Using velocity dispersion to estimate halo mass: Is the Local Group in tension with \( \Lambda \)CDM?

Pascal J. Elahi,1,2* Chris Power,1 Claudia del P. Lagos,1 Rhys Poulton1 and Aaron S. G. Robotham1

1International Centre for Radio Astronomy Research, University of Western Australia, 35 Stirling Highway, Crawley, WA 6009, Australia
2ARC Centre of Excellence for All Sky Astrophysics in 3 Dimensions (ASTRO 3D)

ABSTRACT

Satellite galaxies are commonly used as tracers to measure the line-of-sight (LOS) velocity dispersion \( \sigma_{\text{LOS}} \) of the dark matter halo associated with their central galaxy, and thereby to estimate the halo’s mass. Recent observational dispersion estimates of the Local Group, including the Milky Way and M31, suggest \( \sigma \sim 50 \text{ km s}^{-1} \), which is surprisingly low when compared to the theoretical expectation of \( \sigma \sim 100 \text{ km s}^{-1} \) for systems of their mass. Does this pose a problem for Lambda cold dark matter (\( \Lambda \)CDM)? We explore this tension using the SURFS suite of \( N \)-body simulations, containing over 10000 (sub)haloes with well tracked orbits. We test how well a central galaxy’s host halo velocity dispersion can be recovered by sampling \( \sigma_{\text{LOS}} \) of subhaloes and surrounding haloes. Our results demonstrate that \( \sigma_{\text{LOS}} \) is biased mass proxy. We define an optimal window in \( v_{\text{LOS}} \) and projected distance \( (D_p) \sim 0.5 \lesssim D_p/R_{\text{vir}} \lesssim 1.0 \) and \( v_{\text{LOS}} \lesssim 0.5V_{\text{esc}} \), where \( R_{\text{vir}} \) is the virial radius and \( V_{\text{esc}} \) is the escape velocity – such that the scatter in LOS to halo dispersion is minimized – \( \sigma_{\text{LOS}} = (0.5 \pm 0.1)\sigma_{v, \text{H}} \). We argue that this window should be used to measure LOS dispersions as a proxy for mass, as it minimises scatter in the \( \sigma_{\text{LOS}}-M_{\text{vir}} \) relation. This bias also naturally explains the results from McConnachie (2012), who used similar cuts when estimating \( \sigma_{\text{LOS}, \text{LG}} \), producing a bias of \( \sigma_{\text{LG}} = (0.44 \pm 0.14)\sigma_{v, \text{H}} \). We conclude that the Local Group’s velocity dispersion does not pose a problem for \( \Lambda \)CDM and has a mass of \( \log M_{\text{LG}, \text{vir}}/M_{\odot} = 12.0^{+0.8}_{-2.0} \).

Key words: methods: numerical – Galaxy: halo – galaxies: haloes – dark matter.

1 INTRODUCTION

Currently favoured theories of galaxy formation predict that galaxies are embedded within massive haloes of non-baryonic cold dark matter (CDM; e.g. White & Frenk 1991; Baugh 2006; Benson 2010). These haloes play a fundamental role in regulating galaxy properties, as is evident in scaling relations such as, for example, those between stellar mass and halo mass (e.g. Behroozi, Conroy & Wechsler 2010; Guo et al. 2010; Reddick et al. 2013). While factors such as a halo’s assembly history and its larger scale environment will influence galaxy properties, there are sound physical arguments as to why a halo’s mass should be particularly important. The dark matter mass governs the depth of the gravitational potential well within which galaxies evolve, and it impacts directly the time-scales on which galaxies grow, via gas accretion and mergers, and the efficiency with which feedback from stars and black holes influences gas dynamics within the galaxy (e.g. White & Frenk 1991).

Testing these ideas observationally requires accurate estimates of halo mass that can be determined on a system-by-system basis, which favours the use of satellite galaxies as dynamical tracers. On galaxy cluster mass scales, numerous observational estimators have been published, but it has been shown that velocity dispersion measurements of cluster galaxies in the highest mass systems allow the mass to be recovered with the smallest bias, less than \( \sim 0.2 \text{ dex} \) (e.g. Old et al. 2014, 2015). Using velocity dispersions on galaxy group mass scales is more challenging because individual systems tend to contain fewer dynamical tracers, i.e. satellites, and estimates are likely to be more uncertain. Nevertheless, this approach has been used successfully by Carlberg et al. (e.g. 1997); Schneider (e.g. 2006); Yang et al. (e.g. 2006); Robotham et al. (e.g. 2011).

