff}}} - Ce^{-DP_{\text{eff}}}, \]  

(4.2)

where \( K_\infty, A, B, C, \) and \( D \) are fitting parameters, with \( K_\infty \) equal to the asymptotic value of the bulk modulus at high effective pressure.

At zero effective pressure a sediment will be at critical porosity (Marion et al., 1988; Prasad, 2002; Zimmer, 2003; Vernik and Kachanov, 2010; Saul and Lumley, 2012b), defined as the porosity at which a rock’s mechanical and elastic behaviour is separated into two distinct domains: for porosities less than \( \phi_c \), the grains within the rock are load bearing, while for porosities greater than \( \phi_c \), the grains are in a non-load-bearing suspension (Nur et al., 1995; Mikerji et al., 1995). The bulk modulus of a sediment at critical porosity can be accurately calculated as the Reuss harmonic average of the fluid and mineral constituents (see Figure 4.2):

\[ K_c = \left( \frac{\phi_c}{K_f} + \frac{1 - \phi_c}{K_m} \right)^{-1}, \]  

(4.3)
4.4. Background theory

**Figure 4.2:** Normalised saturated bulk modulus, $K_{\text{sat}}/K_m$, versus porosity, coloured by effective pressure, for unconsolidated samples from Zimmer (2003). The saturated bulk modulus has been calculated from dry ultrasonic velocity measurements and Gassmann (1951) fluid substitution. The blue curve is the Reuss bound (equation 4.3).

where $K_m$ is the mineral grain bulk modulus, and $K_f$ is the bulk modulus of the pore fluid (Reuss, 1929; Marion et al., 1988). For dry conditions, $K_f$ can be specified in various ways as per the discussion in Mavko et al. (1998, p. 274).

Saul and Lumley (2013) incorporate the Reuss bound into their model as a $P_{\text{eff}} = 0$ constraint by setting $(K_{\infty} - A - C)$ equal to $K_c$ within equation 4.2. At critical porosity the shear modulus $G$ of the sediment is approximately zero since the fluid is load bearing and most (non-viscous) fluids have little or no shear strength. A relation analogous to equation 4.2 is used to fit data for the shear modulus $G$.

Saul and Lumley (2013) also propose a similar relationship to describe pressure-dependent porosity:

$$\phi(P_{\text{eff}}) = \phi_{\infty} + \alpha e^{-\xi P_{\text{eff}}} + \gamma e^{-\psi P_{\text{eff}}},$$  \hspace{1cm} (4.4)

where $\alpha$, $\xi$, $\gamma$, and $\psi$ are fitting parameters, determined by a least-squares fit to measured porosity versus effective pressure data. Again, at $P_{\text{eff}} = 0$ the porosity is at the critical porosity value. To describe the change in bulk density with pressure, Equation 4.4 can be
combined with the bulk density relationship:

\[ \rho_b = \phi \rho_f + (1 - \phi) \rho_m, \]  \hspace{1cm} (4.5)

where \( \phi \) is the pressure-dependent porosity, and \( \rho_m \) is the density of the mineral matrix (i.e., effective grain density). Pressure-dependent velocities are then calculated using the standard elastic velocity relations for isotropic media:

\[ V_P = \sqrt{\frac{K + \frac{4}{3}G}{\rho_b}} \quad \text{and} \quad V_S = \sqrt{\frac{G}{\rho_b}}, \]  \hspace{1cm} (4.6)

where \( V_P \) and \( V_S \) are the compressional and shear velocities, respectively. The inclusion of a porosity-pressure relationship, along with the zero effective pressure constraint from equation (4.3), ensures that, upon Gassmann fluid substitution, saturated shear velocity approaches zero, and saturated compressional velocity approaches the value of a suspension (Woods, 1955):

\[ V_P = \sqrt{\frac{K_c}{\rho_b}}, \]  \hspace{1cm} (4.7)

### 4.5 Application to data

#### 4.5.1 Estimating sediment critical porosity

The SL model includes a zero effective pressure constraint on the bulk modulus, given by equation (4.3). The unknown parameter in the equation is the sediment critical porosity. A number of authors (e.g., Beard and Weyl, 1973; Zimmer, 2003) have shown that the primary control on depositional porosity (here assumed to be equivalent to critical porosity) is grain-size distribution (sorting). Figure 4.3a shows depositional (critical) porosity versus the Trask sorting coefficient (Trask, 1931) for 48 unconsolidated samples from Beard and Weyl (1973). Scherer (1987) generalised this data to determine a relationship between critical porosity and sorting:

\[ \phi_c = 20.91 + \frac{22.9}{S_0}, \]  \hspace{1cm} (4.8)
where $S_0$ is the Trask sorting coefficient, calculated from grain size distribution data. Figure 4.3b shows actual versus predicted critical porosity values for samples from Zimmer (2003). Predicted values were calculated using available grain size distribution data and equation 4.8. The predicted values fall close to the 1:1 line, and thus confirm that sorting is the primary control on critical porosity in these unconsolidated samples. The zero effective pressure constraint for bulk modulus can then be calculated using equation 4.3. This has implications for accurate fluid substitution (i.e., to ensure at zero effective pressure the sediment velocity equals that of a suspension), and for when ultrasonic velocity measurements are not available at very low effective pressures, which is typically the case (Bachrach et al., 1998; Duffaut and Landro, 2007).

4.5.2 Fitting the SL model

We demonstrate fitting the SL model to laboratory measurements for 2 unconsolidated samples from Zimmer (2003). Available data for each sample consist of $P$-wave velocity, $S$-wave velocity, and porosity measurements versus effective pressure, as well as grain-size distribution. The Galveston Beach sand sample is well sorted, while the glass bead sample is poorly sorted.

First, we estimate critical porosity using the available grain-size distribution data and equation 4.8. We then use this to estimate the zero effective pressure constraint from equation 4.3. A critical porosity of $\phi_c = 0.428$ has been estimated for the well sorted Galveston Beach sand, with a critical porosity of $\phi_c = 0.339$ estimated for the poorer sorted glass bead sample.

Next, we calculate dry bulk and shear moduli using the available $P$-wave velocity, $S$-wave velocity, and porosity data, and equation 4.6. We select only the data from the loading cycle measurements, because this is equivalent to geological compaction under hydrostatic pressure conditions, and also to unloading due to fluid injection in uncemented sediments (Siggins and Dewhurst 2003; Zimmer 2003; Dutta et al. 2009; Saul and Lumley 2013).

Determining the optimal parameters in equation 4.2 involves fitting two single exponentials to the moduli-pressure data - one for the low effective pressure range (e.g., 0-2 MPa) and one for the high effective pressure range (e.g., 2-20 MPa). Each single exponential has
4.5. Application to data

![Figure 4.3](image)

**Figure 4.3:** (a) Critical porosity versus Trask sorting coefficient for 48 unconsolidated samples from Beard and Weyl (1973). The generalised relationship of Scherer (1987) (equation 4.8) is shown in green. (b) Actual versus predicted critical porosity for unconsolidated samples from Zimmer (2003).

the form:

\[ K = K_{\text{max}} - ae^{-bP_{\text{eff}}}, \quad (4.9) \]

where \(K_{\text{max}}, a,\) and \(b\) are fitting parameters. For each effective pressure range (i.e., low and high) we take the logarithm of equation 4.9 and re-arrange it such that the form is linear in moduli-pressure space. We then perform a least-squares linear fit to determine the fitting parameters in equation 4.9 for each pressure range (see Figure 4.4a for Galveston Beach sand example). In the case of the low effective pressure range (0-2 MPa) \(K_{\text{max}}\) is the bulk moduli value of the next available data point above 2 MPa. Each exponential fit
(i.e., low and high) is then extended over the total pressure range (Figure 4.4b) and the two exponentials are added in a least-squares sense to minimise the data residuals via the following equation:

\[ K_{\text{dry}} = a_0 + a_1f_1 + a_2f_2, \]  \hspace{1cm} (4.10)

where \( f_1 \) and \( f_2 \) are the individual single exponential functions fit to the low and high effective pressure data ranges, respectively, and, \( a_0, a_1, \) and \( a_2 \) are fitting parameters, constrained such that the bulk modulus at zero effective pressure equals the value calculated using equation 4.3. This results in a model with the final form of equation 4.2 (Figure 4.4c).

The porosity-pressure relationship from equation 4.4 is fit with the same process as the moduli data, and along with equation 4.6 allows the calculation of dry compressional and shear velocities versus effective pressure. Fitting the model in this way (i.e., two-stage process for low and high pressures) has beneficial implications when data at low effective pressures is not available, as we will see in the next section.

We go through the same fitting process for the glass bead sample and then using Gassmann fluid substitution predict saturated compressional and shear velocities for both samples. Including a porosity-pressure relationship (equation 4.4), along with the zero effective pressure constraint determined from the estimated critical porosity and equation 4.3 ensures that the saturated compressional velocity approaches the correct suspension value given by Wood’s equation (Figure 4.5a). The constraint also ensures that saturated shear velocity approaches zero at zero effective pressure, as indicated by the \( V_P/V_S \) ratio approaching infinity (Figure 4.5b).

### 4.5.3 Prediction at low effective pressure

In this section we demonstrate the predictive power of the SL model at low effective pressures, by fitting laboratory measurements when data in the low effective pressure range (\( P_{\text{eff}} \approx 0 - 4 \) MPa) is not available. We compare the predicted velocity response at low effective pressure to the actual measurements, and to the velocity prediction \( V \) from a commonly used single exponential model (Yan and Han, 2009):

\[ V(P_{\text{eff}}) = V_{\text{max}} \left( 1 - c_P e^{-\frac{P_{\text{eff}}}{\Delta_P}} \right), \]  \hspace{1cm} (4.11)
where $V_{\text{max}}$, $c_p$, and $b_p$ are fitting parameters.

The process for fitting data with the SL model is the same as that outlined in the previous section; however, in this case $K_{\text{max}}$ in equation 4.9 for the low effective pressure range (where no data is available) is equal to the value of the measurement at the lowest effective pressure available ($\approx 4$ MPa). The zero effective pressure constraint is estimated in the same way using available grain-size distribution data and equation 4.3.

The inclusion of the critical porosity constraint at zero effective pressure, along with the two-stage fitting process, enables reasonably accurate determination of model fitting parameters in the low effective pressure range. Figure 4.6c shows the final predicted re-
4.5. Application to data

Figure 4.5: (a) Saturated compressional velocity versus effective pressure, calculated with SL fit to dry moduli and Gassmann fluid substitution. The velocity approaches the correct suspension velocity given by Wood’s equation (coloured triangles on vertical axis) for the calculated critical porosity of each sample (0.428 for Galveston Beach and 0.339 for glass beads). (b) Saturated $V_p/V_s$ ratio versus effective pressure. Saturated $V_p/V_s$ ratio approaches infinity at zero effective pressure as shear velocity approaches zero.

Response, along with actual data measured in the low effective pressure range. Figure 4.6 shows the predicted porosity-pressure response at low effective pressure, fit with the same two-stage process as used for the dry bulk moduli. Predictions of dry moduli and porosity at low effective pressure match fairly well with the actual measurements in this range, even though we used only data in the range 4 – 20 MPa in the model fit.

Next, we perform Gassmann fluid substitution to calculate saturated compressional and shear velocities. Figure 4.7a shows the saturated compressional velocity versus effective pressure for the SL model along with the comparison of the Yan and Han (2009) model (equation 4.11). Both models were only fit to data in the range of 4 – 20 MPa. The SL
4.5. Application to data

**Figure 4.6:** Fitting SL model to predict moduli and porosity at low effective pressures (0 − 4 MPa) when only data at high effective pressures (4 − 20 MPa) is available. Data used in the fit is shown as blue triangles, with actual data in the low pressure range shown in cyan. (a) to (c) involve the same fitting steps as outlined in Figure 4.4. (d) Porosity versus effective pressure with fit of equation 4.4. The critical porosity at zero effective pressure is estimated using grain size distribution data and equation 4.7.
4.6. Discussion

Figure 4.7: (a) Comparison between SL model (solid line) and the single exponential model of Yan and Han [2009] (dashed line) in predicting water saturated compressional velocity at low effective pressure using only data in the range of 4 – 20 MPa (dark blue triangles). (b) Water saturated $V_p/V_s$ ratio versus effective pressure.

model provides a much improved prediction of the actual data at low effective pressures compared to the model of Yan and Han [2009]. This is particularly evident in Figure 4.7, where predicted $V_p/V_s$ ratio is greatly under predicted at low effective pressures with the Yan and Han [2009] model.

4.6 Discussion

Fitting the SL model with the proposed two-stage approach, which includes a zero effective pressure constraint, enables fairly accurate estimates of sediment elastic properties and porosity at low effective pressures, even when data in this pressure range is not available. Figures 4.8a and 4.8b show dry compressional velocity and porosity, respectively, versus effective pressure for a number of unconsolidated samples [Zimmer, 2003]. It is interesting
to note that the most rapid change in dry compressional velocity occurs in the range $0 - 2$ MPa, with the values then ‘rolling over’ to a shallower trend with increasing effective pressure. The most rapid porosity loss also occurs in the range $0 - 2$ MPa indicating that this is likely the range where significant non-elastic changes in porosity associated with grain re-orientation and packing are occurring (Dutta et al., 2009; Saul and Lumley, 2013).

This consistent ‘rollover’ at approximately 2 MPa can be used to further improve the prediction of elastic properties at low effective pressures using the SL model. For example, the linear fit to the available high effective pressure data in Figure 4.6 can be extended to 2 MPa, with this value then used as the new $K_{\text{max}}$ in equation 4.9 for the low effective pressure range. Figure 4.9 shows the result of applying this approach to the Galveston Beach sand sample, where only data in the range $4 - 20$ MPa has been used to fit the SL model. This approach has been applied to a number of uncemented sediment samples with equal success. The improved fit indicates that one could get a reasonable prediction of the moduli-pressure relationship over the full pressure range, even when only a few data points are available at high effective pressures.
4.6. Discussion

**Figure 4.9:** Dry bulk modulus versus effective pressure. Result of fitting SL model to predict data at low effective pressure when data is only available in the range 4 – 20 MPa. The dashed line shows the result of extending the high pressure exponential fit to 2 MPa within the two-stage fitting process, to exploit the observation that the ‘rollover’ in elastic properties with effective pressure usually occurs at approximately 2 MPa (see Figure 4.8).

To further investigate possible implications of being able to accurately predict sediment elastic properties at low effective pressure we discuss the 4D seismic reservoir characterisation project at Gullfaks (Duffaut et al., 2011). 4D seismic reservoir characterisation requires accurate rock physics models for both static (e.g., mineralogy and cement volume) and dynamic (e.g., saturation and pressure) reservoir parameters, in order to be able to quantify changes in saturation and pressure associated with fluid withdrawal or injection (Tura and Lumley, 1999; Landro, 2001; Lumley, 2001; Duffaut et al., 2011). Duffaut et al. (2011) characterise the static reservoir properties of the Gullfaks oil field using the rock physics diagnostics approach of Avseth et al. (2000, 2009), and show that the reservoir sands are unconsolidated with an average porosity of approximately 33%. At Gullfaks, increasing pore pressure associated with water injection results in a significant decrease in effective pressure, approaching zero. In order to describe observed 4D AVO response around the water injectors, Duffaut and Landro (2007) show that a $V_p/V_S$ ratio of approximately 7 is required. This is consistent with saturated $V_p/V_S$ ratios observed in laboratory measurements of unconsolidated sediments at low effective pressure (Figure 4.1), and also with the low effective pressure prediction of elastic properties given by the SL model fit to higher effective pressure data (e.g., Figure 4.7). Conversely, if we were to use a single ex-
ponential model (e.g., equation 4.11), we could not predict a high enough saturated $V_P/V_S$ ratio at low effective pressure, and therefore could not describe the observed AVO response at Gullfaks. This observation from real 4D seismic data provides further confidence that the SL model can be used accurately to predict the pressure sensitivity of unconsolidated sands at low effective pressure, even when laboratory measurements in this range are not available. It also demonstrates the importance of having accurate rock physics models at low effective pressure. The inclusion of a porosity-pressure relationship in the SL model means we could also account for changes in porosity with pressure, and therefore further improve the 4D characterisation of saturation and pressure changes in the field.

The SL model has obvious implications for improved pore-pressure prediction in unconsolidated sands. This is due to the accurate determination of elastic properties, and porosity, as a function of effective pressure, particularly at low effective pressures when laboratory or well data is not available in this range. Positive implications for AVO-based pore pressure prediction methods are also apparent as the model accounts for variations in elastic properties due to differences in grain-size distribution, accounts for porosity variation with pressure, and accurately models dry and saturated $V_P/V_S$ ratios when combined with Gassmann fluid substitution. These factors are particularly important when predicting overpressure in unconsolidated sands at low effective pressure, as is often associated with SWFs.

