Chapter 5

Sloshing performance in a two-dimensional tank under a combination of horizontal and vertical excitations

Abstract

Sloshing wave asymmetric about the tank centreline is generated in a two-dimensional tank under a combination of horizontal and vertical excitations with amplitude \( b_1 = 10 \text{ mm to 38 mm} \) by an increment of \( 1 \text{ mm} \) for the former and \( b_2 = 1 \text{ mm to 12 mm} \) by an increment of \( 1 \text{ mm} \) for the latter (totally 348 cases were covered). With these tests, the wave patterns inside the tank are classified into four regimes (A to D) and each regime includes two or more types of wave. Regime A is characterised by waves without breaking while regime B is for the gentle wave breaking and breaking with double plungers. Wave tripling breaking is the most striking phenomena in Regime C where four types of wave tripling breaking are identified. Within each type, there exist three wave modes of wave tripling breaking. The main difference among them is the shape of the wave crest. Mode one is characterised by a sharp crest in type 1 and type 2 while an inclining crest and a curving crest are found in types 3 and 4, respectively. As for mode two, from types 1 to 4, a flat crest with gentle double plungers, a sharp and thin crest, an inclining crest and a curving crest are identified, respectively. Mode three has a flat crest in types 1 to 3, while the crest in type 4 is oblique and inclines towards the right side. After applying phase portrait analysis, it is found that the wave crest of mode three is the lowest among the three modes. Hilbert-Huang Transform (HHT) is employed to investigate the wave tripling breaking and the results show that C8 resulting from the Ensemble Empirical Mode Decomposition
(EEMD) is a frequency modulation component for the wave period tripling breaking. Pressure variation during a wave period tripling breaking is studied. The impact pressure is found in mode one, which is due to flip-through and the impact point is higher than the still water level. Regime D includes the chaotic wave breaking without any apparent patterns and a regular breaking wave, termed as wave sextuple breaking, which repeats in every six wave periods.

5.1 Introduction

Sloshing is characterized by oscillation of unrestrained free surface in a tank under external forces. It is very important for several fields of engineering, such as fuel tanks in aircrafts and spaceships and vehicles\(^1\), and the transportation of Liquefied Natural Gas (LNG) in oil and gas industry, which have to operate in various filling levels. Violent liquid motion will be induced by sloshing if the tank is partially-filled, exerting localized impact pressure on the structural walls, which may cause serious structural damage of the tank. For a given tank, the performance of liquid motion depends on the external excitation conditions.

Sloshing has been studied quite extensively both numerically and experimentally due to its significance in engineering application. La Rocca \textit{et al.}\(^2\) investigated a multifluid system inside a sloshing tank under roll motion using experimental method and the Lagrangian variational formulation. They determined the modal damping coefficients by a mixed numerical-experimental technique. Antuono \textit{et al.}\(^3\) built a simple and robust sloshing model for shallow-water condition. They validated their model by experiments under sway and roll motions, respectively. They found that their model was capable of capturing the occurrence of wave breaking. Celebi and Akyildiz\(^4\) employed the volume of fluid technique to simulate a two-dimensional sloshing tank under rolling and moving motion. They found that their calculation became unstable due to the occurrence of turbulence. Kim\(^5\) applied the finite difference method to simulate the sloshing in the tank under a combined surge and sway motion. The calculated pressure showed good agreement with experimental data. However, some local nonlinear phenomenon was neglected in the simulation. Liu and Lin\(^6\) proposed a numerical model NEWTANK to study three-dimensional nonlinear liquid sloshing. They applied the model to simulate sloshing under multi-degree of freedom excitation and found that the model could capture broken free-surface and strong turbulence. Lugni \textit{et al.}\(^7\) adopted Particle Image Velocimetry (PIV) to examine the kinematic of the
flip-through in a two-dimensional sloshing tank by experiment under surge motion. Based on the findings by Lugni *et al.*\(^7\), the influence of air cavity in shallow-water sloshing in a depressurized wave tank was investigated under surge motion experimentally by Lugni *et al.*\(^8,9\). They stated that ullage pressure exerted effect on the air pocket. The sloshing studies based on horizontal motion and rolling are extensive. However, the investigation of sloshing performance under heave motion is not ubiquitous. Therefore, in this study, sloshing performance in a two-dimensional tank under a combination of horizontal and vertical motions is investigated.

Steep standing wave is an important phenomenon of wave motion inside a sloshing tank, which can be excited horizontally or vertically, when the excitation frequency approaches the natural frequency. The one generated by a vertical excitation is known as the ‘Faraday waves’. For a detailed review on Faraday wave, readers are referred to Miles and Henderson\(^10\). Jiang *et al.*\(^11\) studied the steep standing waves via oscillating a small tank vertically. The steep wave they generated was symmetric about the tank centreline. Wave shapes with dimple crest and temporal asymmetry were found. They also indicated that the dimple crest is attributed to the interaction between the first and the second temporal harmonics. Jiang *et al.*\(^12\) extended the work made by Jiang *et al.*\(^11\) by increasing the forcing amplitude from 3.5 mm to 4.7 mm. They reported the phenomenon called wave period tripling breaking, which means that three kinds of wave breaking (i.e. steep crest, flat crest with two plungers and round crest) appear alternatively. Longuet-Higgins and Drazen\(^13\) studied standing waves by considering waves reflected at a vertical wall and confirmed the existence of period tripling breaking. Bredmose *et al.*\(^14\) conducted experiments to generate standing wave and reported the flat crest and sharp crest phenomena. They also employed Boussinesq model in their simulation and reproduced the free-surface motion accurately. Besides experimental studies, a lot of numerical simulations have also been conducted to study the standing waves in a sloshing tank. Mercer and Roberts\(^15\) studied two-dimensional standing waves via a stable numerical method and concluded that the steep wave is unstable to the subharmonic perturbations. Longuet-Higgins and Dommermuth\(^16\) calculated the energetic period wave and highlighted the existence of standing wave profiles with rounded crest and sharp crests. As stated above, in previous studies, the standing wave is mainly excited by a horizontal or a vertical excitation. When the sloshing tank is excited by a combination of horizontal and vertical motions, which represents a more practical case encountered in engineering applications, the standing wave characteristics has not yet been investigated properly. Some phenomena, such as
the wave tripling breaking, initially reported for symmetric wave profile about the tank centreline under vertical excitation, has not been examined under combined horizontal and vertical excitations for asymmetric wave length. This forms the first aim of this study.

Apart from wave motion, the impact pressure is also important in quantifying the forces on the tank wall. Impact pressure is widely studied in past decades. Cooker and Peregrine\(^\text{17}\) employed pressure-impulse theory to analyse large pressure due to violent impacts. Peregrine\(^\text{18}\) reviewed the theoretical work of violent impacts of water waves on tank walls. He indicated that flip-through is a very interesting phenomenon in fluid dynamics of wave impact, which will produce a very high pressure on the tank wall. Lugni \textit{et al.}\(^\text{7}\) studied experimentally via PIV the high pressure of flip-through inside a sloshing tank and confirmed that the impact point of flip-through is higher than the still water level, which is in agreement with that reported by Hull and Muller\(^\text{19}\). Lugni \textit{et al.}\(^\text{8}\) further investigated the influence of air cavity on the pressure of wave impact and concluded that the pressure decay is affected by the air leakage from the cavity. Song \textit{et al.}\(^\text{20}\) conducted experiments to studied the pressure under flip-through and showed that pressure is influenced by trapping air pockets, and the impact point is at the still water level. The high impact pressure induced by flip-through is mainly investigated under horizontal excitation. Whether it occurs or not under the combined horizontal and vertical excitations is another aim of this study.

In this work, the experimental observations of standing steep waves generated in a tank with an aspect ratio of 13:1 under combined horizontal and vertical excitations are reported. The wave elevation is analysed using the recently developed Hilbert-Huang Transform (HHT). HHT is an empirically based data-analysis method. It is adaptive, which is helpful to produce physically meaningful representations of data from nonlinear and non-stationary processes\(^\text{21}\). Huang \textit{et al.}\(^\text{22}\) stated that HHT interprets wave nonlinearity as frequency modulation and the energy remains near the base frequencies. Obviously, in the experiment, especially after wave breaking, the wave characteristics are strongly nonlinear. Therefore, applying HHT could give a more robust interpretation to the complicated physical process.

The experimental setup and methodology are described in Sec. 5.2. Sec. 5.3 is about the experimental results and discussion, including wave without breaking, two types of wave breaking, four types of wave tripling breaking and chaotic wave breaking and wave sextuple breaking. Conclusions are given in the Sec. 5.4.
5.2 Experimental set-up

In the experiment, asymmetric steep standing waves (about the centreline of tank) inside the tank are generated by the coupled horizontal and vertical motions simultaneously.

5.2.1 Description of the experimental facilities and measurements

A rectangular tank is used with a breadth \( B = 100 \) mm, a longitude length \( L = 1300 \) mm and a height \( H = 900 \) mm, which is shown in Figure 5-1. The high aspect ratio \( (L/B = 13:1) \) ensures the waves inside the tank are two-dimensional. The tank is fixed on a hexapod which is capable of generating a vessel's motion in six degree of freedom (6DOF).

The sketch of the experiment is shown in Figure 5-2. The wave profile is recorded by a high-speed camera at a frequency of 240 fps (frame per second). Two wave gauges are used to detect the variation of wave elevations. Inside the tank, there are in total 4 places where the wave crest and trough will occur: left tank wall, right tank wall, approximately 434 mm and 867 mm away from the left side of the tank. In the experiment, the two wave gauges are fixed at 5 mm and 434 mm away from the left side wall of the tank, respectively. Two ultrasonic sensors are used to monitor the horizontal and vertical movements of the tank, respectively. Totally 11 pressure sensors are mounted at the left tank wall. Pressure sensor No.9 is at the still water level (SWL). Pressure sensors No.1 to 8 are located above No.9 with an interval of 15 mm, while pressure sensors No.10 to 11 are placed under No. 9 with the same interval. The sampling frequency for the wave gauges and the ultrasonic sensors is 1 kHz. In order to get the fast varying pressure, a sampling frequency of 10 kHz is employed for the pressure sensors. Prior to the experiment, all the pressure sensors were calibrated carefully.

As shown in Figure 5-3, a synchronization of the hexapod, other instruments including wave gauge and ultrasonic sensors and data acquisition system (DAQ) is built for the sloshing experiment. DAQ is composed of 6 parts including a chassis (U2781A) which is responsible for synchronization of other modules, four modules (U2541A) which are used for acquiring signal from the measurement and a module (U2351A) which is for signal generation. Each module (U2541A) contains simultaneous and differential channels, guaranteeing the synchronization when acquiring data.
5.2.2 Experiment methodology

The water depth in the tank is fixed at approximate 387.4mm. The third sloshing mode wave (one and a half wave length), which is asymmetric about the centreline of the tank, is generated. The driving signal is a coupled surge motion (horizontal) and heave motion (vertical) at the same time. The sinusoidal signal is applied to both horizontal (\( b_1 \sin(\omega_3 t) \)) and vertical motions (\( b_2 \sin(2\omega_3 t) \)). The amplitude of the horizontal movement \( b_1 \) is varied from 10 mm to 38 mm by an increment of 1 mm, while that of the vertical movement \( b_2 \) is from 1 mm to 12 mm by an increment of 1 mm. In total 348 cases have been covered in this study. The frequency of the horizontal excitation is identical to the third mode natural frequency. In order to produce the same wave shape as generated by horizontal excitation, the frequency of the vertical excitation must be twice of the natural frequency of the third mode\(^1\). The natural frequency of the third mode is calculated by the following equation\(^1\):

\[
\omega_n^3 = \pi n \left( \frac{g}{L} \right) \tanh(\pi n \frac{h}{L})
\]  

(5.1)

where \( n \) is the mode number of the internal sloshing, \( l \) is the tank length, \( g \) is acceleration gravity and \( h \) is the filling depth. The result is \( f_3 = \omega_3 / 2\pi = 1.33 \text{Hz} \). Figure 5-4 shows an example for the driving signal. For all experiments reported in this chapter, there are 320 horizontal cycles and 640 vertical cycles included simultaneously in the experiment, so 320 wave periods can be obtained.