Biases arise invariably because assumptions about a halo’s dynamical state and geometry are unavoidable. A typical halo is assumed to be (almost) spherical and virialized, with a satellite population drawn from a dynamically relaxed distribution, but haloes as they might exist are likely to be triaxial spheroids whose axis ratios
axis depend on halo mass (e.g. Elahi et al. 2018) with a virialization state that depends on mass and environment. Major mergers significantly affect a halo’s dynamical state, but its average growth history is dominated by smooth mass accretion and minor mergers (see Rodríguez-Puebla et al. 2016; Elahi et al. 2018). Consequently, using a population of satellites as tracers of the dynamical mass of an object will give reasonable, though possibly biased, mass estimate.

Interestingly, the Local Group (LG) has a velocity dispersion of $\sim 50\, \text{km} \, \text{s}^{-1}$ (McConnachie 2012), much lower than we might expect given the expected mass of the system (see Fig. 1). Our LG is the system in which we might expect the biases arising from incomplete tracer populations, missing satellites below some magnitude limit, to be minimal. Does this low dispersion pose a problem for ΛCDM, imply that the LG is unusual, or might it reflect uncertainties in the mass of, for example, the Milky Way (MW), which could be low? The mass of MW is a contentious estimate (e.g. Watkins, Evans & Harris 2012; Watkins et al. 2015; Garrison-Kimmel et al. 2017; Sawala et al. 2017). This implies that the LG is unusual, or might it reflect uncertainties in the mass of, for example, the Milky Way (MW), which could be low? The mass of MW is a contentious estimate (e.g. Watkins, Evans & Harris 2012; Watkins et al. 2015; Garrison-Kimmel et al. 2017; Sawala et al. 2017).

Figure 1. Halo Dispersions. The mass of a halo given its dispersion from L40N512 (red), L210N1024 (blue), and L210N1536 (orange) simulations. We plot the median and 16 per cent, 84 per cent quantiles as thick lines with contours for each simulation, outliers of the contour region and tails of the mass distribution of well resolved haloes. We also show observational estimates of MW (green circle), M31 (magenta square), and the LG (blue star). Inset shows the distribution of mass within some dispersion range.

2 NUMERICAL DATA

We use the SURFS simulations (Elahi et al. 2018), a suite of N-body simulations of volumes ranging from 40 to 900 $h^{-1}$ Mpc, each containing billions of particles, run assuming a ΛCDM Planck cosmology with $\Omega_m = 0.3121$, $\Omega_b = 0.0459$, $\Omega_{\Lambda} = 0.6879$, a normalization $\sigma_8 = 0.815$, a primordial spectral index of $n_s = 0.9653$, and a Hubble parameter of $h_0 = 0.6751$ (cf. Table of Planck Collaboration XIII 2016). We use a memory lean version of GADGET2 (Springel 2005), storing 200 snapshots evenly spaced in logarithm of the expansion factor between $z = 24$ to $z = 0$ to accurately capture the evolution of dark matter haloes; this temporal spacing ensures that we can follow the freefall time of overdensities of $200\rho_{crit}$, i.e. haloes. Halo catalogues are constructed with the VELOCIRAPTOR phase-space halo finder (Elahi, Thacker & Widrow 2011; Elahi et al. 2013).

We focus on the subset of SURFS simulations with box sizes 40 and $210\, h^{-1}$ Mpc (cf. Table 1) and between ~0.1 and 3 billion particles. This provides us with a sufficient statistical sample of well resolved central haloes – essentially friends-of-friends groups – with virial masses of $\sim 10^{12} \, M_\odot$, and allows us to identify the host haloes of galaxies with stellar masses of $\sim 10^8 \, M_\odot$. Here, we limit our analysis to well-resolved central haloes composed of $\gtrsim 10^9$ particles, with subhaloes and moderately well resolved neighbouring central haloes composed of at least 50 particles, approximately twice the particle limit at which haloes are identified; a summary of these ‘central’ haloes and the tracer population is given in Table 1. With these data, we can estimate reliably central halo velocity dispersion using both the full particle distribution and surrounding sub- and central haloes as discrete tracers of the velocity field. This catalogue spans haloes with virial mass from $10^{11.5} - 10^{15} \, M_\odot$, where we define virial mass as $M_{vir} = 4\pi R_{vir}^3 \rho_{crit}/3$, with $\Delta = 200$, $\rho_{crit}$ is the critical density of the universe. A typical group in our sample consists of $\sim 10$ subhaloes (satellites) and a few nearby but distinct central haloes within $\sim 1.5 R_{vir}$.