4.7 Conclusions

We demonstrate a practical method to predict the elastic properties of unconsolidated sediments at low effective pressure. Our approach is based on a two-stage fit to available calibration data using the SL model. The two-stage fit is consistent with the observation that the transition from elastic to non-elastic geomechanical effects seems to occur around $P_{\text{eff}} = 2\, \text{MPa}$ in the wide variety of rock samples tested. The inclusion of a critical porosity constraint at zero effective pressure, which can be estimated from grain-size distribution information, ensures that at zero effective pressure the sediment compressional velocity approaches the value of a suspension, with zero shear velocity. The constraint, along with the inclusion of a porosity-pressure relationship, means accurate predictions of $V_P/V_S$
ratio can be made at low effective pressure, even when data in this range is not available. The approach presented in this paper also has applications for depth trending studies, where velocity and porosity information needs to be extrapolated to shallower depths using recorded log data over a limited interval. This model also has implications for the improved prediction and interpretation of 3D and 4D seismic data, including for pore-pressure prediction and 4D seismic reservoir characterisation with AVO based methods.

4.8 Acknowledgements

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5 | Modelling the pressure sensitivity of uncremented sediments using a modified grain contact theory

“Make your theory as simple as possible, but no simpler.”
– Albert Einstein

5.1 Foreword

The previous two chapters covered the development and application of the SL model. Chapter 3 discussed briefly that theoretical models for the pressure sensitivity of unconsolidated sediments often do not match laboratory measurements, but did not discuss reasons why. The purpose of this chapter is to investigate the reasons for the observed discrepancies, and to modify existing effective medium theory to incorporate the effects. The proposed modified grain contact theory in this chapter also includes the porosity-pressure model developed in Chapter 3, in order to account for the variation in porosity with changes in effective pressure. The complete text for this chapter is published as a technical paper in Geophysics - (Saul et al., 2013).

5.2 Abstract

We present a modified grain contact theory to better describe the pressure-dependent elastic properties of uncremented sediments. Hertz-Mindlin (HM) theory typically predicts
5.3. Introduction

shear moduli that are much higher than observed laboratory measurements, resulting in inaccurate estimates of the dry bulk to shear modulus ratio ($K_{\text{dry}}/G_{\text{dry}}$). The HM theory further predicts that the $K_{\text{dry}}/G_{\text{dry}}$ ratio is constant with pressure, whereas ultrasonic core measurements typically show an increasing $K_{\text{dry}}/G_{\text{dry}}$ ratio as effective pressure decreases. Laboratory data also suggest that the dry bulk and shear moduli variation with effective pressure is greater than the cube-root power law predicted by the HM theory. We introduce two new pressure-dependent calibration parameters to account for the shortcomings in effective medium theory, and develop a new method to predict pressure-dependent elastic properties. Our calibration parameters agree with the results of published granular dynamics simulations, and they incorporate grain relaxation and porosity effects not included in existing effective medium theories. Our new model provides improved fits to laboratory data when compared to existing models, and it can be used for improved prediction of elastic properties as a function of effective pressure. Our new theory can also be used to model uncemented sediments with values of Poisson’s ratio $> 0.25$, where many existing grain contact and effective medium theories currently fail.

5.3 Introduction

Understanding how the mechanical and elastic properties of uncemented sediments vary as a function of effective pressure is important in many aspects of earth science, including pore pressure prediction, determining borehole stability, modelling production-induced compaction, and 4D seismic monitoring of subsurface fluid-flow (e.g., Lumley, 2001). Owing to the granular nature of unconsolidated sands, the modelling of uncemented sediment properties often invokes an idealised spherical pack approximation, which greatly facilitates the development of theoretical models (e.g., Mindlin, 1949; Walton, 1987) and computational granular dynamics (GD) simulations (e.g., Makse et al., 1999; Sain, 2011). Such models aim to relate the evolution of grain-scale microstructure during deformation to the bulk properties of the granular pack. The predictions of most currently used theoretical models match fairly well with those of laboratory experiments (e.g., Domenico, 1977; Zimmer, 2003) for bulk modulus; however, because theoretical models assume homogeneity in the underlying structure, forces, and stresses, significant discrepancies between the
predicted and measured shear modulus routinely arise (e.g., Makse et al., 1999; Zimmer, 2003; Sain, 2011). Accordingly, errors in the corresponding upscaled velocity/pressure predictions can contribute to any number of reservoir management issues, including borehole breakout and the misestimation of production- and/or injection-induced 4D seismic responses - potentially with significant economic consequences.

Reconciling the discrepancy between measured and theoretical values of pressure-dependent elastic properties in uncemented sediments is an active area of research. Because theoretical models typically do not match measured elastic properties very well, practitioners often rely on empirical regressions (e.g., Eberhart-Phillips et al., 1989; Yan and Han, 2009) to fit ultrasonic velocity measurements from core samples over a limited range of measured pressure values. These empirical regressions lack an underlying physical basis, and thus, they often fail to predict the pressure dependence of elastic properties beyond the typical measured pressure range of 5 – 20 MPa. Saul and Lumley (2013) present a quantitative model based on compaction theory and the concept of critical porosity that can predict velocity-pressure sensitivity beyond these ranges; however, the model requires calibration with core measurements and does not incorporate information regarding the rock microstructure. To include the effects of changes in microstructure, some researchers have modified the existing Hertz-Mindlin (HM) and Walton theories by incorporating additional fitting parameters in the models to better match measured pressure-dependent velocities (e.g., Bachrach and Avseth, 2008; Pride and Berryman, 2009; Dutta et al., 2010; Duffaut et al., 2010).

Effective Medium Theories (EMTs) such as HM are based on calculating normal (compressive) and tangential (shear) contact stiffnesses between two identical spheres in contact. Two major discrepancies typically exist between elastic properties predicted by HM theory and those measured in the laboratory. First, the dynamic bulk and shear moduli tend to vary with effective pressure greater than the predicted cube-root power law. Second, observed $K_{dry}/G_{dry}$ ratios are significantly higher in laboratory measurements (e.g., Domenico, 1977; Makse et al., 1999; Zimmer, 2003).

Makse et al. (1999) and Sain (2011) investigate these two EMT discrepancies by conducting GD simulations of spherical packs. By accounting for increases in coordination
number with pressure, HM predictions of bulk modulus are in good general agreement
with the results of GD simulations; however, shear moduli are still greatly overpredicted,
which these authors attribute to the affine strain approximation underlying EMT. An
affine transformation is a combination of a linear transformation with a translation or ro-
tation, such that originally colinear points are still colinear after an affine transformation,
although they may be scaled (stretched or squeezed), translated, and rotated compared to
their original position. Makse et al. (1999) and Sain (2011) show that for realistic sediment
models with heterogeneous grain contacts, the application of an infinitesimal strain results
in force imbalance and unrelaxed stresses. The GD simulation calculates infinitesimal grain
displacements, referred to as grain relaxations, required to regain force equilibrium and a
relaxed stress state. Because the elastic moduli are calculated from the stress-strain ratio,
calculation of the moduli with the relaxed stresses results in a reduction of elastic moduli,
particularly the shear modulus. These authors find that the grain-relaxed shear moduli
are in better agreement with laboratory measurements.

A second approach to overcoming these two major discrepancies is to heuristically mod-
ify the EMT models directly. Bachrach and Avseth (2008) introduce a binary model in
which some fraction of the grain contacts have zero tangential stiffness, and the remaining
fraction have a tangential stiffness given by the HM model. By inverting for the respective
fraction of contact stiffnesses using measurements of Poisson’s ratio, one can also model
uncemented sediments with nonuniform grain sizes. Although this binary model improves
the fit to laboratory data, it does not directly account for grain relaxation, and it only ap-
plies to granular packs with Poisson’s ratio values between 0 and 0.25. Dutta et al. (2010)
refine this binary model and invert for the coordination number directly from the mea-
sured velocity-pressure data. They propose new relations between coordination number,
porosity, and pressure to predict velocities in uncemented sediments. Although this model
provides improved fits to measured velocity data, it yields nonphysical results including
the requirement of separate coordination number estimates to fit respective compressional
and shear velocity data for the same rock. Moreover, the model is also strictly valid for
sediments with identical, isotropic, elastic, spherical grains.

Duffaut et al. (2010) derive elastic moduli based on Mindlin’s partial-slip model (Mindlin).
which assumes partial slip at homogeneous grain contacts to reduce shear moduli values. Based on GD simulations, Sain (2011) proposes empirical relationships for pressure-dependent grain relaxation to correct for grain heterogeneity effects. The assumption of homogeneous grain contacts in the model of Duffaut et al. (2010) is likely only valid in very special cases, and thus the proposal of heterogeneous grain relaxation in Sain (2011) seems more physically reasonable. The correction factors that Sain (2011) gives also result in improved matches to elastic properties measured in the laboratory; however because the correction factors are based on GD simulations using ideal spherical packs, discrepancies between measured laboratory data for real rocks and the modified grain contact theory (GCT) still remain.

In this paper, we extend previous work to overcome several problems associated with existing modified grain contact models. First, we address how to model uncemented sediments with a dry Poisson’s ratio greater than 0.25 by inclusion of a shear modulus weighting parameter. We invert for this parameter from laboratory data, and, by defining an exponential relationship with pressure, we can accurately predict the variation in the $K_{dry}/G_{dry}$ ratio with effective pressure. This calibration parameter corrects for grain relaxation and the effects of sorting-induced porosity reduction on elastic moduli, not accounted for in EMT or existing modified grain contact models. We also discuss compaction-induced porosity loss and how this results in the discrepancy between the pressure sensitivity observed in laboratory data and the cube-root power law predicted by the HM model. We correct for this discrepancy by inverting for a second calibration parameter from the measured laboratory data.

We begin with a brief overview of the theoretical background for EMT models in granular media, including their limitations in predicting the elastic properties of sediments. We discuss how pressure-dependent porosity change in uncemented sediments leads to observed discrepancies between EMT models and laboratory data. We then propose a modified GCT model to mitigate these issues, resulting in new relations between calibration parameters, porosity, and pressure, that can be used to more accurately predict elastic rock properties in uncemented sediments. Finally, we discuss the limitations of the new model, including comparisons with existing modified GCT models.
Effective medium theory attempts to integrate the behaviour of two representative grains in contact over a full granular pack of spheres to predict the averaged effective medium elastic properties. The contact between the grains can be modelled as welded, unwelded, adhesive, rough, or smooth, and, depending on the specific contact type, different effective medium averaging techniques can be applied (e.g., Mindlin [1949], Digby [1981], Walton [1987]). This section gives a brief overview of the derivation of the HM model as presented, for example, by Bachrach and Avseth (2008).

A single contact between two spheres of radius $R_1$ and $R_2$ can be characterised by its normal and tangential stiffness $S_n$ and $S_t$ defined as (Winkler, 1983)

$$S_n = \frac{\partial F_n}{\partial \delta}, \quad \text{and} \quad S_t = \frac{\partial F_t}{\partial \tau},$$

(5.1)

where $\delta$ and $\tau$ are the displacements along the unit vectors normal and tangential to the contact surface, respectively, resulting from the application of the normal and tangential components of a force acting on the contact surface, $F_n$ and $F_t$. These contact stiffnesses can be expressed in terms of the geometric and material properties of the spherical grains. The normal stiffness is given by

$$S_n = \frac{4a_n G}{1-v},$$

(5.2)

where $G$ is the grain shear modulus, $v$ is the grain Poisson’s ratio, and $a_n$ is the equivalent radius of the contact surface area between the two spheres

$$a_n = \left[ \frac{3F_n \bar{R}(1-v)}{8G} \right]^{\frac{1}{7}}.$$

(5.3)

The effective radius between the two spheres $\bar{R}$ is given by

$$\bar{R} = 0.5 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}.$$

(5.4)

Because the force between the grains is path dependent, we need to assume a specific loading path when deriving the tangential stiffness (Norris and Johnson [1997]). In the case
of Hertzian contacts with infinite friction,

\[ S_t = \frac{8a_t G}{2 - v}, \]  

(5.5)

where \( a_t \) can be interpreted as the equivalent radius of the frictionally locked region of the contact area between the two spheres (Dutta et al., 2010). In the case of the HM model, \( a_t = a_n \) because the entire grain contact is assumed to be frictionally locked. The Walton Smooth model assumes \( S_t = 0 \) because the grain contact is assumed to be frictionless (Walton, 1987).

5.4.1 Bulk modulus

The effective bulk modulus of a random pack of identical spheres can be expressed in terms of the normal stiffness acting at a grain contact averaged over the geometric properties of the sphere pack. The effective bulk modulus (Mavko et al., 1998; Bachrach and Avseth, 2008) is given by

\[ K_{HM} = \frac{C(1 - \phi)}{12\pi R} S_n, \]  

(5.6)

where \( C \) is the coordination number, \( \phi \) is the porosity, and \( R \) is the volumetric averaging radius. We are interested in how the bulk modulus varies with effective pressure \( P_{\text{eff}} \) defined as the difference between overburden pressure \( P_{\text{ov}} \) and pore pressure \( P_p \), such that

\[ P_{\text{eff}} = P_{\text{ov}} - nP_p, \]  

(5.7)

where \( n \) is the effective stress coefficient, typically assumed to be 1.0 for many uncemented sediments (e.g., Hofmann et al., 2005). Because pressure is defined as force/area we can write the normal confining force from equation 5.3 as

\[ F_n = \frac{4\pi R^2 P_{\text{eff}}}{C(1 - \phi)}. \]  

(5.8)

From equations 5.2, 5.4 and 5.6, 5.8, the effective bulk modulus is (Bachrach and Avseth, 2008)

\[ K_{HM} = \left( \frac{(1 - \phi)^2 G^2}{18\pi^2(1 - v)^2} \right)^\frac{1}{2} \left( \frac{C^2 R}{R} \right)^\frac{1}{2} P_{\text{eff}}^\frac{1}{2}. \]  

(5.9)
5.4. Theoretical background for EMT models

5.4.2 Shear modulus

In the HM model, the effective shear modulus (Mavko et al., 1998; Bachrach and Avseth, 2008) is given by

\[ G_{HM} = \frac{C(1 - \phi)}{20\pi R} (S_n + \frac{3}{2} S_r). \]  

(Bachrach and Avseth, 2008) (BA) then extend equation 5.10 to account for nonuniform grain contacts using a binary scheme method. They assume some fraction \((1 - f_t)\) of grain contacts have zero tangential stiffness (no friction at grain contacts), while the remaining contacts have tangential stiffness given by equation 5.5. This leads to an effective shear modulus of

\[ G_{BA} = \frac{C(1 - \phi)}{20\pi R} (S_n + f_t \frac{3}{2} S_r), \]

where \(f_t\) is the fraction of contacts having nonzero tangential stiffness. The scalar \(f_t\) can be interpreted as the fraction of contacts that experience zero slip. The shear modulus as a function of effective pressure can be derived from equations 5.2, 5.5, 7, 8, and 11 to be

\[ G_{BA} = \left[ \frac{1}{10} \left( \frac{12(1 - \phi)^2 G^2}{\pi^2 (1 - v)^2} \right)^{\frac{1}{3}} \left( \frac{C^2 R}{R} \right)^{\frac{1}{3}} P_{eff}^{\frac{1}{3}} \right] + \left[ \frac{3}{10} \left( \frac{12(1 - \phi)^2 G^2 (1 - v)}{\pi^2 (2 - v)^3} \right)^{\frac{1}{3}} \left( \frac{C^2 R}{R} \right)^{\frac{1}{3}} P_{eff}^{\frac{1}{3}} \right] f_t. \]  

Assuming zero tangential stiffness implies that grains slip during wave propagation (e.g., Bachrach and Avseth, 2008; Dutta et al., 2010). The model of Duffaut et al. (2010), based on Mindlin’s partial slip theory, also considers contact slip during dynamic core measurements; however, other authors reject the idea that the low seismic wave strains (of the order of \(10^{-6}\) or smaller) induce slip as a shear wave passes (e.g., Mavko, 1979; Winkler, 1983). The GD simulation results of Sain (2011) show that assuming zero tangential stiffness (no friction) at grain contacts is not required to reduce shear moduli, so long as one accounts for the effect of grain relaxation. Due to the idea that wave propagation is unlikely to induce slip as grain contacts, Sain (2011) suggests that adjusting friction, as in Bachrach and Avseth (2008) and Dutta et al. (2010), may not be a recommended approach to heuristically modifying EMT models.
5.4.3 Poisson’s ratio

Poisson’s ratio can be expressed in terms of the bulk and shear moduli (Mavko et al., 1998):

\[ v_{\text{eff}} = \frac{3K_{\text{eff}} - 2G_{\text{eff}}}{2(3K_{\text{eff}} + G_{\text{eff}})} \]  
(5.13)

where \( K_{\text{eff}} \) and \( G_{\text{eff}} \) could be given by any of the EMT or modified GCT models described above. The HM expression for Poisson’s ratio \( v_{\text{HM}} \) is (Bachrach and Avseth, 2008):

\[ v_{\text{HM}} = \frac{S_n - S_t}{4S_n + S_t} = \frac{v}{10 - 6v} \]  
(5.14)

where the sediment \( v_{\text{HM}} \) is a function of the grain Poisson’s ratio only. From equation 5.14 we can also see that setting \( S_t = 0 \) leads to a Poisson’s ratio of 0.25. In the BA model, the sediment Poisson’s ratio is a function of the grain Poisson’s ratio and the fraction of contacts having tangential stiffness:

\[ v_{\text{BA}} = \frac{S_n - f_tS_t}{4S_n + f_tS_t} = \frac{(2 - v)}{4(2 - v) + 2f_t(1 - v)} - \frac{2f_t(1 - v)}{4(2 - v) + 2f_t(1 - v)}, \]  
(5.15)

where, again, the maximum value of Poisson’s ratio is 0.25 when \( f_t = 0 \).