5.3 Results and discussion

5.3.1 Wave classification

For the given water depth (387.4mm), asymmetric waves are generated by a combined horizontal and vertical excitations of various amplitudes at frequencies of \( \omega_3 \) and \( 2\omega_3 \), respectively. By examining the time series of the wave elevation measured by the wave gauges, the video recorded by the high speed camera and the pressure by the pressure sensors, wave characteristics, such as wave elevation, pressure on the side tank wall and wave crest shape, can be found for different excitation amplitudes. Based on these results, 4 main regimes, namely Regime A to D, can be classified and the results are summarised in Figure 5.
Regime A is characterised with the wave without breaking, including two wave types. The first type is featured with riding wave on the wave crest, which is represented by green hollow circles while the other is characterised by dimple crest, symbolled by magenta left-pointing filled triangles. The dimple crest is similar to the one reported by Jiang et al.\textsuperscript{11} The wave with riding wave occurs when $b_1 = 10 \text{ mm} \text{ to } 12 \text{ mm}$ and $b_2 = 1 \text{ mm}$, and $b_1 = 10 \text{ mm} \text{ to } 11 \text{ mm}$ and $b_2 = 2 \text{ mm}$. It seems that this wave type will occur only when both horizontal and vertical motions are small. Regime B is characterised with wave breaking but without the tripling wave breaking features. Two kinds of wave can be found in this category. The first one is the normal wave breaking (red crosses) and the second one is the wave breaking with double plungers (magenta right-pointing hollow triangles). The plungers are not always symmetric. In regime B, the normal wave takes up the most area. Most of the normal wave breaking occurs when $b_1 > 17 \text{ mm}$, while the wave with double plungers occurs when $b_1 < 20 \text{ mm}$. The cases in regime B are all when $b_2 < 7 \text{ mm}$. Regime C is for wave period tripling breaking, a phenomenon characterised with three kinds of wave breaking repeating every three wave cycles. There are in total four types of tripling wave breaking in the experiment. In general, with the increase of horizontal and vertical excitation amplitudes, wave tripling breaking type 1 (steep tripling breaking), type 2 (colliding tripling breaking), type 3 (inclining tripling breaking) and type 4 (curving tripling breaking) occur gradually. Type 2 is found to be the most common phenomenon in regime C. It is noticeable that there is a box enclosed by blue dashed lines located at the boundary between regimes B and C. Inside the box, three types of wave have been identified. At the right upper corner of the blue box, which is in regime C, two cases belonged to regime B are found. This result indicates that the blue box is a transitional regime. Regime D is characterised with chaotic wave breaking. In this regime, two types of wave are found. The first one is chaotic wave without apparent regular pattern (blue dot) and the other one is featured with wave sextuple breaking (the wave with six types of wave breaking repeating in every six wave cycles).

### 5.3.2 Regime A: two types of wave without breaking

#### 5.3.2.1 Wave with riding wave

In the experiments, wave with riding wave starts to occur when $b_1 = 10 \text{ mm}$ and $b_2 = 1 \text{ mm}$. As is shown in Figure 5-5, there are 5 cases characterised with riding waves, and all of them occur under $b_1 = 12 \text{ mm}$ and $b_2 = 2 \text{ mm}$.
The generation process of the riding wave is shown in Figure 5-6. In the time frame at $t = 0.336$ s, wave crest starts to be formed at $x = 434$ mm. From $t = 0.378$ s to 0.42 s, the riding wave indicated by black arrows can be seen. It becomes more apparent when $t = 0.504$ s. However, the riding wave is hidden in the main wave and cannot be seen clearly sometimes (e.g. at $t = 0.462$ s). It may be because when the main wave forms its peak, the riding wave forms its trough. The riding wave is submerged in the main wave. From $t = 0.546$ s to 0.588 s, when the main body wave descends, the riding wave is still discernible. The riding wave cannot be seen at $t = 0.630$ s. Comparing the riding wave at $t = 0.504$ s and $t = 0.546$ s, it is found that the size of the latter is larger, indicating that the riding wave moves upward when the main body wave moves downward.

In order to investigate the main wave and the riding wave in more details, the time series of the wave elevation measured at $x = 434$ mm is shown in Figure 5-7. The input signals for the tank movement are also shown in the figure for comparison. The deformation of the wave elevation around the trough, as indicated by black arrows, can be seen clearly. The phase of the elevation trough lags that of the corresponding horizontal tank movement. However, there is almost no phase lag between the crest of the horizontal tank movement and wave elevation. Referring back to Figure 5-6, when the main body wave descends from peak to trough at $x = 434$ mm, due to the influence of the riding wave, the position of the wave trough is not at $x = 434$ mm (Figure 5-6, at $t = 0.798$ s, indicated by a blue arrow), resulting in a higher elevation measured by wave gauge 2. When the wave is in the ascending process, the wave trough moves back to $x = 434$ mm without further deviation. This effect should account for the elevation deformation shown in Figure 5-7. It also should be noted that the deformation of the elevation trough is not the same for all periods. This is simply because the deviation between wave gauge position and the wave trough is not the same in every period. Comparing with the horizontal excitation, the vertical tank excitation is too small to influence this wave pattern.

The wave elevation displayed above can be further discussed based on HHT (Hilbert-Huang transform). HHT is an empirically based data-analysis method. It is adaptive, which is helpful to produce physically meaningful interpretations of data for nonlinear and non-stationary processes\textsuperscript{21}. For details of the HHT method, readers can refer to Huang \textit{et al.}\textsuperscript{23}. Briefly, for an arbitrary time series $x(t)$, its Hilbert transform is:

$$y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau,$$

(5.2)
where \( PV \) is the Cauchy principal value. With this definition, \( x(t) \) and \( y(t) \) form a complex conjugate pair, so an analytic signal can be obtained, \( z(t) \)

\[
z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)} \tag{5.3}
\]

\[
a(t) = \sqrt{x(t)^2 + y(t)^2}, \quad \text{and} \quad \theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right), \tag{5.4}
\]

where \( a(t) \) is the instantaneous amplitude, and \( \theta \) is the phase function, which is related with the instantaneous frequency via:

\[
f = \frac{d\theta}{dt}. \tag{5.5}
\]

However, it is difficult to define the instantaneous frequency for arbitrary data because for any function, to have a meaningful instantaneous frequency, the real part of its Fourier transform has to have only positive frequency\(^{24}\). Therefore, Huang et al.\(^{21,24}\) introduced the Empirical Mode Decomposition (EMD) method to deal with data from nonstationary and nonlinear process. The intrinsic mode function (IMF) can be obtained by applying EMD to the original data. Each of the IMF satisfies the following two conditions:

- The number of extreme and zero-crossings must either equal or differ at most by one.
- The mean value of the envelope defined by local maxima and minima is zero.

After getting IMFs, the Hilbert transform will be applied to each IMF to get the amplitude-frequency-time distribution. However, EMD sometimes will lead to mode mixing, which is defined as a single Intrinsic Mode Function (IMF) either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components\(^{25}\). So Huang et al.\(^{25}\) introduced the Ensemble Empirical Mode Decomposition (EEMD) method to overcome the deficiency. The first step of EEMD is to add a white noise to the original data, and then decompose the new data into IMFs. The final step is to obtain the ensemble means of the corresponding IMFs as the result.

The EEMD result of the wave elevation from wave periods 200 to 250 has been decomposed into 11 components. The first 6 components are for high frequency noise and the components C9-C11 are too small to make any significant contribution and therefore will not be discussed here. C7 is the uniform component for the main wave whereas C8 is likely the component with regular variations. Figure 5-8 compares the time series of these two components. C8 actually performs with a repeatability in every twelve wave cycles. Three periods, as marked by I, II and III and divided by blue
dashed lines in Figure 5-8, are shown as examples. Each period is further divided into 4 parts, marked by circled 1, 2, 3, and 4, which last for 4, 3, 2, and 3 wave cycles, respectively. Although the lasting duration is different for different parts, the general shape is similar. Each part is characterized with two high peaks and one trough. For I2, I4 and III4, three peaks can be found in each duration, but the middle one is much lower than its neighbours. Therefore, it can be considered as a trough. It is noted that, strictly speaking, C8 is not perfectly periodical because the patterns of the signals over different periods are not exactly the same. However, the general shape of them are similar, and the lasting duration is the same. For instance, comparing with I2 and II2, there is a small peak in the middle of I2 while no peak is found in II2. Nevertheless, generally, based on the analysis above, it is concluded that the wave with riding wave is periodical instead of random.

5.3.2.2 Wave with dimple crest

In regime A, another type of wave without breaking is the one with dimple crest, as indicated by black arrows when t = 0.462 s and 0.504 s in Figure 5-9. It starts to appear when \( b_1 \) is increased to 13 mm. Figure 5-9 shows 8 successive time frames in one wave period with dimple crest. The water from both sides of wave gauge 2 pushes up the round crest (at t = 0.420 s, marked by two red arrows in Figure 5-9). However, as the speed of the water from both sides is larger than that of the wave at x = 434 mm, it overtakes the upward round crest, resulting in the formation of the dimple crest at t = 0.462 s (Figure 5-9). After the formation of the dimple crest, the water at both sides (at t = 0.504 s, marked by two green arrows) starts to move back to the right and left sides, forming a flat and wide crest (at t = 0.588 s, Figure 5-9).

Figure 5-10 shows the times series of wave elevation measured at x = 434 mm for wave periods from 271 to 279 when \( b_1 = 14 \) mm and \( b_2 = 3 \) mm. It is found that the wave elevation lags the horizontal tank movement. Taking the first wave period in Figure 5-10 as an example, arrow 1 indicates the peak of the wave elevation, which occurs before the horizontal excitation signal reaches its peak. The corresponding moment can be seen in Figure 5-9c (indicated by green arrow). When the tank arrives at the most left position (the peak of the horizontal excitation signal), the elevation of the dimple crest (indicated by arrow 2 in Figure 5-10) becomes lower. The corresponding moment can be seen in Figure 5-9e (indicated by black arrow).

To investigate some more details of the wave with dimple crest, HHT is applied to the wave elevation data measured by wave gauge 2 under \( b_1 = 14 \) mm and \( b_2 = 3 \) mm. It
is found that the component C7 is the main one and the time series of C7, together with its instantaneous frequency are shown in Figure 5-11.

The frequency in one wave period (the area marked by two black vertical dash lines) is illustrated. Before forming the dimple crest, the wave shape is round (at \( t = 0.378 \) s in Figure 5-9 and marked by a black arrow). By checking the video, the round wave rises with an increasing acceleration at the beginning of the wave cycle and a decreasing acceleration in the later part of the wave cycle, resulting in the increase of instantaneous frequency between the time marked by the first vertical line and the red arrow and the decrease of instantaneous frequency between the time marked by the red and black arrows. After the moment marked by the black arrow, there is a fluctuation for instantaneous frequency. It is found that after the formation of dimple crest, the water at both sides of the dimple crest \((x = 434\text{mm})\) moves downward quickly so that the water under dimple crest also decreases quickly. The dimple crest undergoes a sudden drop, so the frequency increases again. However, this sudden drop just lasts for a short period of time because the water at \( x = 434 \) mm meets the main water body soon, giving rise to the decrease of the frequency again.

5.3.3 Regime B: breaking waves

In regime B, there are two types of breaking waves. One is the wave breaking with double plungers and the other is the normal wave breaking. It can be seen from Figure 5-5 that the normal wave breaking takes up the majority of this regime and occurs for \( b_1 \geq 17 \) mm. The wave breaking with double plungers happens when \( b_1 < = 20 \) mm. Some cases can also be found at the lower left corner of regime B. Both of them are found when \( b_2 < 7 \) mm.

5.3.3.1 Wave breaking with double plungers

The wave breaking with double plungers (indicated by two black arrows) is shown in Figure 5-12a. It is noticeable that the double plungers are asymmetric. When the plungers are formed, the water coming from the right side of wave gauge 2 is faster and more energetic than that from the left side.

