Experimental investigation on sloshing behavior in a two-dimensional tank under various external excitations

By

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BEng, MEng

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of

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School of Civil, Environmental and Mining Engineering

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Abstract

The investigations on wave motion inside a partially-filled tank are essential for oil and gas industry, such as the Liquefied Natural Gas (LNG) transportation and aircraft and aviation industries. The first aim of this study is to investigate the relationship between maximum responding wave amplitude and filling levels. For this purpose, a series of sloshing experiments were carried out in a two-dimensional rectangular tank for different filling levels. For each filling level, the tank was under a sway excitation driven by a hexapod with 6-degree-of-freedom (6DOF) at the fundamental sloshing mode. Four excitation amplitudes, namely $b/L = 0.001, 0.002, 0.004$ and $0.005$, were applied. Pressure variation on the wall of the tank and the internal wave surface were measured using four high response pressure sensors and a wave gauge, respectively. To examine the repeatability of the tests, pressure and wave amplitude measurements under the same filling and excitation conditions were repeated for 30 times and the data were then examined using a statistical method, the so-called boxplot. It is found that the repeatability of the impact pressure and wave amplitude can be well expressed by this statistical method. The filling level for the maximum wave amplitude, which is termed as critical filing level, depends on the excitation amplitude. It decreases monotonically with the increase of the excitation amplitude. It is also found that an increase in the damping of the system leads to an increase in the critical filling level and a decrease in the maximum response amplitude. The occurrence of the critical filling level is explained by solving the duffing like equation for the critical depth which categorises the sloshing wave into two types: hard-spring behaviour ($h/L < \text{critical depth}$) and soft-spring behaviour ($h/L > \text{critical depth}$). The theoretical analysis indicates that the critical filling level increases with the decrease of the excitation amplitude, which is consistent with the experimental results.

Another aim in this study is to investigate the asymmetric sloshing wave (asymmetric about the tank centreline) which was excited under horizontal excitation, vertical excitation and the combination of horizontal and vertical excitations at the third mode natural frequency in a two-dimensional tank.
For the experiments under horizontal excitation, it is found that with the increase of the forcing amplitude, the wave profile undergoes a transition from small non-breaking riding wave to period tripling breaking wave. Before wave breaking, a periodical riding wave is observed and analysed. When wave breaking occurs, a transitional periodical breaking wave with a repeatability in seven wave cycles is found. Wave period tripling breaking is observed in the experiment when the external amplitude exceeds a threshold value \((b = 32.5 \text{ mm}, \ b/L = 0.025 \text{ in this study})\). Analysis using phase portrait and Hilbert-Huang Transform (HHT) indicates wave period tripling breaking is related to the frequency modulation. When the forcing amplitude keeps increasing, wave period tripling breaking becomes unstable.

For the experiments under vertical excitation, with the increase of the forcing amplitude, the performance of standing wave changes from a non-breaking state to a breaking state. When the forcing amplitude is small, the temporal asymmetry of the non-breaking wave is found. Wave period tripling breaking is also observed when the forcing amplitude is increased to 11.5 mm \((b/L=0.009)\). However, the modes of the wave period tripling breaking are different from those under the horizontal excitations. Furthermore, the breaking type of mode two is found to be changed when the forcing amplitude continues increasing.

For the experiments under combined horizontal and vertical excitations, it has been found that the wave patterns inside the tank depend on the excitation amplitudes. Based on the results of wave patterns, totally four regimes have been classified and each regime includes two or more types of waves. In total, ten types of wave have been identified. Regime A is for wave without breaking while regime B is for wave with double plungers and normal breaking. Four types of wave tripling breaking are found in Regime C. The main differences among these four types are the shapes of the wave crests appearing in modes one and two. Mode three has a flat crest with the lowest elevation among all the four types. HHT is employed to investigate the wave period tripling breaking and the results show C8 resulting from the Ensemble Empirical Mode Decomposition (EEMD) is a frequency modulation component. Regime D includes the chaotic wave breaking without any patterns and another regular breaking wave called wave sextuple breaking.

The last aim of this study is to investigate the pressure variation during wave tripling breaking under the horizontal excitation and the combined horizontal and vertical excitations. A high pressure peak which is induced by flip-through is found in the wave
period tripling breaking. The impact point is higher than the still water level. Apart from the high pressure peak, other pressure are mainly caused by the hydrostatic force.
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List of Symbols

Non-Greek symbols:

- $A$: Response amplitude
- $B$: Tank breadth
- $b, b_1$: Horizontal excitation amplitude
- $b_2$: Vertical excitation amplitude
- $g$: Gravitational acceleration
- $h$: Filling depth
- $L$: Tank length
- $m$: Nonlinear interaction coefficient
- $n$: Mode number of the internal sloshing
- $t$: Time
- $T$: Period
- $x$: Tank longitudinal direction

Greek Symbols:

- $\alpha$: Damping coefficient
- $\beta$: Forcing coefficient
- $\varepsilon$: Ratio of external forcing amplitude to the tank length
- $\delta$: Detuning parameter
- $\nu$: Viscosity of water
- $\phi$: Phase angle
- $\lambda$: Wave length

Subscripts:

- $\omega$: The lowest natural angular frequency
- $f_3$: The third mode sloshing frequency

Abbreviations:

- $b/L$: Ratio of external forcing amplitude to the tank length
- $a(t)$: Instantaneous amplitude function
- $\theta(t)$: Phase function
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<thead>
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<tr>
<td>$f(t)$</td>
<td>Instantaneous frequency function</td>
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<tr>
<td>$PV$</td>
<td>Cauchy principal value</td>
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<tr>
<td>HHT</td>
<td>Hilbert-Huang Transform</td>
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Chapter 1

Introduction

1.1 Background and brief literature review

Sloshing is a phenomenon which usually occurs in a partially-filled tank when the frequency of the tank motion is close to the natural frequency of the fluid inside the tank. Large-amplitude sloshing wave exerts high hydrodynamic loads on the tank wall, resulting in a high risk of damage as well as the stability concern of the tank. For any reason, the damage and accident induced by sloshing might cause catastrophic economical and human-life losses as well as pollution to the environments. In oil and gas industry, vessels containing LNG or oil (Figure 1-1) will be under various excitation conditions in ocean, which may result in violent sloshing. It will produce localized impact pressure on the tank walls, which may in turn cause structural damages to the tank and may create sufficient moment to destabilize the tank carrier. Hence, it is necessary to investigate sloshing dynamics in order to design the tank walls for extra loads and to stabilise the tank for sloshing effect.

1.1.1 Main factors governing the sloshing in a two-dimensional tank

Sloshing in rectangular tanks is often described as two-dimensional flows when the tank width is much smaller than its length. The sloshing responses inside a tank depend on a number of factors, such as the external excitation frequency, the excitation amplitude and the tank filling level, etc. The sloshing inside a tank has an infinite number of natural frequencies, but it is the lowest few modes that are most likely to be excited\(^1\). To obtain the natural frequency, there are generally two methods. The first one is using the theoretical relationship. Abramson \textit{et al.}\(^2\) studied the natural frequencies inside a rectangular tank via the equation given by Lamb\(^3\). It was found that the value of
the frequency could be approximated by a single curve under a given modal number. Budiansky\textsuperscript{4} investigated sloshing frequencies, mode shapes and hydrodynamic force acting on rigid walls. He calculated the fundamental frequency inside a cylindrical tank. Apart from the theoretical calculations, experimental methods were also used to determine the fundamental natural frequencies. Kobayashi et al.\textsuperscript{5} studied sloshing frequencies and hydrodynamic forces in horizontal cylindrical tanks under small and large sloshing wave amplitudes experimentally and theoretically. The lowest natural frequencies were obtained by sweep tests with small excitation amplitudes. It was shown that the calculated frequencies agreed very well with the test results. Pal et al.\textsuperscript{6} conducted experiments to investigate the relationship between sloshing amplitude and excitation frequencies. They obtained the lowest natural frequencies for different filling levels by experiments and compared them with the calculations via the method proposed by Abramson et al.\textsuperscript{2}. The obtained results using different methods show a discrepancy within 4%.

In the past decades, sloshing under large and medium excitation amplitudes has been studied extensively. The reported results show that the sloshing responses depend significantly on the exciting amplitudes. Ji et al.\textsuperscript{7} conducted experiments on non-resonant sloshing with large amplitude (the ratio of forcing amplitude and tank length $b/L = 0.1$). They classified the free surface waves into four types, namely, (I) 2-D mild waves, (II) 2-D strongly nonlinear waves, (III) 3-D waves with regular longitudinal structure along the z-direction and (IV) chaotic waves. They mainly focused on the second type. Wei et al.\textsuperscript{8} studied slamming pressure inside a sloshing tank with slat-screens under large amplitude excitation ($b/L = 0.102$ to 0.116). They concluded that the slat-screens with high solidity are crucial to decrease the slamming pressure under certain circumstances. Royon-Lebeaud et al.\textsuperscript{9} conducted sloshing experiment in a square tank under a medium excitation amplitude ($b/L = 0.0125$). They investigated wave crest destabilization in detail and stated that the cross-wave instability is interpreted in terms of parametric instability. Some phenomena of sloshing under small excitation amplitudes were also studied previously. Using experimental method, Pal and Bhattacharyya\textsuperscript{6} studied the liquid movement in a sloshing tank under five different excitation amplitudes ($b/L = 0.005, 0.01, 0.015, 0.02, 0.025$, the first one is small, then the rest are median). They found that sloshing wave amplitude follows sinusoidal pattern. Bouscasse et al.\textsuperscript{10} conducted both numerical and experimental studies of sloshing phenomena in shallow water regimes. The experiments covered sway motions from small ($b/L = 0.003$) to large amplitudes ($b/L = 0.1$) for five different filling levels.
The main features of the sloshing motion (wave trains, wave breaking, jet run-up, run-down at the vertical walls, etc.) were described.

Apart from the excitation amplitude, the liquid filling depth inside the tank is another crucial factor that influences sloshing responses. Low filling depths have been paid much attention. Verhagen and Wijngaarden\textsuperscript{11} observed a hydraulic jump, which travels periodically back and forth inside a tank with low filling level under roll excitation. They also compared their theoretical results with those obtained from experiments and part of the discrepancy was found to be due to the absence of viscosity in the theoretical analysis. Antuono \textit{et al.}\textsuperscript{12} extended the mathematical sloshing model proposed by Antuono \textit{et al.}\textsuperscript{13} for rectangular tanks in shallow water conditions to study moderate to strong wave breaking from bores to plunger under both sway and roll motions. However, the model produced some spurious oscillations when the water depth was very shallow and the excitation was large. The role of flip-through in sloshing was studied in shallow water regime by Lugni \textit{et al.}\textsuperscript{14}. Particle Image Velocimetry (PIV) was used to examine the kinematics of the flip-through. Besides, the features of sloshing for high-filling depth, such as the kinematic and dynamic behaviour after water hits the ceiling of the tank, have also been studied. Faltinsen\textsuperscript{15} mentioned that high filling levels can lead to high impact pressure because of the occurrence of sudden flip-through. Abrahamsen and Faltinsen\textsuperscript{16} conducted experiment in a tank with high filling level to study an air pocket entrapped by a free surface water wave. Akyildiz \textit{et al.}\textsuperscript{17} conducted a series of experiments with three different filling levels, namely 25\%, 50\% and 75\%, in a cylindrical tank to elucidate how the filling level influences the responding sloshing wave amplitude. They concluded that the liquid sloshing becomes weaker with the increase of the filling level due to the damping effect. However, the resolution of the filling levels used above may not be fine enough to elaborate the relationship between the filling level and the sloshing wave amplitude responses.

1.1.2 Standing waves and impact pressure in a sloshing tank

As stated above, the performance of liquid motion resulting from sloshing depends on external excitation conditions. Besides, as an important part of sloshing motions, steep standing waves generated in a oscillating container through subharmonic resonance, also draw significant attentions\textsuperscript{18,19}. If the standing wave is generated by vertical motion, it is known as ‘Faraday wave’\textsuperscript{20} which has a frequency equal to half of that of the excitation\textsuperscript{21}. To excite the symmetric standing waves inside a tank, the
forcing frequency should double the even modes natural frequency of the liquid in the tank. To generate the asymmetric standing waves, the forcing frequency should double the odd modes natural frequency of the liquid in the tank. The mode here refers to the sloshing mode.

Jiang et al.\textsuperscript{19} studied the symmetric standing waves about the tank centreline in a vertically oscillated tank. They excited the tank by varying the frequency from 3.15 Hz to 3.34 Hz. Dimple crest and flat wave crest were found when the forcing amplitude was large (3.85 mm). They also indicated that the waves with flat and dimple crest is in temporal asymmetry and the second harmonic is strong in the wave form. Jiang et al.\textsuperscript{22} extended the work reported in Jiang et al.\textsuperscript{19} by increasing the forcing amplitude to 4.60 mm under a forcing frequency of 3.20 Hz. Period tripling breaking phenomenon was observed, which means that three kinds of wave breaking (steep crest, flat crest with two plungers and round crest) appear alternatively. Longuet-Higgins and Drazen\textsuperscript{23} studied standing waves by considering waves reflected at a vertical wall and confirmed the existence of wave period tripling breaking. Bredmose et al.\textsuperscript{24} generated standing waves by oscillating a tank vertically with an amplitude of 30 mm and a frequency of 1.25 Hz. Wave patterns with flat crest and sharp crest were reported. Longuet-Higgins and Dommermuth\textsuperscript{26} reported the existence of standing wave profiles with round and sharp crests using experimental methods. Apart from experimental investigations, a lot of numerical simulations were also conducted to study the standing waves. Bredmose et al.\textsuperscript{24} employed the Boussinesq mode for numerical simulations and reproduced the free-surface motion accurately. Mercer and Roberts\textsuperscript{25} studied two-dimensional standing waves via a stable numerical method and concluded that the steep standing waves are unstable to subharmonic perturbations. Although a lot of previous studies have confirmed the existence of wave breaking shapes reported by Jiang et al.\textsuperscript{22}, there are still problems related to the wave period tripling breaking, a phenomena which was initially found when the wave profile was symmetric about the tank centreline. What will happen if the wave profile is asymmetry about the centreline of a sloshing tank? Bredmose et al.\textsuperscript{24} obtained the standing wave in an asymmetric profile and found sharp and flat wave crests. However, the tank aspect ratio (tank length over tank width) in their study is only 3.71:1, which cannot warrant a two-dimensional flow condition inside the tank. Therefore, the primary aim of this thesis is to examine the existence of the wave period tripling breaking under the excitation of asymmetric wave mode at various excitation amplitudes.
Apart from wave motions, the impact pressure related to it is also important in quantifying the forces on the tank wall. Impact pressure is widely studied in past decades. Cooker and Peregrine\textsuperscript{27} employed pressure-impulse theory to analyse large pressure due to violent impacts. Peregrine\textsuperscript{28} reviewed the theoretical work of violent impacts of water waves on tank walls. He indicated that flip-through is a very interesting phenomenon in fluid dynamics of wave impact, which will produce a very high pressure on the tank wall. Lugni \textit{et al.}\textsuperscript{14} studied experimentally via PIV the high pressure of flip-through inside a sloshing tank and confirmed that the impact point of flip-through is higher than the still water level, which is in agreement with that reported by Hull and Muller\textsuperscript{29}. Lugni \textit{et al.}\textsuperscript{30} further investigated the influence of air cavity on the pressure of wave impact and concluded that the pressure decay is affected by the air leakage from the cavity. Song \textit{et al.}\textsuperscript{31} studied experimentally the pressure under flip-through and showed that pressure is influenced by trapping air pockets, and the impact point is at still water level. From the above review, it seems that there exist controversial conclusions regarding the location of the impact pressure. This forms the second aim of the present thesis.

\subsection*{1.2 Objectives}

The present study aims to:

(a) investigate the relationship between the maximum responding wave amplitude and tank filling levels. For this purpose, a series of experiments were conducted in a two-dimensional rectangular tank which was filled at various filling levels and excited at the fundamental resonant frequency of internal sloshing with excitation amplitude (0.127 cm, 0.254 cm, 0.508 cm and 0.635 cm). The mechanism for the occurrence of the maximum responding wave amplitude was examined by theoretical analysis.

(b) investigate the patterns of wave breaking under horizontal, vertical and combined horizontal and vertical excitations of various amplitudes in the 2D sloshing tank at the third mode natural frequency, corresponding to one and a half wave length inside the tank.

(c) investigate the differences of wave period tripling breaking observed under horizontal excitation, vertical excitation and combined horizontal and vertical excitations when the wave profile is asymmetric and to classify the wave regime under the latter excitations.
(d) examine the pressure variation during wave period tripling breaking under both horizontal and combined horizontal and vertical excitations and to quantify both the location and the magnitude of the impact pressure on the wall.

1.3 Methodologies

Four sets of experiment have been carried out in the present thesis by oscillating the two-dimensional tank fixed on a hexapod with 6-degree-of-freedom (6DOF) for a given format. The experiments for Objective (a) are conducted under the lowest natural frequency while those for the other three objectives are conducted under the third mode natural frequency.

For the experiments related with Objective (a), the responding wave amplitudes measured using a wave gauge are analysed via a statistical tool called box plot. Calculations related with the theoretical analysis are carried out based on the program written using MATLAB.

For the experiments related with wave breaking under horizontal excitation, the wave patterns are examined based on the videos recorded by a high speed camera and the data measured by two wave gauges whereas the impact pressure are measured using 9 pressure sensors. Wave elevation data are also analysed using the Hilbert-Huang Transform (HHT). Phase portrait and Poincare map are examined to investigate the wave period tripling breaking.

For the experiments related with wave breaking under vertical excitations, a small horizontal excitation is applied for the first 10 excitation cycles for each case to provide an initial perturbation to the water inside the tank. This is followed by regular vertical excitations to generate the Faraday wave. Similar to other experiments, wave breaking patterns and wave elevations are obtained by the high speed camera and two wave gauges, respectively. Hilbert-Huang Transform and phase portrait are employed to analyse the data.

For the experiments under coupled horizontal and vertical excitations, the hexapod provided two-degree of freedom excitations to the tank. The data acquisition and analysis methods are similar to the above experiments.

1.4 Outline of the thesis

There are in total 6 chapters in this thesis. Below is the breakdown for each chapter:
Chapter 1 is the introduction part which gives the background and a brief literature review;

Chapter 2 is for the sloshing experiments at different filling ratios (Objective a), aiming at finding the relationship between the maximum responding wave amplitude and the filling level;

Chapter 3 investigates the wave performance including wave tripling breaking when the tank is excited by a horizontal motion at the third mode natural frequency. The results are analysed by HHT method. The magnitude and location of the impact pressure on the tank wall are quantified in this chapter (Objective b, c and d)

Chapter 4 concerns the wave performance including wave tripling breaking when the tank is excited by a sequential combination of horizontal (first 10 cycles) and vertical motions for different excitation amplitudes at the third mode natural frequency. (Objective b and c).