5.4.4 Practical issues

Many practical issues arise when using current modified GCT models to describe the elastic properties of un cemented sediments. In the BA model, the dry Poisson’s ratio is dependent only on the Poisson’s ratio of the grain material and on the volume fraction of nonslip contacts. Figure 5.1a shows the dry Poisson’s ratio plotted against \( P_{\text{eff}} \), coloured by porosity, for un cemented sediment sample measurements (Zimmer, 2003). Poisson’s ratio varies between 0.15 and 0.30 at effective pressures between 3 and 20 MPa and up to 0.40 at \( P_{\text{eff}} < 3 \) MPa. The maximum Poisson’s ratio value that can be predicted by current modified GCT models is 0.25, when grain contacts have zero tangential stiffness (e.g. Walton, 1987; Bachrach and Avseth, 2008). We use equation 5.15 to invert for the fraction of nonslip contacts for each sample in Figure 5.1a. Figure 5.1b shows the inverted
5.4. Theoretical background for EMT models

Figure 5.1: (a) Dry Poisson’s ratio as a function of effective pressure, coloured by porosity, for uncemented sediment samples (Zimmer, 2003). Data are from the loading cycle of the experiments. (b) Fraction of non-slip contacts as a function of effective pressure, coloured by porosity, for the same samples in (a). Negative fractional values are unphysical.

The discrepancy between the Poisson’s ratio values predicted by the models and the experimental data can be attributed to the non-physical negative values of $f_t$ (implying negative tangential stiffnesses) required to fit the data when Poisson’s ratio is greater than 0.25. Therefore, certain types of uncemented sediment samples cannot be modelled accurately with current GCT models. It should be noted that, in general, measurement error can be greatest at low effective pressures, and therefore could result in incorrect high values of Poisson’s ratio. Zimmer (2003) quote velocity errors at low effective pressure (below 100 kPa) of less than 2% for compressional-wave velocities and 4% for shear-wave velocities. We have tested propagating these maximum measured errors on compressional and shear-wave velocities through the calculation of Poisson’s ratio and find that most of samples still exhibit values greater than 0.25 at low effective pressures.

We contend that the discrepancy between the Poisson’s ratio values predicted by the
current modified GCT models and those measured in the laboratory, is due to several assumptions made when deriving the EMT models. First, current modified GCT and EMT models do not predict a porosity pressure dependence on the Poisson’s ratio in uncemented sediments, whereas laboratory measurements suggest this is indeed the case (Figure 5.1). We will show that porosity reduction associated with deteriorating grain sorting has a greater relative effect on the bulk modulus than on the shear modulus, thus resulting in a porosity (grain sorting)-dependent Poisson’s ratio in equation \[5.13\]. We also contend that at low effective pressure, compaction-induced porosity loss has a greater effect on the bulk modulus than on the shear modulus, which would explain the observed pressure dependence of Poisson’s ratio.

### 5.5 Porosity variation in uncemented sediments

Grain-size distribution (sorting) and changes in effective pressure are the primary controls on porosity in uncemented sediments. The former controls the depositional (critical) porosity, whereas the latter controls the compaction or dilation of the pore space. In this section, we give an overview of how porosity changes in uncemented sediments as a function of sorting and effective pressure. We discuss how such porosity change affects the elastic properties of a sedimentary rock, and the implications for the accuracy of current modified GCT and EMT models to predict the pressure sensitivity of uncemented sediments.

#### 5.5.1 Sorting trends

A common model used to describe how bulk and shear moduli of uncemented sediments vary as a function of sorting is the friable-sand model (Dvorkin and Nur [1996]). The elastic properties of the well-sorted, high-porosity end member are calculated using the HM model, whereas the opposite zero-porosity end member takes on the values of the mineral bulk and shear moduli. Poorly sorted sands are represented as the well-sorted end member modified with additional smaller grains deposited in the pore space. These additional grains deteriorate sorting, decrease porosity, and only slightly increase the stiffness of the rock (Avseth et al. [2005]). The moduli of poorly sorted sands with porosities between \( \phi = 0 \) and \( \phi = \phi_c \) are “interpolated” between the the mineral point and the well-sorted end member.
Porosity variation in uncemented sediments

using a modified Hashin-Shtrikman lower bound (MHSLB) \cite{Hashin1963}. The MHSLB represents the elastically softest way to mix two phases, making it a suitable way to simulate the effect on elastic moduli of adding small grains within the pore space due to deteriorating sorting. The bulk and shear moduli of the dry friable sand mixture are \cite{Mavko1998}

\begin{equation}
K_{\text{dry}} = \left(\frac{\phi/\phi_c}{K_{\text{HM}} + \frac{4}{3}G_{\text{HM}}} + \frac{1 - \phi/\phi_c}{K + \frac{4}{3}G_{\text{HM}}} \right)^{-1} - \frac{4}{3}G_{\text{HM}} \quad (5.16)
\end{equation}

and

\begin{equation}
G_{\text{dry}} = \left(\frac{\phi/\phi_c}{G_{\text{HM}} + \frac{G_{\text{HM}}}{6} \left(\frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}}\right)} + \frac{1 - \phi/\phi_c}{G + \frac{G_{\text{HM}}}{6} \left(\frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}}\right)} \right)^{-1} - \frac{G_{\text{HM}}}{6} \left(\frac{9K_{\text{HM}} + 8G_{\text{HM}}}{K_{\text{HM}} + 2G_{\text{HM}}}\right), \quad (5.17)
\end{equation}

where $\phi_c$ is the critical porosity of the well-sorted end member and $\phi$ is the reduced porosity due to deteriorating sorting.

Figure 5.2a and 5.2b shows the application of the friable-sand model to uncemented samples from \cite{Zimmer2003}. Quartz bulk and shear moduli values have been used as the zero-porosity mineral point within the model. The samples were prepared such that the porosity variation between samples is due to changes in grain-size distribution (sorting). We observe that for constant pressure the dry rock moduli increases when sorting deteriorates (porosity decreases). Rather than calculating the well-sorted end member using the HM model, we can choose any of the modified GCT models discussed previously. Figure 5.2c and 5.2d shows the result of using the BA model, in which the bulk modulus is calibrated using the parameter $c = C^2 \bar{R}/R$ in equation 5.9 and the shear modulus is calibrated using the parameter $f_t$ in equation 5.12. For the bulk modulus, even when only calibrated to the high porosity sample, the modified friable-sand model accurately describes the more poorly sorted samples at all effective pressures; however, the modified friable-sand model still overpredicts the effect of sorting on the shear modulus, particularly at low effective pressures. The laboratory data show that deteriorating sorting has a greater effect on the bulk modulus than the shear modulus, resulting in the observed
5.5. *Porosity variation in uncemented sediments*

Figure 5.2: Friable-sand model applied to bulk and shear moduli of uncemented samples from Zimmer (2003), calibrated to the highest porosity sample. Data are from the loading cycle of the experiments. Friable-sand model with the well-sorted end member calculated using the HM model and the coordination number relation of Murphy (1982) for (a) dry bulk and (b) shear moduli. The friable-sand model with the well-sorted end member is calculated using the BA model for (c) dry bulk and (d) shear moduli.

Porosity dependence of Poisson’s ratio. This dependence is not captured by any current modified GCT model, EMT model, or by the modified friable-sand model. It is also worth noting that the poorer sorted samples generally have higher amounts of clay, which if distributed structurally, could have differing effects on the bulk and shear modulus and thus contribute to mechanical weakening and the porosity variation of Poisson’s ratio.

5.5.2 Compaction-induced porosity loss

The other major cause of porosity loss is due to compaction, which to first order follows an exponential depth trend,

\[ \phi(z) = \phi_c e^{-cz}, \quad (5.18) \]
5.5. Porosity variation in uncemented sediments

where $\phi(z)$ is the porosity at depth $z$, $\phi_c$ is the critical porosity at $z = 0$, and $c$ is a compaction rate constant (Athy, 1930).

At shallow burial, grain rotation and reorientation can alter the packing of grains resulting in significant additional porosity loss (Berner, 1980). This added component of porosity loss implies that equation 5.18 is not fully correct at shallow depths (Dutta et al., 2009). Saul and Lumley (2013) present a generalised double exponential porosity-depth relationship to describe compaction-induced porosity loss over the depth range from the surface to the limit of mechanical compaction

$$
\phi(z) = \phi_\infty + ae^{-bz} + ce^{-dz},
$$

where $\phi_\infty$ is the irreducible finite porosity value at the limit of mechanical compaction.

It is reasonable to assume to first order that effective pressure is approximately linear with depth, and thus Saul and Lumley (2013) derive the following porosity-pressure relationship:

$$
\phi(P_{\text{eff}}) = \phi_\infty + \alpha e^{-\xi P_{\text{eff}}} + \gamma e^{-\psi P_{\text{eff}}},
$$

where $\alpha$, $\xi$, $\gamma$, and $\psi$ are fitting parameters. The term $(\phi_\infty + \alpha + \gamma)$ is equal to the critical porosity (which is a function of sorting) at $P_{\text{eff}} = 0$. Figure 5.3 shows porosity versus effective pressure data for uncemented samples from Zimmer (2003), along with the fit of equation 5.20 to the highest porosity sample. We can see that Poisson’s ratio decreases with increasing pressure for a given sample, with more poorly sorted samples generally having higher values.

Porosity loss due to compaction (grain packing), at a constant effective pressure, has a similar trend in moduli-porosity space as changes due to sorting (e.g., Avseth et al., 2005; Mavko et al., 1998). For this reason, the modified friable-sand model is often used to estimate packing-induced changes in elastic moduli (Figure 5.4). The laboratory measurements in Figure 5.4 are from Zimmer (2003), with the packing effect determined by making ultrasonic measurements during a series of loading and unloading pressure cycles. In Figure 5.4, we observe that when using the modified friable-sand model to account for the effects of packing, the bulk modulus is underestimated at all effective pressures.
5.5. **Porosity variation in uncemented sediments**

![Graph showing porosity versus effective pressure, coloured by Poisson's ratio, for uncemented samples from Zimmer (2003). Data are from the loading cycle of the experiments. The highest porosity sample (Galveston Beach sand) has been fit with equation 5.20 (black line).](image)

**Figure 5.3:** Porosity versus effective pressure, coloured by Poisson’s ratio, for uncemented samples from Zimmer (2003). Data are from the loading cycle of the experiments. The highest porosity sample (Galveston Beach sand) has been fit with equation 5.20 (black line).

This is likely because the HM model neither accounts for changes in coordination number with packing, nor for changes in the area of grain contacts. Figure 5.4b shows that the modified friable-sand model better describes the packing effect on the shear modulus at low effective pressures, but again underpredicts the effect at higher effective pressures. In each case, the bulk and shear modulus are greatly underpredicted when changing porosity in the BA model directly (dashed lines in Figure 5.4). Overall, we observe that pressure-induced porosity loss by grain packing has a greater effect on the bulk modulus than the shear modulus at low effective pressures, resulting in a pressure-dependent Poisson’s ratio (Figure 5.1).

Figure 5.5 shows the dry bulk modulus-pressure relationship, coloured by porosity, for the Pomponio Beach sand sample (Zimmer, 2003). The solid red curve is the predicted bulk modulus using the HM model with $C = 8.3$ and $\phi = 0.4$. The bulk modulus evidently varies with effective pressure stronger than the cube-root power law predicted by the HM model. As in Figure 5.3, porosity decreases with effective pressure due to compaction. The blue curve shows the predicted pressure sensitivity calculated using the HM model but with porosity reduction accounted for by a lower Hashin-Shtrikman bound (i.e., friable-
sand model). This shows that packing-induced porosity loss associated with changes in accommodation space, accounts for part of the discrepancy between the pressure sensitivity predicted by HM theory and that measured in the laboratory.

To reduce this remaining discrepancy, a porosity or pressure-dependent coordination number is often introduced into the HM model. Combining the HM model with the coordination number-porosity relationship of Murphy [1982], and the modified friable-sand model to account for changes in elastic moduli associated with packing, leads to the dashed blue curve in Figure 5.5. The remaining discrepancy between the predicted and measured bulk modulus is likely due to the relationship of Murphy [1982] not correctly accounting for changes in coordination number with porosity and/or HM theory not accounting for an increase in grain contact area after compaction. These two factors will increase the sediments’ elastic modulus, as the laboratory data indicate. The HM model combined with a modified friable-sand model therefore constitutes a lower bound for the effect of pressure on the bulk modulus of granular materials.
5.6. Proposed modified GCT model

We have shown that porosity reduction associated with deteriorating sorting and compaction can have significant effects on the elastic moduli of uncemented sediments. These effects explain some of the discrepancies between EMT and laboratory measurements of pressure-dependent elastic properties. In this section, we propose a physically based heuristic modification of the HM model, to incorporate the effects of grain relaxation and porosity on the elastic properties of uncemented sediments. We note that modifying the grain contact theory components of the HM model to make it more physically realistic does not necessarily ensure that our results satisfy EMT integral constitutive stress-strain relationships. It is for this reason we call the proposed model a modified GCT model.

5.6.1 Incorporating grain relaxation and porosity effects

GD simulations have shown that the affine strain assumption causes the HM model to greatly overpredict the shear moduli of realistic heterogeneous sediments. We have further

**Figure 5.5:** Dry bulk modulus versus effective pressure, coloured by porosity, for the Pomponio Beach sand sample [Zimmer, 2003]. Data are from the loading cycle of the experiment. The solid red curve shows the prediction by the HM model. Accounting for changes in coordination number with pressure within the HM model gives the red dashed curve. Blue curves show the result of accounting for compaction-induced porosity loss with the MHSLB as in the friable-sand model. In all cases, quartz mineral elastic properties have been used as input to the HM and friable-sand models.
shown that porosity reduction associated with deteriorating sorting affects the bulk and shear modulus differently, and thus, can explain observed porosity-dependent Poisson’s ratio, as well as values of Poisson’s ratio greater than 0.25. To correct for these effects, we apply a weight factor to the shear modulus within the HM model, which is equivalent to the relaxation correction factor given in Sain (2011). However, rather than determining this factor from GD simulations, we invert for it directly from laboratory data.

From equation 5.10, we define the shear modulus as

\[ G_{GCT} = wG_{HM} = w \left[ \frac{C(1 - \phi)}{20\pi R} (S_n + \frac{3}{2} S_\tau) \right]. \] (5.21)

We call \( w \) a weight factor because it is controls the weight of the shear modulus as predicted by HM. From equation 5.13, we then write the effective Poisson’s ratio as

\[ v_{GCT} = \frac{5S_n - 2wS_n - 3wS_\tau}{10S_n + 2wS_n + 3wS_\tau}. \] (5.22)

When \( w = 0 \) the Poisson’s ratio equals 0.5, whereas assigning \( w = 1 \) reduces Poisson’s ratio to that given by equation 5.14. Note that we can model uncemented sediments with a Poisson’s ratio between 0 and 0.5, rather than being limited to between 0 and 0.25 as with current modified GCT models. Setting \( w = 1 \) implies that the affine strain assumption is valid, and thus the shear modulus is given by the HM model. Figure 5.6 shows the dry Poisson’s ratio versus \( w \) for the proposed modified GCT model, compared to the BA model.