In the descending process of wave elevation, there is a small deformation, which is indicated by a blue arrow. When the wave descends to certain position where the deformation starts, it is deviated from wave gauge 2. As the wave gauge is not exact at the trough of the wave, the elevation measured by the wave gauge is higher than the
trough. During the moment marked by the red arrow, the wave trough starts to move back to location of wave gauge 2. Resulting in the disappearance of the deformation.

HHT analysis to the wave elevation time series shown in Figure 5-12b has been conducted to investigate more details of the breaking wave with double plungers. As explained previously, C7 is the uniform and main component. Figure 5-13 shows the time series of C7 and its instantaneous frequency over the same wave periods with the data shown in Figure 5-12b. It can be seen that the instantaneous frequency is in the range from 0.38 Hz to 1.85 Hz. Taking one wave period, marked by two black vertical dashed lines, as an example, the instantaneous frequency increases with wave elevation till the moment marked by the black arrow, from around 1.5 Hz and peaks at 1.85 Hz. This is similar to that shown in Figure 5-11 (the time from first vertical line to the red arrow). However, there is no fluctuation in the descending process of the instantaneous frequency compared with that in Figure 5-11. After formation of the double plungers (two black arrows in Figure 5-12a), the water at these two plungers is almost in free fall condition, which exerts no influence on the water in the dent (blue arrow in Figure 5-12a). The water in the dent experiences normal descending process. The main water body underneath the dent slows down the falling acceleration. Therefore, the instantaneous frequency decreases without fluctuation.

5.3.3.2 Normal wave breaking

This wave pattern occurs when \( b_1 < 32 \text{ mm} \) and \( b_2 < 5 \text{ mm} \). Images about wave breaking pattern and the time series of the wave elevation and tank movement are shown in Figure 5-14. It can be seen that wave starts to break when it reaches its peak (Figure 5-14a). Correspondingly, there is apparent deformation on the measured wave elevation around the crest. After wave reaches its peak, the tank is still on its way to the largest movement location. As the vertical excitation amplitude is much smaller than the horizontal one, in this case the horizontal excitation is in dominated position. Compared with the results shown in Figure 5-12, it is concluded that the normal wave breaking pattern is likely to happen when the horizontal excitation amplitude is much larger than the vertical one.

In Figure 5-14b, it can also be seen that in the descending process of wave elevation, there is a slight deformation as indicated by a green arrow. The reason is the same as the one accounting for the deformation occurring in Figure 5-12.

The EEMD result for the component C7 and its instantaneous frequency are shown in Figure 5-15 to examine the details of the breaking waves. Similar to the instantaneous
frequency shown in Figure 5-13, the frequency in Figure 5-15 is periodical as well. However, there is one evident difference. During the descending process of the frequency, a small peak is found, as marked by a red arrow. The occurrence of the peak is due to the existence of normal wave breaking. It can be seen that from the time marked by a green arrow, which is also the moment of wave breaking, the frequency starts to increase. The existence of wave breaking makes the wave to accelerate, resulting in the increasing of the frequency. The acceleration process of wave lasts for a short time and then stops as the disappearance of the effect of the wave breaking.

5.3.4 Regime C: wave tripling breaking

With the increase of $b_1$ and $b_2$, regime C (Figure 5) is obtained, which is featured as four different types of wave period tripling breaking. Period tripling breaking is a phenomenon with three types of wave breaking which repeats every three wave cycles. One of them is similar to the finding reported by Jiang et al. There are three wave modes in each type of wave tripling breaking. The shape of the wave modes varies in different types of tripling breaking, which will be elaborated in the following parts.

It is noticed that, in Figure 5, there is a blue box including part of regime C and part of regime B. We define it as a transitional regime, for which, two cases belong to regime B as marked by a small black box.

5.3.4.1 Type 1-steep wave tripling breaking

5.3.4.1.1 Observations of type 1 period tripling breaking

In the wave regime map (Figure 5-5), the wave tripling breaking type 1 is marked by filled upward black triangles. The main features of this type include an energetic and steep wave crest in mode one and flat crest in mode two. The full cycle of the three modes (from trough to trough measured at wave gauge 2) of wave tripling breaking when $b_1 = 18$ mm and $b_2 = 8$ mm are shown in Figure 5-17 - Figure 5-19. From the description in Sec. 5.2.1, it can be seen that, for the current excitation frequency, the wave crest and trough will occur at $x = 0$, 434, 867 and 1300 mm. From Figure 5-17 - Figure 5-19, it is known that the wave crests in the same mode are similar at $x = 0$ and 867, and at $x = 434$ and 1300 mm. Figure 5-16 shows a sketch of the sequence of occurrence of wave breaking modes at $x = 434$ mm and 867 mm. If mode one appears at $x = 434$ mm (at $t = 0.378$ s, Figure 5-17), then the next wave crest occurring at $x = 867$ mm belongs to mode three (at $t = 0.798$ s, Figure 5-18), followed by mode two appearing at $x = 434$ mm (at $t = 1.092$ s, Figure 5-18). Mode one, mode three and mode
two will appear at $x = 867$ mm (at $t = 1.512$ s, Figure 5-19), $434$ mm (at $t = 1.848$ s, Figure 5-19) and $867$ mm (at $t = 0$ s, Figure 5-17) sequentially.

Figure 5-17 shows the formation of mode one at $x = 434$ mm. At $t = 0.294$ s, a sharp crest is formed and it becomes thinner and higher at $t = 0.336$ s. Heavy wave breaking is found at $t = 0.378$ s. The breaking thin crest falls to the main water body, creating a dent as marked by a green arrow at $t = 0.630$ s and a rebounding jet as marked by a black arrow at $t = 0.714$ s. This rebounding jet can be seen clearly at the beginning in mode two (Figure 5-18) at $t = 0.756$ s and $t = 0.798$ s (marked by black arrows). Flat top can be found at $t = 0.966$ s. However, gentle double plungers (marked by two black arrows at $t = 1.050$ s) can be seen from $t = 1.008$ s to $t = 1.092$ s, which is caused by the rebounding jet. In the experiment, the rebounding jet bounces up and down. From $t = 0.798$ s, the jet starts moving down, but the water coming from left and right sides of the tank would create a crest at the location of the rebounding jet, which means that the moving direction of the main water body is upward. When the downward moving rebounding jet meets the upward moving water, the jet splits the upward water, resulting in the gentle double plungers. Figure 5-19 shows the formation of mode three, featured by a flat crest as seen from $t = 1.806$ s to $t = 1.890$ s. This may be induced by the period tripling breaking at the neighbouring crest (at around $x = 867$ mm). Based on the pattern shown in Figure 5-16, when mode three happens at $x = 434$ mm, the previous crest mode in the tank is mode one at $x = 0$ and $867$ mm. Mode one is featured by a high and sharp crest with heavy breaking, which would cause high energy dissipation. The remaining energy in the water is not large enough to push the water to form another high crest wave at $x = 434$ mm.

From the description above, it is noticed that for the first type of wave period tripling breaking, mode two and mode three are similar, both of which have flat crests. The only difference is that mode two has gentle double plungers.

Figure 5-20 and Figure 5-21 show the time series of wave elevation measured by wave gauges 1 (at $x = 0$ mm) and 2 (at $x = 434$ mm), respectively. The numbers marked next to the wave crests represent the modes mentioned above. From the analysis above, it can be known that the elevation in mode one is the highest among the three. Therefore, in Figure 5-20b and Figure 5-21b, the highest elevations should correspond to mode one (Figure 5-17), as marked in the figure. After mode one is determined, the other two modes can be defined correspondingly. From both Figure 5-20 and Figure 5-21, it can be seen that the wave elevations change periodically as the sequence of modes one, two and three. It is noticed that the elevations for modes two and three are
quite comparable. This is simply because the wave crest patterns of modes two and three are similar. In Figure 5-20, because of the reflection of the tank wall, the wave elevations are higher than that in Figure 5-21.

5.3.4.1.2 Phase analysis of period tripling breaking

In order to get a better understanding of period tripling breaking, pseudo-phase-space plot (e.g. Moon\textsuperscript{26}) for wave elevation at \(x = 0\) mm and \(x = 434\) mm are analysed. The three-dimensional pseudo-phase-space plot can be constructed by:

\[
(\eta(t), \eta(t + \Delta t), \eta(t + 2\Delta t))
\]

where \(\eta(t)\) is the measured wave elevation sampled with a frequency of 1000 Hz at \(x = 0\) and 434 mm (wave gauge 1 and 2), \(\Delta t\) is the time delay. Choosing different time delay \(\Delta t\) only affects the shape of orbits shown in Figure 5-22. Here \(\Delta t = 0.1s\) is used to construct the pseudo-phase plot so that the orbits are with clear trajectory.

Figure 5-22a and Figure 5-22c show the pseudo-phase plots for wave elevation displayed in Figure 5-20a and Figure 5-21a. Figure 5-22b and Figure 5-22d are the corresponding Poincare maps when \(\eta(t) = 0\). There are three orbits in Figure 5-22a and Figure 5-22c, standing for the three breaking modes aforementioned. The colors of black, blue and red shown in Figure 5-22 are defined for modes one, two and three based on \(\eta(t)\), respectively. In Figure 5-22a, the black orbit (mode one) is easy to be distinguished, but the other two orbits are mixed with each other. This is because the wave elevation in modes two and three are almost identical. From Figure 5-22b, it can be seen that three clusters are gathering in a small region in the upper right corner and lower left corner, representing three periodical modes. The black dots are obviously higher than the other two colored dots. From Figure 5-22c, the inner orbit is red (mode three), which means that mode three is with the lowest wave elevation. The black orbit is at the outer location, indicating that mode one always has the highest elevation. The blue orbit is mainly between the red and black orbits, but it is also mixed a small part with the red one, indicating that although wave crest in mode two is similar to that in mode three, most of elevations in mode two are higher than those in mode three. From the Poincare map in Figure 5-22d, it also can be known that at the upper right corner, the black clusters are located at the outer space while most of the red clusters are located at the inner space.

It is noticeable that the width of the black orbits in Figure 5-22a and Figure 5-22c is much wider than the others. The width of the orbit represents the magnitude of the
range of wave crest distribution. If the elevation of wave crests vary dramatically in elevation, then the width would be wide, like the black orbits in Figure 5-22a.

5.3.4.1.3 HHT analysis of period tripling breaking

In order to look into the variation of the process of the first type period tripling breaking, HHT analysis is applied to the wave elevation measured at wave gauge 2 (x = 434 mm).

After obtaining the EEMD results of wave elevation data shown in Figure 5-21a, Fast Fourier Transform (FFT) is applied to C7 - C9. The results are shown in Figure 5-23. It is seen clearly that in (a), the peak frequency of C7 is \( f = 1.33 \) Hz which is the same as the excitation frequency. Therefore, it is the most energetic component from the wave elevation at x = 434 mm. In (b), both the excitation frequency (\( f \)) and a modulation frequency (\( 2f/3 \)) are found. They are apparent and comparable. Apart from the two frequencies, another frequency (\( f/3 \)) is also found, but it is much smaller than the other two. Therefore, it is believed that C8 is the frequency modulation component. In (c), only one frequency (\( f/3 \)) is found, indicating that the C9 is a uniform component.

To examine the details of the wave tripling breaking type 1, Figure 5-24 shows part of the time series of the wave elevation shown in Figure 5-21a, time series of C8 (frequency modulation component) from the EEMD result for the wave elevation and its instantaneous frequency.

The numbers marked next to the wave elevation crests represent the period tripling breaking mode. It can be seen clearly that the instantaneous frequency of C8 is periodic with a period the same as the wave tripling breaking. Therefore, it is considered that C8 is a frequency modulation component for the wave period tripling breaking.

The instantaneous frequency in the first duration (from the beginning to the first vertical dash line) will be illustrated hereinafter. It increases for the whole duration of mode one, then followed by a descending process. It reaches the lowest when the wave goes to the peak of mode three. As the wave crest in mode one is the most energetic among the three modes, the wave varying speed is fast, which explains why the instantaneous frequency increases in mode one. In modes two, energy dissipates significantly due to the collision between the rebounding jet and the upward water. In mode three, the heavy energy dissipation is induced by the high and sharp crest in mode one at the neighbouring location (x = 867 mm). As a result, wave varies slowly and the instantaneous frequency decreases. After the peak of mode three, the instantaneous
frequency increases again. This is induced by the accumulating energy from the external excitation and the wave varies fast again.