The focus of this work is on orbits, specifically those in dark matter only simulations. These orbits are reconstructed using ORBWEAVER, a tool that comes the VELOCIRAPTOR package. This code uses the evolution of haloes from merger trees produced by TREEFROG, the VELOCIRAPTOR merger tree builder (see Elahi et al. 2018) for details) and identifies orbits around candidate hosts by tracking objects from $\sim 2 R_{vir}$ and identifying changes in the sign of the radial velocity as peri/apo centres (for more details please see Section A). Approximately 5 per cent of the orbiting (sub)haloes in our sample are short-lived with lifetimes of $\lesssim 1$ Gyrs, most (~85 per cent) of which are poorly resolved with ~50 particles. These short-lived haloes are typically newly formed rather than systems with incorrectly tracked orbits and artificially shortened lifetimes. Based on the reconstructed lifetimes, this orbit catalogue is ~97 per cent complete, i.e. only 5 per cent of subhaloes are short-lived and of those, only 5 per cent are well resolved and artificially short-lived.

This orbit catalogue is extracted from DM only simulations. The addition of baryons, star formation, and associated feedback processes (e.g. supernovae, active galactic nuclei) does alter the dark matter distribution. At low dark matter halo masses, the (sub)halo occupancy is not unity, i.e. there are dark subhaloes (e.g. Sawala et al. 2015; Garrison-Kimmel et al. 2017; Sawala et al. 2017). This

\footnote{This estimate is based on extrapolating the stellar mass to halo mass relation (e.g. Moster et al. 2010; Behroozi et al. 2010; van Uitert et al. 2016).}

\footnote{Haloes composed of fewer particles are more susceptible to artificially shorted lives because they (1) drop below the particle limit at which haloes are identified; and (2) have artificially softened density profiles because of gravitational softening and so are more prone to tidal disruption (e.g. van den Bosch 2017).}
typically occurs below halo masses of $\lesssim 10^{9.5} \, M_\odot$ or maximum circular velocities of $\lesssim 25 \, \text{km} \, \text{s}^{-1}$, the exact scale dependent on the feedback model used. In general, we are looking at scales well above these rough thresholds and so our results are not strongly influenced by the reduced occupancy. Another baryonic affect to consider is subhalo survival.

Hydrodynamical simulations have fewer surviving subhaloes than dark matter only counterparts, a consequence of the stronger tidal field near the central galaxy (e.g. Garrison-Kimmel et al. 2017; Sawala et al. 2017). This tidal field can reduce the total number of subhaloes within 100 kpc of a $\sim 10^{12} \, M_\odot$ halo (near the virial radius) by a factor of 2. The disrupted satellites are more likely to be of the dark variety as subhaloes with stars are more concentrated and less prone to complete disruption. Again, we typically focus on larger subhaloes due to the resolution limits of our simulations.

Orbits themselves can be affected by baryons. Barber et al. (2014) found luminous satellites tended to occupy more radial orbits than the total subhalo population of a $\sim 10^{12} \, M_\odot$ halo, albeit using a semi-analytic model and orbits from a dark matter only simulation. Given haloes are dominated by dark matter save in the central tens of kpc, we might expect pericentres, and hence ellipticities to change due to the presence of baryons. However, the ellipticity of orbits is not critical to this work and will not affect greatly our results.

### 3 Satellites & Halo Mass Estimates

We start by comparing the distribution of halo velocity dispersions, $\sigma_{v, \text{H}}$, to halo masses in Fig. 1; here velocity dispersion is measured using all particles within $R_{\text{vir}}$. Our expectation is that

$$\sigma_{v, \text{H}} \propto (\Delta/2)^{1/6} (H_0 M_{\text{vir}})^{1/3},$$

where $\Delta = 200$ is the virial overdensity, $H_0$ is the Hubble constant, and $G$ is the gravitational constant. This is shown by the dashed black line in Fig. 1 and it provides a reasonable description of the simulation data; the simulated haloes shows little scatter with respect to this expectation, roughly 5 per cent independent of mass. The proportionality constant in equation (1) is 0.68 $\pm$ 0.2/3.

However, the picture is not so simple when considering observational data, which must rely on sparse sampling of the velocity distribution by using satellite galaxies residing in subhaloes as tracers. This is challenging even with high quality spectroscopic data because the numbers of tracers are few. Additionally, observationally assigning group/cluster membership is not a trivial, unlike in simulations where we make use of the full phase-space distribution to separate the virialized phase-space envelope of a halo (central galaxy) and the subhaloes (satellite galaxies) that reside in this region from other subhalo ensembles (galaxies).

A comparison of the true halo velocity dispersion and the observed velocity dispersion inferred from satellites, is presented in Fig. 2. Here, we show the LOS velocity dispersion inferred from satellites, $\sigma_{v, \text{LOS}}$, in our simulations relative to the true underlying halo dispersion $\sigma_{v, \text{H}}$. We calculate the satellite dispersion using LOS motions of all haloes within a projected radius of 2$R_{\text{vir}}$ relative to the host halo in question, whether or not they are true subhaloes within the phase-space envelop or field haloes. We take several LOS and calculate the mean, maximum and minimum $\sigma_{\text{LOS}}$. For clarity, we plot a subset of haloes, although the median, 1$\sigma$ contours and histograms are derived from the entire population. We include also the scatter resulting from different LOS directions. We also place the MW, M31, and LG systems at halo dispersions based on the mean relationship between $M_{\text{vir}}$ and $\sigma_{v, \text{H}}$ seen in Fig. 1.