Chapter 5 focuses on the asymmetric sloshing wave performance under combined horizontal and vertical excitations of various excitation amplitudes. The wave performance inside the tank is classified into four regimes based on the wave patterns and each regime includes two or more types of wave. The magnitude and location of the impact pressure during period tripling breaking are also investigated (Objective b, c and d).

Chapter 6 summaries the main findings from the study and gives some recommendations for future work.

References


Figure 1-1 LNG vessel carrying tanks which may induce sloshing effect
Chapter 2

Sloshing performance in a partially-filled two-dimensional rectangular tank

Abstract

A series of experiments were conducted to investigate the fluid sloshing at different filling levels in a rectangular tank. For each filling level, fluid sloshing was oscillated by forced tank motions in the horizontal plane with variable amplitudes but fixed frequency which is equal to that of the 1st sloshing mode. During these tests, the pressure on the side wall and the water surface elevations inside the tank were recorded synchronously. Tests under one configuration were run for 30 times to check the repeatability. The filling level at which the maximum sloshing amplitude occurs, or the so-called critical filling level, is found to decrease as the excitation amplitude increases. Theoretical analysis is conducted to provide physical understanding of this phenomenon. It is also found that increasing the damping of the system leads to a shift of the critical filling level.

2.1 Introduction

Sloshing is a flow phenomenon that usually occurs in a partially-filled tank when the excitation frequency approaches the natural frequencies of the fluid sloshing modes. When sloshing occurs, the fluid inside tanks is subjected to violent oscillations which may induce significant localized impact pressure on tank walls. It may further induce structural damages and destabilization to a vessel\(^1\) under extreme conditions. Hence, sloshing phenomenon is of practical interest in offshore oil and gas and shipping industries.
Sloshing has been attracting lots of attentions over the past decades. For rectangular tanks, the natural frequencies of the internal sloshing modes can be calculated through the equation (Abramson et al.\textsuperscript{2}) below,

\[ \omega_n^2 = \pi n \left( g \frac{h}{L} \right) \tanh\left( \pi n \frac{h}{L} \right) \]

where \( n \) is the mode number of the internal sloshing, \( l \) is the tank length, \( g \) is the gravitational acceleration and \( h \) is the filling depth of the tank. Specifically, \( \omega_1 \) denotes the fundamental frequency (i.e. \( n = 1 \)). Apart from the theoretical calculation, experiments are also often carried out to determine the natural frequencies through free decay tests (e.g. Kobayashi et al.\textsuperscript{3}). Pal et al.\textsuperscript{4} experimentally investigate the relationship between sloshing amplitude and the excitation frequency. The fundamental sloshing frequency (the first mode) at different filling levels were founded to be different with that calculated by the theoretical method (i.e. Eq. (2.1)), showing a discrepancy around 4%.

Sloshing responses under large and medium excitation amplitudes have been studied extensively in the past decades, for example, Ji et al.\textsuperscript{5}'s experimental study with large amplitude (the ratio between forcing amplitude and tank length \( b/L = 0.1 \)), and Wei et al.\textsuperscript{6}'s study on slamming pressure inside a sloshing tank with large amplitude excitation (\( b/L = 0.102 \) to 0.116). Royon-Lebeaud et al.\textsuperscript{7} conducted sloshing experiment in a square tank under a median excitation amplitude (\( b/L = 0.0125 \)). They investigated wave crest destabilization in detail and interpreted the cross-wave instability in terms of parametric instability. Sloshing under small excitation amplitudes was also studied using experimental methods, e.g. Pal and Bhattacharyya\textsuperscript{4} studied the liquid movement in a sloshing tank under five different excitation amplitudes (\( b/L = 0.005, 0.01, 0.015, 0.02, 0.025 \), the first one is considered as small while the rest are considered as moderate). The sloshing amplitude was found to follow a sinusoidal pattern. i.e. slosh amplitude increases initially with the increase of applied frequency and thereafter it decreases and again with the increase in external frequency slosh amplitude increases.

Apart from the excitation amplitude, the liquid filling depth inside the tank is another crucial factor that influences sloshing responses. Low filling depths have been paid much attention. Verhagen and Wijngaarden\textsuperscript{9} observed a hydraulic jump, which travelled periodically back and forth inside the tank with low filling levels under roll excitation. Antuono et al.\textsuperscript{10} extended the mathematical sloshing model proposed by Antuono et al.\textsuperscript{11} for rectangular tanks under shallow water conditions to study moderate to strong wave breaking from bores to plunger under both sway and roll motions. The
role of flip-through in sloshing was studied in shallow water regime by Lugni et al.\textsuperscript{12}. Particle Image Velocimetry (PIV) was used to examine the kinematics of the flip-through. Besides, the features of sloshing for high-filling depth, such as the kinematic and dynamic behaviour after water hits the ceiling of the tank, have also been studied. High filling levels can lead to high impact pressure because of the occurrence of sudden flip-through\textsuperscript{13}. Abrahamsen and Faltinsen\textsuperscript{14} conducted experiments in a tank with high filling levels to study an air pocket entrapped by a free surface wave. Akyildiz et al.\textsuperscript{15} conducted a series of experiments with three different filling levels, namely 25\%, 50\% and 75\%, in a cylindrical tank to elucidate how the filling level influences the response amplitude. They concluded that the liquid sloshing becomes weaker with the increase of the filling level due to the damping effect. Due to the limited number of filling levels studied, it was not possible to elaborate the relationship between the filling level and the sloshing wave amplitude responses.

The majority of the existing study has been focused on the sloshing phenomena at very low and high filling levels with interest in the local nonlinear phenomena. However, there is practical interest in the relationship between the maximum sloshing motion and filling levels under small excitation amplitudes because it will contribute directly to safe operations at sea. Furthermore, the effect of the excitation amplitude on the natural frequency of the internal sloshing is of great concern. In this study, the critical filling level at which the maximum sloshing response occurs is investigated experimentally, where the rectangular tank is excited horizontally with a given amplitude at the fundamental natural frequency of the internal sloshing over a range of filling levels. The critical filling level at a given excitation amplitude is then quantified and the physical explanation responsible for this phenomenon is explored.

The paper is organized as follows: the experimental set-up, including the calibration of the sensors, is firstly described in Sec. 2.2; Sec. 2.3 shows the results of the measured and calculated natural frequencies, and the discussion of experimental results. The conclusions are summarized in Sec. 2.4.

### 2.2 Experimental set-up

The experiments were conducted in the Sloshing Laboratory at The University of Western Australia. A rectangular tank with a breadth \( B = 100 \) mm, a length \( L = 1300 \) mm and a height \( H = 900 \) mm was used. The tank was fixed on a hexapod which is capable of generating motions in six-degrees-of-freedom (6DOF). A photo of the
hexapod is shown in Figure 2-1, together with the rectangular tank. The tank was excited with a sinusoidal oscillating motion \( x = b \sin(\omega t) \) along the longitudinal direction \( x \), where \( b \) is the excitation amplitude and \( \omega \) is the prescribed angular frequency which is identical to the fundamental sloshing frequency for a given filling level. In total four excitation amplitudes, i.e. \( b/L = 0.001, 0.002, 0.004, 0.005 \), were used to examine the sensitivity of the test results to the excitation amplitude. The filling level covered in this study is in the range from \( h/L = 0.069 \) (the ratio of fluid depth over the tank length) to \( h/L = 0.554 \). With different combinations of the filling levels and the excitation amplitudes, totally 76 cases were tested in this chapter.

In the experiments, the wave amplitude, pressure on the side wall and the tank movement were measured simultaneously by a wave gauge, four pressure sensors and an ultrasonic sensor, respectively (Figure 2-2). In order to eliminate the aliasing signals, a low pass filter was applied before the signals pass into the DAQs. A capacitance wave gauge (with an accuracy of 0.5%) was fixed at 10 mm away from the sidewall. Four Kulite pressure sensors (XCL-8M-100-3.5BARA) of 2.6 mm in diameter with a measurement range of 350 Kpa and an accuracy of 0.1%\(^{16}\) were used to capture the pressure variation on the side wall. The pressure sensor is capable of measuring two-phase flow. To capture the maximum wave pressure acting on the wall, the pressure sensors are arranged in such a way that the first one is 15 mm above the still water level and the rest three are below it with an equal distance of 15 mm between any two adjacent pressure sensors. Such an arrangement of pressure sensors is motivated by the experiences of previous independent studies where the maximum wave pressure acting on the wall usually was found to occur at the still water level or above\(^{12,17}\). A sampling frequency of 5 kHz was employed for all the measurement equipment. The tank movement was measured by an ultrasonic sensor (SU1-B1-0A from Micro Detectors, with an accuracy of 0.2%). According to the investigation of Bulian and Botia-Vera\(^{18}\), temperature control loop exerted little influence on the sloshing experimental results. Consequently, all experiments in the present paper were conducted at a normal room temperature.

The hexapod motion is synchronised with other instruments and the data acquisition system (DAQ). The flow chart of the synchronization was illustrated in Figure 2-3. The DAQ consisted of a chassis (U2781A) which is responsible for acquiring the signal from the measurement and a module (U2351A) responsible for signal generating. The two modules (U2541A) with four simultaneous differential channels in each are responsible for obtaining signals from the pressure sensors, the wave gauge and the
ultrasound sensor. The simultaneous differential channels ensure the synchronization during the data acquisition process.

2.3 Results and discussion

2.3.1 Natural frequencies of sloshing at different filling levels

Prior to the experiments, the instruments were calibrated and the calibration results are shown in Figure 2-4. Excellent correlations have been found between the measurement quantities and voltage as shown in Figure 2-4.

The angular frequency for various sloshing modes \( \omega_n \), or the frequency ratio, \( \omega / \omega_n \), is an important parameter for the study of sloshing, where \( \omega_n \) is the natural frequency for the \( n \)-th mode and \( \omega \) is the forcing frequency imposed. Since all the experiments were conducted under the fundamental frequency \( (\omega / \omega_n = 1 \text{ with } n = 1) \), it is paramount to find the fundamental frequencies accurately for different tank filling levels. In this study, we obtain the natural frequencies experimentally and verify them via the theoretical calculation. In the experiments, the fundamental sloshing frequency for a given filling level is obtained by performing FFT to the measured free decay of the wave elevation excited under a small horizontal stroke. The peak frequency on the spectrum is regarded as the fundamental sloshing frequency. These values for different filling levels are listed in Table 2-I, together with those calculated using Eq. (2.1). It can be seen that the agreement between the calculated natural frequencies and the measured ones is satisfactory with a relative error less than 1.1%.

2.3.2 Repeatability of the impact pressure and wave amplitude

The time histories of the pressure signals and the wave amplitudes have been measured for different filling levels. To examine the repeatability of the extreme impact pressure, a case with filling level \( h/L = 0.242 \) under an excitation amplitude of 0.762 cm and an excitation frequency of 3.865 rad/s (which is equal to the natural frequency as shown in Table 2-I) has been repeated for 30 times and the results are analyzed through a statistical method, the so-called boxplot.

Figure 2-5 shows a typical time history of impact pressure recorded by Sensor No. 1 mounted on the side wall of the tank, together with the corresponding time histories of wave elevation and tank displacement. Please note the water elevation shown in Figure 2-5 (b) is measured relative to mean water level in the tank. Since the pressure sensor 1
is located slightly above the mean water level, the minimum measured pressure is zero when the sensor is exposed to air. The measured pressure shows good periodicity over the testing period. It is seen that within a typical period of tank displacement (e.g. the period between the blue dash lines in Figure 2-5) the pressure variation is characterised by two pressure peaks (marked by ‘1’ and ‘2’) and one trough (marked by ‘3’). A close-up view of the time histories of pressure, water elevation and displacement between the blue dash lines in Figure 2-5 (a) are shown in Figure 2-6, together with illustrations of tank locations and water levels at time instants corresponding to the middle 3 dash lines. It should be noted that Figure 2-6 is plotted in the phase space where the phase angle ($\phi$) of 0° corresponds to the time instant when the tank is at the left-most location (the left blue dash line shown in Figure 2-5c) and $\phi = 180^o$ corresponds to time instant when the tank is at the right-most location (the middle blue dash line shown in Figure 2-5c). It can be seen from Figure 2-6b that the water elevation near the left wall of the tank (25 mm from the wall) remains below zero and the water pressure measured by Sensor 1 fluctuates around zero as the tank moves back from left to right until about $\phi = 156.2^o$. This is because the water level on the right end of the tank is high (above zero) during this period. As the tank continues its right-ward movement from $\phi = 156.2^o$, both water elevation and impact pressure at the left wall start to increase. The pressure at the Sensor 1 location reaches the maximum value at $\phi = 186.0^o$ while the water elevation continues to increase until reaching the maximum at $\phi = 231.0^o$. The maximum pressure would normally be expected when the tank stops $\phi = 180.0^o$. The slight phase difference is believed to be due to the viscous damping induced by the tank walls. The continuation of the water elevation increase for $186.0^o < \phi < 231.0^o$ is due to the inertia of the water mass. It is notice the phase difference between the maximum pressure and the maximum water elevation is about 45°. As the cylinder accelerates back towards the left with $186.0^o < \phi < 231.0^o$, the pressure decreases and reaches to a trough at $\phi = 231.0^o$. This pressure decrease is induced to balance the positive inertia force (in $x$ direction) acting on the water mass due to the left-ward acceleration of the tank. Since the tank acceleration reaches the maximum at $\phi = 231.0^o$ (would have been at $\phi = 225.0^o$ if there were no phase delay), the inertia on the water mass also reaches to the maximum at $\phi = 231.0^o$. That is the reason that the pressure hits a trough at $\phi = 231.0^o$. The inertia force starts to decrease from $\phi = 231.0^o$ and reaches zero at $\phi = 276.7^o$. This is the reason the pressure starts to increase from $\phi = 231.0^o$ and reaches the second peak at $\phi = 276.6^o$. The sharp pressure drop observed from $\phi = 276.6^o$ to $\phi = 301.3^o$ is mainly
due to the rapid water level drop during this period. After that the water level drops below zero till to next period. Similar variations of pressure and water elevation to those observed in this study were also reported elsewhere\textsuperscript{12, 19-21}.

The repeatability of the measured pressure in sloshing tests is examined through the box plot approach. The pressure signals measured by Sensor No.3 for all the 30 tests are plotted in Figure 2-7 using a box plot. This method has been demonstrated to be useful in analysing experimental data with poor repeatability\textsuperscript{18}. The enlargement at the bottom of the figure shows the definition of each of the items. The upper and lower limits of the blue box indicate the third quartile (Q3, 75\% percentile data) and first quartile (Q1, 25\% percentile data) of the sampled data, respectively. The central line (red) is the 50\% percentile (the median). The interquartile range (IQR) means the difference between the values of the third quartile (Q3) and the first quartile (Q1). The red crosses, representing the values which are not in the range between Q1-1.5IQR and Q3+1.5IQR, are called the outliers. The upper and lower whiskers indicate the maximum and minimum of the non-outliers, respectively.

As can be seen, although the upper whiskers vary among different tests, they fluctuate around a mean value of 3 kPa (within a scattering range of \(\pm 13\%\)). The outliers, which are regarded as the peak pressure values, can be very large for some cases. For example, the peak pressure measured in tests 12, 14 and 19 are much larger than those observed in the rest of the tests. The outliers for all tests except 12, 14 and 19 are between 3 kPa and 4 kPa. As a preliminary conclusion, it can be seen that the measured pressure signals show non-satisfactory repeatability in the peak pressures. However, the repeatability of the third quartile, first quartile and the median for different tests is excellent (with a deviation being less than 1%).

The repeatability of the wave surface amplitudes among the 30 tests is examined in Figure 2-8 using the box plot method. The averaged value of the upper whiskers among the 30 tests (\(h/L = 0.242, b = 0.762 \text{ cm}\)) is about 573.5 cm with a standard deviation of about 3.5\%. It is clearly seen that the repeatability of the wave amplitude is good for the upper whiskers, first and third quartile and the median, although the outliers are found in tests 5, 12, 17 and 27. These outliers are within 3\% of the upper whisker. Therefore, it is considered that the repeatability of the outliers is acceptable as well. Based on the results shown in Figure 2-7 and Figure 2-8, it can be seen that the third quartile, first quartile and the median exhibit satisfactory repeatability from the boxplots.
2.3.3 Comparison of sloshing wave amplitude for different filling levels

Sloshing responses in the tank were measured for the filling depth listed in Table 2-I. For each filling level, the tank was excited under sinusoidal movement with the same frequency as the internal sloshing natural frequency. For each case, the test lasted for 180 s to include at least 60 pressure peaks. Figure 2-9 (a) shows the wave amplitude for different filling levels from $h/L = 0.069$ to $h/L = 0.554$ under $b/L = 0.001$. In order to show the completeness of the results, the boxplot of the wave amplitude is also included in the figure. The curve connecting the boxes through the median values shows a general variation trend of water amplitude with the filling level. Obviously, the standing wave amplitude increases with $h/L$ initially, peaks at $h/L = 0.332$ (the critical filling level) and then decreases with $h/L$ till the highest filling level ($h/L = 0.554$) covered in the experiments.

It can be seen in Figure 2-9 (b) ($b/L = 0.002$), the wave amplitude increases mildly from less than 50 mm to 200 mm as the filling level is increased from $h/L = 0.069$ to $h/L = 0.242$, followed by a sharp increase up to 400 mm when the filling level is further increased to $h/L = 0.298$ and peaks at about $h/L = 0.312$. A decreasing trend is then found afterwards. The critical filling level for the case shown in Figure 2-9 (b) is $h/L = 0.312$.

Figure 2-9 (c) shows the sloshing wave amplitude under $b/L = 0.004$ for filling levels from $h/L = 0.069$ to $h/L = 0.415$. For this excitation amplitude, tests were conducted up to $h/L = 0.415$, above which the water jet hits the ceiling of the tank, exerting some influence on the main water body. It is found that the sloshing wave amplitude increases till reaching the peak at the filling level $h/L = 0.287$, followed by a descending trend. Apparently, the wave amplitude increases by almost 100 mm when the external excitation increases from 0.254cm to 0.508cm because of the increase of the excitation energy.