We have shown that at low effective pressures the effect of compaction (packing) is greater on the bulk modulus than the shear modulus, thus resulting in observed pressure-dependent values of Poisson’s ratio (Figure 5.1). We cannot account for the effect of compaction-induced porosity loss with the MHI SBLB as in the friable-sand model because it underestimates the effect on the bulk and shear moduli (Figure 5.4). Sain (2011) shows that as effective pressure increases and sediments compact, the heterogeneity in terms of stress distribution will decrease, and thus the amount of grain relaxation will reduce. Thus, we contend that \( w \) should be stress dependent. This pressure dependence on \( w \) results in a
5.6. Proposed modified GCT model

Figure 5.6: Investigation of the weight factor on the dry Poisson’s ratio for the proposed modified GCT model (solid curve). Comparison to the BA model (dashed curve) shows that the new model can predict Poisson’s ratio values > 0.25.

pressure-dependent Poisson’s ratio as observed in the laboratory measurements. Following the form of the relaxation correction factors given in Sain (2011), we define an exponential relationship for the pressure dependence of $w$:

$$w = \epsilon + \zeta e^{\eta P_{\text{eff}}}, \quad (5.23)$$

where $\epsilon$, $\zeta$, and $\eta$ are fitting parameters determined by a least-squares fit to inverted values of $w$ from laboratory measurements.

We also need to correct the HM prediction of bulk and shear moduli for the remaining effects of compaction-induced porosity loss. This includes the change in accommodation space, coordination number, and the area of grain contacts. To account for these compaction (pressure)-related effects, we invert laboratory measurements of bulk modulus for the calibration parameter $c = C^2 \bar{R}/R$, following Bachrach and Avseth (2008). We propose a simple linear regression between the calibration parameter, $c$, and porosity

$$c = \sigma \phi + c_0, \quad (5.24)$$

where $\sigma$, and $c_0$ are fitting parameters determined by a least-squares fit to the inverted calibration data. By combining equations 5.24 and 5.20 we can calculate how the calibration
5.7. Proposed method for calibrating model parameters

Parameter varies with effective pressure. Including equation 5.20 also allows us to calculate pressure-dependent density and therefore predict compressional and shear velocities outside measured data ranges.

The fits to the two calibration parameters $w$ and $c$ are used in equations 5.9 and 5.21 to determine how bulk and shear moduli vary as a function of effective pressure. The closed-form expressions for the calibration parameters can be used to predict pressure-dependent velocities in sediments with similar grain-size distributions. The calibration parameters provide insight into the microstructure of the sediment: $w$ is a function of heterogeneity and thus related to grain-size distribution and pressure; $c$ provides insight into the general texture and packing of the sediment and is related to sorting, porosity and effective pressure.

5.7 Proposed method for calibrating model parameters

We propose the following procedure to calibrate the modified GCT model to fit bulk and shear moduli as a function of depth/effective pressure:

1. Derive the dry Poisson’s ratio as a function of effective pressure, and use equation 5.22 to invert for weight factor $w$ that minimises the $L_1$ norm of the predicted and observed values at each pressure step. Perform a least-squares fit of $w$ in equation 5.23 to the inverted data.

2. Invert for calibration parameter $c$ by minimising the $L_1$ norm of the predicted and measured bulk modulus using equation 5.9. Perform a least-squares fit of $c$ in equation 5.24 to the inverted data.

3. Perform a least-squares fit of $\phi$ in equation 5.20 to the porosity-pressure data, and combine with equation 5.24 to determine how the constant $c$ varies with effective pressure.

We now guide the reader through the three-step process for three uncemented samples from [Zimmer, 2003] that cover a range of grain-size distributions. Figure 5.7 shows the inverted weight factor $w$ versus pressure, fit with equation 5.23. We can see that $w$
5.7. Proposed method for calibrating model parameters

varies with sorting (porosity) and effective pressure, with lower values of \( w \) associated with lower porosities and pressures. This is consistent with the GD simulation observations of [Sain (2011)], which show that as heterogeneity decreases, there is less grain relaxation, and therefore a smaller relaxation correction (higher value of \( w \)) is required to describe the observed shear modulus. Heterogeneity, in terms of contacts numbers, contact forces, and grain stresses, would likely increase with deteriorating sorting, and thus it explains the porosity dependence observed in the inverted values of \( w \). Figure 5.7b shows the inverted calibration parameter \( c \) versus porosity, fit with equation 5.24. Figure 5.7c shows porosity versus pressure for each sample, fit with equation 5.20. Figure 5.7d shows the corresponding relationship between \( c \) and pressure, obtained by combining equations 5.24 and 5.20. From Figures 5.7b and 5.7c, we can see that \( c \) increases with increasing pressure and deteriorating sorting. This is expected because the coordination number will likely increase due to small grains filling the pore space in the case of deteriorating sorting and as the sediment becomes more tightly packed during compaction.

Figure 5.8 shows the inverted weight factor \( w \), versus effective pressure, for the Galveston Beach sand sample from [Zimmer (2003)]. The plot also shows the grain relaxation correction factor determined from GD simulations of a spherical glass bead pack (Sain, 2011). Even though the grain elastic properties of the Galveston Beach sand and the glass beads are different, there is excellent agreement between the inverted values of \( w \) using our proposed GCT model and the grain relaxation correction factor given by the GD simulation results. Because the Galveston Beach sand is very well sorted, one could assume that the grain contact heterogeneity is similar to that of the spherical pack used in the GD simulation. This provides justification that grain-size distribution is a major control on contact heterogeneity and therefore on the amount of grain relaxation. It also confirms that we can estimate this correction factor accurately from observed laboratory data, at least in this one test case.

Figure 5.9 shows the resulting predictions of the dry bulk, shear, and bulk-to-shear moduli ratio for each sample from Figure 5.7. We observe that the new modified GCT model accurately describes the variation with pressure, even for the poorly sorted sample where \( v > 0.25 \). The new model also captures the variation in \( K_{\text{dry}}/G_{\text{dry}} \) with effective
5.7. Proposed method for calibrating model parameters

Our proposed method can be used to determine new relationships for calibration parameters \( w \) and \( c \) to predict elastic properties in well-sorted sediments where ultrasonic velocity measurements are not available. We determine these relations by applying our proposed three-step calibration process to the Pomponio Beach sand sample from Zimmer (2003). The Pomponio Beach sand is very well sorted with an initial porosity of \( \phi = 0.40 \).

The Pomponio Beach sand is the sample Dutta et al. (2010) use to determine their empirical relations. They invert for the equivalent of the calibration parameter \( c \) in the

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**Figure 5.7:** Calibration parameters for the proposed modified GCT model, for three uncemented samples (Zimmer 2003). Circles are Galveston Beach sand, squares are Santa Cruz aggregate, and triangles are glass beads (GB 35% Tiny 2 from Zimmer (2003)). Data are from the loading cycle of the experiments. (a) Inverted weight factor \( w \), fit with equation 5.23. (b) Inverted calibration parameter \( c \), fit with equation 5.24. (c) Porosity-pressure data fit with equation 5.20. (d) Calibration parameter \( c \) versus \( P_{\text{eff}} \).
5.7. Proposed method for calibrating model parameters

### Figure 5.8: Inverted weight factor \( w \) versus effective pressure, for the Galveston Beach sand sample (Zimmer, 2003). The relationship for the grain relaxation correction factor determined from the GD simulation study of Sain (2011) is shown as the dashed curve.

BA model for \( f_t = 0.6 \), assuming the sediment is perfectly sorted (i.e., \( R/R = 1 \)) and \( c \) is equal to the sample coordination number. They argue that \( f_t = 0.6 \) provides a realistic prediction of \( C \); however, this requires separate coordination number estimates (and therefore different predictions of shear modulus) to fit the respective compressional and shear velocity data for the same rock. Although the model provides good predictions of compressional and shear velocity as a function of effective pressure, it leads to nonphysical fits in moduli space (see Figure 5.10).

We apply our three-step procedure to the Pomponio Beach sand sample, and we compare the results to the model of Dutta et al. (2010). Figure 5.10 shows the comparison between the two models. Figure 5.10a through 5.10c shows the comparison in moduli space, whereas Figure 5.10d through 5.10f shows the comparison in velocity space. Our proposed model provides improved fits in all data spaces while capturing the variation in \( K_{\text{dry}}/G_{\text{dry}} \) (and \( V_P/V_S \)) with effective pressure. The proposed model also requires one less fitting parameter to fit the inverted calibration data. Although the values of \( c \) inverted using our proposed model are higher than those of Dutta et al. (2010), we assert that this is due to an increased grain contact area during compaction that is not accounted for directly in either model.

Based on fitting the proposed modified GCT model to the Pomponio Beach sand, we
Figure 5.9: Fit of proposed modified GCT model to (a) bulk modulus, (b) shear modulus, and (c) bulk to shear modulus ratio, for uncemented samples from Figure 5.7. Circles are Galveston Beach sand, squares are Santa Cruz aggregate, and triangles are glass beads. Data are from the loading cycle of the experiments, and quartz mineral elastic properties have been used as input to the model.
propose the following calibration parameter relations for well sorted sediments:

\[ w(P_{\text{eff}}) = 0.5379 - 0.1897e^{-0.3P_{\text{eff}}} \]  \hspace{1cm} (5.25)

\[ c(\phi) = (-1.534e + 03)\phi + 721.332 \]  \hspace{1cm} (5.26)

These calibration parameter relations can be used for the case when no velocity-pressure data are available. Figure 5.11 shows the results of testing these improved relations on the glass bead sample used by Dutta et al. (2010), where the bulk and shear moduli have been predicted with no knowledge of the ultrasonic velocity-pressure data. Our proposed model
5.7. Proposed method for calibrating model parameters

Figure 5.11: Comparison between the proposed modified GCT model and Dutta et al. (2010) for predicting elastic properties of a glass bead sample (Zimmer, 2003). Data are from the loading cycle of the experiment. (a) Predictions in moduli space. (b) Predictions in velocity space. (c) Log-log plot of dry velocity versus effective pressure.

provides improved predictions in moduli space as well as slightly improved predictions in velocity space, particularly at low effective pressures as evident in the log-log plot (Figure 5.11c).
5.8 Discussion

We have shown that we can describe the elastic properties of uncemented sediments by introducing two pressure-dependent calibration parameters into HM theory. We will always be able to fit bulk and shear moduli when introducing two-pressure dependent calibration parameters; however, the intent is to be able to relate these calibration parameters to measurable sediment properties, such as porosity, grain-size distribution, and pressure. Understanding the controls on the pressure sensitivity of elastic moduli in uncemented sediments also allows these calibration parameters to be related to changes in pore-space microstructure.

The parameter \( w \), which can be inverted from measurements of Poisson’s ratio, accounts for the grain relaxation associated with the infinitesimal strain applied to the sediment during seismic wave propagation. This parameter controls the relative difference between the bulk and shear modulus, and it depends on the degree of heterogeneity of the sediment. As effective pressure increases and porosity decreases, the sediment becomes more tightly packed and the distribution of grain contacts and stresses becomes more homogeneous, leaving \( w \) stress dependent. Because the amount of grain relaxation depends on the degree of heterogeneity, it is also no surprise that \( w \) is related to grain-size distribution. The greater degree of contact heterogeneity associated with poorer sorted sediments means that lower values of \( w \) are required to correct the shear modulus for grain relaxation.

The calibration parameter \( c \) accounts for changes in sediment elastic properties associated with compaction-induced porosity loss. The parameter controls how the bulk and shear moduli vary with changes in porosity (packing), coordination number, and effective grain radius / angularity. We propose not to derive the ratio \( \bar{R}/R \) (angularity parameter) using the coordination number-porosity relation of Murphy (1982), as in Bachrach and Avseth (2008), because it is difficult to isolate how the coordination number varies with porosity/sorting/pressure and therefore to calculate what the effective grain radius/angularity of a given sediment is from the relation \( c = C^2 \bar{R}/R \). Equation 5.24 could also imply that there is a general relation between between porosity and angularity, although it would be difficult to determine due to the same reason of not knowing exactly
5.8. Discussion

how coordination number varies with porosity/sorting/pressure. Although theoretically porosity is independent of grain size for uniformly packed and graded sands, smaller grains are typically more angular and thus have poorer packing and higher porosities (Rogers and Head 1961; Beard and Weyl 1973). The change in elastic moduli due to a change in accommodation space (e.g., friable-sand model) is also not accounted for directly in the BA model, so the value of $\bar{R}/R$ will likely be overestimated even if the coordination number relation of Murphy (1982) were reliable. The calibration parameter $c$ contains information on the general texture, packing, and degree of consolidation of the sediment.

The proposed modified GCT also includes a relationship for how porosity varies as a function of effective pressure. Along with modelling moduli as the primary data parameters rather than velocities, this ensures that the velocity-porosity-pressure relationships are physically realisable. It also improves the accuracy of fluid substitution, because the dry-rock moduli and porosity are key pressure-dependent input parameters to Gassmann fluid substitution equations.

The laboratory data used in this study are from the loading cycles of dry ultrasonic core velocity measurements. As such, the calibration parameter relations presented in this study apply to depositional compaction or loading. Different calibration parameter relations could be required to describe the unloading velocity-pressure response of the presented uncedmented sediments. We use dry measurements to avoid the effects of velocity-frequency dispersion (Mavko et al. 1998).

We have discussed possible porosity-related reasons for observed values of $v > 0.25$, as well as for the observed pressure dependence of $v$. Other factors could also contribute to these observations, including measurement error, variation in clay content, grain contact torque (bending) (Dvorkin and Nur 1996), and stress-anisotropy (e.g., Yin 1992; Xu 2002).

We showed that we can use modified GCT models, such as the one proposed in this paper, to calibrate the high porosity end-member within the friable-sand model in order to estimate the effects of sorting-induced porosity loss on elastic moduli; however, although this modified friable-sand model well predicts changes in bulk modulus due to deteriorating sorting, it still overestimates the shear modulus. In the absence of data to calibrate the
5.9. Conclusions

High-porosity end member within the friable-sand model, one could use Equations 5.25 and 5.26. Figures 5.1 and 5.7 suggest that the calibration parameter \( w \), which is related to the degree of contact heterogeneity, is porosity dependent, with lower values associated with lower porosity, more poorly sorted sediments.

5.9 Conclusions

HM predictions of elastic properties as a function of effective pressure can be corrected by accounting for the effects of grain contact heterogeneity and porosity variation. We introduce two pressure-dependent calibration parameters to account for these effects. The calibration parameter \( w \) can be derived from dry Poisson’s ratio and corrects for grain relaxation associated with heterogeneous grain contacts in real sediments. The variation of \( w \) with effective pressure also agrees with the results of published GD simulations, with further variations in \( w \) associated with grain-size distribution. The calibration parameter \( c \) accounts for changes in sediment elastic properties associated with compaction-induced porosity loss, including correcting for changes in the coordination number and grain contact area. We propose new calibration parameter relations to predict the elastic properties of well-sorted uncemented sediments, and we apply the new model to laboratory measurements made on uncemented sediments. The proposed modified GCT model provides improved predictions compared with existing models, describes the correct variation in \( K_{\text{dry}}/G_{\text{dry}} \) ratio with effective pressure, and can model uncemented sediments with values of Poisson’s ratio > 0.25. This new modified GCT model should help to improve the prediction and interpretation of elastic properties as a function of depth/effective pressure.

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5.10. Acknowledgements
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6 The effects of pressure and cementation on 4D seismic data - a NW Australia example

6.1 Foreword

The previous three chapters dealt with the pressure-dependent elastic properties of unconsolidated sandstones, including the development of new models to more accurately describe such properties. In this chapter we choose a specific data example, with the objective of using the knowledge gained during the development of the models to improve the interpretation of 4D seismic data acquired over a high-pressure water injector in the Enfield oil field, offshore Western Australia. The study presented in this chapter extends work presented in previous chapters, in that we also consider grain cementation effects. We also use the SL model (Chapter 3) to fit porosity-pressure data for an unconsolidated sediment, in order to model a realistic variation in porosity associated with pore-pressure increase. We use the modified grain contact theory presented in Chapter 5 to calibrate the high porosity end-member within the friable-sand model, in order to quantify grain cementation effects using the constant-cement model of [Avseth et al.] (2000). This chapter has been submitted as a technical paper to Geophysics and is currently undergoing review.