Figure 2. **Observed Dispersions versus Halo Dispersions.** LOS velocity dispersion measured using tracers around a central halo along multiple lines of sight as a function of halo dispersion (top panel) and the ratio between these quantities (lower panel) for a random sample of 200 haloes. For each halo, we plot the median LOS dispersion along with error bars encapsulating the minimum and maximum values and colour code points according to number of satellites. In the lower panel, we show the median by a solid black line and 16 per cent, 84 per cent quantiles by shaded region for the entire halo sample, calculated in 10 bins with each bin containing 10 per cent of the population. The inset shows the distribution of ratios in three halo dispersion bins, solid line for the median $\pm 1\sigma$, dashed (dotted) for high (low) dispersion systems, where we have folded in the scatter arising from varying lines of sight. We also show the one-to-one line as a dashed black line to guide the eye. We also place the MW, M31, and LG systems at halo dispersions based on the mean relationship between $M_{\text{vir}}$ and $\sigma_{v, \text{H}}$ seen in Fig. 1.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$N_{\text{main}}$</th>
<th>$M_{\text{vir}}$ ($\log M_\odot$)</th>
<th>$N(r &lt; 2R_{\text{vir}})$</th>
<th>$N_{\text{sub}}$</th>
<th>Surrounding (sub)haloes</th>
<th>$M_{\text{vir}}$ ($\log M_\odot$)</th>
<th>Lifetime (Gyrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L40N512</td>
<td>527</td>
<td>(11.6, 12.1$^{+0.5}_{-0.2}$) 14.3</td>
<td>(1, 10$^{+15}_{-6}$) 1302</td>
<td>(0, 7$^{+15}_{-5}$) 809</td>
<td>(7.8, 10$^{+0.6}_{-0.4}$) 14.3</td>
<td>(0.47, 11.8$^{+0.8}_{-2.2}$) 13.3</td>
<td></td>
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<tr>
<td>L210N1024</td>
<td>3959</td>
<td>(12.1, 13.3$^{+0.4}_{-0.2}$) 14.9</td>
<td>(1, 13$^{+20}_{-7}$) 834</td>
<td>(0, 8$^{+13}_{-5}$) 280</td>
<td>(9.0, 11$^{+3.6}_{-0.4}$) 14.8</td>
<td>(0.46, 10.4$^{+1.2}_{-0.7}$) 13.2</td>
<td></td>
</tr>
<tr>
<td>L210N1536</td>
<td>13551</td>
<td>(11.13, 12.8$^{+0.5}_{-0.2}$) 14.9</td>
<td>(1, 12$^{+24}_{-7}$) 2741</td>
<td>(0, 7$^{+15}_{-5}$) 763</td>
<td>(8.5, 10$^{+0.6}_{-0.3}$) 14.9</td>
<td>(0.46, 10.4$^{+1.5}_{-0.6}$) 13.0</td>
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estimates to only those haloes within $1.25R_{\text{vir}}$, we recover similar results. At face value, satellites appear to be biased tracers of the velocity field; although $\sigma_{\text{LOS}}$ generally underestimates the true dispersion, it also overestimates the dispersions 15–20 per cent of cases. Significant outliers are typically systems undergoing major mergers. Of particular concern is that the observed dispersion and mass estimates of MW, M31, and LG appear significantly colder than expected, a point which we will return to later.

First let us consider the average underestimate seen in Fig. 2, which could be a result of sparse sampling of the velocity field. We test how well the LOS velocity dispersions recover the true 3-dimensional halo dispersion using idealized $N$-body realisations of dark matter haloes following an Navarro–Frenk–White (NFW) profile produced by GALACTICS (Widrow & Dubinski 2005; Widrow, Pym & Dubinski 2008). We do this by randomly sampling 2 million dark matter particle realisations 1000 times, each sample containing only 20 particles from which the LOS dispersion is calculated. We find that, in general, the LOS dispersion has no significant bias, with $\sigma_{\text{LOS}}/\sigma_{\text{HI}} = 0.91^{+0.22}_{-0.24}$ for an NFW halo. This ratio is offset from that seen in Fig. 2 and also has larger scatter, suggesting that our selected tracers, which contain both orbiting subhaloes, infalling haloes, and neighbouring but unassociated haloes, do not perfectly trace the phase-space distribution of the central halo.