When the excitation amplitude is increased to 0.635 cm (Figure 2-9 (d), $b/L = 0.005$) the trend of the curve is very similar to that shown in Figure 2-9 (a, b and c), but the maximum amplitude occurs at $h/L = 0.263$ (the critical filling level), which is smaller than the critical filling level observed for other three cases. To summarize the influence of the excitation amplitude, the variation of the critical filling level with the excitation amplitude is plotted in Figure 2-10 for all the cases tested in this study. It is seen that the critical filling level inside the tank decrease monotonically (almost linearly) with the increase of excitation amplitude. It can be expected when the excitation amplitude
approaches 0, the critical filling level approaches to around 0.34, which is close to the theoretical value reported by Faltinsen et al.\textsuperscript{23}

The variation trends of the response amplitude with the filling level can be explained by using the concept of critical depth\textsuperscript{23-25}, which is defined as the depth of water below which the sloshing response is identified as ‘hard-spring’ and beyond which as ‘soft-spring’\textsuperscript{26}. For the ‘hard-spring’ behaviour, the amplitude of the standing wave increases with the increase of the excitation frequency (or the filling level) whereas for the ‘soft-spring’ behaviour, the wave amplitude decreases with increase of the excitation frequency (or the filling level). Hill\textsuperscript{25} studied two-dimensional weakly-nonlinear standing waves under small excitation amplitude by considering the effects of weak viscosity, general water depth, and transient wave evolution. He used a multiple-scales analysis to obtain an amplitude evolution equation. From such an equation, a critical depth of $h/L = 0.324$ was estimated for infinitesimal amplitude of tank oscillation. Faltinsen\textsuperscript{13} also studied the critical depth and found it is around $h/L = 0.3374$. The present critical depths of $h/L = 0.332, 0.312, 0.287$ and 0.263 for the cases with $b = 0.127 \text{ cm (} b/L = 0.001), \ b = 0.254 \text{ cm (} b/L = 0.002), \ b = 0.508 \text{ cm (} b/L = 0.004) \text{ and } b = 0.635 \text{ cm (} b/L = 0.005) \text{ respectively, are slightly different from the critical depth of } h/L = 0.324 \text{ by Hill}\textsuperscript{25}. \text{ This is attributed to the differences between the experimental setups and the assumptions made in the theoretical analyses (e.g. Hill}\textsuperscript{25}). \text{ The analysis method suggested by Hill}\textsuperscript{25} is applicable to situations where the excitation amplitude is small and influence of viscous damping is weak (because only skin friction due to oscillatory boundary layers is considered), whereas the excitation amplitudes are finite and viscous damping due to tank corners and mixing at the free surface may not be small.

To further illustrate the “hard spring & soft spring” behaviour and the influence of the excitation amplitude on the critical filling level, the method proposed by Hill\textsuperscript{25} is implemented in the present study to predict the steady-state response. For completeness of the discussion, the method proposed by Hill\textsuperscript{25} is briefly outlined here. The dimensionless parameters (indicated by a superscript asterisk) involved in the analysis are defined first, following the same normalization method by Hill\textsuperscript{25},

$$B^* = \frac{B}{L}, \ \omega^* = \frac{h}{\sqrt{g/L}}, \ \delta^* = \frac{\delta}{\sqrt{g/L}}, \ A^* = \frac{A}{\varepsilon^{1/3}L}, \ \nu^* = \frac{\nu}{\varepsilon^{2/3}L^2 \sqrt{g/L}}, \ \varepsilon = \frac{b}{L}$$

where $\delta(= f - f_0)$ is a detuning parameter measuring the difference between the frequency of the external oscillation and the natural frequency of the oscillation mass in the tank, $\varepsilon$ is the ratio of external forcing amplitude to the tank length $L$, $B$ is the tank
breadth and $A$ is the response amplitude, $\nu$ is the viscosity of the fluid inside the tank. Hereafter, the superscript asterisks are dropped out to simplify the expression in the following discussion and expressions.

The equation governing the steady-state response of a two-dimensional wave in the tank is suggested by Hill\(^{25}\) as:

$$m^2 |A|^4 - 2m(\alpha + \delta) |A|^3 + [\alpha^2 + (\alpha + \delta)^2] |A|^2 - \beta^2 = 0 \quad (2.2)$$

where $\alpha$ is the damping coefficient and is defined as

$$\alpha = \frac{1}{n\pi} \sqrt{\frac{\nu \omega}{2}} \left[ 1 + \frac{n\pi(1 - 2h)}{\sinh(2n\pi h)} \right] \quad (2.3)$$

where $n$ is the mode number as already used in Eq. (2.1). In this study, $n$ is set as 1 to focus on the fundamental mode of sloshing in the tank. $\beta$ is the forcing coefficient, which is given by

$$\beta = \left[ 1 + (-1)^{(n-1)} \right] \frac{1}{\sqrt{n\pi}} \left[ \tanh(n\pi h) \right]^{3/2} \quad (2.4)$$

$m$ is a nonlinear interaction coefficient and is defined as

$$m = \frac{\omega n^2 \pi^2}{256 \sinh^5(n\pi h)} \left[ - \cosh(6n\pi h) + 6 \cosh(4n\pi h) + 24 + 7 \cosh(3n\pi h) \right] \quad (2.5)$$

The “hard spring & soft spring” behaviour observed in the tests is illustrated through the model proposed by Hill\(^{25}\). For different values of $\delta$, Eq. (2.2) is solved without damping for a given water filling depth $h/L$. The results for six filling levels, with three lower and three higher than the critical depth ($h/L = 0.324$) obtained by Hill\(^{25}\), are shown in Figure 2-11. For $h/L < 0.324$ (Figure 2-11a), the response wave amplitudes (the branch marked by red arrows) increase with the increase of the external excitation frequency. This behaviour is consistent with that of a ‘hard-spring’. The red ($h/L = 0.20$), blue ($h/L = 0.27$) and black ($h/L = 0.32$) filled circles stand for the intersections between wave amplitudes and the vertical line where detuning factor equals to zero, (i.e. the external excitation frequency is the same as the lowest natural frequency). Obviously, when $h/L$ is lower than the critical depth (0.324), the wave amplitude increases with the increase of the filling level when $\delta = 0$. In Figure 2-11b, for $h/L > 0.324$, the response wave amplitudes (the branch marked by black arrows) increase with the decrease of the external excitation frequency, consistent with the behaviour of a ‘soft-spring’. The red ($h/L = 0.68$), blue ($h/L = 0.38$) and black ($h/L = 0.34$) filled circles stand for the intersections between wave amplitudes and $\delta = 0$. When $h/L$ is higher than the critical depth (0.324), the wave amplitude decreases with the increase of
the filling level when $\delta = 0$. From the analysis above, it is seen that when the filling level equals to the critical depth, the wave amplitude (when $\delta = 0$) would reach the peak value. The model predictions are consistent with the trends observed in our tests.

The variation trend of the critical filling level with the excitation amplitude is also examined through the model proposed by Hill. In order to obtain the results shown in Figure 2-12, a series of excitation amplitudes ($b$) are assumed first. For every value of $b$, by solving Eq. (2) and considering the damping effect ($\alpha$), a series of wave amplitudes at the corresponding filling levels can be obtained. The filling level for the maximum response wave amplitude is identified as the critical filling level. By repeating the above process for all excitation amplitudes, the corresponding critical filling levels are obtained. It can be seen that the critical filling level decreases monotonically with the excitation amplitude ($b/L$). The experimental results shown in Figure 2-9 also confirm that when the external forcing amplitude increases from 0.127 cm to 0.635 cm, the critical filling level decreases from $h/L = 0.332$ to $h/L = 0.263$. It should be noted that although the theoretical analysis predicts the variation trend successfully, the predicted critical filling levels do not agree well with the test results. For example, the calculated results in Figure 2-12 are $h/L = 0.3245, 0.3236, 0.3222, 0.3216$ when the forcing amplitudes are 0.127 cm ($b/L = 0.001$), 0.254 cm ($b/L = 0.002$), 0.508 cm ($b/L = 0.004$) and 0.635 cm ($b/L = 0.005$), respectively, which are larger than the experimental results shown in Figure 2-9. A possible reason is that the damping coefficient in Eq. (2.3) considers only the viscous boundary layer near the wall, whereas in reality, the damping effects due to losses at the tank corners and free surface will also be important. It is anticipated that an increase in damping of the system would be equivalent to a decrease in excitation amplitude $b$ because physically part of the input energy spent on dissipation is increased. Based on the results shown in Figure 2-12, it is expected that the critical filling level would increase with the increase of damping level in the system.

In order to confirm this trend, an additional experiment was conducted, where the damping in the system is deliberately increased by installing five pieces of small wooden baffles with 4 cm$^2$ square cross section on the tank wall and tank bottom (Figure 2-13). One baffle was aligned with the filling level while another one was mounted at 50% of the filling level. The other three baffles were attached to the bottom with equal intervals. No baffle was added on the right hand side wall of the tank because this experiment is aimed only at confirming a trend qualitatively. The experiment was conducted for $b = 0.508$ cm and at the lowest fundamental natural
frequency for filling levels in the range of $h/L = 0.208$ to $h/L = 0.346$. The experimental results with baffles are shown in Figure 2-14 (b). An increasing trend of sloshing amplitude from 200 mm to about 370 mm as the filling level changes from 0.208 to 0.312 is seen, followed by a slight decreasing trend as the filling level increases further. The maximum sloshing amplitude ($A = 350$ mm) is found at $h/L = 0.312$ (the critical filling level). It is seen that the critical filling depth is higher and the maximum response amplitude is smaller than their counterparts ($h/L = 0.287$ and $A = 545$ mm respectively shown in Figure 2-14 (a)) of the case without the baffles. This confirms the earlier inference.

### 2.3.4 Impact pressure for different filling levels

The impact pressure on the walls of the tank is important in quantifying the force experienced by the tank. Therefore, it is important to examine the dependence of the pressure on tank filling levels. Figure 2-15 shows the impact pressure measured by the 4 pressure sensors on the side wall for the case with $b = 0.254$ cm at filling levels from $h/L = 0.069$ to $h/L = 0.554$. With regard to the pressure variation through the median value, Figure 2-15 indicates an overall ascending process with a small fluctuation from $h/L = 0.069$ to $h/L = 0.277$ and a descending process from $h/L = 0.312$ to $h/L = 0.554$. The maximum pressure for all the pressure sensors is found at about $h/L = 0.312$, which is identical to the critical filling level. The variation trends of impact pressure shown in Figure 2-15 share a similar variation trend of the response amplitude measured near the left wall (Figure 2-9a). Figure 2-16 shows the impact pressure measured by the 4 pressure sensors for $b = 0.508$ cm at filling levels from $h/L = 0.069$ to $h/L = 0.415$. All the sensors capture a similar increasing trend from $h/L = 0.069$ to $h/L = 0.277$ and a decreasing trend from $h/L = 0.312$ to $h/L = 0.415$. The measured pressure peaks at $h/L = 0.287$ for all sensors, which is the same as the critical filling level for $b = 0.508$ cm (Figure 2-9b). Comparing the pressure responses shown in Figure 2-15 and Figure 2-16, it is found that the responding pressure for $b = 0.508$ cm is higher than that for $b = 0.254$ cm due to the increase of input energy. It is noticeable that for the two different excitation amplitudes, the filling levels for the maximum impact pressure are equal to those at which the maximum sloshing wave amplitude occurs.

In order to further investigate the pressure, boxplots of the measured pressure signals from the four sensors for four filling levels under $b = 0.254$ cm and $0.508$ cm are compared in Figure 2-17. Pressure sensor No. 4, which is installed at the lowest location
on the side wall, appears to record the largest pressure. The upper whisker and the third quartile for all pressure sensors increase monotonically with the increase of the sensor mounting depth. The present result seems to be different from that reported by Hull and Muller\textsuperscript{29} where the maximum wave impact pressure was found to occur at the still water level. The differences observed in the two sets of experiments are attributed to the differences in the experimental conditions. The present experiments were conducted under small excitation amplitudes and only very minor wave breaking is detected in cases for $h/L = 0.277$ to $0.298$ whereas in the study of Hull and Muller\textsuperscript{29}, significant wave breaking and flip-through were detected. It is noted that the upper limit in Figure 2-17f is an exception, for which, the upper limit of the pressure sensor No.1 is larger than that of No.2. This is most likely caused by wave breaking.

## 2.4 Conclusions

The dependence of the maximum wave amplitude and pressure on tank filling level has been examined experimentally in a rectangular tank under four different excitation amplitudes ($b/L = 0.001$, $0.002$, $0.004$ and $0.005$) at the frequency of the fundamental sloshing mode. The experimental data were analysed using the boxplot method to minimise the influence of lack of repeatability, especially for the time series of the impact pressure. The filling levels for the maximum wave amplitude were found to occur at $h/L = 0.332$, $0.312$, $0.287$ and $0.263$ for $b/L = 0.001$, $0.002$, $0.004$ and $0.005$, respectively, indicating that the critical filling level decreases monotonically with the increase of the excitation amplitude. It is found through a specifically designed test that an increase of the damping in the system is equivalent to the effect of a decreased excitation amplitude, which leads to an increase in the critical filling level and a decrease in the maximum response amplitude.

The occurrence of the critical filling level is explained by solving the duffing like equation for critical depth which categorises the sloshing wave into two types: hard-spring behaviour ($h/L<\text{critical depth}$) and soft-spring behaviour ($h/L>\text{critical depth}$). The theoretical analysis indicates that the critical filling level increases with the decrease of excitation amplitude, which is consistent with the experimental results.

The pressure on the tank sidewall is found to correlate well with the response amplitude of water level in the tank, suggesting that the sloshing pressure measured in this study is mainly caused by the hydrostatic force. This is attributed to the small excitation amplitudes.
Reference


TABLE 2-I. Comparison between measured and calculated natural frequencies and the peaked filling level for different excitation amplitudes

<table>
<thead>
<tr>
<th>$h/L$</th>
<th>Natural Frequency (rad/s)</th>
<th>Discrepancy (%)</th>
<th>Critical filling level for different $b$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
<td></td>
</tr>
<tr>
<td>0.069</td>
<td>2.252</td>
<td>2.277</td>
<td>1.10%</td>
</tr>
<tr>
<td>0.104</td>
<td>2.732</td>
<td>2.756</td>
<td>0.87%</td>
</tr>
<tr>
<td>0.138</td>
<td>3.114</td>
<td>3.116</td>
<td>0.06%</td>
</tr>
<tr>
<td>0.173</td>
<td>3.427</td>
<td>3.416</td>
<td>0.32%</td>
</tr>
<tr>
<td>0.208</td>
<td>3.685</td>
<td>3.667</td>
<td>0.49%</td>
</tr>
<tr>
<td>0.242</td>
<td>3.899</td>
<td>3.865</td>
<td>0.88%</td>
</tr>
<tr>
<td>0.253</td>
<td>3.955</td>
<td>3.925</td>
<td>0.76%</td>
</tr>
<tr>
<td>0.263</td>
<td>4.009</td>
<td>3.985</td>
<td>0.60%</td>
</tr>
<tr>
<td>0.277</td>
<td>4.076</td>
<td>4.075</td>
<td>0.03%</td>
</tr>
<tr>
<td>0.287</td>
<td>4.122</td>
<td>4.135</td>
<td>0.31%</td>
</tr>
<tr>
<td>0.298</td>
<td>4.167</td>
<td>4.165</td>
<td>0.05%</td>
</tr>
<tr>
<td>0.312</td>
<td>4.222</td>
<td>4.195</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.322</td>
<td>4.260</td>
<td>4.254</td>
<td>0.14%</td>
</tr>
<tr>
<td>0.332</td>
<td>4.296</td>
<td>4.284</td>
<td>0.28%</td>
</tr>
<tr>
<td>0.346</td>
<td>4.342</td>
<td>4.314</td>
<td>0.65%</td>
</tr>
<tr>
<td>0.415</td>
<td>4.521</td>
<td>4.494</td>
<td>0.60%</td>
</tr>
<tr>
<td>0.485</td>
<td>4.640</td>
<td>4.614</td>
<td>0.56%</td>
</tr>
<tr>
<td>0.554</td>
<td>4.719</td>
<td>4.734</td>
<td>0.32%</td>
</tr>
</tbody>
</table>
Figure 2-1 Photo of the sloshing experimental facility and the definition of the coordinate system.

Figure 2-2 Sketch of the tank and the arrangements of the sensors.
Figure 2-3 Flow chart of the signal synchronisation for pressure sensors, wave gauge and ultrasound sensor.

Figure 2-4 Calibration of the pressure sensors (a), ultrasonic sensor (b) and the wave gauge (c). The lines are the least squares linear fittings to the experimental data and the symbols are for the results measured by the sensors.
Figure 2-5 Time histories of the pressure signal obtained using sensor No. 1 ($h/L = 0.242, \ b = 0.762 \text{ cm}$) (a); wave elevation (b) and tank displacement (c). The numbers 1 and 2 indicate the pressure peaks, while number 3 indicates the pressure trough in one wave cycle; number 4 indicates the peak for wave amplitude.
Figure 2-6 A close-up view of the time histories of pressure, water elevation and displacement between the blue dash lines in Figure 2-5(a).
Figure 2-7 Check of repeatability of the pressure measured using sensor No.3 \((b = 0.762 \text{ cm})\) by repeating the experiments 30 times. IQR = Q3-Q1; The upper whisker and lower whisker indicate the maximum and minimum value within the range from Q1-1.5IQR to Q3+1.5IQR.

Figure 2-8 Check of repeatability of the wave amplitude measured using the wave gauge \((b = 0.762 \text{ cm})\).
Figure 2-9 Wave amplitude for various tank filling levels under different external excitations. (a) $b = 0.127$ cm, (b) $b = 0.254$ cm, (c) $b = 0.508$ cm and (d) $b = 0.635$ cm. The red ticks at X axis between $h/L = 0.242$ and $h/L = 0.346$ are $h/L = 0.253$, $0.263$, $0.287$, $0.298$, $0.322$ and $0.332$, respectively. The arrows in (a), (b), (c) and (d) indicate the peaked filling levels.
Figure 2-10 The dependence of critical filling level on excitation amplitude. The first red hollow circle is the theoretical value reported by Faltinsen et. al.\textsuperscript{22}.

Figure 2-11 Amplitude-frequency responses for nonlinear sloshing calculated using Eq. (2.3). (a) hard-spring behaviour; (b) soft-spring behaviour.

Figure 2-12 Dependence of critical filling level on external excitation.
Figure 2-13 Sketch of the experimental set-up with baffles mounted in the tank.