6.2 Abstract

Time-lapse seismic has proven to be a useful tool for monitoring production-related fluid-flow effects, identifying undrained resources, and improving overall reservoir management
6.3. Introduction

Understanding how elastic properties of sedimentary rock vary as a function of effective pressure is required to determine the feasibility of time-lapse (4D) seismic monitoring (e.g., Lumley et al. 1997, 2000; Meadows et al. 2002; Avseth et al. 2005; Lumley 2010), and to qualify/quantify observed 4D seismic anomalies (e.g., Lumley 1995; Tura and Lumley 1999; Lumley 2001; Landro 2001; Lumley et al. 2003). The pressure sensitivity of a given reservoir rock is typically determined by fitting empirical regressions to ultrasonic-velocity pressure measurements made on core samples (e.g., Han et al. 1986; Eberhart-Phillips et al. 1989; Lumley 1995, 2001; Meadows et al. 2002; MacBeth 2004; Yan and Han 2009). These empirical regressions often lack an underlying physical basis, and thus often fail to predict the pressure dependence of elastic properties beyond the typical measured pressure range of 5 – 20 MPa. Saul and Lumley (2013) present a quantitative model based on compaction theory and the concept of critical porosity, that can accurately predict velocity-pressure sensitivity in unconsolidated sands beyond these ranges. The model is particularly useful to predict the elastic properties at low effective pressures (below 5 MPa), making it suitable to quantify the time-lapse seismic response associated with large pore-pressure increases, as is often the case during secondary recovery processes such as water
6.4 Field overview

The Enfield oil field is an active water-flood project located in production permit WA-28-L, approximately 40 km northwest of Exmouth, offshore Western Australia (Figure 6.1). The Enfield reservoir is comprised of generally clean, high permeability, unconsolidated to partially consolidated sandstones (Ali et al., 2008; Hamp et al., 2008). Early feasibility work indicated that rock properties were favourable for time-lapse seismic monitoring and so a dedicated baseline seismic survey (Baseline) was acquired prior to the commencement of production in mid-2006 (Ridsdill-Smith et al., 2007; Smith et al., 2008). The field was initially developed via five subsea production wells, six water injection wells, and two gas re-injection wells. Three of the water injection wells are located downdip at the oil-water injection (e.g., Stronen and Diagranes, 2000; Duffaut and Landro, 2007; Smith et al., 2008).

In unconsolidated sediments, it has been shown that both elastic and non-elastic (e.g., packing and porosity) changes occur with variations in effective pressure (e.g., Zimmer, 2003; Saul and Lumley, 2013). Conversely, in cemented rocks it is generally assumed that certain rock properties remain constant with changes in effective pressure, such as porosity and the volume of contact cement (e.g., Avseth and Skjei, 2011). Given these assumptions, it is generally accepted that cemented rocks are less sensitive to changes in effective pressure than uncedmented sediments, and thus weaker 4D seismic anomalies are expected (e.g., Avseth and Skjei, 2011; Saul and Lumley, 2013). If however, changes in effective pressure associated with production/injection were large enough for the reservoir rock to geomechanically weaken or fail (e.g., fracture or break grain contact cement), stronger 4D seismic anomalies could be expected, perhaps even greater than if the reservoir sands were assumed to be uncedmented.

We extend work by Smith et al. (2008, 2010), with the objective to better understand the sensitivity of the Macedon reservoir sandstones to changes in effective pressure. We perform rock physics diagnostics (Dvorkin and Nur, 1996; Avseth et al., 2000) to infer the amount of contact-cement within the sands. We then show how these rock physics diagnostics can guide the analysis of pressure sensitivity within the reservoir sands, and discuss the implications for 4D seismic monitoring of high-pressure water injection.
6.4. Field overview

Figure 6.1: Exmouth Sub-basin and Enfield location map. From Hamp et al. (2008).

Contact (OWC) to provide pressure support (Figure 6.2) (Ali et al., 2008; Hamp et al., 2008). Water was initially injected above fracture pressure of approximately 1000-1500 psi (6.9-10.3 MPa) above the initial reservoir pressure (Wulff et al., 2008; Smith et al., 2010). The first monitor survey (M1) was acquired 7 months after the start of production. Excellent repeatability was achieved, with an average NRMS of less than 20% (Ridsdill-Smith et al., 2007; Ali et al., 2008).

6.4.1 Geology

The Exmouth Sub-basin is the southern-most rift basin within the Carnarvon Basin, containing Jurassic to Early Cretaceous (200-140 Ma) deposits (Ali et al., 2008; Hamp et al., 2008). The Enfield reservoir is within the Upper Jurassic Tithonian to Lower Cretaceous Berriasian age Macedon Member. The Macedon Member is contained within the crest of a north-easterly plunging fault-bounded terrace, with the Enfield trap created by cross-fault juxtaposition of the relatively thin sandstone intervals encased in a thick marine shale sequence. Enfield is highly faulted with a gentle structural dip of 3°.

The Enfield Macedon Member can be divided into two architectural packages: a complex lower channel sequence, and an upper debrite event (see Wulff et al., 2008). These are separated by a correlatable field-wide shale. Generally, the Lower Macedon sandstones
6.4. Field overview

Figure 6.2: Enfield reservoir map and well locations. The area of interest for this study, focused around Injector_1, is in the red square. Adapted from (Hamp et al., 2008).

are very fine to fine grained, relatively clean sandstones up to 50m thick (Ali et al., 2008). The sandstones are highly bioturbated, laminated, cross-bedded, and scarce of primary sedimentary structure. The Upper Macedon sandstones are fine to medium grained and are unconsolidated to poorly cemented, with an average thickness of 11m. Most sandstones are homogeneous, devoid of bioturbation, and clean. Petrophysical analysis of well logs, and core results, suggest that both the Upper and Lower Macedon reservoirs have field averages of 20 – 25% porosity, 0.5 – 1 Darcy permeability, and net-to-gross of 75 – 85% (Ali et al. 2008; Hamp et al. 2008). Siderite nodules and shale beds comprise the non-reservoir facies (Hamp et al. 2008).

The Enfield reservoir is separated into four main compartments: the Main Block North and South, and, the Sliver Block North and South (Figure 6.2). The compartments are characterised by a distinct fault pattern with a dominant SW-NE trend and a less frequent NW-SE to WNW-ESE trend (Ali et al. 2008; Hamp et al. 2008). The dominant SW-NE trending faults are extensional in nature and sub parallel to the current minimum
6.5. Initial 4D seismic feasibility

horizontal stress direction. A WNW-ESE fault in the north-west of the field offsets oil-bearing from water-bearing reservoir sands. Most SW-NE trending faults have relatively small offsets (< 20m), maintaining sand-on-sand juxtaposition. Aside from sealing faults, the largest source of permeability reduction and potential impediment to flow within the reservoir is likely due to observed cementation effects (Ali et al., 2008; Hamp et al., 2008).

6.5 Initial 4D seismic feasibility

In this section, we give an overview of the initial Woodside pressure-sensitive rock physics model used for 4D seismic feasibility work (e.g., Smith et al., 2007; Wulff et al., 2008; Smith et al., 2008), and for the qualification/quantification of observed 4D seismic anomalies associated with high pressure water injection (e.g., Smith et al., 2010). For this study, we focus on the area of interest around Injector_1 (Figure 6.2), and look at Baseline and M1 seismic surveys. Overpressure associated with water injection in Injector_1 is expected to be greater than 10 MPa above initial reservoir pressure (initial pore pressure is 20 MPa) (Smith et al., 2008; Ali et al., 2008).

Initial 4D seismic feasibility work was based on ultrasonic velocity measurements made on a core sample taken from the Macedon sands in Well_2 (Figure 6.3). Data in Figure 6.3 has been normalised to initial baseline velocity and pressure conditions. The blue and red curves in the plot show the fit of the Macbeth empirical regression model (MacBeth, 2004) to the dry compressional and shear velocities, respectively. We can see that both the compressional and shear velocities increase with effective pressure (decreasing pore pressure). For example, an increase in normalised effective pressure from 0.25 to 1 results in an increase in normalised compressional and shear velocity of 18% and 25%, respectively. Synthetic convolutional 4D seismic data, generated using log data from appraisal wells and the fit to the ultrasonic velocity measurements, indicated that a 25 – 30% increase in near-stack (8–19°) amplitude was expected due to the pore pressure increase around Injector_1 (Figure 6.5) (Wulff et al., 2008; Smith et al., 2008). This corresponds to a change in acoustic impedance (or compressional velocity as no change in density is assumed) of approximately 5% (Figure 6.4). 4D time shifts (increase in time thickness between top and base reservoir over time) associated with this modelling are in the order of 1 ms.
6.5. Initial 4D seismic feasibility

**Figure 6.3:** Initial rock physics pressure model. Normalised $V_P$ and $V_S$ as a function of normalised effective pressure. Measurements made on a dry core sample from an Enfield appraisal well are shown as points. Normalised $V_P$, $V_S = 1.0$, and normalised effective pressure = 1.0 represent initial baseline conditions. Curves show $V_P$ and $V_S$ fit using the Macbeth empirical regression model (MacBeth, 2004) for dry and brine-saturated conditions. Brine-saturated conditions are calculated using modified Gassmann fluid substitution, with fluid-pressure effects calculated using equations of Batzle and Wang (1992). Normalised brine velocities are shown relative to dry rock velocities at in-situ effective pressure. Adapted from Wulff et al. (2008).

**Figure 6.4:** We calculate acoustic impedance change (%) in the water leg due to a change in reservoir pore pressure with the initial rock physics pressure model in Figure 6.3.
6.5. Initial 4D seismic feasibility

**Figure 6.5:** We forward model the reservoir amplitude response as a function of pressure change using measured logs in Well_2 (Gamma ray shown in pink) and the rock physics model shown in Figure 6.3. Each seismic trace represents a pore pressure increase of 0.4 MPa above the initial pressure (labelled). In the top panel, wiggles with area fill show the modelled baseline seismic response, while unfilled wiggles show the modelled monitor response (near-stack trace 8–19° where the strongest 4D AVO effects are expected due to pressure). The middle panel shows the top reservoir maximum trough amplitude for the modelled baseline and monitor responses. The bottom panel shows the difference traces (monitor-baseline) at the same scale as the top panel.

Changes in pressure often effect seismic AVO data differently than changes in saturation, and thus 4D AVO analysis can be used to qualitatively and/or quantitatively estimate pressure and saturation from 4D seismic data (e.g., Landro 2001; Lumley 2001; Meadows et al. 2002; Lumley et al. 2003; Grude et al. 2013). At Enfield, 4D AVO feasibility modelling using the rock physics pressure model in Figure 6.3 and Gassmann fluid substitution shows that pressure and saturation effects should be separable, with changes in pressure and saturation plotting in different quadrants in 4D intercept-gradient space (Figure 6.6). (Lumley et al. 2003; Smith et al. 2008). The initial reservoir sands are elastically soft (low impedance relative to encasing shale); therefore, increases in pore pressure should be seen in 4D amplitude difference data as ‘brightening’ anomalies (large increase in amplitude due
6.5. Initial 4D seismic feasibility

**Figure 6.6:** 4D AVO intercept and gradient difference plot, based on our modelling from appraisal wells. Conditions at 0, 0 are for initial reservoir pressure ($P_{\text{eff}} = 20$ MPa, $P_T = 20$MPa) and oil saturation (80%). Pressure and saturation effects plot in different quadrants meaning they may be separable with 4D AVO analysis [Lumley et al. 2003].

to pressure softening) on the near offsets, that dim with increasing offset (Figure 6.7a). Conversely, increases in water saturation from initial oil saturation should be seen in 4D amplitude difference data as ‘dimming’ anomalies (reduction in amplitude due to stiffening effect of increased water saturation) on the near offsets, that increase with increasing offset (Figure 6.7b) [Landro 2001, Lumley et al. 2003]. For example, from Figure 6.7a we can see that an increase in pore pressure of 10MPa results in a 30% increase in near-stack (8 – 19°) amplitude and only a 1% increase in far-stack (31 – 42°) amplitude. Conversely, from Figure 6.7b we see that a change from initial oil saturation to 100% water saturation results in a 20% reduction in near-stack (8 – 19°) amplitude and a 30% reduction in far-stack (31 – 42°) amplitude. This also highlights that, particularly at near-offsets, 4D effects may cancel where water saturation and pore pressure are increasing together.

In summary, the initial 4D feasibility study showed that expected changes in pore pressure associated with Injector_1 should be observed greatest on near-to-mid angle stacks.
6.5. Initial 4D seismic feasibility

**Figure 6.7**: Modelled 4D AVO Gathers using log data from Well_3. (a) Modelled AVO gather for Baseline conditions (20 MPa pore pressure), and an increase in pore pressure of 10 MPa above initial reservoir pressure. The 4D difference shows that pressure effects should be observed greatest at near offsets (angles). (b) Modelled AVO gather for Baseline condition (80% oil saturation), and a change in saturation to water. The 4D difference shows that saturation effects should be observed greatest at far offsets (angles). We also note that in each case, Class III AVO behaviour is observed on the baseline and monitor gathers.
Predicted 4D anomalies consist of a 30% increase in near-stack amplitude at the time of M1, with associated time shifts in the order of 1 ms. In the next section we analyse the real 4D seismic data acquired over Injector_1, and compare the observed 4D anomalies with those predicted from the original Woodside feasibility study.

6.6 4D seismic data analysis

In this section, we analyse Baseline and M1 seismic data acquired over Enfield (Figure 6.8), focusing on the area of interest around Injector_1 (red box). Water is injected into the aquifer meaning the associated 4D response is due to changes in pressure alone, because the salinity of the injected water is similar to the formation water (∼33,000 ppm). This simplifies the study in the aquifer, since we do not need to consider the combination of pressure and saturation effects in the 4D seismic data (e.g., Figure 6.7). At the time of acquisition of M1, down-hole pressure gauges in Injector_1 showed the measured pore-pressure increase associated with water injection to be 1700 psi (11.7 MPa) above initial reservoir pressure (Wulff et al., 2008).

4D AVO analysis by Smith et al. (2008) shows that the 4D amplitude anomaly around Injector_1 (and to the north in the aquifer) is likely due to pore pressure increases rather than gas saturation effects. From Figures 6.7 and 6.9 we can also see that pore-pressure increases are predominantly observed in 4D amplitude data at near-to-mid offsets, with very little effect seen at the far offsets. For this reason we only analyse the near and mid-stack seismic volumes in this study. Figures 6.8 and 6.9 also indicate that pressure effects dominate over saturation effects in the field.
6.6. 4D seismic data analysis

Figure 6.8: Full stack RMS amplitude maps at top reservoir. (a) Baseline survey. (b) M1 survey. (c) 4D difference (M1-Baseline). Area of interest around Injector_1 in red box.
6.6. 4D seismic data analysis

6.6.1 4D amplitudes

We look at 4D difference data between the Baseline and M1 surveys in a thick channel sand to the north of Injector_1 (black polygon in Figure 6.10). We choose to analyse data in this area since it is above tuning thickness and is contained within the aquifer; therefore, we avoid the effects of 4D tuning (Lumley 4D Seismic Course, 2010) and we only need to consider pressure effects. Figure 6.11a and 6.11b show RMS amplitude difference maps (M1-Baseline) at top reservoir, for the near and mid-stack seismic data, respectively. RMS amplitudes were extracted over a 16ms time window around top reservoir for each survey (Baseline and M1), and it is these maps that were differenced to create the 4D difference maps. From Figure 6.11a (near stack 4D difference) and 6.11b (mid stack 4D difference) we can see that areas below tuning thickness (< 12ms time thickness in Figure 6.10) show very large 4D differences associated with large pore pressure increases and 4D tuning. Figure 6.11c and 6.11d show the same RMS amplitude maps but with areas affected by tuning masked out (grey). We can see that the area within the polygon is above tuning thickness, and therefore is a suitable area for this study.

Figures 6.12a and 6.12b show histograms of maximum negative amplitude at top reservoir within the thick-channel sand polygon for Baseline and M1 surveys, for near and mid-stacks respectively. For the near stack, we calculate a 60% increase in the mean maximum negative amplitude at M1 compared to the Baseline mean. This is significantly higher than the 30% increase in amplitude predicted from initial feasibility work (Figure 6.5), which we will show later may be due to mechanical weakening of the reservoir rock due to high pressure water injection. For the mid stack, we calculate a 50% increase in the mean maximum negative amplitude at M1 compared to the Baseline mean.