5.3.4.2 Type 2-colliding wave tripling breaking

5.3.4.2.1 Observations of period tripling breaking

With the increase of \( b_1 \) and \( b_2 \), wave period tripling breaking type 2 (the green squares in Figure 5-5) occurs, which also involves three modes. Figure 5-25 - Figure 5-27 show one full cycle (from trough to trough at wave gauge 2) of the three modes under \( b_1 = 21 \) mm and \( b_2 = 8 \) mm.

Figure 5-25 shows the processes of mode one of tripling breaking type 2 at \( x = 434 \) mm. It is similar to mode one in type 1. From \( t = 0.210 \) s, it can be seen that a sharp crest starts to be formed. The crest in mode one is still the highest among the three. Compared with the one in type 1, crest with heavier breaking in mode one is found in type 2 (at \( t = 0.336 \) s). It is noticed that the sharp crest inclines a little towards to left side. According to the observations, the inclining direction is stochastic. It can be seen clearly that at \( t = 0.294 \) s, the wave elevation measured by wave gauge 2 is not the highest value due to the slight deviation from the wave peak to the location of the wave gauge. After wave breaking starts at \( t = 0.336 \) s, the process of wave motion is almost the same as those described in type 1. The rebounding jet is created as well (at \( t = 0.714 \) s). However, the wave shape of mode 2 in type 2 is totally different from that in type 1. At \( t = 0.996 \) s in Figure 5-26, a dent is created (green arrow) due to the interaction between the rebounding jet and the water coming from left and right sides (marked by two black arrows). This process looks like the one featured with double plungers in type 1. However, from \( t = 1.008 \) s to \( t = 1.092 \) s, instead of forming double plungers, a thin and breaking wave crest occurs. This is because the water from left and right sides is much stronger than the rebounding jet and at \( t = 0.966 \) s, the water at both sides of the dent is still in the upward process. They overwhelm the dent and collide with each other, forming a breaking wave crest. Figure 5-27 shows the images at different instants of mode three. Due to the thin wave crest created in mode two, a rebounding jet is found again starting at \( t = 1.470 \) s in mode two (Figure 5-26) and becoming more apparent at \( t = 1.512 \) s (black arrow). However, the water in mode three is not as energetic as that in mode two and one. As a result, even when the rebounding jet meets with the water coming from both sides, no plungers or sharp wave crests occur. Instead, a flat crest appears in mode three of type 2, which is the same as that in type 1.
Figure 5-28 and Figure 5-29 show the time series of wave elevation measured by wave gauges 1 (x = 0 mm) and 2 (x = 434 mm). The numbers marked next to the wave crest in Figure 5-28b and Figure 5-29b represent the period tripling breaking modes. From the analysis above, it can be known that the elevation for mode three is the lowest among the three modes. Therefore, the lowest elevations should correspond to mode three. From both Figure 5-28 and Figure 5-29, it can be seen that the elevations from modes one and two are comparable due to the sharp crests occurring in these two modes.

5.3.4.2.2 Phase analysis of period tripling breaking

Figure 5-30 shows the pseudo-phase plot and Poincare map using the wave elevation data shown in Figure 5-28a and Figure 5-29a. The color and orbit definitions are the same as those in type 1.

In Figure 5-30a, three orbits can be easily distinguished. The inner red orbit stands for mode three with the lowest wave elevation. The outer orbit is black for mode one. However, a small part of black orbits duplicates the blue ones, meaning that parts of the wave elevation in mode one are identical to those in mode two. From Figure 5-30b, the red dots are obviously lower than the other two colored dots. In Figure 5-30c, the inner orbit is also red (mode three). However, the blue orbits are almost mixed with the black ones and some of the blue ones are even at outer location. This result means that the elevations in mode two are equal or larger than that in mode one at wave gauge 2. In Figure 5-30d, similar conclusion can be obtained. In the upper right corner, the blue clusters are located at the outer space while most of the red clusters are located at the inner space.

From the analysis above, it can be found that at wave gauge 1, the elevation of mode one is the highest while at wave gauge 2, the elevation of mode two is the highest. However, according to the description above, it is known that wave crest in mode one is the highest. The reason is that at wave gauge 2, the wave crest for mode one is not always vertical, such as the one shown in Figure 5-25. It has an inclined crest, resulting in the uncertainties in the measurements. Therefore, the measured values for the wave crest from the wave gauge 2 for mode one is lower than its actual value, resulting in some blue orbits (mode two) in Figure 5-30c being at the outer location.

5.3.4.2.3 HHT analysis of period tripling breaking

In order to look into the details of the process of the second type of period tripling breaking, HHT analysis is applied to the wave elevation measured by wave gauge 2 (x =
The time series of the wave elevation in Figure 5-29a are chosen. The component C8 (frequency modulation component) from this analysis (EEMD results for the wave elevation in Figure 5-29a) is shown in Figure 5-31, together with the wave elevation and the instantaneous frequency of C8.

Again, the numbers marked next to the wave crests represent the period tripling breaking modes, which is the same as those in Figure 5-29b. The variation of instantaneous frequency of C8 is similar to the one in Figure 5-24. The instantaneous frequency of C8 is periodic with a period being the same as the wave tripling breaking (from the beginning to the first vertical dashed line). Therefore, C8 is considered as a frequency modulation component for the wave period tripling breaking.

The instantaneous frequency in period I (from the beginning to the first vertical dashed line) is illustrated. It increases for the whole duration of mode one and then followed by a descending process. Compared with the descending process of the instantaneous frequency during mode two in Figure 5-24, it can be seen that the decreasing trend is a little milder in Figure 5-31 than that in Figure 5-24. It may be because water coming from the left and right sides of the rebounding jet (Figure 5-26, marked by two black arrows) is stronger than that in Figure 5-18 (marked by two black arrows). The instantaneous frequency will go to the lowest when the wave just goes to the peak of mode three.

The crest in mode one is the most energetic among the three and the wave varying speed is fast, which explains why the instantaneous frequency keeps increasing in this mode. In mode two, even though the water coming from the left and right sides of the rebounding jet is stronger than that in type 1, this is not enough to change the wave varying speed. In mode three, energy dissipates due to the rebounding jet and the neighbouring modes, which is similar to that in type 1. The wave varies slowly as well and the instantaneous frequency decreases. The instantaneous frequency increases again
slightly earlier than the wave reaches the peak of mode three. It is induced by the accumulated energy from the outside excitation. After this, the wave varies fast again.

5.3.4.2.4 Pressure variation on the tank side wall during wave period tripling breaking

When the wave period tripling breaking occurs, pressure on the tank side wall varies significantly. In total 11 pressure sensors are mounted on the side wall. Sensors 1-8 are higher than the still water level. No. 9 is at the still water level while No.10 and No. 11 are lower than the still water.

Figure 5-32 is the boxplot of the recorded data from all the 11 pressure sensors for the case when $b_1 = 21$ mm and $b_2 = 8$ mm. The enlargement at the bottom of Figure 5-32 shows the definitions of the boxplot. The upper and lower limits of the blue box indicate the third quartile (Q3, 75% percentile data) and first quartile (Q1, 25% percentile data), respectively. The central line (red) is the 50% percentile (the median). The interquartile range (IQR) means the difference between the values of the third quartile (Q3) and the first quartile (Q1). The red crosses, representing the values which are not in the range between Q1-1.5IQR and Q3+1.5IQR, are called the outliers. The upper and lower whiskers indicate the maximum and minimum of the non-outliers, respectively.

For pressure sensors No.1 to No. 9, it can be seen that the upper whisker, the third quartile and the median of the box are all around zero, indicating that the pressure sensors emerge from water for more than a half time (e.g. the time marked by the red arrows in Figure 5-33, P5). The upper whiskers for all the sensors are under 1 kpa. A lot of outliers can be found from sensor 1 to 8, indicating the occurrence of water impacting which can result in much higher pressure than the normal hydrostatic pressure. The impact pressure detected by pressure sensors No.4 and No.5 are much higher than others. It can therefore be concluded that the impact pressure during wave tripling breaking type 2 occurs around the location of pressure sensors No.4 and No.5.

Figure 5-33 shows the time history of the pressure variation for the three modes of wave period tripling breaking type 2. Only pressure sensors No. 1 to No. 8 are shown here because sensors No. 9 to No. 11 are placed under the still water level and the impact pressure is less likely to be detected. For all the 8 measured pressure signals in Figure 5-33, hydrostatic pressure is the main source for wave tripling breaking modes two and three, as reflected by the slight increase of pressure measured by P5-P8. For pressure sensors No.1 to No.3, the magnitude of the measured pressure is almost zero,
indicating that the water may not hit these sensors. For pressure sensors No. 4 to No. 8, whose mounting locations are deeper, it can be seen that the pressure during modes two and three becomes larger gradually. This also can prove that the pressure in modes two and three are mainly from the hydrostatic pressure. As for the pressure during mode one, a very high pressure peak in P5 (marked by a black arrow) is found. Peaks with much smaller magnitudes can also be found at the same time on P4 and P6 (marked by black arrows). This result indicates that the pressure by P4-P6 is dominated by the impact pressure instead of the hydrostatic pressure and the impact point is around pressure sensor No. 5.

In order to illustrate the high peak pressure in mode one measured at P5 as shown in Figure 5-33, the time frames of mode one are shown in Figure 5-34. The green short bars on the left side of each picture indicate the locations of the pressure sensors. From t = 8.4 ms to 67.2 ms, a wave crest (marked by a blue arrow, t = 8.4 ms) will merge with the wave trough (marked by red arrow, t = 8.4 ms). During this period, the wave crest approaches to the tank wall while the wave trough moves upward. At t = 67.2 ms, the approaching wave crest and the upward wave trough merge together (marked by a black arrow). Then the merged water continues going upward along the tank wall. Finally, a vertical jet is (marked by a black arrow) formed at t = 201.6 ms. All the features of the wave movement described above indicates that it is flip-through, which can exert high pressure on the side wall. The location of the largest pressure (pressure sensor No.5, shown in Figure 5-33) on the wall due to flip-through is higher than the still water level (pressure sensor No.9, shown in Figure 5-2), which is the same as the findings by Lugni et al. and Hull and Muller. It needs to be noted that the flip-through and hence pressure peak occur quite randomly and will not happen in all mode one.

5.3.4.3 Type 3-inclining wave tripling breaking

5.3.4.3.1 Observations of the period tripling breaking

With a further increase of the horizontal and vertical excitation amplitudes to $b_1 = 27$ mm and $b_2 = 9$ mm, wave period tripling breaking type 3 will occur (Figure 5-5), which is featured by inclining wave crests in modes one and two. Figure 5-35 - Figure 5-37 show one full cycle of the three modes from trough to trough occurring at wave gauge 2 (x = 434 mm). In mode one of tripling breaking type 3 (Figure 5-35), an inclining wave crest with gentle breaking at t = 0.336 s can be seen clearly. This is different from that of mode one in type 2, which shows heavy wave breaking and smaller inclining angle (Figure 5-25, t = 0.336 s). After the crest falls into the main
water body (from $t = 0.504$ s to 0.714 s), there is no rebounding jet at $x = 434$ mm, proving again that the crest of mode one in type 3 is not as energetic as the ones in types 1 and 2. In Figure 5-36, a heavily breaking turning crest can be found from $t = 1.050$ s (marked by a black arrow) to 1.134 s (mode two). Compared with mode two in type 2, due to the absence of the rebounding jet in type 3 after mode one, the water coming from left and right sides of wave gauge 2 meets together heavily and pushes the water upward. It is the asymmetry about the wave crest found at $t = 0.966$ s which results in the turning wave crest in mode two of type 3. Obviously, the wave crest of mode two in type 3 is more energetic than that in type 2. At the end of mode two, a rebounding jet at $t = 1.470$ s as marked by a green arrow is found. This rebounding jet plays an important role in the formation of mode three in type 3 as shown in Figure 5-37, which is featured by a flat crest with the lowest wave elevation among the three modes. Based on the pattern shown in Figure 5-16, before the occurrence of mode three at $x = 434$ mm, it is mode one at $x = 0$ and 867 mm. After the high crest in mode one falls into the water, high energy dissipation happens. Then when forming the crest in mode three, there is not enough energy forming a high crest. So a flat and low crest is formed.