Figure 2-14 Wave amplitude without baffles (a) and with baffles (b) mounted on the tank walls for different tank filling ratio under the excitation amplitude $b = 0.508$ cm, $b/L = 0.004$. 
Figure 2-15 Comparison of pressure measured by the four sensors under b = 0.254 cm. Pressure sensor No. 1 is 15 mm above the still water level and the following ones are 15 mm lower than the one before it (as shown in Figure 2-2). The two red ticks at X axis are $h/L = 0.287$ and $h/L = 0.298$, respectively. The vertical arrows in (a) - (d) indicate the filling level for maximum pressure.
Figure 2-16 Comparison of pressure measured by the four sensors under $b = 0.508 \text{ cm}$. Pressure sensor No. 1 is 15 mm above the still water level and the following ones are 15 mm lower than the one before it (as shown in Figure 2). The two red ticks at X axis indicate $h/L = 0.287$ and $h/L = 0.298$, respectively. The vertical arrows in (a) - (d) indicate the filling level for maximum pressure.
Figure 2-17 Comparison using boxplots of pressure measured by different sensors for various tank filling levels and under different excitation amplitudes. (a, c, e, g): $b = 0.254$ cm; (b, d, f, h): $b = 0.508$ cm.
Chapter 3

Wave period tripling breaking inside a two-dimensional tank under horizontal excitations

Abstract

Sloshing wave asymmetric about the tank centreline is generated by oscillating a two-dimensional tank horizontally for forcing amplitude $b$ in the range from 1 mm to 40 mm under the third sloshing mode wave (i.e. one and a half wave length inside the tank). The results show that with the increase of $b$, the wave profile undergoes a transition from small non-breaking riding wave to period tripling breaking wave. When $b = 1$ mm, the wave inside the tank is normal standing wave with low elevation. In the process when $b$ is increased to 5 mm, the standing wave starts to deform. The wave surface becomes rough. When $b$ is increased to 10.5 mm, a small non-breaking riding periodical wave is generated. With a further increase of $b$ to 14.5 mm, a transitional periodical wave breaking starts to occur, which repeats in every seven wave cycles. Wave period tripling breaking occurs when $b$ reaches a threshold value of 32.5 mm, for which three modes, namely sharp crest (mode one), flat crest (mode two) and round crest (mode three), have been identified. Analysis using phase portrait and Hilbert-Huang Transform (HHT) methods indicates that the width of the orbits for each breaking mode is related to the range of magnitude of wave elevation and the tripling breaking is caused by a low frequency modulation. Flip-through occurs during mode three and the impact point is higher than the still water level. When $b$ keeps increasing, wave period tripling breaking becomes unstable without a regular pattern.
3.1 Introduction

Sloshing phenomenon is characterized by violent liquid motion inside a tank excited under an external force. The violent free-surface motion caused by sloshing is a crucial issue in many fields of engineering\(^2\), such as fuel tanks in aircrafts, spaceships, vehicles\(^3\) and floating liquefied natural gas (FLNG) production systems and transport tankers in offshore oil and gas industry, which, in most cases, have to be operated in various filling levels. The violent fluid motion inside the tank will give rise to high pressure, which may result in damage to the tank. Therefore, investigation of sloshing induced by violent free-surface motion is essential for safe operation of the partially filled tanks in the aforementioned areas.

The wave motions inside a sloshing tank have been investigated extensively. Royon-lebeaud \(et\ al.\)^4 conducted experiments in both circular and square-base cylindrical tanks and determined the bounds of different wave regimes including planar wave, swirl wave and chaos. Lugni \(et\ al.\)^5 studied the kinematic flow field of the role of air cavity in a wave impact using Particle Image Velocimetry (PIV) and classified the flow evolution into four stages, namely, the closure of the cavity to the wall (1), to an isotropic compression expansion of the cavity (2), to a subsequent anisotropic compression expansion (3), and to the final rise of the cavity along the wall (4). Ji \(et\ al.\)^6 conducted experiments on non-resonant sloshing with large amplitudes. They classified free surface waves into four categories, namely mild slope 2-D wave (mode-I), strongly nonlinear 2-D wave with hydraulic jump (mode-II), 3D regular wave in a longitudinal direction together with a lateral sloshing (mode-III) and chaotic 3-D wave (mode-IV). In their study, the second category, i.e. strongly nonlinear 2-D wave with hydraulic jump, was studied in details.

As an important phenomenon of wave motions inside a sloshing tank, steep standing waves draw significant attentions in the past years, which can be excited horizontally or vertically when the excitation frequency approaches the natural frequency. The one excited by vertical excitation is well known as the ‘Faraday waves’. For a detailed review on Faraday waves, readers are referred to Miles and Herderson\(^7\). Jiang \(et\ al.\)^8 studied the steep standing waves by oscillating a small tank vertically. The generated steep wave is symmetric with the tank centreline. Dimple crest and temporal asymmetry wave shapes were found. They also indicated that dimple crest is induced by the interaction between the first and the second temporal harmonics. Jiang \(et\ al.\)^9 extended the work of Jiang \(et\ al.\)^8 by increasing the forcing amplitude from 3.5 mm to 4.7 mm.
The phenomenon termed as period tripling breaking was reported, which means that three kinds of wave breaking (i.e. steep crest, flat crest with double plungers and round crest) appear alternatively. Longuet-Higgins and Drazen\cite{9} studied standing waves by considering waves reflected at a vertical wall and confirmed the existence of period tripling breaking. Bredmose \textit{et al.}\cite{2} conducted experiments to generate standing wave and reported the flat crest and sharp crest phenomena. They also employed Boussinesq model in their simulation and reproduced the free-surface motion accurately. Besides experimental studies, a lot of numerical simulations have also been conducted to study the standing waves in a sloshing tank. Mercer and Roberts\cite{10} studied two-dimensional standing waves via a stable numerical method and concluded that the steep wave is unstable to the subharmonic perturbations. Longuet-Higgins and Dommermuth\cite{11} calculated the energetic period wave and highlighted the existence of standing wave profiles with rounded crest and sharp crests. Although the aforementioned studies confirmed the existence of wave breaking shapes reported by Jiang \textit{et al.}\cite{1}, there are still problems relating to the period tripling breaking. For example, whether tripling wave breaking will occur if the wave is asymmetric under horizontal excitation. This is also the first aim of the present study.

Apart from wave motion, its impact pressure is also important in quantifying the forces on the tank wall. Impact pressure is widely studied in past decades. Cooker and Peregrine\cite{12} employed pressure-impulse theory to analyse large pressure induced by violent impacts. Peregrine\cite{13} reviewed the theoretical work of violent impacts of water waves on tank walls. He indicated that flip-through is a very interesting phenomenon in fluid dynamics of wave impact, which will produce a very high pressure on the tank wall. It seems that there exists controversial conclusion about the acting location of the impacting pressure. Lugni \textit{et al.}\cite{14} studied experimentally via PIV the high pressure induced by flip-though inside a sloshing tank and confirmed that the impact point of flip-through is higher than the still water level, which is in agreement with that reported by Hull and Muller\cite{15}. Song \textit{et al.}\cite{16} conducted experiments to study the pressure on the wall under flip-through and showed that pressure was influenced by trapping air pockets, and the impact point was at still water level. As for the location of impacting point, their findings were not consistent. Therefore, studying the pressure variation during the period tripling breaking is another primary aim in this study. For this purpose, asymmetric wave inside a two-dimensional tank is generated under sinusoidal horizontal excitation at the third mode natural frequency. Wave elevation is analysed using the Hilbert-Huang Transform (HHT), which is an empirically based data-analysis
This method is proved to be adaptive and can provide not only a more precise definition of particular events in time-frequency space than wavelet analysis, but also more physically meaningful interpretations of the underlying dynamic processes. HHT interprets wave nonlinearity as frequency modulation and the energy remains near the base frequencies. In the experiment, especially under large external forcing amplitude, the process is in a strong nonlinearity regime. Therefore, applying HHT could give a more robust interpretation to the complicated physical process. The experimental setup and methodology are described in Sec. 3.2. Sec. 3.3 shows the experimental results and discussion, which includes four main parts. Sec. 3.3.1 is about the wave without breaking; Sec. 3.3.2 is about the wave with a transitional periodical breaking; Sec. 3.3.3 and Sec. 3.3.4 are about the stable and unstable wave period tripling breaking. Conclusions are given in Sec. 3.4.

3.2 Experimental set-up

3.2.1 Description of the experimental facilities and measurements

A rectangular tank with a breadth \( B = 100 \text{ mm} \), a longitude length \( L = 1300 \text{ mm} \) and a height \( H = 900 \text{ mm} \) (Figure 3-1), has been used. The 13:1 tank aspect ratio ensures the waves inside the tank are two-dimensional. The tank is fixed on a hexapod which is capable of generating a vessel's motion in six degree of freedom (6DOF).

The sketch about the arrangements of the sensors is shown in Figure 3-2. Wave elevation in the experiments is recorded with a high-speed camera (240 frames per second) and measured by two wave gauges. In the experiments, the two wave gauges are fixed at 5 mm and 434 mm (one third of the tank length) away from the left side wall of the tank, respectively. One ultrasonic sensor is used to monitor the horizontal movement of the tank. Nine pressure sensors are mounted on the left tank wall. They were flush with the inner surface of the tank. Pressure sensor No.5 is at the still water level (SWL). Pressure sensors No.1 to 4 are located above the SWL with an interval of 15 mm while pressure sensors No.6 to 9 are placed below SWL with the same interval. The sampling frequency for wave gauges and ultrasonic sensor is 1 kHz. In order to capture the fast varying pressure signals, a sampling frequency of 10 kHz is employed for the pressure sensors. Prior to the experiment, all the pressure sensors were calibrated satisfactorily.

As shown in Figure 3-3, a synchronization of the hexapod, other instruments including the wave gauges, the ultrasonic sensor and the pressure sensors and the data
acquisition system (DAQ) system is built for the sloshing experiment. The DAQ system is composed of 5 parts including a chassis (U2781A) which is responsible for synchronization of other modules, three modules (U2541A) which are used for acquiring the signal from the sensors and a module (U2351A) which is for signal generation. Each module (U2541A) contains simultaneous and differential channels, guaranteeing the synchronization when acquiring experimental data.

### 3.2.2 Experiment methodology

The water depth inside the tank is fixed at 387.4 mm. A simple estimate of the ratio of depth to tank length (> 0.25) suggests that the phenomenon reported in this study occurs in the finite depth regime based on the criteria given in Faltinsen and Timokha. Sloshing at a particular filling ratio can be excited from the fundamental mode natural frequency to higher ones. The mode represents the number of half wave length excited by the external excitation. In the present study, the third sloshing mode wave (one and a half wave length), which is asymmetric with the tank centreline, will be generated by feeding in a driving sinusoidal signal ($b \sin(\omega t)$) to the 6DOF hexapod.

The amplitude $b$ for horizontal motion varies from 1 mm to 40 mm with an increment of 0.5 mm. The test matrix is shown in Table 3-I. The frequency of excitation is identical to the third mode natural frequency, which can be calculated by the following equation:

$$\omega_n^2 = \pi n \left( \frac{g}{L} \right) \tanh \left( \pi n \frac{h}{L} \right),$$

where $n$ is the mode number of the internal sloshing and for the third mode, it is equal to 3, $L$ is the tank length, $g$ is acceleration gravity and $h$ is the filling depth. For the present experimental setup, $f_3 = \omega_3 / 2 \pi = 1.33 \text{Hz}$, Figure 3-4 shows an example of the driving signal. For all experiments reported in this chapter, the horizontal motion lasts for 320 cycles after the wave becomes stable, and as a result, 320 wave periods can be obtained for the measured signals.

### 3.3 Results and discussion

Asymmetric standing wave is generated by oscillating the tank under the third mode natural frequency. It is found that with the increase of excitation amplitude, the wave inside the tank reveals a trend from non-breaking normal standing wave to period tripling breaking waves.
3.3.1 Non-breaking wave with occurrence of riding wave

When $b = 1$ mm, the wave inside the tank is normal standing wave with low elevation. The wave crest and trough occur at four places, namely, left tank wall, right tank wall and the locations approximate 434 mm and 867 mm away from the left side wall (results are not shown here). In the process when $b$ is increased to 5 mm, the standing wave starts to deform. The wave surface becomes rough. When $b$ is increased to 10.5 mm, a small jet is found on the wave peak at $x = 434$ mm. Six successive time frames from $t = 0.252$ s to 0.462 s are shown in Figure 3-5 to illustrate the formation process of the small jet. It occurs at $t = 0.294$ s (green arrow), $t = 0.378$ s and continues showing in the next two time frames, as indicated by black arrows. The jet looks like a small riding wave over the main one. The riding wave is hidden in the main wave and cannot be seen clearly sometimes (e.g. $t = 0.336$ s). It may be because when the main wave reaches its peak, the riding wave forms its trough. The riding wave is submerged in the main wave and cannot be seen. At $t = 0.378$ s, when the main wave reaches its maximum amplitude and starts to move downward, the riding wave appears and becomes even more evident at $t = 0.42$ s. When $t = 0.462$ s, both the riding wave and the main wave are lower in elevation than that at $t = 0.42$ s, but the riding wave is still discernible. After it merges with the main wave, the riding wave cannot be seen clearly until the next period.

In order to investigate the main wave and the riding wave quantitatively, the time series of the wave elevation measured at $x = 434$ mm is examined and the results for 10 wave cycles are shown in Figure 3-6, together with the corresponding tank movement. Whereas there is a good correspondence between the peaks of the tank movement and wave elevation, the latter around the trough has been deformed apparently, as indicated by black arrows. The phase of the trough of wave elevation lags that of the corresponding tank movement. In contrast, there is almost no phase lag around the crest of the tank movement and wave elevation. The deformation of the wave elevation and phase lag around the trough can be explained as follows. In Figure 3-6, when the main wave is in the descending process, i.e. moving from peak to trough, due to the influence of the riding wave, the position of the wave trough is not at $x = 434$ mm (i.e. the position of wave gauge 2), resulting in overestimated values than the real trough. When the wave is in the ascending process, the wave trough moves back to $x = 434$ mm, resulting in satisfactory correspondence between the real peak and the measured elevation. After moving back to $x = 434$ mm, the wave continues to rise without moving
along the tank length direction. It should also be noted that the deformation of the
elevation trough is not the same for every period. This is simply because the deviation
between wave gauge position and the wave trough is not the same for each wave cycle.

In order to find the patterns of the wave motion, the wave elevation displayed above
is further discussed based on the analysis using Hilbert-Huang transform (HHT). HHT
is an empirically based data-analysis method. It is adaptive, which is helpful to produce
physically meaningful representations of data from nonlinear and non-stationary
processes\textsuperscript{17}. Details of the HHT method can be found in Huang \textit{et al.}\textsuperscript{18} Here only a brief
introduction is provided below.

For an arbitrary time series $x(t)$, its Hilbert transform is:

$$y(t) = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau,$$  \hspace{1cm} (3.2)

where $PV$ is the Cauchy principal value. With this definition, $x(t)$ and $y(t)$ form a
complex conjugate pair, so an analytic signal is obtained, $z(t)$,

$$z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)}$$ \hspace{1cm} (3.3)

with

$$a(t) = \sqrt{x(t)^2 + y(t)^2}, \text{ and } \theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right)$$  \hspace{1cm} (3.4)

where $a(t)$ is the instantaneous amplitude and $\theta$ is the phase function. The
instantaneous frequency is defined as

$$f = \frac{d\theta}{dt}. \hspace{1cm} (3.5)$$

However, it is difficult to define the instantaneous frequency for arbitrary data
because for any function, to have a meaningful instantaneous frequency, the real part
of its Fourier transform has to have only positive frequency\textsuperscript{19}. Therefore, Huang \textit{et al.}\textsuperscript{17,19}
introduced the Empirical Mode Decomposition (EMD) method to solve the problem.
After applying the EMD to the original data, the results can be assured with meaningful
instantaneous frequency. The results are called intrinsic mode function (IMF). Each
IMF should satisfy the following two conditions:

- The number of extreme and zero-crossings must either equal or differ at most by
  one.
- The mean value of the envelope defined by the local maxima and minima is
  zero.

After getting IMFs, Hilbert transform will be applied to each IMF to get the
amplitude-frequency-time distribution. However, EMD sometimes will lead to mode
mixing, which is defined as a single Intrinsic Mode Function (IMF) either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components\textsuperscript{20}. As a result, Huang \textit{et al.}\textsuperscript{20} introduced the Ensemble Empirical Mode Decomposition (EEMD) method to overcome the deficiency. The first step of EEMD is to add a white noise to the original data, and then decompose the new data into IMFs. The final step is to obtain the ensemble means of corresponding IMFs as the result.

The EEMD results of the wave elevation time series are given in Figure 3-7. Totally 11 IMFs are included. The first 6 components are the high frequency noise. C7 is the uniform component for the main wave while C8 is likely the frequency modulation component which will be further discussed later.

Fast Fourier Transform (FFT) analysis is applied to the signals C7-C9 shown in Figure 3-7 and the results are given in Figure 3-8. It is seen clearly that the peak frequency of C7 is 1.328 Hz (Figure 3-8a), which is the same as the excitation frequency. Therefore, it is the most energetic component from the wave elevation at \( x = 434 \) mm. For C8 (Figure 3-8b), apart from the peak at the excitation frequency (1.328 Hz), another peak at 0.778 Hz is apparent and has a comparable height with that at the dominate frequency. For C9 (Figure 3-8c), only one peak at a frequency of 0.229 Hz is found, indicating the C9 is a uniform component. Based on the analysis above, it is believed that C8 is the frequency modulation component. The rest components (C10 and C11) are small in amplitudes and should make negligible contribution to the wave elevation when compared with other components (C7-C9).

To further examine the relationship between the main wave (C7) and the frequency modulation component (C8), the time series of C7 and C8 are shown together in Figure 3-9. It can be seen that C8 actually performs with a repeatability in every twelve wave cycles, as marked by I, II and III and divided by blue dash lines. Each repeatable cycle of C8 is divided into 4 parts, as marked by 1, 2, 3, and 4 respectively. Looking into the parts 1, 2, 3, and 4 in each repeatable cycle (I, II and III), they last for four, three, two, and three wave cycles, respectively. Although the lasting duration is different for different parts, the general shape is similar. Each part is characterized by two high peaks and one trough. For I2, I4 and II2, three peaks can be found, but the middle peak is much lower than its neighbours. Therefore, it can be considered as a trough. Based on the analysis above, it is found that the wave is periodical instead of random as reflected by the variations of C8. This feature can be found when \( b \) is varied from 10 mm to 13 mm. It should be noted that C8 is not strictly periodical because the variation of the
signals are not exactly the same in different cycles. However, the general shape of them is similar and the lasting duration is the same.

### 3.3.2 A transitional periodical wave breaking

When $b$ is increased further to 14.5 mm, a special periodical wave phenomenon is found. The wave shape repeats every seven wave cycles, which can be seen in Figure 3-10. Although this phenomenon can also be seen at the very beginning of the test under other forcing amplitudes (e.g. 15 mm), it is replaced by other stable phenomenon. Only when the forcing amplitude is 14.5 mm, will the periodical results last for the duration of the whole test. Therefore, it is called ‘transitional periodical breaking’.