6.6.2 4D time shifts

Along with interpreting the 4D amplitude data, we also look at 4D time shifts between the Baseline and M1 datasets. Since increased pore pressure decreases the seismic velocity within the reservoir, an increase in travel time can be observed as a ‘push-down’ of events at the base reservoir (and below) on the monitor survey (Figure 6.13) (Landro and Stammeijer 2004; Hatchell and Bourne 2005). In the area of interest, significant 4D time shift...
6.6. 4D seismic data analysis

Figure 6.9: RMS amplitude maps at top reservoir showing AVO response around Injector_1. (a) Baseline near stack. (b) Baseline mid stack. (c) Baseline far stack. (d) M1 near stack. (e) M1 mid stack. (f) M1 far stack. (g) Near stack difference (M1-Baseline). (h) Mid stack difference (M1-Baseline). (i) Far stack difference (M1-Baseline). The strong Class 3 AVO response of the Baseline and M1 surveys is consistent with modelling at the wells (e.g., Figure 6.7). The red 4D AVO anomalies are consistent with increases in pore pressure between Baseline and M1 (bright at near offsets, dimming to far offsets - e.g., Figure 6.7a). Also, the blue 4D AVO anomalies are consistent with increases in water saturation (dim at nears, brightening to far offsets - e.g., Figure 6.7b).
6.6. 4D seismic data analysis

Figure 6.10: Baseline reservoir time-thickness map (ms) showing the thick channel sand (> 20ms) to the north of Injector_1. 4D amplitude and time shift data is analysed within the black polygon (above max tuning thickness of ∼ 12 ms). Approximate OWC shown in pink.

Time shifts are seen at the base reservoir (and below) in the vicinity of the high-pressure water injector (Figure 6.13c).

We first calculate time shifts as the 4D increase in time thickness between top (yellow-dashed event in Figure 6.13) and base (red-dashed event in Figure 6.13) reservoir reflections. We choose to use this time-thickness approach since the source of the time shifts is likely constrained within the reservoir, thus resulting in the most accurate, high resolution time shift estimate. Other methods of estimating time shifts are possible, e.g., correlation-based methods (Landro and Stammeijer, 2004), however these can underestimate time shifts in our situation. This is because to get a stable correlation estimate often a fairly long time window of data is required, meaning the method may essentially be averaging over events that have both large and decreasing time shifts (i.e., the seismic reflection event push-down effects may "heal" with depth below the reservoir as the seismic waves "undershoot" the velocity anomaly). As discussed in Wulff et al. (2008), some difficulty exists in picking the base reservoir reflection, particularly in the area of the thick channel sand. For this reason, we carefully manually pick the base reservoir reflection in this area, rather than relying on peak positive event auto-picking. Using the time-thickness approach between top and
Figure 6.11: 4D RMS amplitude maps at top reservoir. (a) Near-stack difference map (M1-Baseline). (b) Mid-stack difference map (M1-Baseline). (c) Near-stack difference map (M1-Baseline) with areas affected by tuning (< 12ms time thickness between top and base reservoir) masked out (grey). (d) Mid-stack difference map (M1-Baseline) with areas affected by tuning masked out (grey). We also note that the reservoir thickness in Well_2 and Well_3 is above tuning thickness.

base reservoir reflections we generate the time shift map shown in Figure 6.14a.

Uncertainty in 4D time shifts calculated using the time thickness approach can arise when difficulty exists in picking the seismic events used in the calculation (i.e., as with the base reservoir reflection discussed above). For this reason, we also calculate time shifts as the 4D increase in time thickness between top reservoir (yellow-dashed event in Figure 6.13) and the next strong, continuous negative event below the reservoir (cyan-dashed event in Figure 6.13). Using these events we generate the time shift map shown in Figure 6.14b. We calculate an average time shift of 6 ±2 ms within the polygon in Figure 6.14a.
Figure 6.12: Histogram of extracted maximum trough amplitude within the thick channel sand polygon (e.g., Figure 6.10) for (a) near and (b) mid stacks. Baseline amplitudes are shown in blue and M1 amplitudes in red. Amplitude increases between near and mid stacks, consistent with the Class III AVO observed in Figure 6.9 and modelled in Figure 6.7. The near-stack 4D difference (M1-Baseline) is greater than the mid-stack 4D difference, also consistent with modelled 4D AVO behaviour (Figure 6.7). At M1, a 60% increase in mean amplitude is observed compared to Baseline for the near stack, while a 50% increase is observed for the mid stack.

and an average time shift of 4 ± 2 ms within the polygon in Figure 6.14b. The fact that the uncertainty associated with both maps is ±2 ms reflects that in Figure 6.14a we have uncertainty associated with picking the base reservoir reflection, while in Figure 6.14b we have uncertainty in the estimated time shift given that we are picking an event well below the source of the 4D time shifts, and thus the seismic wavefield may have “healed” with depth below the reservoir. In any case, the estimated time shifts in both maps are in the same range (within the uncertainty), and thus together provide added confidence in the estimated time shifts associated with pore pressure increase in the area. We also note that the calculated time shifts are much greater than the time shifts estimated from the initial feasibility study (~ 1 ms) (e.g., Wulff et al., 2008), which we will show later may be due
to mechanical weakening of the reservoir rock due to high pressure water injection.

### 6.6.3 New rock physics model

To describe the amplitude anomalies observed around Injector_1, [Wulff et al. (2008)](Wulffetal2008) and [Smith et al. (2008)](Smithetal2008) propose the ‘fracture/crumbled’ rock physics model (shown in Figure 6.15). Since water is injected well above fracture pressure (1000-1500 psi overpressure), [Wulff et al. (2008)](Wulffetal2008) propose that the rock may have fractured/crumbled. In order to approximate this inelastic rock fracturing or crumbling the authors select extreme parameters in the [MacBeth (2004)](MacBeth2004) empirical regression model, until a model-to-seismic match was achieved close to the injector (i.e., the velocity-pressure model was changed until modelled synthetic seismic data matched observed seismic data around the injector). It should be noted that the [MacBeth (2004)](MacBeth2004) model is purely empirical in that it relies on fitting parameters to specify an exponential curve which is not necessarily physically realisable. Using this approach, [Wulff et al. (2008)](Wulffetal2008) found a set of Macbeth regression model parameters to fit the change in amplitude between Baseline and M1 seismic surveys in the thick channel sands close to the injector.

Since the Macbeth regression model fit of [Wulff et al. (2008)](Wulffetal2008) is purely empirical, and unlikely to be physically realisable, we suggest it may be only suitable to describe the 4D anomalies at or nearby to Injector_1, and only for the pressure change between the Baseline and M1 surveys (i.e., not for a wider range of pressure values). Since their model attempts to describe an inelastic change to the reservoir rock, if the effective pressure were to increase again (decrease in pore pressure), the velocity-pressure response would not reverse back along the Macbeth regression curve, but would follow a flatter trend as shown by the unconsolidated velocity-pressure measurements of [Zimmer (2003)](Zimmer2003) (Figure 6.15). This is because the rock has been irreversibly damaged and is now softer/less cemented than its original state before water injection. Also, the maximum pressure sensitivity of a rock (in an elastic sense) is when the rock is completely uncemented/unconsolidated [Avseth and Skjei (2011); Saul and Lumley (2013)]. Given this, even if the pore pressure were to increase further from that at M1 and the reservoir rock were to completely ‘fracture/crumble’ (become unconsolidated), the velocity at a given pressure should not physically decrease
6.6. 4D seismic data analysis

Figure 6.13: (a) Baseline near-stack seismic section through area of interest and Injector_1. (b) M1 near-stack seismic section. (c) 4D seismic difference section (M1-Baseline) highlighting strong amplitude anomaly around the water injector and the effects of associated time shifts. Red and cyan arrows point to events representing the effects of time shifts in the red and cyan-dashed events, respectively, in (a) and (b). The thick channel sand analysed in this study can be seen just down-dip from the injector.
Figure 6.14: 4D time shifts around Injector_1. (a) Time shift measured as the 4D increase in time thickness between top (yellow dashed interpretation in Figure 6.13) and base reservoir (red dashed interpretation in Figure 6.13) in ms. In the polygon we extract an average time shift of 6 ms. (b) Time shift measured as the 4D increase in time thickness between top reservoir and a deeper negative event below base reservoir (cyan interpretation in Figure 6.13) in ms. In the polygon we extract an average time shift of 4 ms, implying that the source of the 4D time shifts is maximum in the reservoir, and decreases or “heals” in the seismic images with depth below reservoir. The time shifts show a background noise level of ± 2 ms.

below that of a purely unconsolidated sediment. From Figure 6.15 we can see that the Macbeth regression model suggests velocities will decrease below the velocity of a purely unconsolidated sediment for normalised pressures <0.25. This prediction is not physical, and so this regression model is not appropriate to predict 4D responses for further increases or decreases in pore pressure from those at the time of the M1 survey.

In the next section, we use rock physics diagnostics (e.g., Dvorkin and Nur 1996, Avseth et al. 2000) to infer the relative pressure sensitivity of sandstones within the Macedon reservoir interval, and based on the results develop a physical model that can be used to better describe elastic and inelastic changes in rock properties associated with variations in pore pressure. Our objective is to develop a rock physics model that is consistent with observations from log and core data, that can be used to describe the rock mechanical
6.7. Rock physics diagnostics of rock texture and diagenesis

This section investigates the relationship between seismic elastic parameters and rock properties, including grain texture, clay content, and diagenesis, in the Macedon reservoir interval within the Enfield oil field. We apply the technique of rock physics diagnostics (Dvorkin and Nur (1996), Avseth et al. (2000)) to infer rock type and texture from velocity-porosity relations, and based on the results infer the amount of contact cement. This diagnostic

Figure 6.15: Analysis of ‘fracture/crumbled’ velocity-pressure model of Wulff et al. (2008): Normalised $V_P$ and $V_S$ as a function of normalised effective pressure. Measurements made on a dry core sample from an Enfield appraisal well are shown as solid circles. Normalised $V_P$, $V_S = 1.0$, and normalised effective pressure $= 1.0$ represent baseline conditions. Curves show $V_P$ and $V_S$ predictions using the MacBeth (2004) empirical model, where the model fitting parameters have been adjusted to achieve a model-to-seismic match around the water injectors. We also plot data (triangles) from measurements on unconsolidated sands from Zimmer (2003). We normalise data from Zimmer (2003) relative to Enfield dry rock velocities at baseline effective pressure. Due to irreversible changes to the reservoir rock, the velocity-pressure response for a reduction in pore pressure from M1 should follow a flatter trend than the Macbeth model, as indicated by the black arrow. Also, the velocity of a cemented rock cannot decrease below the velocity of an uncemented sample at a given pressure (assuming same porosity and composition).

weakening process due to high pore pressure (and therefore match 4D amplitude and time shift data), but that will also be useful for the modelling and quantification of future monitor surveys.
enables us to gain an understanding of the relative difference in the pressure sensitivity of seismic elastic properties, since more cemented sands should be less sensitive to pressure (Avseth and Skjei [2011] Saul and Lumley [2013]). Achieving this goal will improve the understanding and interpretation of 3D and 4D seismic responses in the field, with the hope of being able to describe the significant 4D amplitude and time shift anomalies discussed in the previous section with a physical (versus empirical) model. Again, we focus on the area of interest around the Injector_1 (Figure 6.2), by analysing log data from the Well_2 and Well_3 appraisal wells (Figure 6.16).

Dvorkin and Nur [1996] introduced two theoretical models to describe the diagenetic and sorting trends for high-porosity sands. The friable-sand model describes porosity loss due to deteriorating sorting, while the contact-cement model describes porosity loss due to the deposition of cement on the surface of grains. Avseth et al. [2000] introduced the constant-cement model to describe sands with constant cement volume, but varying sorting (it is assumed that sands that have a variation in porosity due to sorting have the same amount of contact cement). The model is a combination of the contact-cement model and the friable-sand model (see schematic in Figure 6.17). Firstly, it is assumed porosity reduces from that of the initial sand pack (solid circle in Figure 6.17) to the initial-cement porosity \( \phi_b \) due to the deposition of contact cement (open circle in Figure 6.17). The bulk \( K_b \) and shear \( G_b \) moduli at this initial-cement porosity are calculated with the contact-cement model [see equations in Dvorkin and Nur [1996]]. The elastic moduli at a lower porosity (due to deteriorating sorting) is then calculated using a modified Hashin-Shtrikman lower bound (MHS LB) as in the friable-sand model:

\[
K_{\text{dry}} = \left(\frac{\phi/\phi_b}{K_b + 4G_b/3} + \frac{1 - \phi/\phi_b}{K_m + 4G_b/3}\right)^{-1} - 4G_b/3
\]

\[
G_{\text{dry}} = \left(\frac{\phi/\phi_b}{G_b + z} + \frac{1 - \phi/\phi_b}{G_m + z}\right)^{-1} - z,
\]

where \( z = \frac{G_b}{6} \left(\frac{9K_b + 8G_b}{K_b + 2G_b}\right) \) (6.1)

and \( K_m \) and \( G_m \) are the bulk and shear moduli of the mineral phase, respectively. These models can be superimposed on moduli-porosity cross-plots to diagnose sands into three groups: (1) friable (unconsolidated) sands; (2) sands with porosity variation associated with contact cement; and (3) sands with constant cement volume but varying porosity due
6.7. Rock physics diagnostics of rock texture and diagenesis

**Figure 6.16:** Log data for Well_2 and Well_3 appraisal wells. Vertical axis shows two-way travel time. Well_2 is in the aquifer, while Well_3 is in the oil leg. We calculate Well_3 elastic logs for brine saturation with Gassmann fluid substitution. Different zones of interest are highlighted, including the Upper and Lower Macedon sands.

to deteriorating sorting (or clay content).

Figure [6.18a and 6.18b] show water saturated compressional and shear log velocity versus volume of shale (Vshale), colored by zones of interest from Figure 6.16 for the Macedon reservoir sands, respectively. The Upper Macedon sands are generally cleaner (lower volume of shale) than the Lower Macedon sands, and have higher compressional and shear velocities. We use rock physics diagnostics [Dvorkin and Nur, 1996; Avseth et al., 2000] to determine possible causes for the observed differences (Figure 6.19). We
6.7. Rock physics diagnostics of rock texture and diagenesis

Figure 6.17: Schematic diagram of three theoretical models for high-porosity sands. The thickening of circles represents the addition of contact cement from the initial sand pack. Adapted from Avseth et al. (2005).

can see from Figure 6.19 that the Upper Macedon sands have slightly lower porosity than the Lower Macedon sands; however, the Upper Macedon sands are generally cleaner so rather than this porosity variation being associated with the volume of shale, it is likely due to a combination of poorer sand sorting and increased contact-cement volume. Indeed, the rock physics diagnostics in Figure 6.19 suggests that the Upper Macedon sands have approximately 2.5 – 3% contact cement, while the Lower Macedon sands have closer to 1.5 – 2%. Cleaner sands are generally more susceptible to the formation of contact cement (e.g. Avseth et al. 2005), so the observation of higher cement volume in the cleaner Upper Macedon sands seems plausible. The increased contact-cement volume in the Upper Macedon sands also helps to explain the observed high velocities in the Upper relative to the Lower Macedon sands. We note that variations in clay content, if deposited structurally, could help to explain the lower velocities in the Lower Macedon sands; however, even the very cleanest Lower Macedon sands have compressional and shear velocities lower than the Upper Macedon sands, suggesting there is indeed a variation in the amount of contact cement between the Upper and Lower sands.

Figure 6.19 further highlights that there is a significant variation in the stiffness of
sandstones within the reservoir interval. Sandstones have compressional and shear velocities varying between approximately 2800 - 3500 m/s and 1400 - 2100 m/s, respectively. Based on the rock physics diagnostics in Figure 6.19, the sandstones also vary from nearly unconsolidated to 3% cemented. The log data analysed in Figure 6.19 are from a limited depth range (<100 m) so the observed variation in velocities is inferred to be due to differences in contact-cement volume and sorting. This observation and analysis will have significant implications for pressure sensitivity in the field, since cemented sands should be less sensitive to changes in pressure than unconsolidated sands (Figure 6.20).

The implication of variations in contact-cement volume on pressure sensitivity can be further investigated by looking at the dry-rock pore-space compressibility of the sands.
6.7. Rock physics diagnostics of rock texture and diagenesis

Figure 6.19: Saturated (a) compressional and (b) shear log velocity values versus porosity for the Upper (blue) and Lower (green) Macedon reservoir sands. Also shown is the friable-sand model (Dvorkin and Nur, 1996) which we calibrate to uncemented sediment samples from Zimmer (2003) using the grain contact theory model of Saul et al. (2013), along with the contact-cement (Dvorkin and Nur, 1996) and constant-cement (Avseth et al., 2000) models. The red triangle shows the location of the core sample used in the 4D seismic feasibility study.