Figure 5-38 and Figure 5-39 show the time series of wave elevation measured by wave gauges 1 ($x = 0$ mm) and 2 ($x = 434$ mm). From the time frames shown in Figure 5-35 - Figure 5-37, it is known that the wave crest for mode three is the lowest among the three. Therefore, in Figure 5-38b and Figure 5-39b, the lowest wave crest should be for mode three (Figure 5-37). Correspondingly, modes one and two can be determined on the figures.

In Figure 5-38b, it is seen clearly that the wave elevation in mode two is higher than that in mode one, indicating that the wave crest in mode two is more energetic. In Figure 5-39b, similar to the condition in Figure 5-38b, the wave crest in mode two is the highest. It is noticed that the elevation of the wave crest in mode one is close to that in mode three. However, the former should be higher than the latter. Due to the inclining crest (Figure 5-35, $t = 0.336$ s) in mode one, the elevation measured by the wave gauge is lower than its crest. It is also found that there is a small peak in the wave trough (marked by blue arrows), which is attributed to the rebounding jet created by the heavily breaking crest in mode two (Figure 5-36, $t = 1.470$ s).

### 5.3.4.3.2 Phase analysis of period tripling breaking

Pseudo-phase plot and Poincare map are applied to the wave elevation data shown in Figure 5-38a and Figure 5-39a. Figure 5-40a and c show the pseudo-phase plot, while
Figure 5-40b and Figure 5-40d are the corresponding Poincare map when $\eta(t) = 0$. The definitions for the color and orbit are the same as those in type 2. In Figure 5-40a, the inner red orbit stands for mode three with the lowest wave elevation. The outer blue orbits are for mode two, which are different from the outer orbits for type 1 or 2, indicating that for wave period tripling breaking type 3, the elevation of mode two is higher than those of modes one and three. However, a part of the blue orbits mix with the black ones, meaning that parts of the wave elevation in mode two are identical with those in mode one. In Figure 5-40b, the red dots are obviously the lowest and a part of the blue clusters mix with the black ones.

In Figure 5-40c, the three orbits can be easily distinguished. There is almost no overlap between the blue and the black orbits, meaning that the wave crest from mode two is evidently higher than that from mode one. The Poincare map in Figure 5-40d gives similar information as the phase portrait in Figure 5-40c.

5.3.4.3.3 HHT analysis of period tripling breaking

HHT analysis is applied to the wave elevation measured by wave gauge 2 ($x = 434$ mm) (already shown in Figure 5-39a). The features of the corresponding EEMD results are similar to those for wave tripling breaking type 1. Whereas C7 is the most energetic and almost uniform component representing mainly the external excitation and C9 to C11 are the small components, only the frequency modulation component C8 is shown in Figure 5-41. Also shown in the figure are the instantaneous frequency of C8 and the corresponding wave elevation. The numbers marked next to the wave crest represent the period tripling breaking modes, which is the same as those in Figure 5-39b. The instantaneous frequency of C8 is periodic with a period the same as the wave tripling breaking (from the beginning to the first vertical dash line). This is the reason that C8 is considered as a frequency modulation component for the wave period tripling breaking.

The instantaneous frequency in the period $I$ is illustrated as an example. The variation of the instantaneous frequency of C8 is different from the ones in Figure 5-24 (type 1) and Figure 5-31 (type 2). It increases for the whole duration of modes one and two and peaks at the end of mode two, then followed by a descending process in mode three. It is noticed that the frequency in mode two is higher than that in mode one, indicating that wave varies faster in the former. This is consistent with the description that wave crest in Figure 5-36 (mode two) is more energetic than that in Figure 5-35 (mode one). The frequency in mode three keeps decreasing due to the energy dissipation resulting from the rebounding jet and the neighbour modes.
5.3.4.3.4 Pressure variation during wave period tripling breaking

Figure 5-42 shows the boxplot of the pressure signals measured by the 11 sensors for the case of $b_1 = 27$ mm and $b_2 = 9$ mm. The main features of the boxplot of all pressure sensors are similar to those in Figure 5-32 (type 2). The upper whisker, the third quartile and the median of the box from pressure sensors 1-9 are all around zero, suggesting that these pressure sensors emerge from the water most of the time, which is consistent with the pressure time series shown in Figure 5-43. However, a lot of outliers can be found for sensors 1 to 5, indicating the occurrence of water impacting which will result in higher pressure than the normal hydrostatic pressure. The impact pressure detected by pressure sensors 1, 2 and 3 are much higher than others. Therefore, it can be concluded that the impact pressure during wave tripling breaking type 3 occurs at a location higher than that of pressure sensor No.3.

To further examine the location of the impact pressure, Figure 5-43 shows the pressure time series for the three modes of wave period tripling breaking type 3. Like the sensors displayed for type 2, only the time series from pressure sensors No. 1 to No. 8 are shown in the figure. For all the 8 pressure sensors in Figure 5-43, hydrostatic pressure is also the main source for wave tripling breaking modes two and three. As a result, the measured pressure becomes larger gradually when the mounting location of the sensors becomes lower. In P1, a very high pressure peak (marked by a black arrow) in mode one is observed, which is in the same mode as that in type 2. The peak measured by sensor 2 is much smaller (marked by a blue arrow). This result indicates that the pressure here is dominated by impact pressure instead of hydrostatic pressure and the impact point is around pressure sensor No.1 or higher.

In order to illustrate the high peak pressure in mode one detected by P1 of Figure 5-43, time frames of mode one are shown in Figure 5-44. After check the video carefully, it was realised that the wave motion is analogous to those in type 1 (Figure 5-34). The high impact pressure results from flip-through as well. The impact location is around pressure sensor No.1 (marked by black arrow, $t = 71.3$ ms), which is higher than the still water level (pressure sensor No.9) and is also higher than the one in type 2 (pressure sensor No.5, Figure 5-34). It may be because mode one in type 3 is more energetic than that in type 2.
5.3.4.4 Type 4-curving wave tripling breaking

5.3.4.4.1 Observations of period tripling breaking

When \( b_1 \) and \( b_2 \) keep increasing, wave period tripling breaking type 4 occurs (Figure 5-5). There are also three modes in this type. The main feature of type 4 is the occurrence of curving crest with breaking in modes one and two. Figure 5-45 to Figure 5-47 show a full cycle (from trough to trough at wave gauge 2) of the three modes.

Figure 5-45 shows the formation of mode one of tripling breaking type 4. According to the pattern in Figure 5-16, before the occurrence of mode one at \( x = 434 \) mm, at \( x = 0 \) mm and 867 mm, two wave crests (marked by two blue arrows at \( t = 0 \) s) belonging to mode two, which are characterised by the heavily breaking crests (described in detail in Figure 5-46), are formed. The wave crest at \( x = 0 \) mm (\( t = 0 \) ms) is higher than the one at \( x = 867 \) mm due to the reflection of the wall. During descending process of the both crests, another crest is being formed at \( x = 434 \) mm. The water coming from the lower crest side arrives at \( x = 434 \) mm earlier than that coming from the higher crest side. As a result, the curving crest (\( t = 0.294 \) s and 0.336 s, marked by black arrows) is formed. Similar to mode one in type 3, after the curving crest falls into the main water body (from \( t = 0.504 \) s to 0.714 s), there is no rebounding jet either, indicating that the crest is not as heavy as that in type 1. At \( t = 0.714 \) s, two wave crests (marked by two red arrows) belonging to mode three, occur at \( x = 0 \) and 867 mm. Due to the existence of the tank wall, the crest at \( x = 0 \) mm is higher than the one at \( x = 867 \) mm. When the lower crest moves downward, water is forced to move to \( x = 434 \) mm (marked by a blue arrow, \( t = 0.798 \) s). At the same time, there is no evident water movement from the higher crest side. Till \( t = 0.84 \) s, water from the left side can be observed (marked by a blue arrow). Obviously, water coming from \( x = 867 \) mm arrives at \( x = 434 \) mm earlier than that from \( x = 0 \) mm, resulting in the curving crest in mode two (black arrow, \( t = 1.050 \) s, Figure 5-46). This process is similar to that forming the wave crest in mode one. The curving crest in mode two is much more energetic than that in mode one, which results from the larger horizontal and vertical excitation amplitudes. At the end of mode two, a high crest is formed at \( x = 0 \) mm (marked by a red arrow at \( t = 1.470 \) s). When this crest falls to the main water, a deep trough is formed (marked by a blue arrow at \( t = 1.848 \) s in Figure 5-47). It pushes the water to the right side quickly, resulting in an oblique crest in mode three (marked by a black arrow) at \( t = 1.848 \) s in Figure 5-47. This crest is different from the flat crest appearing in mode three in
previous types. The maximum wave elevation of mode three is still the lowest among the three modes.

Figure 5-48 and Figure 5-49 show the time series of wave elevation measured by wave gauges 1 (x = 0 mm) and 2 (x = 434 mm). The numbers in Figure 5-48b and Figure 5-49b represent the modes. From the analysis above, it is known that the elevation of mode three is the lowest. Therefore, in Figure 5-48b and Figure 5-49b, mode three can be identified easily. Based on the occurring sequence, modes 2 and 3 can be determined subsequently. In Figure 5-48a, the wave crests of modes one and two are comparable. Even in some periods, the wave crest of mode one is higher than that of mode two (e.g. marked by two black arrows in Figure 5-48b). According to Figure 5-16, it has been known that after mode two occurring at x = 434 mm, mode one will happen at x = 0 mm. From Figure 5-46, we know that mode two features with a heavily curving crest. The water from the wave crest in mode two will hit the water severely (t = 1.176 s and t = 1.218 s in Figure 5-46), which will push the water upward along the tank wall (x = 0 mm). Consequently, the elevations in Figure 5-48a for mode one sometimes are equal or higher than that for mode two.

In Figure 5-49b, the elevation of mode two is higher than that of the other two modes, which is consistent with the observation in Figure 5-45 and Figure 5-46. It is noticed that at the end of mode two, the rebounding jet is created (marked by a blue arrow, t = 1.470 s, Figure 5-46). However, in Figure 5-49b, there is no small peak detected which is similar to that shown in Figure 5-39 in the trough of the wave elevation. This is because the location of the rebounding jet is far away from wave gauge 2. As a result, wave gauge 2 cannot detect the elevation change induced by the rebounding jet properly.

5.3.4.4.2 Phase analysis of period tripling breaking

Pseudo-phase plot and Poincare map are applied to the wave elevation data shown in Figure 5-48a and Figure 5-49a and the results are shown in Figure 5-50. The definition of the color and orbit are the same as those for type 2. In Figure 5-50a, the red orbits at the inner location stand for mode three. The black and blue orbits are mixed together, indicating that the elevations from these two modes are comparable, which is consistent with the analysis above (Figure 5-48). This is different from that in type 3 (Figure 5-40a), in which the outer orbits are for mode two. In Figure 5-50b, it is seen clearly that at the upper right corner, the black and blue clusters are mixed while the red dots are lower than them.
In Figure 5-50c, the inner red orbits and outer blue orbits can be distinguished easily, indicating that the lowest and highest elevations are from mode three and mode two, respectively. However, part of the black orbits (mode one) mix with the red ones while other parts mix with the blue ones, indicating that the elevations of mode one sometimes are comparable with that of mode two (e.g. the first wave tripling breaking periods in Figure 5-49b) and sometimes are close to the elevations of mode three (e.g. the second and third wave tripling breaking periods in Figure 5-49b). Compared with the orbits for type 3 (Figure 5-40c), more orbits mixing are found for type 4, which should be due to the larger external amplitudes. The Poincare map in Figure 5-50d reveals similar information with the phase portrait in Figure 5-50c.