As shown in Figure 3-10, a sharp and breaking crest is formed around $x = 434$ mm ($t = 0.5T$), followed by a heavily curving breaking wave crest facing left at $t = 1T$. At $t = 1.5T$, a round wave facing left which is caused by the collapse of the curving crest from $t = 1T$ is found. One crest facing right (at $x = 867$ mm and $t = 2T$) and another crest facing left (at $x = 434$ mm and $t = 2.5T$) are formed, creating high wave elevations at right and left side tank walls as shown at $t = 2.5T$ (green arrow) and $t = 3T$ (green arrow). Two narrow flat crests are formed at $t = 3T$ and $t = 3.5T$ at $x = 867$ mm and $x = 434$ mm, respectively. At $t = 4T$, at around $x = 867$ mm, a sharp crest which is similar to the one at $t = 0.5T$ appears, then followed by a curving crest facing right at around $x = 434$ mm shown at $t = 4.5T$, which is similar to the one at $t = 1T$ ($x = 867$ mm) but with opposite facing direction. The following phenomena shown at $t = 5T$ ($x = 867$ mm), $5.5T$ ($x = 434$ mm), $6T$ ($x = 867$ mm), $6.5T$ ($x = 434$ mm) and $7T$ ($x = 867$ mm) are similar to those at $t = 1.5T$ ($x = 434$ mm), $2T$ ($x = 867$ mm), $2.5T$ ($x = 434$ mm), $3T$ ($x = 867$ mm) and $3.5T$ ($x = 434$ mm). After that, the wave will start another period. From $t = 7.5T$ to $t = 9T$, the shapes are almost the same as the ones shown from $t = 0.5T$ to $t = 2T$.

It is noticed that the curving crests shown at $t = 1T$ ($x = 867$ mm) and $t = 4.5T$ ($x = 434$ mm) are similar in shape but with opposite facing directions. The formation process of the curving crest is elaborated below. The curving crest at $t = 4.5T$ ($x = 434$ mm) is taken as an example to illustrate. Before the occurrence of the curving crest at $t = 4.5T$, a sharp crest ($x = 867$ mm) and a flat crest ($x = 0$ mm) can be found at $t = 4T$. After carefully checking the videos between $t = 4T$ and $4.5T$, it is found that the water from both crests moves in the opposite directions and meet at around $x = 434$ mm. The water from the flat crest side arrives at $x = 434$ mm earlier than that from the sharp crest side. This is because when the sharp crest falls into the water, significant energy dissipation
occurs, and a deeper trough than usual is created. The deeper trough induces the lag for water to arrive at $x = 434$ mm. When the water from the sharp crest side arrives at $x = 434$ mm, the water from the flat crest side has already arrived at $x > 434$ mm. As a result, when the water from both sides collides at this location, the water from the sharp crest side actually hits the bottom of the water from the flat crest side. Then the curving crest forms. The forming process of the curving crest at $t = 1T$ is similar to the one illustrated above.

The time series of the wave elevation measured at wave gauge 1 ($x = 5$ mm) and wave gauge 2 ($x = 434$ mm) for transitional breaking wave are plotted in Figure 3-11. Three regions defined by I, II and III as the three periods for the transitional breaking wave have been marked. Within each region, seven wave cycles are included. The numbers $0.5T - 9T$ shown in Figure 3-11a and b stand for the wave crests corresponding to those in Figure 3-10. The peaks at II-3T and II-6T (Figure 10a) are much higher than other crests in period II. These two high elevations resulting from the thin crests at $x = 0$ mm are shown in Figure 3-10 at $t = 3T$ (green arrow) and $6T$ (blue arrow). According to the wave crests at $x = 434$ mm shown in Figure 3-10, the wave elevations for $t = 0.5T$ and $t = 4.5T$ should be higher than others. However, in Figure 3-11b, II-0.5T and II-4.5T are not prominently higher than others. This is because wave gauge 2 cannot detect the highest elevations for these two crests reliably due to the deviation between the wave gauge and the crest. It is also noticed that the elevation for II-5.5T is almost as high as that for II-4.5T, but in Figure 3-10, the crest at $t = 5.5T$ is lower than that at $t = 4.5T$. The possible reason is that when forming the flat crest in Figure 3-10 at $t = 5.5T$, a splash, as marked by a black arrow, is created. The high elevation (II-5.5T) shown in Figure 3-11b is for the splash which can be detected almost in every period of the transitional breaking wave. Before the flat crest occurring at $t = 5.5T$ ($x = 434$ mm), it is the heavily breaking curving crest ($t = 4.5T, x = 434$ mm). After the curving crest falls down, a rebounding jet is created (marked by green arrow at $t = 5T$ in Figure 3-10). The rebounding jet bounces up and down. When forming the flat crest (5.5T), the rebounding jet moves downward while the water coming from both sides of wave gauge 2 moves upward. They collide and produce the splash.

HHT is applied to the elevation data shown in Figure 3-11. The results are shown in Figure 3-12 and Figure 3-13. There should be 11 components for the HHT results in Figure 3-12 and Figure 3-13. Similar to the EEMD results shown in Figure 3-7, the first 6 components are for high frequency noise. Therefore, only components C7 - C11 are shown in the figures.
FFT has been applied to the signal components C7 to C9 shown in Figure 3-12 and Figure 3-13 and the results are shown in Figure 3-14. It is seen that at both locations (x = 0 mm and 434 mm) (Figure 3-14a and Figure 3-14d), the dominate peak frequency of C7 is 1.328 Hz which is the same as the excitation frequency. Therefore, it is the most energetic component. There are also some other peaks except the higher harmonic peak at 2.64 Hz. For the component C8 (Figure 3-14b), the dominate peaks are at 0.5646 Hz and 0.7477 Hz. The other peaks correspond well with those for C7. For the component C9, two dominate peaks can be identified at 0.3815 Hz and 0.5656 Hz. The features in (d)-(f) are similar to those in (a)-(c). Compared with frequency peaks shown in Figure 8 for the wave with small riding wave (and also those will be discussed later in Figure 24 for regular wave period tripling breaking), it can be seen there are more peaks in Figure 14 than that in Figure 8 (and also Figure 24), indicating the transitional characteristics of the wave at this external forcing amplitude. Based on the analysis above, for the EEMD results at x = 0 and 434 mm, it seems that C8 is the frequency modulation component. The rest components (C10 and C11) are small in amplitudes and should make negligible contribution to the wave elevation when compared with other components (C7-C9).

In order to investigate the effect of the modulation component C8 to the measured wave elevation, instantaneous frequencies of C8 shown in Figure 3-12 and Figure 3-13 are calculated. The results are shown in Figure 3-15 for three periods of the transitional breaking wave (as divided by blue dash lines and marked by I II and III) and each period lasts for seven wave cycles. The corresponding instantaneous frequency of C8 repeats very well in these periods (I, II and III) in both Figure 3-15a and Figure 3-15b, which indicates that C8 modulates the transitional breaking wave periodically. Period II will be illustrated in details as an example.

In Figure 3-15a, the instantaneous frequency of C8 from wave gauge 1 fluctuates around 0.7 Hz from II-1T to II-2T, and then increases up to 0.75 Hz at II-3T. It drops to about 0.5 Hz at II-4T, and increases to the first peak of 1.1Hz at II-5T marked by green line 4 (the same moment as marked by green line 2 in Figure 3-15b). It is noticed that between green line 3 (corresponding to t = 4.5T in Figure 3-10) and green line 4 (corresponding to t = 5T in Figure 3-10), the instantaneous frequency increases by about 0.6 Hz. Checking from t = 4.5T to 5T in Figure 3-10, it is seen that the heavily breaking crest at t = 4.5T (Figure 3-10, blue arrow) falls to the main water and pushes the water up rapidly along the left wall (Figure 3-10, t = 4.5T, red arrow), causing the increase in
the instantaneous frequency. After the peak at \( t = 5T \), the frequency fluctuates around 1.1 Hz until II-6T, and then decreases gradually to around 0.5 Hz.

In Figure 3-15b, the frequency for C8 fluctuates around 0.5 to 0.75 Hz from II-0.5T to II-3.5T. It starts to increase and reaches the first peak of about 1 Hz at II-4.5T as marked by green line 1. It is also the time when the curving breaking wave occurs (Figure 3-10, \( t = 4.5T, x = 434 \) mm). When forming the high and energetic curving crest, the waves coming from both sides of \( x = 434 \) mm move fast, resulting in the increase of the instantaneous frequency of C8. After the peak marked by green line 1, the frequency shows a descending trend in a very short duration, but soon it increases again and reaches another higher peak marked by green line 2. After the heavily breaking wave (Figure 3-10, \( t = 4.5T, x = 434 \) mm), the water at the wave crest starts moving downward. The downward moving wave crest could be divided into two parts. The first part is the direct downward water (Figure 3-10, \( t = 4.5T \), green arrow) while the second part is the breaking crest which is in free fall condition (Figure 3-10, \( t = 4.5T \), blue arrow). The main body water in the tank which is under the downward water (first part) slows down the moving speed of the downward water, resulting in the small drop on the frequency. However, after a short time, the breaking wave water (the second part) falls down and hits on the downward moving water. It provides additional energy to the acceleration of the downward moving water, resulting in the increase in the instantaneous frequency.

In both Figure 3-15a and Figure 3-15b, it is seen clearly that the instantaneous frequencies of C8 repeat in every seven wave cycles, which is the same as the wave shape and elevation for the transitional wave breaking. Therefore, it is believed that C8 is the frequency modulation component and the transitional wave breaking is induced by this frequency modulation.

### 3.3.3 Occurrence of wave period tripling breaking

#### 3.3.3.1 Observation of period tripling breaking

Wave period tripling breaking was first reported by Jiang et al.\(^8\) for the study of symmetric wave about the tank centreline under vertical excitation. It is featured with three types of wave breaking which repeat every three wave cycles. In the experiments, period tripling breaking occurs when \( b = 32.5 \) mm, which includes three modes characterised by different breaking shapes, namely sharp crest (mode one), flat crest (mode two) and round crest (mode three).
After checking the video, it has been found that there are 4 places where the wave crest and trough occur: left tank wall, right tank wall and the locations approximately 434 mm and 867 mm away from the left side wall. Figure 3-16 shows a sketch summarizing the sequence of wave breaking modes occurring at x = 434 mm and 867 mm. If mode one appears at x = 434 mm (Figure 3-17 at t = 0.378 s) then the next crest occurring at x = 867 mm is mode three (Figure 3-18 at t = 0.798 s), followed by mode two appearing at x = 434 mm (Figure 3-18 at t = 1.008 s). Modes one, three and two will appear at x = 867 mm (t = 1.512 s), 434 mm (t = 1.848 s) and 867 mm (t = 0 s) sequentially.

The whole formation process of mode one (from trough to trough) is shown in Figure 3-17. Before forming the sharp crest at x = 434 mm, the wave crest at x = 867 mm is flat (mode two, t = 0 s, from Figure 3-16), wide and low, whereas the wave crest at x = 0 mm is high due to the reflection of tank wall. The water coming from the flat crest arrives at x = 434 mm a little earlier than the water coming from the other side (x = 0 mm, t = 0.168 s), resulting in a left inclination for the sharp crest (from t = 0.252 s onwards). The thin and high crest is formed completely at x = 434 mm when t = 0.336 s, and then wave breaking is found in the next two frames (t = 0.378 s and 0.42 s). The breaking wave falls to the water surface, creating a rebounding jet at t = 0.63 s (marked by a black arrow). It is noticed that the location of the rebounding jet (x = 434 mm, t = 0.63 s) deviates slightly from where the thin wave crest is, resulting from the deviating breaking wave peak. Unlike the observation found by Jiang et al., who stated that the wave peak in mode one tended to left or right randomly, in most of the time (approximately 96% of the time), the results show that the turning direction of the thin wave peak is towards left.

Figure 3-18 shows the formation of mode two of period tripling breaking at x = 434 mm. Unlike the observations from Jiang et al., who stated that there was a crest with double plungers in mode two, in the experiment it is characterised by a flat crest (shown at t = 1.008 s). The rebounding jet (marked by a green arrow, Figure 3-17, t = 0.714 s) resulting from mode one moves up and down, while the water coming from both sides of x = 434 mm moves upward. A weak splash (marked by black arrows) can be found from t = 0.966 s to t = 1.050 s (x = 434 mm). The splash is attributed to the collision between upward moving water and downward rebounding jet. The flat crest moves to the right side and becomes asymmetric about the wave gauge 2 as shown from t = 1.092 s and t = 1.134 s.
Figure 3-19 shows the formation of mode three of the wave period tripling breaking at 434 mm, which is featured by a round crest with a weak breaking (Figure 3-19, time frame $t = 1.890 \text{s to } t = 1.974 \text{s}$). According to Figure 3-16, before mode three occurs at $x = 434 \text{ mm}$, mode one can be found at $x = 0 \text{ mm}$ and 867 mm. The wave crest in mode one is curving and heavily breaking (marked by two green arrows in Figure 3-19). After the heavily breaking crests, the energy is dissipated significantly. Therefore, there is not enough energy to form a sharp crest. This could be the reason that the wave crest at $x = 434 \text{ mm}$ is round (from $t = 1.848 \text{s to } 2.016 \text{s}$ in Figure 3-19).

Figure 3-20 shows the wave elevation time series for wave gauge 1 ($x = 0 \text{ mm}$) and wave gauge 2 ($x = 434 \text{ mm}$) when $b = 32.5 \text{ mm}$. The numbers ‘1’, ‘2’ and ‘3’ represent mode one, two and three, respectively. From the observation from Figure 3-17 to Figure 3-19, mode two (flat crest) has the lowest wave elevation among the three modes. Therefore, the wave with lowest wave height is marked as mode two in Figure 3-20, and then modes one and three can be determined. In Figure 3-20b, it is shown that the wave elevation in mode one is the highest, and that of mode three is higher than that of mode two but lower than that of mode one. However, in Figure 3-20a, sometimes, the elevation of mode three (marked by green arrow) is close to, or even higher than that of mode one (marked by red arrow). Based on the sketch in Figure 3-16, before the occurrence of mode three at $x = 0 \text{ mm}$, it is mode one at $x = 434 \text{ mm}$. Mode one has a high and sharp crest. After the crest falls to the main water, it creates a deep dent ($t = 0.714 \text{s}$, Figure 3-17), which would push the water to a high location along the tank wall. This is the reason that the wave elevation for mode three is higher than that for mode one at $x = 0 \text{ mm}$.

### 3.3.3.2 Phase analysis of period tripling breaking

In order to get a better understanding of period tripling breaking, pseudo-phase-space plots (Moon\textsuperscript{21}) for wave elevation at $x = 0 \text{ mm}$ and $x = 434 \text{ mm}$ are analysed. The three-dimensional pseudo-phase-space plot can be constructed by:

$$\begin{pmatrix} \eta(t) \\ \eta(t + \Delta t) \\ \eta(t + 2\Delta t) \end{pmatrix},$$

where $\eta(t)$ is the measured wave elevation sampled with a frequency of 1000 Hz at $x = 434 \text{ mm}$ (wave gauge 2) and $\Delta t$ is the time delay. In the present study, $\Delta t$ is chosen as 0.1s, which is useful to construct a plot with clear trajectory.

Figure 3-21 shows the pseudo-phase plot and the corresponding Poincare map at $x = 0 \text{ mm}$ and $x = 434 \text{ mm}$ when $\eta(t) = 0$ and $b = 32.5 \text{ mm}$. There are three orbits in Figure
3-21, standing for the three breaking modes aforementioned. The lines in black, blue and red are for modes one, two and three, respectively.

In Figure 3-21a, the inner blue orbit representing mode two with the lowest wave elevation can be clearly seen. The other two modes mix with each other, indicating that the wave elevation for these two modes are comparable, which is consistent with the results discussed for Figure 3-20a above. Figure 3-21b shows the Poincare map for Figure 3-21a by taking $\eta(t) = 0$. The three clusters are evident in the upper right and lower left corners, representing the three periodical modes. For the lower left corner, a small region is overlapped by the blue and black clusters. Figure 3-21c shows the three orbits, which is much more clearly compared with that in Figure 3-21a, representing the three modes. The width of the black orbit is wider than that of the red and blue ones. It can be regarded that the width of the orbit represents the magnitude of the range of wave elevation. For instance, in a time series of wave elevation, if the wave amplitudes are identical, the orbit should be a circle. If the amplitudes of wave crests vary in a large range, the width would be wide, like the black orbit in Figure 3-21c. In Figure 3-21d, it can be seen that the three clusters at the lower left corner take more space than that in Figure 3-21b, indicating that the period tripling breaking is more evident at $x = 434$ mm (wave gauge 2) than that at $x = 0$ mm.

3.3.3.3 HHT analysis of period tripling breaking

HHT analysis is applied to the wave elevation time series at both $x = 0$ and 434 mm when $b = 32.5$ mm (as shown in Figure 3-20). The EEMD results for wave gauges 1 and 2 are shown in Figure 3-22 and Figure 3-23, respectively. As C1 to C6 are the high frequency components induced by added noises, they are not shown in the figures. C7 in Figure 3-22 and Figure 3-23 are uniform and energetic, which are considered as the main components while C8 in Figure 3-22 and Figure 3-23 are regarded as the modulation components. C9 are uniform but smaller than C7. The rest components (C10 and C11) are small in amplitudes and should make negligible contribution to the wave elevation compared with other components (C7 - C9).

FFT analysis is applied to the signal components C7 to C9 shown in Figure 3-22 and Figure 3-23 and the results are shown in Figure 3-24. It is clearly seen that, for C7 (Figure 3-24a), the peak frequency of C7 is $f = 1.33$ Hz which is the same with the excitation frequency. It is the most energetic component from the wave elevation at $x = 0$ mm. The spectrum for C8 (Figure 3-24b) has two dominate peaks. Apart from the peak at the excitation frequency ($f$), another peak at $2f/3$ is found. They are comparable
in peak height. For C9 (Figure 3-24c), only one peak at \( f/3 \) is found, indicating C9 is a uniform component. The features for \( x = 434 \) mm (Figure 3-24d-f) are similar to those in (Figure 3-24a-c). Based on the analysis above, for the EEMD results at \( x = 0 \) and 434 mm, the signals of C8 are the frequency modulation components.

Part of the instantaneous frequency of C8 based on the data shown in Figure 3-22 together with the wave elevation (measured at \( x = 0 \) mm) are shown in Figure 3-25. There are 7 period tripling breaking durations as marked in the figure and each of them includes the three modes mentioned above. Generally, the frequency decreases during mode one, and approaches the minimum value in the descending process of mode one (i.e. the trough in period I as marked by a black arrow), indicating that the wave acceleration for mode one during upward and downward process at \( x = 0 \) mm decreases. As stated in Figure 3-16, before mode one occurs at \( x = 0 \) mm (the wave mode occurring at \( x = 0 \) mm is the same as that at \( x = 867 \) mm), mode two can be found at \( x = 434 \) mm, which is a flat crest with the lowest wave elevation. When it falls down, it cannot provide large enough energy to push the water upward at \( x = 0 \) mm very quickly. So the water in mode one goes up in a decreasing acceleration. This is likely why the frequency decreases during mode one. After the trough of the frequency (black arrow), it starts increasing and peaks during mode three (as marked by a green arrow in period I). There is a flip-through phenomenon (illustrated in detail in Figure 3-28), occurring during mode three, which contributes to the increasing wave acceleration. Apart from the general trend of the frequency of C8, there are two exceptions. Firstly, in periods I, II and VII, another peak can be found during mode two (i.e. marked by a red arrow in periods I, II and VII). In addition, in periods III and VI, it can be seen that the frequency reaches the bottom value during mode two instead of mode one (i.e. marked by a cyan arrow in periods III and VI). It is likely that both of these exceptions are due to the effect of rebounding jet which can move up and down with its own frequency, resulting in a possible modulation to the general instantaneous frequency C8. However, it is difficult to quantify the effect of the rebounding jet because the position of the jet varies with time, making the detection of its frequency difficult.