The compressibility of a dry rock can be expressed as:

$$\frac{1}{K_{\text{dry}}} = \frac{1}{K_m} + \frac{\phi}{K_\phi},$$

where $K_{\text{dry}}$ is the dry-rock bulk modulus, $K_m$ is the mineral (grain) bulk modulus, $\phi$ is the porosity, and $K_\phi$ is the pore-space incompressibility (e.g., Mavko et al., 1998). Rocks that are unconsolidated or are at low effective pressure generally have small values of $K_\phi$ and are therefore highly compressible. Conversely, rocks that are well cemented or
at high effective pressure tend to have large values of $K_\phi$ and are therefore relatively incompressible (Avseth et al., 2005). Since the initial effective pressure of the reservoir sands is the same, lower values of $K_\phi$ indicate more compressible sands. Using equation 6.2, we invert Well_2 and Well_3 log data for dry pore-space incompressibility (Figure 6.21). The plot also includes dry pore-space incompressibility values inverted from unconsolidated sands from Zimmer’s data (Zimmer, 2003). We calculate $K_m$ in equation 6.2 using a Voigt-Reuss-Hill average of shale and quartz mineral properties (e.g., Mavko et al., 1998). We see that sands with higher contact-cement volume have higher values of pore-space incompressibility, providing further justification that these sands should be less sensitive to changes in pressure than sands without contact cement. Here, we have assumed that the variation in pore-space incompressibility is due to variations in cement volume alone; however, we note that variations could also be associated with different pore shapes, among other physical rock properties. Whatever the physical mechanism that is controlling the stiffness of the sands, we still propose that sands with high pore-space incompressibility should be relatively less sensitive to changes in pressure than sands with low pore-space incompressibilities. We choose to quantify the relative difference in stiffness in terms of contact-cement volume since, as we will show later, this enables us to model mechanical
6.7. Rock physics diagnostics of rock texture and diagenesis

**Figure 6.21:** Saturated compressional log velocity versus porosity, coloured by pore-space incompressibility, for the Macedon sands (triangles). The sands diagnosed to have higher contact-cement volume have the highest pore-space incompressibility, and should therefore be less sensitive to changes in pressure. Unconsolidated samples from Zimmer (2003) show much lower pore-space incompressibilities (circles). The highest pressure measurements from Zimmer (2003) track along the friable-sand model curve, and correspond to the in-situ reservoir pressure at Enfield.

Weakening using a modified version of the velocity-pressure-cement model of Avseth and Skjei (2011).

We have discussed that the amount of contact cement will be a control on the pressure sensitivity of rock elastic properties, with more cemented sands being less sensitive to changes in pressure. The location of the core sample from Well_2, used to make the ultrasonic-velocity pressure measurements, was taken from a more consolidated section of the core (Wulff et al., 2008). As such, the measured pressure sensitivity may not be representative of the rest of the field, and may have been underestimated in the initial feasibility study (Wulff et al., 2008). We can use the rock physics diagnostics results to investigate this statement further.

Rock physics diagnostics indeed shows that the sample was taken from a more consolidated section of the reservoir, with the associated log velocity at the location where the core sample was taken plotting close to the 3% cemented-sand line (red triangle in Figure 6.19). We should therefore expect that most of the reservoir sands at Enfield will be more sensitive to pressure than the core sample suggests, since they are generally less consolidated (lower cement volume). However, for the Enfield core sample, the change in velocity
for a given change in effective pressure is equal to or greater than that for a completely unconsolidated (no contact cement) sand sample (same initial porosity) from Zimmer’s data (Figure 6.15). This confirms that the change in velocity with pressure between the two surveys (estimated from the 4D amplitude and travel-time data) cannot be explained by a purely elastic rock physics model. In fact, the velocity change estimated from the 4D data is much larger than the velocity change predicted even if the reservoir rocks were completely unconsolidated instead of partially cemented as they are. This gives evidence that water injection at high pressure is weakening the reservoir rock at Enfield, providing the justification to develop a non-elastic velocity-pressure model that includes the effects of grain cement to better explain the 4D responses. In the next section, we discuss how the results of the rock physics diagnostics study can be used to guide the analysis of pressure sensitivity within the field in terms of grain cementation effects.

### 6.7.1 A new modelling approach for pressure sensitivity

Avseth and Skjei (2011) introduce a heuristic approach to predict the pressure sensitivity of cemented sandstones. They assume that a cemented sandstone consists of a binary mixture of cemented and uncemented grain contacts, where the cemented contacts are pressure-insensitive and the uncemented contacts are pressure-sensitive according to Hertzian contact theory. To model the pressure sensitivity of a cemented sandstone Avseth and Skjei (2011) use a ‘bounding average method’ (see Mavko et al., 1998). They define a weight function, $W$, depending on where the cemented sandstone data plot between Upper and Lower Hashin-Shtrikman bounds:

$$ W = \frac{K_{\text{dry}} - K_{\text{soft}}(P_0)}{K_{\text{stiff}} - K_{\text{soft}}(P_0)}, $$

where $K_{\text{dry}}$ is the dry bulk modulus (modelled or observed) of a cemented sandstone at a given porosity, $K_{\text{soft}}$ is the pressure-sensitive soft (lower bound) bulk modulus at the same porosity at a given reference pressure, $P_0$, and $K_{\text{stiff}}$ is the pressure-insensitive stiff (upper bound) bulk modulus at the same porosity value. $K_{\text{soft}}$ is given by the friable-sand model (pressure sensitive with Hertz-Mindlin theory). $K_{\text{stiff}}$ is calculated by increasing the effective pressure in the Hertz-Mindlin model until the friable-sand model mimics the 10%
constant-cement model. A separate weight factor for shear modulus is then calculated with the same approach.

Any sandstone data point can be inverted for the weight factor $W$, allowing the calculation of pressure sensitivity curves for each data point:

$$K_{\text{dry}}(P_{\text{eff}}) = (1 - W K) K_{\text{soft}}(P_{\text{eff}}) + W K K_{\text{stiff}}, \quad (6.4)$$

$$G_{\text{dry}}(P_{\text{eff}}) = (1 - W G) G_{\text{soft}}(P_{\text{eff}}) + W G G_{\text{stiff}}. \quad (6.5)$$

Avseth and Skjei (2011) simulate synthetic sandstone data for a wide range of contact-cement volumes and porosities using the constant-cement model (Avseth et al., 2000). Contact-cement volumes vary between 0 and 10%, and porosities between 0 and 0.4. They then calculate bulk and shear moduli as a function of effective pressure for each synthetic data point using equations 6.3-6.5. Avseth and Skjei (2011) then perform a nonlinear regression analysis on the simulated data set, for porosities ranging from 0.20 – 0.40. The resulting dry rock velocities for the sandstones representing the stiff (pressure-insensitive) bound are given by the following equations as a function of porosity $\phi$ and effective pressure $P_{\text{eff}}$ (in MPa):

$$V_{P_{\text{stiff}}} = 4992 - 10171\phi + 9548\phi^2 + 65P_{\text{eff}}^{0.2777}, \quad (6.6)$$

$$V_{S_{\text{stiff}}} = 3013 - 5513\phi + 3519\phi^2 + 55P_{\text{eff}}^{0.2851}. \quad (6.7)$$

The resulting dry rock velocities for the sandstones representing the soft (pressure-sensitive) bound are given by the following equations as a function of porosity and effective:

$$V_{P_{\text{soft}}} = 1024 - 6069\phi + 5788\phi^2 + 1117P_{\text{eff}}^{0.1556}, \quad (6.8)$$

$$V_{S_{\text{soft}}} = 769 - 3799\phi + 3562\phi^2 + 648.5P_{\text{eff}}^{0.1582}. \quad (6.9)$$

Finally, Avseth and Skjei (2011) define the effective dry velocities of any point between the soft and stiff bounds as a function of porosity, effective pressure, and cement volume, to
be a weighted average of the soft and stiff bounds with respect to cement volume:

\[ V_{P\text{eff}} = \left[ \frac{(0.09 - C_v)}{0.09} V_{P\text{soft}}^{n(P_{\text{eff}})} + \frac{C_v}{0.09} V_{P\text{stiff}}^{n(P_{\text{eff}})} \right]^{1/n(P_{\text{eff}})}, \]  

(6.10)

\[ V_{S\text{eff}} = \left[ \frac{(0.09 - C_v)}{0.09} V_{S\text{soft}}^{n(P_{\text{eff}})} + \frac{C_v}{0.09} V_{S\text{stiff}}^{n(P_{\text{eff}})} \right]^{1/n(P_{\text{eff}})}, \]  

(6.11)

where \( C_v \) is the contact-cement volume, and \( n(P_{\text{eff}}) = 3.5 + 0.1 P_{\text{eff}} \), reflecting that the weighting average will change with effective pressure. Duffaut et al. (2011) test the above regression formulas on real data from the Gullfaks and Stratfjord fields, and find a good match between predicted and observed 4D responses for changes in effective pressure, as well as for predicted and measured volumes of contact cement (from thin section analysis).

In Avseth and Skjei (2011) and Duffaut et al. (2011), it is assumed that porosity and cement volume are static properties (i.e., do not vary with pressure), and the model is therefore only used to predict the effect that variations in pressure have on cemented-sand velocities. We propose to extend this modelling approach, by allowing cement volume to also be a dynamic property (i.e., to vary with pressure).

We use the results of rock physics diagnostics (Figure 6.19), along with a modified version of the velocity-pressure-cementation model of Avseth and Skjei (2011), to guide the analysis of pressure sensitivity within the reservoir. Figures 6.22a and b show compressional and shear velocity as a function of effective pressure, for varying contact-cement volumes, respectively. The uncedmented sand end-member has been calibrated to data from Zimmer (2003) by varying the fitting constants in equations 6.8 and 6.9 until the data residuals were minimised in a least squares sense. Fitting constants in equations 6.6 and 6.7 were then allowed to vary until the contact-cement volume (\( C_v \)) in equations 6.10 and 6.11 agreed with the average contact-cement volumes estimated in the rock physics diagnostics study (Figure 6.19).

This modelling approach allows us to predict different pressure sensitivities for reservoir sands of varying contact-cement volume, consistent with the physical intuition that cemented sands should be less sensitive to changes in pressure (Figure 6.20). The model also allows us to vary the contact-cement volume within a given sand, in order to model the effect of increasing pore pressure mechanically weakening the rock (effectively breaking...
6.8 4D interpretation and forward modelling

In this section, we develop a physical model-based interpretation of the 4D amplitude and time shifts observed around Injector_1 between Baseline and M1 surveys (Figures 6.11

**Figure 6.22:** Water saturated (a) compressional and (b) shear velocity versus effective pressure for sandstones of varying contact-cement volume, calculated using a modified version of the velocity-pressure-cement model of Avseth and Skjei (2011). The uncemented end-member has been calibrated to data from Zimmer (2003) (circles). The cemented end-member has been calibrated such that the average contact-cement volumes of the Upper and Lower Macedon reservoir sands (cyan circle) agree with the volumes determined from rock physics diagnostics (Figure 6.19).

contact cement). Further to this, upon mechanically weakening the reservoir rock more accurate estimates of velocity-pressure sensitivity for future monitoring surveys are possible, since we can use the velocity-pressure curve predicted for the new estimated contact-cement volume. We use this modelling approach in the next section, to quantify how much the reservoir rock must mechanically weaken in order to explain the observed 4D amplitude and time shift anomalies around Injector_1.
We use the model in Figure 6.22 to quantify what percent volume of contact cement may mechanically weaken near the injector. As in the 4D data analysis section, we only focus on explaining the observed amplitude and time-shift anomalies inside the polygon within the thick channel sand.

### 6.8.1 Amplitudes

We forward model synthetic near and mid-stack seismic data using Well_2 for a range of grain contact-cement volumes, representing variations in the amount of mechanical weakening due to injection. Figures 6.24 and 6.25 show forward modelled near and mid-stack seismic data, respectively, for contact-cement volumes of 2.5% (calculated average initial volume - Figure 6.22), 0.5%, and 0.0%. In each case the pore pressure increase is kept constant as measured downhole in Injector_1 at the time of the M1 survey. Next, we extract near and mid-stack maximum negative amplitudes at top reservoir, calculate the percent increase in amplitude from baseline conditions, and plot the value against the associated contact-cement volume (Figure 6.23). We complete this forward modelling exercise for a wide range of contact-cement volumes, and plot the resulting maximum negative amplitude increase on Figure 6.23.

Forward modelling shows that assuming no mechanical weakening of contact cement (i.e., contact-cement volume remains at initial value of ~ 2.5%, with velocity change purely due to elastic pore-pressure increase) results in near and mid-stack amplitude increases of 16% and 11%, respectively (Figure 6.23). This is significantly less than the measured near and mid-stack 4D amplitude increases of 60% (red dashed line in Figure 6.23) and 50% (black dashed line in Figure 6.23), respectively. Near and mid-stack amplitude increases of 152% and 86%, respectively, are modelled when the contact-cement volume is reduced to 0.0%. This represents the reservoir sands mechanically weakening from sands with initial contact-cement volume of 2.5%, to completely unconsolidated sands. These amplitude increases are much greater than the measured 4D near and mid-stack amplitude changes, suggesting the reservoir sands have not mechanically weakened to the extent of becoming completely unconsolidated.

Near and mid-stack amplitude increases of 60% and 45%, respectively, are modelled
6.8. 4D interpretation and forward modelling

**Figure 6.23:** Forward modelled percent change in maximum trough amplitude at top reservoir (for an 11.7MPa pressure change) versus contact-cement volume. Red and black circles show modelled near and mid-stack amplitudes, respectively. Observed percent change in near and mid-stack amplitude (polygon within thick channel sand - Figure 6.11) between Baseline and M1 surveys are shown as red and black-dashed lines, respectively. Average initial contact-cement volume is interpreted to be 2.5% from rock-physics diagnostics (Figure 6.19).

when the contact-cement volume is reduced to 0.75% (Figure 6.23). This is in excellent agreement with the measured 4D near and mid-stack amplitudes within the thick channel sand to the north of Injector_1. The modelling confirms that velocity variations much greater than purely elastic pressure effects are required to match model the observed 4D amplitude anomalies, again suggesting the reservoir rocks are mechanically weakening.
Figure 6.24: Forward modelled near-stack seismic data from Well_2, for varying amounts of contact-cement weakening using the model in Figure 6.22. In each case the increase in pore pressure is kept constant (increase of 11.7 MPa from initial reservoir pressure, as measured in Injector_1 at the time of M1), and the volume of contact cement is varied. The scale for each synthetic trace is the same as in Panel A.
**Figure 6.25:** Forward modelled mid-stack seismic data from Well_2, for varying amounts of contact-cement weakening using the velocity-pressure-cement model in Figure 6.22.
6.8.2 Time shifts

Along with forward modelling 4D amplitude data, we also calculate corresponding time shifts as the 4D increase in time thickness between top and base reservoir reflections. We calculate time shifts on the forward modelled near-stack synthetics, between initial and M1 pressure conditions, for a range of contact-cement volumes. Figure 6.24 shows forward modelled near-stack seismic data for contact-cement volumes of 2.5% (calculated average initial volume - Figure 6.22), 0.5%, and 0.0%. Again, in each case the pore pressure increase is kept constant as measured downhole in Injector_1 at the time of the M1 survey. Next, we calculate time shifts as the 4D increase in time thickness between top and base reservoir reflections, and plot the value against the associated contact-cement volume (Figure 6.26). We complete this forward modelling exercise for a wide range of contact-cement volumes, and plot the resulting time shift on Figure 6.26.

Forward modelling shows that assuming no mechanical weakening of contact cement (i.e., contact-cement volume remains at initial value of ~ 2.5%, with velocity change purely due to elastic pore-pressure increase) results in a near-stack time shift of 0.5 ms (Figure 6.26). This is significantly less than the average measured near-stack 4D time shift of 6.0 ms (red dashed line in Figure 6.26). A near-stack time shift of 4.2 ms is modelled when the contact-cement volume is reduced to 0.0%. This represents the reservoir sands mechanically weakening from sands with initial contact-cement volume of 2.5%, to completely unconsolidated sands. This modelled time shift is still less than the measured 4D near-stack time shift; however, the sands are thinner in Well_2 than the thick channel sand, as discussed below.