We examine the motions of these candidate tracers in Fig. 3, where we show the distribution of radial, tangential, and total velocities as a function of radial distance for all our haloes. We stack haloes by normalizing distances and velocities by the virial radius $R_{\text{vir}}$ and the maximum circular velocity $V_{\text{max}}$, respectively. One of the notable features of this plots is the non-negligible fraction of candidate tracers that have velocities greater than the escape velocity, some of which reside well within the virial radius. The fraction of escaping subhaloes within one virial radius is $0.25 \pm 0.10$ (where the variance here is the halo-to-halo scatter). These subhaloes are not on bound orbits – they may become backsplash subhaloes or leave entirely. Given the existence of a population of subhaloes on unbound orbits, the naive expectation would be for the LOS velocity based satellite dispersion to overestimate the halo dispersion, rather than underestimate it, if no cuts are applied to this data.

However, this full 3-dimensional information is not observationally accessible. Instead, observations must rely on LOS phase-space of haloes as presented in Fig. 4; that is, the projected radius $D_p$ and the LOS velocity $v_{\text{LOS}}$ (see for instance Gill, Knebe & Gibson 2005; Oman, Hudson & Behroozi 2013; Jaffé et al. 2015; Oman & Hudson 2016; Yoon et al. 2017, for discussions and interpretations of LOS phase-space). Here, the middle panel shows the number density distribution similar to Fig. 3 and the bottom panel shows the mean orbital state. To calculate the mean orbital state, we use our orbit catalogue and identify objects on first infall (no change in sign) and those that have just completed first infall having passed pericentre (half an orbit). These are placed in the same category, the ‘infalling class’. This class also includes interlopers, haloes that are never part of the host group halo with radially outgoing velocities. The other class of objects are those that have completed at least one full orbit. We calculate the mean class of haloes at a given $D_p$ & $v_{\text{LOS}}$ to determine the orbital state.

The number density distribution (middle panel of Fig. 4) shows candidate tracers cover projected distances out to the virial radius. The LOS motion is centred on zero and extents out past the circular velocity threshold and even the escape velocity threshold. However, the distribution is peaked at low velocities with $\lesssim 5$ per cent of tracers on escape velocities with $D_p/R_{\text{vir}} \lesssim 1$, unlike the true value of $\sim 25$ per cent. The projected radial distribution is also more centrally concentrated that the true underlying distribution (see probability distribution in upper left-hand panel, comparing the thick solid purple line to the solid black line). Overall, the $v_{\text{LOS}}$ distribution lies primarily within the circular velocity envelope of a halo over a wide range of projected radii. Applying radial projection cuts can result in 3D radial distributions that differ from the true underlying distribution. An example of this is shown in the upper right-hand panel of Fig. 4, where we have selected haloes close to the virial radius in projected space (see thick dashed line in upper left-hand panel). Despite the distorted radial distribution, the velocity distribution within this projected radial cut is not appreciably different from that of haloes that are true radial distances of $\sim R_{\text{vir}}$ (solid black line compared to solid orange line in upper right-hand panel), nor from orbiting haloes that are within the virial radius (solid black line compared to dashed black line in upper right-hand panel).

The origin of the bias and the scatter seen in Fig. 2 is seen in the lower panel of Fig. 4, where we plot the orbital state at a given LOS phase-space position. At moderate projected radial distances and $v_{\text{LOS}}$, the halo population is dominated by objects on first infall and, as such, will not trace the halo’s phase-space distribution.

Figure 3. Motions of tracers. We show stacked radial, tangential, and total speed distribution as a function of distances for all haloes within 2.0 virial radii composed of $\gtrsim 50$ particles for which we have well defined evolutionary tracks. For each halo, we normalize the distance to group centre, $D_p$, by the virial radius $R_{\text{vir}}$ and the velocities by the maximum circular velocity, $V_{\text{max}}$. Outliers from the distribution are shown as small grey points. Dashed blue line shows the median escape velocity with the shaded region showing the variation in this limit from different halo concentrations, with the dark (light) showing the 1σ (2σ) contour. We also show the circular velocity by the dotted green line, with the associated shaded region giving the 1σ and 2σ contours.
Objects that have completed at least one orbit and are part of the host halo’s phase-space distribution and sample a wide range of velocities are concentrated to within $D_p \lesssim 0.2 R_{\text{vir}}$. Uniformly sampling this LOS phase-space means including both real tracers of the halo’s dispersion, i.e. subhaloes on bound orbits that have virialized, and interlopers, i.e. newly infalling and unassociated haloes. The result is the mild bias but large scatter seen in Fig. 2.