5.3.4.4.3 HHT analysis of period tripling breaking

HHT analysis is applied to the wave elevation time series at wave gauge 2 (x = 434 mm) (data already shown in Figure 5-49a). Whereas C7 is the most energetic and almost uniform component representing mainly the external excitations and C9 to C11 are the small components, only the component C8 and its instantaneous frequency are shown in Figure 5-51. The numbers next to the wave crests represent the period tripling breaking modes, which is the same as those in Figure 5-49b. The instantaneous frequency of C8 is periodic and each period lasts for the same duration as the wave tripling breaking from the beginning to the first vertical dash line). This is the reason that C8 is considered as a frequency modulation component for the wave period tripling breaking.

The instantaneous frequency in period \( I \) is illustrated as an example. The variation of the instantaneous frequency of C8 is similar to the one in type 3 (Figure 5-41). However, there is also some difference between them. The instantaneous frequency increases for the whole duration of mode one and peaks at the end of mode two. Then it keeps decreasing very mildly till the beginning of mode three, followed by a dramatically descending process. It is noticed that the peaking moment of the frequency for type 4 is earlier than that for type 3 (Figure 5-41). It suggests that the wave crest in mode two of type 4 is more heavily than that for type 3. When the water from both sides of wave gauge 2 collides together and forms the curving crest shown in Figure 5-46, significant energy dissipation occurs and the wave crest is in almost free falling process, indicating that there is no additional acceleration or deceleration at x = 434 mm. Only in this condition, can the instantaneous frequency keep relative mild variation.
5.3.4.4 Pressure variation during wave period tripling breaking

Figure 5-52 shows the boxplot of the pressure signals measured by the 11 pressure sensors for \( b_1 = 27 \text{ mm} \) and \( b_2 = 9 \text{ mm} \). The upper whisker, the third quartile and the median of the box from pressure sensors 1-9 are all around zero, suggesting that the pressure sensors emerge from water for more than a half time (e.g. the time marked by red arrows in Figure 5-53 P1). Outliers can be found from sensors 1 to 8, indicating the occurrence of water impacting which can result in higher pressure than the normal hydrostatic pressure. From the outlier distribution for pressure sensors No. 1 to No. 8, it is easy to conclude that the maximum pressure occurs at the location of pressure sensor 1 or higher.

The time series of the measured pressure for the three modes of wave period tripling breaking type 4 are shown in Figure 5-53. Again, only results from pressure sensors No. 1 to No. 8 are shown in the figure. For all the 8 time series in Figure 5-53, the measured pressure increases as the location of the sensors gets lower, indicating that the hydrostatic pressure is the dominated contributor. In P1, a very high pressure peak (marked by a black arrow) in mode one is observed, which is in the same mode as that in type 3. The peak detected by P2 at the same instant becomes much smaller (as marked by a blue arrow). As P2 is 15 mm lower than P1, this result indicates that the measured peak is dominated by the impact pressure instead of the hydrostatic pressure and the impact point is around the location of pressure sensor No. 1 or higher.

In order to illustrate the high peak pressure measured by P1 in Figure 5-53, the time frames of mode one are shown in Figure 5-54. From \( t = 0 \text{ ms} \) to \( t = 29.4 \text{ ms} \), a wave crest marked by yellow arrows is approaching the tank wall. During this process, the wave crest and trough merge together and move towards the wall. When \( t = 41.9 \text{ ms} \), the wave crest marked by a red arrow almost hits the wall at around the location of pressure sensor No. 3 or No. 4. However, from the analysis in Figure 5-53, the highest pressure occurs at pressure sensor No. 1. It is noticed that the wave trough (marked by blue arrows in \( t = 41.9 \text{ ms} \) and \( 58.8 \text{ ms} \)) induced by the falling water of the curving crest in mode two at wave gauge 2 (\( x = 434 \text{ mm} \)) pushes the wave crest up to a higher location (marked by black arrow in \( t = 58.8 \text{ ms} \)). Therefore, the wave crest hits the tank wall at the location of pressure sensor No. 1 (marked by the green arrow) at \( t = 75.6 \text{ ms} \). Finally, a vertical jet is formed when \( t = 92.4 \text{ ms} \) to \( 126 \text{ ms} \).

According to the features described above, the wave motion here is also the flip-through\(^7\), which can exert high pressure on the side wall. However, the location of the high peak pressure from flip-through is influenced by the heavily falling wave crest
Sloshing performance under a combination of horizontal and vertical excitations

from the neighbouring mode two. The location of the largest pressure on the wall is at pressure sensor No.1 or higher, which is higher than both the still water level and the one in type 3 (Figure 5-44).

5.3.5 Regime D: chaotic wave breaking and wave sextuple breaking

5.3.5.1 Chaotic wave breaking

When $b_1$ and $b_2$ are increased to very high values, e.g. $b_1 = 34$ mm and $b_2 = 11$ mm, the phenomenon of wave period tripling breaking reported in Sec. 5.3.4 disappears. As shown in Figure 5-5, in regime D, the blue filled circles stand for the cases without wave tripling breaking. Over this regime, chaotic wave breaking can be observed. By examining the time series of the measured pressure signals and the video, no wave breaking pattern can be identified. For example, the wave elevation of wave gauge 2 with $b_1 = 38$ mm and $b_2 = 10$ mm is shown in Figure 5-55. No apparent wave breaking pattern can be seen from this figure. It is also noticed that the case at the left upper corner in Figure 5 with $b_1 = 38$ mm and $b_2 = 1$ mm also belongs to this type.

5.3.5.2 Wave sextuple breaking

Interestingly, when the vertical amplitude $b_2$ is increased by 1 mm, i.e. with $b_2 = 12$ mm, a new feature, called wave period sextuple breaking occurs, for which, wave breaking repeats every six wave cycles. The case for $b_1 = 38$ mm and $b_2 = 12$ mm is illustrated as an example. The time frames of 12 successive wave crests occurring at $x = 434$ mm are shown in Figure 5-56 for two entire wave sextuple breaking periods, i.e. from $t = 0.42$ s to 4.2 s (the first sextuple breaking period) and from $t = 4.956$ s to 8.736 s (the second sextuple breaking period).

When $t = 0.42$ s and 4.956 s, it is the first crest (defined as mode one) of the wave sextuple breaking. The crest has the lowest elevation and tends to incline to the right side. Mode two ($t = 1.176$ s and 5.712 s) is characterised by a turning crest which is slightly higher than that in mode one. Mode three ($t = 1.932$ and 6.468 s) is featured with a high and sharp crest while mode four ($t = 2.688$ and 7.224 s) has a wide but high crest as well. The wave crests for mode three and mode four incline to the left when break and are higher than the crests for other modes. The crest in mode five ($t = 3.444$ s and 7.98 s) is lower than those in modes three and four with a round shape when break. The last crest (mode six at $t = 4.2$ s and 8.736 s) is similar to the first shape (mode one). It has a very low flat crest.
Figure 5-57 show the time series of the wave elevation measured by wave gauge 2 (x = 434 mm). In Figure 5-57a, it can be seen clearly that the wave elevation varies periodically by every 6 wave cycles. There are in total 7 periods of wave sextuple breaking shown in the figure, which are marked from I to VII. The numbers above the crests in Figure 5-57b represent the modes. From the wave crest profiles shown in Figure 5-56, it can be seen that the elevation for mode one is the lowest, while mode three and mode four are featured with the highest crests. Therefore, in Figure 5-57b, the crests with the lowest elevation are for mode one, and the middle two elevations are for modes three and four. The rest modes can be determined accordingly. Based on the observation, wave crest of mode four should be higher than that of mode five. However, in period III, the wave crest (marked by the black arrow) of mode five is higher than that of mode four. This is because when the heavy wave breaking occurs, the elevation of the wave crest in mode five (e.g. t = 3.444 s, Figure 5-56) is much higher than usual. In such circumstance, wave crest in mode five may be higher than that in mode four.

5.4 Conclusions

Experiments for asymmetric wave under combined horizontal and vertical excitations are conducted in a two-dimensional tank to examine the dependence of sloshing wave patterns on excitation amplitudes. With the increase of both the horizontal and vertical excitation amplitudes, the wave pattern can be classified into four regimes. Each regime includes two or more wave types. In total, 10 types of wave are identified in this study.

Regime A corresponds to small excitation amplitudes in the horizontal and vertical directions. The wave does not break and there are two types of wave in this regime. The first one is with the occurrence of a riding wave. HHT analysis to the time series of wave elevation shows that this type of wave is periodical, repeating in every twelve wave cycles. The other type is the wave with dimple crest. It is formed due to the interaction between the round crest at wave gauge 2 and the water coming from the left and right sides of it.

In Regime B, wave starts to break and two types of wave breaking, namely wave breaking with double plungers and normal wave breaking, are identified. It is noticed that the double plungers are asymmetric. The normal wave breaking pattern is likely to happen when the horizontal excitation amplitude is much larger than the vertical one.
Regime C is characterised with four types of wave period tripling breaking with the increase of the horizontal and vertical excitation amplitudes, namely steep tripling breaking (type 1), colliding tripling breaking (type 2), inclining tripling breaking (type 3) and curving tripling breaking (type 4), among which, type 3 is the most common one of wave tripling breaking. For each type, three modes of wave breaking are identified, depending on the patterns of the wave crest. The thin and sharp wave crests in mode one of types 1 and 2 are similar. The crests in mode one of types 3 and 4 are not as sharp and thin as the ones of types 1 and 2. There is inclined crest in mode one of type 3 whereas a curved crest is found in the same mode of type 4. The differences about the crests of mode two among the four types of wave tripling breaking are evident. Whereas mode two has a flat crest with gentle double plungers in type 1, the crest in type 2 is sharp with breaking due to the strong water movement from its left and right sides. Mode two in types 3 and 4 has heavily inclining and curving crests, respectively, with the latter being more energetic than the former due to the increased input energy. Mode three in types 1, 2, and 3 of wave breaking are very similar, which is featured with a flat crest. However, in type 4, the wave crest is oblique and inclined the right side.

Impact pressure is observed in wave tripling breaking types 2, 3 and 4 in Regime C. For types 2 and 3, high impact pressure is detected in mode one, which results from the flip-through purely. The impact locations are all higher than the still water level. For type 4, flip-through is also the main factor for the impact pressure. However, the falling water from the curving crest in mode two at wave gauge 2 (x = 434 mm) contributes to the high pressure as well. The falling water pushes up the wave crest to a higher location, resulting in a higher impacting location for type 4.

In Regime D, two types of breaking wave are found. The first one is chaotic wave without any apparent patterns whereas the second one is periodic with a period of six times of that of the wave cycle, and has 6 wave breaking modes. Mode one has the lowest flat crest. The crest in mode two turns to the right, and is slightly higher than that in mode one. Mode three has a high, thin and sharp crest while mode four has a wide but high crest. The crest in mode five is lower than those in modes three and four. The shape is round with breaking for most of the time. Mode six has similar crest as mode one but with a slightly higher elevation.
Reference


Figure 5-1 Snapshot of the sloshing experimental facility.

Figure 5-2 Sketch of the experimental set-up
Figure 5-3 Flow chart of the signal measurement and control.

Figure 5-4 Example of the input signals. (a) horizontal excitation; (b) vertical excitation.
Figure 5-5 Wave regime classification based on the horizontal-vertical excitation amplitudes.
Figure 5-6 Twelve successive time frames with time interval of 0.042 s for the wave peak at $x = 434$ mm in Regime A ($b_1 = 12$ mm and $b_2 = 1$ mm). The black arrows indicate the positions of the riding wave. The unit for time is in s.
Figure 5-7 Time series of wave elevation measured by wave gauge 2 (wave period from 221 to 239) when $b_1 = 12$ mm and $b_2 = 1$ mm in Regime A.

Figure 5-8 Comparison between the time series of C7 (black) and C8 (red) (Regime A). The numbers I, II and III stand for the periods of C8. The numbers inside the circles indicate the parts inside the period.
Figure 5-9 Eight successive time frames with time interval of 0.042 s for the wave peak at \( x = 434 \) mm \((b_1 = 14 \text{ mm and } b_2 = 3 \text{ mm, Regime A})\). The unit for time is in s.
Figure 5-10 Time series of wave elevation measured by wave gauge 2 (wave periods from 271 to 279) under $b_1 = 14$ mm and $b_2 = 3$ mm in Regime A.