Part of the instantaneous frequency of C8 from Figure 3-23 together with wave elevation (\( x = 434 \) mm) are displayed in Figure 3-26. Totally seven period tripling breaking durations are shown and each of them includes the three modes mentioned above. The general trend of the instantaneous frequency is that it decreases from the beginning of mode one and reaches the minimum at the end of mode two and then increases during mode three. From Figure 3-16, before forming mode three at \( x = 434 \)
mm, the wave modes at \( x = 0 \) mm and 867 mm are mode one. When the sharp crest in mode one falls down, it would push the water upward quickly, resulting in an increase in the wave acceleration.

Some downward peaks (with zero frequency) on the instantaneous frequency in Figure 3-25 (period II) and Figure 3-26 (periods II and IV) can be seen. During the experiments, noises are inevitable. After applying EEMD, noises included in the signal would still exist. When calculating the instantaneous frequency, the zero frequency would occur but it is meaningless. However, the zero frequency does not influence the general trend analysis.

From Figure 3-25 and Figure 3-26 it can be seen that the instantaneous frequency of C8 varies every three wave cycles, which is the same as that for wave period tripling breaking. It is therefore believed that C8 is the modulation component and the wave period tripling breaking is induced by this frequency modulation.

### 3.3.3.4 Pressure variation during wave period tripling breaking

When the wave period tripling breaking occurs, pressure on the side wall of the tank varies corresponding to the three modes. In total, nine pressure sensors are installed on the side wall to detect the pressure variation (Figure 3-28). Pressure No. 1 is at the highest location and No. 9 is at the lowest location with No. 5 being on the still water line. The gap between two adjacent pressure sensors is 15 mm. Figure 3-27 shows the pressure variation for the three modes of wave period tripling breaking when the \( b = 38 \) mm. The pressure data from sensor number 7 is in poor quality and only the results of other 8 pressure sensors are displayed. For the pressure time series measured by all sensors, apparent pressure variation can be found in each mode (marked by 1, 2 and 3), indicating a correspondence between the wave period tripling breaking and the pressure on the wall. For P1, there is a very high pressure peak occurring in mode three (marked by black arrow). This is because the pressure here is dominated by impact pressure instead of hydrostatic pressure. A similar phenomenon is found in mode three for P2. Although the pressure peak is not as high as that in P1, it is apparently higher than that in other two modes.

Within the three modes, the magnitudes of the pressure measured by sensors No. 5 to No. 9 (except No. 7) which are located below the still water line become larger gradually as the location of the sensors getting lower. This result is consistent with the fact that the pressure on the wall is mainly from the hydrostatic pressure. After the peak pressures in each mode (e.g. marked by blue arrows in P9), an evident pressure trough
(i.e. marked by magenta arrows in P9) is observed before the occurrence of another peak and the trough happens when the wave elevation is the maximum. The explanation for such phenomenon can be found in Sec. 2.3.2.

In order to illustrate the high peak pressure in mode three at P1 in Figure 3-27, the time frames of mode three are shown in Figure 3-28. The green short bars on the left side of each picture indicate the location of the pressure sensors P1 to P9 from top to bottom, respectively. At \( t = 0 \) ms, a curved wave crest (marked by a red arrow) approaching the tank wall can be identified. Then the crest (marked by a blue arrow, \( t = 33.6 \) ms) meets the wave trough (marked by a black arrow, \( t = 33.6 \) ms) and move to the tank wall (\( t = 12.6 \) to 58.8 ms). Finally, a vertical jet (marked by a blue arrow, \( t = 163.8 \) ms) is formed on the wall (\( t = 67.2 \) to 231 ms). This process is consistent with the flip-through\textsuperscript{14} phenomena. From \( t = 83.9 \) ms, it is seen that the actual position where the wave crest hits the tank wall is higher than the location of P1 (marked by green arrow), which is also higher than still water level. It has been checked that flip-through occurs in most mode three of the period tripling breaking event. The location where the water crest hits the tank wall is not the same, but always higher than the still water level. It is therefore believed that the hitting location of a flip-through is higher than the still water level, in agreement with that reported by Lugni \textit{et al.}\textsuperscript{14} and Hull and Muller\textsuperscript{15}.

### 3.3.4 Unstable wave period tripling breaking

When \( b \) is further increased to a threshold value, which is 39.5 mm in the experiments, it is found that the wave period tripling breaking becomes unstable, reflected by the fact that the wave elevation does not follow the pattern of period tripling breaking (i.e. sharp crest for mode one, flat crest for mode two and round crest for mode three) described in Sec. 3.3.3. The wave elevations measured by wave gauges 1 and 2 when \( b = 39.5 \) mm are shown in Figure 3-29. For wave gauge 1 located at \( x = 0 \) mm (Figure 3-29a), regular wave period tripling breaking can be identified till \( t = 189.1 \) s. After that, wave breaking period becomes unstable because the following three wave periods (i.e. wave crests 4, 5 and 6) do not follow the tripling breaking pattern. However, some regular wave period tripling breaking (e.g. the wave crests 7, 8, and 9, wave crests 10, 11, and 12, wave crests 13, 14, and 15, wave crests 16, 17, and 18) can still be found after \( t = 189.1 \)s, revealing an unstable wave period tripling breaking pattern. The unstable wave period tripling breaking pattern at \( x = 434 \) mm occurs a little earlier than that at \( x = 0 \) mm, appearing at \( t = 187.9 \) s. After that, the pattern of the tripling breaking turns into an unstable stage.
Figure 3-30 shows the pseudo-phase plot and the corresponding Poincare map at \( x = 0 \) mm and \( x = 434 \) mm when \( \eta(t) = 0 \) and \( b = 39.5 \) mm. It can be observed that the three orbits are almost mixed with each other but most of the blue orbits (mode two) can still be distinguished at the inner location, which means that some wave elevation for the three modes does not follow the patterns of period tripling breaking closely. In Figure 3-30c, part of the black orbit (mode one) can still be distinguished at the outer region, while a small part of the black orbit is mixed with the other orbits, indicating that some wave elevations for mode one, two and three are not consistent with the patterns of wave tripling breaking. The similar conclusion can also be obtained in Figure 3-30d where some black dots are buried in the red and blue clusters at the bottom left corner, indicating that after the first six period tripling breaking, the rest elevations do not follow the pattern.

### 3.4 Conclusions

Asymmetric sloshing wave is generated by oscillating a two-dimensional tank horizontally at forcing amplitude \( b \) in the range from 1 mm to 40 mm under the third mode sloshing frequency. The results show that the sloshing wave pattern depends on the forcing amplitude. The main conclusions can be summarised as follows.

1. When the \( b = 10 \) mm and 13 mm, the wave is non-breaking with a riding wave on the main body wave being observed at \( x = 434 \) mm. Analysis using HHT to the time series of the wave elevation shows that the wave is periodical with a period of twelve wave cycles.

2. As the forcing amplitude is increased to 14.5 mm, a special periodical wave breaking type called ‘transitional periodical breaking’ is observed at \( x = 434 \) mm and \( x = 867 \) mm. It is only under such a forcing amplitude that the periodical wave breaking lasts for the whole duration of the experiment. After applying HHT to the wave elevation data and analysing the instantaneous frequency, it is found that the instantaneous frequency of C8 repeats in every seven wave cycles, which is the same as the wave elevation for the transitional wave breaking. It is therefore believed that the transitional wave breaking is induced by the frequency modulation.

3. When \( b \) is further increased to 32.5 mm, wave period tripling breaking with three modes is found inside the tank. A thin and sharp crest in mode one at \( x = 434 \) mm is observed which is caused by the collision between the water coming from both sides of \( x = 434 \) mm. The wave crest always tends towards left, which is different from the
previous study by Jiang et al.\textsuperscript{1} where a random tendency of the crest is reported. Mode two is featured with a flat crest instead of double plungers observed previously while mode three is characterised by a round crest with a weak breaking. The FFT results for HHT components C7 - C9 show that C8 is the frequency modulation component. Based on the analysis of the instantaneous frequency of C8 at $x = 0$ mm and $x = 434$ mm, it is found that the frequency varies every three wave cycles which is the same as that for wave period tripling breaking, indicating that wave period tripling breaking is induced by this frequency modulation.

(4) The hydrostatic pressure dominates the pressure measured by sensors 4-9. However, the highest pressure during the period tripling breaking occurs in mode three due to the flip-through phenomenon with the impact location being higher than the still water level, which is consistent with previous studies.

(5) Period tripling breaking becomes unstable when the forcing amplitude reaches 39.5 mm. Based on the phase portrait analysis of wave elevation, it is found that part of the orbits start to mix with each other gradually, indicating that the period tripling breaking starts to become unstable.

\textbf{Reference}


### TABLE 3-I. Test Matrix for the experiment under horizontal excitation (320 excitation periods)

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Excitation amplitude (mm)</th>
<th>Excitation frequency (rad/s)</th>
<th>Case No.</th>
<th>Excitation amplitude (mm)</th>
<th>Excitation frequency (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
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Figure 3-1 Snapshot of the sloshing experimental facility.

Figure 3-2 Sketch of the experimental set-up
Figure 3-3 Flow chart of the signal measurement and control

Figure 3-4 Example of the inputting signal (lasting for 320 cycles)
Figure 3-5 Six successive time frames with 0.042 s time interval for the wave peak at x = 434 mm when b = 10.5 mm. The black and green arrows indicate the positions of the riding wave. The unit for time is in s.
Figure 3-6 Ten wave cycles of time series of wave elevation measured by wave gauge 2 when $b = 10.5$ mm.
Figure 3-7 EEMD results for wave elevation measured at x = 434 mm (wave gauge 2, excitation cycle from 200 to 250) when \( b = 10.5 \) mm.
Figure 3-8 Spectra of signal components C7 - C9 shown in Figure 3-7.
Figure 3-9 Comparison between C7 (black) and C8 (red) from Figure 3-7. The numbers I, II and III represent three periods. The numbers inside the circle indicate the parts inside the period.
Figure 3-10 Continuing periodical wave shape showing the wave crests at different locations in consecutive cycles when $b = 14.5$ mm. Left column: wave crests appearing at $x = 434$ mm (wave gauge 2) and $x = 1300$ mm; right column: wave crests appearing at $x = 0$ mm and $x = 867$ mm. $T$ is wave period. One period of transitional breaking wave is from $0.5T$ to $7T$ while the frames from $7.5T$ to $9T$ are part of the next period.

The arrows are explained in the text.
Figure 3-11 Time series of wave elevation and tank movement for: (a) $x = 0$ mm (wave gauge 1) and (b) $x = 434$ mm (wave gauge 2) when $b = 14.5$ mm. The black thick line is for wave elevation while the red dot-dashed line is for tank movement. I – III represent three periods for the transitional breaking wave. The numbers stand for the instants corresponding to the wave crest shape shown in Figure 3-10.

Figure 3-12 EEMD results when $b = 14.5$ mm for wave elevation at $x = 0$ mm (wave gauge 1, the same data as those shown in Figure 3-11a).
Figure 3-13 EEMD results when $b = 14.5$ mm for wave elevation at $x = 434$ mm (wave gauge 2, the same data as those shown in Figure 3-11b).
Figure 3-14 (a) – (c): Spectra of signal components C7 - C9 at x = 0 mm (i.e. the data shown in Figure 3-12) and (d) – (f): at x = 434 mm (i.e. the data shown in Figure 3-13).
Figure 3-15 Instantaneous frequency of C8 (red) and the corresponding signal (blue) for C8 and wave elevation (black) shown in Figure 3-12 and Figure 3-13. (a) x = 0 mm, wave gauge 1; (b) x = 434 mm, wave gauge 2. The numbers stand for the instants corresponding to the wave crest shape shown in Figure 3-10 and I-III represent three periods for the transitional breaking wave.
Figure 3-16 Sketch on the occurring sequence of wave breaking modes at x = 434 mm and 867 mm.
Figure 3-17 Time frames of wave period tripling breaking mode one (sharp crest) at the location of wave gauge 2 ($x = 434$ mm) when $b = 32.5$ mm. The unit for time is in s.

The arrows indicate the rebounding jet.
Figure 3-18 Time frames of wave period tripling breaking mode two (flat crest) at the location of wave gauge 2 ($x = 434$ mm) when $b = 32.5$ mm. The unit for time is in s.

The arrows are explained in text.
Figure 3-19 Time frames of wave period tripling breaking mode three (round crest) at the location of wave gauge 2 ($x = 434$ mm) when $b = 32.5$ mm. The unit for time is in s.

The green arrows indicate the curving and heavily breaking crests of mode one.
Figure 3-20 Wave elevations for: (a) $x = 0$ mm (wave gauge 1) and (b) $x = 434$ mm (wave gauge 2) when $b = 32.5$ mm from excitation cycles 200 to 250. The black line is for wave elevation while the red dot dashed line is for the tank movement. The numbers marked in (a) and (b) stand for the corresponding wave period tripling breaking mode. The red and green arrows indicate two elevations for mode one and mode three which are close in magnitude.
Figure 3-21 Pseudo phase plots and Poincare map for wave elevation shown in Figure 3-20 (a and b) when $b = 32.5$ mm. Pseudo phase plots: (a) $x = 0$ mm, (c) $x = 434$ mm; Poincare map: (b) $x = 0$ mm, (d) $x = 434$ mm. Black, blue and red stand for modes one, two and three, respectively.

Figure 3-22 EEMD result when $b = 32.5$ mm for wave elevation at $x = 0$ mm (wave gauge 1, the same data as those shown in Figure 3-20a).
Figure 3-23 EEMD result when $b = 32.5$ mm for wave elevation at $x = 434$ mm (wave gauge 2, the same data as those shown in Figure 3-20b).
Figure 3-24 (a) – (c): Spectra of signal components C7 - C9 at x = 0 mm (i.e. the data shown in Figure 3-22) and (d) – (f): at x = 434 mm (i.e. the data shown in Figure 3-23).
Figure 3-25 Instantaneous frequency of C8 (red) and the corresponding signal of C8 (blue) and wave elevation (black) for part of the data shown in Figure 3-22 (x = 0 mm, wave gauge 1). The numbers I – VII represent periods for wave tripling breaking. The numbers next to the peak of wave elevation represent the tripling breaking mode.

Figure 3-26 Instantaneous frequency of C8 (red line) and the corresponding signal of C8 (blue line) and wave elevation (black line) for part of the data shown in Figure 3-23 (x = 434 mm, wave gauge 2). The numbers inside a circle represent periods for wave tripling breaking. The numbers next to the peak of wave elevation represent the tripling breaking mode.
Figure 3-27 Pressure time series (red) from sensors P1-P9 (without P7 due to its poor quality) for the three modes during wave period tripling breaking. The wave elevation (black) and tank movement (blue) are also shown for comparison. The forcing amplitude $b = 38$mm. The numbers in each figure represent tripling breaking mode. The black arrow indicates impact pressure. The blue arrows indicate pressure peaks while the magenta arrows indicate the pressure troughs.
Figure 3-28 Time frames of mode three of wave period tripling breaking for $b = 38\text{mm}$. The green bars represent the location of pressure sensors. The still water line is at $Y = 387.4\text{mm}$.
Figure 3-29 Time series of wave elevations measured at $x = 0$ mm (wave gauge 1) (a) and $x = 434$ mm (wave gauge 2) (b) when $b = 39.5$ mm from excitation cycles from 226 to 276. The black bold line is for wave elevation while the red dot-dashed line is for tank movement. The vertical blue dashed lines indicate the stable tripling breaking modes. The numbers represent tripling breaking modes.
Figure 3-30 Pseudo phase plots and Poincare map for wave elevation shown in Figure 3-29 (a and b) when $b = 39.5$ mm. Pseudo phase plots: (a) $x = 0$ mm, (c) $x = 434$ mm; Poincare map: (b) $x = 0$ mm, (d) $x = 434$ mm. Black, blue and red stand for modes one, two and three, respectively.
Chapter 4

Wave period tripling breaking inside a two-dimensional sloshing tank with asymmetric wave shape under vertical excitations

Abstract

Experiments were conducted in a two-dimensional tank under vertical excitations at a frequency of 2.66 Hz to study the standing wave especially the wave period tripling breaking. This frequency is equal to twice of the third sloshing mode frequency, and as a result, the corresponding wave shape inside the tank is one and a half wave length, which is asymmetric about the tank centreline. It is found that when the vertical forcing amplitude ($b_2$) is increased, the performance of the standing wave changes from a non-breaking state to a breaking state. For small forcing amplitude ($b_2 < 7$ mm), the non-breaking wave at wave gauge 2 is temporally asymmetric around the region close to the wave crest. When the forcing amplitude is increased, the wave asymmetry becomes more apparent, which may be induced by the occurrence of bulge on the forward face of the wave crest. Wave period tripling breaking with three modes, is observed when $b_2$ exceeds a threshold, which is 11.5 mm under the present experimental condition. However, the breaking types are found different from the previous studies. It is also found that the breaking type of mode two depends on the forcing amplitude. The period tripling breaking is further examined using phase portrait and Hilbert-Huang Transform (HHT) analysis. The results reveal that the width of the orbits for each breaking mode is related with the variation range of the elevation of wave crest and tripling breaking is caused by a frequency modulation.
4.1 Introduction

Sloshing phenomenon is a great concern in many engineering aspects, such as the transportation of the liquefied natural gas (LNG) in oil and gas industry, fuel tank in the spaceship or vehicle and so on. Violent liquid motion will be induced by sloshing if the tank is partially-filled, exerting localized impact pressure on the tank walls, which may cause serious structural damage. The performance of liquid motion resulting from sloshing depends on the external excitation condition. For instance, Ji et al.\textsuperscript{1} carried out experiments to study the hydraulic jump from non-resonant sloshing under large amplitude. They classified the free surface waves into four types, namely, (I) 2-D mild waves, (II) 2-D strongly nonlinear waves, (III) 3-D waves with regular longitudinal structure along the z-direction and (IV) chaotic waves. They mainly focused on the second type. Lugni et al.\textsuperscript{2} studied wave impact loads in a two-dimensional tank with shallow water condition. A very interesting phenomenon, named flip-through which will lead to high pressure on tank side walls, was studied via Particle Image Velocimetry (PIV). They also conducted analysis on the impact pressure by means of pressure-impulse theory\textsuperscript{3}. More studies about the liquid sloshing phenomenon can be found in Ibrahim\textsuperscript{4} and Faltinsen and Timokha\textsuperscript{5}.