Ideal tracers reside in the crowded central region; however, sampling this region has several issues. One is simply observational: placing slits to measure spectra in crowded regions and interpreting results is not trivial. The other issue is that objects within small projected radii actually span a large range in radial distances and thus do not sample the same velocity distribution. An example of such a cut is shown in the upper left-hand panel of Fig. 4, where we show the true radial distribution of all haloes identified with $D_p \lesssim R_{\text{vir}}$ by a dashed black line. This selection has a significant fraction of haloes actually located at much larger radii. The observationally tractable outskirts contains a mix of infalling and orbiting haloes. Fortunately, in these outer regions, objects with high LOS motions are dominated by infalling haloes and those not associated with the central halo and only objects with $v_{\text{LOS}} \lesssim V_{\text{esc}}$ are a mix of orbital states. We argue that in order to minimize the dispersion in the true radial distances sampled and to minimize the contributions of interlopers (objects that have not even complete first pericentric passage) both a projected distance and LOS velocity cut needs to be applied.

We search for this optimal window in a grid of projected radial windows centred on some $D_p$ with a width $\Delta D_p$ and maximum LOS velocity threshold $v_{\text{LOS, max}}$ and identify the window with the best fitness. This fitness is defined as

$$F = \left(1 - \frac{|\mu_D - D_p|}{D_p}\right) \left(1 - \frac{\sigma_D - \Delta D_p}{\Delta D_p}\right) \times \left(1 - \frac{v_{\text{esc}}(D_p - \Delta D_p/2) - v_{\text{esc}}(D_p + \Delta D_p/2)}{v_{\text{esc}}(D_p)}\right) \times \frac{N_{\text{win}}}{N_{\text{int}}} \left(1 - \frac{N_{\text{int}}}{N_{\text{win}}}\right) \times \frac{N_{\text{win}}}{N_{\text{int}}} \left(\frac{N_{\text{int}}(N_{\text{int}} > 3)}{N_{\text{H}}}\right) \times (1 - |\delta \sigma_{\text{LOS}}|). \quad (2)$$

Here, the first two terms are associated with the 3-dimensional radial distribution resulting from the projected distance cut. Ideally, this distribution should be similar to the projected one, therefore having the same mean $\mu_D$ and width $\sigma_D$, and so we minimize the fractional difference. We also want to sample regions in which the velocity dispersion does not vary significantly with radius so that tracers probe similar velocity distributions, which is given by the third term. The set of terms relating to tracers given by the 3rd line in equation (2) maximises the number of orbiting objects, $N_{\text{int}}$, relative to the number of interlopers $N_{\text{win}}$ — that is objects that have not even had a pericentric passage — and minimize the number of interlopers relative to the number of objects in the window $N_{\text{win}}$. The next line maximises the number of tracers in the window and the number of haloes that have more than three objects within the window. Finally, we also minimize the halo-to-halo scatter in the LOS velocity dispersion measured using tracers, so as to produce a window that has little scatter in the difference between $\sigma_{\text{LOS}}$ and $\sigma_{v, H}$.

We find that the optimal window is centred on $D_p \approx 0.75 R_{\text{vir}}$ with a width of $\Delta D_p = 0.5 R_{\text{vir}}$, and $v_{\text{LOS, max}} = 1.0 V_{\text{max}} \approx 0.5 V_{\text{esc}}$. This window introduces a bias since it is dominated by objects with small LOS motions, underestimating the dispersion. This bias is seen in Fig. 5, where we have applied these cuts. The bias here is significant; $\sigma_{\text{LOS}}$ underestimates the true dispersion by $\sim 0.5$. We fit the distributions as seen in the inset with Gaussians and find ($\mu$, $\sigma$) = (0.51, 0.10) for halo dispersion of $\sigma_{v, H} \gtrsim 250 \text{ km s}^{-1}$, the bias is independent of halo dispersion (mass), although the scatter increases slightly with decreasing halo mass up to 0.2 for $0.0 \lesssim \sigma_{v, H} \lesssim 150 \text{ km s}^{-1}$ (in part due to decreasing numbers of satellites). The scatter in this ratio arising from variations in LOS is $\approx 0.07$. Applying a similar selection cut to our idealized realisations of haloes gives a bias of 0.55 ± 0.2. This indicates these cuts results in the same dispersion measurement one would expect for true tracers of the halo potential.
3.1 MW, M31, and LG

It is worth noting that the dispersions plotted in Fig. 2 (and the projected distance cuts used obtaining these dispersions) are with respect to a well defined barycentre, analogues to measurements made in spectroscopic surveys and not necessarily those made in estimating MW, M31, and LG dispersions. We discuss each of these caveats in turn, although, we argue that this bias also explains the low velocity dispersion measured by McConnachie (2012) for the MW, M31, and the LG systems (plotted in Figs 2 and 5) as the effective selection cuts used in McConnachie (2012) to measure the dispersion of the LG are similar to the one we propose here.