Figure 5-11 Time series of C7 and its instantaneous frequency (Regime A). The region between the two black vertical dashed lines represents one period of C7. The arrows are explained in the text.
Figure 5-12 (a) Photo for wave breaking with double plungers ($x = 434$ mm) in Regime B; (b) Time series of wave elevation (wave periods from 281 to 290) for $b_1 = 12$ mm and $b_2 = 7$ mm.

Figure 5-13 Time series of C7 and its instantaneous frequency for the data shown in Figure 5-12b (Regime B, wave periods from 281 to 290).
Figure 5-14 (a) Photo for normal wave breaking in regime B; (b) Time series of wave elevation measured by wave gauge 2 (wave periods from 281 to 290) when $b_1 = 27$ mm and $b_2 = 2$ mm.

Figure 5-15 Time series of C7 and its instantaneous frequency for the data shown in Figure 5-14b (Regime B, wave periods from 281 to 290).
Figure 5-16 Sketch on the occurring sequence of wave breaking modes at \( x = 434 \) mm and 867 mm.
Figure 5-17 Time frames for wave period tripling breaking type 1 in Regime C, mode one (location of wave gauge 2), when $b_1 = 18$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-18 Time frames for wave period tripling breaking type 1 in Regime C, mode two (location of wave gauge 2), when $b_1 = 18$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-19 Time frames for wave period tripling breaking type 1 in Regime C, mode three (location of wave gauge 2), when $b_1 = 18$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-20 Time series of wave elevations at $x = 0$ mm (wave gauge 1) when $b_1 = 18$ mm and $b_2 = 8$ mm in Regime C type 1. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by the vertical blue dashed lines in (a). The numbers 1, 2 and 3 marked on the crests in (b) stand for modes one, two and three, respectively.
Figure 5-21 Time series of wave elevations at $x = 434$ mm (wave gauge 2) when $b_1 = 18$ mm and $b_2 = 8$ mm in type 1, Regime C. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by vertical blue dashed lines in (a). The numbers 1, 2 and 3 marked on the crests in (b) stand for modes one, two and three, respectively.
Figure 5-22 Pseudo phase plots and Poincare map for wave elevation shown in Figure 5-20 and Figure 5-21 ($b_1 = 18$ mm and $b_2 = 8$ mm, type 1 in Regime C). Pseudo phase plots: (a) $x = 0$ mm, (c) $x = 434$ mm; Poincare map: (b) $x = 0$ mm, (d) $x = 434$ mm.

Black, blue and red stand for modes one, two and three, respectively.
Figure 5-23 Spectra of signal components C7-C9 of the EEMD results on time series data shown in Figure 5-21a (Regime C, type 1).
Figure 5-24 Time series of wave elevation, EEMD component C8 and its instantaneous frequency (same time length as that in Figure 5-21, Regime C type 1). The black line is for wave elevation. The numbers marked with 1, 2 and 3 stand for modes one, two and three, respectively. The numbers I, II, III stand for the periods of the instantaneous frequency.
Figure 5-25 Time frames for wave period tripling breaking type 2 in Regime C, mode one (location of wave gauge 2), when $b_1 = 21$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-26 Time frames for wave period tripling breaking type 2 in Regime C, mode two (location of wave gauge 2), when $b_1 = 21$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-27 Time frames for wave period tripling breaking type 2 in Regime C, mode three (location of wave gauge 2), when $b_1 = 21$ mm and $b_2 = 8$ mm. The unit for time is in s.
Figure 5-28 Time series of wave elevation at x = 0 mm (wave gauge 1) when \( b_1 = 21 \) mm and \( b_2 = 8 \) mm in Regime C type 2. (a) for horizontal excitation cycles 251 to 300; (b) enlarged area marked by the vertical blue dashed lines in (a).
Figure 5-29 Time series of wave elevation at $x = 434$ mm (wave gauge 2) when $b_1 = 21$ mm and $b_2 = 8$ mm in Regime C type 2. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by the vertical blue dashed lines in (a).
Figure 5-30 Pseudo phase plots and Poincare map for wave elevation shown in Figure 5-28 and Figure 5-29 ($b_1 = 21$ mm and $b_2 = 8$ mm, type 2 in Regime C). Pseudo phase plots: (a) $x = 0$ mm, (c) $x = 434$ mm; Poincare map: (b) $x = 0$ mm, (d) $x = 434$ mm.

Black, blue and red stand for modes one, two and three, respectively.
Figure 5-31 Time series of wave elevation, EEMD component C8 and its instantaneous frequency (same time length as that in Figure 5-29, type 2 in Regime C). The black line is for the wave elevation. The numbers marked with 1, 2 and 3 stand for modes one, two and three, respectively. The numbers I, II, III stand for the periods of the instantaneous frequency.

Figure 5-32 Boxplot of pressure measured by sensors 1 to 11 during horizontal excitation cycles 101 to 300 ($b_1 = 21$ mm and $b_2 = 8$ mm, type 2 in Regime C). The upper whisker and lower whisker indicate the maximum and minimum values within the range from Q1-1.5IQR to Q3+1.5IQR, IQR = Q3-Q1.
Figure 5-33 Time series of pressure measured by sensors P1-P8 for the three modes of type 2 in Regime C during wave period tripling breaking ($b_1 = 21$ mm and $b_2 = 8$ mm). The red line represents the pressure while the blue dashed line and black line indicate the tank movement and wave elevation at $x = 0$ mm, respectively. The horizontal arrows indicate the impacting pressure detected by sensors 4, 5 and 6.
Figure 5-34 Time frames of mode one of wave period tripling breaking type 2 in Regime C when $b_1 = 21$ mm and $b_2 = 8$ mm (pressure sensors 1-11 from top to bottom with sensor 9 being on the SWL).
Figure 5-35 Time frames for wave period tripling breaking type 3 in Regime C, mode one (location of wave gauge 2), when $b_1 = 27$ mm and $b_2 = 9$ mm. The unit for time is in s.
Figure 5-36 Time frames for wave period tripling breaking type 3 in Regime C, mode two (location of wave gauge 2), when $b_1 = 27$ mm and $b_2 = 9$ mm. The unit for time is in s.
Figure 5-37 Time frames for wave period tripling breaking type 3 in Regime C, mode three (location of wave gauge 2), when $b_1 = 27$ mm and $b_2 = 9$ mm. The unit for time is in s.
Figure 5-38 Time series of wave elevation at x = 0 mm (wave gauge 1) when $b_1 = 27$ mm and $b_2 = 9$ mm in type 3, Regime C. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by the blue dashed lines in (a). The numbers 1, 2 and 3 marked on the crests in (b) stand for modes one, two and three, respectively.
Figure 5-39 Time series of wave elevation at x = 434 mm (wave gauge 2) when $b_1 = 27$ mm and $b_2 = 9$ mm in type 3, Regime C. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by blue dashed lines in (a). The numbers 1, 2 and 3 marked on the crests in (b) stand for modes one, two and three. The arrows indicate the fluctuations in wave trough.
Figure 5-40 Pseudo phase plots and Poincare map for wave elevation shown in Figure 5-38 and Figure 5-39 \((b_1 = 27 \text{ mm} \text{ and } b_2 = 9 \text{ mm}, \text{ Regime C, type 3})\). Pseudo phase plots: (a) \(x = 0 \text{ mm}\), (c) \(x = 434 \text{ mm}\); Poincare map: (b) \(x = 0 \text{ mm}\), (d) \(x = 434 \text{ mm}\). Black, blue and red stand for modes one, two and three, respectively.

Figure 5-41 Time series of wave elevation, EEMD component C8 and its instantaneous frequency (same time length as that in Figure 5-39b, Regime C type 3). The black line is for the wave elevation. The numbers marked with 1, 2 and 3 stand for modes one, two and three, respectively. The numbers I, II, III stand for the periods of the instantaneous frequency.
Figure 5-42 Boxplot of pressure measured by sensors 1 to 11 for $b_1 = 27$ mm and $b_2 = 9$ mm in Regime C (type 3) during horizontal excitations 101 to 300. Detailed explanation about the boxplot is given in Figure 5-32.

Figure 5-43 Time series of pressure measured by sensors P1-P8 for the three modes of type 3 in Regime C during wave period tripling breaking ($b_1 = 27$ mm and $b_2 = 9$ mm). The red line represents the pressure while the blue dashed line and black line indicate the tank movement and wave elevation at $x = 0$ mm, respectively. The black and blue arrows indicate the impacting pressure detected by sensors 1 and 2, respectively.
Figure 5-44 Time frames of mode one of wave period tripling breaking for $b_1 = 27$ mm and $b_2 = 9$ mm in type 3 Regime C (pressure sensors 1-11 from top to bottom with sensor 9 being on the SWL).
Figure 5-45 Time frames for wave period tripling breaking type 4 in Regime C, mode one (location of wave gauge 2), when $b_1 = 32$ mm and $b_2 = 10$ mm. The unit for time is in s.
Figure 5-46 Time frames for wave period tripling breaking type 4 in Regime C, mode two (location of wave gauge 2), when $b_1 = 32$ mm and $b_2 = 10$ mm. The unit for time is in s.
Figure 5-47 Time frames for wave period tripling breaking type 4 in Regime C, mode three (location of wave gauge 2), when $b_1 = 32$ mm and $b_2 = 10$ mm. The unit for time is in s.
Figure 5-48 Time series of wave elevations at x = 0 mm (wave gauge 1) when $b_1 = 32$ mm and $b_2 = 10$ mm in type 4 Regime C. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by the blue dashed lines in (a). The numbers 1, 2 and 3 stand for modes one, two and three, respectively.
Figure 5-49 Time series of wave elevations at $x = 434$ mm (wave gauge 2) when $b_1 = 32$ mm and $b_2 = 10$ mm in type 4 Regime C. (a) from horizontal excitation cycles 251 to 300; (b) enlarged area marked by the blue dashed lines in (a). The numbers 1, 2 and 3 stand for modes one, two and three, respectively.
Figure 5-50 Pseudo phase plots and Poincare map for wave elevation shown in Figure 5-48 and Figure 5-49 ($b_1 = 32$ mm and $b_2 = 10$ mm, Regime C type 4). Pseudo phase plots: (a) $x = 0$ mm, (c) $x = 434$ mm; Poincare map: (b) $x = 0$ mm, (d) $x = 434$ mm. Black, blue and red stand for modes one, two and three, respectively.

Figure 5-51 Time series of wave elevation, EEMD component C8 and its instantaneous frequency (same time length as that in Figure 5-49b, Regime C type 4). The black line is for the wave elevation. The numbers marked with 1, 2 and 3 stand for modes one, two and mode three, respectively. The numbers I, II, III stand for the periods of the instantaneous frequency.
Figure 5-52 Boxplot of pressure measured by sensors 1 to 11 for $b_1 = 32$ mm and $b_2 = 10$ mm, in Regime C (type 4) during horizontal excitation 101 to 300. Detailed explanation about the boxplot is given in Figure 5-32.