As an important part of sloshing motions, steep standing waves generated in a vertical oscillating container through subharmonic resonance, which is known as the 'Faraday wave'\textsuperscript{6}, also draw a lot of attention\textsuperscript{7,8}. Faraday wave has a frequency equal to half of that of the excitation\textsuperscript{9}. To excite the symmetric standing wave inside a tank, the forcing frequency should double the even modes natural frequency of the liquid in the tank. If asymmetric standing wave is going to be obtained, the forcing frequency should double the odd modes of the natural frequency of the liquid in the tank. The modes of the natural frequency for a two-dimensional rectangular sloshing tank is given by Eq. (4.1) (Abramson et al.\textsuperscript{10})

\[
\omega_n^2 = \pi n \frac{g}{L} \tanh(\pi n \frac{h}{L})
\]

(4.1)

where \(n\) is the mode number of the internal sloshing, \(L\) is the tank length, \(g\) is the gravity acceleration and \(h\) is the filling depth. Specifically, \(\omega_n\) denotes the \(n\)th mode natural frequency.

Jiang et al.\textsuperscript{8} studied the symmetric standing waves about the tank centreline in a vertical oscillating tank. They oscillated the tank by varying the frequency from 3.15 Hz to 3.34 Hz which corresponds to sloshing mode two. The dimple crest and flat crest
Wave forms were reported when the forcing amplitude was large (3.85 mm). They also indicated that the waves with flat and dimple crests are temporally asymmetric and the second harmonic is strong in the wave form. Jiang et al.\textsuperscript{11} extended the work reported in Jiang et al.\textsuperscript{8} by increasing the forcing amplitude to 4.60 mm under a forcing frequency of 3.20 Hz. Period tripling breaking phenomenon was observed, which means that three kinds of wave breaking, namely steep crest, flat crest with double plungers and round crest, appear alternatively. Longuet-Higgins and Dommermuth\textsuperscript{12} reported the existence of standing wave profiles with rounded crest and sharp crest by experiment. Longuet-Higgins and Drazen\textsuperscript{13} studied standing waves by considering the waves reflected at a vertical wall and confirmed the existence of period tripling breaking. Bredmose et al.\textsuperscript{14} generated standing waves by oscillating a tank vertically with an amplitude of 30 mm at a frequency of 1.25 Hz and reported the wave forms with flat crest and sharp crests. They also employed the Boussinesq mode to simulate wave motion and reproduced the free-surface motion accurately. Apart from the experimental investigations, a lot of numerical simulations were also conducted on standing waves. Mercer and Roberts\textsuperscript{15} studied two-dimensional standing waves via a stable numerical method and concluded that the steep wave is unstable to the subharmonic perturbations. Although a lot of previous studies confirmed the existence of wave breaking shapes reported by Jiang et al.\textsuperscript{11}, there are still problems relating to the period tripling breaking. As tripling breaking was initially found when the wave profile is symmetric about the tank centreline, it is not clear if this phenomenon still exists when the wave profile is asymmetry with the centreline of a sloshing tank. Bredmose et al.\textsuperscript{14} generated the standing wave in an asymmetric profile and found sharp and flat wave crests. However, the tank aspect ratio in their study is 3.71:1, which may be too small to be considered as two-dimensional. Therefore, the primary aim of this study is to examine the steep standing wave, especially the existence of the wave period tripling breaking in a two-dimensional sloshing tank with an aspect ratio of 13:1 and a water depth of 387.4 mm under vertical excitation with amplitudes ranging from 1 mm to 15 mm. The excitation frequency is 2.66 Hz, which corresponds to the third sloshing mode. As a result, the standing wave is asymmetric and has 1.5 wavelengths inside the tank.

The wave elevation is examined via the Hilbert-Huang Transform (HHT) analysis\textsuperscript{16}. HHT is an empirically based data-analysis method. It is adaptive in producing physically meaningful representations of data from nonlinear and non-stationary processes. HHT interprets wave nonlinearity as a frequency modulation and the energy remains near the base frequencies\textsuperscript{17}. Obviously, in this experiment, especially after
wave breaking occurs, the wave evolution process is characterised by a strong nonlinearity. Therefore, by applying HHT analysis to the complicated physical process, a more robust interpretation about wave motion can be obtained.

The paper is organised as follows. After the literature review in Sec. 4.1, the experimental setup and the methodology are described in Sec. 4.2. The experimental observation and discussion are shown in Sec. 4.3, including the standing wave without breaking and the waves with period tripling breaking. HHT analysis is applied to the wave elevation in both wave types, respectively. Finally, conclusions are presented in Sec. 4.4.

4.2 Experimental set-up

4.2.1 Description of the experimental facilities and measurements

A rectangular tank with a breadth $B = 100$ mm, a length $L = 1300$ mm and a height $H = 900$ mm, as shown in Figure 4-1, has been used. The co-ordinate system is defined as $X$ in the length direction and $Y$ in the height direction. The aspect ratio, $L/B$ is 13:1, which is large enough to ensure the two-dimensionality inside the tank. The tank is fixed on a hexapod which is capable of generating a vessel's motion in six degrees of freedom. The hexapod can produce motion up to 0.18 m on heave under the limitation of 0.5 g in acceleration. Two wave gauges were used to detect the variation of wave elevations inside the tank, as sketched in Figure 4-2. The preliminary analysis has shown that when the excitation frequency is double the third mode natural frequency of the water inside the tank, the asymmetric wave with 1.5 wavelength would be formed. Under such condition, inside the tank, there are in total 4 places where the wave crest and trough will occur: left tank wall, right tank wall, approximately 434 mm and 867 mm away from the left side wall. In the experiments, the two wave gauges are fixed at 5 mm and 434 mm away from the left side of the tank, respectively. An ultrasonic sensor was used to monitor the vertical movement of the tank. Wave variation inside the tank was recorded by a high-speed camera with 240 fps (frames per second).

As shown in Figure 4-3, a synchronization of the hexapod motion, other instruments including the wave gauges and the ultrasonic sensor and data acquisition system (DAQ) is built for the sloshing experiment. DAQ is composed of 3 parts including a chassis (U2781A) which is responsible for synchronization of other modules, a module (U2541A) which is used for acquiring the signal from the measurement and a module (U2351A) which is for signal generation. Four simultaneous differential channels are
included in each module (U2541A), guaranteeing the synchronization when acquiring data.

### 4.2.2 Experimental methodology and tank excitation

The water depth inside the tank is 387.4 mm. The third sloshing mode wave, which is asymmetric about the centreline of the tank, is generated. The tank is excited initially by a horizontal motion with the third mode frequency \( f_3 = \omega_3 / 2\pi = 1.33 \text{Hz} \), which is calculated using Eq. (4.1), followed by a vertical motion under a frequency of \( 2f_3 = 2.66 \text{Hz} \). Note that to excite the Faraday wave vertically, the forcing frequency should be twice of the frequency of the wave\(^{18}\). The purpose of giving a horizontal movement to the tank is to provide an initial perturbation to the still water, which is helpful to amplify the vertical motion. The similar method was also adopted by Bredmose et al.\(^{14}\), who generated a steep standing wave by vertical excitation and proved that it was useful to obtain deterministic Faraday waves. Sinusoidal signals are applied to both horizontal (\( b_1 \sin(\omega_1 t) \)) and vertical motions (\( b_2 \sin(2\omega_1 t) \)). The amplitude \( b_1 \) for horizontal movement is 0.254 cm, while for the vertical movement, \( b_2 \) varies from 1 mm to 15 mm by an increment of 0.5 mm. Figure 4-4 shows an example of the driving signals. For all the experiments reported in this chapter, the horizontal motion will initially last for 10 periods, and then followed by 640 periods of excitation in vertical direction at a frequency of \( 2f_3 \) for various amplitudes. The test matrix is shown in Table 4-I.

### 4.3 Results and discussion

When the vertical forcing amplitude \( b_2 \) is increased from 1 mm to 15 mm with an increment of 0.5 mm, the wave amplitudes measured by the wave gauges for each case including 640 vertical oscillating cycles are shown in Figure 4-5 using boxplot. The boxplot method is a useful way to analyse the experimental data statistically and graphically via the following quantities (shown in Figure 4-5). The upper and lower limits of the blue box indicate the third quartile (Q3, 75\% percentile data) and first quartile (Q1, 25\% percentile data), respectively. The central line (red) is the 50\% percentile (the median). The interquartile range (IQR) means the difference between the values of the third quartile (Q3) and the first quartile (Q1). The upper and lower whiskers indicate the maximum and minimum of the range between Q1-1.5IQR and
Q3+1.5IQR, respectively. The values which are not in the range between Q1-1.5IQR and Q3+1.5IQR are called the outliers.

The result shows a general ascending trend of the wave elevation at wave gauges 1 and 2 with the increase of $b_2$. Obviously, in Figure 4-5a and b, the magnitudes of the wave crests are larger than the wave troughs, indicating that the wave shape is asymmetric about the still water level. The following discussion will be illustrated in two parts according to the wave feature (non-breaking wave and breaking wave).

### 4.3.1 Standing wave without breaking

#### 4.3.1.1 Observation and analysis of wave asymmetry under small forcing amplitude

When the vertical exciting amplitude is small, the standing wave inside the tank does not break. Because the forcing frequency is at $2f_c = 2.66 \text{Hz}$, the standing wave is in the third wave mode, which is characterized by asymmetry about the centreline of the tank with one and a half wave length. A temporal symmetry is defined as $\eta(x,t) = \eta(x,-t)$, where $\eta$ is wave elevation. Figure 4-6 shows the wave profiles in ascending and descending processes in the tank when $b_2 = 1$ mm. It is seen clearly that the wave is temporal asymmetry (at wave gauge 2, $x = 434$ mm), meaning that the curves with the same color in the two figures do not have the same wave profile. This result is similar to that reported by Jiang et al.\textsuperscript{11}. For example, the red line in Figure 4-6a marked with ‘A3’ does not match the red line marked with ‘D3’ in Figure 4-6b. According to the numerical simulation conducted by Jiang et al.\textsuperscript{8}, the asymmetry is induced by the different energy dissipation during the ascending and descending processes. To examine the asymmetry found in Figure 4-6, Figure 4-7a shows the time series of the wave elevation (50 wave cycles) measured by wave gauge 2. Over these cycles, the wave elevation is in a steady state. The slight asymmetry can be seen at the wave crest. To see the asymmetry more clearly, Figure 4-7b shows only five wave cycles. It can be seen that the phase of the excitation signal lags that of the wave elevation. There is a slight deformation on the left side of each wave crest, resulting in wave asymmetry described above. If there is no deformation in the ascending process near wave crest, the value of the wave elevation should be larger, which also means that the red line marked with ‘A3’ in Figure 4-6a should be higher.
4.3.1.2 Evolution of wave asymmetry with increasing forcing amplitude

The wave forms shown in Figure 4-6 reveal weak asymmetry, especially around the region close to the wave crest. As \( b_2 \) is increased to 4.5 mm, wave asymmetry becomes more apparent. The wave profiles at different instants under \( b_2 = 4.5 \text{ mm} \), after the wave becomes stable, are shown in Figure 4-8. Compared with the wave profiles shown in Figure 4-6, those shown in Figure 4-8 are apparently temporal asymmetric about \( x = 434 \text{ mm} \). The more apparent asymmetry for larger forcing amplitude may be due to the occurrence of the bulge at the location of wave gauge 2 (indicated by a black arrow).

From the observations, it is found that the bulge always appears on the left or right side of the wave crest. Figure 4-9 shows six successive wave crests at wave gauge 2 (x = 434 mm). In Figure 4-9 (a), the bulge appears on the right side of the crest, while in Figure 4-9 (b) and (c) it is on the left side of the crest. In Figure 4-9 (d), it occurs on the right side again. In the last two photos of Figure 4-9, the bulge is on the same side with those in Figure 4-9 (b) and (c). The periodical change of the bulge location is discussed based on the Hilbert-Huang transform (HHT) analysis as briefly outlined below. Details of this method can be found in Huang et al.\textsuperscript{17}.

HHT is an empirically based data-analysis method. It is adaptive, which is helpful to produce physically meaningful representations of data from nonlinear and non-stationary processes\textsuperscript{16}. Prior to introducing HHT, the Hilbert transform will be illustrated first. For an arbitrary time series \( x(t) \), its Hilbert transform is:

\[
y(t) = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau
\]

(4.2)

where \( \text{PV} \) is the Cauchy principal value. With this definition, \( x(t) \) and \( y(t) \) form a complex conjugate pair, so an analytic signal can be obtained, \( z(t) \)

\[
z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)}
\]

(4.3)

with

\[
a(t) = \sqrt{x(t)^2 + y(t)^2}, \quad \text{and} \quad \theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right),
\]

(4.4)

where \( a(t) \) is the instantaneous amplitude and \( \theta \) is the phase function which is related with the instantaneous frequency via:

\[
f = \frac{d\theta}{dt}.
\]

(4.5)

However, it is difficult to define the instantaneous frequency for arbitrary data because for any function to have a meaningful instantaneous frequency, the real part of its Fourier transform has to have only positive frequency\textsuperscript{19}. Therefore, Huang et al.\textsuperscript{16,19}
introduced the Empirical Mode Decomposition (EMD) method to solve the problem. The intrinsic mode function (IMF) can be obtained by applying EMD to the original data. Each of the IMF which can be assured with meaningful instantaneous frequency satisfies the following two conditions:

- The number of extreme and zero-crossings must either equal or differ at most by one,
- The mean value of the envelope defined by the local maxima and minima is zero.

After getting the IMFs, the Hilbert transform will then be applied to each IMF to get the amplitude-frequency-time distribution. However, EMD sometimes will lead to the mode mixing, which is defined as a single Intrinsic Mode Function (IMF) either consisting of signals of widely disparate scales, or a signal of a similar scale residing in different IMF components\(^{20}\). Therefore, Huang et al.\(^{20}\) introduced the Ensemble Empirical Mode Decomposition (EEMD) method to overcome the deficiency. The first step of EEMD is to add a white noise to the original data, and then decompose the new data into IMFs. The final step is to obtain the ensemble means of the corresponding IMFs as the result.

Figure 4-10 shows the EEMD results of the wave elevation data measured by wave gauge 2 when \(b_2 = 4.5\) mm from vertical excitation cycles 500 to 600. There are in total 11 IMFs. The first 6 decompositions (C1-C6) are the added small but high frequency noises.

Fast Fourier Transform (FFT) is applied to C7-C9 in Figure 4-10. The results are shown in Figure 4-11. It is seen clearly that in (a), the peak frequency of C7 is 1.328 Hz which is the same with the excitation frequency. Therefore, it is the most energetic component from the wave elevation at \(x = 434\) mm. In (b), apart from the peak at excitation frequency (1.328 Hz), another two peaks (0.458 Hz and 0.87 Hz) which are comparable with spectrum of the peak (1.328 Hz) are found. These two peaks are close to the 1/3 and 2/3 of 1.328 Hz. In (c), only one frequency (0.458 Hz) is found, indicating the C9 is a uniform component. The rest components (C9 - C11) are small in amplitudes and should make negligible contribution to the wave elevation when compared with other components (C7-C9). Based on the analysis above, for the EEMD results at \(x = 434\) mm, C8 is the frequency modulation component, which result in the periodical change on the bulge location.
4.3.2 Observation of standing wave with breaking

With further increase in forcing amplitude, the wave profile changes gradually without breaking. Weak wave breaking starts to occur when \( b_2 = 7 \) mm. The corresponding wave steepness is \( h/\lambda = 0.1605 \), where \( h \) represents the wave height. This wave steepness is slightly lower than that in Jiang et al.\textsuperscript{11}. The gentle wave breaking observed when \( b_2 = 7 \) mm will evolve into the double-plunger wave breaking when the \( b_2 = 8 \) mm. The wave steepness then increases to \( h/\lambda = 0.1887 \). Period tripling breaking which was first reported by Jiang et al.\textsuperscript{11}, does not occur in the experiment until \( b_2 = 11.5 \) mm. The experimental results of the period tripling breaking for the asymmetric wave profile will be illustrated in details below.

4.3.2.1 Period tripling breaking for \( b_2 = 11.5 \) mm

Period tripling breaking is a phenomenon with three types of wave breaking which repeats every three wave cycles. Jiang et al.\textsuperscript{11} observed that period tripling breaking consists of three modes, namely mode I (sharp crest), mode II (double plungers) and mode III (round crest). In this experiment, the breaking wave shapes in the wave period tripling breaking are different from their observation. Furthermore, the wave shapes found in the tripling breaking do not always keep the same when the forcing amplitude changes.

The wave period tripling breaking found at the location of wave gauge 2 when \( b_2 = 11.5 \) mm, which corresponds to modes one, two, and three, respectively, are shown from Figure 4-12 - Figure 4-14. One from every ten consecutive images of the original time frames has been selected so that the interval between two adjacent images in these figures is 0.042 s.

Figure 4-12 shows the ascending and descending processes of mode one (sharp crest). From \( t = 0.378 \) s, the crest in mode one occurs, which is characterized by a sharp crest biases towards to the right side. It is observed that the inclining direction of the peak is stochastic, but the shape always keeps sharp. The sharp crest falls to the water body, creating a dent and a rebounding jet (at \( t = 0.714 \) s, black arrow). Figure 4-13 shows the full cycle for the generation of mode two (double plungers). The rebounding jet produced in mode one (at \( t = 0.714 \) s in Figure 4-12, black arrow) can be seen clearly at the beginning of mode two (from \( t = 0.756 \) s to \( t = 0.882 \) s, black arrows). Double plungers occur from \( t = 1.05 \) s (green arrows) to 1.176 s. Interestingly, after the disappearance of the double plungers (at \( t = 1.134 \) s), a solid and sharp crest is formed from \( t = 1.176 \) s (blue arrow) to 1.26 s. This is different from the behaviour of mode II.
Wave period tripling breaking under vertical excitations

reported by Jiang et al.\textsuperscript{11}. This phenomenon is possibly caused by the rebounding jet. From $t = 0.756\, \text{s}$ to $t = 0.882\, \text{s}$, the jet moves upward whereas during the next 4 time frames showing in Figure 4-13 (from $t = 0.924\, \text{s}$ to $t = 1.105\, \text{s}$), the jet moves downward. At the same time, the water coming from left and right sides of the tank creates a wave crest (at $t = 1.050\, \text{s}$), resulting in water moving upward $x = 434\, \text{mm}$. Consequently, the downward moving jet meets the upward moving water. The jet splits the upward moving water resulting in the double plungers (from $t = 1.050\, \text{s}$ to $t = 1.134\, \text{s}$). Afterwards, the plungers start collapsing from $t = 1.134\, \text{s}$ to $t = 1.260\, \text{s}$, while the bouncing jet moves upward again. Therefore, the phenomenon of the solid but sharp crest can occur. Figure 4-14 shows the generation of mode three (flat crest). It is seen that at $t = 1.890\, \text{s}$, the wave shape at wave gauge 2 is featured with a flat crest, which is also the lowest crest among the three modes.