The forward modelling and interpretation of near and mid-stack amplitude data suggested that reservoir sands within the thick channel have mechanically weakened to have an equivalent contact-cement volume of 0.75%. Time shifts associated with this degree of mechanical weakening are ~ 3 ms at Well_2 sand thickness (Figure 6.26). Again, this modelled time shift is less than the measured average 4D near-stack time shift within the thick channel sand (shown as red dashed line in Figure 6.26). To investigate this discrepancy we look at the difference in reservoir thickness between Well_2 (~ 25 m) and the thick channel sand. A change in the arrival time of the base reservoir reflection due to a
6.8. 4D interpretation and forward modelling

Figure 6.26: Forward modelled time shift (4D increase in time thickness between top and base reservoir) in ms versus contact-cement volume for an increase in reservoir pressure of 11.7 MPa. Red circles show modelled time shift on the near-stack using Well_2 (reservoir thickness $\sim 20 - 25$ m). Blue square and blue circle show modelled time shifts using equation [6.12] for $h = 40$ m and $h = 50$ m, respectively. Observed average 4D time shift on near stack (within thick channel sand polygon - Figure 6.14) between Baseline and M1 surveys is shown as red-dashed line. Average initial contact-cement volume is interpreted to be 2.5% from rock-physics diagnostics (Figure 6.19).

Change in compressional velocity can be calculated with the following equation:

$$\Delta T = \frac{2h}{V_0 + \Delta V} - \frac{2h}{V_0}, \quad (6.12)$$

where $h$ is the reservoir thickness, $V_0$ is the Baseline compressional velocity, and $\Delta V$ is the 4D change in compressional velocity at the time of M1. If we assume the thickness of the channel sand is 40 – 50m, the quoted thickest parts of the reservoir [Ali et al., 2008; Hamp et al., 2008], then the change in velocity associated with a reduction in contact-cement volume to 0.75% ($V_0 = 3133$ m/s and $\Delta V = 517$ m/s) results in a calculated time shift of $\sim 5 - 6$ ms (Figure 6.26). This is in agreement with the $6\pm 2$ ms time shifts measured from the 4D seismic data (Figure 6.14).
6.9 Discussion

We have shown that high-pressure water injection is likely to mechanically weaken reservoir rock at Enfield, thus explaining the large observed 4D amplitude and time-shift anomalies that cannot be explained by purely elastic rock physics models alone. We quantify this inelastic weakening using a modified version of the velocity-pressure-cement model of Avseth and Skjei (2011), constrained by rock physics diagnostics. We have developed a physical model to explain the observed injection-induced weakening of reservoir rock as mechanical weakening of the grain cement bonds, but it could also be due to other sources of weakening, such as chemical reaction in the rock grains or cement, which is beyond the scope of this thesis.

Forward modelling with our rock physics model shows that high-pressure water injection into the cemented channel sands at Enfield weakens the sandstones to a state that they are almost unconsolidated in nature. This has a number of implications for further seismic reservoir characterisation and monitoring. For example, the feasibility and interpretation of future monitor surveys should not be based on either the initial state or empirical (e.g., Macbeth or 'fractured/crumbled’ model) velocity-pressure models, but on a physical and potentially time-varying velocity-pressure model based on the interpreted volume of contact-cement at the time of M1. This is because the injection-induced weakening of contact cement is an inelastic time-varying process, and thus the associated velocity-pressure response is not reversible.

The biggest short-coming of the contact-cement model (Dvorkin and Nur, 1996) is that it does not include pressure-sensitivity. This is because the model assumes all grain contacts have the same amount of cement, and that grains immediately lose pressure sensitivity once cementation begins. However, this study, along with others (e.g., Avseth et al., 2009; Avseth and Skjei, 2011), has shown that cemented reservoirs can still have significant pressure sensitivity, probably because cement is not usually deposited evenly on all grain contacts. The short-coming of the contact-cement model means that the absolute volume of contact cement may be overestimated from rock physics diagnostics. We are not so much concerned with the absolute volume of contact cement, but rather the relative
variation within the sands, since this tells us about the relative difference in expected pressure sensitivity.

A question raised during this study was that if the reservoir rock is mechanically weakening due to high pore pressure increases, then why doesn’t the rock fail equally in the laboratory during the ultrasonic-velocity pressure measurements? This may be due to the fact that the core sample used to make the measurements is from a more consolidated section of the reservoir (as shown in Figure 6.19), meaning it does not mechanically weaken to the same extent as the weaker reservoir rocks. We do however think that some mechanical weakening may have occurred to the core sample during the ultrasonic-velocity pressure measurements, as evidence by the pressure sensitivity of velocity relative to a completely uncemented sample (Figure 6.15). We have shown that cemented sands should be less sensitive to changes in pressure than uncemented sands (Figure 6.20), so for the pressure sensitivity of the core sample (cemented) to be equal to or greater than that of the uncemented sediment means the core may have mechanically weakened in the laboratory.

Since the reservoir sands at Enfield are unconsolidated to partially consolidated, it is also possible that porosity is increasing (i.e., pore expansion) during pore pressure increase in the reservoir (e.g., Zimmer 2003, Saul and Lumley 2013). Figure 6.27 shows porosity versus effective pressure data for an uncemented sediment sample from Zimmer (2003). The measurements are from the loading cycle, directly analogous to porosity loss due to compaction (e.g., Zimmer 2003, Dutta et al. 2009, 2010), or inversely, porosity increase due to geomechanical dilation during pore pressure increase (e.g., Siggins and Dewhurst 2003, Saul and Lumley 2013). This pressure-induced increase in porosity could also contribute to the velocity reduction in the reservoir rock, and thus to the observed 4D amplitude and time shifts. We test modelling a porosity increase of 1.5% (realistic variation based on pressure difference and Figure 6.27) on the elastic properties of the Macedon reservoir sands using the friable-sand model, and find that the effect is negligible (< 2% reduction in $V_P$ and $V_S$).

Effective stress is generally defined as confining pressure minus a fraction $n$ of the pore pressure. For unconsolidated rocks, the effective stress coefficient $n$ is typically close to 1 (e.g., Siggins and Dewhurst 2003, Hofmann et al. 2005); however, in more consolidated
Porosities versus effective pressure for an uncemented sand sample from Zimmer (2003). Data are from the loading cycle of the measurements. The data has been fit with the SL porosity-pressure model (Saul and Lumley, 2013). Based on this model, a realistic porosity variation associated with the pore-pressure increase around Injector_1 is $1 - 2\%$ (porosity units). This is strictly for an unconsolidated sediment, so the porosity increase at Enfield could be less; therefore, $1 - 2\%$ is an upper-bound on the porosity change.

rocks it has been shown that $n$ varies with porosity (e.g., Hofmann et al., 2005). Since the rocks at Enfield are unconsolidated to partially consolidated in nature, we have assumed $n = 1$ in this study. We do however note that this assumption may not always be valid in general.

We discussed in the initial 4D seismic feasibility section that changes in pressure and saturation (from initial oil saturation) can be separated with 4D AVO analysis (e.g., Landro, 2001; Lumley, 2001; Smith et al., 2008). Figure 6.28 shows 4D intercept and gradient modelling for changing contact-cement volume (Figure 6.28b) and pressure (Figure 6.28a), from in-situ conditions near Injector_1. We can see that changes due to pressure and contact cement plot in the same quadrant in 4D intercept and gradient space, meaning away from well control ambiguity may exist when pressure-induced mechanical weakening of rocks in present. The ambiguity means that it may be difficult to quantify inelastic pressure effects using 4D AVO attributes (e.g., Tura and Lumley, 1999; Landro, 2001; Lumley, 2001; Lumley et al., 2003), since a given anomaly could be described by either an elastic pore pressure effect, a contact-cement effect, or a combination of both. It will however still be possible to separate saturation and pressure-related effects. Figure 6.28 also shows that,
as with pore pressure effects, contact-cement effects will be most observable at near-to-mid offsets (angles).

As shown in Figure 6.22, our new velocity-pressure-cement model makes it possible to predict a different velocity-pressure sensitivity depending on the contact-cement volume of a given sand, as well as varying amounts of contact-cement. In this chapter, we model the situation where contact-cement fails the same amount in both the Upper and Lower Macedon sands; however, we could model various situations where the amount of contact-cement failure varies between the sands, and attempt to improve the match to the observed top and base reservoir amplitudes, and to the time shifts. We tried this approach near Injector_1; however, owing to the difficulty in picking the base reservoir in this area, our analysis is much more confident in matching the top reservoir amplitudes and time shifts.

6.10 Conclusions

Understanding the effects that injected fluids and pressure have on rock elastic properties is crucial in order to determine whether time-lapse seismic is feasible at a given site, and in order to interpret/quantify observed time-lapse seismic anomalies. In this chapter, we have shown that high pressure water injection can inelastically mechanically weaken reservoir rock at Enfield, thus explaining the large observed 4D amplitude and time-shift anomalies that cannot be explained by elastic rock physics models alone. We quantify this inelastic effect using a modified velocity-pressure-cement model, constrained by rock
physics diagnostics. Forward modelling with our rock physics model shows that high-pressure water injection into the cemented channel sands at Enfield weakens the sandstones to a state that they are almost unconsolidated in nature. This has a number of implications for further seismic reservoir characterisation and monitoring in Macedon reservoir sands, as well as in other geologically analogous systems.

6.11 Acknowledgements

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6.11. Acknowledgements
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7 | Conclusions

Reservoir characterisation and monitoring using 3D and 4D seismic data is common practise for many earth science problems, particularly in hydrocarbon and CO$_2$ sequestration reservoirs. In order to model and interpret the data, a knowledge of the link between rock properties and seismic elastic parameters is required. The properties of weakly cemented sandstones are particularly sensitive to pressure; therefore, understanding the pressure dependence of sandstone elastic parameters is a key aspect of the 3D/4D seismic problem. Currently, the two most common methods to determine the pressure sensitivity of a given sandstone are: (1) predict pressure-dependent elastic properties from theoretical equations, e.g., with effective medium theory (EMT); or (2) fit an empirical regression to measured elastic property data made on core samples in the laboratory. In this thesis, we discuss the benefits and limitations of both methods, and develop new theoretical models that describe the pressure-dependent rock and elastic properties of unconsolidated to partially consolidated sandstones. We test the models on a wide range of laboratory measurements, showing improved fits/predictions compared to existing models. We also show that we can use the models, along with rock physics diagnostics, to improve the interpretation of a real 3D/4D seismic field case study.

7.1 Summary

Theoretical models, such as the Hertz-Mindlin effective medium theory (EMT), often do not accurately predict the pressure-sensitivity of unconsolidated sandstones, as evidenced by the significant mis-match to measured data shown throughout this thesis (Chapters 3 and 5). Conversely, existing empirical models lack an underlying physical basis, and thus often fail to predict pressure sensitivity outside measured laboratory data ranges, particu-
larly at low effective pressures where it is difficult to make measurements (Chapters 3 and 4). In Chapter 3, we develop a new theoretical model to describe the pressure sensitivity of the bulk and shear moduli for unconsolidated sedimentary rocks. The model incorporates effects of sedimentary compaction and critical porosity, including a relationship to account for porosity and density changes with pressure. In Chapter 4, we show that we can estimate critical porosity using grain-size distribution data and, along with the two-stage fitting procedure, accurately predict pressure sensitivity at low effective pressures outside normally measured data ranges. The new velocity-pressure-compaction model should be useful for improved prediction and interpretation of pressure-dependent rock properties and seismic data.

In Chapter 5, we further investigate the causes of observed discrepancies between theoretical predictions of unconsolidated rock elastic properties and those measured in the laboratory. We show that grain heterogeneity and porosity variations (e.g., sorting and compaction) help to explain the discrepancies, and then modify existing EMT to incorporate these effects. We conduct a detailed study into the effects of porosity variation on observed elastic moduli, and find that sorting and compaction can explain observations of Poisson’s ratio > 0.25. To account for observed discrepancies, we introduce two pressure-dependent calibration parameters into the Hertz-Mindlin theory that can be inverted from measured laboratory measurements. We further show that the calibration parameters agree with the results of published granular dynamics (GD) simulations. This places increased confidence in the proposed model, since the GD simulations solve a constitutive set of physics relations. The proposed modified grain contact theory (GCT) model provides improved predictions compared with existing models, describes the correct variation in $K_{\text{dry}}/G_{\text{dry}}$ ratio with effective pressure, and can model uncemented sediments with values of Poisson’s ratio > 0.25 which current models fail to do. This new modified GCT model should help to improve the prediction and interpretation of elastic properties as a function of depth/effective pressure. As an example, we show in Chapter 6 that the modified GCT model can be used to calibrate the high porosity end-member within the friable-sand model. This results in improved rock physics diagnostics, in order to estimate the effects of texture and cementation on elastic moduli.
In Chapter 6 we analyse a real data example with the objective of improving the interpretation of 4D seismic data acquired over a high-pressure water injector in the Carnarvon Basin, offshore Western Australia. From the analysis of 4D seismic data acquired at the site we observe significantly larger amplitude and time-shift anomalies than were predicted from the initial rock physics feasibility work. Using rock physics diagnostic techniques we are able to quantify the relative amount of contact cement within the reservoir sands, and therefore the relative difference in pressure sensitivity since more cemented sands should be less sensitive to pressure. This provides the basis for a new velocity-pressure-cementation model, in which we model elastic properties as a function of pressure and cementation. Using our new technique, we quantitatively determine that high-pressure water injection is likely to mechanically weaken reservoir rock around the water injectors, which explains the significant 4D amplitude and time-shift anomalies. Additional forward modelling shows that high-pressure water injection in the thick channel sands at Enfield leads to sandstones that become almost unconsolidated in nature. The observations are important for quantifying further time-lapse seismic feasibility and interpretation in weakly cemented Macedon reservoir sands, as well as in other geologically analogous rocks. The results may also have implications for other possible reservoir management issues, including geomechanical effects, compaction/dilation of the reservoir, sanding, and damage to boreholes.

Throughout this thesis, we discuss the importance of accounting for changes in porosity as a function of effective pressure. We develop a new porosity-depth-pressure model in Chapter 3 that accounts for variations in porosity associated with compaction (packing and grain re-orientation) and sorting (grain-size distribution). We use this model throughout the thesis. In Chapter 4, we show that we can use grain-size distribution data to estimate the zero effective pressure critical porosity constraint, allowing us to predict porosity/density outside measured data ranges. We use the porosity-pressure model again in Chapter 5 to see how the calibration parameters of the proposed GCT model vary with porosity and pressure. We also use the porosity model in Chapter 6 to model a realistic increase in porosity associated with high-pressure water injection. The inclusion of the porosity-pressure model also improves the accuracy of fluid substitution, because porosity is a key pressure-dependent input parameter to Gassmann and other fluid substitution
7.2 Contribution and future research

The work presented in this thesis should be useful for the improved prediction and interpretation of pressure-dependent rock properties and seismic data. The implications span a wide range of earth science problems, including for the analysis of rocks in sedimentary basins, the prediction of pore pressure and geomechanical effects, and for time-lapse seismic monitoring of hydrocarbon, groundwater, geothermal, and CO\textsubscript{2} sequestration reservoirs.

The contribution to knowledge of this thesis is based largely around developing a better understanding of the relationship between sediment elastic properties and effective pressure. The newly developed models address long-standing research problems in that theoretical relationships, such as the Hertz-Mindlin and Walton models, often do not accurately fit laboratory core measurement data, and alternately empirical regression models that can be adapted to fit the data often do not accurately predict the correct velocity-pressure behaviour outside of the range of measured pressure values. In contrast, the models developed in this thesis accurately fit measured velocity-pressure data over a wide range of sample types and pressure ranges for unconsolidated to partially consolidated sedimentary rocks. Including physical constraints within the developed models helps ensure that accurate predictions can be made outside available calibration data. We also show the importance of accounting for porosity variations with changes in effective pressure, and that diagnosing sands in terms of diagenetic and depositional trends is important for understanding a rock’s pressure sensitivity.

One could extend the work presented in this thesis in a number of ways, including: (1) extending the stress-dependent models to account for anisotropy; 2) establishing a link between dynamic and static moduli, and therefore between the predicted velocity-porosity changes and geomechanical effects such as compaction; and (3) the developed models could also be used in a new 4D inversion approach to better quantify observed 4D seismic anomalies in terms of saturation, pressure, porosity, and cementation changes.

The work presented in this thesis allows for the improved analysis, prediction, and interpretation of pressure-dependent rock properties and their effects in seismic data, as
7.2. Contribution and future research

a result of our development and analysis of new, more accurate relationships between stress-dependent elastic properties and seismic data.