First, 3D radial cuts are used, not projected radial cuts. However, so long as subhaloes within a small projected distance are removed, the resulting 3D distribution is similar to that which would result from a 3D cut. For LG, dispersion reported in McConnachie (2012) uses satellites that are within 3D distance of $\approx 2R_{\text{vir}}$ (assuming a mass of $\approx 5 \times 10^{12} M_\odot$). Our optimal projected distance cut includes objects out to these distance. However, objects at these distances should be treated with caution as those with radial velocities $\approx 0.25 \times V_{\text{circ}}$ tend to be on first infall or not orbiting the host (see Fig. B1). Overall, these 3D radial cuts do not greatly affect the biased dispersion.

While M31 observations are more in keeping with LOS measurements presented in Fig. 5, the MW LOS velocities are nearly radial velocity wrt to the centre of MW. We find using radial velocities relative to the central halo instead of a uniform LOS introduces no significant bias. When using all objects within 3D distances of $\approx 1-2R_{\text{vir}}$, we find $\sigma_{\text{LOS}} = 0.92 \pm 0.25\sigma_R$, where $\sigma_R$ is the radial velocity dispersion. Placing a tighter radial cut does not change this relation significantly. Thus, the bias is present in MW observations using radial velocities.

For the LG system, the velocities are neither LOS velocities nor radial velocities wrt the barycentre. Observations only measure radial velocities of satellites wrt to MW and this must be correct to the LG barycentre frame. This correction is done by removing the velocity of MW relative to LG from velocities of each satellite and using the resulting velocity in the direction of the barycentre to calculate dispersions. Another caveat to consider is the fact that the LG system is not a single virialized halo, but an early stage merger.

Let’s address the issue of mergers first. For late-stage mergers, that is one where the largest subhalo of the host dark matter halo is $>0.5$ times mass of host, we find simply using an arbitrary LOS gives $\sigma_{\text{LOS}} = 0.58 \pm 0.17\sigma_{\text{circ}}$. For early stage mergers, that is one where two dark matter haloes are infalling but separated by $\approx 1.5R_{\text{vir}}$ and are of similar masses, $\sigma_{\text{LOS}} = 0.55 \pm 0.17\sigma_{\text{circ}}$ (assuming a dispersion based on the combined masses of the two merging haloes using equation 1). Thus, the relation holds reasonably well even for early stage mergers.

If we additionally mimic LG observations, i.e. using LOS velocities wrt to the primary (MW) in the direction of the barycentre (LG) while accounting for the motion of the primary towards the barycentre, the result is also biased. We find a barycentre dispersion to halo dispersion relation of $\sigma_{\text{bary}} = 0.44 \pm 0.14\sigma_{\text{circ}}$ as seen in Fig. 6. Although we do not have many merging systems at LG mass scales, the bias show no dependence on mass and the expectation is that the LG dispersion underestimates the merging system’s dispersion.

Though difficult to optimize the selection criteria for merging systems, we argue against using satellites with large positive velocities located $\approx 1.5R_{\text{vir}}$ away, like Tucana, as this portion of phase-space is dominated by unassociated objects (see orbital state plot in Fig. B1). If we remove all outward going objects with velocities of $V_{\text{LOS}} \approx (0.1V_{\text{MW}}\text{, escape},)$, which at $2R_{\text{vir}} \approx 42\text{ km s}^{-1}$ (assuming $V_{\text{MW}}\text{, circ}$ follows an NFW profile with a density concentration of $c \approx 10$, $R_{\text{vir}} \approx 500\text{ kpc}$ and a maximum circular velocity of $\approx 350\text{ km s}^{-1}$) and keep objects at distances $\approx 600\text{ kpc}$, we get a mean velocity of $\langle V_{\text{LG}} \rangle = -7 \pm 17\text{ km s}^{-1}$ and $\sigma_{\text{LG}} = 48 \pm 35\text{ km s}^{-1}$, giving a true halo dispersion of $\approx 100\text{ km s}^{-1}$.

4 DISCUSSION AND CONCLUSION

Using the SURFS suite that provides a sample of tens of thousands of (sub)haloes with accurate, well-tracked orbits, we have explored the relationship between a dark matter halo’s intrinsic, 3D velocity dispersion, which is a reliable proxy for its mass, and the LOS velocity dispersion deduced from subhaloes (satellite galaxies). In particular, we have analysed the key assumption, namely that neighbouring galaxies are satellite galaxies orbiting the central and are therefore reliable tracers of the underlying halo’s phase-space distribution. We showed that the average halo velocity dispersion is tightly correlated with halo mass, with a scatter of 7 per cent, but the same is not true when using the LOS motions of surrounding haloes within a three dimensional distance of $\lesssim 2R_{\text{vir}}$.