The three modes in the experiment are characterised by sharp crest (mode one), double plungers (mode two) and flat crest (mode three). As stated in Sec. 5.2.1, apart from the location at the tank walls, the wave crests will also occur at the locations of wave gauges 1 ($x = 434\, \text{mm}$) and 2 ($x = 867\, \text{mm}$) alternatively. Figure 4-15 shows a sketch of the sequence of occurrence of wave breaking modes at $x = 434\, \text{mm}$ and $867\, \text{mm}$. If mode one appears at $x = 434\, \text{mm}$ (at $t = 0.378\, \text{s}$, Figure 4-12) then the next crest occurring at $x = 867\, \text{mm}$ is mode three (at $t = 0.798\, \text{s}$, Figure 4-13), followed by mode two appearing at $x = 434\, \text{mm}$ (at $t = 1.092\, \text{s}$, Figure 4-13). Mode one, mode three and mode two will appear at $x = 867\, \text{mm}$ (at $t = 1.512\, \text{s}$, Figure 4-14), $434\, \text{mm}$ (at $t = 1.848\, \text{s}$, Figure 4-14) and $867\, \text{mm}$ (at $t = 0\, \text{s}$, Figure 4-12) sequentially.

Based on the patterns shown in Figure 4-15, there is a possible reason attributing to the occurrence of flat crest showing in mode three. When mode three happens at $x = 434\, \text{mm}$, the previous crest mode in the tank is mode one at $x = 0$ and $867\, \text{mm}$. Mode one is featured with high and sharp crest, forming a rebounding jet, which causes high energy dissipation. It is therefore conjectured that the remaining energy in the water is not large enough to push another high crest at $x = 434\, \text{mm}$.

4.3.2.2 Period tripling breaking for $b_2 = 15\, \text{mm}$

When the forcing amplitude is increased further, the shapes of the wave crest in the three modes of period tripling breaking vary with it. In Figure 4-16, the crest in mode one appearing at $x = 434\, \text{mm}$ (at $t = 0.42\, \text{s}$) is similar to that in Figure 4-12 but with an opposite inclining direction. Mode two shown in Figure 4-17 is different from that in Figure 4-13. It is featured with a sharp crest with heavily breaking (at $t = 1.134\, \text{s}$, Figure

4-10
4.3.3 Analysis of wave period tripling breaking

4.3.3.1 Phase analysis of wave period tripling breaking

In order to get a better understanding of the period tripling breaking, pseudo-phase-space plot for wave elevation at x = 434 mm is examined based on the method proposed by Moon.²² The three-dimensional pseudo-phase-space plot can be constructed by:

\[ (\eta(t), \eta(t + \Delta t), \eta(t + 2\Delta t)) \]  

(4.6)

where \( \eta(t) \) is the measured wave elevation sampled with a rate of 1000 Hz at x = 434 mm (wave gauge 2) and \( \Delta t \) is the time delay increment. Jiang et al.¹¹ applied this method to analyse their symmetric tripling breaking. Choosing different time delay \( \Delta t \) only affects the shape of the orbits shown in Figure 4-19. Here \( \Delta t = 0.1s \) is used to construct the pseudo-phase plot so that the orbits are with clear trajectory.

Figure 4-19 shows the pseudo-phase plots and the corresponding Poincare maps at x = 434 mm when \( \eta(t) = 0 \) under different forcing amplitudes. There are three orbits in Figure 4-19, standing for the aforementioned three breaking modes. Mode one, two and three are defined by black, red and blue, respectively.

Figure 4-19a and Figure 4-19b are the results for waves without breaking \( (b_2 = 1\text{mm}) \). It can be seen that only one orbit exists in Figure 4-19a. As a result, the Poincare map in Figure 4-19b shows only two dots, indicating that the wave is in periodical condition. As the forcing amplitude is increased to \( b_2 = 11.5 \text{ mm} \) (Figure 4-19c and d), period tripling breaking starts to appear. In Figure 4-19c, the outer orbit (black for mode
one, which is featured with sharp and the highest wave crest as shown in Figure 4-15, at $t = 0.420s$) is easy to be distinguished, whereas parts of the other two orbits are mixed with each other. This is because some of the wave elevations in modes two and three are almost identical (e.g. the elevations from mode two at $t = 1.134$ s in Figure 4-13 and mode three at $t = 1.890$ s in Figure 4-14 are close to each other). In Figure 4-19d, the blue cluster is in the most inner position, which generally indicates that the elevation of the wave crest in mode three is the lowest. Figure 4-19e shows the period tripling breaking when $b_2 = 15$ mm. The inner blue orbit is for mode three with the lowest wave elevation (flat crest at $t = 1.890$ s in Figure 4-18). It is noticeable that the red orbit is mixed with the black ones, indicating that the wave elevations in mode two (red orbits) are comparable with those in mode one (black orbits). This result is attributed to the heavily breaking wave crest of mode two (e.g. at $t = 1.134$ s in Figure 4-17) described in Sec. 4.3.2.2. It seems that, generally, the wave elevation in mode two (red) is higher than that in mode one. This is because sometimes the sharp crest in mode one occurs with inclination (e.g. Figure 4-16, at $t = 0.42$ s). Consequently, the measured wave elevation is lower than its actual value. The widths of the black orbits are much wider than those in Figure 4-19c, indicating that the range of magnitude of elevation from mode one under $b_2 = 15$ mm is larger than that under $b_2 = 11.5$ mm. In Figure 4-19f, compared with Figure 4-19d, the three clusters in the upper right corner stretch over a larger space, indicating that the period tripling breaking phenomenon is more evident under a larger forcing amplitude.

4.3.3.2 HHT analysis of wave period tripling breaking

In order to look into the details of the process of period tripling breaking, HHT analysis is conducted to the wave elevation signals measured at $x = 434$ mm for $b_2 = 11.5$ mm and $b_2 = 15$ mm. In the first case, period tripling breaking appears from the middle of the duration, whereas in the second case, period tripling breaking is found during the entire time series, which is in a stable state.

Figure 4-20 shows the results of EEMD for the time series measured at wave gauge 2 when the vertical excitation cycle is from 400 to 500 for $b_2 = 11.5$ mm. C1 to C6 are the high frequency noise components and therefore are not shown in the figure.

FFT is applied to C7-C9 in Figure 4-20. The results are shown in Figure 4-21. It is seen clearly that in (a), the peak frequency of C7 is $f = 1.33$ Hz which is the same with the excitation frequency. Therefore, it is the most energetic component from the wave elevation at $x = 434$ mm. In (b), both the excitation frequency ($f$) and a modulation
frequency \((2f/3)\) are found. They are apparent and comparable. Apart from the two frequencies, another frequency \((f/3)\) is also found, but it is much smaller than the other two. So C8 is the frequency modulation component. In (c), only one frequency \((f/3)\) is found, indicating C9 is a uniform component. The rest components (C10 and C11) are small in amplitudes and should make negligible contribution to the wave elevation when compared with other components (C7-C9).

Figure 4-22 shows the instantaneous frequency of the components C8 (Figure 4-20). To have a better visualisation of the variation of the instantaneous frequency, only parts of the results are shown in the figure. As stated above, period tripling breaking starts at around \(t = 166.4\) s (marked by a black arrow). There is no apparent pattern on the instant frequency of C8 before that moment. After \(t = 166.4\) s, the wave period tripling breaking happens. Four tripling breaking periods are shown in Figure 4-22. Each period includes three wave cycles, and the numbers marked above the wave cycle indicate the mode for tripling breaking. The wave cycle with the highest elevation is for mode one (Figure 4-12), as marked by ‘1’. Then the next two wave cycles are marked by ‘2’ (mode two, Figure 4-13) and ‘3’ (mode three, Figure 4-14), respectively.

The instantaneous frequency of C8 in the period \(I\) is illustrated. The instantaneous frequency increases for the whole duration of mode one and peaks at the end of it, then followed by a descending process. It goes to the lowest during mode three. There is a high and thin crest in mode one (Figure 4-12), which means that it is the most energetic mode among the three. Therefore, the wave varies fast in this mode, which explains why the instantaneous frequency increases in mode one. The wave varies slowly and the instantaneous frequency decreases. In modes two, energy dissipates significantly due to the collision between the rebounding jet and the upward water. In mode three, the heavy energy dissipation is induced by the high and sharp crest in mode one at the neighbouring location \((x = 867\) mm). As a result, wave varies slowly and the instantaneous frequency decreases during these two modes. After the peak of mode three, the instantaneous frequency increases again. This is induced by the accumulating energy from the external excitation and the wave varies fast again.

Figure 4-23 shows the results of EEMD for the time series from wave gauge 2 under \(b_2 = 15\) mm. Period tripling breaking can be always observed in this case. Therefore, there is no sudden variation on C8 and C9 compared with those in Figure 4-20. C1 to C6 are the high frequency noise component which are not shown in this figure.

FFT (Fast Fourier Transform) is applied to C7-C9 in Figure 4-23. The results are shown in Figure 4-24. It is clearly seen that in (a), the peak frequency of C7 is \(f = 1.33\)
Hz which is the same with the excitation frequency. Therefore, it is the most energetic component. In (b), apart from the peak at excitation frequency ($f$), another peak at a modulation frequency ($2f/3$) is found. They are apparent and comparable. In (c), only one frequency ($f/3$) is found, indicating the C9 is a uniform component. Based on the analysis above, for the EEMD results at $x = 434$ mm, C8 is the frequency modulation component. The rest components (C10 and C11) are small in amplitudes and should make negligible contribution to the wave elevation when compared with other components (C7-C9).

Part of the instantaneous frequency of C8 and the corresponding wave elevation are shown in Figure 4-25. The numbers marked on wave crest represent the period tripling breaking modes. Totally five tripling breaking periods are shown in Figure 4-25. The wave cycle with the lowest elevation is for mode three, which is marked by ‘3’ (Figure 4-18). Then the previous two wave cycles are marked by ‘2’ (mode two, Figure 4-17) and ‘1’ (mode one, Figure 4-16), respectively. The instantaneous frequency of C8 is periodic and each period lasts the same duration as the wave tripling breaking. Therefore, C8 is considered as a frequency modulation component for the wave period tripling breaking. The variation of the instantaneous frequency of C8 is very similar to the one shown in Figure 4-22 (from the time marked by a black arrow). The instantaneous frequency in period $I$ is illustrated below. The instantaneous frequency of C8 keeps increasing for the whole duration of mode one and peaks at the end of mode one, then followed by a descending process, and reaches the lowest value in mode three. It has been shown that the crest in mode one is the most energetic among the three modes. So the wave varies fast in this mode, which explains why the instantaneous frequency keeps increasing in mode one. In mode two, even though the water coming from left and right sides of the rebounding jet is stronger than that when $b_2 = 11.5$ mm (Figure 4-13), it may not be strong enough to change the wave varying speed. As a result, the frequency keeps decreasing. In mode three, significant energy dissipation occurs due to the heavy wave breaking in the neighbouring mode, which is the same as that when $b_2 = 11.5$ mm. The wave varies slowly as well and the instantaneous frequency decreases.

4.4 Conclusions

Experiments under an initial horizontal and followed by a vertical excitation were conducted in a two-dimensional sloshing tank to study the dependence of the wave form
on vertical excitation amplitude. The horizontal excitation acts as a ‘seed’ to produce perturbation to the water whereas the forcing amplitude $b_2$ of the vertical excitation changes from 1 mm to 15 mm. The excitation is at the third mode sloshing frequency ($2f_3 = 2.66Hz$). The main results are summarised as follows.

1. When $b_2 = 1$ mm, the wave forms are slightly temporal asymmetry at the region close to the wave crest ($x = 434$ mm) without breaking. The temporal asymmetry is intensified when $b_2$ is increased to 4.5 mm. This may be due to the occurrence of the bulge on the side of the wave crest. Interestingly, the location of the bulge changes periodically. It is on the right side of the wave crest once and then on the left side twice at the next two successive crests and then repeating this format.

2. Wave starts to break when $b_2 = 7$ mm and the corresponding wave steepness is $h/\lambda = 0.1605$. Wave period tripling breaking with three modes, first reported by Jiang et al.\textsuperscript{11} is also observed in this experiment as $b_2$ is increased to 11.5 mm. However, the breaking modes two and three in this study are different from the previous ones. Mode one is similar to the observation of Jiang et al.\textsuperscript{11}, which is characterised by a steep and sharp crest with random inclined direction. Mode two in this experiment is featured with double plungers. Mode three is a flat crest instead of a round one. It is also noticed that the wave shape of mode two varies with the increase of the forcing amplitude. When $b_2$ is increased up to 15 mm, the shape of mode two becomes a steep crest which is similar to mode one but with heavily wave breaking resulting from wave collision.

3. Phase portrait is applied to the non-breaking ($b_2 = 1$ mm) and period tripling breaking waves when $b_2 = 11.5$ mm and 15 mm, respectively. The width of the orbits stands for the magnitude of the range of the wave crest. The results for the non-breaking wave show only one orbit whereas three orbits can be found for period tripling breaking. For $b_2 = 11.5$ mm, the orbits show a partial mixing between modes two and three, revealing that the wave elevation of modes two and three are comparable. For $b_2 = 15$ mm, the mixing between modes one and two is also detected.

4. The measured wave elevations for $b_2 = 11.5$ mm and 15 mm are also analysed using HHT. The results suggest that the frequency modulation from HHT component 8 (C8) induces the period tripling breaking.
Reference


TABLE 4-I. Test Matrix for the experiment under horizontal excitation (10 horizontal excitation periods and 640 vertical excitation periods)

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<th>Case No.</th>
<th>Horizontal Excitation amplitude (mm)</th>
<th>Horizontal Excitation frequency (rad/s)</th>
<th>Vertical Excitation amplitude (mm)</th>
<th>Vertical Excitation frequency (rad/s)</th>
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Figure 4-1 Snapshot of the sloshing experimental facility.

Figure 4-2 Sketch of the experimental set-up and arrangement of wave gauges.
Figure 4-3 Flow chart of the signal synchronisation and control

Figure 4-4 The initial 10-period input horizontal excitation signal followed by the vertical excitation signal for 640 periods.
Figure 4-5 Boxplot of wave elevation at (a) wave gauge 1; and (b) wave gauge 2 when forcing amplitude increases from 1mm to 15mm. The upper whisker and lower whisker indicate the maximum and minimum value within the range from Q1-1.5IQR to Q3+1.5IQR. IQR = Q3-Q1.
Figure 4-6 Wave profiles under vertical exciting amplitude of 1 mm: (a) ascending process ($x = 434$ mm, wave gauge 2); (b) descending process ($x = 434$ mm, wave gauge 2). The time interval between two neighbouring wave profiles is 0.04s. The sequence of the wave profile is: A1-A2-A3-A4-wave peak-D4-D3-D2-D1.

Figure 4-7 Time series of wave elevation measured by wave gauge 2 under $h_2 = 1$ mm: (a) from vertical exciting cycle 500 to 600; (b) enlarged part of (a) from $t = 206.7$ s to 210.5 s.
Figure 4-8 Wave profiles under $b_2 = 4.5$ mm: (a) ascending process (at $x = 434$ mm, wave gauge 2); (b) descending process (at $x = 434$ mm, wave gauge 2). The time interval between two neighbouring wave profiles is 0.04s. The sequence of the wave profile is: A1-A2-A3-A4-wave peak-D4-D3-D2-D1.
Figure 4-9 Wave frames for six successive crests with the occurrence of bulge at the location of wave gauge 2 ($b_2 = 4.5$ mm). The wave proceeding direction for (b), (c), (e), (f) is towards right, while the proceeding direction for (a) and (d) is towards left.
Figure 4-10 EEMD results for wave elevation at wave gauge 2 when $b_2 = 4.5$ mm from vertical exciting cycles 500 to 600.
Figure 4-11 Spectra of signal components C7 – C9 shown in Figure 4-10
Figure 4-12 Time frames for wave period tripling breaking mode one (location of wave gauge 2), when $b_2 = 11.5$ mm. The unit for time is in s.
Figure 4-13 Time frames for wave period tripling breaking mode two (location of wave gauge 2), when $b_2 = 11.5$ mm. The unit for time is in s.
Figure 4-14 Time frames for wave period tripling breaking mode three (location of wave gauge 2), when $b_2 = 11.5$ mm. The unit for time is in s.
Figure 4-15 Sketch of the sequence of occurrence of wave breaking modes at $x = 434$ mm and 867 mm
Figure 4-16 Time frames for wave period tripling breaking mode one (location of wave gauge 2), when $b_2 = 15$ mm. The unit for time is in s.
Figure 4-17 Time frames for wave period tripling breaking mode two (location of wave gauge 2), when $b_2 = 15$ mm. The unit for time is in s.
Figure 4-18 Time frames for wave period tripling breaking mode three (location of wave gauge 2), when $b_2 = 15$ mm. The unit for time is in s.
Figure 4-19 Pseudo phase plot (a, c, e) and Poincare map by taking  \( \eta(t) = 0 \) (b, d, f) for wave elevation measured at \( x = 434 \) mm at different forcing amplitude. (a) and (b): \( b_2 = 1 \) mm; (c) and (d): \( b_2 = 11.5 \) mm; (e) and (f): \( b_2 = 15 \)mm. Black, red and blue represent modes one, two and three.
Figure 4-20 EEMD results for wave elevation measured at $x = 434$ mm (wave gauge 2) when $b_2 = 11.5$ mm from vertical excitation cycle 400 to 500. The black arrow indicates the beginning of wave tripling breaking (around 166.4 s).
Figure 4-21 Spectra of signal components C7-C9 shown in Figure 4-20
Figure 4-22: Part of the instantaneous frequency of C8 (red dot dash) obtained based on the data from Figure 4-20. The blue dashed line is the corresponding wave elevation. The numbers ‘1’, ‘2’ and ‘3’ stand for period tripling breaking modes one, two and three, respectively.

Figure 4-23: EEMD results for wave elevation at x = 434 mm (wave gauge 2) when $b_2 = 15$ mm from vertical exciting cycle 400 to 500.
Figure 4-24 Spectra of signal components C7-C9 shown in Figure 4-23
Figure 4-25 Part of the instantaneous frequency for C8 (red dot dash) obtained from the data shown in Figure 4-23. The blue dashed line is the corresponding wave elevation. The numbers ‘1’, ‘2’ and ‘3’ stand for period tripling breaking modes one, two and three, respectively.