Figure 5-53 Time series of pressure measured by sensors P1-P8 for the three modes of type 4 in Regime C during wave period tripling breaking ($b_1 = 32$ mm and $b_2 = 10$ mm). The red line represents the pressure while the blue dashed line and the black line indicate the tank movement and wave elevation at $x = 0$ mm, respectively. The black and blue arrows indicate the impacting pressure detected by sensors 1 and 2.
Figure 5-54 Time frames of mode one of wave period tripling breaking for $b_1 = 32$ mm and $b_2 = 10$ mm in type 4 Regime C (pressure sensors 1-11 from top to bottom with sensor 9 being on the SWL).
Figure 5-55 Time series of wave elevations at \( x = 434 \text{ mm} \) (wave gauge 2) \( b_1 = 38 \text{ mm} \) and \( b_2 = 10 \text{ mm} \) in Regime D (chaotic wave breaking) from horizontal excitation cycles 201 to 250.
Figure 5-56 Time frames for 12 successive wave crests (corresponding to two periods) at wave gauge 2 ($x = 434$ mm) for wave period sextuple breaking. The first column is for period I and the second one is for period II ($b_1 = 38$ mm and $b_2 = 12$ mm, in Regime D). The unit for time is in s.
Figure 5-57 Time series of wave elevations at \( x = 434 \) mm (wave gauge 2) when \( b_1 = 38 \) mm and \( b_2 = 12 \) mm in Regime D. (a) from horizontal excitation cycles 201 to 250; (b) enlarged area marked by the blue dashed lines in (a). The numbers I-VII stand for the seven periods of wave sextuple breaking. The arrow in period III indicates mode five.
Chapter 6

Conclusions and Recommendations for future work

6.1 Conclusions of the study

This study focuses on the sloshing behaviour in a two-dimensional tank under various external excitations. Through the experiments, the dependence of critical filling level on the excitation amplitude, the asymmetric wave performance including non-breaking wave, breaking wave as well as the wave period tripling breaking under horizontal excitation, vertical excitation and a combination of them, are investigated systematically. The main conclusions of this research are summarised as follows.

6.1.1 The relationship between filling levels and maximum sloshing response in a two-dimensional tank

(1) The dependence of the maximum wave amplitude and pressure on tank filling level has been examined experimentally in a two-dimensional rectangular tank under 4 different excitation amplitudes \( b/L = 0.001, 0.002, 0.004 \) and 0.005) at an excitation frequency equal to the natural frequency of the lowest mode. The experimental data were analysed using the boxplot method to minimise the influence of lack of repeatability, especially for the time series of the impact pressure, on the experimental results. The filling levels for the maximum wave amplitude were found to occur at \( h/L = 0.332, 0.312, 0.287 \) and 0.263 for external excitation amplitudes \( b/L = 0.001, 0.002, 0.004 \) and 0.005, respectively, indicating that the critical filling level decreases monotonically with the increase of \( b \). It is found through a specifically designed test that an increase of the damping in the system is equivalent to the effect of a decreased
excitation amplitude, which leads to an increase in the critical filling level and a
decrease in the maximum response amplitude.

(2) The occurrence of the critical filling level is explained by solving the duffing
like equation for critical depth which categorises the sloshing wave into two types:
hard-spring behaviour \((h/L < \text{critical depth})\) and soft-spring behaviour \((h/L > \text{critical}
\text{depth})\). The theoretical analysis indicates that the critical filling level increases with the
decrease of excitation amplitude, which is consistent with the experimental results.

(3) The pressure on the tank sidewall is found to correlate well with the response
amplitude of water level in the tank, suggesting that the sloshing pressure measured in
this study is mainly caused by the hydrostatic force. This is attributed to the small
excitation amplitudes tested in this study.

6.1.2 Asymmetric wave performance under horizontal excitations

Based on the experiments for asymmetric wave under horizontal excitation,
different phenomena are obtained via varying the forcing amplitude \(b\) from 1 mm to
40 mm.

(1) When \(b = 10\) mm \((b/L = 0.008)\) to 13 mm \((b/L = 0.010)\), the wave in the tank is
non-breaking with a riding wave being observed at \(x = 434\) mm. After applying HHT to
the time series of the measured wave elevation data, it is found that the wave is
periodical with a period of twelve wave cycles.

(2) As \(b\) is increased to 14.5 mm \((b/L = 0.011)\), a special periodical wave breaking
type called ‘transitional periodical breaking’ is observed at \(x = 434\) mm and \(x = 867\)
mm. It is only under such a forcing amplitude that the periodical wave breaking lasts for
the whole duration of the case. After applying HHT to the wave elevation data and
analysing the instantaneous frequency, it is found that the instantaneous frequency of
C8 repeats in every seven wave cycles which is the same as the wave elevation for the
transitional wave breaking. It is therefore believed that the transitional wave breaking is
induced by the frequency modulation.

(3) When \(b\) is further increased to 32.5 mm \((b/L = 0.025)\), wave period tripling
breaking is found inside the tank. A thin and sharp crest in mode one at \(x = 434\) mm is
observed which is caused by the collision between the water coming from both sides of
\(x = 434\) mm. The wave crest always tends towards left instead of being random. Mode
two is featured with a flat crest instead of double plungers observed previously while
mode three is characterised by a round crest with a weak breaking. HHT is applied to
the wave elevation at \(x = 0\) and 434 mm. The FFT results for HHT components C7 - C9
show that C8 is the frequency modulation component. Based on the analysis of instantaneous frequency of C8 at x = 0 mm and x = 434 mm, it is found that the frequency varies every three wave cycles which is the same as that for wave period tripling breaking, indicating that wave period tripling breaking is induced by this frequency modulation.

(4) The hydrostatic pressure dominates the pressure measured by sensors 4-9. However, the highest pressure during the period tripling breaking occurs in mode three due to the flip-through phenomenon with the impact location being higher than the still water level, which is consistent with previous studies.

(5) Period tripling breaking becomes unstable when \( b \) reaches 39.5 mm \((b/L = 0.030)\). Based on the phase portrait analysis of wave elevation, it is found that part of the orbits starts to mix with each other gradually, indicating that the period tripling breaking starts to become unstable.

### 6.1.3 Asymmetric wave performance under vertical excitations

Experiments under an initial horizontal and followed by a vertical excitation are conducted in a two-dimensional sloshing tank. The vertical excitation is at the third mode sloshing frequency \((2\omega_3 = 2.66 \text{ Hz})\). The wave form is dependent on vertical excitation amplitudes \((b_2 = 1 \text{ to } 15 \text{ mm}, b_2/L = 0.0007 \text{ to } 0.0115)\).

(1) When \( b_2 = 1 \text{ mm} \((b_2/L = 0.0007)\), the wave forms are slightly temporal asymmetry at the region close to the wave crest (x = 434 mm) without breaking. The temporal asymmetry is intensified when \( b_2 \) is increased to 4.5 mm \((b_2/L = 0.0035)\). This may be due to the occurrence of the bulge on the side of the wave crest. Interestingly, the location of the bulge changes periodically. It is on the right side of the wave crest once and then on the left side twice at the next two successive crests. The above sequence repeats afterwards.

(2) Wave starts to break when \( b_2 = 7 \text{ mm} \((b_2/L = 0.0054)\), and the corresponding wave steepness is \( h/\lambda = 0.1605 \). Wave period tripling breaking is observed in this experiment as \( b_2 \) is increased to 11.5 mm \((b_2/L = 0.0088)\). Mode one is characterised by a steep and sharp crest with random changing inclined direction, while mode two is featured with double. Mode three has a flat crest. It is also noticed that the wave shape of mode two varies with the increase of forcing amplitude. When \( b_2 \) is increased up to 15 mm \((b_2/L = 0.0115)\), the shape of mode two becomes a steep crest which is similar to mode one but with heavily wave breaking resulting from wave collision.
(3) Phase portrait is applied to the non-breaking ($b_2 = 1 \text{ mm}$, $b_2/L = 0.0007$) and period tripling breaking waves when $b_2 = 11.5 \text{ mm}$ ($b_2/L = 0.0088$) and $15 \text{ mm}$ ($b_2/L = 0.0115$), respectively. The width of the orbits stands for the magnitude of the range of the wave crest. The results for the non-breaking wave show only one orbit whereas three orbits can be found for period tripling breaking. For $b_2 = 11.5 \text{ mm}$ ($b_2/L = 0.0088$), the orbits show a partial mixing between modes two and three, revealing that the wave elevation of modes two and three are comparable. For $b_2 = 15 \text{ mm}$ ($b_2/L = 0.0115$), the mixing between modes one and two is also detected.

(4) The measured wave elevations for $b_2 = 11.5 \text{ mm}$ ($b_2/L = 0.0088$) and $15 \text{ mm}$ ($b_2/L = 0.0115$) are also analysed using HHT. The results suggest that the frequency modulation from HHT component 8 (C8) induces the period tripling breaking.

6.1.4 Asymmetric wave performance under the combination of horizontal and vertical excitations

Experiments for asymmetric wave under combined horizontal and vertical excitations are conducted to examine the dependence of wave patterns in the tank on excitation amplitudes. With the increase of both the horizontal and vertical excitation amplitudes, the wave pattern can be classified into four regimes. Each regime includes two or more wave types. In total, 10 types of wave are identified in this study.

(1) Regime A corresponds to small excitation amplitudes in the horizontal and vertical directions. The wave does not break and there are two types of wave in this regime. The first one is with the occurrence of a riding wave, repeating in every twelve wave cycles. The other type is the wave with dimple crest which is due to the interaction between the round crest at wave gauge 2 and the water coming from left and right sides of it.

(2) In Regime B, wave starts to break and two types of wave breaking, namely wave breaking with double plungers and normal wave breaking, are identified.

(3) Regime C is characterised with four types of wave period tripling breaking with the increase of the horizontal and vertical amplitudes, namely steep tripling breaking (type 1), colliding tripling breaking (type 2), inclining tripling breaking (type 3) and curving tripling breaking (type 4), among which, type 3 is the most common type of wave tripling breaking. For each type, three modes of wave breaking are identified, depending on the patterns of the wave crest. The thin and sharp wave crests in mode one of types 1 and 2 are similar. The crests in mode one of types 3 and 4 are not as sharp and thin as the ones of types 1 and 2. There is inclined crest in mode one of type 3.
whereas a curved crest is found in the same mode of type 4. The differences about the crests of mode two among the four types of wave tripling breaking are evident. Whereas mode two has a flat crest with gentle double plungers in type 1, the crest in type 2 is sharp with breaking due to the strong water movement from its left and right sides. Mode two in types 3 and 4 has heavily inclining and curving crests, respectively, with the latter being more energetic than the former due to the increased input energy. Mode three in types 1, 2, and 3 of wave breaking are very similar, which is featured with a flat crest. However, in type 4, the wave crest is oblique and inclined towards the right side.

(4) Impact pressure is observed in wave tripling breaking types 2, 3 and 4 in Regime C. For types 2 and 3, the high impact pressure is detected in mode one, which results from the flip-through purely. The impact locations are all higher than the still water level. For type 4, flip-through is also the main factor for the impact pressure. However, the falling water from the curving crest in mode two at wave gauge 2 (x = 434 mm) contributes to the high pressure as well. The falling water pushes up the wave crest to a higher location, resulting in a higher impacting location for type 4.

(5) In Regime D, two types of breaking wave are found. The first one is violent and chaotic without any apparent patterns whereas the second one is periodic with a period of six times of that of the wave cycle, and has 6 wave breaking modes. Mode one has the lowest flat crest. The crest in mode two turns to the right, and is slightly higher than that in mode one. Mode three has a high, thin and sharp crest while mode four has a wide but high crest. The crest in mode five is lower than those in modes three and four. The shape is round with breaking for most of the time. Mode six has similar crest as mode one but with a slight higher elevation.

6.2 Suggestions for future work

First of all, our work for investigating the relationship between the maximum sloshing response and filling level is done via the horizontal excitation. However, if the tank is under vertical motion/excitation and/or roll motion, the reported relationship may be different. In addition, the present experiments were conducted at the lowest mode of natural frequency. When the exciting frequency is increased to a higher mode, such as the second or third mode, the relationship between the filling level and sloshing reaction needs to be clarified.

Secondly, the wave performance captured by a high speed camera in Chapters 3 - 5 are just for qualitative analysis. Some physical process is still difficult to explain.
Therefore, experiments using PIV could be employed aiming at analysing quantitatively the flow filed of the whole process of the wave performance.

Thirdly, as the impact pressure is highly transitional and highly localized, the resolution and the cover range of the pressure measurement should be increased.

Finally, the wave performances discussed in Chapters 3 - 5 are obtained when the excitation frequency equals to the third mode natural frequency, which assures that the wave profile is asymmetric inside the tank. When the tank is excited at frequencies rather than the third mode natural frequency, the wave performances inside the tank are still unclear. Therefore, experiments under other excitation frequencies could be conducted so that full understanding about the sloshing wave performances in a two-dimensional tank can be obtained.