Comparing the LOS dispersion to the true halo dispersions, we find it underestimates the dispersion on average but has significant scatter. The large scatter is a result of the orbits sampled when uniformly sampling the projected phase-space populated by

3 This list includes Aquarius, SagDIR, UGC4879, LeoA, WLM, LeoT, PegDIR, and Cetus.
(sub)haloes surrounding a larger central halo (cf. Fig. 4). Much of the phase-space is dominated by interlopers or newly infalling haloes that are not tracers of the halo’s phase-space.

Our data allow us to identify the optimal window in projected distance and relative LOS velocity where dispersion measurements should be made to minimize the scatter in the σ_{LOS}−σ_v, H relation and keep any systematic bias mass independent. Applying a projected distance cut of 0.5 ≤ D/ R_vir ≤ 1.0, where R_vir is the virial radius and a LOS velocity cut of 0.5V esc, where V esc is the escape velocity, ensures that one has a significant fraction of orbiting subhaloes, some newly infalling objects and few interlopers. The resulting LOS dispersion σ_{LOS} is a mass-independent, biased estimate of the true σ_v, H with a small amount of scatter: σ_{LOS} = (0.5 ± 0.1)σ_v, H. The small scatter means that LOS dispersion measurements within this window can be trivially corrected to produce halo dispersion and hence virial halo masses.

Approximately 60 per cent of all haloes in our catalogue have at least three haloes within this window, with most haloes in our catalogue being low mass groups. The typical satellites had accretion masses of log M_{vir}/ M_⊙ = 10.9^{+0.5}_{−0.3} i.e. catalogues need to be complete to stellar masses of ∼10^8 M_⊙. For higher stellar mass cuts (M_* ≥ 10^9 M_⊙, M_{vir} ≥ 10^{11} M_⊙), the fraction of host haloes for which a dispersion can be measured within this optimal window drops significantly. Only 27 per cent of groups with M_{vir} ≥ 10^{13} M_⊙ have at least three satellites with masses ≥10^{11} M_⊙, though by M_{vir} ≥ 10^{13.5} M_⊙ this percentage increases to 82 per cent. Introducing a larger window naturally increases the haloes for which a σ_{LOS} can be measured but these should be flagged as low fidelity estimates.

Observational group catalogues, like Yang et al. (2006); Robotham et al. (2011), or cluster mass estimates, like those compared in Old et al. (2015, 2018), could be improved using this window, producing a high fidelity catalogue with mass uncertainties of ≤0.5 dex (similar to the uncertainty reported in Li et al. 2017, who used far more detailed modelling, albeit making numerous assumptions and requiring full 3D velocities). Groups and clusters with more than 10 members would have significantly reduced error bars and methods that use iterative cleaning to remove interlopers would benefit from using such a window.

This bias also naturally explains the results from McConnachie (2012), who effectively used similar cuts when estimating dispersions. Mimicking observations gives a bias of σ_{LOS} = (0.44 ± 0.14)σ_v, H. No longer is the LG unusually cold but instead lies comfortably within the 1σ scatter. Using our LG dispersion with the LOS correction or using the LG analogues correction, we predict a halo velocity dispersion of 95 ± 36 km s^{-1} instead of 26 ± 20 km s^{-1} (Elahi et al., in preparation), ORBWEAVER (Elahi et al., in preparation), TREEFOG (Elahi et al., in preparation), for which a dispersion can be measured within this window can be trivially corrected to produce halo dispersion and hence virial halo masses.

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**Facilities**

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**Software**

*Python, Matplotlib (Hunter 2007), Scipy (Jones et al. 2001), Scikit (Pedregosa et al. 2011), Gadget (Springel 2005), Velociraptor (Elahi et al. 2011, Elahi et al., in preparation), Treefrog (Elahi et al., in preparation), Orbweaver (Elahi et al., in preparation)."
APPENDIX A: ORBWEAVER

ORBWEAVER is part of the VELOCIRAPTOR tool-kit. It traces the relative motions of (sub)haloes around other haloes using the halo merger tree of TREEFROG combined with the halo catalogues of VELOCIRAPTOR. This python code (soon to be translated to c++ and make use of the MPI API) calculates a variety of orbital properties, from positions of apsides, ellipticity, orbital period, orbital angular momentum, etc. Orbiting haloes are traced forwards and backwards in time along the merger tree to identify changes in the sign of the radial velocity corresponding to pericentric and apocentric passages. First pericentric passage is defined as the first from negative radial velocities to positive radial velocities that occurs within $2R_{\text{vir}}$. We determine the apsides by linearly interpolating the orbiting halo’s relative position and velocities between the transition points. Due to the high cadence of our halo catalogue, linear interpolation is a reasonable approximation, though quadratic interpolation of positions is possible.

Figure B1. Orbital state. We show the typical orbital state (similar to Fig. 4) at a given radial, tangential and total speed distribution as a function of distances (similar to Fig. 3)

APPENDIX B: ORBITS IN 3D

Orbital state information as a function of 3D distance from halo centre.